## B BYJU'S

## GATE 2023

## Engineering Sciences (XE)

## Questions \& Solutions

Memory Based

## GATE 2023 Engineering Sciences (XE): Major Highlights

> Overall Difficulty Level: Moderate
> MSQ Weightage: 4 Qs.
> NAT Weightage: 39 Qs.
> MCQ Weightage: 22 Qs.
> Thermodynamics was easy to moderate.
> Few questions from PYQs of XE and ME.
> Fluid Mechanics: 22 Qs. (1 Mark - 9 Qs., 2 Marks - 13 Qs.)
> Strength of Materials: 22 Qs. (1 Mark-9 Qs., 2 Marks - 13 Qs.)
> Engineering Mathematics: 15 Qs. (1 Mark - 7 Qs., 2 Marks - 4 Qs.)

GATE 2023 Engineering Sciences (XE): Marking Scheme

| Subject | Total Marks |
| :---: | :---: |
| General Aptitude | 15 |
| Engineering Mathematics (XE-A) | 15 |
| XE-B/C/D/E/F/G/H (Any-One) | 35 |
| XE-B/C/D/E/F/G/H (Any-One) | 35 |
| Total | 100 |

GATE 2023 Engg. Sciences (XE): Last 3 Year's Comparison

| GATE XE | 2023 | 2022 | 2021 | 2020 |
| :---: | :---: | :---: | :---: | :---: |
| MCQ | 39 | 45 | 51 | 52 |
| MSQ | 4 | 6 | 1 | 0 |
| NAT | 22 | 36 | 35 | 35 |

## GATE 2023 Engg. Sciences (XE): Exam Pattern \& Marking Scheme

| Subjects/Sections | 1 Mark Qs. | 2 Mark Qs. | Total Qs. | Total Marks |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| General Aptitude | 5 (1 to 5) | 5 (6 to 10) | 10 | 15 | Compulsory |
| Engineering Mathematics (XE-A) | 7 (11 to 17) | 4 (18 to 21) | 11 | 15 | Compulsory |
| Fluid Mechanics (XE-B) | 9 (22 to 30) | 13 (31 to 43) | 22 | 35 | Any Two Subjects |
| Material Science (XE-C) | 9 (44 to 52) | 13 (53 to 65) | 22 | 35 |  |
| Solid Mechanics (XE-D) | 9 (66 to 74) | 13 (75 to 87) | 22 | 35 |  |
| Thermodynamics (XE-E) | 9 (88 to 96) | 13 (97 to 109) | 22 | 35 |  |
| Polymer Science and Engineering (XE-F) | 9 (110 to 118) | 13 (119 to 131) | 22 | 35 |  |
| Food Technology (XE-G) | 9 (132 to 140) | 13 (141 to 153) | 22 | 35 |  |
| Atmospheric and Ocean Science (XE-H) | 9 (154 to 162) | 13 (163 to 175) | 22 | 35 |  |

1. The village was nested in a green spot
$\qquad$ the ocean and hills
A. through
B. in
C. between
D. at
[MCQ - 1 Mark]
Ans. C
Sol. between
2. A frabjous number is 3 digit number with all odd digits, and no two (adjacent) digits being the same. For e.g., 137 is frabjous and 133 is not frabjous. How many frabjous numbers exist?
A. 125
B. 720
C. 60
D. 80
[MCQ - 1 Mark]
Ans. D
Sol. Odd digits $=1,3,5,7,9$
Total odd digits $=5$
Let the 3 digit numbers be abc
$\rightarrow$ a can be any of the 5 choices.
Since adjacent digits do not repeat
$\rightarrow b$ can be any of the remaining 4 choices.
$\rightarrow c$ can be any of the remaining 4 choices.
Note: Numbers such as 757 will also be frabjous since the criteria is that only adjacent digits are not same. Alternate digits ( $a$ and $c$ ) can be same.
$\therefore$ Total frabjous numbers $=5 \times 4 \times 4=80$
3. Disagree: Protest : : Agree : $\qquad$ (By word meaning)
A. Pretext
B. Refuse
C. Refute
D. Recommend
[MCQ-1 Mark]
Ans. D
Sol. Pretext means an excuse.
Refuse means to reject.
Refute means to disprove.
Recommend means to suggest.
4. Which among the following statements must be TRUE about the mean and the median of the scores of all candidates appearing for GATE 2023?
A. The median is at least as large as the mean.
B. Almost half the candidates have a score that is larger than the mean.
C. The mean is at least as large as the median
D. Almost half the candidates have a score that is larger than the median
[MCQ - 2 Marks]
Ans. D
Sol. 1. Mean can be greater, equal to or less than the median. So, statements (a) and (c) are not accurate.
5. Mean is the average of the values given (in this case, scores of candidates) 3. Median is the middle value among the given set of values (scores of candidates) obtained by arranging the values in ascending or descending order and choosing the middle value. Therefore, almost half the candidates have a score larger than the median (since it is the middle value)
Option (D) is correct.
6. Ankita has to climb 5 stairs starting at the ground while respecting the following rules
7. At any stage, Ankita can move either one or two stairs up.
8. At any stage Ankita cannot move to a lower step.
Let $F(N)$ denote the number of possible ways in which Ankita can reach $N^{\text {th }}$ stair. $F(1)=1, F(2)=2, F(3)=3$ then $F(5)=$
A. 8
B. 5
C. 6
D. 7
[MCQ - 2 Marks]
Ans. A
Sol. According to the rules, number of possible ways for Ankita to climb 5
stairs-
$1+1+1+1+1$
$2+1+1+1$
$1+2+1+1$
$1+1+2+1$
$1+1+1+2$
$2+2+1$
$2+1+2$
$1+2+2$
Note: Number denotes the total number of stairs climbed in one go.
Total possible ways $=8$

## Engineering Mathematics (XE-A)

6. $P$ (telling truth) $=\frac{4}{6}$. An unbiased dice is thrown by the same person twice and the person reports the number appeared in both the throws are same probability rat actual number appeared in both the throws are same $\qquad$ .
[NAT - 2 Marks]
Ans. 0.285
Sol. $P($ truth $)=\frac{4}{6}=\frac{2}{3}$
$\mathrm{P}($ lie $)=\frac{2}{6}=\frac{1}{3}$
Now two condition exist on tossing the dice twice
7. getting same outcome

$$
P(\text { same })=\frac{6}{36}=\frac{1}{6}
$$

2. getting different outcome
$P($ different $)=\frac{30}{36}=\frac{5}{6}$
So,

$P($ of getting same $)=P($ same when reported truth) $+P$ (different when reported lie)

$$
=\frac{1}{6} \times \frac{2}{3}+\frac{5}{6} \times \frac{1}{3}=\frac{7}{18}
$$

Now,

$$
12
$$

$P($ actual same $)=\frac{\frac{1}{6} \times \frac{1}{3}}{\frac{7}{18}}=\frac{2}{7}=0.285$
7. Let $A(3 \times 3)$ eigen value 1,2 and $3, B=$ $A^{2}+2 A+I$ eigen value of $B$
A. 1, 2, 3
B. $4,16,25$
C. 1, 4, 9
D. $4,9,16$

Ans.
Sol. $\lambda_{1}=1$
$\lambda_{2}=2$
$\lambda_{3}=3$
Given $B=A^{2}+2 A+I$
$A=3 \times 3$ matrix
Eigen value of $A$ will be satisfying equation $B$ as well

Hence,
$B\left(\lambda_{1}=1\right)=A^{2}+2 A+I=4$
$B\left(\lambda_{2}=2\right)=A^{2}+2 A+I=9$
$B\left(\lambda_{3}=3\right)=A^{2}+2 A+I=16$
So eigen value of $B$ is $4,9,16$
8. The value of vector field
$\vec{F}(x, y)=\left(4 x^{m} y^{2}=2 x y^{m}\right) \hat{i}+\left(2 x^{4} y-3 x^{2} y^{2}\right) \hat{j}$ is a conservative vector field $\qquad$ _.
[NAT]

Ans. 3
Sol. $\vec{F}(x, y)=\left(4 x^{m} y^{2}=2 x y^{m}\right) \hat{i}+\left(2 x^{4} y-3 x^{2} y^{2}\right) \hat{j}$ For conservative field

$\hat{i}[0]-\hat{j}[0-0]+\hat{k}\left[\left(8 x^{3} y-6 x y^{2}\right)-\left(8 x^{m} y-2 m x y^{m-1}\right)\right]$ $8 x^{3} y-6 x y^{2}=8 x^{m} y-2 m x y^{m-1}$ $m=3$.
9. If the quadrature formula
$\int_{-1}^{1} f(x) d x=\frac{1}{9}\left(C_{1} f(-1)+C_{2} f\left(\frac{1}{2}\right)+C_{3} f(1)\right)$
is exact for all polynomial degree less than or equal 2 then.
A. $C_{1}+\frac{C_{2}}{3}+C_{3}=4$
B. $C_{1}+\frac{C_{2}}{4}+C_{3}=6$
C. $C_{1}+\frac{C_{2}}{2}+C_{3}=2$
D. $C_{1}+C_{2}+C_{3}=5$
[MCQ]
Ans. B
Sol. Let, $f(x)=a+b x+c x^{2}$
$\int_{-1}^{1} f(x) d x$
$=\frac{1}{9}\left[c_{1}(a-b+c)+c_{2}\left(a+\frac{b}{2}+\frac{c}{4}\right)+C_{3}(a+b+c)\right]$
$2 a+\frac{2 c}{3}=\frac{1}{9}\left[\left(c_{1}+c_{2}+c_{3}\right) a+\left(-c_{1}+\frac{c_{2}}{2}+c_{3}\right) b+\right.$
$\left.\left(\mathrm{c}_{1}+\frac{\mathrm{c}_{2}}{4}+\mathrm{c}_{3}\right) \mathrm{c}\right]$
Comparing both side

$$
\begin{array}{l|l}
\frac{C_{1}+C_{2}+C_{3}}{9}=2 & \frac{1}{9}\left(C_{1}+\frac{C_{2}}{4}+C_{3}\right)=\frac{2}{3} \\
C_{1}+\frac{C_{2}}{4}+C_{3}=6
\end{array}
$$

## Fluid Mechanics (XE-B)

10. A water jet $\left(\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}\right)$ is approaching a vertical plate, having an orifice at centre as shown. While a part of the jet passes through orifice and remainder flow along the plate. Neglect friction and assume both the inlet and exits jet to have circular cross section. If $\mathrm{V}=5 \mathrm{~m} / \mathrm{sec}, \mathrm{D}=100 \mathrm{~mm}, \mathrm{~d}=25 \mathrm{~mm}$, magnitude of horizontal force required to hold the plate in its position is

[NAT- 2 Marks]
Ans. 184.07 N

## Sol.



Given,
Diameter, $D=100 \mathrm{~mm}$
$\mathrm{d}=25 \mathrm{~mm}$
Velocity, $V=5 \mathrm{~m} / \mathrm{s}$
Density of water $\rho=100 \mathrm{~kg} / \mathrm{m}^{3}$
Force $=$ change in lines momentum
$F=\rho Q_{1} V-\rho Q_{2} V$
$=\rho V^{2} \frac{\pi}{4}\left[D^{2}-d^{2}\right]$
$=1000 \times 5^{2} \times \frac{\pi}{4} \times\left[0.1^{2}-0.025^{2}\right]$
$=184.07 \mathrm{~N}$
11. Axial velocity profile $u(r)$ for an axisymmetric flow through circular tube of radius R is given as
$\frac{u(r)}{U}=\left(1-\frac{r}{R}\right)^{1 / n}$
where U is centerline velocity. If V refers to area averaged velocity (vol. flow rate per unit area) then the ratio of $V / U$ for $n$ $=1$ is $\qquad$
[NAT - 2 Marks]
Sol. $\frac{u(r)}{U}=\left(1-\frac{r}{R}\right)^{1 / n}$
$\mathrm{U} \rightarrow \mathrm{U}_{\text {maximum }} \rightarrow$ centerline velocity Now,
Discharge $Q=\pi R^{2} V_{\text {avg }}=\int_{0}^{R} u(r) 2 \pi r d r$

$$
R^{2} V_{a v g}=2 \int_{0}^{R} U(1-r / R)^{1 / n} r d r
$$

$R^{2} V_{a v g}=2 U\left[\int_{0}^{R}(1-r / R)^{1 / n} r d r\right]$
As $\mathrm{n}=1$ given
$R^{2} V_{a v g}=2 U\left[\int_{0}^{R} r d r-\int_{0}^{R} \frac{r^{2}}{R} d r\right]$
$=2 U\left(\frac{R^{2}}{2}-\frac{R^{3}}{3 R}\right)$
$R^{2} V_{a v g}=2 U\left(\frac{R^{2}}{6}\right)$
$\frac{V_{\mathrm{avg}}}{\mathrm{U}}=\frac{1}{3}$
12. In steady $2 D$ incompressible flow, $u$ and $v$ are $x$ and $y$ component of flow velocity and $\rho$ is density. Among the following pair of relations, which one satisfies definition of stream function ( $\Psi$ )
[MSQ-2 Marks]
A. $\rho \mathbf{u}=-\frac{\partial \psi}{\partial \mathbf{y}}$ and $\rho \mathbf{v}=\frac{\partial \psi}{\partial \mathbf{x}}$
B. $u=\frac{\partial \psi}{\partial y}$ and $v=-\frac{\partial \psi}{\partial x}$
C. $\rho u=\frac{\partial \psi}{\partial \mathbf{y}}$ and $\rho \mathbf{v}=-\frac{\partial \psi}{\partial x}$
D. $u=-\frac{\partial \psi}{\partial x}$ and $v=-\frac{\partial \psi}{\partial y}$
[MSQ - 2 Marks]
Ans. A, B, C
Sol. Given, steady 2D incompressible flow Stream function is defined for 2-D incompressible flow such that,

1. $u=\frac{\partial \psi}{\partial y}$ and $v=-\frac{\partial \psi}{\partial x}$
2. $u=-\frac{\partial \psi}{\partial y}$ and $v=\frac{\partial \psi}{\partial x}$
3. $\rho \mathbf{u}=-\frac{\partial \psi}{\partial \mathbf{y}}$ and $\rho \mathbf{v}=\frac{\partial \psi}{\partial x}$
4. $\rho \mathrm{u}=\frac{\partial \psi}{\partial \mathrm{y}}$ and $\rho \mathrm{v}=-\frac{\partial \psi}{\partial \mathrm{x}}$
5. The momentum thickness expressed as
[MCQ - 1 Mark]
A. $\int_{0}^{\infty} \frac{u}{u_{\infty}} d y$
B. $\int_{0}^{\infty}\left(1-\frac{\mathrm{u}}{\mathrm{u}_{\infty}}\right) \mathrm{dy}$
C. $\int_{0}^{\infty}\left(1-\frac{u^{2}}{u_{\infty}^{2}}\right) d y$
D. $\int_{0}^{\infty} \frac{\mathrm{u}}{\mathrm{u}_{\infty}}\left(1-\frac{\mathrm{u}}{\mathrm{u}_{\infty}}\right) \mathrm{dy}$

Ans. D
Sol. Momentum thickness: It is the distance perpendicular to the boundary layer of the body over which flow occurs by which the boundary should be displaced to compensate for the reduction in the momentum of the flowing fluid due to boundary layer formation.
It is given by
$\theta=\int_{0}^{\infty} \frac{\mathrm{u}}{\mathrm{u}_{\infty}}\left(1-\frac{\mathrm{u}}{\mathrm{u}_{\infty}}\right) \mathrm{dy}$
14. Water $\left(\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}\right)$ and alcohol ( $\mathrm{G}=$ 0.7 ) enter $Y$ shaped channel at a $Q$ of 0.2 $\mathrm{m}^{3} / \mathrm{sec}$ and $0.3 \mathrm{~m}^{3} / \mathrm{sec}$. Their mix leaves through other end of channel as shown average density of mix is $\qquad$
[NAT - 2 Marks]


Sol.
alcohol ( $\rho_{2}=700 \mathrm{~kg} / \mathrm{m}^{3}$ )
$\mathrm{Q}_{2}=0.3 \mathrm{~m}^{3} / \mathrm{s}$


Applying mass conversation
$\mathrm{m}_{1}+\mathrm{m}_{2}=\mathrm{m}_{3}$
$\rho_{1} Q_{1}+\rho_{2} Q_{2}=\rho_{\mathrm{avg}} \times\left(\mathrm{Q}_{1}+\mathrm{Q}_{2}\right)$
$1000 \times 0.2+700 \times 0.3=\rho_{\text {avg }} \times(0.2+$ 0.3)
$\rho_{\mathrm{avg}}=820 \mathrm{~kg} / \mathrm{m}^{3}$
15. For a potential flow, the fluid velocity is given by
$\vec{V}(x, y)=u \hat{i}+v \hat{j}$
Slope of potential line at $(x, y)$ is
[MCQ - 2 Marks]
A. $\frac{u}{v}$
B. $-\frac{\mathrm{v}}{\mathrm{u}}$
C. $\frac{\mathrm{v}}{\mathrm{u}}$
D. $-\frac{\mathrm{u}}{\mathrm{v}}$

Ans. D
Sol. Potential function $=\phi$
$\mathrm{u}=-\frac{\partial \phi}{\partial \mathbf{x}}$
$v=-\frac{\partial \phi}{\partial y}$
$\mathrm{d} \phi=\frac{\partial \phi}{\partial \mathrm{x}} \mathrm{dx}+\frac{\partial \phi}{\partial \mathrm{y}} \mathrm{dy}$
For equipotential line $d \phi=0$
0 = -udx - vdy
$\frac{d y}{d x}=-\frac{u}{v}$
16. Consider steady fully developed incompressible flow of Newtonian fluid between two infinite parallel plate. The plate moves in the opposite direction as shown. In absence of body force and pressure grad ratio of $\boldsymbol{\tau}_{\text {top }} / \boldsymbol{\tau}_{\text {bottom }}$ ?

[MCQ-2 Marks]
A. $U_{1}+U_{2}$
B. $\mathrm{U}_{1 / 2}$
C. $\frac{U_{1}-U_{2}}{U_{2}}$
D. 1

Ans. D

## Sol.



Relative velocity $=\mathrm{U}_{1}+\mathrm{U}_{2}$
Shear stress

$$
\begin{aligned}
& \tau=\mu \frac{\mathrm{du}}{\mathrm{dy}} \\
& \tau=\frac{\mu\left[\mathrm{U}_{1}+\mathrm{U}_{2}\right]}{\mathrm{H}}=\text { constant } \\
& \frac{\tau_{\text {top }}}{\tau_{\text {bottom }}}=1
\end{aligned}
$$

17. A stationary circular pipe of radius $R=0.5$ $m$ is half filled with water ( $\rho=1000$ $\mathrm{kg} / \mathrm{m}^{3}$ ) whereas upper half is filled with air at atmospheric pressure as shown. $\mathrm{g}=$ $9.81 \mathrm{~m} / \mathrm{sec}^{2}$. Magnitude of force per unit length (in $\mathrm{kN} / \mathrm{m}$ ) applied by water on pipe section $A B$ is

[NAT-2 Marks]

Ans. 2.28

## Sol.


$F_{H}=\rho g(0.5 \times 1) \times \frac{0.5}{2}=1.226 \mathrm{kN}$
$F_{V}=\rho g \times\left(\frac{1}{4} \times \pi \times(0.5)^{2}\right)=1.926 \mathrm{kN}$
$F_{n}=\sqrt{F_{H}^{2}+F_{V}^{2}}$
$=2.28 \mathrm{kN} / \mathrm{n}$
18. Water flow around a thin flat ( 0.25 m long, 2 m wide) with $U_{\infty}=1 \mathrm{~m} / \mathrm{sec}$. Consider linear velocity profile ( $\mathrm{U} / \mathrm{U}_{\infty}=$ $y / \delta)$ for which the laminar boundary layer thick is expressed as $\delta=3.5 x / \sqrt{\operatorname{Re}_{x}}$. For water ( $\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}$ ) and $\mu=0.001$ $\mathrm{kg} / \mathrm{m}$ sec. Net drag force acting on plate, Neglecting the end effects is


Sol. Given,
Length, $L=0.25 \mathrm{~m}$,
Wide, $b=2 \mathrm{~m}$
Velocity, $\mathrm{U}_{\infty}=1 \mathrm{~m} / \mathrm{s}$


Given,
$\delta=\frac{3.5 \mathrm{x}}{\sqrt{\mathrm{Re}_{\mathrm{x}}}}$
$\delta=\frac{3.5 \mathrm{~L}}{\sqrt{\mathrm{Re}_{\mathrm{L}}}}$
$R e_{L}=\frac{\rho U_{\infty} L}{\mu}=\frac{10^{3} \times 1 \times 0.25}{10^{-3}}=250000$
$\delta=\frac{3.5 \times 0.25}{\sqrt{250000}}=1.75 \times 10^{-3}$

We know, $\frac{\tau_{0}}{\rho \mathrm{U}_{\infty}^{2}}=\frac{\mathrm{d} \theta}{\mathrm{dx}}$
$\theta=\int_{0}^{\delta} \frac{U}{U_{\infty}}\left(1-\frac{U}{U_{\infty}}\right) d y=\int_{0}^{\delta}\left(\frac{\mathrm{y}}{\delta}-\frac{\mathrm{y}^{2}}{\delta^{2}}\right) \mathrm{dy}$
$\theta=\frac{\delta}{2}-\frac{\delta}{3}=\frac{\delta}{6}$
From eq. (i)
$\tau_{0}=\frac{1}{6} \rho \mathrm{U}_{\infty}^{2} \frac{\mathrm{~d} \delta}{\mathrm{dx}}$
$\mathrm{F}_{\mathrm{D}}=\int \tau_{0} \times \mathrm{bdx}=\frac{1}{6} \int \rho \mathrm{U}_{\infty}^{2} \frac{\mathrm{~d} \delta}{\mathrm{dx}} \times \mathrm{bdx}$
$=\frac{1}{6} \rho \mathrm{U}_{\infty}^{2} \mathrm{~b} \delta$
$F_{D}=\frac{1}{6} \times 10^{3} \times 1^{2} \times 2 \times 0.00175$
$=0.583 \mathrm{~N}$
19. Water $\left(\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}\right)$ flow steadily with $\mathrm{Q}=0.05 \mathrm{~m}^{3} / \mathrm{s}$ through venturimeter having $\mathrm{d}=100 \mathrm{~mm}$. The pipe diameter D $=200$ mm, and losses are negligible, pressure drop between an upstream location in pipe and the throat is ( kPa )
$\qquad$
[NAT - 2 Marks]
Ans. (18.8 to 19.2)
Sol. Using Bernoulli's equation
$\underbrace{\frac{P_{1}}{p g}+\frac{V_{1}^{2}}{2 g}+Z_{1}}_{\text {upstream }}=\underbrace{\frac{P_{2}}{p g}+\frac{V_{2}^{2}}{2 g}+Z_{2}}_{\text {throat }}$
As, $\mathrm{Q}=0.005 \mathrm{~m}^{3} / \mathrm{sec}=\mathrm{Q}_{1}=\mathrm{Q}_{2}$
Pipe diameter $\Rightarrow d_{1}=200 \mathrm{~mm}$
Throat diameter $\Rightarrow d_{2}=100 \mathrm{~mm}$
Losses are negligible.
$\Delta \mathrm{P}=\mathrm{P}_{1}-\mathrm{P}_{2}=\mathrm{P}\left(\frac{\mathrm{V}_{2}^{2}}{2}-\frac{\mathrm{V}_{1}^{2}}{2}\right)$
$=\mathrm{P}\left(\frac{\mathrm{Q}_{2}^{2}}{2 \mathrm{~A}_{2}^{2}}-\frac{\mathrm{Q}_{1}^{2}}{2 \mathrm{~A}_{1}^{2}}\right)$
$=\frac{\mathrm{PQ}^{2} \times 16}{2 \times \pi^{2}}\left(\frac{1}{\mathrm{~d}_{2}^{4}}-\frac{1}{\mathrm{~d}_{1}^{4}}\right)$
$(\Delta P)=\frac{P^{2} \times 8}{\pi^{2}}\left(\frac{1}{d_{2}^{4}}-\frac{1}{d_{1}^{4}}\right)$
$=\frac{1000 \times 0.05^{2} \times 8}{\pi^{2}} \times\left(\frac{1}{(0.1)^{4}}-\frac{1}{(0.2)^{4}}\right)$
$=\frac{1000 \times 9375 \times 0.05^{2} \times 8}{\pi^{2}}$
$\Delta \mathrm{P}=18997.72 \mathrm{~N} / \mathrm{m}^{2} \mathrm{~Pa}$
$=18.9 \mathrm{kPa}$
20. Consider steady incompressible flow of a Newtonian fluid over a horizontal flat plate Boundary Layer Theory is proportional to
[MCQ - 2 Marks]
A. $x^{1 / 2}$
B. $\mathrm{x}^{-1 / 2}$
C. $x^{2}$
D. $x^{1 / 4}$

Ans. A
Sol. Boundary layer thickness is given by

$$
\begin{aligned}
& \delta=\frac{5 x}{\sqrt{R e_{x}}} \\
& \operatorname{Re}_{\mathrm{x}}=\frac{\rho U_{\infty} \mathrm{x}}{\mu} \\
& \delta=5 \sqrt{\mathrm{x}} \times \sqrt{\frac{\mu}{\rho U_{\infty}}} \\
& \delta \propto \sqrt{x}
\end{aligned}
$$

21. Which is true
(i) Conservation of mass for an unsteady incompressible flow can be represented as $\nabla \cdot \vec{v}=0$.
(ii) Circulation is defined as the line integral of vorticity about a closed loop
(iii) For some fluids, t can be nonlinear function of shear strain rate.
(iv) Integration of Bernoulli's along a streamline under steady state leads to Euler's equation
A. (i), (ii) and (iii)
B. (i) and (ii)
C. (i), (ii) and (iv)
D. (ii) and (iv)
[MSQ]

## Ans. A

Sol. (i), (ii) and (iii) are true.
Integration of Euler's along a streamline under steady state leads to Bernoulli's equation.
22. Among shear stress versus shear strain rate curves, which one corresponds to shear thinning fluids.

[MCQ]
Sol. Pseudoplastic (R) $\rightarrow$ corresponds to shear thinning fluids ( n less than 1 ).
23. A stationary objective fully subject in a static fluid. Which statement is correct?

A. object is in neutral equation if $y_{C G}=y_{C B}$
B. Unstable equation if $y_{C G}=y_{C B}$
C. stable equation if $y_{C G}<y_{\text {CB }}$
D. stable equation if $y_{c G}>y_{c B}$
[MSQ]
Ans. A and D
Sol. Stable $\rightarrow$ CB should be above CG
Unstable $\rightarrow$ CB should be below CG
Neutral $\rightarrow$ CB should coincide CG
24. For a 2D flow field given as $\vec{V}=-x \hat{i}+y \hat{j}$, a streamline passes through points $(2,1)$ and $(5, p)$, value of $p$ is
A. 5
B. $5 / 2$
C. 2
D. $2 / 5$

Ans. B
Sol. Equation of streamline

$$
\begin{aligned}
& \frac{d x}{u}=\frac{d y}{v} \\
& \quad y=-x \hat{i}+y \hat{j} \\
& \text { As } \quad \downarrow \quad \downarrow \\
& u \quad v \\
& -\frac{d x}{x}=\frac{d y}{y} \\
& -\ln x=\ln y+\ln c \\
& \ln (x)+\ln (y)=\ln (c) \\
& \ln (x y)=\ln c \\
& x y=c
\end{aligned}
$$

At point $(2,1)$ and ( $5, \mathrm{p}$ )
$2 \times 1=5 \times p$
Or $p=2 / 5$
25.

A. Bernoulli equation cannot be applied in Region I
B. Bernoulli equation can be applied in Region I only along streamline.
C. Bernoulli equation cannot be applied in Region I
D. Bernoulli equation can be applied in Region I between any two arbitrary points.
[MCQ]
Ans. B
Sol. Bernoulli equation can be applied in Region I only along streamline.

## Solid Mechanics (XE-D)

26. A this walled, closed cylinder vessel of inside diameter $d$ and wall thickness $t$ coteries fluid under pressure p. fig shows part of cylindrical vessel perpendicular to axis of cylinder $\sigma_{1}$ and $\sigma_{2}$ are

A. $\sigma_{1}=\mathrm{pd} / \mathrm{t} \sigma_{2}=\mathrm{pd} / 2 \mathrm{t}$
B. $\sigma_{1}=\mathrm{pd} / 2 \mathrm{t}, \sigma_{2}=\mathrm{pd} / 4 \mathrm{t}$
C. $\sigma_{1}=\mathrm{pd} / 4 \mathrm{t}, \sigma_{2}=\mathrm{pd} / 2 \mathrm{t}$
D. $\sigma_{1}=p d / 2 t, \sigma_{2}=0$
[MCQ - 2 Marks]
Ans. B
Sol. $S_{2}=$ parallel to longitudinal axis
$\mathrm{s}_{2}=$ Longitudinal stress
$=\frac{\mathrm{pd}}{4 \mathrm{t}}$
$\sigma_{1}=$ Hoop stress $=\frac{\mathrm{Pd}}{2 \mathrm{t}}$
27. State of strain at point in machine component is $\epsilon_{x x}=2.5 \times 10^{-4}, \epsilon_{y y}=20 \times$ $10^{-4} \epsilon_{z z}=-1.5 \times 10^{-4}, \epsilon_{x y}=2.5 \times 10^{-4}, \epsilon_{y z}$ $=-0.5 \times 10^{-4}, \epsilon_{y z}=-1 \times 10^{-4}$ volumetric strain is $\qquad$
A. $3 \times 10^{-4}$
B. $-5 \times 10^{-4}$
C. $-3 \times 10^{-4}$
D. $4 \times 10^{-4}$
[MCQ-2 Marks]
Ans. A
Sol. Given,
$\varepsilon_{x x}=2.5 \times 10^{-4} \quad \varepsilon_{y y}=2.0 \times 10^{-4}$
$\varepsilon_{z z}=-1.5 \times 10^{-4}$
$\varepsilon_{x y}=2.5 \times 10^{-4} \quad \varepsilon_{y z}=-0.5 \times 10^{-4}$
$\varepsilon_{\mathrm{zx}}=-1 \times 10^{-4}$
Volumetric strain
$e_{v}=e_{x x}+e_{y y}+e_{z z}$
$=2.5 \times 10^{-4}+2 \times 10^{-4}-1.5 \times 10^{-4}$
$e_{v}=3 \times 10^{-4}$
28. A block of mass $m=10 \mathrm{~kg}$ is lying on inclined plane PQ. The mass is restrained from sliding down by a force F. Coefficient of friction between block and plane is 0.3. The minimum force $F$ required is $\qquad$ N. Take $\mathrm{g}=10 \mathrm{~m} / \mathrm{sec}^{2}$.

[MCQ - 2 Marks]
Ans. (23 to 25)
Sol. Free body diagram of block,

$F+F_{F}-m g \sin 30=0$
$F=m g \sin 30-\mu N$
$F=m g(\sin 30-\mu \cos 30)$
$F=10 \times 10\left[\frac{1}{2}-0.3 \times \frac{\sqrt{3}}{2}\right]$
$\mathrm{F}=24.01 \mathrm{~N}$
29. Consider a solid cylindrical shaft \& a hollow cylinder shaft. Both shafts are axisymmetric \& elastic. Both shafts have same cross section. Hollow shaft has $D_{\text {outside }}=150 \mathrm{~mm}$ and $\mathrm{D}_{\text {inside }}=120 \mathrm{~mm}$. When both shafts twisted by same twisting moment, then ratio of $\tau$ hollow max/ $\tau_{\text {solid max }}$ is $\qquad$ .
[NAT - 2 Marks]
Ans. (0.30 to 0.40)
Sol. Given a solid cylindrical shaft and a hollow cylindrical shaft have same crosssection area and same elastic properties. Hollow $\rightarrow D_{o}=150 \mathrm{~mm}$
$D_{i}=120 \mathrm{~mm}$
$A_{H}=A_{s}$
$\frac{\pi}{4}\left(D_{0}^{2}-D_{i}^{2}\right)=\frac{\pi}{4} D_{S}^{2}$
$150^{2}-120^{2}=D s^{2}$
$\mathrm{D}_{\mathrm{s}}=90 \mathrm{~mm}$
Applying torsional equation
$\frac{\mathrm{T}}{\mathrm{J}}=\frac{\tau_{\max }}{\mathrm{R}}$
Both shafts are twisted by same twisting moment hence
$\tau_{\max } \propto \frac{1}{\mathrm{Z}_{\mathrm{p}}}$
$\frac{\tau_{H, \max }}{\tau_{S, \max }}=\frac{\left(z_{P}\right)_{S}}{\left(z_{P}\right)_{H}}=\frac{\frac{\pi}{16} \times D_{S}^{3}}{\frac{\pi}{16}\left(\frac{D_{o}^{4}-D_{i}^{4}}{D_{o}}\right)}$
Or, $\frac{\tau_{\mathrm{H}, \max }}{\tau_{\mathrm{S}, \max }}=\frac{90^{3} \times 150}{\left(150^{4}-120^{4}\right)}=0.365$
30. For a plane stress problem, the principal stresses are 100 MPa and 50 MPa . The magnitude of maximum shear stress in the material is $\qquad$ _.
[NAT - 1 Mark]
Ans. (50 to 50)
Sol. Given principal stresses,
$\sigma_{1}=100 \mathrm{MPa}$
$\sigma_{2}=50 \mathrm{MPa}$
Maximum shear stress in the material,
abs. $\tau_{\max }=\frac{\sigma_{1}}{2}=\frac{100}{2}=50 \mathrm{MPa}$
31. A slender uniform elastic rod of length 1 m and of solid circular cross section of diameter $D=50 \mathrm{~mm}$ is originally straight. It is then loaded by equal and opposite end moments as shown. The resulting lateral displacement of mid point of the rod is 100 mm . The maximum longitude strain in the rod is $\mathrm{p} \times 10^{-3}, \mathrm{p}=$ ?

[NAT - 2 Marks]

Ans. (1.8 to 2.2)
Sol. Applying bending equation

$$
\begin{align*}
& \frac{\sigma}{y}=\frac{E}{R} \\
& \frac{\sigma}{E}=\frac{y}{R} \\
& \epsilon=\frac{d}{2 R} \tag{i}
\end{align*}
$$


from the properties of the chord of a circle
$\delta(2 R-\delta)=\frac{L^{2}}{4}$
$2 R \delta-\delta=\frac{L^{2}}{4}$
Neglecting $\delta^{2}$ term as compared to $2 R \delta$
$2 R \delta=\frac{L^{2}}{4}$
$2 R=\frac{L^{2}}{4 \delta}$
From equations (i) and (ii)
$\epsilon=\frac{d}{2 R}=\frac{4 \delta d}{L^{2}}=\frac{4 \times 0.01 \times 0.05}{1^{2}}$
$\epsilon=2 \times 10^{-3}$
32. A uniform cantilever beam has flexural rigidity EI and length L . It is subjected to a concentrated force $F \&$ moment $M=2 F L$ at free end as shown. $\delta$ at free end is

A. $\frac{4 \mathrm{FL}^{3}}{3 \mathrm{EI}}$
B. $\frac{7 \mathrm{FL}^{3}}{6 \mathrm{EI}}$
C. $\frac{11 \mathrm{FL}^{3}}{12 \mathrm{EI}}$
D. $\frac{8 \mathrm{FL}^{3}}{9 \mathrm{EI}}$
[MCQ-2 Marks]
Ans. A

## Sol.


$\delta=\delta_{\text {due to }} \mathrm{F}+\delta_{\text {due to }} \mathrm{M}$
$=\frac{F L^{3}}{3 E I}+\frac{\mathrm{ML}^{2}}{2 E I}=\frac{\mathrm{FL}^{3}}{3 E I}+\frac{(2 \mathrm{FL}) \mathrm{L}^{2}}{2 E I}$
$=\frac{4}{3} \frac{\mathrm{FL}^{3}}{\mathrm{EI}}$
33. A plane truss is simply supported at $P$ and $R$ as shown. A downward force $F$ is applied at hinge Q . The axis force developed in member PS is

A. $\frac{\sqrt{5}}{2} F$ (comp.)
B. $\frac{\sqrt{5}}{2} F$ (Tensile)
C. $\sqrt{5} \mathrm{~F}$ (comp.)
D. $\sqrt{5} \mathrm{~F}$ (Tensile)

Ans. A
Sol. $R_{p}$ and $R_{R}$ consider as reaction forces at support.
Taking moment of R .
$F \times L=R p \times 2 L$
$R_{P}=F / 2$


Free body diagram of point P

$\Sigma \mathrm{V}=0$
$F_{P S} \sin \theta=R_{p}$
$F_{P S}\left(\frac{L}{\sqrt{L^{2}+(2 L)^{2}}}\right)=\frac{F}{2}$
$F_{P S} \times \frac{1}{\sqrt{5}}=\frac{F}{2}$
$F_{P S}=\frac{\sqrt{5}}{2} F$ (comp)
34. A steel ball of $m=10 \mathrm{~kg}$ is suspended from the ceiling of a moving carriage by 2 inextensible strings making $60^{\circ}$ angle from horizontal. The carriage has acceleration (a) such that the tension in the string on the right is double the tension in string on left, $\mathrm{g}=10 \mathrm{~m} / \mathrm{sec}^{2}$. $a=$ ?

[MCQ-2 Marks]
Ans. (1.75 to 2.05)

## Sol.


$\Sigma \mathrm{V}=0$
$\mathrm{T} \sin 60^{\circ}+2 \mathrm{~T} \sin 60^{\circ}=\mathrm{mg}$
$3 \mathrm{~T} \sin 60^{\circ}=\mathrm{Mg}$
As per Newton's second law in horizontal direction
$2 \mathrm{~T} \cos 60^{\circ}-\mathrm{T} \cos 60^{\circ}=\mathrm{ma}$
$\mathrm{T} \cos 60^{\circ}=\mathrm{Ma}$
Divide equation (i) by (ii)
$3 \tan 60^{\circ}=\mathrm{g} / \mathrm{a}$

$$
\begin{aligned}
& \mathrm{a}=\frac{\mathrm{g}}{3 \times \tan 60^{\circ}} \\
& \mathrm{a}=\frac{10}{3 \times \sqrt{3}}=1.92 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

35. A massless rigid rod OP of length $L$ is hinged frictionlessly at O . A concentrated mass $m$ is attached to end $P$ of the road. Initially rod $O P$ is horizontal. Then it released from the rest. There is gravity as shown. The rod acquires an angular velocity as it swings. The CW angular velocity of the rod when it first reaches the vertical position as shown.
A. $2 \sqrt{\frac{g}{\mathrm{~L}}}$
B. $\sqrt{\frac{\mathrm{g}}{\mathrm{L}}}$
C. $\sqrt{\frac{2 \mathrm{~g}}{\mathrm{~L}}}$
D. $\frac{1}{2} \sqrt{\frac{\mathrm{~g}}{\mathrm{~L}}}$
[MCQ-2 Marks]
Ans. C
Sol. As per conservation law of energy
$[(P . E)+K . E]_{1}=[P . E+K . E]_{2}$
$(\mathrm{MgL}+0)=0+\frac{1}{2} \mathrm{I} \omega^{2}$
$\mathrm{MgL}=\frac{1}{2} \times \mathrm{ML}^{2} \omega^{2}$
$\omega=\sqrt{\frac{2 g}{L}}$
36. A spherical rigid ball of mass 10 kg moving with $V=2 \mathrm{~m} / \mathrm{s}$ in the direction as shown in figure. Ball collides with a rigid frictionless wall and rebound at an angle $a$ with a speed $u$ as shown. Coefficient of restitution is 0.9. $\alpha=$ ?

[MCQ-2 Marks]
Ans. (56.5 to 58.5)
Sol. Given
Initial velocity of ball, $\mathrm{V}=2 \mathrm{~m} / \mathrm{s}$
Mass of ball, $\mathrm{m}=10 \mathrm{~kg}$
Coefficient of restitution
$\mathrm{e}=0.9=\frac{\mathrm{u} \sin \alpha}{\mathrm{V} \sin 60^{\circ}}$
$0.9=\frac{u \sin \alpha}{2 \times \frac{\sqrt{3}}{2}}$
$u \sin \alpha=0.9 \sqrt{3}$

These in no External force in horizontal direction, so momentum will be conserved in horizontal direct so
$\mathrm{mv} \cos 60^{\circ}=\mathrm{mu} \cos \alpha$
$2 \times \frac{1}{2}=u \cos \alpha$
$\mathrm{u} \cos \alpha=1$
Equations (1) divide by (2)
$\frac{\sin \alpha}{\cos \alpha}=0.9 \sqrt{3}$
$\tan \mathrm{a}=1.558$
$a=57.32^{\circ}$
37. Two equivalent descriptions of state of stress at critical point are shown in figure. The normal stresses $\sigma_{1}$ and $\sigma_{2}$ as shown on the right figure must be, respectively.

A. $-t_{0}$ and $t_{0}$
C. $\frac{\tau_{0}}{\sqrt{2}}$ and $-\frac{\tau_{0}}{\sqrt{2}}$
D. $\tau_{0}$ and $-\tau_{0}$
[MCQ-2 Marks]
Ans. D
Sol. AB plane is diagonal of initial plane state of stress.
Initial plane state of stress is of pure shear stress condition. Hence Its diagonal gives principal stresses.

$\sigma_{1,2}=\frac{\sigma_{x}+\sigma_{y}}{2} \pm \sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau_{x y}^{2}}$
$\because \sigma_{x}=\sigma_{y}=0, \tau_{x y}=\tau_{0}$
$\sigma_{1,2}=\frac{0+0}{2} \pm \sqrt{0+\tau_{0}^{2}}$
$\sigma_{1,2}= \pm \tau_{0}$
$=\left(\tau_{0},-\tau_{0}\right)$
38. A simply supported beam of length 3 m is shown is figure. Shear force at mid point of beam is $\qquad$ kN.


Ans. (4.99 to 5.01)
Sol. Taking moment about A
$\Sigma M_{A}=0$
$30 \times 1-10+(10 \times 1) \times 2.5=R_{D} \times 3$
$\mathrm{R}_{\mathrm{D}}=15 \mathrm{kN}$


For force equilibrium
$\Sigma \mathrm{V}=0$
$30+10=R_{A}+R_{D}$
$40=R_{A}+15$
$\mathrm{R}_{\mathrm{A}}=25 \mathrm{kN}$
Shear force at mid point of beam
( $\mathrm{x}=1.5 \mathrm{~m}$ )

$\mathrm{R}_{\mathrm{A}}=\mathbf{2 5} \mathbf{k N}$
$F_{v}=R_{A}-30$
$F_{v}=25-30$
$\mathrm{Fv}_{\mathrm{v}}=-5 \mathrm{kN}$
$\mathrm{F}_{\mathrm{V}}=5 \mathrm{kN}$ upward
39. A thin steel plate is loaded in the $x-y$ plane. Take Poisson ratio $\mu=0.3, \mathrm{E}=200$ GPa. Strain along the $z$ direction is $\mathrm{E}_{\mathrm{zz}}=$ $-3 \times 10^{-4} \times 5 y=$ ?

120 MPa

[MCQ - 2 Marks]
Ans. (80 to 80)
Sol. Given $\sigma_{x x}=120 \mathrm{MPa}$

$$
\begin{aligned}
& \sigma_{z z}=0 \\
& \varepsilon_{z z}=-3 \times 10^{-4} \\
& \mathrm{E}=200 \mathrm{GPa} \\
& \varepsilon_{z z}=\frac{\sigma_{z z}}{\mathrm{E}}-\mu\left(\frac{\sigma_{x x}}{\mathrm{E}}+\frac{\sigma_{\mathrm{yy}}}{\mathrm{E}}\right) \\
& -3 \times 10^{-4}=0-\frac{0.3}{200 \times 10^{3}}\left(120+\sigma_{y y}\right) \\
& \sigma_{y y}=80 \mathrm{MPa}
\end{aligned}
$$

40. A composite rod made of steel and copper is fixed immovably at its end as shown in figure. The cross section of both portion is same, $\mathrm{E}_{\mathrm{s}}=200 \mathrm{GPa}, \mathrm{E}_{\mathrm{c}}=100 \mathrm{GPa}, \alpha_{\mathrm{s}}=$ $12 \times 10^{-6} /{ }^{\circ} \mathrm{C}, \alpha_{c}=18 \times 10^{-6} /{ }^{\circ} \mathrm{C}$. Composite rods are initially stress free. Then, the temp is increased by $100^{\circ} \mathrm{C}$. The magnitude of axial stress developed in Rod (in MPa)

[MCQ - 2 Marks]
Ans. (198 to 202)
Sol. Given
$\mathrm{Es}_{\mathrm{s}}=200 \mathrm{GPa}$
$\mathrm{E}=100 \mathrm{GPa}$
$\alpha_{\mathrm{s}}=12 \times 10^{-6} /{ }^{\circ} \mathrm{C}$
$\alpha_{c}=18 \times 10^{-6} /{ }^{\circ} \mathrm{C}$
$\Delta \mathrm{T}=100^{\circ} \mathrm{C}$


Total strain in composite rod is zero $\varepsilon_{\mathrm{st}}+\varepsilon_{\mathrm{cu}}=0$
$\left(\alpha_{\mathrm{st}} \Delta \mathrm{T}-\frac{\mathrm{R}}{\mathrm{AE}_{\mathrm{St}}}\right)+\left(\alpha_{\mathrm{Cu}} \Delta \mathrm{T}-\frac{\mathrm{R}}{\mathrm{AE}_{\mathrm{cu}}}\right)=0$
$\therefore \frac{\mathrm{R}}{\mathrm{A}}=$ stress $=\sigma$
$\left(12 \times 10^{-6} \times 100-\frac{\sigma}{200 \times 10^{9}}\right)$
$+\left(18 \times 10^{-6} \times 100-\frac{\sigma}{100 \times 10^{9}}\right)=0$
$30 \times 10^{-4}=\sigma\left(\frac{1}{200 \times 10^{9}}+\frac{1}{100 \times 10^{9}}\right)$
$\sigma=20 \times 10^{7} \mathrm{~Pa}$
$\sigma=200 \mathrm{MPa}$
41. Consider an electric pole with dimensions as shown. Let the end $R$ be subjected to a vertical force $F$. The flexural rigidity of both vertical \& horizontal bars is EI. Neglect axial deflection of vertical bar and all effects of self-weight. The vertical deflection at end $R$ is $\qquad$ _.

A. $10 \mathrm{FL}^{3} / 3 \mathrm{EI}$
B. $7 \mathrm{FL}^{3} / 3 \mathrm{EI}$
C. $5 \mathrm{FL}^{3} / 3 E I$
D. $8 \mathrm{FL}^{3} / 3 E I$
[MCQ - 2 Marks]
Ans. B
Sol. Apply strain energy theorem, $\mathrm{U}=\mathrm{U}_{\mathrm{QR}}+\mathrm{U}_{\mathrm{PQ}}$


Neglecting axial load condition

$$
U=\int_{0}^{L} \frac{(F x)^{2} d x}{2 E I}+\int_{0}^{2 L} \frac{(F L)^{2} d x}{2 E I}
$$

$=\frac{F^{2}}{2 E I}\left(\frac{x^{3}}{3}\right)_{0}^{L}+\frac{F^{2} L^{2}}{2 E I}(x)_{0}^{2 L}$
$U=\frac{F^{2} L^{3}}{6 E I}+\frac{2 F^{2} L^{3}}{2 E I}=\frac{7}{6} \frac{E^{2} I^{3}}{E I}$
$\delta=\frac{\partial \mathrm{U}}{\partial \mathrm{F}}$
$\delta=\frac{14}{6} \frac{\mathrm{FL}^{3}}{\mathrm{EI}}=\frac{7}{3} \frac{\mathrm{Fl}^{3}}{\mathrm{EI}}$
42. A solid uniform rigid disk of mass $m$ and radius $R$ without slipping along $a$ horizontal surface PQ. Speed of centre of disk is $V$. Then disk sticks to a hurdle of height $3 R / 20$ at point $S$. During the impact, there is no rebound or slip at $S$ and no impulse from the surface PQ . Magnitude of velocity of centre of disk immediately after impact is

A. 0.1 V
B. 0.7 V
C. 0.3 V
D. 0.9 V
[MCQ - 2 Marks]
Ans. D
Sol. Conservation law of angular momentum
$r \times M V+I \omega_{1}=I_{2} \omega_{2}$
$M V\left(R-\frac{3 R}{20}\right)+\frac{M R^{2}}{2} \omega_{1}=\left(\frac{M R^{2}}{2}+M R^{2}\right) \omega_{2}$
$M V \times \frac{17 R}{20}+\frac{M R^{2}}{2} \omega_{1}=\frac{3 M R^{2}}{2} \omega_{2}$
$\frac{17 \mathrm{~V}}{20}+\frac{\mathrm{V}}{2}=\frac{3}{2} \mathrm{~V}_{2}$
$\frac{27 \mathrm{~V}}{20}=\frac{3}{2} \mathrm{~V}_{2}$
$\mathrm{V}_{2}=0.9 \mathrm{~V}$
43. The state of stress at the critical location in a structure is $\sigma_{x x}=420 \mathrm{MPa} \sigma_{y y}=100$ $\mathrm{MPa}, \sigma_{z z}=\sigma_{y z}=\sigma_{z x}=\sigma_{x y}=0$. The yield stress of the material in uniaxial tension is 400 MPa . Which statement is correct?
A. Structure is safe by both Tresca ( $\tau_{\max }$ ) theory and Von-mises (Distortion energy) B. Structure is unsafe by Tresca ( $\tau_{\max }$ ) \& safe by Von-mises (Distortion energy)
C. Structure is safe by Tresca ( $\tau_{\max }$ ) \& unsafe by Von-mises (Distortion energy)
D. Structure in unsafe by both

## Ans. B

Sol. The state of stress at the critical location

$$
\begin{array}{ll}
\sigma_{x x}=420 \mathrm{MPa} & \sigma_{y t}=100 \mathrm{MPa} \\
\sigma_{y y}=100 \mathrm{MPa} &
\end{array}
$$

There is no shear stress, $\sigma_{x x}$ and $\sigma_{y y}$ are principal stresses.

$$
\sigma_{1}=420 \mathrm{MPa} \quad \sigma_{2}=100 \mathrm{MPa}
$$

MSST
$\tau_{\text {max }}<\frac{\sigma_{y}}{2}$
$\max \left[\frac{\left|\sigma_{1}-\sigma_{2}\right|}{2}, \frac{\left|\sigma_{1}-\sigma_{3}\right|}{2}, \frac{\left|\sigma_{2}-\sigma_{3}\right|}{2}\right] \leq \frac{\sigma_{y}}{2 \mathrm{~N}}$
$\frac{\sigma_{1}}{2} \leq \frac{\sigma_{y}}{2 N}$
$\frac{420}{2} \leq \frac{400}{2 N}$
$N<\frac{400}{420}$
$\mathrm{N}<1$
$\Rightarrow$ Hence, structure is not safe.
MDET
$\frac{1}{12 \mathrm{G}}\left[\begin{array}{l}\left(\sigma_{1}-\sigma_{2}\right)^{2} \\ +\left(\sigma_{2}-\sigma_{3}\right)^{2}+\left(\sigma_{3}-\sigma_{1}\right)^{2}\end{array}\right] \leq \frac{1}{6 \mathrm{G}}\left(\frac{\sigma_{y}}{\mathrm{~N}}\right)^{2}$
$\frac{1}{2}\left[\begin{array}{l}(420-100)^{2} \\ +(420-0)^{2}+(100-0)^{2}\end{array}\right] \leq\left(\frac{400}{N}\right)^{2}$
$N>\frac{400}{380} \Rightarrow N>1$
Hence structure is safe.

## Thermodynamics (XE-E)

44. $(C O P)_{\text {actual }}=60 \%(C O P)_{\text {maximum }}$

Ratio of $\frac{W_{\text {actual }}}{W_{\text {maximum }}}$ ?


Ans. (1.6 to 1.7)
Sol. Given,
$(C O P)_{\text {actual }}=60 \%(C O P)_{\text {maximum }}$
$(\mathrm{COP})_{H P}=\frac{T_{H}}{T_{H}-T_{L}}$
$(C O P)_{\text {ideal }}=\frac{300}{300-280}=\frac{300}{20}=15$
$(C O P)_{\text {actual }}=15 \times 0.6=9$
$\mathrm{W}_{\text {actual }}=\frac{\mathrm{Q}_{1}}{(\mathrm{COP})_{\text {actual }}}$
$W_{\min }=\frac{Q_{1}}{(C O P)_{\text {ideal }}}$
$\frac{\mathrm{W}_{\text {actual }}}{\mathrm{W}_{\text {minimum }}}=\frac{(\mathrm{COP})_{\text {ideal }}}{(\mathrm{COP})_{\text {actual }}}$
$\frac{W_{\text {actual }}}{W_{\text {minimum }}}=\frac{15}{9}=1.67$
45. Which among the following is extensive property
A. Total mass
B. Density
C. Pressure
D. Temperature

## Ans. A

Sol. Extensive property: The extensive properties are defined as the properties which depends on the amount of matter present.
Ex. $\rightarrow$ Volume, mass.
46. For same compression ratio,
A. $\eta_{\text {otto }}>\eta_{\text {diesel }}$
B. $\eta_{\text {otto }}=\eta_{\text {diesel }}$
C. $\eta_{\text {otto }}<\eta_{\text {diesel }}$
D. None of these

Ans. A
Sol. $\eta_{\text {otto }}=1-\frac{1}{(r)^{\gamma-1}}$
$\eta_{\text {Diesel }}=1-\frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$
Because the term $\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$, the value of $\frac{1}{(r)^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]$ will be more than $\frac{1}{(r)^{\gamma-1}}$. For the same compression ratio, the efficiency of an Otto cycle is higher than that of the diesel cycle.
$\eta_{\text {otto }} \gg \eta_{\text {Diesel }}$
47. $\overline{c_{V}}=a+b T+c T^{2}$ at $T_{1}=400 \mathrm{~K}$ and $T_{2}=600 \mathrm{~K}$, Find $(\Delta \mathrm{U})$ internal energy (in $\mathrm{kJ} / \mathrm{kmol}$ )
$a, b, c$ is given as
$a=19.650, b=0.002, c=5 \times 10^{-6}$
Ans. (4389 to 4391)
Sol. Given, $\overline{c_{V}}=a+b T+c T^{2}$
Temperature, $\mathrm{T}_{1}=400 \mathrm{~K}$
$\mathrm{T}_{2}=600 \mathrm{~K}$,
$\mathrm{a}=19.686 \mathrm{~kJ} / \mathrm{kmol}$
$b=0.002$
$c=5 \times 10^{-6}$
internal energy $(\Delta U)=$ ?
$\int d u=\int_{T_{1}}^{T_{2}} \overline{C_{V}} d T$
$\Delta \bar{U}=\int_{T_{1}}^{T_{2}}\left(a+b T+c T^{2}\right) d T$
$=\mathrm{a}\left(\mathrm{T}_{2}-\mathrm{T}_{1}\right)+\mathrm{b} \frac{1}{2}\left(\mathrm{~T}_{2}^{2}-\mathrm{T}_{1}^{2}\right)+\frac{\mathrm{c}}{3}\left(\mathrm{~T}_{2}^{3}-\mathrm{T}_{1}^{3}\right)$
$=19.686(600-400)$
$+\frac{0.002}{2}\left(600^{2}-400^{2}\right)$
$+\frac{5 \times 10^{-6}}{3}\left(600^{3}-400^{3}\right)$
$=3937.2+200+253.33$
$=4390.53 \mathrm{~kJ} / \mathrm{kmol}$
48. For diesel cycle cut off ratio $=1.5, r=20$, $\mathrm{T}_{1}=25^{\circ} \mathrm{C}$. Calculate $\mathrm{W}_{\text {net }}(\mathrm{kJ} / \mathrm{kg})=$ ?
Ans. (331 to 331.5)
Sol. Given,
Cut-off ratio, $\rho=1.5$
Compression ratio, $\mathrm{r}=20$
Temperature $\mathrm{T}_{1}=25^{\circ} \mathrm{C}=298 \mathrm{~K}$
Pressure, $\mathrm{P}_{1}=100 \mathrm{kPa}$
$Y=1.4, C_{P}=1.005 \mathrm{~kJ} / \mathrm{kgK}$
$\eta_{\text {diesel }}=1-\frac{1}{\mathbf{r}^{\gamma-1}} \frac{\left[\rho^{\gamma}-1\right]}{[\gamma(\rho-1)]}$
$\eta_{\text {diesel }}=1-\frac{1}{20^{1.4-1}} \frac{\left[1.5^{1.4}-1\right]}{[1.4(0.5)]}=67.06 \%$

$\frac{T_{2}}{T_{1}}=r^{\gamma-1}=20^{1.4-1}$
$\mathrm{T}_{2}=3.314 \times 298=987.70 \mathrm{~K}$
Cut-off ratio, $\rho=\frac{V_{3}}{V_{2}}=\frac{T_{3}}{T_{2}}$
$T_{3}=\rho T_{2}=1.5 \times 987.7=1481.56 \mathrm{~K}$
Heat supplied, $\mathrm{Q}_{\mathrm{s}}=\mathrm{CP}_{\mathrm{P}}\left(\mathrm{T}_{3}-\mathrm{T}_{2}\right)$
$=1.005[1481.56-987.70]$
$=493.86 \mathrm{~kJ} / \mathrm{kg}$
$\eta_{\text {diesel }}=\frac{W_{\text {net }}}{Q_{s}}$
$0.6706=\frac{W_{\text {net }}}{493.86}$
$W_{\text {net }}=331.18 \mathrm{~kJ} / \mathrm{kg}$
49. Match the column.

|  | List-I |  | List-II |
| :--- | :---: | :--- | :--- |
| A. | Helmholtz <br> function | I. | $\left(\frac{\mathrm{dP}}{\mathrm{dT}}\right)_{\text {sat }}=\frac{\mathrm{h}_{\mathrm{g}}-\mathrm{h}_{\mathrm{f}}}{\mathrm{RT}^{2}}$ |
| B. | Gibs <br> function | II. | $\mathrm{u}-\mathrm{Ts}$ |
| C. | Tds <br> equation | III. | $\mathrm{h}-\mathrm{Ts}$ |
| D. | Clausius <br> Clapeyron <br> equation | IV. | Tds $=\mathrm{dh}-\mathrm{vdp}$ |

## Sol.

| List-I | List-II |
| :---: | :---: |
| Helmholtz function | $\mathrm{u}-\mathrm{Ts}$ |
| Gibs function | $\mathrm{h}-\mathrm{Ts}$ |
| Tds equation | $\mathrm{dh}-\mathrm{vdP}$ |
| Clausius Clapeyron <br> equation | $\left(\frac{\mathrm{dP}}{\mathrm{dT}}\right)_{\text {sat }}=\frac{\mathrm{h}_{\mathrm{g}}-\mathrm{h}_{\mathrm{f}}}{\mathrm{RT}^{2}}$ |

50. In one question $\left(\frac{\partial \mathrm{P}}{\partial \mathrm{T}}\right)_{\text {sat }}=3579 \mathrm{~Pa} / \mathrm{k}$ at $100^{\circ} \mathrm{C}$. Calculate the value of $\mathrm{h}_{\mathrm{g}}-\mathrm{h}_{\mathrm{f}}$. In the question value of $v_{g}-v_{f}$ was given.
Ans. *
Sol. Classius - Clayepron Equation
$\left(\frac{d P}{d T}\right)_{\text {sat }}=\frac{h_{g}-h_{f}}{T\left(v_{g}-v_{f}\right)}$
Put the value of $\left(v_{g}-v_{f}\right)$ and get the value of $\mathrm{h}_{\mathrm{g}}-\mathrm{h}_{\mathrm{f}}$.
51. A block of mass $5 \mathrm{~kg}(\mathrm{c}=0.5 \mathrm{~kJ} / \mathrm{kgK})$ is at a temperature of 373 K is dropped in water of mass $10 \mathrm{~kg}(\mathrm{c}=4.2 \mathrm{~kJ} / \mathrm{kgk})$ at a temperature of 293 K . Calculate entropy change of the system $(\mathrm{kJ} / \mathrm{K})$.
Ans. ( 0.070 to 0.075 )

## Sol.



Let final temperature of block and water is $\mathrm{T}_{\mathrm{f}}$,
as per conservation of energy
$\delta \mathrm{Q}=\mathrm{W}_{\text {net }}+\mathrm{dU}$
heat transfer and work transfer is zero.
$(\mathrm{du})_{\text {block }}+(\mathrm{du})_{\text {water }}=0$
$\mathrm{m}_{\mathrm{b}} \mathrm{Cb}\left(\mathrm{T}_{\mathrm{f}}-373\right)+\mathrm{m}_{w} \mathrm{~T}_{\mathrm{w}}\left(\mathrm{T}_{\mathrm{f}}-293\right)$
$5 \times 0.5\left(T_{f}-373\right)+10 \times 4.2\left(T_{f}-293\right)$
$373-T_{f}=16.8 T_{f}-16.8 \times 293$
$\mathrm{T}_{\mathrm{f}}=297.49 \mathrm{~K}$
Entropy
$(\mathrm{ds})_{\text {sys }}=(\mathrm{ds})_{\text {block }}+(\mathrm{ds})_{\text {water }}$
$=m_{b} c_{b} \ln \frac{T_{f}}{T_{b}}+m_{w} c_{w} \ln \frac{T_{f}}{T_{w}}$
$=5 \times 0.5 \ln \frac{297.49}{373}+10 \times 4.2 \ln \frac{297.49}{293}$
$=0.07324 \mathrm{~kJ} / \mathrm{K}$

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