Working Stress Method

The Working Stress Method is a design methodology used in civil engineering for the analysis and design of reinforced concrete structures. It involves calculating the stresses and strains induced in the structure due to the applied loads and comparing them to the allowable stresses for the materials used in the construction of the structure.

The Working Stress Method assumes that both concrete and steel behave linearly elastic, meaning that their stresses are proportional to their strains. The allowable stresses for concrete and steel are determined based on their respective strengths and a factor of safety is applied to account for uncertainties in the material properties, construction methods, and other factors.

The methodology involves the selection of appropriate sections and reinforcement for a structure, based on the stress and strain analysis, and ensuring that the stresses induced in the materials under the expected loads are within the allowable limits. The method is widely used for the design of reinforced concrete structures such as buildings, bridges, and dams.

While the Working Stress Method is a well-established methodology, it has some limitations and disadvantages. It does not consider the ultimate strength of the materials or the possibility of failure due to non-linear behavior, which has led to the development of more advanced design methodologies such as the Limit State Design method.

Modular Ratio

$m = E_S/E_C$

where,

- m = Modular ratio
- E_S = Modulus of elasticity of steel
- E_C = Modulus of elasticity of concrete

Equivalent Area of Concrete

The concept of Equivalent Area of Concrete is used in the design of reinforced concrete beams using the Working Stress Method. It involves replacing the actual concrete section of the beam with an equivalent area of concrete that represents the same strength as the actual section but is easier to work with mathematically.

The equivalent area of concrete is determined by dividing the actual area of concrete into two parts: the area of concrete above the neutral axis (compression zone) and the area of concrete below the neutral axis (tension zone). The area of concrete in the compression zone is multiplied by a factor called the modular ratio, which is the ratio of the modulus of elasticity of steel to that of concrete.

$$A_C = mA_S$$

Here,

- A_C = Area of concrete
- A_S = Area of steel

Stress in Concrete, $p_c = p_s/m$

Where,

- p_C = stress in concrete(depends on various properties of concrete)
- p_S = stress in steel

Critical Depth of Neutral Axis (X_c)

The Critical Depth of the Neutral Axis is an important parameter used in the Working Stress Method for the design of reinforced concrete beams. It is the depth of the neutral axis or the location where the stress in the concrete changes from compression to tension, at which the maximum allowable moment occurs. The Critical Depth of the Neutral Axis can be calculated by the limit state method using the following equation:

$$d = (0.87\ f_y\ As) / (0.36\ f_{ck}\ b)$$

where:

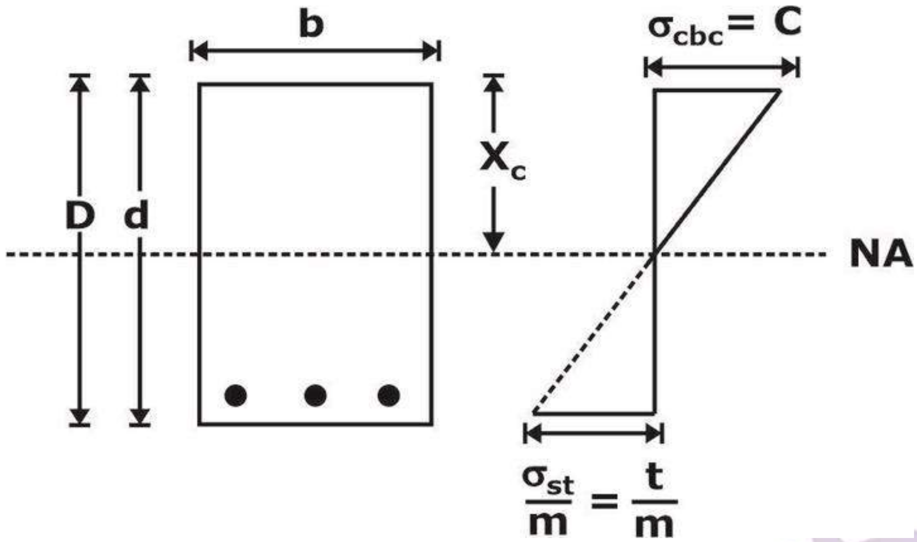
- d = Critical Depth of Neutral Axis
- f_y = Yield strength of steel reinforcement
- As = Area of steel reinforcement
- f_{ck} = Characteristic compressive strength of concrete
- b = Width of the beam.

In this equation, the numerator represents the resisting moment provided by the steel reinforcement, and the denominator represents the moment of resistance provided by the concrete. The value of 0.87 is a factor that accounts for the reduced strength of the steel reinforcement due to possible yielding under load, and the value of 0.36 is a factor that accounts for the reduced strength of the concrete due to uncertainties in its compressive strength.

If the actual depth of the neutral axis is greater than the critical depth, then the beam is considered under-reinforced and the steel reinforcement is not fully utilized. If the actual depth of the neutral axis is less than the critical depth, then the beam is considered over-reinforced and the concrete is not fully utilized.

The depth of the neutral axis as per the working state method can be calculated as follows:

$X_c = \{mc/(t+mc)\}d$



Here,

- D = Overall depth
- d = Effective depth
- σ_{cbc} = c = permissible stress in concrete
- σ_{st} = t = permissible stress in steel

Actual Depth of the Neutral axis (X_a)

$BX_a^2/2 = mA_{st}(d-X_a)$

Special case

(i) when $X_a = X_c$ for balanced section

(ii) when $X_a > X_c$ for over-reinforced section

(iii) when $X_a < X_c$ for under-reinforced section

Moment of Resistance (M_r)

The moment of resistance is a critical parameter used in the design of reinforced concrete beams using the Working Stress Method. It represents the maximum moment that the beam can resist without experiencing failure or reaching its maximum allowable stress.

The moment of resistance is calculated based on the strength of the materials used in the construction of the beam, including the concrete and steel reinforcement. The formula for calculating the moment of resistance is:

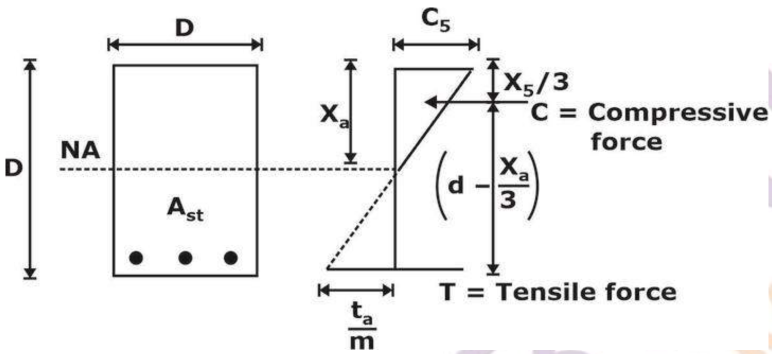
$$\textbf{MOR} = 0.87 \times f_y \times A_s \times (d - 0.42 \times x_u) + 0.36 \times f_{ck} \times (b \times x_u) \times (d - 0.42 \times x_u)$$

In this equation, the first term represents the moment of resistance provided by the steel reinforcement, while the second term represents the moment of resistance provided by the concrete. The factor of 0.87 is used to account for the reduction in strength of the steel due to possible yielding under load, while the factor of 0.36 is used to account for the reduction in strength of the concrete due to uncertainties in its compressive strength.

The moment of resistance is compared to the maximum bending moment expected in the beam to ensure that it is sufficient to resist the applied loads. If the moment of resistance is less than the expected bending moment, then the section of the beam needs to be redesigned with a larger effective depth or a higher amount of reinforcement to increase the moment of resistance.

MOR by the working stress method can be calculated as follows:

(i) For balanced section ($X_a = X_C$)

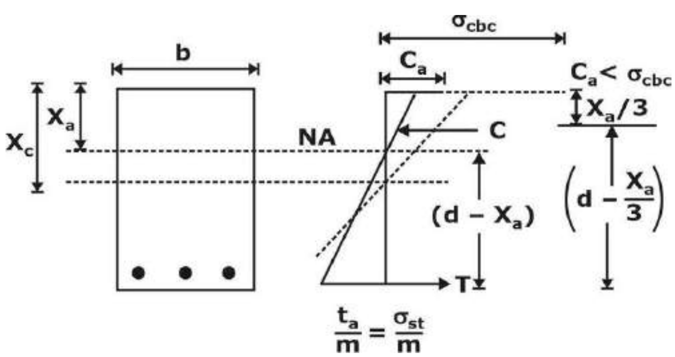


Section Stress diagram

$$M_r = bX_a \cdot \frac{\sigma_{cbc}}{2}\left(d - \frac{X_a}{3}\right) \quad \text{or} \quad M_r = \sigma_{st} \cdot A_{st}\left(d - \frac{X_a}{3}\right)$$

$$\left(d - \frac{X_a}{3}\right) = \text{Lever Arm}$$

(ii) For under reinforced section ($X_a < X_C$)

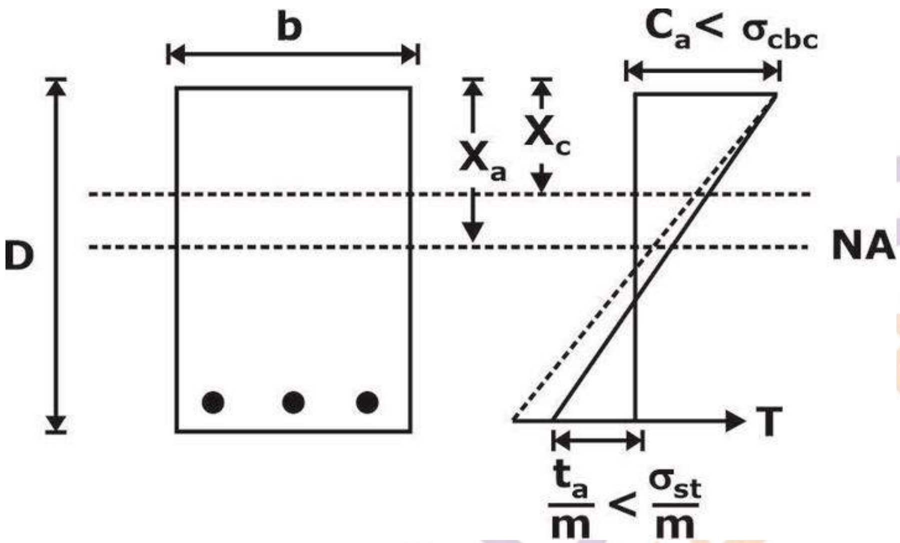


Section　　　　Stress diagram

$$\boxed{M_r = BX_a \cdot \frac{C_a}{2}\left(d - \frac{X_a}{3}\right)} \text{ or } \boxed{M_r = \sigma_{st} \cdot A_{st}\left(d - \frac{X_a}{3}\right)}$$

$$\boxed{C_a = \frac{\sigma_{st} \cdot X_a}{m(d - X_a)}} \quad \text{Here, } C_a < \sigma_{cbc}$$

(iii) For over-reinforced section ($X_a > X_C$)



$$\frac{\sigma_{cbc}}{X_a} = \frac{t_a / m}{d - x_a}$$

$$M_r = bX_a \cdot \frac{\sigma_{cbc}}{2}\left(d - \frac{x_a}{3}\right)$$

$$M_r = t_a A_{st}\left(d - \frac{x_a}{3}\right)$$