BYJU'S EXAM PREP

GATE 2023

Civil Engineering

Afternoon Shift

Questions with Detailed Solutions

General Aptitude

1. The line ran ________ the page, right through the centre, and divided the page into two.

A. Across B. of

C. Between D. About

[MCQ – 1 Mark]

Ans. A

Sol. The line ran across the page, right through the centre, and divided the page into two.

2. Kind : _________ : : Often : Seldom

(By word meaning)

A. Cruel B. Variety

C. Type D. Kindred

[MCQ – 1 Mark]

Ans. A

Sol. Kind : Cruel : : Often : Seldom

Often means regular repetition.

Seldom means very rarely occurring.

Hence, often and seldom are the opposite.

The correct opposite of kind is cruel.

Kindred means similar or related.

3. In how many ways can cells in a 3×3 grid be shaded, such that each row and each column have exactly one shaded cell? An example of one valid shading is shown.

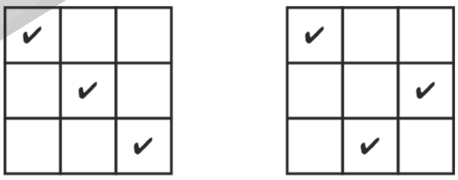
A. 2 B. 9

C. 3 D. 6

[MCQ – 1 Mark]

Ans. D

Sol.



$3 \times 2 \times 1 = 6$

4. There are 4 red, 5 green, and 6 blue balls inside a box. If N number of balls are picked simultaneously, what is the smallest value of N that guarantees there will be at least two balls of the same colour?

One cannot see the colour of the balls until they are picked.

A. 4 B. 15

C. 5 D. 2

[MCQ – 1 Mark]

Ans. A

Sol. 3 varieties of balls are there i.e. R, G & B

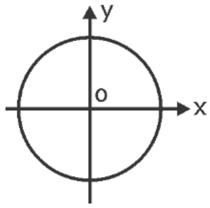
If we pick up 4 balls, one ball will repeat.

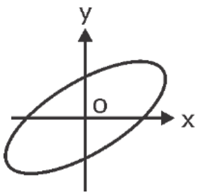
5. Consider a circle with its centre at the origin (O), as shown. Two operations are allowed on the circle.

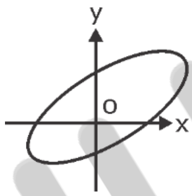
Operation 1: Scale independently along the x and y axes.

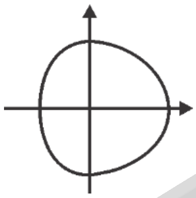
Operation 2: Rotation in any direction about the origin.

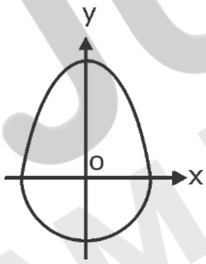
Which figure among the options can be achieved through a combination of these two operations on the given circle?



A. 

B. 

C. 

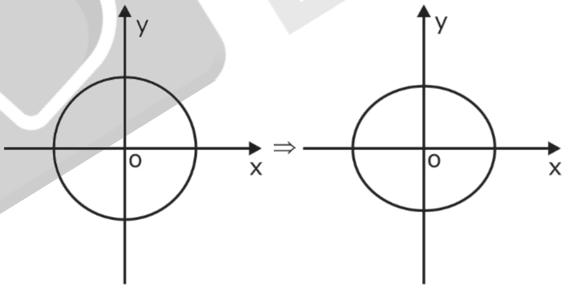
D. 

[MCQ – 1 Mark]

Ans. A

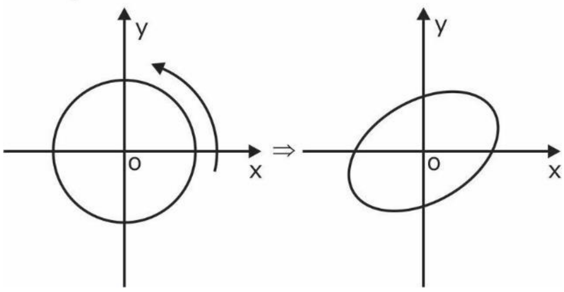
Sol.

Operation 1:



Suppose $S_{x\text{-axis}} > S_{x\text{-axis}}$

Operation 2:



6. Elvesland is a country that has peculiar beliefs and practices. They express almost all their emotions by gifting flowers. For instance, if anyone gifts a white flower to someone, then it is always taken to be a declaration of one's love for that person. In a similar manner, the gifting of a yellow flower to someone often means that one is angry with that person.

Based only on the information provided above, which one of the following sets of statement(s) can be logically inferred with certainty?

(i) In Elvesland, one always declares one's love by gifting a white flower.

(ii) In Elvesland, all emotions are declared by gifting flowers.

(iii) In Elvesland, sometimes one expresses one's anger by gifting a flower that is not yellow.

(iv) In Elvesland, sometimes one expresses one's love by gifting a white flower.

A. only (ii)
B. (i), (ii) and (iii)
C. (i), (iii) and (iv)
D. only (iv)

[MCQ – 2 Marks]

Ans. D

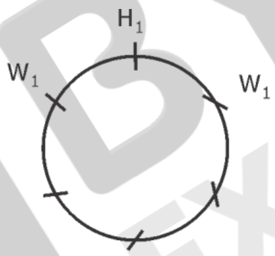
7. Three husband-wife pairs are to be seated at a circular table that has six identical chairs. Seating arrangements are defined only by the relative position of the people. How many seating arrangements are possible such that every husband sits next to his wife?

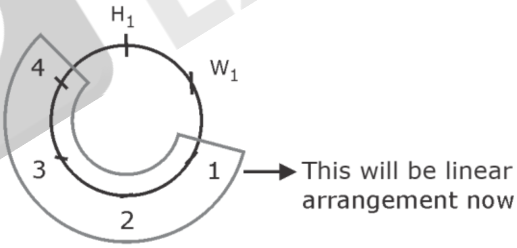
A. 16
B. 4
C. 120
D. 720

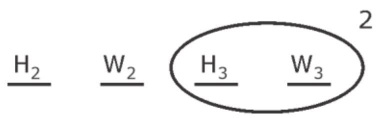
[MCQ – 2 Marks]

Ans. C

Sol.



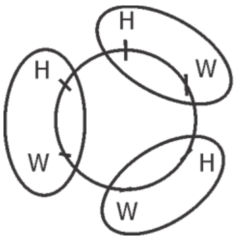




$= 2 \times 2 \times 2$

So, total no. = 2 × 2 × 2 × 2

Or



$= (3! - 1) \times 2 \times 2 \times 2$

$= 16$

8. Based only on the following passage, which one of the options can be inferred with certainty?

When the congregation sang together, Apenyo would also join, though her little screams were not quite audible because of the group singing. But whenever there was a special number, trouble would begin; Apenyo would try singing along, much to the embarrassment of her mother. After two or three such mortifying Sunday evenings, the mother stopped going to church altogether until Apenyo became older and learnt to behave.

At home too, Apenyo never kept quiet; she hummed or made up silly songs to sing by herself, which annoyed her mother at times but most often made her become pensive. She was by now convinced that her daughter had inherited her love of singing from her father who had died unexpectedly away from home.

A. The mother was embarrassed about her daughter's singing at home.

B. The mother's feelings about her daughter's singing at home were only of annoyance.

C. The mother was not sure if Apenyo had inherited her love of singing from her father.

D. When Apenyo hummed at home, her mother tended to become thoughtful.

[MCQ – 2 Marks]

Ans. B

9. If x satisfies the equation $4^{8^x} = 256$, then x is equal to ________ .

A. $\frac{1}{2}$

B. $\log_{16}8$

C. $\frac{2}{3}$

D. $\log_{4}8$

[MCQ – 2 Marks]

Ans. C

Sol.

$$4^{8^x} = 256$$

$$4^{8^x} = 4^4$$

or

$$8^x = 4$$

$$\left(2^3\right)^x = 2^2$$

$$2^{3x} = 2^2$$

or $$3x = 2$$

or $$x = \frac{2}{3}$$

10. Consider a spherical globe rotating about an axis passing through its poles. There are three points P, Q, and R situated respectively on the equator, the north pole, and midway between the equator and the north pole in the northern hemisphere. LetP, Q, and R move with speeds v_P, v_Q and v_R, respectively.

Which one of the following options is CORRECT?

A. $v_P < v_R < v_Q$

B. $v_P < v_Q < v_R$

C. $v_P < v_R < v_Q$

D. $v_P = v_R \neq v_Q$

[MCQ – 2 Marks]

Ans. C

Civil Engineering

11. Let ϕ be a scalar field, and u be a vector field. Which of the following identities is true for div(ϕu)?

A. div (ϕu) = ϕdiv(u) + u grad (ϕ)

B. div (ϕu) = ϕdiv(u) + u × grad (ϕ)

C. div (ϕu) = ϕgrad(u) + u grad (ϕ)

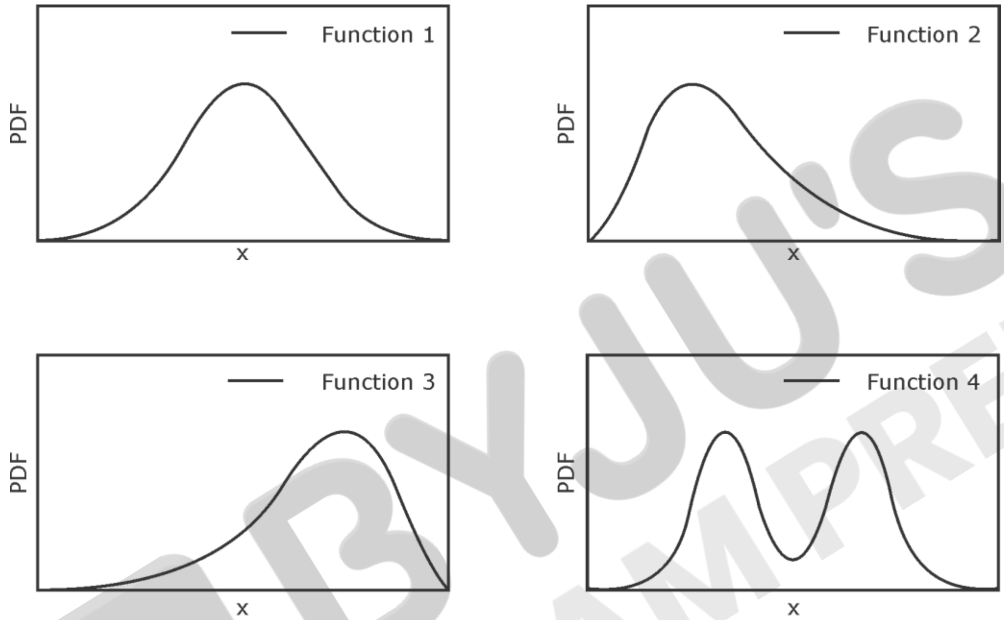
D. div (ϕu) = ϕgrad(u) + u × grad (ϕ)

[MCQ – 1 Mark]

Ans. A

Sol. div (φu) = φdiv(u) + u.grad (φ) is a special property of vector.

12. Which of the following probability distribution functions (PDFs) has the mean greater than the median?



A. Function 1

B. Function 2

C. Function 3

D. Function 4

[MCQ – 1 Mark]

Ans. B

Sol. For positively skewed PDF, mean > median (Function 2)

For negatively skewed PDF, mean < median (Function 3)

For symmetrical PDF, mean = median (Function 1)

13. A remote village has exactly 1000 vehicles with sequential registration numbers starting from 1000. Out of the total vehicles, 30% are without pollution clearance certificate. Further, even-and odd-numbered vehicles are operated on even-and odd-numbered dates, respectively.

If 100 vehicles are chosen at random on an even-numbered date, the number of vehicles expected without pollution clearance certificate is________.

A. 15

B. 30

C. 50

D. 70

[MCQ – 1 Mark]

Ans. B

Sol. Since 30% of the total vehicles are without pollution clearance certificate.

∴ The number of vehicles expected without pollution clearance certificate out of 100 vehicles,

$$= 100 \times 0.3$$
$$= 30$$

14. A circular solid shaft of span L = 5 m is fixed at one end and free at the other end. A torque T = 100 kN.m is applied at the free end. The shear modulus and polar moment of inertia of the section are denoted as G and J, respectively. The torsional rigidity GJ is 50,000 kN.m2/rad. The following are reported for this shaft:

Statement–I: The rotation at the free end is 0.01 rad

Statement–II: The torsional strain energy is 1.0 kN.m

With reference to the above statements, which of the following is true?

A. Both the statements are correct

B. Statement (I) is correct, but Statement II) is wrong

C. Statement (I) is wrong, but Statement II) is correct

D. Both the statements are wrong

[MCQ – 1 Mark]

Ans. B

Sol. Given

Length of shaft = 5m, Applied torque = 100 kNm, Torsional Rigidity = 50,000kN.m^2/rad

Statement-I

Rotation at the free end (θ) = $\frac{\tau L}{GJ} = \frac{100 \times 5}{50,000}$ = 0.01 rad

Statement -II

Torsional strain energy = $\frac{1}{2} \times T \times \theta = \frac{1}{2}$ × 100 × 0.01 = 0.5 kN/m.

15. M20 concrete as per IS 456: 2000 refers to concrete with a design mix having __________

A. an average cube strength of 20 MPa

B. an average cylinder strength of 20 MPa

C. a 5-percentile cube strength of 20 MPa

D. a 5-percentile cylinder strength of 20 MPa

[MCQ – 1 Mark]

Ans. C

Sol. M20 concrete as per IS 456:2000 refers to the concrete with a design mix having a 5-percentile cube strength of 20 MPa.

16. When a simply-supported elastic beam of span L and flexural rigidity EI (E is the modulus of elasticity and I is the moment of inertia of the section) is loaded with a uniformly distributed load w per unit length, the deflection at the mid-span is $\Delta_0 = \frac{5}{384}\frac{wL^4}{EI}$.

If the load on one half of the span is now removed, the mid–span deflection _____ .

A. reduces to $\Delta_0/2$

B. reduces to a value less than $\Delta_0/2$

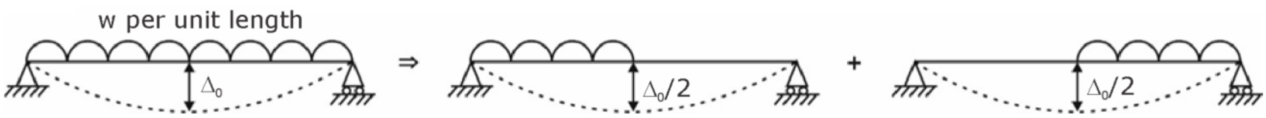
C. reduces to a value greater than $\Delta_0/2$

D. remains unchanged at Δ_0

[MCQ – 1 Mark]

Ans. A

Sol.



Total deflection due to loading on entire span = Deflection due to load on left half + deflection due to load on right half

Total deflection on SSB due to UDL over the entire span $\Delta_0 = \frac{5}{384}\frac{WL^4}{EI}$

Deflection due to half portion loading. $\Delta' = \frac{\Delta_0}{2} = \frac{1}{2}\left(\frac{5}{384}\frac{WL^4}{EI}\right)$

So, midspan deflection reduced to $\Delta_0/2$

17. Muller-Breslau principle is used in analysis of structures for ______________.

A. drawing an influence line diagram for any force response in the structure

B. writing the virtual work expression to get the equilibrium equation

C. superposing the load effects to get the total force response in the structure

D. relating the deflection between two points in a member with the curvature diagram in-between

[MCQ – 1 Mark]

Ans. A

Sol. Muller-Breslau principle is used to draw ILD for any force response in the structure.

18. A standard penetration test (SPT) was carried out at a location by using a manually operated hammer dropping system with 50% efficiency. The recorded SPT value at a particular depth is 28. If an automatic hammer dropping system with 70% efficiency is used at the same location, the recorded SPT value will be _______.

A. 28

B. 20

C. 40

D. 25

[MCQ – 1 Mark]

Ans. B

Sol. Efficiency $(\eta) \propto \frac{1}{\text{SPT value}(N)}$

$$\eta \times N = \text{constant}$$

$$\eta_1 \times N_1 = \eta_2 \times N_2$$

$$0.3 \times 28 = 0.7 \times N_2$$

$$N_2 = 20$$

19. A vertical sheet pile wall is installed in an anisotropic soil having coefficient of horizontal permeability, k_H and coefficient of vertical permeability, k_V. In order to draw the flow net for the isotropic condition, the embedment depth of the wall should be scaled by a factor of ______, without changing the horizontal scale.

A. $\sqrt{\frac{k_H}{k_V}}$

B. $\sqrt{\frac{k_V}{k_H}}$

C. 1.0

D. $\sqrt{\frac{k_H}{k_V}}$

[MCQ – 1 Mark]

Ans. A

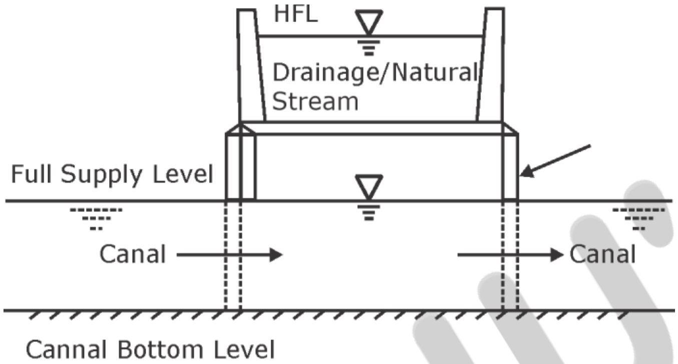
Sol. As we know to change the horizontal distance in an anisotropic medium,

$$X_T = X\sqrt{\frac{k_V}{k_H}}$$

But as per the question we cannot change the horizontal scale, hence we have to change the vertical length. So, we have to multiply depth with $\sqrt{\frac{k_H}{k_V}}$ to get the desired result.

Note: In the question, first they have mentioned soil as anisotropic then asked for isotropic condition. To transform the depth, we have to multiply $\sqrt{\frac{k_H}{k_V}}$. In isotropic conditions, $k_H = k_V$, hence the factor becomes 1. So, the correct answer can be A and C.

20. Identify the cross-drainage work in the figure.



A. Super passage
B. Aqueduct
C. Siphon aqueduct
D. Level crossing

[MCQ – 1 Mark]

Ans. A

Sol. Cross drainage works where the bed level of the stream is sufficiently above the FSL of canal is called super-passage.

21. Which one of the following options provides the correct match of the terms listed in Column-1 and Column-2?

Column–1		Column–2	
P	Horton equation	I	Precipitation
Q	Muskingum method	II	Flood frequency
R	Penman method	III	Evapotranspiration
		IV	Infiltration
		V	Channel routing

A. P-IV, Q-V, R-III
B. P-III, Q-IV, R-I
C. P-IV, Q-III, R-II
D. P-III, Q-I, R-IV

[MCQ – 1 Mark]

Ans. A

Sol. Horton's equation → used in Infiltration calculation.
Muskingum method → used in channel routing.
Penman's equation → used in evapotranspiration calculation.

22. In the context of Municipal Solid Waste Management, 'Haul' in 'Hauled Container System operated in conventional mode' includes the __________.

A. time spent by the transport truck at the disposal site
B. time spent by the transport truck in traveling between a pickup point and the disposal site with a loaded container
C. time spent by the transport truck in picking up a loaded container at a pickup point
D. time spent by the transport truck in driving from the depot to the first pickup point

[MCQ – 1 Mark]

Ans. B

Sol. 'Haul' in 'Hauled Container System operated in conventional mode' includes the time spent by the transport truck in traveling between a pickup point and the disposal site with a loaded container. The time spent at the disposal site isn't included in this.

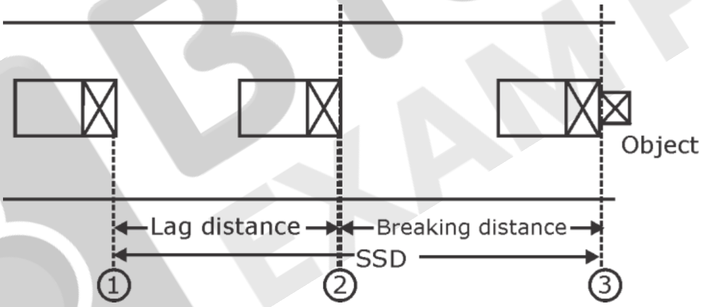
23. Which of the following is equal to the stopping sight distance?

A. (braking distance required to come to stop) + (distance travelled during the perception-reaction time)
B. (braking distance required to come to stop) – (distance travelled during the perception-reaction time)
C. (braking distance required to come to stop)
D. (distance travelled during the perception-reaction time)

[MCQ – 1 Mark]

Ans. A

Sol.



SSD= Lag Distance + Breaking Distance
Breaking Distance =Braking distance required to come to stop
Lag Distance = Distance travelled during the perception-reaction time

24. The magnetic bearing of the sun for a location at noon is 183° 30′. If the sun is exactly on the geographic meridian at noon, the magnetic declination of the location is.

A. 3°30′W
B. 3°30′E
C. 93°30′W
D. 93°30′E

[MCQ – 1 Mark]

Ans. A

Sol. When the sun is exactly on the geographic meridian at noon, True bearing= 180°

Now, Magnetic Declination = True bearing – Magnetic bearing
= 180° – 183°30′
= −3°30′ = 3°30′ W

25. For the matrix

$$[A] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

which of the following statements is/are TRUE?

A. [A]{x}={b} has a unique solution

B. [A]{x}={b} does not have a unique solution

C. [A] has three linearly independent eigenvectors

D. [A] is a positive definite matrix

[MSQ – 1 Mark]

Ans. B, C

Sol.

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix} = 0$$

$$(1 - \lambda)\,[(2 - \lambda)\,(1 - \lambda) - 1] + 1\,[\lambda - 1] = 0$$

$$(1 - \lambda)\,[(2 - \lambda)\,(1 - \lambda) - 1 - 1] = 0$$

$$\lambda\,(1 - \lambda)\,(3 - \lambda) = 0$$

$$\lambda = 0,\ 1,\ 3$$

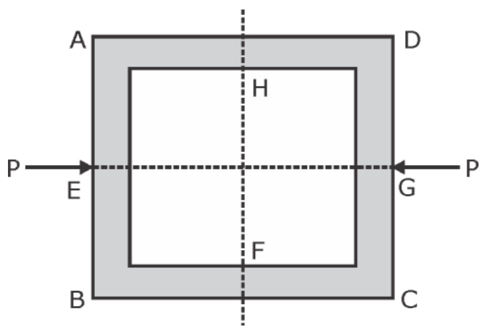
As there are three eigen values so number of linearly independent eigenvectors are 3.

Since, $|A| = 0$

So, [A]{x} = {b} does not have a unique solution

Now, for a positive matrix, all the eigenvalues of the matrix have to be positive but one of the $\lambda = 0$. So, [A] is not a positive definite matrix.

26. In the frame shown in the figure (not to scale), all four members (AB, BC, CD, and AD) have the same length and same constant flexural rigidity. All the joints A, B, C, and D are rigid joints. The midpoints of AB, BC, CD, and AD, are denoted by E, F, G, and H, respectively. The frame is in unstable equilibrium under the shown forces of magnitude P acting at E and G. Which of the following statements is/are TRUE?



A. Shear forces at H and F are zero

B. Horizontal displacements at H and F are zero

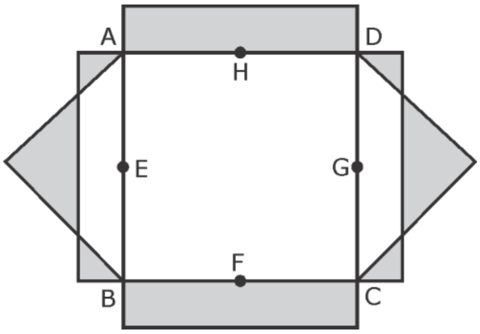
C. Vertical displacements at H and F are zero

D. Slopes at E, F, G, and H are zero

[MSQ – 1 Mark]

Ans. A, B and D

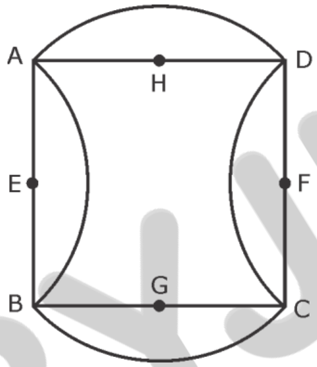
Sol. BMD for symmetrical box frame–



At H and F, BM is constant.

⇒ SF = 0 (option A is correct)

Deflection diagram for symmetrical box frame



From the diagram, it is clear that-

→ δ_v at H and F ≠ 0 (option B is incorrect)

→ θ at E, F, G, H = 0 (option C is correct)

→ δ_H at H and F = 0 (option D is correct)

27. With regard to the shear design of RCC beams, which of the following statements is/are TRUE?

A. Excessive shear reinforcement can lead to compression failure in concrete

B. Beams without shear reinforcement, even if adequately designed for flexure, can have brittle failure

C. The main (longitudinal) reinforcement plays no role in the shear resistance of beam

D. As per IS456:2000, the nominal shear stress in the beams of varying depth depends on both the design shear force as well as the design bending moment

[MSQ – 1 Mark]

Ans. A, B and D

Sol.

1. Excessive shear reinforcement can cause compressive failure in concrete. So, option A is correct.
2. Beams need to be provided with minimum shear reinforcement to avoid brittle failure. So, option B is correct.
3. Design shear strength of concrete τ_c (without shear reinforcement) depends upon–
   (i) grade of concrete
   (ii) % main reinforcement

So, option C is incorrect.

4. Nominal shear stress for beam of varying depth, $\tau_{v_u} = \dfrac{V_u + \dfrac{M_u}{d}\tan\beta}{B.d}$

where, V_u = design shear force

M_u = design bending moment

So, option D is correct.

28. The reason(s) of the nonuniform elastic settlement profile below a flexible footing, resting on a cohesionless soil while subjected to uniform loading, is/are:

A. Variation of friction angle along the width of the footing

B. Variation of soil stiffness along the width of the footing

C. Variation of friction angle along the depth of the footing

D. Variation of soil stiffness along the depth of the footing

[MSQ – 1 Mark]

Ans. A, B and D

Sol. For a flexible footing placed on cohesionless soil, the variation of soil stiffness along the width of the footing results in a non-uniform settlement of the footing.

29. Which of the following is/are NOT active disinfectant(s) in water treatment?

A. OH (hydroxyl radical)

B. O_3 (ozone)

C. OCl^- (hypochlorite ion)

D. Cl^- (chloride ion)

[MSQ – 1 Mark]

Ans. D

Sol. Hydroxyl radicals are highly reactive and can destroy pathogens quickly. Ozone & hypochlorite ions are well-known disinfectants used to kill pathogens. However, chloride ions by themselves are not active disinfectants. They can help to stabilize and activate other disinfectants.

30. As per the Indian Roads Congress guidelines (IRC 86: 2018), extra widening depends on which of the following parameters?

A. Horizontal curve radius

B. Superelevation

C. Number of lanes

D. Longitudinal gradient

[MSQ – 1 Mark]

Ans. A, C

Sol. Extra widening = $\dfrac{nl^2}{2R} + \dfrac{V}{9.5\sqrt{R}}$

So extra widening depends on number of lane and radius.

31. The steady-state temperature distribution in a square plate ABCD is governed by the 2-dimensional Laplace equation. The side AB is kept at a temperature of 100 °C and the other three sides are kept at a temperature of 0°C. Ignoring the effect of discontinuities in the boundary conditions at the corners, the steady-state temperature at the center of the plate is obtained as T_0 °C. Due to symmetry, the steady-state temperature at the center will be same (T_0 °C), when any one side of the square is kept at a temperature of 100 °C and the remaining three sides are kept at a temperature of 0 °C. Using the principle of superposition, the value of T_0 is ________ (rounded off to two decimal places).

[NAT – 1 Mark]

Ans. 19.93

Sol. We know the, Laplace equation is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ this is also known as the harmonic function

The solution of the function is

$$u\,(x, y) = (C_1 \cos\lambda x + C_2 \sin\lambda x)\,(C_3 e^{\lambda y} + C_4 e^{-\lambda y})$$

Let

$$AB = CD = 1$$

Using u (x, 0) = 0

$$\Rightarrow C_3 = -C_4$$

Using (0, y) = 0

$$\Rightarrow C_1 = 0$$

$$u\,(x, y) = (0 + C_2 \sin\lambda x)\,(C_2 e^{\lambda y} - C_3 e^{-\lambda y})$$

$$u\,(x, y) = a_x \sin\lambda\, x\,(e^{\lambda y} - e^{-\lambda y})$$

Now using u (1, y) = 0

$$\sin \lambda = 0 \Rightarrow \lambda = n\pi$$

Hence,

$$u(x, y) = \Sigma a_n \sin (n\pi n)[e^{n\pi y} - e^{-n\pi y}]$$

Using u(x, 1) = 100

$$\Rightarrow a_1 = \frac{100}{\sin \pi x[e^{\pi} - e^{-\pi}]}$$

Using equation (iii),

$$u(x, y) = \frac{100}{o^{\pi} \quad o^{-\pi}}(e^{\pi y} - e^{-\pi y})$$

At mid-point, i.e.,

$$u\left(\frac{1}{2}, \frac{1}{2}\right) = T_0$$

$$\Rightarrow T_0 = \frac{100}{(e^{\pi} - e^{-\pi})}(e^{\pi/2} - e^{-\pi/2})$$

$$= \frac{100}{(e^{\pi/2} + e^{-\pi/2})} = \frac{100}{2\cos\frac{\pi}{2}} = \frac{50}{2.509} = 19.928 = 19.93°C$$

32. An unconfined compression strength test was conducted on a cohesive soil. The test specimen failed at an axial stress of 76 kPa. The undrained cohesion (in kPa, in integer) of the soil is ______.

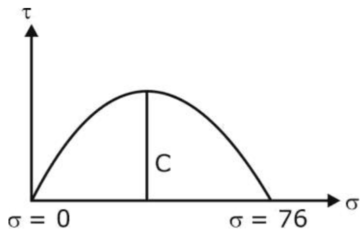
[NAT – 1 Mark]

Ans. 76

Sol. Test done on soil – UCS

Axial stress =76 kPa

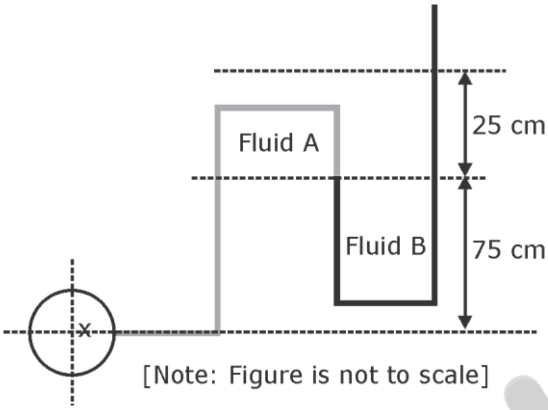
For UCS test



$$C = \text{Radius} = \frac{76}{2} = 38 \text{ kPa}$$

33. The pressure in a pipe at X is to be measured by an open manometer as shown in figure. Fluid A is oil with a specific gravity of 0.8 and Fluid B is mercury with a specific gravity of 13.6. The absolute pressure at X is kN/m^2 (round off to one decimal place).

[Assume density of water as 1000 kg/m^3 and acceleration due to gravity as 9.81 m/s^2 and atmospheric pressure as 101.3 kN/m^2]



[Note: Figure is not to scale]

[NAT – 1 Mark]

Ans. 140.5

Sol. $P_x - (800 \times 9.81 \times 0.75) - (13600 \times 9.81 \times 0.25) = P_{atm}$

$P_x = 101.3 \times 10^3 + 800 \times 9.81 \times 0.75 + 13600 \times 9.81 \times 0.25$

$P_x = 140.54 \text{ kN/m}^2$

34. For the elevation and temperature data given in the table, the existing lapse rate in the environment is __________ °C/100 m (round off to two decimal places).

Elevation from ground level (m)	Temperature (°C)
5	14.2
325	16.9

[NAT – 1 Mark]

Ans. 0.84

Sol.

Elevation above ground level	Temperature
5 m	14.2°C
325 m	16.9°C

$$ELR = \frac{16.9 - 14.2}{(325 - 5)}$$

= 0.00843 °C/m

= 0.84 °C/100 m

35. If the size of the ground area is 6 km × 3 km and the corresponding photo size in the aerial photograph is 30 cm × 15 cm, then the scale of the photograph is 1 : (in integer).

[NAT – 1 Mark]

Ans. 20,000

Sol. Ground area $= 6 \text{ km} \times 3 \text{ km}$

Photograph size $= 30 \text{ cm} \times 15 \text{ cm}$

Scale $= \sqrt{\dfrac{\text{photograph size}}{\text{ground size}}}$

$= \sqrt{\dfrac{30 \times 15}{6 \times 3 \times \left(10^5\right)^2}}$

$= 1 : 20000$

36. The solution of the differential equation

$$\frac{d^3y}{dx^3} - 5.5\frac{d^2y}{dx^2} + 9.5\frac{dy}{dx} - 5y = 0$$

is expressed as $y = C_1e^{2.5x} + C_2e^{\alpha x}$, where C_1, C_2, C_3, α, and β are constants, with α and β being distinct and not equal to 2.5. Which of the following options is correct for the values of α and β?

A. 1 and 2

B. −1 and −2

C. 2 and 3

D. −2 and −3

[MCQ – 2 Marks]

Ans. A

Sol. Auxiliary equation is,

$$m^3 - 5.5m^2 + 9.5m - 5 = 0$$

By solving above equation, we get

$M = 2.5, 1, 2$

So, m_1 and m_2 are 1 and 2.

37. Two vectors $[2 \ \ 1 \ \ 0 \ \ 3]^T$ and $[1 \ \ 0 \ \ 1 \ \ 2]^T$ belong to the null space of a 4×4 matrix of rank 2. Which one of the following vectors also belongs to the null space?

A. $\begin{bmatrix}1 & 1 & -1 & 1\end{bmatrix}^T$

B. $\begin{bmatrix}2 & 0 & 1 & 2\end{bmatrix}^T$

C. $\begin{bmatrix}0 & -2 & 1 & -1\end{bmatrix}^T$

D. $\begin{bmatrix}3 & 1 & 1 & 2\end{bmatrix}^T$

[MCQ – 2 Marks]

Ans. A

Sol. Nullity of matrix

= No. of variables (n) – rank of A

= 4 – 2 = 2

and

Nullity is no. of linearly independent vectors in the null space.

After seeing the given matrix, these two vectors are already independent.

i.e., $x = \begin{bmatrix}2\\1\\0\\3\end{bmatrix}$ & $y = \begin{bmatrix}1\\0\\1\\2\end{bmatrix}$

Any other vectors in null space

$$z = \alpha x + \beta y$$

Where, α, β can be any real value.

For $\alpha = 1$ and $\beta = -1$

$$z = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 & 1 \end{bmatrix}^T$$

Option A is correct.

38. Cholesky decomposition is carried out on the following square matrix [A].

$$[A] = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$$

Let l_{ij} and a_{ij} be the $(l,j)^{th}$ elements of matrices [L] and [A], respectively. If the element l_{22} of the decomposed lower triangular matrix [L] is 1.968, what is the value (rounded off to the nearest integer) of the element a_{22}?

A. 5
B. 7
C. 9
D. 11

[MCQ – 2 Marks]

Ans. B

Sol. Cholesky decomposition

$$A = LL^*$$

Where,
L = Lower triangular matrix with real & positive diagonal elements.
L* = Transpose of conjugate

$$LL^* = A$$

$$\begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix}\begin{bmatrix} L_{11} & L_{12} \\ 0 & L_{22} \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -5 & a_{22} \end{bmatrix}$$

L_{11} & L_{22} → positive

and L_{22} = 1.968 (given)

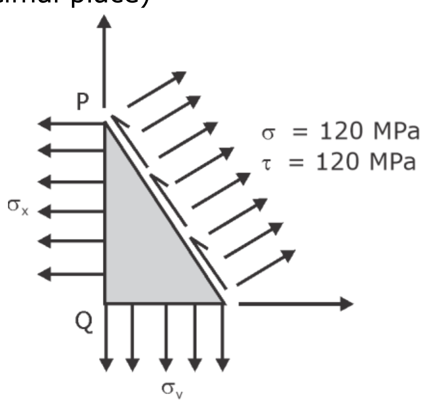
(i) $L_{11}^2 = 8 \Rightarrow L_1 = \sqrt{8}$

(ii) $L_{11}\,L_{21} = -5 \Rightarrow L_{21} = \dfrac{-5}{\sqrt{8}}$

(iii) $L_{21}^2 + 1.968^2 = a_{22}$

$\Rightarrow a_{22} = \dfrac{25}{8} + 1.968^2 \cong 7$

39. In a two-dimensional stress analysis, the state of stress at a point is shown in the figure. The values of length of PQ, QR, and RP are 4, 3, and 5 units, respectively. The principal stresses are _______ (round off to one decimal place)



A. σ_x = 26.7 MPa, σ_y = 172.5 MPa

B. σ_x = 54.0 MPa, σ_y = 128.5 MPa

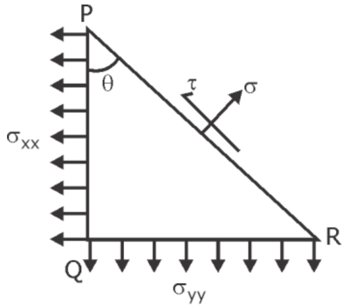
C. σ_x = 67.5 MPa, σ_y = 213.3 MPa

D. σ_x = 16.0 MPa, σ_y = 138.5 MPa

[MCQ – 2 Marks]

Ans. C

Sol. Given,



PQ = 4 QR = 3 PR = 5

From fig. $\sin\theta = \frac{3}{5}$, $\cos\theta\ \frac{4}{5}$

σ = 120 MPa

τ = 70 MPa

We have,

$$\sigma_\theta = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\sin\theta.\cos\theta$$

$$120 = \sigma_{xx}\left[\frac{4}{5}\right]^2 + \sigma_{yy}\left(\frac{3}{5}\right)^2 + 0$$

$$120 = \frac{16}{25}\sigma_{xx} + \frac{9}{25} \times \sigma_{yy} \quad \text{...(i)}$$

$$\tau_\theta = -\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$70 \times 2 = -(\sigma_{xx} - \sigma_{yy}) \times 2\sin\theta\cos\theta$$

$$70 \times 2 = -(\sigma_{xx} - \sigma_{yy}) \times 2 \times \frac{3}{5} \times \frac{4}{5}$$

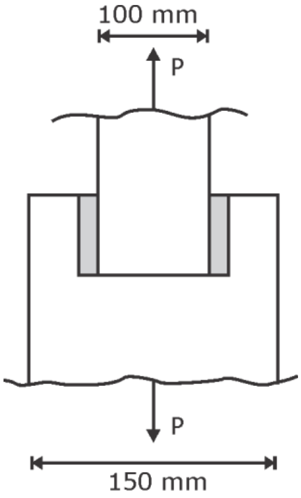
$$12\sigma_{xx} - 12\sigma_{yy} = -70 \times 25 \quad \text{...(ii)}$$

By solving equation (i) and (ii)

σ_{xx} = 67.5 MPa

σ_{yy} = 213.33 MPa

40. Two plates are connected by fillet welds of size 10 mm and subjected to tension, as shown in the figure. The thickness of each plate is 12 mm. The yield stress and the ultimate stress of steel under tension are 250 MPa and 410 MPa, respectively. The welding is done in the workshop (partial safety factor, γ_{mw} = 1.25). As per the Limit State Method of IS 800: 2007, what is the minimum length (in mm, rounded off to the nearest higher multiple of 5 mm) required of each weld to transmit a factored force P equal to 275 kN?

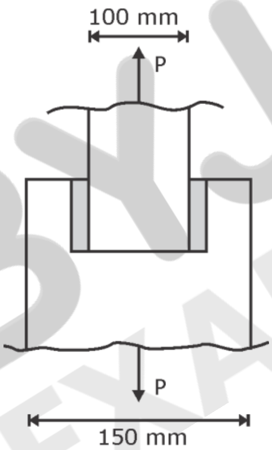


A. 100
B. 105
C. 110
D. 115

[MCQ – 2 Marks]

Ans. B

Sol.



Fillet weld:

Size = 10 mm

Fusion angle = 90°

Effective Throat thickness = 0.7 × size= 0.7 × 10= 7mm

So, t_e = 7 mm

Design strength of fillet weld:

$$P_{dw} = \frac{f_u}{\sqrt{3} \cdot \gamma_{mw}} \times l_e \times t_e$$

Let's take limiting condition:

$$P_{dw} = 275 \times 10^3 N = \frac{410}{\sqrt{3} \times 1.25} \times 7\,mm \times l_e$$

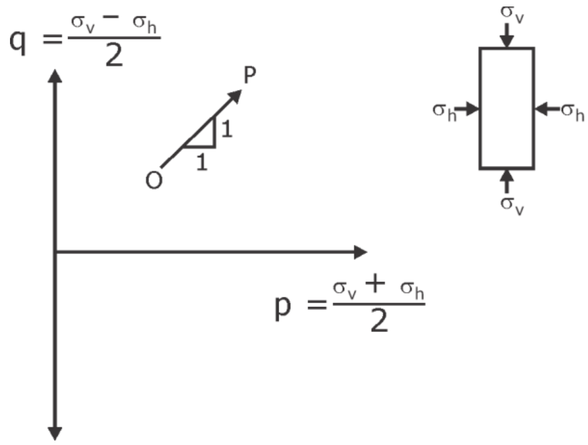
l_e = 207.45 mm

So, the required length on each side = $\frac{l_e}{2} = \frac{207.45}{2}$

= 103.72 mm

Hence, nearest higher multiple of 5mm is 105 mm.

41. In the given figure, Point O indicates the stress point of a soil element at initial non-hydrostatic stress condition. For the stress path (OP), which of the following loading conditions is correct?



A. σ_v is increasing and σ_h is constant

B. σ_v is increasing and σ_h is increasing

C. σ_v is increasing and σ_h is decreasing

D. σ_v is decreasing and σ_h is increasing

[MCQ – 2 Marks]

Ans. A

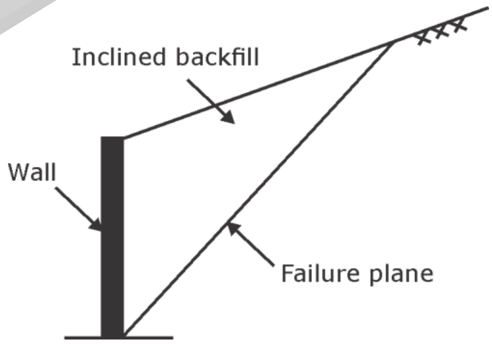
Sol.

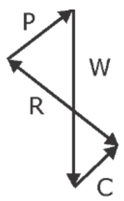
$$q = \frac{\sigma_v - \sigma_h}{2}$$

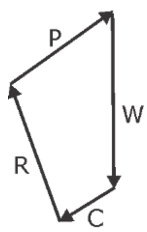
$$p = \frac{\sigma_v + \sigma_h}{2}$$

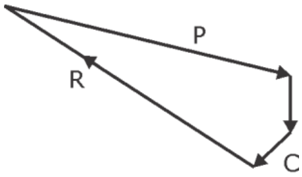
When σ_v increasing and σ_h is constant, then the ordinates of both x-axis and y-axis will increase. It will make the stress path as given in the question. Hence option A is correct.

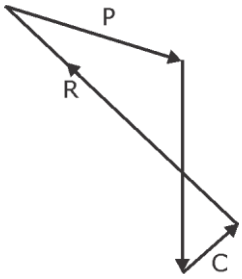
42. The figure shows a vertical retaining wall with backfill consisting of cohesive-frictional soil and a failure plane developed due to passive earth pressure. The forces acting on the failure wedge are: P as the reaction force between the wall and the soil, R as the reaction force on the failure plane, C as the cohesive force along the failure plane and W as the weight of the failure wedge. Assuming that there is no adhesion between the wall and the wedge, identify the most appropriate force polygon for the wedge.



A.


B.


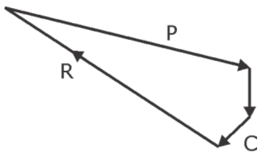
C. 

D. 

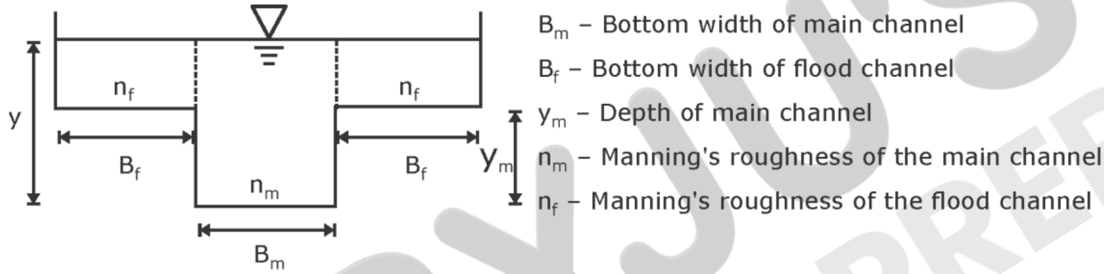
[MCQ – 2 Marks]

Ans. C

Sol. Force polygon for passive state:



43. A compound symmetrical open channel section as shown in the figure has a maximum of ________ critical depth(s).



A. 3
B. 2
C. 1
D. 4

[MCQ – 2 Marks]

Ans. A

Sol. Consider the following three cases:

Case 1:

When $Q = Q_1$ and depth of flow (y) < y_m

Then, y_{c1} is the critical depth.

Case 2:

When $Q = Q_2$ and depth of flow (y) > y_m

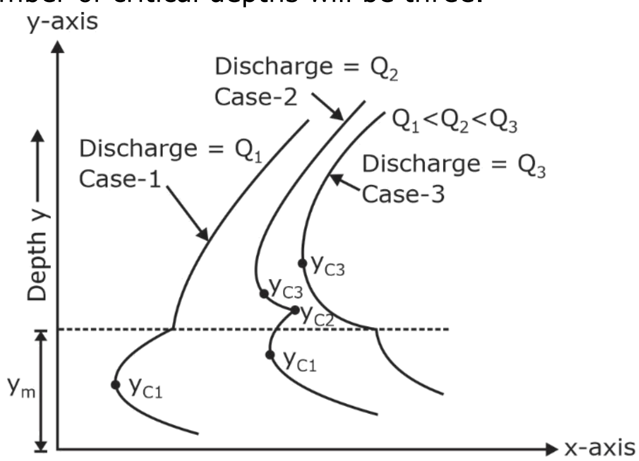
Then, y_{c1}, y_{c2} and y_{c3} be the critical depth.

Case 3:

When $Q = Q_3$

Only y_{c3} will exist.

So, the maximum number of critical depths will be three.



44. The critical flow condition in a channel is given by _______ .

[Note: α – kinetic energy correction factor; Q – discharge; A_c – cross-sectional area of flow at critical flow condition; T_c – top width of flow at critical flow condition; g – acceleration due to gravity]

A. $\frac{\alpha Q^2}{g} = \frac{A_c^3}{T_c}$

B. $\frac{\alpha Q}{g} = \frac{A_c^3}{T_c^2}$

C. $\frac{\alpha Q^2}{g} = \frac{A_c^3}{T_c^2}$

D. $\frac{\alpha Q}{g} = \frac{A_c^3}{T_c}$

[MCQ - 2 Marks]

Ans. A

Sol. For critical flow condition in channel of any shape

$$\frac{Q^2 T_c}{g A_c^3} = 1$$

If kinetic energy factor α is considered.

$$\frac{\alpha Q^2 T_c}{g A_c^3} = 1$$

$$\frac{\alpha Q^2}{g} = \frac{A_c^3}{T_c}$$

45. Match the following air pollutants with the most appropriate adverse health effects:

Air pollutant		**Health effect to human and/or test animal**	
P.	Aromatic hydrocarbons	I.	Reduce the capability of the blood to carry oxygen
Q.	Carbon monoxide	II.	Bronchitis and pulmonary emphysema
R	Sulfur oxides	III.	Damage of chromosomes
S	Ozone	IV.	Carcinogenic effect

A. (P) – (II), (Q) – (I), (R) – (IV), (S) – (III)
B. (P) – (IV), (Q) – (I), (R) – (III), (S) – (II)
C. (P) – (III), (Q) – (I), (R) – (II), (S) – (IV)
D. (P) – (IV), (Q) – (I), (R) – (II), (S) – (III)

[MCQ – 2 Marks]

Ans. D

Sol. Aromatic hydrocarbons - Carcinogenic effect
Carbon monoxide - Reduce the capability of the blood to carry oxygen
Sulfur oxides - Bronchitis and pulmonary emphysema
Ozone - Damage of chromosomes

46. A delivery agent is at a location R. To deliver the order, she is instructed to travel to location P along straight-line paths of RC, CA, AB and BP of 5 km each. The direction of each path is given in the table below as whole circle bearings. Assume that the latitude (L) and departure (D) of R is (0, 0) km. What is the latitude and departure of P (in km, rounded off to one decimal place)?

Paths	RC	CA	AB	BP

Directions (in degrees)	120	0	90	240

A. L = 2.5; D = 5.0
B. L = 0.0; D = 5.0
C. L = 5.0; D = 2.5
D. L = 0.0; D = 0.0

[MCQ – 2 Marks]

Ans. B

Sol. Latitude and departure of starting point, R= (0, 0) km

Latitude of location P = Latitude of R + Σ Latitude for each line

= 0 + 5 cos 120° + 5 cos 0° + 5 cos 90° + 5 cos 240°

= 0 km

Departure of location P = Departure of R + Σ Departure for each line

= 0 + 5 sin 120° + 5 sin 0° + 5 sin 90° + 5 sin240°

= 5 km

Hence, L = 0 km, D = 5 km

47. Which of the following statements is/are TRUE?

A. The thickness of a turbulent boundary layer on a flat plate kept parallel to the flow direction is proportional to the square root of the distance from the leading edge

B. If the streamlines and equipotential lines of a source are interchanged with each other, the resulting flow will be a sink

C. For a curved surface immersed in a stationary liquid, the vertical component of the force on the curved surface is equal to the weight of the liquid above it

D. For flow through circular pipes, the momentum correction factor for laminar flow is larger than that for turbulent flow

[MSQ – 2 Marks]

Ans. C and D

Sol.

1. Vertical component of force on curved surface immersed in a stationary liquid is equal to the weight of the liquid above the curved surface upto the free surface of the liquid. So, option A is true.
2. Momentum correction factor for flow in circular pipe

Laminar flow = 1.33

Turbulent flow = 1.015

So, option B is true.

3. Sink will form if you reverse the direction of the streamlines in the source. So, option C is false.
4. Turbulent boundary layer thickness δ relation with distance x from the leading edge of plate is given as

$$\frac{\delta}{x} = \frac{0.376}{Re^{1/5}}$$

where, $Re = \frac{\rho v x}{\mu}$

$$\frac{\delta}{x} = \frac{0.376}{\left(\frac{\rho v x}{\mu}\right)^{1/5}} \Rightarrow \delta = \frac{0.376x^{4/5}}{\left(\frac{\rho v}{\mu}\right)}$$

$\therefore\ \delta \propto x^{4/5}$

So, option D is false.

48. In the context of water and wastewater treatments, the correct statements are:

A. particulate matter may shield microorganisms during disinfection

B. ammonia decreases chlorine demand

C. phosphorous stimulates algal and aquatic growth

D. calcium and magnesium increase hardness and total dissolved solids

[MSQ – 2 Marks]

Ans. A, C, D

Sol.

1. Particulate matter shields microorganisms during disinfection. So, option A is correct.
2. Ammonia increases chlorine demand. So, option B is incorrect.
3. Phosphorous stimulates algal and aquatic growth. So, option C is correct.

Calcium and magnesium increase hardness and total dissolved solids in water. So, option D is correct.

49. Which of the following statements is/are TRUE for the aerobic composting of sewage sludge?

A. Bulking agent is added during the composting process to reduce the porosity of the solid mixture

B. Leachate can be generated during composting

C. Actinomycetes are involved in the process

D. In-vessel composting systems cannot be operated in the plug-flow mode

[MSQ – 2 Marks]

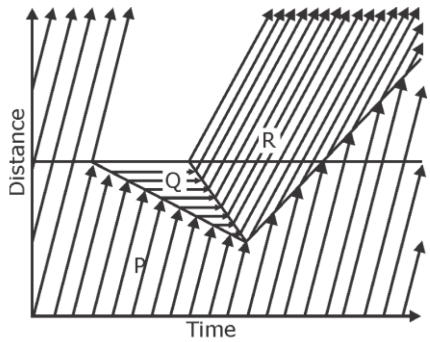
Ans. B, C

Sol.

1. Bulking agent is added to increase the volume. So, option A is incorrect.
2. Leachate can be generated during compositing. So, option B is correct.
3. Antinocytes are involved in the process. So, option C is correct.

In-vessel composting, systems can be operated in plug-flow mode. So, option D is incorrect.

50. The figure presents the time-space diagram for when the traffic on a highway is suddenly stopped for a certain time and then released. Which of the following statements are true?



A. Speed is higher in Region R than in Region P

B. Volume is lower in Region Q than in Region P

C. Volume is higher in Region R than in Region P

D. Density is higher in Region Q than in Region R

[MSQ – 2 Marks]

Ans. B, C and D

Sol.

- Slope of distance v/s time plot (represents speed) for region P is more than region R. So, option A is incorrect.
- Slope in region Q is parallel to x-axis. It means the vehicles are not moving. So, the volume for region Q is zero and density will be maximum. So, option B and D are correct.

Since, speed in region P is more than region R, region R will have a higher volume. So, option C is correct.

51. Consider the Marshall method of mix design for bituminous mix. With the increase in bitumen content, which of the following statements is/are TRUE?

A. the Stability decreases initially and then increases

B. the Flow increases monotonically

C. the air voids (VA) increases initially and then decreases

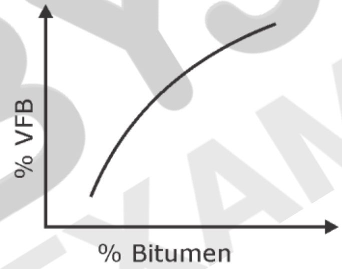
D. the voids filled with bitumen (VFB) increases monotonically

[MSQ – 2 Marks]

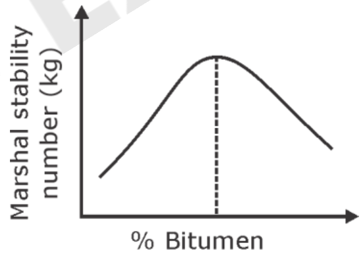
Ans. B, D

Sol.

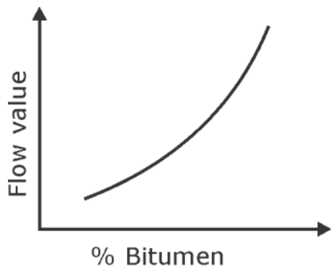
- With increase with Bitumen content, Air voids decrease.
- With increase with Bitumen content, VFB increases.



- As Bitumen content increases, the stability increases initially then decreases.



- As Bitumen content increases, the flow will increase.



52. A 5 cm long metal rod AB was initially at a uniform temperature of T_0 °C. Thereafter, temperature at both the ends are maintained at 0 °C. Neglecting the heat transfer from the lateral surface of the rod, the heat transfer in the rod is governed by the one-dimensional diffusion equation $\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$, where D is the thermal diffusivity of the metal, given as 1.0 cm^2/s.

The temperature distribution in the rod is obtained as

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$$

where x is in cm measured from A to B with $x = 0$ at A, t is in s, C_n are constants in °C, T is in °C, and β is in s^{-1}.

The value of β (in s^{-1}, rounded off to three decimal places) is ____________.

[NAT – 2 Marks]

Ans. 0.395

Sol. $\frac{\partial T}{\partial t} = \Delta\frac{\partial^2 T}{\partial x^2}$; $T(0, t) = 0, T(5, t) = 0, T(x, 0) = T_0, D = 1$ ...(i)

Put D = 1, it's general solution using separation of variables methods in

$T(x, t) = (C_1 \cos px + C_2 \sin px)C_3 e^{-p2t}$ ...(ii)

Using $T(0, t) = 0 \Rightarrow C_1 = 0$

Using $T(5, t) = 0 \Rightarrow C_2C_3 \sin 5p e^{-p2t} = 0$

Or $\sin 5p = 0 \Rightarrow p = \frac{n\pi}{5}, n \in I$

Hence by (2), $T(x, t) = b_n \sin\left(\frac{n\pi x}{5}\right)\cdot e^{\frac{-n^2\pi^2 t}{25}}$ Where $b_n = C_2C_3$

Hence most general solution is

$$T(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{5}\right)\cdot e^{\frac{-n^2\pi^2 t}{25}} \quad ...(iii)$$

Now using T(x, 0) = T0

$$\Rightarrow T_0 = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{5}\right)$$

Which is half range fourier sine series for T_0

Hence, $$b_n = \frac{2}{5}\int_0^5 T_0 \sin\left(\frac{n\pi x}{5}\right)dx = \begin{cases} 0 & n \text{ even} \\ \frac{4T_0}{n\pi} & n \text{ odd} \end{cases}$$

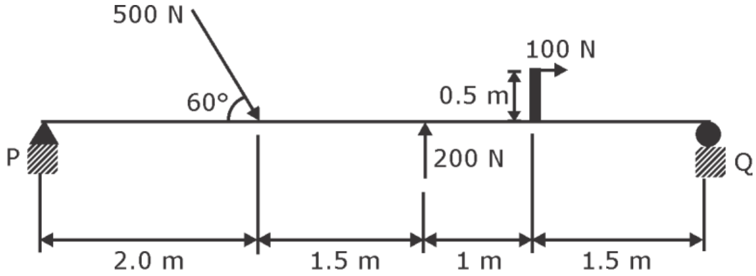
By (3) $$T(x, t) = \sum_{n=\text{odd}} \frac{4T_0}{n\pi}\sin\left(\frac{n\pi x}{5}\right)e^{\frac{-n^2\pi^2 t}{25}} \quad ...(iv)$$

On comparison with,

$$T(x, t) = \sum_{n=1,3,5} C_n \sin\left(\frac{n\pi x}{5}\right)e^{-\beta n^2 t}$$

$$\Rightarrow \quad C_n = \frac{4T_0}{n\pi} \text{ and } \beta = \frac{\pi^2}{25} = 0.3947 \approx 0.395$$

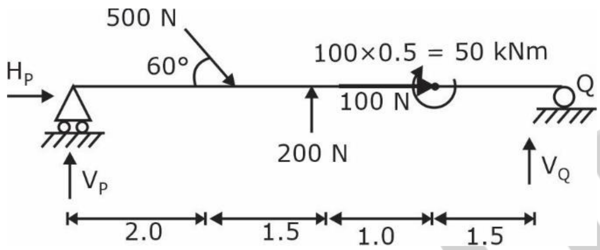
53. A beam is subjected to a system of coplanar forces as shown in the figure. The magnitude of vertical reaction at Support P is N (round off to one decimal place).



[NAT – 2 Marks]

Ans. 197.06

Sol. Let vertical reaction at support P = V_P



Sum of vertical force ΣV = 0

$$V_P + V_Q = 500 \sin 60° - 200 \text{ N}$$

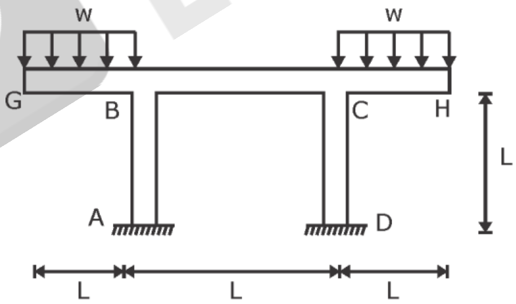
$$V_P + V_Q = 2330.01 \text{ N} \quad \text{...(i)}$$

Moment about point Q, $\Sigma M_Q = 0$

$$V_P \times 6 + 200 \times 2.5 - 500 \sin 60° \times 4 + 50 = 0$$

$$\boxed{V_P = 197.06 \text{ N}}$$

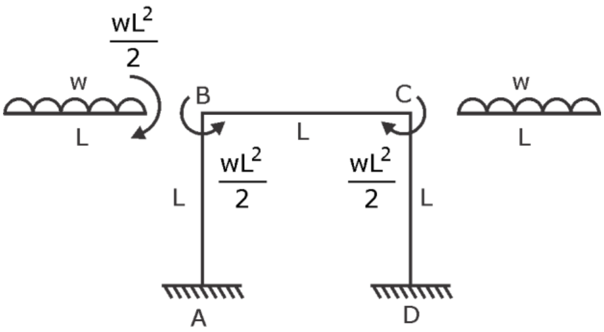
54. For the frame shown in the figure (not to scale), all members (AB, BC, CD, GB, and CH) have the same length, L and flexural rigidity, EI. The joints at B and C are rigid joints, and the supports A and D are fixed supports. Beams GB and CH carry uniformly distributed loads of w per unit length. The magnitude of the moment reaction at A is wL^2/k. What is the value of k (in integer)?



[NAT – 2 Marks]

Ans. 6

Sol.



Stiffness of BC = $\frac{2EI}{L}$

(Since, the beam is bending in a symmetrical mode)

Stiffness of BA = $\frac{4EI}{L}$

$$\text{D.F. for BA} = \frac{\frac{4EI}{L}}{\frac{4EI}{L}+\frac{2EI}{L}} = \frac{2}{3}$$

$$M_{BA} = \frac{wL^2}{2} \times \frac{2}{3} = \frac{2wL^2}{6}$$

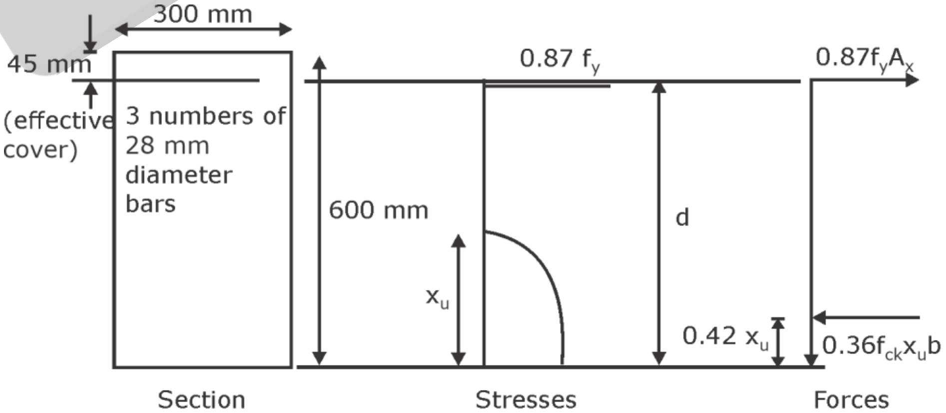
Carry over moment at A, $M_{AB} = \frac{1}{2} \times \frac{2wL^2}{6}$

$$= \frac{wL^2}{6}$$

$\Rightarrow k = 6$

55. Consider the singly reinforced section of a cantilever concrete beam under bending, as shown in the figure (M25 grade concrete, Fe415 grade steel). The stress block parameters for the section at ultimate limit state, as per IS 456: 2000 notations, are given. The ultimate moment of resistance for the section by the Limit State Method is _______________ kN.m (round off to one decimal place).

[**Note:** Here, A_s is the total area of tension steel bars, b is the width of the section, d is the effective depth of the bars, f_{ck} is the characteristic compressive cube strength of concrete, f_y is the yield stress of steel, and x_u is the depth of neutral axis.]



[NAT – 2 Marks]

Ans. 300.9

Sol. B = 300 mm

A_{st} = 3 – 28 mm

Effective cover = 45 mm

D = 600 mm

d = 600 – 45 = 555 mm

$x_{u,lim}$ = kd {for Fe 415, k = 0.48}

= 0.48 d

= 0.48 × 555

= 266.4 mm

C = T

$0.36\ f_{ck}\ Bx_u = 0.87\ f_y\ A_{st.}$

$$0.36 \times 25 \times 300 \times x_u = 0.87 \times 415 \times \frac{\pi}{4} \times 28^2 \times 3$$

x_u = 247 mm

Now, $x_u < x_{ulim}$

So, it's an under-reinforced section.

MOR = $0.36\ f_{ck}\ Bx_u\ (d - 0.42\ x_u)$

= 0.36 × 25 × 300 × 247 [555 – (0.42 × 247)]

= 300.9 kNm

56. A 2D thin plate with modulus of elasticity, E = 1.0 N/m^2, and Poisson's ratio, μ = 0.5, is in plane stress condition. The displacement field in the plate is given by $u = Cx^2y$ and v = 0 and v = 0, where u and v are displacements (in m) along the X and Y directions, respectively, and C is a constant (in m^{-2}). The distances x and y along X and Y, respectively, are in m. The stress in the X direction is $\sigma_{xx} = 40xy$ N/m^2, and the shear stress is $\tau_{xy} = \alpha x^2$ N/m^2. What is the value of α (in N/m^4, in integer)? ___________.

[NAT – 2 Marks]

Ans. 7

Sol. We have given.

$$E = 1\ \text{N/m}^2,\ \mu = 0.5,\ \sigma_{xx} = 40xy\ \text{N/m}^2 \text{ and } \tau_{xy} = \alpha x^2\ \text{N/m}^2$$

The stress strain variation is assumed to be linear.

Strain in X- direction $\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_x}{E}$

Strain in Y- direction $\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

Also, $G = \frac{E}{2(1+\mu)}$

$$\epsilon_x + \mu\,\epsilon_y = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} + \frac{\mu\sigma_y}{E} - \frac{\mu^2\sigma_x}{E}$$

$$\epsilon_x + \mu\,\epsilon_y = \frac{\sigma_x}{E}\left(1-\mu^2\right)$$

$$\sigma_x = \frac{E}{1-\mu^2}\left[\epsilon_x + \mu\,\epsilon_y\right]$$

$$\sigma_x = \frac{E}{1-\mu^2}\left[\frac{\partial u}{\partial x} + \mu\frac{\partial v}{\partial y}\right]$$

We have $u = Cx^2y$

$$\frac{\partial u}{\partial x} = 2Cxy$$

$$\sigma_x = \frac{2ECxy}{1-\mu^2}$$

$$\Rightarrow \frac{2ECxy}{1-\mu^2} = 40xy$$

$$\frac{2EC}{1-\mu^2} = 40 \qquad \text{...(i)}$$

And $\gamma_{xy} = \dfrac{\tau_{xy}}{G}$

$$G\gamma_{xy} = \tau_{xy}$$

Shear strain, $\gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} = 0 + Cx^2$

$$\frac{E}{2(1+\mu)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] = \tau_{xy}$$

$$\frac{E}{2(1+\mu)}[Cx^2 + 0] = ax^2$$

$$a = \frac{CE}{2(1+\mu)}$$

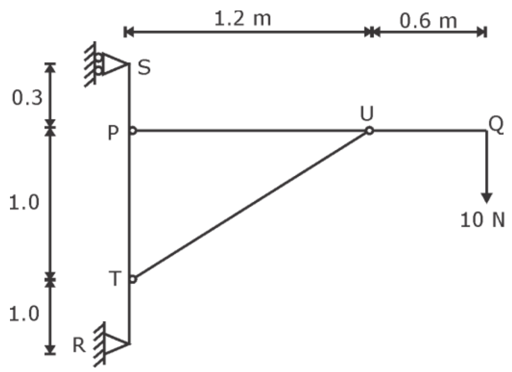
Put the value of CE from equation (i)

$$a = \frac{40(1-\mu^2)}{2E}\frac{E}{2(1+\mu)}$$

$$a = 10(1+\mu) = 10(1-0.5) = 5$$

$$a = 5$$

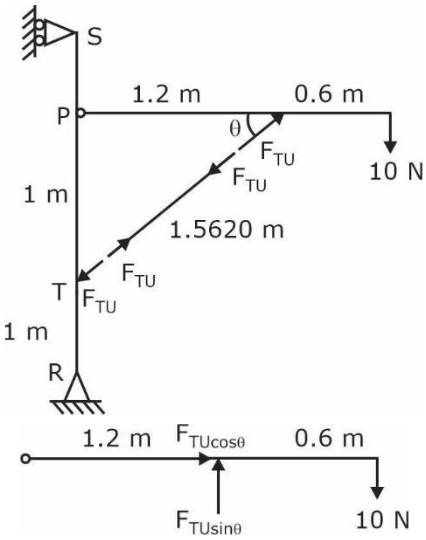
57. An idealised frame supports a load as shown in the figure. The horizontal component of the force transferred from the horizontal member PQ to the vertical member RS at P is N (round off to one decimal place).



[NAT – 2 Marks]

Ans. 18

Sol.



$\Sigma M_P = 0$

$-10 \times (1.2 + 0.6) + F_{TU} \sin\theta \times 1.2 = 0$

$1.2 \times F_{TU} \times \dfrac{1}{1.5620} = 10(1.8)$

$F_{TU} = 23.43$ N

Horizontal component is $F_{TU} \cos\theta = 23.43 \times \dfrac{1.2}{1.5620} = 18$ N

58. A square footing is to be designed to carry a column load of 500 kN which is resting on a soil stratum having the following average properties: bulk unit weight = 19 kN/m^3; angle of internal friction = 0° and cohesion = 25 kPa. Considering the depth of the footing as 1 m and adopting Meyerhof's bearing capacity theory with a factor of safety of 3, the width of the footing (in m) is ______________ (round off to one decimal place)

[Assume the applicable shape and depth factor values as unity; ground water level at greater depth.]

[NAT – 2 Marks]

Ans. 3 to 3.4

Sol. As per Meyerhof's method

$q_u = C N_c d_c s_c i_c + \gamma D_f N_q d_q s_q i_q + 0.5 B \gamma N_\gamma d_\gamma s_\gamma i_\gamma$

$N_q = 1$, $N_\gamma = 0$, $N_c = 5.14$

$C = 25$ kPa, $\phi = 0°$, $\gamma = 19$ kN

Putting the value in equation

$q_u = 147.5$ kN/m^2

$q_{nu} = q_u - \gamma D_f$

$q_{nu} = 147.5 - 19 \times 1 = 128.5$ kN/m^2

Area $= \dfrac{\text{Load} \times \text{FOS}}{q_{nu}} = \dfrac{500 \times 3}{128.5} = 11.67$ m^2

Since it's a square footing

Area = (width)^2

Width $= \sqrt{11.67} = 3.4$ m

59. A circular pile of diameter 0.6 m and length 8 m was constructed in a cohesive soil stratum having the following properties: bulk unit weight = 19 kN/m^3; angle of internal friction = 0° and cohesion = 25 kPa.

The allowable load the pile can carry with a factor of safety of 3 is _________ kN (round off to one decimal place).

[Adopt: Adhesion factor, $\alpha = 1.0$ and Bearing capacity factor, $N_c = 9.0$]

[NAT – 2 Marks]

Ans. 146.5 to 146.9

Sol.

$$Q_{up} = q_s A_s + q_b A_b$$

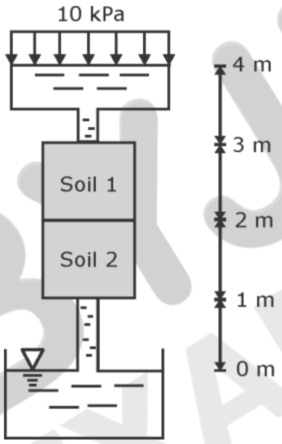
$$Q_{up} = \bar{\alpha}\bar{C}(\pi d L) + C N_c \frac{\pi}{4} d^2$$

$$Q_{up} = (1 \times 25 \times \pi \times 0.6 \times 8) + (25 \times 9 \times \frac{\pi}{4} \times 0.6^2)$$

$$Q_{up} = 440.60 \text{ kN}$$

$$Q_{allowable} = Q_{up}/\text{ FOS } = 440.60/3 = 146.87 \text{ kN}$$

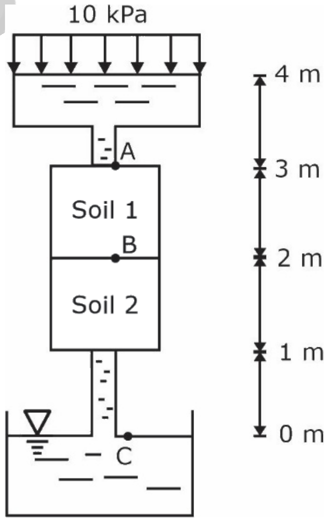
60. For the flow setup shown in the figure (not to scale), the hydraulic conductivities of the two soil samples, Soil 1 and Soil 2, are 10 mm/s and 1 mm/s, respectively. Assume the unit weight of water as 10 kN/m^3 and ignore the velocity head. At steady state, what is the total head (in m, rounded off to two decimal places) at any point located at the junction of the two samples?



[NAT – 2 Marks]

Ans. 4.54

Sol.



Note: Unless mentioned downstream end is taken as datum.

Pressure at A = 10 kPa

$\gamma \times H = 10$

Since, $\gamma = 10 \text{ kN/m}^3$

$\Rightarrow H = 1 \text{ m}$

PH at A = 1 + 1 = 2 m

	V.H	D.H	P.H	T.H
A	0	3	2	5
C	0	0	0	0

$H_L = 5 - 0 = 5 \text{ m}$

As flow is perpendicular to bedding plane ∴ Q = constant

$$K_{total} \times \left(\frac{H_L}{L}\right)_{Total} A = \frac{k_1 h_{L1}}{L_1} A$$

$$\frac{1+1}{\frac{1}{10}+\frac{1}{1}} \times \frac{5}{2} \times A = 10 \times \frac{h_{L1}}{1} \times A$$

$h_{L1} = 0.4545$

$(T_H)_B = 5 - 1 \times .4545 = 4.54 \text{ m}$

61. A consolidated drained (CD) triaxial test was carried out on a sand sample with the known effective shear strength parameters, c′ =0 and ϕ′=30°. In the test, prior to the failure, when the sample was undergoing axial compression under constant cell pressure, the drainage valve was accidentally closed. At the failure, 360 kPa deviatoric stress was recorded along with 70 kPa pore water pressure. If the test is repeated without such error, and no back pressure is applied in either of the tests, what is the deviatoric stress (in kPa, in integer) at the failure? ____________

[NAT – 2 Marks]

Ans. 500

Sol.

$$\bar{\sigma}_1 = \bar{\sigma}_3 \tan^2\left(45+\frac{\phi}{2}\right) + 2c'\tan\left(45+\frac{\phi}{2}\right)$$

$\because \bar{\sigma}_3 = \sigma_3 - 70$

$\bar{\sigma}_1 = \sigma_1 - 70 = \sigma_3 + \sigma_d - 70 = (\sigma_3 + 360 - 70)$

$\Rightarrow (\sigma_3 + 360 - 70) = (\sigma_3 - 70) \tan^2 (45 + 30/2)$ [∵ C = 0]

$\Rightarrow \sigma_3 = 250 \text{ kPa}$

Now, $\sigma_1 = \sigma_3 \tan^2 (45 + \phi/2)$ [∵ C = 0]

$\Rightarrow (\sigma_3 + \sigma_d) = \sigma_3 \tan^2 (45 + 30/2)$

$\Rightarrow (250 + \sigma_d) = 250 \times 3$

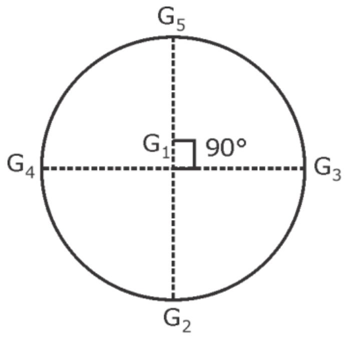
$\Rightarrow \sigma_d = 500 \text{ kPa}$

62. A catchment may be idealized as a circle of radius 30 km. There are five rain gauges, one at the center of the catchment and four on the boundary (equi-spaced), as shown in the figure (not to scale).

The annual rainfall recorded at these gauges in a particular year are given below.

Gauge	G_1	G_2	G_3	G_4	G_5
Rainfall (mm)	910	930	925	895	905

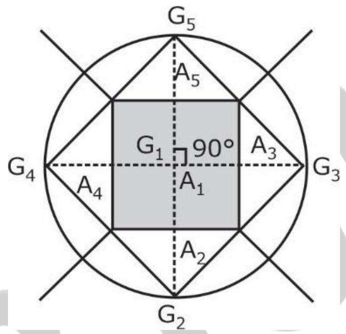
Using the Thiessen polygon method, what is the average rainfall (in mm, rounded off to two decimal places) over the catchment in that year? ______________



[NAT – 2 Marks]

Ans. 912.55

Sol.



$$A_1 = 30 \times 30 = 900 \text{ km}^2$$

$$A_2 = A_3 = A_4 = A_5 = \left(\frac{\pi \times 30^2 - 900}{4}\right)$$

$$= 481.85 \text{ km}^2$$

$$P_{avg} = \left(\frac{P_1A_1 + P_2A_2 + P_3A_3 + P_4P_4 + P_5P_5}{A_1 + A_2 + A_4 + A_5}\right)$$

$$\frac{900 \times 910 \times 930 \times 481.85 + 925 \times 481.85 + 895 + 481.85 + 905 \times 481.89}{900 + 4 \times 481.85}$$

$$\boxed{P_{avg} = 912.55 \text{ mm}}$$

63. The cross-section of a small river is sub-divided into seven segments of width 1.5 m each. The average depth, and velocity at different depths were measured during a field campaign at the middle of each segment width. The discharge computed by the velocity area method for the given data is ________________m^3/s (round off to one decimal place).

Segment	**Average depth (D) (m)**	**Velocity (m/s) at different depths**		
		0.2D	**0.6D**	**0.8D**
1	0.40	–	0.40	–
2	0.70	0.76	–	0.70
3	1.20	1.19	–	1.13

4	1.40	1.25	–	1.10
5	1.10	1.13	–	1.09
6	0.80	0.69	–	0.65
7	0.45	–	0.42	–

[NAT – 2 Marks]

Ans. 8.4 to 8.6

Sol.

Segment	Avg. Depth (Y)	Avg. width(W)	$V_{0.2}$	$V_{0.6}$	$V_{0.8}$	Q= Y x W x Mean Velocity(m^3/s)
(i)	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**
1	0.40	1.6875		0.40		$0.4 \times 1.6875 \times .4 = 0.27$
2	0.70	1.5	0.76	-	0.70	$0.70 \times 1.5 \times \left[\frac{0.76+0.70}{2}\right]$ $= 0.767$
3	1.20	1.5	1.19	-	1.13	$1.20 \times 1.5 \times \left[\frac{1.19+1.13}{2}\right]$ $= 2.088$
4	1.40	1.5	1.25	-	1.10	$1.40 \times 1.5 \times \left[\frac{1.25+1.10}{2}\right]$ $= 2.467$
5	1.10	1.5	1.13	-	1.09	$1.10 \times 1.5 \times \left[\frac{1.13+1.09}{2}\right]$ $= 1.8315$
6	0.80	1.5	0.69	-	0.65	$0.80 \times 1.5 \times \left[\frac{0.09+0.05}{2}\right]$ $= 0.804$
7	0.45	1.6876		0.42		$0.45 \times 1.6875 \times 0.42 = 0.319$
						Total Discharge = 8.546

Average width calculation column (iii)

$$(W_2)_{avg.} = \frac{1.5}{2} + \frac{1.5}{2} = 1.5$$

$W_2 = W_3 = W_4 = W_5 = W_6 = 1.5$

$$W_{avg.} = \frac{W_1 + \frac{W_2}{2}}{2W_1} = \frac{1.5 + \left(\frac{1.5}{2}\right)^2}{2 \times 1.5} = 1.6875$$

Discharge calculation, column (vii)

Discharge = Average depth × average width x mean velocity

64. The theoretical aerobic oxidation of biomass ($C_5H_7O_2N$) is given below:

$$C_5H_7O_2N + 5O_2 \rightarrow 5CO_2 + NH_3 + 2H_2O$$

The biochemical oxidation of biomass is assumed as a first-order reaction with a rate constant of 0.23/d at 20ºC (logarithm to base e). Neglecting the second-stage oxygen demand from its biochemical oxidation, the ratio of BOD5 at 20ºC to total organic carbon (TOC) of biomass is (round off to two decimal places).

[Consider the atomic weights of C, H, O and N as 12 g/mol, 1 g/mol, 16 g/mol and 14 g/mol, respectively]

[NAT – 2 Marks]

Ans. 1.82

Sol. $C_5H_7O_2N = 5 \times (12) + 7 \times (1) + 2 \times (16) + 14 = 113$ g

113gm of $C_5H_7O_2N \rightarrow 5 \times 12$ gm carbon

1 mole of $C_5H_7O_2N \rightarrow \frac{5 \times 12}{113} = 0.53$ gm carbon

BOD_5 at 20°C = BOD_u [$1 - e^{-Kd \times t}$]

= BOD_u ($1 - e^{-0.23 \times 5}$)

113 gm $C_5H_7O_2N$ required 5×32 gm of O_2

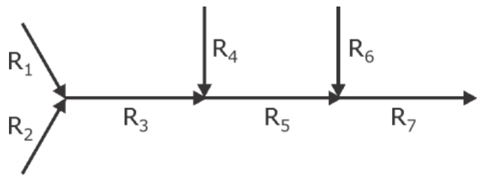
So, 1 gm $C_5H_7O_2N$ requires.

$\frac{5 \times 32}{113} = 1.415$ gm of O_2

BOD_u= 1.4159 ($1 - e^{-0.23 \times 5}$) = 0.967 g/l

$$\frac{BOD_{5,20^\circ C}}{TOC} = \frac{0.967}{0.53} = 1.82$$

65. A system of seven river segments is shown in the schematic diagram. The R_i's, Q_i's, and C_i's (i = 1 to 7) are the river segments, their corresponding flow rates, and concentrations of a conservative pollutant, respectively. Assume complete mixing at the intersections, no additional water loss or gain in the system, and steady state condition. Given: Q_1 = 5 m^3/s ; Q_2 = 15 m^3/s; Q_4 = 3 m^3/s ; Q6 = 8 m^3/s ; C_1 = 8 kg/m^3 ; C_2 = 12 kg/m^3 ; C_6 = 10 kg/m^3. What is the steady state concentration (in kg/m^3, rounded off to two decimal place) of the pollutant in the river segment 7? ___________



[NAT – 2 Marks]

Ans. 10.68

Sol. $Q_1 + Q_2 = Q_3$

$Q_3 = 5 + 15 = 20$ m^3/s

$Q_3 - Q_4 = Q_5$

$Q_5 = 20 - 3 = 17\ m^3/s$

$Q_5 + Q_6 = Q_7$

$Q_7 = 17 + 8 = 25\ m^3/s$

$$C_3 = \frac{C_1Q_1 + C_2Q_2}{Q_1 + Q_2}$$

$$= \frac{(8 \times 5) + (12 \times 15)}{5 + 15} = 11\ kg/m^3$$

$C_3 = C_5 = 11\ kg/m^3$

$$C_7 = \left(\frac{C_5Q_5 + C_6Q_6}{Q_5 + Q_6}\right) = \frac{(11 \times 17 + 10 \times 8)}{(17 + 8)}$$

$= 10.68\ kg/m^3$

***

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