BYJU'S EXAM PREP

Important Formulae

AVERAGES, MIXTURES, ALLEGATIONS

- Avg of x1, x2, x3 ……. xn

  A = x1 + x2 + + x3 …..xn/n

- Average of two or more quantities necessarily lies between the lowest and the highest quality.

Weighted average:
WA = k1 a1 + k2 a2 + …..kn an/k1 + k2 + ……kn / k1 + k2 + …. Kn
K1, k2, ……kn – no of elements of different groups
A1, a2 …… an – averages of respective groups

Average speed **=** total dist / total time

2 ppl s = 2uv/u+v
2 ppl s = 3uvw/uv + vw + uw
Average of first ‘n’ natural numbers = n + 1/2
Average of first ‘n’ even nos = n + 1
Average of first ‘n’ odd nos = n

Milk and water formula:
Let q be the volume of vessel
Q – Quantity of mixture of milk & water to be removed each time from the Mixture
N – No. of times operation is done
A – Final quantity of milk in mixture

$A = (1 - g/q)^n \times q$

- Allegation rule:
  The ratio of weights of a items mixed will inversely proportional to the deviation of attributes of these two theme from the average attribute of the resultant mixture w1/w2 = x2 – x/x – x1

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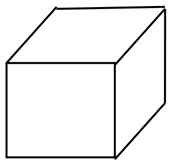
x1     x2
  \   /
    n
  /   \
x2-n   x-x1
  w1 ;  w2
```

Cubes



Euler's theorem - > F +C =E+2

Faces = 6

Corners = 8

Edges = 12

2) To cut a cube into minimum number of pieces, all the cuts should be along the same direction. To cut a cube into maximum number of pieces all cuts should be as equal as possible in the 3 directions.

3) No pieces into which a cube can be cut = (x +1) (y+1)(z+1)

X,y,z - no of cuts in the corresponding $\perp$r direction.

4) Min no. of cuts required to cut a cube into $n_x*n_y*n_z$ Smaller cubes

$= (n_x *n_y*n_z) – 3$

- ➔ Three faces painted = 8
- ➔ Exactly two faces painted = $(n_x-2)\ 4 + (n_y-2)\ 4 +(n_z-2)\ 4$
- ➔ Exactly 1 face painted = $(n_x-2)(n_y-2)2 + (n_y-2)(n_z-2)\ 2 + (n_x-2)(n_z-2)2$
- ➔ No face painted = $(n_x-2)\ )(n_y-2)\ )(n_z-2)$

5) If $n_x = n_y = n_z = n$

Exactly 3 faces painted -> 8

Exactly 2 faces painted -> (n-2) (12)

Exactly 1 face painted -> $(n-2)^2 6$

No face painted -> $(n-2)^3$

6) When a large cube is split into n*n*n smaller cubes, no. of cubes that are cut by

	n = even	n = odd
1. Diagonal cut	n^2	n^2
2. Diagonal cuts	$2n^2$	$2n^2-n$

EQUATION, RATIO PREPARATION & VARIATION

1) Consider a ratio a:b
   Duplicate ratio a2:b2
   Triplicate ratio a3:b3
   Sub duplicate ratio $\sqrt{a} : \sqrt{b}$

2) For a positive number x, a/b > a + x/ b + x if a >b
   a/b = a + x/b + x if a:b
   a/b < a + x/b + x if a < b

3) If a/b = c/d = e/f = …. K
   Then k = a + c + e+…../b + d + f+……
   $= [p_0 a^n + p_1c^n + p_2e^n + …./p_0b^n + p_1d^n + p_2f^n + …]^{1/n}$

4) If $a_1x + b_1y + c_1z = 0$
   $a_2x + b_2y + c_2z = 0$
   then $x/b_1c_2 – b_2c_1 = y/c_1a_2 – c_2a_1 = z/a_1b_2 – a_2b_1$

5) If a/b = c/d
   a + b/b = c + d/d component
   a –b /b = c – d /d dividend
   a + b/a – b = c + d/c-d component – dividend

PROPORTION

1) When two ratios are equal, the four quantities are said to be in proportion.
   If a/b = c/d   a:b: c:d extremes   b. c means

2) If a/b = b/c,   d, b, c are in continued proportion
   b –means proportional   c – third proportional

3) If a, b, c, d are in continued proportion
   a/b = b/c = c/d = ……. k

VARIATION

a * b   a = kb   direct variation
a * 1/b   a = k/b inverse variation
a * b/c a = kb/c   joint variation

Two Digit Numbers:

N = 10a + b
R = 10b + a
N + r = 11 (a + b)
N ~ r = 11 (a ~ b)

Three digit numbers:

n = 100a + 10b + c
r = 100c + 10b + c
n ~ r = 99 (a ~ c)

Special Equations:

1) If ax + by + = c and a, b, c are the, then x will increase / decrease as the coefficient of y & vice versa successive values of (x + y) differ by 1a – b1

2) If ax + by = c
   If a ∠ b
   Y = a k + r (c/a)/r(b/a)
   if b ∠ a
   x = b k + r (c/b)/r (a/b)

3) If ax – by = c, both x and y increase
   X increases by the coefficient of y
   Y increases by the coefficient of x
   (x = y) by the of (a + b)
   If a1/b1, a2/b2, an/bn are unequal fractions of which all the denominators are of the same sign, then the fraction a1 + a2 + ….an / b1 + b2 + ….bn
   lies in magnitude between the greatest and least of them.

4) A special equation should be solved only in its simplest possible form.
   Ax + by = c
   A, b → co prime

TIME & DISTANCE

1) d = s x t s x 1/t for a constant distance

2) average speed = total distance/total time
   If a person travels a distance ‘d’ at speed ‘p’ and the same distance at a speed ‘q’
   Average speed = 2 p q/p + q

3) If a body covers a part of the journey at speed p and another part of the journey at speed q and the distances are in the ratio m:n
   Average speed for entire journey = (m + n) pq/mq + np
   - Average speed if time is constant = $s_1 + s_2/2$

4) If a person travelling between 2 points reaches p hours late (at a speed of u km ph) and q hours early (at a speed of v km ph)
   Distance between 2 points = vu/v-u (p + q)

5) Relative speed
   → u → ←
   → v R = u + v
   R = (u – v)

6) Boats and streams
   U = speed of boat against stream = speed of boat – speed of stream
   In still water
   V = speed of boat with the stream = speed of boat + speed of stream
   In still water
   Speed of boat in still water = u + v/2
   Speed of water current = u – v/2

7) If two persons start from same pt and run in opposite directions along a ok track, meet for 1st time at starting pt. after completing n1 & n2 rounds respectively, n1 & n2 → co prime
   Rel. speed = a + b (opp. direction)
   Rel. speed = a – b (same direction)

   - 2 people sun around a circular track with speeds a, b same direction opp. Direction

Time taken to meet for the 1^{st} time → at any point L/a – b   L/a + b
At starting point LCM [L/a, L/b]   LCM [L/a, L/b]

- 3 people run around a circular track with speeds a, b, c
  Time taken to meet for the 1^{st} time → at any point LCM of [L/a-b, L/b-c]
  Starting point LCM of [L/a, L/b, L/c]

- If speeds of two people travelling in the same direction is min, then they will meet at (m – n) points and if they travel in opposite directions they will meet at (m + n) points. [m, n co prime]

8) Clocks:
- 11 coincidences of hour & minute hand in 12 hours
- Minute hand covers 360^0/hr   or 6^0/min.
- Hour hand covers 360^0/12 hrs   or $1/2^0$/min.
  Relative speed = 5 $1/2^0$/min.

In a period of 12 hours:
- In a period of 12 hours, the two hands make an angle of 0^0 with each other 11 times.
  Time interval between
  Two successive coincidences = 12/11 hrs = 65 5/11 min.
- They make 180^0 with each other 11 times.
- 90^0 or any other angle 22 times
  If an event starts at a:y min and ends at b:x min and the hour and minute hands interchange,
  Time difference = (b – a) 12/13 hrs
  Y = 60/143(12b + a)
  X = 60/143 (12a + b)

9) In a race, if a beats b by 'm' meters and b beats c by 'n' meters, then a beats c by L (n – m)/L – m meters

10) If a person travels three equal distances with speeds x, y, z respectively
Average speed = 3d/d/x + d/y + d/z = 3 xyz/xy + yz + zx

11) If walking at 'p' fraction ($p<1$) of usual speed, one reaches a place 'n' hours late, the usual time taken to travel ie $n\ p/1 - p$
Similarly if one reaches 'n' hours early ($p<1$),
Usual time of travel is $hp/p - 1$

12) Vehicle breakdown:
After a car breakdown, a person travels at 'p' fraction of his original speed. Had the breakdown happened 'd' km earlier / later he would have reached 'h' hours later / earlier.
Original speed = $d/n\ (1/p - 1)$

13) Let two people start simultaneously from a to b and b to a respectively. After meeting at a point after 'x' they take t_1, t_2 to reach their destination.

```
          V2 t2
0------------------\------------0
A                  P   v1 t1   B
```

$Ap/v_1 = pb/v_2 \qquad v_1/v_2 = \sqrt{t_2/t_1}$

$X = \sqrt{t_1/t_2}$

14) For a constant distance, if speeds are in AP, then time taken would be in HP and vice versa.

15) Relative speed:
For trains → time = $L_1 + L_2/S_1 \pm S_2$

TIME AND WORK

1) $M_1\ D_1\ H_1/W_1 = M_2\ D_2\ H_2/W2$

2) A & B → x days
A → x + a days
B → x + b days
$x = \sqrt{ab}$

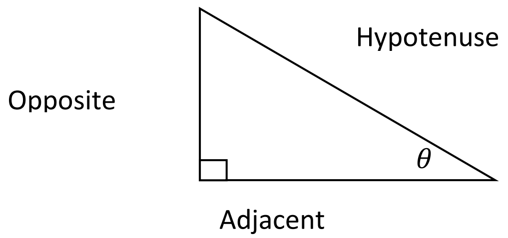
Trig Cheat Sheet

Definition of the Trig functions

Right triangle definition

For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0^0 < \theta < 90^0$



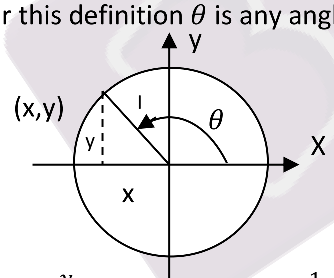
$\text{Sin } \theta = \frac{opposite}{hypotenuse}$ $\csc \theta = \frac{hypotenuse}{opposite}$

$\cos \theta = \frac{adjacent}{hypotenuse}$ $\sec \theta = \frac{hypotenuse}{adjacent}$

$\tan \theta = \frac{opposite}{adjacent}$ $\cot \theta = \frac{adjacent}{opposite}$

Unit circle definition

For this definition θ is any angle.



$\text{Sin } \theta = \frac{y}{1} = y$ $\csc \theta = \frac{1}{y}$

$\cos \theta = \frac{x}{1} = x$ $\sec \theta = \frac{1}{x}$

$\tan \theta = \frac{y}{x}$ $\cot \theta = \frac{x}{y}$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \neq (n + \frac{1}{2})\pi$, n = 0, ± 1, ± 2,.....

$\csc\theta$, $\theta \neq n\pi$, n = 0, ± 1, ± 2,..........

$\sec\theta$, $\theta \neq (n+\frac{1}{2})\pi$, n = 0, ± 1, ± 2,......

$\cot\theta$, $\theta \neq n\pi$, n = 0, ± 1, ± 2,......

Range

The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1$ $\quad$ $\csc\theta \ge 1$ and $\csc\theta \le -1$

$-1 \le \cos\theta \le 1$ $\quad$ $\sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty < \tan\theta < \infty$ $\quad$ $-\infty < \cot\theta < \infty$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So if ω is a fixed number and θ is any angle we have the following periods.

$\sin(\omega\theta) \longrightarrow T = \frac{2\pi}{\omega}$

$\cos(\omega\theta) \longrightarrow T = \frac{2\pi}{\omega}$

$\tan(\omega\theta) \longrightarrow T = \frac{\pi}{\omega}$

$\csc(\omega\theta) \longrightarrow T = \frac{2\pi}{\omega}$

Sec $(\omega\theta) \longrightarrow T = \frac{2\pi}{\omega}$

Cot $(\omega\theta) \longrightarrow T = \frac{\pi}{\omega}$

Formulas and Identities

Tangent and Cotangent Identitites

Tan $\theta = \frac{\sin\theta}{\cos\theta}$ $\quad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Recriprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\quad$ $\sin\theta = \frac{1}{\csc\theta}$

sec $\theta = \frac{1}{\cos\theta}$ cos $\theta = \frac{1}{\sec\theta}$

cot $\theta = \frac{1}{\tan\theta}$ tan $\theta = \frac{1}{\cot\ \theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Even / Odd Formulas

Sin $(-\theta)$ = - sin θ csc $(-\theta)$ = - csc θ

cos $(-\theta)$ = cos θ sec $(-\theta)$ = sec θ

tan $(-\theta)$ = - tan θ cot $(-\theta)$ = - cot θ

Periodic Formulas

If n is an interger.

sin $(\theta + 2\pi n)$ = sin θ csc $(\theta + 2\pi n)$ = csc θ

cos $(\theta + 2\pi n)$ = cos θ sec $(\theta + 2\pi n)$ = sec θ

tan $(\theta + \pi n)$ = tan θ cot $(\theta + \pi n)$ = cot θ

Double Angle Formulas

Sin $(2\theta) = 2\sin\theta\ cos\ \theta$

Cos $(2\ \theta) = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

tan $(2\ \theta) = \frac{2\tan\theta}{1 - tan^2\theta}$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} => t = \frac{\pi x}{180}\ and\ x = \frac{180t}{\pi}$$

Half Angle Formulas

$\text{Sin}^2\,\theta = \frac{1}{2}\,(1-\cos(2\,\theta))$

$\text{Cos}^2\,\theta = \frac{1}{2}\,(1+\cos(2\,\theta))$

$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Sum and Difference Formulas

$\text{Sin}\,(\alpha \pm \beta) = \sin \alpha \cos\beta \pm \cos\alpha \,\text{Sin}\,\beta$

$\text{Cos}\,(\alpha \pm \beta) = \cos\alpha \cos\beta \pm \sin\alpha \sin\beta$

$\tan(\alpha \pm \beta) = = \frac{\tan\alpha \pm \tan\beta}{1 \pm \tan\alpha \tan\beta}$

Product to Sum Formulas

$\sin\alpha \,\text{Sin}\,\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

Sum to Product Formulas

$\text{Sin}\,\alpha + \text{Sin}\,\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\text{Sin}\,\alpha - \text{Sin}\,\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Co function Formulas

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad$ $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ $\quad$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Inverse Properties

$\cos(\cos^{-1}(x)) = x \qquad \cos^{-1}(\cos(\theta)) = \theta$

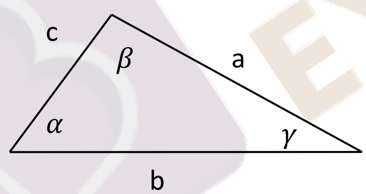
$\sin(\sin^{-1}(x)) = x \qquad \sin^{-1}(\sin(\theta)) = \theta$

$\tan(\tan^{-1}(x)) = x \qquad \tan^{-1}(\tan(\theta)) = \theta$

Domain and Range

Function	Domain	Range	Alternate Notation
$y = \sin^{-1}x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$	$\sin^{-1} x = \arcsin x$
$y = \cos^{-1}x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$	$\cos^{-1} x = \arccos x$
$y = \tan^{-1}x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$	$\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



Law of sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{1}{2R}$$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

Mollweide's Formula

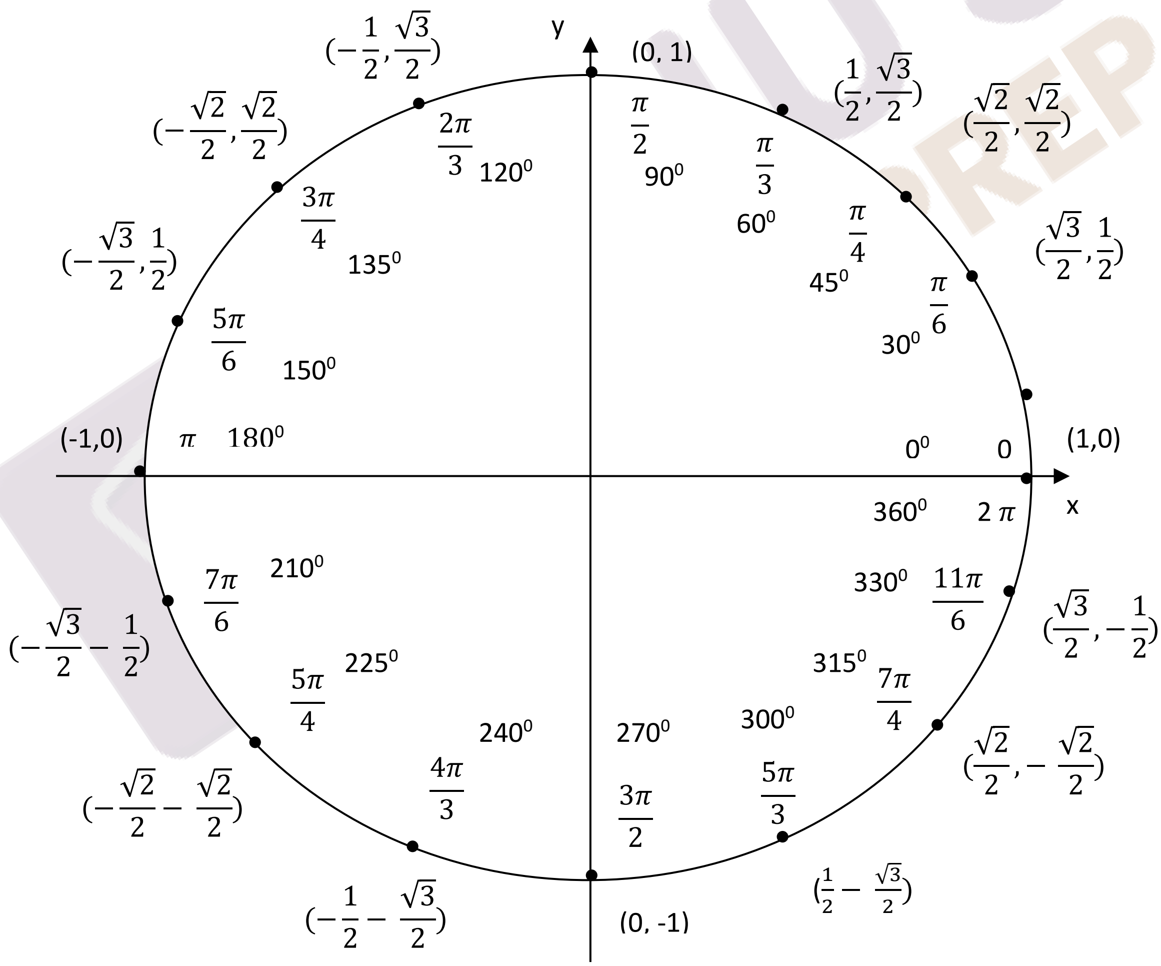
$$\frac{a+b}{c} = \frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of tangents

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta}$$

$$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$



For any ordered pair on the unit circle (x, y) : cos $\theta = x$ and sin $\theta = y$

Example

cos ($\frac{5\pi}{3}$) = $\frac{1}{2}$ sin ($\frac{5\pi}{3}$) = $-\frac{\sqrt{3}}{2}$

In any △le

a = b cos c + c cos B

b = c cos A + a cos C

c = a cos B + b cos A

tan ($\frac{A}{2}$) = $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

tan ($\frac{B}{2}$) = $\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$

tan ($\frac{C}{2}$) = $\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

FUNCTIONS & GRAPHS

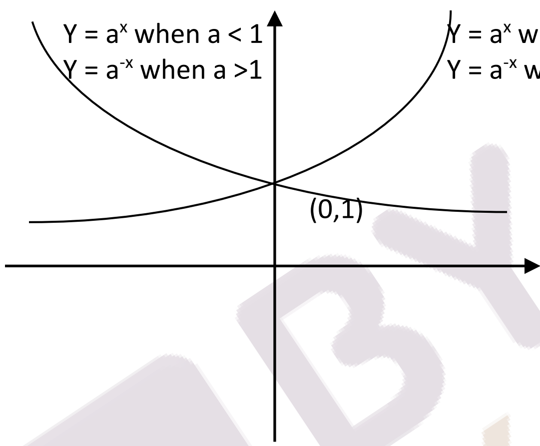
1) Sum of 2 odd functions is always odd,

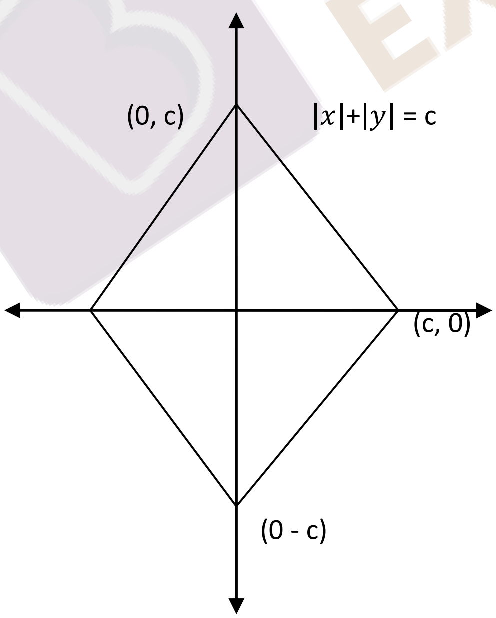
2) For any function f (x), f (x) – f (-x)/2 is always odd.

3) Even function f (x) = f (-x)
   Odd function f (x) = - f (-x)

4) f (x) = $|x|$
   Greatest integer function – largest integer less than or equal to x

Y = ax





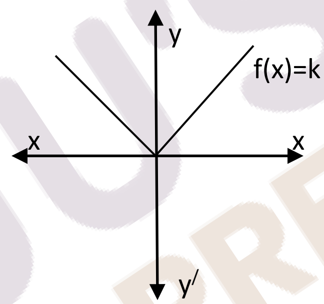
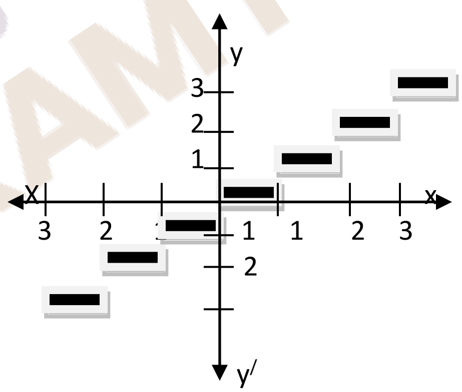
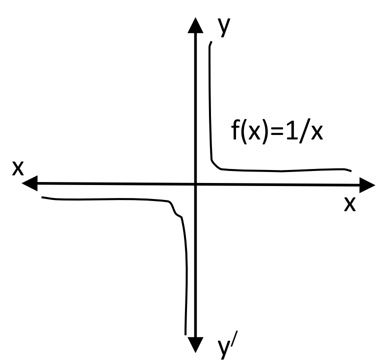
Important: If f and g are two functions defined from set A in to set B, then

1. sum / difference of two functions is (f ± g) (x) = f(x) ± g (x)
2. product of two functions is (f * g) (x) = f(x) * g (x)
3. division of two functions is [f/g] (x) = f(x)/g(x)

Points on graph

- If the graph is symmetrical about y-axis then it is even function.
- If the graph is symmetrical about origin then it is odd function.
- For graph questions it is always better to take values and check the options.

Some important graphs

Modulus Function f(x) = or f(x) = -x when x < 0 X when x ≥ 0 Domain : R Range :R^+	
Greatest Integer Functions or Step Function f(x) = $\lfloor x \rfloor$ Domain: R Range : Integer	
Reciprocal function f(x) = 1/x Domain: R – (0) Range : R – (0)	

One to one – Injective – every element of co domain is mapped to almost one element of the domain
one-one into / onto
Bijective - one – one and onto
Surjective – onto

Geometry

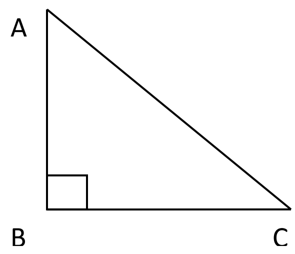
Triangles

1) Area of a triangle = $\frac{abc}{4R} = \frac{1}{2}$ bh = rs

$= \sqrt{s(s-a)(s-b)(s-c)}$

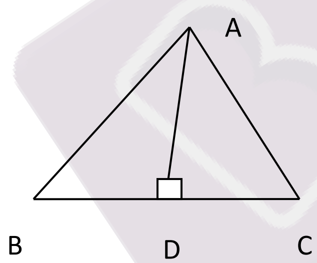
$= \frac{1}{2}ab\sin C = \frac{1}{2}\ bc\ Sin\ A = \frac{1}{2}\ ca\ Sin\ B$

2) Rt angled ∆le



$AB^2 + BC^2 = AC^2$

Acute angled ∆le



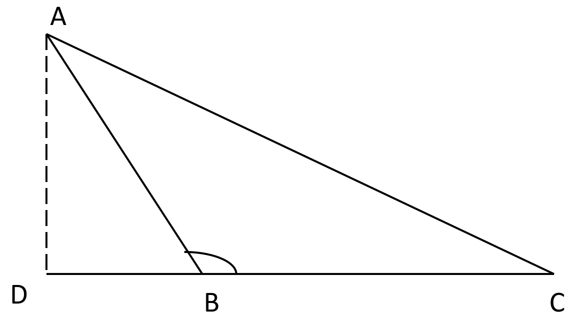
$AC^2 = AB^2 + BC^2 - 2BC.BD$

$Ac^2 < AB^2 + BC^2$

$BC^2 < AC^2 + AB^2$

$AB^2 < AC^2 + BC^2$

Obtuse ∆le



$AC^2 = AB^2 + BC^2 + 2BC.BD$

$AC^2 > AB^2 + BC^2$

3. In a ∆le, a +b > c

b + c > a

c + a > b

a – b < c

a – c < b

a – b < c

Circum centre: Meeting pt of ⊥r bisectors

In centre: Meeting point of angle bisectors

Ex-centre: Meeting point of internal bisector of one angle and external bisectors of other two angles.

Orthocenter: Meeting point of altitudes

Centroid: Meeting point of medians

Circumcentre

Equidistant from vertices

Distance is the circum radius

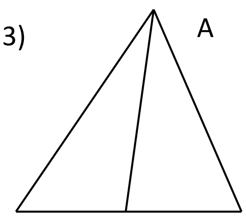
Acute ∆le → circum centre is inside the ∆le

Right ∆le → circum centre is the mid pt of hypotenuse

For a rt ∆le, Median = circum radius =

$\frac{hypotenuse}{2}$

Medians & Centroids

1) A median divides a Δle into 2 Δles of equal area. 3 medians divide a Δle into 6 Δles of equal area
2) Centroid is the point of concurrence. Centroid divides the median in the ratio 2:1
3) 

Apollonius theorem

$AB^2 + AC^2 = 2 (BD^2 + DC^2)$

4) Median to hypotenuse in a right Δle is half the hypotenuse

5) Shortest median is to the longest side and vice versa.

6)

$$\frac{Sum\ of\ squares\ of\ medians}{Sum\ of\ squares\ of\ sides} = \frac{3}{4}$$

7) In an equilateral Δ, In radius = $\frac{Median}{3}$

8) In any Δle, centroid, orthocenter and circum centre are collinear and the centroid divides line joining circum centre or orthocenter in the ratio 2 : 1 (orthocenter & circum centre?

In-centre and In-radius

1. In radius is equidistant from the sides
2. For any Δle, In radius <

$$\frac{Shortest\ Altitude}{2}$$

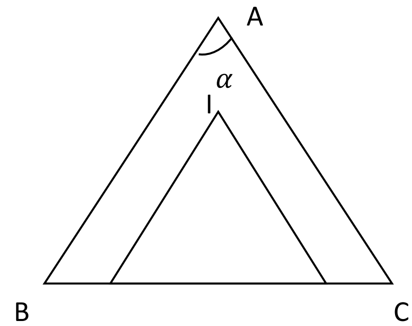
3. In an equilateral Δle,

   All the centres coincide

$$\frac{circumradius}{In\ radius} = \frac{2}{1}\ ;\ \frac{Area\ of\ Circumcircle}{Area\ of\ in\ circle} = \frac{4}{1}$$

4) In a right angled Δle with integer sides, in-radius is an integer.

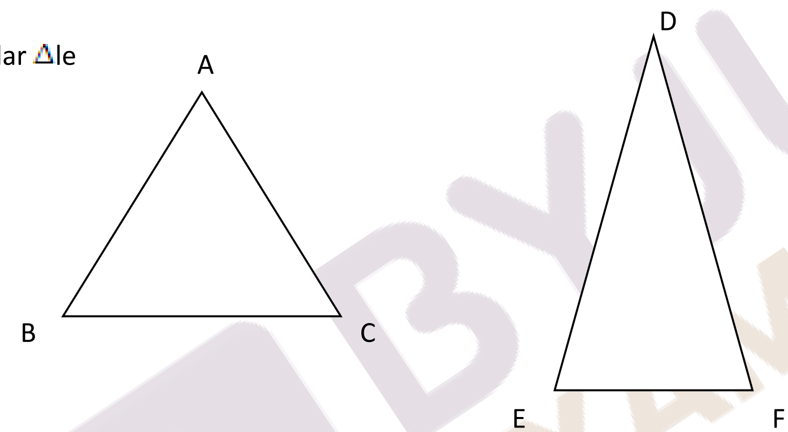
5) In an isosceles Δle, centroid, orthocenter, circum centre and in-centre all lie on the median to the base.

6) 

If I is the in-radius

∠BIC = $90^0 + \alpha/2$

Similar Δle



$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

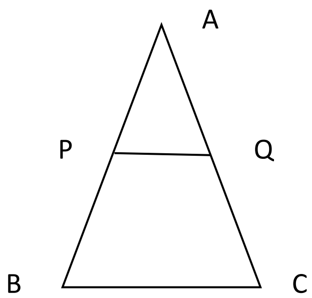
1) Two triangles are similar if
   - The three angles of one Δle are equal to the three angles of the other
2) In two similar Δles

   Ratio of sides = Ratio of heights = Ratio of medians

   = Ratio of in-radii = Ratio of circum radii = Ratio of length of angle bisectors = Ratio of perimeters

   $= \sqrt{\frac{A_1}{A_2}};\ \frac{A_1}{A_2} = Ratio\ of\ Areas$
3) Two similar Δles of equal areas are congruent

Angle Bisector theorem



If PQ II BC

$\frac{AP}{PB} = \frac{AQ}{QC} = \frac{m}{n}$

$PQ = \frac{m}{m+n} BC$

Also

$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$

Line segment joining mid points of 2 ∆les is

$\frac{1}{2}$ the third side & parallel to it.

Equilateral ∆les

1. Side = a

Area = $\frac{\sqrt{3}}{4} a^2$

Altitude = $\frac{\sqrt{3}}{2} a$

In-radius = $\frac{a}{2\sqrt{3}}$

Circum radius = $\frac{a}{\sqrt{3}}$

2. If area of equilateral Δles = Area of isosceles Δles then,

   Perimeter of equilateral Δle

   Perimeter of isosceles Δle

3. For a given perimeter, equilateral Δle has the greatest area.

Congruency rules for Δles

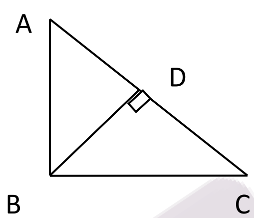
SAS RHS (hypotenuse side)

SSS RAS (one acute angle one side is equal to corresponding angle and side)

ASA

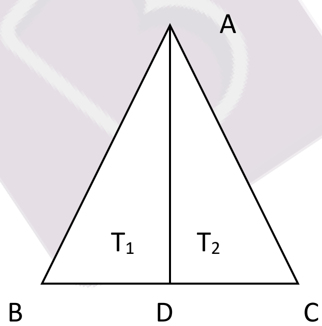
Trivia

1.



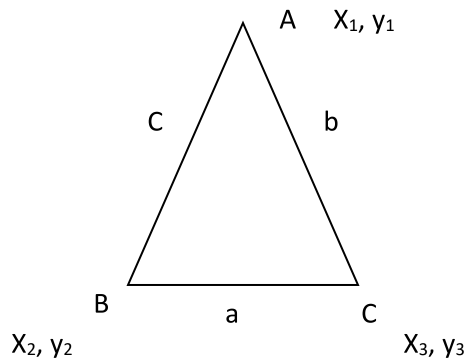
$BD^2 = AD.DC$

2.



If AD divides BC in the ratio m:n, then area of triangles T_1, T_2 is also in the ratio m:n.

3)



In centre =

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

Ex-centre E_1

$$\left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$$

4) When the internal angle bisector and external angle bisector of a Δle, the angle formed is half the angle not bisected.

5) Perimeter of a right angled

Δle = 2r + 4R

R = in-radius

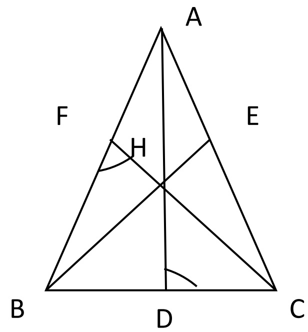
R = Circum radius

- For a right angled Δle , $\frac{R}{r} \geq \sqrt{2} + 1$
  Area = $r^2 + 2Rr$

In an ‘n’ sided figure, the no of Δle that can be formed such that 2 sides of the polygon = n

In an 'n' sided polygon, the number of Δle that can be formed such that 1 side of the Δle coincides with 1 side of the polygon = n(n-4)

6)



H – orthocenter

AFHE, CDHE and BDHF are all cyclic quadrilaterals

$$\frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C} = 2R$$

$$Cos\ A = \frac{b^2+c^2-a^2}{2bc}$$

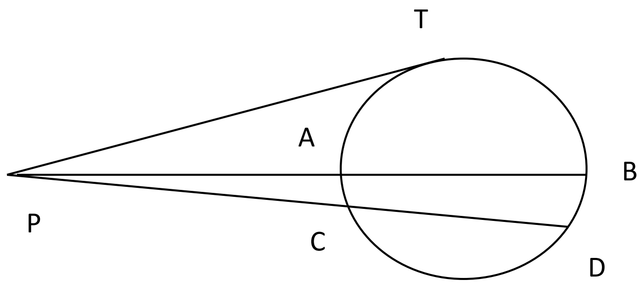
$$Cos\ B = \frac{a^2+c^2-b^2}{2ac}$$

$$Cos\ C = \frac{a^2+b^2-c^2}{2ab}$$

7. Area of Δle formed by joining alternate vertices of a regular hexagon will be half that of the regular hexagon.

Geometry

Circles



PA * PB = PC* PD = PT^2

2) Two tangents can be drawn to a circle from a point and they are equal in length.

A tangent is ⊥ r to the radius at the point of tangency.

3) A perpendicular drawn from the centre of circle to a chord bisects the chord.

->⊥ r bisector of a chord of a circle passes through the centre.

-> The line drawn from the centre to the midpoint of a chord is ⊥ r to the chord.

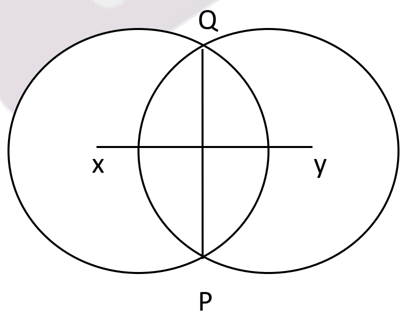
-> Equal chords are equidistant from the centre of a circle.

-> If two chords are equidistant from the centre of a circle, they are equal in length.

-> Equal chords subtend equal angles at the centre.

4) Only one circle passes through three non – collinear points.

5)

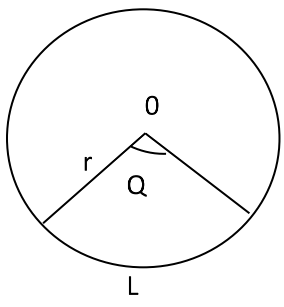


X, Y – centers

PQ – line joining the point of intersection

XY bisects PQ

6) Sector



Area of a sector = $\frac{\vartheta}{360} * \Pi r^2$

$\frac{lr}{2}$

Length of Arc l = $\frac{\vartheta}{360} * 2\Pi r$

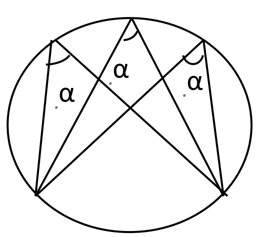
7) All parallelogram inscribed inside a circle is a rectangle.

A trapezium inscribed inside a circle is isosceles and its diagonals are equal.

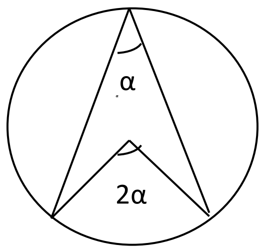
Angle in a semicircle] = 90^0

Area of the largest right angle Δ le in a semicircle = r^2

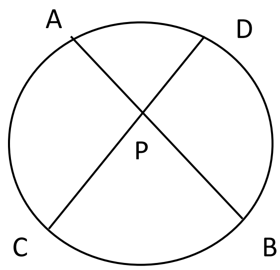
8) Angles in the same segment are equal.



9) Angle at circumference is half the angle at the centre



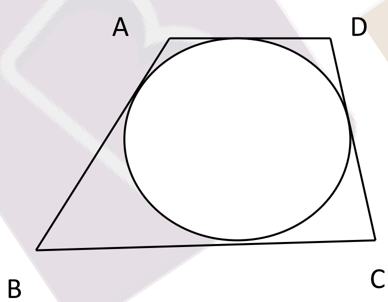
10)



PA * PB = PC *PD

11) Angle in the major segment is acute and in the minor segment is obtuse.

12) If a quadrilateral circumscribes a circle, then sum of opposite sides of the quadrilateral are equal.



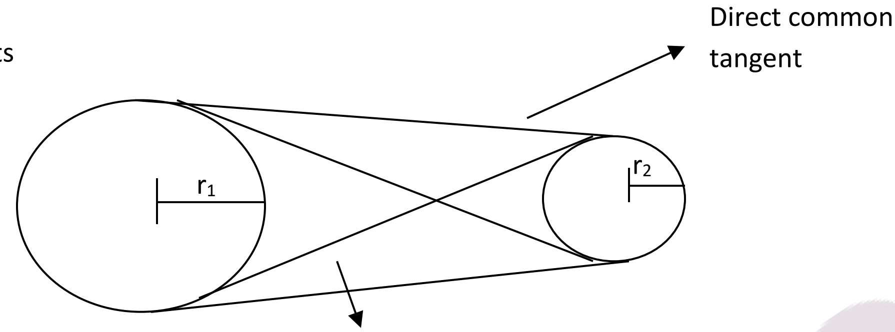
AB + CD = BC + AD

13) If two chords AB and AC are equal, then the bisector of $\angle$BAC passes through the centre of the circle.

- If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel.
- In two concentric circles the chord of the larger circle that is tangent to the smaller circle is bisected at the point of contact.

- When two circles touch each other externally, the point of contact and centres are collinear.
- Two circles can have 0 – 4 common tangents.

Tangents



Length of direct common

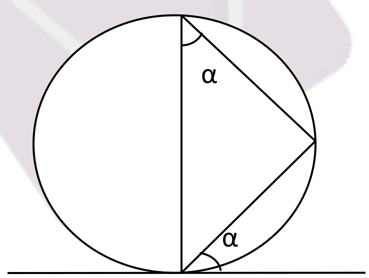
Tangent = $\sqrt{(\textit{Distance between centres})^2 - (r_1 - r_2)^2}$

Length of transverse Common tangent

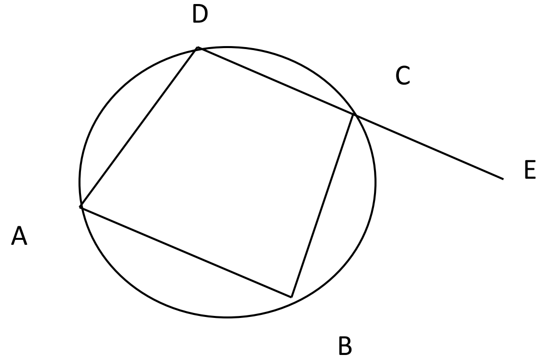
= $\sqrt{(\textit{Distance between centres})^2 - (r_1 + r_2)^2}$

Alternate segment theorem

Angle made by a tangent with a chord at the point of contact is equal to the angle is the alternate segment.



Cyclic Quadrilateral



$\angle A + \angle C = \angle B + \angle D = 180^0$

$\angle BCE + \angle BAD$

Area = $\sqrt{(S - a)(S - b)(S - c)(s - d)}$

S = $\frac{a + b + c + d}{2}$

$d_1d_2 = ac + bd$

d_1d_2 - diagonals

a, b, c, d – sides

(For a convex quadrilateral)

Important Formulae for Geometry and Algebra

3-D Figures:

Prism: $V = Bh$

Pyramid: $V = \frac{1}{3}Bh$

Cylinder: $V = \pi r^2 h$; $SA = 2\pi rh + 2\pi r^2$

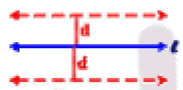
Cone: $V = \frac{1}{3}\pi r^2 h$; $SA = s\pi r + \pi r^2$

Sphere: $V = \frac{4}{3}\pi r^3$; $SA = 4\pi r^2 = \pi d^2$

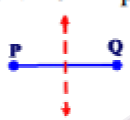
Locus Theorems:

Fixed distance from point.

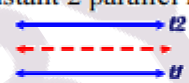
Fixed distance from a line.



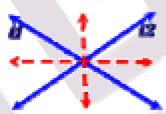
Equidistant from 2 points.



Equidistant 2 parallel lines.



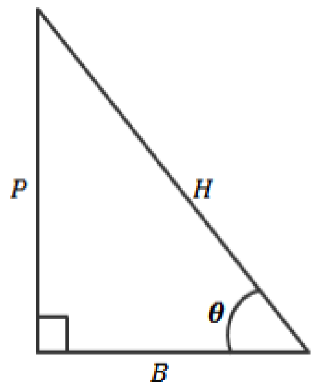
Equidistant from 2 intersecting lines



Slopes and Equations:	Coordinate Geometry Formulas:	Circles:
$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$. $y = mx + b$ slope-intercept $y - y_1 = m(x - x_1)$ point-slope	Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Midpoint Formula: $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Equation of circle center at origin: $x^2 + y^2 = r^2$ where r is the radius. Equation of circle not at origin: $(x-h)^2 + (y-k)^2 = r^2$ where (h,k) is the center and r is the radius.

Trigonometric Ratios

For a right triangle, if P is the length of perpendicular, B is the length of base, H is the length of hypotenuse and θ is the angle between base and hypotenuse,



$$\sin\theta = \frac{P}{H}$$

$$\cos\theta = \frac{B}{H}$$

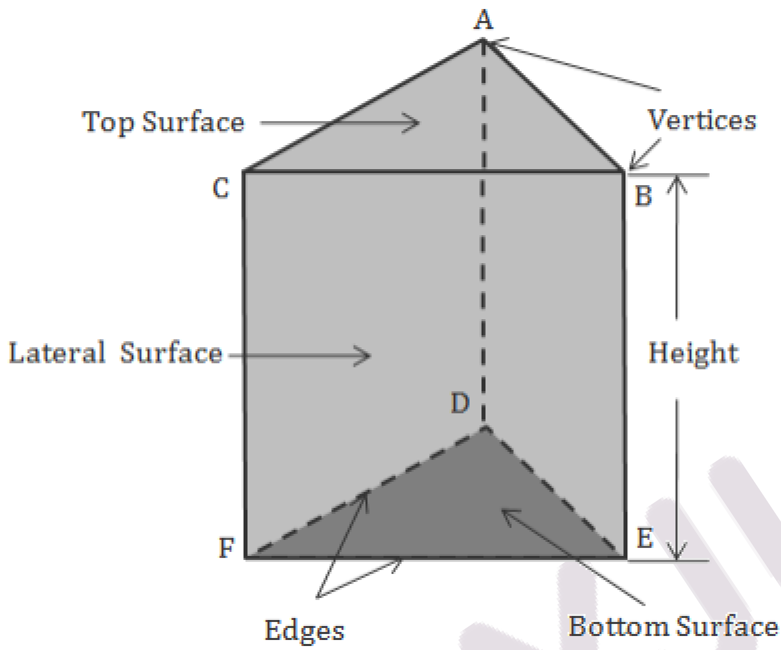
$$\tan\theta = \frac{P}{B}$$

Distance between Points

Distance between two points A (x_1, y_1) and B (x_2, y_2) is given by

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Right Prism

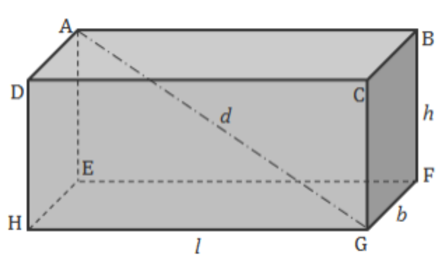


Lateral Surface Area (L.S.A.) = Perimeter of base × height

Total Surface Area (T.S.A.) = L.S.A. + 2 × Area of base

Volume (V) = Area of base × height

Cuboid



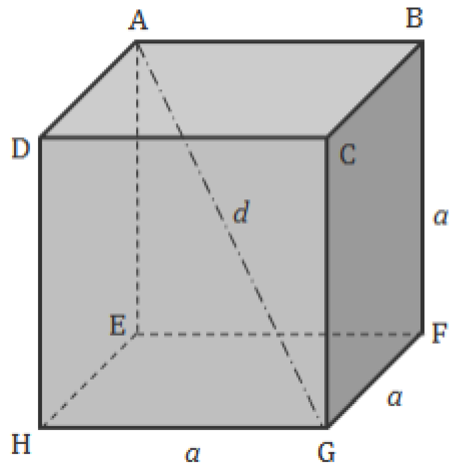
L.S.A. = $2(lh + bh)$

T.S.A. = $2(lh + bh + lb)$

Volume (V) = lbh

Body diagonal (d) = $\sqrt{l^2 + b^2 + h^2}$

Cuboid



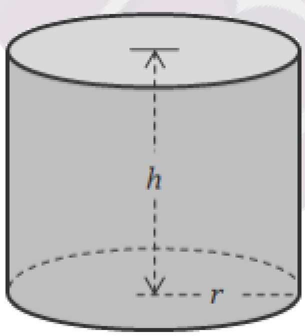
L.S.A. = $4a^2$

T.S.A. = $6a^2$

Volume (V) = a^3

Body diagonal (d) $= a\sqrt{3}$

Cylinder

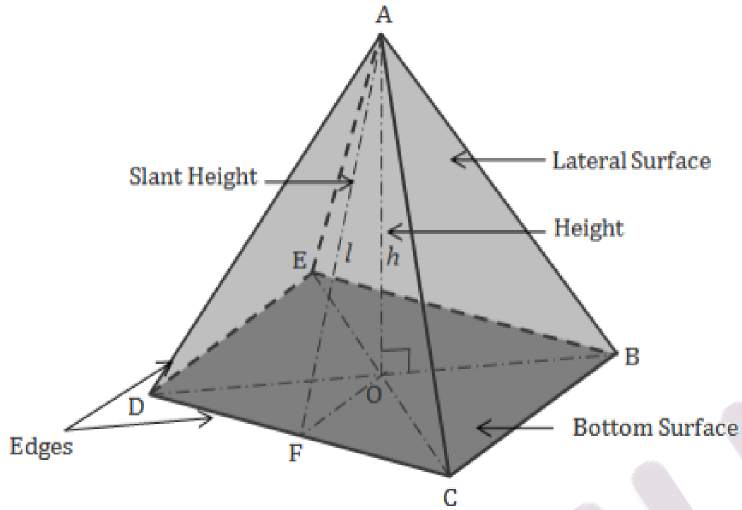


Curved Surface Area (C.S.A.) = $2\pi rh$

T.S.A. = $2\pi rh + 2\pi r^2$

Volume (V) = $\pi r^2 h$

Right Pyramid

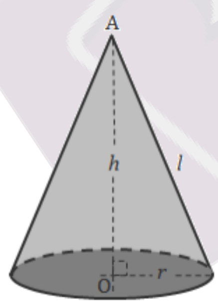


$$\text{L. S. A.} = \frac{1}{2} \times \text{Perimeter of Base} \times \text{Slant Height}$$

T.S.A. = L.S.A. + Area of base

$$\text{Volume } (V) = \frac{1}{3} \times \text{Area of Base} \times \text{Height}$$

Cone



C.S.A. = πrl

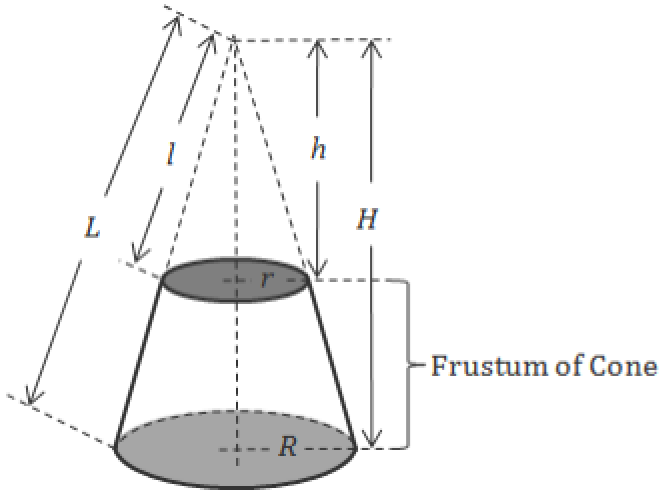
T.S.A. = $\pi rl + \pi r^2$

$$\text{Volume } (V) = \frac{1}{3}\pi r^2 h$$

$$\text{Slant height } (l) = \sqrt{(r^2 + h^2)}$$

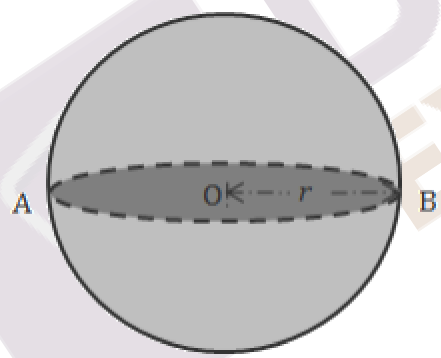
Frustum of Cone

Frustum of a cone



$$\frac{\text{Volume of the original Cone}}{\text{Volume of the removed Cone}} = \frac{V}{v} = \left(\frac{R}{r}\right)^3 = \left(\frac{H}{h}\right)^3 = \left(\frac{L}{l}\right)^3$$

Spehere



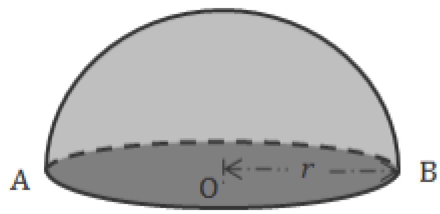
C.S.A. = $4\pi r^2$

T.S.A. = $4\pi r^2$

Volume $(V) = \frac{4}{3}\pi r^3$

Hemisphere

Hemisphere

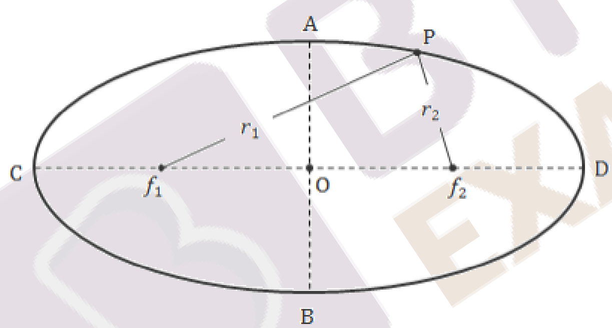


C.S.A. = $2\pi r^2$

T.S.A. = $3\pi r^2$

$$\text{Volume } (V) = \frac{2}{3}\pi r^3$$

Ellipse



If semi-major axis (OD) = a and semi-minor axis (OA) = b,

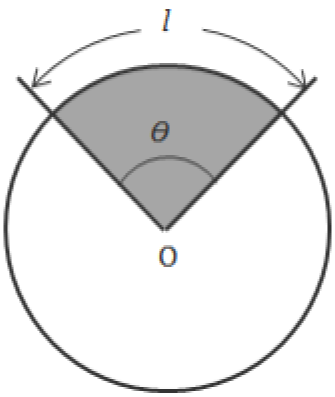
Perimeter of the ellipse

$P_e = \pi(a + b)$

Area of the ellipse

$A_e = \pi ab$

Circle



Circumference

$C = 2\pi r$

Area

$A = \pi r^2$

Length of Arc

$l = 2\pi r\left(\frac{\theta}{360°}\right)$

Area of Sector

$A_S = \pi r^2\left(\frac{\theta}{360°}\right)$

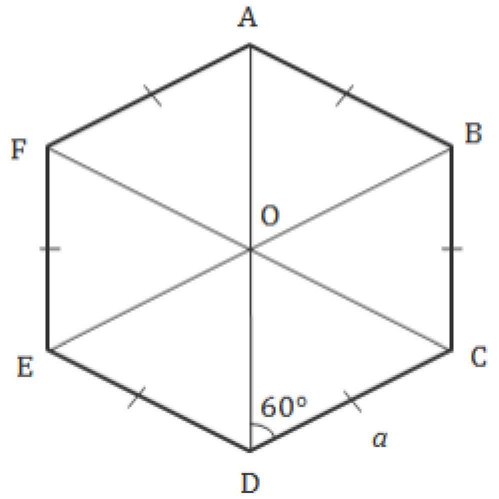
Or,

$A_S = \frac{1}{2} lr$

Perimeter of Sector

$P_S = l + 2r$

Regular Hexagon



$$\text{Area} = \frac{3\sqrt{3}}{2}a^2$$

Polygons

Number of Diagonals

$$N_d = \frac{n(n-3)}{2}$$

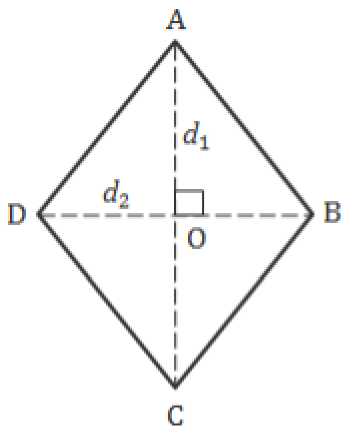
The sum of all the interior angles

$$\sum A_i = (n-2)180^{\circ}$$

The sum of all the exterior angles

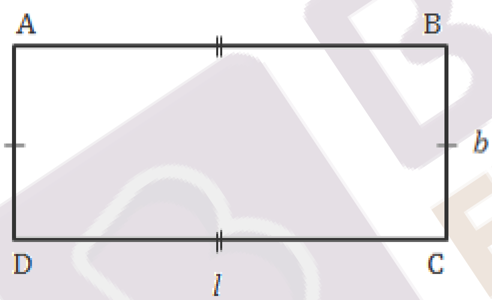
$$\sum A_e = 360^{\circ}$$

Rhombus



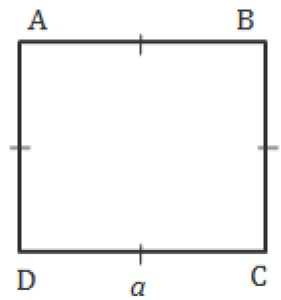
$$\text{Area} = \frac{1}{2}d_1 d_2$$

Rectangle



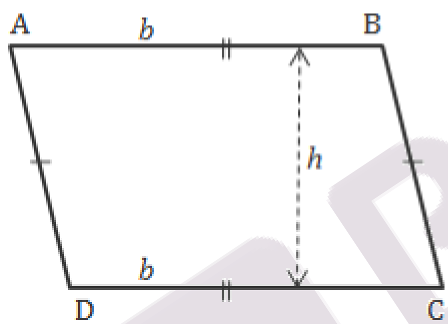
$$\text{Area} = lb$$

Square



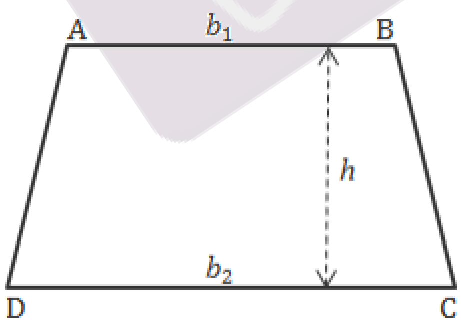
$$\text{Area} = a^2$$

Parallelogram



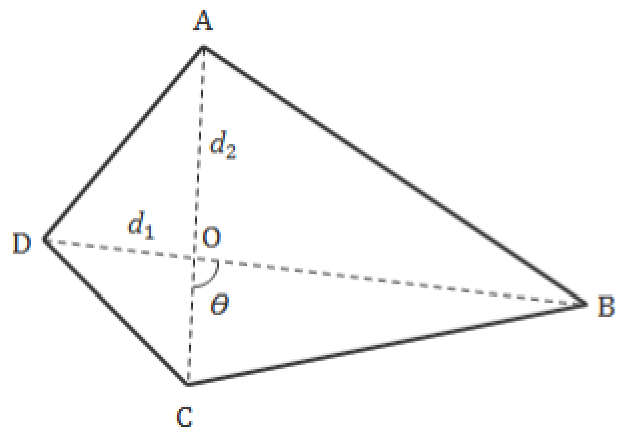
$$\text{Area} = bh$$

Trapezium



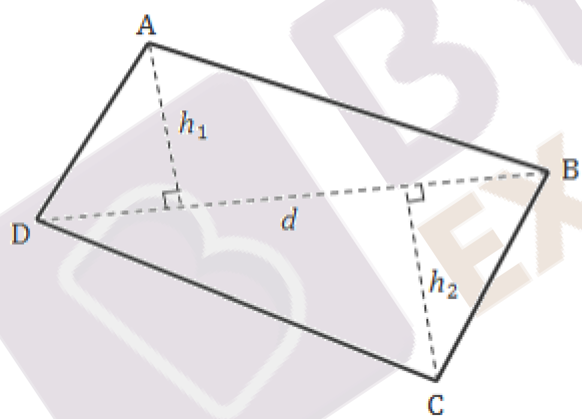
$$\text{Area} = \frac{1}{2}(b_1 + b_2)h$$

If lengths of two diagonal & included angle are given



$$\text{Area} = \frac{1}{2} d_1 d_2 \sin\theta$$

If lengths of one angle & two offsets are given



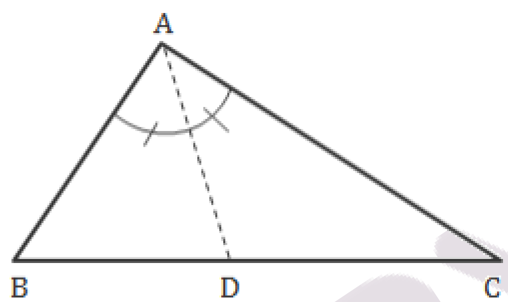
$$\text{Area} = \frac{1}{2} d(h_1 + h_2)$$

Area of Cyclic Quadrilateral

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$\text{where, semiperimeter } (s) = \frac{a+b+c+d}{2}$$

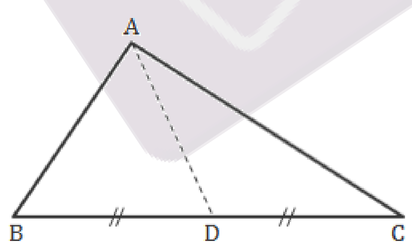
Angle bisector theorem



If AD is the angle bisector for angle A, then:

$$\frac{AB}{BD} = \frac{AC}{CD}$$

Apollonius Theorem

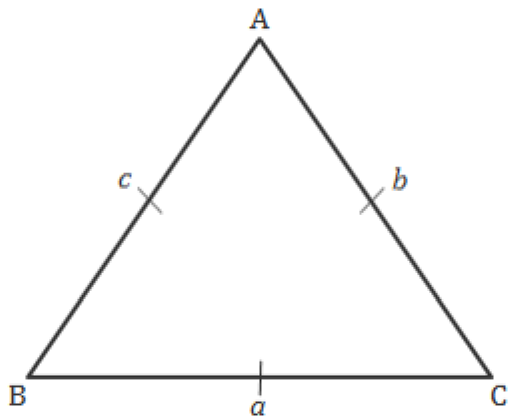


If AD is the median, then:

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

Area of triangle

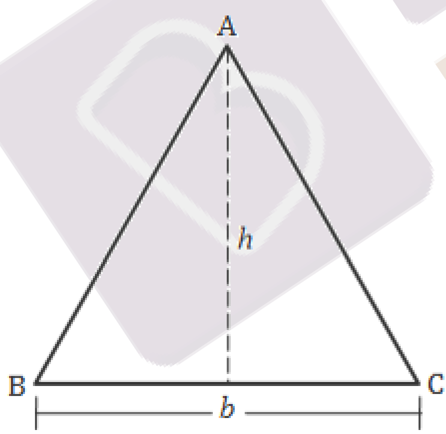
When lengths of the sides are given



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

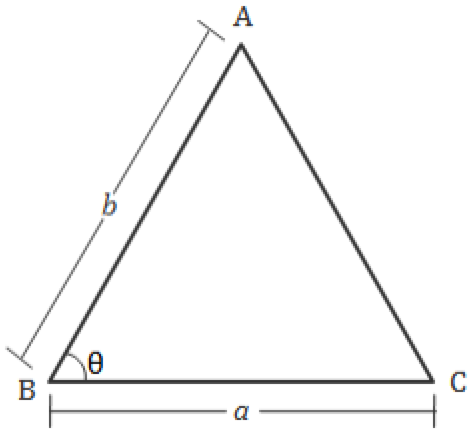
$$\text{where, semiperimeter } (s) = \frac{a+b+c}{2}$$

When lengths of the base and altitude are given



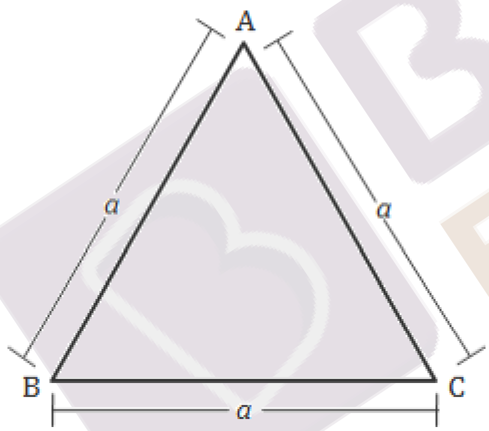
$$\text{Area} = \frac{1}{2}bh$$

When lengths of two sides and the included angle are given



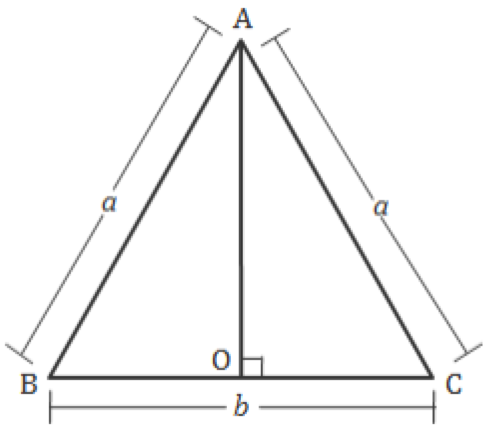
$$\text{Area} = \frac{1}{2}ab\sin\theta$$

For Equilateral Triangle



$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

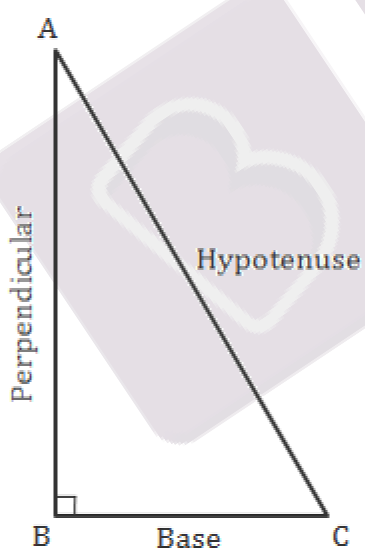
For Isosceles Triangle



$$\text{Area} = \frac{b}{4} \times \sqrt{4a^2 - b^2}$$

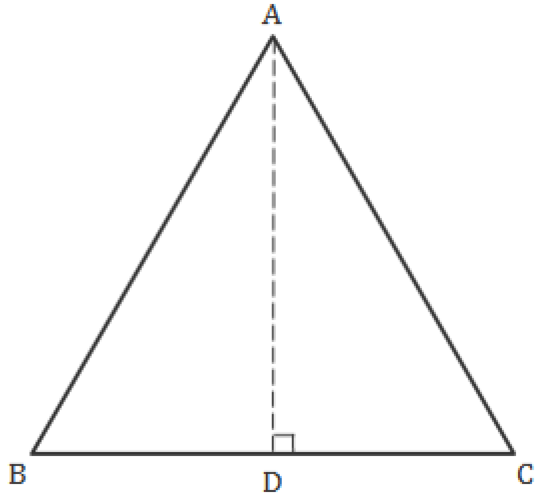
Pythagoras Theorem

For right triangle ABC



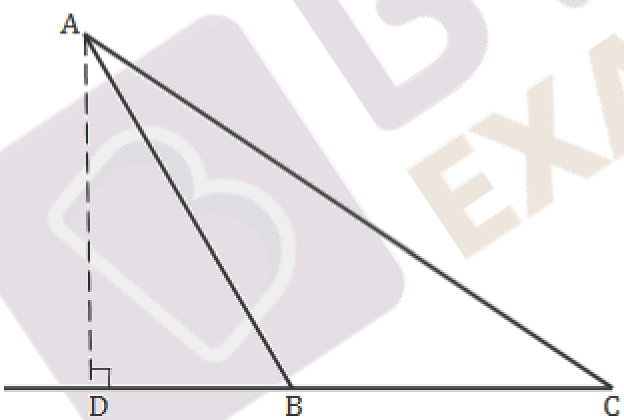
$$AC^2 = AB^2 + BC^2$$

For acute triangle ABC



$$AC^2 = AB^2 + BC^2 - 2 \times BC \times BD$$

For obtuse triangle ABC



$$AC^2 = AB^2 + BC^2 + 2 \times BC \times BD$$

Probability

$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}$$

$$\text{Odds in favour} = \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}}$$

$$\text{Odds against} = \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcomes}}$$

Partition Rule

Number of ways of distributing n identical things among r persons when each person may get any number of things = ${}^{n+r-1}C_{r-1}$

Permutation & Combinations

Permutation

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

Combination

$${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

Properties

$${}^{n}C_{r} = {}^{n}C_{n-r}$$

$${}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + {}^{n}C_{3} + \dots + {}^{n}C_{n} = 2^{n}$$

Factorial

$n! = 1 \times 2 \times 3 \times \ldots \times (n-1) \times n$

$n! = n \times (n-1)!$

Sum of first *n* natural numbers

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

Sum of the squares of the first *n* natural numbers

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of the cubes of the first *n* natural numbers

$$1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

AP

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

GP

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{(r-1)}$$

$$S_\infty = \frac{a}{1-r}, \text{for } r < 1$$

HP

$$T_n = \frac{1}{a + (n - 1)d}$$

Algebraic Formulae

$(a + b)(a - b) = a^2 - b^2$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

HCF & LCM of Fractions

$$\text{H. C. F. of fractions} = \frac{\text{H. C. F. of numerators of all fractions}}{\text{L. C. M. of denominators of all fractions}}$$

$$\text{L. C. M. of fractions} = \frac{\text{L. C. M. of numerators of all fractions}}{\text{H. C. F. of denominators of all fractions}}$$

Races & Clocks

Linear Races

Winner's distance = Length of race

Loser's distance = Winner's distance – (beat distance + start distance)

Winner's time = Loser's time – (beat time + start time)

Deadlock / dead heat occurs when beat time = 0 or beat distance = 0

Circular Races

Two people are running on a circular track of length L with speeds a and b in the same direction

- ⇨ Time for 1st meeting = $\frac{L}{a-b}$
- ⇨ Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a},\frac{L}{b}\right)$

Two people are running on a circular track of length L with speeds a and b in the opposite direction

- ⇨ Time for 1st meeting = $\frac{L}{a+b}$
- ⇨ Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a},\frac{L}{b}\right)$

Three people are running on a circular track of length L with speeds a, b and c in the same direction

- ⇨ Time for 1st meeting = LCM $\left(\frac{L}{a-b},\frac{L}{a-c}\right)$
- ⇨ Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a},\frac{L}{b},\frac{L}{c}\right)$

Clocks To solve questions on clocks, consider a circular track of length 360°. The minute hand moves at a speed of 6° per min and the hour hand moves at a speed of ½° per minute.

Time & Work

$$\text{Number of days to complete the work} = \frac{1}{\text{Work done in one day}}$$

Relative Speed for Boats & Streams

$$S_{downstream} = S_{boat} + S_{stream}$$

$$S_{upstream} = S_{boat} - S_{stream}$$

For trains

$$\text{Time} = \frac{\text{Sum of the lengths}}{\text{Relative speed}} = \frac{L_1 + L_2}{S_1 \pm S_2}$$

Average Speed

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{d_1 + d_2 + d_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

If the distance is constant, then average speed is given by harmonic mean of two speeds:

$$S_{avg} = \frac{2S_1S_2}{S_1 + S_2}$$

If the time is constant, then average speed is given by arithmetic mean of two speeds:

$$S_{avg} = \frac{S_1 + S_2}{2}$$

TIME SPEED & DISTANCE

Speed = Distance / Time

1 kmph = 5/18 m/sec; 1 m/sec = 18/5 kmph

$\text{Speed}_{\text{Avg}} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}} = \frac{d_1 + d_2 + d_3 \ldots d_n}{t_1 + t_2 + t_3 \ldots t_n}$

If the distance covered is constant then the average speed is Harmonic Mean of the values ($s_1, s_2, s_3 \ldots s_n$)

⇨ $\text{Speed}_{\text{Avg}} = \frac{n}{1/s_1 + 1/s_2 + 1/s_3 \ldots 1/s_n}$

⇨ $\text{Speed}_{\text{Avg}} = \frac{2s_1 s_2}{s_1 + s_2}$ (for two speeds)

If the time taken is constant then the average speed is Arithmetic Mean of the values ($s_1, s_2, s_3 \ldots s_n$)

⇨ $\text{Speed}_{\text{Avg}} = \frac{s_1 + s_2 + s_3 \ldots s_n}{n}$

⇨ $\text{Speed}_{\text{Avg}} = \frac{s_1 + s_2}{2}$ (for two speeds)

For Trains, time taken = $\frac{\text{Total length to be covered}}{\text{Relative Speed}}$

For Boats,

$\text{Speed}_{\text{Upstream}} = \text{Speed}_{\text{Boat}} - \text{Speed}_{\text{River}}$

$\text{Speed}_{\text{Downstream}} = \text{Speed}_{\text{Boat}} + \text{Speed}_{\text{River}}$

$\text{Speed}_{\text{Boat}} = (\text{Speed}_{\text{Downstream}} + \text{Speed}_{\text{Upstream}}) / 2$

$\text{Speed}_{\text{River}} = (\text{Speed}_{\text{Downstream}} - \text{Speed}_{\text{Upstream}}) / 2$

For Escalators,The difference between escalator problems and boat problems is that escalator can go either up or down.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Important Conversion Factors:

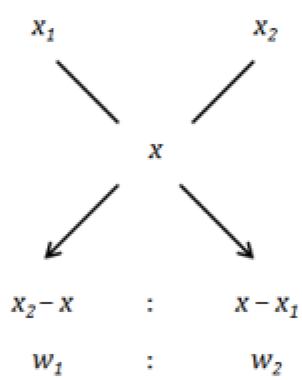
$$1\ \text{km/hr} = \frac{5}{18}\ \text{m/s} \ \text{and} \ 1\ \text{m/s} = \frac{18}{5}\ \text{km/hr}$$

Alligation Rule

The ratio of the weights of the two items mixed will be inversely proportional to the deviation of attributes of these two items from the average attribute of the resultant mixture.

$$\frac{w_1}{w_2} = \frac{(x_2 - x)}{(x - x_1)}$$

Alligation Cross:

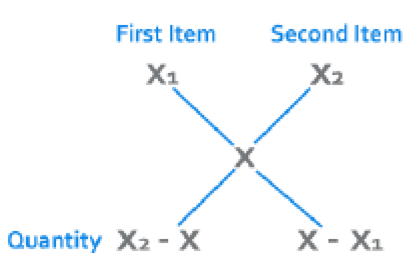


Successive Replacement – Where *a* is the original quantity, *b* is the quantity that is replaced and *n* is the number of times the replacement process is carried out, then

$$\frac{\text{Quantity of original entity after n operation}}{\text{Quantity of mixture}} = \left(\frac{a-b}{a}\right)^n$$

Alligation – The ratio of the weights of the two items mixed will be inversely proportional to the deviation of attributes of these two items from the average attribute of the resultant mixture

$$\Rightarrow \frac{\text{Quantity of first item}}{\text{Quantity of second item}} = \frac{x_2 - x}{x - x_1}$$



Proportions

If $a:b::c:d$ or $\frac{a}{b}=\frac{c}{d}$, then

$\frac{a}{c}=\frac{b}{d}$ ... Alternendo Law

$\frac{b}{a}=\frac{d}{c}$ ... Invertendo Law

$\frac{a+b}{b}=\frac{c+d}{d}$ ... Componendo Law

$\frac{a-b}{b}=\frac{c-d}{d}$ ... Dividendo Law

$\frac{a+b}{a-b}=\frac{c+d}{c-d}$ ... Componendo and Dividendo Law

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\cdots=k$, then $\frac{a+c+e+\cdots}{b+d+f+\cdots}=k$

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\cdots=k$ and p,q,r are real numbers, then $\frac{pa^n+qc^n+re^n+\cdots}{pb^n+qd^n+rf^n+\cdots}=k^n$

Ratios

If $a:b=c:d$, then $a:b=c:d=(a+c):(b+d)$

If $a<b$, then for a positive quantity x,

$\frac{a+x}{b+x}>\frac{a}{b}$ and $\frac{a-x}{b-x}<\frac{a}{b}$

If $a>b$, then for a positive quantity x,

$\frac{a+x}{b+x}<\frac{a}{b}$ and $\frac{a-x}{b-x}>\frac{a}{b}$

Discounts

Discount = Marked Price – Selling Price

$$\text{Discount Percentage} = \frac{\text{Discount}}{\text{Marked Price}} \times 100$$

Buy x and Get y Free

If articles worth Rs. x are bought and articles worth Rs. y are obtained free along with x articles, then the discount is equal to y and discount percentage is given by

$$\text{Percentage discount} = \frac{y}{x+y} \times 100$$

Successive Discounts

When a discount of $a\%$ is followed by another discount of $b\%$, then

$$\text{Total discount} = \left(a + b - \frac{ab}{100}\right)\%$$

Profit & Loss

Profit = SP – CP

Loss = CP – SP

$$\text{Percentage Profit} = \frac{\text{Profit}}{\text{CP}} \times 100 = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100$$

$$\text{Percentage Loss} = \frac{\text{Loss}}{\text{CP}} \times 100 = \frac{\text{CP} - \text{SP}}{\text{CP}} \times 100$$

False Weights

If an item is claimed to be sold at cost price, using false weights, then the overall percentage profit is given by

$$\text{Percentage Profit} = \left(\frac{\text{Claimed weight of item}}{\text{Actual weight of item}} - 1\right) \times 100$$

Growth

Absolute Growth = Final Value – Initial Value

$$\text{Growth rate for a year} = \frac{\text{Final Value} - \text{Initial Value}}{\text{Initial Value}} \times 100$$

$$\text{S. A. G. R. or A. A. G. R.} = \frac{\text{Growth Rate}}{\text{Number of Years}} \times 100$$

$$\text{C. A. G. R.} = \left(\frac{\text{Final Value}}{\text{Initial Value}}\right)^{\frac{1}{\text{Number of Years}}} - 1$$

[Here, S. A. G. R. = Simple Annual Growth Rate, A. A. G. R. = Average Annual Growth Rate and C. A. G. R. = Compound Annual Growth Rate]

Population

$$P' = P \times \left(1 \pm \frac{r}{100}\right)^n$$

[Here, P = Original population, P' = population after n years, r% = rate of annual change]

Depreciation

$$P' = P \times \left(1 - \frac{r}{100}\right)^n$$

[Here, P = original value, P' = final value after n years, r% = rate of annual depreciation]

Interest

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100} = \frac{P \times R \times N}{100}$$

$$\text{Compound Interest} = P \times \left(1 + \frac{R}{100}\right)^N - P$$

Amount = Principal + Interest

Percentages

Fractions and their percentage equivalents:

Fraction	%age	Fraction	%age
1/2	50%	1/9	11.11%
1/3	33.33%	1/10	10%
1/4	25%	1/11	9.09%
1/5	20%	1/12	8.33%
1/6	16.66%	1/13	7.69%
1/7	14.28%	1/14	7.14%
1/8	12.5%	1/15	6.66%

$$\text{Percentage Change} = \frac{\text{Final Value} - \text{Initial Value}}{\text{Initial Value}} \times 100$$

For two successive changes of $a\%$ and $b\%$,

$$\text{Total Percentage Change} = \left(a + b + \frac{ab}{100}\right)\%$$

Mean

$$\text{Arithmetic Mean} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

$$\text{Geometric Mean} = \sqrt[n]{x_1 \times x_2 \times x_3 \times \ldots \times x_n}$$

$$\text{Harmonic Mean} = \frac{n}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_n}\right)}$$

For two numbers a and b,

$$\text{Harmonic Mean} = \frac{2ab}{a + b}$$

Averages

$$\text{Simple Average} = \frac{\text{Sum of Elements}}{\text{Number of Elements}}$$

$$\text{Weighted Average} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + \cdots + w_nx_n}{w_1 + w_2 + w_3 + \cdots + w_n}$$

Laws of Indices

Laws of Indices

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^{\left(\frac{1}{m}\right)} = \sqrt[m]{a}$

$a^{-m} = \frac{1}{a^m}$

$a^{\left(\frac{m}{n}\right)} = \sqrt[n]{a^m}$

$a^0 = 1$

If $a^m = a^n$, then $m = n$

If $a^m = b^m$ and $m \neq 0$;
Then $a = b$ if m is Odd
Or $a = \pm b$ if m is Even

Last digit of a^n

n(Right) a(Down)	1	2	3	4	Cyclicity
0	0	0	0	0	1
1	1	1	1	1	1
2	2	4	8	6	4
3	3	9	7	1	4
4	4	6	4	6	2
5	5	5	5	5	1
6	6	6	6	6	1
7	7	9	3	1	4
8	8	4	2	6	4
9	9	1	9	1	2

Divisibility Rules

A number is divisible by:

2, 4 & 8 when the number formed by the last, last two, last three digits are divisible by 2,4 & 8 respectively.

3 & 9 when the sum of the digits of the number is divisible by 3 & 9 respectively.

11 when the difference between the sum of the digits in the odd places and of those in even places is 0 or a multiple of 11.

6, 12 & 15 when it is divisible by 2 and 3, 3 and 4 & 3 and 5 respectively.

7, if the number of tens added to five times the number of units is divisible by 7.

13, if the number of tens added to four times the number of units is divisible by 13.

19, if the number of tens added to twice the number of units is divisible by 19.

INDICES & SURDS

1) $a^m * a^n = a^{m+n}$

2) $a^m/a^n = a^{m-n}$

3) $(a^m)^n = a^{mn}$

4) $a^{-m} = 1/a^m$

5) $(ab)^m = a^m b^m$

6) $(a/b)^n = a^n/b^n$

7) $a^0 = 1$; $a^1 = a$

8) If $a^x = b^y = (ab)^z$
   $1/x + 1/y = 1/z$

9) If $a^k > b^l$
   and k = b/bna
   then l = a/bna

10) To compare two nos., take HCF of powers and manipulate the bases

11) $2 \leq (1 + 1/n)^n \leq 2.8$

12) Rationalizing factor is a number that has to be multiplied with a surd to make it a rational no.

13) If a > b, then
    $\sqrt{a+d} - \sqrt{a} < \sqrt{b+d} - \sqrt{b}$

14) $y = a + a^{1/3} + a^{2/3}$
    $Y^3 - 3ay^2 + 3a(a-1)y - a(a-1)^2 = 0$

15) For a surd $a + \sqrt{b}$, a conjugate will be of the form $\pm(a-\sqrt{b})$

16) A square root for the surd $a \pm\sqrt{b}$ will be of the form $\sqrt{x} \pm \sqrt{y}$
    If $\sqrt[3]{a + \sqrt{b}} = x + \sqrt{y}$, then $\sqrt[3]{a - \sqrt{b}} = x - \sqrt{y}$

INEQUALITIES & MODULUS

1) If $a > b$ then
   $a + c > b + c$
   $a – c > b-c$
   For positive c
   $ac > bc$
   $a/c > b/c$

2) If $a_1 > b_1$, $a_2 > b_2$, $a_3 > b_3$ …… $a_n > b_n$
   $a_1 + a_2 + …. a_n > b_1 + b_2 + ….. b_n$
   $a_1 a_2 a_3 ……a_n > b_1 b_2 ….. b_n$

3) If $a > b$, and if p and q are 2 positive integers, then
   $a^{p/q} > b^{p/q}$

4) If a2 + b2 = 1
   x2 + y2 = 1
   then ax + by < 1
   for $a\neq b\neq x\neq y$

5) $N^n > 1*3*5*7 …. (2n-1)$

6) $a^m + b^m/2 > (a+b/2)^m$
   If $a \neq b$ and $m > 1$ (ie) m is not a positive proper fraction.

7) If x, a, b are positive integers and $a > b$
   $(1 + x/a)^a > (1 + x/b)^b$

8) $a^m b^n c^p$ will be maximum when
   $a/m = b/n = c/p$

9) $(n!)^2 > n^n$ for $n > 2$

10) $a^2 b + b^2 c + c^2 a \geq 3abc$

11) a4 +b4 +c4 +d4 ≥ 4 abcd

12) $x + 1/X \geq 2$
    for any positive x
    $x + 1/x = (\sqrt{x})^2 + (1/\sqrt{x})^2 – 2 + 2$

$= (\sqrt{x} - 1/\sqrt{x})^2 + 2$
Minimum value = 0

13) a/b + b/c + c/d + d/a ≥ 4

14) (a +b + c) (1/a + 1/b + 1/c) ≥ 9

15) $GM^2 = AM * HM$

16) If a + b + c + d = constant, then $a^p * b^q * c^r * d^s$ is maximum of
a/p = b/q = c/r = d/s

General rules for inequalities & modulus

1) AM ≥GM ≥ HM
a + b/2 ≥ $\sqrt{ab}$ ≥ 2ab/a + b

2) For any positive number x>0
X + 1/x ≥ 2 ; when x = 1 => x +1/x = 2

3) For x ≥ 1
2 ≤ (1 + 1/x)x < e ≈ 2.78

4) For a given sum, the product is maximum when the variables are equal.

a + b		8
a	b	ab
1	7	7
2	6	12
3	5	15
4	4	16

5) For a given product, the sum is minimum when the variables are equal.

ab = 16	a b	b + b
	1 * 16	17
	2 * 8	10
	4 * 4	8

6) For positive numbers a and b where $a > b$
   $1/a < 1/b$
   $a/c < b/c$ if c is negative
   $a/c > b/c$ if c is positive
   $a^2 > b^2$
   $a^n > b^n$ if n is positive

   Modulus:

1) $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & x \leq 0 \end{cases}$

2) $|x| \geq 0$ $\quad - |x| \leq 0$

3) $|x + y| \leq |x| + |y|$

4) $\big||x| - |y|\big| \leq |x + y|$

5) $- |x| \leq x \leq |x|$

6) $|x * y| = |x| * |y|$

7) $|x/y| = |x| / |y|$

8) $|x|^2 = x^2$

General Rules for Quadratic Inequalities:

1) A quadratic inequality of the type $(x – p)(x – q) < 0$ is satisfied by all values of x for $p < x < q$.
   A quadratic inequality of the type $(x – p)(x – q) > 0$ is satisfied by all values of x for $- \infty < x < p$ and $q < x < \infty$.

2) For $p > 0$ $\quad |x + y| = p$ then $\quad x = a + p/a – p$

3) If $ax + by = k$, where a, b, x, y, k are all positive then maximum value of $x^m y^n$ is obtained when $ax/m = by/n$
   conversely if $x^m y^n = k$, where $x>0$, $y>0$ and m, n are positive integers, the minimum value of $ax + by$ is realized when $ax/m = by/n$

4) The A, M of M power (m is an integer) of the positive numbers ≥ m^{th} power of the A.M of the numbers

(ie) $x_1^m + x_2^m + x_3^m + \ldots . x_n^m / n \geq (x_1 + x_2 + x_3 \ldots x_n/n)^m$

Where $m \geq 1$ or $m \leq 0$

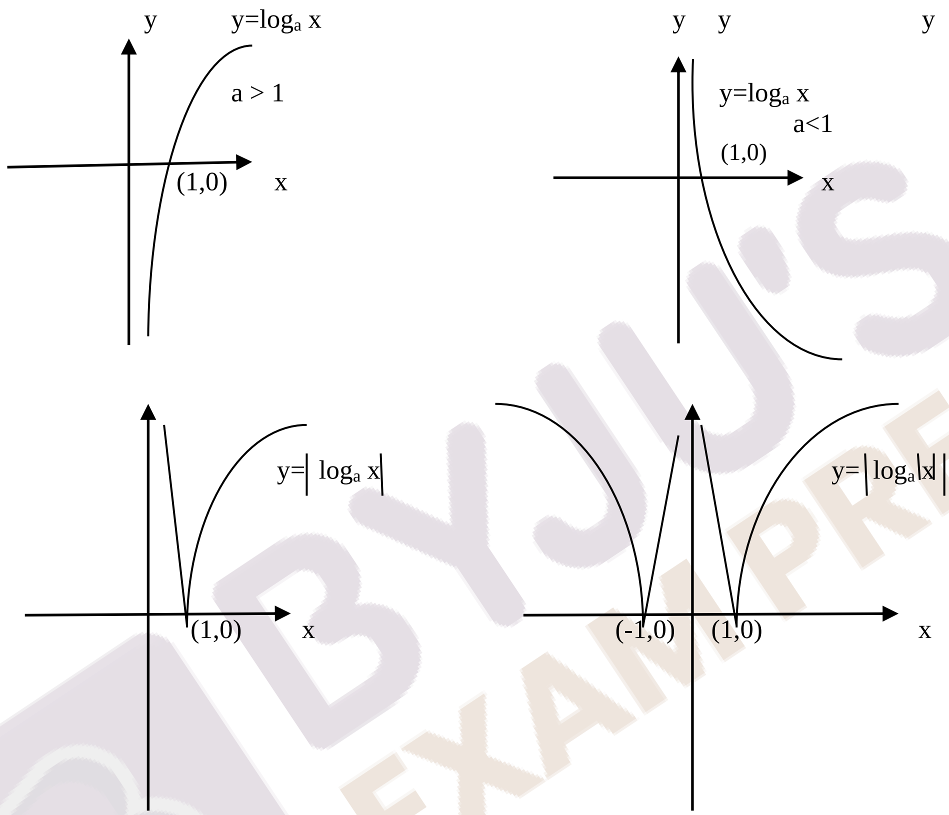
When $0 < m < 1$

The AM of M power ≤ M^{th} power of AM

$X_1^m + x_2^m + \ldots x_n^m/n \leq (x_1 + x_2 + \ldots . xm/n)^m$

LOGARITHMS

1) Logarithms of a number is the exponent of a number to a base.
   If $\log_a N = x$ then $a^x = N$ $N > 0, a > 0 \quad a \neq 1$
   $- \infty \leq x \leq \infty$



Log x is a monotonically increasing or decreasing function depending on the value of x .
Properties of logarithms

1) $\text{Log}_c\, a + \log_c b = \log_c ab$; $\log_c a - \log_c b = \log_c (a/b)$
   Let $\log_c a = x$ $\quad \log_c b = y$ $\quad \log_c ab = z$
   $c^x = a$ $\quad c^y = b$ $\quad c^z = ab$
   $c^{x+y} = ab = c^z \Rightarrow x + y = z$
   (Thirdly $\log p + \log q + \log r = \log pqr$)

2) $\log_b a^m = m \log_b a$

3) $\log_b a \log_c b = \log_c a$

4) $\log_b a = 1/\log_a b \Rightarrow \log_b a \log_a b = 1$

5) $\mathrm{Log}_b n\ a^m = m \log_b n\ ^a = m/n \log_b a$

6) $\mathrm{Log}c^a/\log c^b = \log_c a \log_b c = \log_b a$

7) If $\log_a x = \log_a y$ then $x=y$

8) If $\log_a x = \log_b x$ then $a=b$ if $x \# 1$

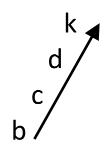
9) $a \log a^x = x$

Let $y = a\log_a x$

$\mathrm{Log}_a y = \log_a a \log a^x = \log a^x$

$\therefore x + y$

10) If $\log_d \log_c \log_b \log_a x = k$



then $x = a$

11) Characteristic : integral part of a logarithm

Mantissa : decimal part of a logarithm

Mantissa is always positive

(for eg) $\bar{4}.3013 = -4 + 0.3013 = -3.6987$

To fin log $\sqrt[5]{0.00000165}$ given log 165 = 2.2174839

And log 697424 = 5.8434968

Log x 1/5 log (0 .00000615) = 1/5 ($\bar{6}$.2174839)

= 1/5 ($\overline{10}$ + 4.2174839 = $\bar{2}$.8434968

= 0. 0697424

12) If logarithm of N is the base ‘a’ is known, then logarithm of N is base ‘b’ can be found by

$\mathrm{Log}_b N = 1/\log_a b \times \log_a N$

13) If ax = by then $\log_b a = y/x$

Exponentials

1. $e = 1 + 1/2! + 1/3! + 1/4! + \ldots\ldots \infty$

2. $e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! + \ldots\ldots. \infty$

3. $e^{ax} = 1 + ax + \alpha 2\ x2/2! + a3\ x3/3! + \ldots\ldots\ \infty$

4. $(1 + x/n)^{n} = 1 + x + x^{2}/2! + x^{3}/3! + x^{4}/4! + \ldots\ldots\ \infty$

5. Logarithmic series
   Loge $(1 + x) = x - x^{2}/2 + x^{3}/3 - x^{4}/4 + \ldots\ldots\ \infty$

NUMBERS

1) Rational Numbers:
   A number which can be represented of the form p/q where p and q are integers and q # 0.
   Any terminating or recurring decimal (eg) 0.333, 0.1667

2) Irrational Numbers:
   A non – terminating non recurring decimal
   $\sqrt{2}$, $\sqrt{3}$, $\sqrt[4]{5}$, π, e

3) Perfect numbers:
   Sum of all factors of number = 2 * number

4) Recurring decimals
   $0.\overline{37} = 37/99$ $0.\overline{225} = 225/999$

   KI.abc$\overline{def}$ = (KIabcdef – Kiabc)/999000

   D = 0.P QQQ ….

   10^p*D = P, QQQ

   $(0^{p+q} - 10^p)$ D = PQ – P

   D = PQ – P/$(10^q - 1)10^p$

   P – figures not recurring and p in number

   Q – figures recurring & consisting of q figures

   $0.1\overline{56}$ = 156 – 1/990; $0.7\overline{3}$ = 73 – 7/90 = 11/15

5) Number of factors of a number:

   N = $a^p b^q c^r$ ….
   no of factors = (p + 1) (q + 1) (n + 1) …..
   a, b. c – distinct prime factors
   no of ways expressing a given no. as a product of two numbers
   = 1/2[(p + 1) (q + 1) ….]

6) For a square number
$= 1/2 [(p + 1) (q + 1) (r + 1) \ldots.. 1]$

7) Sum of all factors of a number

$= (a^{p+1} – 1/a – 1) (b^{q+1} – 1/b – 1) (c^{r+1} – 1/c-1) \ldots.$

No of ways of writing a number as a product of two cop rims = 2^{n-1}

n – no of different prime factor

8) No of co primes to N that are less than N

$\emptyset = N (1 – 1/a) (1 – 1/6) (1-1/c) \ldots\ldots$

Sum of those co primes = $N/2*\emptyset$

9) $(a + b)^2 = a^2 + 2ab + b^2$

$(c – b)^2 = a^2 – 2ab + b^2$

$(a + b+ c)^2 = a^2 + b^2 + c^2 + 2 (ab + bc +ca)$

$(a + b)^3 = a^3 + b^3 + 3ab (a + b)$

$(a – b)^3 = a^3 – b^3 – 3ab (a – b)$

$(a + b) (a – b) = a^2 – b^2$

$a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

$a^3 – b^3 = (a - b) (a^2+ ab + b^2)$

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 +c^2 – ab – bc – ca)$

$a^3 + b^3 + c^3 = 3abc$ if $a + b + c = 0$

$(a^n – b^n) = (a – b) (a^{n-1} + a^{n-2} b + a^{n-3} b^2 \ldots. b^n)$

10) HCF & LCM

$N_1 = Ha$ $N_2 = Hb$ HCF = H LCM = Hab

a, b – co prime to each other

HCF of fractions = HCF of numerators/LCM of denominators

LCM of tractions = LCM of numerators/HCF of denominators

The fractions should be in their simplest form

11) Reminder theorem:

When a polynomial function in n, f(n) is divided by x-a, than the reminder is f(a) Not applicable for exponential/logarithmic/trigonometric functions

12) Fermat' theorem:

- Reminder when a^{p-1} is divided by 'p' is '1' provided 'p' is prime and 'a' is not a multiple for 'p'.
- Remainder when a (p-1)/2 is divided by 'p' could be '±1'.

- Reminder when a (p-1)/4 is divided by 'p' could be ±1.

13) Wilson's theorem

Reminder when (p-1)!+1 is divided by p is Zero if p is a prime no.

14) Successive Division:

The quotient of division is used as the divided in the next division.

```
3 | 800
4 | 263   - 2
5 | 65    - 3
6 | 13    - 0
7 | 2     - 1
    0     - 2
```

Largest power of a number in n!

Largest power of 5 that can divide 216!

```
5 | 216
 5 | 43
  5 | 8
      1
```

43 + 8 + 1 =52 => 5^{52}

Applicable only for prime nos.

For non-prime nos., write it as a product of 2 relative primer

Fermat's last theorem :

There is no natural no. x, whose n^{th} power can be expressed as a sum of two numbers y^n and z^n, where y, z, n are natural nos. >2

15) Rules pertaining to $a^n + b^n$

- never divisible by a – b

- when n is odd, divisible by a + b
- when n is even, not divisible by a + b

Rules pertaining to $a^n - b^n$

- always divisible by a – b
- when n is even, also divisible by a + b
- when n is odd, not divisible by a + b

16) Prime numbers

Any prime no greater than 3 can be expressed of the form 6k ± 1.
(converse is not true)
25 prime nos. from 1-100
21 prime nos. from 101-200

17) Division:

A number divided by a divisor

- $N = d_q + r1$ & $kr_1 = dq_1 + r_2$
  $KN = kdq + kr_1$
  $KN/d = k\ dq/d + kr/d \Rightarrow dq_1 = k\ r_1 - r_2$
  $d > r_1, r_2$
  $N = d\ q_1 + r_1$ $Nk = dq2 + r2$
  $N^k = (dq_1 + r_1)k$
  $dq_1 = r_1^k - r_2$
  $d > r_1, r_2$
- A number N divided by divisors $d_1, d_2 \ldots d_n$ leaves reminder $r_1, r_2 \ldots r_n$ such that
  $d_1 - r_1 - d_2 - r_2 = d_3 - r_3 = n$
  $N = LCM\ [d_1\ d_2 \ldots d_n]\ k - d$

18) Remainders

$R(40/6) = 4$ $= 2\ R(20/3)$
$R(2^{66}/68) = 4 * R(2^{64}/17)$

19) If 'p' is a prime no., the coefficient of every term in $(a + b)^p$ (except the first and the last) is divisible by p.

20) Euler's theorem

$N = P_1^a * P_2^b * P_3^c$, then the number of numbers less than and prime to
$N = Ø(N)$
$Ø(N) = N(1-1/P_1)(1-1/P_2)(1-1/P_3)$

21) Bezant's Identity:

If a & b are non – zero integers with greatest common division d, there exists integers x, y such that ax + by = d

Trivia:

1) Product of factors of N = $x^a y^b z^c$

$= N^{(a + 1)(b + 1)(c + 1)/2}$

2) $1^p + 2^p + 3^p + \ldots\ldots. n^p$ is divisible by (1 + 2 + 3 ……n) if p is odd.

3) Any single digit no. written 3^n times will be divisible by 3^n

4) Fermat's theorem corollary:

If 'p' is prime and 'N' is co prime to p, then $N^p – N$ is divisible by p.

TRIVIA

1) The product of 'r' consecutive numbers is divisible by r!

2) The product of an 'm' digit and 'n' digit number has (m + n – 1) / (m + n) digits.

3) m digit no * n digit no * p digit no

(m + n + p) or (m + n + p – 1) or (m + n + p – 2) digits

4) n^{th} root of an M digit no. will have M/n digits if M/n is an integer or [M/n] + 1digits if M/n is not an integer, where [M/n] is the greatest integer less than M/n.

5) Any number which is a power of 5 greater than 2, last 3 digits – 125 odd power 3 digits – 625 even power.

6) For integers solution $a^2 – b^2 – n$ if n is even, 'n' should be multiple of 4 odd, 'set n' should be possible always.

7) Co primes => 2 nos. whose HCF is unity if x is co prime to N, (N – x) is also co prime to N and (N + x)

8) If the highest power of a prime no ‘p’ in n! is exactly x , ‘n’ can take ‘p’ values. Highest power of a prime no. ‘p’ in ‘n!’ can never be ‘p’

9) Digital sum of a perfect square = 1/4/7/9 (converse is not true)

R (abc abc abc …. 6k) digits/1001 = 0
R (abc abc ……abc) 12k digits/101 = 0

10) A number p = x (x + 3) (x + 6) (x + 9) has a minimum of 2 prime factors.

11) Only 2, 3, 4, 6, 8, 12 & 24 are divisible by all the nos. less than their square root.

12) 14 641 is a perfect square in any base > 6.

13) If x is prime, then for any natural no. $p^x - p$ is divisible by x

14) No of 2 digit nos. divisible by num of their digits = 23
No of 2 digit nos. divisible by product of digits = 5

15) Units digit of xy
- When y is not a multiple of 4
  Y = 4q + r -> remainder when y is divided by 4 units digit of xy is same as xr
- When y is multiple of 4
  Even nos. 2, 4, 6, 8 raised to multiple of 4 gives unit digit of 6
  Odd nos. 1, 3, 7, 9 revised to power of 4 gives unit digit of 1.

16) (abcd … f 5)pqr
If r and f both/either are even, the no ends in 25, if both are odd, the no. ends in 75.

17) The digital sum of a perfect square odds up to 1/4/7/9 but the converse is not true.

NUMBER SYSTEMS

1) If the radix of a number is n, then
$1_2 + 2_3 + \ldots\ldots (n-2)_{n-1} + (n-1)_n = [n(n-1)/2]10$

2) $(abc)_n = an^2 + bn + c$ base – radix

3) Greatest 'n' light no. in a system of radix is $r^n - 1$

4) A no. in the base N is divisible by N – 1 when the sum of the lights of N is divisible by N – 1.

5) When the lights of k light number N_1, written in the base N are rearranged in any order to form a new k digit number N_2, the difference $(N_1 - N_2)$ is divisible by N-1

6) If a no. has an even no. of digits in base N and is a palindrome, it will be divisible by N + 1

Divisibility rules

Test for divisibility by 2 The last digit should be divisible by 2

Test for divisibility by 3 The sum of digits should be divisible by 3

Test for divisibility by 4 The number formed with its last 2 digits should be divisible by 4

Test for divisibility by 5 The last number should be divisible by 5

Test for divisibility by 6 it should be divisible by both 2 and 3

Test for divisibility by 7 subtract two times the unit digit from the remaining number. If it is divisible by 7, then the number is also divisible by7.

Test for divisibility by 8 The number formed by its last 3 digits should be divisible by 8.

Test for divisibility by 9 The sum of the digits should be divisible by 9

Test for divisibility by 10 The last digit should be 0.

Test for divisibility by 11 subtract the unit digit from the remaining number.

Test for divisibility by 13 Add four times the unit digit to the remaining number.

Test for divisibility by 17 subtract five times the unit digit from the remaining number.

Test for divisibility by 19 Add two times the unit digit to the remaining numbers.

Test for divisibility by 23 Add seven times the unit digit to the remaining number.

Question	Approach
Find the least number, which is exactly divisible by x, y, z.	LCM (x, y, z)
Find the least number, which when divided by x, y, z leaves a remainder 'r' in each case.	LCM (x, y, z) + r
Find the least number, which when divided by x, y, z leaves remainders a, b, c respectively.	Observe, if x – a = y – b = z – c = k (say) Then LCM (x, y, z) - k

Find the greatest number, that will exactly divide x, y, z.	HCF (x, y, z)
Find the greatest number, that will divided x, y, z leaving remainders a, b, c respectively.	HCF (x – a, y – b, z – c)
Find the greatest number, that will divide x, y, z leaving the same remainder in each case.	HCF (x – y, y – z, z – x)

Trivia

1) Any perfect square of the form 4k/4k + 1.

2) Perfect square have odd no of factor and prime squares have three factors.

3) Only 5 digit perfect square of the form a b c b a is 111^2 = 12321

4) Only 4 digit perfect square of the form a a b b is 7744.

5) 6 digit no in cyclic form is 142857

6) If a perfect square ends in an odd digit the preceding digit must be even, if a perfect square ends in even the preceding digit must be odd.

7) In every 'n' consecutive numbers there is exactly one multiple of 'n'.

8) K (k + 1) (k + 2) (k + 3) + 1 is always a perfect square

9) If the sum of 'n' natural nos. is a constant then the sum of their squares is minimum when the nos. are as close to each other as possible and maximum when one of the nos. is as close to the constant (ie. Sum) as possible.

PERMUTATIONS AND COMBINATIONS

1)

$1C_1$
1

$2C_0$ $2C_1$ $2C_2$
1 2 1

$3C_0$ $3C_1$ $3C_2$ $3C_3$
1 3 3 1

$4C_0$ $4C_1$ $4C_2$ $4C_3$ $4C_4$
1 4 6 4 1

2)

P/C
/ \
Distinct Non-distinct
/ \ / \
Repetition No Repetition Repetition No Repetition

3) If 'm' distinct items have to be arranged with 'n' forming a pattern, => m! /n!

4) $nC_r = n!/r!\,(n-r)!$ $nP_r = n!/(n-r)!$

$nC_r = n\,C_{n-r}$

$nC_0 + nC_1 + nC_2 + \ldots\ldots$ $nC_n = 2^n$

5) $(n+1)\,C_r = nC_r + nC_{r-1}$
$nP_r = r * (n-1)P_{(r-1)} + (n-1)P_n$

6) No. of diagonals of a 'n' sided figure = nC2-n = n(n-3)/2
In a pentagon, no of sides = no of diagonals.

7) No. of arrangements of 'n' items in which, 'p' are of one type, 'q' are of one type, 'r' are of one type.
= n! /p! q! r!

8) Circular arrangements
* of 'n' items = (n – 1)!
* of 'n' items where clockwise/anticlockwise
Arrangement does not matter = (n – 1)!/2
= n pr/2r (if no diff between clockwise/anti)
(eg) flowers in a garland, beads in a necklace
No. of circular permutations of 'n' different things taken 'r' at a time = nP_r/r

9) No. of ways of answering 'n' questions that have 'k' choices each,

$nc_0 + knc_1 + k^2. nc_2 + ...k^n. nc_n = (1 + k)^n$

10) The sum of all the 'n' digit nos. that can he formed using the digits a, b, c, d .....

Without repetition

S = (sum of digits) * (n – 1)! * (11.........11) n times

S = (n – 2)! (sum of all digits) * [(n – 1) (111 ..... n times) – (111 ..... n-1 times)]

With repetition

S = sum of digits * n^{n-1} * (11 .... 1) n times

11) No. of non-negative integral solutions of

$x_1 + x_2 + x_3 .... x_n = n$

= (n + r – 1) (r – 1)

r – non-identical items   n – identical items

12) No. of ways of selecting 3 numbers from 1 to n + 1 such that they are in AP is n^2.

13) Total no of ways of selecting some or all out of (p + q + r .....) things, where 'p' are alike of one kind, q are alike of one kind, n are alike of one kind = [(p + 1) (q + 1) (r +1) – 1]

14) The no. of ways of selecting 'n' items out of 'n' items, in which 'p' items always occur is (n – p) $c_{(n-p)}$ and no of ways in which 'p' items never occur is (n – p) C_r.

15) No of ways of dividing 'n' items into 'p' groups of 'm' items each is

= n!/m! m! .... m! p!

'p' times

16) The no of different ways of dividing 'mn' items into 'n' equal groups is $(m\ n)!/(m\ !)^2\ n!$

17) No of ways of selecting at least one book from 'n' different books, each of which has 'p' copies is

= $(p + 1)^n – 1$

18) No of ways of selecting at least one item from 'n' items of which 'p' are alike of one kind and 'q' are alike of one kind and 'q' are alike of other kind

= $(p+1) (q+1) 2^{n-(p+q)} – 1$

19) Binomial expansion

$(a + b)^n = n\ C_0a^n + n\ C_1a^{n-1}b^1 + n\ C_2a^{n-2}b^2 + ... b^n$

$n\ c_0 + n\ c_2 + n\ c_4 + ...\ = n\ c_1 + n\ c_3 + n\ c_5 + ...$

20) n c_r is always divisible by n

21) If there are 'n' letters and 'n' envelopes, with each letter corresponding its just one envelope, the no of ways in which the letters can be put into envelopes such that none of the letters go into the correct envelope

$n! – n!/1! + n!/2! – n!/3! + \dots\dots\dots\ n!/n! = \sum_{k=0}^{n}\ (-1)k/k!$

22) Fundamental principle of counting
If Event A happens in m ways
Event B happens in n ways
Event A or Event B can happen in (m + n) ways
Event A and Event B can happen in m * n ways

PROBABILITY

1) Compound events – two one nine events in relation with each other.

2) $P(E) + P(\overleftarrow{E}) = 1$

3) P (x success) = $nc_x\ p^x\ q^{n-x}$ Binomial distribution

4) If 'n' fair coins are tossed/a fair coin is tossed 'n' times
Total no of outcomes = 2n
Probability of getting exactly
'r' heads / tails = $nc_r/2n$

5) Addition theorem of probability
$P(A \cup B) = P(A) + P(B) – P(A \cap B)$
For mutually exclusive events $P(A \cap B) = 0$

6) Independent events $P(A \cap B) = P(A) - P(B)$

7) Expected value = ∈ [probability (∈;) * monetary value associated with event E;]

8) If m is the favourable chances of an event and n is the total chances of the event
odds in favour = m:(n – m)
odds against = (n – m):m

Percentage

A percentage is a ratio expressed in terms of a unit being 100. A percentage is usually denoted by the symbol”%”

- To express a% as a fraction, divide it by 199 => a% = a/100
- To express a fraction as %, multiply it by 100 => a/b = [a/b] *100] %
- X% of y is given by $\frac{x}{100}$ y

Conversion of fractions to percentage:-

$\frac{1}{1}$ = 100% $\frac{1}{2}$ = 50% $\frac{1}{3}$ = 33.33% $\frac{1}{4}$ = 25% $\frac{1}{5}$ = 20% $\frac{1}{6}$ = 16.66% $\frac{1}{7}$ = 14.28% $\frac{1}{8}$ = 12.5%

$\frac{1}{9}$ = 11.1% $\frac{1}{10}$ = 10% $\frac{1}{11}$ = 9.09% $\frac{1}{12}$ = 8.33%

Percentage increases / Decrease

- X increased by 10% is given by x +0.1x = 1.1x
  Similarly 20% more of x = x+0.2x = 1.2x
  10% less of x = x – 0.1x =0.9x
  20% less of x = x-0.2x =0.8x
- If X is n times of y, it means x is (n-1) * 100% more than y.
- Percentage increase = [increase / Original value] * 100%
- Percentage Decrease = [Decrease / Original value] * 100%
- If A is x% more /less, then B is $\frac{100\,x}{100 \pm x}$ %. Less/more than A.

If any number (quantity) is changed (increased/decreased) by p%, then

New quantity = Original quantity *($\frac{100+p*}{100}$)

*p is (-) ve, when the original quantity is reduced by p%.

New value = original value + increase

Or New value = original value – decrease

Percentage change in product of two quantities

Consider a product of two quantities A = a *b

If a and b change (increase or decrease) by a certain percentage say x & y respectively, then the overall % age change in their product is given by the formula:

$x + y + \frac{xy}{100}$

This formula also holds true if there are successive changes as in the case of population increase or decrease. But when there are either more than 2 successive changes or there is a product of more than 2 quantities as in the case of volume, then we have to apply the same formula twice.

This formula can be used for following questions:

- If A is successively increased by X % and Y %, find the percentage increase.
- If there is successive discount of X % and Y %, find the total discount.
- If there is X% increase and y% decrease, find the total change is X - Y - $\frac{XY}{100}$
- If the sides of a rectangle increases by X % and Y %, Find the percentage increase in its area.

Population Increase / Decrease

Let the present population of a town be "p" and let there be an increase / decrease at X % per annum. Then

(i) Population after n year = p [1+ (X/100)]^n

(ii) Population n year ago = p [1 + (X /100)] ^{n}

[X is positive if population is increasing annually and negative if decreasing]

Income Comparison

(i) If A's income is r % more than B's then B's income is [r/(r + 100)] * 100% less than A's

(ii) If A's income is r % less than B's then B's income is [r / (100 - r)] * 100% more than A's

Mixture problems

If X % of a quantity is taken by the first person, y % of the remaining quantity is taken by the second person and z% if the remaining is taken by the third person and if A is left, then initial quantity was.

$= \frac{A*100*100*100}{(100-x)(100-y)(\,100-z)}$

The same concept we can use, if we add something, then the initial quantity was

$= \frac{A*100*100*100}{(100+x)(100+y)(\,100+z)}$

Profit, Loss and Discount

1. Gain or profit = S.P – C.P
2. Profit % = $\frac{S.P-C.P}{C.P}$ * 100 (S.P. is sold price. C.P is cost price)

3. Discount = M.P – S.P (M.P is marked price)
4. Discount % = $\frac{M.P-S.P}{M.P}$*100
5. If the product is constant, and if one quantity increases / decreases by x %, then the other quantity decreases / increases by $\frac{100\,x}{100\pm x}$ %.
6. If the price of an item increases by x%, the consumption has to be reduced by $\frac{100}{100+x}$ % to keep the expenditure constant.
7. If two articles are sold at the same price, and on the first one a shopkeeper makes a profit of p% and on the other suffers a loss of p%, overall he will suffer a loss and it is given by

$$\text{Loss} = \frac{p^2}{100}\%$$

Stocks and shares

- % dividend = $\frac{dividend\ amount}{face\ value}$ * 100
- Average annual growth rate = = $\frac{Growth\ rate}{no\ of\ years}$ * 100
- Compounded Annual Growth rate = $(\frac{Final\ value}{Inital\ value})^{1/\text{no. of years}}$ -1
- If articles worth Rs. X are bought and articles worth Rs. Y are obtained free along with them, Discount = y Discount % = $\frac{y}{x+y}$ * 100
- Successive discounts
- a % discount followed by a b% discount total discount = (a +b - $\frac{ab}{100}$)%

POLYGONS

1) For a polygon of n sides.

   Sum of interior angles } = $(n-2)\ 180^0$

2) For a regular polygon,

   each interior angle} = $\frac{(n-2)180^0}{n}$

3) Sum of all exterior angles = 360^0

4) In a regular polygon, each exterior angle = $\frac{360^0}{n}$

5) No. of diagonals = ${}^nc_2 - n$

   $= \frac{n(n-3)}{2}$

   Holds for convex polygons

6) Area of regular polygon of n sides

   $= n * \frac{a^2}{4} \cot\left(\frac{180}{n}\right)$

Quadrilaterals

TCY online

'N' straight lines on a plane. The no. of regions it can be divided into:

	Total	Unbounded	Bounded
Min	n + 1	n+1	O
Max	$\frac{n(n+1)}{2} + 1$	2n	$\frac{(n-1)(n-2)}{2}$

PROGRESSIONS

1) Harmonic mean of (x_1, x_2, x_3 ……x_n) = $n/1/x_1+1/x_2+…1/x_n$

2) To insert 'n' arithmetic means between a and b

   a + b-a/n+1, a+2(b-a)/n+1, a+3(b-a)/n+1 …… a + n(b-a)/n+1

   A1 A2 A3 An

   Sum of the n arithmetic = n/2(a + b) : common difference

   = n/2(A1+An); d=b-a/n+1

3) If the p^{th}, q^{th},r^{th} terms of an AP are a, b, c, respectively
   (q-r)a + (r-p) b + (p-q) c = 0

4) If the p^{th} term of an AP is 'q' and the q^{th} term is 'p' then the m^{th} term is p + q - m

5) Sum of 'm' terms of an AP/Sum of 'n' terms of an AP=m^2/n^2
   tm/tn = 2m-1/2n-1
   Proof:
   m/2[2a+m-1d]/n/2[2a+n-1d=m^2/n^2
   simplify 2a=d
   tm/tn=d/2+(m-1)d/d/2+(n-1)d

6) Sum of terms of an AP where the no of terms is odd.
   Sn=n t_m, t_n-middle term

7) If the sum up to p terms is same as the sum up to q terms in an AP, than the sum up to (p + q) terms is zero (ie) If $s_p=s_q$ then $s_{p+q}=0$

8) If m, n, p are in AP, then the m^{th} term, n^{th} term, p^{th} term of an AP are also in AP and the m^{th} term, n^{th} term, p^{th} term of a GP are in GP

9) If the positive res a, b. c are in GP, then log a, log b and log c are in AP.

10) S=nx1+(n-1)x2+(n-2)x3 ………2x(n-1)+1xn
S=n(n+1) (n+2)/6
In an AP,
* For a given sum, n can take 2 values if t_1 and common difference are of different signs.

11) An AP:- a, a + d, a+2d, a+3d ….. a+(n-1)d
$t_1+t_n=t_2+t_{n-1} = t_3+t_{n-2} = \ldots\ldots = t_n+t_a\text{-}r+1$
$S=n/2(t_1+t_n)= n/2(t_2+t_{n-1})=n/2(t_n+t_n\text{-}r+1)$
S=n/2[2a+(n-1)d]

12) If the sums up to 'n' terms of two AP are in the ratio $a_n + b/c_n+2$, then the ratio of the k^{th} terms of the two AP, can be obtained by substituting n=2k-1

13) In an AP, if the sum of the first 'm' terms is 'n' and the sum of the first 'n' terms is 'm' then sum of first (m + n) terms = - (m + n)

$S_m = n$
$S_n = m$
$S_m + n = -(m + n)$

14) In an AP, if the no of terms is even,
ε even no terms – ε odd no terms = nd/2

HARMONIC PROGRESSIONS

1) Three quantities a, b, c are in HP when
a/c=a-b/b-c: 1/c-1/b = 1/b-1/a
HM=2ab/a + b for 2 numbers a and b

2) If a, b, c are in HP
a/a-b = a + c/a-c; also 1/b-a+1/b-c=1/a+1/c

3) If the p^{th} term, q^{th} term and r^{th} term of HP are a, b, c respectively
(q-r)bc + (r-p)ca + (p-q)ab = 0

4) AM ≥ GM ≥ HM
$a + b/2 \geq \sqrt{ab} \geq 2ab/a + b$

5) If the m^{th} term of a HP is n, n^{th} term is m, then t_{m+n}=mn/m + n

GEOMETRIC PROGRESSIONS

1) $S_n = a+ar+ar^2+ \ldots\ldots\ldots ar^{n-1}$

   $rS_n = ar+ar^2+ar^3+ \ldots\ldots ar^n$

   $(1-r)s_n = a-ar^n$

   $S_n = a(1-r^n)/1-r$ or $a(r^n-1)/r-1$

   1r1<1 1r1>1

   $S_n = r\, t_n-t_1/r-1$ or $r=s_n-t_1/s_n-t_n$

2) GM of two numbers a, b GM=$\sqrt{ab}$

   GM of 'n' numbers, a_1, a_2 … a_n GM = $\sqrt[n]{a_1 a_2 \ldots\ldots a_n}$

3) If the no of terms of a GP are even

   $R=s_{even}/s_{odd}$=sum of the even no terms/sum of odd no terms

4) Insertion of 'n' geometric means between two quantities a and b

   b – the $(n+2)^{th}$ term → $b=ar^{n+1}$

   $r^{n+1} = b/a$ or $r = (b/a)^{1/n+1}$

   ∴ the G Ms are a, ar, ar^2 ……..ar^n

5) If a, b, c, d are in GP, they are also in continued proportion

   a/b = b/c = c/d = …….1/r

6) If the p^{th}, q^{th} and r^{th} terms of a GP are a, b, c respectively, then

   $a^{q-r}\ b^{r-p}\ c^{p-q} = 1$

7) If a, b. c. d are in GP, then

   $(b-c)^2+(c-a)^2+(d-b)^2=(a-d)^2$

Infinite GP

8) Sum up to infinite numbers of terms for a decreasing GP,

   S=a/1-r

   Sum of the squares of the terms = $a^2/1-r^2$

   Sum of the cubes of the terms = $a^3/1-r^3$

9) If 1x1<1
   $1+x+x^2+$………….= 1/1-x
   $1+2x+3x^2+$ ………= $1/(1-x)^2$
   $1+2x+3x^2+$……….= n terms = $1-a^n/(1-a)^2 - na^n/(1-a)$
10) In an infinite GP, if every term is 'k' times the sum of the terms that follow it.
    r=1/k+1

11) Sum of an in finite GP commencing at any term (say the $(n+1)^{th}$ term) is equal to the preceding term multiplied by r/1-r
    $ar^n+ar^{n+1}+ar^{n+2} = ar^n/1-r=(ar^{n-1})\ r/1-r$

OTHERS

1) ∈ n =n(n+1)/2; ∈ n^2 = n(n+1)(2n+1)/6; ∈ n3 = $[n(n+1)/2]^2 = (\in n)^2$

2) If a1/d1 x d2 + a2/d2 x d3 + a3/d3 x d4 + ……..
   and d2-d1/a1 = d3-d2/a2 = d4-d3/a3 = k
   then multiply and divide the fraction by 'k'

ARITHMETIC GEOMETRIC PROGRESSIONS (AGP)

$a,\ (a + d)r,\ (a + 2d)r^2,\ (a+3d)r^3 +$ …… $(a + (n-1)d)r^{n-1}$

$s = a/1-r + dn(1-r^{n-1})/(1-r)^2 - (a+(n-1)d)r^n/1-r$

COROLLARY

If 1r/<1 and n= ∞
$S = a/1-r + dr/(1-r)^2$

1) $1+4x+9x^2+16x^3+25x^4+$………. = $1+x/(1-x)^3$

GENERAL CASE

2) $S_n = 1+(k-1)x + (2k+1-x)x^2 + (3k+1)x^3 +$ ……..

   S_n = 1+(k-1)x/(1-x)2; applicable only if t1 = 1

3) If the average of 'n' terms if an AP, is a term in the AP, then n is odd. If 'n' is even, the average of the n terms will not be a term in the AP.

4) If a constant k is added to / subtracted from / multiplies or divided every term of an AP, the resulting sequence is also on AP.
5) Sum of any 'n' successive terms in an AP should contain n^2 (for eg.) if ratio of sums of 2 AP's is (2n+3)/(4n+2), how to find the AP's?
   Multiply numerator & denominator by n, $(2n^2+3n)/(4n^2+2n)$
   Sum of 'n' terms of AP_1 = n2 d/2 + d[n(n-1)]/d2 = $n^2d/2 + n(a-d/2)$

TRIVIA

Proof for 7

$Sp = sq$ let $p < q$
$T_{p+1} + t_{p+2} + t_{p+3} + \ldots.. t_q = 0$
q-p/2[a + pd + a+(q-1)d] = 0
q-p/2 [2a + (p+q-1)d] = 0 → 2a+(p+q-1)d = 0
∴ s_{p+q} = p+q/2 [2a + (p-q-1)d] = 0

Proof for 13

S_n = n/2[2a+(n-1)d] = m; sm = m/2 [2a+(m-1)d] = n
$S_n - s_m$ = (n-m)2a + [n(n-1) – m(m-1)d] d = 2(m-1)
2a (n-m) + (n^2-m^2)d – (n-m)d = 2(m-1)
2a + (n+m-1)d = -2
Sm + n = m+n/2 [2a+(n+m-1)d]
= -(m+n)

QUADRATIC EQUATIONS

1) In a quadratic equation, the highest power of the variable is 2, with almost 2 roots standard from $ax^2 + bx + c = 0$

   $x^2 + (b/a) x + (c/a) = 0$
   $x^2 + 2(b/2a)x + (c/a) = 0$
   $x^2 + 2(b/2a)x + (b/2a)^2 – (b/2a)^2 + c/a = 0$
   $(x + b/2a)^2 – b^2/4a^2 + c/a = 0$
   $(x + b/2a)^2 = b^2 – 4ac/4a^2$
   $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

2) Any quadratic equation can be written of the form $k (x – \alpha) (x – \beta) = 0$

   $\alpha + \beta = - b/a$ $\alpha\beta = c/a$ $\alpha - \beta = \frac{\sqrt{b^2-4ac}}{a}$
   X2 – (SR) x + PR = 0

3) Discriminate:

   $b^2 – 4ac < 0$ complex roots
   $b^2 – 4ac = 0$ real, rational & equal
   $b^2 – 4ac > 0$ & a perfect square – rational & unequal
   $b^2 – 4ac > 0$ & not a perfect square – a rational & unequal

4) → A quadratic equation whose roots are k more / less than roots of given equation is obtained by substituting x – k/x + k in place of x.
   → A quadratic equation whose roots are k times / 1/k times the root of an equation is obtained by substituting x/k / kx in place of x
   → A quadratic equation whose roots are reciprocals of the given equation is obtained by substituting 1/x in place of x.
   → To find the equation whose roots are square roots of $ax^2 + bx + c = 0$ (is) $ax^4 + bx^2 + c = 0$
   Not applicable for squares of the equation replace x with $\sqrt{x}$ and square the equation.

5) If a + $\sqrt{b}$ /a + ib is a root of the equation px2 + qx + r = 0, than a - $\sqrt{b}$ /a + ib is the other root provided p, q, r are rational/real.

6) Reciprocal equations:
$ax^4 + bx^3 + cx^2 + bx + a = 0$
Two sets of two roots, where each root is reciprocal to each other.
x = 0 is not a solution.

7) Cubic equations:
$x^3 - (\propto + \beta + \gamma)x^2 + (\propto \beta + \beta \gamma + \beta + \propto \gamma)x - \propto \beta \gamma = 0$

Fourth degree equations:

$x^4 - (sr)x^3 + (spr_2)x^2 - (spr_3)x + pr = 0$
spr_2 – sum of product of roots taken two at a time
spr_3 – sum of product of roots taken three at a time

n^{th} degree equation:

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots a_{n-1}x + a_n = 0$
$\propto_1 + \propto_2 + \propto_3 \ldots\ldots \propto_n = -a_1/a_0$ sum of roots
$\propto_1\propto_2 + \propto_2\propto_3 + \propto_1\propto_3 \ldots\ldots \propto_{n-1}\propto_n = a_2/a_0$
Sum of product of roots two at a time
$\propto_1\propto_2\propto_3 + \propto_1\propto_3\propto_4 + \ldots\ldots \ldots\ldots \propto_{n-2}\propto_{n-1}\propto_n = -a_3/a_0$ SPR_3
$\propto_1\propto_2\propto_3 \ldots\ldots \propto_n = (-1)^n \; a_n/a_0$ PR

8) The expression ax2 + bx + c will have its minimum value at x = -b/2a if a is positive, maximum value cannot be found.
The expression will have its maximum value at x = -b/2a if a is negative, minimum value cannot be found.
   - To find maxima / minima, dy/dx = 0. Because at the point of maxima/minima, the slope is parallel to x axis
   - Maximum value of a
     Quadratic equation = $4ac - b^2/4a$

9) Common roots of two equations:
The equations have common roots only when their curves intersect at x axis.
To find common roots of $f_1(x)$, $f_2(x)$
   (1) $f_1(x) - f_2(x) = 0$
   (2) Solve for x
   (3) Substitute for x in $f_1(x)$, $f_2(x)$

(4) If $f_1(x) = f_2(x) = 0$, then they have common roots otherwise no common roots.

10) Consider $a_0 x^n + a_1 x^{n-1} + \dots\dots\dots$ an = 0
If the no of sign charges = g, then almost of positive roots replace 'x' by '-x' , if the no. of sign charges = n, then almost 'n' negative roots, complex roots brat in pairs

11) A liter of step 7
$x^n + k_1x^{n-1} + k_2x^{n-2} + \dots. K_n = 0$
$k_1 = (-1)$ sum of all roots
$k_2 = (-1)^2 \in$ product of 2 roots
$k_3 = (-1)^3 \in$ product of 3 roots
$k_n = (-1)^n$ product of roots

12) If x2 + an + b = 0, x2 + cn + d = 0 share a common root,
Then the root $\propto$ = b-d/c-a

Simple Interest & Compound Interest:

1) $SI = pnr/100$ $CI = p(1 + r/100)^n - p$

2) If compounding is done continuously $A = pe^{nR/100}$

3) For two years $CI_2 - SI_2 = (r/100)\ SI_1$
(for the same rate of interest 'n')

4) $CI_{k+1} - Ci_k = (r/100)\ CI_k$

5) Present value under CI = final amount/$(1 + r/100)^n$

6) If compounding is done 'k' times a year, (ie once every 12/k months),
$A = p(1 + r/100k)^{kn}$

7) Present value under SI = $A/1 + na/100$

8) Amount to be paid in each installment = $x = Ar/100[1 - (100/100 + r)^n]$
A => Amount that is borrowed (to be repaid 'n' installments)
$x/(1 + r/100) + x/(1 + r/100)^2 + \ldots\ldots x/(1 + r/100)^n = P$

9) Let P the present value; A – the amount
Banker's discount = Anr
True discount = Anr/(1 + nr)

CI = P*(1+R/100)^N-p

The calculation get very tedious when N>2 (more than 2 years). The method suggested below is elegant way to get C1/Amount after 'N' years.

You need to recall the good old Pascal's Triangle in following way:

Code:

Number of years (N)

____

1   1

2   1 2 1

3   1 3 3 1

4   1 4 6 4 1

1……………...1

Example P = 1000, R = 10%, and N=3 years. What is C1 & Amount?

10% of 1000 = 100, Again 10% of 100 = 10 and 10% of 10=1

We did this three times because N = 3

Now amount after 3 years = 1*1000 + 3*100 + 3*10 + 1*1 = Rs.1331/-

The coefficients – 1, 3, 3, 1 are lifted from the Pascal's triangle above.

C1 after 3 years = 3*100 + 3*10 + 3*1 = Rs.331/- (leaving out first term in step 2)

If N = 2, we would have had, Amt = 1*1000 + 2*100 + 1*10 = Rs.1210/-

C1 = 2*100 + 1*10 = Rs.210/-

This method is extendable for any 'N' and it avoids calculations involving higher powers on 'N' altogether!

A variant to this short cut can be applied to find depreciating value of some property, (Example, A property worth 100,000 depreciates by 10% every year, find its value after 'N' years).