BYJU'S EXAM PREP

ESE Mains 2023

Electrical Engineering

Questions & Solutions

PAPER-1

S. No.	Subjects	Difficulty Level 2023	2023 Marks	2022 Marks	2021 Marks
1	Engineering Mathematics	Difficult	72	72	72
2	Electrical Materials	Tough	64	76	54
3	Electrical Circuits & Fields	Easy	156	94	104
4	Electrical & Electronic Measurements	Moderate	72	102	92
5	Computer Fundamentals		44	64	64
6.	Basic Electronics Engineering	Moderate	72	72	94
Total			480	480	480

ELECTRICAL ENGINEERING

Paper-1

SACTION - A

1. a. Suppose A is a 3 × 3 diagonalizable matrix. Then-

(i) show that each eigenvalue of A is 0 or 1, if $A^2 = A$;

(ii) find the trace of the matrix $B = A + A^3 + A^{-1}$, if the eigenvalues of A are 2, 3, -2.

Sol. 1a. Let A is a 3 × 3 diagonalizable matrix with eigen value $\lambda_1, \lambda_2, \lambda_3$ and respective eigen vectors, x_1, x_2, x_3

So, A can be expressed as

$A = PDP^{-1}$

Where $P = [x_1\ x_2\ x_3]$

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

(i) Now, $A = PDP^{-1}$

$\Rightarrow AP = PD$

Or, $A(AP) = A(PD)$

$A^2P = (AP)D = (PD)D$

Or, $A^2 = PD^2P^{-1}$

$$\text{Where, } D^2 = \begin{bmatrix} \lambda_1^2 & 0 & 0 \\ 0 & \lambda_2^2 & 0 \\ 0 & 0 & \lambda_3^2 \end{bmatrix}$$

So, eigen values of A^2 are $\lambda_1^2, \lambda_2^2, \lambda_3^2$ with same eigen vectors

In general, eigen values of A^n are λ^n

Given $A^2 = A$

So, $\lambda^2 = \lambda$

$\lambda(\lambda - 1) = 0$

$\Rightarrow \lambda = 0$ or 1

So, eigen values for (A) is 0 or 1.

(ii) Now, $B = A + A^3 + A^{-1}$

So, eigen values of B are $\lambda + \lambda^3 + \lambda^{-1}$

given eigen values of A are 2, 3, −2

So, eigen values of B are

$\lambda_1 = 2 + 2^3 + 2^{-1} = 10.5$

$\lambda_2 = 2 + 3^3 + 3^{-1} = 30.33$

$\lambda_3 = -2 + (-2)^3 + (-2)^{-1} = -10.5$

Now, trace matrix is equal to sum of it's eigen values

So, trace of B = 10.5 + 30.33 + (−10.5)

= 30.33

1. b. Evaluate the linear density in atoms per mm in the following directions in BCC iron, which has lattice constant of 2.89 A:

(i) [1 0 0]

(ii) [1 1 0]

(iii) [1 1 1]

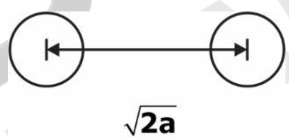
Sol. 1b. BCC structure, $4R = \sqrt{3}a$

a = 2.89 Å

(i) Linear density in [100]

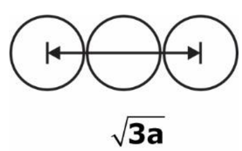
$$LD_{[100]} = \frac{1}{a} = \frac{1}{2.89} \text{ atoms/Å}$$

(ii) Linear density in [110]



$$LD_{[110]} = \frac{1}{\sqrt{2}a} = \frac{1}{\sqrt{2} \times 2.89} \text{ atoms/Å}$$

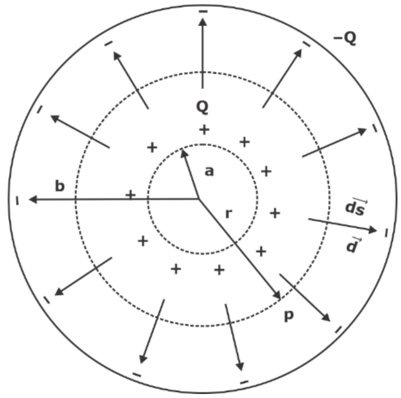
(iii) Linear density



$$LD_{[111]} = \frac{2}{\sqrt{3}a} = \frac{1}{\sqrt{3} \times 2.89} \text{ atoms/Å}$$

1. c. Derive an expression for capacitance (C) of concentric spheres having radii a and b(a < b) respectively with single dielectric.

Sol. 1c. Spherical Capacitors



$$V_{21} = -\int_2^1 \vec{E} \cdot d\vec{l}$$

$$\oint \vec{D} \cdot d\vec{s} = Q$$

$$\oint D \cdot ds \cdot \cos 0° = Q$$

$$\varepsilon E \oint ds = \phi$$

$$\varepsilon E\left(4\pi r^2\right) = Q$$

$$E = \frac{Q}{4\pi\varepsilon r^2} \cdot a_r$$

$$V_{21} = -\int_b^a \frac{Q}{4\pi\varepsilon r^2} a_r \cdot dr \cdot a_r$$

$$V_{21} = -\int_b^a \frac{Q}{4\pi E r^2} dr$$

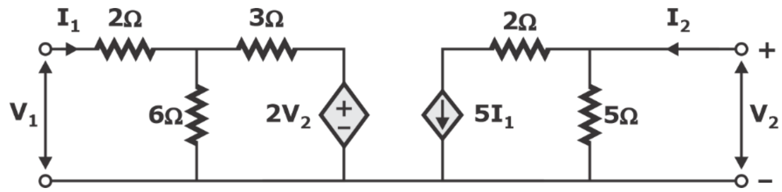
$$V_{21} = \int_a^b \frac{Q}{4\pi\varepsilon r^2} dr = \left[\frac{-Q}{4\pi\varepsilon r}\right]_a^b = \frac{-Q}{4\pi\varepsilon}\left[\frac{1}{b} - \frac{1}{a}\right]$$

$$C = \frac{Q}{V_{21}}$$

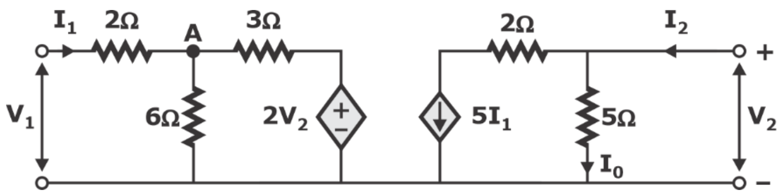
$$C = \frac{Q}{\frac{-Q}{4\pi\varepsilon}\left[\frac{1}{b} - \frac{1}{a}\right]} = \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$C = \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

1. **d.** Find the hybrid parameters of the following circuit :



Sol. 1d.



hybrid parameter equation are

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}v_2$$

From the above circuit $I_o = I_2 - 5I_1$

So, $V_2 = 5\left[I_2 - 5I_1\right] \Rightarrow I_2 = 5I_1 + \frac{V_2}{5}$ (1)

Let voltage at point A is V_A and $V_A = V_1 - 2I_1$

According to KCL at point A

$$I_1 = \frac{V_1 - 2I_1}{6} + \frac{V_1 - 2I_1 - 2V_2}{3}$$

$$\Rightarrow \quad 6I_1 = V_1 - 2I_1 + 2V_1 - 4I_1 - 4V_2$$

$$6I_1 = V_1 - 2I_1 + 2V_1 - 4I_1 - 4(5)\left(I_2 - 5I_1\right)$$

$$6I_1 = V_1 - 2I_1 + 2I_1 - 4I_1 - 20I_2 + 100I_1$$

$$\Rightarrow \quad 3v_1 = -88I_1 + 2 = I_2$$

$$V_1 = -\frac{88I_1}{3} + \frac{20}{3}\left(5I_1 + \frac{V_2}{5}\right)$$

$$V_1 = 4I_1 + \frac{4}{3}V_2 \qquad (2)$$

From equation (1) and (2)

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 4\Omega & 4/3 \\ 5 & 1/5 \end{bmatrix}$$

1. e. Construct full-

(i) Conjunctive normal form for the statement $P \rightarrow Q$;

(ii) disjunctive normal form for the statement $(P \rightarrow (Q \vee R)) \wedge (P \vee Q)$.

Sol. 1e. (i) To convert the statement "P → Q" into conjunctive normal form (CNF), we need to break it down into a conjunction (AND) of one or more clauses, where each clause is a disjunction (OR) of literals.

The implication "P → Q" can be rewritten as "¬P ∨ Q" using the logical equivalence of the implication operator.

Here's the step-by-step breakdown and explanation:

Start with the original implication: P → Q

Apply the logical equivalence of the implication: ¬P ∨ Q

Explanation: The implication "P → Q" is equivalent to saying "if P is false (¬P), then Q can be either true or false (Q)". This is represented by the disjunction "¬P ∨ Q".

Therefore, the conjunctive normal form (CNF) of "P → Q" is (¬P ∨ Q).

(ii) To find the disjunctive normal form (DNF) of the expression "(P → (Q ∨ R)) ∧ (P ∨ Q)", we need to distribute the conjunction (∧) over the implication (→) and simplify the resulting expression.

Here's the step-by-step breakdown:

Distribute the conjunction (∧) over the implication (→): ((¬P ∨ (Q ∨ R)) ∧ (P ∨ Q))

Apply the distributive property to simplify: ((¬P ∨ Q ∨ R) ∧ (P ∨ Q))

Simplify the expression by distributing (∧) over (∨): ((¬P ∧ (P ∨ Q)) ∨ (Q ∧ (P ∨ Q)) ∨ (R ∧ (P ∨ Q)))

Further simplify each term: ((¬P ∧ P) ∨ (¬P ∧ Q) ∨ (Q ∧ P) ∨ (Q ∧ Q) ∨ (R ∧ P) ∨ (R ∧ Q))

Remove redundant terms and simplify: (False ∨ (¬P ∧ Q) ∨ (Q ∧ P) ∨ Q ∨ (R ∧ P) ∨ (R ∧ Q))

Simplify further: (¬P ∧ Q) ∨ (Q ∧ P) ∨ Q ∨ (R ∧ P) ∨ (R ∧ Q)

Therefore, the disjunctive normal form (DNF) of "(P → (Q ∨ R)) ∧ (P ∨ Q)" is "(¬P ∧ Q) ∨ (Q ∧ P) ∨ Q ∨ (R ∧ P) ∨ (R ∧ Q)".

2. a. (i) Obtain the half-range cosine series for the function f(x) = sin x in $0 \le x \le \pi$ and hence, find the value of $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$

(ii) Evaluate the integral $\iint_R (x-y)^4 \cos^2(x+y)\,dxdy$, where R is the rhombus with successive vertices at $(\pi, 0), (2\pi, \pi), (\pi, 2\pi), (0, \pi)$.

Sol. 2a. (i) Half-range cosine series for f(x) is [0, L] is represented as:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

$$\text{where, } a_0 = \frac{1}{L}\int_0^L f(x)dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

Here, f(x) = sin x is [0, π]

L = π

$$\Rightarrow a_0 = \frac{1}{\pi}\int_0^\pi \sin x\,dx = \frac{1}{\pi}[\cos x]_0^\pi = \frac{-1}{\pi}[-1-1]$$

$$\Rightarrow a_0 = \frac{2}{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^\pi \sin x\left(\cos\left(\frac{n\pi x}{\pi}\right)\right)dx$$

$$= \frac{1}{\pi}\int_0^\pi 2\sin x\cos(nx)\,dx$$

$$= \frac{1}{\pi}\left[\frac{-\cos(1-n)x}{(1+n)} - \frac{\cos(1-n)x}{1-n}\right]_0^\pi$$

$$= -\frac{1}{\pi}\left[\left(\frac{\cos(\pi r\pi n)}{1+n} + \frac{\cos(\pi-\pi_n)}{1-n}\right) - \left(\frac{1}{1+n} + \frac{1}{1-n}\right)\right]$$

$$= -\frac{1}{\pi}\left[\left(\frac{-\cos\pi n}{1+n} - \frac{\cos\pi n}{1-n}\right) - \left(\frac{2}{1+n^2}\right)\right]$$

If n = odd, cos (nπ) = – 1

$$\text{So, } a_n = \frac{1}{\pi}\left[\frac{-2}{1-n^2} + \frac{2}{1-n^2}\right] = 0$$

If n = even, cos (nπ) = + 1

$$\text{So, } a_n = \frac{1}{\pi}\left[\frac{2}{1-n^2} + \frac{2}{1-n^2}\right] = \frac{1}{\pi}\left(\frac{4}{1-n^2}\right)$$

Now, we have $f(x) = a_0 + \sum_{n-1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$

$$f(x) = \frac{2}{\pi} + \sum_{n=\text{even}} \frac{4}{\pi(1-n^2)}\cos(nx)$$

$$\Rightarrow f(x) = \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{\cos 2x}{5} + \frac{\cos 4x}{15} + \frac{\cos 6x}{35} + \ldots\right)$$

Put x = 0 in above,

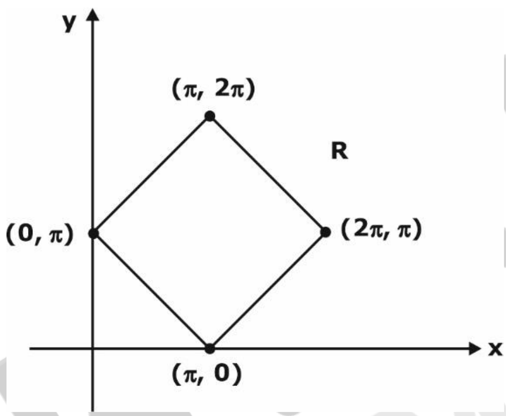
$$f(x) = \sin x = 0 = \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \ldots\right)$$

$$\Rightarrow \frac{1}{(2-1)(2+1)} + \frac{1}{(4-1)(4+1)} + \frac{1}{(6-1)(6+1)} \ldots = \frac{1}{2}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}$$

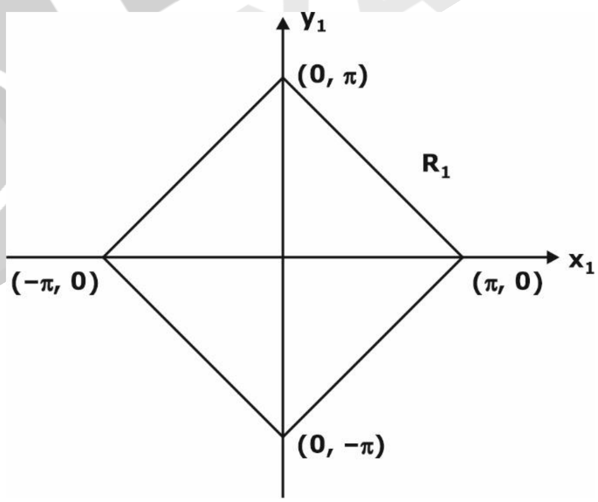
Or, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(4n^2-1)} = \frac{1}{2}$

(ii) $\displaystyle\iint_R (x-y)^4 \cos^2(x+y)dxdy$



Translate the rhombus to origin

Let $x_1 = x - \pi$ and $y_1 - y - \pi$



$$\iint_R (x-y)^4 \cos^2(x+y)dxdy$$

$$= \iint_{R_1} (x_1 - y_1)^4 \cos^2(x_1 + y_1 + 2\pi)dx_1dy_1$$

$$= \iint_{R_1} (x_1 - y_1)^4 \cos^2(x_1 + y_1)dx_1dy_1$$

Now, change coordinate system:

$x_1 - y_1 = 4,\ x_1 + y_1 = V$

So, $I = \iint 4^4 \cos^2(V) J(u, v) du dV$

Where, J will be $\frac{1}{2}$

So, $I = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} u^4 \cos^2 V\, du dV$

$= \frac{1}{2}\int_{-\pi}^{\pi} u^4 du \int_{-\pi}^{\pi} \cos^2 V dV$

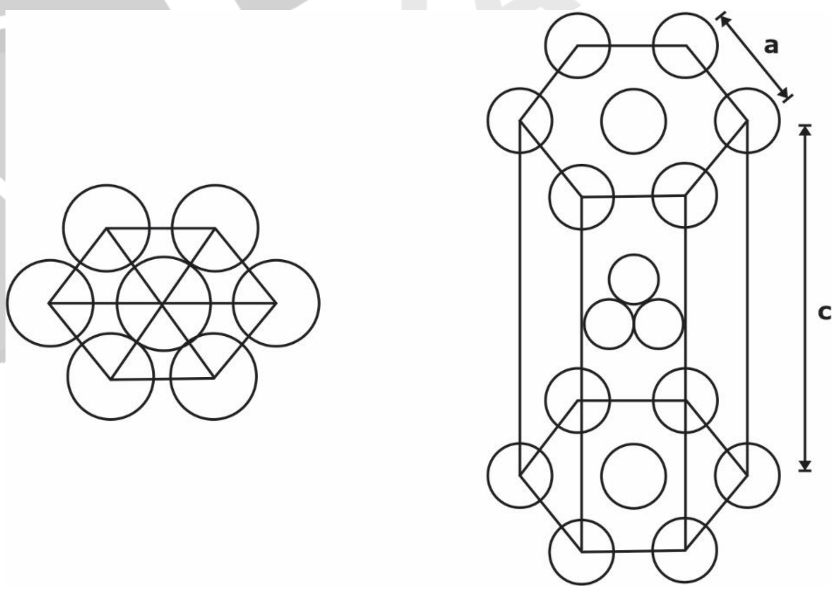
$= \frac{1}{2} \times \left[2 \times \left(\frac{u^5}{5}\right)_0^{\pi}\right] \times \left[2 \times 2 \times \frac{1}{2} \times \frac{\pi}{2}\right]$

So, $I = \frac{\pi^6}{5}$

2. b. (i) Determine the volume of an HCP unit cell in terms of its a and c lattice parameters.

(ii) Copper has an atomic radius of 0.13 nm, an FCC crystal structure and an atomic weight of 63.5 g/mol. Evaluate its theoretical density and compare the answer with its measured density. (Take Avogadro number, $N_A = 6.022 \times 10^{23}$ atoms/mol)

Sol. 2b. (i) HCP structure



Volume of HCP = Abase × Height

$= 6 \times \frac{\sqrt{3}}{4} a^2 \times c$

Note: Area of hexagon base plane is divided into 6 equilateral triangles.

(ii) Copper, Crystal structure

FCC, n = 4, $4R = \sqrt{2}a$

R = 0.13 mm, $V_{uc} = a^3 = \left(2\sqrt{2}R\right)^3 = \left(2\sqrt{2}\times 0.13\right)^3 \times 10^{-21} cm^3$

AW = 63.5 g/mol

$N_A = 6.022 \times 10^{23}$ atoms/mol

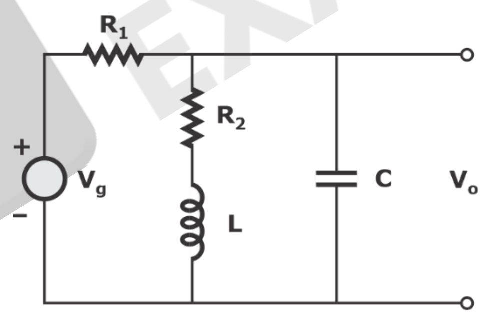
$$\text{Theoretical density} = \rho = \frac{n\times AW}{AN\times V_{uc}} = \frac{4\times 63.5}{6.022\times 10^{23}\times(2\sqrt{2}\times 0.13)^3\times 10^{-21}}$$

$$= \frac{4\times 63.5}{6.022\times 0.0497\times 10^2} = 8.489/cc$$

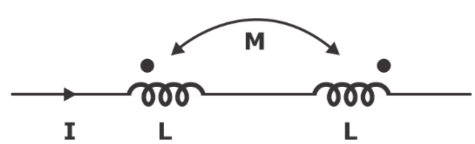
Measured density of copper (Near room temperature) is = 8.969/cc

2. C. (i) Two inductive coils having same self-inductance when connected in series carrying a current of I amperes store W joules of magnetic energy in their fields. When the connections of one of the coils are interchanged and the current is reduced to $\left(\frac{I}{3}\right)$ amperes, the stored energy remains the same. Calculate the ratio of mutual to self-inductance.

(ii) Determine the transfer function $\frac{V_o(s)}{V_g(s)}$ for the circuit shown below for R_1 = 500 Ω, R_2 = 50 Ω, L = 10 mH and C = 2 μ F:



Sol. 2c. (i)



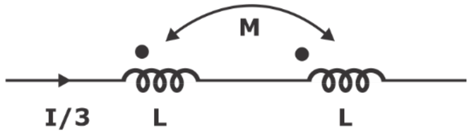
Let mutual inductance is M

Energy stored in coupled coils $\frac{1}{2}LI^2 + \frac{1}{2}LI^2 \pm MI^2$

Let two coils are connected in opposite passions (as shown above)

So, $\varepsilon_1 = \frac{1}{2}LI^2 + \frac{1}{2}LI^2 - MI^2 = LI^2 - MI^2$

If one coil is interchanged then connection will be



$$\varepsilon_2 = \frac{1}{2}L\left(\frac{I}{3}\right)^2 + \frac{1}{2}L\left(\frac{I}{3}\right)^2 + M\left(\frac{I}{3}\right)^2$$

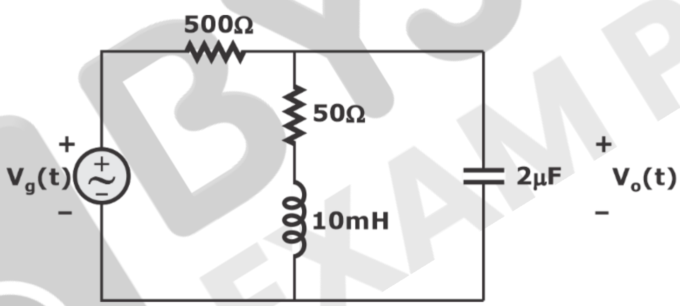
$$= \frac{LI^2}{9} + \frac{MI^2}{9}$$

According to statement $\varepsilon_1 = \varepsilon_2$

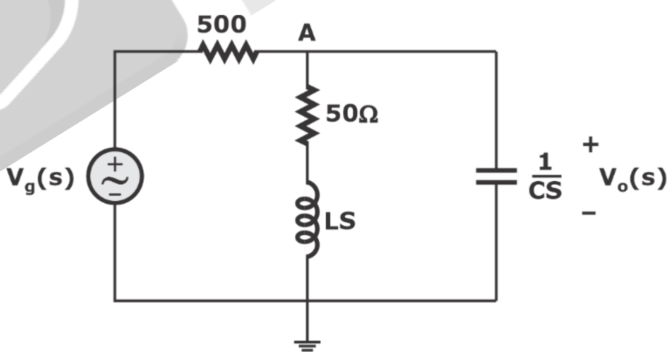
$$LI^2 - MI^2 = \frac{LI^2}{9} + \frac{MI^2}{9}$$

$$\Rightarrow \frac{8LI^2}{9} = \frac{10MI^2}{9} \Rightarrow \frac{M}{L} = \frac{8}{10} = 0.8$$

(ii)



if above circuit is drawn in Laplace domain then



Voltage at point A with respect to go rt refence point is $V_0(s)$

So according to nodal

$$V_0(s)\left[\frac{1}{500} + \frac{1}{50 + Ls} + cs\right] = V_g\frac{(s)}{500}$$

$$\Rightarrow \frac{V_0(s)}{V_g(s)} = \frac{1}{500}\left[\frac{500(50+Ls)}{50+Ls+500+Cs(500)(50+Ls)}\right]$$

$$= \frac{50+Ls}{500LCs^2+(L+500C(50))s+550}$$

After putting the value of L and C

$$\frac{V_0(s)}{V_g(s)} = \frac{50+0.01s}{10^{-5}s^2+0.06s+550}$$

3. a. (i) Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = e^{x+2y} + \sin(4x+3y) + y\cos x$$

(ii) Compute the following integral by residue theorem :

$$\int_0^{2\pi} \frac{\sin\theta}{3-2\sin\theta}\, d\theta$$

Sol. 3a. (i) Given

$$\frac{d^2z}{dx^2} + \frac{d^2z}{dxdy} - 6\frac{d^2z}{dy^2} = e^{x+2y} + \sin(4x+3y) + y\cos x$$

Let $\frac{d}{dx} = D$

$\frac{d}{dy} = D'$

$\Rightarrow D^2z + DD^1z - 6D^{1^2}z = g(x,y)$

Or $(D^2 + DD^1z - 6D^{1^2})z = g(x,y)$ ...(1)

Or $f(D,D')z = g(x,y)$ ...(2)

<u>Complementary function (Z_C)</u>

First find auxiliary equation (AE)

By putting D = M & D′ = 1

So, $AE \Rightarrow m^2 + m - 6 = 0$

$\Rightarrow (m+3)(m-2) = 0$

Or, $m = -3, +2 \rightarrow$ real & different

CF is of the form $Z_C = f(y + mx)$

$\Rightarrow z_c = f_1(y-3r) + f_2(y+2x)$

<u>Particular integral (y_P)</u>

From (2), $f(D,D')z = g(x,y)$

$$z_p = \frac{1}{f(D,D')} g(x,y)$$

$$g(x,y) = e^{x+2y} + \sin(4x+5y) + y\cos x$$

$$= g_1 + g_2 + g_3$$

For $g_1 = e^{x+2y}$

PI is $\dfrac{e^{x+2y}}{D^2 + DD' - 6D'^2}$

Put D = 1, D' = 2

$$PI = \frac{e^{x+2y}}{1^2 + 1\times 2 - 6(2)^2} = \frac{e^{x+2y}}{-21}$$

For $g_2 = \sin(4x+3y)$

PI is $\dfrac{\sin(4x+3y)}{D^2 + DD' - 6D'^2}$

Put $D^2 = -4^2$

$D' = -3^2$

$DD' = -(4\times 3)$

$$PI = \frac{\sin(4x+3y)}{-16-12-6(-9)} = \frac{\sin(4x+3y)}{26}$$

For g_3 = y cosx

PI is $\dfrac{y\cos x}{D^2 + DD' - 6D'^2} = \dfrac{y\cos x}{(D-2D')(D+3D')}$

$$= \frac{1}{D-2D'}\left(\frac{1}{D+3D'} y\cos x\right)$$

Now, $\dfrac{1}{D+mD'}(y\cos x) = \left[\int (y\cos x dx)\right]_{y=c+mx}$

So, $\dfrac{1}{D-2D'}\left(\dfrac{1}{D+3D'}(y\cos x)\right)$

$$= \frac{1}{D-2D'}\left[\int (c+3x)\cos x dx\right]$$

Solving by Integration by Parts,

$$\Rightarrow \quad \frac{1}{D-2D'}\left[(c+3x)\sin x + 3\cos x\right]$$

$$= \frac{1}{D-2D'}[y\sin x + 3\cos x]$$

$$= \int [(c-2x)\sin x + 3\cos x]dx$$

$= (c-2x)(-\cos x)-(-2)(-\sin x)+3\sin x$

$PI \Rightarrow -y\cos x+\sin x$

So, Total PI is

$$z_p = \frac{e^{x+2y}}{-21}+\frac{\sin(4x+2y)}{26}+(-y\cos x+\sin x)$$

So, complete solution is:

$z = z_c + z_p$

$$Z = f_1(y-3x)+f_2(y+2x)-\frac{e^{x+2y}}{21}+\frac{\sin(4x+3y)}{26}+\sin x - y\cos x$$

(ii) $I = \int_0^{2\pi}\frac{\sin\theta}{3-2\sin\theta}d\theta$

Put $\sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$

$$\Rightarrow I = \int \frac{\frac{e^{i\theta}-e^{-i\theta}}{2i}}{3-2\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)}d\theta$$

$$I = \int \frac{e^{i\theta}-e^{-i\theta}}{6i-2e^{i\theta}+2e^{-i\theta}}d\theta \qquad (1)$$

Now, put $z = e^{i\theta} = re^{i\theta}$

So, $r = |z| = 1$ (2)

Also, $dz = ie^{i\theta}d\theta = iz\,d\theta$

So, $d\theta = \dfrac{dz}{iz}$

$$\text{So, } I = \int \frac{z-\frac{1}{z}}{6i-2z+\frac{2}{z}}\left(\frac{dz}{iz}\right)$$

$$= \frac{1}{i}\int \frac{z^2-1}{\left(6iz-2z^2+2\right)\times z}dz$$

$$\text{Or, } I = i\int_c \frac{z^2-1}{z\left(2z^2-6iz-2\right)}dz$$

Where C is the curve |z| = 1 (as per equation 2)

Poles : $z\left(2z^2-6iz-2\right)=0$

$\Rightarrow z = 0, 0.38i, 2.62i$

Or, $z \approx 0, 0.4i, 2.6i$

Out of which z = 0, 0.4i are inside unit circle

Now, $f(z) = \dfrac{z^2 - 1}{z\left(2z^2 - 6iz - 2\right)} = \dfrac{z^2 - 1}{2z(z - 0.4i)(z - 2.6i)}$

$\text{Res}(z = 0) = \lim_{z \to 0} zf(z) = \dfrac{-1}{2 \times 0.4 \times 2.62^2} \approx \dfrac{1}{2} = 0.5$

$\text{Res}(z = 0.4i) = \lim_{z \to 0.4i} \left(z - 0.4i\right) f(z)$

$= \dfrac{(0.4i)^2 - 1}{2(0.4i)(0.4i - 2.6i)}$

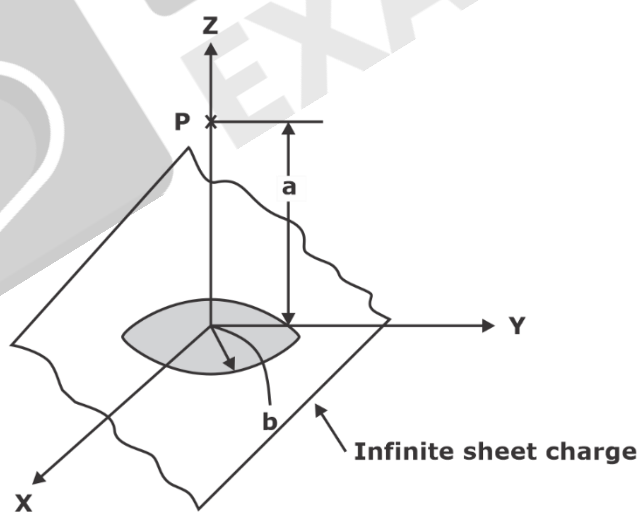
≈ -0.65

So, $I = i[2\pi i(0.5 - 0.65)] \approx 0.942$

3. b. What is the electric field intensity $\vec{E}_1 V/m$ due to an infinite sheet of uniform charge density σ C/m² ?

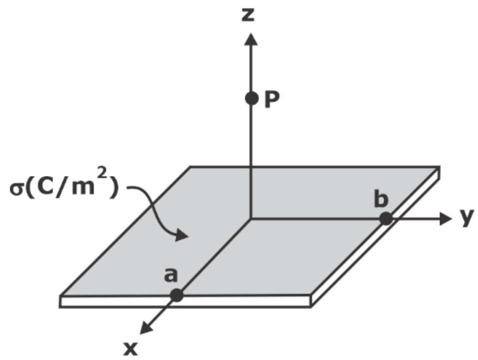
(i) Derive the electric field intensity $\vec{E}_2 V/m$ at P contributed by the circular portion of this infinite sheet charge of radius b metre on the perpendicular axis at a metre from the sheet as shown in the figure below.

(ii) Find b, if a = 0.5 m and $\vec{E}_2 = \dfrac{\vec{E}_1}{2}$.



Sol. 3b. (i) The electric field intensity due to infinite sheet is given by,

$\vec{E}_1 = \dfrac{\sigma}{2\epsilon_0} \hat{a}_z$



Consider an element of area ds of the disk. The contribution due to ds =ρdϕ dρ is

$$dE = \frac{\rho_s ds}{4\pi\varepsilon_0 r^2} = \frac{\rho_s ds}{4\pi\varepsilon_0\left(\rho^2 + h^2\right)}$$

The sum of the contribution along ρ given zero.

$$E_z = \frac{\rho_s}{4\pi\varepsilon_0}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\frac{z\rho d\rho d\phi}{\left(\rho^2+h^2\right)^{3/2}} = \frac{h\rho_s}{2\varepsilon_0}\int_{\rho-0}^{a}\frac{\rho d\rho}{\left(\rho^2+h^2\right)^{3/2}}$$

$$= \frac{h\rho_s}{4\varepsilon_0}\int_0^a\left(\rho^2+h^2\right)^{3/2} d\left(\rho^2\right) = \frac{h\rho_s}{2\varepsilon_0}\left(-2\left(\rho^2+h^2\right)^{-1/2}\right|_0^a$$

$$= \frac{\rho_s}{2\varepsilon_0}\left[1 - \frac{h}{\left(h^2+a^2\right)^{1/2}}\right]$$

(ii) Given that, $\vec{E}_2 = \frac{\vec{E}_1}{2}$

$$\frac{\sigma}{2\varepsilon_0}\left[1 - \frac{a}{\left(a^2+b^2\right)^{1/2}}\right]\hat{a}_2 = \frac{1}{2}\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{a}_2$$

$$1 - \frac{a}{\left(a^2+b^2\right)^{1/2}} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{a}{\left(a^2+b^2\right)^{1/2}}$$

Squaring both sides,

$$\frac{1}{4} = \frac{a^2}{a^2+b^2}$$

$a^2 + b^2 = 4a^2$

$b_2 = 3a_2$

$b = \sqrt{3}a$

$\because \quad a = \frac{1}{2} \quad b = \frac{\sqrt{3}}{2} = 0.866$

3. c. (i) In low-voltage Schering bridge designed for measurement of permittivity, the branch ab consists of two electrodes between which the specimen under test may be inserted; arm bc is a non-reactive resistor R_3 in parallel with a standard capacitor C_3 and cd is a non-reactive resistor R_4 in parallel with a standard capacitor C_4; arm da is a standard air capacitor of capacitance C_2. Without the specimen between the electrodes, balance is obtained with the following values :

C_3 = 150 pF

C_4 = 200 pF

C_2 = 250 pF

$R_3 = 5 \times 10^3\ \Omega$

$R_4 = 10 \times 10^3\ \Omega$

With specimen inserted, these values become

C_3 = 200 pF

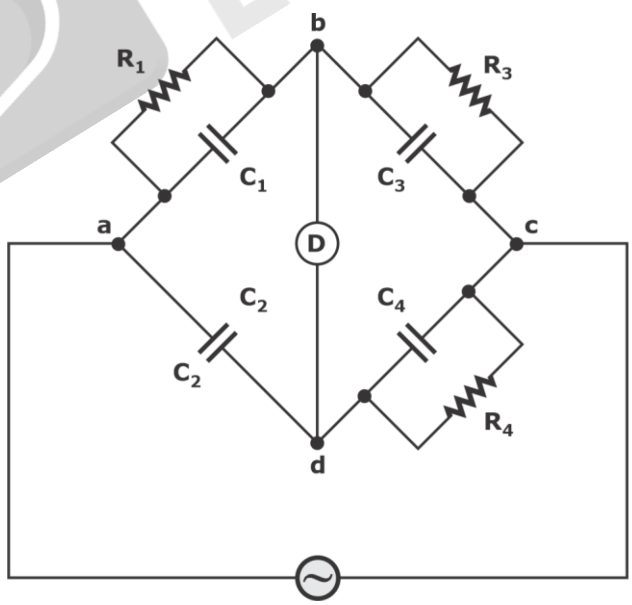
C_4 = 1200 pF

C_2 = 1000 pF

and R_3 and R_4 remain as previous. In each case, the frequency is $\omega = 10 \times 10^3$ rad/s. Determine the relative permittivity of the specimen.

(ii) Draw the connections and phasor diagram of Anderson's bridge along with its advantages and disadvantages.

Sol. 3c. (i) The voltage circuit is shown in Figure.



For balance $Y_1 Y_4 = Y_2 Y_3$

$$\text{or}\left(\frac{1}{R_1}+j\omega C_1\right)\left(\frac{1}{R_4}+j\omega C_4\right)=\left(j\omega C_2\right)\left(\frac{1}{R_3}+j\omega C_3\right)$$

$$\text{or}\left(\frac{1}{R_1R_4}-\omega^2C_1C_4\right)+j\omega\left(\frac{C_4}{R_1}+\frac{C_1}{R_4}\right)=j\omega\frac{C_2}{R_3}-\omega^2C_2C_3$$

Equating the real and imaginary parts, we have

$$\frac{1}{R_1R_4}-\omega^2C_1C_4=-\omega^2C_2C_3 \qquad \text{...(i)}$$

$$\text{And } \frac{C_4}{R_1}+\frac{C_1}{R_4}=\frac{C_2}{R_3} \qquad \text{...(ii)}$$

From (i) and (ii), we have

$$C_1=\frac{\frac{C_2R_4}{R_3}+\omega^2C_2C_3C_4R_4^2}{1+\omega^2C_4^2R_4^2}$$

Now $\omega^2C_2C_3C_4R_4^2 \ll \frac{C_2R_4}{R_3}$

And $\omega^2C_4^2R_4^2 \ll 1$

Hence we can write

$$C_1=C_2\frac{R_4}{R_3}$$

Assume without specimen, $C_1 = C_0$

$$\therefore\ C_0=C_2-\frac{R_4}{R_3}$$

$$=250\times10^{-12}\times\frac{10\times10^3}{5\times10^3}$$

C_0 = 500 pF

Now, with specimen, $C_1 = C_S$ (assume)

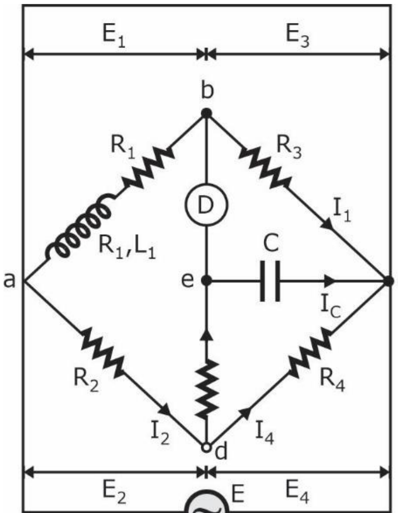
$$\therefore\ C_s=C_2\frac{R_4}{R_3}$$

$$=1000\times10^{-12}\times\frac{10\times10^3}{5\times10^3}$$

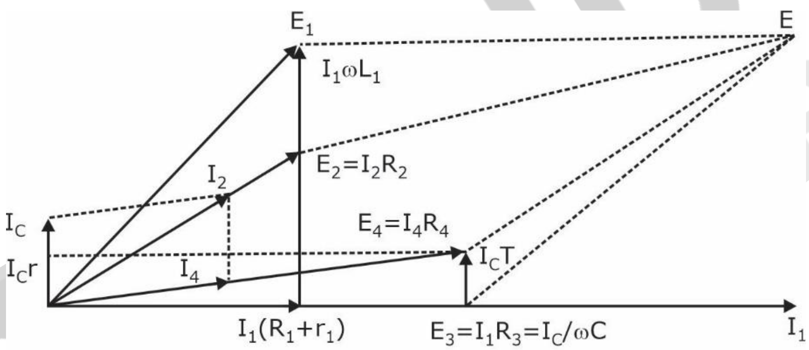
C_S =2000 pF

Hence, $\varepsilon_r=\frac{C_S}{C_0}=\frac{2000\,\text{pF}}{500\,\text{pF}}=4$

(ii)



(a)



(b)

Let

L_1 = Self–inductance to be measured,

R_1 = Resistance of self–inductor,

r_1 = Resistance connected in series with self–inductor, r, r_2, R_3

R_4 = Known non–inductive resistance,

And C = Fixed standard capacitor.

Advantages :

1. In case adjustments are carried out by manipulating control over r_1 and r, they become independent of each other. This is a marked superiority over sliding balance conditions met with low Q coils when measuring with Maxwell's bridge. A study of convergence

conditions would reveal that it is much easier to obtain balance in the case of Anderson's bridge than in Maxwell's bridge for low Q-coils.

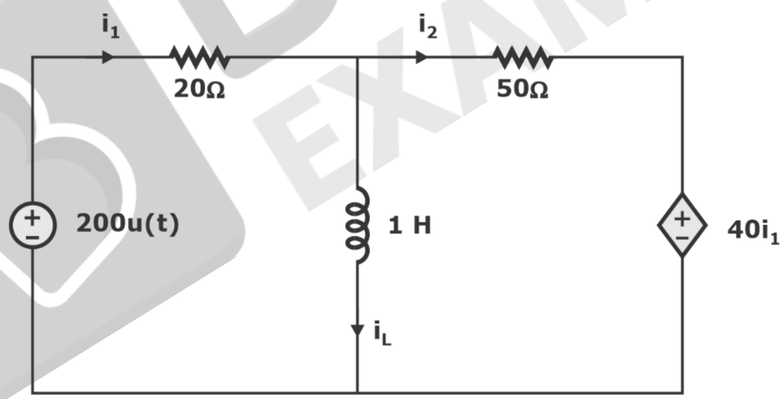
2. A fixed capacitor can be used instead of a variable capacitor as in the case of Maxwell's bridge.
3. This bridge may be used for accurate determination of capacitance in terms of inductance.

Disadvantages :

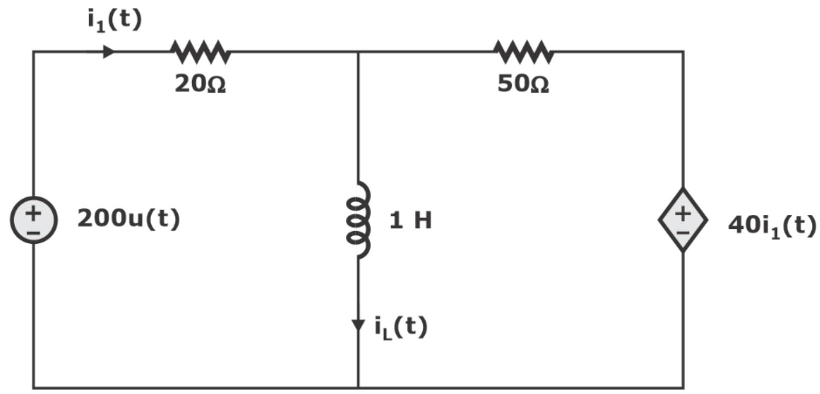
1. The Anderson's bridge is more complex than its prototype Maxwell's bridge. The Anderson's bridge has more parts and is more complicated to set up and manipulate. The balance equations are not simple and in fact are much more tedious.
2. An additional junction point increases the difficulty of shielding the bridge. Considering the above complication of the Anderson's bridge, in all the cases where a variable capacitor is permissible the more simple Maxwell's bridge is used instead of Anderson's bridge.

4. a. (i) For the network shown in the following figure, compute $i_L(t)$ and $i_1(t)$, if the initial current through the inductor is 0 ampere :

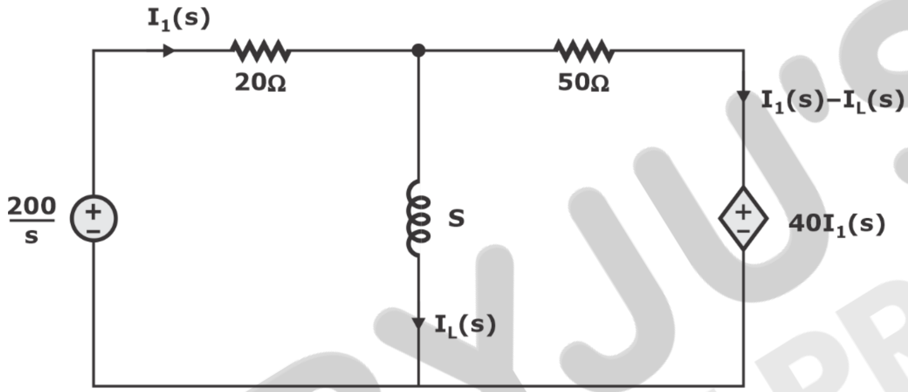
(ii) Determine the current at t > 0, if AC voltage V is applied, when switch S is moved from position 2 to position 1 at t = 0, for the network shown in the following figure. Assume steady-state current of 1 ampere in the network, when the switch is at position 1:



Sol. 4a. (i)



The above circuit can be drawn in Laplace domain as shown below.



The lower point is assume as reference

According to KVL

$$\frac{200}{s} - 20I_1(s) - sI_2(s) = 0 \qquad (1)$$

$$\frac{200}{s} - 20I_1(s) - 50\left[I_1(s) - I_L(s)\right] - 40I_1(s) = 0$$

$$\frac{200}{s} - 110I_1(s) + 50I_2(s) = 0 \qquad (2)$$

From equation (1) and (2)

$20I_1(s)+SI_L(s) = 110I_1(s) - 50I_L(s)$

$90I_1(s) = (s + 50)I_L(s)$

$$I_L(s) = \frac{90}{s+50}I_1(s)$$

Put $I_L(s)$ value in equation (1)

$$\frac{200}{s} = 20I_1(s) + s\left(\frac{90}{s+50}\right)I_1(s)$$

$$\frac{200}{s} = \left(20 + \frac{90s}{s+50}\right)I_1(s)$$

$$\left(\frac{200}{s}\right)\left(\frac{s+50}{20s+1000+90s}\right)=I_1(s)$$

$$\frac{200(s+50)}{s(110s+1000)}=I_1(s)=\frac{1.82(s+50)}{s(s+9.1)}$$

Using $I_1(s)$ we can find $I_L(s)$ from equation (3)

$$I_L(s)=\frac{200(90)}{s(110s+1000)}=\frac{163.64}{s(s+9.1)}$$

Using partial function

$$I_1(s)=\frac{1.82(s+50)}{\rho(s+9.1)}=\frac{A}{s}+\frac{B}{s+9.1}$$

A = 10 B = −8.18

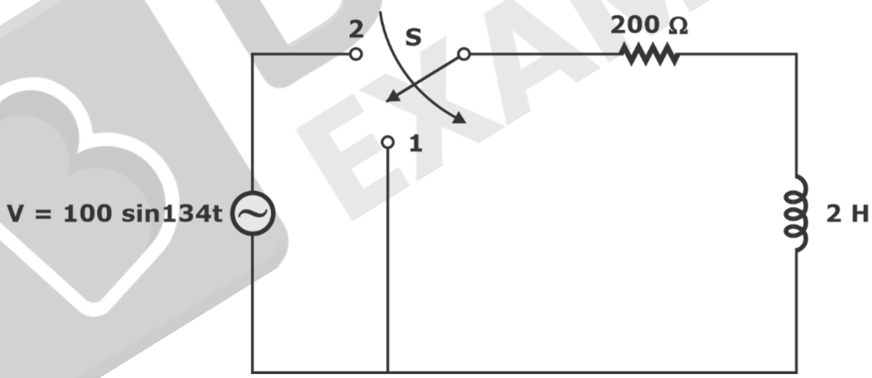
$$i_1(t)=10u(t)-8.18e^{-9.1t}u(t)$$

$$=\left(10-8.18e^{-9.1t}\right)u(t)$$

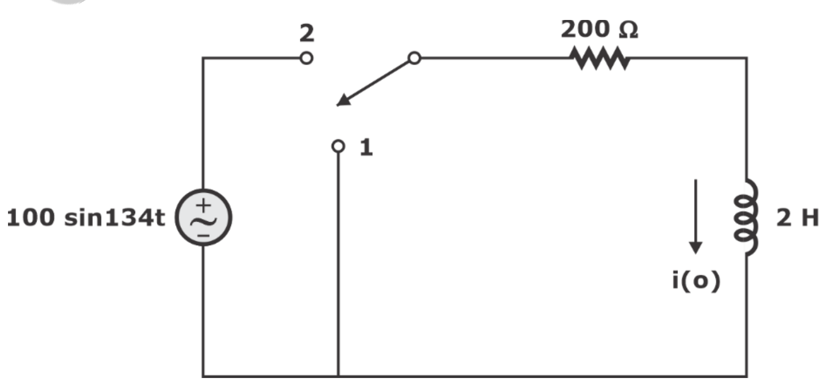
$$I_L(s)=\frac{163.64}{s(s+9.1)}=\frac{A}{s}+\frac{B}{s+9.1}$$

$$=\frac{17.982}{s}+\frac{17.982}{s+9.1}$$

$$i_L(t)=17.982\left[1-e^{-9.1t}\right]u(t)$$

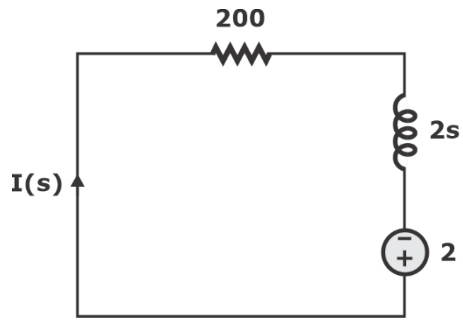


(ii) For t > 0 the circuit will be



$i(o) = 1$ Amp.

Using Laplace the circuit will be



$$I(s) = \frac{2}{2s + 200}$$

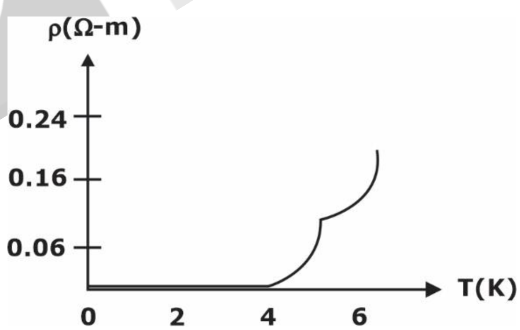
$$I(s) = \frac{1}{s + 100}$$

$i(t) = e^{-100t}u(t)$

4. b. (i) Give the variation of resistivity of purified mercury with temperature. Also, represent the resistivity of normal metal as a function of temperature (T) along with pure and impure superconductors.

(ii) Compute the drift mobility and mean scattering time of conduction electrons in copper at room temperature, given that the density of copper is 8.98 g/cm^3, the conductivity of copper is $5.95 \times 10^5\ \Omega^{-1}\ cm^{-1}$ and the atomic mass of copper is 63.5 g/mol. Take Avogadro number, $N_A = 6.02 \times 10^{23}$ and charge on electron (e) = 1.6×10^{-19} coulomb, mass of electron (m^e) = 9.1×10^{-31} kg.

Sol. 4b. (i)



The graph shows the variation of resistance of mercury in the temperature range.

OK to 4.12 K $\rightarrow$ Super conductor, $\rho = 0$

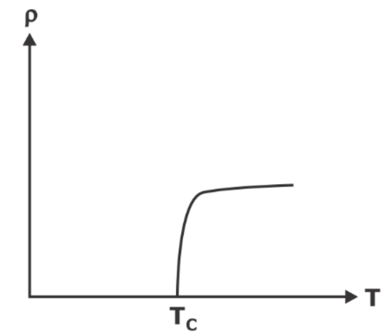
4.12K to Room Temperature $\rightarrow$ Normal conductor

$R \propto aT^5$

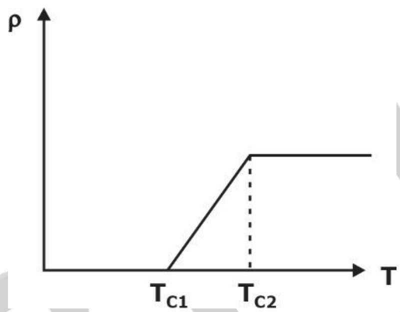
Form room temperature to high temperature, resistance follow linear curve.

$RT_2 = RT_1[(1 + \alpha)(T_2 - T_1)]$

Pure metal super conductor: Super conductivity of material suddenly loses at critical temperature.



Impure metal super conductor: The super conductivity losses gradually from 1st critical temperature to 2nd critical temperature.



(ii) Given data,

Density if copper = ρ = 8.98 g/cm^3 = 8.98 × 10^6 g/m^3

Conductivity of copper = s = 5.95 × 10^5 $(\Omega - \text{cm})^{-1}$

Atomic weight = AW = 63.5 g/mol

Avogadro's number = AN = 6.02 × 10^{13}

e^- = 1.6 × 10^{-19}

m_c = 9.1 × 10^{-31} kg

$\sigma = ne\mu$

$$\mu = \frac{\sigma}{ne}$$

$$\text{Theoretical density} = \rho = \frac{n \times AW}{AN \times V_{UC}}$$

$$N = \text{no. of atoms/m}^3 = \frac{\rho AN}{AW} = \frac{8.98 \times 6.02 \times 10^{23} \times 10^6}{63.5}$$

= 8.51 × 10^{28} atoms/m^3

Let us assume, one effective moving electron in one atom no. of electron/m^3 = n = 1 × 8.51 × 10^{28} e^-/m^3

Mobility of e–

$$= \frac{5.95 \times 10^5 \times \frac{1}{10^{-2}}}{8.51 \times 10^{28} \times 1.6 \times 10^{-19}} = 0.436 \times 10^{-2} m^2/V\text{-}sec$$

= 0.00436 m^2/V-sec

Mean scattering time (t)

$$\mu = \frac{et}{m}$$

$$t = \frac{\mu m}{e} = \frac{0.00436 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}$$

= 0.02479 × 10^{-12} seconds

4. c. (i) A moving-coil instrument has a resistance of 5 Ω between terminals and full-scale deflection is obtained with a current of 15 mA. This instrument is to be used with a manganin shunt to measure 100 A at full scale. Calculate the error caused by a 10 °C rise in temperature-

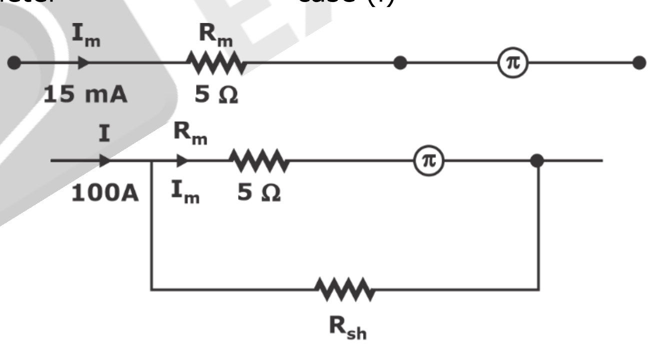
(1) when the internal resistance of 5 Ω is due to copper only;

(2) when a 3 Ω manganin swamping resistance is used in series with a copper coil of 2 Ω resistance.

The resistance temperature coefficients of copper and manganin are 0.004/ °C and 0.000015/ °C respectively.

(ii) Draw the block diagram of ramp-type digital voltmeter and explain its functioning.

Sol. 4c. (i) Meter coil meter case (i)



$$m = \frac{I}{I_m} = \frac{100}{15m}$$

$$m = \frac{20}{3}k$$

$$\therefore R_{Sh} = \frac{R_m}{m-1} = \frac{5}{\left(\frac{20}{3}k - 1\right)}$$

$R_{Sh} = 7.5 \times 10^{-4}\Omega$

The change in meter resistance for 10°C rise.

$R_2 = R_1(1 + \alpha_0 \Delta T)$

= 5(1 + 0.004 x 10)

$R_2 = 5.2\Omega = R_{mnew}$

Here α_0 is consider for copper.

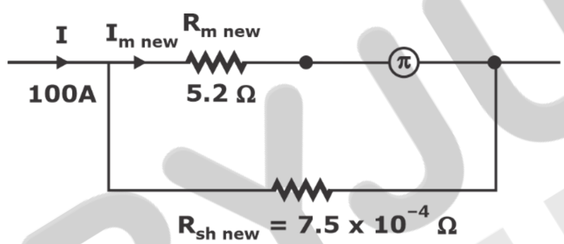
Similarly, change in R_{sh} for 10°C rise.

$R_{sh\ new}$ = 7.5 x 10^{-4}(1 + 0.000015 x 10)

$R_{sh\ new}$ = 7.5 x $10^{-4}\,\Omega$

Here α_0 is consider for managing.

The meter with new resistance will be



Now, $I_{m\ new}$ while measuring 100 A will be.

$$I_{mnew} = 100 \times \frac{7.5 \times 10^{-4}}{7.5 \times 10^{-4} + 5.2}$$

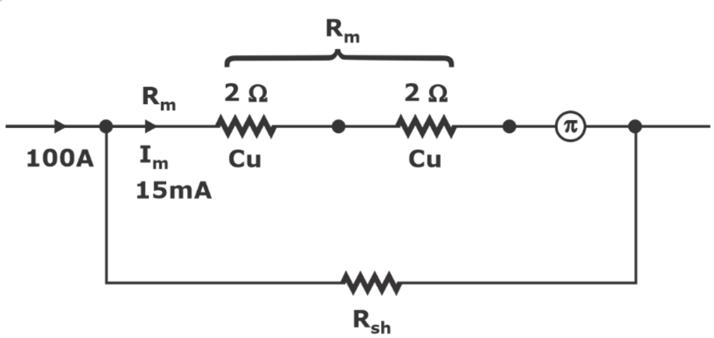
$I_{m\ new}$ = 14.42 mA

∴ Error due to 10° C rise in temperature is

$$\varepsilon = \frac{(14 - 42 - 15}{15}\Big) \times 100$$

$\varepsilon = -3.86\%$

Moving coil meter case (ii)



$$R_{sh} = \frac{R_m}{m-1} = \frac{5}{\left(\frac{100}{15\times10^{-3}} - 1\right)}$$

$$R_{sh} = 7.5\times10^{-4}\Omega$$

The change in R_m for 10°C rise temperature.

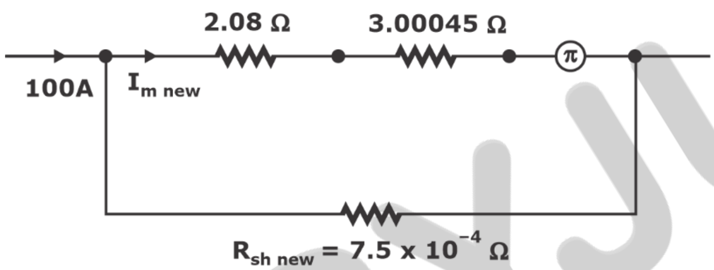
$$R_{mew} = [2(1+0.004\times10) + 3(1+0.000015\times10)]$$

$$R_{mnew} = 5.08045\Omega$$

Similarly, change in R_{sh} for 10°C rise temperature is same as case (i) i.e.,

$$R_{shnew} = 7.5\times10^{-4}\Omega$$

The meter with new resistance will be



$$I_{mnew} = 100\times\frac{7.5\times10^{-4}}{7.5\times10^{-4}+5.08045}$$

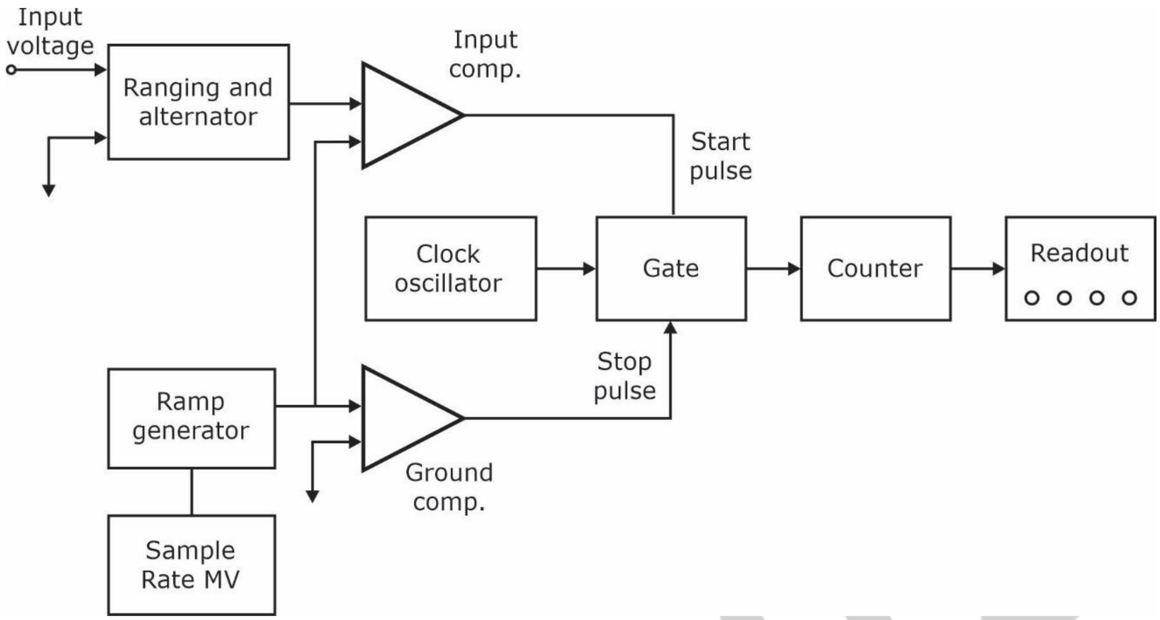
$I_{m\,new}$ = 14.76 mA

∴ Error due to 10°C rise.

$$\%\varepsilon = \frac{14.76-15}{15}\times100$$

$$\%\varepsilon = -1.6\%$$

(ii)

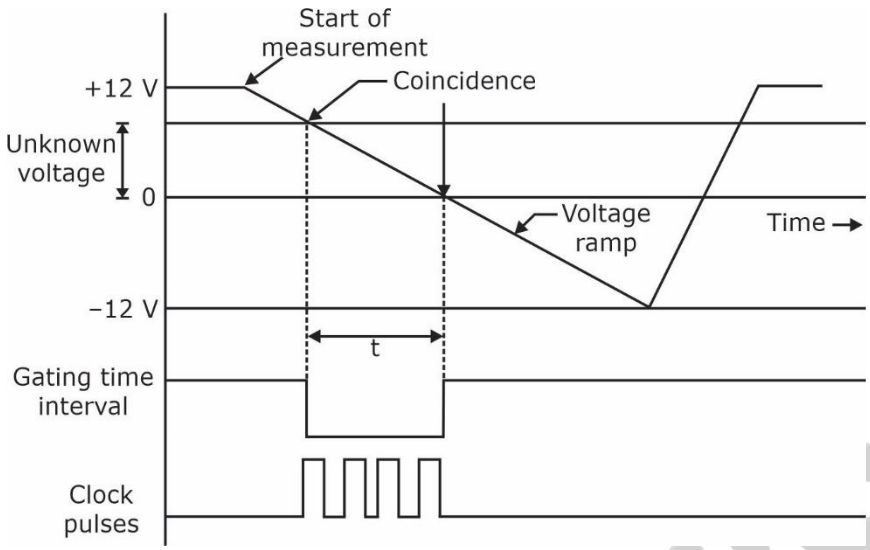


Ramp Type Digital Voltmeter

The operating principle of a ramp type digital voltmeter is to measure the time that a linear ramp voltage takes to change from level of input voltage to zero volt-ge (or vice versa). This time interval is measured with an electronic time interval counter and the count is displayed as a number of digits on electronic indicating tubes of the output readout of the voltmeter.

The conversion of a voltage value of a time interval is shown in the timing diagram of Fig. At the start of measurement a ramp voltage is initiated. A negative going ramp shown in Fig. but a positive going ramp may also be used. The ramp voltage value is continuously compared with the voltage being measured (unknown voltage). At the instant the value of ramp voltage is equal to that of unknown voltage a coincidence circuit, called an input comparator, generates a pulse which opens a gate (See Fig.). The ramp voltage continues to decrease till it reaches ground level (zero voltage). At this instant another comparator called ground comparator generates a pulse and closes the gate.

The time elapsed between opening and closing of the gate is t as indicated in Fig. During this time interval pulses from a clock pulse generator pass through the gate and are counted and displayed.

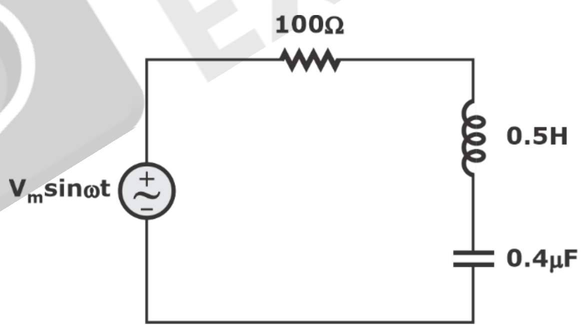


The decimal number as indicated by the readout is a measure of the value of input voltage.

The sample rate multivibrator determines the rate at which the measurement cycles are initiated. The sample rate circuit provides an initiating pulse for the ramp generator to start its next ramp voltage. At the same time it sends a pulse to the counters which sets all of them to 0. This momentarily removes the digital display of the readout.

5. a. For a series R-L-C circuit excited from an AC source, find the resonant frequency, bandwidth and quality factor, if R = 100 Ω, L = 0.5 H and C = 0.4 μF,

Sol. 5a.



At resonance frequency $z_{eq} = 100\Omega$

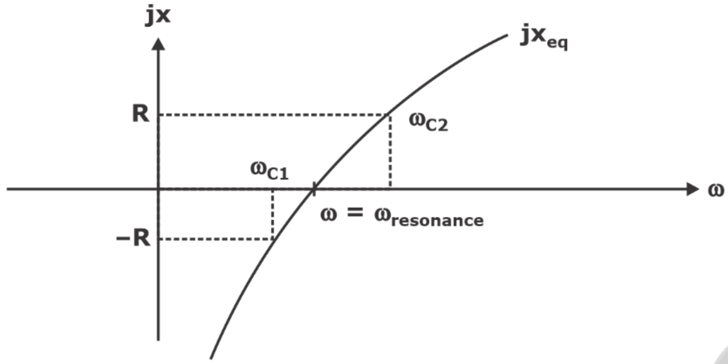
$$z_{eq} = 100 + j\left(\frac{\omega}{2} - \frac{10^6}{\omega(0.4)}\right)$$

$$\Rightarrow \frac{\omega}{2} - \frac{10^6}{\omega(0.4)} = 0$$

$$\omega^2 = \frac{10^6}{0.2} = 5 \times 10^6$$

$\omega = \sqrt{5\times10^6} = \sqrt{5}\,\text{k rad/sec}$

The reactance diagram is shown below



$x_{eq} = \omega L - \frac{1}{\omega C}$

ω_{C_2} = upper cut-off frequency

ω_{C_1} = lower cut-off frequency

$\Rightarrow \omega_{C_2}L - \frac{1}{\omega_{C_2}C} = R$

$\omega_{C_1}L - \frac{1}{\omega_{C_1}C} = -R$

$\Rightarrow \omega_{C_2} - \omega_{C_1} = \frac{R}{2}$

$\text{Bandwidth} = \frac{100}{0.5} = 200\,\text{rad/sec}$

$\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{1}{100}\sqrt{\frac{0.5}{0.4}\times10^6} = 11.18$

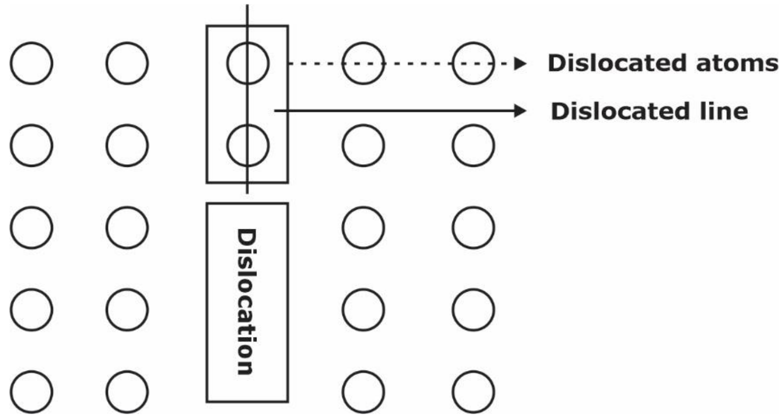
5. b. Define line defects in materials. Explain different types of line defects and compare them. Also, explain their cause of creation.

Sol. 5b. Line Defects (or) Dislocation: (1-D-defect)

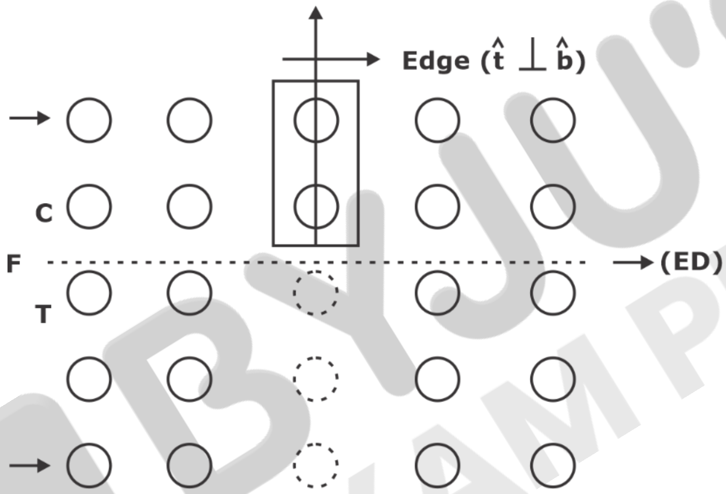
They are generated by accumulation of more than one point defect in a line (or) missing of line of atoms. It is the disturbed region between two perfect crystals. Based on dislocation motion, the defect are of two types:

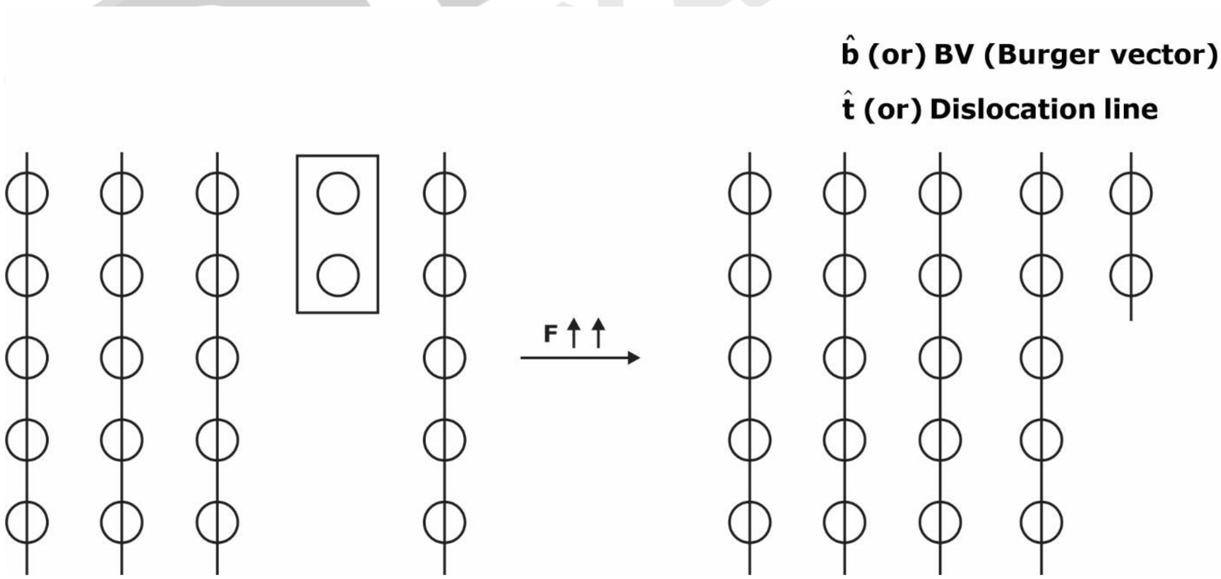
(i) Edge dislocation

(ii) Screw dislocation



Screw ($\hat{t} \parallel \hat{b}$)





Edge Dislocation	Screw Dislocation

• In this, the dislocated atoms are moving parallel to applied force	• In this, the dislocation is moving perpendicular to applied force.
• It is mainly because of compressive and tensile stresses	• It is mainly because of shear stresses.
• In this, the burger's vector is parallel to applied force (or) perpendicular to dislocation line $(\hat{t} \perp \hat{b})$.	• In this, the burger's vector is perpendicular to applied force (or) parallel to dislocation line. $(\hat{t} \,\|\, \hat{b})$
• In edge dislocation movement require less force because of only translating motion of atoms	• Screw dislocation requires more force because of rotational and translatory motion of atoms.

Burger's Vector:

It is used to find direction and magnitude to dislocation motion (or) slip.

$$\hat{b} = \frac{a}{2} < hkl >$$

$$\hat{b} = \frac{a}{2}\sqrt{h^2 + k^2 l^2}$$

A = lattice parameter

(h, k, l) = miller indices (direction)

5. **c.** Design a circuit that accepts a 3-bit number and gives an output 0, if input represents even decimal number and gives an output 1, if input represents an odd decimal number.

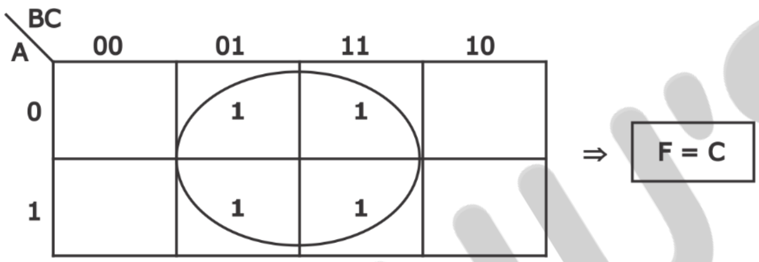
Sol. 5c. The Truth Table is

A	B	C	Decimal Value	Output F
0	0	0	0	0
0	0	1	1	1
0	1	0	2	0
0	1	1	3	1
1	0	0	4	0

1	0	1	5	1
1	1	0	6	0
1	1	1	7	1

The canonical SOP expression of the output is

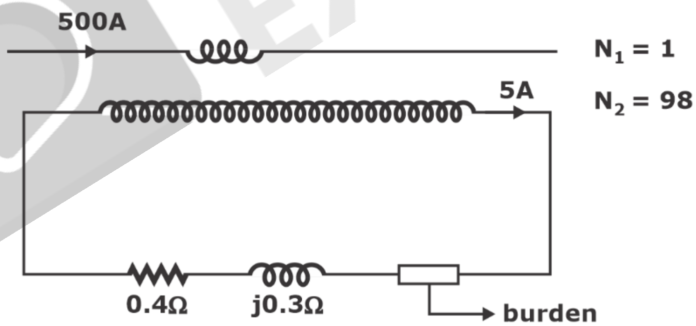
$F(A, B, C) = \sum m(1, 3, 5, 7)$



Map for output 'F'

5. d. A current transformer having a bar primary is rated at 500/5 A, 50 Hz with an output of 20 VA. At rated load with non-inductive burden, the inphase and quadrature components (referred to the flux) of the exciting mmf are 8 A and 10 A respectively. The number of turns in the secondary winding is 98 and the impedance of the secondary winding is (0.4 + j 0.3) Ω. Calculate the ratio and phase angle errors.

Sol. 5d. Current Transformer



Turns ratio, $n = \frac{98}{1} = 98$

Nominal ratio, $K_n = \frac{500}{5} = 100$

Magnetising current, I_μ = 8 A

Loss current, I_e = 10 A

Impedance of secondary load burden

$$Z_L = \frac{VA}{I_S^2} = \frac{20}{5^2} = 0.8\Omega$$

∴ Resistance of total secondary circuit is, 0.4 + 0.8 = 1.2 Ω

& Reactance of total secondary circuit is j0.3 Ω

∴ Secondary phase angle, δ

$$\delta = \tan^{-1}\frac{0.3}{1.2} = 14.03°$$

So, $\cos\delta = \cos 14.03° = 0.97$

And $\sin\delta = \sin 14.03° = 0.24$

Now, actual ratio,

$$R = n + \frac{I_e \cos\delta + I_\mu \sin\delta}{I_s}$$

$$= 98 + \frac{10 \times 0.97 + 8 \times 0.24}{5}$$

R = 100.32

∴ Ratio error $= \frac{K_n - R}{R} \times 100$

$$= \frac{100 - 100.32}{100.32} \times 100$$

= –0.32%

Phase angle error,

$$\theta = \frac{180°}{\pi}\left[\frac{I_\mu \cos\delta - I_e \sin\delta}{nI_s}\right]$$

$$= \frac{180°}{\pi}\left[\frac{8 \times 0.97 - 10 \times 0.24}{98 \times 5}\right]$$

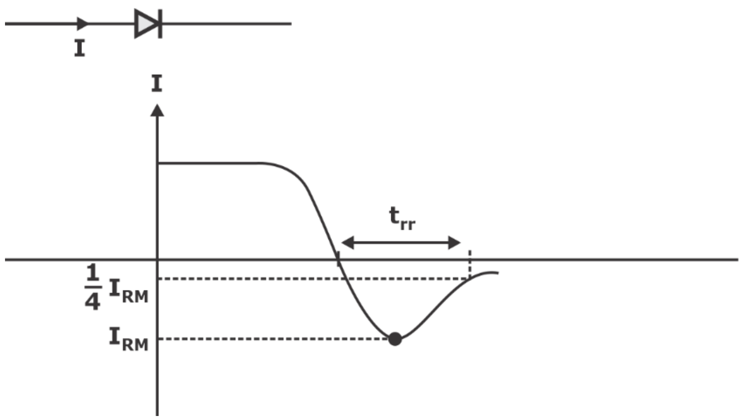
$\theta = 0.626°$

5. e. (i) The reverse recovery time t_{rr} of a diode is 2 μs. In the conducting mode to reverse blocking mode operation, the diode needs the rate of fall of forward current of 50 amperes/μs. Determine the storage charge and the peak reverse current.

(ii) A diode with 500 mW power dissipation at 25° C has 5 mW/°C derating factor. If the forward voltage drop remains constant at 0.7 V, calculate the maximum forward current at 50°C.

Sol. 5e. (i) t_{rr} = 2μ sec

Rate of fall of forward current, $\frac{di}{dt} = 50A / \mu sec$



I_{RM} = Peak inverse current

Q_R =storage charge$= \frac{1}{2} I_{RM} \cdot t_{rr}$

$$I_{RM} = \frac{2Q_R}{t_{rr}}$$

$$t_{RM} = t_{rr}\left(\frac{di}{dt}\right) = \frac{2Q_R}{t_{rr}}$$

$$\Rightarrow Q_R = \frac{t^2}{2}(di/dt)$$

Storage charge,

$$Q_R = \frac{1}{2}\left(2\times10^{-6}\right)^2\left(\frac{50}{10^{-6}}\right) = 100\times10^{-6}C$$

= 100 μC

Peak reverse current,

$$I_{RM} = \left(2\times10^{-6}\right)\left(\frac{50}{10^{-6}}\right) = 100A$$

(ii) Temperature **=** 25°C

Power dissipation = 500 mW at 25°C

Derating factor (D) = 5mW/°C

Increment derating = 5mW x 25 = 125 mw

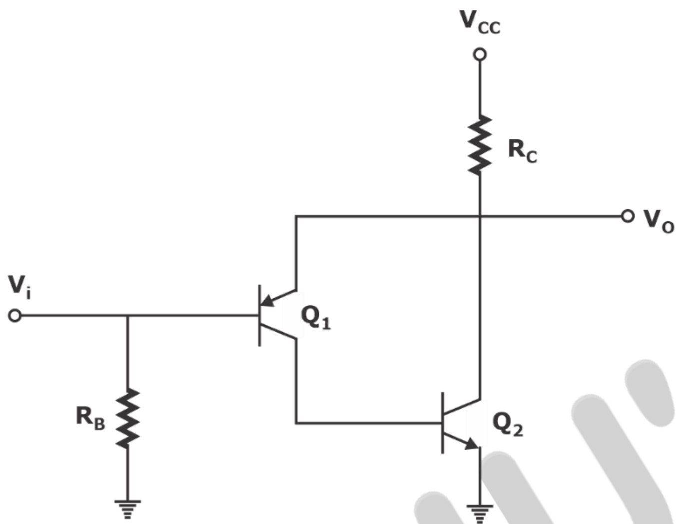
So, at 50°C the power dissipation = (500 – 125) mW = 375 mW

Voltage across diode = 0.7 V

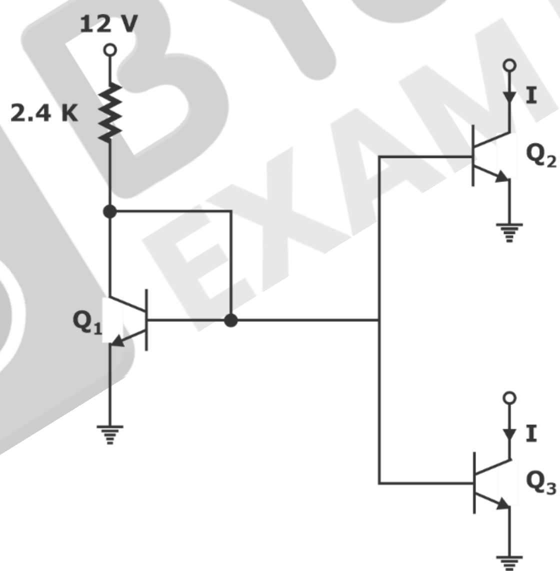
$$I_{max}(0.7) = 375\times10^{-3} \text{ watt}$$

$$I_{max} \text{ at } 50^{\circ}C = \frac{375\times10^{-3}}{0.7} = 0.536 \text{ A}$$

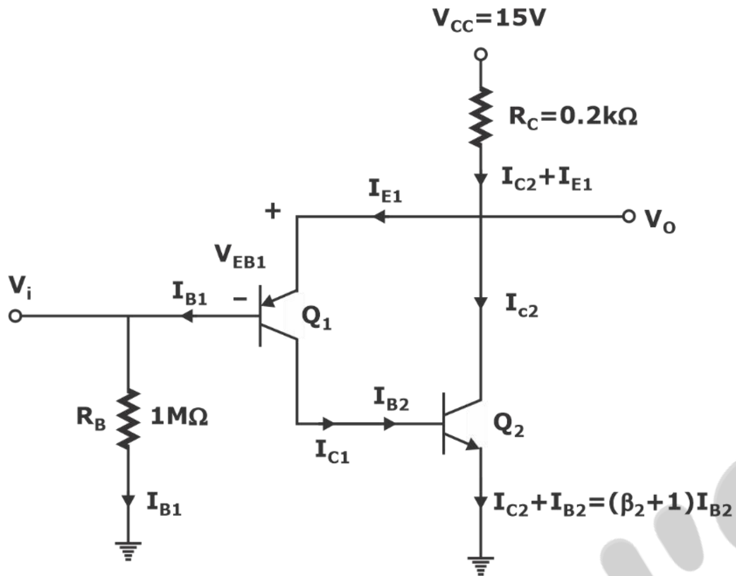
6. a. (i) Calculate the DC voltages V_o and V_i, and the bias currents for the feedback pair of transistors shown in the figure below, given that $\beta_1 = 100$, $\beta_2 = 150$, $V_{CC} = 15$ volts, $R_C = 200\ \Omega$ and $R_B = 1\ M\Omega$:



(ii) In the current mirror circuit shown in the figure below, the current is mirrored in two transistors. All the three transistors are identical. Calculate the load current I assuming $\beta = 100$:



Sol. 6a. (i)



$$V_{CC} = \left(I_{C_2} + I_{E_1}\right)R_C + V_{EB_1} + R_B I_{B_1} \text{ ....(1)}$$

$$I_{C_2} = \beta_2 I_{B_2} = \beta_2 I_{C_1} \qquad \because I_{B_2} = I_{C_1}$$

$$I_{C_1} = \beta_1 I_{B_1} \qquad I_{E_1} = \left(1+\beta_1\right) I_{B_1}$$

$$\therefore \; I_{C_1} = 100 I_{B_1} \quad I_{E_1} = 101 I_{B_1} \quad \therefore \; I_{C_2} = \beta_2\beta_1 I_{B_1}$$

Put in (1)

$$V_{CC} = \left(\beta_2\beta_1 I_{B_1} + \left(\beta_1+1\right) I_{B_1}\right)R_C + V_{EB_1} + R_B I_{B_1}$$

$$I_{B_1} = \frac{V_{CC} - V_{EB_1}}{\left[\beta_2\beta_1 + \left(\beta_1+1\right)\right]R_C + R_B}$$

$$= \frac{15-0.7}{(15000+101)0.2+10^3} = 3.55\times10^{-3}\text{mA}$$

$$\therefore \; V_i = I_{B_1} R_B = 3.55\times10^{-3}\times10^3 \text{Volt}$$

V_i = 3.55 volt

$$I_{E_1} = \left(1+\beta_1\right) I_B = 101\times3.55\times10^{-3}$$

= 0.359 mA

$$I_{C_2} = \beta_2\beta_1 I_{B_1} = 15000\times3.55\times10^{-3}$$

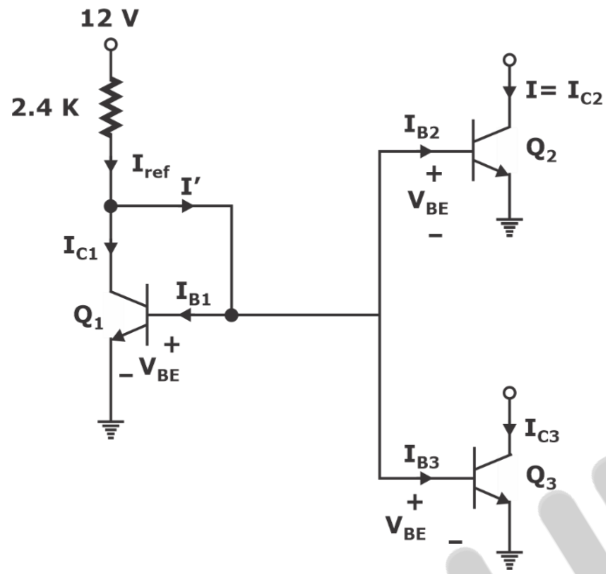
= 53.25 mA

$$V_0 = V_{CC} - \left[I_{C_2} + I_{E_1}\right]\times R_c$$

$$= 15 - [53.25 + 0.359]\times0.2$$

$$V_0 = 4.278 \text{ volt}$$

(ii)



$$12 = 2.4\ I_{ref} + V_{BE}$$

$$I_{ref} = \frac{12 - V_{BE}}{2.4} = \frac{12 - 0.7}{2.4} = 4.708\,mA$$

$$I_{ref} = I_{C_1} + I'$$

Here $I' = I_{B_1} + I_{B_2} + I_{B_3}$

$\therefore\ \beta_1 = \beta_2 = \beta_3 \qquad V_{BE_1} = V_{BE_2} = V_{BE_3} = 0.7\,V$

$\therefore\ I_{B_1} = I_{B_2} = I_{B_3} = I_B \quad \& \quad I_{C_1} = I_{C_2} = I_{C_3}$

$$I_{ref} = I_{C_1} + 3I_{B_1}$$

$$I_{ref} = I_{C_1} + 3I_{B_1}$$

$$I_{ref} = I_{C_2} + \frac{3I_{C_2}}{\beta}$$

$$I_{C_2} = I_{C_3} = I = \frac{I_{ref}}{1 + 3/\beta}$$

$$I = \frac{4.708}{1 + \frac{3}{100}} = 4.57\,mA$$

6. **b.** State Gauss divergence theorem. Let R be the region bounded by the closed cylinder $x^2 + y^2 = 4$, $z = 0$ and $z = 2$. Verify this theorem, if $\vec{F} = 3x^2\hat{i} + y^2\hat{j} + z\hat{k}$.

Sol. 6b. From the definition of the divergence of F,

$$\oint_s \vec{F} \cdot d\vec{s} = \int_v \nabla \cdot \vec{F} dv$$

This is called the divergence theorem, otherwise known as the Gauss-Ostrogradsky theorem.

The divergence theorem states that the total outward flux of a vector field F through the closed surface S is the same as the volume integral of the divergence of F.

Given: $\vec{F} = 3x^2\hat{i} + y^2\hat{j} + z\hat{k}$

The gauss divergence theorem says,

$\oint_s \vec{F}\cdot d\vec{s} = \int_v \nabla\cdot\vec{F}dv$

$\nabla\cdot\vec{F} = \frac{\partial}{\partial x}\left(3x^2\right) + \frac{\partial}{\partial y}\left(y^2\right) + \frac{\partial}{\partial z}(z)$

= 6x + 2y + 1

Use cylindrical coordinates,

x = r cosθ, y = r sinθ, z = z with $0 \le \theta \le 2\pi$, $0 \le r \le 2$, $0 \le z \le 2$

so, $\int_v (\nabla\cdot\vec{F})dv = \int_0^2\int_0^2\int_0^{2\pi}(6r\cos\theta + 2r\sin\theta + 1)rdrd\theta dz$

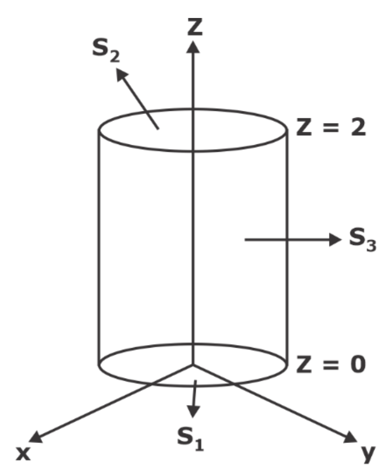
$$= \int_0^2\int_0^{2\pi}\left[\frac{6}{3}\left(r_0^3\right)_0^2\cos\theta + \frac{2}{3}\left(\frac{r^3}{3}\right)_0^2\sin\theta + \left(\frac{r^2}{2}\right)_0^2\right]d\theta dz$$

$$= \int_0^2\int_0^{2\pi}\left(16\cos\theta + \frac{16}{9}\sin\theta + 2\right)d\theta dz$$

$$= \int_0^2\left[16(\sin\theta)_0^{2\pi} - \frac{16}{9}(\cos\theta)_0^{2\pi} + 2(\theta)_0^{2\pi}\right]dz$$

$$= \int_0^2 -4\pi dz = 4\pi(z)_0^2$$

= 8 π (1)



On $S_1(z=0); \hat{n} = -\hat{k}$. So

$\vec{F}\cdot n = \left(3x^2, y^2, z\right)\cdot(0,0,-1)$

= −z = 0

Since $z = 0$, so $\int_{s_1}\vec{F}\cdot d\vec{s} = 0$

On $S_2(z=2); \hat{n} = \hat{k}$, so

$\vec{F}\cdot\hat{n} = \left(3x^2, y^2, z\right)\cdot(0,0,1) = z = 2$

Since z = 2, so

$$\int_{s_2}\vec{F}\cdot d\vec{s} = \int_{s_2} Zds = 2\int_0^{2\pi}\int_0^2 rdrd\theta = 2\left(\frac{r^2}{2}\right)_0^2(2\pi) = 8\pi$$

Finally on $S_3 \cdot \left(x^2 + y^2 = 4\right)$: Parameterise cylinder as x = 2cosθ , y = 2 sinθ, z = z, 0

$0 \le \theta \le 2\pi, 0 \le z \le 2$

So, r(θ, Z) = (2cos θ, 2sin θ, z). Also

$$\vec{r}_\theta \times \vec{r}_z = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2\sin\theta & 2\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = (2\cos\theta, 2\sin\theta, 0)$$

And $\vec{F}[r(\theta, z)] = (12\cos^2\theta, 4\sin^2\theta, z)$

$$\int_{S_3} \vec{F}\cdot\vec{ds} = \iint_R \vec{F}(r(\theta,z))\cdot\left(\vec{r}_\theta \times \vec{r}_z\right) d\theta\, dz$$

$$= \int_0^2 \int_0^{2\pi} \left(24\cos^3\theta + 8\sin^3\theta\right) d\theta dz$$

$$= 2\int_0^{2\pi} \left(24\cos^3\theta + 8\sin^3\theta\right) d\theta$$

= 0

Thus, $\oint_s \vec{F}\cdot\vec{ds} = \int_{s_1} \vec{F}\cdot\vec{ds} + \int_{s_2} \vec{F}\cdot\vec{ds} + \int_{s_3} \vec{F}\cdot\vec{ds}$

$= 0 + 8\pi + 0$

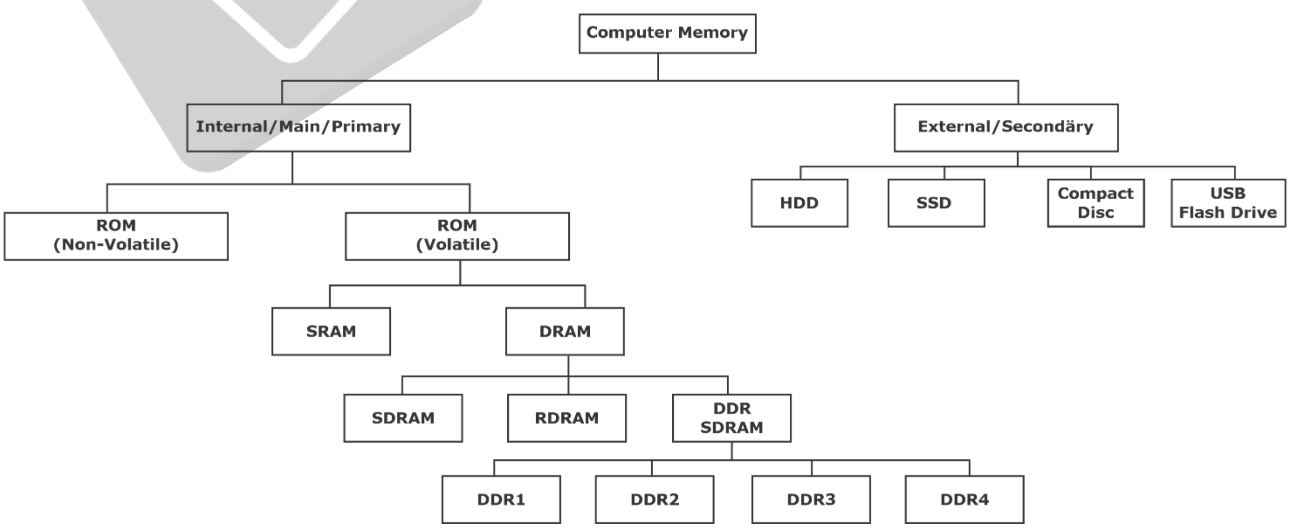
$= 8\pi$ (2)

From equation (1) and (2) it is verified.

6. c. (i) What is Random Access Memory? Explain Static Random Access Memory and Dynamic Random Access Memory.

(ii) Write a program in C to find whether the given number is even or odd and if it is odd, find whether it is prime or not.

Sol. 6c. (i)



Computer memory is generally classified as either internal or external memory.

Internal memory, also called "main or primary memory" refers to memory that stores small amounts of data that can be accessed quickly while the computer is running.

External memory, also called "secondary memory" refers to a storage device that can retain or store data persistently. They could be embedded or removable storage devices. Examples include hard disk or solid state drives, USB flash drives, and compact discs.

There are basically two kinds of internal memory: ROM and RAM.

Once the operating system is loaded, the computer uses RAM, which stands for random-access memory, which temporarily stores data while the central processing unit (CPU) is executing other tasks. With more RAM on the computer, the less the CPU has to read data from the external or secondary memory (storage device), allowing the computer to run faster. RAM is fast but it is volatile, which means it will not retain data if there is no power. It is therefore important to save data to the storage device before the system is turned off.

There are two main types of RAM: Dynamic RAM (DRAM) and Static RAM (SRAM).

DRAM, is widely used as a computer's main memory. Each DRAM memory cell is made up of a transistor and a capacitor within an integrated circuit, and a data bit is stored in the capacitor. Since transistors always leak a small amount, the capacitors will slowly discharge, causing information stored in it to drain; hence, DRAM has to be refreshed (given a new electronic charge) every few milliseconds to retain data.

SRAM is made up of four to six transistors. It keeps data in the memory as long as power is supplied to the system unlike DRAM, which has to be refreshed periodically. As such, SRAM is faster but also more expensive, making DRAM the more prevalent memory in computer systems.

Common types of DRAM:

Synchronous DRAM (SDRAM) "synchronizes" the memory speed with CPU clock speed so that the memory controller knows the exact clock cycle when the requested data will be ready. This allows the CPU to perform more instructions at a given time. Typical SDRAM transfers data at speeds up to 133 MHz.

Rambus DRAM (RDRAM) takes its name after the company that made it, Rambus. It was popular in the early 2000s and was mainly used for video game devices and graphics cards, with transfer speeds up to 1 GHz.

Double Data Rate SDRAM (DDR SDRAM) is a type of synchronous memory that nearly doubles the bandwidth of a single data rate (SDR) SDRAM running at the same clock frequency by employing a method called "double pumping," which allows transfer of data on both the rising and falling edges of the clock signal without any increase in clock frequency.

(ii) /* Program to check number is even or odd, if odd find it is prime or not */

```
#include<stdio.h>
void main( )
{
int i,count=0,num;
printf("Enter the number");
scanf("%d",&n);
if(n%2!=0)
{
printf("\nThe given number is Odd number");
for(i=1;i<=n;i++)
{
if(n%i==0) count++;
}
if(count==2)
printf("The given number is prime number");
else
printf("The given number is not a prime number");
}
else
printf("The given number is Even Number");
}
```

Sample Output:

Enter the number: 17

The given number is odd number

The given number is prime number

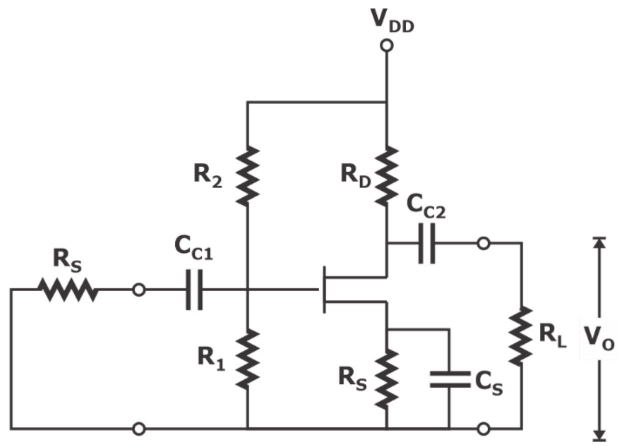
7. a. For the JFET amplifier circuit shown in the figure below, g_m = 2 mS, r_d = 200 k Ω, C_{gs} = 10 pF, C_{gd} = 2 pF, R_S = 1 kΩ, R_1 = 10 MΩ, R_2 = 100 kΩ, R_D = 5 kΩ, C_{C1} = C_{C2}= 0.1 μF. Assume output capacitor C_0 = 10 pF, C_S and R_L to be very large. Find-

(i) mid-frequency gain;

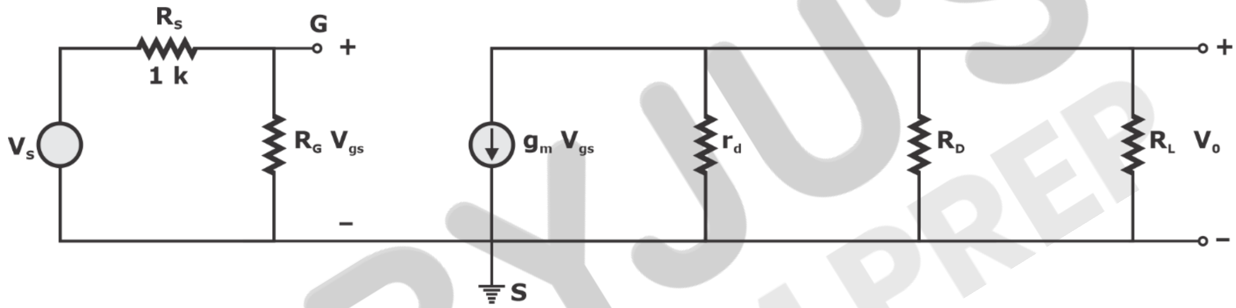
(ii) lower cut-off frequency;

(iii) higher cut-off frequency;

(iv) gain-bandwidth product.



Sol. 7a. (i) In mid frequency C_{C1} C_{C2} C_S → S.C and all internal capacitor act as O.C.



$$R_G = R_1 \,||\, R_2 = 10 \times 10^3 \,||\, 100 = 99 k\Omega$$

$$\frac{V_{gs}}{V_s} = \frac{R_G}{R_G + R_S} = \frac{99}{99+1} = 0.99$$

$$V_0 = -g_m V_{gs} \left(r_d \,||\, R_D \,/\, R_L\right) \cong -g_m V_g R_D'$$

$$\because R_L \to \text{ V.Large}$$

$$R_D' = r_d \,||\, R_D = 200 \,||\, 5 = 4.88 k\Omega$$

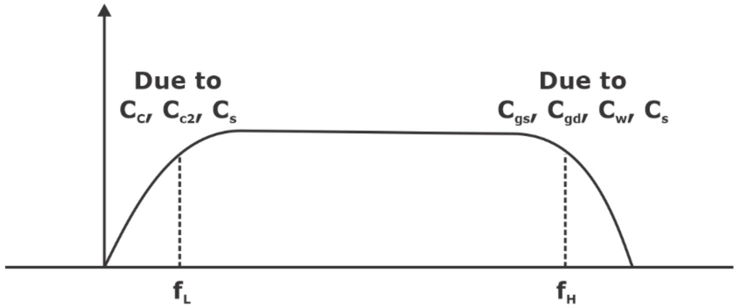
$$A_V = \frac{V_0}{V_{gs}} = -g_m R_D' = -2 \times 4.88 = -9.756$$

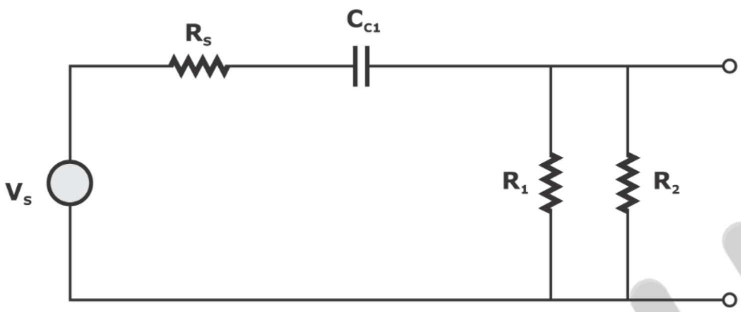
$$A_{V_S} = \frac{V_0}{V_S} = \frac{V_0}{V_{gs}} \times \frac{V_{gs}}{V_S} = -9.756 \times 0.99$$

$$A_{V_S} = -9.658$$

$$\phi = 180^\circ$$

(ii) Lower cut-off frequency f_2





$$f_{L_1} = \frac{1}{2\pi\left(R_1 \parallel R_2 + R_s\right)C_{C_1}}$$

$$= \frac{1}{2\pi \times 100 \times 10^3 \times 0.1 \times 10^{-6}}$$

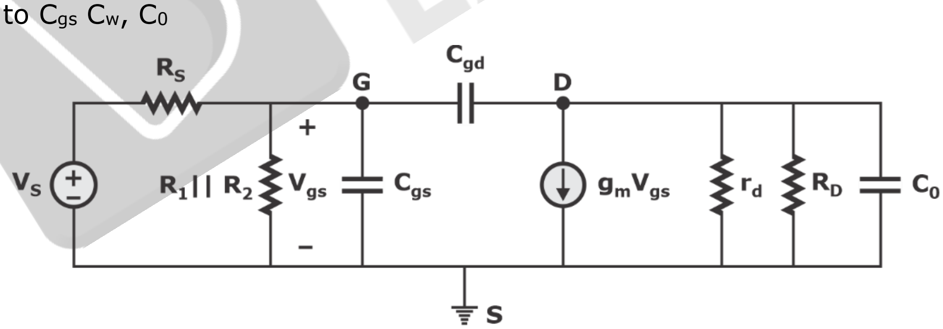
$$= \frac{100}{2\pi}\,\text{Hz} = 15.91\ \text{Hz due to } C_{C1}$$

$\because R_L \to \infty$ [given in question]

f_L due to C_{C2} is $\cong 0\,\text{Hz}$

(iii) Higher cut-off frequency

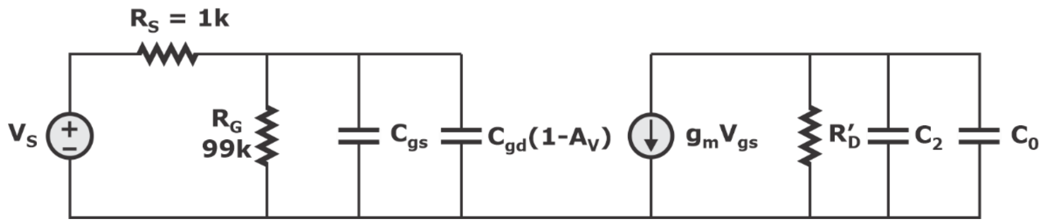
Due to C_{gs} C_w, C_0



Note: 1. At high frequency C_{C1} C_{C2} $C_S \to$ Short Circuit

2. $R_L \to \infty$

Apply miller's Theorem



$$\text{Where } C_2 = \frac{C_{g_d}(A_v - 1)}{A_v} \cong C_{gd} \because |A_v| >> 1$$

$$C_0' = C_2 \,||\, C_0 = C_{gd} \,||\, C_0 = C_{gd} + C_0 = 2 + 10 = 12\,pF$$

$$\because A_V = \frac{V_0}{V_{gs}} \cong -g_m R_D' = -9.756$$

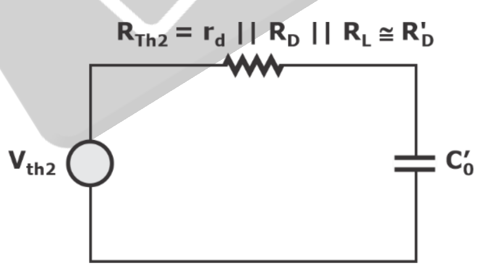
$$C_{gd}(1 - A_v) = 2 \times 10.756 = 21.512\,pF$$

Thevenin's equivalent at input

$C_{in} = C_{gs} + C_{gd}(1 - A_V) = 10 + 21.512 = 31.512$ pF

$$f_{H_1} = \frac{1}{2\pi R_{Th_1} C_{in}} = \frac{1}{2\pi \times 0.99 \times 10^3 \times 31.512 \times 10^{-12}}$$

$$f_{H_1} = 5.1 \times 10^6 Hz = 5.1\,MHz$$



Thevenin's Equivalent at output

$$f_{H_2} = \frac{1}{2\pi R_D' C_0'} = \frac{1}{2\pi \cdot 4.878 \times 12 \times 10^{-12} \times 10^3}$$

$$f_{H_2} = 2.71 \times 10^6 Hz \text{ or } 2.71\,MHz$$

(iv) Gain bandwidth product = Mid band gain x BW

$BW = f_H - f_L \cong f_H \because f_H >> f_L$

GBW = $|A_{VS}|BW$ = 9.658 x 2.71 = 26.17 MHz

7. **b.** A balanced 240 V, 3-phase voltage is applied to an unbalanced delta-connected load having the following phase impedances :

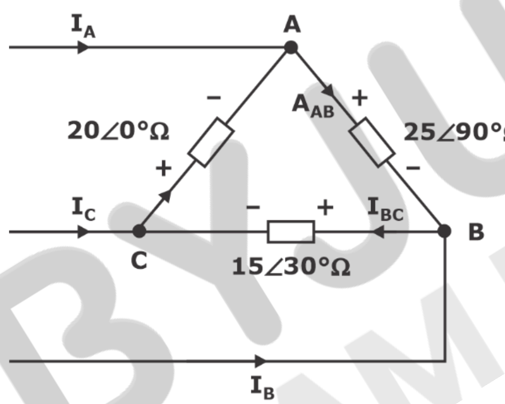
$Z_{AB} = 25\angle 90°\Omega, \quad Z_{BC} = 15\angle 30°\Omega, \quad Z_{CA} = 20\angle 0°\Omega$

(i) Calculate the line currents.

(ii) Obtain the readings of the two wattmeters whose current coils are connected in the lines A and B, and the voltage coils are connected across the line C.

Consider ABC system for supply voltage and V_{BC} as reference.

Sol. 7b. (i)



$V_{BC} = 240\angle 0°$ V

$V_{CA} = 240\angle -120°$ V

$V_{AB} = 240\angle 120°$ V

From above connection, $I_{AB} = \dfrac{240\angle 120}{25\angle 90} = 9.6\angle 30°$ A

$I_{BC} = \dfrac{240\angle 0}{15\angle 30} = 16\angle -30°$ A

$I_{CA} = \dfrac{240\angle -120°}{20\angle 0°} = 12\angle -120°$ A

According to KCL,

$I_A + I_{CA} = I_{AB}$

$I_A = I_{AB} - I_{CA}$

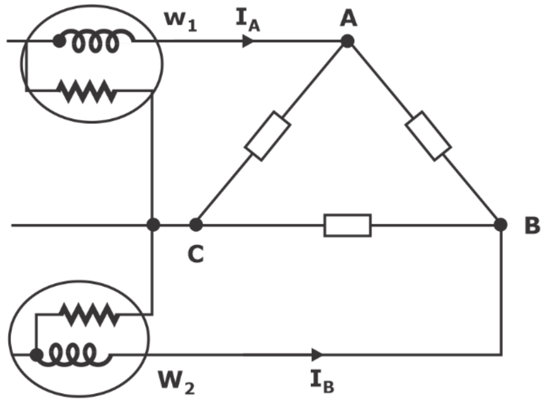
$= 9.6\angle 30° - 12\angle -120° = 20.87\angle 46.705$ A

$I_B = I_{BC} - I_{AB}$

$= 16\angle -30° - 9.6\angle 30°$

$= 20\angle -173.13°$ A

(ii) Wattmeter are connected as shown below



Reading of ω_1 = I_A, V_{AC} $\cos\theta$

θ is angle between I_A and V_{AC}

$I_A = 20.87\angle 46.705$

$V_{AC} = -V_{CA} = -240\angle -120^\circ$

$= 240\angle 60^\circ$

$W_1 = (20.87)(240)\cos[60 - 46.705]$

= 4874.56 watt

Reading of $W_2 = I_B V_{BC} \cos\phi$

ϕ is angle between I_B and V_{BC}

$I_B = 13.95\angle 66.59^\circ$

$V_{BC} = 240\angle 0^\circ$

$W_2 = (13.95)(240)\cos(66.59^\circ)$ watt

= 1330.187 watt

7. c. (i) Suppose that X and Y are independent random variables having the common density function

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

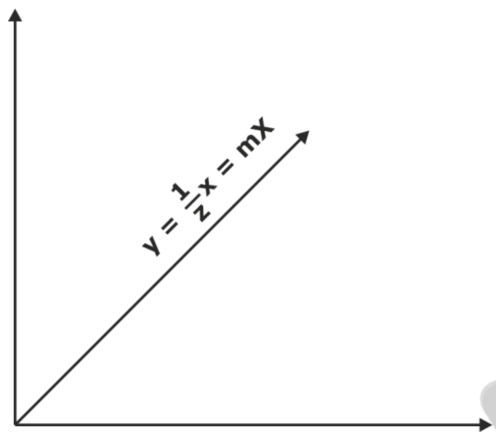
Find the density function of the random variable X | Y.

(ii) A root of the equation $xe^x - 1 = 0$ lies in the interval (0.5, 1.0). Determine this root correct to three decimal places using regula-falsi method. First find how many least decimal digits are required for three decimal places accuracy.

Sol. 7c. (i) Let $z = \frac{x}{y}$

First find the CDF $G(z) = P(z \le z)$

$$G(z) = P\left(\frac{x}{y} \le z\right) = P\left(y \ge \frac{1}{z}x\right)$$



$P\left(y \ge \frac{1}{z}x\right)$ = area above the line $y = \frac{1}{z}x = \frac{z}{z+1}$

So, PDF of Z is $g(z) = \frac{d}{dz}G(z)$

$$= \frac{d}{dz}\left[\frac{z}{(z+1)}\right]$$

$$\Rightarrow g(z) = \frac{1}{(z+1)^2}, z \ge 0$$

(ii) Regula-falsi method: The root lies in (0.5, 1.0). We have $x_0 = 0.5$, $x_1 = 1.0$, $f_0 = -0.17564$, $f_1 = 1.71828$

$$k = 1; \quad x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0} = 0.54637, |x_2 - x_1| = 0.45363$$

$$f_2 = f(0.54637) = -0.05643$$

Since $f_1 f_2 < 0$, the root lies in the interval (x_2, x_1).

$$k = 2: \quad x_3 = \frac{x_1 f_2 - x_2 f_1}{f_2 - f_1} = 0.56079, |x_3 - x_2| = 0.01442$$

$$f_3 = f(0.56079) = -0.01746$$

Since $f_1 f_3 < 0$, the root lies in the interval (x_3, x_1).

$$k = 3; \quad x_4 = \frac{x_1 f_3 - x_3 f_1}{f_3 - f_1} = 0.56521, |x_4 - x_3| = 0.00442$$

$$f_4 = f(0.56521) = -0.00533$$

Since, $f_1 f_4 < 0$, the root lies in the interval (x_4, x_1).

$k = 4: \quad x_5 = \dfrac{x_1 f_4 - x_4 f_1}{f_4 - f_1} = 0.56654, x_5 = x_4| = 0.00133,$

$f_5 = f(0.56654) = -0.00167$

Since $f_1 f_5 < 0$, the root lies in the interval (x_5, x_1).

$k = 5: \quad x_6 = \dfrac{x_1 f_5 - x_5 f_1}{f_5 - f_1} = 0.56696, |x_6 - x_5| = 0.00042 < 0.0005$

Hence, the root correct to three decimal places is 0.567. Note that the right end point x_1, of the initial interval, is fixed in all iterations.

8. a. (i) The power in a single-phase circuit is measured by an electrodynamometer wattmeter. The voltage across the load is 100 V and the load current is 10 A at a power factor of 0.2 lagging. The wattmeter circuit has a resistance of 3500 Ω and an inductance of 30 mH. Estimate the percentage error in the wattmeter reading when the pressure coil is connected (1) on the supply side and (2) on the load side. The current coil has a resistance of 0.1 Ω and negligible inductance. The supply frequency is 50 Hz.

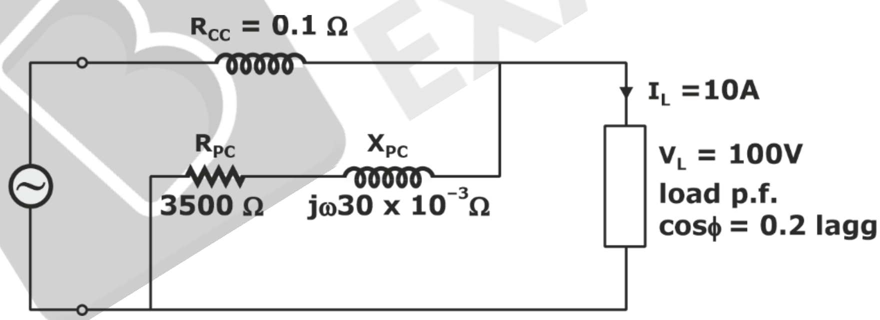
(ii) The limiting errors for a four-dial resistance box are :

Units : ± 0.2 % & Hundreds: ± 0.05 %

Tens: ± 0.1 % & Thousands : ± 0.02 %

If the resistance value is set at 3525 Ω, calculate the limiting error in the resistance value.

Sol. 8a. (i) Pressure coil on supply side.



The True load power, $P_t = V_L I_L \cos\phi$

= 100 x 10 x 0.2

P_t = 200 W

$\phi = \cos^{-1}(0.2) = 78.46$

$\therefore \tan\phi = 4.89$

Reactance of P.C. $X_{PC} = 2\pi \times 50 \times 30 \times 10^{-3}$

$= 9.42\,\Omega$

$$\therefore \tan\beta = \frac{X_{pc}}{R_{pc}} = \frac{9.42}{3500} = 0.00269\,\text{rad}$$

Now, the wattmeter reading is

$$= P_t + I_{cc}^2 R_{cc}$$

$$= 200 + (10)^2 \times 0.1$$

= 210 watts

The impedance of load, $X_L = \frac{V_L}{I_L} = \frac{100}{10} = 10\,\Omega$

$$\therefore R_L = Z_L \cos\phi = 10 \times 0.2 = 2\Omega$$

$$X_L = Z_L \sin\phi = 10 \times 0.97 = 9.7\,\Omega$$

Now, Impedance of load including current coil impedance.

$$Z_{total} = \left(R_L + R_{cc} + j\left(X_L + X_{cc}\right)\right)$$

$$= (2 + 0.1 + j9.7 + 0)$$

$$Z_{total} = (2.1 + j9.7)\Omega$$

$$\therefore \phi = \tan^{-1}\frac{9.7}{2.1} = 77.78^\circ$$

$$\therefore \tan\phi = 4.61$$

Updated readieg of wattmeter due to current coil impedance will be.

$$\text{Wattmeter reading}(1 + \tan\phi\tan\beta)$$

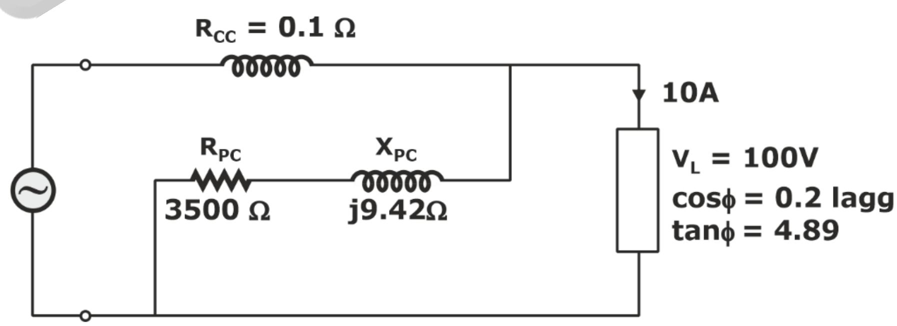
= 210(1+4.61 x 0.00269)

P_m = 212.6 watts

$$\therefore \%\ \text{Error} = \frac{P_m - P_t}{P_t} = \frac{212.6 - 200}{200} \times 100$$

$$\%\ \text{Error} = 6.3\%$$

Case (ii): Pressure coil on load side.



We know, P_t = 200 W

Now, wattmeter reading, $= P_t(1 + \tan\phi\tan\beta)$

$= 200(1 + 4.89 \times 0.00269)$

= 202.63 watts.

Updated reading of wattmeter due to pressure coil loss will be.

$$= 202.63 + \frac{V^2}{R_{pC}} = 202.63 + \frac{(100)^2}{3500} = 205.48 \text{ watts}$$

$$\% \text{ Error } = \frac{205.48 - 200}{200} \times 100$$

$\% E = 2.74\%$

Conclusion: If the P.C. is used on load side, the % E is less.

(ii) Given resistance value is 3525 Ω and setted on decade resistance box.

3525 = 3000 + 500 + 20 + 5

$$\text{Error at 3k} = 3000 \times \pm \frac{0.02}{100} = \pm 0.6\,\Omega$$

$$\text{Error at 500} = 500 \times \pm \frac{0.05}{100} = \pm 0.25\Omega$$

$$\text{Error at 20} = 20 \times \pm \frac{0.1}{100} = \pm 0.02\Omega.$$

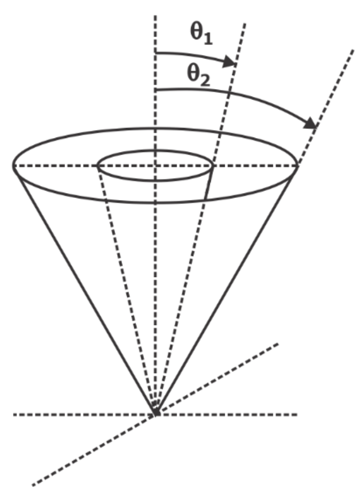
$$\text{Error at 5} = 5 \times \pm \frac{0.2}{100} = \pm 0.01\Omega$$

∴ Total absolute Error = ± (0.6 + 0.25 + 0.02 + 0.01)

= ± 0.88 Ω

$$\therefore \% \text{ Relative Error} = \frac{\delta \in}{A_t} = \frac{0.88}{3525} \times 100 = 0.024\%$$

8. **b.** In the region between the two coaxial cones with insulated vertices as shown in the figure below, the voltage at θ_1 = 30° is 0 volt and at θ_2 = 45° is 125.5 volts :

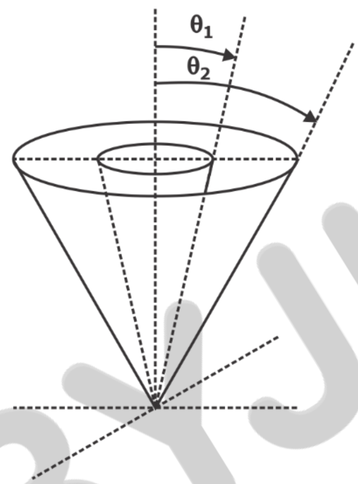


(i) Calculate the angle θ at which the voltage is 75 volts. Assume air as the dielectric in the region between the two coaxial cones.

(ii) Find the charge distribution on the conducting plane at $\theta_2 = 90°$

Sol. 8b. (i) Consider the coaxial cone of Figure, where the gap serves as an insulator between the two conducting cones. V depends only on θ, so Laplace's equation in spherical coordinates becomes

$$\nabla^2 V = \frac{1}{r^2 \sin\theta}\frac{d}{d\theta}\left[\sin\theta \frac{dV}{d\theta}\right] = 0$$



Since r = 0 and θ = 0, π are excluded, we can multiply by $r^2 \sin\theta$ to get

$$\frac{d}{d\theta}\left[\sin\theta \frac{dV}{d\theta}\right] = 0$$

Integrating once gives

$$\sin\theta \frac{dV}{d\theta} = A$$

Or

$$\frac{dV}{d\theta} = \frac{A}{\sin\theta}$$

Integrating this results in

$$V = A\int \frac{d\theta}{\sin\theta} = A\int \frac{d\theta}{2\cos\theta/2\sin\theta/2}$$

$$= A\int \frac{1/2\sec^2\theta/2d\theta}{\tan\theta/2}$$

$$= A\int \frac{d(\tan\theta/2)}{\tan\theta/2}$$

$$= A\ln(\tan\theta/2) + B$$

We now apply the boundary conditions to determine the integration constants A and B.

$$V(\theta=\theta_1)=0 \rightarrow 0 = A\ln(\tan\theta_1/2)+B$$

Or

$$B=-A\ln(\tan\theta_1/2)$$

$$V = A\ln\left[\frac{\tan\theta/2}{\tan\theta_1/2}\right]$$

$$V(\theta=\theta_2)=V_o \rightarrow V_o = A\ln\left[\frac{\tan\theta_2/2}{\tan\theta_1/2}\right]$$

$$A=\frac{V_o}{\ln\left[\frac{\tan\theta_2/2}{\tan\theta_1/2}\right]}$$

$$V=\frac{V_o\ln\left[\frac{\tan\theta/2}{\tan\theta_1/2}\right]}{\ln\left[\frac{\tan\theta_2/2}{\tan\theta_1/2}\right]}$$

(i) $$V=V_0\left[\frac{\ln\left(\frac{\tan\theta_2}{\tan\theta_1/2}\right)}{\ln\left(\frac{\tan\theta_2/2}{\tan\theta_1/2}\right)}\right]$$

Given, $\theta_1=30°,\ \theta_2=45°,\ V_0=125.5V,\ V=75V$

$$75=125.5\left[\frac{\ln\left(\frac{\tan\theta/2}{\tan 15°}\right)}{\ln\left(\frac{\tan 22.5}{\tan 15°}\right)}\right]$$

$$\ln\left[\tan\left(\frac{\theta}{2}\right)\right]-\ln(\tan 15)=0.597[\ln(1.545)]$$

$$\ln\left[\tan\left(\frac{\theta}{2}\right)\right]=0.259+\ln(\tan 15°)$$

$$\ln\left[\tan\left(\frac{\theta}{2}\right)\right]=0.259-1.32$$

$$\ln\left[\tan\frac{\theta}{2}\right]=-1.058$$

$$\tan\frac{\theta}{2}=0.347$$

$$\theta=2\tan^{-1}(0.347)$$

$$\theta=38.27°$$

(ii) $E = -\nabla V = -\frac{1}{r}\frac{dV}{d\theta}a_\theta = -\frac{A}{r\sin\theta}a_\theta$

The surface charge density on the lower conducts is

$$\sigma_2 = \frac{1}{r}\frac{A}{\sin\theta_2}$$

Where, $A = \dfrac{V_0}{\ln\left[\dfrac{\tan\theta_2/2}{\tan\theta_1/2}\right]} = \dfrac{125.5}{\ln\left[\dfrac{\tan(22.5^\circ)}{\tan(15^\circ)}\right]}$

A = 288.12

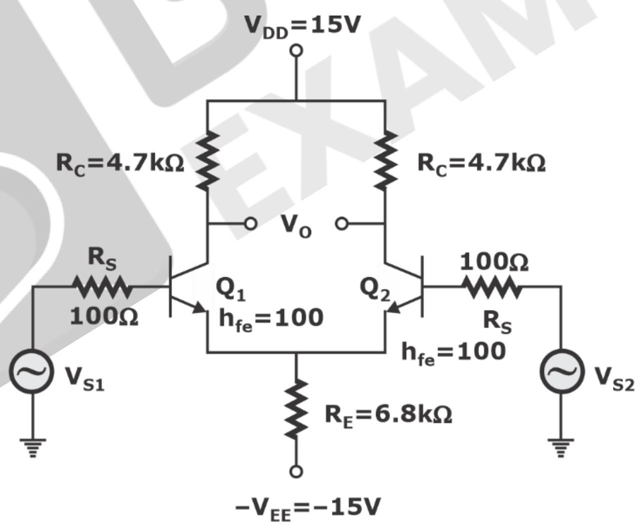
$$\sigma_2 = \frac{288.12}{r\sin\theta_2}$$

At θ_2 = 90°, $\sigma_2 = \dfrac{288\cdot 12}{r}$

8. c. A dual input, balanced output differential amplifier is configured using silicon transistors which are identical having h_{ie} = 2.8 kΩ as shown in the figure below.

(i) Calculate the differential gain, common mode gain and CMRR.

(ii) What is the peak-to-peak output voltage V_o, if V_{S1} is 50 mV peak-to-peak at 2 kHz and V_{S2} is 30 mV peak-to-peak at 2 kHz ?



Sol. 8c. (i) AC analysis DC Sources → 0

$h_{ie} = 2.8\ k\Omega \quad h_{fe} = 100$

$V_0 = A_d V_d + A_c V_c$ (1)

$A_d \rightarrow$ differential gain.

$A_C \rightarrow$ Common mode again

$$\text{CmRR} = \left|\frac{A_d}{A_C}\right| \qquad (2)$$

$$V_d = V_{s_1} - V_{s_2} \qquad (3)$$

$$V_c = \frac{V_{s_1} + V_{s_2}}{2} \qquad (4)$$

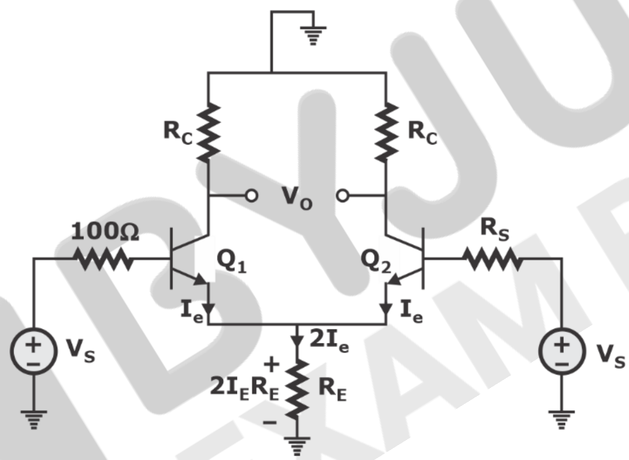
For Ac Let $V_{s_1} = V_{s_2} = V_s$

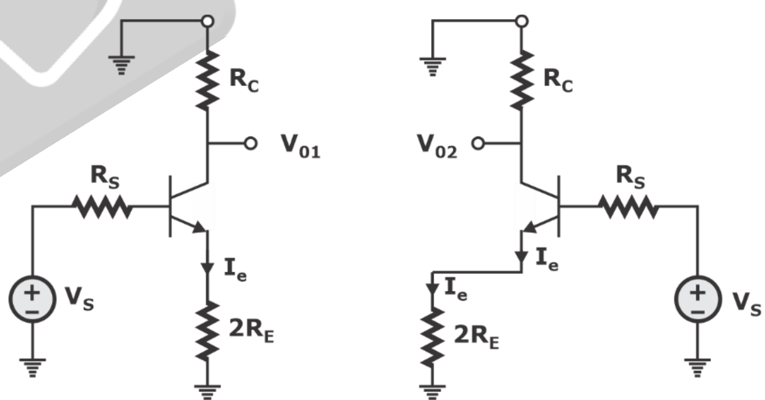
$\therefore V_d = 0 \quad V_c = V_s$

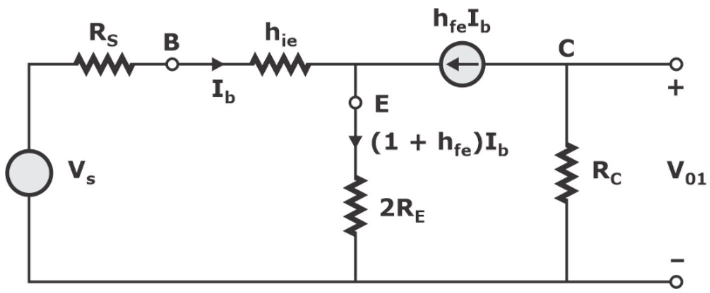
Put in (1)

$V_0 = A_d \times 0 + A_c V_S$

$$A_C = \frac{V_0}{V_S} \qquad (5)$$







Unbalance output

$$V_{01} = -h_{fe}I_bR_c$$

$$V_s = I_b\left(R_S + h_{ie}\right) + \left(1 + h_{fe}\right)I_b 2R_E$$

$$\therefore \ A_{c_1} = \frac{V_{01}}{V_s} = -\frac{h_{fe}R_c}{R_s + h_{ie} + \left(1 + h_{fe}\right) \cdot 2R_E}$$

If $\left(1 + h_{fe}\right)2R_E >> R_S + h_{ie}, h_{fe} >> 1$

$$A_{c_1} = \frac{-h_{fe}R_c}{\left(1 + h_{fe}\right)2R_E} \cong \frac{-R_c}{2R_E} \quad (7)$$

Unbalanced common mode gain.

Similarly $A_{C_2} = \frac{V_{0_2}}{V_s} = \frac{-h_{fe}R_c}{R_s + h_{ie} + \left(1 + h_{fe}\right)2R_E}$ (8)

Balanced common mode again

$$A_c = \frac{V_{01} - V_{02}}{V_s} = \frac{V_{0_1}}{V_s} - \frac{V_{02}}{V_s} = Ac_1 - A_{c_2}$$

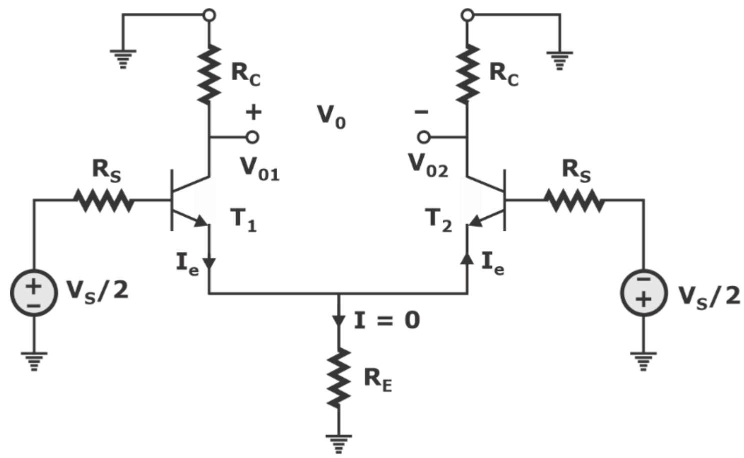
$A_C = 0$ (9)

Differential gain

$V_{s_1} = V_s / 2 \quad V_{s_2} = -V_s / 2$ Put in (3) & (4)

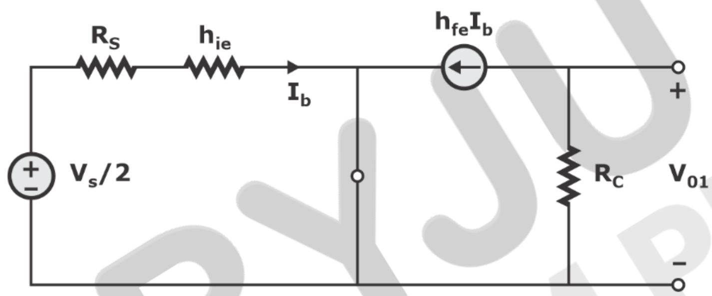
$\therefore V_d = V_s \quad V_c = 0$ Put in (1)

$$V_0 = A_dV_s + A_c \times 0 = A_dV_s \quad (10)$$



$\because IR_E = 0$ short circuit R_E

For transistor T_1



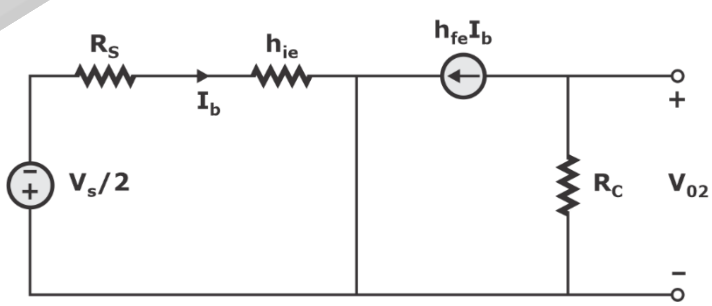
$$\frac{V_{01}}{V_s/2} = \frac{-h_{fe}I_bR_c}{I_b\left[R_s + h_{ie}\right]}$$

$$\frac{2V_{01}}{V_s} = \frac{-h_{fe}R_c}{R_s + h_{ie}}$$

$$\therefore A_{d_1} = \frac{V_{01}}{V_s} = \frac{-h_{fe}R_c}{2\left[R_s + h_{ie}\right]}$$

A_{d_1} difference gain for unbalanced output.

For transistor T_2



$$\frac{V_{02}}{-V_S/2} = \frac{-h_fI_bR_c}{I_b\left[R_s + h_{ie}\right]} = \frac{-h_{fe}R_C}{R_s + h_{ie}}$$

$$A_{d_2} = \frac{V_{02}}{V_s} = \frac{h_{fe}R_C}{2\left[R_s + h_{ie}\right]}$$

Difference gain for balanced output

$$A_d = \frac{V_{0_1} - V_{02}}{V_s} = \frac{V_{01}}{V_s} - \frac{V_{02}}{V_s}$$

$$A_d = A_{d_1} - A_{d_2}$$

$$A_d = \frac{-h_{fe}Rc}{R_s + h_{ie}}$$

$$A_d = \frac{-100 \times 4.7}{0.1 + 2.8} = -162.06$$

For unbalance o/p

$$A_d = \frac{-h_{fe}R_c}{2\left[R_s + h_{ie}\right]} = -81.03$$

$$A_C = \frac{-h_{fe}R_C}{R_S + h_{ie} + (1 + h_{fe}) \cdot 2R_E} = \frac{-100 \times 4.7}{0.1 + 2.8 + (1 + 100) \times 2 \times 6.8} = -0.341$$

$|A_C| = .341$

$$CMRR = \frac{|A_d|}{|A_c|}$$

For unbalanced op-amp

$$CMRR = \frac{81.03}{0.341} = 237.31$$

$$(CMRR)_{dB} = 20\log(237.31) = 47.5 \text{ dB}$$

For balanced op-amp

$$CMRR = \frac{|A_d|}{|A_C|} = \frac{162.06}{0} = \infty$$

Ideal differential Amplifier with $A_C = 0$ CMRR $\rightarrow \infty$.

(ii) $V_{S_1} = 50\,mV_{(P-P)}$ at $2\,kHz$

$V_{s_2} = 30\,mV_{(P-P)}$ at 2kHz

$$V_0 = A_dV_d + A_cV_c$$

$$V_{op\text{-}p} = -162.06\left[V_{s_1} - V_{s_2}\right] + 0 \times \left[\frac{V_{s_1} + V_{s_2}}{2}\right] = -162.06[50 - 30] \times 10^{-3}$$

= –3.24 volt at 2kHz

****