## ESE Mains 2023

## Civil Engineering

## Questions \& Solutions

PAPER-1

| ESE CE Paper 1 : Marks Distribution |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S.NO. | Subjects | Difficulty Level 2023 | $2023$ <br> Marks | $2022$ <br> Marks | $2021$ <br> Marks |
| 1 | Strength of Materials | Easy | 44 | 44 | 64 |
| 2 | Structural Analysis | Easy | 112 | 112 | 96 |
| 3 | Design of Steel Structures | Easy | 84 | 84 | 52 |
| 4 | Design of Concrete structures | Easy | 84 | 84 | 144 |
| 5 | Building Materials | Moderate | 96 | 96 | 52 |
| 6 | Construction Planning Management | Easy | 60 | 60 | 52 |
| 7 | Eng. Mechanics | - | 0 | 0 | 20 |
|  | Total | Easy | 480 | 480 | 480 |

## CIVIL ENGINEERING <br> Paper-1

## SECTION-'A'

1. (a) (i) Explain the types of glazing used for clay products.
[6 Marks]
Sol. (i) A glassy layer is provided on the surface of clay products or ceramics. The Layer is called the glazing layer, and this process which involves the fusing of the glazing layer to a ceramic body at high temperature is called glazing.

## Benefits

1. Increase durability.
2. Provide an aesthetic appearance.
3. It makes surfaces smooth.
4. Resist the action of chemicals.


## Examples

| Lead glazing | Terracotta, fire clay products |
| :--- | :--- |
| Salt glazing | Stoneware, sanitary pipes |
| Opaque glazing | Sanitary pipes |

1. (a) (ii) What are the causes and remedies of efflorescence in bricks?
[6 Marks]
Sol. (ii) Efflorescence in bricks refers to the white or greyish deposits that can appear on the surface of bricks. It occurs when soluble salts in the brick or surrounding materials dissolve in water, migrate to the surface, and crystallize as the water evaporates. The main causes of efflorescence in bricks are:
2. Presence of excessive moisture: Water acts as a carrier for soluble salts, and when bricks are exposed to excessive moisture, it dissolves the salts within the bricks or in the surrounding soil.
3. Presence of alkalis: Alkalis can activate the process of efflorescence by reacting with soluble salts present in the bricks or the surrounding materials.
4. Poor Drainage: Insufficient drainage or improper grading around the building can lead to water accumulation, increasing the likelihood of efflorescence.
5. Porous Bricks: Bricks with high porosity or insufficient firing can absorb more water, increasing the potential for salt dissolution and subsequent efflorescence.

The conditions are categorised in the table below based on the amount of surface affected by efflorescence.

| Surface affected | Efflorescence. |
| :---: | :---: |
| $0 \%$ | Nil |
| $0-10 \%$ | Mild |
| $10-50 \%$ | Moderate |
| Above $50 \%$ | Heavy |

To address and prevent efflorescence in bricks, the following remedies can be applied:

1. Proper Curing: Adequate curing of bricks during manufacturing can minimize the presence of soluble salts, reducing the chances of efflorescence.
2. Improved Masonry Practices: Ensuring that bricks are properly cleaned, handled, and laid with appropriate mortar mixtures can minimize water infiltration and salt migration.
3. Moisture Control: Implementing effective moisture control measures, such as proper grading, waterproofing, and adequate drainage systems, can help prevent water accumulation and reduce the potential for efflorescence.
4. Sealing and Coating: Applying a suitable sealer or coating to the surface of the bricks can create a barrier that inhibits the migration of moisture and salts, thereby reducing the occurrence of efflorescence.

Cleaning: In cases where efflorescence has already occurred, cleaning the affected bricks using appropriate methods (such as brushing or light pressure washing) can help remove the deposits. However, care should be taken to avoid damaging the brick surface or exacerbating the problem.

1. (b) Explain the specific reasons for the following:
(i) For prestressed concrete, the Code recommends to use high tensile steel and high strength of concrete.
[6 Marks]
Sol. (i)

- Prestressed concrete requires high-strength concrete. The material exhibits exceptional tensile, shear, and bearing resistance.
- High-strength concrete is less susceptible to shrinkage cracks, has a lower modulus of elasticity, and has a lower ultimate creep strain, resulting in lesser steel prestress loss.
- The use of high-strength concrete reduces the cross-sectional area of prestressed concrete structural elements, allowing for longer spans that are both technically and economically feasible.
- The tensile strength of high tensile steel is in the range of 1000 to $2000 \mathrm{~N} / \mathrm{mm}^{2}$, and if initially stressed up to $1000 \mathrm{~N} / \mathrm{mm}^{2}$, there will still be large stress in the high tensile reinforcement after making a deduction for the loss of prestress.
- Therefore, high-tensile steel is made for prestressed concrete.

1. (b) (ii) Helically reinforced circular columns have better compressive strength than that of similar columns with lateral ties.
[6 Marks]

Sol. (ii)The code permits larger load in compression members with helical reinforcement because columns with helical reinforcement have greater ductility or toughness when loaded concentrically or with small eccentricity. As per the Code, the strength of compression members with helical reinforcement satisfying the requirement given below shall be taken as 1.05 times the strength of similar members with lateral ties.
Requirement: The ratio of the volume of helical reinforcement to the volume of the core shall not be less than $0.36\left(\frac{A_{g}}{A_{k}}-1\right) \frac{f_{c k}}{f_{y}}$
Where $A_{g}=$ gross Area of the section
$A_{k}=$ Area of the core of the helically reinforced column measured to the outside diameter of the helix.
$D_{k}=$ diameter of concrete core, measured from outside of helix $=D-2 \times$ clear cover
$\mathrm{f}_{\mathrm{yh}}=$ characteristic strength of helical reinforcement, not exceeding 415 MPa .
As per IS 456-2000:
Load carrying capacity of circular column with tie bar is:
$\mathrm{P}_{\mathrm{u}}=0.4 \mathrm{f}_{\mathrm{ck}} \mathrm{A}_{\mathrm{c}}+0.67 \mathrm{f}_{\mathrm{y}} \mathrm{A}_{\mathrm{sc}}$


Load carrying capacity of circular column with helical reinforcement is$\mathrm{P}_{\mathrm{u}}=1.05 \times\left(0.4 \mathrm{ff}_{\mathrm{ck}} \mathrm{A}_{\mathrm{c}}+0.67 \mathrm{ff}_{\mathrm{y}} \mathrm{Asc}_{\mathrm{sc}}\right)$
1.(c) The rafter member of a truss consists of two angles ISA $75 \times 75 \times 8$ placed (back-to-back) both sides of the gusset of thickness 10 mm . It carries factored axial compressive force of 200 kN . Determine the number of 16 mm diameter, 4.6 grade ordinary bolts for the joint. Assume E250
grade of steel and the cross-sectional Area in the threaded part for 16 mm diameter bolt is $157 \mathrm{~mm}^{2}$. Use $\mathrm{K}_{\mathrm{b}}=0.49, \mathrm{Ymb}=1.25$. Use limit state method of design.
[12 Marks]
Sol. For 16 mm diameter ( 4.6 grade) bolts

$$
\begin{aligned}
& A_{s b}=201.1 \mathrm{~mm}^{2} \text { [Shank Area] } \\
& A_{\mathrm{nb}}=156.8 \mathrm{~mm}^{2} \simeq 157 \mathrm{~mm}^{2} \text { [Net Area] } \\
& \left(A_{\mathrm{nb}}=0.78 \mathrm{~A}_{\mathrm{sb}}\right) \\
& {\left[\begin{array}{l}
\mathrm{f}_{\mathrm{ub}}=400 \mathrm{~N} / \mathrm{mm}^{2} \\
\mathrm{f}_{\mathrm{yb}}=240 \mathrm{~N} / \mathrm{mm}^{2}
\end{array}\right]\left[\mathrm{f}_{\mathrm{yb}}=0.6 \mathrm{f}_{\mathrm{ub}}\right]}
\end{aligned}
$$

ISA $75 \times 75 \times 8$ placed back-to-back on both sides of gusset plate.
Number of shear planes $=2$
(Assuming both pass through net/threaded Area)
$n_{n}=2$ (net areas)
$\mathrm{V}_{\mathrm{dsb}}$ (Design shear strength of bolt)
$=\left(\frac{f_{u b}}{\sqrt{3} \gamma_{m b}}\right) n_{n} A_{n b}$
$=\frac{400}{\sqrt{3}(1.25)} \frac{2 \times 157}{10^{3}} \mathrm{kN}$
$=58 \mathrm{kN}$
$\mathrm{V}_{\mathrm{dpb}}$ (Design bearing strength of bolt)
$=2.5 \mathrm{~kb} \mathrm{f}_{\mathrm{u}} \mathrm{dt} / \mathrm{Ymb}$
$\mathrm{k}_{\mathrm{b}}=0.49$ (Given),
For E250 steel, $f_{u}=410 \mathrm{~N} / \mathrm{mm}^{2}$
$\mathrm{t}=10 \mathrm{~mm}$ (for the gusset plate)
So, $V_{\mathrm{dpb}}=2.5 \times 0.49 \times 410 \times 16 \times 10 /\left(1.25 \times 10^{3}\right)$
$\mathrm{V}_{\mathrm{dpb}}=64.3 \mathrm{kN}$
Strength of bolt $=\min \left(\mathrm{V}_{\mathrm{dsb}}, \mathrm{V}_{\mathrm{dpb}}\right)=58 \mathrm{kN}$
Required number of bolts $=\frac{200 \mathrm{kN}}{58 \mathrm{kN}}=3.45$
Hence 4 bolts are to be provided.
1.(d) A bar specimen of 38 mm diameter was subjected to a pull of 98 kN during a tensile test. The extension on a gauge length of 200 mm was measured to be 0.092 mm and the change in
diameter of 0.0048 mm . Determine the Poisson's ratio, modulus of elasticity, modulus of rigidity and bulk modulus of the material of bar specimen.
[12 Marks]

## Sol.



## Poisson's ratio

$$
\begin{aligned}
& \mu=-\frac{\text { lateral strain }}{\text { longitudinal strain }}=-\frac{(-.0048 / 38)}{(.092 / 200)} \\
& \mu=0.2745
\end{aligned}
$$

## Modulus of Elasticity

$E=\frac{\sigma}{\epsilon}$ (For unidirectional loading, we can apply Hooke's law)
$\sigma=\frac{\text { Load }}{\text { Area }}=\frac{98000 \mathrm{~N}}{\frac{\pi}{4} \times 38^{2}}=\frac{98000}{1134.11}$
$\sigma=86.41 \mathrm{MPa}$
$E=\frac{86.41}{.0092 / 200}=1.88 \times 10^{5} \mathrm{MPa}$

## Modulus of Rigidity

$$
\begin{aligned}
& \mathrm{E}=2 \mathrm{G}(1+\mu) \\
& 1.88 \times 10^{5}=2 \mathrm{G}(1+0.2745) \\
& \mathrm{G}=7.37 \times 10^{4} \mathrm{MPa}
\end{aligned}
$$

## Bulk Modulus

$$
\begin{aligned}
& E=3 K(1-2 \mu) \\
& 1.88 \times 10^{5}=3 K(1-2 \times 0.2745) \\
& 1.88 \times 10^{5}=3 K(.451) \\
& K=1.39 \times 10^{5} \mathrm{Mpa}
\end{aligned}
$$

$$
\begin{gathered}
\mu=0.2745 \\
\mathrm{E}=1.88 \times 10^{5} \mathrm{Mpa} \\
\mathrm{G}=7.37 \times 10^{4} \mathrm{Mpa} \\
\mathrm{~K}=1.39 \times 10^{5} \mathrm{MPa}
\end{gathered}
$$

1.(e) (i) With the help of a neat sketch of a typical grading curve, describes the term 'gap-graded aggregate' and the adverse effects of using such type of aggregates in concrete.
[6 Marks]

## Sol. (i) Gap-graded aggregate'



Fig. gap-graded curve

- A flat or horizontal portion in the curve indicates gap-graded aggregates certain sizes of Aggregates are missing in such gradation.
- Although gap gradation leads to less cement and lower w/c ratios, when they are used in high workability mixes segregation may be a problem.
If gap gradation of fines is missing, concrete becomes difficult to handle and compact develops low strength and high permeability.
1.(e) (ii) How is PPC different from OPC on the basis of their ingredients? Describe the advantages of using PPC in comparison to OPC.
[6 Marks]
Sol. (ii) Portland pozzolana cement (PPC) is prepared by adding a certain proportion of fly ash (or other pozzolanic material). It can be produced by inter-grinding Portland cement clinker and pozzolana (15-35\% by mass) together with a small amount of gypsum.
- Pozzolanic materials are silicious in nature and react with calcium hydroxide in the presence of water (in cement) to form cementitious compounds.


## Advantages of PPC over OPC

1. Less heat of hydration at a slower rate of hydration.
2. More resistance to sulfate attack.
3. Can be used in sewers, sewage disposal works, marine structures, mass concrete structures like dams, piers, and thick foundations where OPC is not recommended.
4. Reduces shrinkage and bleeding.
5. Presents calcium hydroxide leaching.

Better workability.
2.(a) A room of effective span $16.50 \mathrm{~m} \times 11.00 \mathrm{~m}$ is surrounded by brick walls. In order to lay the RCC slab over it, the room is divided in four equal panels by providing two central beams. The slab is simply supported on all the four walls as shown in the figure below. Using limit state design, determine and provide main reinforcement in a single panel, using 12 mm diameter steel bars of $\mathrm{Fe}-415$ grade. Consider the grade of concrete as $\mathrm{M}-20$. Draw the reinforcement detail of a panel. Use the following additional data:

Total factored load on slab (dead load + live load) $=16 \mathrm{kN} / \mathrm{m}^{2}$
Thickness of slab $=175 \mathrm{~mm}$
Effective depth of slab $=150 \mathrm{~mm}$
Note: Refer Annex D of IS 456: 2000 for finding the moments at different locations. The Annex is reproduced at Page Nos. 11 and 12.

[20 Marks]

## Sol.



Aspect ratio $=\frac{\text { Larger span }}{\text { Smaller span }}=\frac{L_{y}}{L_{x}}$

$$
=\frac{8.25}{5.5}=1.5<2
$$

So, it is a two-way slab with two adjacent edges discontinuous.
Effective depth $=\mathrm{d}=150 \mathrm{~mm}$.

$$
\mathrm{w}_{\mathrm{u}}=16 \mathrm{kN} / \mathrm{m}^{2}
$$

From Annex D of IS 456 (Table 26)
For aspect ratio $=1.5$ and two adjacent sides discontinuous

|  | $\alpha$ | Equation | $M(k N . m)$ | $A_{\text {st }}\left(\mathrm{mm}^{2}\right)$ | $S(\mathrm{~mm})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{x}(-)$ | 0.075 | $0.075 \times 16 \times 5.5^{2}$ | 36.3 | 748 | 151.1 |
| $\mathrm{x}(+)$ | 0.056 | $0.056 \times 16 \times 5.5^{2}$ | 27.1 | 543.4 | 207.8 |
| $\mathrm{y}(-)$ | 0.047 | $0.047 \times 16 \times 5.5^{2}$ | 22.75 | 493.5 | 228.8 |
| $\mathrm{y}(+)$ | 0.035 | $0.035 \times 16 \times 5.5^{2}$ | 16.94 | 360 | 314.2 |

$$
M=\alpha W_{u} L_{x}^{2}
$$

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{st}}=0.5 \frac{\mathrm{f}_{\mathrm{ck}}}{\mathrm{f}_{\mathrm{y}}}\left[1-\sqrt{1-\frac{4.6 \mathrm{M}_{\mathrm{u}}}{\mathrm{bd}^{2} \mathrm{f}_{\mathrm{ck}}}}\right] \mathrm{bd} \\
& \mathrm{~d}_{\mathrm{x}}=150 \mathrm{~mm}, \mathrm{~b}=1 \mathrm{~m}=1000 \mathrm{~mm}, \mathrm{f}_{\mathrm{ck}}=20 \mathrm{MPa}, \mathrm{f}_{\mathrm{y}}=415 \mathrm{MPa} \\
& \mathrm{~d}_{\mathrm{y}}=150-12=138 \mathrm{~mm}
\end{aligned}
$$

(As 12 mm diameter bars are given)
Final reinforcements:

$$
\begin{aligned}
& \mathrm{A}_{\text {stx }(-)}=12 \mathrm{~mm} @ 150 \mathrm{~mm} \mathrm{c} / \mathrm{c} \\
& \mathrm{~A}_{\text {st } \times(+)}=12 \mathrm{~mm} @ 200 \mathrm{~mm} \mathrm{c} / \mathrm{c} \\
& \mathrm{~A}_{\text {sty }(-)}=12 \mathrm{~mm} @ 210 \mathrm{~mm} \mathrm{c} / \mathrm{c} \\
& \mathrm{~A}_{\text {sty }(+)}=12 \mathrm{~mm} @ 300 \mathrm{~mm} \mathrm{c} / \mathrm{c} \\
& \mathrm{~S}=\frac{1000}{\mathrm{~A}_{\text {st }}} \times \frac{\pi}{4} \times(12)^{2}
\end{aligned}
$$

Maximum spacing $=\min \left\{\begin{array}{l}3 \mathrm{dx} \\ 300\end{array}=\min \left\{\begin{array}{l}450 \mathrm{~mm} \\ 300 \mathrm{~mm}\end{array}=300 \mathrm{~mm}\right.\right.$

2.(b) Explain the mechanism of alkali-aggregate reaction in concrete. How can it be controlled?
[20 Marks]
Sol. The alkali-aggregate reaction (AAR) in concrete refers to a chemical reaction between the alkaline components in cement (such as sodium and potassium) and certain reactive minerals or aggregates present in the concrete mixture. This reaction can lead to the expansion and cracking of concrete structures over time, compromising their durability and structural integrity.

The two main types of AAR are the alkali-silica reaction (ASR) and the alkali-carbonate reaction (ACR).

## Alkali-Silica Reaction (ASR):

- Map at random Concrete spalling, cracking, and closed joints are signs of alkali-silica interactions.
- Petrographic examination can identify alkali-silica reactions.
- It occurs broadly because aggregates containing reactive silica materials are more common.
- Alkali-silica reaction generates enough expansive pressure to damage concrete.
- Moisture-rich regions, such as those near the ground behind retaining walls, near the waterline in piers, or in piers or columns vulnerable to wicking activity, are where cracking first appears.
- It can be controlled using proper portions of supplementary cementitious materials like silica fume, fly ash, and ground granulated blast-furnace slag.
- Alkali-silica reactions can be reduced by using lithium compounds.


## Alkali-Carbonate Reaction (ACR):

- It has been noted with several dolomitic rocks.
- It might result in significant expansion.
- ACR is relatively uncommon compared to alkali-silica reactions because aggregates prone to this phenomenon are less frequent.
- The introduction of additional cementing materials does not stop harmful expansion brought on by ACR.
- Therefore, it is advised against using ACR-sensitive aggregates in concrete.

Controlling Alkali-Aggregate Reaction: To control alkali-aggregate reaction in concrete, several preventive measures can be taken:

1. Aggregate Selection: Use non-reactive or low-reactive aggregates that have been tested for potential reactivity with alkalis. These aggregates should have low levels of reactive silica or carbonate minerals.
2. Cement Selection: Opt. for low-alkali cement, as the alkaline content in cement is a significant factor contributing to AAR. Check the alkali content of the cement and ensure it is within acceptable limits.
3. Blended Cement: Consider using blended cement that contains supplementary cementitious materials (such as fly ash, slag, or silica fume). These materials can help mitigate the risk of AAR by reducing the availability of alkalis for reaction.
4. Chemical Admixtures: Incorporate chemical admixtures, such as lithium-based compounds, that can help control alkali reactivity in concrete.
5. Proper Mix Design: Ensure a well-balanced concrete mix design with an appropriate watercement ratio and adequate air entrainment. The right mix design can enhance durability and reduce the potential for AAR.
6. Moisture Control: Control the moisture content in concrete during its curing and service life, as higher moisture levels can accelerate AAR. Proper drainage and waterproofing measures should be implemented to minimize moisture ingress.
Testing and Monitoring: Conduct aggregate petrographic examination and laboratory testing to assess the potential reactivity of aggregates and to identify reactive materials.
2.(c) Draw the bending moment and shearing force diagrams for the overhanging beam loaded as shown in the figure below. Determine the positions of maximum bending moment, maximum shearing force and locate the locations of zero bending moment:

[20 Marks]
Sol.


Step 1: Calculating support reaction

$$
\begin{align*}
& \Sigma V=0 \\
& R_{B}+R_{E}=25 \times 0.5+45+35=92.5 \mathrm{kN} \tag{i}
\end{align*}
$$

Taking moment about point E

$$
\begin{aligned}
& \Sigma M_{E}=0 \\
& -25 \times 0.5 \times 4.75+R_{B} \times 4.5-95-45 \times 1.5+35 \times 0.5=0 \\
& R_{B}=45.416 \mathrm{kN}
\end{aligned}
$$

From equation (i)
$R_{E}=92.5-45.416=47.08 \mathrm{kN}$
Step 2: Calculation of shear force and bending moment


## Calculation of shear force

Calculation of bending moment

Segment AB
(SF) xx $=-25 x$
(SF) at $x=0=0$
(SF) at $x=0.5$ (just left of support B)
$=-25 \times 0.5=-12.5 \mathrm{kN}$

## Segment AB

(BM) ${ }_{x-x}=-25 \cdot x \cdot \frac{x}{2}=-\frac{25 x^{2}}{2}$
(Parabolic variation)
(BM) at $\mathrm{x}=0=0 \mathrm{kN}-\mathrm{m}$
(BM) ${ }_{\text {at }} x=0.5$ (just left of $B$ ) $=$ $\frac{-25}{2} \times(0.5)^{2}=-3.125 \mathrm{kN}-\mathrm{m}$

## Segment BC

(SF) at $\mathrm{x}=0.5$ (just right of support B)
$=-25 \times 0.5+45.416=32.916 \mathrm{kN}$
(SF remains constant in segment BC )

|  | $45.416(x-0.5)$ <br> (Linear variation) <br> $(B M)_{\text {at }}=0.5$ (just right of $\left.B\right)=-3.125$ <br> kN-m <br> (BM) at $\mathrm{x}=1.5$ ( just left of C ) $=29.791$ <br> kN-m |
| :---: | :---: |
| Segment CD $(\mathrm{SF})_{\mathrm{xx}}=32.916 \mathrm{kN}$ <br> (SF remains constant in segment CD) | Segment CD [1.5 $\leq x \leq 3.5]$ $\begin{aligned} & (B M)_{x x}=-25 \times 0.5 \times(x-0.25)+ \\ & 45.416(x-0.5)-95 \end{aligned}$ <br> (Linear variation) <br> $(B M)_{\text {at }} x=1.5$ (just right of $C$ ) $=-65.215$ <br> kN-m <br> $(B M)$ at $x=3.5$ (just left of $D)=+0.623$ <br> kN-m |
| Segment DE $\begin{aligned} & (S F)_{x x}=32.916-45 \\ & =-12.09 \mathrm{kN} \end{aligned}$ <br> (SF remains constant in segment DE) | $\begin{aligned} & \text { Segment DE }[3.5 \leq x \leq 5] \\ & (B M)_{x x}=-25 \times 0.5 \times(x-0.25) \\ & +45.416(x-0.5)-95-45(x-3.5) \\ & (\text { Linear variation }) \\ & \left.(B M)_{\mathrm{at} x}=3.5 \text { (just right of } D\right)=0.623 \\ & \mathrm{kN}-\mathrm{m} \\ & \left.(B M)_{\mathrm{at} x}=5 \text { (just left of } E\right)=-17.50 \mathrm{kN}- \\ & m \end{aligned}$ |
| Segment EF $(S F)_{x x}=-12.09+47.08=35 \mathrm{kN}$ <br> (SF remains constant in segment EF) | Segment EF $\begin{aligned} & (B M)_{x x}=-25 \times 0.5(x-0.25)+ \\ & 45.416(x-0.5)-95-45(x-3.5)+ \\ & 47.08(x-5) \\ & (\text { Linear variation }) \\ & \left.(B M)_{x=5} \text { (just right of } E\right)=-17.50 \mathrm{kN}- \\ & m \\ & \left.(B M)_{x=5.5} \text { (just left of } F\right)=0 \mathrm{kN}-\mathrm{m} \end{aligned}$ |



1. Maximum bending moment $=65.215 \mathrm{kN}-\mathrm{m}$ which is just right of C and 1.5 m from A .
2. Maximum shear force is 35 kN in segment EF.
3. Position of zero bending moment: $B M$ is zero in segment $B C, C D, D E$ and at $A$ and $F$.

## In segment $B C$ :

$(B M)_{x x}=-25 \times 0.5\left(x-\frac{0.5}{2}\right)+45.416(x-0.5)=0$
$=-12.5 x+3.125+45.416 x-22.708=0$
$x=0.594$ from end $A$
and 0.094 from support $B$.

## In segment CD:

$(B M)_{x x}=-25 \times 0.5 \times(x-0.25)+45.416(x-0.5)-95=0$
$-12.5 x+3.125+45.416 x-22.708-95=0$
$x=3.48 \mathrm{~m}$ from left end $A$

## In segment $D E:$

$(B M)_{x x}=-25 \times 0.5 \times(x-0.25)+45.416(x-0.5)-95-45(x-3.5)=0$

```
x = 3.55 m from left end A
```

3.(a) (i) What is gel-space ratio? How is it estimated? Discuss its effect on the strength of concrete.

Sol. (i) In cement concrete, the reaction between cement and water leads to the formation of a gellike substance called cement gel or hydration gel. This gel occupies a certain volume within the concrete. At the same time, concrete contains various pores, including capillary pores, gel pores, and larger voids, which make up the total pore space.

- The gel-space ratio is a measure of the relative amount of gel present in comparison to the total pore space. It provides an indication of the compactness of the cementitious matrix and can be used to assess the quality and durability of concrete.

$$
\text { Gel space ratio }=\frac{\text { Volume of gel (including gel pores) }}{\text { Volume of gel }+ \text { Volume of capillary pores }}
$$

- The strength of cement/concrete is governed by its porosity which is affected by the gel/space ratio.
- A higher gel/space ratio reduces porosity and increases strength.
- A higher water/cement ratio decreases the gel/space ratio, thus increasing porosity and decreasing strength.


Fig. Curve represents strength vs gel/space ratio.

## Estimation of gel/ space ratio

Gel space ratio $=\frac{0.657 \mathrm{C}}{0.319 \mathrm{C}+\mathrm{W}_{0}}$
$\mathrm{C}=$ Weight of cement (in gm)
$\mathrm{W}_{0}=$ Volume of mixing water (in ml)
For example, if $\mathrm{C}=600 \mathrm{~g}$ and $\mathrm{w} / \mathrm{c}$ ratio $=0.65$
$\mathrm{W}_{0}=0.65 \times 600=390 \mathrm{ml}$
Gel space ratio $=\frac{0.657 \times 600}{0.319 \times 600+390}$
$=0.68$
3.(a) (ii) What are the factors affecting durability of concrete?
[10 Marks]
Sol. (ii) Factors affecting the durability of concrete.

1. Environmental such as extreme temperature, abrasion, and electrostatic actions.
2. Internal causes such as alkali-aggregate reaction, volume changes, etc
3. Chemical attack

- Carbonation
- Acidic water
- Chloride ion penetration
- Marine environment
- Sulfate attack
- Carbonation is the reaction of $\mathrm{CO}_{2}$ in the atmosphere with water in concrete which turns water acidic. Concrete loses its pH and steel becomes prone to oxidation and corrosion.
- Chloride ion penetration leads to swelling in volume, and loss of strength of concrete and makes steel vulnerable to corrosion.
Sulfates lead to an increase in concrete volume and consequent cracking. Calcium sulfate reacts with calcium aluminate forming an expansive ettringite.
3.(b) A rectangular beam of size $300 \mathrm{~mm} \times 600 \mathrm{~mm}$ is used over a simply supported effective span of 7 m . The beam supports a live load of $12 \mathrm{kN} / \mathrm{m}$. A straight tendon is provided at an eccentricity of 100 mm below the centroid of the beam section. Find the minimum prestressing force required for no tension condition at mid-span under live load. Also, show the stress distribution under self-weight only at mid-span and at the ends of the member.
[20 Marks]
Sol. Live load $=w=12 \mathrm{kN} / \mathrm{m}$
Size of beam:
$b=300 \mathrm{~mm}, \mathrm{~d}=600 \mathrm{~mm}$
Effective span $=l_{e}=7 \mathrm{~m}$
Eccentricity= $\mathrm{e}=100 \mathrm{~mm}$


Bending moment at mid span due to live load $=M=\frac{12 \times 7^{2}}{8}=73.5 \mathrm{kN} . \mathrm{m}$
For, no tension condition at midspan under live load
$\sigma($ at bottom of midspan $)=0$

$$
\frac{P}{A}-\frac{M}{Z}+\frac{P e}{Z}=0
$$

(Considering P in kN )

$$
\begin{aligned}
& \frac{P \times 10^{3}}{b d}-\frac{M}{\left(b d^{2} / 6\right)}+\frac{P e}{\left(b d^{2} / 6\right)}=0 \\
& \frac{P \times 10^{3}}{300 \times 600}-\frac{73.5 \times 10^{6} \times 6}{300 \times 600^{2}}+\frac{P \times 10^{3} \times 100 \times 6}{300 \times 600^{2}}=0 \\
& \frac{P}{180}-4.0833+\frac{P}{180}=0 \\
& \frac{P}{90}=4.0833 \\
& P=367.5 \mathrm{kN}
\end{aligned}
$$

Stress at top and bottom of midspan due to dead load only (self-weight)
Dead load $=$ bd. $y_{c}=0.3 \times 0.6 \times 23.5 \mathrm{kN} / \mathrm{m}=4.23 \mathrm{kN} / \mathrm{m}$
(Unit weight of Pre-stressed concrete $=23.5 \mathrm{kN} / \mathrm{m}^{3}$ as per IS 875 Part I

$$
\begin{aligned}
& M=\frac{4.23 \times 7^{2}}{8}=25.90875 \mathrm{kN} . \mathrm{m} \\
& \sigma(\text { top })=\frac{P}{A}+\frac{M}{Z}-\frac{P e}{Z} \\
& =\frac{P}{180}+\frac{25.90875 \times 10^{6} \times 6}{300 \times 600^{2}}-\frac{P}{180} \\
& =1.44 \mathrm{~N} / \mathrm{mm}^{2} \\
& \sigma(\text { bottom })=\frac{P}{A}-\frac{M}{Z}+\frac{P e}{Z} \\
& =\frac{P}{90}-1.44=2.643 \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$


2.643 MPa

At mid span (due to self-weight)
Stress at top and bottom of end (supports) due to dead load only (self-weight)

$$
\begin{aligned}
& M=0(\text { Simply supported }) \\
& \sigma(\text { top })=\frac{P}{A}-\frac{P e}{Z}=\frac{P}{180}-\frac{P}{180}=0 \\
& \sigma(\text { bottom })=\frac{P}{A}+\frac{P e}{Z}=\frac{P}{90}=4.083 \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$


3.(c) A uniformly distributed load of $45 \mathrm{kN} / \mathrm{m}$ longer than the span rolls over a simply supported girder of 35 m span. Using influence line diagram for shear force and bending moment, determine the maximum shear force and maximum bending moment at a section 14 m from left-hand support.
[20 Marks]
Sol.


## ILD for SFC



For the maximum value of SF at $C$, the load should be on span $B C$.

$$
\left(\mathrm{SF}_{\mathrm{c}}\right)_{\max }=45 \times \frac{1}{2} \times 0.6 \times 21=283.5 \mathrm{kN}
$$

## ILD for BMC



The length of UDL is greater than the span length. So, for maximum BM the load should be on the entire span.

$$
(B M)_{\max }=45 \times \frac{1}{2} \times 8.4 \times 35=6615 \mathrm{kN}-\mathrm{m}
$$

4.(a) (i) For the vibrating system shown in the figure below, determine the following parameters:
(1) Natural frequency of the vibrating system
(2) Critical damping of the vibrating system
(3) Damping ratio
(4) Damped natural frequency of the vibrating system

[10 Marks]
Sol. (i) Given data


Stiffness of spring $\mathrm{k}=2400 \mathrm{~N} / \mathrm{m}$
Viscous damping C $=77 \mathrm{~N}-\mathrm{S} / \mathrm{m}$

## 1. Natural frequency ( $\omega_{n}$ )

$\omega_{n}=\sqrt{\frac{k}{m}}=\sqrt{\frac{2400}{10}}=15.5$ radian $/ \mathrm{sec}$
2. Critical damping coefficient (Ccritical)
$C_{\text {critical }}=\frac{2 \mathrm{k}}{\omega_{\mathrm{n}}}=\frac{2 \times 2400}{15.5}=309.67 \mathrm{~N}-\mathrm{s} / \mathrm{m}$
3. Damping ratio ( $\xi$ )
$\xi=\frac{C}{C_{\text {critical }}}=\frac{77}{309.67}=0.248$
4. Damped natural frequency of the vibration system ( $\omega \mathrm{D}$ )

$$
\begin{aligned}
& \omega_{D}=\omega_{n} \sqrt{1-\xi^{2}} \\
& =15.5 \sqrt{1-0.248^{2}}=15.015 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

4.(a) (ii) A short braced reinforced concrete column has unsupported length of 3.5 m and size of $300 \mathrm{~mm} \times 360 \mathrm{~mm}$. Verify the applicability of simplified formula of $\mathrm{P}_{u}$ (i.e., ultimate loadcarrying capacity of a short axially loaded column) as given in the Code. Also, determine the design moments due to minimum eccentricity to be considered for this case if the column is subjected to an ultimate axial load of 1600 kN .
[10 Marks]

Sol. (ii) Unsupported length $=3.5 \mathrm{~m}=3500 \mathrm{~mm}$
Size $300 \mathrm{~mm} \times 360 \mathrm{~mm}$.
Least lateral dimension $\mathrm{b}=300 \mathrm{~mm}$.
$\mathrm{P}_{\mathrm{u}}=1600 \mathrm{kN}$
Assuming leff $=3.5 \mathrm{~m}=3500 \mathrm{~mm}$ (both ends pinned)


## Check for slenderness ratio:

Slenderness ratio $=\frac{l_{\text {eff }}}{b}=\frac{3500}{300}=11.67<12$.
It is hence a short column.

## Check for eccentricity:

Minimum eccentricity (about 300 mm axis)
$=\frac{\mathrm{L}}{500}+\frac{300}{30}=17 \mathrm{~mm}$ or $20 \mathrm{~mm}(\max )=20 \mathrm{~mm}$.
Minimum eccentricity (about 360 mm )
$=\frac{\mathrm{L}}{500}+\frac{360}{30}=19 \mathrm{~mm}$ or $20 \mathrm{~mm}(\max )=20 \mathrm{~mm}$.
$20 \mathrm{~mm}>0.05 \times 300=15 \mathrm{~mm}$.
$20 \mathrm{~mm}>0.05 \times 360=18 \mathrm{~mm}$.
So, the column is designed as eccentrically loaded column subjected to biaxial bending.
Design moment $=$ P.e $=1600 \mathrm{kN} \times 0.02 \mathrm{~m}=32 \mathrm{kN} . \mathrm{m}$.
4.(b) (i) What are the factors affecting rheological properties of concrete?
[10 Marks]
Sol. (i) Rheology of fresh concrete like workability includes parameters of stability, mobility, and compatibility. It also includes forces involved in the transmission of mechanical stresses on concrete and the deformation curve of fresh concrete.
The Bingham model is used to represent flow properties of a material. As shown in the diagram, $\tau_{0}$ (yield stress) is the minimum stress necessary for flow to occur $\mu$ is called plastic viscosity.


Fig. Bingham model
The behavior of concrete varies from the Bingham model as yield stress is not well defined and the flow curve is not linear.

Factors affecting rheological behavior of concrete.

## 1. Mix proportion

| Excess coarse aggregate | Harsh mix - greater effort to place and <br> compact due to loss of cohesion |
| :--- | :--- |
| Excess fine aggregate or entrained air | Increase in cohesion making concrete <br> difficult to move |
| Lower w/c ratio and high cement content | Reduction of workability of rich mixes. |

## 2. Consistency

Consistency is measured by a slump test.

| High slump | Greater fluidity and reduced cohesion increases the chances of <br> segregation of bleeding |
| :--- | :--- |
| Low slump | Reduction in mobility and compatibility. Also, difficulty in placement |

3. Shape of aggregates

| Rough and angular aggregates | A higher percentage of voids and an increase in internal <br> friction require higher water content than in the case of <br> round aggregates |
| :--- | :--- |

4. Aggregate gradation

Well-graded aggregates give good workability.

| Finer fine aggregate | More water requirement sticky mixture |
| :--- | :--- |
| Coarser fine aggregate | Harsh mix and tendency for bleeding |

5. Admixtures

| Plasticizers and <br> superplasticizer | Reduce water content for the same amount of workability so more <br> strength |
| :--- | :--- |
| Air entraining agents | Increased workability |
| Accelerators | Increase rate of hardening |
| Retarders | Delays the setting |

4.(b) (ii) How are the properties of concrete affected by seawater and industrial wastewater, if they are used for making the concrete?
[10 Marks]

Sol. (ii) Salts present in seawater reduce the ultimate strength of concrete (around 10 to 20 percent reduction).

1. Chlorides in seawater increase the risk of corrosion of reinforcing steel.
2. Chlorides also cause efflorescence.
3. The effluent from sewerage works, gas works, and from the paint, textile, sugar, and fertilizer industries is harmful to concrete.
4. Dissolved salts in industrial water reduce strength by 10 to $30 \%$ (Salt of $\mathrm{Zn}, \mathrm{Cu}, \mathrm{Pb}, \mathrm{Sn}$ and Mn )
5. $\mathrm{ZnCl}_{2}$ retard the setting of concrete.
6. $\mathrm{Pb}\left(\mathrm{NO}_{3}\right)_{2}$ [Lead nitrate] is destructive.
7. Salts of sodium like $\mathrm{NaI}, \mathrm{Na}_{3} \mathrm{PO}_{4}, \mathrm{Na}_{3} \mathrm{AsO}_{4}, \mathrm{Na}_{2}\left[\mathrm{~B}_{4} \mathrm{O}_{5}(\mathrm{OH})_{4}\right] .8 \mathrm{H}_{2} \mathrm{O}$ (Sodium iodide, sodium phosphate, sodium arsenate, and sodium borate respectively) reduces the initial strength of concrete.
8. Sodium and potassium carbonates may cause rapid setting and reduces concrete strength.
9. Acids present in industrial wastewater are unsuitable for concrete construction as it decreases pH and leads to the risk of corrosion of reinforced steel.
10. Algae may be present in some industrial water and has the effect of entraining large amounts of air in concrete and reduces the strength of concrete. Algae combine with cement and reduce the bond between aggregates and cement paste.
Mineral oils when present more than eight percent lead to a reduction in strength. When it is up to two percent by weight of cement, an increase in strength has been noticed.
4.(c) Analyze the portal frame shown in the figure below by moment distribution method. The frame is fixed at $A$ and $D$, and has rigid joints at $B$ and $C$. Draw the bending moment diagram and sketch the deflected shape of the structure. Take EI as constant:


## Sol.



Step (1) Fixed end moments

$$
\begin{aligned}
& \overline{M_{A B}}=\overline{M_{B A}}=\overline{M_{C D}}=\overline{M_{D C}}=0 \\
& \overline{M_{B C}}=\frac{-w L^{2}}{12}=-\frac{16 \times 4^{2}}{12}=-21.33 \mathrm{kNm} \\
& \overline{M_{C B}}=+\frac{w L^{2}}{12}=\frac{+16 \times 4^{2}}{12}=+21.33 \mathrm{kNm}
\end{aligned}
$$

## Step (2) Distribution factor

| Joint | Member | K | гK | DF |
| :---: | :---: | :---: | :---: | :---: |
| B | BA | $\frac{4 \mathrm{EI}}{3}$ |  | $\frac{4}{7}$ |
|  | BC | $\frac{4 \mathrm{EI}}{4}$ | $\frac{7 \mathrm{EI}}{3}$ | $\frac{3}{7}$ |
|  | CB | $\frac{4 \mathrm{EI}}{4}$ |  | $\frac{3}{7}$ |
|  | CD | $\frac{4 \mathrm{EI}}{3}$ | $\frac{7 E I}{3}$ | $\frac{4}{7}$ |

Step (3) Moment distribution table



## SECTION-'B'

5.(a) (i) Describe the mechanical properties of ceramics.
[6 Marks]

## Sol. (i) Mechanical Properties of Ceramics

A ceramic is a non-metallic solid material comprising an inorganic compound of metal, nonmetal atoms that are bonded by ionic or covalent bonds.

Table: Brief summary of mechanical properties of ceramics

| Property | Degree |
| :---: | :---: |
| Brittle | High |
| Hardness | High |
| Elastic modulus | High |
| Density | Low |
| Ductility | Low |

1. Elastic modulus of ceramics is higher than that of steel.
2. Density of ceramics is $20-60 \%$ of the density of steel.
3. Ceramics have good thermal insulating properties.
4. Ceramics have good compressive strength.
5. They are chemically inert (although it's a chemical property) making ceramics applicable in orthopedic and dental implants.
6. Ceramics have a high melting point.
7. Ceramics have low electrical conductivity.
8. Ceramics have low tensile strength.
9. Ceramics resist penetration by scratching or indentation.
5.(a) (ii) Explain roller-compacted concrete. What are the advantages of roller compacted concrete?

Sol. (ii) Roller-compacted concrete is a mixture of aggregates, cement, and water, such that it has zero slumps. The stiff (dry and lean) concrete is mixed in a pug mill mixer and spread with a modified asphalt paver and compacted with a roller.

1. It requires only 60 to $70 \%$ cement in comparison to conventional concrete thus reducing the heat of hydration.
2. It has a low-water-cement ratio.
3. Aggregate interlocking is good.
4. It exhibits lower shrinkage.
5. It is economical as compared to conventional mass concreting due to a reduction in the quality of cement, reduction in manual work, etc.
They find applications in dams, off-highway pavement projects, parking, storage area carriageways of airports.
5.(b) A steel cable of 12 mm diameter is stretched across two poles 80 m apart. If the central dip is 1.10 m at normal temperature, determine the stress intensity in the cable. Also, determine the change in temperature necessary to raise the stress to 80 MPa . Take unit weight of steel $\gamma=78 \mathrm{kN} / \mathrm{m}^{3}$ and $\alpha=12 \times 10^{6} /{ }^{\circ} \mathrm{C}$.
[12 Marks]

## Sol.



$$
\begin{aligned}
& \gamma=\text { unit weight of steel }=78 \mathrm{kN} / \mathrm{m}^{3} \\
& \alpha=12 \times 10^{6} /{ }^{\circ} \mathrm{C} . \\
& \mathrm{d}=12 \mathrm{~mm}
\end{aligned}
$$

Let us consider w is self-weight of cable.

$$
\mathrm{w}=\gamma \times \frac{\pi}{4} \times \mathrm{d}^{2}=78 \times \frac{\pi}{4} \times\left(12 \times 10^{-3}\right)=8.821 \times 10^{-3} \mathrm{kN} / \mathrm{m}=8.82 \mathrm{~N} / \mathrm{m}
$$

Maximum tension will occur at any support, because both the support are at same level.

$$
\begin{aligned}
& \mathrm{T}_{\max }=\sqrt{\mathrm{V}_{\mathrm{A}}^{2}+\mathrm{H}_{\mathrm{A}}^{2}} \\
& \mathrm{H}_{\mathrm{A}}=\frac{\mathrm{wL}^{2}}{8 \mathrm{~h}}=\frac{8.82 \times 80^{2}}{8 \times 1.1}=6414.54 \mathrm{~N} \\
& \mathrm{~V}_{\mathrm{A}}=\frac{\mathrm{WL}}{2}=\frac{8.82 \times 80}{2}=352.8 \mathrm{~N} \\
& \mathrm{~T}_{\max }=\sqrt{352.8^{2}+6414.54^{2}}=6424.23 \mathrm{~N}
\end{aligned}
$$

Stress intensity in cable $=\frac{T_{\max }}{A}=\frac{6424.23}{\frac{\pi}{4} \times 12^{2}}=56.80 \mathrm{~N} / \mathrm{mm}^{2}$
Due to change in temperature, the central dip will also change.
Let us consider $\mathrm{t}^{\circ} \mathrm{C}$ is the fall in temp.
$\delta \mathrm{h}=$ Change in dip due to change in temp

$$
\begin{aligned}
& \delta h=\frac{3}{16} \frac{L^{2}}{h} \alpha t=\frac{3}{16} \times \frac{80^{2}}{1.1} \times 12 \times 10^{-6} t=0.013090 t \\
& \frac{\delta f}{f}=\frac{\delta h}{h}
\end{aligned}
$$

where, $\delta \mathrm{f}$ - Change in stress

$$
\begin{aligned}
& \frac{80-56.80}{56.80}=\frac{0.013090 t}{1.1} \\
& t=34.32^{\circ} \mathrm{C}
\end{aligned}
$$

5.(c) A T- beam is continuous over a span of 10 m . The sectional parameters of the beam are as below:

Width of web $=250 \mathrm{~mm}$
Width of flange $=1100 \mathrm{~mm}$
Effective depth of beam $=460 \mathrm{~mm}$
Area of steel in tension $=1800 \mathrm{~mm}^{2}$
Area of steel in compression $=1000 \mathrm{~mm}^{2}$
Use M-20 grade of concrete and Fe-415 grade of steel. Estimate the safety of the beam for deflection control using the empirical method given in the Code IS 456: 2000. The corresponding graphs are reproduced at Page Nos. 13-15.
[12 Marks]
Sol. Given:
$\mathrm{B}_{\mathrm{w}}=250 \mathrm{~mm}$
$\mathrm{~B}_{\mathrm{f}}=1100 \mathrm{~mm}$
$\mathrm{~d}=460 \mathrm{~mm}$
$\mathrm{~A}_{\mathrm{st}}=1800 \mathrm{~mm}^{2}$
$\mathrm{~A}_{\mathrm{sc}}=1000 \mathrm{~mm}^{2}$
As IS $456-2000$

Clause 23.2.1

$$
\begin{aligned}
& \frac{\text { Span }}{\mathrm{d}} \leq A \text { value (up to } 10 \mathrm{~m} \text { span) } \\
& \frac{\text { Span }}{\mathrm{d}} \leq A \text { value } \times \frac{10}{\text { span }} \times M F_{1} \times M F_{2} \times R F
\end{aligned}
$$

Multiplied with $\frac{10}{\text { Span }}$ when span is $\geq 10 \mathrm{~m}$.
$M F_{1}=$ Modification factor 1 based on tension reinforcement
$M F_{2}=$ Modification factor 2 based on compression reinforcement
RF $=$ Reduction factor for flange section
$\mathrm{d} \geq \frac{\text { Span } \times \text { Span }}{\text { A value } \times 10 \times \mathrm{MF}_{1} \times \mathrm{MF}_{2} \times \mathrm{RF}}$
Span = 10 m
$\mathrm{d} \geq \frac{\text { Span } \times 10}{\text { A value } \times 10 \times \mathrm{MF}_{1} \times \mathrm{MF}_{2} \times \mathrm{RF}}$
(i) For continuous beam, A value $=26$
(ii) $\mathrm{MF}_{1}$

$$
\begin{aligned}
& \mathrm{p}_{\mathrm{t}} \%=\frac{A_{\text {st provided }}}{B_{w} d} \times 100=\frac{1800}{250 \times 460} \times 100=1.5652 \% \\
& \mathrm{f}_{\mathrm{s}}=0.58 \mathrm{f}_{\mathrm{y}} \times \frac{A_{\text {st required }}}{A_{\text {st provided }}}=0.58 \times 415 \times \frac{1800}{1800}=240.7 \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$

As per the graph / figure 4 for $\mathrm{MF}_{1}$
For $p_{t}=1.5652 \%$, and $f_{s}=240.7 \mathrm{~N} / \mathrm{mm}^{2}$
$M F_{1}=0.9$ (approx.)
(iii) $\mathrm{MF}_{2}$
$\% p_{c}=\frac{A_{s c}}{B_{w} d} \times 100=\frac{1000}{250 \times 460} \times 100=0.87 \%$
As per the graph/fig 5 for MF2
For $\mathrm{p}_{\mathrm{c}}=0.87 \%$,
$M F_{2}=1.22$
(iv) RF
$\frac{\mathrm{B}_{\mathrm{w}}}{\mathrm{B}_{\mathrm{f}}}=\frac{250}{1100}=0.227$
So, RF $=0.8$ (As per figure 6)
$d \geq \frac{10000}{26 \times 0.9 \times 1.22 \times 0.8}$
$\mathrm{d}_{\text {required }} \geq 437.85 \mathrm{~mm}$
$\mathrm{d}_{\text {provided }}=460 \mathrm{~mm}>\mathrm{d}$ required
So, beam is safe as per deflection criteria.
5.(d) A single angle ISA $100 \times 100 \times 10$ is connected to a gusset plate of thickness 10 mm by weld along two parallel edges. The size of weld (fillet) is 6 mm . The member is subjected to an axial compressive load of 150 kN (factored). Find the weld length along two parallel edges. Assume E250 grade of steel and shop welded. For ISA $100 \times 100 \times 10, c_{y}=c_{z}=27.6 \mathrm{~mm}$. Use limit state method.
[12 Marks]
Sol. Factored load $=150 \mathrm{kN}$
So, the strength of the weld should be equal to 150 kN .
Size of weld $=6 \mathrm{~mm}=\mathrm{S}$
Effective throat thickness $=\mathrm{t}_{\mathrm{t}}=0.7 \mathrm{~S}=4.2 \mathrm{~mm}$
Let the total length of weld $=L_{w}$
For E250 grade of steel, $f_{u}=410 \mathrm{~N} / \mathrm{mm}^{2}$
Partial safety factor for weld $=Y_{m w}=1.25$ [Since shop welded]

$$
\begin{aligned}
& 150 \times 10^{3}=\left(\frac{f_{u}}{\sqrt{3} \gamma_{m w}}\right) L_{w} t_{t} \\
& 150 \times 10^{3}=\frac{410}{\sqrt{3} \times 1.25} L_{w} \times 4.2 \\
& L_{w}=188.6 \mathrm{~mm}
\end{aligned}
$$

Total weld length is distributed along two parallel edges as shown
$L_{w 1}+L_{w}=L_{w}$


5.(e) Determine the maximum principal stress developed in a cylindrical shaft 10 cm in diameter, subjected to BM of $3.0 \mathrm{kN}-\mathrm{m}$ and twisting moment of $4.50 \mathrm{kN}-\mathrm{m}$. If the yield stress of the shaft material is $230 \mathrm{MN} / \mathrm{m}^{2}$, determine the factor of safety according to the maximum shearing stress theory of failure.
[12 Marks]

## Sol. When only bending moment (M) is acting



$$
\sigma_{\max }=\frac{32 \mathrm{M}}{\pi D^{3}}
$$

## When only twisting moment ( $T$ ) is acting



$$
\tau_{\max }=\frac{16 \mathrm{~T}}{\pi \mathrm{D}^{3}}
$$

## When both M and T are acting


$\sigma_{x}=\frac{32 \mathrm{M}}{\pi \mathrm{D}^{3}}, \sigma_{y}=0$
$\tau_{x y}=\frac{16 T}{\pi D^{3}}$
$\sigma_{1}=\frac{\sigma_{x}+\sigma_{y}}{2}+\sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau_{x y}^{2}}$
$=\frac{32 \mathrm{M}}{\pi \mathrm{D}^{3} \times 2}+\sqrt{\left(\frac{32 \mathrm{M}}{\pi \mathrm{D}^{3} \times 2}\right)^{2}+\left(\frac{16 \mathrm{~T}}{\pi \mathrm{D}^{3}}\right)^{2}}$
$\sigma_{1}=\frac{16}{\pi D^{3}}\left[M+\sqrt{M^{2}+T^{2}}\right]$
Similarly, $\sigma_{2}=\frac{16}{\pi D^{3}}\left[M-\sqrt{M^{2}+T^{2}}\right]$
Given:

$$
\begin{aligned}
& M=3 \mathrm{kN}-\mathrm{m} \\
& \mathrm{~T}=4.5 \mathrm{kN}-\mathrm{m} \\
& \mathrm{D}=10 \mathrm{~cm}=100 \mathrm{~mm} \\
& \sigma_{1}=\frac{16 \times 10^{6}}{\pi \times 100^{3}}\left[3+\sqrt{3^{2}+4.5^{2}}\right]=42.82 \mathrm{~N} / \mathrm{mm}^{2} \\
& \sigma_{2}=\frac{16 \times 10^{6}}{\pi \times 100^{3}}\left[3-\sqrt{3^{2}+4.5^{2}}\right]=-12.26 \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$

According to maximum shear stress theory
$\frac{\sigma_{1}-\sigma_{2}}{2} \leq \frac{\sigma_{y} / \text { FOS }}{2}$
$42.82-(-12.26) \leq \frac{230}{\text { FOS }}$
$55.08 \leq \frac{230}{\text { FOS }}$
FOS $=\frac{230}{55.08}$
$=4.17$
6.(a) Determine the vertical and horizontal deflection at the free end of the frame shown in the figure below. Take EI $=12 \times 10^{4} \mathrm{kN}-\mathrm{m}^{2}$ :


Sol.


## Vertical deflection at D



Let us apply V - Pseudo force at D in vertically downward direction.
As per Castigliano's II theorem

$$
U_{T}=\int \frac{M^{2} d x}{2 E I}
$$

For $\Delta_{v D}$ take partial derivative w.r.t V

$$
\left.\frac{\partial \mathrm{U}_{\mathrm{T}}}{\partial \mathrm{~V}}\right|_{\mathrm{V}=0}=\Delta_{\mathrm{VD}}=\int \frac{\mathrm{M} \frac{\partial \mathrm{M}}{\partial \mathrm{~V}}}{\mathrm{EI}} \mathrm{dx}
$$

| Members | Section | BM | $\frac{\partial \mathrm{M}}{\partial \mathrm{V}}$ | Limit of integration |
| :---: | :---: | :---: | :---: | :---: |
| $C D$ |  | 0 | 0 | 0 to 2 |
| CB |  | $-V x-\frac{35 x^{2}}{2}$ | -X | 0 to 4 |
| AB |  | $\begin{gathered} -4 V-\frac{35 \times 4^{2}}{2}-45 x \\ =(-4 V-45 x- \\ 280) \end{gathered}$ | -4 | 0 to 3.5 |

$\Delta_{V D}=\int \frac{\frac{M \partial M}{\partial V} d x}{E I}$
$\left.\Delta_{\mathrm{VD}}\right|_{\mathrm{V}=0}=0+\int_{0}^{4} \frac{\left(-\mathrm{Vx}-\frac{35 \mathrm{x}^{2}}{2}\right)(-\mathrm{x}) \mathrm{dx}}{\mathrm{EI}}+\int_{0}^{3.5} \frac{(-4 \mathrm{x}-45 \mathrm{x}-280)(-4) \mathrm{dx}}{\mathrm{EI}}$
$\Delta_{V D}=\int_{0}^{4}+\frac{35 x^{3}}{2 E I} d x+\int_{0}^{3.5}+\frac{180 x+1120}{E I} d x$
$\Delta_{\mathrm{VD}}=\left.\right|_{0} ^{4} \frac{35 \mathrm{x}^{4}}{8 \mathrm{EI}}+\left.\right|_{0} ^{3.5} \frac{\frac{180 \mathrm{x}^{2}}{2}+1120 \mathrm{x}}{\mathrm{EI}}$
$\Delta_{\mathrm{VD}}=\frac{35 \times 4^{4}}{8 \times 12 \times 10^{4}}+\frac{180 \times 3.5^{2}}{2 \times 12 \times 10^{4}}+\frac{1120 \times 3.5}{12 \times 10^{4}}$
$\Delta \mathrm{vD}=0.0511875 \mathrm{~m}$ or 51.19 mm

## Horizontal deflection at $\mathbf{D}$



Let us apply H - pseudo force at D towards the right direction As per Castigliano's II theorem

$$
\begin{aligned}
& U_{T}=\int \frac{M^{2} d x}{2 E I} \\
& \left.\frac{\partial U_{T}}{\partial H}\right|_{H=0}=\int \frac{M \frac{\partial M}{\partial H} d x}{E I}
\end{aligned}
$$

| Members | Section | BM | $\frac{\partial \mathrm{M}}{\partial \mathrm{H}}$ | Limit of integration |
| :---: | :---: | :---: | :---: | :---: |
| CD |  | + Hx | X | 0 to 2 |
| CB |  | $+2 \mathrm{H}-\frac{35 \mathrm{x}^{2}}{2}$ | 2 | 0 to 4 |
| AB |  | $\begin{gathered} +(2-x) H-\frac{35 \times 4^{2}}{2} \\ -45 x \\ (2-x) H-280- \\ 45 x \end{gathered}$ | (2-x) | 0 to 3.5 |

$$
\begin{aligned}
& \left.\Delta_{\mathrm{HD}}\right|_{\mathrm{H}=0}=\int \frac{\frac{\mathrm{M} \partial \mathrm{M}}{\partial \mathrm{H}} \mathrm{dx}}{\mathrm{EI}} \\
& ==0+\int_{0}^{4} \frac{\left(2 \mathrm{H}-\frac{35 \mathrm{x}^{2}}{2}\right) 2 \mathrm{dx}}{\mathrm{EI}}+\int_{0}^{3.5} \frac{[(2-\mathrm{x}) \mathrm{H}-280-45 \mathrm{x}](2-\mathrm{x}) \mathrm{dx}}{\mathrm{EI}} \\
& \Delta_{\mathrm{HD}}=\int_{0}^{4} \frac{-35 \mathrm{x}^{2} \mathrm{dx}}{\mathrm{EI}}+\int_{0}^{3.5} \frac{\left(-560-90 \mathrm{x}+280 \mathrm{x}+45 \mathrm{x}^{2}\right)}{\mathrm{EI}} \\
& \Delta_{\mathrm{HD}}=\left.\right|_{0} ^{4} \frac{-35 \mathrm{x}^{3}}{3 \mathrm{EI}}+\left.\right|_{0} ^{3.5} \frac{-560 \mathrm{x}}{\mathrm{EI}}-\frac{90 \mathrm{x}^{2}}{2 \mathrm{EI}}+\frac{280 \mathrm{x}^{2}}{2 \mathrm{EI}}+\frac{45 \mathrm{x}^{3}}{3 \mathrm{EI}} \\
& \Delta_{\mathrm{HD}}=\frac{-35 \times 4^{3}}{3 \times 12 \times 10^{4}}-\frac{560 \times 3.5}{12 \times 10^{4}}-\frac{90 \times 3.5^{2}}{2 \times 12 \times 10^{4}}+\frac{280 \times 3.5^{2}}{2 \times 12 \times 10^{4}}+\frac{45 \times 3.5^{3}}{3 \times 12 \times 10^{4}} \\
& \Delta_{\mathrm{HD}}=-7.49 \times 10^{-3} \mathrm{~m}=-7.49 \mathrm{~mm}
\end{aligned}
$$

6.(b) A cantilever beam (ISMB 500) is connected to the flange of the column (ISHB 450) by fillet weld of size 5 mm . The beam is subjected to a vertical load $P$ and a horizontal load $P / 2$ at a distance of 200 mm from the flange of the column as shown in the figure below. Find the factored $P$ that can be applied for the joint. Assume E250 grade of steel, site weld. Given, $f_{e}=\sqrt{f_{a}^{2}+3 q^{2}}$. Use limit state method. Assume that the beam section is safe:

[20 Marks]
Sol. $S=5 \mathrm{~mm}$ (Size), Effective throat thickness $\left(\mathrm{t}_{\mathrm{t}}\right)=0.7 \mathrm{~S}=3.5 \mathrm{~mm}$. Let $P$ is the factored load in $k N$.

$$
A=(180+400+400+180) 3.5=4060 \mathrm{~mm}^{2}
$$

## About vertical direction

$$
\begin{aligned}
\text { Shear stress } & =q_{v}=\frac{\text { Vertical force }}{\text { Area of weld }}=\frac{P \times 1000}{4060} . \\
& =0.2463 \mathrm{P} \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$



Bending moment $=P \times 10^{3} \times 200$ N.mm

$$
=2 \times 10^{5} \mathrm{P} \mathrm{N.mm} .
$$

Bending stress calculation:

$$
I_{x x}=\left[180 \times \frac{3.5^{3}}{12}+(180 \times 3.5) \times\left(250+\frac{3.5}{2}\right)^{2}\right] \times 2+7 \times \frac{400^{3}}{12}
$$

$$
\begin{aligned}
& =117.19 \times 10^{6} \mathrm{~mm}^{4} \\
& Z_{x x}=\frac{I_{x x}}{Y_{\max }}=\frac{117.19 \times 10^{6}}{253.5}=46.23 \times 10^{4} \mathrm{~mm}^{3}
\end{aligned}
$$

So, $\quad f_{b}=\frac{M}{Z_{x x}}=\frac{2 \times 10^{5} \mathrm{P}}{46.23 \times 10^{4}}=0.4326 \mathrm{P} \mathrm{N} / \mathrm{mm}^{2}$
Resultant shear stress along vertical direction:

$$
\begin{aligned}
& \left(q_{R}\right)_{v}=\sqrt{f_{b}^{2}+3 q^{2}} \\
& =\left(\sqrt{0.4326^{2}+3 \times 0.2463^{2}}\right) \mathrm{P} \\
& =0.6076 \mathrm{P} \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$

## About horizontal direction

Shear stress $=\mathrm{q}_{\mathrm{H}}=\frac{\text { Horizontal force }}{\text { Area }}=\frac{\mathrm{P} \times 10^{3}}{2 \times 4060}$

$$
=0.12315 \mathrm{P} \mathrm{~N} / \mathrm{mm}^{2}
$$

Bending moment $=\frac{P}{2} \times 1000 \times 200=1 \times 10^{5} \mathrm{P} \mathrm{Nmm}$.
Bending stress calculation:

$$
\begin{aligned}
& I_{y y}=\frac{3.5 \times 180^{3}}{12} \times 2+\left[400 \times \frac{3.5^{3}}{12}+(400 \times 3.5) \times 6.75^{2}\right] \times 2 \\
& =3.5324 \times 10^{6} \mathrm{~mm}^{4} \\
& Z_{y y}=\frac{I_{y y}}{x_{\max }}=\frac{3.5324 \times 10^{6}}{90}=3.925 \times 10^{4} \mathrm{~mm}^{3} . \\
& f_{b}=\frac{M}{Z_{y y}}=\frac{1 \times 10^{5} \mathrm{P}}{3.925 \times 10^{4}}=2.5478 \mathrm{P} \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$

Resultant shear stress about horizontal direction:

$$
\begin{aligned}
& \left(q_{R}\right)_{H}=\sqrt{f_{b}^{2}+3 q^{2}} \\
& =\sqrt{2.5478^{2}+3 \times 0.12315^{2}} \mathrm{P}=2.5567 \mathrm{P} \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
$$

Resultant shear stress

$$
\begin{aligned}
& q_{R}=\sqrt{\left(q_{R}\right)_{H}^{2}+\left(q_{R}\right)_{V}^{2}} \\
& =\sqrt{2.5567^{2}+0.6076^{2}} \mathrm{P} \\
& =2.628 \mathrm{P} \mathrm{~N} / \mathrm{mm}^{2} \\
& q_{R}=\text { Design shear strength of weld. } \\
& 2.628 \mathrm{P}=\frac{f_{u}}{\sqrt{3} \mathrm{Y}_{\mathrm{mw}}} \\
& 2.628 \mathrm{P}=\frac{410}{\sqrt{3}(1.25)} \\
& P=72.06 \mathrm{kN}
\end{aligned}
$$

6.(c) The following table gives the details of various activities of a construction project:

| Activity | Optimistic time <br> (months) | Most likely time <br> (months) | Pessimistic time <br> (months) |
| :---: | :---: | :---: | :---: |
| $1-2$ | 2 | 2 | 8 |
| $1-3$ | 2 | 5 | 8 |
| $1-4$ | 3 | 3 | 9 |
| $2-5$ | 2 | 2 | 4 |
| $3-5$ | 3 | 6 | 15 |
| $4-6$ | 3 | 6 | 9 |
| $5-6$ | 2 | 7 | 16 |
| $6-7$ | 2 | 2 | 2 |

(i) Draw the network for the project.
(ii) Find the expected duration and variance of each activity.
(iii) What is the expected project length?
(iv) What is the probability that the project will be completed at least 3 months earlier than expected?
(v) What will be the time required for a $95 \%$ probability of its completion?
[20 Marks]
Sol. $\quad \mathrm{t}_{0}=$ Optimistic time
$\mathrm{t}_{\mathrm{m}}=$ Most likely time
$t_{p}=$ Pessimistic time
$\mathrm{t}_{\mathrm{E}}=$ Expected duration $=\frac{\mathrm{t}_{0}+4 \times \mathrm{t}_{\mathrm{m}}+\mathrm{t}_{\mathrm{p}}}{6}$
$\sigma=$ standard dentation $=\frac{t_{p}-t_{0}}{6}$
$\sigma^{2}=$ Variance

| Activity <br> Number | $\mathrm{t}_{\mathrm{o}}$ | $\mathrm{t}_{\mathrm{m}}$ | $\mathrm{t}_{\mathrm{p}}$ | $\mathrm{t}_{\mathrm{E}}$ | $\sigma$ | $\sigma^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1-2$ | 2 | 2 | 8 | 3 | 1 | 1 |
| $1-3$ | 2 | 5 | 8 | 5 | 1 | 1 |
| $1-4$ | 3 | 3 | 9 | 4 | 1 | 1 |
| $2-5$ | 2 | 2 | 4 | 2.33 | 0.33 | 0.11 |
| $3-5$ | 3 | 6 | 15 | 7 | 2 | 4 |
| $4-6$ | 3 | 7 | 2 | 16 | 8 | 2 |
| $5-6$ | 2 | 2 | 2 | 2 | 0 | 4 |
| $6-7$ | 2 |  | 2 |  | 0 |  |

## (i) Network diagram


(ii) Expected duration and variance of each activity mentioned in Table above
(iii) Expected project length

Project duration along different paths
Path 1-2-5-6-7, total duration $3+2.33+8+2=15.33$ months

Path 1-3-5-6-7, total duration $5+7+8+2=22$ months
Path 1-4-6-7, total duration $4+6+2=12$ months
Expected project length $=T_{E}=22$ months and critical path is 1-3-5-6-7.
(iv) Probability that the project will be completed at least 3 months earlier than expected.
$T_{s}=T_{E}-3=22-3=19$ months
$Z=\frac{T_{S}-T_{E}}{\sigma}=\frac{19-22}{3}=-1$
$[\sigma=\sqrt{1+4+4}=3$ months $]$
For $Z=1 P(Z)=84.13 \%$
For $Z=-1 P(Z)=100-84.13=15.87 \%$
(v) Time required for $\mathbf{9 5 \%}$ probability of its completion

For $\mathrm{P}(Z)=95 \%$
$Z=1.645$
$\frac{T_{S}-T_{E}}{\sigma}=1.645$
$T_{s}=1.645 \sigma+T_{E}$
$=1.645 \times 3+22$
$=26.935$ months
7.(a) Determine the forces in the member of the braced frame as shown in the figure below. Also, determine the drift due to shear in each storey. Areas of diagonals and horizontal girders are shown in $\mathrm{mm}^{2}$. brackets and they are in mm . Take $\mathrm{E}=205 \mathrm{kN} / \mathrm{mm}^{2}$ :

[20 Marks]

## Sol.



$$
\begin{aligned}
& D_{s}=D_{s e}+D_{s i} \\
& D_{s e}=3-3=0 \\
& D_{s i}=m-(2 j-3) \\
& =13-(2 \times 8-3)=0 \\
& \Rightarrow D_{s}=0+0=0
\end{aligned}
$$

Truss is statically determinate.

$$
\Sigma F_{x}=0
$$

$$
\begin{aligned}
& \Rightarrow H_{A}-25-25-25=0 \\
& H_{A}=75 \mathrm{kN} \\
& \Sigma F_{X}=0 \\
& V_{A}+V_{E}=0 \\
& \Sigma M_{E}=0 \\
& V_{A} \times 4.8+25 \times 3+25 \times 6+25 \times 9=0
\end{aligned}
$$

(In analysis of truss, the applied moments will not be considered)

$$
\begin{aligned}
& V_{A}=-93.75 \mathrm{kN} \\
& -93.75+V_{E}=0 \\
& V_{E}=93.75 \mathrm{kN}
\end{aligned}
$$

In the given structure

$$
\tan \theta=\frac{3}{4.8} \Rightarrow \theta=32^{\circ}
$$

## Member forces

Using method of joint

## Joint E



$$
\begin{aligned}
& \Sigma F_{x}=0 \\
& F_{E A}=0 \\
& \Sigma F_{y}=0 \\
& F_{E F}=93.75 \mathrm{kN} \text { (Compressive) }
\end{aligned}
$$

## Joint A



$$
\begin{aligned}
& \Sigma F_{\mathrm{X}}=0 \\
& 75=F_{\mathrm{AF}} \cos 32 \\
& \mathrm{~F}_{\mathrm{AF}}=88.44 \mathrm{kN} \text { (Tension) } \\
& \Sigma \mathrm{F}_{\mathrm{y}}=0 \\
& \Rightarrow \mathrm{~F}_{\mathrm{AB}}+\mathrm{F}_{\mathrm{AF}} \sin 32=93.75 \\
& \mathrm{~F}_{\mathrm{AB}}+88.44 \sin 32=93.75 \\
& \mathrm{~F}_{\mathrm{AB}}=46.88 \mathrm{kN} \text { (Tension) }
\end{aligned}
$$

## Joint F



$$
\begin{aligned}
& \Sigma F_{y}=0 \\
& F_{B F}=F_{A F} \cos 32=88.44 \sin 32^{\circ} \\
& F_{B F}=75 \mathrm{kN} \text { (Compression) } \\
& \Sigma F_{Y}=0 \\
& F_{G F}+F_{A F} \cos 32=F_{E F} \\
& F_{G F}=46.88 \mathrm{kN} \text { (Compressive) }
\end{aligned}
$$

## Joint B



$$
\Sigma F_{x}=0
$$

$$
25+F_{B G} \cos 32=F_{B F}
$$

$$
25+\mathrm{F}_{\mathrm{BG}} \cos 32=75
$$

$\mathrm{F}_{\mathrm{BG}}=58.96 \mathrm{kN}$ (Tension)
$\Sigma \mathrm{F}_{\mathrm{y}}=0$
$F_{B C}+F_{B G} \sin 32=F_{A B}$
$\mathrm{F}_{\mathrm{BC}}+58.96 \sin 32=46.88$
$\mathrm{F}_{\mathrm{BC}}=15.63$ (Tension)

## Joint G



$$
\begin{aligned}
& \Sigma \mathrm{F}_{\mathrm{X}}=0 \\
& \mathrm{~F}_{\mathrm{CG}}=\mathrm{F}_{\mathrm{GB}} \cos 32=58.96 \cos 32 \\
& \mathrm{~F}_{\mathrm{CG}}=50 \mathrm{kN}(\text { Compression }) \\
& \Sigma \mathrm{F}_{\mathrm{Y}}=0 \\
& \mathrm{~F}_{\mathrm{GF}}=\mathrm{F}_{\mathrm{GH}}+\mathrm{F}_{\mathrm{GB}} \sin 32 \\
& 46.88=\mathrm{F}_{\mathrm{GH}}+58.96 \sin 32 \\
& \mathrm{~F}_{\mathrm{GH}}=15.63 \mathrm{kN} \text { (Compression) }
\end{aligned}
$$

## Joint D



$$
\begin{aligned}
& \Sigma \mathrm{F}_{\mathrm{x}}=0 \\
& \mathrm{~F}_{\mathrm{DH}}=25 \mathrm{kN} \text { (Compression) } \\
& \Sigma \mathrm{F}_{\mathrm{y}}=0 \\
& \mathrm{~F}_{\mathrm{CD}}=0
\end{aligned}
$$

## Joint H


$\Sigma F_{y}=0$
$\mathrm{F}_{\mathrm{GH}}=\mathrm{F}_{\mathrm{CH}} \sin 32$
$\mathrm{F}_{\mathrm{CH}}=29.49 \mathrm{kN}$ (Tension)


## Drift due to shear

Apply unit load at joint F and find member forces. ( K - value of forces)
There will be member forces only in AF and EF. Force in all other members will be zero.


## Considering Joint F


$\Sigma F_{X}=0$
$K_{\text {AF }} \cos 32=1$
$\mathrm{K}_{\mathrm{AF}}=1.18$ (Tension)
$\Sigma F_{y}=0$
$\mathrm{K}_{\mathrm{AF}} \sin 32=\mathrm{K}_{\mathrm{FE}}$
$K_{\text {FE }}=0.625$ (Compression)
$L_{A F}=\sqrt{A E^{2}+E F^{2}}=\sqrt{4.8^{2}+3^{2}}=5.66 m$
Deflection at F,

$$
\begin{aligned}
& \Delta_{F}=\sum \frac{\mathrm{PKL}}{\mathrm{AE}} \\
& =\frac{88.44 \times 1.18 \times 5660}{300 \times 205}+\frac{-93.75 \times-0.625 \times 3000}{200 \times 205} \\
& =13.9 \mathrm{~mm}
\end{aligned}
$$

By linear interpolation

$$
\begin{aligned}
& \Delta_{G}=2 \times 13.9=27.8 \mathrm{~mm} \\
& \Delta_{H}=3 \times 13.9=41.7 \mathrm{~mm}
\end{aligned}
$$

7.(b) A member of a transmission tower is composed of two angles ISA $75 \times 75 \times 8$ in star configuration as shown in the figure below. The angles are tack welded by a gusset plate of thickness 10 mm suitably. Find the axial compressive load-carrying capacity of the member under dead and live load condition. The nodal length of the member is 3 m . Assume $\mathrm{K}=0.85$ and E250 grade of steel. The properties of ISA $75 \times 75 \times 8$ are as follows:
$A=1140 \mathrm{~mm}^{2}, \mathrm{I}_{\mathrm{yy}}=\mathrm{I}_{\mathrm{zz}}=59 \mathrm{~cm}^{4}, \mathrm{C}_{y}=\mathrm{C}_{\mathrm{z}}=21.4 \mathrm{~mm}, \mathrm{r}_{u u}=28.8 \mathrm{~mm}, \mathrm{r}_{\mathrm{vv}}=14.5 \mathrm{~mm}$
Given:

| $\frac{\mathrm{KL}}{\mathrm{r}}$ | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{\mathrm{cr}}(\mathrm{MPa})$ | 168 | 152 | 136 | 121 | 107 | 94.6 | 83.7 | 74.3 |



Use limit state method.
[20 Marks]

## Sol.



Each angle area $=11.4 \mathrm{~cm}^{2}, \mathrm{I}_{\mathrm{zz}}=\mathrm{I}_{\mathrm{yy}}=59 \mathrm{~cm}^{4}$
$\mathrm{I}_{z z}$ (Built up section)

$$
\begin{aligned}
& =2\left[\mathrm{I}_{\mathrm{zz}}(\text { Angle })+\text { Area } \times \text { Distance }^{2}\right] \\
& =2\left[59+11.4 \times 2.64^{2}\right] \\
& =276.907 \mathrm{~cm}^{4}
\end{aligned}
$$

$r_{\text {zz }}$ (Built up section)

$$
=\sqrt{\frac{\mathrm{I}_{\mathrm{zz}}}{\text { Area }}}=\sqrt{\frac{276.907}{2 \times 11.4}}=3.485 \mathrm{~cm}
$$

From symmetry

$$
r_{y y}=r_{z z}=3.485 \mathrm{~cm}
$$



$$
\begin{aligned}
& I_{v v}=r_{v v}^{2} \text { Area }=1.45^{2} \times 11.4 \mathrm{~cm}^{4} \\
& =23.9685 \mathrm{~cm}^{4}
\end{aligned}
$$

Since uu of built-up section and individual angles are same.
ruu (built up section)
$=r_{u u}($ Angle $)=2.88 \mathrm{~cm}$

Ivv (built-up section)

$$
\begin{aligned}
& =2\left[\mathrm{I}_{\mathrm{vv}}(\text { Angle })+\text { Area } \times \text { distance }^{2}\right] \\
& =2\left[23.9685+11.4 \times 3.734^{2}\right] \\
& =365.832 \mathrm{~cm}^{4}
\end{aligned}
$$

$r_{v v}$ (Built-up section) $=\sqrt{\frac{I_{v v}}{A}}=\sqrt{\frac{365.832}{2 \times 11.4}}=4 \mathrm{~cm}$
For Built-up section:

$$
\begin{aligned}
& r_{z z}=r_{y y}=3.485 \mathrm{~cm} \\
& r_{u u}=2.88 \mathrm{~cm} \\
& r_{v v}=4 \mathrm{~cm}
\end{aligned}
$$

So, the minimum radius of gyration is 2.88 cm .
Buckling will happen about $u-u$.
Slenderness ratio $=\frac{l_{\text {eff }}}{r_{\text {min }}}=\frac{K L}{r_{\text {min }}}=\frac{0.85 \times 300}{2.88}=88.542$
From table of $\frac{\mathrm{KL}}{\mathrm{r}}$ and $\mathrm{f}_{\mathrm{cr}}$

$$
\begin{aligned}
& \text { For } \frac{\mathrm{KL}}{\mathrm{r}}=88.542 \\
& \begin{aligned}
& \mathrm{f}_{\mathrm{cr}}=121+\left(\frac{136-121}{90-80}\right)(90-88.542) \\
& \quad=123.187 \mathrm{~N} / \mathrm{mm}^{2}
\end{aligned}
\end{aligned}
$$

Factored axial load carrying capacity:

$$
=\frac{(123.187 \times 2 \times 1140)}{10^{3}}=280.87 \mathrm{kN}
$$

Axial load capacity $=\frac{280.87}{1.5}=187.24 \mathrm{kN}$
7.(c) (i) What are the precautions to be taken for labour safety during formwork construction?
[10 Marks]

Sol. (i) Provisions for labour safety during formwork construction.

1. Scaffolding used for various works should have proper fastening at joints and buildings. In the care of movable platforms, the brake system should be in order and should be in lock position during work.
2. There should be an overhead net covering to prevent any accident due to the falling of materials or tools.
3. Vertical supports to formwork should be braced so that they don't fail when a load is released or in case any object hits them.
4. Shuttering frames must be tied together with sufficient braces to make rigid struts and diagonal braces should be installed in proper position so that frames develop loadcarrying capacity.
5. An emergency escape route and plan shall be provided.
6. Guard rails should be provided where needed.
7. Workers should be protected from wet concrete by the use of protective gloves and boots.
8. Other safety wear like helmets, aprons, and eyeglasses should be provided as necessary.
9. Fall arrests or catch plate forms should be provided.

Ladders should be provided at the appropriate location for labour maneuvering.
7.(c) (ii) Discuss the parameters influencing the degree of compaction achieved by a vibratory roller.
[10 Marks]

Sol. (ii) The degree of compaction achieved by a vibratory roller is influenced by several parameters, including:

1. Roller Weight: The weight of the roller has a significant impact on compaction. A heavier roller exerts more force on the soil, increasing the degree of compaction. The weight is typically measured in terms of static linear load or static drum load.
2. Frequency and Amplitude: Vibratory rollers generate vibrations through their drum or plate, which helps in compacting the soil. The frequency refers to the number of vibrations per minute usually ( 1500 to 3000 vibrations/min), while the amplitude refers to the vertical movement of the drum or plate. Higher frequencies and larger amplitudes generally lead to better compaction.
3. Speed of Operation: The speed at which the roller moves over the soil affects compaction. A slower speed allows for more effective compaction, ensuring that the vibrations have sufficient time to penetrate and compact the soil. However, excessively slow speeds may cause over-compaction or damage to the soil. An optimum roller speed varies between $3 \mathrm{~km} / \mathrm{hr}$ to $6 \mathrm{~km} / \mathrm{hr}$.
4. Moisture Content: Moisture content affects soil compaction significantly. The optimum moisture content (OMC) is the moisture level at which maximum compaction can be achieved. Insufficient moisture can lead to inadequate compaction, while excessive moisture can result in soil instability. The roller should be operated at the appropriate moisture content to achieve desired compaction levels.
5. Layer Thickness: The thickness of the soil layer being compacted also influences the compaction process. Thicker layers may require multiple passes of the roller to achieve the desired degree of compaction. Thinner layers are generally easier to compact uniformly.
6. Rolling Pattern: The pattern in which the roller moves across the soil surface can impact compaction. Common rolling patterns include straight-line, circular, or overlapping
patterns. The choice of the rolling pattern depends on the specific project requirements and soil conditions.
7. Surface Conditions: The condition of the soil surface, such as its smoothness or roughness, can affect compaction. Rough or uneven surfaces may require additional passes or adjustments to achieve uniform compaction.
8. Number of passes: Generally, multiple passes are made to ensure uniform compaction throughout the entire depth of the soil layer. During each pass, the vibratory roller exerts a force on the soil, causing compaction. However, due to factors such as variations in soil density and irregularities in the compaction process, additional passes are often needed to achieve the desired level of compaction.
9. Centrifugal force: The centrifugal force generated by the rotation of the vibratory roller's drum or plate contributes to the overall compaction effort along with other factors such as roller weight, vibrations, and other compaction parameters.
It is important to note that while centrifugal force aids in the compaction process, it is not the sole parameter responsible for achieving compaction. The combined effect of centrifugal force, roller weight, vibrations, and other compaction parameters mentioned earlier work together to achieve the desired degree of compaction during the operation of a vibratory roller.
8.(a) (i) A mass of 1000 kg is placed at the free end of a cantilever beam of span 3 m . Assume that the beam is massless compared to applied mass. The flexural rigidity of the beam is $10^{3} \mathrm{kN}$ $\mathrm{m}^{2}$. Determine the natural time period of the system. (Assume $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$ )
[10 Marks]
Sol. (i)


So, the stiffness of beam $K=\frac{3 E I}{L^{3}}$

$$
\mathrm{K}=\frac{3 \times 10^{3}}{3^{3}}=111.11 \mathrm{kN} / \mathrm{m}
$$

Time period $(T)=2 \pi \sqrt{\frac{\mathrm{~m}}{\mathrm{k}}}=2 \pi \sqrt{\frac{1000}{111.11 \times 10^{3}}}=0.596 \mathrm{sec}$
8.(a) (ii) A trapezoidal combined footing supports two columns of sizes $450 \mathrm{~mm} \times 450 \mathrm{~mm}$ and 550 $\mathrm{mm} \times 550 \mathrm{~mm}$ carrying service load of 750 kN and 1250 kN respectively. The CG of smaller column lies at 0.45 m from the property line. The centre-to-centre distance of two columns is 4.4 m . The total length of footing is to be restricted to 5.6 m . Determine and show the layout plan of the above footing. Consider safe bearing capacity of soil $=155 \mathrm{kN} / \mathrm{m}^{2}$.
[10 Marks]

Sol. (ii)


## Given:

Distance of CG of 450 mm column from the property line $=0.45 \mathrm{~m}$

$$
=450 \mathrm{~mm}
$$

Centre-to-centre distance between columns $=4.4 \mathrm{~m}=4400 \mathrm{~mm}$
Total length of footing $=5600 \mathrm{~mm}$
Safe bearing capacity of soil $=155 \mathrm{KPa}$.
Let $B_{1}$ and $B_{2}$ are the smaller and larger widths of footing respectively.
Load on smaller column $=P_{1}=750 \mathrm{kN}$
Load on bigger column $=\mathrm{P}_{2}=1250 \mathrm{kN}$
Distance of C.G. of the footing from the property line:

$$
\begin{align*}
& =\left(\frac{\text { Length of footing }}{3}\right) \times\left(\frac{B_{1}+2 B_{2}}{B_{1}+B_{2}}\right) \\
& =\frac{5.6 m}{3}\left(\frac{B_{1}+2 B_{2}}{B_{1}+B_{2}}\right) \tag{1}
\end{align*}
$$

Now area of footing $=\frac{1}{2}\left(B_{1}+B_{2}\right)(5.6)=2.8\left(B_{1}+B_{2}\right)$
Area of footing required as per loading $=\frac{\text { Total load }}{\text { Safe bearing capacity of soil }}$
Assuming self-weight of footing $=10 \%$ of column load
So, Total load=1.1×(750+1250) $=2200 \mathrm{kN}$
Hence, Area of footing as per loading =

$$
\begin{equation*}
=\frac{2200}{155}=14.19 \mathrm{~m}^{2} \tag{ii}
\end{equation*}
$$

Equating (i) and (ii)

$$
\begin{align*}
& 2.8\left(B_{1}+B_{2}\right)=14.19 \\
& \Rightarrow B_{1}+B_{2}=5.067 \tag{iii}
\end{align*}
$$

Now distance of C.G. of loads from the property line:
Let $P_{1}=750 \mathrm{kN}$
$\mathrm{y}_{1}=$ Distance of C.G. of $\mathrm{P}_{1}$ from property line $=0.45 \mathrm{~m}$
Let $\mathrm{P}_{2}=1250 \mathrm{kN}$
$\mathrm{y}_{2}=$ Distance of C.G. of $\mathrm{P}_{2}$ from property line $=(0.45+4.4)=4.85 \mathrm{~m}$.
So, the distance of C.G. of the resultant load from property line:

$$
\begin{equation*}
=\frac{P_{1} y_{1}+P_{2} y_{2}}{P_{1}+P_{2}}=\frac{750(0.45)+1250(4.85)}{750+1250}=3.2 \mathrm{~m} \tag{2}
\end{equation*}
$$

Now, for uniform base pressure, the C.G. of the footing from property line and the C.G. of the resultant load from property line should coincide with each other.
$\Rightarrow$ Equating (1) and (2)

$$
\frac{5.6}{3} \frac{\left(\mathrm{~B}_{1}+2 \mathrm{~B}_{2}\right)}{\mathrm{B}_{1}+\mathrm{B}_{2}}=3.2
$$

[Now, we know, $B_{1}+B_{2}=5.067$ (iii)].

So, $\frac{5.6}{3} \frac{\left(\mathrm{~B}_{1}+2 \mathrm{~B}_{2}\right)}{5.067}=3.2$
$\Rightarrow B_{1}+2 B_{2}=8.687$
And, $B_{1}+B_{2}=5.067$
On solving, we get:

$$
\begin{aligned}
& \mathrm{B}_{1}=1.447 \mathrm{~m} \\
& \mathrm{~B}_{2}=3.62 \mathrm{~m}
\end{aligned}
$$

Now, calculating the distance "x" as shown in the layout plan above:

$$
\begin{aligned}
& \text { Length of footing }=450+4400+x \\
& \qquad \begin{array}{l}
\Rightarrow 5600=4850+x \\
\Rightarrow x=750 \mathrm{~mm}
\end{array}
\end{aligned}
$$

8.(b) Find the designed plastic moment for the portal frame as shown in the figure below under collapse condition for the factored (applied) loads. Assume that the frame has uniform crosssection. Also, find the minimum section required for the frame for E250 grade of steel.
Given:

| Section | ISMB <br> 125 | ISMB <br> 150 | ISMB <br> 175 | ISMB <br> 200 | ISMB <br> 225 | ISMB <br> 250 | ISMB <br> 300 | ISMB <br> 350 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Plastic section <br> modulus $\left(\mathrm{cm}^{3}\right)$ | 81 | 110 | 184 | 255 | 348 | 466 | 651 | 889 |



## Sol.



## Beam mechanism: Mechanism-1


$\mathrm{W}_{\mathrm{u}}=50 \mathrm{kN}$
$\mathrm{L}=6 \mathrm{~m}$
$W_{u}=8 M_{p} / L$
So, $\quad M_{p}=\frac{W_{\mathrm{u}} \mathrm{L}}{8}=\frac{50 \times 6}{8}=37.5 \mathrm{kNm}$
Sway mechanism: Mechanism - 2


$$
5 \theta=3 \alpha \quad \alpha=\frac{5}{3} \theta
$$

Internal work $=M_{p} \theta+M_{p} \alpha$

$$
=M_{p}\left(\theta+\frac{5 \theta}{3}\right)=\frac{8}{3} M_{p} \theta
$$

External work $=20 \mathrm{kN} \times 5 \theta=100 \theta$
Internal work $=$ External work

$$
\begin{aligned}
& \frac{8}{3} M_{p} \theta=100 \theta \\
& M_{p}=\frac{300}{8}=37.5 \mathrm{kNm}
\end{aligned}
$$

## Combined mechanism: Mechanism-3



$$
\frac{5}{3} \theta=\alpha
$$

Internal work done $=3 M_{p} \theta+M_{p} \alpha$

$$
=M_{p} \theta\left(3+\frac{5}{3}\right)=\frac{14}{3} M_{p} \theta
$$

External work done:

$$
=50 \times 3 \theta+20 \times 5 \theta=250 \theta
$$

Internal work done $=$ External work done

$$
\frac{14}{3} M_{p} \theta=250 \theta
$$

$$
M_{p}=\frac{750}{14}=53.57 \mathrm{kNm}
$$

From the above 3 mechanisms, maximum moment capacity is selected.
So, $\quad M_{p}=53.57 \mathrm{kN} . \mathrm{m}$

$$
M_{p}=f_{y} z_{p}
$$

$$
z_{p}=\frac{53.57 \times 10^{6}}{250 \mathrm{~N} / \mathrm{mm}^{2}} \mathrm{Nmm}
$$

$$
=214280 \mathrm{~mm}^{3}
$$

$$
=214.28 \mathrm{~cm}^{3}
$$

ISMB $200\left(z_{p}=255 \mathrm{~cm}^{3}\right)$ shall be selected.
8.(c) (i) Explain different types of contracts. Discuss the importance of each type of contract.
[10 Marks]
Sol. (i) Type of contract and their importance

|  | Description |  | Advantages |  |
| :---: | :---: | :---: | :---: | :---: |
| (i) Lump sum contract | (i) | Fixed sum for complete work | (i) | Total cost is known beforehand |
|  | (ii) | Detailed measurement of different items of work is not essential | (ii) | Time is saved in avoiding detailed measurements |
|  | (iii) | The schedule of rates comes into picture in case of any changes in original work | (iii) | Completion of project on time becomes a priority for contractor |
|  | (iv) | Work is to be completed in stipulated time for payment |  |  |
| (ii) Item rate contract | (i) | Work is executed in item rate basis based on schedule of rates and quantities also known as a bill of quantities | (i) | Changes made in design and drawings and their payments can be done more smoothly |
|  | (ii) | Payment is based on actual work done by the contractor | (ii) | Detailed drawings are not required beforehand. They can be done during project |
|  |  |  | (iii) | Comparison of rates becomes easy |
| (iii) <br> Percentage rate contract | (i) | The contractor can carry out work as per rates in bill of quantities or some percentage above or some percentage below it | (i) | A contractor cannot quote their own rate, giving an advantage to client to avoid an unbalanced tender |


|  | (ii) | The client draws the bill of quantities | (ii) | It becomes easy to <br> sort the tenders of <br> contractors |
| :--- | :--- | :--- | :--- | :--- |

Apart from the above three, we have
(iv)Cost plus percentage rate contract
(v) Cost plus fixed fee contract.

In cost plus percentage rate, the contractor works on the actual cost of work plus an agreed percentage.
In cost plus fixed fee, the contractor works on actual cost work plus a fixed lump sum amount. This fixed lump sum won't vary with the actual cost.
8.(c). (ii) One homogenous embankment, compacting rollers are used to compact silty clay soil. Determine the quantity of earth compacted if the sheep-foot roller travels at $4 \mathrm{~km} / \mathrm{hr}$, the time of rolling is 50 min , the length of the drum is 2.4 m , the number of drums is one, a fraction of overlap is $1 / 8$, the layer thickness is 0.45 m and number of passes given is 5 .
[10 Marks]
Sol. (ii) Quantity of earth compacted $=\frac{V \mathrm{thLn}(1-\mathrm{p})}{\mathrm{N}}$
Where,
$\mathrm{V}=$ Speed of roller
h = Layer thickness
$t=$ time of rolling
$L=$ Length of each drum
$\mathrm{n}=$ Number of drums
$p=$ Fraction of overlap
$\mathrm{N}=$ Number of passes
Vt gives distance travelled.


For each drum volume compacted $=(\mathrm{Vt}) \mathrm{Lh}$
For $n$ drums and $P$ overlapping volume compacted $=(V t)$ Lh $n(I-P)$
For each pass
Volume compacted $=\frac{(\mathrm{Vt}) \operatorname{Lh} \mathrm{n}(1-\mathrm{p})}{\mathrm{N}}$
Volume compacted.

$$
\begin{aligned}
& =\frac{\left(\frac{4 \times 1000}{60} \frac{\mathrm{~m}}{\mathrm{~min}} \times 50 \mathrm{~min}\right) 2.4 \mathrm{~m} \times 0.45 \mathrm{~m} \times 1\left(1-\frac{1}{8}\right)}{5} \\
& \quad=630 \mathrm{~m}^{3}
\end{aligned}
$$

## Outstanding performance by our students in GATE 2021

CONGRATULATIONS TO OUR TOPPERS



Poojasree (ECE)


Harshit (IN)


Munish (ME)


Amit (CE)


Divakar (IN)


Vatsal (ME)


Parag (ECE)


Hemant (EE)


Rajat (ME)


Abhishek (ECE)

