Laminar and Turbulent Flow

Laminar and turbulent flow are the flow conditions described based on the Reynolds number of the flow.

Laminar Flow and Turbulent Flow

- Laminar flow occurs in the form of lamina or layers with no intermixing between the layers.
- Laminar flow is also referred to as streamlined or viscous flow.
- In the case of turbulent flow, fluid particles are intermixed.

Reynold's Number

- The dimensionless Reynolds number plays a prominent role in foreseeing the patterns in a fluid's behaviour. It is referred to as R_e and is used to determine whether the fluid flow is laminar or turbulent.

Reynold's no, $R_e = \rho VD/\mu = VD/v$

V=mean velocity of flow through a pipe

D = Characteristic length of the geometry

μ = dynamic viscosity of the liquid (N–s /m^2)

v = Kinematic viscosity of the liquid (m^2/s)

$D=4A_c/P$

Where,

- A_c = Cross–section area of the pipe
- P = Perimeter of the pipe

Pipe	Plate
$R_e < 2000$ laminar	$R_e < 5 \times 10^5$ Laminar
$2000 < R_e < 4000$ Transient	$R_e > 5 \times 10^5$ turbulent
$R_e > 4000$ turbulent	[transient is small, so neglected]

Laminar Flow in a Pipe

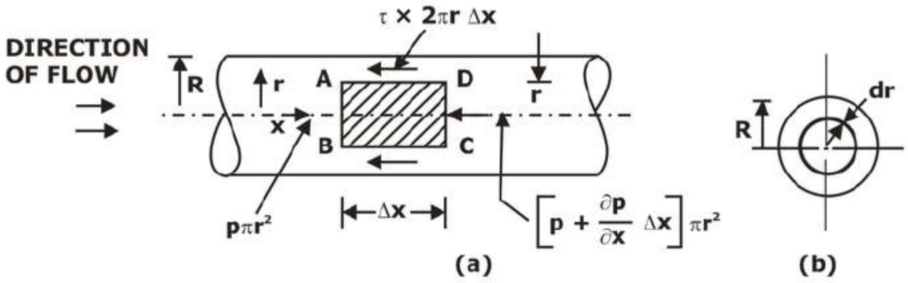


Fig. 9.1 Viscous flow through a pipe

Now, the forces acting on the fluid element are:

(a). The pressure force, $P \times \pi R^2$ on face AB.

(b). The pressure force on the face CD. = $[p + (dp/dx)\Delta x]\pi r^2$

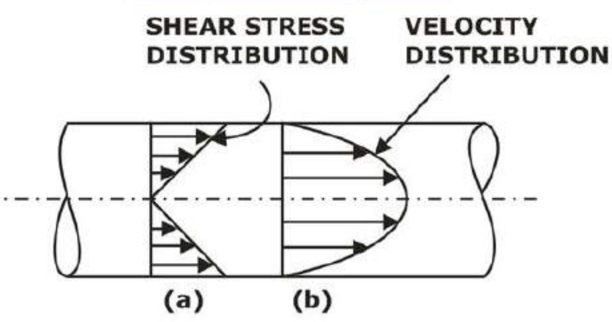
(c). The shear force, $\tau \times 2\pi r\Delta x$ on the surface of the fluid element. As there is no acceleration hence:

Net force in the x direction =0

$\Sigma F_x = 0$ results in

$\tau = -(dp/dx) \times (r/2)$ and τ_{max} at r=R $\Rightarrow -(dp/dx) \times (r/2)$

As $\partial P/\partial x$ across a section is constant, thus the shear stress τ varies linearly with the radius r, as shown in the Figure.



The Ratio of Maximum Velocity to the Average Velocity

Thus, the Average velocity for Laminar flow through a pipe is half of the maximum velocity of the fluid at the center of the pipe.

$$\frac{u_{max}}{u_{avg}} = \frac{-\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)R^2}{-\frac{1}{8\mu}\left(\frac{\partial P}{\partial x}\right)R^2} = 2$$

Pressure Variation in Laminar Flow Through a Pipe of Length L

$U_{avg} = (-1/8\mu)(\partial P/\partial x)R^2 \Rightarrow -\partial P = 8\mu u_{avg}\partial x/R^2$

On integrating the above equation on both sides

$-\Delta P = 8\mu u_{avg}L/R^2 \Rightarrow P_1 - P_2 = 32\mu u_{avg}L/D^2$

Head loss in Laminar flow through a pipe over length L

$(P_1 - P_2)/\rho g = h_f \Rightarrow P_1 - P_2 = \rho g h_f$

$P_1 - P_2 = 32\mu u_{avg}L/D^2 \Rightarrow h_f = 32\mu u_{avg}L/\rho g D^2$

The above equation is the **Hagen Poiseuille Equation**.

As we know,

$h_f = fLu^2_{avg}/2gd$ and $h_f = 32\mu u_{avg}L/\rho g D^2$

Thus,

$fLu^2_{avg}/2gd = 32\mu u_{avg}L/\rho g D^2$

$f = 64/(\rho u D/\mu) = 64/R_e$ where f is the friction factor

$f' = f/4 = 16/R_e$ where f' is the friction coefficient

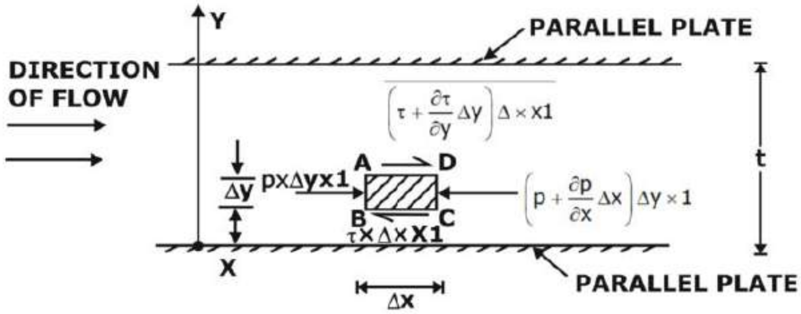
Radial distance from the pipe axis at which the velocity equals the average velocity

$$u_{avg} = -\frac{1}{8\mu}\left(\frac{\partial P}{\partial x}\right)R^2 \quad u = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left(R^2 - r^2\right)$$

$$-\frac{1}{8\mu}\left(\frac{\partial P}{\partial x}\right)R^2 = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left(R^2 - r^2\right)$$

$$r = \frac{R}{\sqrt{2}} = 0.707R$$

Laminar Flow Between Two Fixed Parallel Plates



For steady and uniform flow, there is no acceleration; hence, the resultant force in the flow direction is zero.

$\partial\tau/\partial y = \partial P/\partial x$

Velocity Distribution (u)

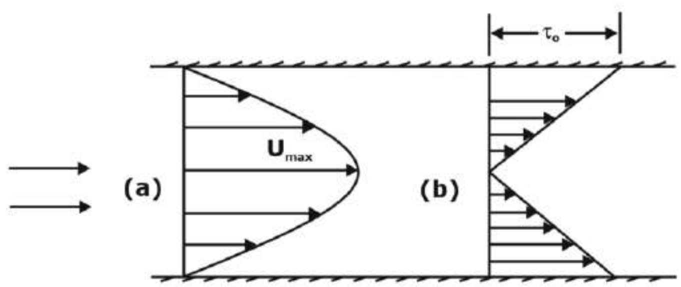
the value of shear stress is given by

$\tau = \mu\, du/dy$ and $\partial\tau/\partial y = \partial P/\partial x$

Boundary condition, at y = 0 u = 0

$$u = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\left(ty - y^2\right)$$

Thus, velocity varies parabolically as we move in the y-direction, as shown in Figure.



Velocity and shear stress profile for turbulent flow

At $y = t/2$; $u = U_{max}$

Umax = (1/8μ)(-∂P/∂x)t^2

Discharge (Q) between two parallel fixed plates

The average velocity is obtained by dividing the discharge (Q) across the section by the area of the section t ×1.

$$dQ = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\left(ty - y^2\right) \times dy \times 1$$

$$Q = \int_0^t dQ = \int_0^t \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\left(ty - y^2\right) dy$$

$$Q = \frac{-1}{12\mu}\left(\frac{\partial P}{\partial x}\right)t^3$$

The ratio of Maximum velocity to average velocity:

$$u_{avg} = \frac{Q}{A} = \frac{-1}{12\mu}\left(\frac{\partial P}{\partial x}\right)t^2 \qquad u_{max} = \frac{-1}{8\mu}\left(\frac{\partial P}{\partial x}\right)t^2$$

Thus,

$$u_{avg} = \frac{2}{3}u_{max} \Rightarrow \frac{u_{avg}}{u_{max}} = \frac{2}{3}$$

The pressure difference between two parallel fixed plates

$$-\partial P = \frac{12\mu V_{avg}}{t^2}\partial x \Rightarrow \int_{P_1}^{P_2} -\partial P = \int_0^L \frac{12\mu V_{avg}}{t^2}\partial x$$

$$P_1 - P_2 = \frac{12\mu V_{avg} L}{t^2}$$

Different Correction Factors

Two correction factors include the momentum correction factor and the kinetic energy correction factor. These correction factors relate the flow's actual behavior with the flow's behavior based on the average velocity and are important for the GATE exam. These are described below:

1. Momentum correction factor (β)
2. Kinetic energy correction factor (α)

Momentum correction factor (β)

It is defined as the ratio of momentum per second based on actual velocity to the momentum per second based on average velocity across a section. It is denoted by β.

$$\beta = \frac{P_{actual}}{P_{avg}} = \frac{\int_0^R \rho u dA \times u}{\rho AV \times V}$$

$$\beta = \frac{1}{AV^2}\int_0^R u^2 dA$$

Kinetic energy correction factor (α)

It is defined as the ratio of the kinetic energy of flow per second based on actual velocity to the flow's kinetic energy per second based on average velocity across the same section.

Let p = momentum

P = momentum/sec.

$$\alpha = \frac{\dot{K}E_{actual}}{\dot{K}E_{avg}} = \frac{\int_0^R \rho dA \frac{u^3}{2}}{\frac{\rho AV^3}{2}}$$

$$\alpha = \frac{1}{AV^3}\int_0^R u^3 dA$$

For flow through pipes values of α and β:

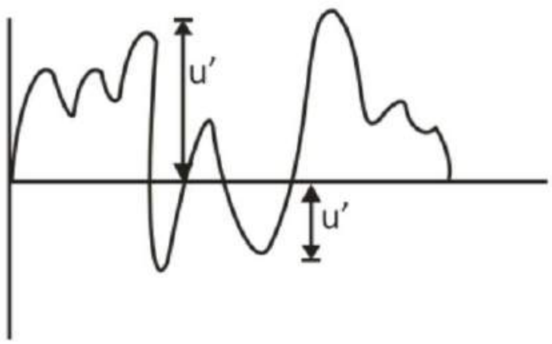
Correction-factor	Laminar	Turbulent
α	2	1.33
β	1.33	1.2

From the above, we can say that the value of the correction factor for Laminar flow is more than that for turbulent flow.

Turbulent Flow Characteristics

Turbulent flow is a fluid flow categorized based on the Reynolds number of the flow. In turbulent flow, fluid particles are mixed with each other. Here a few turbulent flow characteristics are described.

Shear stress in turbulent flow



In the case of turbulent flow, there are huge orders of intermission fluid particles; due to this, various fluid properties will change with space and time.

Difference Between Laminar and Turbulent Flow

Laminar and turbulent flow is the flow characteristics categorized based on the Reynolds number of the flow. In the laminar flow, fluid particles move in the layered form and in the turbulent flow, fluid particles are mixed with each other during its motion.

For the pipe flow, If the Reynolds number is less than 500, it will be considered as the laminar flow, and if the Reynolds number is greater than 2000, it will be considered as the turbulent flow, and for the in-between value of Reynolds number flow will be considered as the transition flow.

Average Velocity and Fluctuating Velocity in Turbulent Flow

Average velocity is the value for which the total discharge through a section remains the same as the actual discharge. And fluctuating velocity is the actual velocity in the turbulent flow. It is said to be fluctuating velocity because the direction and magnitude of the velocity vary in nature.

Boussinesq Hypothesis

Similar to the expression for viscous shear, J. Boussinesq expressed the turbulent shear mathematical form as

$\tau_t = \eta(du_{avg}/dy)$

where τ_t = shear stress due to turbulence

η = eddy viscosity

u_{avg} = average velocity at a distance y from the boundary. The ratio of η (eddy viscosity) and (mass density) is known as kinematic eddy viscosity and is denoted by ϵ (epsilon). Mathematically it is written as

$\varepsilon = \eta/\rho$

If the shear stress due to viscous flow is also considered, then …. shear stress becomes as

$\tau = \tau_v + \tau_t = \mu(du/dy) + \eta(du_{avg}/dy)$

The value of $\eta = 0$ for laminar flow.

Reynolds Expression for Turbulent Shear Stress

Reynolds developed an expression for turbulent shear stress between two layers of a fluid at a small distance apart, which is given as:

$\tau = \rho u'v'$

where u', v' = fluctuating velocity component in the direction of x and y, respectively, due to turbulence.

As u' and v' vary, τ will also vary.

Hence, the time average on both sides of the equation to find the shear stress.

$\tau_{avg} = (\rho u'v')_{avg}$

The turbulent shear stress given by the above equation is known as Reynold stress.

Prandtl Mixing Length Theory

According to Prandtl, the mixing length L is the distance between two layers in the transverse direction such that the lumps of fluid particles from one layer could reach the other layer and the particles are mixed in the other layer in such a way that the momentum of the particles in the direction of x is same.

Velocity Distribution in Turbulent Flow

Velocity distribution in turbulent flow follows the logarithmic distribution. In the turbulent flow, the velocity at the boundary of the pipe is zero and maximum at the center of the pipe. It is shown in the below diagram.

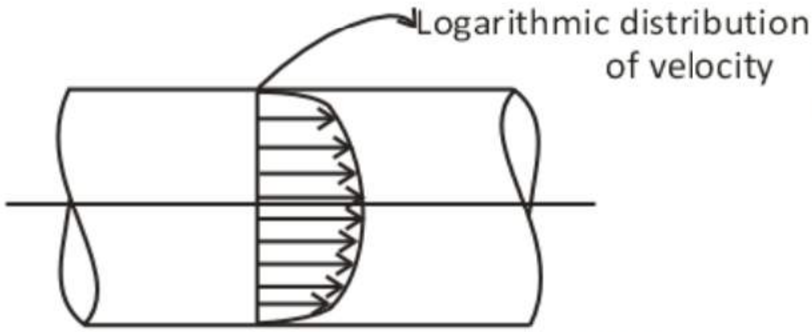
$(u_{max} - u)/u^* = 2.5 \log_e(R/y)$

$(u_{max} - u)/u^* = 2.5 \times 2.303 \log_{10}(R/y)$

$(u_{max} - u)/u^* = 5.75 \log_{10}(R/y)$

In the above equation, the difference between the maximum velocity u_{max}, and local velocity u at any point, i.e. $(u_{max} - u)$, is known as a 'velocity defect'.

Velocity distribution in turbulent flow through a pipe



Laminar sublayer thickness

Laminar sublayer thickness (δ') = 11.6v/u*