Kirchhoff's Law

In 1847, Gustav Robert Kirchhoff's introduced a pair of laws based on the law of conservation of charge and energy in an electrical circuit. The equivalent impedance of any complex network or circuit can easily be calculated using Kirchhoff's Law.

Define Kirchhoff's Law

The current or voltage of any circuit branch can also be calculated using Kirchhoff's Law. These laws are valid in AC and DC networks at low frequencies.

Types of Kirchhoff's Law

Kirchhoff's Law is defined based on the usage and application of the law. Kirchhoff's laws are classified into two types:

- Kirchhoff's Current Law (KCL)
- Kirchhoff's Voltage Law (KVL)

Kirchhoff's Current Law

Kirchhoff's current law is also known as Kirchhoff's First law or Kirchhoff's Law of the junction, but the most used term is Kirchhoff's Current Law or KCL. KCL is based on the law of conservation of charge.

Define Kirchhoff's Current Law

Kirchhoff's current law states that the algebraic sum of currents entering a node or a closed boundary equals zero.

If there are N number of branches connected to a node and it is the current of the nth branch, then mathematically, KCL states,

$$\sum_{n=1}^{N} i_n = 0$$

In this law, current entering a node is taken as positive, and current leaving a node is taken as negative or vice versa. Assume a set of currents ikt where k = 1, 2, 3 . . . . n flow into a node. Let $i_T(t)$ be the total current then,

$$i_T(t) = i_1(t) + i_2(t) + i_3(t) + \ ......\ i_n(t)$$

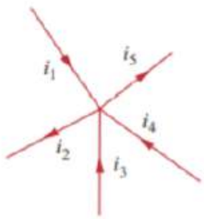
On integrating both sides,

$$Total\ charge = q_T(t) = q_1(t) + q_2(t) + q_3(t) + \ ......\ q_n(t)$$

An electrical node doesn't store any charge. So,

$$q_T(t) = q_1(t) + q_2(t) + q_3(t) + \ ......\ q_n(t) = 0$$

This shows that KCL is based on the law of conservation of charge. Consider the node shown below,



Applying KCL to the above node,

$$-i_1 + i_2 - i_3 - i_4 + i_5 = 0$$

$$i_2 + i_5 = i_1 + i_3 + i_4$$

Current leaving = current entering

So, an alternate statement for KCL is that the sum of currents entering a node equals the sum of currents leaving the same node.

- Download Formulas for GATE Electronics & Communication Engineering - Control System
- Download Formulas for GATE Electronics & Communication Engineering - Digital Circuits
- Download Formulas for GATE Electronics & Communication Engineering - Electronic Devices

Application of Kirchhoff's Current Law

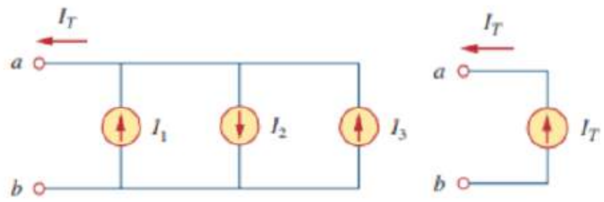
KCL is used to combine the current sources present in parallel. The overall equivalent current is the algebraic sum of individual currents present in parallel, as shown below:

Applying KCL at node a,

$$I_T - I_1 + I_2 - I_3 = 0$$

$$I_T = I_1 - I_2 + I_3$$

So, the above circuit can be modified into an equivalent circuit with a single current source as shown below:



Validity of Kirchhoff's Current Law

There exist some conditions where KCL is valid, and in some cases, it is not valid. Those conditions are:

- KCL is independent of the variation in temperature in the circuit.
- KCL is valid for linear, non-linear, bilateral, unilateral, passive, and active elements.
- KCL is valid for lumped electrical networks only, not for distributed electrical networks. At high frequencies, the circuit is treated as distributed and not lumped, and the effect of parasitic resistance cannot be ignored, so KCL is invalid at high frequencies.
- Kirchhoff's law is not valid for time-varying magnetic fields.

Kirchhoff's Voltage Law

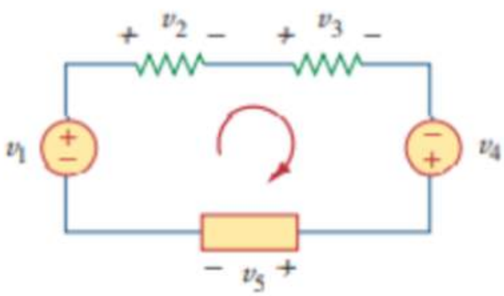
Kirchhoff's Voltage Law is also known as Kirchhoff's Second law or KVL. KVL is based on the law of conservation of energy.

Define Kirchhoff's Voltage Law

Kirchhoff's Voltage Law states that the algebraic sum of voltages around a closed path or loop in a circuit equals zero. If there are M number of voltages in a loop and V_m is the m^{th} voltage, then mathematically, KVL can be written as:

$$\sum_{n=1}^{M} V_m = 0$$

The sign of each voltage is taken as the polarity of the terminal encountered first as we travel around the loop. We can go around the loops in either clockwise or anti-clockwise directions. Consider the figure shown below:



From the figure, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_2 + v_3 + v_5 = v_1 + v_4$$

$$Sum\ of\ voltage\ drop = Sum\ of\ voltage\ rises$$

This is an alternate statement of KVL that in a closed loop the sum of voltages drop across elements is equal to the sum of voltage rises from the element.

Now,

$$v = \frac{dW}{dq}\ , \quad \frac{Joule}{Coulomb} = volt$$

So, from the above circuit, KVL equation can be modified as,

$$-\frac{dW_1}{dq} + \frac{dW_2}{dq} + \frac{dW_3}{dq} - \frac{dW_4}{dq} + \frac{dW_5}{dq} = 0$$

On integrating,

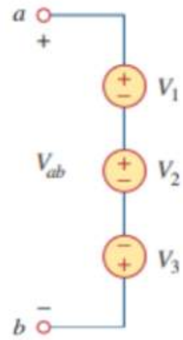
$$-W_1 + W_2 + W_3 - W_4 + W_5 = 0$$

$$W_2 + W_3 + W_5 = W_1 + W_4$$

This shows that KVL is based on the law of conservation of energy.

Application of Kirchhoff's Voltage Law

KVL is used to combine the voltage sources present in series. The overall equivalent voltage is the algebraic sum of individual voltage present in series as shown below:

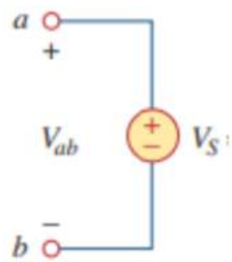


Applying KVL in the above loop,

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

$$V_{ab} = V_1 + V_2 - V_3$$

So, the above circuit can be modified into an equivalent circuit with a single voltage source as shown below:



Validity of Kirchhoff's Voltage Law

There exist some conditions where KVL is valid, and in some cases, it is not valid. Those conditions are:

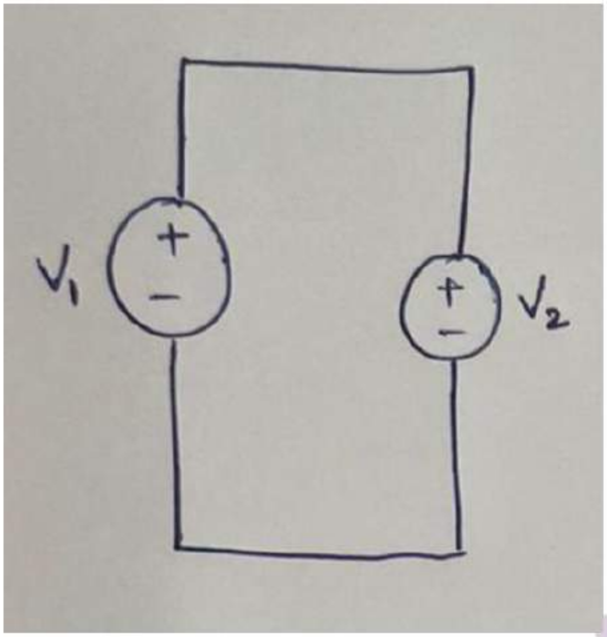
- KVL is independent of the variation in temperature in the circuit.
- KVL is valid for linear, non-linear, bilateral, unilateral, passive, and active elements.
- KVL is only valid for lumped electrical networks, not for distributed electrical networks.

Validity of Kirchhoff's Law

Kirchhoff's Laws are valid for lumped networks, not for distributed networks. There are some hypothetical cases where KCL and KVL are not satisfied.

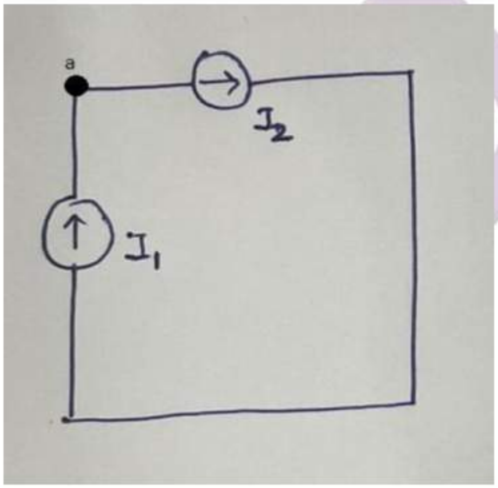
- Two unequal voltage sources are placed in parallel to each other.

If $V_1 \neq V_2$, then the KVL violates because KVL states that the sum of voltages in a closed loop must be zero.



- Two unequal current sources are placed in series to each other.

If $I_1 \neq I_2$, the KCL violates because the current entering at node a will not be equal to the current leaving at node a.



Kirchhoff's Law Formula

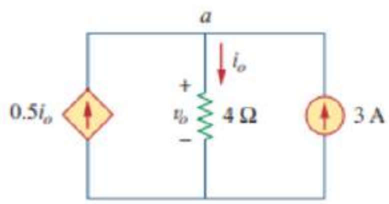
Kirchhoff's law comprises KVL and KCL. KVL states that the algebraic sum of voltages around a closed loop equals zero. KCL states that the algebraic sum of currents entering a node is equal to zero. Mathematically, KCL and KVL can be written as:

$$KVL, \quad \sum_{m=1}^{M} V_m = 0 \quad where\ M = number\ of\ voltages\ in\ a\ closed\ loop$$

$$KCL, \quad \sum_{n=1}^{N} i_n = 0 \quad where\ N = number\ of\ branches\ connected\ to\ a\ node$$

Kirchhoff's Law Examples

Example 1: Find current i_0 and voltage v_0 in the circuit shown below



Solution: Applying KCL at node a,

$$-0.5i_0 + i_0 - 3 = 0$$

$$0.5i_0 = 3$$

$$i_0 = 6\,A$$

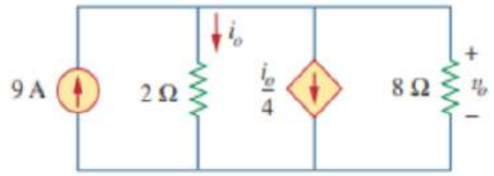
From the ohm's law,

$$v_0 = 4\,i_0$$

$$v_0 = 4\times6$$

$$v_0 = 24\,V$$

Example 2: Find current i_0 and voltage v_0 in the circuit shown below



Solution: Applying KCL equation at upper node,

$$-9 + i_0 + \frac{i_0}{4} + \frac{v_0}{8} = 0$$

$$i_0 + \frac{i_0}{4} + \frac{v_0}{8} = 9$$

Also from the ohm's law,

$$v_0 = 2i_0$$

Put the value of v_0 in the above equation,

$$i_0 + \frac{i_0}{4} + \frac{2i_0}{8} = 9$$

$$\frac{8i_0 + 2i_0 + 2i_0}{8} = 9$$

$$\frac{12\, i_0}{8} = 9$$

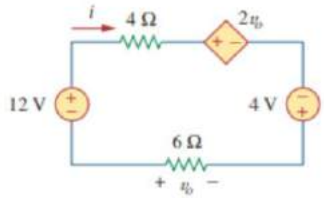
$$i_0 = \frac{9 \times 2}{3}$$

$$i_0 = 6\, A$$

So,

$$v_0 = 2\, i_0 = 2 \times 6 = 12\, V$$

Example 3: Determine current i and voltage v_0 in the circuit shown below



Solution: Applying KVL to the loop,

$$12 - 4i - 2v_0 + 4 + v_0 = 0$$

$$16 = 4i + v_0$$

From the ohm's law,

$$v_0 = -6i$$

Put the value of v_0 in the above equation,

$$16 = 4i + -6i$$

$$16 = -2i$$

$$i = -8\,A$$

This negative sign implies that the direction of current is opposite of the direction which is given or assumed.

So,

$$v_0 = -6i$$

$$v_0 = -6 \times -8$$

$$v_0 = 48\,V$$