## GATE 2023

## Electronics

## Engineering

## Questions \& Solutions

## Memory Based

## GATE 2023 Electronics \& Communication Engineering: Major Highlights

> Overall Difficulty Level: Easy to Moderate
> Electronic Devices \& Signals and Systems Questions were easy but tricky.
> MSQ weightage: 4
> NAT weightage: 16
> MCQ weightage: 45
> Questions from General Aptitude were easy but lengthy.

GATE 2023 Electronics \& Communication Engineering Comparison with last 3 Years' Data

| S.No. | Subject Name | 2023 | 2022 | 2021 | 2020 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Control Systems | 7 | 7 | 8 | 12 |
| 2 | Digital Circuits | 8 | 11 | 9 | 7 |
| 3 | Electronic Devices | 7 | 8 | 6 | 10 |
| 4 | Signals \& Systems | 11 | 6 | 8 | 8 |
| 5 | Communication Systems | 10 | 13 | 16 | 12 |
| 6 | Network Theory | 8 | 10 | 9 | 13 |
| 7 | Electromagnetics | $\mathbf{7}$ | $\mathbf{8}$ | 8 | 8 |
| 8 | Engineering Mathematics | 11 | 6 | 8 | 6 |
| 9 | General Aptitude | 16 | 13 | 10 |  |
| 10 | Total | 15 | 15 | 15 |  |
|  |  | 100 | 100 | 100 | 100 |

GATE 2023 Electronics \& Communication Engineering: Subject-Wise Marks Distribution

| Subjects | Questions |  | Total Marks |
| :---: | :---: | :---: | :---: |
|  | 1 Mark | 2 Marks |  |
| Control Systems | 1 | 3 | 7 |
| Digital Circuits | 4 | 2 | 8 |
| Electronic Devices | 3 | 2 | 7 |
| Signals \& Systems | 3 | 4 | 11 |
| Communication Systems | 2 | 4 | 10 |
| Analog Circuits | 2 | 3 | 8 |
| Network Theory | 4 | 2 | 8 |
| Electromagnetics | 1 | 5 | 11 |
| Engineering Mathematics | 5 | 5 | 15 |
| General Aptitude | 5 | 5 | 15 |
| Total | 30 | 35 | 100 |



## Section-A: General Aptitude

1. Which of the following options represent the given graph?

A. $f(x)=x^{2} e^{2-|x|}$
B. $f(x)=x^{2} e^{-|x|}$
C. $f(x)=x^{2} e^{2-|x|}$
D. $f(x)=x^{2} e^{-2|x|}$
[MCQ, 2 Marks]
Ans. D
Sol. Check for even symmetry.
$f(t)=f(-t)$
So, correct answer is $\mathrm{x}^{2} \mathrm{e}^{-2|\mathrm{x}|}$
2. How many rectangles are present in the given figure?

A. 10
B. 8
C. 9
D. 12
[MCQ, 2 Marks]
Ans. A
Sol.

$(1+2)(1+2)=9$
Total $\Rightarrow 9+1=10$
3. In a class of 100 students
(i) There are 30 students who neither like romantic movie nor comedy movies.
(ii) Number of students who like romantic movies is twice the number of students who like comedy moves and
(iii) The number of students who like both romantic movies and comedy moves is 20 How many students in the class like romantic movies
A. 60
B. 30
C. 40
D. 20
[MCQ, 1 Mark]
Ans. A
Sol.

$x+20+y+30=100$
$x+y=50$
$R=2 C$
$(x+20)=2(Y+20)$
$x+20=2 Y+40$
$x+2(50-x)+20$
$x=40$
So that $x+20=60$
4. What is the smallest number with distinct digits whose digits add upto 45.
A. 99999
B. 123456789
C. 123555789
D. 123457869
[MCQ, 2 Marks]
Ans. B
Sol. Distinct digits mean all the digits should be different.
$\therefore \quad 123456789 \& 123457869$ So, $123456789<123457869$ Correct answer is 123456789
5. A $100 \mathrm{~cm} \times 32 \mathrm{~cm}$ rectangular sheet is folded 5 times. Each time the sheet is folded, the long Edge aligns with its opposite side. Eventually, the folded sheet is a rectangle of dimension 100 cm $\times 1 \mathrm{~cm}$. The total no. of creases visible when the sheet is unfolded is.
A. 32
B. 63
C. 31
D. 5
[MCQ, 1 Mark]
Ans. C
Sol. $\Rightarrow 1+2^{1}+2^{2}+2^{3}+2^{4}$
$\Rightarrow 1+2+4+8+16$
$\Rightarrow 31$
6. Courts : $\qquad$ : : Parliament : Legislature
A. Executive
B. Governmental
C. Judiciary
D. Legal
[MCQ, 2 Marks]
Ans. C
Sol. As Legislature functions in Parliament Judiciary functions in courts.
7. When I was a kid, I was partial to stories about other worlds and interplanetary travel. I used to imagine that I could just gaze off into space and be whisked to another planet.
A. It is an adult's memory of what he or she liked as a child.
B. The child in the passage read stories about interplanetary travels only in parts.
C. It is a child's description of what he or she likes.
D. It teaches us that stories are good for children.
[MCQ, 2 Marks]
Ans. A
Sol. It is an adult's memory of what he or she liked as a child.
8. I cannot support this proposal.

My $\qquad$ will not permit it.
A. Conscious
B. Constant
C. Consensus
D. Conscience

Ans. D
Sol. Option D is correct.
[MCQ, 1 Mark]

## Section-B: Technical

11. A closed loop system is shown with $k>0$ $a>0$.

Find the steady state error due to ramp input.

A. $2 \mathrm{a} / \mathrm{k}$
B. $a / 2 \mathrm{k}$
C. $a / k$
D. $a / 4 \mathrm{k}$
[MCQ, 2 Marks]
Ans. A
Sol. $G(s)=\frac{k}{s(s+2)} \rightarrow$ Type '1'

$$
\mathrm{K}_{\mathrm{v}}=\underset{\mathrm{sim} s \mathrm{~S}(\mathrm{~s})}{\operatorname{Lin}}=\frac{\mathrm{k}}{2}
$$

So, ess for input a[t u (t)]
$=\alpha\left(\frac{1}{\mathrm{~K}_{\mathrm{v}}}\right)=\alpha\left(\frac{2}{\mathrm{k}}\right)$
$\mathrm{e}_{\mathrm{ss}}=\frac{2 \alpha}{\mathrm{k}}$
12. In the following block diagram, $R(s)$ and $P(s)$ are two inputs.
The output $\mathrm{Y}(\mathrm{s})$ is expressed as $\mathrm{Y}(\mathrm{s})=$ $\mathrm{G}_{1}(\mathrm{~s}) \mathrm{R}(\mathrm{s})+\mathrm{G}_{2}(\mathrm{~s}) \mathrm{D}(\mathrm{s})$. Then $\mathrm{G}_{1}(\mathrm{~s})$ and $\mathrm{G}_{2}(\mathrm{~s})$ are given by

A. $\mathrm{G}_{1}(\mathrm{~s})=\frac{\mathrm{G}(\mathrm{s})}{1+\mathrm{G}(\mathrm{s})-\mathrm{H}(\mathrm{s})}$ and

$$
G_{2}(s)=\frac{G(s)}{2+G(s)-G(s) H(s)}
$$

B. $G_{1}(s)=\frac{G(s)}{1+G(s)+G(s) H(s)}$ and

$$
\mathrm{G}_{2}(\mathrm{~s})=\frac{\mathrm{G}(\mathrm{~s})}{1+\mathrm{G}(\mathrm{~s})+\mathrm{G}(\mathrm{~s}) \mathrm{H}(\mathrm{~s})}
$$

C. $\mathrm{G}_{1}(\mathrm{~s})=\frac{\mathrm{G}(\mathrm{s})}{2+\mathrm{G}(\mathrm{s})+\mathrm{H}(\mathrm{s})}$ and
$\mathrm{G}_{2}(\mathrm{~s})=\frac{\mathrm{G}(\mathrm{s})}{2+\mathrm{G}(\mathrm{s})+\mathrm{G}(\mathrm{s}) \mathrm{H}(\mathrm{s})}$
D. $G_{1}(s)=\frac{G(s)}{1-G(s)+H(s)}$ and
$\mathrm{G}_{2}(\mathrm{~s})=\frac{\mathrm{G}(\mathrm{s})}{1-\mathrm{G}(\mathrm{s})+\mathrm{G}(\mathrm{s}) \mathrm{H}(\mathrm{s})}$
[MCQ, 2 Marks]
Ans. B
Sol.


So, $Y=G(D+R-y-H y)$
$Y(1+G+G H)=G R+G D$
$Y=\frac{G}{1+G+G H} R+\frac{G}{1+G+G H}$
13. The open loop transfer function of a unity negative feedback system is

$$
\mathrm{G}(\mathrm{~s})=\frac{\mathrm{K}}{\mathrm{~s}\left(1+\mathrm{sT} \mathrm{~T}_{1}\right)\left(1+\mathrm{sT} \mathrm{~T}_{2}\right)}
$$

Where $K, T_{1}$ and $T_{2}$ are positive constants. The phase crossover frequency, in rad/s is.
A. $\frac{1}{T_{2} \sqrt{T_{1}}}$
B. $\frac{1}{\mathrm{~T}_{1} \mathrm{~T}_{2}}$
C. $\frac{1}{\mathrm{~T}_{1} \sqrt{\mathrm{~T}_{2}}}$
D. $\frac{1}{\sqrt{T_{1} \mathrm{~T}_{2}}}$
[MCQ, 1 Mark]
Ans. D
Sol. $G(\mathrm{j} \omega)=\frac{K}{\mathrm{j} \omega(1+\mathrm{j} \omega \pi)\left(1+\mathrm{j} \omega \mathrm{T}_{2}\right)}$
$\angle \mathrm{G}(\mathrm{j} \omega)=-90^{\circ}-\tan ^{-1}(\omega \pi)-\tan ^{-1}\left(\omega \mathrm{~T}_{2}\right)$
$=-180^{\circ}$
$\tan ^{-1}\left(\omega \mathrm{~T}_{1}\right)+\tan ^{-1}\left(\omega \mathrm{~T}_{2}\right)=90^{\circ}$
$\tan ^{-1}\left(\frac{\omega \mathrm{~T}_{1}+\omega \mathrm{T}_{2}}{1-\omega^{2} \mathrm{~T}_{1} \mathrm{~T}_{2}}\right)=90^{\circ}$
$\Rightarrow 1-\omega^{2} \mathrm{~T}_{1} \mathrm{~T}_{2}=0 \Rightarrow \omega_{\mathrm{p}}=\frac{1}{\sqrt{\mathrm{~T}_{1} \mathrm{~T}_{2}}}$
14. In the circuit shown below; $P$ and $Q$ are the inputs. The logical function realized by the circuit.

A. $\overline{P+Q}$
B. PQ
C. $P+Q$
D. $\overline{P Q}$
[MCQ, 2 Marks]
Ans. B
Sol. $Y=\bar{S} I_{0}+S I_{1}$
$\mathrm{Y}=\overline{\mathrm{Q}} \cdot \mathrm{O}+\mathrm{Q} \cdot \mathrm{P}=\mathrm{PQ}$
15. In the given sequence circuit initial states are $Q_{1}=1, Q_{2}=0$. For clock frequency of 1 MHz ; frequency of $\mathrm{Q}_{2}$ (in kHz ).

[NAT, 2 Marks]
Ans. 250
Sol.

$\mathrm{Q}_{2}^{+}=\overline{\mathrm{Q}}_{1}$
$\mathrm{Q}_{1}^{+}=\mathrm{Q}_{2}$
MOD - 4
$\mathrm{f}_{\mathrm{Q}_{2}}=\frac{\mathrm{f}_{\mathrm{clk}}}{4}=\frac{1 \mathrm{MHz}}{4}=250 \mathrm{kHz}$
16. The synchronous sequential circuit shown below works at a clock frequency of 1 GHz. The throughput in M bits/s and latency in ns respectively
A. $333.3,1$
B. $33.3,3$
C. 2000,3
D. 1000,3

[MCQ-2 Marks]
Ans. D
Sol. $f_{\text {clk }}=1 \mathrm{GHz}$
$\mathrm{T}_{\mathrm{clk}}=1 \mathrm{~ns}$
So, $t_{p d}$ for each flip flop to satisfy.
$\mathrm{t}_{\mathrm{pd}} \leq 1 \mathrm{~ns}$
warts case $\mathrm{t}_{\mathrm{pd}}=1 \mathrm{~ns}$
So, to transmit $1^{\text {st }}$ bit serially
delay $=3 \mathrm{~ns}$
but for $2^{\text {nd }}$ bit $\Rightarrow 4 \mathrm{~ns}$
$3^{\text {rd }}$ bits $\Rightarrow 5$ ns
:
$n$ bits $=(n+2) n s$
$(n+2) \times 10^{-9} s \rightarrow n$ bits
let $(\mathrm{n}+2) \times 10^{-9}=1$
$\mathrm{n}=10^{9}-2 \approx 10^{9}$

So, $1 \mathrm{sec} \rightarrow 10^{9}$ bits $=1000$ Mbits
throughput $=1000$ Mbits
17. The SNR of an ADC with a full scale sinusoidal input is 61.96 dB . \% Resolution of ADC $\qquad$
[NAT, 1 Mark]
Ans. 0.097
Sol. $\operatorname{SNR}=1.763+6.02 \mathrm{n}=61.96$
$\mathrm{n} \approx 10$
$\%$ resolution $=\frac{1}{2^{n-1}} \times 100$
$\approx 0.097 \%$
18. For a circuit shown below, the propagation delay of each NAND gate is 1ns. The critical path delay $\qquad$ ns.

[NAT, 1 Mark]
Ans. 2
Sol.


Critical path delay $=2 \mathrm{~ns}$
19. In an external semiconductor, the hole concentration given by $1.5 n_{i}$, where $n_{i}$ is intrinsic carrier concentration of $1 \times 10^{10}$ $\mathrm{cm}^{-3}$. The ratio of electron to hole mobility for equal hole and electron drift current is given as $\qquad$
[NAT, 2 Marks]

Ans. 2.25
Sol. Electron drift current.
$I_{n}=A J_{n}=A \sigma E=A n q \mu_{n} E$
Hole drift current $I_{D}=A p q \mu_{p} E \ldots$ (2)
$\frac{I_{n}}{I_{p}}=\frac{n}{p} \cdot \frac{\mu_{n}}{\mu_{p}}=1$
$\because \mathrm{I}_{\mathrm{n}}=\mathrm{I}_{\mathrm{p}}$
$\frac{\mu_{\mathrm{n}}}{\mu_{\mathrm{p}}}=\frac{\mathrm{p}}{\mathrm{n}}$
$p=1.5 n_{i} \quad n=\frac{n_{i}^{2}}{p}=\frac{n_{i}}{1.5}$
$\therefore \frac{\mu_{n}}{\mu_{\mathrm{p}}}=\frac{1.5 \mathrm{n}_{\mathrm{i}}}{\mathrm{n}_{\mathrm{i}} / 1.5}=1.5^{2}=2.25$
20. In a semiconductor, if fermi energy level lies in conduction band, then the semiconductor is known as.
A. Degenerative n-type
B. Non-degenerative n-type
C. Degenerative p-type
D. Non-degenerative n-type
[MCQ, 2 Marks]
Ans. A
Sol. Option A is correct.
21. For an intrinsic semiconductor at $T=0 \mathrm{~K}$, which of the following statement is true.
A. All energy states in the conduction band is filled with electrons and valence is empty
B. All energy states in conduction and valence bands are filled with electrons
C. All energy states in the conduction and valence band are filled with holes
D. All energy states in the valence band are filled with electrons and conduction band is empty
[MCQ, 2 Marks]
Ans. B
Sol. At $=0 \mathrm{~K}$, semiconductor is insulator
$\therefore$ There will be no intrinsic excitation.
22. In a semiconductor device, the fermienergy level is 0.35 eV above the valence band energy, the effective density state in valence band at $T=300 \mathrm{~K}$ is $1 \times 10^{19}$ $\mathrm{cm}^{-3}$. The thermal equilibrium hole concentration in Si at 400 K is $\qquad$ $\times$ $10^{13} \mathrm{~cm}^{-3}$.
[NAT, 2 Marks]
Ans. 60.4
Sol. $\mathrm{N}_{\mathrm{v}}=1 \times 10^{19} \mathrm{~cm}^{-3}$ at 300 K

$E_{F}-E_{V}=k T \ln \frac{N_{V}}{P}$
From this,
$\mathrm{p}=\mathrm{N}_{\mathrm{v}} \mathrm{e}^{-\left[\mathrm{E}_{\mathrm{F}}-\mathrm{E}_{\mathrm{v}}\right] / \mathrm{k} T}$
$N_{V_{2}}=N_{V_{1}} \times\left(\frac{T_{2}}{T_{1}}\right)^{3 / 2}$
$=1 \times 10^{19} \times\left(\frac{400}{300}\right)^{3 / 2}=1.539 \times 10^{19} / \mathrm{cm}^{3}$
kT at $\mathrm{T}=300 \mathrm{~K}$ is 0.0259 eV
kT at $\mathrm{T}=400 \mathrm{~K}$ is
$=0.0259 \times \frac{400}{300}=0.0345 \mathrm{eV}$
$p=1.539 \times 10^{19} e^{-0.35 / 0.0345}$
$p=6.04 \times 10^{14} / \mathrm{cm}^{3}$
23. The h-parameters of a two-port network are shown below. The condition for the maximum small signal voltage gain Vout/Vs is.
A. $\mathrm{h}_{11}=0, \mathrm{~h}_{12}=$ very high, $\mathrm{h}_{21}=$ very high and $h_{22}=0$
B. $\mathrm{h}_{11}=0, \mathrm{~h}_{12}=0 \mathrm{~h}_{21}=$ very high, $\mathrm{h}_{22}=$ 0
C. $\mathrm{h}_{11}=$ very high, $\mathrm{h}_{12}=0, \mathrm{~h}_{21}=$ very high $h_{22}=0$
D. $h_{11}=0, h_{12}=0, h_{21}=$ very high, $h_{22}=$ very high

[MCQ, 2 Marks]
Ans. B
Sol.

$\mathrm{V}_{1}=\mathrm{h}_{11} \mathrm{I}_{1}+\mathrm{h}_{12} \mathrm{~V}_{2}$
$\mathrm{I}_{2}=\mathrm{h}_{21} \mathrm{I}_{1}+\mathrm{h}_{22} \mathrm{~V}_{2}$
$V_{2}=-I_{2} R_{L}$
$\frac{I_{2}}{I_{1}}=\frac{h_{21}}{1+h_{22} R_{L}}$
Put (6) in (4)
$V_{1}=h_{11} I_{1}-\frac{h_{12} R_{2} h_{21}}{1+h_{22} R_{L}} I_{1}$
$\left.\mathrm{V}_{\mathrm{S}}=\mathrm{V}_{1}=\left[\mathrm{h}_{11}\left(1+\mathrm{h}_{22} \mathrm{R}_{\mathrm{L}}\right)\right)-\mathrm{h}_{12} \mathrm{~h}_{21} \mathrm{R}_{\mathrm{L}}\right] \mathrm{I}_{1}$
$A_{v s}=\frac{V_{0}}{V_{S}}=\frac{-I_{2} R_{L}}{I_{1}\left(h_{11}\left(1+h_{22}+R_{L}\right)-h_{12} h_{21} R_{L}\right)} \ldots$
From (6) and (8)
$A_{v s}=\frac{V_{0}}{V_{s}}=\frac{h_{21}}{1+h_{22} R_{L}} \cdot \frac{R_{L}}{\left[h_{11}\left(1+h_{22} R_{L}\right)-h_{12} h_{21} R_{L}\right]}$
From this,
if $h_{11}=0$ $h_{22}=0$
the $A_{\text {vs }} \cong-\frac{1}{h_{12}}$
$\mathrm{h}_{12}=0$
$A_{\text {vs }} \cong \infty$ very high
B. $h_{11}=0 \quad h_{12}=0 \quad h_{22}=V$. Low $h_{21}=$ very high
24. If $D_{1} \& D_{2}$ are silicon diodes and cut off voltage is given as 0.7 V . Then the transfer characteristics is.

A.

B.

C.
I
[
A. $I-R_{4} / R_{3}$
B. $R_{4} / R_{3}$
C. $-R_{4} / R_{3}$
D. $1+R_{4} / R_{3}$
[MCQ, 2 Marks]
Ans. C

## Sol.



$$
\begin{equation*}
V_{x}=-R_{2} / R_{1} V_{\text {in }} \tag{1}
\end{equation*}
$$

$V_{y}=\left(1+R_{2} / R_{1}\right) V_{\text {in }}$
$V_{0}=-\frac{R_{4}}{R_{3}} V_{x}-\frac{R_{4}}{R_{3}} V_{y}$
$V_{0}=-\frac{R_{4}}{R_{3}}\left[V_{x}+V_{y}\right]$
$V_{0}=-\frac{R_{4}}{R_{3}}$
[from (1) (2) (3)
26. In the circuit below, the voltage $V_{L}$ is?

[NAT, 2 Marks]
Ans. 2

## Sol.


$M_{2} \& M_{4}$ are in series
$I_{2}=I_{4}=\frac{(W / L)}{(W / L)} \times I_{\text {ref. }}$
$=\frac{10}{1} \times 1=10 \mathrm{~mA}$
$\mathrm{I}_{3}=\frac{(\mathrm{W} / \mathrm{L})_{3}}{(\mathrm{~W} / \mathrm{L})_{1}} \times \mathrm{I}_{\text {ref }}=\frac{7}{1} \times 1=7 \mathrm{~mA}$
$I_{5}=\frac{(W / L)_{5}}{(W / L)_{4}} I_{4}$
$I_{5}=\frac{5}{10} \times 10=5 \mathrm{~mA}$
$\mathrm{I}_{3}=\mathrm{I}_{5}+\mathrm{IL}$
$7=5+I_{L}$
$\mathrm{I}_{\mathrm{L}}=7-5=2 \mathrm{~mA}$
$V_{L}=I_{L} R_{L}=2 \times 1=2 \mathrm{Volt}$
27. Consider the signal $x(t)$ and $y(t)=x\left(e^{t}\right)$ output. The system is $\qquad$ .
A. Causal and TIV
B. Non-Causal and TV
C. Causal and TV
D. Non-Causal and TIV
[MCQ, 2 Marks]
Ans. B
Sol. $y(t)=x\left(e^{t}\right)$

## (i) Causality test:

Put, $\mathrm{t}=0$
$y(0)=x\left(e^{0}\right)=x(1)$
present output depends on future input samples.
$\therefore$ System is non-causal.

## (ii) Test for time invariance:

$\because y(t)=x\left(e^{t}\right)$
Argument has time function $e^{t}$ which represents time variant system.
$\therefore$ System is non-causal and time variant.
Correct answer (B)
28. FT of $x(t)=e^{-t^{2}}$ is $\qquad$
A. $\sqrt{\pi} \mathrm{e}^{\frac{-\omega^{2}}{4}}$
B. $\sqrt{\pi} \mathrm{e}^{\frac{\omega^{2}}{2}}$
C. $\frac{e^{\frac{-\omega^{2}}{4}}}{2 \sqrt{\pi}}$
D. $\sqrt{\pi} \mathrm{e}^{\frac{-\omega^{2}}{2}}$
[MCQ, 2 Marks]
Ans. A
Sol. $x(t) \stackrel{\text { CTFT }}{\longleftrightarrow} X(\omega)$

$$
\because \mathrm{e}^{-\mathrm{at}}{ }^{2} \stackrel{\text { CTFT }}{\longleftrightarrow} \sqrt{\frac{\pi}{\mathrm{a}}} \mathrm{e}^{-\frac{\omega^{2}}{4 \mathrm{a}}}
$$

$$
\text { Put, } a=1
$$

$$
\mathrm{e}^{-\mathrm{t}^{2}} \stackrel{\text { CTFT }}{\longleftrightarrow} \sqrt{\pi} \mathrm{e}^{-\frac{\omega^{2}}{4}}
$$

Correct option (A)
29. Match the following

| Signal types |  | Spectral <br> characteristics |  |
| :---: | :---: | :---: | :---: |
| 1. | Continuous <br> and <br> aperiodic | a. | Continuous and <br> aperiodic |
| 2. | Continuous <br> and <br> Periodic | b. | Continuous and <br> Periodic |
| 3. | Discrete and <br> aperiodic | c. | Discrete and <br> aperiodic |
| 4. | Discrete and <br> Periodic | d. | Discrete and <br> Periodic |

A. 1-a, 2-b, 3-c, 4-d
B. 1-a, 2-c, 3-d, 4-b
C. 1-a, 2-c, 3-b, 4-d
D. 1-d, 2-b, 3-c, 4-a
[MCQ - 2 Marks]
Ans. C
Sol. Duality table

| Time domain |  | Frequency domain |
| :--- | :---: | :--- |
| Continuous | $\rightarrow$ | Aperiodic |
| Aperiodic | $\rightarrow$ | Continuous |
| Discrete | $\rightarrow$ | Periodic |
| Periodic | $\rightarrow$ | Discrete |

30. If input $x(n)$ having DTFT
$X\left(\mathrm{e}^{\mathrm{j} \Omega}\right)=1-\mathrm{e}^{-\mathrm{j} \Omega}+2 \mathrm{e}^{-3 j \Omega}$ be passed through as LTI system of frequency response $H\left(e^{j \Omega}\right)=1-\frac{1}{2} e^{-2 j \Omega}$

The output $y(n)$ of the system
A. $\delta(n)-\delta(n-1)-0.5 \delta(n-2)+2.5 \delta(n$
$-3)-\delta(n-5)$
B. $\delta(n)+\delta(n-1)-0.5 \delta(n-2)-2.5 \delta(n$
$-3)+\delta(n-5)$
C. $\delta(n)-\delta(n-1)-0.5 \delta(n-2)-2.5 \delta(n$
$-3)+\delta(n-5)$
D. $\delta(n)+\delta(n-1)+0.5 \delta(n-2)+2.3 \delta(n$ $-3)+\delta(n-5)$
[MCQ-2 Marks]
Ans. A
Sol. $X[n] \stackrel{\text { DTFT }}{\longleftrightarrow} X\left(e^{+j \Omega}\right)$
$X\left(e^{j \Omega}\right)=1-e^{-j \Omega}+2 e^{-j 3 \Omega}$
$x(z)=1-z^{-1}+2 z^{-3}$
$\& H\left(e^{j \Omega}\right)=1-\frac{1}{2} e^{-j 2 \Omega}$
$H(z)=1-\frac{1}{2} z^{-2}$
Now, $Y(z)=X(z) \cdot H(z)$
$Y(z)=\left(1-z^{-1}+2 z^{-3}\right) \cdot\left(1-\frac{1}{2} z^{-2}\right)$
$Y(z)=1-z^{-1}+2 z^{-3}-\frac{1}{2} z^{-2}+z^{-3}-z^{-5}$
$Y(z)=1-z^{-1}-\frac{1}{2} z^{-2}+\frac{5}{2} z^{-3}-z^{-5}$
Taking I.Z.T.
$y[n]=\delta(n)-\delta(n-1)-0.5 \delta(n-2)+$ $2.5 \delta(n-3)-\delta(n-5)$
Correct answer (A)
31. Let $m(t)$ be a bandlimited signal with bandwidth $B$ and energy $E$. Let $\omega_{0}=10 B$, the energy of signal $m(t) \cos \omega_{0} t$
A. $\frac{E}{4}$
B. E
C. 2 E
D. $\frac{E}{2}$
[MCQ, 2 Marks]
Ans. D
Sol. $m(t) \stackrel{\text { CTFT }}{\longleftrightarrow} M(\omega)$
Let $M(\omega)$ be as follows

$E=\frac{1}{2 \pi} \int_{-B}^{B}|M(\omega)|^{2} \cdot d \omega$
$E=\frac{1}{2 \pi} \int_{-B}^{B} 1 \cdot d \omega$
$E=\frac{1}{2 \pi} 2 B$
$\Rightarrow E=\frac{B}{\pi}$
Now, $y(t)=m(t) \cdot \cos (\omega o t) \leftrightarrow y(\omega)$

$E_{y}=\frac{1}{2 \pi} \int_{-\infty}^{\infty}|Y(\omega)|^{2} \cdot d \omega$
$=2 \cdot \frac{1}{2 \pi} \int_{9 B}^{11 B}\left(\frac{1}{2}\right)^{2} \cdot d \omega$
$=\frac{1}{4 \pi}(11 B-9 B)=\frac{2 B}{4 \pi}=\frac{B}{2 \pi}$
$E_{y}=\frac{E}{2}$
32. Let $x_{1}(t)$ and $x_{2}(t)$ be two band limited signals having bandwidth.
$B=4 \pi \times 10^{3} \mathrm{rad} / \mathrm{s}$ each. In the figure below, the Nyquist sampling frequency in rad/s, required to sample $y(t)$ is?
[MCQ, 2 Marks]

A. $8 \square \times 10^{3}$
B. $20 \pi \times 10^{3}$
C. $40 \pi \times 10^{3}$
D. $32 \pi \times 10^{3}$

Ans. D
Sol.





Signal $\mathrm{y}(\mathrm{t})$ is bandlimited to 8 kHz .
Nyquist rate, $\mathrm{f}_{\mathrm{s}}=2 \mathrm{f}_{\mathrm{m}}=2 \times 8 \times 10^{3}=16$ kHz
$\omega_{\mathrm{s}}=2 \pi \mathrm{f}_{\mathrm{s}}=32 \mathrm{n} \times 10^{3} \mathrm{rad} / \mathrm{sec}$
33. Let $X(t)$ be a white gaussian noise with power spectral density $1 / 2 \mathrm{~W} / \mathrm{H}_{2}$. If $\mathrm{X}(\mathrm{t})$ is input to an LTI system with impulses response $e^{-t} u(t)$. The average power of the system output is $W$. (Upto two decimal places)
[NAT, 2 Mark]
Ans. 0.25

## Sol.

$$
\begin{aligned}
& S_{x}(\omega)=\frac{1}{2} \frac{W}{H z} \\
& h(t)=e^{-t} u(t) \longrightarrow S_{y}(\omega) \\
& S_{y}(\omega)=S_{x}(\omega)|H(\omega)|^{2} \\
& =\frac{1}{2}\left|\frac{1}{1+j \omega}\right|^{2} \\
& =\frac{1}{2}\left(\frac{1}{1+\omega^{2}}\right)
\end{aligned}
$$

Output average power $=\frac{1}{2 \pi} \int_{-\infty}^{\infty} S_{y}(\omega) d \omega$
$=\frac{1}{2 \pi} \times 2 \int_{-\infty}^{\infty} \frac{1}{2} \frac{1}{\left(1+\omega^{2}\right)} d \omega$
$=\frac{1}{2 \pi} \times\left[\tan ^{-1} \omega\right]_{0}^{\infty}=\frac{1}{2 \pi} \times \frac{\pi}{2}$
$=0.25$
34. For a real signal, which of the following is /are valid power spectral density/densities?
A.

B.
C. $\mathrm{S}_{\mathrm{x}}(\omega)=\mathrm{e}^{-\omega^{2}} \cos ^{2} \omega$
D. $S_{x}(\omega)=\frac{2}{9+\omega^{2}}$

Ans. C, D
Sol. Properties of power spectral density:
(1) $S_{x}(\omega)$ should be real and positive
(2) $S_{x}(\omega)=S_{x}(-\omega)$
$S_{x}(\omega)=\mathrm{e}^{-\omega^{2}} \cos ^{2} \omega$
$S_{x}(-\omega)=\mathrm{e}^{-(-\omega)^{2}} \cos ^{2}(-\omega)=\mathrm{e}^{-\omega^{2}} \cos ^{2} \omega$
(Hence even symmetry)
$S_{x}(\omega)=\frac{2}{9+\omega^{2}}$
$S_{x}(\omega)=\frac{2}{9+\left(-\omega^{2}\right)^{2}}=\frac{2}{9+\omega^{2}}=S_{x}(\omega)$
(even symmetry)
Option (C) and (D) are correct.
35. A random variable $X$, distributed normally as $N(G 1)$ undergoes the transformation $y$ $=n(x)$, given in the figure. The form of probability density function of $y$ is ( $a, b, c$ are non zero) constants and $g(y)$ is piecewise continuous function)

A. $a \delta(y+2)+b \delta(y)+c \delta(y-2)+g(y)$
B. $a \delta(y-1)+b \delta(y+1)+g(y)$
C. $a \delta(y+2)+b \delta(y-2)+g(y)$
D. $a \delta(y+1)+b \delta g(y)+c \delta(y-1)+g(y)$
[MCQ - 1 Mark]
Ans. D

## Sol.



For $-1<x<1, y=0$
For $x>2, y=+1$
For $1<x<2$ and $-2<x<-1, y=$ variable

The PDF of $Y$ will have form,
$f_{y}(y)=\sum_{i=0}^{n} p\left(y_{i}\right) \delta\left(y-y_{i}\right)+g(y)$
Where, $g(y)=$ piece-wise continuous function because for $1<x<2$ and $-2<$ $x<-1$ it is nootropics increasing hence it is following continuous function which will have form $\frac{f_{x}(x)}{\left(\frac{d y}{d x}\right)}$.
$f_{y}(y)=a \boldsymbol{\delta}(y+1)+b \boldsymbol{\delta} g(y)+c \boldsymbol{\delta}(y-1)$ $+\mathrm{g}(\mathrm{y})$
option (D).
36. Let a frequency modulated (FM) signal $x(t)=A \cos \left(\omega_{c t}+k f_{f}\right.$ where $m(t)$ is a message signal of band width W . It is passed through a non-linear system with output $y(t)=2 x(t)+5(x(t))^{2}$, Let $B_{T}$ denote the FM bandwidth. The minimum value of $\omega c$ required to recover $x(t)$ from $y(t)$ is $\qquad$ .
[MCQ-2 Mark]
A. $B_{T}+\omega$
B. $\frac{3}{2} B_{T}$
C. $\frac{5}{2} B_{T}$
D. $2 B_{T}+\omega$

Ans. B
Sol. $y(t)=2 x(t)+5 x^{2}(t)$

$$
\begin{aligned}
& =2 A \cos \left(\omega_{c} t+k_{f} \int_{-\infty}^{t} m(d) d \lambda+5 A^{2} \cos ^{2}\right. \\
& {\left[\omega_{c} t+k_{f} \int_{-\infty}^{t} m(\lambda) d \lambda\right]} \\
& =\underbrace{2 a \cos \left[\omega_{c} t+k_{f} \int_{-\omega}^{t} m(\lambda) d \lambda\right]}_{F M} \\
& +\frac{\underbrace{}_{D C}}{5 A^{2}}[\underbrace{\left[1+\cos \left(2 \omega_{c} t+2 k_{f} \int_{-\omega}^{t} m(\lambda) d \lambda\right]\right.}_{F M}
\end{aligned}
$$

Bandwidth of FM is gives by, $\mathrm{BT}_{\mathrm{T}}=2(\Delta \mathrm{f}+$ $f_{m}$ )

After squaring $f_{c}$ and $\Delta f$ will get doubled and $f_{m}$ unchanged.

$$
\text { i.e., }\left(B_{T}\right)_{\text {new }}=2\left[2 \Delta f+f_{m}\right]
$$



To recover original signal $x(t)$ the following condition should be satisfied.
$\mathrm{f}_{\mathrm{c}}+\Delta \mathrm{f}+\mathrm{f}_{\mathrm{m}}<2 \mathrm{f}_{\mathrm{c}}-2 \mathrm{nf}-\mathrm{f}_{\mathrm{m}}$
$\mathrm{fc}_{\mathrm{c}}>3 \Delta \mathrm{f}+2 \mathrm{ff}_{\mathrm{m}}$
$\Rightarrow f_{c}>\frac{3}{2}\left[2\left(\Delta f+f_{m}\right)\right]-f_{m}$
$f_{c}>\frac{3}{2} B_{T}-f_{m}$
Option (B)
37. The switch $S_{1}$ was closed and $S_{2}$ was open for a long time. At $t=0$, switch $S_{1}$ is opened and $\mathrm{S}_{2}$ closed. The value of $\mathrm{i}_{c}\left(\mathrm{O}^{+}\right)$ (in A).

[NAT, 2 Marks]

## Ans. -1

## Sol.


$\mathrm{S}_{1} \rightarrow$ closed and $\mathrm{S}_{2} \rightarrow$ open for long time.
i.e., $\mathrm{S}_{1} \rightarrow$ closed from $\mathrm{t}=-\infty$ to $\mathrm{t}=0^{-}$
and $S_{2} \rightarrow$ opened from $t=-\infty$ to $t=0^{-}$
At $t=0^{-}$, the network is in steady state and $\mathrm{i}_{\mathrm{c}}\left(\mathrm{O}^{-}\right)$will be zero.
$\therefore$ capacitor is O.C. and inductor is S.C.

$\mathrm{i}_{1}\left(0^{-}\right)=\frac{1}{100}\left[\begin{array}{cc}2 & -1 \\ -1 & 4 / 3\end{array}\right]$ S. $=0.2$ Amp's
$\mathrm{i}_{\mathrm{c}}\left(0^{-}\right)=\mathrm{i}_{\mathrm{L}}\left(0^{-}\right) \times 100=0.2 \times 100=20$
volts
At $t=0^{+}$, The network is in transient state,
$\therefore$ capacitor acts as S.C and inductor acts as O.C.


$$
-\mathrm{i}_{\mathrm{c}}\left(0^{+}\right)=0.2+0.8
$$

$$
\mathrm{I}_{\mathrm{c}}\left(\mathrm{O}^{+}\right)=-1 \mathrm{~A}
$$

38. The maximum magnitude of $V_{R}$ in $\qquad$ Volts.

[NAT, 1 Mark]
Ans. +4
Sol. At $t=0^{-}$, the network is in steady state $\therefore$ A inductor acts S.C.

$\mathrm{i}_{\mathrm{L}}\left(0^{-}\right)=\frac{2}{1}=2 \mathrm{Amps}$
At $t=0^{+}$; the network is in transient
state
$\therefore$ inductor acts as O.C


By KVL; $-\mathrm{V}^{\mathrm{L}}\left(0^{+}\right)=\mathrm{V}_{\mathrm{R}}=0$
$\left|\mathrm{V}_{\mathrm{L}}\left(\mathrm{O}^{+}\right)\right|=\mathrm{V}_{\mathrm{R}}=2 \times 2=4 \mathrm{Volts}$
39. For a two-port network shown below, the
[ $\lambda$ ] parameters is given as [y] = $\frac{1}{100}\left[\begin{array}{cc}2 & -1 \\ -1 & 4 / 3\end{array}\right]$ S. The value of load impedance $Z_{L}$ in $\Omega$, for maximum power transfer will be $\qquad$

[NAT - 2 Marks]
Ans. 82.19
Sol. For maximum power transfer to load $Z_{L}$
$=Z_{\text {Th }}$
$Z_{\text {th }}$ calculation:

$Y=\left[\begin{array}{cc}0.02 & -0.01 \\ -0.01 & 0.013 a\end{array}\right]$
We know, $\quad I_{1}=Y_{11} V_{1}+Y_{12} V_{2}$
$\mathrm{I}_{2}=\mathrm{Y}_{21} \mathrm{~V}_{1}+\mathrm{Y}_{12} \mathrm{~V}_{2}$
$\mathrm{I}_{1}=0.02\left[-10 \mathrm{I}_{1}\right]+(-0.01) \mathrm{V}_{2}$
$1.2 \mathrm{I}_{1}=-0.01 \mathrm{~V}_{2}$
$\mathrm{I}_{2}=-0.01\left[-10 \mathrm{I}_{1}\right]+0.013 \mathrm{~V}_{2}$
$\mathrm{I}_{2}=0.1 \mathrm{I}_{\mathrm{I}}+0.013 \mathrm{~V}_{2}$
Sub (3) in (4), we get
$\mathrm{I}_{2}=0.1\left[\frac{-0.01 \mathrm{~V}_{2}}{1.2}\right]+0.013 \mathrm{~V}_{2}$
$\frac{\mathrm{V}_{2}}{\mathrm{I}_{2}}=\mathrm{Z}_{22}=\mathrm{Z}_{\mathrm{th}}=82.19 \Omega$
40. Find current I flowing through $200 \Omega$ resistor in mA.

[MCQ - 2 Marks]

Ans. 1.67
Sol. By source transformation.


By Nodal analysis

$\frac{\mathrm{V}-2}{2 \mathrm{k}}+\frac{\mathrm{V}-2}{2 \mathrm{k}}+\frac{\mathrm{V}+1}{1200}=0$
$\mathrm{V}\left[\frac{1}{2 \mathrm{k}}+\frac{1}{2 \mathrm{k}}+\frac{1}{1200}\right]=\frac{\mathrm{k}}{2 \mathrm{k}}+\frac{2}{2 \mathrm{k}}+\frac{1}{1200}$
$V\left[1.833 \times 10^{-3}\right]=1.83 \times 10^{-3}$
$\mathrm{V}=1$ volts
$\therefore \mathrm{i}=\frac{1+1}{1 \mathrm{k}+200}=\frac{2}{1200}=1.67 \mathrm{~mA}$
41. A series RLC circuit has $\theta=1000$ at a centre frequency $10^{6} \mathrm{rad} / \mathrm{sec}$. Possible values of $R, L$ and $C$.
A. $L=1 \mu H, C=1 \mu F, R=0.01 \Omega$
B. $L=1 \mu H, C=1 \mu F, R=0.001 \Omega$
C. $L=1 \mu H, C=1 \mu F, R=0.1 \Omega$
D. $L=1 \mu H, C=1 \mu F, R=1 \Omega$
[MCQ, 1 Mark]
Ans. B
Sol. Series RLC circuit
$Q=\frac{1}{R} \sqrt{\frac{L}{C}}$
$1000=\frac{1}{\mathrm{R}} \sqrt{\frac{1}{1}}$
$\mathrm{R}=0.001 \Omega$
$\omega=10^{6} \mathrm{rad} / \mathrm{sec}$
42. A transparent dielectric coating is applied to glass ( $\varepsilon_{r}=4, \mu_{r}=1$ ) to eliminate the reflection of red light $\left(\lambda_{0}=0.75 \mu \mathrm{~m}\right)$. The minimum thickness of the dielectric coating in $\mu \mathrm{m}$ that can be used Is (round off to 2 decimal place)
Sol. $\eta_{2}=\sqrt{\eta_{1} \eta_{3}}=\sqrt{\varepsilon_{r 1} \varepsilon_{r_{3}}}=2$
$\lambda_{2}=\frac{0.75}{\sqrt{\varepsilon_{r_{2}}}}=0.530 \mu \mathrm{~m}$
$\mathrm{d}=\frac{\lambda_{2}}{4}=0.133 \mu \mathrm{~m}$
43. The electric field of a plane electromagnetic wave is $E=a_{x} C_{1 x} \cos (\omega t$ $\left.-\beta_{z}\right)+a_{y} C_{1 y} \cos (\omega t-\beta z+\theta) V / m$. Which of the following combination(s) will give rise to a left-handed elliptically polarized (LHEP) wave?
A. $C_{1 x}=1, C_{1 y}=2, \theta=3 \pi / 2$
B. $C_{1 x}=2, C_{1 y}=1, \theta=\pi / 2$
C. $C_{1 x}=2, C_{1 y}=1, \theta=3 \pi / 4$
D. $C_{1 x}=1, C_{1 y}=1, \theta=\pi / 4$

Ans. B, C, D
Sol. $\left.\left.E=C_{i x} \cos \mid \omega t-\beta z\right) \hat{i}+C_{i y} \cos \mid \omega t-\beta z+0\right] \hat{j}$
$\left.E_{x}=C_{i x} \cos \mid \omega t-\beta z\right]$
$\left.E_{y}=C_{i y} \cos \mid \omega t-\beta z+\theta\right]$
At $z=0$ and $t=0$,
$\mathrm{E}_{\mathrm{x}}=\mathrm{Cix}_{\mathrm{ix}}$
$\mathrm{E}_{\mathrm{y}}=\mathrm{C}_{\mathrm{iy}} \cos \theta$
At $\mathrm{z}=0$ and $\mathrm{t}=\mathrm{t}_{1}$,
$\mathrm{E}_{\mathrm{x}}=\mathrm{C}_{\mathrm{ix}} \cos \omega \mathrm{t}_{1}$
$E_{y}=C_{i y} \cos \left[\omega t_{1}+\theta\right]$
Option A,
$\mathrm{E}_{\mathrm{x}}(0,0)=1$
$\mathrm{E}_{\mathrm{y}}=(0,0)=2 \cos \frac{\pi}{2}=0$
$\mathrm{E}_{\mathrm{x}}\left(0, \mathrm{t}_{1}\right)=1 \cos \omega \mathrm{t}_{1}$
$E_{y}(0, t)=2 \cos \left[\omega t_{1}+\frac{3 \pi}{2}\right]=$ positive and
less than 2


This is right elliptical.
Option (B)
At $\mathrm{t}=0$,
Option B:
$E_{x}(0,0)=1$
$E_{x}\left(0, t_{1}\right)=2 \cos \omega t_{1}$
$\mathrm{E}_{\mathrm{y}}(0,0)=0$
$\mathrm{E}_{\mathrm{y}}\left(0, \mathrm{t}_{1}\right)=\cos \left[\omega \mathrm{t}_{1}+\frac{\pi}{2}\right]$

This is left elliptical.
Option (C)
$\mathrm{E}_{\mathrm{x}}=(0,0)=2$
$E_{y}(0,0)=1 \cos \frac{3 \pi}{4}=-\frac{1}{\sqrt{2}}$
$E_{x}=\left(0, t_{1}\right)=2 \cos \omega t_{1}$
$E_{y}\left(0, t_{1}\right)=1 \cos \left[\omega t_{1}+\frac{3 \pi}{4}\right]$


It is left elliptical.

Option (D)
$\mathrm{E}_{\mathrm{x}}(0,0)=1$
$E_{y}(0,0)=1 \cos \frac{\pi}{4}=\frac{1}{\sqrt{2}}$
$\mathrm{E}_{\mathrm{x}}\left(0, \mathrm{t}_{1}\right)=1 \cos \omega \mathrm{t}_{1}$
$E_{y}\left(0, t_{1}\right)=\cos \left(\frac{\pi}{4}+\omega t_{1}\right)$


This is left elliptical.
So, option B,C, D are correct.
44. Consider a narrow band signal, propagating in a lossless dielectric material ( $\epsilon_{r}=4, \mu_{r}=1$ ), with phase velocity $v_{p}$ and group velocity $v_{g}$, which of the following statements are true ( $c$ is velocity of light in vacuum)
A. $v_{p}>c, v_{g}>c$
B. $v_{p}>c, v_{g}>c$
C. $v_{p}<c, v_{g}>c$
D. $\mathrm{v}_{\mathrm{p}}<\mathrm{c}, \mathrm{v}_{\mathrm{g}}<\mathrm{c}$

## Ans. *

Sol. $V_{P}=\frac{c}{\sqrt{\varepsilon_{r} \mu_{r}}}=\frac{3 \times 10^{8}}{\sqrt{4 \times 1}}=1.5 \times 10^{8}<c$
$\mathrm{V}_{\mathrm{P}} \mathrm{V}_{\mathrm{g}}=\mathrm{c}^{2}$
$V_{g}=\frac{V}{V_{p}}=\frac{9 \times 10^{16}}{1.5 \times 10^{8}}=6 \times 10^{8}>c$
45. The following circuits representing an lumped element equivalent of an infinitesimal section of a transmission line is/are.
A.

B.

C.

D.


Ans. A, C, D
Sol. From a reciprocity and symmetry property of transmission lines, options A, C, D are correct.
46. Given $S$ parameter, $S=\left[\begin{array}{ll}\mathrm{S}_{11} & \mathrm{~S}_{12} \\ \mathrm{~S}_{21} & \mathrm{~S}_{22}\end{array}\right]$ with reference to $\mathrm{Z}_{0}$. Two lossless transmission line section of length $\theta_{1}=\beta \mathrm{I}_{1}, \theta \mathrm{I}_{2}$, are added to input and output port as shown. The resultant $S$ parameter $S^{\prime}$ is.

A. $\left[\begin{array}{ll}\mathrm{S}_{11} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} & \mathrm{S}_{12} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} \\ \mathrm{S}_{21} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} & \mathrm{S}_{21} \mathrm{e}^{-\mathrm{j}\left(\theta_{2}\right)}\end{array}\right]$
B. $\left[\begin{array}{ll}\mathrm{S}_{11} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} & \mathrm{S}_{12} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} \\ \mathrm{S}_{21} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} & \mathrm{S}_{21} \mathrm{e}^{\mathrm{j}\left(\theta_{2}\right)}\end{array}\right]$
C. $\left[\begin{array}{ll}\mathrm{S}_{11} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}\right)} & \mathrm{S}_{12} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} \\ \mathrm{S}_{21} \mathrm{e}^{-j\left(\theta_{1}\right)} & \mathrm{S}_{21} \mathrm{e}^{\mathrm{j}\left(\theta_{2}\right)}\end{array}\right]$
D. $\left[\begin{array}{ll}\mathrm{S}_{11} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} & \mathrm{S}_{12} \mathrm{e}^{-\mathrm{j}\left(\theta_{2}\right)} \\ \mathrm{S}_{21} \mathrm{e}^{\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} & \mathrm{S}_{21} \mathrm{e}^{\mathrm{j}\left(\theta_{2}\right)}\end{array}\right]$

Ans. None of the above

## Sol.


$\left(V_{1}^{+}\right)=\left(V_{1}^{+}\right)^{\prime} e^{-j \theta_{1}}$
$\left(V_{1}^{-}\right)^{\prime}=\left(V_{1}^{-}\right) e^{-j \theta_{1}} \Rightarrow\left(V_{1}^{-}\right)=\left(V_{1}^{-}\right)^{\prime} \mathrm{e}+j \theta_{1}$
$\left(V_{2}^{-}\right)^{\prime}=\left(V_{2}^{-}\right) \mathrm{e}^{-j \theta_{2}} \Rightarrow\left(V_{2}^{-}\right)=\left(V_{2}^{-}\right)^{\prime} \mathrm{e}+j \theta_{2}$
$\left(\mathrm{V}_{2}^{+}\right)=\left(\mathrm{V}_{2}^{+}\right)^{\prime} \mathrm{e}^{-\mathrm{j} \theta_{2}}$
$\mathrm{V}_{1}^{-}=\mathrm{S}_{11} \mathrm{~V}_{1}^{+}+\mathrm{S}_{12} \mathrm{~V}_{2}^{+}$
$\mathrm{V}_{2}^{-}=\mathrm{S}_{2} \mathrm{~V}_{1}^{+}+\mathrm{S}_{22} \mathrm{~V}_{2}^{+}$
From Equation (i)
$\left(V_{1}^{-}\right)^{\prime} e^{j \theta 1}=S_{11}\left(V_{1}^{+}\right)^{\prime} e^{-j \theta_{1}}+S_{12}\left(V_{2}^{+}\right)^{\prime} e^{-j \theta_{2}}$
$\left(\mathrm{V}_{1}^{-}\right)^{\prime}=\mathrm{S}_{11} \mathrm{e}^{-\mathrm{j} 2 \theta_{1}}\left(\mathrm{~V}_{1}^{+}\right)^{\prime}+\mathrm{S}_{12} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)}\left(\mathrm{V}_{2}^{+}\right)^{\prime}$
From Equation (ii)
$\left(V_{2}^{-}\right)^{\prime} e^{+j \theta_{2}}=S_{21}\left(V_{1}^{+}\right) e^{-j \theta_{1}}+S_{22}\left(V_{2}^{+}\right) e^{-j \theta_{2}}$
$\left(\mathrm{V}_{2}^{-}\right)^{\prime}=\mathrm{S}_{21} \mathrm{e}^{+\mathrm{j}\left(\theta_{1}+\theta_{2}\right)}\left(\mathrm{V}_{1}^{+}\right)^{\prime}+\mathrm{S}_{22} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)}\left(\mathrm{V}_{2}^{+}\right)^{\prime}$
$\left[\begin{array}{l}\left(\mathrm{V}_{1}^{-}\right)^{\prime} \\ \left(\mathrm{V}_{2}^{-}\right)^{\prime}\end{array}\right]=\left[\begin{array}{cc}\mathrm{S}_{11} \mathrm{e}^{-\mathrm{j}\left(2 \theta_{1}\right)} & \mathrm{S}_{12} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} \\ \mathrm{S}_{21} \mathrm{e}^{-\mathrm{j}\left(\theta_{1}+\theta_{2}\right)} & \mathrm{S}_{21} \mathrm{e}^{-\mathrm{j} 2\left(\theta_{2}\right)}\end{array}\right]\left[\begin{array}{l}\left(\begin{array}{l}\left.\mathrm{V}_{1}^{+}\right)^{\prime} \\ \left(\mathrm{V}_{2}^{+}\right)^{\prime}\end{array}\right]\end{array}\right]$
47. Contour integral $\oint_{c}\left(\frac{z+2}{z^{2}+2 z+2}\right) d z$ where the contour $C$ is $\left\{C:\left|z+1-\frac{3}{2} j\right|=1\right\}$ taken in the contour clockwise direction is -
A. $\pi(1+j)$
B. $-\pi(1+j)$
C. $-\pi(1-j)$
D. $\pi(1-j)$
[MCQ - 2 Marks]
Ans. A
Sol. C: $\left|z+1-\frac{3}{2} i\right|=1$

$$
\left|(x+1)+i\left(y-\frac{3}{2}\right)\right|=1
$$

$(x+1)^{2}+\left(y-\frac{3}{2}\right)^{2}=1$
Centre $\left(-1, \frac{3}{2}\right)$, radians $=1$


Poles: $z^{2}+2 z+z=0$
$z=-1 \pm i$
$R=\lim _{z \rightarrow-1+i}[z-(-1+i)] \frac{(z+2)}{[z-(-1+i)][z-(-1-i)]}$
$R=\frac{1+i}{2 i}$
Integral $(\mathrm{I})=2 \pi \mathrm{i}(\mathrm{R})$
$=2 \pi \mathrm{i} \times\left(\frac{1+\mathrm{i}}{2 \mathrm{i}}\right)$
$=\pi(1+\mathrm{i})$
48. $V_{1}=\left[\begin{array}{l}1 \\ 2 \\ 0\end{array}\right], V_{2}=\left[\begin{array}{l}2 \\ 1 \\ 3\end{array}\right]$ are two vectors. The value of coefficient $a$ in the expression $V_{1}$ $=\alpha V_{2}+e$, which minimized the length of error vectors $C$ is.
[MCQ, 1 Mark]
A. $\frac{7}{2}$
B. $-\frac{2}{7}$
C. $\frac{2}{7}$
D. $-\frac{7}{2}$

Ans. C
Sol. $V_{1}=\left[\begin{array}{l}1 \\ 2 \\ 0\end{array}\right], V_{2}=\left[\begin{array}{l}2 \\ 1 \\ 3\end{array}\right]$
$\because \mathrm{V}_{1}=\alpha \mathrm{V}_{2}+\mathrm{e}$
$\mathrm{e}=\mathrm{V}_{1}-\alpha . \mathrm{V}_{2}$
$\mathrm{e}=\left[\begin{array}{l}1 \\ 2 \\ 0\end{array}\right]-\left[\begin{array}{c}2 \alpha \\ \alpha \\ 3 \alpha\end{array}\right]$
$\mathrm{e}=\left[\begin{array}{c}1-2 \alpha \\ 2-\alpha \\ -3 \alpha\end{array}\right]$
Length of $\mathrm{e}=|\mathrm{e}|=$
$\sqrt{(1-2 \alpha)^{2}+(2-\alpha)^{2}+(-3 \alpha)^{2}}$
$e^{2}=(1-2 \alpha)^{2}+(2-\alpha)^{2}+(-3 \alpha)^{2}$
$=4 \alpha^{2}+\alpha^{2}+9 \alpha^{2}-4 \alpha-4 \alpha+5$
$e^{2}=14 \alpha^{2}-8 \alpha+5$
let, $f(x)=14 \alpha^{2}-8 \alpha+5$
(i) $f^{\prime}(x)=28 \alpha-8$
(ii) From max/min

$$
\begin{aligned}
& f^{\prime}(\alpha)=0 \\
& 28 \alpha-8=0
\end{aligned}
$$

$$
\alpha=\frac{2}{7}
$$

(iii) $f^{\prime \prime}(\alpha)=28$

$$
\because f^{\prime \prime}\left(\frac{2}{7}\right)=28>0
$$

$\because f(\alpha)$ will minimum for $\left(\alpha=\frac{2}{7}\right)$
49. Let $\omega^{4}=16 \mathrm{j}$. Which of the following cannot be value of $\omega$.
A. $2 \mathrm{e}^{\mathrm{j} / 8}$
B. $2 \mathrm{e}^{\mathrm{j} 5 \pi / 8}$
C. $2 \mathrm{e}^{\mathrm{j} 2 \pi / 8}$
D. $2 \mathrm{e}^{\mathrm{j} \pi / 8}$
[MCQ, 2 Marks]
Ans. C
Sol. $\omega^{4}=16 \mathrm{j}$
$\omega=(16 \mathrm{j})^{1 / 4}$
$\omega=2(\mathrm{j})^{1 / 4}$
$\omega=2\left(e^{j \frac{\pi}{2}}\right)^{1 / 4}$
$\omega=2 \cdot e^{j \frac{\pi}{8}}$
(i) Option A: $\omega=2 \mathrm{e}^{\mathrm{j} \frac{\pi}{8}}$
(ii) Option B: $2 e^{j \frac{5 \pi}{8}}=2 e^{j \frac{\pi}{8}}$
(iii) Option C: $2 e^{j \frac{2 \pi}{8}} \neq 2 e^{j \frac{\pi}{8}}$
(iv) Option D: $2 e^{j \frac{9 \pi}{8}}=2 e^{j \frac{\pi}{8}}$

Correct option (C)
50. The value of line integral $\int_{P}^{0}\left[\left(z^{2} d x+3 y^{2} d y+2 x^{3} d z\right)\right]$ along the straight line joint the point $P(1,1,2)$ and $Q(2,3,1)$ is
A. -5
B. 24
C. 20
D. 29
[MCQ, 2 Marks]
Ans. B
Sol. $\int_{p}^{Q}\left[\left(z^{2} d x+2 x z d z\right)+3 y^{2} d y\right]$
$\int d\left(z^{2} x\right)+d\left(y^{3}\right)=\int d\left(x z^{2}+y^{3}\right)$
$=\left[\mathrm{xz}^{2}+\mathrm{y}^{3}\right]_{\mathrm{P}(1,1,2)}^{\mathrm{Q}(2,3,1)}$
$=(2 \times 1+27)-(1 \times 4+1)$
$=24$
51. The value of integral $\iint x y d x d y$ over the region R given in the figure is

[NAT, 2 Marks]
Ans. 0
Sol.

$I=\int_{x=-1}^{0} \int_{y=-x}^{x+2} x y d y d x+\int_{x=0}^{1} \int_{y=x}^{2-x} x y d y d x$
$=\int x\left[\frac{y^{2}}{2}\right]_{-x}^{x+2} d x+\int x\left[\frac{y^{2}}{2}\right]_{x}^{2-x} d x$
$=\int_{x=-1}^{0}\left(2 x+2 x^{2}\right) d x+\int_{x=0}^{1}\left(2 x-2 x^{2}\right) d x$
$=\int_{-1}^{0} 2 x+\int_{-1}^{0} 2 x^{2} d x-\int_{0}^{1} 2 x^{2} d x$
$=0+\mathrm{I}+\mathrm{I}$
$=0$
52. Let $x$ be an $n \times 1$ real column vector with length $\ell=\sqrt{x^{\top} x}$. The trace of the matrix $P=x x^{\top}$ is
A. $\frac{\ell^{2}}{2}$
B. $\ell^{2}$
C. $\frac{\ell^{2}}{4}$
D. $\ell$
[MCQ, 2 Marks]
Ans. B

## Sol.

$$
\begin{aligned}
P & =x x^{\top}=\left[\begin{array}{l}
x_{1} \\
x_{2} \\
\vdots \\
x_{n}
\end{array}\right]\left[\begin{array}{llll}
x_{1} & x_{2} & \cdots & x_{n}
\end{array}\right] \\
& =\left[\begin{array}{lllll}
x_{1} & x_{1} x_{2} & x_{1} x_{3} & \cdots & x_{1} x_{n} \\
x_{2} & x_{2}^{2} & x_{2} x_{1} & \cdots & x_{2} x_{n} \\
\vdots & & \\
x_{n} & \cdots \cdots \cdots \cdots \cdots \cdots & \cdots & \cdots & x_{n}
\end{array}\right]
\end{aligned}
$$

So, Trace $=x_{1}^{2}+x_{2}^{2}+\cdots \cdots \cdots x_{n}^{2}=\ell^{2}$

## GATE 2023 Electronics Engineering: Expected Topper's Marks

> 80+/100 Marks Expected for AIR under 10
> 70+/100 Marks Expected for AIR under 100

GATE 2023 Electronics Engineering: Expected Cut-Off

| Category | 2021 | 2022 | Expected 2023 |
| :---: | :---: | :---: | :---: |
| General | 25 | 25 | 26 Marks |
| OBC | 25.5 | 22.5 | 23.5 Marks |
| SC/ST | 16.6 | 16.6 | 17.5 Marks |

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Rank 03 Munish (ME)


Rank 08
Hemant (EE)


Rank 06 Ghanendra (EC)


Rank 09
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Rank 39
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Parag (EC)


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