# Force Method of Structural Analysis

The forced method of structural analysis is one of the methods of analysis of indeterminate structures. In this method, reaction forces are considered as the unknown parameter. Here are a few points of the forced method of analysis important for the GATE CE exam are given.

- For determinate structures, the force method allows us to find internal forces (using equilibrium i.e. based on Statics) irrespective of the material information. Material (stress-strain) relationships are needed only to calculate deflections.
- However, for indeterminate structures, Statics (equilibrium) alone is insufficient to conduct structural analysis. Compatibility and material information are essential.
- The flexibility method is based upon the solution of "equilibrium equations and compatibility equations". There will always be as many compatibility equations as redundant ones. It is called the flexibility method because flexibilities appear in the compatibility equations. Another name for the method is the force method because forces are unknown quantities in compatibility equations.

The method is based on transforming a given structure into a statically determinate primary system and calculating the magnitude of statically redundant forces required to restore the geometric boundary conditions of the original structure.

## Steps Involved in the Forced Method of Structural Analysis

The forced method of structural analysis is the method of analysing indeterminate structures. These steps are important to score marks from this section of the GATE CE syllabus. The analysis should be carried out in proper order. It consists of the following steps:
(a) Determine the structure's degree of static indeterminacy, "n".
(b) Transform the structure into a statically determinate system by releasing some static constraints equal to the degree of static indeterminacy, n. This is accomplished by releasing external support conditions or by creating internal hinges. The system thus formed is called the basic determinate structure.
(c) For a given released constraint j, introduce an unknown redundant force $R_f$ corresponding to the type and direction of the released constraint.
(d) Apply the given loading or imposed deformation to the basic determinate structure. Use a suitable method to calculate displacements at each released constraint in the basic determinate structure.
(e) Solve for redundant forces $R_f$ ( j =1 to n ) by imposing the compatibility conditions of the original structure. These conditions transform the basic determinate structure back to the original structure by finding the combination of redundant forces that make displacement at each released constraint equal to zero.

## Types of Force Method of Structural Analysis

The forced method of structural analysis is also called the flexibility and compatibility method. In this method, forces are considered unknown parameters. The forced method of structural analysis can be categorized into various methods; these are listed below:

1. Virtual work/Unit load Method
2. Method of consistent deformation
3. Three-moment theorem
4. Castigliano's theorem of minimum strain energy
5. Maxwell-Mohr's equation
6. Column Analogy Method

## Strain Energy Method for Analysis of Indeterminate Structures

The strain energy method is the method used for the analysis of indeterminate structures. This method is based on the principle of strain energy, and it can be related to Castigliano's theorem. The idea about the strain energy due to different combinations of loading over the structure is required to analyse indeterminate structures. Here formula for the strain energy for a few types of loading is given below:

### Strain energy stored due to axial load

$U_i=\int(P^2dx/2AE)$

where,

- P = Axial load
- dx = Elemental length
- AE = Axial rigidity

### Strain energy stored due to bending

$U_i=\int(M_x^2ds/2EI)$

where

- $M_X$ = Bending moment at section x-x
- ds = Elemental length
- EI = Flexural rigidity
- or E = Modulus of elasticity
- I = Moment of inertia

Strain energy stored due to shear

$U_i=\int(q^2/2G)dv$

where,

- $q$ = Shear stress
- $G$ = Modulus of rigidity
- $dv$ = Elemental volume

Strain energy stored due to shear force

$U_i=\int(S^2 ds/2GA_s)$

where,

- $A_s$ = Area of shear
- $S$ = Shear force
- $G$ = Modulus of rigidity
- $ds$ = Elemental length

Strain energy stored due to torsion

$U_i=\int(T^2 dx/2GI_P)$

where

- $T$ = Torque acting on a circular bar
- $dx$ = Elemental length
- $G$ = Modulus of rigidity
- $I_P$ = Polar moment of inertia

Strain energy is stored in terms of maximum shear stress

$U_i= \tau^2_{max}V/4G$

Where

- $T_{max}$ = Maximum shear stress at the surface of the rod under twisting.
- $G$ = Modulus of rigidity
- $V$ = Volume

Strain energy stored in a hollow circular shaft is,

$$U_i = \int \frac{\tau_{max}^2}{4G} \cdot V \cdot \left( \frac{D^2 + d^2}{D^2} \right)$$

Where

- $D$ = External dia of hollow circular shafts

- d = Internal dia of the hollow circular shaft
- $T_{max}$ = Maximum shear stress

### Castigliano's first Theorem

$$\frac{\partial U}{\partial \Delta} = P \& \frac{\partial U}{\partial \theta} = M$$

where,

- U = Total strain energy
- Δ = Displacement in the direction of force P.
- θ = Rotation in the direction of moment M.

### Castiglianos Second Theorem

$$\frac{\partial U}{\partial P} = \Delta, \frac{\partial U}{\partial M} = \theta$$