Unsteady State Conduction Study Notes for Chemical Engineering

Unsteady state conduction heat transfer refers to the transfer of heat through a solid medium when the temperature within the medium is changing with time. In this process, the temperature distribution within the medium is not constant and varies over time.

Unsteady state conduction occurs when there is a time-varying temperature gradient within a solid, leading to the flow of heat from regions of higher temperature to regions of lower temperature. This phenomenon is governed by the Fourier's law of heat conduction, which states that the heat flux (rate of heat transfer per unit area) is proportional to the negative gradient of temperature.

Unsteady State Conduction Heat Transfer

Transient State Systems

The process of heat transfer by conduction where the temperature varies with time and with space coordinates is called 'unsteady or transient'. All transient state systems may be broadly classified into two categories:

- Non-periodic Heat Flow System - the temperature at any point within the system changes as a nonlinear function of time.
- Periodic Heat Flow System - the temperature within the system undergoes periodic changes which may be regular or irregular but definitely cyclic.

There are numerous problems where changes in conditions result in transient temperature distributions and they are quite significant. Such conditions are encountered in - manufacture of ceramics, bricks, glass and heat flow to boiler tubes, metal forming, heat treatment, etc.

Biot and Fourier Modulus

- For an initially heated long cylinder (L >> R) placed in a moving stream of fluid at, $T_\infty < T_s$ (Ts= surface temperature of the solid cylinder) as shown In Fig. 3.1(a).
- The convective heat-transfer coefficient at the surface is h, we have,

$$Q = hA\,(T_s - T_\infty)$$

- This energy must be conducted to the surface, and therefore,

$$\mathbf{Q = -kA(dT / dr)_{r=R}}$$

or,

$$h(T_s - T_\infty) = -k(dT/dr)_{r=R} \approx -k(T_c - T_s)/R$$

where T_c is the temperature at the axis of the cylinder

By rearranging,

$$\mathbf{(T_s - T_c) / (T_s - T_\infty) = hR/k}$$

- The term, **hR/k**, is called the '**BIOT MODULUS**' or **BIOT NUMBER** abbreviated as **Bi**.
- It is a **dimensionless number** and is the **ratio of internal heat flow resistance to external heat flow resistance** and plays a fundamental role in transient conduction problems involving surface convection effects.
- It provides a measure of the temperature drop in the solid relative to the temperature difference between the surface and the fluid.

Note: For **Bi <<1**, it is reasonable to assume a uniform temperature distribution across a solid at any time during a transient process.

Fourier Modulus

- It is also a dimensionless number and is defined as

$$\mathbf{F_o = \alpha t/L^2}$$

where L is the characteristic length of the body, α is the thermal diffusivity, and t is the time

- The Fourier modulus measures the magnitude of the rate of conduction relative to the change in temperature, i.e., the unsteady effect.
- If Fo<< 1, the change in temperature will be experienced by a region very close to the surface.

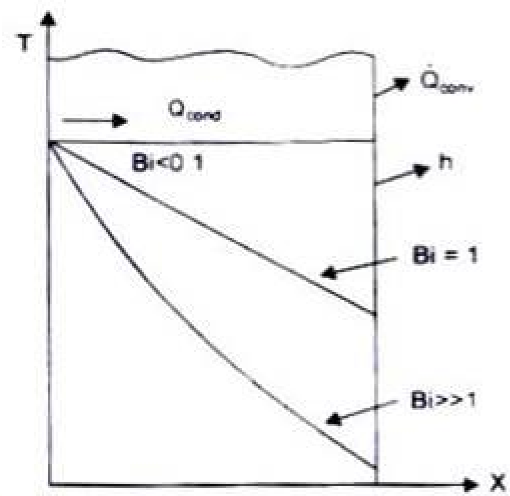

Fig. Effect of Biot Modulus on steady state temperature distribution in a plane wall with surface convection.

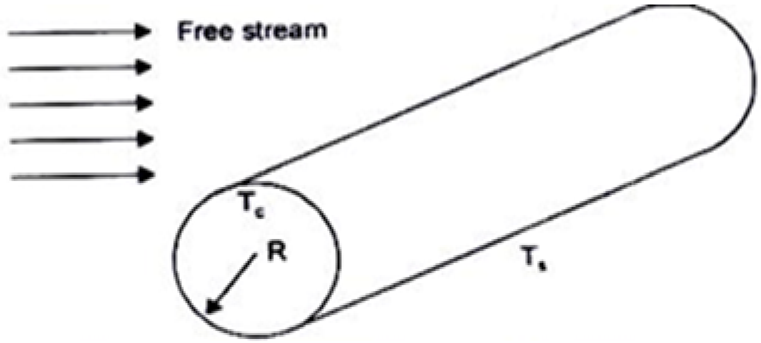

Fig.: Nomenclature for Biot Modulus

Lumped Capacity System Assumptions

- We know that a temperature gradient must exist in a material if heat energy is to be conducted into or out of the body.
- When **Bi < 0.1**, it is assumed that the internal thermal resistance of the body is very small in comparison with the external resistance and the transfer of heat energy is primarily controlled by the convective heat transfer at the surface T the temperature within the body is approximately uniform. This idealized assumption is possible, if
    - the physical size of the body is very small,
    - the thermal conductivity of the material is very large
    - the convective heat transfer coefficient at the surface is very small and there is a large temperature difference across the fluid layer at the interface.

Expression for the Temperature Variation in Solid Using Lumped Capacity

- Let us consider a small metallic object which has been suddenly immersed in a fluid during a heat treatment operation. By applying the first law of Thermodynamics

Heat flowing out of the body during time dt = Decrease in the internal thermal energy during time dt

$$hA_s(T - T_\infty) = -\rho VC\frac{dT}{dt}$$

where A_s is the surface area of the body, V is the volume of the body and C is the specific heat capacity.

or,

$$(hA/\rho CV)dt = -dT/(T - T_\infty)$$

initial condition being: at **t = 0, T = T_0**

final condition at **t=t, T =T,** integration gives $\frac{T-T_\infty}{T_o-T_\infty} = \exp(\frac{-hA}{\rho VC}t)$

The above equation can be used to calculate the temperature of the object after time t.

Fig. depicts the cooling of a body (temperature distribution time) using lumped thermal capacity system. The temperature history is seen to be an exponential decay.

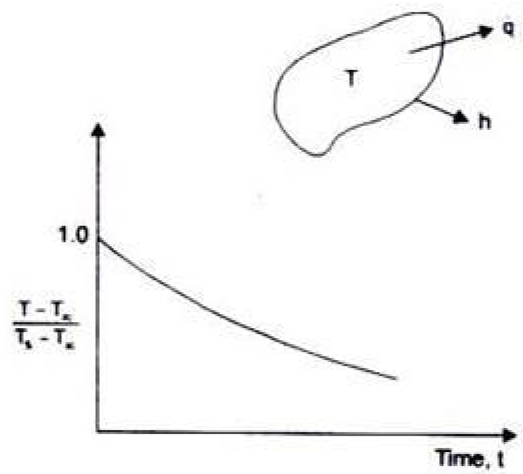

Figure: cooling of a body for Bi < 0.1

We can express

$$\mathbf{Bi \times Fo = (hL/k)\times(\alpha t/L^2) = (hL/k)(k/\rho C)(t/L^2) = (hA/\rho CV)t}$$

where **V / A is the characteristic length L**.

And, the solution describing the temperature variation of the object with respect to time is given by

$$\frac{T-T_\infty}{T_o-T_\infty} = \exp(-B_i \times F_o)$$

Overall Heat Transfer Coefficient

The heat transfer rate

- **q = UA (T_1-T_2)**

Where U= overall heat transfer coefficient (analogous to heat transfer h coefficient by convection)

- For **Composite wall** (resistance in series) of figure overall heat transfer coefficient is defined as

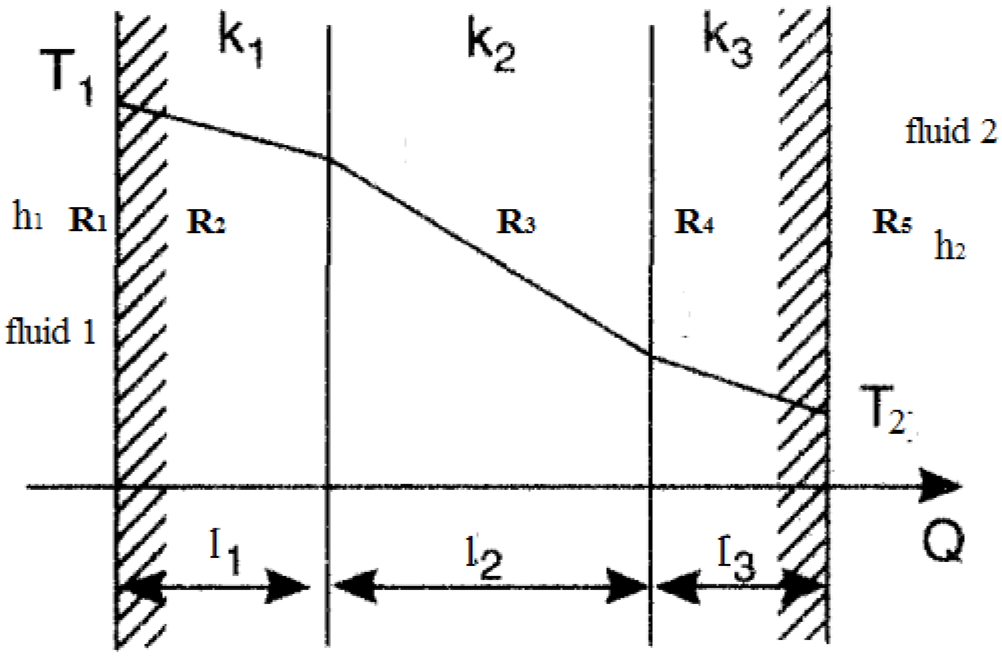

Figure: thermal resistances in series (composite wall)

- U = $\frac{1}{\left[\frac{1}{h_1}+\frac{l_1}{K_1}+\frac{l_2}{K_2}+\frac{l_3}{K_3}+\frac{1}{h_2}\right]}$

- $\frac{1}{U} = \frac{1}{h_1} + \frac{l_1}{k_1} + \frac{l_2}{k_2} + \frac{l_3}{k_3} + \frac{1}{h_2}$

In the situation of heat transfer through the composite wall area (A) of heat transfer is uniform,

Steps to calculate the overall heat transfer coefficient U are as follows

- Calculate the total thermal resistance i.e.
  - **$R_{th} = R_1+R_2+R_3+R_4+R_5$**
  - $R_{th} = \frac{1}{A}\left[\frac{1}{h_1} + \frac{l_1}{k_1} + \frac{l_2}{k_2} + \frac{l_3}{k_3} + \frac{1}{h_2}\right]$

- Multiply both side by heat transfer area A and take reciprocal (1/ R_{th} A) to get U

○ U = $\frac{1}{R_{th}A}$= $\frac{1}{\left[\frac{1}{h_1}+\frac{l_1}{K_1}+\frac{l_2}{K_2}+\frac{l_3}{K_3}+\frac{1}{h_2}\right]}$

Note:

- The above equation and steps for the calculation of overall heat transfer coefficient are true for **the constant area of heat transfer A**.
- If **A is varying with the radius** we will have to specify the area on which heat transfer coefficient is expressed.
- To understand this better, let us take an example of the composite cylinder as shown in figure below:

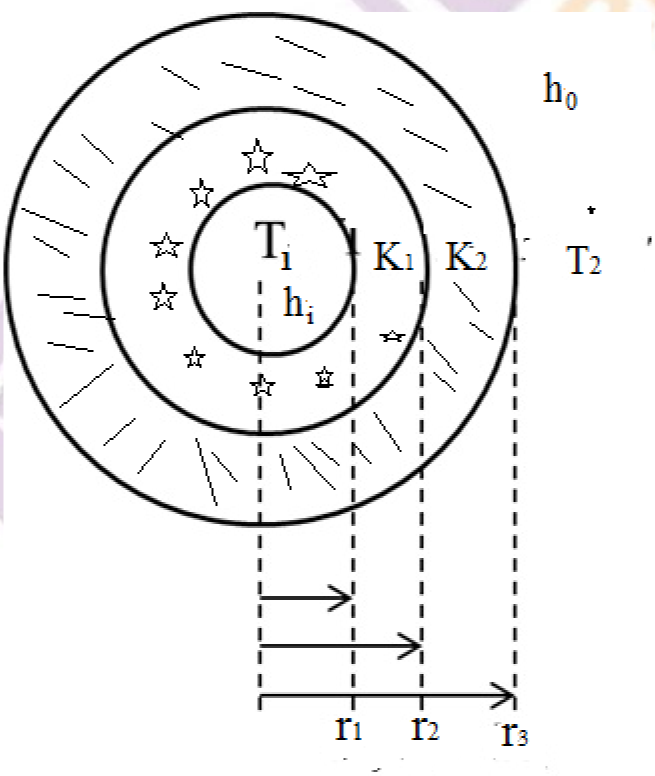

Figure: thermal resistances in series (composite hollow cylinders in series)

Here

- L = length of the composite cylinder
- T_1= temperature inside the hollow cylinder
- T_2= temperature outside the hollow cylinder

- h_i = heat transfer coefficient of inner surface and
- = heat transfer coefficient of outer surface

So, the total thermal resistance of hollow composite cylinder,

$$R_{th} = \frac{1}{(2\pi r_1 L)h_i} + \frac{\ln(r_2/r_1)}{2\pi k_1 L} + \frac{\ln(r_3/r_2)}{2\pi k_2 L} + \frac{1}{(2\pi r_3 L)h_0}$$

Now the heat transfer rate based on inner area $(2\pi\, r_1 L)$

$$\mathbf{Q_{r1} = U_{r1} \,.\, (2\pi r_1 L)\ \ (T_1 - T_2)}$$

So the overall heat transfer coefficient based on inner area ($2\ \pi r_1 L$) is calculated by multiplying ($2\ \pi\ r_1 L$) to both sides of equation and taking reciprocal i.e

$$\frac{1}{Ur_1} = \left[\frac{1}{h_i} + \frac{r_1}{k_1}\ ln(r_2/r_1) + \frac{r_1}{k_2}\ ln\left(\frac{r_3}{r_2}\right) + \frac{r_1}{r_3}\frac{1}{h_0}\right]$$

Thanks!

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