

GATE 2023

Electrical Engineering

Questions with Detailed
Solutions



General Aptitude

1. Rafi told Mary, "I am thinking of watching a film this weekend." The following reports the above statement in indirect speech: Rafi told Mary that he _____ of watching a film that weekend.
- A. thought
B. is thinking
C. am thinking
D. was thinking

[MCQ - 1 Mark]

Ans. D

Sol. As it is in incorrect speech it would be was thinking instead of is thinking.

2. Permit : _____ : : Enforce : Relax
(By word meaning)
- A. Allow
B. Forbid
C. License
D. Reinforce

[MCQ - 1 Mark]

Ans. B

Sol. As enforce has the opposite meaning of relax so forbid will be the answer as it is the along of permit.

3. Given a fair six-faced dice where the faces are labelled '1', '2', '3', '4', '5', and '6', what is the probability of getting a '1' on the first roll of the dice and a '4' on the second roll?
- A. 1/36
B. 1/6
C. 5/6
D. 1/3

[MCQ - 1 Mark]

Ans. A

Sol. Probability of getting 1 on 1st roll and 4 on 2nd roll

$$= \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

4. A recent survey shows that 65% of tobacco users were advised to stop consuming tobacco. The survey also shows that 3 out of 10 tobacco users attempted to stop using tobacco.
- Based only on the information in the above passage, which one of the following options can be logically inferred with certainty?
- A. A majority of tobacco users who were advised to stop consuming tobacco made an attempt to do so.
B. A majority of tobacco users who were advised to stop consuming tobacco did not attempt to do so.
C. Approximately 30% of tobacco users successfully stopped consuming tobacco.
D. Approximately 65% of tobacco users successfully stopped consuming tobacco.

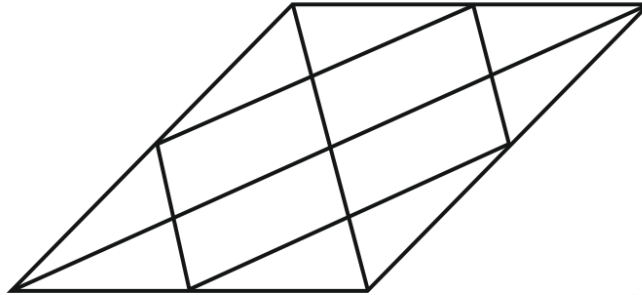
[MCQ - 1 Mark]

Ans. B

Sol. A is wrong as 65% of tobacco users were asked to stop but any 30% made an effort to stop which is not the majority.

- B – It is correct as only 30% of the tobacco users tried to stop consuming.
- C – Incorrect as the question does not specify if they were successful or not it only says they tried.
- D – Wrong due to same reason as C.

5. How many triangles are present in the given figure?



- A. 12
- B. 16
- C. 20
- D. 24

[MCQ - 1 Mark]

Ans. C

Sol. The number of triangles are 20.

6. Students of all the departments of a college who have successfully completed the registration process are eligible to vote in the upcoming college elections. However, by the time the due date for registration was over, it was found that surprisingly none of the students from the Department of Human Sciences had completed the registration process.

Based only on the information provided above, which one of the following sets of statement(s) can be logically inferred with certainty?

- (i) All those students who would not be eligible to vote in the college elections would certainly belong to the Department of Human Sciences.
- (ii) None of the students from departments other than Human Sciences failed to complete the registration process within the due time.
- (iii) All the eligible voters would certainly be students who are not from the Department of Human Sciences.

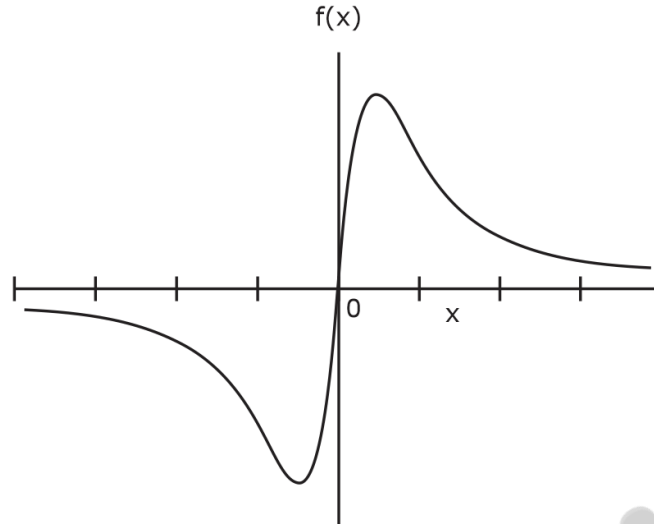
- A. (i) and (ii)
- B. (i) and (iii)
- C. only (i)
- D. only (iii)

[MCQ - 2 Marks]

Ans. D

Sol. (i) This is not mentioned in the question so cannot be said with certainty so it is wrong.
(ii) It is wrong as it is given in the question that all the students of department of human science have not completed the registration process.
(iii) As all the students of department of human science have not filled the registration so all eligible voters would. Certainly, be not from the Department of Human Sciences.

7. Which one of the following options represents the given graph?



A. $f(x) = x^2 2^{-|x|}$

B. $f(x) = x 2^{-|x|}$

C. $f(x) = |x| 2^{-x}$

D. $f(x) = |x| 2^{-x}$

[MCQ - 2 Marks]

Ans. D

Sol.

$$f(x) = -f(-x)$$

Odd symmetry

So, by options,

$$f(x) = x 2^{-|x|}$$

$$f(-x) = -x 2^{-|x|}$$

This is only option satisfying even figure.

8. Which one of the options does NOT describe the passage below or follow from it? We tend to think of cancer as a 'modern' illness because its metaphors are so modern. It is a disease of overproduction, of sudden growth, a growth that is unstoppable, tipped into the abyss of no control. Modern cell biology encourages us to imagine the cell as a molecular machine. Cancer is that machine unable to quench its initial command (to grow) and thus transform into an indestructible, self-propelled automaton.

[Adapted from The Emperor of All Maladies by Siddhartha Mukherjee]

- A. It is a reflection of why cancer seems so modern to most of us
- B. It tells us that modern cell biology uses and promotes metaphors of machinery.
- C. Modern cell biology encourages metaphors of machinery, and cancer is often imagined as a machine.
- D. Modern cell biology never uses figurative language, such as metaphors, to describe or explain anything.

[MCQ - 2 Marks]

Ans. D

Sol. As mentioned in the passage. Modern cell biology encourages us to imagine all as a molecular machine.

9. The digit in the unit's place of the product $3^{999} \times 7^{1000}$ is _____ .

- A. 7
- B. 1
- C. 3
- D. 9

[MCQ - 2 Marks]

Ans. A

Sol. \Rightarrow

$$3^{999} \times 7^{1000}$$

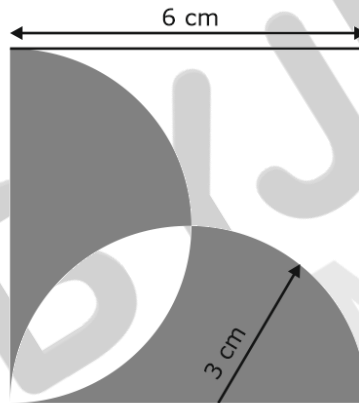
$$= 3^{4(249)} \times 3^3 \times 7^{4(250)}$$

$$= 3^0 \times 3^3 \times 7^0 = 1 \times 7 \times 1 = 7$$

Unit place = 7

10. A square with sides of length 6 cm is given. The boundary of the shaded region is defined by two semi-circles whose diameters are the sides of the square, as shown.

The area of the shaded region is _____ cm^2 .

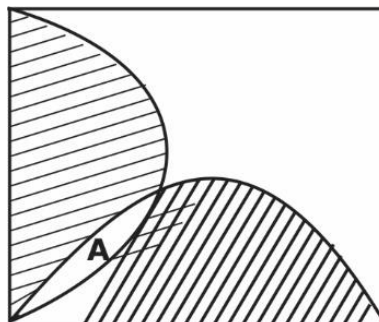


- A. 6π
- B. 18
- C. 20
- D. 9π

[MCQ - 2 Marks]

Ans. B

Sol. Area of shaded = $(2 \times \text{area of semi-circle}) - 2(\text{unshaded common area})$



\therefore

$$\text{Area} = \frac{2\pi r^2}{2} - 2A$$

$$= \pi(3)^2 - 2A$$



$$\begin{aligned}\text{Area of top portion} &= \frac{\pi r^2}{4} - \frac{1}{2}(3)(3) \\ &= \frac{9\pi}{4} - \frac{9}{2}\end{aligned}$$

$$\therefore \text{Total unshaded area} = 2 \left[\frac{9\pi}{4} - \frac{9}{2} \right]$$

$$\therefore \text{Total shaded area} = 9\pi - 2 \times 2 \left[\frac{9\pi}{4} - \frac{9}{2} \right] = 18$$



Technical

11. For a given vector $\mathbf{w} = [1 \ 2 \ 3]^T$, the vector normal to the plane defined by $\mathbf{w}^T\mathbf{x} = 1$ is

- A. $[-2 \ -2 \ 2]^T$
- B. $[3 \ 0 \ -1]^T$
- C. $[3 \ 2 \ 1]^T$
- D. $[1 \ 2 \ 3]^T$

[MCQ - 1 Mark]

Ans. D

Sol.

$$W = [1 \ 2 \ 3]^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Let

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

\therefore

$$W^T \cdot x = 1$$

$$[1 \ 2 \ 3] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$$

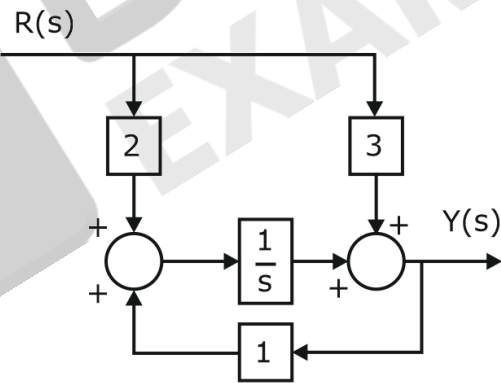
$$x + 2y + 3z - 1 = 0$$

Let $\phi = x + 2y + 3z - 1$

$$\bar{\nabla}\phi = \hat{i} + 2\hat{j} + 3\hat{k}$$

Normal vector to plane $\phi = [1 \ 2 \ 3]^T$

12. For the block diagram shown in the figure, the transfer function $\frac{Y(s)}{R(s)}$ is

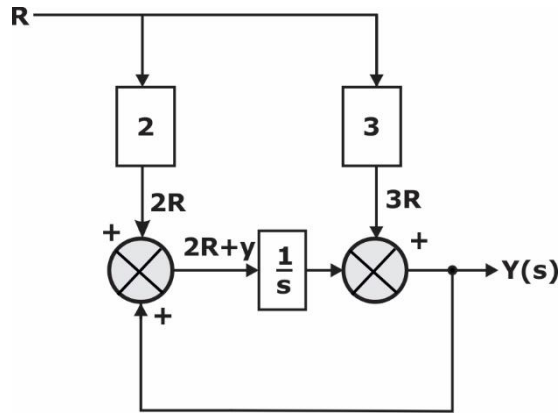


- A. $\frac{2s+3}{s+1}$
- B. $\frac{3s+2}{s-1}$
- C. $\frac{s+1}{3s+2}$
- D. $\frac{3s+2}{s+1}$

[MCQ - 1 Mark]

Ans. B

Sol.



$$Y = 3R + \frac{1}{s}(2R + Y)$$

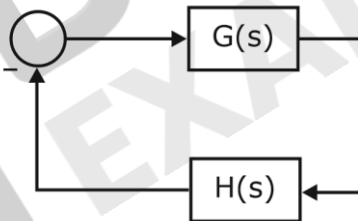
$$Y\left(1 - \frac{1}{s}\right) = R\left(3 + \frac{2}{s}\right)$$

$$\frac{Y}{R} = \frac{3s + 2}{s - 1}$$

13. In the Nyquist plot of the open-loop transfer function

$$G(s)H(s) = \frac{3s + 5}{s - 1}$$

corresponding to the feedback loop shown in the figure, the infinite semi-circular arc of the Nyquist contour in s-plane is mapped into a point at



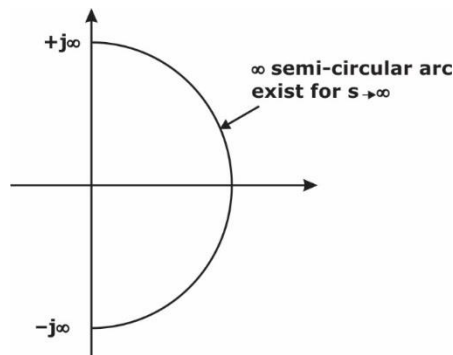
- A. $G(s) H(s) = \infty$
- C. $G(s) H(s) = 3$

- B. $G(s) H(s) = 0$
- D. $G(s) H(s) = -5$

[MCQ - 1 Mark]

Ans. C

Sol. So, $\lim_{s \rightarrow \infty} G(s) H(s) = \lim_{s \rightarrow \infty} \frac{3s + 5}{s - 1} = \frac{3}{1} = 3$



14. Consider a unity-gain negative feedback system consisting of the plant $G(s)$ (given below) and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

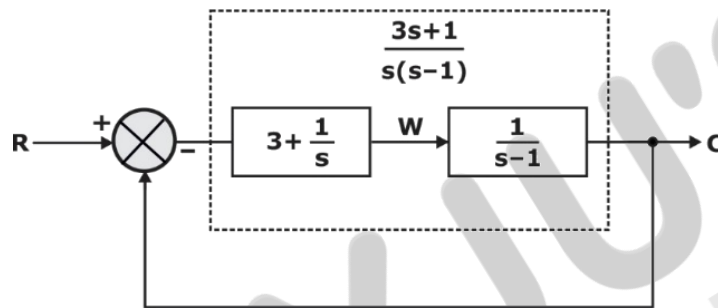
$$G(s) = \frac{1}{s-1}$$

- A. ∞, ∞
- B. 1, 0
- C. 1 -1
- D. -1, 1

[MCQ - 1 Mark]

Ans. D

Sol.



$$\frac{C}{R} = \frac{3s+1}{s^2+2s+1}$$

$$R(s) = 1/s$$

$$\text{Plant output } C(s) = \frac{1}{s} \left[\frac{3s+1}{s^2+2s+1} \right]$$

$$\text{Steady state value } C_{ss} = \lim_{s \rightarrow 0} sC(s) = \left(\frac{1}{1} \right) = 1$$

Controller output $W(s)$

$$W(s) \times \frac{1}{s-1} = C(s)$$

$$W(s) = (s-1)C(s)$$

$$W(s) = (s-1) \times \frac{1}{s} \left(\frac{3s+1}{s^2+2s+1} \right)$$

$$\text{Steady state value } \lim_{s \rightarrow 0} sW(s) = (-1) \left(\frac{1}{1} \right) = -1$$

15. The following columns present various modes of induction machine operation and the ranges of slip

A	B
Mode of operation	Rang of Slip
a. Running in generator mode	(p) From 0.0 to 1.0
B. Running in motor mode	(q) From 1.0 to 2.0
c. Plugging in motor mode	(r) From -1.0 to 0.0

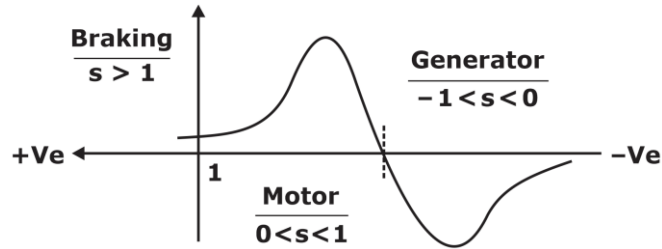
The correct matching between the elements in column A with those of column B is

- | | |
|----------------------|----------------------|
| A. a-r, b-p, and c-q | B. a-r, b-q, and c-p |
| C. a-p, b-r, and c-q | D. a-q, b-p, and c-r |

[MCQ - 1 Mark]

Ans. A

Sol. The operation range of slip of induction machine.



Option (A) is correct.

16. A 10-pole, 50 Hz, 240 V, single phase induction motor runs at 540 RPM while driving rated load.

The frequency of induced rotor currents due to backward field is

- | | |
|-----------|----------|
| A. 100 Hz | B. 95 Hz |
| C. 10 Hz | D. 5 Hz |

[MCQ - 1 Mark]

Ans. B

Sol. Given data of 1- ϕ induction motor.

$$P = 10$$

$$f = 50$$

$$V = 240V$$

$$N_r = 540 \text{ rpm}$$

$$f_r = ?$$

$$N_s = \frac{120 \times 50}{10} = 600 \text{ rpm}$$

$$f_r = s_b f$$

$$s_b = 2 - s_f$$

$$s_f = \frac{N_s - N_r}{N_s} = \frac{600 - 540}{600} = 0.1$$

$$s_b = 2 - 0.1 = 1.9$$

$$\therefore f_r = 1.9 \times 50 = 95\text{Hz}$$

Option (B)

17. A continuous-time system that is initially at rest is described by

$$\frac{dy(t)}{dt} + 3y(t) = 2x(t)$$

where $x(t)$ is the input voltage and $y(t)$ is the output voltage. The impulse response of the system is

A. $3e^{-2t}$

B. $\frac{1}{3}e^{-2t}u(t)$

C. $2e^{-3t}u(t)$

D. $2e^{-3t}$

[MCQ - 1 Mark]

Ans. C

Sol.

$$\frac{dy(t)}{dt} + 3y(t) = 2x(t)$$

$$sY(s) + 3Y(s) = 2X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{2}{s+3}$$

$$H(s) = \frac{2}{s+3}$$

$$h(t) = 2L^{-1}\left\{\frac{1}{s+3}\right\}$$

$$h(t) = 2e^{-3t} \cdot u(t)$$

18. The Fourier transform $X(\omega)$ of the signal $x(t)$ is given by

$$X(\omega) = 1, \text{ for } |\omega| < \omega_0$$

$$= 0, \text{ for } |\omega| > \omega_0$$

Which one of the following statements is true?

A. $x(t)$ tends to be an impulse as $\omega_0 \rightarrow \infty$.

B. $x(0)$ decreases as ω_0 increases.

C. At $t = \frac{\pi}{2\omega_0}$, $x(t) = -\frac{1}{\pi}$

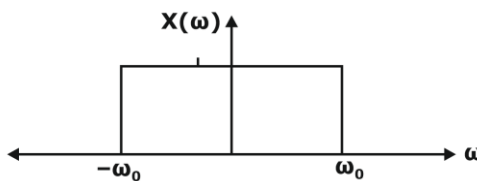
D. At $t = \frac{\pi}{2\omega_0}$, $x(t) = \frac{1}{\pi}$

[MCQ - 1 Mark]

Ans. B

Sol.

$$X(\omega) = \begin{cases} 1 & ; |\omega| < \omega_0 \\ 0 & ; |\omega| > \omega_0 \end{cases}$$



$$X(\omega) = \text{rect}\left(\frac{\omega}{2\omega_0}\right)$$

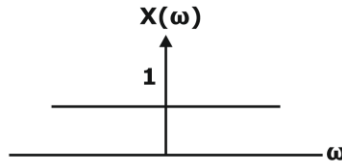
$$x(t) = F^{-1}\{X(\omega)\} = \frac{\sin(\omega_0 t)}{\pi t}$$

$$(i) \because x(0) = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} |X(\omega)|^2 \cdot d\omega$$

$$x(0) = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} 1 \cdot d\omega = \frac{2\omega_0}{2\pi} = \frac{\omega_0}{\pi}$$

as ω_0 increase then $x(0)$ also increase

(ii) As $\omega_0 \rightarrow \infty$ $x(t) =$ impulse signal



$$(iii) \text{ at } t = \frac{\pi}{2\omega_0} \Rightarrow x(t) = \frac{\sin\left(\omega_0 \frac{\pi}{2\omega_0}\right)}{\pi \cdot \frac{\pi}{2\omega_0}} = \frac{2\omega_0}{\pi^2}$$

19. The Z-transform of a discrete signal $x[n]$ is

$$X(z) = \frac{4z}{\left(z - \frac{1}{5}\right)\left(z - \frac{2}{3}\right)(z - 3)}$$

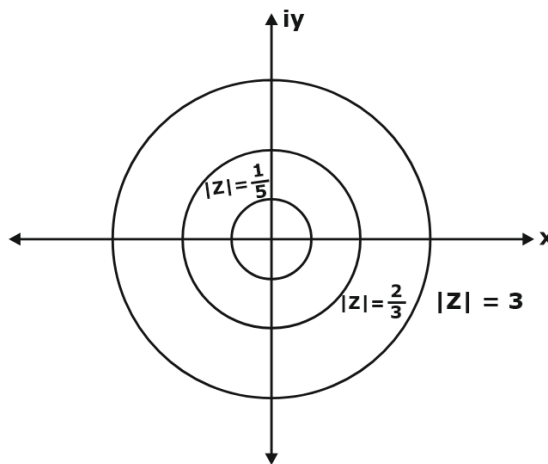
Which one of the following statements is true?

- A. Discrete-time Fourier transform of $x[n]$ converges if R is $|z| > 3$
- B. Discrete-time Fourier transform of $x[n]$ converges if R is $\frac{2}{3} < |z| < 3$.
- C. Discrete-time Fourier transform of $x[n]$ converges if R is such that $x[n]$ is a left sided sequence
- D. Discrete-time Fourier transform of $x[n]$ converges if R is such that $x[n]$ is a right sided sequence

[MCQ - 1 Mark]

Ans. B

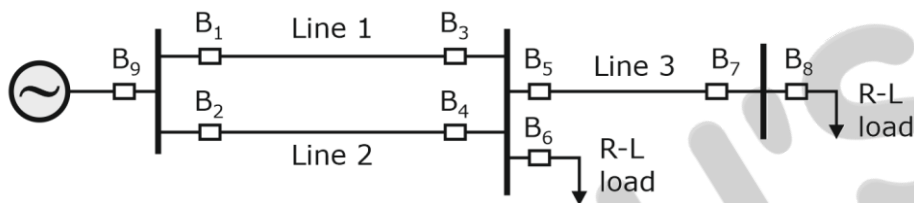
Sol. $Z = \frac{1}{5}, Z = \frac{2}{3}, Z = 3$



For this ROC : $\frac{2}{3} < |Z| < 3$

System is stable as it includes unit circle $|Z| = 1$
 also, DTFT will coverage for this ROC

20. For the three-bus power system shown in the figure, the trip signals to the circuit breakers B₁ to B₉ are provided by overcurrent relays R₁ to R₉, respectively, some of which have directional properties also. The necessary condition for the system to be protected for short circuit fault at any part of the system between bus 1 and the R-L loads with isolation of minimum portion of the network using minimum number of directional relays is

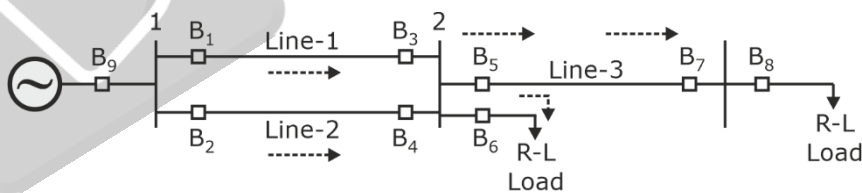


- A. R₃ and R₄ are directional overcurrent relays blocking faults towards bus 2
- B. R₃ and R₄ are directional overcurrent relays blocking faults towards bus 2 and R₇ is directional overcurrent relay blocking faults towards bus 3
- C. R₃ and R₄ are directional overcurrent relays blocking faults towards Line 1 and Line 2, respectively, R₇ is directional overcurrent relay blocking faults towards Line 3 and R₅ is directional overcurrent relay blocking faults towards bus 2
- D. R₃ and R₄ are directional overcurrent relays blocking faults towards Line 1 and Line 2, respectively.

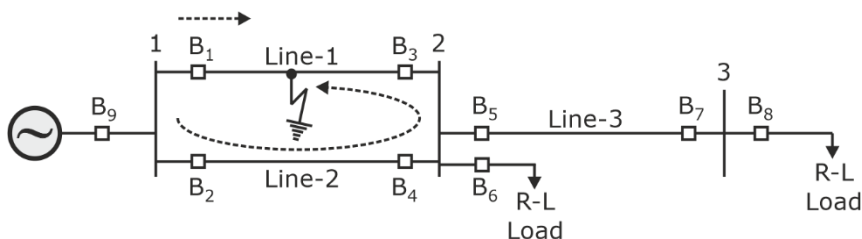
[MCQ - 1 Mark]

Ans. D

Sol. (i) General current direction: (without fault)

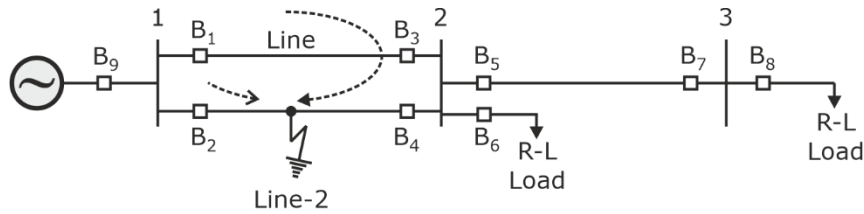


(ii) If fault is in Line - 1



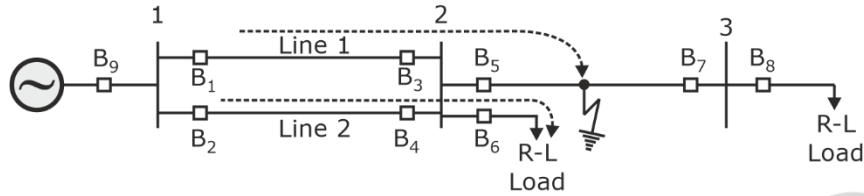
As current through opposite to original direction. Hence, Relay R₃ should be directional.

(iii) Let fault in line - 2



As current through B4 is in opposite to original direction. Hence, Relay R4 should be directional.

(iv) If fault is in Line - 3



None of the relays have current the opposite direction.

21. The expressions of fuel cost of two thermal generating units as a function of the respective power generation P_{G1} and P_{G2} are given as

$$F_1(P_{G1}) = 0.1aP_{G1}^2 + 40P_{G1} + 120Rs / \text{hour} \quad 0 \text{ MW} \leq P_{G1} \leq 350 \text{ MW}$$

$$F_2(P_{G2}) = 0.2P_{G2}^2 + 30P_{G2} + 100Rs / \text{hour} \quad 0 \text{ MW} \leq P_{G2} \leq 350 \text{ MW}$$

where a is a constant. For a given value of a , optimal dispatch requires the total load of 290 MW to be shared as $P_{G1} = 175 \text{ MW}$ and $P_{G2} = 115 \text{ MW}$. With the load remaining unchanged, the value of a is increased by 10% and optimal dispatch is carried out. The changes in P_{G1} and the total cost of generation, $F (= F_1 + F_2)$ in Rs/hour will be as follows

- A. P_{G1} will decrease and F will increase
- B. Both P_{G1} and F will increase
- C. P_{G1} will increase and F will decrease
- D. Both P_{G1} and F will decrease

[MCQ - 1 Mark]

Ans. A

Sol.

$$0 \leq P_{G1} \leq 350 \text{ MW}$$

$$F_1(P_{G1}) = 0.1aP_{G1}^2 + 40P_{G1} + 120Rs / \text{hr}$$

$$F_2(P_{G2}) = 0.2P_{G2}^2 + 30P_{G2} + 100Rs / \text{hr}$$

$$0 \leq P_{G2} \leq 350 \text{ MW}$$

$$a \begin{cases} P_D = 290, P_{G1} = 175 \text{ MW} \\ P_{G2} = 115 \text{ MW} \end{cases}$$

$$a' = 1.1a \begin{cases} P_D = 290, P_{G1} = ? \\ P_{G2} = ? \end{cases}$$

Changes in P_G , and total cost of generation $F(F_1 + F_2)$

$$\frac{dF_1}{dP_{G1}} = 0.2a'P_{G1} + 40$$

$$\frac{dF_2}{dP_{G2}} = 0.4P_{G2} + 30$$

$$\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}}$$

$$0.2aP_{G1} + 40 = 0.4P_{G2} + 30$$

$$(0.2 \times 175)a + 40 = 0.4 \times 115 + 30$$

$$a = \frac{36}{0.2 \times 175} = 1.028$$

$$a' = 1.013$$

$$0.2 \times 1.13 P_{G1} + 40 = P_{G2} + 30$$

$$P_{G1} + P_{G2} = 290$$

$$0.626 P_{G1} + 10 = 0.4 (290 - P_{G1})$$

$$P_{G1} = 169.329 \text{ MW}$$

$$P_{G2} = 120.67 \text{ MW}$$

Hence, P_{G1} decreases and F is increase.

- 22.** The four stator conductors (A, A', B and B') of a rotating machine are carrying DC currents of the same value, the directions of which are shown in the figure (i). The rotor coils a-a' and b-b' are formed by connecting the back ends of conductors 'a' and 'a'' and 'b' and 'b'', respectively, as shown in figure (ii). The e.m.f. induced in coil a-a' and coil b-b' are denoted by $E_{a-a'}$ and $E_{b-b'}$, respectively. If the rotor is rotated at uniform angular speed ω rad/s in the clockwise direction then which of the following correctly describes the $E_{a-a'}$ and $E_{b-b'}$?

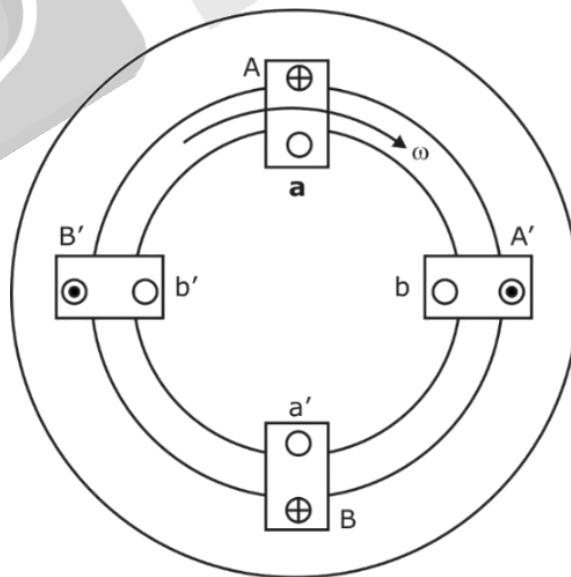


figure (i): cross-sectional view

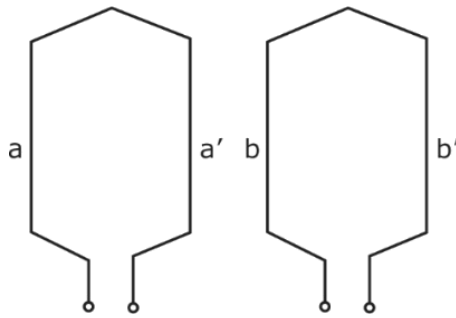


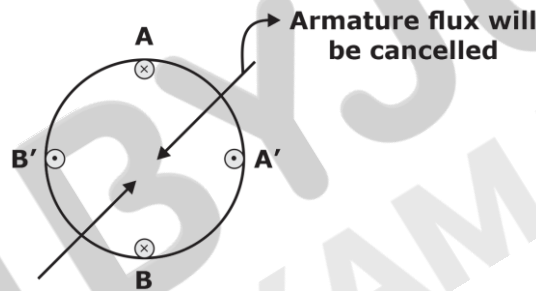
figure (ii): rotor winding connection diagram

- A. $E_{a-a'}$ and $E_{b-b'}$ have finite magnitudes and are in the same phase.
- B. $E_{a-a'}$ and $E_{b-b'}$ have finite magnitudes with $E_{b-b'}$ leading $E_{a-a'}$
- C. $E_{a-a'}$ and $E_{b-b'}$ have finite magnitudes with $E_{a-a'}$ leading $E_{b-b'}$
- D. $E_{a-a'} = E_{b-b'} = 0$

[MCQ - 1 Mark]

Ans. D

Sol. The flux produced by armature coils will be cancelled and hence the EMF induced in the rotor coils will be zero.



23. The chopper circuit shown in figure (i) feeds power to a 5 A DC constant current source. The switching frequency of the chopper is 100 kHz. All the components can be assumed to be ideal. The gate signals of switches S_1 and S_2 are shown in figure (ii). Average voltage across the 5 A current source is

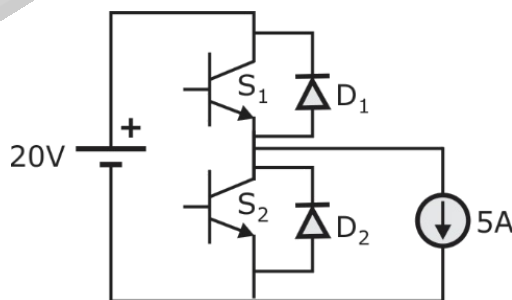


Figure (i)

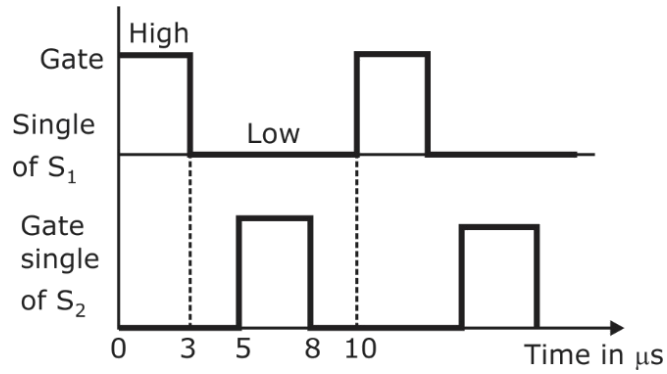


Figure (ii)

- A. 10 V
- B. 6 V
- C. 12 V
- D. 30 V

[MCQ - 1 Mark]

Ans. B

Sol.

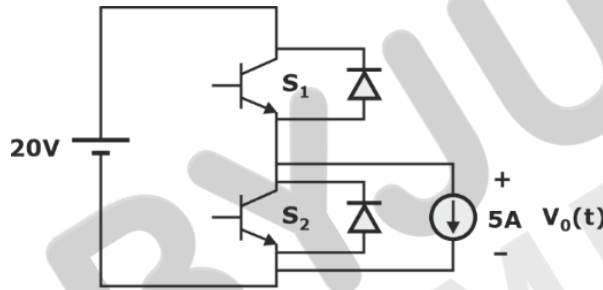
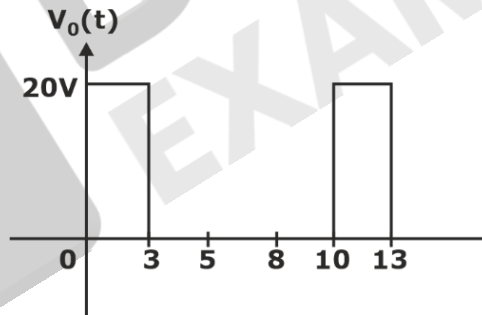
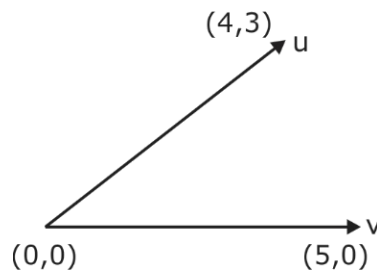


Fig. (i)



$$V_0(t)_{avg} = \frac{(20)(3)}{10} = 6 \text{ volt}$$

24. In the figure, the vectors \mathbf{u} and \mathbf{v} are related as: $\mathbf{A}\mathbf{u} = \mathbf{v}$ by a transformation matrix \mathbf{A} . The correct choice of \mathbf{A} is



A. $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix}$

B. $\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$

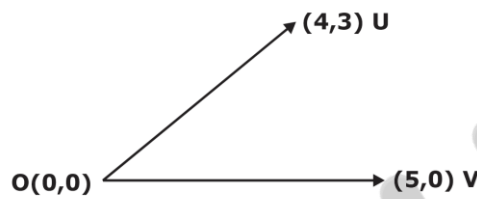
C. $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$

D. $\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{bmatrix}$

[MCQ - 1 Mark]

Ans. B

Sol.



∴

$$AU = V$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$$4a + 3b = 5 \quad \dots(1)$$

$$4c + 3d = 0 \quad \dots(2)$$

Checking with options

Option B satisfies equation (1) and (2)

25. One million random numbers are generated from a statistically stationary process with a Gaussian distribution with mean zero and standard deviation σ_0 .

The σ_0 is estimated by randomly drawing out 10,000 numbers of samples (x_n). The estimates $\hat{\sigma}_1, \hat{\sigma}_2$ are computed in the following two ways.

$$\hat{\sigma}_1^2 = \frac{1}{10000} \sum_{n=1}^{10000} x_n^2$$

$$\hat{\sigma}_2^2 = \frac{1}{9999} \sum_{n=1}^{10000} x_n^2$$

A. $E(\hat{\sigma}_2^2) = \sigma_0^2$

B. $E(\hat{\sigma}_2) = \sigma_0$

C. $E(\hat{\sigma}_1^2) = \sigma_0^2$

D. $E(\hat{\sigma}_1) = E(\hat{\sigma}_2)$

[MCQ - 1 Mark]

Ans. C

Sol. Given, a gaussian distance with mean = $0 = \mu = E(x_n)$

And standard deviation = $\sigma = \sigma_0$

Now,

$$\sigma_0^2 = \text{Var}(x_n) = E(x_n^2) - [E(x_n)]^2$$

$$\sigma_0^2 = E(x_n^2) - 0$$

$$\sigma_0^2 = E(x_n^2) \quad \dots(1)$$

Given,

$$\hat{\sigma}_0^2 = \frac{1}{10000} \sum_{n=1}^{10000} x_n^2$$

$$E[\hat{\sigma}_1^2] = \frac{1}{10000} \sum_{n=1}^{10000} E(x_n^2) = \frac{1}{10000} \sum_{n=1}^{10000} \sigma_0^2 = \frac{\sigma_0^2}{10000} \sum_{n=1}^{10000} 1$$

$$= \frac{\sigma_0^2}{10000} \times 10000$$

$$E[\hat{\sigma}_1^2] = \sigma_0^2 \quad \dots(2)$$

Also,

$$\hat{\sigma}_2^2 = \frac{1}{9999} \sum_{n=1}^{10000} x_n^2$$

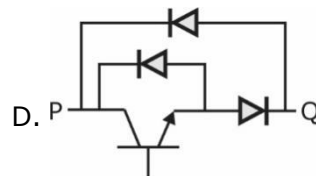
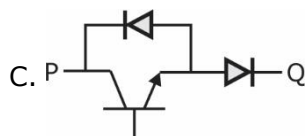
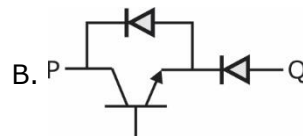
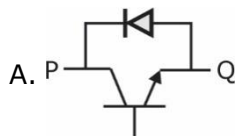
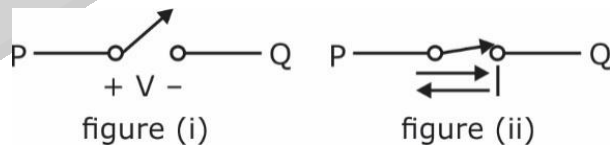
$$E(\hat{\sigma}_2^2) = \frac{1}{9999} \sum_{n=1}^{10000} E(x_n^2)$$

$$E(\hat{\sigma}_2^2) = \frac{1}{9999} \sum_{n=1}^{10000} \sigma_0^2$$

$$= \frac{\sigma_0^2}{9999} \sum_{n=1}^{10000} 1$$

$$E(\hat{\sigma}_2^2) = \sigma_0^2 \left(\frac{10000}{9999} \right) \quad \dots(3)$$

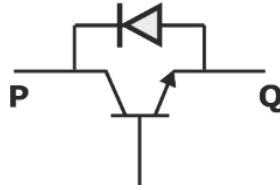
26. A semiconductor switch needs to block voltage V of only one polarity ($V > 0$) during OFF state as shown in figure (i) and carry current in both directions during ON state as shown in figure (ii). Which of the following switch combination(s) will realize the same?



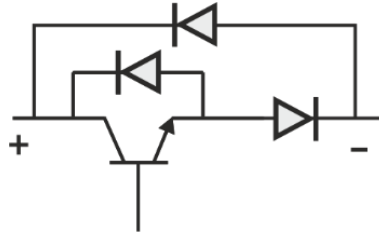
[MSQ - 1 Mark]

Ans. A, D

Sol. Option A D are current in option- A & D current can flow in both the direction.



And Blocking voltage shown in Figure.



- 27.** Which of the following statement(s) is/are true?
- A. If an LTI system is causal, it is stable.
 - B. A discrete time LTI system is causal if and only if its response to a step input $u[n]$ is 0 for $n < 0$.
 - C. If a discrete time LTI system has an impulse response $h[n]$ of finite duration the system is stable.
 - D. If the impulse response $0 < |h[n]| < 1$ for all n , then the LTI system is stable.

[MSQ - 1 Mark]

Ans. B

Sol. (i) Option A is wrong

As causality does not guarantee stability

(ii) Option B is correct,

Because a discrete time LTI System is causal if its impulse response $h[n] = 0$ for $n < 0$

\Rightarrow step-response $h[n] = 0$ for $n < 0$

(iii) Time limited (finite duration) signals may be stable/unstable depending on the amplitude

\therefore option C is wrong.

(iv) $0 < h[n] < 1$

Also does not guarantee stability as if this $h[n]$ is up to $-\infty$ or ∞ may result in an unstable system

option d is wrong.

28. The bus admittance (Y_{bus}) matrix of a 3-bus power system is given below.

$$\begin{matrix}
 & \begin{matrix} 1 & 2 & 3 \end{matrix} \\
 \begin{matrix} 1 \\ 2 \\ 2 \end{matrix} & \begin{bmatrix} -j15 & j10 & j5 \\ j10 & -j13.5 & j4 \\ j5 & j4 & -j8 \end{bmatrix}
 \end{matrix}$$

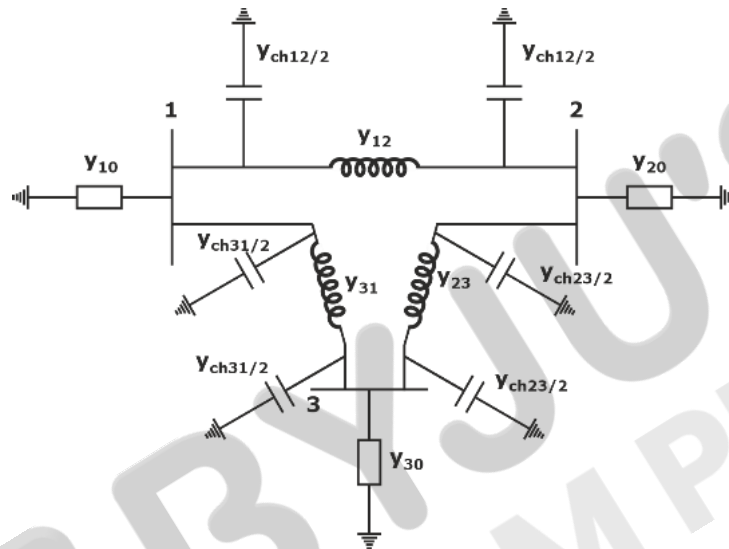
Considering that there is no shunt inductor connected to any of the buses, which of the following can NOT be true?

- A. Line charging capacitor of finite value is present in all three lines.
- B. Line charging capacitor of finite value is present in line 2-3 only.
- C. Line charging capacitor of finite value is present in line 2-3 only and shunt capacitor of finite value is present in bus 1 only.
- D. Line charging capacitor of finite value is present in line 2-3 only and shunt capacitor of finite value is present in bus 3 only.

[MSQ - 1 Marks]

Ans. A, B, C, D

Sol. Let all buses have shunt capacitors and all lines have all charging capacitance.



$$Y_{12} = -y_{12} = j10 \text{ P } y_{12} = -j10$$

$$Y_{31} = -y_{31} = j5 \text{ P } y_{31} = -j5$$

$$Y_{23} = -y_{23} = j4 \text{ P } y_{23} = -j4$$

$$Y_{11} = y_{10} + \frac{Y_{ch12}}{2} + \frac{Y_{ch31}}{2} + (-j10) + (-j5) = -j15$$

$$y_{10} + \frac{Y_{ch12}}{2} + \frac{Y_{ch31}}{2} = 0 \tag{1}$$

To satisfy equation (1), shunt element y_{10} should be inductor which is not possible.

Hence, $\frac{Y_{ch12}}{2}$ and $\frac{Y_{ch31}}{2}$ should be zero.

$$Y_{22} = y_{20} + y_{12} + y_{23} + \frac{Y_{ch12}}{2} + \frac{Y_{ch23}}{2} = -j13.5$$

$$\frac{Y_{ch12}}{2} + \frac{Y_{ch23}}{2} + y_{20} = j0.5 \tag{2}$$

Conclusion:

From equation (1): Bus-1 has no shunt elements and $y_{ch12} = y_{ch31} = 0$

Equation (2): $y_{20} + \frac{Y_{ch23}}{2} = j0.5$ (as $y_{ch12} = 0$)

Possibilities: $y_{20} = 0$ and $y_{ch23} = j1$

$y_{ch23} = 0$ and $y_2 = j0.5$

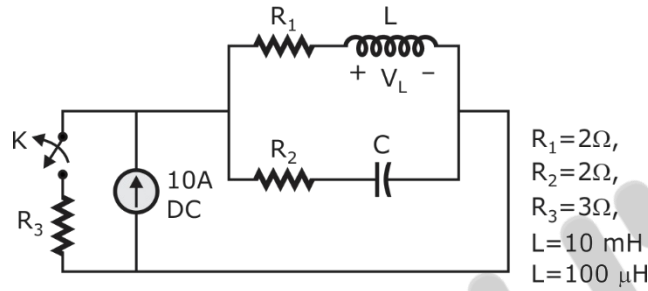
Means at bus-2, line charging and shunt capacitances may or may not present

So, they asked wrong statements, answer will be A, B, C, D

29. The value of parameters of the circuit shown in the figure are

$R_1 = 2\Omega, R_2 = 2\Omega, R_3 = 3\Omega, L = 10 \text{ mH}, C = 100 \mu\text{F}$

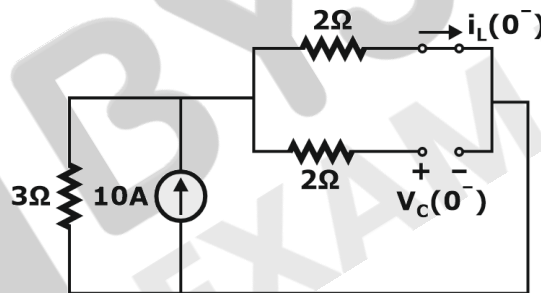
For time $t < 0$, the circuit is at steady state with the switch 'K' in closed condition. If the switch is opened at $t = 0$, the value of the voltage across the inductor (V_L) at $t = 0^+$ in Volts is _____ (Round off to 1 decimal place).



[NAT - 1 Mark]

Ans. 8

Sol. For $t = 0^-$; inductor behaves as short circuit capacitor behaves as open circuit

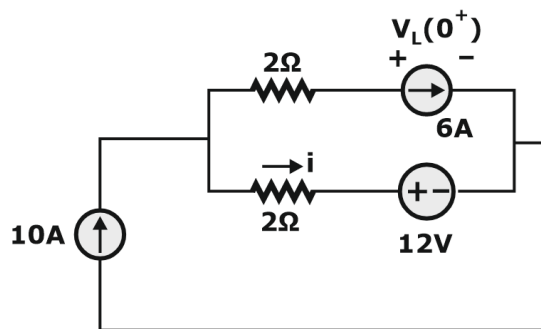


By current division rule,

$$i_L(0^-) = \frac{10 \times 3}{3 + 2} = 6\text{A}$$

$$V_C(0^-) = 2 \times i_L(0^-) = 2 \times 6 = 12\text{V}$$

At $t = 0^+$



$$V_C(0^+) = V_C(0^-) = 12\text{V}$$

$$i_L(0^+) = i_L(0^-) = 6\text{A}$$

$$i = 10 - 6 \text{ (KCL)}$$

$$i = 4A$$

By KVL,

$$2 \times i + 12 - V_L(0^+) - 2 \times 6 = 0$$

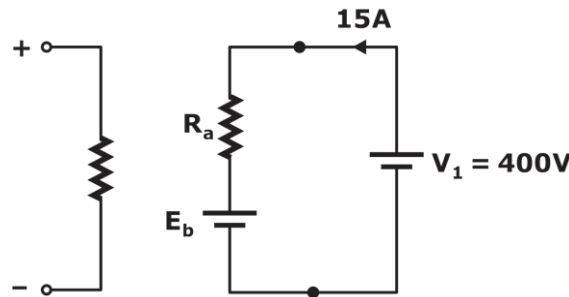
$$V_L(0^+) = 2 \times 4 = 8V.$$

- 30.** A separately excited DC motor rated 400 V, 15 A, 1500 RPM drives a constant torque load at rated speed operating from 400 V DC supply drawing rated current. The armature resistance is 1.2 Ω. If the supply voltage drops by 10% with field current unaltered then the resultant speed of the motor in RPM is _____ (Round off to the nearest integer).

[NAT - 1 Marks]

Ans. 1342.93

Sol.



Given separately excited motor

$$N_1 = 1500 \text{ rpm}$$

$$R_a = 1.2\Omega$$

$$E_{b1} = 400 - 15 \times 1.2 = 382 \text{ Volts}$$

If the voltage drops 10% and, I_f unchanged.

$$V_2 = 400 \times 0.9 = 360 \text{ Volts}$$

$$\therefore E_{b2} = 360 - I_{a2} \times 1.2$$

Since, constant torque load, $T_2 = T_1$

$$I_{a2} = I_{a1} = 15A$$

$$\therefore E_{b2} = 342 \text{ Volts}$$

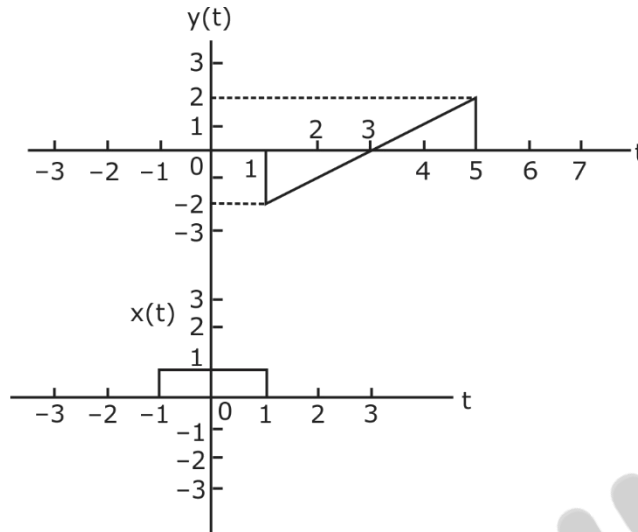
$$N \propto E_b$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

$$\frac{N_2}{1500} = \frac{342}{382}$$

$$N_2 = 1342.93 \text{ rpm}$$

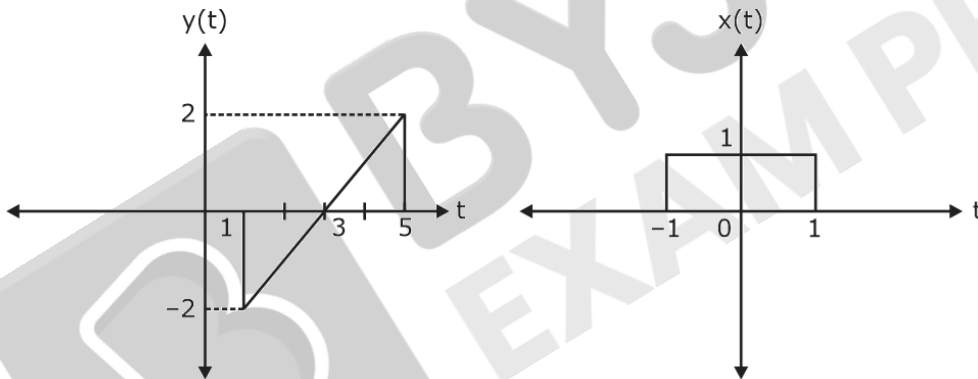
31. For the signals $x(t)$ and $y(t)$ shown in the figure, $z(t) = x(t) * y(t)$ is maximum at $t = T_1$. Then T_1 in seconds is _____ (Round off to the nearest integer).



[NAT - 1 Mark]

Ans. 4

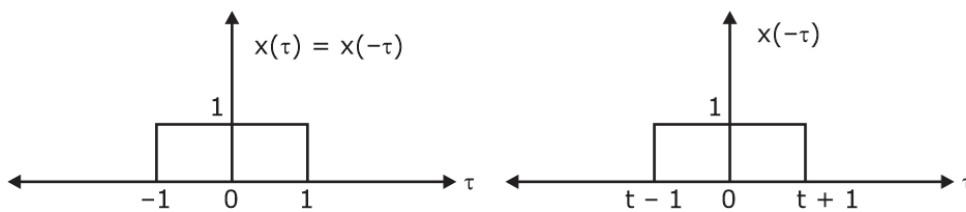
Sol.



∴

$$z(t) = x(t) * y(t) = y(t) * x(t)$$

$$z(t) = \int_{-\infty}^{\infty} y(\tau) \cdot x(t - \tau) \cdot d\tau \quad \dots(i)$$



By equation (i) $z(t)$ will be maximum when,

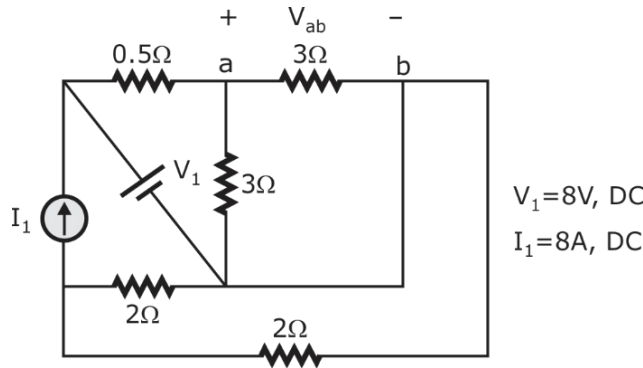
$$t - 1 = 3 \quad \text{or} \quad t + 1 = 5$$

$$t = 4 \quad \quad \quad t = 4$$

Hence

$$T_1 = 4$$

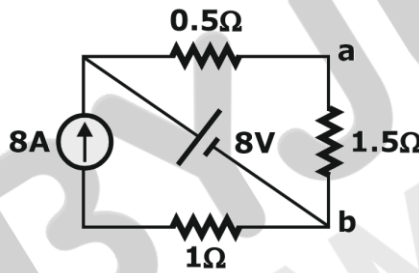
32. For the circuit shown in the figure, $V_1 = 8\text{ V}$, DC and $I_1 = 8\text{ A}$, DC. The voltage V_{ab} in Volts is _____ (Round off to 1 decimal place).



[NAT - 1 Mark]

Ans. 6

Sol. → 2Ω and 2Ω resistor are in parallel
 → 3Ω and 3Ω resistor are in parallel
 After simplifying,



By voltage division rule

$$V_{ab} = \frac{8 \times 1.5}{0.5 + 1.5} = 6\text{V}$$

33. A 50 Hz, 275 kV line of length 400 km has the following parameters:
 Resistance, $R = 0.035\ \Omega/\text{km}$;
 Inductance, $L = 1\ \text{mH}/\text{km}$;
 Capacitance, $C = 0.01\ \mu\text{F}/\text{km}$;
 The line is represented by the nominal- π model. With the magnitudes of the sending end and the receiving end voltages of the line (denoted by V_S and V_R , respectively) maintained at 275 kV, the phase angle difference (θ) between V_S and V_R required for maximum possible active power to be delivered to the receiving end, in degree is _____ (Round off to 2 decimal places).

[NAT - 1 Mark]

Ans. 83.64°

Sol. Given data,

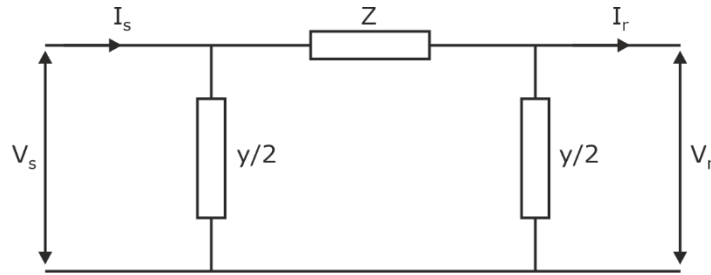
$$R = 0.035\ \Omega/\text{km},\ l = 400\ \text{km}.$$

$$L = 1\ \text{mH}/\text{km}$$

$$C = 0.01\ \mu\text{F}/\text{km}$$

$$|V_s| = 275 \text{ kV}, |V_r| = 275 \text{ kV}$$

$$V_s = 275\angle\theta \text{ and } V_r = 275\angle 0^\circ$$



$$Z = (R + j \omega L) \times 400$$

$$= (0.035 + j 2 \times 3.14 \times 50 \times 1 \times 10^{-3}) \times 400$$

$$= 14 + j125.6W$$

$$y = (j\omega c) \times l = j314 \times 0.01 \times 10^{-6} \times 400 = j0.00129675$$

The real power received at the receiving end.

$$P_r = \frac{|V_s||V_r|}{|B|} \cos(\beta - \delta) - \frac{|A|}{|B|} |V_r|^2 \cos(\beta - \alpha)$$

Nominal-p model

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{YZ}{2}\right) & z \\ y\left(1 + \frac{YZ}{4}\right) & \left(1 + \frac{YZ}{2}\right) \end{bmatrix}$$

For

$$P_{rmax}, \delta = \beta$$

$$\therefore \delta = \theta$$

Hence,

$$\theta = \beta = \tan^{-1} \frac{125.6}{14}$$

$$\theta = \tan^{-1} 8.97$$

$$\theta = 83.64^\circ$$

34. In the following differential equation, the numerically obtained value of $y(t)$, at $t = 1$, is _____ (Round off to 2 decimal places).

$$\frac{dy}{dt} = \frac{e^{-at}}{2 + at}, \quad a = 0.01 \text{ and } y(0) = 0$$

[NAT - 1 Mark]

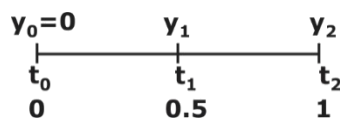
Ans. 0.7462

Sol.

$$\frac{dy}{dt} = \frac{e^{-at}}{2 + at}$$

$$a = 0.01$$

$$y_0 = 0$$



$$y_1 = y_0 + hf(t_0, y_0)$$

$$y_1 = 0 + 0.5 \left(\frac{e^0}{2 + 0} \right) = \frac{1}{4} = 0.25$$

$$y_1 = 0.25$$

and

$$y_2 = y_1 + h f(t_1, y_1)$$

$$y_2 = 0.25 + \frac{e^{-0.01 \times 0.5}}{2 + 0.01 \times 0.5}$$

$$y_2 = 0.7462 = y(1)$$

35. Three points in the x-y plane are (-1, 0.8), (0, 2.2) and (1, 2.8). The value of the slope of the best fit straight line in the least square sense is _____ (Round off to 2 decimal places).

[NAT - 1 Mark]

Ans. 1

Sol.

$$x : -1 \quad 0 \quad 1$$

$$y : 0.8 \quad 2.2 \quad 2.8$$

$$y = a + bx$$

$$N.E.1 \Rightarrow \Sigma y = na + b\Sigma x$$

$$N.E.2 \Rightarrow \Sigma xy = a\Sigma x + b\Sigma x^2$$

x	Y	xy	x ²
-1	0.8	-0.8	1
0	2.2	0	0
1	2.8	2.8	1
$\Sigma x = 0$	$\Sigma y = 5.8$	$\Sigma xy = 2$	$\Sigma x^2 = 2$

$$3a + 0 = 5.8 \Rightarrow a = 1.933$$

and

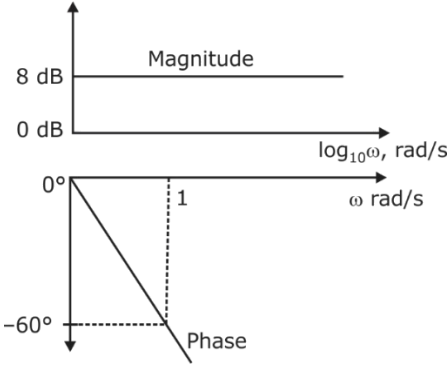
$$2 = 1.933 \times 0 + b \times 2$$

$$b = 1$$

$$y = 1.933 + x$$

$$\text{Slope} = 1$$

36. The magnitude and phase plots of an LTI system are shown in the figure. The transfer function of the system is



A. $2.51 e^{-0.032s}$

B. $\frac{e^{-2.514s}}{s + 1}$

C. $10.4 e^{-2.514s}$

D. $2.51 e^{-1.047s}$

[MCQ - 2 Marks]

Ans. D

Sol. Magnitude = constant

Phase = linear function

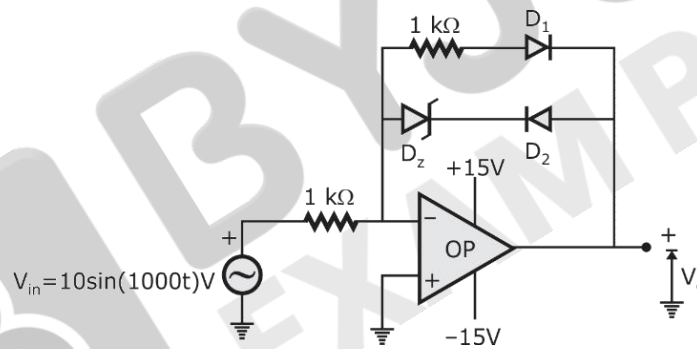
So, $G(j\omega) = ke^{-j\omega T}$
 $20 \log k = 8$
 $k = 2.51$

Phase = $-\omega T$

$-\frac{\pi}{3} = -(1)T \Rightarrow T = \frac{\pi}{3} = 1.05$

So, $G(j\omega) = 2.51e^{-j1.05\omega}$

37. Consider the OP AMP based circuit shown in the figure. Ignore the conduction drops of diodes D_1 and D_2 . All the components are ideal and the breakdown voltage of the Zener is 5 V. Which of the following statements is true?



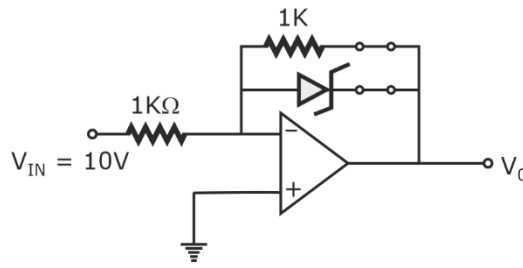
- A. The maximum and minimum values of the output voltage V_o are +15 V and -10 V, respectively.
- B. The maximum and minimum values of the output voltage V_o are +5 V and -15 V, respectively.
- C. The maximum and minimum values of the output voltage V_o are +10 V and -5 V, respectively.
- D. The maximum and minimum values of the output voltage V_o are +5 V and -10 V, respectively.

[MCQ - 2 Marks]

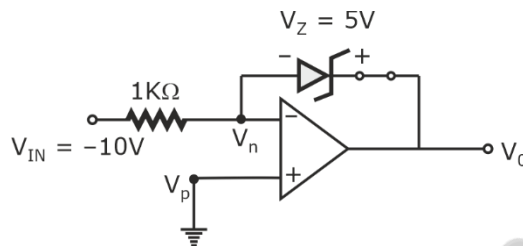
Ans. D

Sol.

V_{IN}	D_1	D_2	D_z	D_o
$V_{IN} = 10V$	On	Off	Not conduct	$V_{0min} = -10V$
$V_{IN} = -10$	Off	On	Break down	$V_{0max} = 5V$



$$V_0 = \frac{-R_f}{R_1} V_{IN} = -\frac{1}{1} \times 10 = -10V$$



$$V_p = V_n = 0$$

$$\therefore V_0 = V_z + V_n = 5 + 0 = 5V$$

38. Consider a lead compensator of the form

$$K(s) = \frac{1 + \frac{s}{\alpha}}{1 + \frac{s}{\beta\alpha}}, \beta > 1, \alpha > 0$$

The frequency at which this compensator produces maximum phase lead is 4 rad/s. At this frequency, the gain amplification provided by the controller, assuming asymptotic Bode-magnitude plot of $K(s)$, is 6 dB. The values of α, β , respectively, are

- A. 1, 16
- B. 2, 4
- C. 3, 5
- D. 2.66, 2.25

[MCQ - 2 Marks]

Ans. B

Sol.

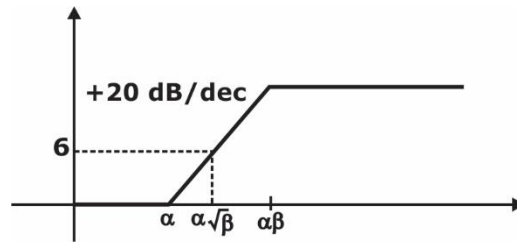
$$K(s) = \frac{1 + \frac{s}{\alpha}}{1 + \frac{s}{\alpha\beta}}$$

Comer frequency

$$s = \underset{\text{Zero}}{\alpha}, \quad \underset{\text{Pole}}{\alpha\beta}$$

$$\omega_n = 4 = \text{GM of } (\alpha, \alpha\beta) = \sqrt{\alpha(\alpha\beta)} = \alpha\sqrt{\beta}$$

So, $\alpha\sqrt{\beta} = 4$



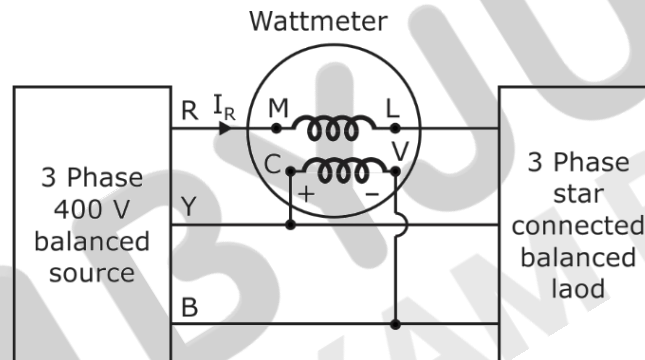
$$\text{Slope} = 20 = \frac{6 - 0}{\log \alpha\sqrt{\beta} - \log \alpha} = \frac{6}{\log \sqrt{\beta}}$$

$$\log \sqrt{\beta} = \frac{6}{20} = 0.3$$

$$\beta = 4$$

$$\alpha\sqrt{\beta} = 4 \quad \Rightarrow \alpha = 2$$

39. A 3-phase, star-connected, balanced load is supplied from a 3-phase, 400 V (rms), balanced voltage source with phase sequence R-Y-B, as shown in the figure. If the wattmeter reading is -400 W and the line current is $I_R = 2$ A (rms), then the power factor of the load per phase is

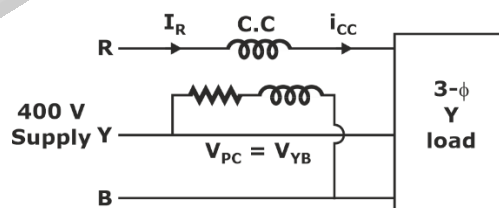


- A. Unity
- B. 0.5 leading
- C. 0.866 leading
- D. 0.707 lagging

[MCQ - 2 Marks]

Ans. C

Sol. Wattmeter reading, $W = V_{PC} \cdot I_{CC} \cdot \cos \angle V_{PC} \text{ \& } I_{CC}$.

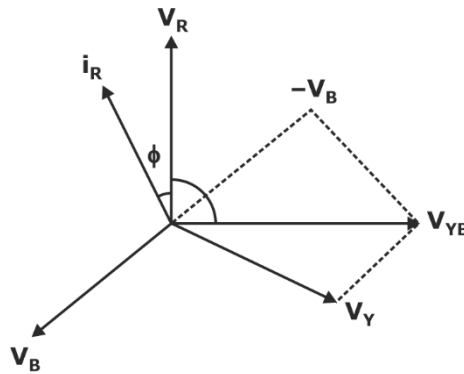


Given,

Wattmeter reading

$$W = 400 \text{ Watts.}$$

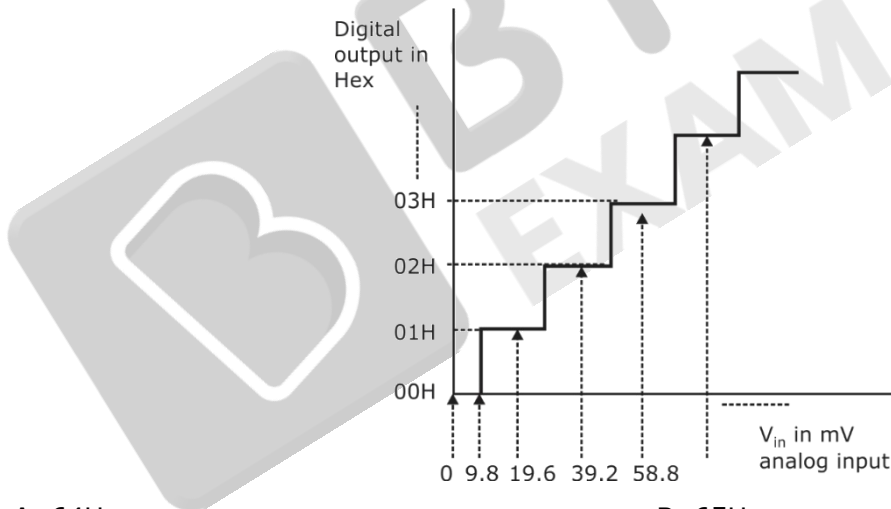
$$I_R = i_{cc} = 2A$$



For leading load

$$\begin{aligned}
 W &= V_{PC} \cdot i_{CC} \cos \angle V_{PC} \text{ \& \ } i_{CC} \\
 &= V_{YB} \cdot I_R \cdot \cos \angle V_{YB} \text{ \& \ } I_R \\
 -400 &= 400 \times 2 \times \cos(90 + \phi) \\
 -\frac{1}{2} &= -\sin \phi \\
 \phi &= 30^\circ \\
 \cos 30^\circ &= 0.866 \text{ leading}
 \end{aligned}$$

40. An 8-bit ADC converts analog voltage in the range of 0 to +5V to the corresponding digital code as per the conversion characteristics shown in figure. For $V_{in} = 1.9922 \text{ V}$, which of the following digital output, given in hex, is true?



- A. 64H
- B. 65H
- C. 66H
- D. 67H

[MCQ - 2 Marks]

Ans. C

Sol. Resolution,

$$\begin{aligned}
 V_{LSB} &= \frac{V_{FL}}{2^n - 1} = \frac{5}{2^8 - 1} \\
 &= 0.0196 \text{ V} \\
 &= 19.6 \text{ mV}
 \end{aligned}$$

As per graph

$$0 < V_{in} < 9.8, \text{ output} = 00 \text{ H}$$

$$9.8 < V_{in} < 19.6, \text{ output} = 01 \text{ H}$$

$$19.6 < V_{in} < 29.4, \text{ output} = 01 \text{ H}$$

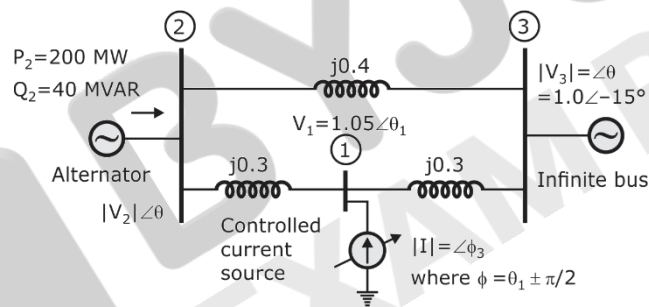
$$29.4 < V_{in} < 39.8, \text{ output} = 02 \text{ H}$$

$$V_{in} = 1.9922 \text{ V}$$

$$\frac{1.9922}{0.0196} \text{ V} = 101.63 > 101.5$$

So, take (102)
 If it is less than 101.5
 Take 101
 So, $(102)_{10} = (66)_H$

41. The three-bus power system shown in the figure has one alternator connected to bus 2 which supplies 200 MW and 40 MVAR power. Bus 3 is infinite bus having a voltage of magnitude $|V_3| = 1.0$ p.u. and angle of -15° . A variable current source, $|I| \angle \phi$ is connected at bus 1 and controlled such that the magnitude of the bus 1 voltage is maintained at 1.05 p.u. and the phase angle of the source current, $\phi = \theta_1 \pm \frac{\pi}{2}$, where θ_1 is the phase angle of the bus 1 voltage. The three buses can be categorized for load flow analysis as



- | | | | |
|----------|-----------|----------|-----------|
| Bus 1 | Slack bus | Bus 1 | P- V bus |
| A. Bus 2 | P- V bus | B. Bus 2 | P- V bus |
| Bus 3 | P-Q bus | Bus 3 | Slack bus |
| Bus 1 | P-Q bus | Bus 1 | P- V bus |
| C. Bus 2 | P-Q bus | D. Bus 2 | P-Q bus |
| Bus 3 | Slack bus | Bus 3 | Slack bus |

[MCQ - 2 Marks]

Ans. D

Sol. BUS - 1 = PV bus

BUS - 2 = PQ bus

BUS - 3 = slack bus

PV Bus is a bus which has at least one generator connected and if there is additional voltage control capability its called a special PV Bus. Bus 1 is voltage controlled bus.

Bus- 2 has real and reactive power specified, hence, it is a PQ bus.

Bus 3 is infinite bus. Where voltage magnitude and phase angle is specified hence it is a slack bus.

42. Consider the following equation in a 2-D real-space.

$$|x_1|^p + |x_2|^p = 1 \text{ for } p > 0$$

Which of the following statement(s) is/are true.

- A. When $p = 2$, the area enclosed by the curve is π .
- B. When p tends to ∞ , the area enclosed by the curve tends to 4.
- C. When p tends to 0, the area enclosed by the curve is 1.
- D. When $p = 1$, the area enclosed by the curve is 2.

[MSQ - 2 Marks]

Ans. A, B & D

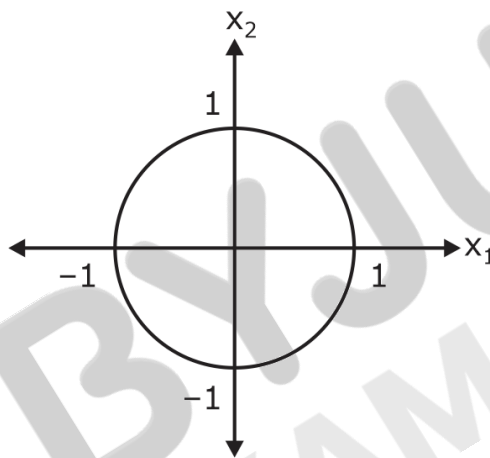
Sol. Given,

$$|x_1|^p + |x_2|^p = 1 ; P > 0$$

(i) when

$$P = 2$$

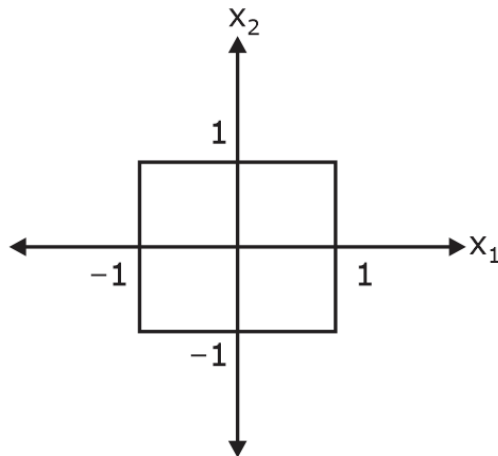
$$|x_1|^2 + |x_2|^2 = 1$$



is a circle with centre (0, 0) & radius = 1
Bounded Area = $\pi (1)^2 = \pi$
Option A is correct

(ii) when $P \rightarrow \infty$

$$|x_1|^p + |x_2|^p = 1$$



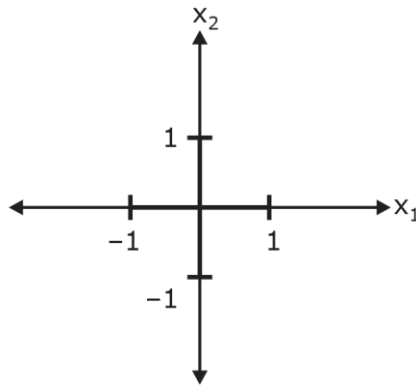
represents a square

Bounded Area = [side]² = 2² = 4

Option B is correct

(iii) when $P \rightarrow 0$

$|x_1|^P + |x_2|^P = 1$



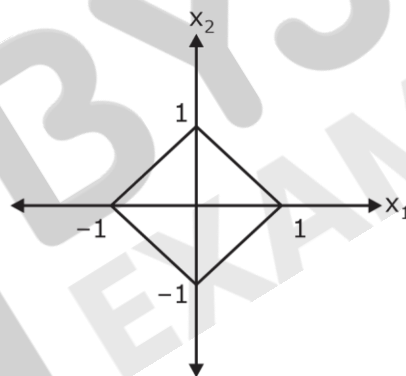
does not bound any area = 0

Bound area = 0

(iv) when $P = 1$,

$|x_1| + |x_2| = 1$

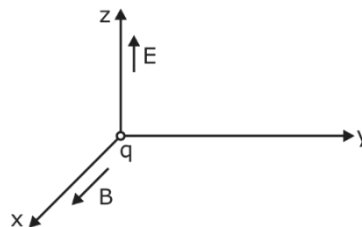
represents a square as shown in figure



Bound area = $\sqrt{2}^2 = 2$

Option D is correct

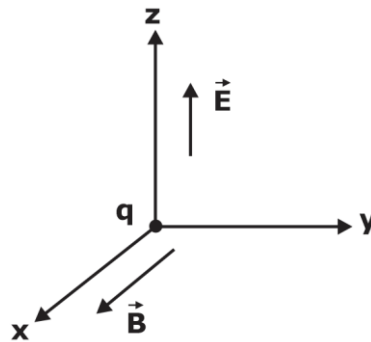
- 43.** In the figure, the electric field **E** and the magnetic field **B** point to x and z directions, respectively, and have constant magnitudes. A positive charge 'q' is released from rest at the origin. Which of the following statement(s) is/are true.



- A. The charge will move in the direction of z with constant velocity.
- B. The charge will always move on the y-z plane only.
- C. The trajectory of the charge will be a circle.
- D. The charge will progress in the direction of y.

Ans. B, D

Sol.



Charge is at rest $\vec{v} = 0$

Force on charge due to electric field,

$$\vec{F}_E = q\vec{E}$$

Force on charge due to magnetic field

$$\vec{F}_M = q(\vec{v} \times \vec{B}) = 0$$

So, the charge will move the direction of electric field

$$\vec{F}_E = ma \neq 0$$

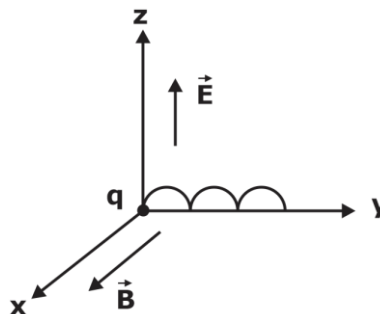
$$a \neq 0$$

$$\frac{dv}{dt} \neq 0$$

The charge will move in direction of z. since now the charge is moving, magnetic field will also exist

$$\begin{aligned} \text{i.e. } \vec{F}_M &= q(\vec{v} \times \vec{B}) \\ &= q(v\hat{z} \times B_0\hat{x}) \\ &= qvB_0\hat{y} \end{aligned}$$

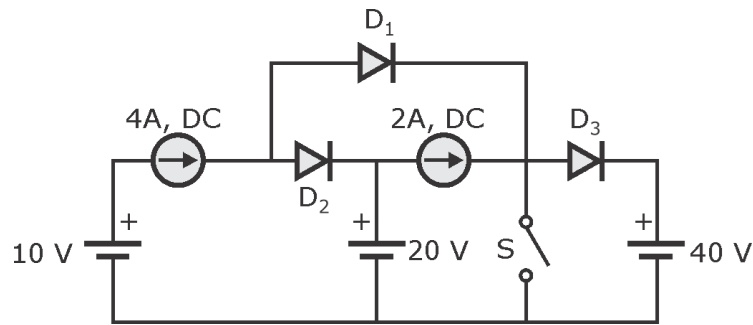
Because of magnetic field it will follow cycloid path in the direction of y.



The charge will move in the x-y plane.

B, D are correct.

44. All the elements in the circuit shown in the following figure are ideal. Which of the following statements is/are true?



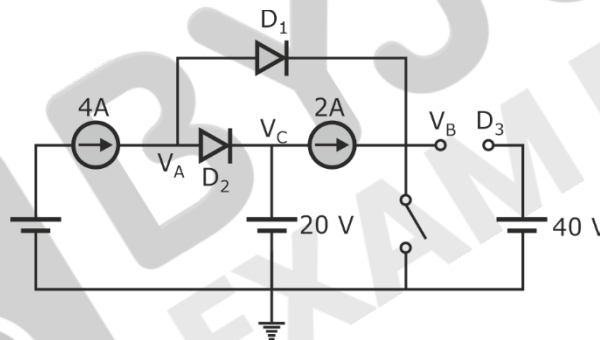
- A. When switch S is ON, both D₁ and D₂ conduct and D₃ is reverse biased
- B. When switch S is ON, D₁ conducts and both D₂ and D₃ are reverse biased
- C. When switch S is OFF, D₁ is reverse biased and both D₂ and D₃ conduct
- D. When switch S is OFF, D₁ conducts, D₂ is reverse biased and D₃ conducts

[MSQ - 2 Marks]

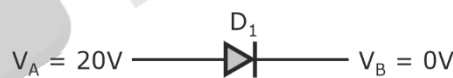
Ans. B, C

Sol. Switch → On

D₃ reverse biased by 40V battery ∴ V_B = 0V



Now if D₂ is on then V_A = V_C = 20



∴ D₁ is also on & V_A = V_B = 0V

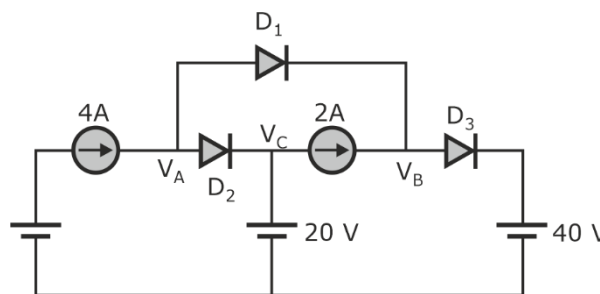
Two values of V_A i.e. 20 V & 0V are not possible.

∴ D₁ is on D₂ is off.

$$V_A = V_B = 0V \quad V_C = 20V$$

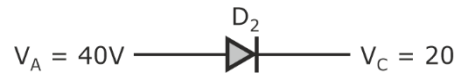
Case II

When switch is off



Now D_3 is always on $\therefore 4A$ & $2A$ current will pass through D_3 , $\therefore V_B = 40V$

Let D_1 is on then $V_A = V_B = 40V$



Now D_2 is forward biased by $20V$.

So D_2 is on

$\therefore V_A = 20V$

Two values of V_A i.e. $20V$ & $40V$ are not possible.

So $D_1 \rightarrow$ off D_2 on

$$V_A = V_C = 20V$$

- 45.** The expected number of trials for first occurrence of a "head" in a biased coin is known to be 4. The probability of first occurrence of a "head" in the second trial is _____ (Round off to 3 decimal places).

[NAT - 2 Marks]

Ans. 0.1875

Sol. We are looking for the 1st success.

\therefore given problem is related with geometric distribution.

$$\text{given, } E(x) = 4 = \frac{1}{p}$$

$$\Rightarrow p = \frac{1}{4} = \text{prob. of success}$$

$$\Rightarrow q = 1 - p = \frac{3}{4} = \text{probability of failure}$$

$$\text{Now, the probability of 1st occurrence of a "head" in 2nd trail} = q \cdot p = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16} = 0.1875$$

- 46.** Consider the state-space description of an LTI system with matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [3 \quad -2], D = 1$$

For the input, $\sin(\omega t)$, $\omega > 0$, the value of ω for which the steady-state output of the system will be zero, is _____ (Round off to the nearest integer).

[NAT - 2 Marks]

Ans. 2

Sol. Given form is CCF model.

$$A = \begin{bmatrix} 0 & 1 \\ -a_v & -a_1 \end{bmatrix} B = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$C = [C_0 \quad C_1] \quad D = [d]$$

So,

$$T.F = \frac{b(c_1s + c_0)}{s^2 + a_1s + a_0} + d$$

$$= \frac{1(-2s + 3)}{s^2 + 2s + 1} + 1 = \frac{s^2 + 4}{(s^2 + 2s + 1)}$$

$$T(s) = \frac{s^2 + 4}{(s + 1)^2}$$

$$T(j\omega) = \frac{4 - \omega^2}{(1 + j\omega)^2}$$

At $\omega = 2, |T(j\omega)| = 0$
 So, output = 0

47. A three-phase synchronous motor with synchronous impedance of $0.1 + j0.3$ per unit per phase has a static stability limit of 2.5 per unit. The corresponding excitation voltage in per unit is _____ (Round off to 2 decimal places).

[NAT - 2 Marks]

Ans. 1.107

Sol. For synchronous motor

$$(P_{in})_{max} = \frac{E_f V_t}{|Z|} - \frac{V_t R}{|Z|^2} = 2.5$$

$$(P_{in})_{max} = 2.5$$

$$Z_s = 0.1 + j(0.3)$$

$$|Z_s| = \sqrt{0.1^2 + 0.3^2} = 0.32$$

$$V_t = 1 \text{ pu}$$

$$\frac{(E_f)(1)}{0.32} - \frac{(1)(0.1)}{(0.32)^2} = 2.5$$

$$E_f = 1.107$$

48. A three phase 415 V, 50 Hz, 6-pole, 960 RPM, 4 HP squirrel cage induction motor drives a constant torque load at rated speed operating from rated supply and delivering rated output. If the supply voltage and frequency are reduced by 20%, the resultant speed of the motor in RPM (neglecting the stator leakage impedance and rotational losses) is _____ (Round off to the nearest integer).

[NAT - 2 Marks]

Ans. 760 rpm

Sol. As voltage and frequency are reduced by 20%. The ratio of voltage to frequency is constant.

$$\text{We know, } T_{em} = \frac{180}{2\pi N_s} \cdot \frac{SE_2^2}{R_2}$$

$$= \frac{180}{2\pi N_s} \cdot \frac{N_s - N_r}{N_s} \cdot \frac{E_2^2}{R_2}$$

$$T_{em} \propto \frac{E_2^2}{N_s^2} (N_s - N_r)$$

$$T_{em} \propto \underbrace{\left(\frac{V}{f}\right)^2}_{\text{Constant}} (N_s - N_r)$$

$\therefore T_{em} \propto (N_s - N_r)$

Given constant torque load $T_2 = T_1$

$$(N_{s_2} - N_{r_2}) = (N_{s_1} - N_{r_1})$$

$$= \left(\frac{120 \times 50}{6} - 960\right)$$

$$= \left[\frac{120 \times 50 \times 0.8}{6} - N_{r_2}\right] = 40$$

$$N_{r_2} = 760 \text{ rpm}$$

49. The period of the discrete-time signal $x[n]$ described by the equation below is $N =$ _____ (Round off to the nearest integer)

$$x[n] = 1 + 3 \sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right) - 5 \sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)$$

[NAT - 2 Marks]

Ans. 48

Sol. $x[n] = 1 + 3 \sin\left(\frac{15\pi}{8}n + \frac{3\pi}{4}\right) - 5 \sin\left(\frac{\pi}{3}n - \frac{\pi}{4}\right)$

(1) $\Omega_1 = \frac{15\pi}{8}$

$$N_1 = \left(\frac{2\pi}{\Omega_1}\right) \cdot k$$

$$N_1 = \left(\frac{2\pi}{\left(\frac{15\pi}{8}\right)}\right) \cdot k$$

for $k = 15$

$$N_1 = 16$$

(2) $\Omega_2 = \frac{\pi}{3}$

$$N_2 = \left(\frac{2\pi}{\Omega_2}\right) \cdot k$$

$$N_2 = \left(\frac{2\pi}{\left(\frac{\pi}{3}\right)}\right) \cdot k$$

$$N_2 = 6.k$$

$$\text{For } k = 1$$

$$N_2 = 6$$

$$N = \text{LCM}(16, 6) = 48$$

50. The discrete-time Fourier transform of a signal $x[n]$ is $X(\Omega) = (1 + \cos \Omega)e^{-j\Omega}$. Consider that $x_p[n]$ is a periodic signal of period $N = 5$ such that

$$x_p[n] = x[n], \text{ for } n = 0, 1, 2$$

$$= 0, \text{ for } n = 3, 4$$

Note that $x_p[n] = \sum_{k=1}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$. The magnitude of the Fourier series coefficient a_3 is _____ (Round off to 3 decimal places)

[NAT - 2 Marks]

Ans. 0.038

Sol. Given,

$$x[n] \xrightarrow{\text{DTFT}} x(\Omega) = (1 + \cos \Omega)e^{-j\Omega}$$

∴

$$X(\Omega) = (1 + \cos \Omega)e^{-j\Omega}$$

$$x(\Omega) = \left(1 + \frac{e^{j\Omega} + e^{-j\Omega}}{2}\right) e^{-j\Omega}$$

$$x(\Omega) = e^{-j\Omega} + \frac{1}{2} + \frac{1}{2} e^{-j2\Omega}$$

$$x(\Omega) = \frac{1}{2} + e^{-j\Omega} + \frac{1}{2} e^{-j2\Omega}$$

taking I.D.T.F.T,

$$x[n] = \left\{ \frac{1}{2}, 1, \frac{1}{2} \right\}$$

Given,

$$x_p[n] = \begin{cases} x[n]; & n = 0, 1, 2 \\ 0; & n = 3, 4 \end{cases}$$

$$x_p[n] = \left\{ \frac{1}{2}, 1, \frac{1}{2}, 0, 0, \frac{1}{2}, 1, \frac{1}{2}, 0, 0, \dots \right\}$$

One period of $x_p[n]$ has 5 samples

As $\left\{ \frac{1}{2}, 1, \frac{1}{2}, 0, 0 \right\}$

∴

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p[n] \cdot e^{-j\frac{2\pi}{N}nk}$$

$$N = 5$$

$$\Rightarrow a_k = \frac{1}{5} \sum_{n=0}^4 x_p[n] \cdot e^{-j\frac{2\pi}{5}nk}$$

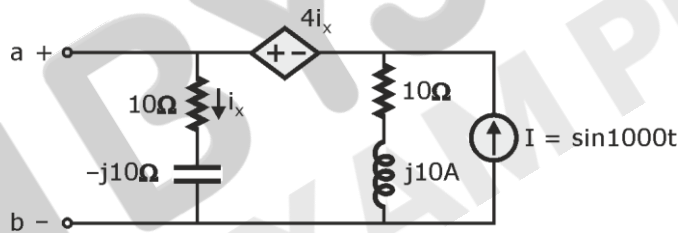
Put $k = 3$.

$$\begin{aligned} \Rightarrow a_3 &= \frac{1}{5} \sum_{n=0}^4 x_p[n] \cdot e^{-j\frac{6\pi n}{5}} \\ \Rightarrow a_3 &= \frac{1}{5} \left[\frac{1}{2} + 1 \cdot e^{-j\frac{6\pi}{5}} + \frac{1}{2} e^{-j\frac{12\pi}{5}} \right] \\ \Rightarrow a_3 &= \frac{1}{5} \left[\frac{1}{2} e^{-j\frac{6\pi}{5}} \left\{ e^{j\frac{6\pi}{5}} + e^{-j\frac{6\pi}{5}} \right\} + e^{-j\frac{6\pi}{5}} \right] \\ \Rightarrow a_3 &= \frac{1}{5} \left[\frac{1}{2} e^{-j\frac{6\pi}{5}} 2 \cos \frac{6\pi}{5} + e^{-j\frac{6\pi}{5}} \right] \\ \Rightarrow a_3 &= \frac{1}{5} \left[1 + \cos \left(\frac{6\pi}{5} \right) \right] e^{-j\frac{6\pi}{5}} \end{aligned}$$

Now magnitude of a_3

$$|a_3| = \frac{1 + \cos \left(\frac{6\pi}{5} \right)}{5} = 0.0382$$

- 51.** For the circuit shown, if $i = \sin 1000t$, the instantaneous value of the Thevenin's equivalent voltage (in Volts) across the terminals a-b at time $t = 5$ ms is _____ (Round off to 2 decimal places).



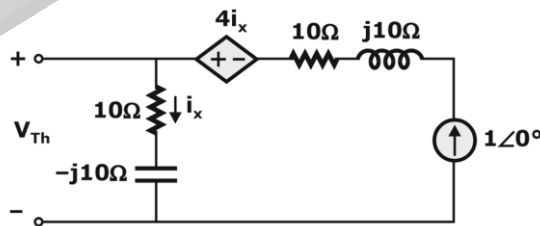
[NAT - 2 Marks]

Ans. -11.98

Sol.

$$I = \sin (1000t) = 1 \angle 0^\circ \text{ A}$$

By source transformation,



By KVL,

$$\begin{aligned} -(10 + j10) + (10 + j10 + 10 - j10) I_x - 4I_x &= 0 \\ 16I_x &= 10 + j10 \\ I_x &= \frac{10 + j10}{16} \\ V_{Th} &= I_x(10 - j10) \end{aligned}$$

$$V_{Th} = \frac{10 + j10}{16} \times (10 - j10)$$

$$V_{Th} = \frac{200}{16} V$$

$$V_{Th} = \frac{200}{16} \sin(1000t)$$

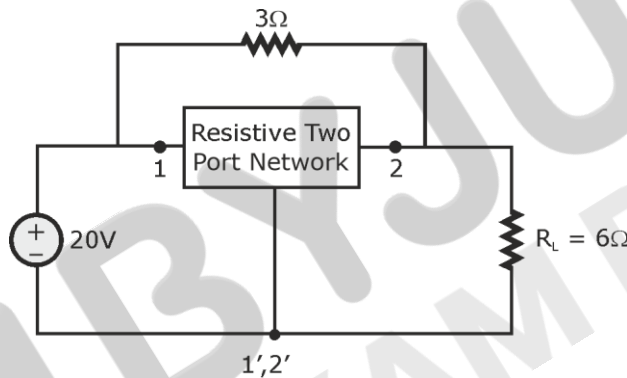
By

$$t = 5 \text{ ms,}$$

$$V_{Th} = \frac{200}{16} \sin(5)$$

$$V_{Th} = -11.98 \text{ V}$$

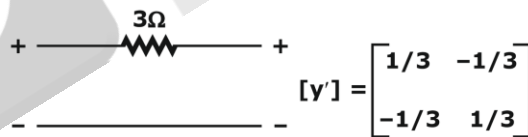
- 52.** The admittance parameters of the passive resistive two-port network shown in the figure are $y_{11} = 5\text{s}$, $y_{22} = 1\text{s}$, $y_{12} = y_{21} = -2.5\text{s}$
The power delivered to the load resistor R_L in Watt is _____ (Round off to 2 decimal places)



[NAT - 2 Marks]

Ans. 242.69

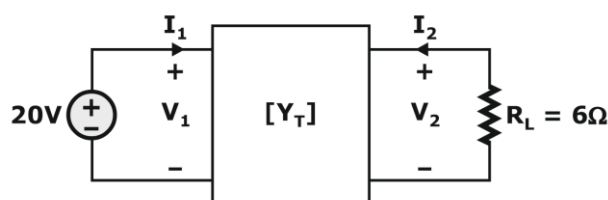
Sol. For network



Total Y-parameter;

$$[Y_T] = [Y'] + [Y] = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} + \begin{bmatrix} 5 & -2.5 \\ -2.5 & 1 \end{bmatrix}$$

$$[Y_T] = \begin{bmatrix} 5.3 & -2.8 \\ -2.8 & 1.3 \end{bmatrix}$$



$$V_1 = 20 \quad \dots (1)$$

$$V_2 = -6I_2 \quad \dots (2)$$

Also,

$$I_1 = 5.3 V_1 - 2.8V_2 \quad \dots (3)$$

$$I_2 = -2.8V_1 + 1.3V_2 \quad \dots (4)$$

Put equation 2 in 4

$$I_2 = -2.8V_1 + 1.3 \times (-6I_2)$$

$$8.8 I_2 = -2.8 V_1$$

$$I_2 = \frac{-2.8}{8.8} \times 20$$

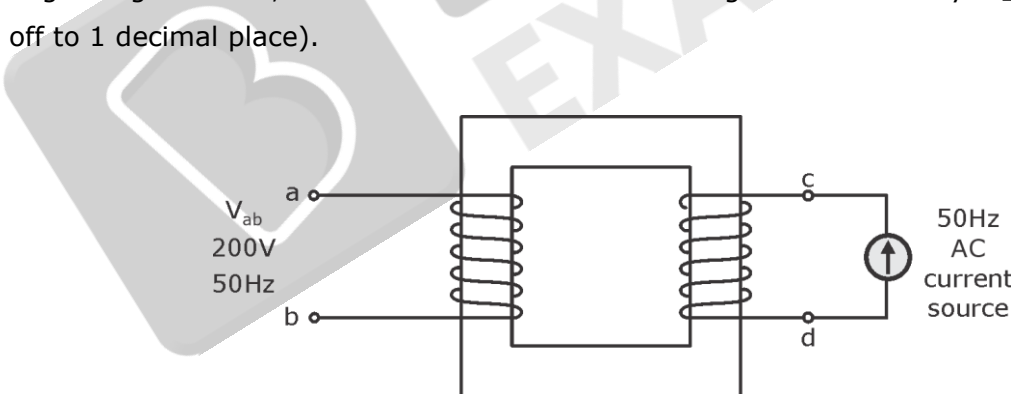
$$I_2 = -6.36 \text{ A}$$

$$P = I_2^2 R_L$$

$$P = (6.36)^2 \times 6$$

$$P = 242.69 \text{ W}$$

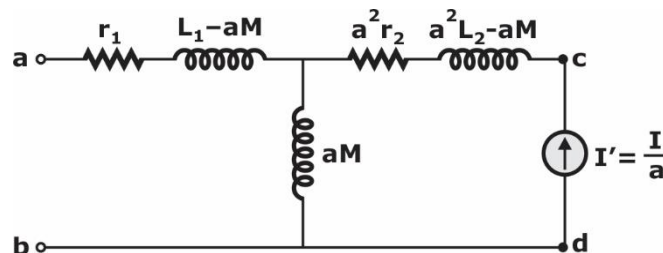
- 53.** When the winding c-d of the single-phase, 50 Hz, two winding transformer is supplied from an AC current source of frequency 50 Hz, the rated voltage of 200 V (rms), 50 Hz is obtained at the open-circuited terminals a-b. The cross sectional area of the core is 5000 mm² and the average core length traversed by the mutual flux is 500 mm. The maximum allowable flux density in the core is $B_{max} = 1 \text{ Wb/m}^2$ and the relative permeability of the core material is 5000. The leakage impedance of the winding a-b and winding c-d at 50 Hz are $(5 + j100p \times 0.16) \text{ W}$ and $(11.25 + j100 p \times 0.36) \text{ W}$, respectively. Considering the magnetizing characteristics to be linear and neglecting core loss, the self-inductance of the winding a-b in millihenry is _____ (Round off to 1 decimal place).



[NAT - 2 Marks]

Ans. 2197.18

Sol. Equivalent circuit



$$a = \frac{N_1}{N_2}$$

Self-inductance of coil a - b = L_1

We know, leakage inductance = $L_1 - aM$

Given leakage reactance = 0.16Ω

$$L_1 - aM = 160 \text{ mH}$$

And mutual inductance $M = \frac{N_1 \cdot N_2}{R}$

$$aM = \frac{N_1}{N_2} \left[\frac{N_1 N_2}{R} \right]$$

$$aM = \frac{N_1^2}{R}$$

$$200 = E(\text{rms}) = \sqrt{2}\pi f N_1 \phi_m$$

$$200 = \sqrt{2}\pi [50][N_1][BA_c]$$

$$200 = \sqrt{2}\pi [50][N_1][5000 \times 10^{-6} \times 1]$$

$$N_1 = 180.063$$

Reluctance

$$R = \frac{\ell}{\mu_0 \mu_r A_c} = \frac{500 \times 10^{-3}}{4\pi \times 10^{-7} (5000)(5000 \times 10^{-6})}$$

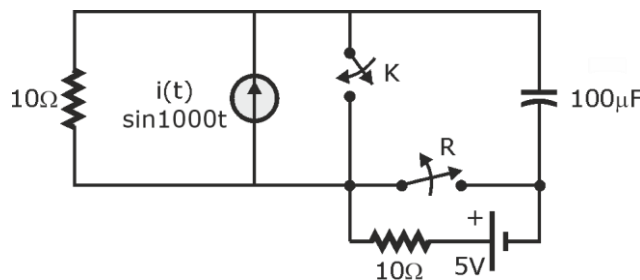
$$R = 15915.47$$

$$aM = \frac{N_1^2}{R} = \frac{(180.063)^2}{15915.47} = 2.03718 \text{ H}$$

$$aM = 2037.18 \text{ mH}$$

\therefore self-inductance $L_1 = (160 + 2037.18) \text{ mH}$
 $= 2197.18 \text{ mH}$

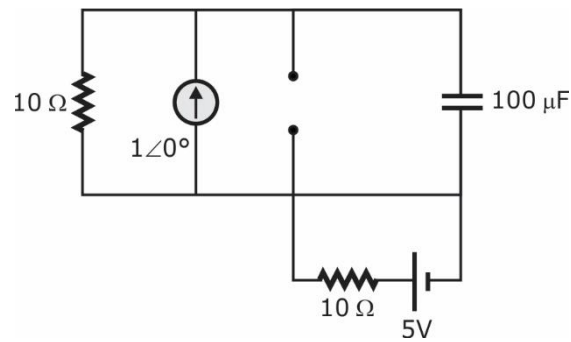
- 54.** The circuit shown in the figure is initially in the steady state with the switch K in open condition and \bar{K} in closed condition. The switch K is closed and \bar{K} is opened simultaneously at the instant $t = t_1$, where $t_1 > 0$. The minimum value of t_1 in milliseconds, such that there is no transient in the voltage across the $100 \mu\text{F}$ capacitor, is _____ (Round off to 2 decimal places).



[NAT - 2 Marks]

Ans. 1.58

Sol. Before SW operate



$$i(t) = \sin(1000t) = 1 \angle 0^\circ$$

$$X_c = \frac{1}{\omega C} = \frac{1}{1000 \times 100 \times 10^{-6}}$$

$$= 10 \Omega$$

$$V_c = 1 \angle 0^\circ (10 \parallel -j10) = 1 \times \frac{10 - j10}{10 - j10}$$

$$V_c = 5\sqrt{2} \angle -45^\circ$$

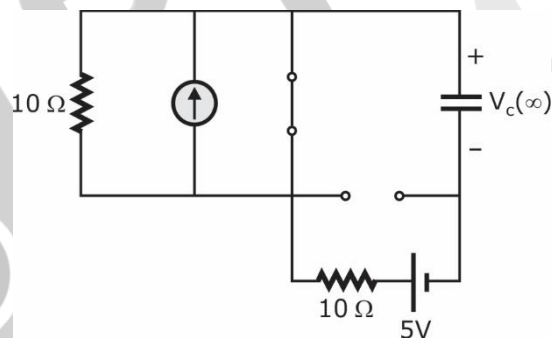
$$V_c = 5\sqrt{2} \sin(1000t - 45^\circ)$$

At

$$t = t_1$$

$$V_c(t_1) = 5\sqrt{2} \sin(1000t_1 - 45^\circ)$$

After SW operate



$$V_c(\infty) = 5$$

$$V_c(t) : 5 + (V_c(t_1) - 5) e^{-t/\tau}$$

For no transient,

$$V_c(t_c) = 5$$

$$5\sqrt{2} \sin\left(1000t_1 - \frac{\pi}{4}\right) = 5$$

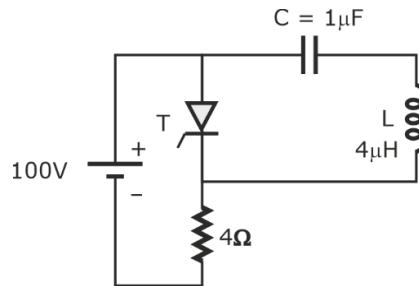
$$\sin\left(1000t - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$1000t - \frac{\pi}{4} = \frac{\pi}{4}$$

$$t = \frac{\pi}{2 \times 1000}$$

$$t = 1.57 \text{ ms}$$

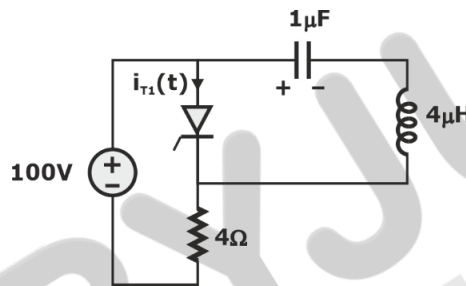
55. The circuit shown in the figure has reached steady state with thyristor 'T' in OFF condition. Assume that the latching and holding currents of the thyristor are zero. The thyristor is turned ON at $t = 0$ sec. The duration in microseconds for which the thyristor would conduct, before it turns off, is _____ (Round off to 2 decimal places).



[NAT - 2 Marks]

Ans. 7.33

Sol.



Under steady state capacitor is charged to 100 volt with polarity shown in fig. Let thyristor is fired at $t = 0$

Current through thyristor will be

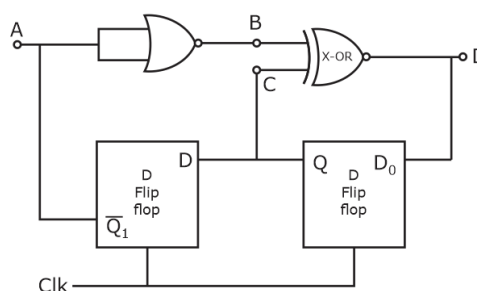
$$\frac{V_s}{R} + V_s \sqrt{\frac{C}{L}} \sin \frac{1}{\sqrt{LC}} t$$

$$25 + 50 \sin \frac{1}{\sqrt{LC}} t = i_{T1}(t)$$

$$i_{T1}(t) \text{ is zero at } t = \pi\sqrt{LC} + \frac{\pi}{6}\sqrt{LC} = \frac{7\pi}{6} [2 \times 10^{-6}] \text{ sec}$$

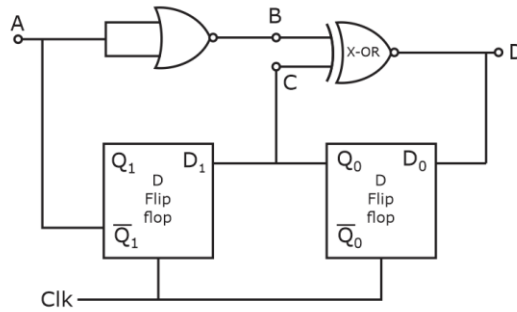
$$= \frac{7\pi}{3} \mu \text{ sec} = 7.33 \mu \text{ sec}$$

56. Neglecting the delays due to the logic gates in the circuit shown in figure, the decimal equivalent of the binary sequence [ABCD] of initial logic states, which will not change with clock, is _____.



Ans. 8

Sol.



Present state		A	B	C	D	D ₁	D ₀	Next state	
Q ₁	Q ₀	\bar{Q}_1	Q ₁	Q ₀	$Q_1 \oplus Q_0$	Q ₀	$Q_1 \oplus Q_0$	Q ₁ ⁺	Q ₀ ⁺
0	0	1	0	0	0	0	0	0	0

When 00 is the initial state, next state is also 00. It continues to be 00 even through we apply clock which we discuss a lock out condition our counter circuits.

ABCD is 1000 in all clock

∴ Decimal equivalent of 1000 is 8.

57. In a given 8-bit general purpose micro-controller there are following flags. C-Carry, A-Auxiliary Carry, O-Overflow flag, P-Parity (0 for even, 1 for odd) R0 and R1 are the two general purpose registers of the micro-controller. After execution of the following instructions, the decimal equivalent of the binary sequence of the flag pattern [CAOP] will be _____.

MOV R0, +0x60

MOV R1, +0x46

ADD R0, R1

[NAT - 2 Marks]

Ans. 2

Sol.

MOV	R0,	0 × 60	$\xrightarrow{60}$	0 1 1 0	0 0 0 0
MOV	R1,	0 × 46	$\xrightarrow{46}$	0 1 0 0	0 1 1 0
ADD	R0,	R1		1 0 1 0	0 1 1 0

Here number of 1s = 4. Therefore parity = 0(even)

Since no carry generated during addition of these numbers carry = 0

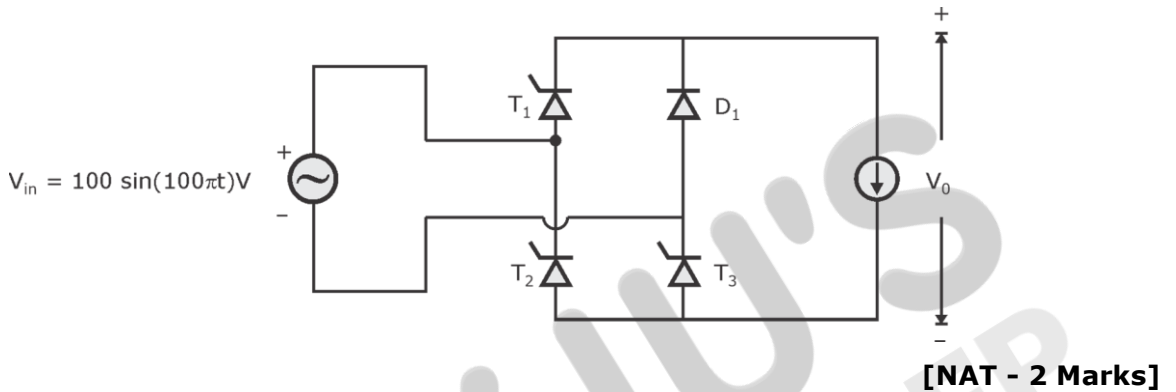
Auxiliary carry is the carry during the addition of lower nibble. Here 0000 + 0100 = 0100 and no auxiliary carry

Two numbers are positive ie 01100000 and
01000110

But the resultant is 1010 0110, this is due to over flow.

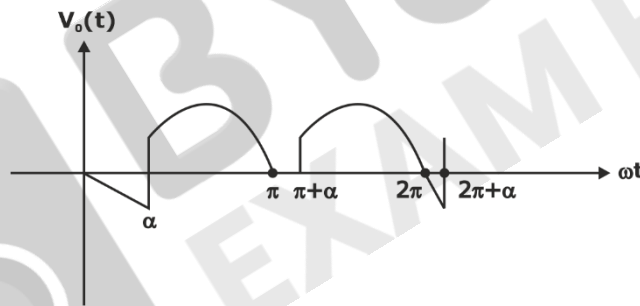
∴ CAOP = 0010, decimal equivalent = 2.

- 58.** The single phase rectifier consisting of three thyristors T_1, T_2, T_3 and a diode D_1 feed power to a 10 A constant current load. T_1 and T_3 are fired at $\alpha = 60^\circ$ and T_2 is fired at $\alpha = 240^\circ$. The reference for α is the positive zero crossing of V_{in} . The average voltage V_0 across the load in volts is _____ (Round off to 2 decimal places).



Ans. 39.79

Sol.

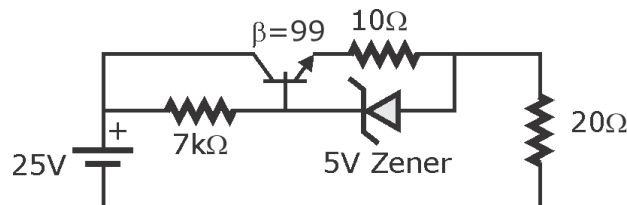


$$V_0(t)_{avg} = \frac{1}{2\pi} \left[\int_{\alpha}^{\pi} V_m \sin \omega t d(\omega t) + \int_{\pi+\alpha}^{2\pi+\alpha} -V_m \sin \omega t d(\omega t) \right]$$

$$\frac{V_m}{2\pi} [1 + \cos \alpha + \cos \alpha + \cos \alpha] = \frac{V_m}{2\pi} [1 + 3 \cos \alpha]$$

$$V_0(t) = \frac{100}{2\pi} \left(1 + \frac{3}{2} \right) = 39.79 \text{ volt}$$

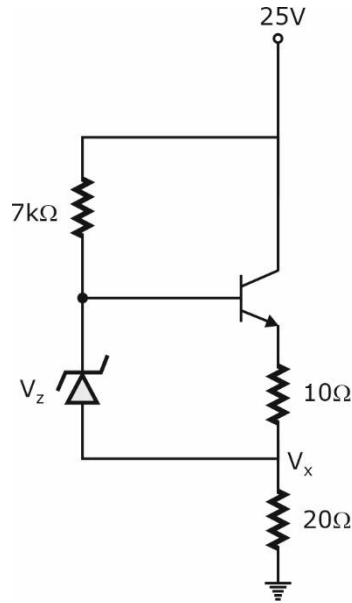
- 59.** The Zener diode in circuit has a breakdown voltage of 5 V. The current gain β of the transistor in the active region is 99. Ignore base-emitter voltage drop V_{BE} . The current through the 20 Ω resistance in milliamperes is _____ (Round off to 2 decimal places).



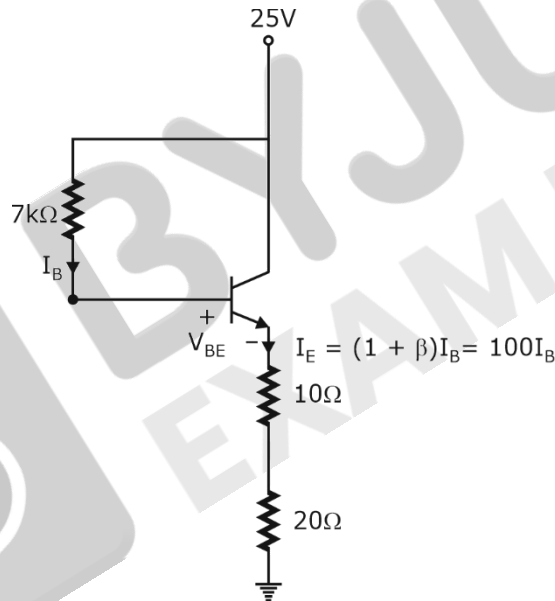
[NAT - 2 Marks]

Ans. 250

Sol.



Let Diode is in RB and it acts as an open circuit.

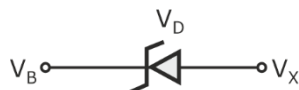


$$25 = 7000I_B + V_{BE} + 30 \times 100I_B$$

$$25 = 10000I_B + 0$$

$$I_B = \frac{25}{10000} = 2.5\text{mA}$$

$$I_E = 100I_B = 250\text{mA}$$



Note:

$$V_B = 25 - I_B \times 7 = 25 - 7 \times 2.5$$

$$V_B = 7.5 \text{ volt}$$

$$V_X = 20\Omega \times I_E = 0.02 \times 250 = 5\text{V}$$

Drop across diode.

$$V_D = 7.5 - 5 = 2.5 \text{ V} < V_z$$

∴ Diode is in RB and it is not in Breakdown Region.

60. The two-bus power system shown in figure (i) has one alternator supplying a synchronous motor load through a Y-Δ transformer. The positive, negative and zero-sequence diagrams of the system are shown in figures (ii), (iii) and (iv), respectively. All reactances in the sequence diagrams are in p.u. For a bolted line-to-line fault (fault impedance = zero) between phases 'b' and 'c' at bus 1, neglecting all pre-fault currents, the magnitude of the fault current (from phase 'b' to 'c') in p.u. is _____ (Round off to 2 decimal places).

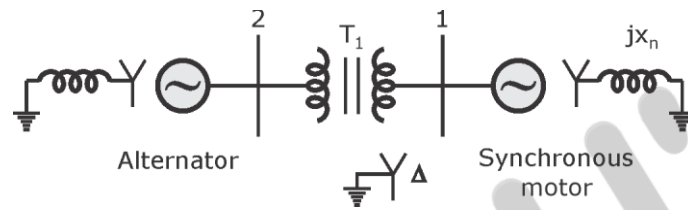


figure (i): Single-line diagram of the power system

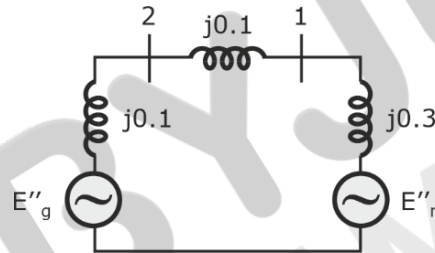


figure (ii): Positive-sequence network

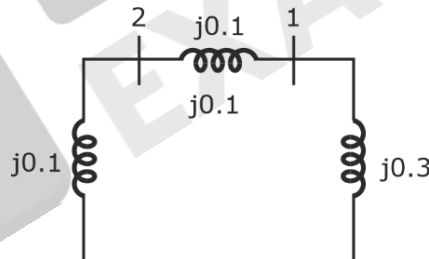


figure (iii): Negative-sequence network

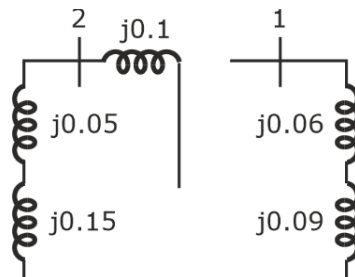


figure (iv): Zero-sequence network

[NAT - 2 Marks]

Ans. 7.2

Sol. In case of line-to-line fault the ground is not involved, hence zero sequence will not be present.

$$I_f = \frac{-j\sqrt{3}E_a}{Z_{1eq} + Z_{2eq}}$$

$$Z_{1eq} = (j0.1 + j0.1) || j0.3$$

$$= \frac{j0.2 \times j0.3}{j0.2 + j0.3} = \frac{j0.06}{0.5} = j0.12pu$$

$$Z_{2eq} = Z_{1eq} = j0.12$$

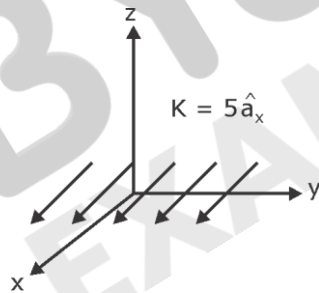
Hence,

$$I_f = \frac{-j\sqrt{3} \times 1}{Z_{1eq} + Z_{2eq}}$$

$$= \frac{-j\sqrt{3} \times 1}{j0.12 + j0.12} = \frac{-j\sqrt{3}}{j0.24}$$

$$I_f = 7.22 pu$$

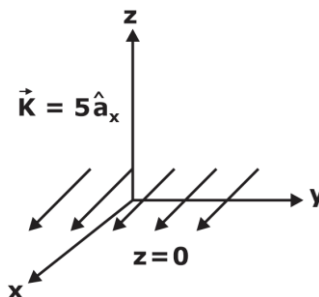
- 61.** An infinite surface of linear current density $K = 5\hat{a}_x$ A / m exists on the x-y plane, as shown in the figure. The magnitude of the magnetic field intensity (H) at a point (1,1,1) due to the surface current in Ampere/meter is _____ (Round off to 2 decimal places).



[NAT - 2 Marks]

Ans. +2.5 mA

Sol.



Magnetic field due to infinite charge sheet is given by

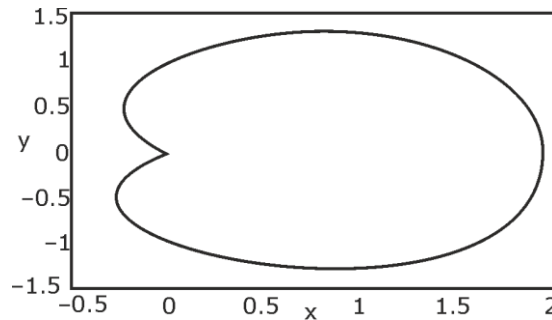
$$\vec{H} = \frac{1}{2}(\vec{K} \times \hat{a}_n) = \frac{1}{2}(5\hat{a}_x \times \hat{a}_z)$$

$$= -2.5\hat{a}_y$$

Magnitude of \vec{H} will be +2.5 A/m.

62. The closed curve shown in the figure is described by $r = 1 + \cos \theta$, where $r = \sqrt{x^2 + y^2}$, $x = r \cos \theta$, $y = r \sin \theta$

The magnitude of the line integral of the vector field $F = -y\hat{i} + x\hat{j}$ around the closed curve is _____ (Round off to 2 decimal places).



[NAT - 2 Marks]

Ans. 9.42

Sol. Given,

$$\vec{F} = -y\hat{i} + x\hat{j}$$

$$\vec{F} \cdot d\vec{r} = -ydx + x \cdot dy = Mdx + Ndy$$

By Green's Theorem,

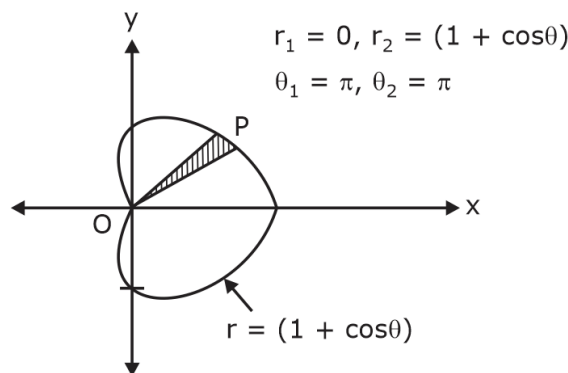
$$\oint_C \vec{F} \cdot d\vec{r} = \oint_C Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$$

$$= \oint_C -ydx + x \cdot dy = \iint_R [1 - (-1)] dx dy$$

$$= 2 \iint_R dx dy$$

$$= 2 \iint_R r \cdot dr \cdot d\theta$$

$$= 2 \int_{-\pi}^{\pi} \int_{r_1=0}^{r_2=(1+\cos\theta)} r \cdot dr \cdot d\theta$$



$$= 2 \int_{-\pi}^{\pi} \left[\frac{r^2}{2} \right]_{r_1=0}^{r_2=(1+\cos\theta)}$$

$$\begin{aligned}
 &= \int_{-\pi}^{\pi} [(1 + \cos \theta)^2 - 0] d\theta \\
 &= \int_{-\pi}^{\pi} \left[1 + 2 \cos \theta + \left(\frac{1 + \cos 2\theta}{2} \right) \right] \cdot d\theta \\
 &= \int_{-\pi}^{\pi} \left[\frac{3}{2} + 2 \cos \theta + \frac{\cos 2\theta}{2} \right] d\theta \\
 &= \left[\frac{3}{2} \theta + 2 \sin \theta + \frac{\sin 2\theta}{4} \right]_{-\pi}^{\pi} \\
 &= \frac{3}{2} (2\pi) + 0 + 0 \\
 &= 3\pi = 9.4248
 \end{aligned}$$

63. A signal $x(t) = 2\cos(180nt)\cos(60nt)$ is sampled at 200 Hz and then passed through an ideal low pass filter having cut-off frequency of 100 Hz.

The maximum frequency present in the filtered signal in Hz is _____ (Round off to the nearest integer).

[NAT - 2 Marks]

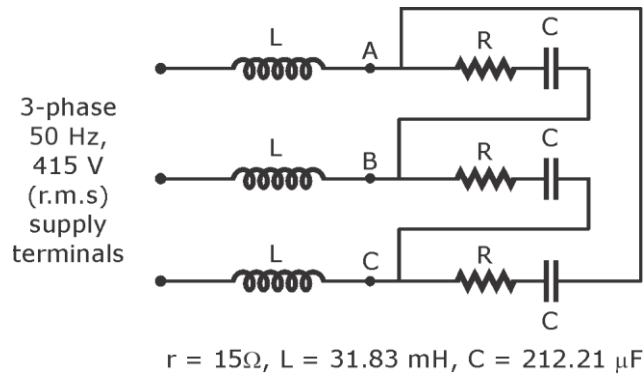
Ans. 80

Sol.

$$\begin{aligned}
 x(t) &= 2 \cos(180nt) \cdot \cos(60nt) \\
 f_s &= 200 \text{ Hz} \\
 x(t) &= \cos(240nt) + \cos(120nt) \\
 f_c &= 100 \text{ Hz} \\
 f_1 &= 120 \text{ Hz} & f_2 &= 60 \text{ Hz} \\
 nf_s \pm f_m & \\
 200n \pm 120 & & 200n \pm 60 & \\
 n = 0 : 120\text{Hz} & & n = 0; 60 \text{ Hz} & \\
 n = 1 : 320, 80\text{Hz} & & n = 1; 260, 140\text{Hz} &
 \end{aligned}$$

maximum frequency component at output = 80 Hz

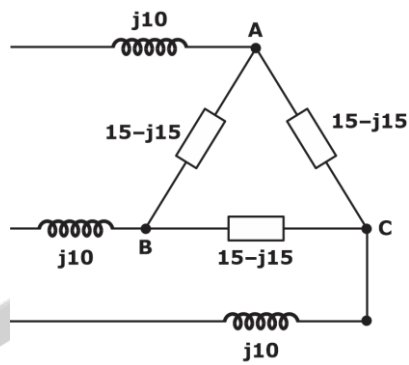
64. A balanced delta connected load consisting of the series connection of one resistor ($R = 15 \text{ W}$) and a capacitor ($C = 212.21 \text{ mF}$) in each phase is connected to three-phase, 50 Hz, 415 V supply terminals through a line having an inductance of $L = 31.83 \text{ mH}$ per phase, as shown in the figure. Considering the change in the supply terminal voltage with loading to be negligible, the magnitude of the voltage across the terminals V_{AB} in Volts is _____ (Round off to the nearest integer).



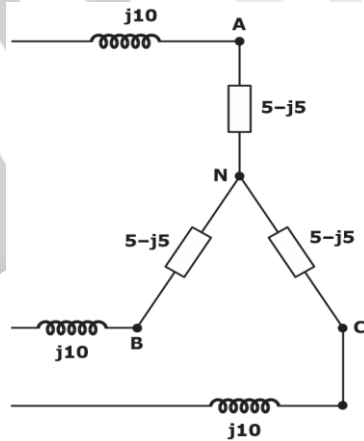
[NAT - 2 Marks]

Ans. 415

Sol.



Equivalently we can represent it



$$\text{Supply phase voltage} = \frac{415}{\sqrt{3}} \angle 0^\circ$$

$$\text{Phase current} = \left[\frac{\frac{415}{\sqrt{3}} \angle 0^\circ}{5 + j5} \right]$$

$$V_{AN} = \left[\frac{415}{\sqrt{3}} \angle 0^\circ - \left(\frac{\frac{415}{\sqrt{3}} \angle 0^\circ}{5 + j5} \right) j10 \right]$$

$$\frac{415}{\sqrt{3}} \angle 0^\circ - \frac{415\sqrt{2}}{\sqrt{3}} \angle -45^\circ = 239.6 \angle 90^\circ$$

$$V_{AN} = 239.6\sqrt{3} \angle 120^\circ \\ = 415 \angle 120^\circ$$

Line voltage = 415 volts

65. A quadratic function of two variables is given as

$$f(x_1, x_2) = x_1^2 + 2x_2^2 + 3x_1 + 3x_2 + x_1x_2 + 1$$

The magnitude of the maximum rate of change of the function at the point (1,1) is _____
(Round off to the nearest integer).

[NAT - 2 Marks]

Ans. 10

Sol.

$$f(x_1, x_2) = x_1^2 + 2x_2^2 + 3x_1 + 3x_2 + x_1x_2 + 1$$

$$\bar{\nabla}f = \hat{i} \frac{\partial f}{\partial x_1} + \hat{j} \frac{\partial f}{\partial x_2}$$

$$\bar{\nabla}f = \hat{i}(2x_1 + 3 + x_2) + \hat{j}(4x_2 + 3 + x_1)$$

at

$$P(1, 1)$$

$$\bar{\nabla}f|_{(1,1)} = \hat{i}(6) + \hat{j}(8)$$

Now, magnitude of the maximum rate of change of the function $f(x, y)$ at (1,1) is

$$|\bar{\nabla}f| = \sqrt{6^2 + 8^2} = 10$$

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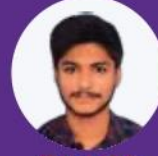
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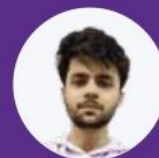
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