Representation of Continuous and Discrete-Time Signals-1 Study Notes

Continuous-time signals and discrete-time signals are two fundamental types of signals used in signal processing and communications. Continuous-time signals are represented as functions of time, while discrete-time signals are represented as sequences of values at discrete time instants. Continuous-time signals are defined for all values of time, while discrete-time signals are defined only at specific points in time. Both signal types can be analyzed and processed in various domains, such as the time domain, frequency domain, or z-domain.

Here you will find the study notes on **Representation of Continuous and Discrete-Time Signals** which will cover the topics such as **Different types of Basic Continuous/Discrete Signals, Energy & Power Signal.**

Properties of Signals

A signal can be classified as periodic or aperiodic; discrete or continuous time; discrete or continuous-valued; or as a power or energy signal. The following defines each of these terms. In addition, the signal-to-noise ratio of a signal corrupted by noise is defined.

Periodic / Aperiodic:

A periodic signal repeats itself at regular intervals. In general, any signal $x(t)$ for which

$$x(t) = x(t+T)$$

for all t is said to be *periodic*.

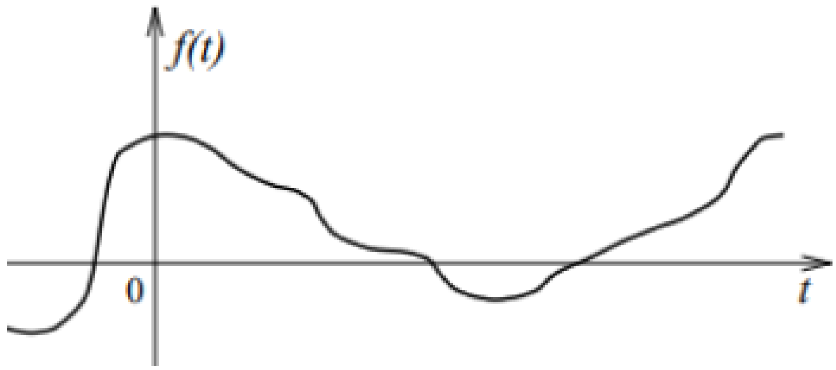
The fundamental period of the signal is the minimum positive, non-zero value of T for which the above equation is satisfied. If a signal is not periodic, then it is *aperiodic*.

Symmetric / Asymmetric:

There are two types of signal symmetry: odd and even. A signal $x(t)$ has *odd symmetry* if and only if $x(-t) = -x(t)$ for all t. It has *even symmetry* if and only if $x(-t) = x(t)$.

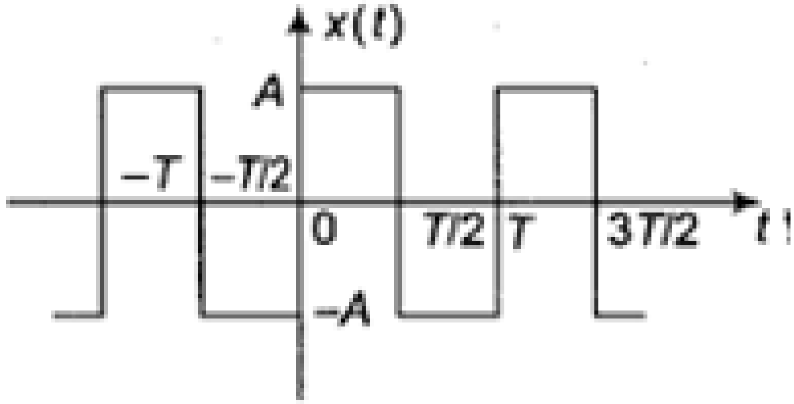
Continuous and Discrete Signals and Systems

A continuous signal is a mathematical function of an independent variable, which represents a set of real numbers. It is required that signals are uniquely defined in except for a finite number of points.



- A continuous time signal is one which is defined for all values of time. A continuous time signal does not need to be continuous (in the mathematical sense) at all points in time. A continuous-time signal contains values for all real numbers along the X-axis. It is denoted by x(t).
- Basically, the Signals are detectable quantities which are used to convey some information about time-varying physical phenomena. some examples of signals are human speech, temperature, pressure, and stock prices.
- Electrical signals, normally expressed in the form of voltage or current waveforms, they are some of the easiest signals to generate and process.

Example: A rectangular wave is discontinuous at several points but it is continuous time signal.



A rectangular wave

Discrete / Continuous-Time Signals:

A continuous time signal is defined for all values of *t*. A discrete time signal is only defined for discrete values of $t = ..., t_{-1}, t_0, t_1, ..., t_n, t_{n+1}, t_{n+2}, ...$ It is uncommon for the spacing between t_n and t_{n+1} to change with *n*. The spacing is most often some constant value referred to as the sampling rate,

$$T_s = t_{n+1} - t_n.$$

It is convenient to express discrete time signals as $x(nT_s)=x[n]$.

That is, if $x(t)$ is a continuous-time signal, then $x[n]$ can be considered as the n^{th} sample of $x(t)$.

Sampling of a continuous-time signal $x(t)$ to yield the discrete-time signal $x[n]$ is an important step in the process of digitizing a signal.

Energy and Power Signal:

When the strength of a signal is measured, it is usually the signal power or signal energy that is of interest.

The signal power of *x*(*t*) is defined as

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt$$

and the signal energy as

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \qquad\qquad E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

- A signal for which P_x is finite and non-zero is known as a *power signal*.
- A signal for which E_x is finite and non-zero is known as an *energy signal*.
- P_x is also known as the *mean-square* value of the signal.
- Signal power is often expressed in the units of decibels (dB).
- The power of signal in decibel is defined as $P_{xdB} = 10\log\left(\frac{P_x}{P_0}\right)$
- where P_0 is a reference power level, usually equal to one squared SI unit of the signal.
- For example if the signal is a voltage then the P_0 is equal to one square Volt.

- A Signal can be Energy Signal or a Power Signal but it can not be both. Also a signal can be neither a Energy nor a Power Signal.
- As an example, the sinusoidal test signal of amplitude *A,*

x(t)=Asin(ωt)

has energy E_x that tends to infinity and power , $P_x = \frac{1}{2}A^2$

or in decibels (dB): 40log(*A*)-6

The signal is thus a power signal.

Signal to Noise Ratio:

Any measurement of a signal necessarily contains some random noise in addition to the signal. In the case of additive noise, the measurement is

x(t)=s(t)+n(t)

where *s*(*t*) is the signal component and *n*(*t*) is the noise component.

The signal to noise ratio is defined as

$$SNR_x = \frac{P_s}{P_n}$$

or in decibels, $SNR_x = 10\log\left(\frac{P_s}{P_n}\right)$

The signal to noise ratio is an indication of how much noise is contained in a measurement.

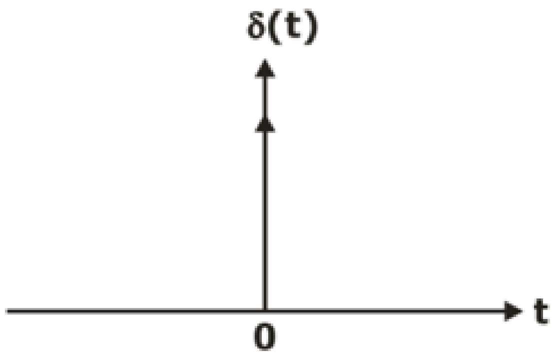
Standard Continuous Time Signals

- **Impulse Signal**

$$\delta(t) = \begin{cases} \infty \ ; \ t = 0 \\ 0 \ ; \ t \neq 0 \end{cases}$$

where ∞ is the height of an impulse signal having unit area.

and $\int_{-\infty}^{\infty} \delta(t)\, dt = A$ Where A = 1 (unit impulse Area)



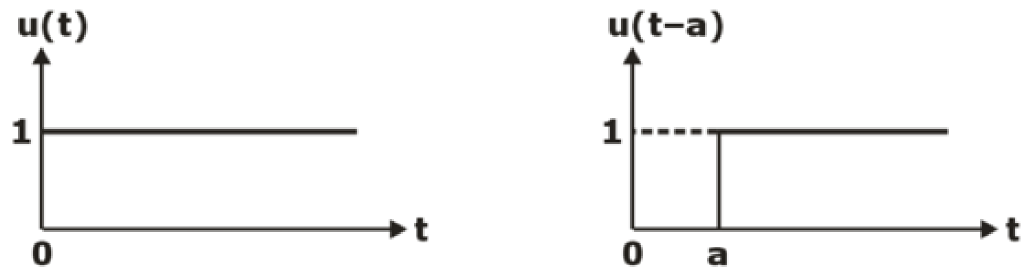
Unit impulse function

- **Step Signal**

$$x(t) = \begin{cases} A\,;\, t \geq 0 \\ 0\,;\, t < 0 \end{cases}$$

Unit Step Signal if A =1,

$$x(t) = u(t) = \begin{cases} 1\,;\, t > 0 \\ 0\,;\, t < 0 \end{cases}$$



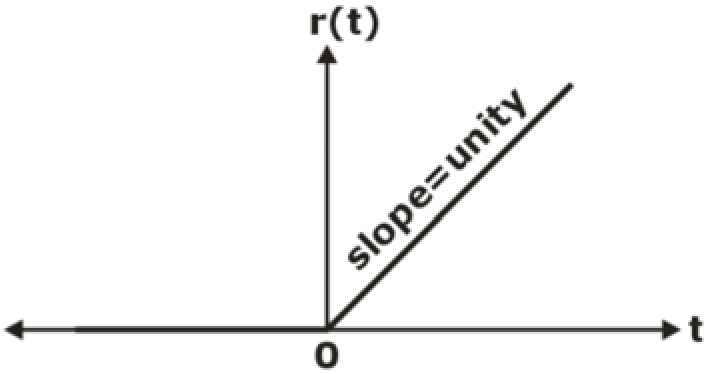
(a) unit step function (b) Shifted Unit Step Function

- **Ramp Signal**

$$x(t) = \begin{cases} At \; ; \; t \geq 0 \\ 0 \; ; \; t < 0 \end{cases}$$

Unit Ramp Signal (A=1)

$$x(t) = r(t) = \begin{cases} t \; ; \; t \geq 0 \\ 0 \; ; \; t < 0 \end{cases}$$



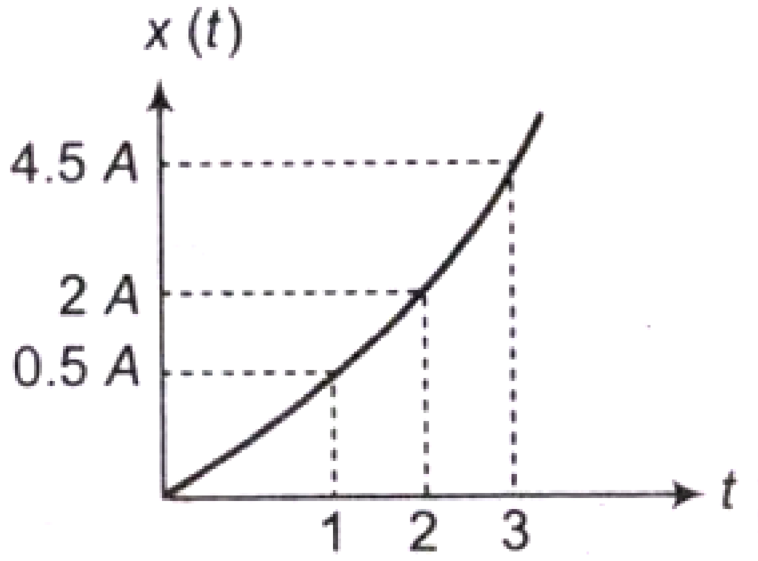
Unit Ramp Function

- **Parabolic Signal**

$$x(t) = \begin{cases} \dfrac{At^2}{2} \; ; \; t \geq 0 \\ 0 \; ; \; t < 0 \end{cases}$$

Unit Parabolic Signal when A = 1,

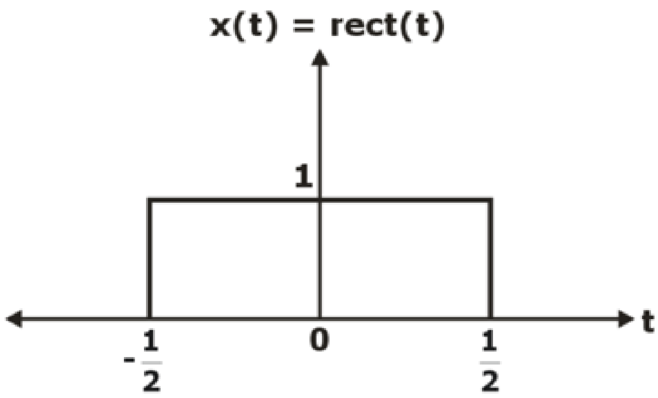
$$x(t) = \begin{cases} \dfrac{t^2}{2} \; ; \; t \geq 0 \\ 0 \; ; \; t < 0 \end{cases}$$



Parabolic function

- **Unit Pulse Signal**

$$x(t) = \pi(t)$$
$$= u(t + 1/2) - u(t - 1/2)$$



Unit Rectangular Pulse Function

Sinusoidal Signal

- **Co-sinusoidal Signal:**

$$x(t) = A\cos(\omega_0 t + \phi)$$

Where, ω_0 is the angular frequency in rad/sec

f_0 = frequency in cycle/sec or Hz

T = time period in second

When $\phi = 0, x(t) = A\cos\omega_0 t$

When ϕ = positive, $x(t) = A\cos(\omega_0 t + \phi)$

When ϕ = negative, $x(t) = A\cos(\omega_0 t - \phi)$

- **Sinusoidal Signal:**

$$x(t) = A\sin(\omega_0 t + \phi)$$

Where, $\omega_0 = 2\pi f_0 = \frac{2\pi}{T}$ is the Angular frequency in rad/sec

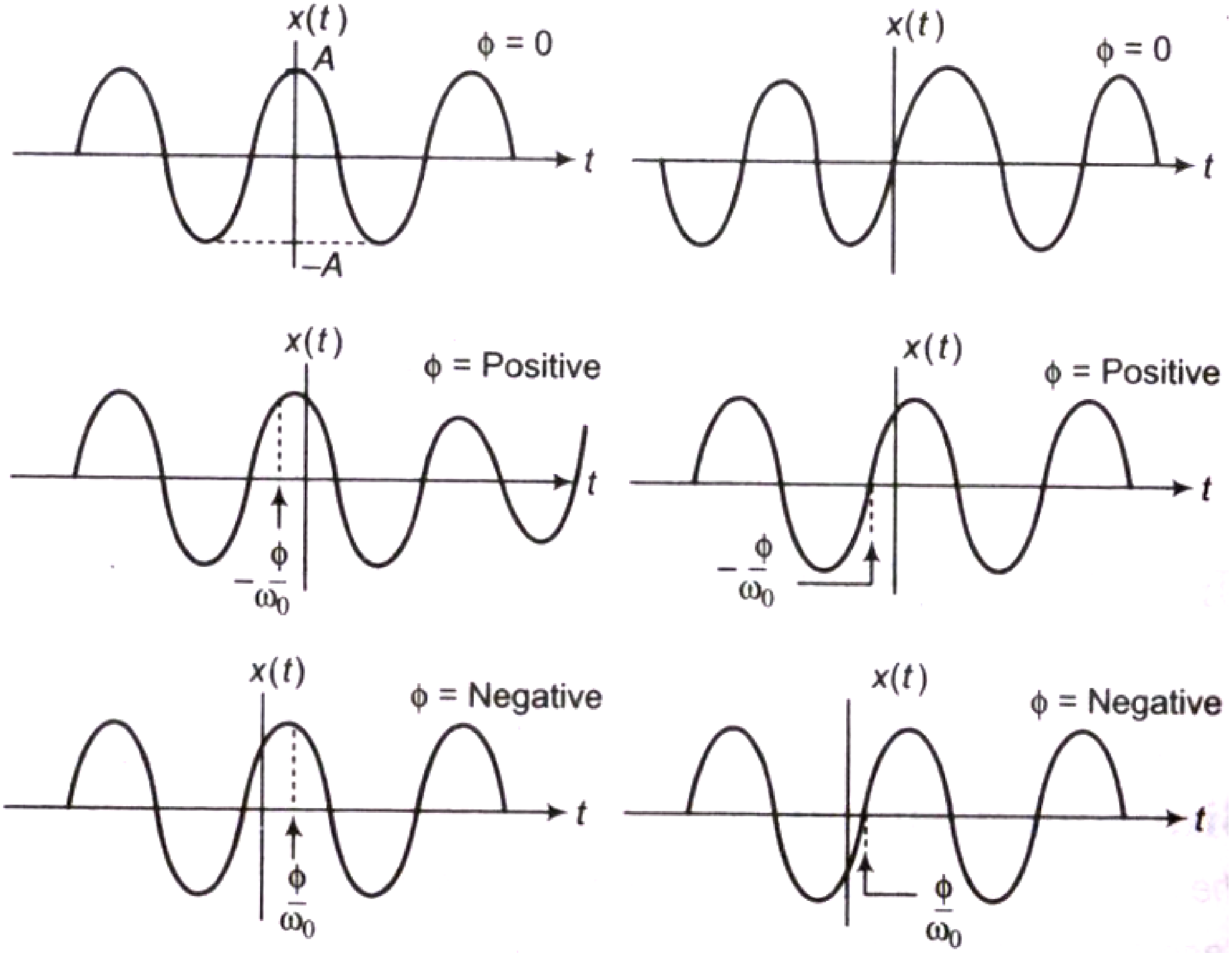
f_0 = frequency in cycle/sec or Hz

T = time period in second

When $\phi = 0, x(t) = A\sin(\omega_0 t)$

When ϕ = positive, $x(t) = A\sin(\omega_0 t + \phi)$

When ϕ = negative, $x(t) = A\sin(\omega_0 t - \phi)$

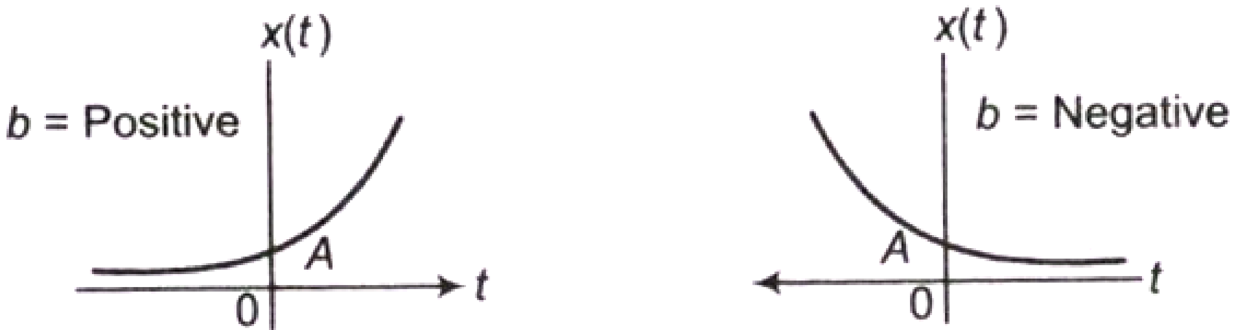


Co-sinusoidal signal   Sinusoidal signal

Exponential Signal:

- **Real Exponential Signal**

$x(t) = A\, e^{bt}$; where, *A* and *b* are real.

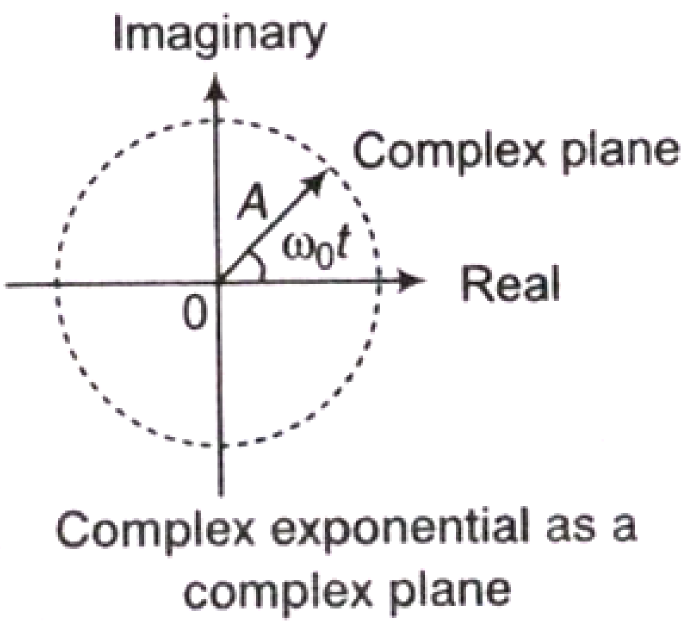


Exponential signal   Exponential signal when $b < 0$

- **Complex Exponential signal**

$x(t) = A\, e^{j\omega_0 t}$

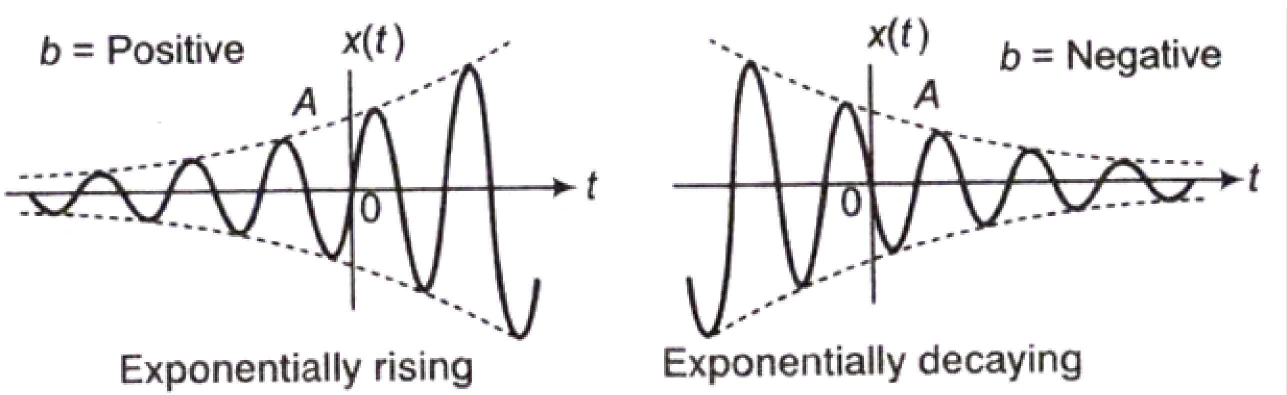
The complex exponential signal can be represented in a complex plane by a rotating vector, which rotates with a constant angular velocity of ω_0 rad/sec.



Complex exponential as a complex plane

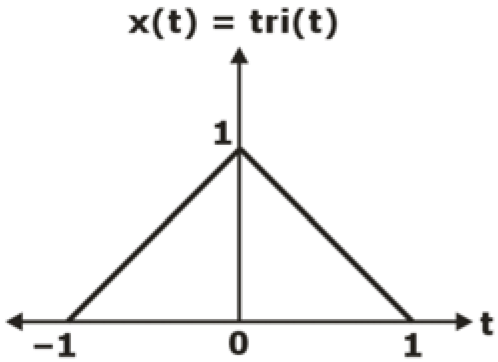
- **Exponentially Rising/Decaying Sinusoidal Signal**

$$x(t) = A\, e^{bt} \sin \omega_0 t$$



- **Triangular Pulse Signal**

$$x(t) = \triangle a(t) = \begin{cases} 1 - \dfrac{|t|}{a};\ |t| \le a \\ 0\ ;\ |t| > a \end{cases}$$



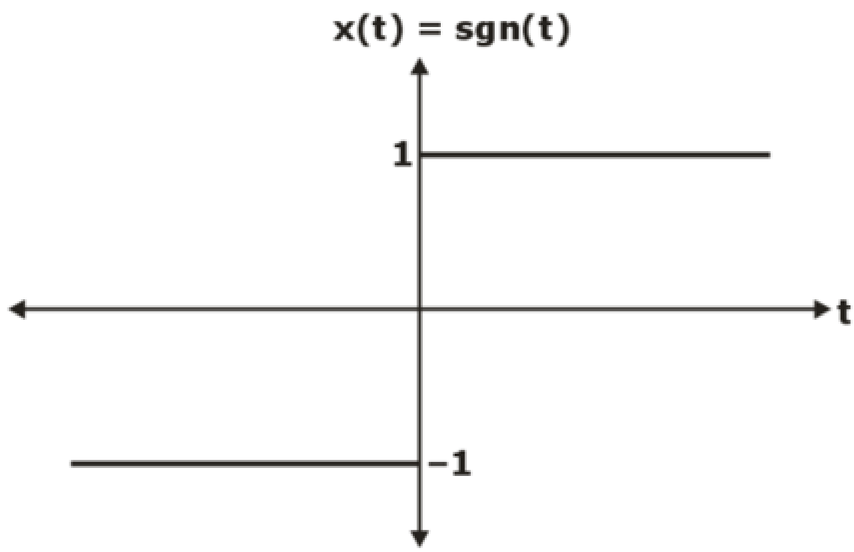
Unit Triangular Function

- **Signum Signal**

$$x(t) = Sgn(t) = \begin{cases} 1\ ;\ t > 0 \\ -1\ ;\ t < 0 \end{cases}$$

$$Sgn(t) = 2u\,(t) - 1$$

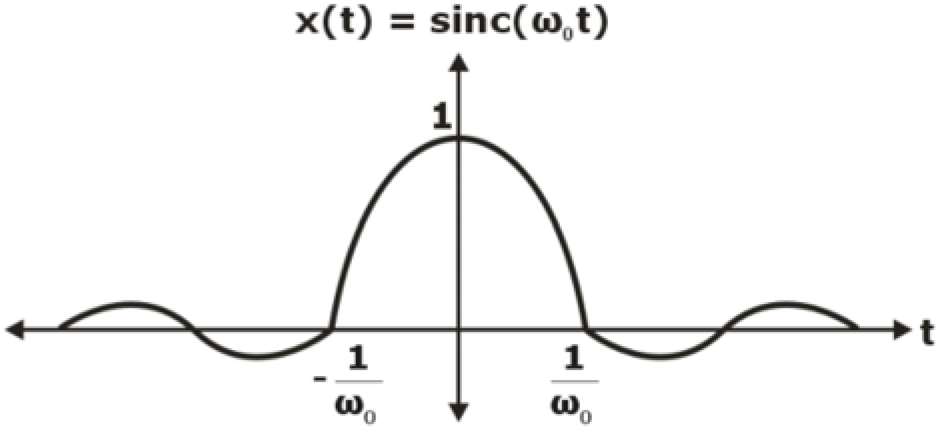
$$Sgn(t) = u(t) - u(-t)$$



Unit Signum Function

- **SinC Signal**

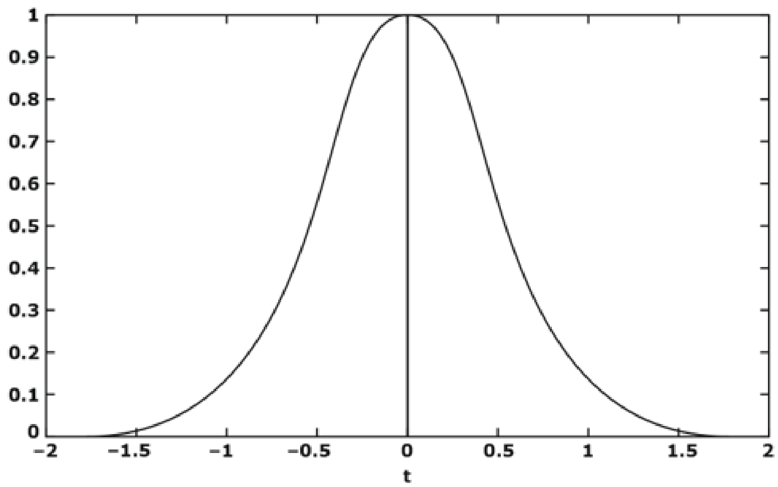
$$x(t) = \sin C\,(t) = \frac{\sin t}{t}$$



Sinc Function

- **Gaussian Signal**

$$x(t) = g_a(t) = e^{-a^2t^2}$$



Gaussian function

Important points:

- The sinusoidal and complex exponential signals are always periodic.
- The sum of two periodic signals is also periodic if the ratio of their fundamental periods is a rational number.

- Ideally, an impulse signal is a signal with infinite magnitude and zero duration.
- Practically, an impulse signal is a signal with large magnitude and short duration.

Classification of Continuous Time Signal: *The continuous time signal can be classified as,*

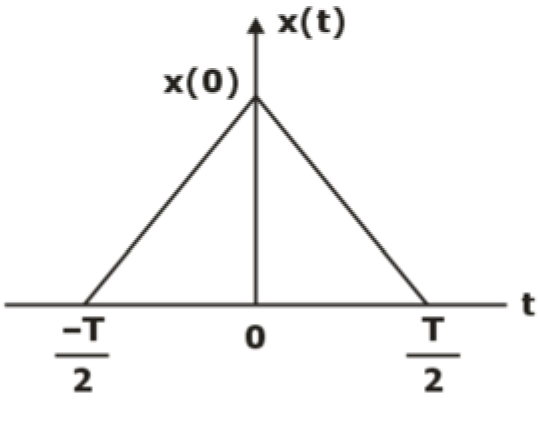
1. **Deterministic and Non-deterministic Signals:**
    - The signal that can be completely specified by a mathematical equation is called a deterministic signal. The step, ramp, exponential and sinusoidal signals are examples of deterministic signals.
    - The signal whose characteristics are random in nature is called a non-deterministic signal. The noise signals from various sources like electronic amplifiers, oscillators etc., are examples of non-deterministic signals.
    - Periodic and Non-periodic Signals
    - A periodic signal will have a definite pattern that repeats again and again over a certain period of time.

x(t+T) = x(t)

2. **Symmetric (even) and Anti-symmetric (odd) Signals**

When a signal exhibits symmetry with respect to *t* = 0, then it is called an **even signal.**

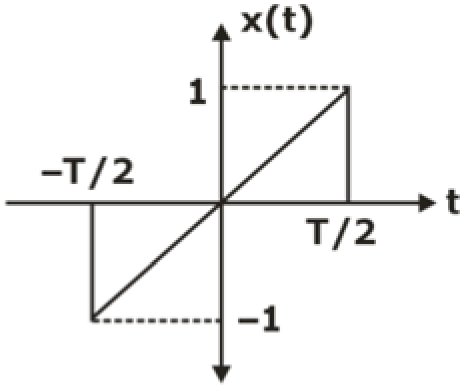
x(-t) = x(t)



Even Signal

When a signal exhibits anti-symmetry with respect to *t* = 0, then it is called an **odd signal.**

x(-t) = -x(t)



Odd signal

Let $X(t) = X_e(t) + X_0(t)$

Where, $X_e(t) =$ even part of $X(t)$

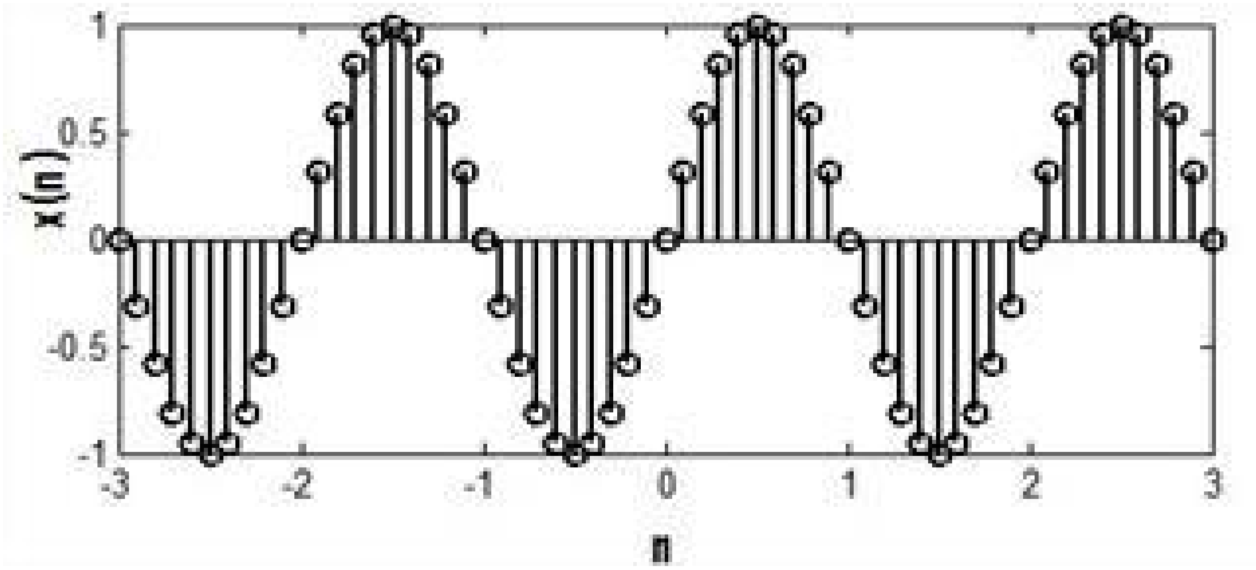
$X_0(t) =$ odd part of $X(t)$

$$X_e(t) = \frac{1}{2}\left[X(t) + X(-t)\right]$$

$$X_0(t) = \frac{1}{2}\left[X(t) - X(-t)\right]$$

Discrete-Time Signals

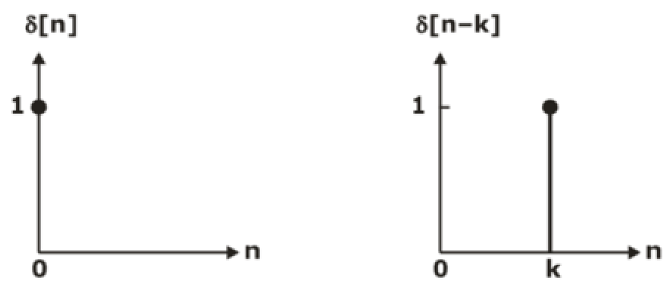
The discrete signal is a function of a discrete independent variable. In a discrete time signal, the value of discrete time signal and the independent variable time are discrete. The digital signal is same as discrete signal except that the magnitude of the signal is quantized. Basically, discrete time signals can be obtained by sampling a continuous-time signal. It is denoted as x(n).



Standard Discrete Time Signals

- **Digital Impulse Signal or Unit Sample Sequence**

Impulse signal, $$\delta(n)=\begin{cases}1; & n=0\\ 0; & n\neq 0\end{cases}$$



(a) DT Unit Impulse Function (b) DT Shifted Unit Impulse Function

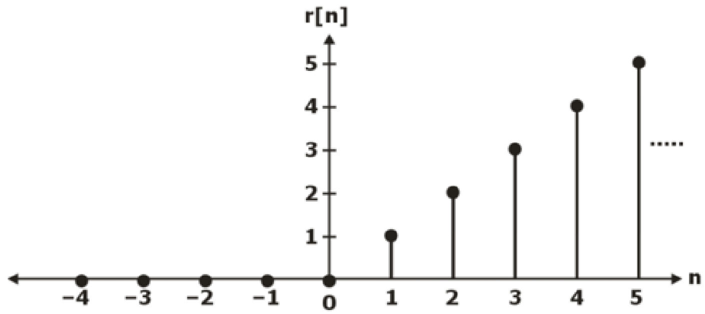
- **Unit Step Signal**

$$u(n)=\begin{cases}1; & n\geq 0\\ 0; & n<0\end{cases}$$

(a) DT Unit Impulse Function,(b) Shifted DT Unit Impulse Function

- **Ramp Signal**

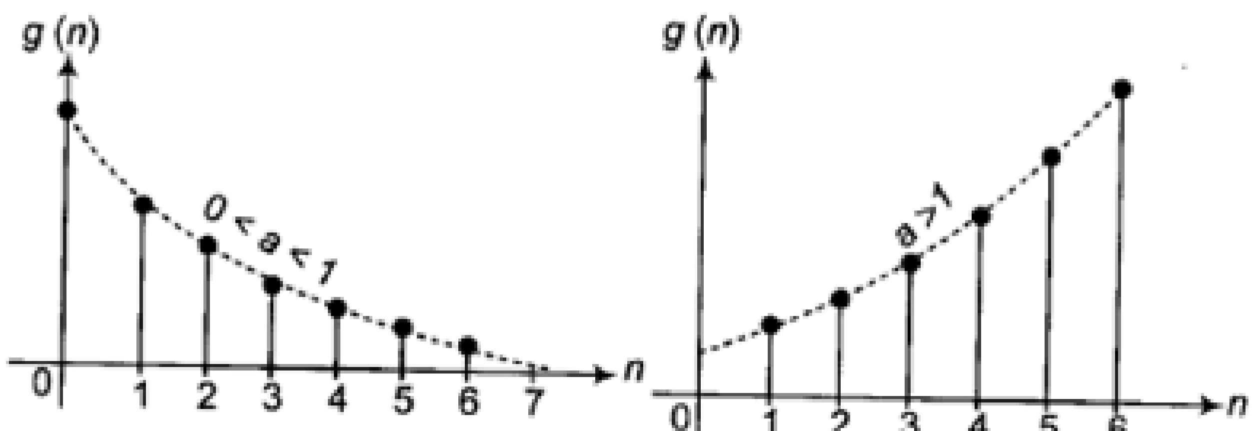
Ramp signal, $u_r(n) = \begin{cases} n; & n \geq 0 \\ 0; & n < 0 \end{cases}$



DT unit Ramp Function

- **Exponential Signal**

Exponential Signal, $g(n) = \begin{cases} a^n; & n \geq 0 \\ 0; & n < 0 \end{cases}$



(a) Decreasing exponential signal (b) Increasing exponential signal

- **Discrete Time Sinusoidal Signal**

$$x[n] = A\cos(\omega_0 n + \theta)\text{ ; For n in the range } -\infty < n < \infty$$

$$x[n] = A\sin(\omega_0 n + \theta)\text{ ; For n in the range } -\infty < n < \infty$$

- A discrete-time sinusoid is periodic only if its frequency is a rational number.
- Discrete-time sinusoids whose frequencies are separated by an integer multiple of 2π are identical.

Thanks!

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