BYJU'S EXAM PREP

GATE 2023

Mechanical Engineering

Questions with Detailed Solutions

General Aptitude

1. He did not manage to fix the car himself, so he _______ in the garage.

A. got it fixed
B. getting it fixed
C. gets fixed
D. got fixed

[MCQ - 1 Mark]

Ans. A

Sol. The given statement is past tense.

2. Planting : Seed :: Raising : _____

(By word meaning)

A. Child
B. Temperature
C. Height
D. Lift

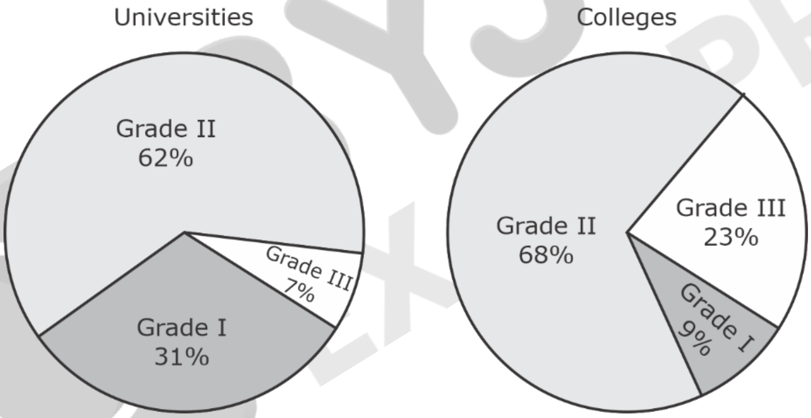
[MCQ - 1 Mark]

Ans. A

Sol. As seed is grown into a plant through planting. Similarly, a child is grown in a nature being through raising.

3. A certain country has 504 universities and 25951 colleges. These are categorized into Grades I, II, and III as shown in the given pie charts.

What is the percentage, correct to one decimal place of higher education institutions (colleges and universities) that fall into Grade III?



A. 22.7
B. 23.7
C. 15.0
D. 66.8

[MCQ - 1 Mark]

Ans. A

Sol. Total number of university = 504

Total number of colleges = 25951

For universities:

Grade III = 7% of university

$$= \frac{7}{100} \times 504 = 25.28$$

For collages

Grade III = 23% colleges

$$= \frac{23}{100} \times 25951 = 5968.73$$

Total percentage of Grade III (college & university)

$$= \frac{5968.73 + 25.28}{25951 + 504} = 22.7\%$$

4. The minute-hand and second-hand of a clock cross each other ________ times between 09:15:00 AM and 09:45:00 AM on a day.

A. 30 | B. 15
C. 29 | D. 31

[MCQ - 1 Mark]

Ans. A

Sol. 9 : 15 : 00 – 9 : 45 : 00
One crossing in each minute.
So, total = 30

5. The symbols ○ , *, Δ, and ◻ are to be filled, one in each box, as shown below.
The rules for filling in the four symbols are as follows.
1. Every row and every column must contain each of the four symbols.
2. Every 2 × 2 square delineated by bold lines must contain each of the four symbols.
Which symbol will occupy the box marked with '?' in the partially filled figure?

	?		△
	○	*	
		□	*
*		△	○

A. ○ | B. *
C. Δ | D. ◻

[MCQ - 1 Mark]

Ans. B

Sol.

□	*	○	Δ
Δ	○	*	□
○		□	*
*		Δ	○

Hence, * will be come at ?.

6. In a recently held parent-teacher meeting, the teachers had very few complaints about Ravi. After all, Ravi was a hardworking and kind student. Incidentally, almost all of Ravi's friends at school were hardworking and kind too. But the teachers drew attention to Ravi's complete lack of interest in sports. The teachers believed that, along with some of his friends who showed similar disinterest in sports, Ravi needed to engage in some sports for his overall development.
Based only on the information provided above, which one of the following statements can be logically inferred with certainty?
A. All of Ravi's friends are hardworking and kind.
B. No one who is not a friend of Ravi is hardworking and kind.
C. None of Ravi's friends are interested in sports.
D. Some of Ravi's friends are hardworking and kind.

[MCQ - 2 Marks]

Ans. D

Sol. The given statement is exactly matching with the information given in the passage.

7. Consider the following inequalities

$$p^2 - 4q < 4$$
$$3p + 2q < 6$$

where p and q are positive integers.

The value of $(p + q)$ is ________.

A. 2 B. 1

C. 3 D. 4

[MCQ - 2 Marks]

Ans. A

Sol.

$p^2 - 4q < 4$

$3p + 2q < 6 \Rightarrow 6p + 4q < 12$

$4q < 12 - 6p$

$p^2 - 4 < 4q < 12 - 6p$

$p^2 + 6p - 16 < 0$

$(p + 8)(p - 2) < 0$

$-8 < p < 2$

So, $p = 1$

$4q < 12 - 6$

$4q < 6$

$q < 3/2$

$q = 1$

$p + q = 2$

8. Which one of the sentence sequences in the given options creates a coherent narrative?

(i) I could not bring myself to knock.

(ii) There was a murmur of unfamiliar voices coming from the big drawing room and the door was firmly shut.

(iii) The passage was dark for a bit, but then it suddenly opened into a bright kitchen.

(iv) I decided I would rather wander down the passage.

A. (iv), (i), (iii), (ii) B. (iii), (i), (ii), (iv)

C. (ii), (i), (iv), (iii) D. (i), (iii), (ii), (iv)

[MCQ - 2 Marks]

Ans. C

Sol. Sentence-3 follows sentence-4

9. How many pairs of sets (S, T) are possible among the subsets of {1, 2, 3, 4, 5, 6} that satisfy the condition that S is a subset of T?

A. 729 B. 728

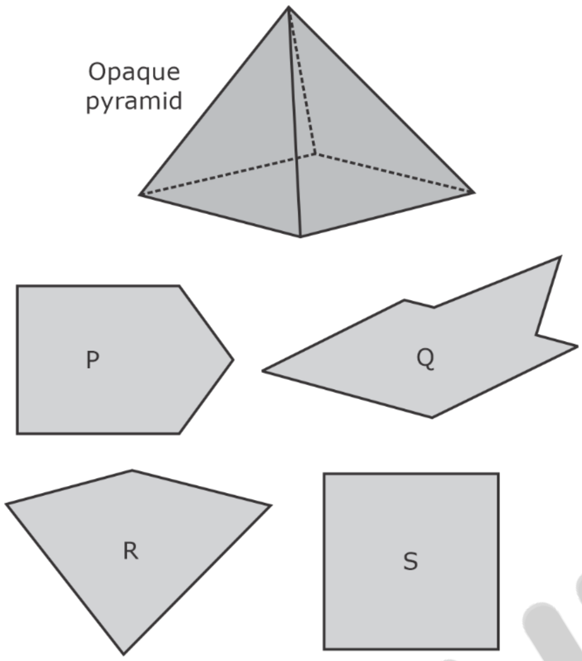
C. 665 D. 664

[MCQ - 2 Marks]

Ans. A

Sol. *

10. An opaque pyramid (shown below), with a square base and isosceles faces, is suspended in the path of a parallel beam of light, such that its shadow is cast on a screen oriented perpendicular to the direction of the light beam. The pyramid can be reoriented in any direction within the light beam. Under these conditions, which one of the shadows P, Q, R, and S is NOT possible?

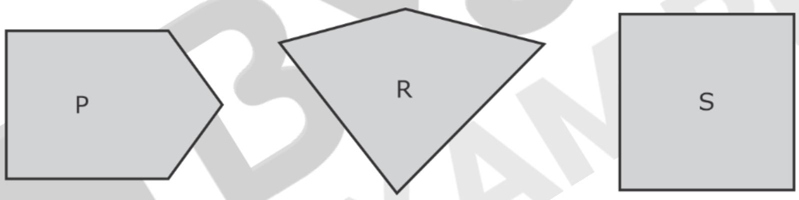


A. P
B. Q
C. R
D. S

[MCQ - 2 Marks]

Ans. B

Sol.



An opaque pyramid, with a square base and isosceles faces, is suspended in path of a parallel beam of light, such that its shadow is cast on a screen oriented perpendicular to the direction of the light beam. While changing orientation of opaque pyramid, we can obtain three shadow P, R and S. So, shadow Q is not possible, because shadow will not appear from corner in diagram Q.

11. A machine produces a defective component with a probability of 0.015. The number of defective components in a packed box containing 200 components produced by the machine follows a Poisson distribution. The mean and the variance of the distribution are.

A. 3 and 3, respectively
B. $\sqrt{3}$ and $\sqrt{3}$, respectively
C. 0.015 and 0.015, respectively
D. 3 and 9, respectively

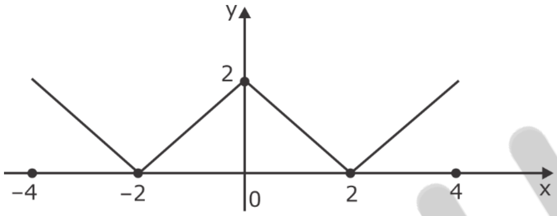
[MCQ - 1 Mark]

Ans. A

Sol. Mean = λ = nP = 200 × 0.015 = 3

Variance = λ = 3.

12. The figure shows the plot of a function over the interval [-4, 4]. Which one of the options given CORRECTLY identifies the function?

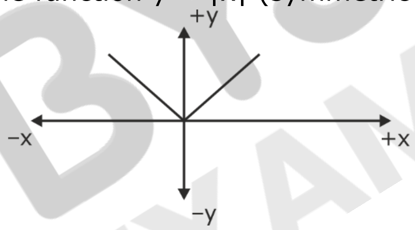


A. $|2 - x|$
B. $|2 - |x||$
C. $|2 + |x||$
D. $2 - |x|$

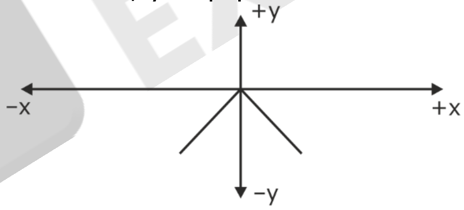
[MCQ - 1 Mark]

Ans. B

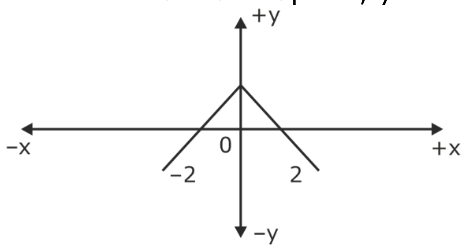
Sol. We know that the graph for the function y = |x| (symmetric about y-axis)



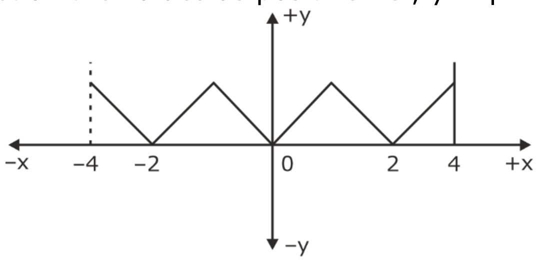
Taking the reflection about x-axis i.e., y = |x|



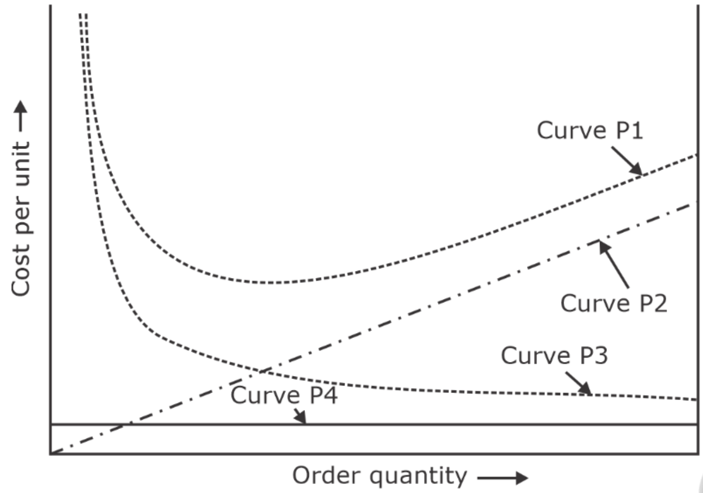
As given in questions the curve is shifted 2 unit up i.e., y = 2 − |x|



Take modulus again to get all the values as positive i.e., y = |2 − |x||



13. With reference to the Economic Order Quantity (EOQ) model, which one of the options given is correct?



A. Curve P1: Total cost, Curve P2: Holding cost, Curve P3: Setup cost, and Curve P4: Production cost.

B. Curve P1: Holding cost, Curve P2: Setup cost, Curve P3: Production cost, and Curve P4: Total cost.

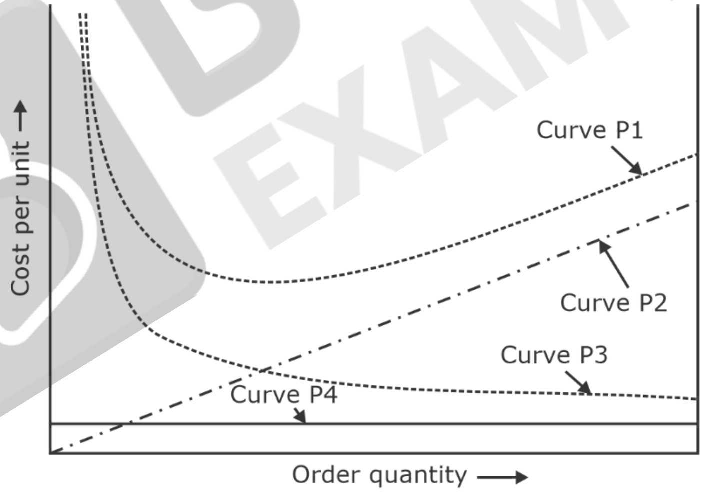
C. Curve P1: Production cost, Curve P2: Holding cost, Curve P3: Total cost, and Curve P4: Setup cost.

D. Curve P1: Total cost, Curve P2: Production cost, Curve P3: Holding cost, and Curve P4: Setup cost.

[MCQ - 1 Mark]

Ans. A

Sol.



Curve P_1 = Total cost

Curve P_2 = Holding cost

$\because$ Holding $= \frac{Q}{2} C_h$

Holding cost $\propto Q$

Curve P_3 = Setup cost

Cost $= \frac{D}{Q} \times C_0$

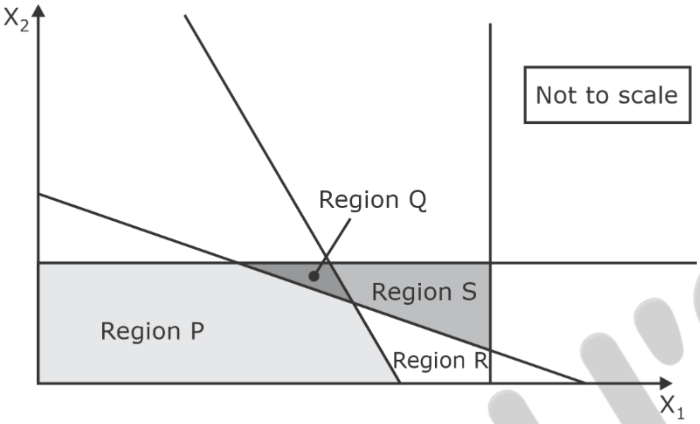
$\boxed{\text{Setup cost} \propto \frac{1}{Q}}$

Curve P_4 = Production cost

14. Which one of the options given represents the feasible region of the linear programming model:

Maximize $45X_1 + 60X_2$

$$X_1 \le 45$$
$$X_2 \le 50$$
$$10X_1 + 10X_2 \ge 600$$
$$25X_1 + 5X_2 \le 750$$



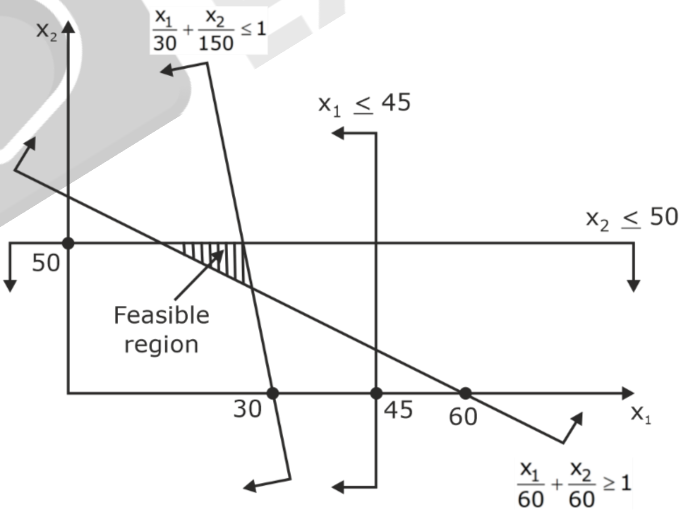
A. Region P
B. Region Q
C. Region R
D. Region S

[MCQ - 1 Mark]

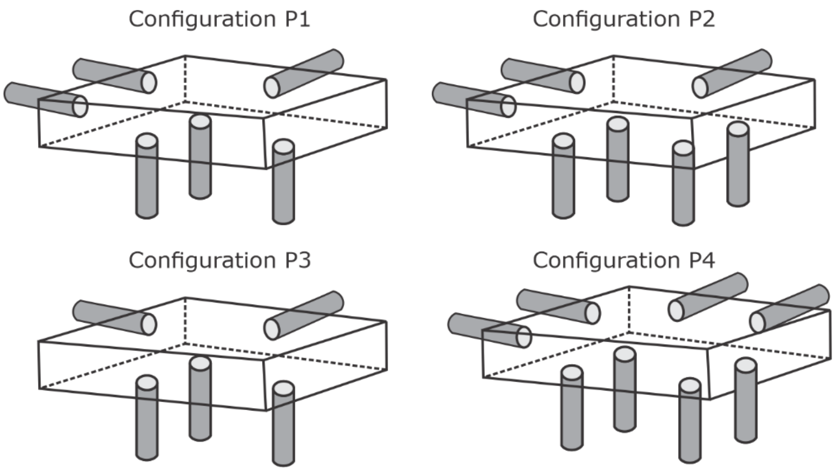
Ans. B

Sol.

$$x_1 \le 45$$
$$x_2 \le 50$$
$$\frac{x_1}{60} + \frac{x_2}{60} \ge 1$$
$$\frac{x_1}{30} + \frac{x_2}{150} \le 1$$



15. A cuboidal part has to be accurately positioned first, arresting six degrees of freedom and then clamped in a fixture, to be used for machining. Locating pins in the form of cylinders with hemi-spherical tips are to be placed on the fixture for positioning. Four different configurations of locating pins are proposed as shown. Which one of the options given is correct?



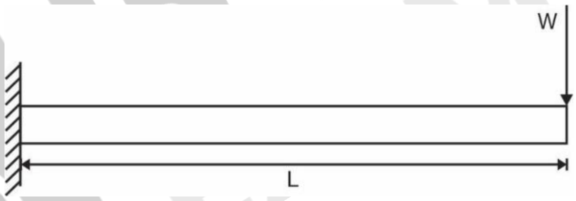
A. Configuration P1 arrests 6 degrees of freedom, while Configurations P2 and P4 are over-constrained and Configuration P3 is under-constrained.
B. Configuration P2 arrests 6 degrees of freedom, while Configurations P1 and P3 are over-constrained and Configuration P4 is under-constrained.
C. Configuration P3 arrests 6 degrees of freedom, while Configurations P2 and P4 are over-constrained and Configuration P1 is under-constrained.
D. Configuration P4 arrests 6 degrees of freedom, while Configurations P1 and P3 are over-constrained and Configuration P2 is under-constrained.

[MCQ - 1 Mark]

Ans. A

Sol. Configuration P3 arrests 6 degrees of freedom because it follows 3-2-1 principle of jigs and fixture and configure P2 and P4 are over constrained and configuration P3 is under constrained.

16. The effective stiffness of a cantilever beam of length L and flexural rigidity EI subjected to a transverse tip load W is



A. $\frac{3EI}{L^3}$

B. $\frac{2EI}{L^3}$

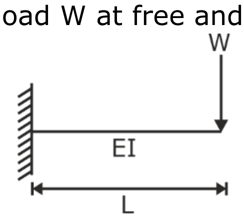
C. $\frac{L^3}{2EI}$

D. $\frac{L^3}{3EI}$

[MCQ - 1 Mark]

Ans. A

Sol. Given a cantilever beam with point load W at free and deflection at free end,



$$\delta = \frac{WL^3}{3EI}$$

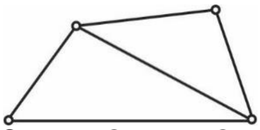
We know,

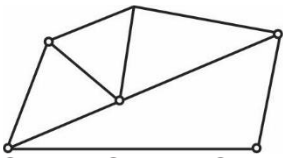
$$k\delta = W$$

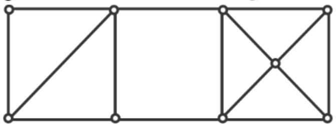
$$k \times \frac{WL^3}{3EI} = W$$

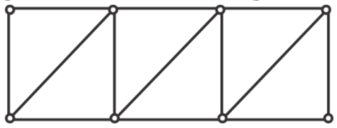
$$k = \frac{3EI}{L^3}$$

17. The options show frames consisting of rigid bars connected by pin joints. Which one of the frames is non-rigid?

A. 

B. 

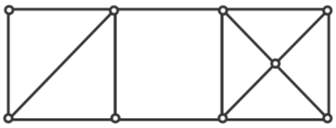
C. 

D. 

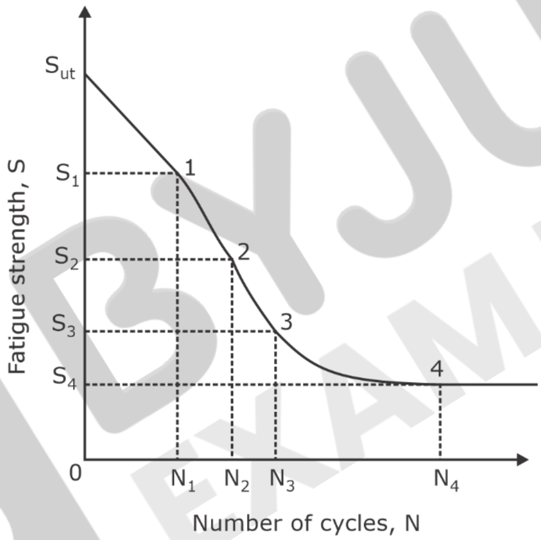
[MCQ - 1 Mark]

Ans. C

Sol.



18. The S-N curve from a fatigue test for steel is shown. Which one of the options gives the endurance limit?



A. S_{ut}

B. S_2

C. S_3

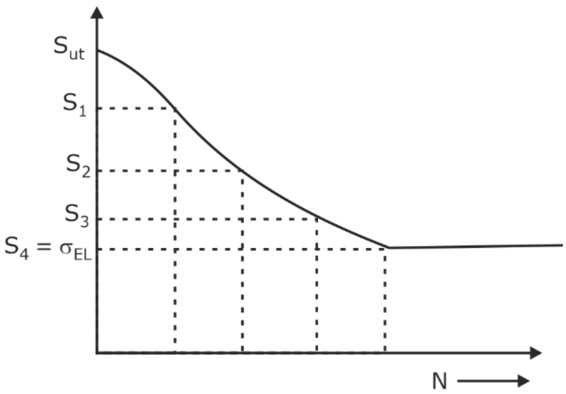
D. S_4

[MCQ - 1 Mark]

Ans. D

Sol. The endurance limit of a material is defined as the stress below which a material can endure an infinite number of repeated load cycles without exhibiting failure. In other words, when a material is subjected to a stress that is lower than its endurance limit, it should theoretically be able to withstand an indefinite amount of load cycles.

Hence correct endurance limit is S_4.



19. Air (density = 1.2 kg/m^3, kinematic viscosity = 1.5×10^{-5} m^2/s) flows over a flat plate with a free-stream velocity of 2 m/s. The wall shear stress at a location 15 mm from the leading edge is τ_w. What is the wall shear stress at a location 30 mm from the leading edge?

A. $\tau_w/2$

B. $\sqrt{2}\tau_w$

C. $2\tau_w$

D. $\tau_w / \sqrt{2}$

[MCQ - 1 Mark]

Ans. D

Sol. For laminar boundary layer over a flat plate $\tau_0 \propto \frac{1}{\sqrt{x}}$

So, $$\frac{\tau_{0_2}}{\tau_{0_1}} = \sqrt{\frac{x_1}{x_2}}$$

$$\Rightarrow \tau_{0_2} = \tau_w \times \sqrt{\frac{15}{30}}$$

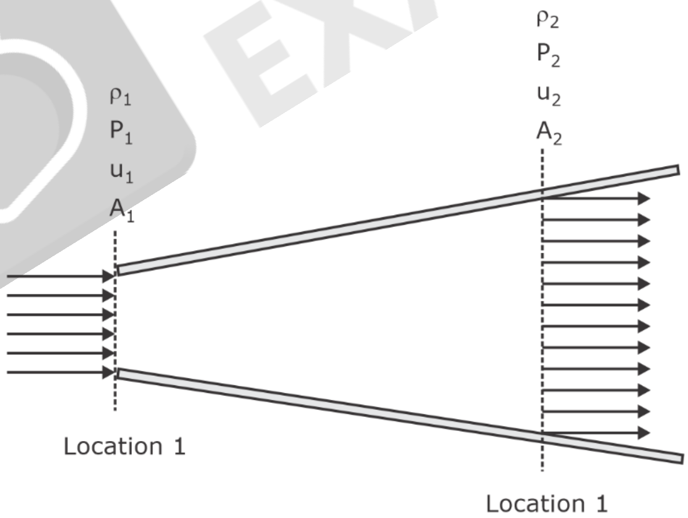
$$\tau_{0_2} = \frac{\tau_w}{\sqrt{2}}$$

20. Consider an isentropic flow of air (ratio of specific heats = 1.4) through a duct as shown in the figure.

The variations in the flow across the cross-section are negligible. The flow conditions at Location 1 are given as follows:

P_1 = 100 kPa, ρ_1 = 1.2 kg/m^3, u_1 = 400 m/s

The duct cross-sectional area at Location 2 is given by $A_2 = 2A_1$, where A_1 denotes the duct cross-sectional area at Location 1. Which one of the given statements about the velocity u_2 and pressure P_2 at Location 2 is TRUE?



A. $u_2 < u_1$, $P_2 < P_1$

B. $u_2 < u_1$, $P_2 > P_1$

C. $u_2 > u_1$, $P_2 < P_1$

D. $u_2 > u_1$, $P_2 > P_1$

[MCQ - 1 Mark]

Ans. C

Sol. p_1 = 100 KPa ρ_1 = 1.2

u_1 = 400 m/s

The ideal gas equation is

$$P_1 = \rho_1 R T_1$$

$$T_1 = \frac{100}{1.2 \times 0.287} = 290.36\text{ K}$$

Mach number at inlet is

$$M = \frac{400}{\sqrt{1.4 \times 287 \times 290.36}} = 1.17$$

Mach number at the inlet of duct is greater than one so flow is supersonic. And divergent cross section area behaves like a nozzle so velocity increase and pressure decreases.

$$\Rightarrow u_2 > u_1,\ P_2 < P_1$$

21. Consider incompressible laminar flow of a constant property Newtonian fluid in an isothermal circular tube. The flow is steady with fully developed temperature and velocity profiles. The Nusselt number for this flow depends on

A. neither the Reynolds number nor the Prandtl number

B. both the Reynolds and Prandtl numbers

C. the Reynolds number but not the Prandtl number

D. the Prandtl number but not the Reynolds number

[MCQ - 1 Mark]

Ans. A

Sol. For laminar flow in pipe Nusselt number remains constant which is independent of Prandtl number and Reynold number.

22. A heat engine extracts heat (Q_H) from a thermal reservoir at a temperature of 1000 K and rejects heat (Q_L) to a thermal reservoir at a temperature of 100 K, while producing work (W). Which one of the combinations of [Q_H, Q_L and W] given is allowed?

A. Q_H = 2000 J, Q_L = 500 J, W = 1000 J

B. Q_H = 2000 J, Q_L = 750 J, W = 1250 J

C. Q_H = 6000 J, Q_L = 500 J, W = 5500 J

D. Q_H = 6000 J, Q_L = 600 J, W = 5500 J

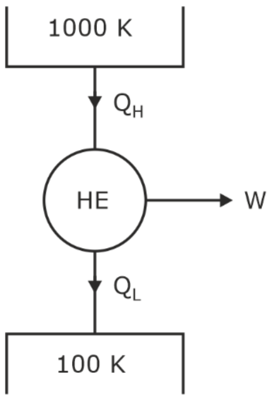
[MCQ - 1 Mark]

Ans. B

Sol. Given,

$$T_H = 1000\text{ K}$$

$$T_L = 100\text{ K}$$



A) Q_H = 2000 J, Q_L = 750J, W = 1250 J

We know that,

$$W = Q_H - Q_L = 2000 - 750 = 1250 \text{ J}$$

$$(\eta_{carnot})_{HE} = 1 - \frac{T_L}{T_H} = 1 - \frac{100}{1000} = 0.9 \text{ or } 90\%$$

$$(\eta)_{\text{for this option}} = \frac{W}{Q_H} = \frac{1250}{2000} = 62.5\%$$

∴ This is the correct option.

B) ∵ $Q_H - Q_L = 6000 - 600$

$= 5400 \neq W(5500 \text{ J})$

So, option B is incorrect.

C) ∵ $Q_H - Q_L = 2000 - 500$

$= 1500 \text{ J} \neq W(1000 \text{ J})$

So, option C is incorrect.

D) $Q_H - Q_L = 6000 - 500$

$= 5500 \text{ J} = W(5500 \text{ J})$

Now, $\eta = \frac{5500}{6000} = 0.9166 \text{ or } 91.67\% > (\eta)_{\text{carnot efficiency}}$

So, option D is incorrect.

23. Two surfaces P and Q are to be joined together. In which of the given joining operation(s), there is no melting of the two surfaces P and Q for creating the joint?

A. Arc welding B. Brazing

C. Adhesive bonding D. Spot welding

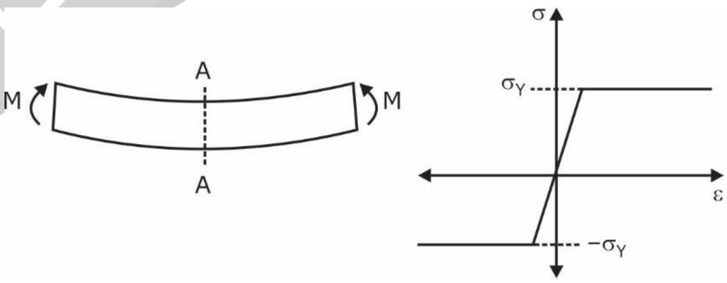
[MSQ - 1 Mark]

Ans. B,C

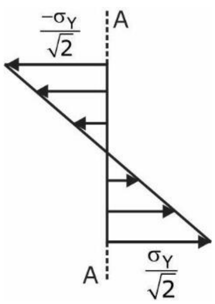
Sol. Brazing, Adhesive bonding

24. A beam is undergoing pure bending as shown in the figure. The stress (σ)-strain (ε) curve for the material is also given. The yield stress of the material is σ_Y.

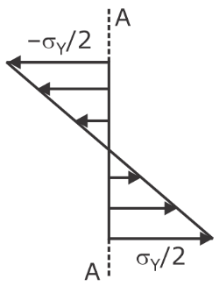
Which of the option(s) given represent(s) the bending stress distribution at cross-section AA after plastic yielding?

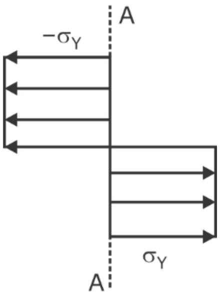


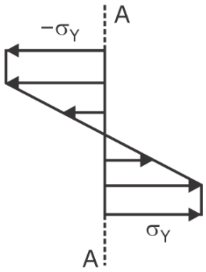
A.



B.



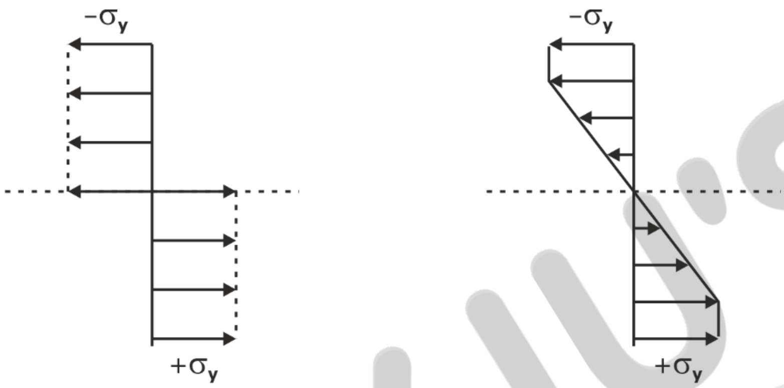
C. 

D. 

[MSQ - 1 Mark]

Ans. C,D

Sol.



If complete yielding happened If yielding just started

25. In a metal casting process to manufacture parts, both patterns and moulds provide shape by dictating where the material should or should not go. Which of the option(s) given correctly describe(s) the mould and the pattern?

A. Mould walls indicate boundaries within which the molten part material is allowed, while pattern walls indicate boundaries of regions where mould material is not allowed.

B. Moulds can be used to make patterns.

C. Pattern walls indicate boundaries within which the molten part material is allowed, while mould walls indicate boundaries of regions where mould material is not allowed.

D. Patterns can be used to make moulds.

[MSQ - 1 Mark]

Ans. A, B, D

Sol. Mould wall indicates boundaries within which the molten part material is allowed while pattern wall indicate boundaries where mould material is not allowed.

Moulds can be used to make patterns.

Pattern can be used to make mould.

26. The principal stresses at a point P in a solid are 70 MPa, −70 MPa and 0. The yield stress of the material is 100 MPa. Which prediction(s) about material failure at P is/are CORRECT?

A. Maximum normal stress theory predicts that the material fails

B. Maximum shear stress theory predicts that the material fails

C. Maximum normal stress theory predicts that the material does not fail

D. Maximum shear stress theory predicts that the material does not fail

[MSQ - 1 Mark]

Ans. B,C

Sol. Given principal stresses.

$$\sigma_1 = 70 \text{ MPa}$$
$$\sigma_2 = -70 \text{ MPa}$$
$$\sigma_3 = 0$$

Yield stress $\sigma_y = 100$ MPa

According to maximum shear stress theory

$$\left|\frac{\sigma_1 - \sigma_2}{2}\right| \le \frac{\sigma_y}{2N}$$

$$\left|\frac{70+70}{2}\right| \le \frac{100}{2N}$$

$$70 \le \frac{50}{N}$$

$$N < 1$$

Hence material fails.

According to maximum normal stress

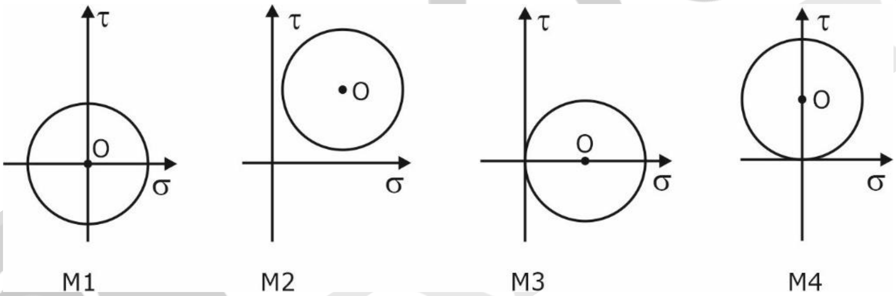
$$\sigma_1 \le \sigma_y/N$$

$$70 \le 100/N$$

$$N > 1$$

Hence material does not fail.

27. Which of the plot(s) shown is/are valid Mohr's circle representations of a plane stress state in a material? (The center of each circle is indicated by O.)

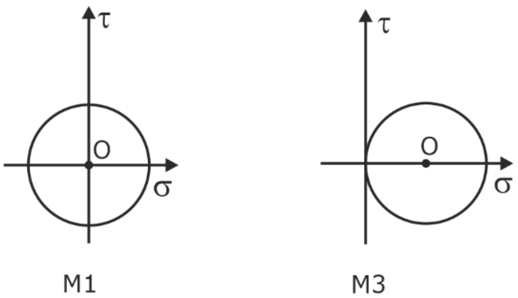


A. M1
B. M2
C. M3
D. M4

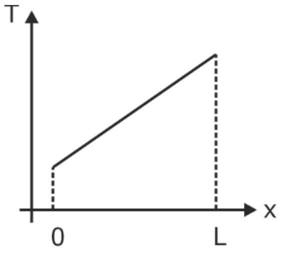
[MSQ - 1 Mark]

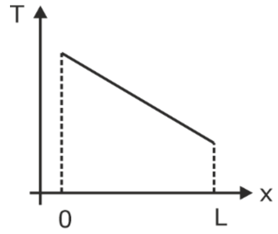
Ans. A,C

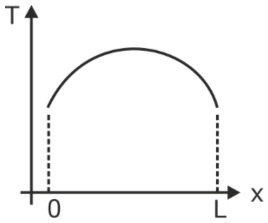
Sol. Centre of Mohr's circle always lies on σ-axis. Hence M_1 and M_3 is correct representation of a plane stress state.

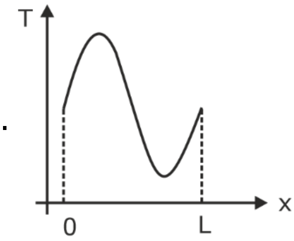


28. Consider a laterally insulated rod of length L and constant thermal conductivity. Assuming one-dimensional heat conduction in the rod, which of the following steady-state temperature profile(s) can occur without internal heat generation?

A.


B.


C.


D.


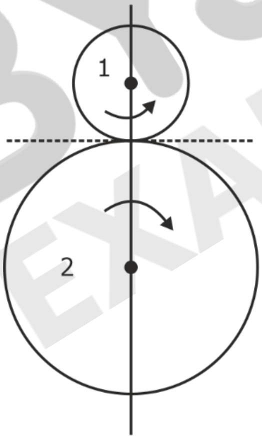
[MSQ - 1 Mark]

Ans. A,B

Sol. Temperature profile is linear for steady state conduction without heat generation.

29. Two meshing spur gears 1 and 2 with diametral pitch of 8 teeth per mm and an angular velocity ratio $|\omega_2|/|\omega_1| = 1/4$, have their centers 30 mm apart. The number of teeth on the driver (gear 1) is ________ .

(Answer in integer)

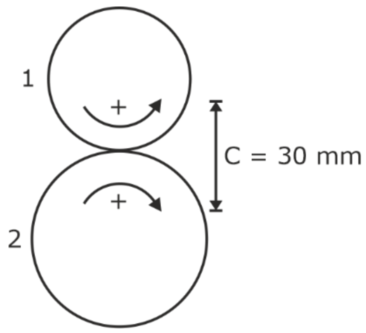


[NAT - 1 Mark]

Ans. 95.999 to 96.001

Sol. P_d = 8 teeth/mm

$$\frac{\omega_2}{\omega_1} = \frac{1}{4}$$



$$\boxed{P_d = \frac{T}{D}}$$

$$\frac{T_1}{D_1} = \frac{T_2}{D_2}$$

$$C = \frac{D_1 + D_2}{2}$$

$60 = D_1 + D_2$

$D_1 = 12$

$D_2 = 48$

$$\frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} = \frac{D_1}{D_2}$$

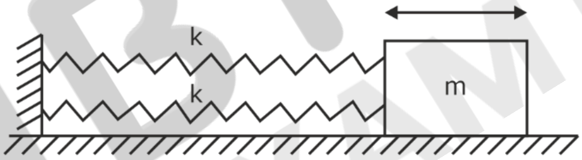
$$\frac{1}{4} = \frac{D_1}{D_2} \Rightarrow \boxed{D_2 = 4D_1}$$

$$\frac{T_1}{D_1} = 8$$

$T_1 = 96$

30. The figure shows a block of mass m = 20 kg attached to a pair of identical linear springs, each having a spring constant k = 1000 N/m. The block oscillates on a frictionless horizontal surface. Assuming free vibrations, the time taken by the block to complete ten oscillations is ________ seconds. (Rounded off to two decimal places)

Take π = 3.14.



[NAT - 1 Mark]

Ans. 6.27 to 6.29

Sol.

m = 20 kg

k = 1000 N/m

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

$$= \sqrt{\frac{2k}{m}}$$

$$= \sqrt{\frac{2 \times 1000}{20}} = 10 \text{ rad/s}$$

$$T = \frac{2\pi}{\omega_n}$$

$$= \frac{2\pi}{10} \text{ sec/ oscillation}$$

For 10 oscillations

$$= 10 \times \frac{2\pi}{10} \text{ sec}$$

$= 2\pi$

= 6.28 sec

31. A vector field

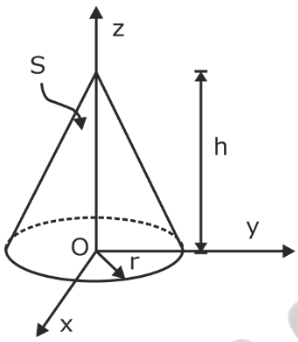
$$B(x, y, z) = x\hat{i} + y\hat{j} - 2z\hat{k}$$

is defined over a conical region having height h = 2, base radius r = 3 and axis along z, as shown in the figure. The base of the cone lies in the x-y plane and is centered at the origin.

If n denotes the unit outward normal to the curved surface S of the cone, the value of the integral

$$\int_S B.n\ dS$$

equals __________ . (Answer in integer)



[NAT - 1 Mark]

Ans. -0.001 to 0.001

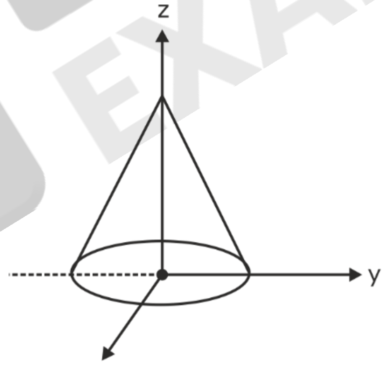
Sol. Given

$$\vec{B} = x\,\hat{i} + y\,\hat{j} - 2z\,\hat{k}$$

Given S is curved surface. Let S_1 is base surface.

Hence S + S_1 is total cone surface.

So, we can apply Gauss-divergence theorem to (S + S_1) surface.



$$\int_{S+S_1} \vec{B}.n\,ds = \iiint \vec{\nabla}.\vec{B}\,dxdydz$$

$$= \iiint \vec{\nabla}.\left(x\hat{i} + y\hat{j} + 2z\hat{k}\right)dxdydz$$

$$= \iiint (1 + 1 - 2)\,dxdydz$$

$$= 0$$

Now,

$$\int_{S_1} \vec{B}.\vec{n}\,ds = \int \left(x\hat{i}.y\hat{j} - 2z\hat{k}\right)\left(-\hat{k}\right)ds \qquad \left(\text{here } \hat{n} = -\hat{k}\right)$$

$$= \int 2z\,ds$$

But S_1 is base surface, So z = 0

$$\int \vec{B}.\hat{n}\,ds = 0$$

Now,

$$\int_S \vec{B}.n\,ds = \int_{S+S_1} \vec{B}.n\,ds - \int_{S_1} \vec{B}.n\,ds$$

$$= 0 - 0$$

$$= 0$$

32. A linear transformation maps a point (x, y) in the plane to the point $(\hat{x}, \hat{y})$ according to the rule

$$\hat{x} = 3y,\ \ \hat{y} = 2x.$$

Then, the disc $x^2 + y^2 \leq 1$ gets transformed to a region with an area equal to __________ . (Rounded off to two decimals)

Use π = 3.14.

Ans. 18.80 to 18.90

Sol.

$$x = 3y$$

$$y = 2x$$

$$x^2 + y^2 \leq 1$$

$$\Rightarrow \left(\frac{y}{2}\right)^2 + \left(\frac{x}{3}\right)^2 \leq 1$$

$$\frac{(x)^2}{3^2} + \frac{(y)^2}{2^2} \leq 1$$

→ Area of ellipse = πab

$$= \pi(3)\,(2) = 6\pi$$

$$= 18.84$$

33. The value of k that makes the complex-valued function

$f(z) = e^{-kx}$ (cos 2y − i sin 2y)

analytic, where z = x + iy, is __________.

(Answer in integer)

[NAT - 1 Mark]

Ans. 1.999 to 2.001

Sol.

$$u = e^{-kx}\cos 2y$$

$$v = -e^{-kx}\sin 2y$$

For analytic solution

$$u_x = v_y$$

$$-ke^{-kx}\cos xy = -e^{-kx}(2\cos 2y)$$

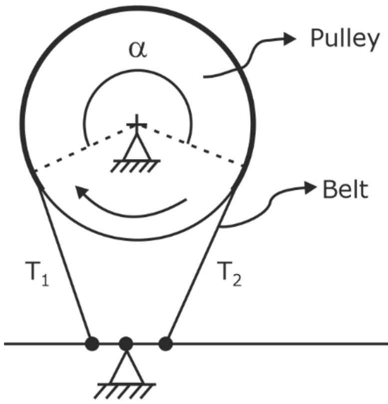
$$k = 2$$

34. The braking system shown in the figure uses a belt to slow down a pulley rotating in the clockwise direction by the application of a force P. The belt wraps around the pulley over an angle α = 270 degrees. The coefficient of friction between the belt and the pulley is 0.3. The influence of centrifugal forces on the belt is negligible.

During braking, the ratio of the tensions T_1 to T_2 in the belt is equal to ___________.

(Rounded off to two decimal places)

Take π = 3.14.



[NAT - 1 Mark]

Ans. 4.05 to 4.15

Sol. Given,

$$m = 0.3$$

$$\theta = 270° = 270 \times \frac{\pi}{100} = \frac{3\pi}{2}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3\times 3\pi/2} = 4.11$$

35. Consider a counter-flow heat exchanger with the inlet temperatures of two fluids (1 and 2) being $T_{1,\,in}$ = 300 K and $T_{2,\,in}$ = 350 K. The heat capacity rates of the two fluids are C_1 = 1000 W/K and C_2 = 400 W/K, and the effectiveness of the heat exchanger is 0.5. The actual heat transfer rate is ______ kW.

(Answer in integer)

[NAT - 1 Mark]

Ans. 9.999 to 10.001

Sol. $T_{1,in}$ = 300 K and $T_{2,in}$ = 350 K.

We know that,

$$\in = \frac{q_{act}}{q_{max}}$$

$$\Rightarrow q_{act} = \in \times q_{max} \quad (\text{As } C_2 = C_{min} = 400 \text{ W/k})$$

$$= \in \times C_2(T_{2,\,in} - T_{1,in})$$

$$= 0.5 \times 400(350 - 300)$$

$$= 10000 \text{ W}$$

$$= 10 \text{ kW}$$

36. Which one of the options given is the inverse Laplace transform of $\frac{1}{S^3 - S}$?

u(t) denotes the unit-step function.

A. $\left(-1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^{t}\right)u(t)$

B. $\left(\frac{1}{3}e^{-t} - e^{t}\right)u(t)$

C. $\left(-1 + \frac{1}{2}e^{-(t-1)} + \frac{1}{2}e^{(t-1)}\right)u(t-1)$

D. $\left(-1 - \frac{1}{2}e^{-(t-1)} - \frac{1}{2}e^{(t-1)}\right)u(t-1)$

[MCQ - 2 Marks]

Ans. A

Sol.

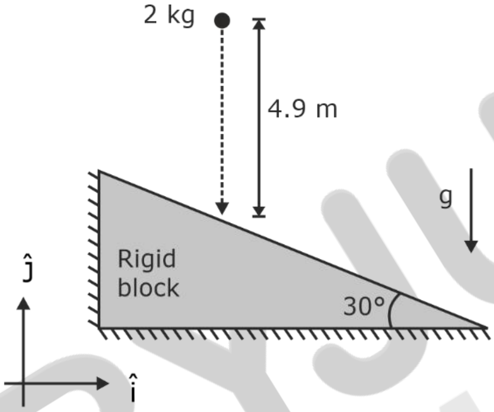
$$F(s) = \frac{1}{s(s^2-1)} = -\left(\frac{1}{s} - \frac{s}{s^2-1}\right)$$

$$= \frac{s}{s^2-1} - \frac{1}{s}$$

$$f(t) = \cosh t - 1 = \left(\frac{e^t + e^{-t}}{2}\right) - 1$$

37. A spherical ball weighing 2 kg is dropped from a height of 4.9 m onto an immovable rigid block as shown in the figure. If the collision is perfectly elastic, what is the momentum vector of the ball (in kg m/s) just after impact?

Take the acceleration due to gravity to be g = 9.8 m/s^2. Options have been rounded off to one decimal place.



A. $19.6\ \hat{i}$

B. $19.6\ \hat{j}$

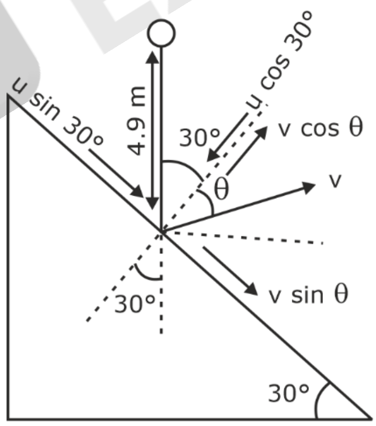
C. $17.0\ \hat{i} + 9.8\ \hat{j}$

D. $9.8\ \hat{i} + 17.0\ \hat{j}$

[MCQ - 2 Marks]

Ans. C

Sol.



Initial velocity

$$u = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 4.9} = 9.8\,m/s$$

Conservation of momentum along inclined path

$$m_1 u \sin 30 = m_1 v \sin\theta$$

$$9.8 \times \frac{1}{2} = v \sin\theta$$

$$4.9 = v \sin\theta \qquad \text{...(i)}$$

Given that collision is perfect elastic hence co-efficient of restitution is 1.

$$e = \frac{\text{Velocity of seperation}}{\text{Velocity of approach}} = 1$$

velocity at separation = velocity of approach

$$u \cos 30° = v \cos \theta$$

$$9.8 \times \frac{\sqrt{3}}{2} = v \cos \theta$$

$$4.9\sqrt{3} = v \cos \theta \qquad \text{...(ii)}$$

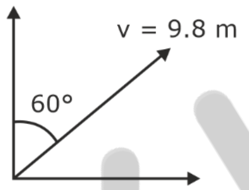
By dividing equation (i) and (ii), we get

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30°$$

Put the value of θ in equation (ii), we get,

$$v = 9.8 \text{ m/s}$$



Total angle of v from vertical = 30° + 30° = 60°

Total momentum after collision

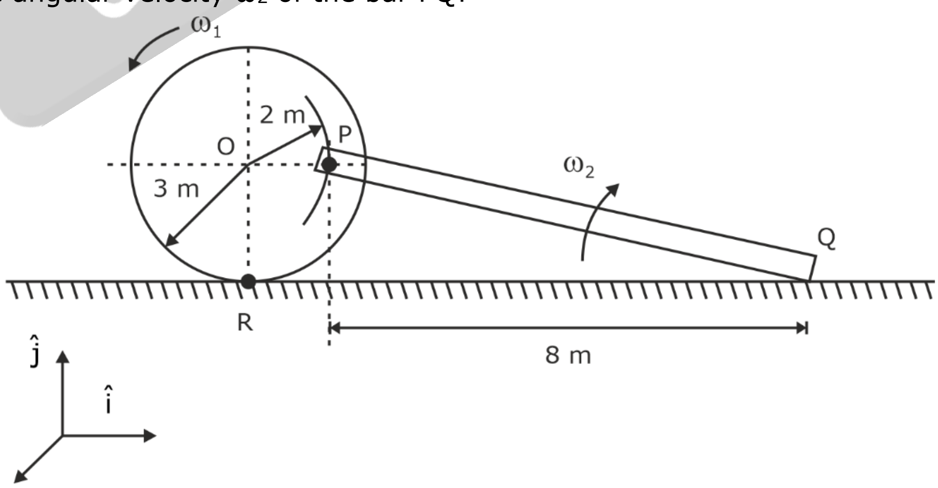
$$\vec{P} = m_2\vec{v}_2 = m\left(v \sin 60° \hat{i} + v \cos 60° \hat{j}\right)$$

$$= 2 \times 9.8\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$$

$$= 9.8\sqrt{3}\,\hat{i} + 9.8\,\hat{j}$$

$$= 16.98\,\hat{i} + 9.8\,\hat{j} \approx 17\,\hat{i} + 9.8\,\hat{j}$$

38. The figure shows a wheel rolling without slipping on a horizontal plane with angular velocity ω_1. A rigid bar PQ is pinned to the wheel at P while the end Q slides on the floor. What is the angular velocity ω_2 of the bar PQ?



A. $\omega_2 = 2\omega_1$

B. $\omega_2 = \omega_1$

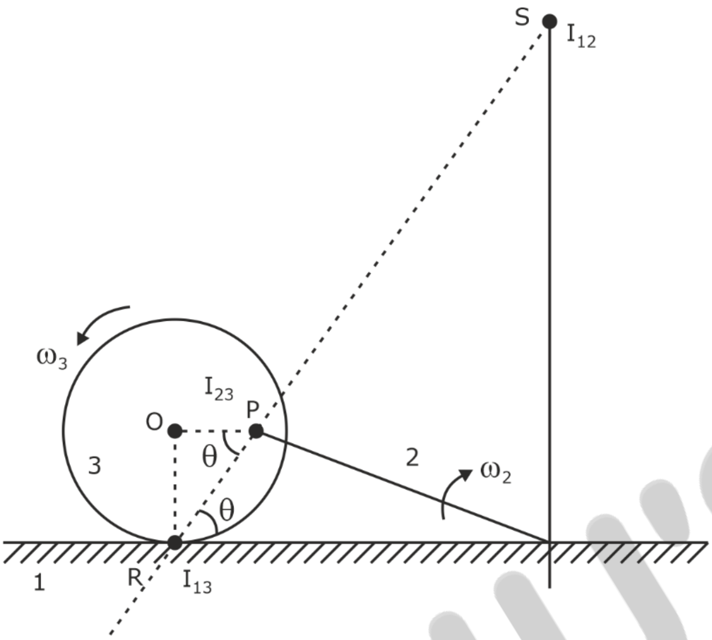
C. $\omega_2 = 0.5\,\omega_1$

D. $\omega_2 = 0.25\,\omega_1$

[MCQ – 2 Marks]

Ans. D

Sol.



$\because$ there is relative motion, hence using Kennedy's theorem their I-center's will always line on a straight line.

$$\omega_3 (I_{13} I_{23}) = \omega_2 (I_{12} I_{23})$$

$$\omega_3 (PR) = \omega_2 (PS) \quad \text{...(i)}$$

From ΔPOR

$$PR = \sqrt{2^2 + 3^2}$$

$$= \sqrt{13}$$

$$\tan\theta = \frac{3}{2} \qquad \cos\theta = \frac{2}{\sqrt{13}}$$

From ΔRSQ

$$\cos\theta = \frac{RQ}{RS}$$

$$RS = \frac{10}{\cos\theta}$$

$$= 5\sqrt{13}$$

PS = RS – PR

$$= 5\sqrt{13} - \sqrt{13}$$

$$= 4\sqrt{13}$$

From equation (i)

$$\omega_3\sqrt{13} = \omega_2 \times 4\sqrt{13}$$

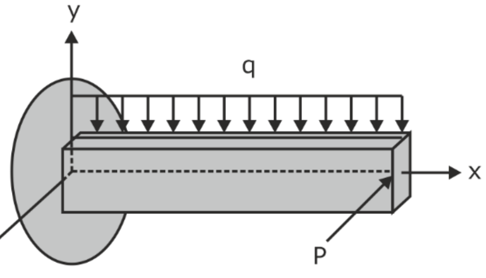
$$\omega_3 = 4\,\omega_2$$

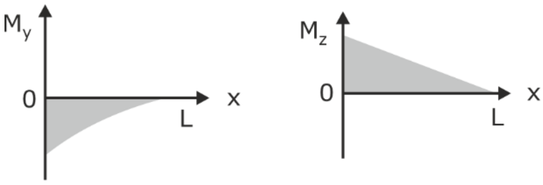
But according to question

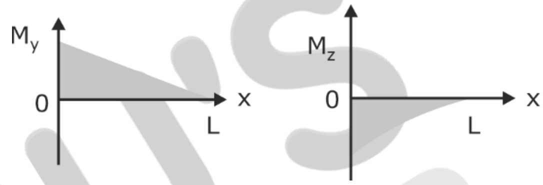
$$\boxed{\omega_2 = 0.25\ \omega_1}$$

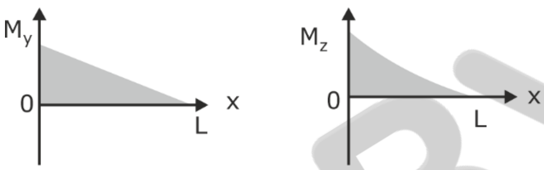
39. A beam of length L is loaded in the xy −plane by a uniformly distributed load, and by a concentrated tip load parallel to the z −axis, as shown in the figure. The resulting bending moment distributions about the y and the z axes are denoted by M_y and M_z, respectively.

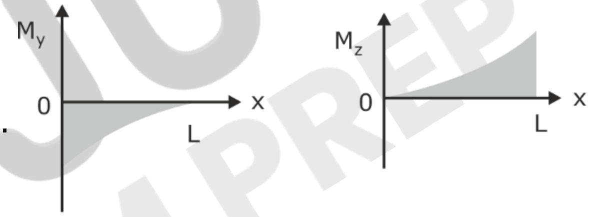
Which one of the options given depicts qualitatively CORRECT variations of M_y and M_z along the length of the beam?



A. 

B. 

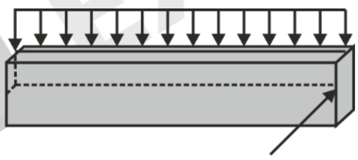
C. 

D. 

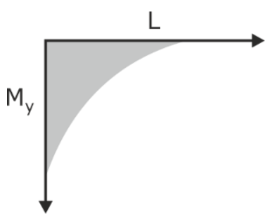
[MCQ - 2 Marks]

Ans. B

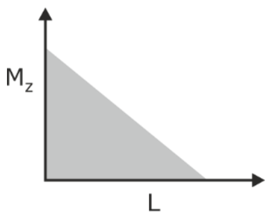
Sol.



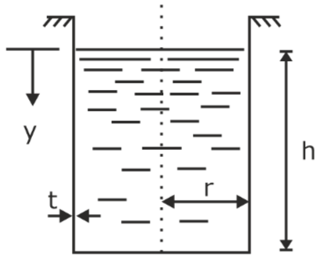
Due to uniformly distributed load

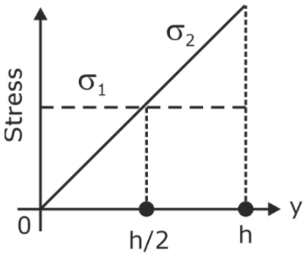


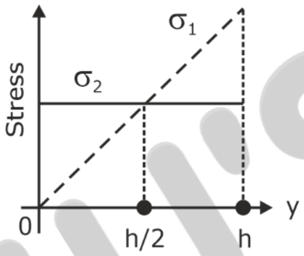
Due to concentrated load

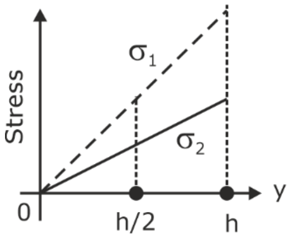


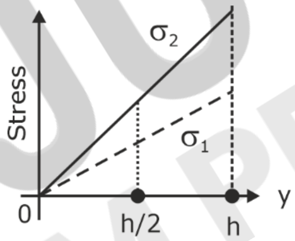
40. The figure shows a thin-walled open-top cylindrical vessel of radius r and wall thickness t. The vessel is held along the brim and contains a constant-density liquid to height h from the base. Neglect atmospheric pressure, the weight of the vessel and bending stresses in the vessel walls. Which one of the plots depicts qualitatively CORRECT dependence of the magnitudes of axial wall stress (σ_1) and circumferential wall stress (σ_2) on y?



A. 

B. 

C. 

D. 

[MCQ - 2 Marks]

Ans. A

Sol. As we all know that

$$\sigma_L = \frac{p_{max} r}{2t} \qquad p_{max} = \rho g h$$

$$(\sigma_h)_{x-x} = \frac{p_{x-x} r}{t} = \frac{\rho g x \cdot r}{t}$$

When x = h/2

$$\boxed{(\sigma_h) = \frac{\rho g h r}{t}}$$

So, C is correct.

41. Which one of the following statements is FALSE?

A. For an ideal gas, the enthalpy is independent of pressure.

B. For a real gas going through an adiabatic reversible process, the process equation is given by PV^{γ} = constant, where P is the pressure, V is the volume and γ is the ratio of the specific heats of the gas at constant pressure and constant volume.

C. For an ideal gas undergoing a reversible polytropic process $PV^{1.5}$ = constant, the equation connecting the pressure, volume, and temperature of the gas at any point along the process is $\frac{P}{R} = \frac{mT}{V}$, where R is the gas constant and m is the mass of the gas.

D. Any real gas behaves as an ideal gas at sufficiently low pressure or sufficiently high temperature.

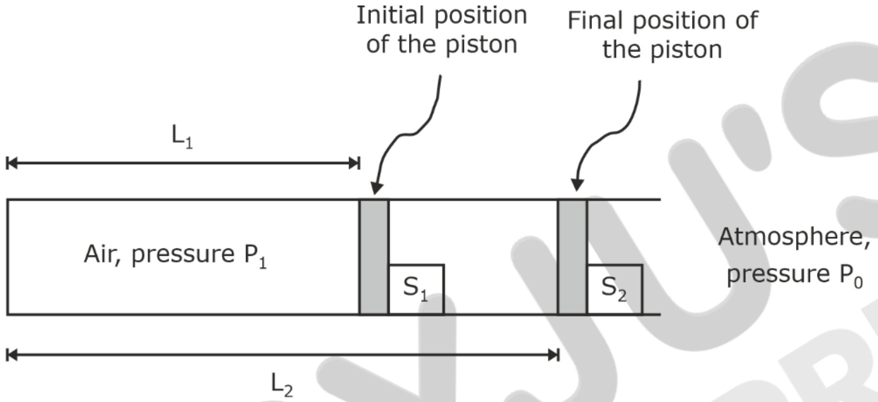
[MCQ - 2 Marks]

Ans. B

Sol. $PV^{\gamma} = C$ is valid for reversible adiabatic process of the ideal gas. so, this statement is not correct, and the correct answer is B.

42. Consider a fully adiabatic piston-cylinder arrangement as shown in the figure. The piston is massless and cross-sectional area of the cylinder is A. The fluid inside the cylinder is air (considered as a perfect gas), with γ being the ratio of the specific heat at constant pressure to the specific heat at constant volume for air. The piston is initially located at a position L_1. The initial pressure of the air inside the cylinder is $P_1 \gg P_0$, where P_0 is the atmospheric pressure. The stop S_1 is instantaneously removed and the piston moves to the position L_2, where the equilibrium pressure of air inside the cylinder is $P_2 \gg P_0$.

What is the work done by the piston on the atmosphere during this process?



A. 0

B. $P_0A(L_2 - L_1)$

C. $P_1AL_1 \ln\frac{L_1}{L_2}$

D. $\frac{(P_2L_2 - P_1L_1)A}{(1-\gamma)}$

[MCQ - 2 Marks]

Ans. B

Sol. The work done by piston on surrounding can be given as

$$w_{1-2} = \int p_{ex}.dv = P_o\int_1^2 dv = P_o(V_2 - V_1)$$

$$w_{1-2} = P_o(AL_2 - AL_1) = P_oA(L_2 - L_1)$$

43. A cylindrical rod of length h and diameter d is placed inside a cubic enclosure of side length L. S denotes the inner surface of the cube. The view-factor $F_{S\text{-}S}$ is

A. 0

B. 1

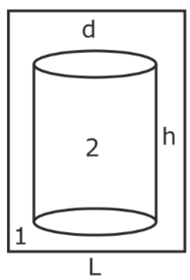
C. $\frac{(\pi dh + \pi d^2/2)}{6L^2}$

D. $1 - \frac{(\pi dh + \pi d^2/2)}{6L^2}$

[MCQ - 2 Marks]

Ans. D

Sol.



By Enclosure theorem (summation rule)

$$F_{21} + F_{22} = 1$$

$$\Rightarrow F_{21} = 1$$

($F_{22} = 0$: Convex and flat surface)

Also $F_{11} + F_{12} = 1$ (Enclosure theorem)

$$\Rightarrow F_{11} = 1 - F_{12}$$

$$= 1 - \frac{A_2F_{21}}{A_1} \text{ (reciprocity theorem)}$$

$$= 1 - \frac{A_2}{A_1}$$

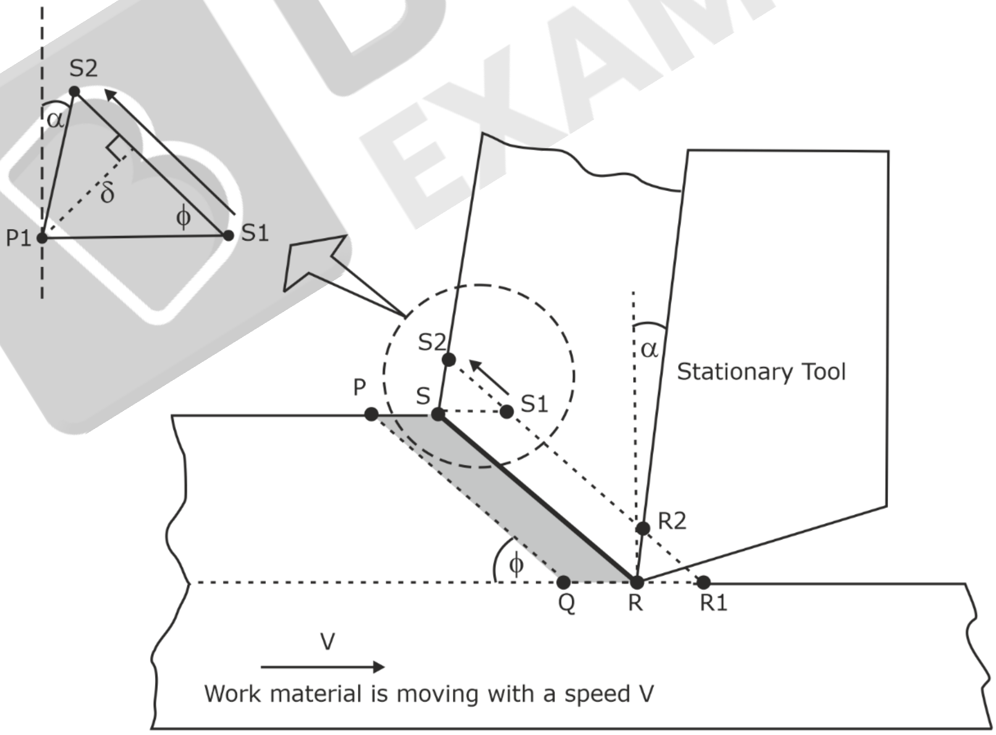
$$= 1 - \frac{\left(2 \times \frac{\pi}{4}d^2 + \pi dh\right)}{6L^2}$$

$$= 1 - \left(\frac{\frac{\pi}{2}d^2 + \pi dh}{6L^2}\right)$$

44. In an ideal orthogonal cutting experiment (see figure), the cutting speed V is 1 m/s, the rake angle of the tool α = 5°, and the shear angle, ϕ, is known to be 45°.

Applying the ideal orthogonal cutting model, consider two shear planes PQ and RS close to each other. As they approach the thin shear zone (shown as a thick line in the figure), plane RS gets sheared with respect to PQ (point R1 shears to R2, and S1 shears to S2).

Assuming that the perpendicular distance between PQ and RS is δ = 25 μm, what is the value of shear strain rate (in s^{-1}) that the material undergoes at the shear zone?



A. 1.84×10^4

B. 5.20×10^4

C. 0.71×10^4

D. 1.30×10^4

[MCQ - 2 Marks]

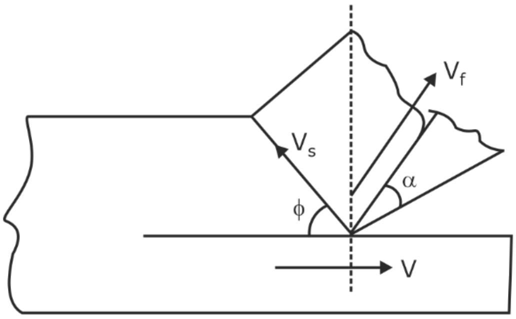
Ans. B

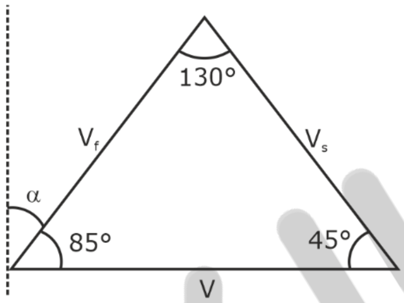
Sol. Given,

Shear angle, $\varphi = 45°$

Rake angle, $\alpha = 5°$

Cutting velocity, V = 1 m





From the above velocity triangle,

$$\frac{V_s}{\sin 85°} = \frac{V}{\sin 130°}$$

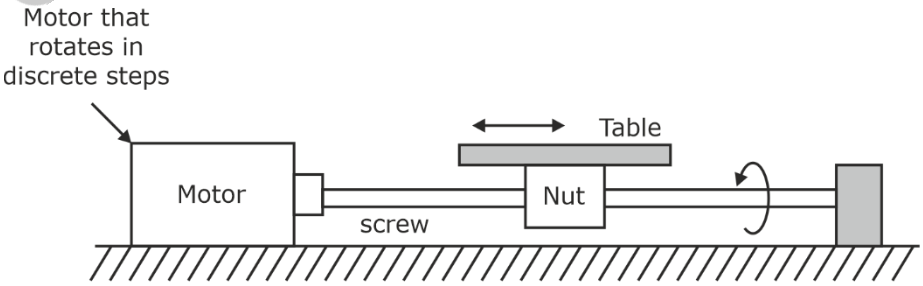
$$V_s = 1.3\ \text{m/sec}$$

$$\text{Strain rate} = \frac{V_s}{t_s}$$

$$= \frac{1.3}{25 \times 10^{-6}}$$

$$= 5.2 \times 10^{-4}\ s^{-1}$$

45. A CNC machine has one of its linear positioning axes as shown in the figure, consisting of a motor rotating a lead screw, which in turn moves a nut horizontally on which a table is mounted. The motor moves in discrete rotational steps of 50 steps per revolution. The pitch of the screw is 5 mm and the total horizontal traverse length of the table is 100 mm. What is the total number of controllable locations at which the table can be positioned on this axis?



A. 5000

B. 2

C. 1000

D. 200

[MCQ - 2 Marks]

Ans. C

Sol. Number of steps per revolution, $n_s = 50$

Pitch $= 5$ mm

BLU $= \frac{5}{50} = \frac{1}{10}$ mm

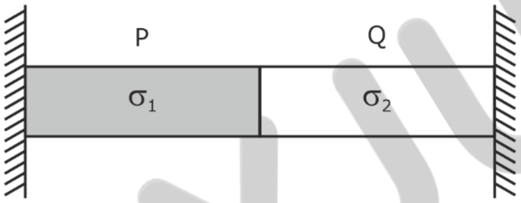
Length of the table $= 100$ mm

Number of controllable positions $= \frac{100}{BLU} = \frac{100}{(1/10)} = 1000$

46. Cylindrical bars P and Q have identical lengths and radii, but are composed of different linear elastic materials. The Young's modulus and coefficient of thermal expansion of Q are twice the corresponding values of P. Assume the bars to be perfectly bonded at the interface, and their weights to be negligible.

The bars are held between rigid supports as shown in the figure and the temperature is raised by ΔT. Assume that the stress in each bar is homogeneous and uniaxial. Denote the magnitudes of stress in P and Q by $\sigma_1$and σ_2, respectively.

Which of the statement(s) given is/are CORRECT?



A. The interface between P and Q moves to the left after heating

B. The interface between P and Q moves to the right after heating

C. $\sigma_1 < \sigma_2$

D. $\sigma_1 = \sigma_2$

[MSQ - 2 Marks]

Ans. A,D

Sol. Given,

$L_P = L_Q$

$A_P = A_Q$

$\alpha_Q = 2\alpha_P$

$E_Q = 2E_P$

For equilibrium

$R_P = R_Q = R$

$\sigma_1 = \frac{R}{A}$ and $\sigma_2 = \frac{R}{A}$

Hence, $\sigma_1 = \sigma_2$

$\Delta L_P = L(\alpha)\Delta T - \frac{RL}{AE}$

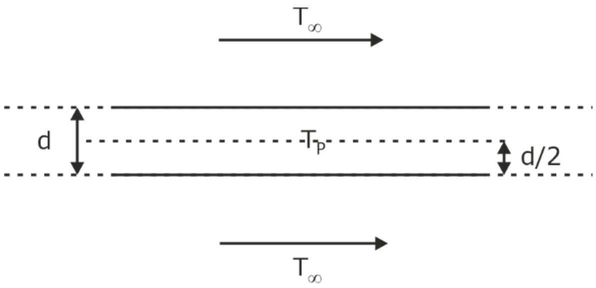
$\Delta L_Q = L(2\alpha)\Delta T - \frac{RL}{2AE}$

From above two values,

$\Delta L_Q > \Delta L_P$

Hence, interface between P and Q moves left.

47. A very large metal plate of thickness d and thermal conductivity k is cooled by a stream of air at temperature T_∞ = 300 K with a heat transfer coefficient h, as shown in the figure. The centerline temperature of the plate is T_P. In which of the following case(s) can the lumped parameter model be used to study the heat transfer in the metal plate?



A. h = 10 $Wm^{-2}K^{-1}$, k = 100 $Wm^{-1}K^{-1}$, d = 1 mm, T_P = 350 K
B. h = 100 $Wm^{-2}K^{-1}$, k = 100 $Wm^{-1}K^{-1}$, d = 1 m, T_P = 325 K
C. h = 100 $Wm^{-2}K^{-1}$, k = 1000 $Wm^{-1}K^{-1}$, d = 1 m, T_P = 325 K
D. h = 1000 $Wm^{-2}K^{-1}$, k = 1 $Wm^{-1}K^{-1}$, d = 1 m, T_P = 350 K

[MSQ - 2 Marks]

Ans. A,C

Sol. For lumped system analysis to be valid Biot number ≤ 0.1.

$$B_i = \frac{hL_c}{k} \le 0.1$$

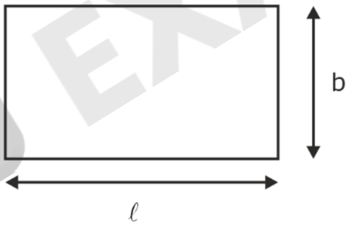
48. The smallest perimeter that a rectangle with area of 4 square units can have is _______ units. (Answer in integer)

[NAT - 2 Marks]

Ans. 7.999 to 8.001

Sol. Given,

Area = 4 $unit^2$



Area, $A = \ell b = 4\,unit^2$

$$b = \frac{4}{\ell}\text{ unit}$$

Perimeter $= 2(\ell + b)$

$$P = 2\left(\ell + \frac{4}{\ell}\right)$$

$$\frac{dP}{d\ell} = 2 - \frac{8}{\ell^2}$$

For minimum perimeter,

$$\frac{dP}{d\ell} = 0$$

$$2 = \frac{8}{\ell^2}$$

$\ell^2 = 4$

$\ell = \pm 2$ (neglect negative term)

$\frac{dP}{d\ell} = \frac{+16}{\ell^3} > 0$ so minimum

$\ell = 2$ unit, $b = \frac{4}{2} = 2$ unit

Smallest rectangle is square perimeter $= 2(\ell + b) = 2(2+2) = 8$ units

49. Consider the second-order linear ordinary differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0,\ x \geq 1$$

with the initial conditions

$$y(x=1) = 6,\ \left.\frac{dy}{dx}\right|_{x=1} = 2.$$

The value of y at x = 2 equals __________.
(Answer in integer)

[NAT - 2 Marks]

Ans. 8.999 to 9.001

Sol. $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

⇒ Euler-Cauchy form

Let, $x = e^t$

$x\frac{dy}{dx} = Dy$

$x^2\frac{d^2y}{dx^2} = D(D-1)y$

$D(D - 1)y + Dy - y = 0$

$(D^2 - 1)y = 0$

Auxiliary Equation

$m^2 - 1 = 0$

$m = \pm 1$

$y = C_1e^t + C_2e^{-t} = C_1x + \frac{C_2}{x}$

$y(1) = 6$ $\quad 6 = C_1 + C_2$ ...(i)

$y' = C_1 - \frac{C_2}{x^2}$

$y'(1) = 2 = C_1 - C_2$ ...(ii)

From equation (i) and (ii)

$C_1 = 4$

$C_2 = 2$

$y = 4x + \frac{2}{x}$

$y(2) = 8 + \frac{2}{2} = 9.$

50. The initial value problem $\frac{dy}{dt} + 2y = 0,\ y(0) = 1$ is solved numerically using the forward Euler's method with a constant and positive time step of Δt.

Let y_n represent the numerical solution obtained after n steps. The condition $|y_n + 1| \le |y_n|$ is satisfied if and only if Δt does not exceed _____________.

(Answer in integer)

[NAT - 2 Marks]

Ans. 0.999 to 1.001

Sol.

$$\frac{dy}{dt} + 2y = 0$$

$$\frac{dy}{dt} = -2y = f(x, y)$$

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(-2y_n)$$

$$y_{n+1} = y_n(1 - 2h)$$

$$|y_{n+1}| \le |y_n|$$

$$|y_n(1 - 2h)| \le |y_n|$$

$$|-2h| \le 1$$

$$-1 \le 1 - 2h \le 1$$

$$-2 \le -2h \le 0$$

$$2 \ge 2h \ge 0$$

$$1 \ge h \ge 0$$

$$0 \le h \le 1$$

Hence = 1

51. The atomic radius of a hypothetical face-centered cubic (FCC) metal is $\left(\frac{\sqrt{2}}{10}\right)$ nm. The atomic weight of the metal is 24.092 g/mol. Taking Avogadro's number to be 6.023×10^{23} atoms/mol, the density of the metal is _____________ kg/m^3.

(Answer in integer)

[NAT - 2 Marks]

Ans. 2499.999 to 2500.001

Sol. Given,

Atomic radius of FCC material, $r = \frac{\sqrt{2}}{10}$ nm

Atomic weight (W) = 24.092 g/mole

Avogadro number (N) = 6.023×10^{23} atoms/mole

Find density = ?

We know,
$$\rho = \frac{N_{avg} \times \text{Atomic weight}}{6.023 \times 10^{23} \times (a)^3}$$

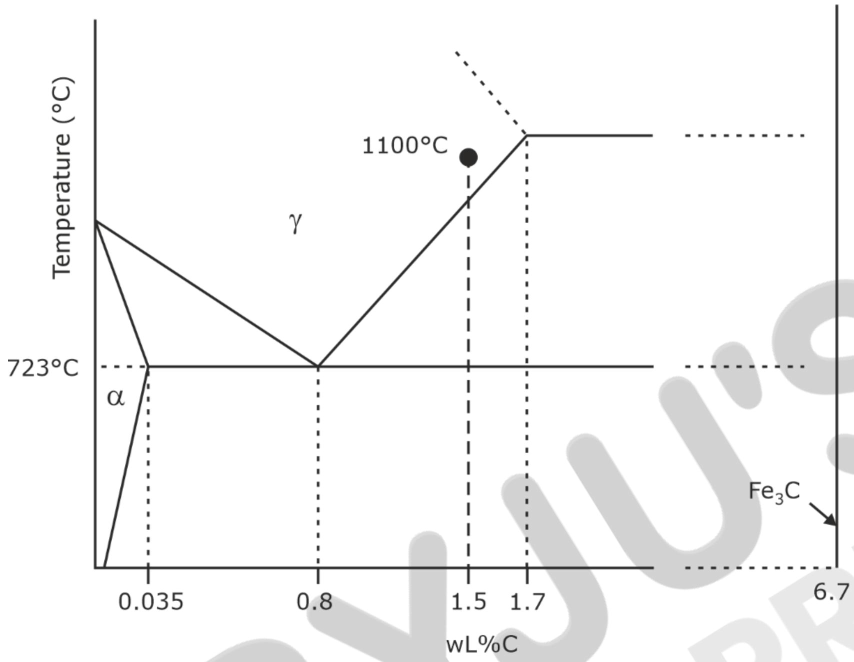
For FCC,
$$a = 2\sqrt{2}\,r \ \&\ N_{avg} = 4$$

$$\therefore \rho = \frac{4 \times 24.092 \times 10^{-3}}{6.023 \times 10^{23} \times \left(2\sqrt{2} \times \frac{\sqrt{2}}{10} \times 10^{-9}\right)^3}$$

$$= 2500\ kg/m^3$$

52. A steel sample with 1.5 wt.% carbon (no other alloying elements present) is slowly cooled from 1100 °C to just below the eutectoid temperature (723 °C). A part of the iron-cementite phase diagram is shown in the figure. The ratio of the pro-eutectoid cementite content to the total cementite content in the microstructure that develops just below the eutectoid temperature is ________.

(Rounded off to two decimal places)



[NAT - 2 Marks]

Ans. 0.53 to 0.55

Sol.

$$m_{pro-Fe_3C} = \frac{1.5-0.8}{6.67-0.8} = 0.1192$$

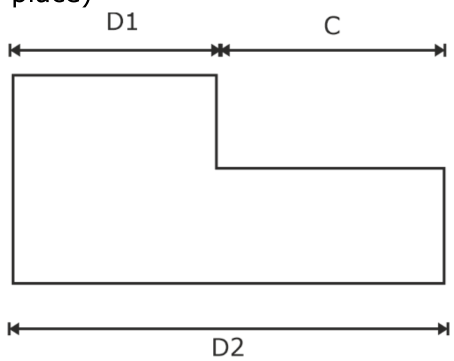
$$m_{total-Fe_3C} = \frac{1.5-0.02}{6.67-0.02} = 0.2225$$

$$\therefore \frac{m_{pro-Fe_3C}}{m_{total-Fe_3C}} = \frac{0.1192}{0.2225} = 0.53$$

53. A part, produced in high volumes, is dimensioned as shown. The machining process making this part is known to be statistically in control based on sampling data. The sampling data shows that D1 follows a normal distribution with a mean of 20 mm and a standard deviation of 0.3 mm, while D2 follows a normal distribution with a mean of 35 mm and a standard deviation of 0.4 mm. An inspection of dimension C is carried out in a sufficiently large number of parts.

To be considered under six-sigma process control, the upper limit of dimension C should be ______ mm.

(Rounded off to one decimal place)



[NAT - 2 Marks]

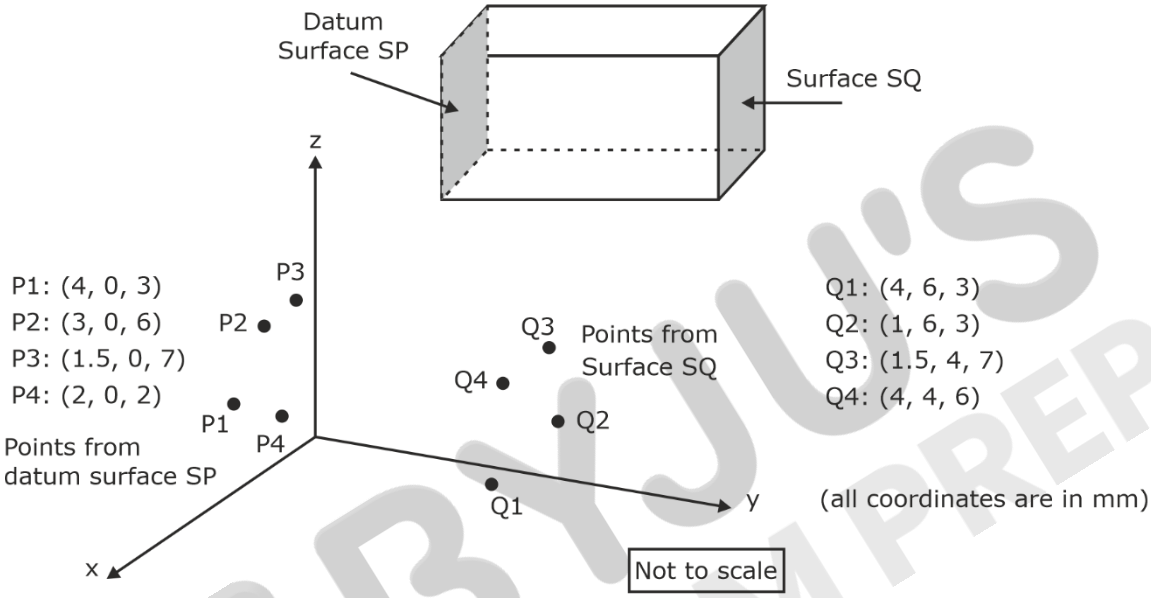
Ans. 16.4 to 16.6

Sol. *

54. A coordinate measuring machine (CMM) is used to determine the distance between Surface SP and Surface SQ of an approximately cuboidal shaped part. Surface SP is declared as the datum as per the engineering drawing used for manufacturing this part. The CMM is used to measure four points P1, P2, P3, P4 on Surface SP, and four points Q1, Q2, Q3, Q4 on Surface SQ as shown. A regression procedure is used to fit the necessary planes.

The distance between the two fitted planes is ___________ mm.

(Answer in integer)



[NAT - 2 Marks]

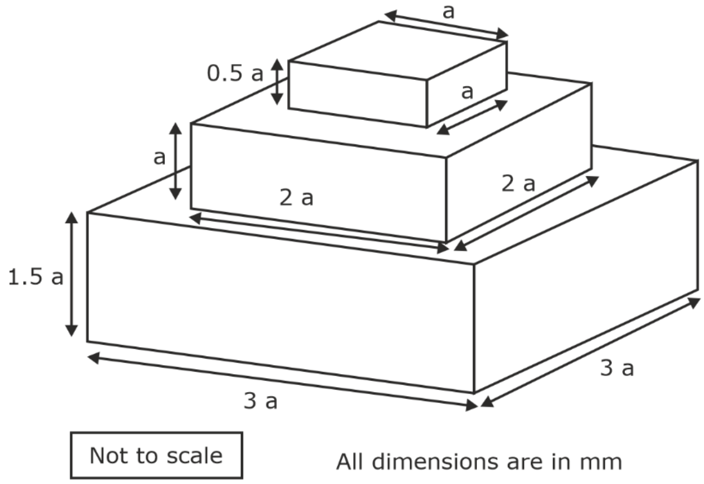
Ans. 4.999 to 5.001

55. A solid part (see figure) of polymer material is to be fabricated by additive manufacturing (AM) in square-shaped layers starting from the bottom of the part working upwards. The nozzle diameter of the AM machine is a/10 mm and the nozzle follows a linear serpentine path parallel to the sides of the square layers with a feed rate of a/5 mm/min.

Ignore any tool path motions other than those involved in adding material, and any other delays between layers or the serpentine scan lines.

The time taken to fabricate this part is ___________ minutes.

(Answer in integer)

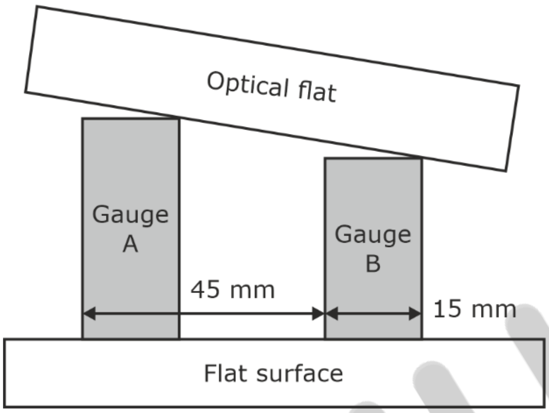


[NAT - 2 Marks]

Ans. 8999.999 to 9000.001

Sol. *

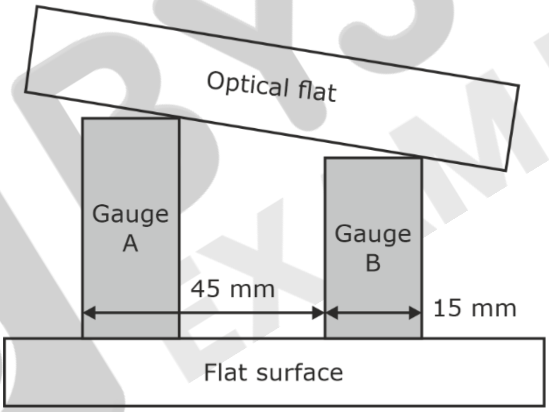
56. An optical flat is used to measure the height difference between a reference slip gauge A and a slip gauge B. Upon viewing via the optical flat using a monochromatic light of wavelength 0.5 μm, 12 fringes were observed over a length of 15 mm of gauge B. If the gauges are placed 45 mm apart, the height difference of the gauges is ______________ μm.
(Answer in integer)



[NAT - 2 Marks]

Ans. 8.999 to 9.001

Sol.



Given, $\lambda = 0.5\ \mu m = 0.5 \times 10^{-6}\ m = 0.5 \times 10^{-6}$

$\ell = 45\ mm = 4.5\ cm$

$n = \dfrac{12}{1.5}\ cm$

$\Delta h = \dfrac{n\lambda\ell}{2}$

$= \dfrac{12}{1.5} \times \dfrac{0.5 \times 10^{-6} \times 4.5}{2}$

$= 9 \times 10^{-6}\ m$

$\Delta h = 9\ \mu m$

57. Ignoring the small elastic region, the true stress (σ) – true strain (ε) variation of a material beyond yielding follows the equation $\sigma = 400\varepsilon^{0.3}$ MPa. The engineering ultimate tensile strength value of this material is _________ MPa.
(Rounded off to one decimal place)

[NAT - 2 Marks]

Ans. 206.4 to 206.6

Sol.
$$\sigma_T = 400\,\epsilon_T^n$$
$$\epsilon = n = 0.3$$
$$\sigma_T = 400(0.3)^{0.3}$$
$$\sigma_T = 278.74 \text{ MPa}$$

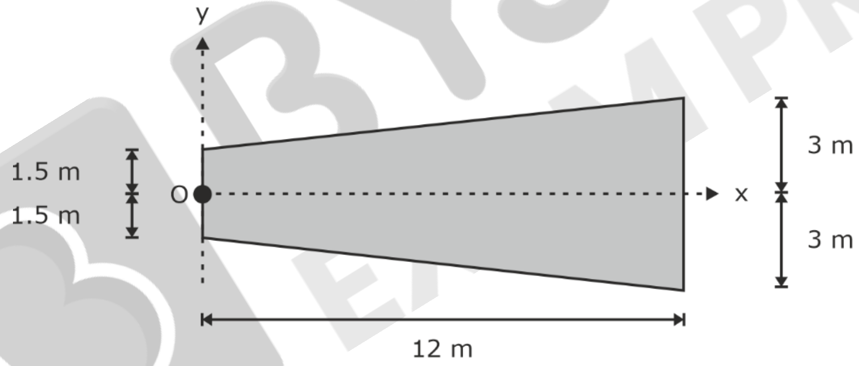
As we know that
$$\sigma_T = \sigma_E\,(1+\epsilon_E) \quad \text{...(i)}$$
$$\epsilon_T = \ln(1+\epsilon_E)$$
$$(1+\epsilon_E) = e^{\epsilon T} \quad \text{...(ii)}$$

Now equation (i) becomes,
$$\sigma_T = \sigma_E\, e^{\epsilon T}$$
$$278.74 = \sigma_E\, e^{0.3}$$
$$\sigma_E = 206.38 \text{ MPa}$$

Hence engineering ultimate stress σ_{ult} = 206.38 MPa

58. The area moment of inertia about the y-axis of a linearly tapered section shown in the figure is ______________ m^4.

(Answer in integer)

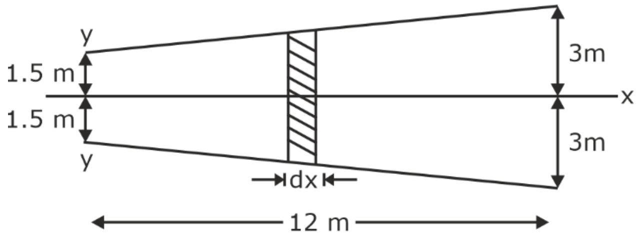


[NAT - 2 Marks]

Ans. 3023.999 to 3024.001

Sol. Let us consider a small strip of length dx as shown in the figure.

The area of small strip can be given as
$$dA = 2y.dx$$
area moment of inertia of small strip about y axis can be given as
$$dI_y = x^2 \cdot 2y \cdot dx$$



y can be given as

$$y = \left(\frac{3-1.5}{12}\right)x + 1.5$$

$$\Rightarrow \frac{1.5}{12}x + 1.5 = \frac{x}{8} + 1.5$$

So the area moment of inertia is

$$dI_y = 2\left(\frac{x}{8} + 1.5\right) \times x^2 \cdot dx$$

$$\Rightarrow x^2\left(\frac{2x}{8} + 3\right)dx = \left(\frac{x^3}{4} + 3x^2\right)dx$$

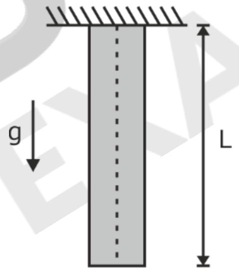
Total moment of inertia about y axis is

$$I_y = \int_0^{12}\left(\frac{x^3}{4} + 3x^2\right)dx = \left[\frac{x^4}{16} + \frac{3x^3}{3}\right]_0^{12}$$

$$= \frac{12 \times 12 \times 12 \times 12}{16} + 12 \times 12 \times 12$$

$$= 1296 + 1728 \Rightarrow 3024 \text{ m}^4$$

59. A cylindrical bar has a length L = 5 m and cross section area S = 10 m^2. The bar is made of a linear elastic material with a density ρ = 2700 kg/m^3 and Young's modulus E = 70 GPa. The bar is suspended as shown in the figure and is in a state of uniaxial tension due to its self-weight. The elastic strain energy stored in the bar equals __________ J.
(Rounded off to two decimal places)
Take the acceleration due to gravity as g = 9.8 m/s^2.



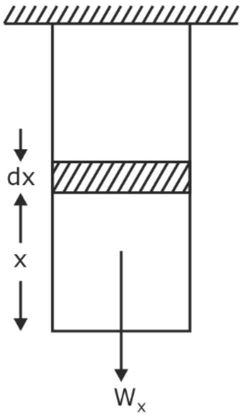
[NAT - 2 Marks]

Ans. 2.00 to 2.16

Sol. Given,

L = 5 m
A = 10 m^2
E = 70 GPa
ρ = 2700 kg/m^3

Consider a strip dx at a distance x from the bottom.



Total load, $W = \rho gAL$

Strain energy

$$U = \int_0^L \frac{(W_x)^2\,dx}{2AE}$$

$$U = \int_0^L \frac{(\rho gAx)^2\,dx}{2AE}$$

$$U = \frac{(\rho g)^2 A}{2E}\left[\frac{x^3}{3}\right]_0^L = \frac{(\rho g)^2 A}{2E}\left(\frac{L^3}{3}\right)$$

$$U = \frac{(2700\times 9.8)^2\times 10}{2\times 70\times 10^9}\times\left(\frac{5^3}{3}\right)$$

$U = 2.083$ J

60. A cylindrical transmission shaft of length 1.5 m and diameter 100 mm is made of a linear elastic material with a shear modulus of 80 GPa. While operating at 500 rpm, the angle of twist across its length is found to be 0.5 degrees.
The power transmitted by the shaft at this speed is _______kW.
(Rounded off to two decimal places)
Take π = 3.14.

[NAT - 2 Marks]

Ans. 237 to 240

Sol. Length, $L = 1.5$ m

Diameter, $d = 100$ mm

Shear modulus, $G = 80$ GPa

Speed, $N = 500$ rpm

Angle of twist, $\theta = 0.5°$

From torsional equation

$$\theta = \frac{TL}{GJ}$$

$$0.5\times\frac{\pi}{180} = \frac{T\times 1.5}{80\times 10^9\times\frac{\pi}{32}\times 0.1^4}$$

$T = 4569.26$ N-m

Power transmitted, $P = \frac{2\pi NT}{60}$

$$P = \frac{2\pi \times 500 \times 4569.26}{60} = 239245 \text{ W}$$

$$P = 239.24 \text{ kW}$$

61. Consider a mixture of two ideal gases, X and Y, with molar masses $\bar{M}x = 10$ kg/kmol and $\bar{M}y = 20$ kg/kmol, respectively, in a container. The total pressure in the container is 100 kPa, the total volume of the container is 10 m^3 and the temperature of the contents of the container is 300 K. If the mass of gas-X in the container is 2 kg, then the mass of gas-Y in the container is ____ kg. (Rounded off to one decimal place)

Assume that the universal gas constant is 8314 J $kmol^{-1}K^{-1}$.

[NAT - 2 Marks]

Ans. 3.9 to 4.1

Sol. Given data,

$$\bar{M}_x = 10 \text{ kg/k-mol}$$

$$\bar{M}_y = 20 \text{ kg / k} - \text{mol}$$

$$P = 100 \text{ kPa}$$

$$V = 10 \text{ m}^3$$

$$T = 300 \text{ K}$$

$$m_x = 2 \text{ kg}$$

$$m_y = ?$$

Ideal gas equation for the gas mixture can be given as

$$PV = mRT$$

$$PV = n\bar{R}T$$

By substituting the value of total number of moles we get ideal gas equation

$$100 \times 10 = \left(\frac{m_x}{M_x} + \frac{m_y}{M_y}\right) 8.314 \times 300$$

$$1000 = \left(\frac{2}{10} + \frac{m_y}{20}\right) 8.314 \times 300$$

$$m_y = 4.02 \text{ kg}$$

62. The velocity field of a certain two-dimensional flow is given by

$$V(x, y) = k\left(x\hat{i} - y\hat{j}\right)$$

where k = 2 s^{-1}. The coordinates x and y are in meters. Assume gravitational effects to be negligible.

If the density of the fluid is 1000 kg/m^3 and the pressure at the origin is 100 kPa, the pressure at the location (2 m, 2 m) is ____________ kPa.

(Answer in integer)

[NAT - 2 Marks]

Ans. 83.999 to 84.001

Sol. Given,

$$v(x, y) = k\left(x\hat{i} - y\hat{j}\right)$$

$$V_x = kx = u$$

$$V_y = -ky = v$$

Using Navier-Stokes equation in x-direction

$$\frac{\partial P}{\partial x}+\mu\frac{\partial^2 u}{\partial x^2}=\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)$$

$$-\frac{\partial P}{\partial x}=\rho\left(kx\times k+(-ky)0\right)$$

$$-\frac{\partial P}{\partial x}=k^2\rho x$$

$$P(x)=\frac{-k^2\rho x^2}{2}+c_1 \quad \text{...(i)}$$

Using Navier-Stokes equation in y-direction

$$-\frac{\partial P}{\partial y}+\mu\frac{\partial^2 u}{\partial y^2}=\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)$$

$$-\frac{\partial P}{\partial y}=\rho\left(kx\times 0+(-ky)(-k)\right)$$

$$-\frac{\partial P}{\partial y}=k^2\rho y$$

$$P(y)=\frac{-k^2\rho y^2}{2}+c_2 \quad \text{...(ii)}$$

From equation (i) and (ii), we get,

$$P(x,y)=-\frac{k^2\rho x^2}{2}-\frac{k^2\rho y^2}{2}+c$$

Now, it is given that

P(0, 0) = 100 kPa

100 = 0 + c

c = 100

then,

$$P(x,y)=\frac{-\rho k^2}{2}\left(x^2+y^2\right)+100$$

$$P(2,2)=\frac{-4}{2}\left(2^2+2^2\right)+100$$

P(2, 2) = −16 + 100 = 84 kPa

63. Consider a unidirectional fluid flow with the velocity field given by $V(x,y,z,t)=u(x,t)\hat{i}$ where u(0, t) = 1. If the spatially homogeneous density field varies with time $\rho(t) = 1 + 0.2e^{-t}$ the value of u(2, 1) is _______________. (Rounded off to two decimal places)
Assume all quantities to be dimensionless.

[NAT - 2 Marks]

Ans. 1.13 to 1.15

Sol. Continuity equation for 1D compressible flow.

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)=0$$

$$\Rightarrow -0.2e^{-t} + (1 + 0.2e^{-t})\frac{\partial u}{\partial x}=0$$

$$\Rightarrow \frac{\partial u}{\partial x}=\frac{0.2e^{-t}}{1+0.2e^{-t}}$$

Integrate

$$\Rightarrow u = \left(\frac{0.2e^{-t}}{1+0.2e^{-t}}\right)x + C$$

At, $x = 0 \Rightarrow u = 1 \Rightarrow C = 1$

$$\Rightarrow u = \left(\frac{0.2e^{-t}}{1+0.2e^{-t}}\right)x + 1$$

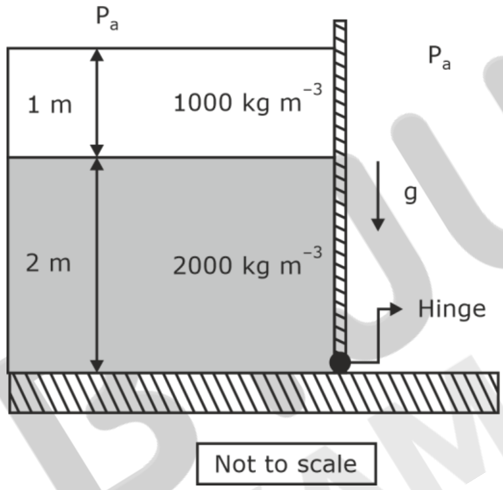
At, $x = 2,\ t = 1$

$$u = \left(\frac{0.2e^{-1}}{1+0.2e^{-1}}\right)\times 2 + 1$$

$= 1.137$ m/s

64. The figure shows two fluids held by a hinged gate. The atmospheric pressure is P_a = 100 kPa. The moment per unit width about the base of the hinge is _____________ kNm/m. (Rounded off to one decimal place)

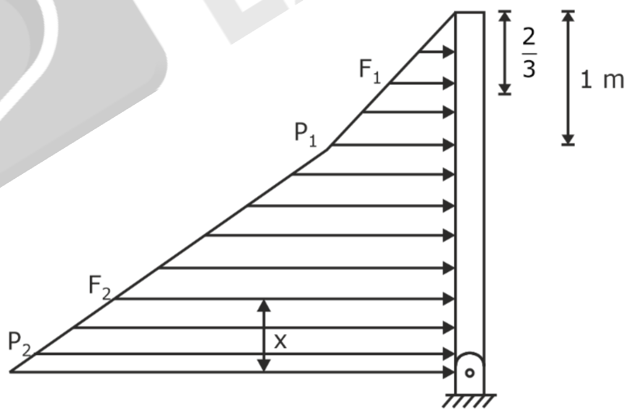
Take the acceleration due to gravity to be $g = 9.8\ m/s^2$.



[NAT - 2 Marks]

Ans. 57.1 to 57.3

Sol. Making the pressure prism for the system



$P_1 = 10^3 \times 9.81 \times 1$

$P_1 = 9.81$ kN

$P_2 = P_1 + [2000 \times 9.81 \times 2]$

$P_2 = 49.05$ kN

The resultant force F_1 and F_2 due to pressure P_1 and P_2 respectively are given as:

F_1 = Volume of triangular pressure prism

$$= \frac{1}{2} \times P_1 \times 1 \times 1 = 4.9 \text{ kN}$$

F_2 = Volume of trapezoidal pressure prism

$$= \frac{1}{2} \times \left[P_1 + P_2\right] \times 2 \times 1 = 58.86 \text{ kN}$$

Now, the moment due to forces is given as

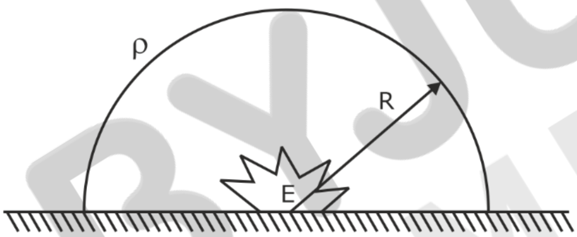
$$M = F_1 \times \left[2 + \frac{1}{3}\right] + F_2 \times \left[\frac{2P_1 + P_2}{P_1 + P_2}\right] \times \frac{2}{3}$$

Putting the values, we get

$$M = 57.225 \text{ kN-m}$$

65. An explosion at time t = 0 releases energy E at the origin in a space filled with a gas of density ρ. Subsequently, a hemispherical blast wave propagates radially outwards as shown in the figure. Let R denote the radius of the front of the hemispherical blast wave. The radius R follows the relationship $R = k\, t^q E^b \rho^c$, where k is a dimensionless constant. The value of exponent a is ____________.

(Rounded off to one decimal place)



[NAT - 2 Marks]

Ans. 0.39 to 0.41

Sol. Given,

$$R = k\, t^a E^b \rho^c$$

$$E = \text{Energy} = J = Nm = \frac{kg-m}{s^2}.m = kg\frac{m^2}{s^2} = \left[ML^2T^{-2}\right]$$

$$\rho = \text{density} = \frac{kg}{m^3} = \left[ML^{-3}\right]$$

t = time = s = [T]

K = constant = $[M^0L^0T^0]$

R = Radius = m = [L]

By applying dimensional analysis

$$\left[L\right] = \left[M^0L^0T^0\right]\left[T\right]^a\left[ML^2T^{-2}\right]^b\left[ML^{-3}\right]^c$$

$$\left[M^0L^1T^0\right] = \left[M^{0+b+c}\right]\left[L^{0+2b-3c}\right]\left[T^{0+a-2b}\right]$$

b + c = 0 ...(i)

2b − 3c = 1 ...(ii)

a − 2b = 0 ...(iii)

From equations (i) and (ii)

2b + 3b = 1

b = 1/5

c = -1/5

a = 2(1/5) = 0.4

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