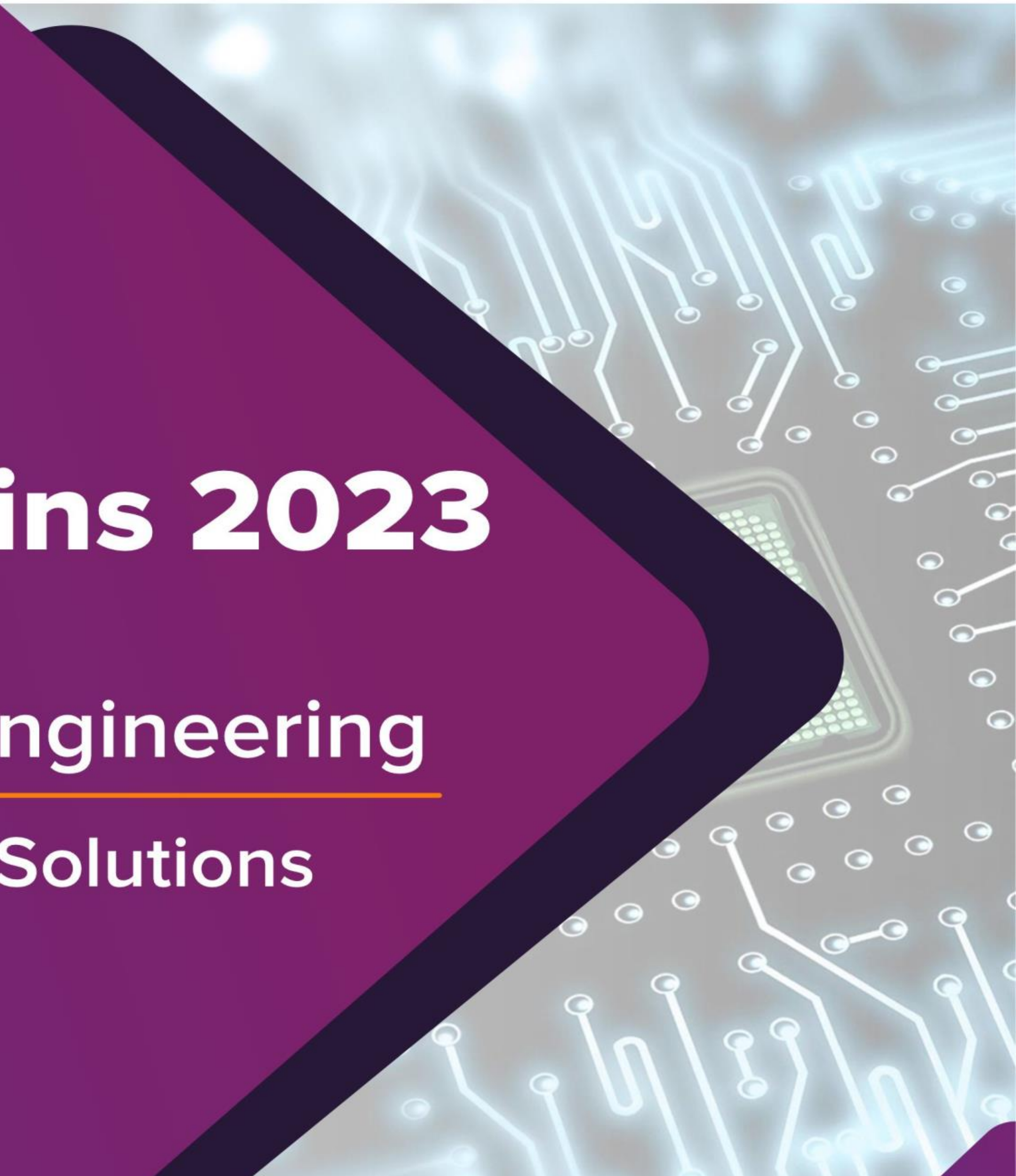


ESE Mains 2023

Electrical Engineering

Questions & Solutions

PAPER-2

Electrical Engineering Paper 2 : Marks Distribution					
S. No.	Subjects	Difficulty Level 2023	2023 Marks	2022 Marks	2021 Marks
1	Analog and Digital Electronics	Moderate	84	46	58
2	Systems and Signal Processing	Moderate	84	82	66
3	Control Systems	Moderate	72	92	104
4	Electrical Machines	Moderate	84	124	104
5	Power Systems	Easy	84	72	64
6.	Power Electronics and Drives	Moderate	72	64	84
Total			480	480	480

ELECTRICAL ENGINEERING

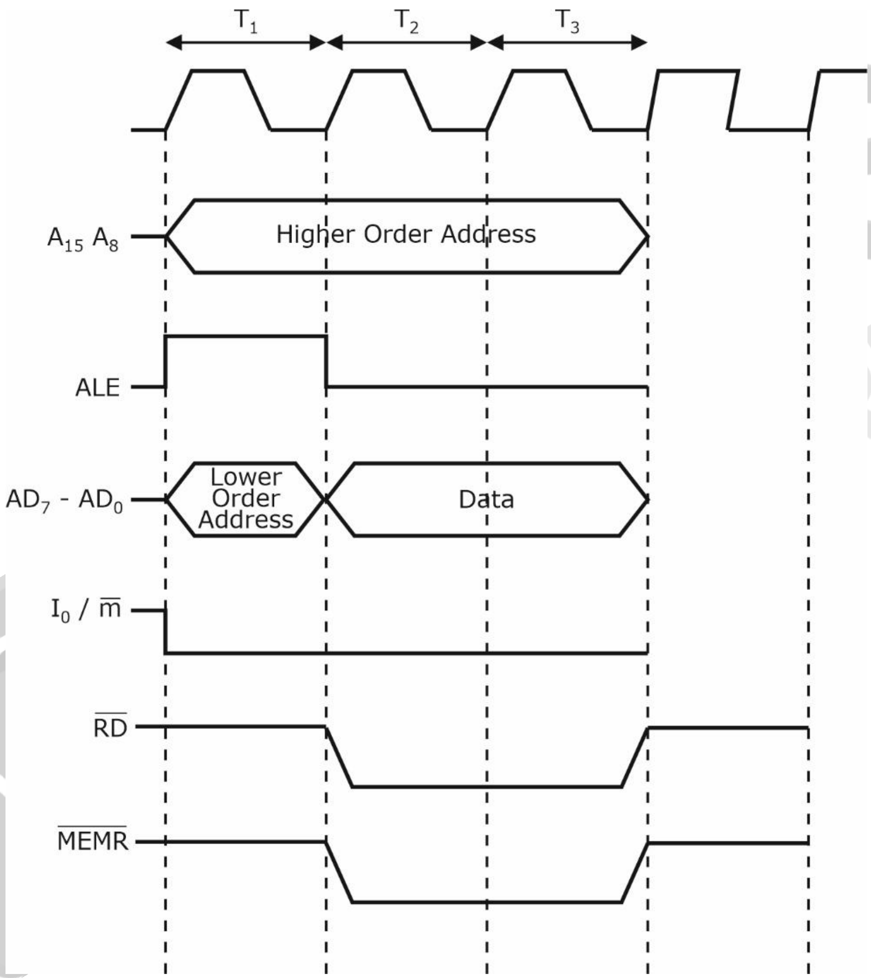
Paper-2

SACTION - A

1. a. Draw memory read machine cycle of 8085 microprocessor and explain.

[12 Marks]

Sol. 1a.



In 8085 microprocessor

Machine cycles

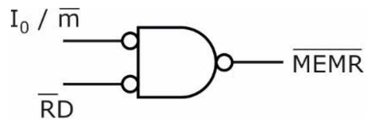
1. Opcode fetch

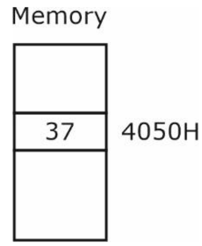
2. memory Read

3. Memory

Memory Read

$$\left.\begin{aligned} I_0 / \bar{m} &= 0 \\ \overline{R_D} &= 0 \end{aligned}\right\} \text{then } \overline{MEMR} = 0$$





Let memory location 4050H

has 37H as data.

copy 37H to Accumulator.

3T states are required for memory Read

<u>1st t State</u>

ALE → 1

$A_{15} - A_B$ → 40H Higher order address

$AD_7 \to AD_0$ → 50H Lower order Address

Lower order address [50H] will store in D latch as ALE → 1. $I_0 / \overline{m} = 0$

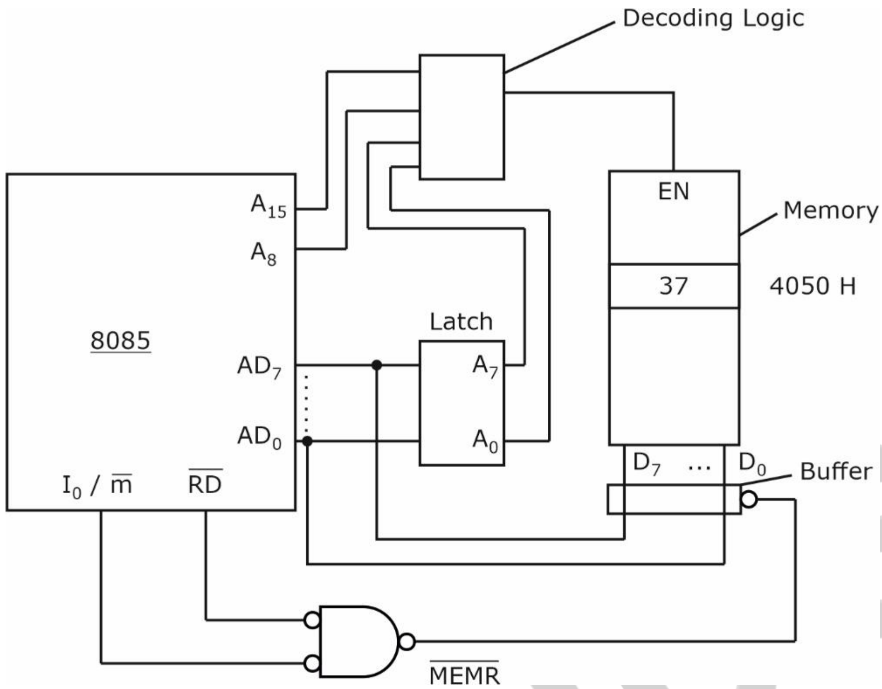
<u>2nd T State</u>

ALE → 0 now $AD_7 - AD_0$ in act as data line but $A_{15} - A_8$ remain same.

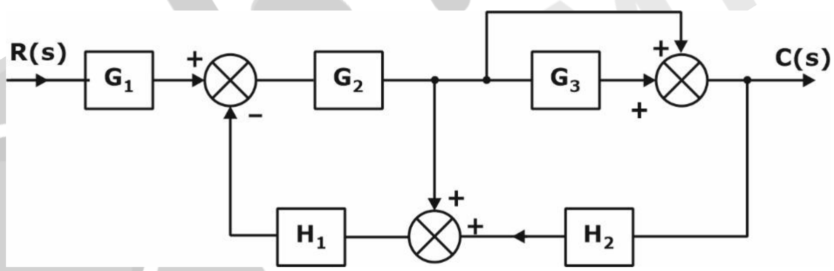
now read pin goes low

$\because \overline{RD} = 0 \quad I_0 / \overline{m} = 0 \;:\; \overline{MEMR} = 0$

It will active the output butter of memory. Data will transfer through. Data bus to microprocessor.

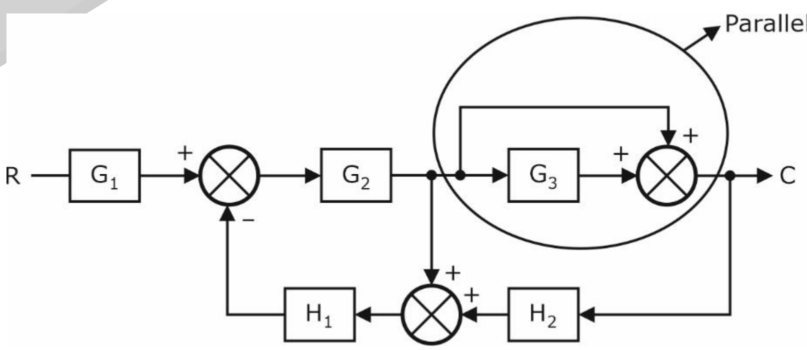


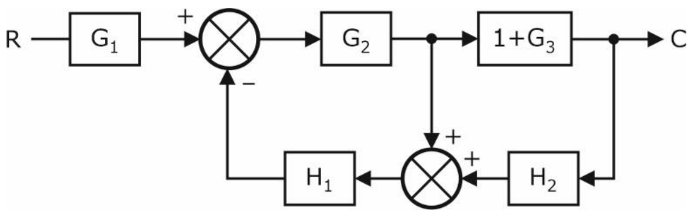
1. b. Reduce the block diagram shown below, using block diagram reduction technique and find the transfer function $\frac{C(s)}{R(s)}$.



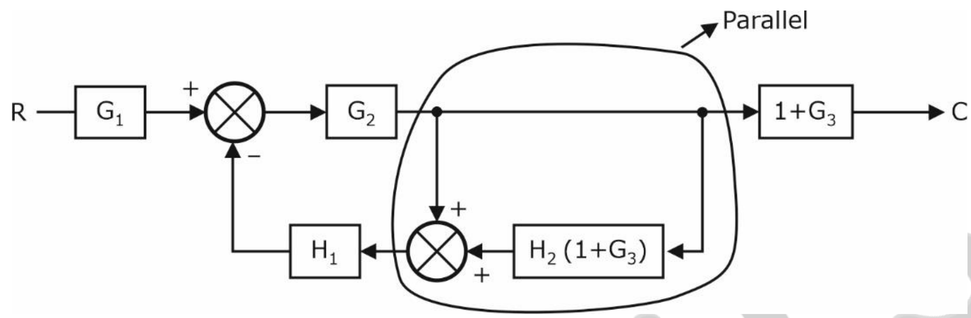
[12 Marks]

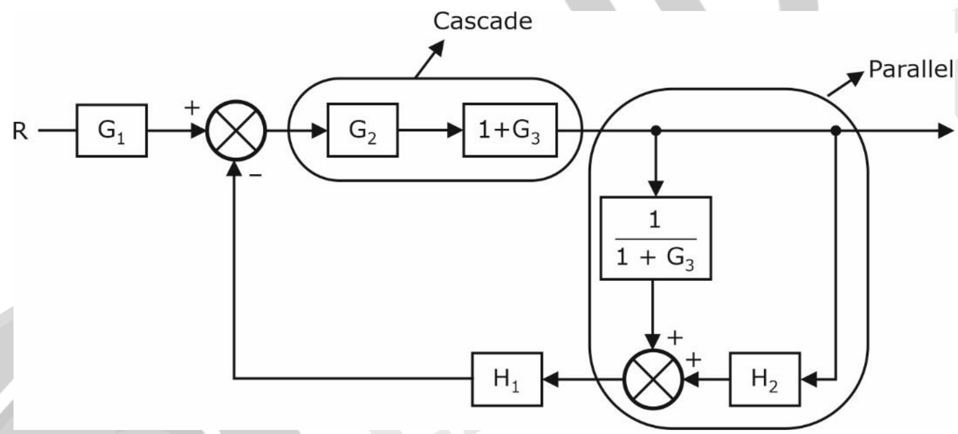
Sol. 1b.



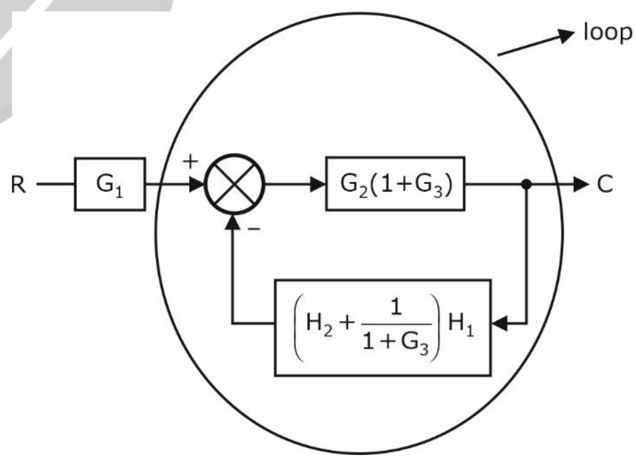


Shift Take-off point before (1 + G_3) to after it





Solving cascade and parallel blocks :



Closed loop gain $\dfrac{G_2\left(1+G_3\right)}{1+\left(\dfrac{H_2(1+G_3)+1}{1+G_3}\right)H_1G_2(1+G_3)}$

$$= \frac{G_2\left(1+G_3\right)}{1+H_1G_2\left(H_2(1+G_3)+1\right)}$$

$$R \rightarrow \boxed{G_1} \rightarrow \boxed{\frac{G_2\left(1+G_3\right)}{1+H_1G_2\left[H_2(1+G_3)+1\right]}} \rightarrow C$$

So, $\dfrac{C}{R} = G_1 \times \dfrac{G_2\left(1+G_3\right)}{1+H_1G_2\left[H_2(1+G_3)+1\right]}$

$$\boxed{\frac{C}{R} = \frac{G_1G_2\left(1+G_3\right)}{1+G_2H_1H_2+G_2G_3H_1H_2+G_2H_1}}$$

1. c. The maximum efficiency of a 500 KVA, 3300/500 V, 50 Hz single-phase transformer is 97% and occurs at $\frac{3}{4}$ full load, unity power factor. If the impedance is 10%, find the voltage regulation at full load, power factor 0.8 leading.

[12 Marks]

Sol. 1c. Given η_{max} = 97% at $\frac{3}{4}$ of full load at UPF

$$\eta_{max} = \frac{\frac{3}{4}\times S_{out}\times 1}{\frac{3}{4}S_{out}\times 1+\text{losses}}$$

$$0.97 = \frac{\frac{3}{4}\times 500}{\frac{3}{4}\times 500+\text{losses}}$$

$$\text{losses} = \frac{375}{0.97}-375 = 11.597\ \text{kW}$$

At maximum efficiency, copper loss = Iron loss = $\dfrac{11.597}{2}$ kW $= 5.798$ kW

$x^2 P_{cu,FL}$ = 5.798 kW

$$P_{cu,\,FL} = \left(\frac{4}{3}\right)^2 \times 5.798\ \text{kW} = 10.308\ \text{kW}$$

We know, % R = % Cu loss at full load

$$= \frac{10.308}{500} \times 100 = 2.061\%$$

$\%Z(\text{given}) = 10\%$

$\%X = \sqrt{(\%Z)^2 - (\%R)^2}$

$= \sqrt{(10)^2 - (2.061)^2} = 9.785\%$

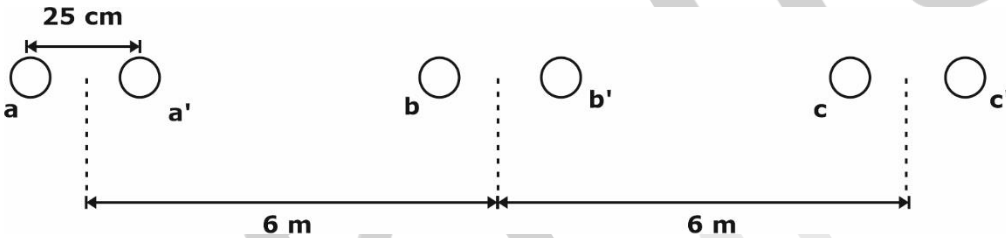
$\therefore$ Voltage regulation at full load and 0.8 leading

= %Rcosϕ – %Xsinϕ

= (2.061) (0.8) – (9.785) (0.6)

= –4.22%

1. **d.** Calculate the inductance and capacitance of the single-circuit, two-bundle conductor, 200 km long line as shown below. The diameter of each conductor is 5 cm.



[12 Marks]

Sol. 1d. As we know for bundled conductions

$$L = 2 \times 10^{-7} \ln\left(\frac{GMD}{GMR}\right) \text{H/m}$$

From given figure,

$GMD = \sqrt[3]{6 \times 6 \times 12}$

= 7.56 m

$\text{radius} = \frac{D}{2} = \frac{5}{2} = 2.5 \text{ cm}$

From figure,

Daa' = Da'a = 25 cm

Daa = Da'a' = 0.7788 × 2.5 × 10^2

= 1.947 × 10^{-2} m

$GMR = \sqrt[4]{\text{Daa Daa' Da'a Da'a'}}$

$= \sqrt[4]{1.947 \times 10^{-2} \times 25 \times 10^{-2} \times 25 \times 10^{-2} \times 1.947 \times 10^{-2}}$

= 6.97 × 10^{-2} m

$$L = 2 \times 10^{-7} \ln\left(\frac{7.56}{6.97 \times 10^{-2}}\right)$$

$$= 9.37 \times 10^{-7} \text{ H/m}$$

For 200 km,

L = 200 × 9.37 × 10^{-7} × 10^{3}

= 0.187 H

Capacitance calculation

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{GMD}{GMR}\right)}$$

We know,

GMD = 7.56 m

$$GMR = \sqrt{25 \times 2.5 \times 25 \times 25 \times 10^{-2} \times 10^{-2} \times 10^{-2} \times 10^{-2}}$$

GMR = 7.9 × 10^{-2} m

$$C = \frac{2\pi \times 8.854 \times 10^{-12}}{\ln\left(\frac{7.56}{7.9 \times 10^{-2}}\right)}$$

C = 12.2 × 10^{-12} F/m

For 200 km

C = 200 × 10^{3} × 12.2 × 10^{-12}

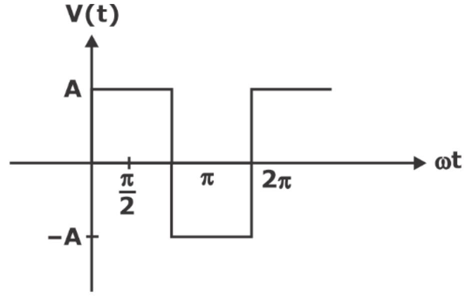
C = 2.44 μF

1. e. Explain the concept of Pulse Width Modulation. How is it used in the reduction of harmonics in a single-phase full bridge Inverter?

[12 Marks]

Sol. 1e. PWM stands for Pulse Width Modulation. It is a modulation technique commonly used in electronic systems to control the average power delivered to a load by varying the width of the pulses of a periodic signal. In PWM, the continuous waveform, typically a square wave, is divided into two parts: a high state and a low state. The width or duration of the high state, known as the pulse width, is varied to achieve the desired output. By changing the duty cycle (the ratio of the pulse width to the total period), the average power delivered to the load can be controlled.

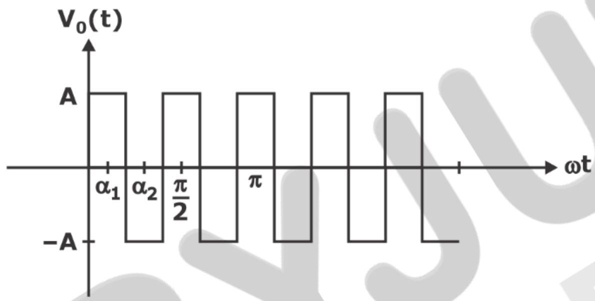
Using pule width modulation, the width of the pule is Modulated to reduce the harmonics content. Lets take an example



the fourier expression of $V(t) = \sum_{\substack{\eta_{e1} \\ n \to odd}}^{\infty} \frac{4A}{n\pi} \sin n\omega_0 t$

So above wave contains odd harmonics

Now lets modulate the above wave as shown below.



Above signal is modulated (have taken two switching per quarter cycle) keeping the odd half wave symmetry and odd nature of wave intact.

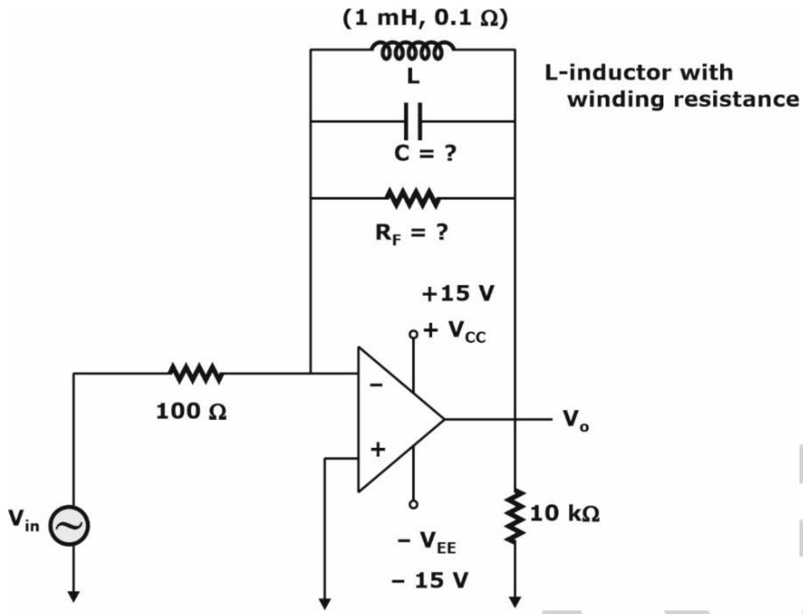
Now harmonic content can be shown using its fourier expression.

$$V_0(t) = \sum_{\substack{n=1 \\ n \to odd}}^{\infty} \frac{4A}{\eta\pi} \left[1 - \cos n\alpha_1 + \cos n\alpha_2\right] \sin n\omega_0 t$$

by controlling α_1 and α_2 we can eliminate few harmonics. If we can increase number of switching, per quarter then we can eliminate higher number of harmonics.

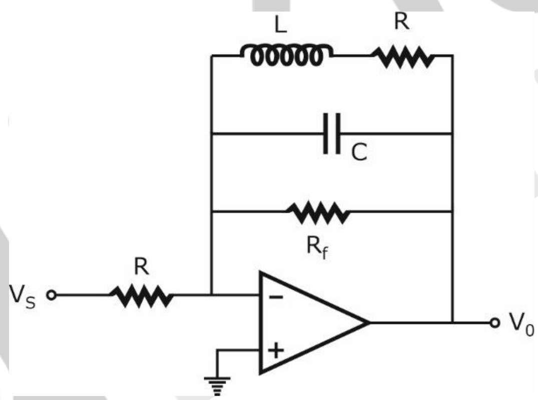
So from above discussion it is evident that PWM technique is used to minimise the harmonic content of the ware.

2. **a.** For the circuit given below, find the value of the components. Gain is 5 at a frequency of 32 kHz.



[20 Marks]

Sol. 2a. Gain = 5 at 32 KHz



$Z_L = j\omega l$

$|Z_L| = \omega L$

$|Z_L| = 2\pi \times 32 \times 10^3 \times 10^{-3} = 64\pi = 201.06\Omega$

$|Z_L| >> R$; nelgect R

R is internal resistance of coil

feedback path is parallel RLC circuit and at resonant freq. LC combination act or open circuit

$$f_0 = f_r = \frac{1}{2\pi\sqrt{LC}} = 32 \times 10^3$$

we are assuming 32 KHz as resonant freq.

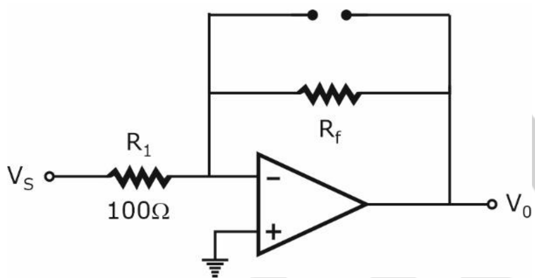
$$\frac{1}{2\pi\sqrt{10^{-3}C}} = 32\times10^{3}$$

$$\frac{1}{2\pi\times32\times10^{3}} = \sqrt{10^{-3}C}$$

$$C = \frac{1}{10^{-3}}\times\left[\frac{1}{2\pi\times32\times10^{3}}\right]^{2}$$

$C = 2.47 \times 10^{-8}$ F

$\boxed{C = 24.7\times10^{-9}\text{ F} = 24.7\text{ nF}}$



$$A_V = \frac{V_0}{V_S} = -\frac{R_f}{R_1}$$

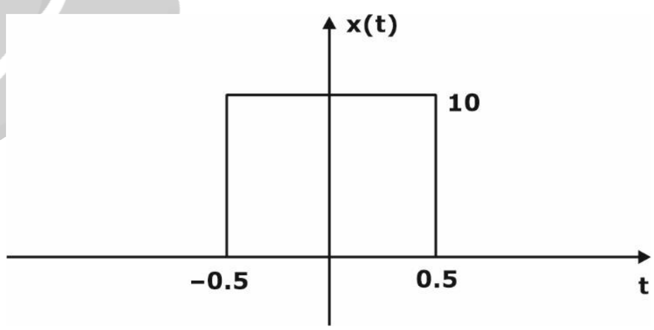
$$|A_V| = \frac{R_f}{R_1} = 5$$

$R_f = 5R_1 = 5 \times 100$

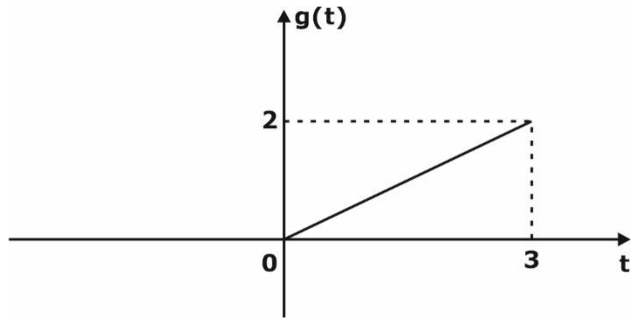
$\boxed{R_f = 500\ \Omega}$

2. **b.** Find x(t) * g(t), using graphical convolution.

x(t)



g(t)

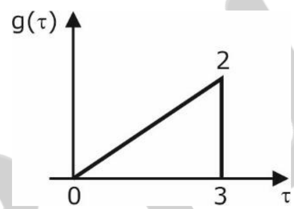


[20 Marks]

Sol. 2b. Given, y(t) = x(t) * g(t)

So, $y(t) = \int_{-\infty}^{\infty} g(\tau) \cdot x(t-\tau) d\tau$

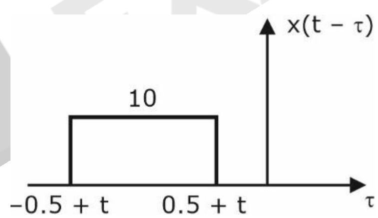
Case-I:



When 0.5 + t < 0

⇒ t < −0.5

then, y(t) = 0



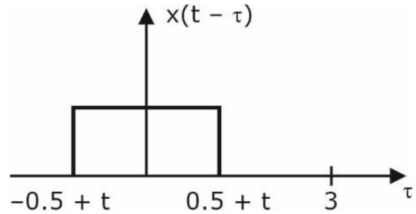
Case-II:

When −0.5 + t < 0 but 0.5 + t > 0

i.e. −0.5 < t < 0.5

then, $y(t) = \int_{0}^{0.5+t} t_0 \cdot \frac{2}{3}\tau \, d\tau = \frac{20}{3}\left[\frac{\tau^2}{2}\right]_0^{0.5+t}$

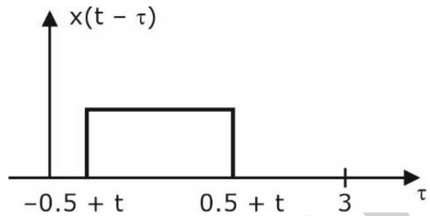
$$= \frac{10}{3}(0.5+t)^2$$



Case-III:

When −0.5 + t > 0 but 0.5 + t < 3

i.e 0.5 < t < 2.5



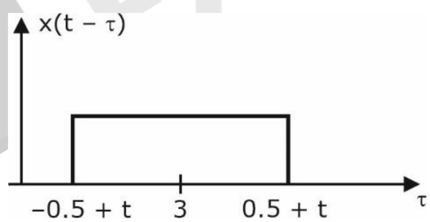
$$y(t) = \int_{-0.5+t}^{0.5+t} 10 \cdot \frac{2}{3} \tau d\tau = \frac{20}{3}\left[\frac{\tau^2}{2}\right]_{-0.5+t}^{0.5+t}$$

$$= \frac{10}{3}\left[(0.5+t)^2 - (-0.5+t)^2\right]$$

$$= \frac{20}{3} t$$

Case-IV:

When −0.5 + t < 3 but 0.5 + t > 3



i.e. 2.5 < t < 3.5

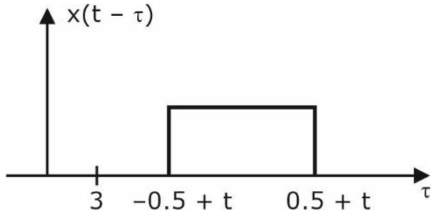
then, $y(t) = \int_{-0.5+t}^{3} 10 \cdot \frac{2}{3} \tau \, d\tau = \frac{20}{3}\left[\frac{\tau^2}{2}\right]_{-0.5+t}^{3}$

$$= \frac{10}{3}\left[9 - (-0.5+t)^2\right] = \frac{10}{3}\left[8.75 - t^2 + t\right]$$

Case-V:

When −0.5 + t > 3,

i.e. t > 3.5



then $y(t) = 0$

$$\text{Thus, } y(t) = \begin{cases} 0, & \text{for } t < -0.5 \\ \frac{10}{3}(0.5+t)^2, & \text{for } -0.5 < t < 0.5 \\ \frac{20t}{3}, & \text{for } 0.5 < t < 2.5 \\ \frac{10}{3}\left[8.75 - t^2 + t\right], & \text{for } 2.5 < t < 3.5 \\ 0, & \text{for } t > 3.5 \end{cases}$$

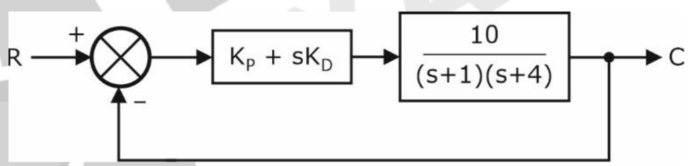
2. c. Design a PD controller for a unity feedback system whose open loop transfer function.

$$G(s)H(s) = \frac{10}{(s+1)(s+4)}$$

Will have poles at $s = -4 \pm j4$.

[20 Marks]

Sol. 2c. Let PD controller gain be $(K_P + sK_D)$



Now, Characteristic equation, CE, is

$$1 + (K_P + sK_D) \times \frac{10}{(s+1)(s+4)} = 0$$

$\Rightarrow s^2 + 5s + 4 + 10K_P + 10sK_D = 0$

$s^2 + (5 + 10K_D)s + (10K_P + 4) = 0$ ...(1)

we need poles at $s = -4 \pm j4$

So, characteristic equation desired is

$(s - (-4 + j4))\,(s - (-4 - j4)) = 0$

$(s + 4 - j4)\,(s + 4 + j4) = 0$

$(s + 4)^2 + 4^2 = 0$

$\Rightarrow s^2 + 8s + 32 = 0$ ...(2)

Compared (1) & (2),

$5 + 10K_D = 8 \Rightarrow K_D = 0.3$

$10K_P + 4 = 32 \Rightarrow K_P = 2.8$

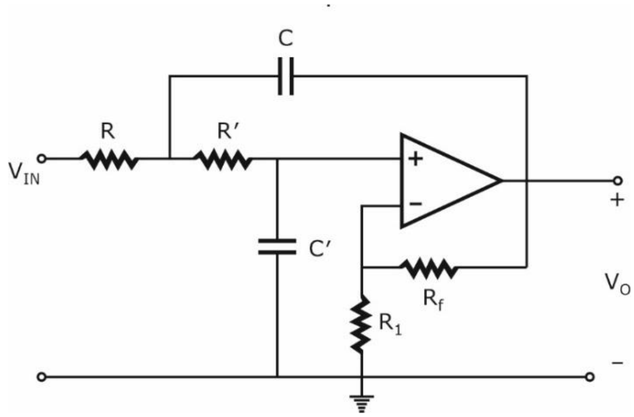
So, DD controller gain, is

$\boxed{G_C(s) = 2.8 + 0.3s}$

3. **a.** Design a second order low pass filter using Op-Amp with feedback gain 1.586. High cut-off frequency is 10 kHz. Assume capacitor 0.1 μF and R_1 = 10kΩ (resistor connected between input source to input terminal of Op-Amp). Draw the circuit diagram and plot the frequency response.

[20 Marks]

Sol. 3a.



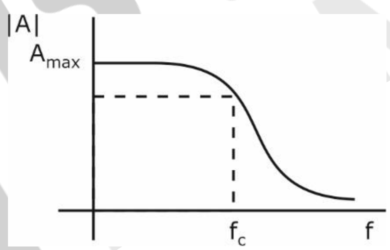
$$\text{Cutoff frequency } f_c = \frac{1}{2\pi\sqrt{R\,R'\,C\,C'}}$$

if R = R′ and C = C′

$$f_c = \frac{1}{2\pi RC} = 10\text{ KHz}$$

$$R = \frac{1}{2\pi C \times 10 \times 10^3} = \frac{1}{2\pi 0.1 \times 10^{-6} \times 10^4}$$

$$\boxed{R = \frac{100}{2\pi} = 159.15\ \Omega}$$



$$A_{max} = 1 + \frac{R_f}{R_1} = 1.586$$

R_1 = 10 KΩ (given)

$$1 + \frac{R_f}{10} = 1.586$$

R_f = 0.586

$$\boxed{R_f = 0.586 \times 10 = 5.86\ K\Omega}$$

3. **b.** A DC motor is mechanically connected to a constant torque load. When the armature is connected to a 120 volt DC supply, it draws an armature current of 10 amperes and runs at 1800 rpm. The armature resistance is R_a=0.1 Ω. Accidentally, the field circuit breaks and the flux drops to the residual flux, which is only 5% of the original flux.

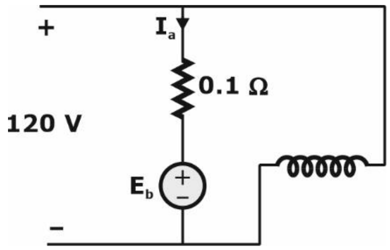
(i) Determine the value of the armature current immediately after the field circuit breaks (i.e. before the speed has had time to change from 1800 rpm).

(ii) Determine the hypothetical final speed of the motor after the field circuit breaks.

Neglect the inductance of the armature circuit.

[20 Marks]

Sol. 3b. (i)



I_0 = 10A

According to KVL, E_b = 120 – 10(0.1) = 119 V

$$k_e\phi\left(1800\times\frac{2\pi}{60}\right)=119$$

As the field has dropped to 0.05 ϕ, the new back emf will be

119(0.05) = 5.95 V

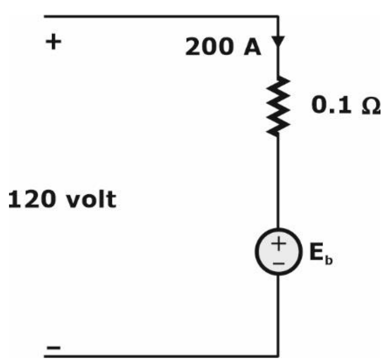
So, armature current = $\frac{120-5.95}{0.1}=1140.5$ A

(ii) Under steady state, torque remain constant

$$K_e\phi I_a = T_L$$

$$K_e\phi(10) = K_e\phi(0.05)I_a'$$

$$I_a' = \frac{10}{0.05} = 200\text{ A}$$



E_b = 120 – 200(0.1)

= 100 V

$$100 = K_e\phi\ (0.05)\ N\left(\frac{2\pi}{60}\right)$$

From initial stable condition

$$119 = K_e\phi\ (1800)\left(\frac{2\pi}{60}\right)$$

$$\Rightarrow \frac{119}{100} = \frac{1800}{0.05\,N} \Rightarrow N = \frac{1800 \times 100}{119(0.05)}$$

= 30252.10 rpm

3. c. A 250 km long, three-phase, 50 Hz, transmission line has the following line constants:

$A = D = 0.9\angle 1°$

$B = 120\angle 72°\ \Omega$

$C = 0.001\angle 90°\ \Omega^{-1}$

The sending end voltage is 230 kV.

Find

(i) Line charging current

(ii) Maximum active power that can be transferred at 220 kV, and also the corresponding reactive power.

[20 Marks]

Sol. 3c. (i) Line charging current

$$I_C = \frac{V}{X_c} = V_{\beta c}$$

X_C = C (From ABCD parameter)

$= 0.001\underline{|90°}$

$$I_C = \frac{230 \times 10^3}{\sqrt{3}} \times 0.001\underline{|90°}$$

=133 A

(ii) Maximum power at the receiving end

$$P_{R(max)} = \frac{3\,|V_S V_R|}{|B|} - 3\left|\frac{A}{B}\right| V_R^2 \cos(\theta - \alpha)$$ [$\because$ V_S and V_R are per phase voltages]

$$Q_{R(max)} = -3\left(\frac{A}{B}\right) V_R^2 \sin(\theta - \alpha)$$

Given

$\alpha = 1°$ $\theta = 72°$

$\theta - \alpha = 72° - 1° = 71°$

Per phase voltages

$$V_S(ph) = \frac{230}{\sqrt{3}} = 132.8 \text{ kV}$$

$$V_R(ph) = \frac{220}{\sqrt{3}} = 127 \text{ kV}$$

Substitute all the data into the given formula

$$P_{R(max)} = \frac{3 \times 132.8 \times 127 \times 10^6}{120} - \frac{3 \times 0.9}{120} \times (127)^2 \times \cos 71°$$

= 303.5 MW

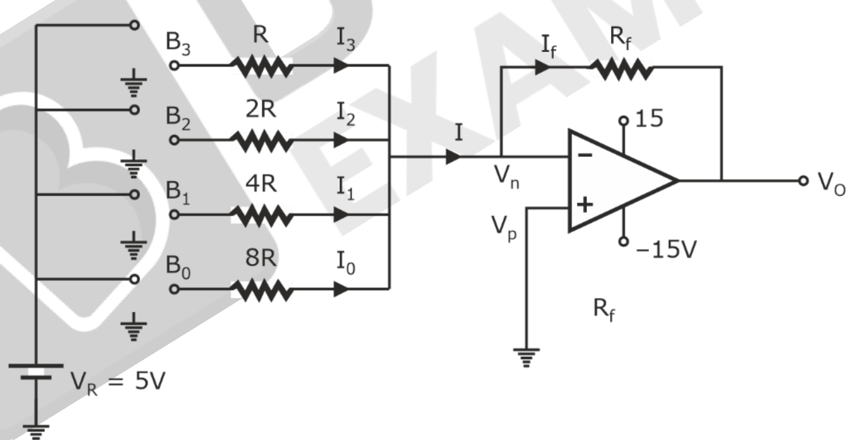
$$Q_{R(max)} = \frac{3(0.9)}{120} \times (127)^2 \times \sin 71°$$

= 343.13 MVAR

4. a. Draw a 4-bit digital to analog (D – A) convert circuit diagram using Op-Amp and binary weighted resistors. Derive the output voltage equation to get bidirectional signal output. Assume digital input 5 V and bias power supply ± 15V.

[20 Marks]

Sol. 4a.



$V_p = V_n = 0$ Virtual Ground process

$I_f = I = I_3 + I_2 + I_1 + I_0$

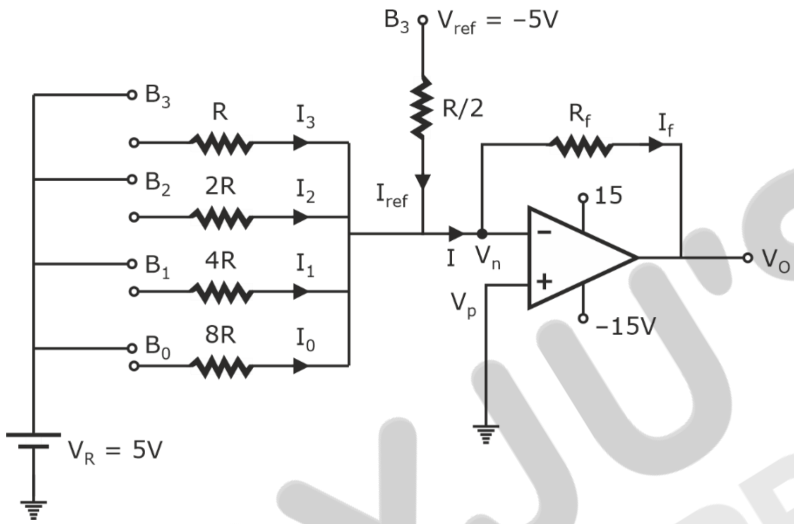
$$\frac{0 - V_O}{R_f} = \frac{B_3 V_R}{R} + \frac{B_2 V_R}{2R} + \frac{B_1 V_R}{2R} + \frac{B_0 V_R}{8R}$$

$$V_0 = -\frac{R_f}{8R} V_R \left[8B_3 + 4B_2 + 2B_1 + B_0\right]$$

$$V_0 = \underbrace{\left[\frac{V_R}{2^3}\right]}_{\text{Resolution}}\underbrace{\left[\frac{-R_f}{R}\right]}_{\text{gain}}\underbrace{\sum_{i=0}^{i=3} 2^i B_i}_{\text{Decimal Equivalent of Binary number}}$$

$\because V_R = 5V$

for 2's complement representation



$V_P = V_n = 0 \quad VG_P$

$I_f = I = I_{ref} + I_3 + I_2 + I_1 + I_0$

$$\frac{-V_0}{R_f} = B_3\frac{V_{ref}}{R/2} + B_3\frac{V_R}{R} + B_2\frac{V_R}{2R} + \frac{B_1V_R}{4R} + \frac{B_0V_R}{8R}$$

$\because V_{ref} = -V_R$ & it is connected as

i/p if $B_3 = 0$ otherwise it is connected to ground.

$$V_O = \left[\frac{V_R}{2^3}\right]\left[\frac{-R_f}{R}\right]\left[-16B_3 + 8B_3 + 4B_2 + 2B_1 + B_0\right]$$

$$V_O = \underbrace{\left[\frac{V_R}{2^3}\right]}_{\text{Resolution}}\underbrace{\left[\frac{-R_f}{R}\right]}_{\text{gain}}\underbrace{\left[-8B_3 + 4B_2 + 2B_1 + B_0\right]}_{\text{Decimal Equivalent for 2's complement No.}}$$

B_3	B_2	B_1	B_0	$-8B_3 + 4B_2 + 2B_1 + B_0$
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	–8
1	0	0	1	–7
1	0	1	0	–6
1	0	1	1	–5
1	1	0	0	–4
1	1	0	1	–3
1	1	1	0	–2
1	1	1	1	–1

∴ V_O is proportional to 2's complement representation. we are getting bidirectional signal output.

4. b. A unity feedback control system has

$$G(s) = \frac{10 * K}{s\left(\frac{s}{2}+1\right)(s+10)}$$

(i) Find gain and phase margin for K = 1.

(ii) If a phase-lag element with transfer function of $\frac{(1+2s)}{(1+5s)}$ is added in the forward path, find the new value of K to keep the same gain margin.

[20 Marks]

Sol. 4b. (i) K = 1

$$G(j\omega)H(j\omega) = \frac{10}{j\omega\left(1+\frac{j\omega}{2}\right)(10+j\omega)}$$

$$M = \left|G(j\omega)H(j\omega)\right| = \frac{10}{\omega\sqrt{\left(1+\frac{\omega^2}{4}\right)\left(100+\omega^2\right)}}$$

$$\phi = \angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

Phase Crossover Frequency, ω_p

at $\omega = \omega_p$, $\phi = -180°$

$$\Rightarrow -90^\circ - \tan^{-1}\left(\frac{\omega_p}{2}\right) - \tan^{-1}\left(\frac{\omega_p}{10}\right) = -180^\circ$$

$$\tan^{-1}\left(\frac{\frac{\omega_p}{2} + \frac{\omega_p}{10}}{1 - \frac{\omega_p}{2} \cdot \frac{\omega_p}{10}}\right) = 90^\circ$$

$$\Rightarrow 1 - \frac{\omega_p^2}{20} = 0 \Rightarrow \omega_p^2 = 20$$

$$\omega_p = \sqrt{20}$$

ω_p = 4.47 rad/sec

Gain Margin

$$GM = \frac{1}{\left|G(j\omega_p)H(j\omega_p)\right|}$$

$$\left|G(j\omega_p)H(j\omega_p)\right| = \frac{10}{\sqrt{20}\sqrt{\left(1 + \frac{20}{4}\right)(100 + 20)}}$$

$$= \frac{10}{\sqrt{20 \times 6 \times 120}} = \frac{10}{120} = \frac{1}{12}$$

$$\text{So, GM} = \frac{1}{1/12} = 12$$

In dB, GM = 20 log (12) = 21.58 dB

Gain crossover frequency, ω_g

at $\omega = \omega_g$, M = |G(jω) H(jω)| = 1

$$\frac{10}{\omega_g\sqrt{\left(1 + \frac{\omega_g^2}{4}\right)(100 + \omega_g^2)}} = 1$$

$$\Rightarrow \omega_g^2\left(1 + \frac{\omega_g^2}{4}\right)(100 + \omega_g^2) = 10^2 = 100$$

let $\omega_g^2 = x$

$$x\left(\frac{x+4}{4}\right)(x+100)=100$$

$x(x^2 + 104x + 400) = 400$

$x^3 + 104x^2 + 400x - 400 = 0$

So, $x = \omega_g^2 = 0.82, -99.96m, -4.86$

but x = ω_g^2 must be positive

So, ω_g^2 = 0.82 ⇒ $\omega_g \approx 0.9$ rad/sec

Phase Margin PM

$$PM = 180° + \angle G(j\omega_g)H(j\omega_g)$$

$$= 180° + \left(-90° - \tan^{-1}\left(\frac{0.9}{2}\right) - \tan^{-1}\left(\frac{0.9}{10}\right)\right)$$

PM = 60.63°

(ii) Gain margin is same as case (i) = 12 (or) 21.58dB

$$\text{New } G(S) = \left(\frac{1+2s}{1+5s}\right)\frac{10k}{S\left(\frac{s}{2}+1\right)(S+10)}$$

Phase cross over frequency $G(s) = -180|_{\omega=\omega_{pc}}$

$$= -90 + \tan^{-1}(2\omega) - \tan^{-1}(5\omega) - \tan^{-1}(\omega/2) - \tan^{-1}(\omega/10) = -180$$

$$= -\tan^{-1}\left[\underbrace{\frac{5\omega - 2\omega}{1+10\omega^2}}_{A}\right] - \tan^{-1}\left(\underbrace{\frac{\frac{\omega}{2}+\frac{\omega}{10}}{1-\frac{\omega^2}{20}}}_{B}\right) = -90$$

$$\tan^{-1}\left(\frac{A+B}{1-AB}\right) = 90°$$

$$\Rightarrow 1 - AB = 0 \text{ and } AB = 1$$

$$\left(\frac{3\omega}{1+10\omega^2}\right)\left(\frac{12\omega}{20-\omega^2}\right) = 1$$

$$36\omega^2 = 20 - \omega^2 + 200\omega^2 - 10\omega^4$$

$$10\omega^4 - 163\omega^2 - 20 = 0$$

By solving $\omega = \omega_{pc} = 4.05$ rad/s

$$\text{Gin margin } = 12 = \frac{1}{|G(s)|_{\omega_{pc}}}$$

$$|G(S)|_{\omega_c \omega_{pc}} = \frac{\sqrt{1+(2\omega)^2}}{\sqrt{1+(5\omega)^2}} \cdot \frac{k}{\omega\sqrt{\left(\frac{\omega}{2}\right)^2+1}\sqrt{\omega^2+10^2}}$$

$$= \frac{\sqrt{1+(2\times 4.05)^2}}{\sqrt{1+(5\times 4.05)^2}} \cdot \frac{k}{4.05\sqrt{\left(\frac{4.05}{2}\right)^2+1}\sqrt{(4.05)^2+100}}$$

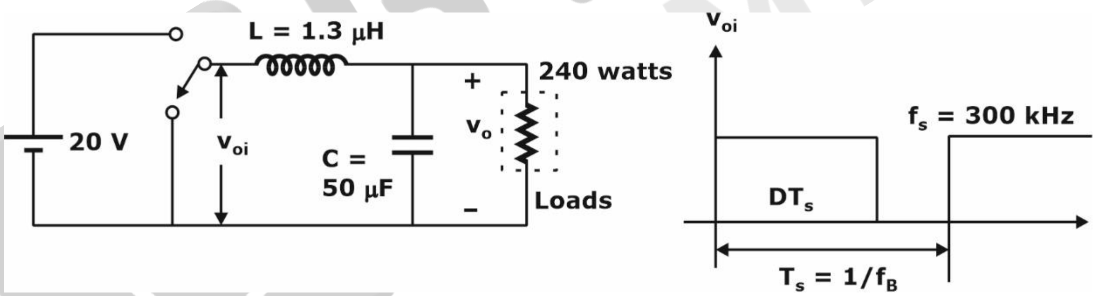
$$= \frac{8.161}{20.274} \times \frac{k}{4.05\times 2.258\times 10.788} \quad = k/244.99$$

$$\therefore GM = \frac{1}{(k/244.99)}$$

$$12 = \frac{244.99}{k}$$

$$k = 20.416$$

4. c. The equivalent circuit and is associated voltage waveform for a switched mode DC power supply is show below.



(i) Assuming a pure dc V_o = 15 V at the output across a load of 240 watts, calculate and draw the waveforms of volage and current associated with the filter inductor 'L' and current through 'C'. Let switch duty ratio D = 0.75 in this condition.

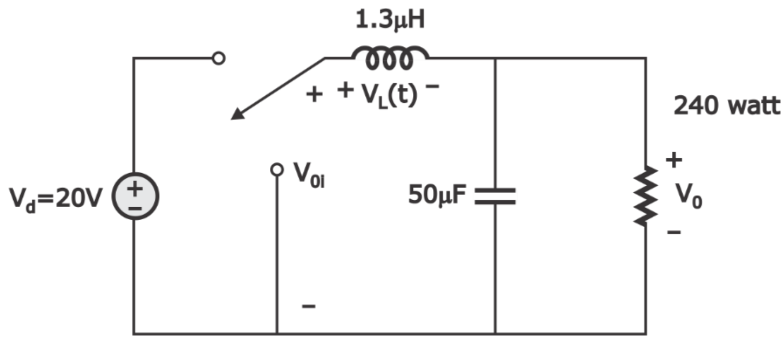
(ii) Estimate the peak-to-peak ripple in the voltage across capacitor.

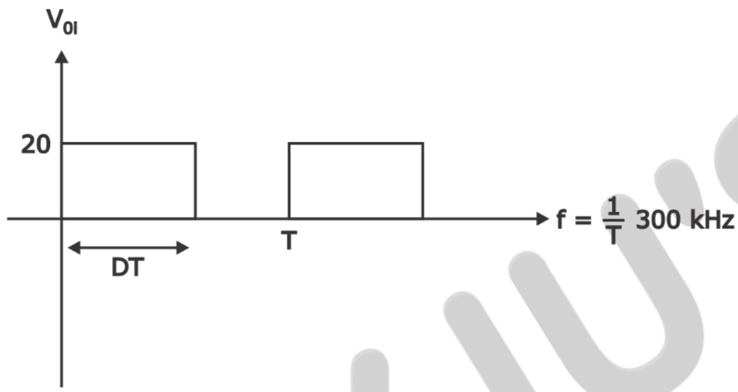
(iii) Calculate the harmonic voltage of v_{oi}.

(iv) Calculate the attenuation in decibels of ripple voltage in v_{oi} at harmonic frequency.

[20 Marks]

Sol. 4c.





(i) $V_0 = 15$ volt

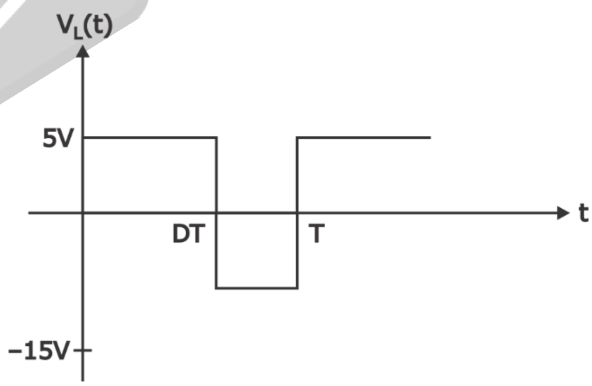
$$I_0 = \frac{240}{15} = 16A$$

$$D = 0.75 = \frac{V_0}{V_A}$$

So above circuit is buck convence

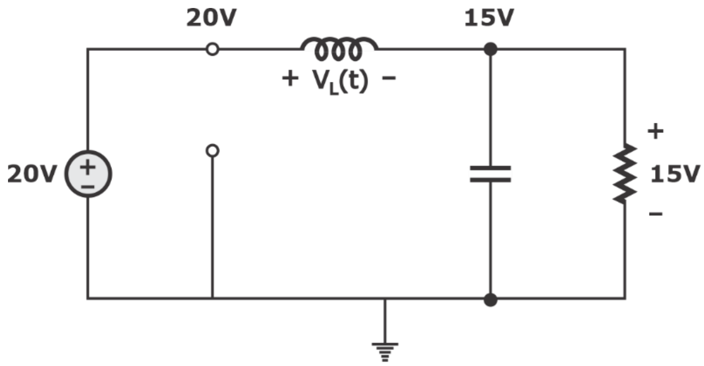
Since $\frac{V_0}{V_d} = D$ (duty ratio) so there is continues conduction.

Waveshape of $V_L(t)$ and $i_L(i)$ are shown below.



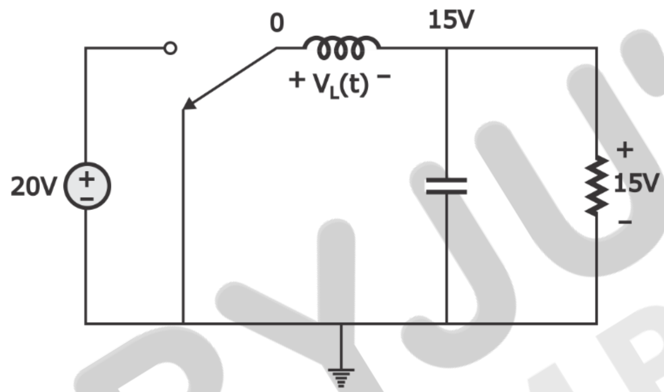
During on condition

$0 \le t \le DT$



During on condition

$DT \le t \le T$



During ON condition

$V_L(t)$ = 5 Volt

$$i_L(t) = i_{Lmin} + \frac{1}{L}\int_0^t 5\,dt$$

$$i_L(t) = i_{L\,min} + \frac{5}{L}t$$

$0 \le t \le DT$

At $t = DT$ $\quad i_L(t) = i_{L\ max} = i_{L\ min} + \frac{5}{L}DT$ (1)

During off condition

$V_L(t) = -15$ volt

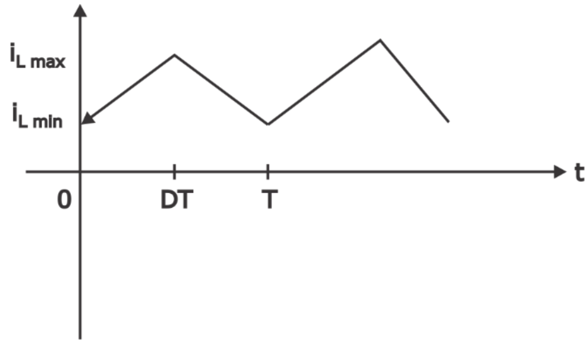
$$i_L(t) = i_{L\,max} + \frac{1}{L}\int_{DT}^t -15\,dt$$

$$i_L(t) = i_{L\,max} - \frac{15}{L}(t - DT) \quad , \quad T \le t \le T$$

at t = T $i_L(t) = i_{L\,min} = i_{Lmax}$IL

$$i_L(t) = i_{L\,min} = i_{L\,max} - \frac{15}{L}(T - DT) \qquad (2)$$

The current waveshape is shown below.



$$16 = i_{imin} + \frac{i_{L\,max} - i_{L\,min}}{2} = \frac{i_{L\,max} + i_{L\,min}}{2} \Rightarrow i_{L\,min} + i_{L\,max} = 32$$

Using equation (1) $i_{L\,min} + i_{L\,min} + \frac{5}{L}DT = 32$

$$2i_{L\,min} = 32 - \frac{5DT}{L} \Rightarrow 2i_{L\,min} = 32 - \frac{(5)(0.75)}{300\times10^3\left(1.3\times10^{-6}\right)}$$

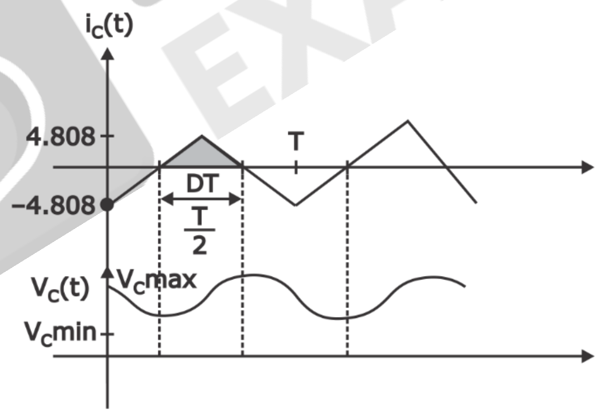
$$\Rightarrow i_{L\,min} = 2211.192\,\text{Amp}$$

$$\Rightarrow i_{L\,max} = 11.192 + \frac{5(0.75)}{300\times10^3\left(1.3\times10^{-6}\right)} = 20.80\text{ Amp}$$

(ii) Capacitor current

$i_c(t) = i_L(t) - I_0$

Wave shape of $i_c(t)$ is shown below.



Peak to peak ripple in capacitor voltage $= \Delta V_c = V_{c\,max} - V_{c\,min}$

$$\Delta V_c = \frac{\Delta Q}{c}$$

$\Delta V =$ Area of shaded region

$$= \frac{1}{2}[4.808]\left[\frac{T}{2}\right]$$

$$\Delta V_c = \frac{1}{4}\left(\frac{4.808)}{50\times10^{-6}}\right)\left(\frac{1}{300\times10^{3}}\right) = 0.0801\,\text{volt}$$

(iii) $V_{0i}(t) = V_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$

$v_0 = 15\,\text{volt}$

$$a_1 = \frac{2}{T}\int_0^T V_0 i \cos n\omega_0 t\,dt = \frac{2}{T}\int_0^{DT} 20\cos n\omega_0 t\,dt$$

$$= \frac{40}{T(n\omega_0)}\left[\sin n\omega_0 t\Big|_0^{DT}\right] \qquad \omega_0 T = 2\pi$$

$$= \frac{40}{2n\pi}\left[\sin\frac{3}{4}(2n\pi)\right] = \frac{20}{n\pi}\sin\frac{3n\pi}{2}$$

$$b_n = \frac{2}{T}\int_0^T V_{0i}\sin n\omega_0 t\,dt = \frac{2}{T}\int_0^{DT} 20\sin n\,\omega_0 t\,dt$$

$$\frac{40}{T(n\omega_0)}\left[-\cos n\omega_0 t\Big|_0^{DT}\right] = \frac{40}{2n\pi}\left[1-\cos\frac{3}{2}n\pi\right] = \frac{20}{n\pi}\left(1-\cos\frac{3}{2}n\pi\right)$$

$$V_{0i}(t) = 15 + \sum_{n=1}^{\infty}\frac{20}{n\pi}\sin\frac{3n\pi}{2}\cdot\cos n\omega_0 t + \sum_{n=1}^{\infty}\frac{20}{n\pi}\left(1-\cos\frac{3n\pi}{2}\right)\cdot\sin n\omega_0 t$$

R.M.S value of $V_{0i}(t)$

$$= \sqrt{\frac{1}{T}\int_0^{DT} 20^2\,dt} = \sqrt{\frac{20^2(DT)}{T}} = \sqrt{20^2\left(\frac{3}{4}\right)} = 20\sqrt{\frac{3}{4}}\text{ volt}$$

R.M.S value of harmonic content

$$= \sqrt{V_{0i\,rms}^2 - V_0^2}$$

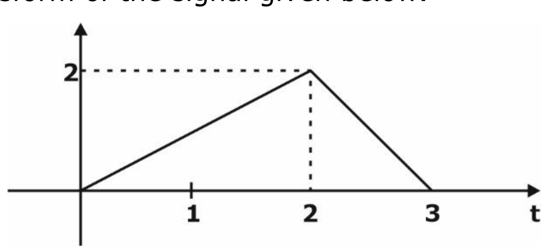
$$= \sqrt{400\left(\frac{3}{4}\right) - 15^2} = \sqrt{300-225} = 8.66\text{ volt}$$

R.M.S value of ripple voltage = 8.66 volt

Attenuation is $\text{dB} = 20\log\frac{8.66}{15} = -4.77\,\text{dB}$

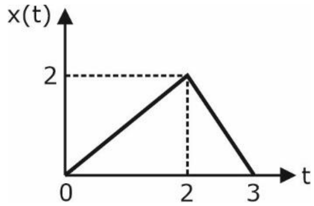
SECTION - B

5. a. Find the Laplace transform of the signal given below.



[12 Marks]

Sol. 5a.



x(t) = r(t) − 3r(t − 2) + 2r (t − 3)

Taking LT, $x(s) = \frac{1}{s^2} - \frac{3e^{-2s}}{s^2} + \frac{2e^{-3s}}{s^2}$

$$x(s) = \frac{1}{s^2}\left[1 - 3e^{-2s} + 2e^{-3s}\right]$$

5. b. Find the time response, initial value and final value of the given function

$$F(s) = \frac{12(s+1)}{s(s+2)^2(s+3)}$$

[12 Marks]

Sol. 5b. $F(s) = \frac{12(s+1)}{s(s+2)^2(s+3)}$

By partial fraction

$$F(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{D}{(s+3)}$$

F(s) = A(s + 2)^2 (s + 3) + Bs(s + 2) (s + 3) + C(s + 3)s + D(s)(s + 2)^2

put s = −3, 12(−3 +1) = −3D

D = 8

put s = 0, 12 = 12A

A = 1

puts = −2, −12 = −2C(−2+3)

C = 6

puts = 1, 24 = 9A^2 × 4 + B × 12 + 4c + 9D

24 = 36 + 12B + 24 + 72

B = −9

$$F(s) = \frac{1}{s} - \frac{9}{s+2} + \frac{6}{(s+2)^2} + \frac{8}{(s+3)}$$

Taking ILT,

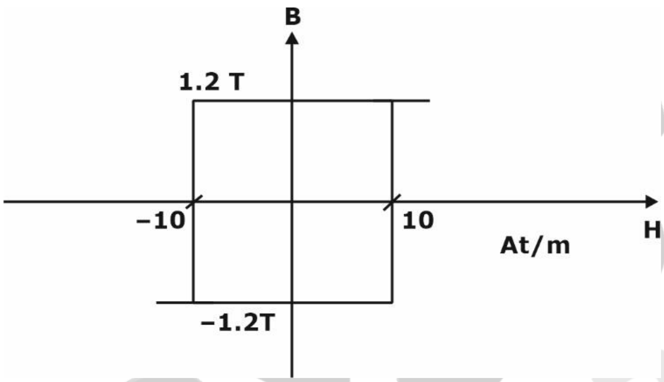
f(t) = 1 − 9e^{-2t} + 6te^{-2t} + 8e^{-3t}

Initial value theorem, $f(t)|_{t=0} = \lim_{s\to\infty} sF(s) = 0$

Final value theorem, $f(t)|_{t\to\infty} = \lim_{s\to 0} sF(s) = 1$

5. c. A toroidal core of mean length 15 cm and cross-sectional area 10 cm^2 has a uniformly distributed winding of 300 turns.

The B-H characteristic of the core can be assumed to be of rectangular form, as shown in the figure below. The coil is connected to a 100 V, 400 Hz supply. Determine the hysteresis loss in the core.



[12 Marks]

Sol. 5c. Hysteresis loss = (Volume of cone) (Area of BH curve) (f)

Volume of core = (0.15) (10 × 10^{-4}) = 1.5 × 10^{-4} m^3

Area of BH curve = 2(1.2) (20) = 48 m^2

Hysteresis loss = (1.5 × 10^{-4}) (48) (400) = 2.88 watt

5. d. The incremental fuel cost for a generating plant having two units are

$IC_1 = 20 + 0.1P_1$ Rs/MWhr

$IC_2 = 15 + 0.12P_2$ Rs /MWhr

If the total demand P_D = 200 MW, determine the division of load between the units for the most economical operation.

[12 Marks]

Sol. 5d. Given,

$P_1 + P_2 = 200$ MW ...(1)

$I_{C_1} = I_{C_2}$

$0.1P_1 + 20 + 0.12P_2 + 15$

$0.1P_1 - 0.12P_2 = -5$ ...(2)

$2 \div 0.1$

$P_1 - 1.2P_2 = -50$ ...(3)

(1) – (3)

$\Rightarrow P_2 = 113.64$ MW

Given $P_1 + P_2 = 200$

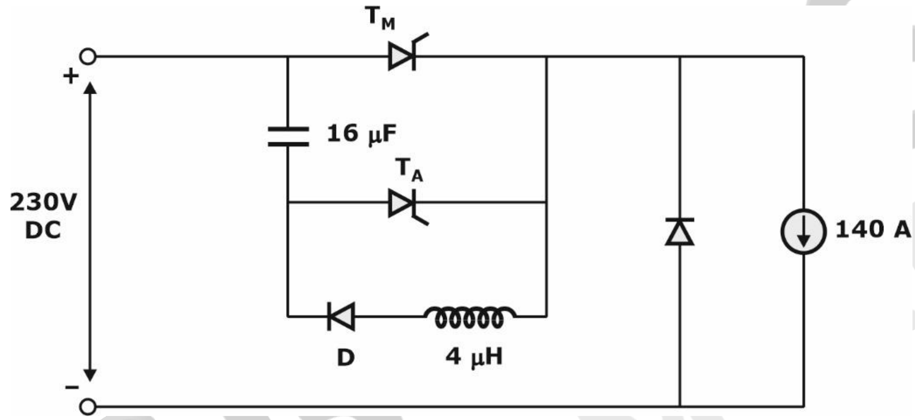
$P_1 = 200 - 113.64$

$P_1 = 86.34$ MW

5. e. For a class-D communication circuit shown below, calculate

(i) peak currents through Main and Auxiliary thyristors

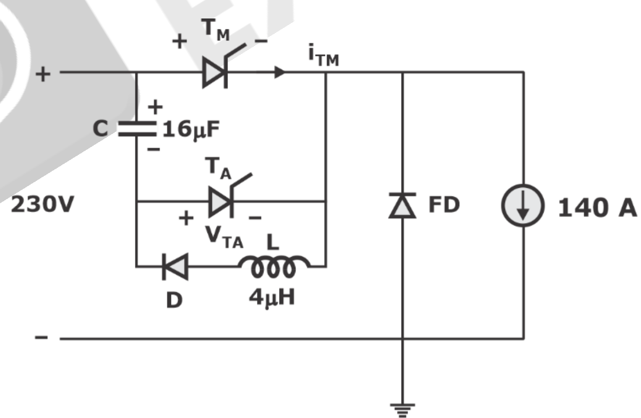
(ii) turn-off time (s) for Main and Auxiliary thyristors



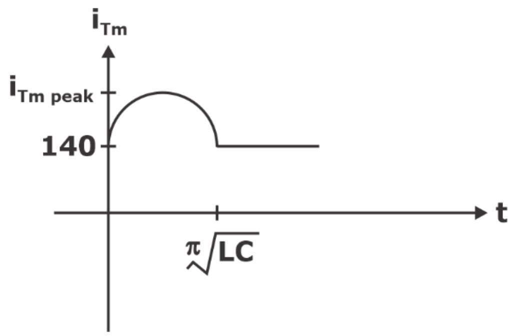
Where T_M is main thyristors and T_A is Auxiliary thyristors.

[12 Marks]

Sol. 5e.



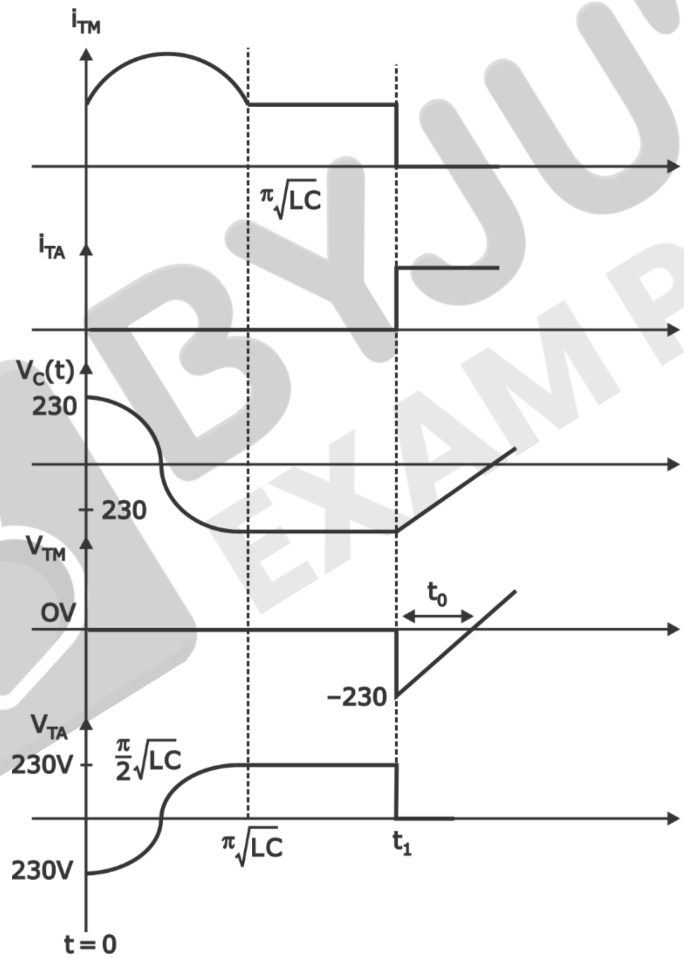
(i) before t = 0 capacitor is charged at 23000 t with polarity shown above. Output current is flowing through FD and no thyristor is conducting. V_{TM} = 230 volt. It is under forward blocking made. At t = 0 T_M is fired. Waveshape of i_{TM} is shown below i_{TM}.



because t > 0 FD gets reverse biased and LC circuit current will also How through T_M

$$i_{TM\ peak} = 140 + 230\sqrt{\frac{C}{L}} = 140 + 230\sqrt{\frac{16}{4}} = 140 + 460 = 600\,\text{Amp}$$

(ii) Waveshape of V_{TM} and V_{TA} are shown below.



AT t = 0 T_M is fired. During T_m ON $V_{TA} = -V_C(t)$

At t = t_1 auxilary thyristor T_A get fired. At the save instant T_m gets reversed biased by capacitor voltage and then load current will start flowing through capacitor and auxiliary thyristor. So capacitor will charge. Linearly as shown above from above waveshape we can conclude that T_A. will get reversed biased for minimum interval of $\frac{\pi}{2}\sqrt{LC}$.

So t_c (circuit term-off time) for $T_A = \frac{\pi}{2}\sqrt{64}\ \mu sec$.

$$= \frac{\pi}{2}(8)\mu\,sec$$

$$= 4\pi\mu\,sec$$

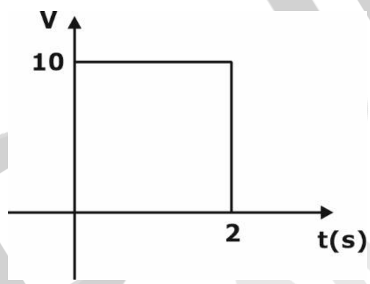
t_L (circuit tern-off time) for main thyristor = t_0 from capacitor

$$0 = -230 + \frac{1}{c}\int_0^{t_0} 140\,dt$$

$$\Rightarrow 230 = \frac{140t_0}{c}$$

$$t_0 = \frac{230C}{140} = 26.28\ \mu sec$$

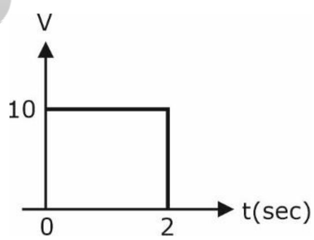
6. a. Determine the Fourier transform of a pulse shown below.



Find the magnitude at $\omega = 2\pi$.

[20 Marks]

Sol. 6a.



$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}dt$$

$$= \int_0^2 10\, e^{-j\omega t}dt = \frac{10}{-j\omega}\left[e^{-j\omega t}\right]_0^2$$

$$= \frac{10}{-j\omega}\left[e^{-j\omega(2)} - 1\right] = \frac{\left(1 - e^{j2\omega}\right)10}{j\omega}$$

$$= \frac{10\left(1-e^{j2\omega}\right)}{j\omega}$$

Magnitude at $\omega = 2\pi$

$$X(\omega = 2\pi) = \frac{10}{j2\pi}\left(1 - e^{j4\pi}\right)$$

$$= 0$$

6. b. For a single machine infinite bus shown below, if δ_c is the critical clearing angle for a three-phase short circuit 'F', prove that the clearing time 't_c' of the circuit breaker CB must satisfy the following:

Where P_d is the mechanical power input,

δ_0 is the initial power angle,

F is the frequency and

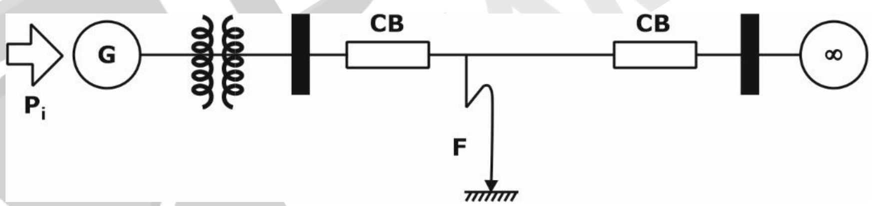
H is the machine inertia constant and is given by

$$H = \frac{\pi f}{G} J \left(\frac{2}{P}\right)^2 \omega_e \times 10^{-6}$$

J is moment of inertia of rotor (kg-m^2)

ω_e is synchronous speed in electrical rad/sec.

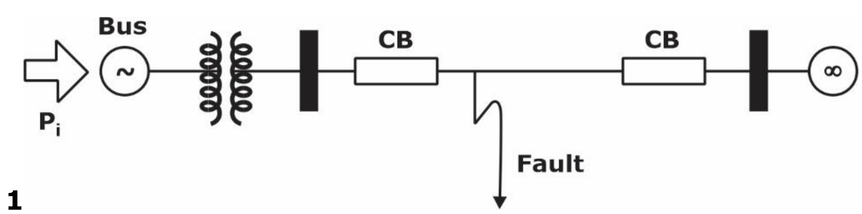
G is three-phase MVA rating (base) of machine.



Also express the relation of δ_{cr} with δ_0, where δ_{cr} is the critical clearing angle for corresponding critical clearing time.

[20 Marks]

Sol. 6b. Single machine connected to an infinite bus



P_i = Mechanical input

P_{e1} = Electrical output before fault

P_{e2} = Electrical output during the fault

There is only one network between the generation and infinite bus.

Hence $P_{e2} = 0$ (During the fault)

Swing equation is used for the transient stability study

$$M\frac{d^2\delta}{dt^2} = P_a = P_i - P_{e2}$$

$$M\frac{d^2\delta}{dt^2} = P_i - 0 = P_i(\text{constant})$$

$$\frac{d^2\delta}{dt^2} = \frac{P_i}{M}(\text{constant})$$

M is the angular momentum of synchronous machine

Integrate w.r.t. dt on both sides

$$\frac{d\delta}{dt} = \frac{P_i}{M}t + A$$

When t = 0, before acceleration period

The synchronous machine is running $\omega = \omega_s$ at δ_0

$$\frac{d\delta}{dt} = 0$$

$$0 = \frac{P_i}{M} \times 0 + A$$

A = 0

$$\frac{d\delta}{dt} = \frac{P_i}{M}t$$

Integrate on both sides w.r.t. dt on both sides

$$\delta = \frac{P_i}{M}\frac{t^2}{2}t$$

Assume t is t_c and δ is δ_c

t_c = critical time

δ_c = critical time

$$\delta_c = \frac{P_i}{M}\frac{t_c^2}{2} + B$$

When $t_c = 0$ before acceleration period $\omega = \omega_s$ at δ_0

$\delta_c = \delta_0$

$$\delta_0 = \frac{P_i}{M} \times 0 + B$$

$B = \delta_0$

$$\delta_c = \frac{P_i}{M}\frac{t_c^2}{2} + \delta_0$$

$$(\delta_c - \delta_0) = \frac{P_i}{M}\frac{t_c^2}{2}$$

$$t_c = \sqrt{\frac{2M(\delta_c - \delta_0)}{P_i}}$$

$$t_c = \sqrt{\frac{2\frac{SH}{\pi f}(\delta_c - \delta_0)}{P_i}}$$

$$\left[M = \frac{SH}{\pi f}\right]$$

$$\left[M = \frac{GH}{\pi f}\right]$$

$$S = G$$

$$t_c = \sqrt{\frac{2GH(\delta_c - \delta_0)}{\pi f P_i}}$$

However, the time taken by CB shall be $t_c \le \sqrt{\frac{2GH(\delta_c - \delta_0)}{\pi f P_i}}$

To clear the fault

P_i = Mechanical input

P_{e1} = Electrical output before fault

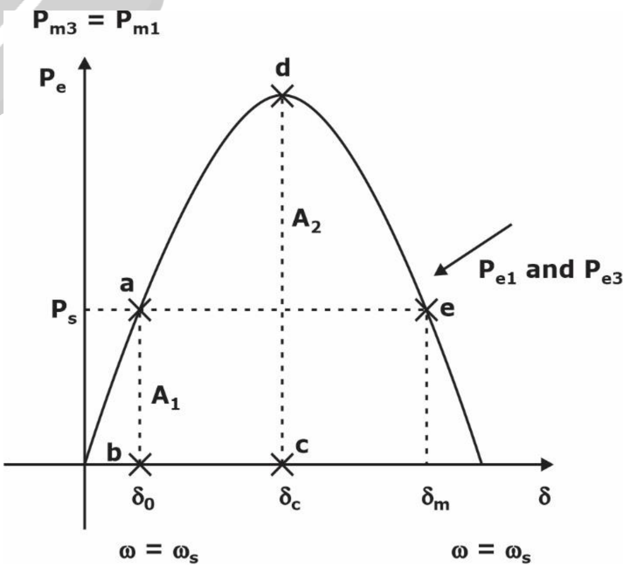
P_{e1} = $P_{m1}\sin\delta_0$

P_{e2} = Electrical output during fault

P_{e2} = 0

P_{e3} = Electrical output after fault cleared

P_{e2} = $P_{m3}\sin\delta$



$$\int_{\delta_0}^{\delta_m} P_a\, d\delta = 0$$

$$\int_{\delta_0}^{\delta_c} P_a\, d\delta + \int_{\delta_c}^{\delta_m} P_a\, d\delta = 0$$

$$\int_{\delta_0}^{\delta_c} (P_i - P_{e2})d\delta + \int_{\delta_c}^{\delta_m} (P_i - P_{e3})d\delta = 0$$

$$[P_i\delta]_{\delta_0}^{\delta_c} + [P_i\delta + P_{m3}\cos\delta]_{\delta_c}^{\delta_m} = 0$$

$$P_i\delta_c - P_i\delta_0 + P_i\delta_m = P_i\delta_c + P_{m3}\cos\delta_m - P_{m3}\cos\delta_c = 0$$

$$P_i(\delta_m - \delta_0) + P_{m3}\cos\delta_m = P_{m3}\cos\delta_c$$

$$\delta_c = \cos^{-1}\left[\frac{P_i(\delta_m - \delta_0) + P_{m3}\cos\delta_m}{P_{m3}}\right]$$

$$\delta_c = \sin^{-1}\left(\frac{P_i}{P_{m1}}\right)$$

$$\delta_m = 180 - \sin^{-1}\left(\frac{P_i}{P_{m3}}\right)$$

$$P_{m3} = P_{m1}$$

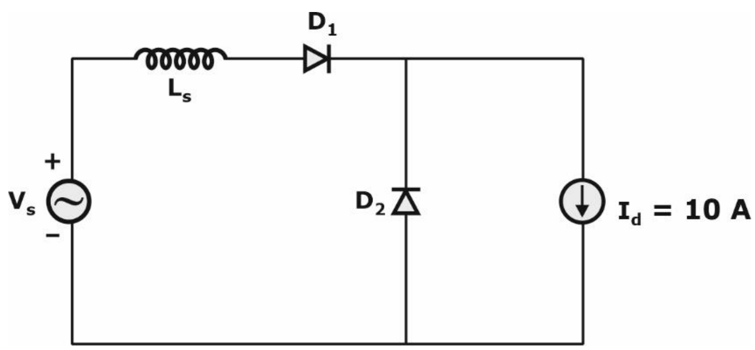
$$\delta_m = 180 - \delta_0$$

$$\delta_m = 180 - \sin^{-1}\left(\frac{P_i}{P_{m1}}\right)$$

$$\delta_m = 180 - \delta_0$$

$$\delta_c = \cos^{-1}\left[\frac{P_i(180 - \delta_0 - \delta_0) + P_{m3}\cos(180 - \delta_0)}{P_{m3}}\right]$$

6. c. A half-wave uncontrolled rectifier circuit is fed from ac source with source inductance 'L_s'. It is driving a dc load at a constant I_d as shown in figure below.



Calculate average output voltage V_d. average power P_d, communication overlap angle μ and plot the wave form of source current is, if

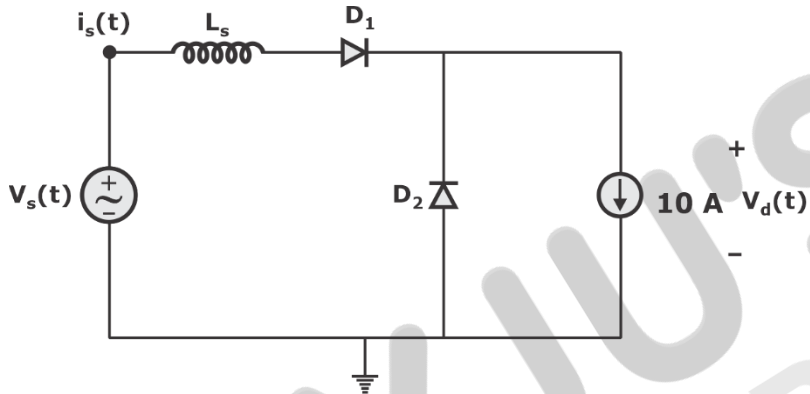
(i) V_s = 310 sin (314 t) and L_s = 0

(ii) V_s = 310 sin (314 t) and L_s = 5 mH

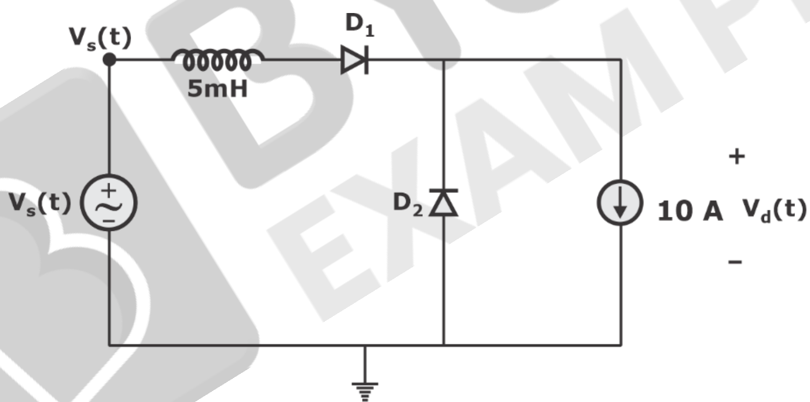
(iii) V_s is a square wave of 310 V and 50 Hz a source inductance L_s = 5 mH.

[20 Marks]

Sol. 6c.



(i) $v_s(t) = 310\sin(314t)$ $L_s = 0$



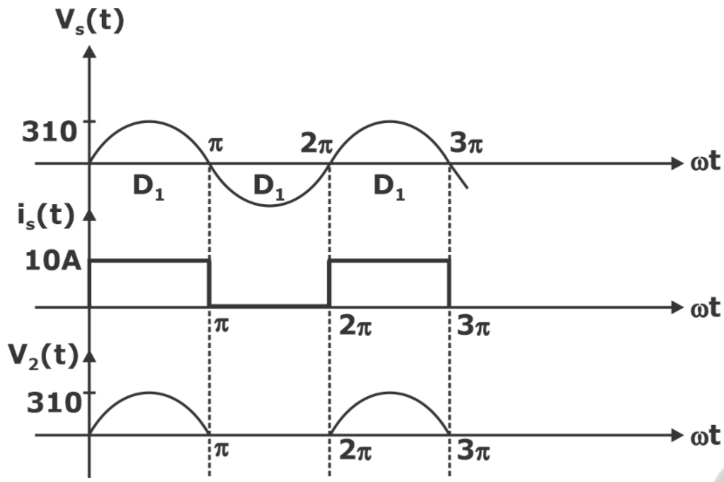
D_1 and D_2 are in common cathode connection

At anode of D_1 voltage is $V_s(t)$

At anode of D_2 Voftye is zero.

If $V_s(t) > 0$ D_1 will be ON and D_2 will be off

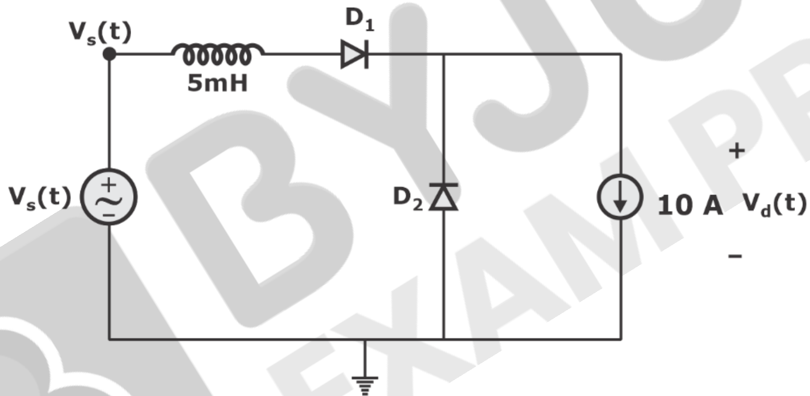
If $V_5(t) < 0$ D_1, will be off and D_2 will be ON ware shapes ax shown below



Average value of $V_d(t) = \frac{1}{2\pi}\int_0^{\pi} 310 \sin \omega t \, d(\omega t) = \frac{310}{\pi}$ volt $= 98.67$ volt.

Average power absorbed by current source = 98.67 (10) = 986.7 watt.

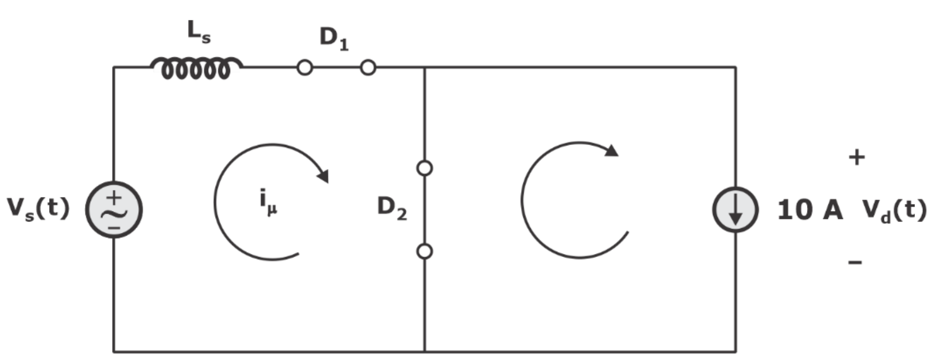
(ii)



before t = 0 D_2 is conducting so entire current (l0 A) is flowing through D_2 and current through D_1 is zero.

After t = 0 $V_s(t) > 0$. So now D_1 will also start conducting. Due to inductor there will be overlapping interval. During overlapping internal (μ) both diodes will conduct. Current in D_1 will increase and current in D_2 will decrease as shown below

$0 \le \omega t \le \mu$



i_μ chages fron 0 to 10 A duing aveslapph intenal μ

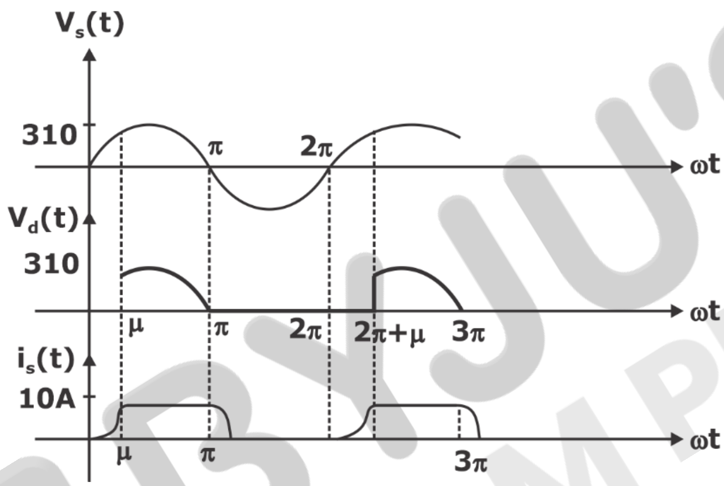
$$V_s(t) - L_3 \frac{di_\mu}{dt} = 0 \Rightarrow 310 \sin \omega t = L_3 \frac{di_\mu}{dt}$$

$$\Rightarrow \quad \frac{310}{WL_s} \int_0^\mu \sin \omega t d(\omega t) = \int_0^{10} di_\mu$$

$$5 \times 10^{-3} \frac{310}{314} [1 - \cos \mu] = 10 \Rightarrow 1 - \cos \mu = \frac{314 \times 5 \times 10^{-3}}{31}$$

$1 - \cos \mu = 0.0506 \Rightarrow \cos \mu = 0.9493 \Rightarrow \mu = 18.31° = 0.3196$ rad

Waveshape are shown below

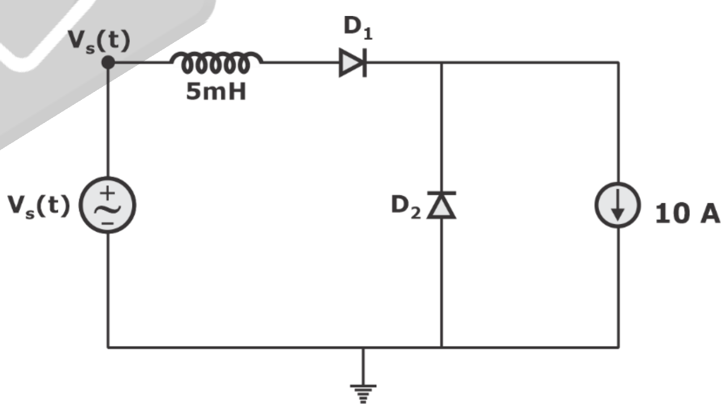


$$V_d(t)_{ay} = \frac{1}{2\pi} \int_\mu^\pi V_m \sin \omega t \, d(\omega t) = \frac{V_M}{2\pi} \left[1 + \cos \mu\right] = \frac{310}{2\pi} [1 + 0.9493] = 96.175 \text{ volt}$$

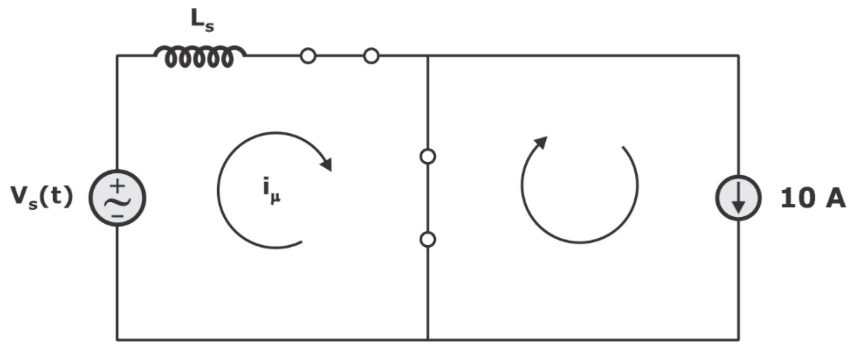
Power absorbed by current source = 96.175 (10)

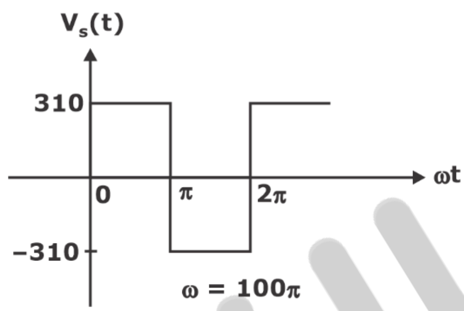
= 961.75 watt

(iii)



$0 \le \omega t \le \mu$ circuit is shown below





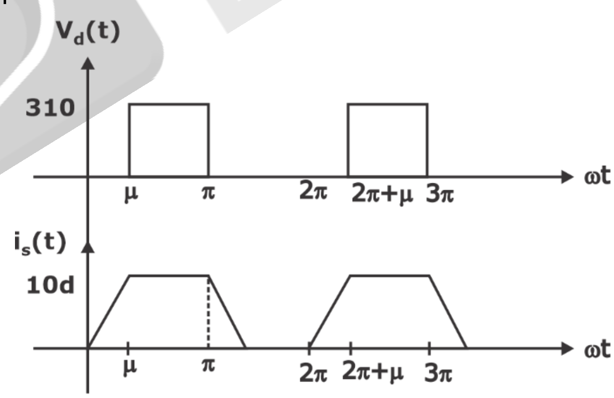
According to K.V.L

$$310 - L_s \frac{d_{i\mu}}{dt} = 0$$

$$\Rightarrow \quad \frac{310}{\omega L_s}\int_0^{\mu} d(\omega t) = \int_0^{10} di_\mu \Rightarrow \frac{310\mu}{100\pi\left(5\times 10^{-3}\right)} \Rightarrow 10$$

$$\mu = \frac{10(100\pi)\left(5\times 10^{-3}\right)}{310} = 0.0506\,\text{rad}$$

The waveshapes will look like



$$V_d(t)_{ay} = \frac{1}{2\pi}\left[310(\pi - 0.0506)\right] = 152.503\,\text{volt}$$

Power absorbed by current source = 152.503 (10) = 152.503 watt.

7. a. A certain system is described by

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} u(t)$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Determine the transformation matrix [P] so that if [x] = [P][Z]; the state matrices [A], [B], [C] and [D] describing the dynamics of [Z] are in control canonical form.

[20 Marks]

Sol. 7a. Given

$$\left.\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \begin{pmatrix} -2 & 1 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} 4 \\ y &= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{aligned}\right\} \ldots(1)$$

Finding the transfer function, by T(s)=CQ(s)B+D

Where $Q(s) = (sI - A)^{-1}$

$$(sI - A) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -2 & 1 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} s+2 & -1 \\ 2 & s \end{pmatrix}$$

$$(sI - A)^{-1} = \frac{\text{adj}(sI - A)}{|sI - A|} = \frac{\begin{pmatrix} s & 1 \\ -2 & s+2 \end{pmatrix}}{s(s+2)+2}$$

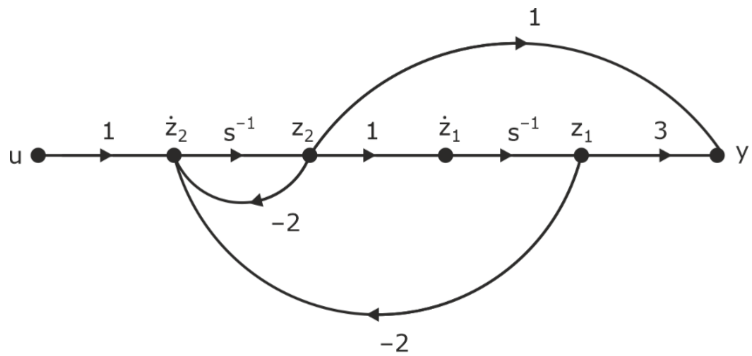
T(s) = CQ(s) B + D

$$= \begin{pmatrix} 1 & 0 \end{pmatrix} \frac{\begin{pmatrix} s & 1 \\ -2 & s+2 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix}}{s^2 + 2s + 2} + 0$$

$$T(s) = \frac{\begin{pmatrix} s & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix}}{s^2 + 2s + 2} = \frac{s+3}{s^2 + 2s + 2}$$

$$\text{or, } T(s) = \frac{s^{-1} + 3s^{-2}}{1 - (-2s^{-1} - 2s^{-2})} = \frac{P_1 \times 1 + P_2 \times 1}{1 - (L_1 + L_2)}$$

Drawing signal flow graph for it, (as per controllable canonical form)



Taking output of integrators (s^{-1}) as state variables,

$\dot z_1 = z_2$

$\dot z_2 = -2z_1 - 2z_2 + 4$

$y = 3z_1 + z_2$

$$\left.\begin{aligned} &\text{So, } \begin{pmatrix} \dot z_1 \\ \dot z_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} 4 \\ &\quad y = \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \end{aligned} \right\} \ldots(2)$$

$$\left.\begin{aligned} \text{or, } \dot z. &= \bar A z + \bar B y \\ y &= \bar c z \end{aligned} \right\} \ldots(3)$$

But given $\dot x. = Ax + By$

$y = Cx$

put x + Pz

$P\dot z. = APz + B4$

y = CPz

$$\left.\begin{aligned} \Rightarrow \dot z. &= P^{-1}APz + P^{-1}B4 \\ y &= CPz \end{aligned} \right\} \ldots(4)$$

Comparing (3) and (4)

$\bar A = P^{-1}AP$

$\bar B = P^{-1}B$

$\bar C = CD$

So, $P\bar{A} = AP$ ...(5)

$P\bar{B} = B$ ..(6)

$\bar{C} = CP$ ...(7)

Let $P = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

Using (6), $P\bar{B} = B$

$$\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$\Rightarrow q = 1$

$s = 3$

Using (5), $P\bar{A} = AD$

$$\begin{pmatrix} p & 1 \\ r & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -2 & -2 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} p & 1 \\ r & 3 \end{pmatrix}$$

$-2 = -2p + 4$

$p - 2 = -2 + 3 = 1 \Rightarrow p = 3$

So, $r = 2p - 2 = 2(3) - 2 = 4$

So, $P = \begin{pmatrix} 3 & 1 \\ 4 & 3 \end{pmatrix}$

7. b. A 3-phase, 440 V, 50 Hz, four pole wound rotor induction motor develops full load torque at a slip of 0.04 (i.e. 4%) when the slip rings are short circuited. The maximum torque it can develop is 2.5 per unit. The stator leakage impedance is negligible. The rotor resistance measured between two slip rings is 0.5 Ω.

(i) Determine the speed of the motor at maximum torque. Derive the formula used.

(ii) Determine the starting torque in per unit. (Full load torque is one per unit torque)

(iii) Determine the value of resistance to be added to each phase of the rotor circuit so that maximum torque is developed at the starting condition.

(iv) Determine the speed at full-load torque with the added rotor resistance of part (iii).

20 Marks]

Sol. 7b. $V_e = 440$ volt

$f = 50$ Hz

$P = 4$

$S_R = 0.04$

$T_{em} = 2.5\ T_{eff}$

Stator impedance is neglected

Rotor resistance between two slip rings = 0.5 W

Rotor resistance $= \frac{0.5}{2} = 0.250\ \Omega$

As we know

$$T = \frac{m}{\omega_s} \frac{V_e^2}{\left(R_e + \frac{r_2}{s}\right)^2 + (X_2 + X_e)^2} \frac{r_2}{s}$$

Since stator impedance is neglected

$$T_{eff} = \frac{mV_e^2}{\omega_s \left(\frac{r_2}{s}\right)^2 + X_2^2)} \frac{r_2}{s}$$

T_{em} (maximum torque) occurs at S_{MT}

$$S_{MT} = \frac{r_2}{\sqrt{R_e^2 + (X_2 + X_e)^2}}$$

So, $$T_{em} = T_e |_{S=S_{MT}} = \frac{mV_e^2}{\omega_s [R_e + \sqrt{R_e^2 + (X_e + X_2)^2}]}$$

Since stator impedance is neglected

$$T_{em} = \frac{mV_e^2}{\omega_s X_2} \quad \text{....(2)}$$

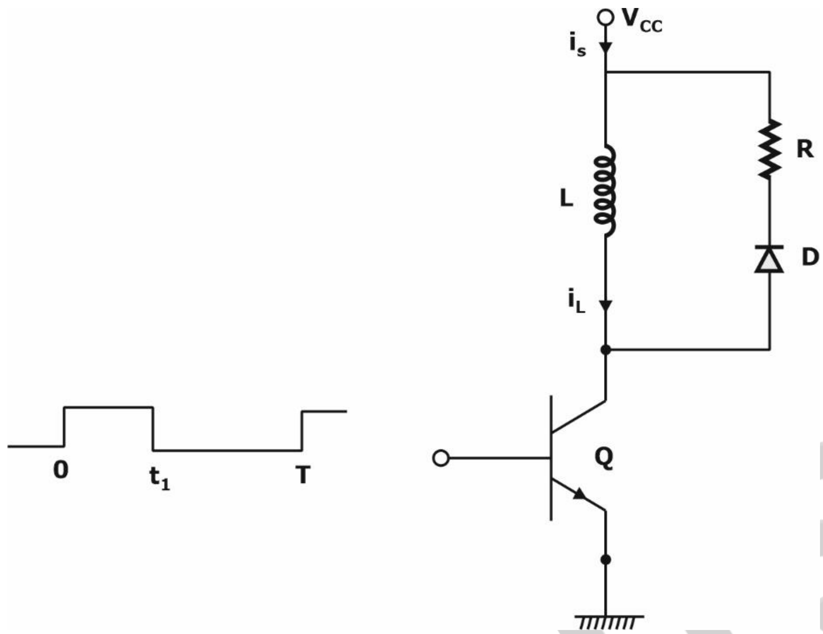
And $$S_{MT} = \frac{r_2}{X_2} \quad \text{...(3)}$$

From eq. (1) and (2)

$$\frac{T_e}{T_{em}} = \frac{2X_2}{\left(\frac{r_2}{s}\right)^2 + X_2^2} \cdot \frac{r_2}{s} \qquad X_2 = \frac{r_2}{s_{MT}}$$

$$\Rightarrow \quad \frac{T_e}{T_{em}} = \frac{2}{\frac{s_{MT}}{s} + \frac{s}{s_{MT}}} = \frac{2ss_{MT}}{s_{MT}^2 + s^2}$$

7. c. For the Figure shown above, the transistor 'Q' is excited by a pulse of duration 't_1' with a periodicity of $\frac{1}{T}$.



(i) Draw the current waveform of 'i_s' and 'i_L'.

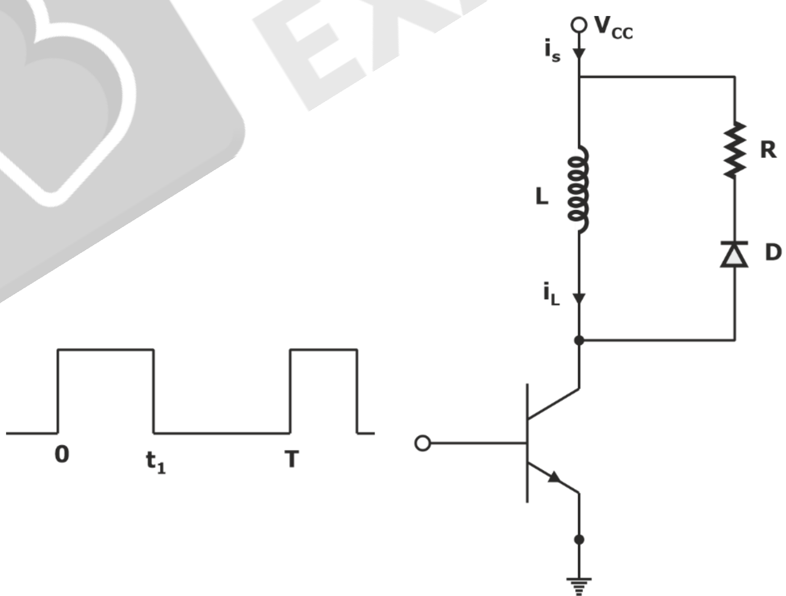
(ii) Expression for absorbed average power by resistor 'R' in the circuit.

Assume $\frac{L}{R}$ ratio to be too small in comparison to 'T'.

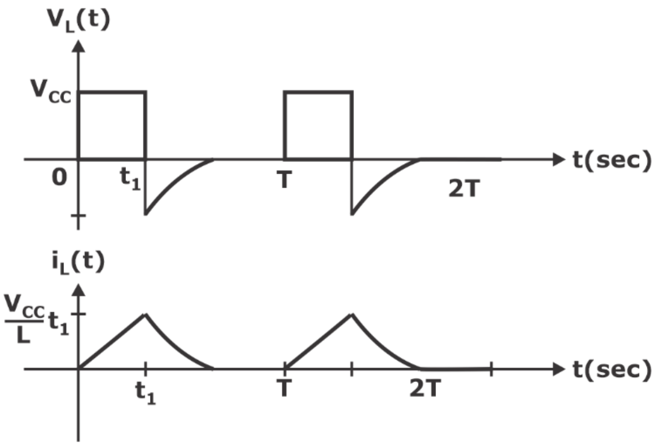
(iii) Expression for $i_L(t)$, the current through inductor 'L'.

[20 Marks]

Sol. 7c.



(i) During 0 to t_1 transistor is ON and from t_1 to T transistor is off. Waveshape of $V_L(t)$ will be

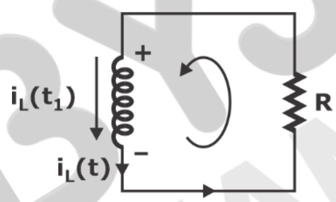


$0 \le t \le t_1 \quad V_L(t) = V_{cc}$

So $i_L(t) = 0 + \frac{1}{L}\int_0^t V_{cc}dt = \frac{V_{cc}t}{L} \quad 0 \le t \le t_1$

$i_L(t)_{max} = \frac{V_{cc}t_1}{L}$

$t_1 \le t \le T$ inductor will discharge through R



$i_L(t) = i_L(t_1)e^{-\frac{R}{L}(t-t_1)}U(t-t_1)$

$V_L(t) = -V_R(t)$

$= -Ri_L(t_1)e^{\frac{-R}{L}(t-t_1)}U(t-t_1)$

$i_L(t_1) = \frac{V_{c_c t_1}}{L}$

(ii) Energy stored in inductor at $t = t_1$

$= \frac{1}{2}L\left(\frac{V_{cc}t_1}{L}\right)^2 = \frac{V_{cc}^2 t_1^2}{2L}$

This energy is dissipated in R.

average power absorbed by $R = \frac{V_{cc}^2 t_1^2}{2LT}$

(iii) From the above calculation

$i_L(t) = \frac{V_{cc}t}{L}\left[U(t) - U(t-t_1)\right] + \frac{V_{cc}t_1}{L}e^{-\frac{R}{L}(t-t_1)}U(t-t_1)$

(iv) With addition of rotor resistance maximum torque will not effect

$\Rightarrow T_{max} = 2.5\ T_{fl}$

$$\frac{3}{w_s}\frac{E_2^2}{2X_2} = 2.5\frac{3}{W_s}\frac{S_{fl}E_2^2}{R_2^2+(SX_2)^2}.R_2$$

$$\frac{1}{2\times 1.3089} = \frac{2.5\times S_{fl}}{R_{zeq}^2+(S_{fl}\times 1.3089)^2}R_{zeq}$$

$$\frac{1}{2\times 1.31} = \frac{2.5\times S_{fl}}{131^2+1.31^2\ S_{fl}^2}$$

$$\frac{1}{2} = \frac{2.5S_{fl}}{1+S_{fl}^2}$$

$$S_{fl}^2+1 = 5S_{fl} \Rightarrow S_{fl}^2-5S_{fl}+1 = 0$$

$\therefore S_{fl} = 0.2087$

$\therefore$ Full load speed $N_{r,fl} = N_s(1 - S_{fl})$

$= 1500\ (1 - 0.2087)$

$= 1186.95$ rpm

8. a. For a causal system $H(z) = \frac{z}{z-0.5}$, find the zero state response to input

$$x(n) = \left(\frac{1}{4}\right)^n u(n)+5(3)^n\ u[-(n+1)]$$

[20 Marks]

Sol. 8a. Given:-

$$H|z| = \frac{z}{z-0.5};\ |z|>0.5$$

$$x(n) = \left(\frac{1}{4}\right)^n u(n)+5(3)^n u(-n+1)$$

z transform of a^n u(n) is $\frac{1}{1-az^{-1}}; |z|>a$

z-transform of -a^n u(−n−1) is $\frac{1}{1-az^{-1}}; |z|>a$

$$x(z) - \frac{1}{1-\frac{1}{4}z^{-1}} - \frac{5}{1-3z^{-1}}; \frac{1}{4}<|z|<3$$

For ZSR, initial conditions are zero.

Y(z) = X(z) H(z)

$$= \frac{1}{\left(1-0.5^{-1}\right)}\left[\frac{1}{1-\frac{1}{4}z^{-1}} - \frac{5}{1-3z^{-1}}\right]$$

$$= \frac{1}{\left(1-0.52^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)} - \frac{5}{\left(1-0.5z^{-1}\right)\left(1-3z^{-1}\right)}$$

After partial fraction,

$$Y(z) = \frac{2}{1-0.5z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-0.5z^{-1}} + \frac{6}{1-3z^{-1}}$$

$$= \frac{3}{1-0.5z^{-1}} - \frac{1}{1-\frac{1}{4}z^{-1}} - \frac{6}{1-3z^{-1}}$$

Taking IZT,

$$y(n) = 3(0.5)^n\, u(n) - \left(\frac{1}{4}\right)^n u(n) + 6(3)^n u(-n-1)$$

8. b. Two identical 250 KVA, 230/460 volt transformers are connected in open delta to supply a balanced 3-phase star connected load at 460 volt and at answer the following:

(i) Draw the phasor diagram of the open-delta condition.

(ii) Find the maximum secondary line current without overloading the transformers.

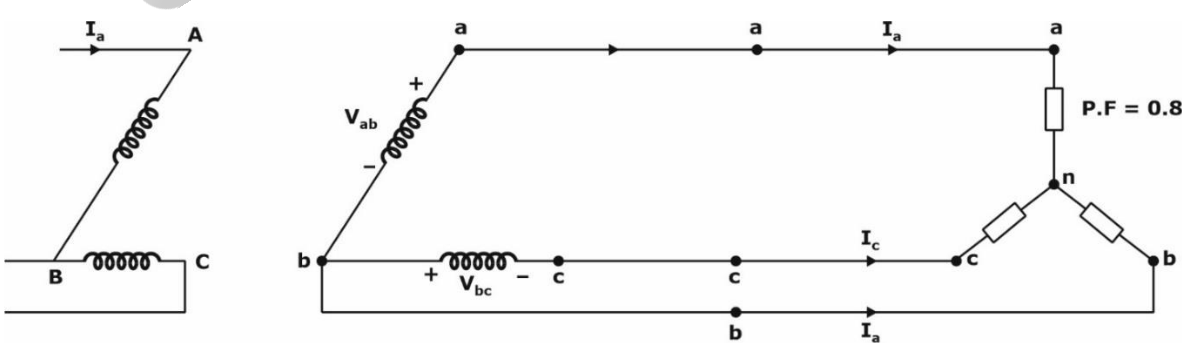
(iii) Find the real power delivered by each transformer and the total real power delivered.

(iv) Find the primary line currents.

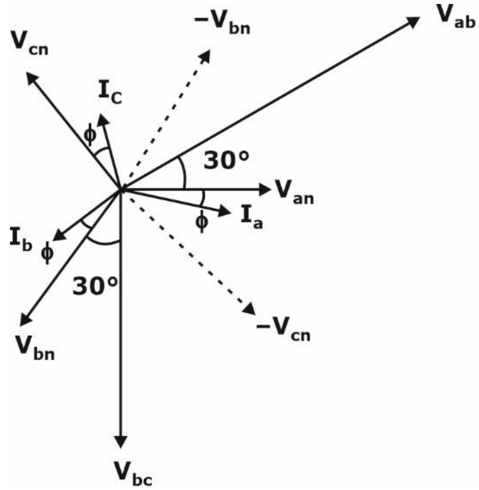
(iv) If a similar transformer is now added to complete the Δ, find the percentage increase in real power that can be supplied. Assume that the load voltage and power factor remain unchanged at 460 volt and 0.8 lagging, respectively.

[20 Marks]

Sol. 8b.



(i) Phasor diagram



$\phi = \omega_s = (0.8) \;=\; 36.86°$

$I_a + I_b + I_c = 0$

(ii) maximum secondary current = Maximum secondary phase current of open delta

$= \frac{250 \times 10^3}{460}$

$= 543.48$ A

(iii) Real power delivered by transformer coil a.b

$= V_{ab}.I_a \cos[30 + \phi]$

$= 460(543.48) \cos[30 + 36.86°]$

= 98.245 kW

Real power delivered by transformer coil BC.

$= V_{bc}(-I_C) \cos[q]$

θ is angle between V_{bc} and I_2 and is $\theta = 30 - \phi$

$= 460(543.86) \cos[30 - \phi]$

= 248.211 kW

Total real power delivered = 98.245 +248.211 = 346.456 kW

(iv) Primary line current

$I_A = (543.48)\left(\frac{60}{230}\right)$

= 1086.96 A & $I_B = I_C$

If it is now closed delta then real power output = $(3)\left(\frac{460}{\sqrt{3}}\right)(543.48\sqrt{3})(0.8)$ = 600 kW

(v) Increase is real power = (600 – 246.456) kW = 253.54 kW

% increase in real power = $\frac{253.544}{346.456} \times 100 = 73.18\%$

8. **c.** The positive, negative and zero sequence reactances of a 25 MVA, 13.2 kV synchronous generator are 0.3 pu, 0.2 pu and 0.1 pu respectively. The generator is star connected and neutral is solidly grounded. When it is unloaded, find the fault current and line-line voltages when a fault of

(i) Line-line occurs,

(ii) Double line to ground occurs.

[20 Marks]

Sol. 8c. **(i)** For line to line fault

$$|I_{f(Pu)}| = \left|\frac{jV_3 E}{Z_1 + Z_2}\right|$$

$$= \frac{jV_3(1)}{j0.3 + j0.2}$$

$$If_{pu} = 2V_3 \text{ Pu}$$

$$I_{base} = \frac{(MVA)_{base}}{V_3(KV)_{base}}$$

$$= \frac{25 \times 10^6}{V_3 \times 13.2 \times 10^3}$$

I_{base} = 1093.5 Amp

I_{act} = I_b × I_{pu}

= 1093.5 × $2V_3$

I_{act} ≅ 3788 Amp

(ii) Double line to ground fault

$$I_f = 3I_0 = 3\left(I_1 \frac{Z_2}{Z_2 + Z_0}\right)$$

$$= +3\left(\frac{1}{0.3 + \frac{0.1 \times 0.2}{0.3}}\right)\left(\frac{0.2}{0.3}\right)$$

I_f = 5.454 Pu

$I_{f(actual)}$ = 5.454 × I_{base}

= 5.545 × 1039.5

= 5964.54 Amp

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