GATE 2023

Civil Engineering

Forenoon Shift

Questions with Detailed Solutions

General Aptitude

1. "I have not yet decided what I will do this evening; I _______ visit a friend."

A. Mite
B. Would
C. Might
D. Didn't

[MCQ – 1 Mark]

Ans. C

Sol. "I might visit a friend"

2. Eject: Insert :: Advance : _______

(By word meaning)

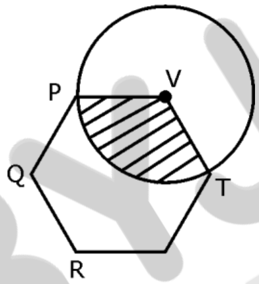
A. Advent
B. Progress
C. Retreat
D. Loan

[MCQ – 1 Mark]

Ans. C

Sol. Eject: Insert :: Advance: Retreat

3. In the given figure, PQRSTV is a regular hexagon with each side of length 5 cm. A circle is drawn with its centre at V such that it passes through P. What is the area (in cm^2) of the shaded region? (The diagram is representative)



A. $\frac{25\pi}{3}$
B. $\frac{20\pi}{3}$
C. 6π
D. 7π

[MCQ – 1 Mark]

Ans. A

Sol. Interior angle sum = $(n - 2) \times 180°$

Each angle of regular Hexagon.

$$\frac{(n-2)\times180^\circ}{n} = 120°$$

$$\text{Required area} = \frac{\theta}{360^\circ}\times\pi\times25 = \frac{25\pi}{3}$$

4. A duck named Donald Duck says "All ducks always lie."

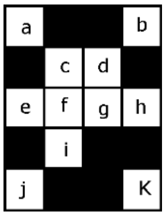
Based only on the information above, which one of the following statements can be logically inferred with certainty?

A. Donald Duck always lies.
B. Donald Duck always tells the truth.
C. Donald Duck's statement is true.
D. Donald Duck's statement is false.

Ans. D

5. A line of symmetry is defined as a line that divides a figure into two parts in a way such that each part is a mirror image of the other part about that line.

The figure below consists of 20 unit squares arranged as shown. In addition to the given black squares, upto 5 more may be coloured black. Which one among the following options depicts the minimum number of boxes that must be coloured black to achieve two lines of symmetry? (The figure is representative)



A. d
B. c, d, i
C. c, i
D. c, d, i, f, g

[MCQ – 1 Mark]

Ans. B

Sol. Both c, d, i and C, d, i, f, g makes the figure symmetrical about two lines, but as minimum number is asked option (b) is correct.

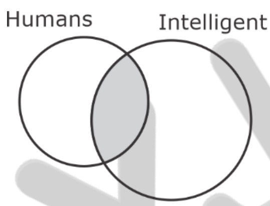
6. Based only on the truth of the statement 'Some humans are intelligent', which one of the following options can be logically inferred with certainty?

A. No human is intelligent
B. All humans are intelligent
C. Some non-humans are intelligent
D. Some intelligent beings are humans

[MCQ – 2 Marks]

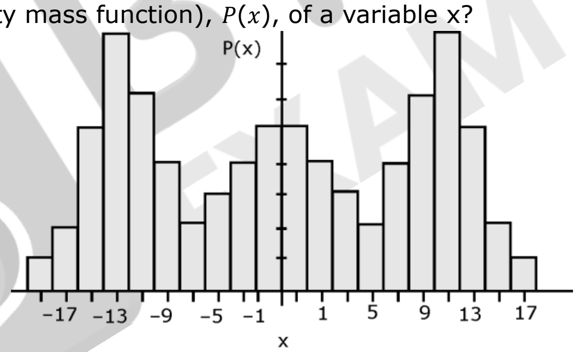
Ans. D

Sol.



Therefore, Option D is correct

7. Which one of the options can be inferred about the mean, median, and mode for the given probability distribution (i.e, probability mass function), $P(x)$, of a variable x?



A. mean = median ≠ mode
B. mean = median = mode
C. mean ≠ median = mode
D. mean ≠ mode = median

[MCQ – 2 Marks]

Ans. A

8. The James Webb telescope, recently launched in space, is giving humankind unprecedented access to the depths of time by imaging very old stars formed almost 13 billion years ago. Astrophysicists and cosmologists believe that this odyssey in space may even shed light on the existence of dark matter. Dark matter is supposed to interact only via the gravitational interaction and not through the electromagnetic-, the weak- or the strong-interaction. This may justify the epithet "dark" in dark matter.

Based on the above paragraph, which one of the following statements is FALSE?

A. No other telescope has captured images of stars older than those captured by the James Webb telescope.

B. People other than astrophysicists and cosmologists may also believe in the existence of dark matter

C. The James Webb telescope could be of use in the research on dark matter

D. If dark matter was known to interact via the strong-interaction, then the epithet "dark" would be justified.

[MCQ – 2 Marks]

Ans. D

9. Let a = 30!, b = 50!, and C =100! . Consider the following numbers:

$\log_a c$ $\quad$ $\log_c a$, $\quad$ $\log_b a$, $\quad$ $\log_a b$

Which one of the following inequalities is CORRECT?

A. $\log_c a < \log_b a < \log_a b < \log_a c$

B. $\log_c a < \log_a b < \log_b a < \log_b c$

C. $\log_c a < \log_b a < \log_a c < \log_a b$

D. $\log_b a < \log_c a < \log_a b < \log_a c$

[MCQ – 2 Marks]

Ans. A

Sol.

$$w = \log_a c = \frac{\log c}{\log a} = \frac{\log 100!}{\log 30!}$$

$$x = \log_c a = \frac{\log a}{\log c} = \frac{\log 30!}{\log 100!}$$

$$y = \log_a b = \frac{\log b}{\log a} = \frac{\log 50!}{\log 30!}$$

$$z = \log_b a = \frac{\log a}{\log b} = \frac{\log 30!}{\log 50!}$$

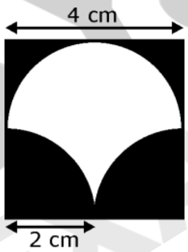
clearly

$$x < z < y < w$$

$$\log_c a < \log_b a < \log_a b < \log_a c$$

10. A square of side length 4 cm is given. The boundary of the shaded region is defined by one semi-circle on the top and two circular arcs at the bottom, each of radius 2 cm, as shown.

The area of the shaded region is ____ cm^2.



A. 8

B. 4

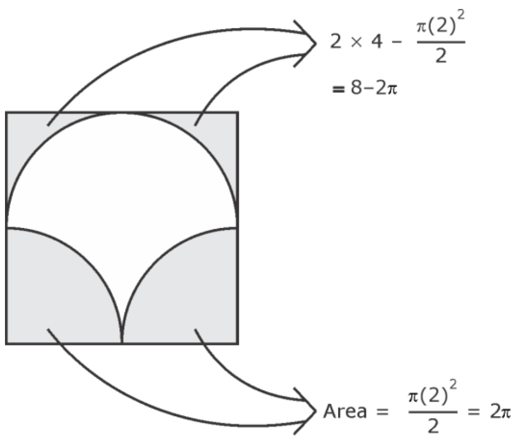
C. 12

D. 10

[MCQ – 2 Marks]

Ans. A

Sol.



$$\text{Total} = 2\pi + (8 - 2\pi)$$
$$= 8$$

Civil Engineering

11. For the integral $I = \int_{-1}^{1} \frac{1}{x^2} dx$ which of the following statements is TRUE?

A. I = 0
B. I = 2
C. I = −2
D. The integral does not change

[MCQ – 1 Mark]

Ans. D

Sol. $\int_{-1}^{1} \frac{1}{x^2} dx \rightarrow$ Improper Integral of 2nd Kind at x = 0,

$\frac{1}{x^2}$ is discontinuous

$$I = \int_{-1}^{1} \frac{1}{x^2} dx = \int_{-1}^{0} \frac{1}{x^2} dx + \int_{0}^{1} \frac{1}{x^2} dx = I_1 + I_2$$

→ It is convergent if both integrals are convergent.

$$\int_{0}^{1} \frac{1}{x^2} dx = \left[-\frac{1}{x}\right]_0^1$$

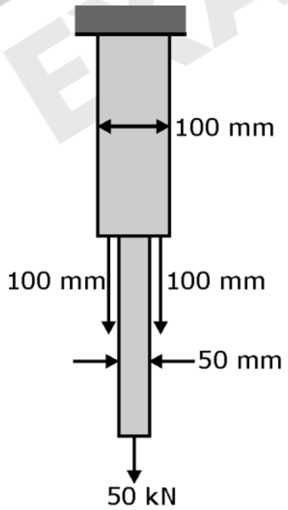
$$= -\left[\frac{1}{1} - \frac{1}{0}\right]$$

$$= -(1 - \infty)$$

= undefined

so, I is divergent.

12. A hanger is made of two bars of different sizes. Each bar has a square cross-section. The hanger is loaded by three-point loads in the mid vertical plane as shown in the figure. Ignore the self-weight of the hanger. What is the maximum tensile stress in N/mm² anywhere in the hanger without considering stress concentration effects?



A. 15.0
B. 25.0
C. 35.0
D. 45.0

[MCQ – 1 Mark]

Ans. B

Sol. For a bar of 100 mm square cross-section A = 100 × 100

P = 100 + 100 + 50 = 250 kN

$$\sigma = \frac{P}{A} = \frac{250 \times 10^3}{100 \times 100} = 25\ N/mm^2$$

For a bar of 50 mm square cross-section

$$A = 50 \times 50$$

$$P = 50 \text{ kN}$$

$$\sigma = \frac{P}{A} = \frac{50 \times 10^3}{50 \times 50} = 20 \text{ N/mm}^2$$

The maximum tensile stress is 25 N/ mm^2

13. Creep of concrete under compression is defined as the

A. increase in the magnitude of strain under constant stress

B. increase in the magnitude of stress under constant strain

C. decrease in the magnitude of strain under constant stress

D. decrease in the magnitude of stress under constant strain

[MCQ – 1 Mark]

Ans. A

Sol. Creep under compression is defined as increase in the magnitude of strain under constant stress.

14. A singly reinforced concrete beam of balanced section is made of M20 grade concrete and Fe415 grade steel bars. The magnitudes of the maximum compressive strain in concrete and the tensile strain in the bars at ultimate state under flexure, as per IS 456: 2000 are, respectively. (Round off to four decimal places)

A. 0.0035 and 0.0038

B. 0.0020 and 0.0018

C. 0.0035 and 0.0041

D. 0.0020 and 0.0031

[MCQ – 1 Mark]

Ans. A

Sol. For M20, Fe415

As per IS 456, the maximum compressive strain in concrete = 0.0035

Maximum compressive strain in steel

$$= 0.002 + 0.87\frac{F_y}{E_s}$$

We know $E_s = 2 \times 10^5$ MPa

Maximum compressive strain in steel

$$= 0.002 + \frac{0.87 \times 415}{2 \times 10^5} = 0.0038$$

15. In cement concrete mix design, with the increase in water-cement ratio, which one of the following statements is TRUE?

A. Compressive strength decreases but workability increases

B. Compressive strength increases but workability decreases

C. Both compressive strength and workability decrease

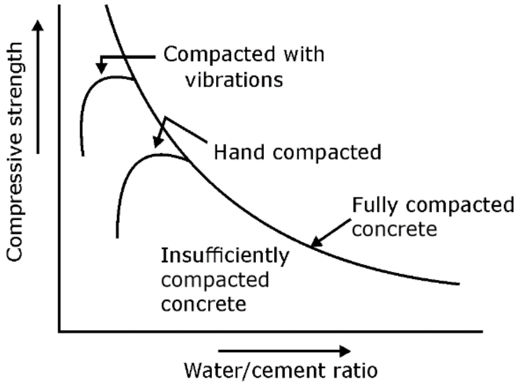
D. Both compressive strength and workability increase

[MCQ – 1 Mark]

Ans. A

Sol. **Effect of water-cement ratio on the compressive strength of concrete-**

As the water-cement ratio increases, the compressive strength decreases.



Effect of water-cement ratio on the workability of concrete-

As the water-cement ratio increases, the workability of concrete decreases.

16. The specific gravity of a soil is 2.60. The soil is at 50% degree of saturation with a water content of 15%. The void ratio of the soil is.

A. 0.35　　B. 0.78
C. 0.87　　D. 1.28

[MCQ – 1 Mark]

Ans. B

Sol.

S = 0.5
G = 2.60
W = 0.15

As we know,

Se = WG
0.5 × e = 0.15 × 2.60
e = 0.78

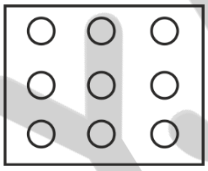
17. A group of 9 friction piles are arranged in a square grid maintaining equal spacing in all directions. Each pile is of diameter 300 mm and length 7 m. Assume that the soil is cohesionless with effective friction angle ϕ'=32°. What is the center-to-center spacing of the piles (in m) for the pile group efficiency of 60%?

A. 0.582　　B. 0.486
C. 0.391　　D. 0.677

[MCQ – 1 Mark]

Ans. A or B

Sol.



$$\eta_g = \left(1 - \frac{\theta}{90}\left[\frac{m(n-1) + n(m-1)}{mn}\right]\right) \times 100$$

$$0.6 = \left(1 - \frac{\theta}{90}\left[\frac{3(3-1) + 3(3-1)}{3 \times 3}\right]\right) \times 100$$

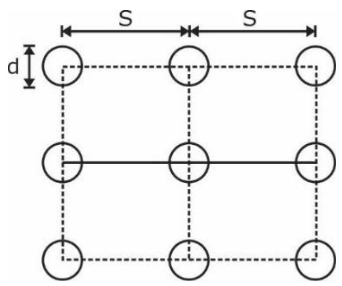
θ = 27°

$$\theta = \tan^3\left(\frac{d}{s}\right)$$

$$27 = \tan^{-1}\left(\frac{0.3}{S}\right)$$

$$S = 0.587 \text{ m}$$

Alternate Solution



$$\text{Group efficiency} = \frac{Q_{ug}}{nQ_{us}}$$

$$0.6 = \frac{\frac{1}{2}(\gamma l)\tan\delta\ (2S + d) \times l \times 4}{9 \times \frac{1}{2}(\gamma l)\tan\delta(pdl)}$$

$$0.6 = \frac{(2S + d) \times 4}{9 \times \pi d}$$

$$S = \frac{1}{2}\left[\frac{0.6 \times 9 \times \pi \times 0.3}{4} - 0.3\right] = 0.486$$

18. A possible slope failure is shown in the figure. Three soil samples are taken from different locations (I, II and III) of the potential failure plane. Which is the most appropriate shear strength test for each of the sample to identify the failure mechanism?

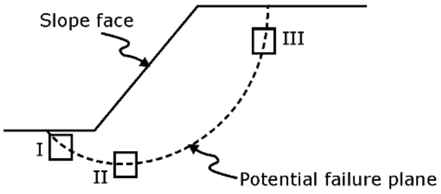
Identify the correct combination from the following options:

P: Triaxial compression test

Q: Triaxial extension test

R: Direct shear or shear box test

S: Vane shear test



A. I-Q, II-R, III-P

B. I-R, II-P, III-Q

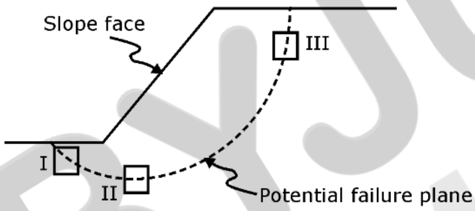
C. I-S, II-Q, III-R

D. I-P, II-R, III-Q

[MCQ – 1 Mark]

Ans. A

Sol.



I. $\sigma_H > \sigma_V \rightarrow$ Triaxial extension (As σ_v decreases depth from ground level is less)

II. Failure plane is same as direct shear box test i.e perpendicular to vertical plane

III. $\sigma_v > \sigma_h \rightarrow$ Triaxial compression test

19. When a supercritical stream enters a mild-sloped (M) channel section, the type of flow profile would become _______ .

A. M_1

B. M_2

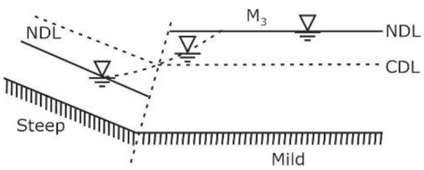
C. M_3

D. M_1 and M_2

[MCQ – 1 Mark]

Ans. C

Sol.



When a super-critical steam enters a mild slope (M) channel section, type of flow profile will be M_3.

20. Which one of the following statements is TRUE for Greenhouse Gas (GHG) in the atmosphere?

A. GHG absorbs the incoming short wavelength solar radiation to the earth surface, and allows the long wavelength radiation coming from the earth surface to pass through

B. GHG allows the incoming long wavelength solar radiation to pass through to the earth surface, and absorbs the short wavelength radiation coming from the earth surface

C. GHG allows the incoming long wavelength solar radiation to pass through to the earth surface, and allows the short wavelength radiation coming from the earth surface to pass through

D. GHG allows the incoming short wavelength solar radiation to pass through to the earth surface, and absorbs the long wavelength radiation coming from the earth surface

[MCQ – 1 Mark]

Ans. D

Sol. GHG allows the incoming short wavelength solar radiation to pass through to the earth surface and absorbs the long wavelength radiation coming from the earth surface.

21. G_1 and G_2 are the slopes of the approach and departure grades of a vertical curve, respectively.

Given $|G_1|<|G_2|$ and $|G_1|\neq|G_2|\neq 0$

Statement 1: + G_1 followed by + G_2 results in a sag vertical curve.

Statement 2: − G_1 followed by − G_2 results in a sag vertical curve.

Statement 3: + G_1 followed by − G_2 results in a crest vertical curve.

Which option amongst the following is true?

A. Statement 1 and Statement 3 are correct; Statement 2 is wrong

B. Statement 1 and Statement 2 are correct; Statement 3 is wrong

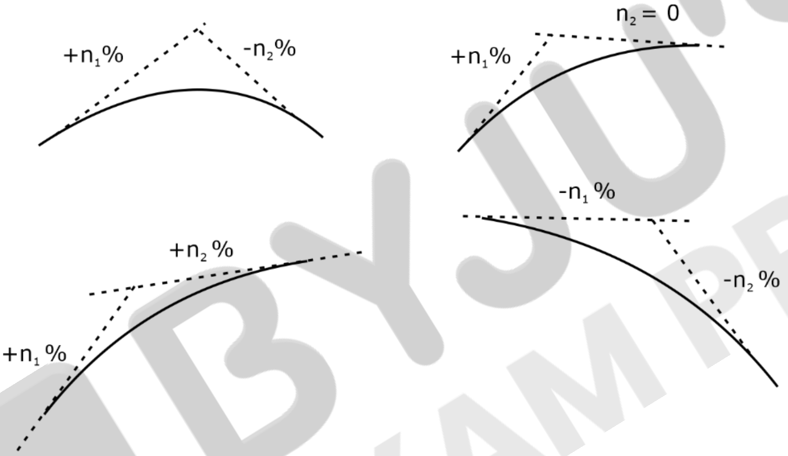
C. Statement 1 is correct; Statement 2 and Statement 3 are wrong

D. Statement 2 is correct; Statement 1 and Statement 3 are wrong

[MCQ – 1 Mark]

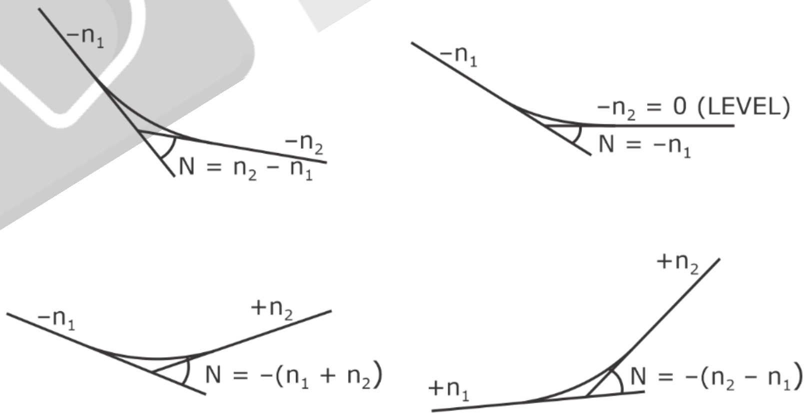
Ans. A

Sol. **Conditions for summit curve**



Types of Summit Curve

Condition for valley curve



Types of Valley Curve

Only statement 1 and 3 will fulfil the desired condition

22. The direct and reversed zenith angles observed by a theodolite are 56° 00' 00" and 303° 00' 00", respectively. What is the vertical collimation correction?

A. +1° 00' 00"

B. −1° 00' 00"

C. −0° 30' 00"

D. +0° 30' 00"

[MCQ – 1 Mark]

Ans. D

Sol. Direct zenith angle = 56°

Reversed zenith angle = 303°

Vertical collimation Error $= \dfrac{360^\circ - (\text{sum of direct and reversed zenith angle})}{2}$

$= \dfrac{360^\circ - (56^\circ + 303^\circ)}{2}$

= 0.5° = 30′

23. A student is scanning his 10 inch × 10 inch certificate at 600 dots per inch (dpi) to convert it to raster. What is the percentage reduction in number of pixels if the same certificate is scanned at 300 dpi?

A. 62

B. 88

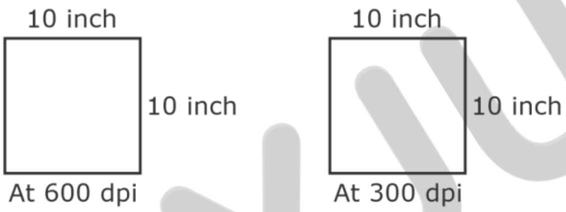
C. 75

D. 50

[MCQ – 1 Mark]

Ans. C

Sol.



1 dpi = 12 pixel /ft

⇒ 600 dpi = (600 × 12)

For 10 inch × 10 inch,

We get $(600 \times 12)^2$

Similarly, for 300 dpi, we get $(300 \times 12)^2$

% Reduction in pixels $= \dfrac{(600\times12)^2 - (300\times12)^2}{(600\times12^2)} \times 100 = 75\%$

24. If M is an arbitrary real n × n matrix, then which of the following matrices will have non-negative eigenvalues?

A. M^2

B. MM^T

C. M^TM

D. $(M^T)^2$

[MSQ – 1 Mark]

Ans. A, B, C, D

25. The following function is defined over the interval $[-L, L]$:

$$f(x) = px^4 + qx^3$$

If it is expressed as a Fourier series,

$$F(x) = a_0 + \sum_{n=1}^{\infty}\left\{a_n \sin\left(\frac{\pi x}{L}\right) + b_n \cos\left(\frac{\pi x}{L}\right)\right\},$$

which options amongst the following are true?

A. a_n, n = 1,2, ⋯ ,∞ depend on p

B. a_n, n = 1,2, ⋯ ,∞ depend on q

C. b_n, n = 1,2, ⋯ ,∞ depend on p

D. b_n, n = 1,2, ⋯ ,∞ depend on q

[MSQ – 1 Mark]

Ans. B,C

Sol.

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{(n\pi x)}{L}dx$$

$$= \frac{1}{L}\int_{-L}^{L} px^4 \cos\frac{(n\pi x)}{L}dx + 0$$

$$\left(qx^5 \cos\frac{n\pi x}{L} \text{ is odd function}\right)$$

$\therefore$ b_n depends on p.

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{(n\pi x)}{L}dx$$

$$= 0 + \frac{1}{L}\int_{-L}^{L} qx^5 \sin\frac{(n\pi x)}{L}dx$$

$$\left(px^4 \sin\frac{n\pi x}{L} \text{ is odd function}\right)$$

$\therefore$ a_n depends on q.

26. Consider the following three structures:

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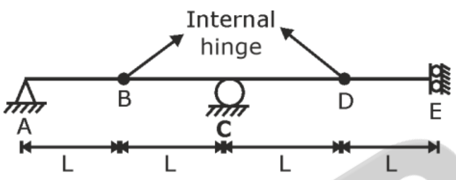
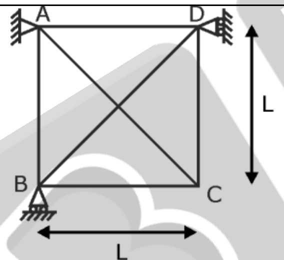
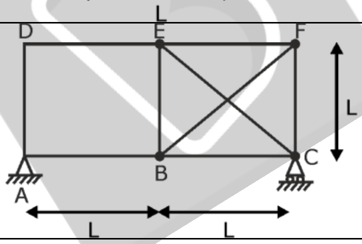
<tr><td>
</td><td>Structure I: Beam with hinge support at A, roller at C, guided roller at E, and internal hinges at B and D</td></tr>
<tr><td>
</td><td>Structure II: Pin-jointed truss, with hinge support at A, and rollers at B and D</td></tr>
<tr><td>
</td><td>Structure III: Pinpointed truss, with hinge support at A and roller at C</td></tr>
</table>

Which of the following statements is/are TRUE?

A. Structure I is unstable

B. Structure II is unstable

C. Structure III is unstable

D. All three structures are stable

[MSQ – 1 Mark]

Ans. A, B and C

Sol. **Structure I → Unstable**

Internal hinge at D can cause rigid body rotation.

Structure II → Unstable

Since all reactions are concurrent at point A, rigid body rotation can take place.

Structure III → Unstable

Even if m = 2j – 3 criteria is satisfied, there is an improper arrangement of members in 1st panel. Hence, shear in 1st panel is not resisted by any member. So, it is unstable.

27. Identify the waterborne diseases caused by viral pathogens

A. Acute anterior poliomyelitis
B. Cholera
C. Infectious hepatitis
D. Typhoid fever

[MSQ – 1 Mark]

Ans. A, C

Sol. Waterborne diseases caused by Viral pathogens are-

1. Acute anterior poliomyelitis
2. Infectious hepatitis

Waterborne diseases caused by Bacterial pathogens are-

1. Cholera
2. Typhoid fever

28. Which of the following statements is/are TRUE for the Refuse-Derived Fuel (RDF) in the context of Municipal Solid Waste (MSW) management?

A. Higher Heating Value (HHV) of the unprocessed MSW is higher than the HHV of RDF processed from the same MSW
B. RDF can be made in the powdered form
C. Inorganic fraction of MSW is mostly converted to RDF
D. RDF cannot be used in conjunction with oil

[MSQ – 1 Mark]

Ans. B

Sol. A major inorganic fraction of Municipal solid waste can be converted into refuse-derived fuel (RDF) but not all of it. The process involves sorting, shredding, and compacting the inorganic waste materials into fuel pellets or briquettes. These RDF pellets can be burned as a fuel source in industries, such as cement and power plants, as a substitute for fossil fuels. The conversion of inorganic waste into RDF helps to reduce waste going to landfills, reduces greenhouse gas emissions, and provides a more sustainable energy source.

The HHV of RDF is generally higher than HVV of unprocessed MSW.

29. The probabilities of occurrences of two independent events A and B are 0.5 and 0.8, respectively. What is the probability of occurrence of at least A or B (rounded off to one decimal place)? ________

[NAT – 1 Mark]

Ans. 0.9

Sol. P (at least one of A or B)

$$P(A \cup B) = p(A) + p(B) - p(A \cap B)$$
$$= p(A) + p(B) - p(A)\,p(B)$$
$$= 0.5 + 0.8 - 0.5 \times 0.8$$
$$= 0.9$$

30. In the differential equation $\frac{dy}{dx}$ + a x y = 0 positive constant. If y =1.0 at x = 0.0, and y = 0.8 at x = 1.0, the value of ɑ is ______(rounded off to three decimal places).

[NAT – 1 Mark]

Ans. 0.446

Sol.

$$\frac{dy}{dx} = -\alpha xy$$

$$\frac{dy}{y} = -\alpha x\,dx$$

$$\log_e y = -\alpha\frac{x^2}{2} + \log_e c$$

$$\frac{y}{c} = e^{-\frac{\alpha x^2}{2}}$$

$$y = ce^{-\alpha\frac{x^2}{2}} \quad ...(1)$$

$\therefore$ $y(0) = 1$

$(1) \Rightarrow 1 = c \Rightarrow \quad c = 1$

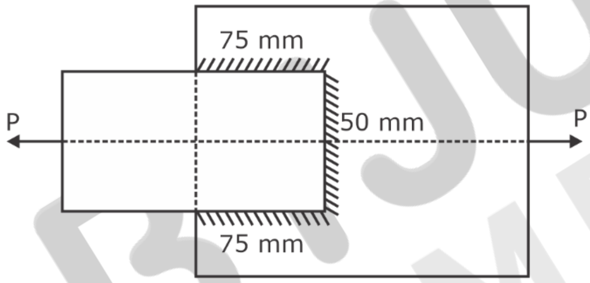
& $y(1) = 0.8$

$(1) \Rightarrow \; 0.8 = 1 \cdot e^{-\frac{\alpha}{2}}$

$$\Rightarrow \log_e 0.8 = \frac{-\alpha}{2}$$

$$\alpha = -2\log_e 0.8 = 0.4463$$

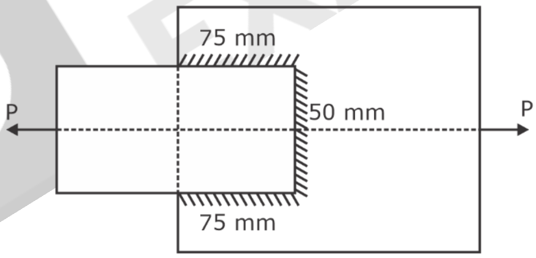
31. Consider the fillet-welded lap joint shown in the figure (not to scale). The length of the weld shown is the effective length. The welded surfaces meet at right angle. The weld size is 8 mm, and the permissible stress in the weld is 120 MPa. What is the safe load P (in kN, rounded off to one decimal place) that can be transmitted by this welded joint?



[NAT – 1 Mark]

Ans. 133 to 136

Sol.



Given,

Size of weld = S = 8 mm

Permissible stress = 120 MPa

Effective throat thickness = 0.7 S = 0.7 × 8 = 5.6 mm

Effective length = $l_{eff.}$ = 75 + 50 + 75 = 200 mm

Hence, safe load = P = Permissible stress × Effective throat thickness × Effective length of weld

= 120 × 5.6 × 200

= 134400 N

= 134.4 kN

32. A drained direct shear test was carried out on a sandy soil. Under a normal stress of 50 kPa, the test specimen failed at a shear stress of 35 kPa. The angle of internal friction of the sample is degree (round off to the nearest integer).

[NAT – 1 Mark]

Ans. 35°

Sol.

$$\sigma_n = 50 \text{ kPa}$$

$$\tau = 35 \text{ kPa}, C = 0$$

$$\tau = C + \sigma_n \tan\phi$$

$$35 = 0 + 50 \tan\phi$$

$$\tan\phi = \frac{35}{50}$$

$$\phi \cong 35°$$

33. A canal supplies water to an area growing wheat over 100 hectares. The duration between the first and last watering is 120 days, and the total depth of water required by the crop is 35 cm. The most intense watering is required over a period of 30 days and requires a total depth of water equal to 12 cm. Assuming precipitation to be negligible and neglecting all losses, the minimum discharge (in m^3/s, rounded off to three decimal places) in the canal to satisfy the crop requirement is __________.

[NAT – 1 Mark]

Ans. 0.046

Sol. Area = A = 100 Ha

Base period = B = 120 days

Delta = Δ = 35 cm

More intense watering is required for = 30 days and the depth of water required (Delta for 30 days)

= 12 cm

So, 12 cm is required in the first 30 days,

Hence, (35 – 12) = 23 cm will be required for (120 – 30) = 90 days (next 90 days)

So, Discharge for the first 30 days:

$$= \frac{A}{\Delta} = \frac{A}{864\frac{B}{\Delta}} = \frac{100}{864 \times \frac{30}{12}} = 0.046 \text{ cumecs}$$

Discharge for the next 90 days:

$$= \frac{A}{\Delta} = \frac{A}{864\frac{B}{\Delta}} = \frac{100}{864 \times \frac{90}{23}} = 0.029 \text{ cumecs}$$

Hence, minimum design discharge required = Max. (0.046, 0.029)

= 0.046 cumecs

Note:

If we chose 0.029 cumecs, then the discharge requirement for the first 30 days will not be satisfied.

34. The ordinates of a one-hour unit hydrograph for a catchment are given below:

t (hour)	0	1	2	3	4	5	6	7
Q(m^3/s)	0	9	21	18	12	5	2	0

Using the principle of superposition, a D-hour unit hydrograph for the catchment was derived from this one-hour unit hydrograph. The ordinates of the D-hour unit hydrograph were obtained as 3 m^3/s at t = 1 hour and 10 m^3/s at t = 2 hour. The value of D (in integer) is ______.

[NAT – 1 Mark]

Ans. 3

Sol.

Time	Q	S-curve addition	S-Curve ordinate	S-curve lagged (3 hr)	$\Delta y \times \left(\frac{1}{\Delta T}\right)$
0	0	–	0	–	0
1	9	0	9	–	$(9 - 0) \times \frac{1}{3} = 3\ m^3/sec$
2	21	9	30	–	$(30 - 0) \times \frac{1}{3} = 10\ m^3/sec$
3	18	30	48	0	
4	12	48	60	9	
5	5	60	65	30	
6	2	65	67	48	

$\therefore$

$$\Delta T = 3$$

$$\Delta T = \frac{D}{1\,hr} \Rightarrow D = 3\ hr$$

35. For a horizontal curve, the radius of a circular curve is obtained as 300 m with the design speed as 15 m/s. If the allowable jerk is 0.75 m/s^3, what is the minimum length (in m, in integer) of the transition curve___________?

[NAT – 1 Mark]

Ans. 15

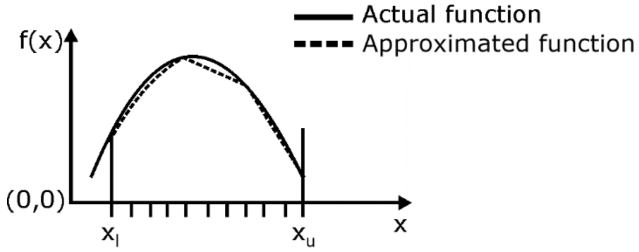
Sol. Given,

$$R = 300\ m$$
$$V = 15\ m/sec$$
$$C = 0.75\ m/sec^2$$
L_s = Length of transition curve

$$L_s = \frac{V^3}{CR} = \frac{15^3}{0.75 \times 300} = 15\ m$$

Note: Since we don't know anything about the terrain hence, We cannot find out the length by formula given by IRC

36. A function (x), that is smooth and convex shaped between interval (x_l, x_u) is shown in the figure. This function is observed at odd number of regularly spaced points. If the area under the function is computed numerically, then______.



A. The numerical value of the area obtained using the trapezoidal rule will be less than the actual
B. The numerical value of the area obtained using the trapezoidal rule will be more than the actual
C. The numerical value of the area obtained using the trapezoidal rule will be exactly equal to the actual
D. With the given details, the numerical value of area cannot be obtained using trapezoidal rule

[MCQ – 2 Marks]

Ans. A

37. Consider a doubly reinforced RCC beam with the option of using either Fe250 plain bars or Fe500 deformed bars in the compression zone. The modulus of elasticity of steel is 2×10^5 N/mm^2. As per IS456:2000, in which type(s) of the bars, the stress in the compression steel (c) can reach the design strength ($0.87 f_y$) at the limit state of collapse?

A. Fe250 plain bars only

B. Fe500 deformed bars only

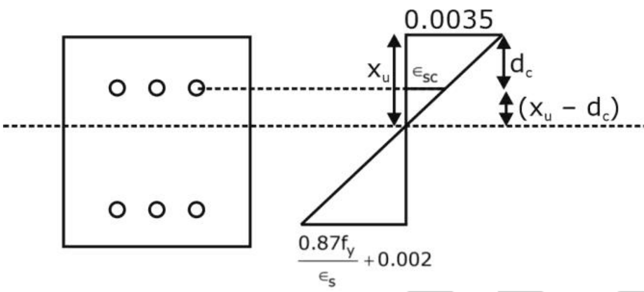
C. Both Fe250 plain bars and Fe500 deformed bars

D. Neither Fe250 plain bars nor Fe500 deformed bars

[MCQ – 2 Marks]

Ans. A

Sol.

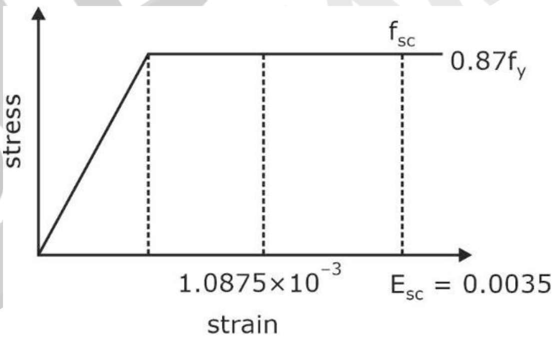


So, $E_{sc} < 0.0035$ $E_{sc} = 0.0035 \times \frac{(x_u - d_c)}{x_u}$

But we can consider $E_{sc} \approx 0.0035$.

For Fe250

The permissible strain is $\frac{0.87 f_y}{\epsilon_s} = \frac{0.87 \times 250}{2 \times 10^5} = 1.0875 \times 10^{-3}$



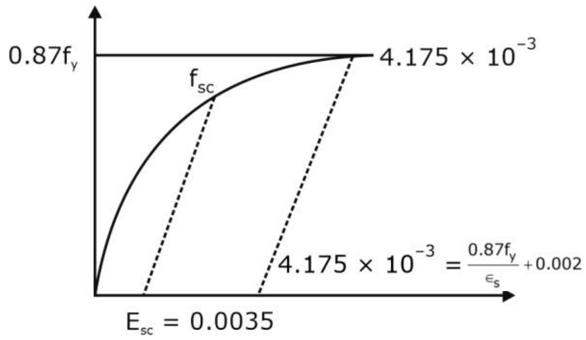
When $E_{sc} = 0.0035$ $f_{sc} = 0.87 f_y$

For Fe500

The permissible strain $= \frac{0.87 f_y}{\epsilon_s} + 0.002$

$= \frac{0.87 \times 500}{2 \times 10^5} + 0.002$

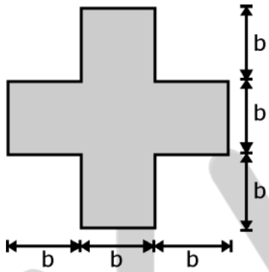
$= 4.175 \times 10^{-3}$



Hence, $f_{sc} < 0.87\ f_y$

Fe250 will reach to $0.87f_y$

38. Consider the horizontal axis passing through the centroid of the steel beam cross-section shown in the figure. What is the shape factor (rounded off to one decimal place) for the cross-section?



A. 1. 5

B. 1.7

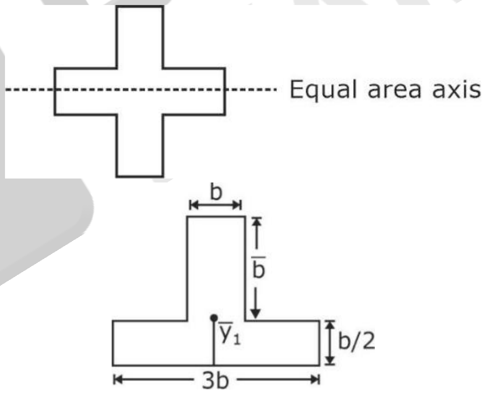
C. 1.3

D. 2.0

[MCQ – 2 Marks]

Ans. B

Sol.



Plastic Section modulus:

$$\bar{y}_1 = \frac{(3b)\left(\frac{b}{2}\right)\left(\frac{b}{4}\right) + (b)(b)\left(\frac{b}{2} + \frac{b}{2}\right)}{(3b)\left(\frac{b}{2}\right) + (b)(b)}$$

$$\bar{y}_1 = \frac{\frac{3b^3}{8} + b^3}{\frac{3b^2}{2} + b^2}$$

$$\bar{y}_1 = \frac{1.375\ b^3}{2.5\ b^2} = 0.55\ b$$

$$Z_P = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = \frac{5b^2}{2}(0.55\text{ b} + 0.55\text{ b}) = 2.75\text{ b}^3$$

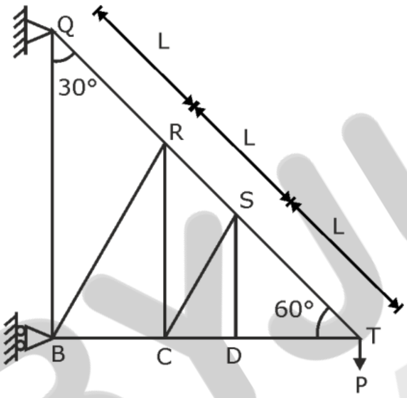
Elastic section modulus:

$$Z = \frac{I}{y_{max}} = \frac{\frac{(b)(3b)^3}{12} + \frac{(2b)(b)^3}{12}}{3\text{ b}/2}$$

$$Z_\ell = 1.611\text{ b}^3$$

$$\text{Shape factor} = \frac{Z_p}{Z_e} = \frac{2.75\text{ b}^3}{1.611\text{ b}^3} = 1.7$$

39. Consider the pin-jointed truss shown in the figure (not to scale). All members have the same axial rigidity, AE. Members QR, RS, and ST have the same length L. Angles QBT, RCT, SDT are all 90°. Angles BQT, CRT, DST are all 30°. The joint T carries a vertical load P. The vertical deflection of joint T is $k\frac{PL}{AE}$. What is the value of k?



A. 1.5

B. 4.5

C. 3.0

D. 9.0

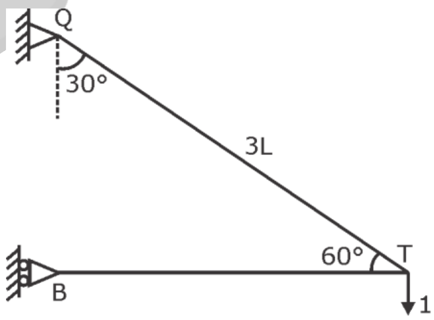
[MCQ – 2 Marks]

Ans. B

Sol. Member SD, SC, CR, RB will be zero force members. (At a joint, if 3 members are meeting, and 2 members are collinear, then force in the third member is zero)

Member BQ will also be zero since the reaction at B and member BT are collinear.

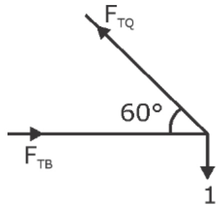
So, the structure reduces to



$$BT = 3L\cos 60 = \frac{3L}{2}$$

For P-values of forces:

At joint T



$\Sigma F_V = 0$

$F_{TQ}\sin 60 = P$

$$F_{TQ} = \frac{2P}{\sqrt{3}}(T)$$

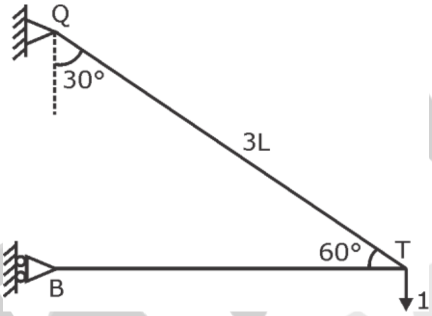
$\Sigma F_H = 0$

$F_{TQ}\cos 60 = F_{TB}$

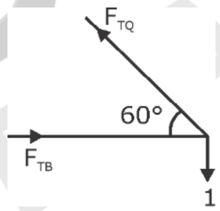
$$\frac{2P}{\sqrt{3}} \times \frac{1}{2} = F_{TB} \Rightarrow F_{TB} = \frac{P}{\sqrt{3}} \;(C)$$

For K-values of forces:

Apply a unit load at T in the downward direction.



At joint T



$\Sigma F_V = 0$

$F_{TQ}\sin 60 = 1$

$$F_{TQ} = \frac{2}{\sqrt{3}}(T)$$

$\Sigma F_H = 0$

$F_{TQ}\cos 60 = F_{TB}$

$$\frac{2}{\sqrt{3}} \times \frac{1}{2} = F_{TB} \Rightarrow F_{TB} = \frac{1}{\sqrt{3}} \;(C)$$

Member	P-value	K-value	Length	PKL/AE
TB	$-\frac{P}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$\frac{3L}{2}$	$\frac{PL}{2AE}$
TQ	$\frac{2P}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$	3L	$\frac{4PL}{AE}$

$$\Delta_{VT} = \sum \frac{PKL}{AE} = \frac{9PL}{2AE} = \frac{4.5PL}{AE}$$

$\Rightarrow k = 4.5$

40. With reference to the compaction test conducted on soils, which of the following is INCORRECT?

A. Peak point of the compaction curve gives the maximum dry unit weight and optimum moisture content

B. With increase in the compaction effort, the maximum dry unit weight increases

C. With increase in the compaction effort, the optimum moisture content decreases

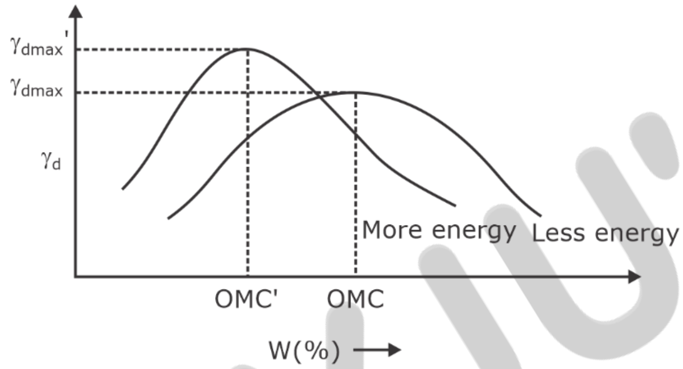
D. Compaction curve crosses the zero-air-voids curve

[MCQ – 2 Marks]

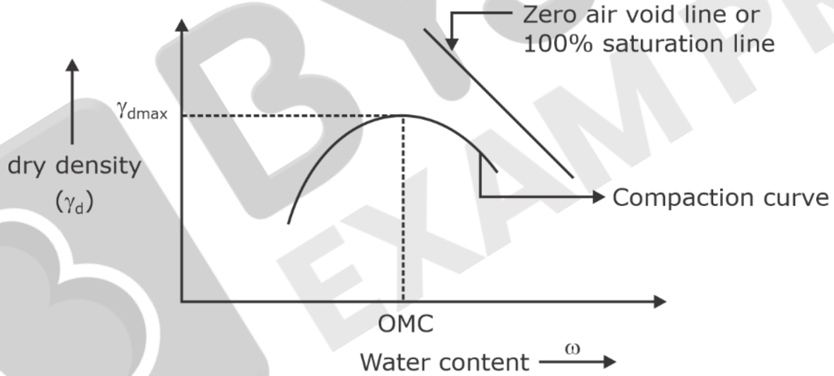
Ans. D

Sol.

- Peak point of the compaction curve gives the maximum dry unit weight and optimum moisture content.
- With increase in the compaction effort, the maximum dry unit weight increases
- With increase in the compaction effort, the optimum moisture content decreases



- Compaction curve do not cross the zero-air-voids line



41. Consider that a force P is acting on the surface of a half-space (Bossiness's problem). The expression for the vertical stress (σz) at any point (r, z), within the half-space is given as,

$$\sigma_z = \frac{3P}{2\pi}\frac{z^3}{\left(r^2+z^2\right)^{\frac{5}{2}}}$$

where, r is the radial distance, and z is the depth with downward direction taken as positive. At any given r, there is a variation of σ_z along z, and at a specific z, the value of σ_z will be maximum. What is the locus of the maximum σz?

A. $z^2 = \frac{3}{2}r^2$

B. $z^3 = \frac{3}{2}r^2$

C. $z^2 = \frac{5}{2}r^2$

D. $z^3 = \frac{5}{2}r^2$

[MCQ – 2 Marks]

Ans. A

Sol.

$$\sigma_z = \frac{3P}{2\pi}\frac{z^3}{\left(r^2+z^2\right)^{\frac{5}{2}}}$$

$$\frac{d\sigma_z}{dz} = 0$$

$$\frac{3P}{2\pi}\left[\frac{\left(r^2+z^2\right)^{\frac{5}{2}}3z^2 - z^3\frac{5}{2}\left(r^2+z^2\right)^{\frac{3}{2}}2z}{\left(r^2+z^2\right)^5}\right] = 0$$

$$3r^2 + 3z^2 - 5z^2 = 0$$

$$z^2 = \frac{3}{2}r^2$$

42. A square footing of size 2.5 m × 2.5 m is placed 1.0 m below the ground surface on a cohesionless homogeneous soil stratum. Considering that the groundwater table is located at the base of the footing, the unit weights of soil above and below the groundwater table are 18 kN/m^3 and 20 kN/m^3, respectively, and the bearing capacity factor Nq is 58, the net ultimate bearing capacity of the soil is estimated as 1706 kPa (unit weight of water = 10 kN/m^3).
Earlier, a plate load test was carried out with a circular plate of 30 cm diameter in the same foundation pit during a dry season, when the water table was located beyond the plate influence zone. Using Terzaghi's bearing capacity formulation, what is the ultimate bearing capacity (in kPa) of the plate?

A. 110.16
B. 61.20
C. 204.00
D. 163.20

[MCQ – 2 Marks]

Ans. A

Sol.

$$q_{nu} = q_u - \bar{\sigma}$$

$$q_{nu} = q_u - \gamma D_f$$

$$q_{nu} = \{1.3\,CN_c + \gamma D_f N_q + 0.4\,B\,\gamma\,N_\gamma\} - \gamma D_f$$

For cohesionless homogeneous soil stratum C=0

$$q_{nu} = \{\gamma D_f N_q + 0.4\,B\,\gamma\,N_\gamma\} - \gamma D_f$$

$$1706 = \{18 \times 1 \times 58 + 0.4 \times 2.5 \times 10 \times N_\gamma\} - 18$$

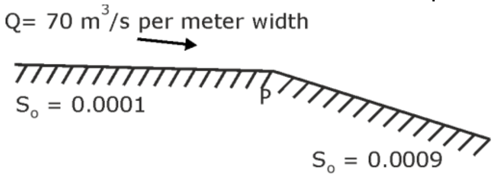
$$N_\gamma = 68$$

For circular Plate
For cohesionless homogeneous soil stratum C=0 and depth of footing= 0

$$q_u = 0.3 \times 0.3 \times 18 \times 68 = 110.16\,\text{kPa}$$

Note: Surcharge at the plate level is zero

43. A very wide rectangular channel carries a discharge (Q) of 70 m^3/s per meter width. Its bed slope changes from 0.0001 to 0.0009 at a point P, as shown in the figure (not to scale). The Manning's roughness coefficient of the channel is 0.01. What water surface profile(s) exist(s) near the point P?

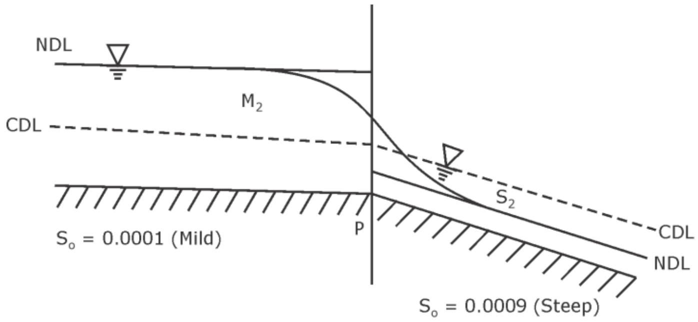


A. M_2 and S_2
B. M_2 only
C. S_2 Only
D. S_2 and hydraulic jump

[MCQ – 2 Marks]

Ans. A

Sol.



CDL will remain same.

But since, NDL varies inversely with slope, the NDL in mild slope is above the CDL, but in steep slope, NDL is below the CDL.

So, as clear from the diagram, flow profile will be M_2 – S_2.

44. A jet of water having a velocity of 20 m/s strikes a series of plates fixed radially on a wheel revolving in the same direction as the jet at 15 m/s. What is the percentage efficiency of the plates? (round off to one decimal place)

A. 37.5
B. 66.7
C. 50.0
D. 88.9

[MCQ – 2 Marks]

Ans. A

Sol. Efficiency of plates,

$$\eta = \frac{2u(V-u)}{V^2} \times 100$$

u = velocity of plate = 15 m/s

V = velocity of jet = 20 m/s

$$\eta = \frac{2 \times 15 \times (20-15)}{20^2} \times 100 = 37.5\%$$

45. In the following table, identify the correct set of associations between the entries in Column-1 and Column-2.

Column-1	**Column-1**
P: Reverse Osmosis	I: Ponding
Q: Trickling Filter	II: Freundlich Isotherm
R: Coagulation	III: Concentration Polarization
S: Adsorption	IV: Charge Neutralization

A. P-II, Q-I, S-III
B. Q-III, R-II, S-IV
C. P-IV, R-I, S-II
D. P-III, Q-I, R-IV

[MCQ – 2 Marks]

Ans. D

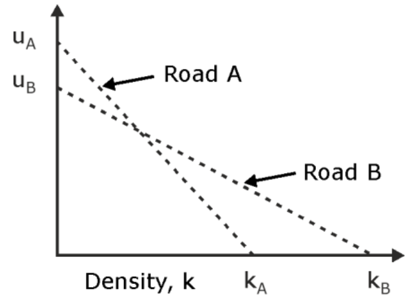
Sol. Reverse Osmosis – Concentration Polarization

Trickling Filter – Ponding

Coagulation– Charge Neutralization

Adsorption- Freundlich Isotherm

46. A plot of speed-density relationship (linear) of two roads (Road A and Road B) is shown in the figure.



If the capacity of Road A is CA and the capacity of Road B is CB, what is $\frac{C_A}{C_B}$

A. $\frac{k_A}{k_B}$

B. $\frac{u_A}{u_B}$

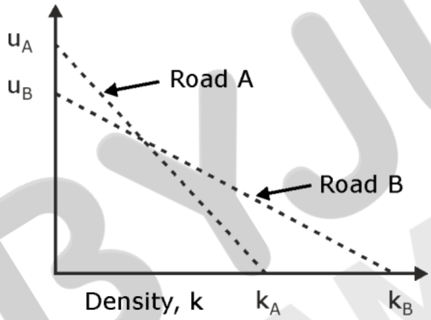
C. $\frac{k_A u_A}{k_B u_B}$

D. $\frac{k_A u_B}{k_B u_A}$

[MCQ – 2 Marks]

Ans. C

Sol.



As per Greenshield model, we know.

$$q_{max} = \frac{V_{sf} k_j}{4}$$

$$\frac{\text{Capcity A}}{\text{Capcity B}} = \frac{\frac{(k_J)_A \times (V_{SF})_A}{4}}{\frac{(k_J)_B \times (V_{SF})_B}{4}} = \frac{K_A \times U_A}{K_B \times U_B}$$

47. For the matrix

$$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

which of the following statements is/are TRUE?

A. The eigenvalues of $[A]^T$ are same as the eigenvalues of [A]

B. The eigenvalues of $[A]^{-1}$ are the reciprocals of the eigenvalues of [A]

C. The eigenvectors of $[A]^T$ are same as the eigenvectors of [A]

D. The eigenvectors of $[A]^{-1}$ are same as the eigenvectors of [A]

[MSQ – 2 Marks]

Ans. A, B, D

Sol. By standard properties:

Eigen values of $[A]^T$ and [A] are same.

Eigen vectors of [A] and $[A]^T$ are not same.

Eigen values of $[A]^{-1}$ is reciprocal of Eigen value of [A].

Eigen vectors of $[A]^{-1}$ are same as the eigenvectors of [A]

48. For the function $f(x)=e^x|\sin x|$; $x \in R$, which of the following statements is/are TRUE?

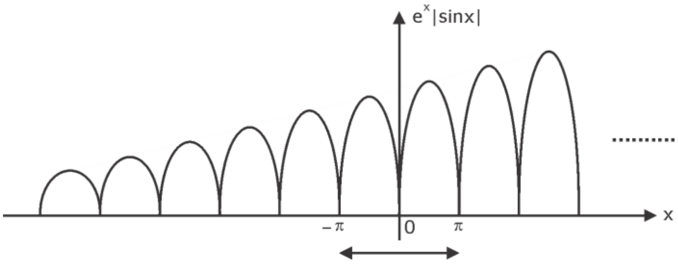
A. The function is continuous at all x
B. The function is differentiable at all x
C. The function is periodic
D. The function is bounded

[MSQ – 2 Marks]

Ans. A

Sol. $y(x) = e^x \mid \sin x \mid$

$e^x \rightarrow$ Aperiodic & unbounded



Consider, $-\pi < x < \pi$

$$f(x) = \begin{cases} e^x \sin x, & 0 < x < \pi \\ -e^x \sin x, & -\pi < x < 0 \end{cases}$$

At $x = 0,$

LHL $= -e^0 \sin 0 = 0$

RHL $= e^0 \sin 0 = 0$

LHL = RHL = f(0) $\rightarrow$ continuous

$$f'(x) = \begin{cases} e^x \cos + e^x \sin x, & 0 < x < \pi \\ -e^x \cos x - e^x \sin x, & -\pi < x < 0 \end{cases}$$

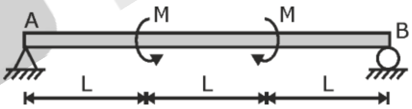
At $x = 0,$

LHD $= -e^0 \times \cos 0 - e^0 \sin 0 = -1$

RHD $= e^0 \cos 0 + e^0 \sin 0 = +1$

LHD $\neq$ RHD $\rightarrow$ Not differentiable.

49. Consider the beam shown in the figure (not to scale), on a hinge support at end A and a roller support at end B. The beam has a constant flexural rigidity, and is subjected to the external moments of magnitude M at one-third spans, as shown in the figure. Which of the following statements is/are TRUE?

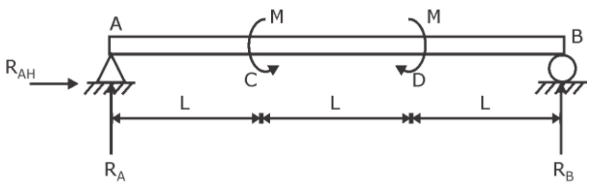


A. Support reactions are zero
B. Shear force is zero everywhere
C. Bending moment is zero everywhere
D. Deflection is zero everywhere

[MSQ – 2 Marks]

Ans. A, B

Sol.



$\Sigma V = 0$

$R_A + R_B = 0$

$\Sigma M_A = 0$

$+R_B \times 3L - M + M = 0$

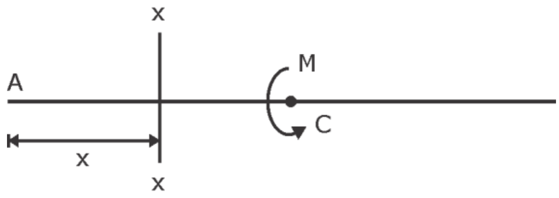
$R_B = 0$

$\therefore$ $R_A = 0$

Since there is no vertical loading the shear force will also be zero anywhere on the beam.

Calculating bending moment:

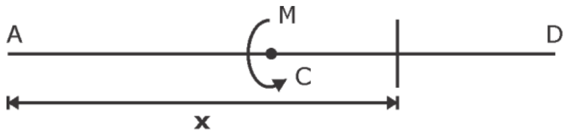
Bending moment at span AC



$M_{xx} = 0$

From A to left of C, no bending moment.

Bending moment at span CD

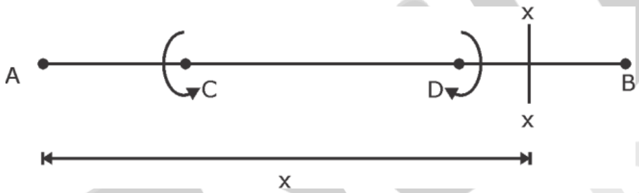


$M_{xx} = -M$ (constant)

$M_C = -M$

$M_D = -M$ (Left of span)

Bending moment of span DB

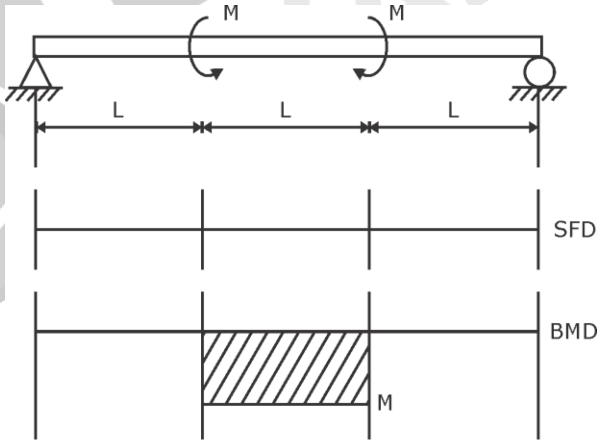


$M_{xx} = -M + M = 0$

$M_D = 0$ (Right of D)

$M_B = 0$

Shear force and bending moment diagram.



Deflection anywhere on the beam $= \dfrac{M}{EI}$ diagram × centroid.

Since the bending moment is not zero, so the deflection will exist.

50. Which of the following statements is/are TRUE in relation to the Maximum Mixing Depth (or Height) 'D_{max}' in the atmosphere?

A. D_{max} is always equal to the height of the layer of unstable air

B. Ventilation coefficient depends on D_{max}

C. A smaller D_{max} will have a smaller air pollution potential if other meteorological conditions remain same

D. Vertical dispersion of pollutants occurs up to D_{max}

[MSQ – 2 Marks]

Ans. B and D

Sol. The ventilation coefficient depends on the maximum mixing depth. The ventilation coefficient is a measure of the rate of air exchange between indoor and outdoor environments. It represents the fraction of outdoor air that is brought into a building through natural or mechanical ventilation systems. The more the maximum mixing depth, the more complete the air exchange between indoor and outdoor air will be.

Vertical dispersion refers to the vertical mixing of pollutants in the atmosphere. A smaller vertical dispersion means that pollutants are less likely to spread out vertically and become diluted in the air, leading to a higher concentration of pollutants at the ground level. This can lead to a higher potential for air pollution and negative impacts on human health and the environment. Therefore, smaller vertical dispersion is generally considered an indicator of a higher air pollution potential.

The maximum mixing depth is not always equal to half of the layer of unstable air. The maximum mixing depth refers to the height at which air is well mixed between the indoor and outdoor environments and is determined by several factors, including atmospheric stability, wind speed, and building design. The layer of unstable air is determined by the temperature and moisture profile of the atmosphere and refers to the height at which vertical air mixing occurs.

In certain atmospheric conditions, the maximum mixing depth may be equal to half of the layer of unstable air, but this is not always.

51. Which of the following options match the test reporting conventions with the given material tests in the table?

Test reporting convention	**Material test**
(P) Reported as ratio	(I) Solubility of bitumen
(Q) Reported as percentage	(II) Softening point of bitumen
(R) Reported in temperature	(III) Los Angeles abrasion test
(S) Reported in length	(IV) Flash point of bitumen
	(V) Ductility of bitumen
	(VI) Specific gravity of bitumen
	(VII) Thin film oven test

A. (P)-(VI); (Q) - (I); (R) -(II);(S)- (VII)

B. (P)-(VI);(Q)-(III); (R) - (IV); (S) - (V)

C. (P)-(VI); (Q)- (I); (R) - (II); (S) - (V)

D. (P)-(VI);(Q)-(III); (R)- (IV); (S) (VII)

[MSQ – 2 Marks]

Ans. B, C

Sol. Specific gravity of bitumen is the ratio of mass of given volume of substance to the mass of equal volume of water, the temperature of both being specified.

(P) - (VI); (Q) - (I, III, VII); (R) - (II, IV); (S) - (V)

52. The differential equation,

$$\frac{du}{dt} + 2tu^2 = 1$$

is solved by employing a backward difference scheme within the finite difference framework. The value of u at the $(n-1)^{th}$ time-step, for some n, is 1.75. The corresponding time (t) is 3.14 s. Each time step is 0.01 s long. Then, the value of $(u_n - u_n - 1)$ is (round off to three decimal places).

[NAT – 2 Marks]

Ans. −0.151

Sol.

$$\frac{du}{dt} = 1 - 2tu^2$$

$$y_n = y_{n-1} + h_f (x_n, y_n)$$

$$u_n = u_{n-1} + h\, f(t_n, u_n)$$

Given: $u_{n-1} = 1.75$, $t_{n-1} = 3.14$ s, $h = 0.01$ s

So, $t_n = 3.14 + 0.01 = 3.15$ s

$$u_n = 1.75 + 0.01\,[1 - 2 \times 3.15\, u_n^2\,]$$

$$u_n = 1.75 + 0.01 - 0.063\,u_n^2$$

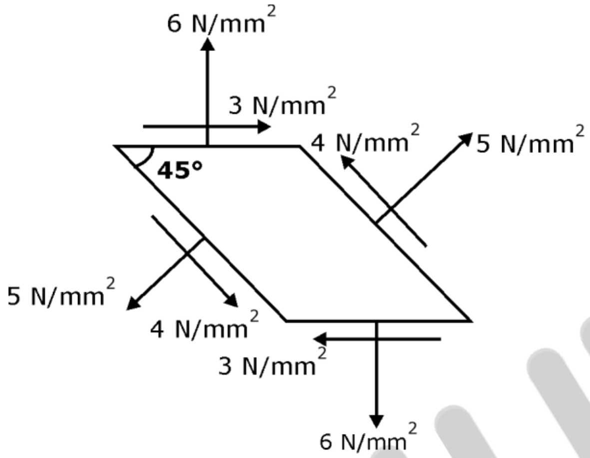
$$0.063u_n^2 + u_n - 1.76 = 0$$

$u_n = 1.5989$ ✓

$u_n = -17.47$ ✗

$u_n - u_{n-1} = -0.1511$

53. The infinitesimal element shown in the figure (not to scale) represents the state of stress at a point in a body. What is the magnitude of the maximum principal stress (in N/mm², in integer) at the point?



[NAT – 2 Marks]

Ans. 7

Sol. $\sigma_{yy} = 6$ N/mm² and $\tau_{xy} = 3$N/mm²

We know.

$$\sigma_\theta = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$5 = \frac{\sigma_{xx} + 6}{2} + \frac{\sigma_{xx} - 6}{2}\cos 90 + 3\sin 90^\circ$$

$$\sigma_{xx} = -2 \text{ N / mm}^2$$

Maximum Principal Stress

$$\sigma_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

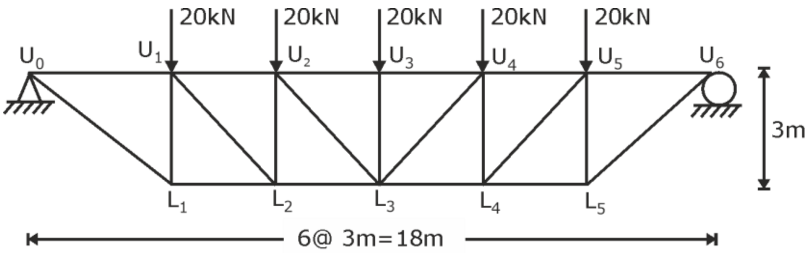
$$= \frac{-2+6}{2} + \sqrt{\left(\frac{-2-6}{2}\right)^2 + 3^2}$$

$$= 2 + \sqrt{4^2 + 3^2}$$

$= 2 + 5$

$= 7$ N/mm²

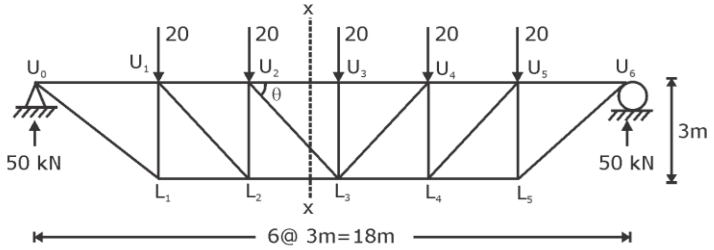
54. An idealised bridge truss is shown in the figure. The force in Member U_2L_3 is _______ kN (round off to one decimal place).



[NAT – 2 Marks]

Ans. 14.1

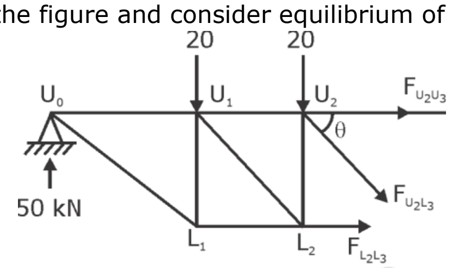
Sol.



Vertical reaction at support at U_0 and U_6

$$= \frac{20 \times 5}{2} = 50\,\text{kN}$$

Cut a section x-x as shown in the figure and consider equilibrium of LHS.



$$\tan\theta = \frac{3}{3} = 1 \;\Rightarrow \theta = 45^\circ$$

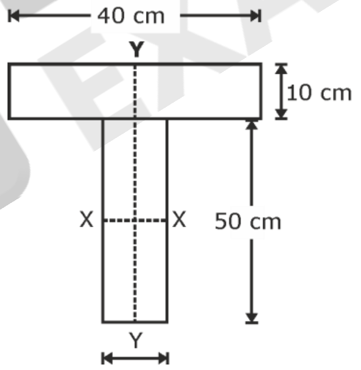
$$\Sigma F_V = 0$$

$$50 - 20 - 20 - F_{U_2L_3}\sin\theta = 0$$

$$F_{U_2L_3}\sin 45 = 10$$

$$F_{U_2L_3} = \frac{10}{\sin 45} = 10\sqrt{2} = 14.14 \text{ kN}$$

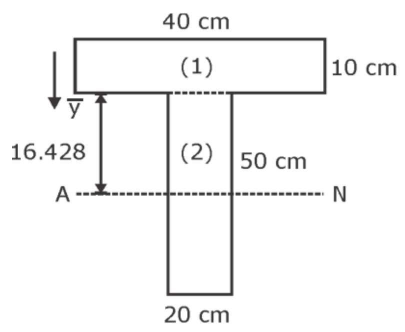
55. The cross-section of a girder is shown in the figure (not to scale). The section is symmetric about a vertical axis (Y-Y). The moment of inertia of the section about the horizontal axis (X-X) passing through the centroid is cm^4 (round off to nearest integer).



[NAT – 2 Marks]

Ans. 468775 to 468810

Sol.



Taking $\bar{y}$ from top

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$\bar{y} = \frac{40 \times 10 \times 5 + 50 \times 20 \times 35}{40 \times 10 + 50 \times 20} = 26.428\,\text{cm}$$

$$I_{NA} = \frac{bd^3}{12} + Ah^2$$

$$I_{NA} = \frac{40 \times 10^3}{12} + 40 \times 10 \times (26.428 - 5)^2$$

$$+ \frac{20 \times 50^3}{12} + 50 \times 20 \times (26.428 - 10 - 25)^2$$

$$\cong 468775.2\,\text{cm}^2 = 468775\ \text{cm}^2$$

56. A soil having the average properties, bulk unit weight = 19 kN/m^3; angle of internal friction = 25° and cohesion = 15 kPa, is being formed on a rock slope existing at an inclination of 35° with the horizontal. The critical height (in m) of the soil formation up to which it would be stable without any failure is (round off to one decimal place).
[Assume the soil is being formed parallel to the rock bedding plane and there is no ground water effect.]

[NAT – 2 Marks]

Ans. 5.02 to 5.05

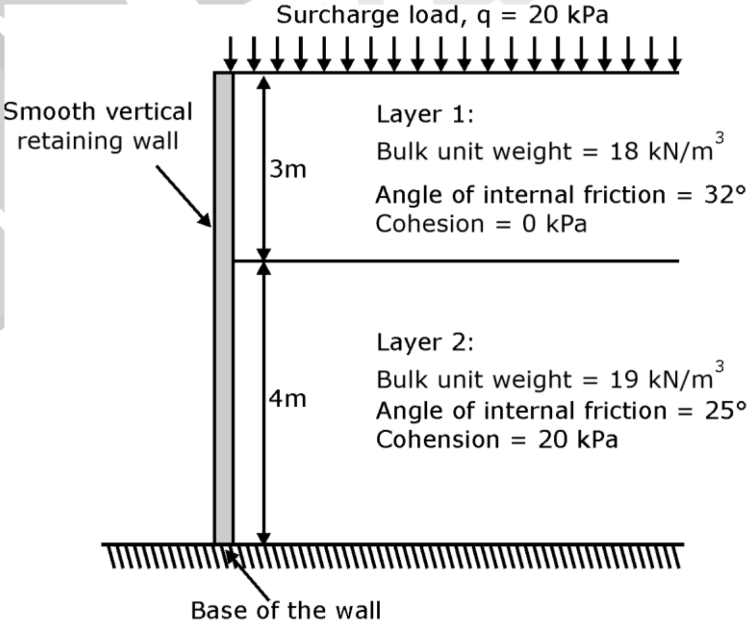
Sol.

$$C + \gamma H_C \cos^2\beta \tan\phi = \gamma H_C \sin\beta \cos\beta$$

$$15 + 19 H_C \cos^2 35 \tan 25 = 19 H_C \sin 35 \cos 35$$

H_C = 5.03 m

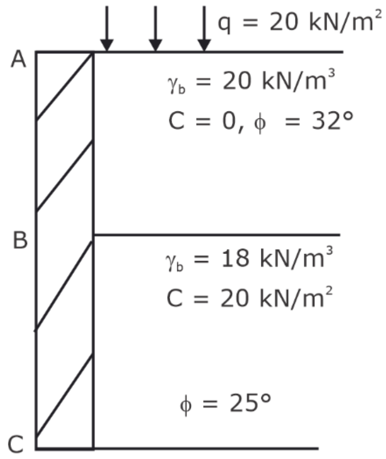
57. A smooth vertical retaining wall supporting layered soils is shown in figure. According to Rankine's earth pressure theory, the lateral active earth pressure acting at the base of the wall is kPa (round off to one decimal place).



[NAT – 2 Marks]

Ans. 35.3 to 35.8

Sol.



$$\sigma_H = K_a\sigma_v - 2C\sqrt{K_a}$$

$$K_a = \frac{1-\sin 25}{1+\sin 25} = 0.408$$

$$\sigma_H = 0.408[20 + 18\times 3 + 19\times 4] - 2\times 20\times\sqrt{0.408}$$

= 61.2 – 25.55

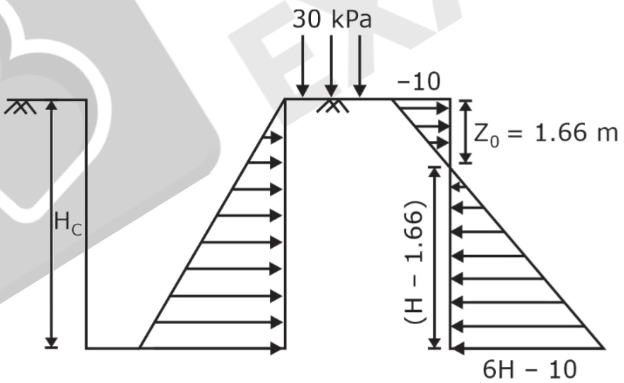
= 35.7 kPa

58. A vertical trench is excavated in a clayey soil deposit having a surcharge load of 30 kPa. A fluid of unit weight 12 kN/m^3 is poured in the trench to prevent collapse as the excavation proceeds. Assume that the fluid is not seeping through the soil deposit. If the undrained cohesion of the clay deposit is 20 kPa and saturated unit weight is 18 kN/m^3, what is the maximum depth of unsupported excavation (in m, rounded off to two decimal places) ______?

[NAT – 2 Marks]

Ans. 3.33

Sol.



Now, active pressure at depth z,

$$\sigma_z = k_a\gamma z - 2C\sqrt{k_a} - 12z + kq$$

At $z = 0$

[∵ $k = 1$ for $\varphi = 0°$, $q = 30$ kN/m^2; $C = 20$ kN/m^2]

$\sigma_z = -2 \times C + q$

$\sigma_z = 2 \times 20 + 30$

$\sigma_z = -40 + 30$

$\sigma_z = -10$ kN/m^2

At z_0, $\sigma_z = 0$

$18z_0 - 40 - 12z_0 + 30 = 0$

$6z_0 - 10 = 0$

$z_0 = 1.66$ m

At depth H,

$$\sigma_H = k_a \gamma z - 2C\sqrt{k_a} - 12H + k_q$$

$\sigma_H = 18H - 40 - 12H + 30$

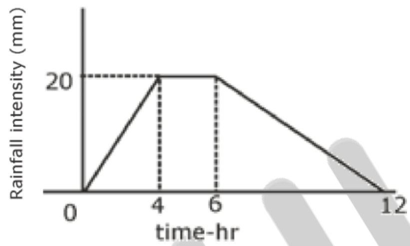
$\sigma_H = 6H - 10$

For unsupported depth of excavation, total active thrust must be zero. So,

$$\frac{1}{2} \times 10 \times 1.66 = \frac{1}{2} \times (H - 1.66) \times (6H - 10)$$

$$H = \frac{10}{3} = 3.33 \text{ m}$$

59. A 12-hour storm occurs over a catchment and results in a direct runoff depth of 100 mm. The time-distribution of the rainfall intensity is shown in the figure (not to scale). The φ-index of the storm is (in mm, rounded off to two decimal places) ________.



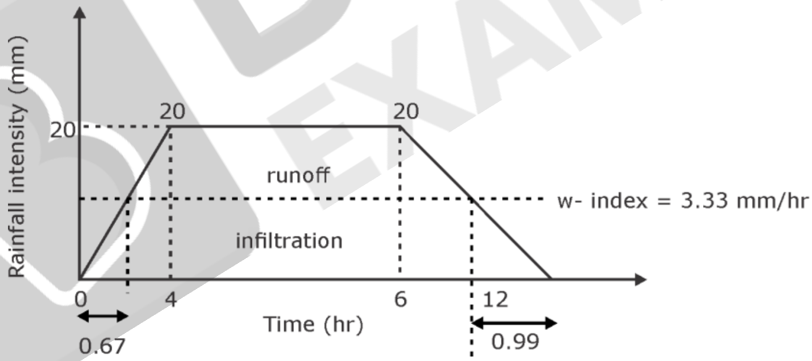
[NAT – 2 Marks]

Ans. 3.60

Sol.

$$P = \frac{1}{2}(12 + 2) \times 20 = 140 \text{ mm}$$

$$\text{w-index} = \frac{P - R}{t} = \frac{140 - 100}{12} = 3.33 \text{ mm/hr}$$



$$\frac{20}{4} = \frac{3.33}{t_1}$$

$t_1 = 0.67$

$$\frac{20}{6} = \frac{3.33}{t_2}$$

$t_2 = 0.99$

$$\varphi = \frac{P_e - R}{t_e}$$

$$= \frac{\left[140 - \left[\frac{1}{2} \times 3.33 \times 0.67 + \frac{1}{2} \times 3.33 \times 0.99\right]\right] - 100}{12 - 0.67 - 0.99} = 3.60 \text{ mm/hr}$$

60. A hydraulic jump occurs in a 1.0 m wide horizontal, frictionless, rectangular channel, with a pre-jump depth of 0.2 m and a post-jump depth of 1.0 m. The value of g may be taken as 10 m/s^2. The values of the specific force at the pre-jump and post-jump sections are same and are equal to (in m^3, rounded off to two decimal places) _____________

[NAT – 2 Marks]

Ans. 0.60 to 0.64

Sol. Given, B = 1 m = width of channel

Pre-jump depth = y_1 = 0.2 m

Post-jump depth = y_2 = 1.0 m

As we know,

$$\text{Specific force} = F_S = \frac{F_1 + M_1}{\rho g}$$

$$= A_1.z_1 + \frac{Q^2}{g.A_1} = A_2.z_2 + \frac{Q^2}{g.A_2}$$

$$\text{Now, } y_c^3 = \frac{y_1.y_2\left(y_1 + y_2\right)}{2}$$

$$= \frac{\left(0.2 \times 1\right)\left(0.2 + 1\right)}{2}$$

$$\Rightarrow y_c^3 = 0.12$$

$$\frac{q^2}{g} = 0.12$$

So, $q^2 = 0.12 \times 10 = 1.2$

$Q^2 = q^2 \times B^2 = (1.2) \times (1)^2 = 1.2$

$$F_S = B.y_1.\frac{y_1}{2} + \frac{Q^2}{g.B.y_1}$$

$$= 1 \times 0.2 \times \frac{0.2}{2} + \frac{1.2}{10 \times 1 \times 0.2}$$

= 0.62

61. In Horton's equation fitted to the infiltration data for a soil, the initial infiltration capacity is 10 mm/h; final infiltration capacity is 5 mm/h; and the exponential decay constant is 0.5 /h. Assuming that the infiltration takes place at capacity rates, the total infiltration depth (in mm) from a uniform storm of duration 12 h is ________. (round off to one decimal place)

[NAT – 2 Marks]

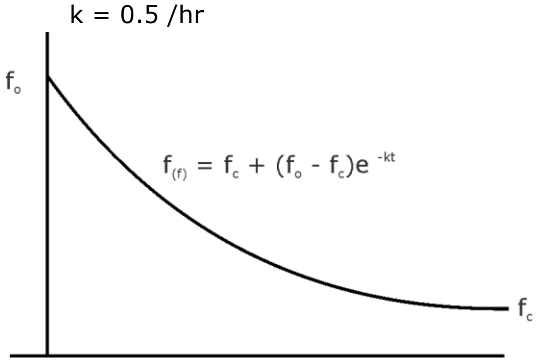
Ans. 70.0

Sol. We have given.

f_o = 10 mm/hr

f_c = 5 mm/hr

k = 0.5 /hr



∴ Total infiltration depth

$$= \int_0^{12} 5 + (10-5)e^{-0.5t}dt$$

$$= \left| 5t + \frac{5e^{-0.5t}}{-0.5} \right|_0^{12}$$

$= (5 \times 12 - 10e^{-0.5 \times 12}) - (5 \times 0 - 10e^{-0.5*0})$

$= 59.97 + 10 = 69.97$mm

$\simeq 70.0$ mm

62. The composition and energy content of a representative solid waste sample are given in the table. If the moisture content of the waste is 26%, the energy content of the solid waste on dry-weight basis is MJ/kg (round off to one decimal place).

Component	Percent by mass	Energy content as-discarded basis (MJ/kg)
Food waste	20	4.5
Paper	45	16.0
Carboard	5	14.0
Plastics	10	32.0
Others	20	8.0

[NAT – 2 Marks]

Ans. 18.4

Sol.

Component	Percent by mass	Energy content as-discarded basis (MJ/kg)	Energy
Food waste	20	4.5	20 × 4.5 = 90
Paper	45	16.0	45 × 16.0 = 720
Cardboard	5	14.0	5 × 14.0 = 70
Plastics	10	32.0	10 × 32.0 = 320
Others	20	8.0	20 × 8.0 = 160
			1360 MJ

Assume 100 kg of solid waste

Moisture content = 26%

Energy content on dry basis $= \frac{1360}{(100-26)} = 18.38$ MJ/kg

63. A flocculator tank has a volume of 2800 m^3. The temperature of water in the tank is 15°C, and the average velocity gradient maintained in the tank is 100/s. The temperature of water is reduced to 5□C, but all other operating conditions including the power input are maintained as the same. The decrease in the average velocity gradient (in %) due to the reduction in water temperature is (round off to nearest integer).

[Consider dynamic viscosity of water at 15°C and 5°C as 1.139×10^{-3} N-s/m^2 and 1.518×10^{-3} N-s/m^2, respectively]

[NAT – 2 Marks]

Ans. 13

Sol. **Case I:**

$T_1 = 15°C$

$P_1 = G_1^2 \mu_1 V$

$P_1 = G_1^2 \times 1.139 \times 10^{-3} \times V$

Case II:

$T_2 = 5°C$

$$P_2 = G_2^2 \mu_2 V$$

$$P_2 = G_2^2 \times 1.518 \times 10^{-3} \times V$$

Now, Power is the same in both the cases

$P_1 = P_2$

$$G_1^2 \times 1.139 \times 10^{-3} \times V = G_2^2 \times 1.518 \times 10^{-3} \times B$$

$$\left(\frac{G_1}{G_2}\right)^2 = \frac{1.518 \times 10^{-3}}{1.139 \times 10^{-3}}$$

$$\left(\frac{G_1}{G_2}\right)^2 = 1.3327$$

$$\frac{G_1}{G_2} = 1.154$$

$$G_2 = \frac{G_1}{1.154} = 0.8665\, G_1$$

∴ % decrease

$$= \frac{G_1 - G_2}{G_1} \times 100\%$$

$$= \frac{G_1 - 0.8665\, G_1}{G_1} \times 100\%$$

= 13.34%=13%

64. The wastewater inflow to an activated sludge plant is 0.5 m^3/s, and the plant is to be operated with a food to microorganism ratio of 0.2 mg/mg-d. The concentration of influent biodegradable organic matter of the wastewater to the plant (after primary settling) is 150 mg/L, and the mixed liquor volatile suspended solids concentration to be maintained in the plant is 2000 mg/L. Assuming that complete removal of biodegradable organic matter in the tank, the volume of aeration tank (in m^3, in integer) required for the plant is _______ .

[NAT – 2 Marks]

Ans. 16200

Sol. Given,

Q= 0.5 m^3/sec

F/M = 0.2 mg/mg-day

S_0 = 150 mg/L

X = 2000 mg/L

V = ?

$$\frac{F}{M} = \frac{QS_0}{Vx}$$

$$\frac{0.20\text{ mg}}{\text{mg-d}} = \frac{\frac{0.5\text{ m}^3}{\text{s}} \times 86400 \times \frac{150\text{ mg}}{\text{l}}}{V(\text{m}^3) \times 2000\text{ mg/l}}$$

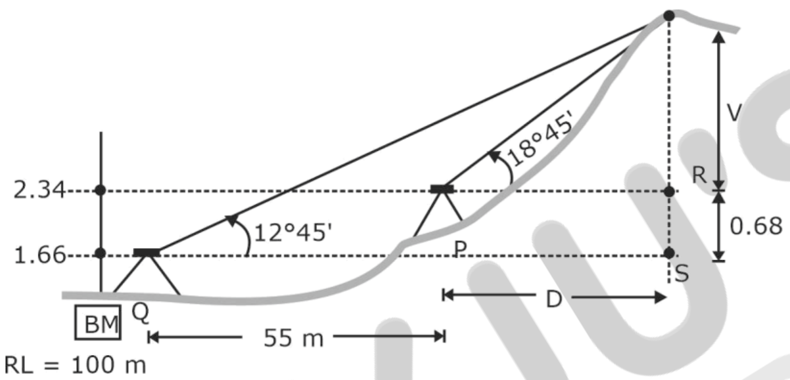
V = 16200 m^3

65. Trigonometric levelling was carried out from two stations P and Q to find the reduced level (R. L.) of the top of hillock, as shown in the table. The distance between Stations P and Q is 55 m. Assume Stations P and Q, and the hillock are in the same vertical plane. The R. L. of the top of the hillock (in m) is ______ (round off to three decimal places).

Station	Vertical angle of the top of hillock	Staff reading on benchmark	R. L. of benchmrk
P	18°45′	2.340 m	100.000 m
Q	12°45′	1.660 m	

[NAT – 2 Marks]

Ans. 137.627



We have,

$$\tan 12°45' = \frac{V + 0.68}{D + 55}$$

And, $$\tan 18°45' = \frac{V}{D}$$

On solving (i) and (ii), we get

V = 35.287 m

D = 103.954 m

Now, R.L of hill top = R.L of BM + 2.34 + V

= 100 + 2.34 + 35.28 = 137.627m

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Vatsal (ME)
Rank 08
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Rank 08
Hemant (EE)
Rank 08
Rajat (ME)
Rank 09
Vamsi (EC)
Rank 13
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