# Electronics \& Communication Engineering 

## GATE 100 Most Important Questions with Solutions

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1. Consider the CMOS circuit shown in the figure (substrates are connected to their respective sources). The gate width (W) to gate length (L) ratios (W/L) of the transistors are as shown. Both the transistors have the same gate oxide capacitance per unit area. For the pMOSFET, the threshold voltage is -1 V and the mobility of holes is $40 \mathrm{~cm}^{2} / \mathrm{Vs}$. For the nMOSFET, the threshold voltage is 1 V and the mobility of electrons is $300 \mathrm{~cm}^{2} / \mathrm{Vs}$. The steady state output voltage $\mathrm{V}_{0}$ is $\qquad$ _.

A. equal to 0 V
B. more than 2 V
C. less than 2 V
D. equal to 2 V

Ans. C
Sol. Given,
$I_{D}=I_{D n}=I_{D P}$
$V_{D S} \geq V_{G S}-V_{t}$, always true.
From the given diagram $V_{G}=V_{D}$ hence both MOSFET are in saturation
$I_{D n}=\frac{1}{2} \mu_{n} C_{C x}\left(\frac{W}{L}\right)_{n}\left(V_{G S}-V_{t}\right)^{2}$
$I_{D P}=\frac{1}{2} \mu_{P} C_{O X}\left(\frac{W}{L}\right)_{P}\left(V_{G S}-\left|V_{t}\right|\right)^{2}$
$I_{D}=I_{D n}=I_{D p}$
$\frac{1}{2} \mu_{n} C_{O X}\left(\frac{W}{L}\right)_{n}\left(V_{G S}-V_{t}\right)^{2}=\frac{1}{2} \mu_{P} C_{O X}\left(\frac{W}{L}\right)_{P}\left(V_{G S}-\left|V_{t}\right|\right)^{2}$
$\mu_{n} C_{o x}\left(\frac{W}{L}\right)_{n}\left(V_{0}-V_{s}-V_{t}\right)^{2}=\mu_{p} C_{o x}\left(\frac{W}{L}\right)_{p}\left(V_{s}-V_{0}-\left|V_{t}\right|\right)^{2}$
$300 \times 1\left(V_{0}-0-1\right)^{2}=40 \times 5\left(4-V_{0}-1\right)^{2}$
$3\left(V_{0}-1\right)^{2}=2\left(3-V_{0}\right)^{2}$
$3 \mathrm{~V}_{0}^{2}+3-6 \mathrm{~V}_{0}=18+2 \mathrm{~V}_{0}^{2}-12 \mathrm{~V}_{0}$
$\mathrm{V}_{0}^{2}+6 \mathrm{~V}_{0}-15=0$
$V_{0}=\frac{-6 \pm \sqrt{36+60}}{2}=1.89 \mathrm{~V},-789 \mathrm{~V}$
At $\mathrm{V}_{0}=1.89 \mathrm{~V}$, both MOSFET will be in saturation.
Hence, the correct option is (C).
2. The Fourier transform $X(j \omega)$ of the signal $x(t)=\frac{t}{\left(1+t^{2}\right)^{2}}$ is $\qquad$ -.
A. $\frac{\pi}{2 \mathrm{j}} \omega \mathrm{e}^{-|\omega|}$
B. $\frac{\pi}{2} \omega \mathrm{e}^{-|\omega|}$
C. $\frac{\pi}{2 \mathrm{j}} \mathrm{e}^{-|\omega|}$
D. $\frac{\pi}{2} \mathrm{e}^{-|\omega|}$

Ans. A
Sol. Consider, $x(t)=e^{-|t|}$
By taking Fourier transform,
$X(\mathrm{j} \omega)=\frac{2}{1+\omega^{2}}$
$e^{-|t|} \stackrel{\text { F.T. }}{\longleftrightarrow} \frac{2}{1+\omega^{2}}$
By differentiation in frequency domain property,
$\mathrm{tx}(\mathrm{t}) \stackrel{\text { F.T. }}{\longleftrightarrow} \mathrm{j} \frac{\mathrm{d}}{\mathrm{d} \omega} \mathrm{X}(\omega)$
$t e^{-|t|} \stackrel{\text { F.T. }}{\longleftrightarrow} \mathrm{j}\left[\frac{\mathrm{d}}{\mathrm{d} \omega}\left(\frac{2}{1+\omega^{2}}\right)\right]$
$t e^{-|t|} \stackrel{\text { F.T. }}{\longleftrightarrow} \frac{-4 \mathrm{j} \omega}{\left(1+\omega^{2}\right)^{2}}$
Apply duality property,
$\frac{-4 \mathrm{jt}}{\left(1+\mathrm{t}^{2}\right)^{2}} \stackrel{\text { F.T. }}{\longleftrightarrow} 2 \pi(-\omega) \mathrm{e}^{-|-\omega|}$
$\frac{\mathrm{t}}{\left(1+\mathrm{t}^{2}\right)^{2}} \stackrel{\text { F.T. }}{\longleftrightarrow} \frac{-2 \pi \omega \mathrm{e}^{-|\omega|}}{-4 \mathrm{j}}$
$\frac{\mathrm{t}}{\left(1+\mathrm{t}^{2}\right)^{2}} \stackrel{\text { F.T. }}{\longleftrightarrow} \frac{\pi}{2 \mathrm{j}} \omega \mathrm{e}^{-|\omega|}$
3.


for what range of $V_{i}(t)$ diode in the Breakdown region.
A. $\mathrm{V}_{\mathrm{i}}(\mathrm{t})=10 \mathrm{~V}$
B. $\mathrm{V}_{\mathrm{i}}(\mathrm{t})>12 \mathrm{~V}$
C. $\mathrm{V}_{\mathrm{i}}(\mathrm{t})>11 \mathrm{~V}$
D. None

## Ans. B

Sol. From given V-I graph if is clear that diode breakdown voltage is -8 volt.

$V_{a b}=8-2$
$V_{a b}=6 \mathrm{~V}$
$V_{a b}=\frac{V_{i}(t) \times 2 k \Omega}{2 k \Omega+2 k \Omega}$
$\mathrm{V}_{\mathrm{ab}}=\frac{\mathrm{V}_{\mathrm{i}}(\mathrm{t})}{2}$
$6=\frac{V_{i}(t)}{2}$
$\mathrm{V}_{\mathrm{i}}(\mathrm{t})=12$ Volt
So, diode will go in breakdown if $\mathrm{V}_{\mathrm{i}}(\mathrm{t})>12 \mathrm{~V}$
4. The system with input $x(t)$ and output $y(t)$ described as $\frac{d y(t)}{d t}+4 t^{2} y(t)=2 t x(t)$
A. Linear and non causal
B. Non linear and causal
C. Linear and causal
D. non linear and non causal

Ans. C

Sol. Linearity:
For input $\mathrm{x}_{1}(\mathrm{t})$
$x_{1}(t) \rightarrow \frac{d y_{1}(t)}{d t}+4 t^{2} y_{1}(t)=2 t x_{1}(t)$
For input $\mathrm{x}_{2}(\mathrm{t})$
$\mathrm{x}_{2}(\mathrm{t}) \rightarrow \frac{\mathrm{dy}}{2}(\mathrm{t}) \mathrm{dt}+4 \mathrm{t}^{2} \mathrm{y}_{2}(\mathrm{t})=2 \mathrm{t} \mathrm{x}_{2}(\mathrm{t})$
Multiply equn (1) by a and equn (2) by band then adding

$$
\frac{a d y_{1}(t)}{d t}+4 a t^{2} y_{1}(t)+\frac{b d y_{2}(t)}{d t}+4 b t^{2} y_{2}(t)=2 a t x_{1}(t)+2 b t x_{2}(t)
$$

$$
\frac{\mathrm{d}}{\mathrm{dt}}\left[\mathrm{ay}_{1}(\mathrm{t})+\mathrm{by}_{2}(\mathrm{t})\right]+4 \mathrm{t}^{2}\left[\mathrm{ay}_{1}(\mathrm{t})+\mathrm{by}_{2}(\mathrm{t})\right]=2 \mathrm{t}\left[\mathrm{ax}_{2}(\mathrm{t})+\mathrm{bx}_{2}(\mathrm{t})\right]
$$

This equ ${ }^{n}$ shows that for an input $a x_{1}(t)+b x_{2}(t)$ output is $a y_{1}(t)+b y_{2}(t)$, so system is linear.
Causality: The output depends on the present input only therefore the system is causal.
5. A magnetic field strength of $10 \mu \mathrm{~A} / \mathrm{m}$ is required at point on $\theta=\frac{\pi}{2}, 4 \mathrm{Km}$ from an antenna in air. Neglect ohmic loss.How much power (in Watt) must antenna transmitted if it is $\frac{\pi}{2}$ dipole?

Ans. [2 to 2.5]

## Sol.

$$
\begin{aligned}
& \left|H_{o s}\right|=\frac{I_{0} \cos \left(\frac{\pi}{2} \cos \theta\right)}{2 \pi \mathrm{r} \sin \theta} \\
& 10 \times 10^{-6}=\frac{I_{0} \cos \left(\frac{\pi}{2} \cos \frac{\pi}{2}\right)}{2 \pi \times 4 \times 10^{3} \times \sin \frac{\pi}{2}} \\
& 10 \times 10^{-6}=\frac{I_{0} \cos (0)}{2 \pi \times 4 \times 10^{3} \times 1} \\
& 10 \times 10^{-6}=\frac{I_{0}}{8 \pi \times 10^{3}} \\
& I_{o}=80 \pi \times 10^{-6} \times 10^{3} \\
& I_{o}=80 \pi \mathrm{~mA} \\
& P_{r a d}=\frac{1}{2} I_{o}^{2} R_{\mathrm{rad}} \\
& P_{\mathrm{rad}}=\frac{1}{2}(80 \pi)^{2} \times 73 \\
& P_{\mathrm{rad}}=\frac{4606405.12 \times 10^{-6}}{2} \\
& P_{\mathrm{rad}}=2.3 \mathrm{w}
\end{aligned}
$$

6. Find value of $k$ under resonance cond ${ }^{n}$.

A. $\frac{1}{\sqrt{17}}$
B. $\frac{1}{\sqrt{42}}$
C. $\frac{1}{\sqrt{37}}$
D. $\frac{1}{\sqrt{53}}$

Ans. B
Sol. At resonance $X_{\text {eq }}=15$

$$
\begin{aligned}
& 2 \pi f\left(M=k^{L_{1} L_{2}}\right) \\
& X_{M}=k \sqrt{X_{1} X_{2}} \\
& 2 \pi f\left(L_{e q}=L_{1}+L_{2}+2 M\right) \\
& X_{e q}=X_{1}+X_{2}+2 X_{M} \\
& X_{e q}=X_{1}+X_{2}+2 k \sqrt{X_{1} X_{2}} \\
& 15=6+7+2 k \sqrt{6 \times 7} \\
& k=\frac{1}{\sqrt{42}}
\end{aligned}
$$

7. Determine DTFT for given signal
$x(n)=\{-4,-1,0,1,4\}$ origin at 0
A. $2 \mathrm{j}(4 \sin 2 \Omega-\sin \Omega)$
B. $-2 \mathrm{j}(\sin \Omega-4 \sin 2 \Omega)$
C. $2 \mathrm{j}(\sin \Omega+4 \sin 2 \Omega)$
D. $-2 \mathrm{j}(4 \sin 2 \Omega+\sin \Omega)$

## Ans. D

Sol. $X\left(\mathrm{e}^{\mathrm{j} \Omega}\right)=-4 \mathrm{e}^{2 \mathrm{j} \Omega}-1 . \mathrm{e}^{\mathrm{j} \Omega}+0+\mathrm{e}^{-\mathrm{j} \Omega}+4 \mathrm{e}^{-2 j \Omega}$
$=-4\left[e^{2 j \Omega}-e^{-2 j \Omega}\right]+(-1)\left[e^{j \Omega}-e^{j \Omega}\right]$
$=-4 \frac{\left[e^{2 j \Omega}-e^{-2 j \Omega}\right]}{2 j} \times 2 j-1.2 j \frac{\left[e^{j \Omega}-e^{-j \Omega}\right]}{2 j}$
$=-8 \mathrm{j} \sin 2 \Omega-2 \mathrm{j} \sin \Omega$
$=-2 \mathrm{j}[4 \sin 2 \Omega+\sin \Omega]$
8. $\mathrm{Nd}=10^{16} / \mathrm{cm}^{3}$
$\mathrm{V}_{\mathrm{T}}=26 \mathrm{mV}$
$D_{n}=35.1 \mathrm{~cm}^{2} / \mathrm{s}$
$D_{p}=12.48 \mathrm{~cm}^{2} / \mathrm{s}$
$\mathrm{n}_{\mathrm{i}}=1.5 \times 10^{10} / \mathrm{cm}^{3}$
$A=1 \mathrm{~nm}^{2}$
Find the minority carries in $N$-Type semiconductor in length of semiconductor is 2 nm ?
A. $10^{10}$
B. 0
C. $10^{12}$
D. $10^{-3}$

Ans. B
Sol. Total minority carriers
$=\frac{n_{i}^{2}}{N_{D}} \times V$
$=\frac{n_{i}^{2}}{N_{D}} \times \mathrm{A} \times \mathrm{L}$
$=\frac{2.25 \times 10^{20}}{10^{16}} \times 10^{-14} \times 2 \times 10^{-7}$
$=4.5 \times 10^{-17}$
$=0$
$\left\{\mathrm{L}=2 \mathrm{~nm} ; \mathrm{L}=2 \times 10^{-7} \mathrm{~cm}\right.$
$A=1 \mathrm{~nm}^{2} ; \mathrm{A}=1 \times\left(10^{-7}\right)^{2} \mathrm{~cm}^{2}$
$\left.A=10^{-14}\right\}$
9.


Find io
A. 1 mA
B. 2 mA
C. 5 mA
D. -2 mA

Ans. B
Sol. It is a comparator
$\mathrm{V}^{+}=8 \mathrm{~V}$
$\mathrm{V}^{-}=\frac{10 \times 5}{5+5}=\frac{10}{2}=5 \mathrm{~V}$
If, $\mathrm{V}^{+}>\mathrm{V}^{-} ; \mathrm{V}_{\text {out }}=+\mathrm{V}_{\text {sat }}=10 \mathrm{~V}$
$\mathrm{V}^{+}<\mathrm{V}^{-} ; \mathrm{V}_{\text {out }}=-\mathrm{V}_{\text {sat }}=-10 \mathrm{~V}$
Here, $\mathrm{V}^{+}>\mathrm{V}^{-}$
So, $\mathrm{V}_{\text {out }}=10 \mathrm{~V}$
$I_{0}^{\prime}=\frac{10}{5 \mathrm{k} \Omega}=2 \mathrm{~mA}$
10. The Laplace transform of a signal $x(t)$ is
$x(s)=\frac{d^{2}}{d s^{2}}\left(\frac{1}{s-8}\right), R O C: R_{e}(s)>8$
The signal $x(t)=\left(t e^{k t}\right)^{2} U(t)$, where constant $k$ is
A. 2
B. 6
C. 4
D. 3

Ans. C

## Sol.

$\frac{1}{S-8} \stackrel{L^{-1}}{\longleftrightarrow} e^{8 t} U(t)$
$\frac{d^{2}}{d s^{2}}\left\{\frac{1}{s-8}\right\} \stackrel{L^{-1}}{\longleftrightarrow} t^{2} e^{8 t} U(t)$
$\frac{d^{2}}{d s^{2}}\left\{\frac{1}{s-8}\right\} \stackrel{L^{-1}}{\longleftrightarrow}\left(t e^{k t}\right)^{2} U(t)$
$2 \mathrm{k}=8$
$K=4$
11.


Find $\mathrm{V}_{\mathrm{EC}}=$ ?
A. 6.2
B. 3.43
C. 4.5
D. 5

## Ans. B

Sol. $10-10 \times(1+\beta) I_{B}-0.7-10 I_{B}+2=0$
$10-10 \times(1+75) I_{B}-0.7-10 I_{B}+2=0$
$10-760 I_{B}-10 I_{B}+1.3=0$
$770 \mathrm{I}_{\mathrm{B}}=11.3$
$I_{B}=\frac{11.3}{770}=14.6 \mu \mathrm{~A}$
$I_{C}=\beta I_{B}=75 \times 14.6 \mu \mathrm{~A}=1.095 \mathrm{~mA}$
$I_{E}=(1+\beta) I_{B}$
$\mathrm{I}_{\mathrm{E}}=76 \times 14.6 \mu \mathrm{~A}$
$\mathrm{I}_{\mathrm{E}}=1.109 \mathrm{~mA}$
Write KVL
$10-10 \mathrm{k} \Omega \times 1.109 \mathrm{~mA}-\mathrm{V}_{\mathrm{EC}}-5 \mathrm{k} \times 1.095 \mathrm{~mA}+10=0$
$20-11.09-5.475=\mathrm{V}_{\mathrm{EC}}$
$\mathrm{V}_{\mathrm{EC}}=3.435 \mathrm{Volt}$
12. A signal uniformly distributed on $[-1,1]$ is baseband modulated using PCM with 128 levels. Calculate the resulting SQNR?
A. 36 dB
B. 42 dB
C. 48 dB
D. 30 dB

Ans. B
Sol. The pdf of the signal given will be:


The power of the signal is given by
$P_{s}=\int_{-\infty}^{\infty} x^{2} f_{X}(x) d x$
$\Rightarrow P_{S}=\int_{-1}^{1} \frac{1}{2} x^{2} d x=\frac{1}{3}$
$\mathrm{n}=\log _{2} 128=7$ bits
SQNR $=P_{s} / P_{N}$
Where $P_{N}=\frac{\Delta^{2}}{12}$ and $\Delta=\frac{2 A_{m}}{L}=\frac{2}{128}=\frac{1}{64}$
$\Rightarrow P_{N}=\frac{1}{64^{2} \times 12}$
$\Rightarrow$ SQNR $=16384$
$\Rightarrow$ SQNR $(\mathrm{dB})=10 \log { }_{10}($ SQNR $)=42 \mathrm{~dB}$
13. Two $\lambda / 4$ transformer lines are used to connect a $50 \Omega$ line to a $70 \Omega$ load. Determine characteristic impedance $Z_{01}$ if $Z_{02}=30 \Omega$.
Assume there is no reflection to the $t$ of $A$.

A. $24.5 \Omega$
B. $20.5 \Omega$
C. $50 \Omega$
D. $25.35 \Omega$

## Ans. D

## Sol.



$$
Z_{\mathrm{in}_{2}}=\frac{Z_{02}^{2}}{Z_{\mathrm{L}}}=\frac{(30)^{2}}{70}
$$

$$
Z_{\mathrm{in}_{1}}=\frac{\left(Z_{01}\right)^{2}}{Z_{\mathrm{in}_{2}}}
$$

$$
\begin{aligned}
\left(Z_{01}\right)^{2} & =Z_{\text {in }_{1}} \times Z_{\text {in }_{2}} \\
& =Z_{\text {in }_{1}} \times \frac{(30)^{2}}{70}
\end{aligned}
$$

There is no reflection to left to A so
$Z_{\mathrm{in}_{1}}=\mathrm{Z}_{0}$
$Z_{i i_{1}}=50$
So,
$\left(Z_{01}\right)^{2}=50 \times \frac{(30)^{2}}{70}$
$Z_{01}=\sqrt{\frac{(30)^{2} \times 50}{70}}$
$Z_{01}=30 \sqrt{\frac{50}{70}}$
$Z_{01}=30 \times .85$
$Z_{01}=25.35 \Omega$
14. For circuit Diode is ideal plot $\mathrm{V}_{0}$ versus Vi is

A.

B.

C.

D. None

Ans. B
Sol. Voltage at cathode terminal $=\frac{10 \times 2}{4}=5 \mathrm{~V}$
So, for $\mathrm{V}_{\mathrm{i}}$ (anode)< 5 V
Diode is open circuit
$V_{0}=V_{i}$
For $\mathrm{V}_{\mathrm{i}}$ (anode)>5V diode is short circuit
So, Circuit will be


Write KCL,
$\frac{V_{0}}{2 K}+\frac{V_{0}-10}{2 k \Omega}+\frac{V_{0}-V_{i}}{1 k \Omega}=0$
$\mathrm{V}_{0}+\mathrm{V}_{0}-10+2 \mathrm{~V}_{0}-2 \mathrm{~V}_{\mathrm{i}}=0$
$4 V_{0}-2 V_{i}=10$
$4 \mathrm{~V}_{0}=2 \mathrm{~V}_{\mathrm{i}}+10$
$V_{0}=.5 V_{i}+2.5$
So, If $\mathrm{Vi}=10$
$\mathrm{V}_{\mathrm{o}}=7.5 \mathrm{v}$
15. A sum of money is to be distributed among $P, Q, R$, and $S$ in the proportion $5: 2: 4: 3$, respectively. If R gets ₹ 1000 more than S , what is the share of Q (in ₹) ?
A. 500
B. 1000
C. 1500
D. 2000

Ans. D
Sol. P: Q : R:S = 5: 2: 4:3
Money of $P=5 x$
Money of $\mathrm{Q}=2 \mathrm{x}$
Money of $R=4 x$
Money of $S=3 x$
Money of $\mathrm{R}=1000+$ Money of S
i.e. $4 x=1000+3 x$
$x=1000$
Now, Money of $\mathrm{Q}=2 \mathrm{x}$
$=2000$
16. If the vectors $(1.0,-1.0,2.0),(7.0,3.0, x)$ and $(2.0,3.0,1.0)$ in $R^{3}$ are linearly dependent the value of $x$ is $\qquad$
Ans. 8
Sol. Given vectors are
$\mathrm{x}_{1}=\left[\begin{array}{lll}1 & -1 & 2\end{array}\right]$
$x_{2}=[7,3 x]$
$x_{3}=\left[\begin{array}{ll}2 & 3\end{array}\right]$
are linearly dependent
Let,
$A=\left[X_{1}^{\top} X_{2}^{\top} X_{3}^{\top}\right]$
$A=\left[\begin{array}{ccc}1 & 7 & 2 \\ -1 & 3 & 3 \\ 2 & x & 1\end{array}\right]_{3 \times 3}$
Rank (A) < order of 3
$\because$ rank (A) < 3
$\Rightarrow A$ should be singular matrix $|A|=0$
$\Rightarrow\left[\begin{array}{ccc}1 & 7 & 2 \\ -1 & 3 & 3 \\ 2 & x & 1\end{array}\right]=0$
$\Rightarrow 1 \times(3-3 x)-7 x(-1-6)+2 \times(-x-6)=0$
$\Rightarrow-5 x+40 \Rightarrow x=8$
17. A digital transmission system uses a $(7,4)$ systematic linear Hamming code for transmitting data over a noisy channel. If three of the message -codeword pairs in this code ( $m_{i} ; c_{i}$ ) where $c_{i}$ is the codeword corresponding to the $\mathrm{i}^{\text {th }}$ message $\mathrm{m}_{\mathrm{i}}$, are known to be $(1100 ; 0101100)$, $(1110 ; 0011110)$ and $(0110 ; 1000110)$, then which of the following is a valid codeword in this code?
A. 0110100
B. 1011010
C. 0001011
D. 1101001

## Ans. C

Sol. Given code is systematic linear hamming code of order ( 7,4 )
Given message and code word pairs are
1100; 0101100
1110; 0011110
0110; 1000110
The code word is of the form
$p_{1} p_{2} p_{3} d_{1} d_{2} d_{3} d_{4}$
where $\mathrm{P}_{1}=\mathrm{d}_{1} \oplus \mathrm{~d}_{2} \oplus \mathrm{~d}_{4}$
$\mathrm{P}_{2}=\mathrm{d}_{2} \oplus \mathrm{~d}_{3} \oplus \mathrm{~d}_{4}$
$\mathrm{P}_{3}=\mathrm{d}_{1} \oplus \mathrm{~d}_{2} \oplus \mathrm{~d}_{3}$
The code word which satisfies this pattern is 0001011
$\therefore$ option 'C' (or) option ' 3 ' is answer.
18. Find resp. of $\mathrm{V}_{\mathrm{c}} \mathrm{ic}, \& \mathrm{~V}_{\mathrm{x}}$ when initial voltage of cap. Is 3 V .

A. $V_{c}(t)=3 e^{-t / 2}, i_{c}(t)=-0.75 e^{-t / 2}$
B. $V_{C}(t)=6 e^{-t / 2}, i_{c}(t)=2 e^{-t / 2}$
$V_{x}(t)=\frac{3}{4} e^{-t / 2}$
$V_{x}(t)=3 e^{-t / 2}$
C. $V_{C}(t)=4 e^{-t / 2} \quad i_{C}(t)=1.5 e^{-t / 2}$
D. None
$v_{x}(t)=1.5 \mathrm{e}^{-t / 2}$

Ans. A

## Sol.



In steady state cap. Without source
$\mathrm{V}_{\mathrm{c}}=0$ (cap. Discharge through R )
$\mathrm{V}_{\mathrm{c}}\left(0^{-}\right)=\mathrm{V}_{\mathrm{c}}\left(0^{+}\right)=3 \mathrm{~V}$ (given)
$R_{\text {eq }}=8 \| 8=4$
Cap. Is without source.
$V_{c}=3 e^{-t / 4 \times .5}$
$V_{C}(t)=3 e^{-t / 2}$
$\mathrm{i}_{\mathrm{C}}=\frac{\mathrm{CdV}}{\mathrm{dt}}=.5 \times 3 \frac{\mathrm{~d}}{\mathrm{dt}} \mathrm{e}^{-\mathrm{t} / 2}$
$\mathrm{i}_{C}=-\frac{1.5}{2} e^{-\mathrm{t} / 2}$
$i_{C}=-.75 e^{-t / 2}$
$V_{x}=\frac{V_{C} \times 2}{6+2}$
$V_{x}=\frac{V_{C}}{4}=\frac{3}{4} e^{-t / 2}$
19. An 8 bit digital data 10101100 is fed to an DAC. The reference voltage is +5 V . Analog output voltage will be:
A. 1.05
B. 3.372
C. 4.5
D. 5.15

## Ans. B

Sol. $\mathrm{n}=8$
$i / p=10101100=(172)_{10}$
$V_{\text {out }}=\frac{V_{R}}{2^{n}-1} \times 172$
$=\frac{5 \times 172}{2^{8}-1}=\frac{5 \times 172}{255}$
$=\frac{860}{255}=3.372$
20. The state equation Of LTI system is represented by $\dot{X}=\left[\begin{array}{cc}0 & 1 \\ -4 & 0\end{array}\right] X+\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right] U$

Eigen value are
A. $-1,+1$
B. $-1,-1$
C. $\pm 2 \mathrm{j}$
D. None

Ans. C
Sol. For Eigen value,

$$
|S I-A|=0
$$

$(S I-1)=\left[\begin{array}{cc}s & -1 \\ 4 & s\end{array}\right]$
$|S I-A|=S^{2}+4$
$|S I-A|=0$
$S^{2}+4=0$
$S^{2}=-4, S= \pm 2 j$
21. Given $L^{-1}\left[\frac{3 s+4}{s^{2}+4 s+(k-2) s}\right]$
if $\lim _{x \rightarrow \infty} f(t)=1$, then value of $k$ is
A. 1
B. 2
C. 3
D. 0

Ans. B
Sol. $\lim _{t \rightarrow \infty} f(t)=\lim _{s \rightarrow 0} s F(s)$
$\lim _{s \rightarrow 0} s \cdot\left[\frac{(3 s+4)}{s[s+4+(k-2)]}\right]=1$
$\lim _{s \rightarrow 0} \frac{3 s+4}{s+4+k-2}=1$
$\frac{4}{4+k-2}=1$
$4=4+k-2 \Rightarrow k=2$
22. A ideal long silicon $p n$ junction diode is shown in fig. The n-region is doped with $10^{16}$ organic atoms per $\mathrm{cm}^{3}$ and the $p$ - region is doped with $5 \times 10^{16}$ boron atoms per $\mathrm{cm}^{3}$. The minority carrier lifetime of holes are $10^{-8}$ seconds and diffusion constant is $D_{n}=23 \mathrm{~cm}^{2} / \mathrm{s}$ and $D_{p}=8 \mathrm{~cm}^{2} / \mathrm{s}$.
The forward-bias voltage is $\mathrm{V}_{\mathrm{a}}=0.61 \mathrm{~V}$


The excess hole concentration is
A. $6.8 \times 10^{12} \mathrm{e}^{-246 \mathrm{x}} \mathrm{cm}^{-3}, \mathrm{x} \geq 0$
B. $6.8 \times 10^{12} \mathrm{e}^{246 \mathrm{x}} \mathrm{cm}^{-3}, \mathrm{x} \geq 0$
C. $3.8 \times 10^{14} \mathrm{e}^{-353 \mathrm{x}} \mathrm{cm}^{-3}, \mathrm{x} \geq 0$
D. $3.8 \times 10^{14} \mathrm{e}^{3534 \mathrm{x}} \mathrm{cm}^{-3}, \mathrm{x} \geq 0$

Ans. D
Sol. $\delta p_{n}=p_{n}-p_{n o}=p_{\text {no }}\left[e^{\frac{e V_{a}}{k T}}-1\right]\left[e^{-\left(\frac{x}{L_{P}}\right)}\right]$
$p_{n o}=\frac{n_{i}^{2}}{N_{d}}=\frac{\left(1.5 \times 10^{10}\right)^{2}}{10^{16}}=2.25 \times 10^{4} \mathrm{~cm}^{-3}$
$L_{P}=\sqrt{D_{p} \tau_{p o}}=\sqrt{8 \times 10^{-8}}=2.83 \times 10^{-4} \mathrm{~cm}$
$\delta p_{n}=2.25 \times 10^{4}\left[e^{\left.\left(\frac{0.61}{0.0259}\right)-1\right)}\right]\left[e^{-\left(\frac{x}{2.83 \times 10^{-4}}\right)}\right]=3.8 \times 10^{14} e^{-3534 x} \mathrm{~cm}^{-3}$
23. The electrical system shown in the figure converts input source current is $(t)$ to output voltage $\mathrm{V}_{\mathrm{o}}(\mathrm{t})$


Current $i_{L}(\mathrm{t})$ in the inductor and voltage $\mathrm{v}_{\mathrm{c}}(\mathrm{t})$ across the capacitor are taken as the state variables, both assumed to be initially equal to zero, i.e., $\mathrm{i}_{\mathrm{L}}(0)=0$ and $\mathrm{v}_{0}(0)=0$ The system is
A. neither state controllable nor observable
B. completely state controllable but not observable
C. completely observable but not state controllable
D. completely state controllable as well as completely observable

Ans. A
Sol. $i_{s}=i_{L}+\frac{V_{L}}{1}$
$\mathrm{i}_{\mathrm{s}}=\mathrm{i}_{\mathrm{L}}+\mathrm{L} \frac{\mathrm{di}}{\mathrm{dt}}$
$\mathrm{i}_{\mathrm{s}}=\mathrm{i}_{\mathrm{C}}+\frac{\mathrm{V}_{\mathrm{C}}}{1}$
$i_{c}=C \frac{d V_{C}}{d t}$
Let $\mathrm{i}_{\mathrm{L}}=\mathrm{x}_{1}$
$\mathrm{V}_{\mathrm{c}}=\mathrm{x}_{2}$
$\mathrm{i}_{\mathrm{s}}=\mathrm{u}=$ input

1. $\Rightarrow \mathrm{L} \frac{\mathrm{d}}{\mathrm{dt}} \mathrm{X}_{1}+\mathrm{X}_{1}=u$

$$
\dot{x}_{.1}=\frac{-1}{L} x_{1}+\frac{1}{L} u
$$

2. $\Rightarrow C \frac{d}{d t} x_{2}+x_{2}=u$

$$
\dot{x}_{2}=-\frac{1}{C} x_{2}+\frac{1}{C} u
$$

Output, $\mathrm{y}(\mathrm{t})=\mathrm{V}_{\mathrm{c}}(\mathrm{t})=\mathrm{x}_{2}(\mathrm{t})$
So, we have
$\dot{x}=\left(\begin{array}{cc}-\frac{1}{L} & 0 \\ 0 & -\frac{1}{C}\end{array}\right) x+\binom{1 / L}{1 / C} u$
$y=x_{2}$
$y=\left(\begin{array}{ll}0 & 1\end{array}\right) x$
Putting $L=1 \mathrm{H}$ and $\mathrm{C}=1 \mathrm{~F}$ we get
$\dot{x}=\left(\begin{array}{cc}-1 & 0 \\ 0 & -1\end{array}\right) x+\binom{1}{1} u$
$y=x_{2}$
$y=(01) x$
Controllability Matrix
$S=\left[\begin{array}{ll}B & A B\end{array}\right]=\left[\begin{array}{ll}1 & -1 \\ 1 & -1\end{array}\right]$
$|S|=0 \Rightarrow$ uncontrollable
Observability Matrix
$V=\left[\begin{array}{c}C \\ C A\end{array}\right]=\left[\begin{array}{cc}0 & 1 \\ 0 & -1\end{array}\right]$
$|\mathrm{V}|=0 \Rightarrow$ unobservable
24.
$20 \Omega$


Find $\mathrm{i}\left(\mathrm{O}^{-}\right)$and $\mathrm{i}\left(\mathrm{O}^{+}\right)$
A. $2 \mathrm{~A}, 10 / 7 \mathrm{~A}$
B. $6 \mathrm{~A}, 20 / 7 \mathrm{~A}$
C. $2.5 \mathrm{~A}, 20 / 7 \mathrm{~A}$
D. $4.5 \mathrm{~A}, 10 / 7 \mathrm{~A}$

Ans. C

## Sol.

at $\mathrm{t}=0$ -

$L$ is $S . C$.

$i\left(0^{-}\right)=\frac{100}{40}$
$=2.5 \mathrm{~A}$
$\mathrm{i}_{\mathrm{L}}\left(\mathrm{O}^{-}\right)=2.5 \mathrm{~A}$
at $t=0^{+}$

$\frac{V_{a}-100}{30}+2.5+\frac{V_{a}}{40}=0$
$V_{a}=\frac{100}{7}$
$i\left(0^{+}\right)=2.5+\frac{100}{7 \times 40}$
$=2.5\left(1+\frac{1}{7}\right)$
$=\frac{8 \times 2.5}{7}$
$\mathrm{i}\left(\mathrm{O}^{+}\right)=\frac{20.0}{7} \mathrm{~A}$
25. An air filled rectangular wave guide is operating at 6 GHz with dominant propagating inside it. If the wavelength inside the wave guide is 6 cm , then the wave impedance of the mode is given by $\qquad$ $\Omega$.
A. 245.6
B. 345.6
C. 452.4
D. 568.6

Ans. C
Sol. Given, $f=6 \mathrm{GHz}$
$\Rightarrow \lambda=\frac{c}{f}=5 \mathrm{~cm}$
$\lambda_{s}=6 \mathrm{~cm}$

Mode $=\mathrm{TE}_{10}$
Wave impedance
$\eta_{T z}=\eta \frac{\lambda_{s}}{\lambda}=120 \pi \times \frac{6}{5}$
$\eta_{T E}=452.4 \Omega$
26. A message signal $m(t)$ and a carrier signal $C(t)$ are applied to an amplitude modulation shown


The message signal $m(t)$ is band-limited to $f_{m}$, where $f_{c} \gg 3 f_{m}$. The BPF has a unity passband gain over a Bandwidth of $2 f_{m}$ centred at $f_{c}$. The input-output characteristic of non-linear device is $y(t)$ $=8 x(t)+2 x^{2}(t)$. If average power of message signal $m(t)$ is 16 W , find the $\%$ transmission efficiency of resultant AM signal.
A. 40
B. 60
C. 80
D. 90

## Ans. C

Sol. $x(t)=m(t)+\cos 2 \pi f c t$
$y(t)=8 x(t)+2 x^{2}(t)$
$=8\left[m(t)+\cos 2 \pi f_{c} t\right]+2\left[m(t)+\cos 2 \pi f_{c} t\right]^{2}$
$=8 m(t)+8 \cos 2 \pi f_{c} t+2 m^{2}(t)+2 \cos ^{2} 2 \pi f_{c} t+4 m(t) \cos 2 \pi f_{c} t$
After passing through BPF, we get
$y(t)=8 \cos 2 \pi f_{c} t+4 m(t) \cos 2 n f c t$
$=8\left[1+\frac{1 m(t)}{2}\right] \cos 2 \pi f_{c} t$
The amplitude sensitivity of resultant AM signal is $K_{a}=\frac{1}{2} \mathrm{~V}^{-1}$
The transmission efficiency of resultant AM signal is
$\% \eta=\left(\frac{K_{a}^{2} P_{m}}{1+K_{a}^{2} P_{m}} \times 100\right) \%$
$\mathrm{P}_{\mathrm{m}}=$ message signal power $=16 \mathrm{~W}$
$\% \eta=\left(\frac{\left(\frac{1}{2}\right)^{2} \times 16}{1+\left(\frac{1}{2}\right)^{2} \times 16}\right) \times 100 \%=80 \%$
27. How many 1 's are present in binary representation of $7 \times 64+5 \times 8+3$
A. 8
B. 9
C. 7
D. 10

Ans. C
Sol. $\left(2^{2}+2+1\right) \times 2^{6}+\left(2^{2}+1\right) \times 2^{3}+(2+1)$
$2^{8}+2^{7}+2^{6}+2^{5}+2^{3}+2^{1}+2^{0}$
111101011
So, Ans is 7
28. $X$ is random variable with uniform probability density function in the interval [-2, 10]. For $Y=2 X-6$, the conditional probability $P(Y \leq 7 \mid X \geq 5)$ (rounded off to three decimal places) is
$\qquad$
Ans. 0.3
Sol. Given (0.3 to 0.3)
$f_{x}(x)=\left\{\begin{array}{cc}1 / 12 & -2 \leq x \leq 10 \\ 0 & \text { otherwise }\end{array}\right.$
As $\mathrm{y}=2 \mathrm{x}-6$
So,
$f_{y}(y)=\left\{\begin{array}{cc}1 / 24 & -10 \leq x \leq 14 \\ 0 & \text { otherwise }\end{array}\right.$
If $x \geq 5$ then $y \geq 4$
So, $P(y \leq 7 / x \geq 5)=P(Y \leq 7 / y \geq 4)$
$=\frac{P(4 \leq y \leq 7)}{P(4 \leq y \leq 14)}=\frac{3}{10}=0.3$

29. The block diagram of a closed-loop control system is shown in the figure. $R(s), Y(s)$, and $D(s)$ are the Laplace transforms of the time-domain signals $r(t), y(t)$, and $d(t)$, respectively. Let the error signal be defined as $e(t)=r(t)-y(t)$. Assuming the reference input $r(t)=0$ for all $t$, the steady-state error $e(\infty)$, due to a unit step disturbance $\mathrm{d}(\mathrm{t})$, is $\qquad$ (rounded off to two decimal places).


Ans. [- 0.11 to - 0.09]

## Sol.


$G_{1}(s)=10, \quad G_{2}(s)=\frac{1}{s(s+10)}$
$\frac{E(s)}{D(s)}=\frac{-G_{2}(s)}{1+G_{1}(s) G_{2}(s)}$
$=\frac{\frac{-1}{s(s+10)}}{1+\left(10 \times \frac{1}{s(s+10)}\right)}$
$=\frac{-1}{s^{2}+10 s+10}$
$D(s)=\frac{1}{s}$ (Given in the question)
$e_{s s}=\lim _{t \rightarrow \infty} \mathrm{e}(\mathrm{t})=\lim _{\mathrm{s} \rightarrow 0} \mathrm{~s} \mathrm{E}(\mathrm{s})$
$e_{s s}=\lim _{s \rightarrow 0} \frac{s\left(\frac{1}{s}\right)(-1)}{s^{2}+10 s+10}$
$e_{s 5}=\frac{-1}{10}$
$e_{s 5}=-0.1$
30. The magnetic field of a uniform plane wave in vacuum is given by
$\vec{H}(x, y, z, t)=\left(\hat{a}_{x}+2 \hat{a}_{y}+b \hat{a}_{z}\right) \cos (\omega t+3 x-y-z)$
The value of $b$ is $\qquad$
Ans. 1

Sol. Given,
$\vec{H}(x, y, z, t)=\left(\hat{a}_{x}+2 \hat{a}_{y}+b+\hat{a}_{z}\right) \cdot \cos (\omega t+3 x-y-z) A / m$
For a uniform wave,
$\overrightarrow{\mathrm{k}} \cdot \overrightarrow{\mathrm{H}}_{0}=0, \overrightarrow{\mathrm{k}} \cdot \overrightarrow{\mathrm{E}}_{0}=0, \overrightarrow{\mathrm{E}}_{0} \cdot \overrightarrow{\mathrm{H}}_{0}=0$
i.e., $\vec{E}, \vec{H}$ and $\vec{K}$ are mutually perpendicular to each other.
( $\vec{K}$ is the vector along the direction of wave propagation)
Comparing the given expression of $\overrightarrow{\mathrm{H}}$ with the standard expression.
$\vec{k}=3 \hat{a}_{x}-\hat{a}_{y}-\hat{a}_{z}$
And,
$\vec{H}_{0}=\left(\hat{a}_{x}+2 \hat{a}_{y}+b \hat{a}_{z}\right)$
Then,
$\overrightarrow{\mathrm{k}} \cdot \overrightarrow{\mathrm{H}}_{0}=3-2-\mathrm{b}=0$
$\Rightarrow \mathrm{b}=1$
31. A sinusoidal message signal having root mean square value of 4 V and frequency of 1 kHz is fed to a phase modulator with phase deviation constant 2 rad/volt. If the carrier signal is $c(t)=2 \cos \left(2 \pi 10^{6} t\right)$, the maximum instantaneous frequency of the phase modulated signal (rounded off to one decimal place) is $\qquad$ KHz .
Ans. [1010 to 1013]
Sol. $m(t)_{r m s}=4 V$
$\frac{\mathrm{d}}{\mathrm{dt}} \mathrm{m}(\mathrm{t})=4 \sqrt{2} \times 2 \pi \times 10^{3} \cos \left(2 \pi \times 10^{3} \mathrm{t}\right)$
$m(t)=4 \sqrt{2}$
$\frac{d \mathrm{~m}(\mathrm{t})}{\mathrm{dt}}=4 \sqrt{2} \times 2 \pi \times 10^{3}$
$\mathrm{K}_{\mathrm{p}}=2 \mathrm{rad} / \mathrm{volt}$
$P M: \theta(t)=w_{c} t+k_{p} m(t)$
$w_{i}(t)=w_{c}+k_{p} \frac{d m(t)}{d t}$
$f_{\text {Maximum }}=f_{c}+\frac{K_{p}}{2 \pi} \cdot\left[\frac{d m(t)}{d t}\right]$
$f_{\max }=1000 \times 10^{3}+\frac{2}{2 \pi} \times 4 \sqrt{2} \times 2 \pi \times 10^{3} \mathrm{~Hz}$
$f_{\text {Maximum }}=(1000+8 \sqrt{2}) \mathrm{kHz}$
$f_{\text {Maximum }}=(1000+11.3137) \mathrm{kHz}$
$\mathrm{f}_{\text {Maximum }}=1011.3137 \mathrm{kHz}$
32. A germanium sample of dimensions $1 \mathrm{~cm} \times 1 \mathrm{~cm}$ is illuminated with a $20 \mathrm{~mW}, 600 \mathrm{~nm}$ laser light source as shown in the figure. The illuminated sample surface has a 100 nm of loss-less Silicon dioxide layer that reflects one-fourth of the incident light. From the remaining light, onethird of the power is reflected from the Silicon dixodie-Germanium interface, one-third is absorbed in the Germanium layer, and one-third is transmitted through the other side of the sample. If the absorption coefficient of Germanium at 600 nm is $3 \times 10^{4} \mathrm{~cm}^{-1}$ and the bandgap is 0.66 eV , the thickness of the Germanium layer, rounded off to 3 decimal places, is $\qquad$ $\mu \mathrm{m}$.


Ans. [0.230 to 0.231]
Sol. Pabsorbed $=$ Pincident $\left(1-\mathrm{e}^{-\alpha \times}\right)$
$1 / 3=(2 / 3)\left(1-\mathrm{e}^{-\alpha \times}\right)$
$\mathrm{e}^{-\alpha x}=0.5$
now $\propto=3 \times 10^{4} \mathrm{~cm}^{-1}$
$\therefore x=\frac{-\ln (0.5)}{3 \times 10^{4}}$
$\therefore \mathrm{x}=0.231 \mu \mathrm{~m}$
33. Consider the circuit shown with an ideal OPAMP. The output voltage $\mathrm{V}_{0}$ is $\qquad$ $V$ (rounded off to two decimal places).


Ans. [-0.55 to -0.45]

## Sol.



$$
\begin{array}{r}
V_{\text {in }}=\frac{2^{0} b_{0} \times 1.6+2^{1} b_{1} \times 1.6+2^{2} b_{2} \times 1.6+2^{3} b_{3} \times 1.6}{2^{4}} \\
=\frac{(1 \times 1.6 \mathrm{~V})+(4 \times 1.6 \mathrm{~V})}{16}=0.5 \mathrm{~V}
\end{array}
$$

Now we get,

$V_{0}=\frac{-3 R}{3 R} \times 0.5=-0.5 V$
Hence, the correct answer is -0.5 V .
34. Consider a unity feedback system as in the figure shown

with transfer function as $G(s)=\frac{1}{s^{2}+3 s+2}$, where $K>0$,
Find the positive value of $K$ for which there are exactly two poles of the unity feedback system on the $\mathrm{j} \omega$ axis is equal to $\qquad$ (rounded off to two decimal places).
Ans. 6
Sol. Overall
$G_{C}(s)=\frac{K}{s\left(s^{2}+3 s+2\right)}$
$\therefore q(s)=s^{3}+3 s^{2}+2 s+k=0$

| $s^{3}$ | 1 | 2 |
| :---: | :---: | :---: |
| $s^{2}$ | 3 | $k$ |
| $s^{1}$ | $\frac{6-k}{3}$ |  |
| $s^{0}$ | $k$ |  |

Auxiliary equation is $3 s^{2}+k=0$
And for roots on imaginary axis $s^{1}$ row $=0$
$\therefore \frac{6-\mathrm{k}}{3}=0$
$\therefore \mathrm{k}=6$
35. The switch in the circuit in the figure is in position $P$ for a long time and moved to position $Q$ at time $\mathrm{t}=0$


The value of $\frac{d v(t)}{d t}$ at $t=0^{+}$is
A. $-5 \mathrm{~V} / \mathrm{s}$
B. $3 \mathrm{~V} / \mathrm{s}$
C. $-3 \mathrm{~V} / \mathrm{s}$
D. $0 \mathrm{~V} / \mathrm{s}$

Ans. C
Sol. At $\mathrm{t}=\mathrm{O}^{-}$

$\mathrm{i}_{L}\left(0^{-}\right)=\frac{20}{20}=1 \mathrm{~mA}$
$\mathrm{V}_{\mathrm{c}}\left(0^{-}\right)=20 \times \frac{10}{20}=10 \mathrm{volt}$

$i_{c}\left(0^{+}\right)+\frac{10}{5}+1=0$
$\Rightarrow \mathrm{i}_{\mathrm{c}}\left(0^{+}\right)=-3 \mathrm{~mA}$
$\frac{d V_{c}\left(0^{+}\right)}{d t}=\frac{i_{c}\left(0^{+}\right)}{C}=\frac{-3 \mathrm{~mA}}{1 \mathrm{mF}}=-3 \mathrm{volt} / \mathrm{sec}$
36. A simple closed path $C$ in the complex plane is shown in the figure. If

$$
\oint_{c} \frac{2^{z}}{Z^{2}-1} d z=-i \pi A
$$

Where ${ }^{i}=\sqrt{-1}$, then the value of $A$ is $\qquad$ (rounded off to two decimal places).


Ans. [0.5 to 0.5]

Sol. $\oint \frac{2^{z}}{z^{2}-1} d x=-i \pi A$
LHS $\oint_{c} \frac{2^{z}}{z^{2}-1} d z=\frac{1}{2} \oint_{c} \frac{2^{z}}{z^{2}-1} d z-\frac{1}{2} \oint_{c} \frac{2^{z}}{z^{2}-1} d z$
For pole $z=1$ does not lie inside the close path counter so apply cauchy's integral theorem
$\frac{1}{2} \oint \frac{2^{z}}{z-1} d z=0$
$Z=-1$ lie inside the close path C. So, $-\frac{1}{2} \oint \frac{2^{z}}{z+1} d z=-\frac{1}{2} \times 2 \pi i \times 2^{-1}=\frac{-1}{2} \pi i$
$\oint_{\mathrm{c}} \frac{2^{z}}{z^{2}-1} d z=\frac{1}{2} \oint_{\mathrm{c}} \frac{2^{z}}{z^{2}-1} d z-\frac{1}{2} \oint_{\mathrm{c}} \frac{2^{z}}{z^{2}-1} d z=0+\frac{-1}{2} \pi i$
$A=0.5$
37. The stack pointer of an 8085 micro-processor is $A B C D H$. At the end of execution of the sequence of instructions, what will be the content of the stack pointer?
PUSH PSW
XTHL
PUSH D
JMP FC70H
A. ABCBH
B. ABCAH
C. ABC 9 H
D. ABC 8 H

Ans. C
Sol. Push instruction decreases the stack pointer by 2.
$\therefore$ Two push instruction in programme decrement it by 4.
$\therefore \mathrm{SP}=\mathrm{ABCD}-4=\mathrm{ABC} 9 \mathrm{H}$
38. The Nyquist sampling rate of the signal $x(t)=4 \operatorname{Sinc}^{2}\left(10^{4} t\right) \operatorname{sinc}^{2}\left(10^{6} t\right)$ is $\qquad$ MHz (upto two decimal points)
Ans. [2.01 to 2.09]

## Sol.

$\sin \mathrm{ct} \stackrel{\mathrm{cHI}}{\longleftrightarrow} \operatorname{rect}(f)$




$f_{\max }=1000+10$
$=1010 \mathrm{KHz}$
$=1.01 \mathrm{MHz}$
Nyquist sampling rate $f_{s(\min )}=2 f_{\max }$
$=2(1010) \mathrm{KHz}$
$=2.02 \mathrm{MHz}$
39. The electric field in free space is given by $E=100 \cos \left(10^{8}+\frac{1}{3} . X\right) a_{y} \mathrm{~V} / \mathrm{m}$. Calculate the time it take to travel a distance of $\lambda / 2$.
A. 30 ns
B. 31 ns
C. 31.42 ns
D. 32 ns

## Ans. C

Sol. Distance $=\frac{\lambda}{2}$
Wave Traveling at speed of light $C=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$.
So, Distance $=$ Speed $\times$ Time
$\frac{\lambda}{2}=C \times T$
$\mathrm{T}=\frac{\lambda}{2 \mathrm{C}}$
$\lambda=\frac{2 \pi}{\beta}$
$E=100 \cos \left(10^{8} t+\frac{1}{3} x\right)$ ay
So, $\beta=\frac{1}{3}$
$\lambda=\frac{2 \pi}{\frac{1}{3}}=6 \pi$
$\lambda=6 \pi$
$T=\frac{6 \pi}{2 \times\left(3 \times 10^{8}\right)}$
$\mathrm{T}=31.42 \mathrm{~ns}$
40. The state space representation of a control system is given by
$\left[\begin{array}{l}\dot{x}_{1} \\ \dot{x}_{2}\end{array}\right]=\left[\begin{array}{cc}0 & 1 \\ -2 & -3\end{array}\right]\left[\begin{array}{l}x_{1} \\ x_{2}\end{array}\right]+\left[\begin{array}{l}0 \\ 1\end{array}\right] u$;
$x(0)=\left[\begin{array}{l}1 \\ 1\end{array}\right]$;
$y=\left[\begin{array}{ll}1 & 0\end{array}\right]\left[\begin{array}{l}x_{1} \\ x_{2}\end{array}\right]$
The transfer function of above system will be
A. $\frac{1}{s^{2}+3 s+2}$
B. $\frac{1}{s^{2}+6 s+4}$
C. $\frac{4}{s^{2}+3 s+8}$
D. $\frac{6}{s^{2}+6 s+8}$

Ans. A
Sol. Transfer function
$\frac{Y(s)}{u(s)}=T(s)=c[S I-A]^{-1} B+D$
$=\left[\begin{array}{ll}1 & 0\end{array}\right]\left[\begin{array}{cc}s & -1 \\ 2 & s+3\end{array}\right]\left[\begin{array}{l}0 \\ 1\end{array}\right]$
$=\frac{\left[\begin{array}{ll}1 & 0\end{array}\right]\left[\begin{array}{cc}s+3 & 1 \\ -2 & s\end{array}\right]\left[\begin{array}{l}0 \\ 1\end{array}\right]}{(s+1)(s+2)}$
$=\frac{1}{s^{2}+3 s+2}$
41. A silicon NMOS has gate width $95 \mu \mathrm{~m}$ length $1.5 \mu \mathrm{~m}$, $\mathrm{t}_{\mathrm{ox}}=0.2 \mu \mathrm{~m}$. Relative permittivity of oxide is $3.9, \mu_{n}=0.08 \mathrm{~m}^{2} / \mathrm{v}-\mathrm{s}$. Calculate the trans conductance of device in triode region for drain voltage of 2 V .
A. $6.9 \times 10^{-4} \mathrm{~S}$
B. $6.9 \times 10^{-2} \mathrm{~S}$
C. $1.749 \times 10^{-3} \mathrm{~S}$
D. $1.45 \times 10^{-5} \mathrm{~S}$

Ans. C
Sol. Given that,
$\mathrm{L}=1.5 \mu \mathrm{~m}$
$W=95 \mu \mathrm{~m}$
$t_{o x}=0.2 \mu \mathrm{~m}$
$\because C_{o x}=\frac{\varepsilon_{0 x}}{t_{\mathrm{ox}}}=\frac{3.9 \times 8.85 \times 10^{-12}}{0.2 \times 10^{-6}}$
$C_{o x}=1.7265 \times 10^{-4} \mathrm{~F} / \mathrm{m}^{2}$
In triode region
$g=\mu_{\mathrm{n}} C_{o x} \frac{W}{L} \cdot V_{D S}$
$=0.08 \times 1.7265 \times 10^{-4} \times \frac{95}{1.5} \times 2$
$g=1.749 \times 10^{-3} \mathrm{~S}$
42. The value of $\int_{0}^{1} \frac{e^{x}}{e^{2 x}+1} d x$ $\qquad$ -.
A. $\tan ^{-1}(e)+\frac{\pi}{4}$
B. $\tan ^{-1}(e)-\frac{\pi}{4}$
C. $\frac{\pi}{4}$
D. $\tan ^{-1}(e)+\frac{\pi}{2}$

Ans. B
Sol. Put $\mathrm{e}^{\mathrm{x}}=\mathrm{t}$
$e^{x} d x=d t$
limit
$x=0, t=1$
$x=1, t=e$

So, $\int_{1}^{e} \frac{1}{t^{2}+1} d t$
$=\left[\tan ^{-1} \mathrm{t}\right]_{1}^{\mathrm{e}}$
$=\tan ^{-1}(e)-\tan ^{-1}(1)$
$=\tan ^{-1}(\mathrm{e})-\pi / 4$
43. In the voltage regulator shown below, $\mathrm{V}_{1}$ is the unregulated at 15 V . Assume $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}$ and the base current is negligible for both the BJTs. If the regulated output $\mathrm{V}_{0}$ is 9 V , the value of $R_{2}$ is $\qquad$ $\Omega$.


Ans. 800

## Sol.



Voltage $\mathrm{V}_{\mathrm{B}}=\mathrm{V}_{\mathrm{z}}+\mathrm{V}_{\mathrm{BE}}$
$=3.3+0.7$
$V_{B}=4 V \ldots$ (i)
$\therefore \mathrm{I}=\frac{9-4}{1 \mathrm{~K}}$
$\mathrm{I}=5 \mathrm{~mA}$
Since base cement is negligible,
$V_{B}=9 \times \frac{R_{2}}{R_{1}+R_{2}}$
$4=\frac{9 R_{2}}{1 K+R_{2}} \Rightarrow R_{2}=800 \Omega$
44. Using the incremental low frequency small-signal model of the MOS device, the Norton equivalent resistance of the following circuit is

A. $r_{d s}+R+g_{m} r_{d s} R$
B. $\frac{r_{d s}+R}{1+g_{m} r_{d s}}$
C. $r_{d s}+R$
D. $r_{d s}+\frac{1}{g_{m}}+R$

Ans. B
Sol.

$\mathrm{V}_{\text {test }}=-\mathrm{V}_{\mathrm{gs}}$
$=r_{\text {ds }}\left(I_{\text {test }}-g_{m} V_{\text {test }}\right)+I_{\text {test }} R$
$V_{\text {test }}\left(1+g_{m} r_{d s}\right)=I_{\text {test }}\left(r_{d s}+R\right)$
$\frac{V_{\text {test }}}{I_{\text {test }}}=\operatorname{Req}=\frac{r_{d s}+R}{1+g_{m} r_{d s}}$
45. A solar cell of area $1.0 \mathrm{~cm}^{2}$, operating at 1.0 sun intensity, has a short circuit current of 20 mA , and an open circuit voltage of 0.65 V . Assuming room temperature operation and thermal equivalent voltage of 26 mV , the open circuit voltage (in volts, correct to two decimal places) at 0.2 sun intensity is $\qquad$ -.
Ans. [0.59 to 0.63]

Sol. For solar cell open circuit voltage is given by,

$$
V_{O C}=V_{T} \ln \left(\frac{I_{S C}}{I_{0}}\right)
$$

Since, the Current through the solar cell is directly proportional to intensity of light,

$$
\begin{aligned}
V_{O C 2}-V_{O C 1} & =V_{T} \ln \left(\frac{I_{S C 2}}{I_{S C 1}}\right)=V_{T} \ln \left(\frac{0.20}{1.0}\right) \\
V_{O C 2} & =V_{O C 1}-0.026 \ln (5) \\
& =0.65-0.041845=0.608 \mathrm{~V}
\end{aligned}
$$

$\mathrm{V}_{\mathrm{oc} 2}=0.608 \mathrm{~V}$.
46. Let $x_{1}(t)=e^{-t} u(t)$ and $x_{2}(t)=u(t)-u(t-2)$, where $u($.$) denotes the unit step function. If y(t)$ denotes the convolution of $x_{1}(t)$ and $x_{2}(t)$, then $\lim _{t \rightarrow \infty} y(t)=$ $\qquad$ . (Rounded off to one decimal place).

Ans. [0 to 0]
Sol. $x_{1}(t)=e^{-t} u(t)$
$\mathrm{x}_{2}(\mathrm{t})=\mathrm{u}(\mathrm{t})-\mathrm{u}(\mathrm{t}-2)$
$y(t)=x_{1}(t) * x_{2}(t)$
By applying Laplace transform
$Y(s)=X_{1}(s) \cdot X_{2}(s)=\frac{1}{(s+1)} \frac{1-e^{-2 s}}{s}$
By applying final value theorem,
$\left.y(t)\right|_{t-\infty}=\lim _{s \rightarrow 0} s Y(s)=\lim _{s \rightarrow 0}\left(\frac{1-e^{-2 s}}{s+1}\right)$
$=0$
Alternate Method:
$\mathrm{y}(\mathrm{t})=\mathrm{x}_{1}(\mathrm{t}) * \mathrm{x}_{2}(\mathrm{t})$
$=\mathrm{e}^{-\mathrm{t}} \mathrm{u}(\mathrm{t}) *[\mathrm{u}(\mathrm{t})-\mathrm{u}(\mathrm{t}-2)]$
$=\mathrm{e}^{-\mathrm{t}} \mathrm{u}(\mathrm{t}) * \mathrm{u}(\mathrm{t})-\mathrm{e}^{-\mathrm{t}} \mathrm{u}(\mathrm{t}) * \mathrm{u}(\mathrm{t}-2)$
$=\int_{-\infty}^{t} e^{-t} u(t) d t-\int_{-\infty}^{t-2} e^{-t} u(t) d t \quad[u(t)$ is the impulse response of an integrator]
$y(t)=\left[1-e^{-t}\right] u(t)-\left[1-e^{-(t-2)}\right] u(t-2)$
$y(\infty)=[1-0] 1-[1-0] 1=0$
47. In the circuit shown below, the (W/L) value for $M_{2}$ is twice that for $M_{1}$. The two NMOS transistors are otherwise identical. The threshold voltage $\mathrm{V}_{\mathrm{T}}$ for both transistors is 1.0 V . Note that $V_{G S}$ for $M_{2}$ must be $>1.0 \mathrm{~V}$.


Current through the nMOS transistors can be modeled as
$I_{D S}=\mu C_{0 x}\left(\frac{W}{L}\right)\left(\left(V_{G S}-V_{T}\right) V_{D S}-\frac{1}{2} V_{D S}^{2}\right)$ for $V_{D S} \leq V_{G S}-V_{T}$
$\mathrm{I}_{\mathrm{DS}}=\mu \mathrm{C}_{\mathrm{ox}}\left(\frac{\mathrm{W}}{\mathrm{L}}\right) \frac{\left(\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}\right)}{2}$ for $\mathrm{V}_{\mathrm{DS}} \geq \mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}$
The voltage (in volts, accurate to two decimal places) at $\mathrm{V}_{\mathrm{x}}$ is
Ans. [0.3 to 0.5]
Sol. The device constant $K_{n}$,

$$
K_{n}=\frac{\mu_{n} C_{o x}}{2}\left(\frac{W}{L}\right)
$$

Given that,
$\left(\frac{W}{L}\right)_{2}=2\left(\frac{W}{L}\right)_{1}$
Then,
$K_{n 2}=2 K_{n 1}$
For $\mathrm{M}_{1}$
$V_{G S 1}-V_{T}=2-1=1 \mathrm{~V}$
Now, for $M_{2}$

$$
\begin{aligned}
V_{G S 2}-V_{T} & =2-V_{x}-1=1 V-V_{x}<1 V \\
V_{D S 2} & =\left(3.3-V_{x}\right)>\left(V_{G S 2}-V_{T}\right)
\end{aligned}
$$

Here, clearly $M_{1}$ will be in linear region and $M_{2}$ will be in saturation region. But current across them would be same,

$$
\begin{gathered}
I_{D_{1}}=I_{D_{2}} \\
K_{n 1}\left[2\left(V_{G S 1}-V_{T}\right) V_{D S 1}-V_{D S 1}^{2}\right]=K_{n 2}\left(V_{G S 2}-V_{T}\right)^{2} \\
K_{n 1}\left[2(2-1) V_{x}-V_{x}^{2}\right]=2 K_{n 1}\left(2-V_{x}-1\right)^{2} \\
2 V_{x}-V_{x}^{2}=2\left(1+V_{x}^{2}-2 V_{x}\right)=2 V_{x}^{2}-4 V_{x}+2 \\
3 V_{x}^{2}-6 V_{x}+2=0 ; V_{x}^{2}-2 V_{x}+\frac{2}{3}=0 \\
V_{x}=1 \pm \sqrt{\frac{4-\frac{8}{3}}{43}}=1 \pm \sqrt{\frac{1}{3}} V \\
V_{G S 2}=\left(2-V_{x}\right) \geq V_{T} \Rightarrow\left(1-V_{x}\right) \geq 0
\end{gathered}
$$

So, the only valid value, $\mathrm{V}_{\mathrm{x}}=1-\sqrt{\frac{1}{3}}=0.4226 \mathrm{~V}$
48. A sinusoidal input $x(t)=2 \sin (2 t)$ is applied to system with transfer function $P(s)=\frac{2}{s(s+2)}$. Determine steady state o/p.
A. $2 \sin \left(2 t+45^{\circ}\right)$
B. $.707 \sin \left(2 t-135^{\circ}\right)$
C. $.5 \sin \left(2 t-90^{\circ}\right)$
D. $707 \sin \left(2 t-90^{\circ}\right)$

Ans. B
Sol. $X(t)=A \operatorname{Sin}(W t)$
$W=2, A=2$
Calculate magnitude and phase at $w=2$ of system
Magnitude $=\frac{2}{2 \sqrt{(2)^{2}+(2)^{2}}}$
$=\frac{1}{\sqrt{8}}$
Phase $=-90^{\circ}-\tan ^{-1}(2 / 2)$
$=-90^{\circ}-45^{\circ}=-135^{\circ}$
So, o/p will be
$y(t)=2 \times \frac{1}{\sqrt{8}} \sin \left(2 t-135^{\circ}\right)$
$Y(t)=.707 \sin \left(2 t-135^{\circ}\right)$
49. The function $f(x)=x^{2}-x-2$ the maximum value of $f(x)$ at value of $x$ in closed internal $[-2,2]$ is
A. . 5
B. -2
C. 2
D. 0

## Ans. B

Sol. $f(-2)=(-2)^{2}-(-2)-2$
$f(-2)=4+2-2$
$f(-2)=4$
$f(2)=(2)^{2}-2-2$
$=4-4$
$f(2)=0$
So, maximum value of $f(x)$ at value of $x=-2$
50. A causal LTI system described by the difference equation:
$y(n)=y(n-1)+y(n-2)+x(n-1)$
The impulse response of the system is $h(n)$. Then the value of $h(1)=$ $\qquad$ .
Ans. [0.95 to 1.05]
Sol. Given difference equation is:

$$
y(n)=y(n-1)+y(n-2)+x(n-1)
$$

Applying Z-transform to this equation,
$\Rightarrow \mathrm{Y}(\mathrm{z})=\mathrm{z}^{-1} \mathrm{Y}(\mathrm{z})+\mathrm{z}^{-2} \mathrm{Y}(\mathrm{z})+\mathrm{z}^{-1} \mathrm{X}(\mathrm{z})$
$\Rightarrow \mathrm{H}(\mathrm{z})=\frac{\mathrm{Y}(\mathrm{z})}{\mathrm{X}(\mathrm{z})}=\frac{\mathrm{z}^{-1}}{1-\mathrm{z}^{-1}-\mathrm{z}^{-2}}$
$\Rightarrow \mathrm{H}(\mathrm{z})=\frac{\mathrm{z}}{\mathrm{z}^{2}-\mathrm{z}-1}$
$\Rightarrow \mathrm{H}(\mathrm{z})=\frac{0.447 \mathrm{z}}{\mathrm{z}-1.618}-\frac{0.447 \mathrm{z}}{\mathrm{z}+0.618}$
$\Rightarrow \mathrm{h}(\mathrm{n})=0.447(1.618)^{\mathrm{n}} \mathrm{u}(\mathrm{n})-0.447(-0.618)^{\mathrm{n}} \mathrm{u}(\mathrm{n})$
$\Rightarrow \mathrm{h}(1)=0.447 * 1.618-0.447(-0.618)$
$h(1) \approx 1$
51. The voltage and current associated with a load is $V=100 \angle 30^{\circ}, \mathrm{I}=5 \angle-30^{\circ}$.

Then which of the following are true?
A. the power factor of the load is 0.5 lag
B. the power factor of the load is 0.5866 lag
C. the active power of load is 250 W
D. the active power of load is 500 W

Ans. A, C
Sol. Given: $V=100 \angle 30^{\circ}, I=5 \angle-30^{\circ}$
$\Rightarrow \mathrm{S}=\mathrm{VI}^{*}$
$\Rightarrow \mathrm{S}=\left(100 \angle 30^{\circ}\right)\left(5 \angle 30^{\circ}\right)$
$\Rightarrow S=500 \angle 60^{\circ}$
Here $\varphi=60^{\circ}$
$\Rightarrow \mathrm{pf}=\cos \varphi=\cos 60^{\circ}=0.5$ lag
Since, current lags the voltage in this load.
The active power of the load is given by:
$\mathrm{P}=\mathrm{S} \cos \varphi=500 \times 0.5=250 \mathrm{~W}$
52. An engineer needs to make an $R C$ high pass filter. He has one 10PF capacitor, one 30PF capacitor, one $1.8 \mathrm{~K} \Omega$ resistor and one $3.3 \mathrm{~K} \Omega$ resistor available. The greatest cutoff frequency (in MHz ) possible is $\qquad$
Ans. [18 to 19]
Sol. Cut-off frequency of HP filter is given by:
$\mathrm{f}=\frac{1}{2 \pi \mathrm{RC}}$
So, to get the maximum cutoff frequency, the values of $R$ and $C$ should be as low as possible.
So the two resistors should be connected in parallel.
$\Rightarrow \mathrm{R}_{\mathrm{eq}}=\frac{1.8 \times 3.3}{1.8+3.3}=1.1647 \mathrm{~K} \Omega$
Also, the two capacitors should be connected in series.
$\Rightarrow \mathrm{C}_{\text {eq }}=\frac{10 \times 30}{10+30}=7.5 \mathrm{PF}$
So, the greatest cutoff frequency is given by:
$\mathrm{f}=\frac{1}{2 \pi \mathrm{R}_{\text {eq }} \mathrm{C}_{\mathrm{eq}}}=\frac{1}{2 \pi * 1.1647 * 10^{3} * 7.5 * 10^{-12}}$
$\Rightarrow \mathrm{f}=18.2 \mathrm{MHz}$
53. Let $\mathrm{x}(\mathrm{t})=\frac{1}{\mathrm{t}}\left[1-\mathrm{e}^{-\mathrm{t}}\right] \mathrm{u}(\mathrm{t})$. Then the value of Laplace transform of function at $\mathrm{s}=1$ is $\qquad$
Ans. [0.67 to 0.71]
Sol. Let, $\mathrm{y}(\mathrm{t})=\left[1-\mathrm{e}^{-\mathrm{t}}\right] \mathrm{u}(\mathrm{t})$
$\Rightarrow \mathrm{x}(\mathrm{t})=\frac{1}{\mathrm{t}} \mathrm{y}(\mathrm{t})$
$\Rightarrow \mathrm{X}(\mathrm{s})=\int_{\mathrm{s}}^{\infty} \mathrm{Y}(\mathrm{s}) \mathrm{ds}$
$\mathrm{Y}(\mathrm{s})=\frac{1}{\mathrm{~s}}-\frac{1}{\mathrm{~s}+1}$
$\Rightarrow X(s)=\int_{s}^{\infty} \frac{1}{s} d s-\int_{s}^{\infty} \frac{1}{s+1} d s$
$\Rightarrow \mathrm{X}(\mathrm{s})=-\ln (\mathrm{s})+\ln (\mathrm{s}+1)$
$X(s)=\ln \left(\frac{s+1}{s}\right)$
$\Rightarrow X(\mathrm{~s})_{\mathrm{s} \rightarrow 1}=\ln \left(\frac{2}{1}\right)=\ln (2)$
$\Rightarrow \mathrm{X}(\mathrm{s}=1)=0.693$
54. The value of $R$ for which the circuit resonates is?

A. $8 \Omega$
B. $12 \Omega$
C. $16 \Omega$
D. $6 \Omega$

Ans. D
Sol. When a circuit resonates at a particular frequency, at that frequency the imaginary part of impedance or admittance of the network is zero.
$\Rightarrow Y_{A B}=\frac{1}{10+j 10}+\frac{1}{R-j 2}$
$Y_{A B}=\frac{10-j 10}{10^{2}+10^{2}}+\frac{R+j 2}{R^{2}+2^{2}}$
$\mathrm{Y}_{\mathrm{AB}}=\frac{10}{200}+\frac{\mathrm{R}}{\mathrm{R}^{2}+4}-\mathrm{j}\left(\frac{10}{200}-\frac{2}{\mathrm{R}^{2}+4}\right)$
Now equate the imaginary part to zero.
$\Rightarrow \frac{10}{200}-\frac{2}{\mathrm{R}^{2}+4}=0$
$\Rightarrow R^{2}+4=40$
$\Rightarrow R^{2}=36$
$\Rightarrow \mathrm{R}=6 \Omega$
55. A unity feedback system has open transfer function $G(s)=\frac{50}{(s+4)(s+5)}$. A PID controller having transfer function $G_{c}(s)=\left(k_{p}+s k_{d}+\frac{k_{i}}{s}\right)$ is introduced to improve the steady state as well as transient performance. For what value of $k_{i}$, the steady state error is $10 \%$ for unit ramp input?
A. 1
B. 2
C. 3
D. 4

Ans. D
Sol. The open loop transfer function with controller is given by:
$\mathrm{G}(\mathrm{s})_{\text {with compensator }}=\frac{50}{(\mathrm{~s}+4)(\mathrm{s}+5)} *\left(\mathrm{k}_{\mathrm{p}}+\mathrm{sk}_{\mathrm{d}}+\frac{\mathrm{k}_{\mathrm{i}}}{\mathrm{s}}\right)$
$\mathrm{G}(\mathrm{s})_{\mathrm{wc}}=\frac{50\left(\mathrm{~s}^{2} \mathrm{k}_{\mathrm{d}}+\mathrm{sk}_{\mathrm{p}}+\mathrm{k}_{\mathrm{i}}\right)}{\mathrm{s}(\mathrm{s}+4)(\mathrm{s}+5)}$
$\Rightarrow \mathrm{e}_{\mathrm{ss}}=\frac{\mathrm{A}}{\mathrm{K}_{\mathrm{v}}}=\frac{1}{\frac{50 * \mathrm{k}_{\mathrm{i}}}{4 * 5}}$
Given $\% \mathrm{e}_{\mathrm{ss}}=10 \%$
$\Rightarrow 0.1=\frac{20}{50 \mathrm{k}_{\mathrm{i}}}$
$\Rightarrow 5 \mathrm{k}_{\mathrm{i}}=20$
$\Rightarrow \mathrm{k}_{\mathrm{i}}=4$
56. The position of a particle $y(t)$ is described by the differential equation:
$\frac{d^{2} y}{d t^{2}}=-\frac{d y}{d t}-\frac{5 y}{4}$
The initial conditions are $y(0)=1$ and $\left.\frac{d y}{d t}\right|_{t=0} 0$. The position (accurate to two decimal places) of the particle at $t=\pi$ is $\qquad$ .
Ans. - 0.21

Sol. Given condition,

$$
\begin{aligned}
\frac{d^{2} y}{d t^{2}}+\frac{d y}{d t}+\frac{5 y}{4} & =0 \\
y(0) & =1 \\
y^{\prime}(0) & =0
\end{aligned}
$$

This can be solved easily in laplace domain,

$$
\begin{aligned}
& s^{2} Y(s)-s(1)+s Y(s)-1+\frac{5}{4} Y(s)=0 \\
& Y(s)=\frac{s+1}{s^{2}+s+\frac{5}{4}}=\frac{s+1}{\left(s+\frac{1}{2}\right)^{2}+1} \\
&=\frac{\left(s+\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^{2}+1}+\frac{\frac{1}{2}}{\left(s+\frac{1}{2}\right)^{2}+1}
\end{aligned}
$$

By taking inverse Laplace transform we get $y(t)$,

$$
y(t)=e^{-t / 2}\left[\cos (t) \frac{1}{2} \sin (t)\right] ; t>0
$$

Now its value at $t=\pi$,
$y(t=\pi)=e^{-\pi / 2}[(-1)+(0)]=e^{-\pi / 2}$
$=-0.2078 \sim-0.21$
57. A curve passes through the point $(x=1, y=0)$ and satisfies the differential equation $\frac{d y}{d x}=\frac{x^{2}+y^{2}}{2 y}+\frac{y}{x}$. The equation that describes the curve is
A. $\ln \left(1+\frac{y^{2}}{x^{2}}\right)=x-1$
B. $\frac{1}{2} \ln \left(1+\frac{\mathrm{y}^{2}}{\mathrm{x}^{2}}\right)=\mathrm{x}-1$
C. $\ln \left(1+\frac{y}{x}\right)=x-1$
D. $\frac{1}{2} \ln \left(1+\frac{y}{x}\right)=x-1$

Ans. A
Sol. Given Differential equation,

$$
\frac{d y}{d x}=\frac{x^{2}+y^{2}}{2 y}+\frac{y}{x}
$$

We need to use suitable substitution here,
Put, $\frac{y}{x}=t$

$$
\begin{aligned}
& \frac{d y}{d x}=t+x \frac{d t}{d x} \\
& 1+x \frac{d t}{d x}=\frac{x}{2 t}+\frac{t x}{2}+t \\
& x \frac{d t}{d x}=x\left(\frac{1}{2 t}+\frac{t}{2}\right) \\
& x \frac{d t}{d x}=x\left(\frac{1+t^{2}}{2 t}\right) \\
& \int \frac{2 t}{1+t^{2}} d t=\int d x+C \\
& \ln \left(1+t^{2}\right)=x+C \\
& \mathrm{t}=\frac{\mathrm{y}}{\mathrm{x}}
\end{aligned}
$$

After simplification we obtain the following relation,

$$
\ln \left(1+\frac{y^{2}}{x^{2}}\right)=x+C
$$

Given that the curve passes through points, $x=1, y=0$, we can obtain the value of constant $C$.

$$
\begin{aligned}
\ln \left(1+\frac{0}{1}\right) & =\ln (1)=0=1+C \\
C & =-1
\end{aligned}
$$

So, $\ln \left(1+\frac{y^{2}}{x^{2}}\right)=x-1$
58. Consider a white Gaussian noise process $N(t)$ with two-sided power spectral density $S_{N}(f)=0.5 \mathrm{~W} / \mathrm{Hz}$ as input to a filter with impulse response $0.5 \mathrm{e}^{\frac{-t^{2}}{2}}$ (where $t$ is in seconds) resulting in output $Y(t)$. The power in $Y(t)$ in watts is
A. 0.11
B. 0.22
C. 0.33
D. 0.44

## Ans. B

Sol. Power Spectral Density of noise input,
$S_{N}(f)=0.5 \mathrm{~W} / \mathrm{Hz}$
Power of $y(t)$,

$$
\begin{aligned}
& P_{y}=\int_{-\infty}^{\infty} S_{N}(f)|H(f)|^{2} d f \\
& =0.50 \int_{-\infty}^{\infty}|H(f)|^{2} d f=0.50 \int_{-\infty}^{\infty}|h(f)|^{2} d t
\end{aligned}
$$

Given the impulse response of the filter being used,
$h(t)=\frac{1}{2} e^{-t^{2} / 2}$
So,

$$
\begin{aligned}
P_{y} & =\frac{1}{2} \int_{-\infty}^{\infty}\left(\frac{1}{2} e^{-t^{2} / 2}\right)^{2} d t=\frac{1}{8} \int_{-\infty}^{\infty} e^{-t^{2}} d t \\
& =\frac{\sqrt{\pi}}{8}=0.22 \mathrm{~W}
\end{aligned}
$$

59. Let $c(t)=A_{c} \cos \left(2 \pi f_{c} t\right)$ and $m(t)=\cos \left(2 \pi f_{m} t\right)$. It is given that $f_{c} \gg 5 f_{m}$. The signal $c(t)+m(t)$ is applied to the input of a non-linear device, whose output $v_{0}(t)$ is related to the input $v_{i}(t)$ as $v_{0}(t) a v_{i}(t)+b v_{i}^{2}(t)$, where $a$ and $b$ are positive constants. The output of the nonlinear device is passed through an ideal band-pass filter with center frequency $f_{c}$ and bandwidth $3 f_{m}$, to produce an amplitude modulated (AM) wave. If it is desired to have the sideband power of the AM wave to be half of the carrier power, then $a / b$ is
A. 0.25
B. 0.5
C. 1
D. 2

## Ans. D

Sol. According to given input signal, we can obtain an output signal as follows,
$v_{i}(t)=A_{c} \cos \left(2 \pi f_{c} t\right)+\cos \left(2 \pi f_{m} t\right)$
$v_{0}(t)=a v_{i}(t)+b v_{i}^{2}(t)$
$=\left[a A_{c} \cos \left(2 \pi f_{c} t\right)+a \cos \left(2 \pi f_{m} t\right)\right]+b$
$\left[A_{c}^{2} \cos ^{2}\left(2 \pi f_{c} t\right)+\cos ^{2}\left(2 \pi f_{m} t\right)+2 A_{c} \cos \right.$
$\left.\left(2 \pi f_{c} t\right) \cos \left(2 \pi f_{m} t\right)\right]$
When the signal is passed through given Band Pass Filter,

$$
\begin{aligned}
& y(t)=a A_{c} \cos 2 \pi f_{c} t+2 b A_{c} \cos \left(2 \pi f_{c} t\right) \\
& \cos \left(2 \pi f_{m} t\right) \\
& \quad=a A_{c}\left[1+\frac{2 b}{a} \cos \left(2 \pi f_{m} t\right)\right] \cos \left(2 \pi f_{c} t\right)
\end{aligned}
$$

The Modulation index can be obtained through output of the BPF, $\mu=\frac{2 \mathrm{~b}}{\mathrm{a}}$
We have been given in the problem statement that Side Band contains half the carrier power,
$P_{S B}=\frac{\mu^{2}}{2} P_{c}=\frac{1}{2} P_{c}$
So, $\mu^{2}=1 \Rightarrow \mu=1$
Comparing with the value obtained in form of $a$ and $b$,
$\frac{2 b}{a}=1$
$\frac{a}{b}=2$
60. The distance (in meters) a wave has to propagate in a medium having a skin depth of 0.1 m so that the amplitude of the wave attenuates by 20 dB , is
A. 0.12
B. 0.23
C. 0.46
D. 2.3

Ans. B
Sol. Attenuation constant is related with skin depth as follows, And according to given condition of 20 dB attenuation we can get required depth by following calculation,

$$
\alpha=\frac{1}{\text { skin depth }}=10 \mathrm{~Np} / \mathrm{m}
$$

$20 \log _{10}\left(\frac{E_{0}}{E_{x}}\right)=20 d B$

$$
\begin{aligned}
\frac{E_{0}}{E_{x}} & =10 \Rightarrow\left(E_{x}\right)=\frac{E_{0}}{10} \\
E_{x} & =E_{0} e^{-\alpha x}=E_{0} e^{-10 x}=\frac{E_{0}}{10} \\
e^{-10 x} & =\frac{1}{10} \\
x & =\frac{1}{10} \operatorname{In}(10)=0.23 \mathrm{~m}
\end{aligned}
$$

61. A circuit and the characteristics of the diode ( $D$ ) in it are shown. The ratio of the minimum to the maximum small signal voltage gain $\frac{\partial \mathrm{V}_{\text {out }}}{\partial \mathrm{V}_{\text {in }}}$ in $\qquad$ (rounded off to two decimal places).


Ans. [0.7 to 0.8]
Sol. Given circuit is shown below,


And diode characteristics is,


Figure (b)
Replacing the circuit in figure (a) with the small signal equivalent


Case 1: When diode ON
As $r_{d}(O N)=0$, the $2 \mathrm{k} \Omega$ resistor in parallel to the diode becomes short circuit.

$$
\begin{aligned}
& \therefore \mathrm{V}_{\text {out }}=\frac{\mathrm{V}_{\text {input }} \times 2}{4}=\frac{\mathrm{V}_{\text {input }}}{2} \\
& \left.\therefore \frac{\partial \mathrm{~V}_{\text {out }}}{\partial \mathrm{V}_{\text {in }}}\right|_{\text {mex. }}=\frac{1}{2}
\end{aligned}
$$

Case 2: When diode OFF
As $r_{d}(\mathrm{OFF})=$ infinite, the equivalent resistance will $2 \mathrm{k} \Omega+2 \mathrm{k} \Omega+2 \mathrm{k} \Omega=6 \mathrm{k} \Omega$
$\therefore \mathrm{V}_{\text {out }}=\frac{\mathrm{V}_{\text {input }} \times 4}{2+2+2}=\frac{2 \mathrm{~V}_{\text {input }}}{3}$
$\left.\frac{\partial \mathrm{V}_{\text {out }}}{\partial \mathrm{V}_{\text {in }}}\right|_{\text {trin. }}=\frac{2}{3}$
$\frac{\left.\frac{\partial \mathrm{V}_{\text {out }}}{\partial \mathrm{V}_{\text {in }}}\right|_{\text {min. }}}{\left.\frac{\partial \mathrm{V}_{\text {out }}}{\partial \mathrm{V}_{\text {in }}}\right|_{\text {max. }}}=\frac{\frac{1}{2}}{\frac{2}{3}}=\frac{1}{2} \times \frac{3}{2}=0.75$
Hence, Correct answer is 0.75
62. A lossy transmission line has resistance per unit length $R=0.05 \Omega / m$. The line is distortionless and has characteristic impedance of $50 \Omega$. The attenuation constant (in Np/m, correct to three decimal places) of the line is $\qquad$ -.

Ans. 0.001
Sol. The following condition is true for a distortion-less transmission line,
$\frac{L}{R}=\frac{C}{G}$
Propagation constant is given by,

$$
\begin{aligned}
\gamma & =\alpha+j \beta=\sqrt{(R+j \omega L)(G+j \omega C)} \\
& =\sqrt{R G}\left(1+j \omega \frac{L}{R}\right)
\end{aligned}
$$

And the attenuation constant, which is real part of the propagation constant, $\alpha=\sqrt{R G}$

Characteristic impedance,

$$
\begin{aligned}
& Z_{o}=\sqrt{\frac{(R+j \omega L)}{(G+j \omega C)}}=\sqrt{\frac{R}{G}} \\
& \sqrt{G}=\frac{\sqrt{R}}{Z_{0}}
\end{aligned}
$$

So,

$$
\begin{aligned}
\alpha & =\sqrt{R} \cdot \frac{\sqrt{R}}{Z_{o}}=\frac{R}{Z_{0}}=\frac{0.05}{50}=\frac{0.01}{10} \\
& =0.001 \mathrm{~Np} / \mathrm{m}
\end{aligned}
$$

63. A p-type semiconductor with zero electric field is under illumination (low level injection) in steady state condition. Excess minority carrier density is zero at $x= \pm 2 \ell_{\mathrm{n}}$, where $\ell_{\mathrm{n}}=10^{-4} \mathrm{~cm}$ is the diffusion length of electrons. Assume electronic charge, $\mathrm{q}=-1.6 \times 10^{-19} \mathrm{C}$. The profiles of photogeneration rate of carriers and the recombination rate of excess minority carriers ( $R$ ) are shown. Under these conditions, the magnitude of the current density due to the photo-generated electrons at $x=+2 \ln$ is $\qquad$ $\mathrm{mA} / \mathrm{cm}^{2}$ (rounded off to two decimal places).


Ans. [0.57 to 0.61]

## Sol.

$$
\begin{aligned}
& \delta n(x)=R \tau_{n}=10^{20} e^{-\mid x / / / h} \tau_{n} \\
& \delta n(\ln )=1020 e^{-1} \tau_{n} \ldots(i) \\
& I_{n} \leq x \leq 21_{n}
\end{aligned}
$$

Continuity equation in steady state,
$D_{n} \frac{\partial^{2} \delta n}{\delta x^{2}}+G-R=0$
Since, $\left.\begin{array}{rl}G & =0 \\ R & =0\end{array}\right\} I_{n} \leq x \leq 2 I_{n}$
$\therefore D_{n} \frac{\partial^{2} \delta \mathrm{n}}{\partial \mathrm{x}^{2}}=0$
Whose solution is,
$\delta n(x)=A x+B$
Since at $x=2 \ln _{n}$ :
$\delta n\left(\left.2\right|_{n}\right)=0$ (given)
$0=A\left(2 l_{n}\right)+B$
$A=-\frac{B}{2 I_{n}}$
$\therefore \delta n(x)=-\frac{B}{2 I_{n}} x+B=B\left(1-\frac{x}{2 I_{n}}\right)$..
$\therefore$ At $\mathrm{x}=\mathrm{I}_{\mathrm{n}}$ : equation (i) = equation (ii)
$10^{20} e^{-1} \tau_{n}=B\left(1-\frac{I_{n}}{2 I_{n}}\right)$
$\therefore B=2 \times 10^{20} e^{-1} \tau_{n}$
$\therefore \delta n(x)=2 \times 10^{20} e^{-1} \tau_{n}\left(1-\frac{x}{2 I_{n}}\right) I_{n} \leq x \leq\left. 2\right|_{n}$
$\therefore$ Electron diffusion current density:
$\left|J_{n}\right|_{d f f}=q D_{n} \frac{d n}{d x}=q D_{n} \times 2 \times 10^{20} \times e^{-1} \times \tau_{n}\left(0-\frac{1}{2_{n}}\right)$
$=\frac{1.6 \times 10^{-19} \times \mathrm{I}_{n}^{2} \times 2 \times 10^{20} \times \mathrm{e}^{-1}}{21_{\mathrm{n}}}$
$=1.6 \times 10^{-19} \times I_{n} \times 10^{20} \times \mathrm{e}^{-1}$
$=1.6 \times 10^{1} \times 1 \times 10^{-4} \times \mathrm{e}^{-1} \quad\left(I_{\mathrm{n}}=10^{-4} \mathrm{~cm}\right)$
$=0.588 \mathrm{~mA} / \mathrm{cm}^{2}=0.59$
64. In the circuit shown in figure below, identical transistors with large $\beta$ value and $V_{A 1}=V_{A 2}=100 \mathrm{~V}$ at a thermal voltage of 26 mV are used. The approximate small signal output resistance of the circuit in $K \Omega$ is $\qquad$ —.


Ans. [8.8 to 9.2]
Sol. Apply KCL at node $\mathrm{C}_{1}$ :
$\Rightarrow \mathrm{I}_{\mathrm{ref}}=\mathrm{I}_{\mathrm{C} 1}+\mathrm{I}_{\mathrm{B} 1}+\mathrm{I}_{\mathrm{B} 2}$
But given both transistors are identical,
$\Rightarrow \mathrm{I}_{\text {ref }}=\mathrm{I}_{\mathrm{C} 2}+2 \mathrm{I}_{\mathrm{B} 2}$
$\mathrm{I}_{\mathrm{ref}}=\mathrm{I}_{\mathrm{C} 2}\left[1+\frac{2}{\beta}\right]$
$\mathrm{I}_{\mathrm{ref}} \approx \mathrm{I}_{\mathrm{C} 2}$
Now $\mathrm{r}_{\mathrm{o} 2}=\frac{\mathrm{V}_{\mathrm{A} 2}}{\mathrm{I}_{\mathrm{C} 2}}=\frac{100}{1 \mathrm{~m}}=100 \mathrm{~K} \Omega$
$\Rightarrow \mathrm{R}_{\mathrm{o}}=\frac{10 \mathrm{~K} * 100 \mathrm{~K}}{10 \mathrm{~K}+100 \mathrm{~K}}=9.09 \mathrm{~K} \Omega$
65. An intrinsic semiconductor is doped with an impurity concentration of $10^{16} / \mathrm{cm}^{3}$. Assume the intrinsic carrier concentration $n_{i}=1.5 \times 10^{10} / \mathrm{cm}^{3}$ and the thermal voltage $\mathrm{V}_{\mathrm{T}}=26 \mathrm{mV}$. The Fermi level in doped semiconductor
A. Goes down by 0.35 eV if the impurity is 3 rd group element
B. Goes up by 0.35 eV if the impurity is 3 rd group element
C. Goes up by 0.35 eV if the impurity is 5 th group element
D. Goes down by 0.35 eV if the impurity is 5 th group element

Ans. A, C
Sol. Shift in Fermi-level is given by:
shift $=\mathrm{kTln}\left(\frac{\text { Doping concentration }}{\text { Intrinsic concentration }}\right)$
$\Rightarrow$ shift $=0.026 \ln \left(\frac{10^{16}}{1.5 \times 10^{10}}\right)$
$\Rightarrow$ shift $=0.35 \mathrm{eV}$
If the impurity is $3^{\text {rd }}$ group element, the resultant is a P-type semiconductor and the Fermi-level shifts downwards.
If the impurity is $5^{\text {th }}$ group element, the resultant is an $N$-type semiconductor and the Fermilevel shifts upwards.
66. In the following circuit $\mathrm{V}_{c}\left(0^{-}\right)=5 \mathrm{~V}$ and $\mathrm{V}_{\mathrm{c}}(\mathrm{t})=\frac{5}{e} \mathrm{~V}$ at $\mathrm{t}=0.1 \mathrm{sec}$, then the value of C in $\mu \mathrm{F}$ is $\qquad$ _.


Ans. [2.4 to 2.6]
Sol. It is a source free RC circuit; voltage across capacitor is given by:
$\mathrm{V}_{\mathrm{c}}(\mathrm{t})=\mathrm{V}_{\mathrm{c}}\left(0^{-}\right) \mathrm{e}^{-\frac{\mathrm{t}}{\tau}}$
Where, $\tau=\mathrm{CR}$
Given $V_{c}(0.1)=\frac{5}{e}=5 e^{-\frac{0.1}{R C}}$
$\Rightarrow 5 \mathrm{e}^{-1}=5 \mathrm{e}^{-\frac{0.1}{\mathrm{RC}}}$
$\Rightarrow \mathrm{RC}=0.1$
$\Rightarrow \mathrm{C}=\frac{0.1}{40 \mathrm{~K}}=2.5 \mu \mathrm{~F}$
67. A photo diode has a quantum efficiency of $80 \%$ when photons of energy $1.6^{*} 10^{-19} \mathrm{~J}$ are incident upon it. Then the incident optical power in $\mu \mathrm{W}$ required to obtain a photocurrent of $4 \mu \mathrm{~A}$ is $\qquad$ -.
Ans. [5 to 5]
Sol. Given data is:
$\eta=80 \%=0.8$
$\mathrm{E}=\mathrm{hf}=1.6 * 10^{-19} \mathrm{~J}$
Responsivity of a photo diode is given by:
$\mathrm{R}=\frac{\eta \mathrm{q}}{\mathrm{hf}}$
$\Rightarrow \mathrm{R}=\frac{0.8 * 1.6 * 10^{-19}}{1.6 * 10^{-19}}=0.8$
Responsivity is also given by:
$R=\frac{I_{p}}{P}=\frac{\text { photocurrent }}{\text { incident optical power }}$
$\Rightarrow \mathrm{P}=\frac{4 * 10^{-6}}{0.8}=5 \mu \mathrm{~W}$
68. Consider a CMOS inverter fabricate in a $0.13 \mu \mathrm{~m}$ process for which $\mathrm{V}_{\mathrm{DD}}=1.2 \mathrm{~V}$, $\mathrm{V}_{\mathrm{tn}}=-\mathrm{V}_{\mathrm{tp}}=0.4, \frac{\mu_{\mathrm{n}}}{\mu_{\mathrm{p}}}=4$ and $\mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}}=430 \mu \mathrm{~A} / \mathrm{V}^{2}$. If the NMOS and PMOS transistor are matched, then find the noise margin for high level and noise margin for low level respectively.
A. 0.55 V and 0.65 V
B. 0.55 V and 0.55 V
C. 0.65 V and 0.55 V
D. 0.65 V and 0.65 V

## Ans. B

Sol. Noise margin for high level is given by:
$\mathrm{NM}_{\mathrm{H}}=\mathrm{V}_{\mathrm{OH}}-\mathrm{V}_{\mathrm{IH}}$
But $\mathrm{V}_{\mathrm{OH}}=\mathrm{V}_{\mathrm{DD}}=1.2 \mathrm{~V}$
Also $\quad \mathrm{V}_{\mathrm{IH}}=\frac{1}{8}\left(5 \mathrm{~V}_{\mathrm{DD}}-2 \mathrm{~V}_{\mathrm{t}}\right)=\frac{1}{8}(5 * 1.2-2 * 0.4)=0.65 \mathrm{~V}$
$\Rightarrow \mathrm{NM}_{\mathrm{H}}=1.2-0.65=0.55 \mathrm{~V}$

Noise margin for high level is given by:
$\mathrm{NM}_{\mathrm{L}}=\mathrm{V}_{\mathrm{IL}}-\mathrm{V}_{\mathrm{OL}}$
But $\mathrm{V}_{\mathrm{OL}}=0 \mathrm{~V}$
Also $\mathrm{V}_{\mathrm{IL}}=\frac{1}{8}\left(3 \mathrm{~V}_{\mathrm{DD}}+2 \mathrm{~V}_{\mathrm{t}}\right)=\frac{1}{8}(3 * 1.2+2 * 0.4)=0.55$
$\Rightarrow \mathrm{NM}_{\mathrm{L}}=0.55-0=0.55 \mathrm{~V}$
Since NMOS and PMOS are matched $\mathrm{NM}_{\mathrm{H}}=\mathrm{NM} \mathrm{M}_{\mathrm{L}}$
69. The impulse response of a unity feedback control system is $c(t)=t e^{-t} u(t)$, and then which of the following are correct?
A. $\mathrm{CLTF}=\frac{1}{(s+1)^{2}}$
B. $\mathrm{OLTF}=\frac{1}{\mathrm{~s}(\mathrm{~s}+2)}$
C. Closed loop system is stable
D. Type of closed loop system is zero

Ans. A, B, C
Sol. $\quad \mathrm{CLTF}=\mathrm{L}[$ impulse response $]=\mathrm{L}\left[\mathrm{te}^{-\mathrm{t}} \mathrm{u}(\mathrm{t})\right]=\frac{1}{(\mathrm{~s}+1)^{2}}$
As the poles of closed loop system are in the left half of s-plane, it is stable.
OLTF can be found as below:
$\mathrm{CLTF}=\frac{\mathrm{C}(\mathrm{s})}{\mathrm{R}(\mathrm{s})}=\frac{\mathrm{G}(\mathrm{s})}{1+\mathrm{G}(\mathrm{s})}=\frac{1}{(\mathrm{~s}+1)^{2}}=\frac{1}{\mathrm{~s}^{2}+2 \mathrm{~s}+1}$
$\Rightarrow \frac{1+\mathrm{G}(\mathrm{s})}{\mathrm{G}(\mathrm{s})}=\mathrm{s}^{2}+2 \mathrm{~s}+1$
$\Rightarrow \frac{1}{\mathrm{G}(\mathrm{s})}+1=\mathrm{s}^{2}+2 \mathrm{~s}+1$
$\Rightarrow \mathrm{G}(\mathrm{s})=\frac{1}{\mathrm{~s}(\mathrm{~s}+2)}$
$\Rightarrow \mathrm{OLTF}=\frac{1}{\mathrm{~s}(\mathrm{~s}+2)}$
Only one pole at $s=0$, so the type of the closed loop system is 1
70. For the system shown in figure below, the DC gain due to noise is $\qquad$


Ans. [0.6 to 0.7]
Sol. The transfer function is given by:
$\frac{C}{N}=\frac{\frac{4}{s+2}}{1-\left[\frac{-1}{s+1} * \frac{4}{s+2}\right]}$
$\Rightarrow \frac{\mathrm{C}}{\mathrm{N}}=\frac{4(\mathrm{~s}+1)}{(\mathrm{s}+1)(\mathrm{s}+2)+4}$
$\Rightarrow \frac{\mathrm{C}}{\mathrm{N}}=\frac{4(\mathrm{~s}+1)}{\mathrm{s}^{2}+3 \mathrm{~s}+6}$
DC gain is obtained by putting $s=0$ in the TF,
$\Rightarrow$ DC gain $=\frac{4(0+1)}{0+0+6}=\frac{4}{6}$
$\Rightarrow$ DC gain $=0.67$
71. Given that $V_{T}=1 V, \mu_{n} C_{o x}=60 \mu \mathrm{~A} / \mathrm{V}^{2}, \frac{\mathrm{~W}}{\mathrm{~L}}=40$, and $\mathrm{I}_{\mathrm{D}}=0.3 \mathrm{~mA}$ and $\mathrm{V}_{\mathrm{D}}=0.4 \mathrm{~V}$. Then the value of $R_{s}$ is?

A. $3.3 \mathrm{k} \Omega$
B. $4.2 \mathrm{k} \Omega$
C. $2.6 \mathrm{k} \Omega$
D. $3.8 \mathrm{k} \Omega$

## Ans. A

Sol. The transistor is in saturation region, so $I_{D}$ is given by:

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{D}}=\frac{1}{2} \mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}}\left(\frac{\mathrm{~W}}{\mathrm{~L}}\right)\left(\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}\right)^{2} \\
& \Rightarrow 0.3 * 10^{-3}=\frac{1}{2} * 60 * 10^{-6} * 40 *\left(\mathrm{~V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}\right)^{2} \\
& \Rightarrow \mathrm{~V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}=0.5 \mathrm{~V} \\
& \Rightarrow \mathrm{~V}_{\mathrm{GS}}=1.5 \mathrm{~V} \\
& \mathrm{~V}_{\mathrm{G}}-\mathrm{V}_{\mathrm{S}}=1.5 \mathrm{~V} \\
& \operatorname{But} \mathrm{~V}_{\mathrm{G}}=0 \mathrm{~V} \\
& \Rightarrow \mathrm{~V}_{\mathrm{S}}=-1.5 \mathrm{~V} \\
& \Rightarrow \mathrm{R}_{\mathrm{S}}=\frac{-1.5-(-2.5)}{0.3 \mathrm{~m}}=3.3 \mathrm{~K} \Omega
\end{aligned}
$$

72. The real conjugate symmetric part of signal $x(t)=e^{(5+7 j) t}$ is
A. $\sinh (5 t) \cdot \sin (7 t)$
B. $\sinh (5 t) \cdot \cos (7 t)$
C. $\cosh (5 t) \cdot \cos (7 t)$
D. $\cosh (5 t) \cdot \sin (7 t)$

Ans. C
Sol. We know that conjugate symmetric part is given by:
$\mathrm{x}_{\mathrm{cs}}(\mathrm{t})=\frac{\mathrm{x}(\mathrm{t})+\mathrm{x}^{*}(-\mathrm{t})}{2}$
$\Rightarrow \mathrm{x}_{\mathrm{cs}}(\mathrm{t})=\frac{\mathrm{e}^{(5+7 \mathrm{j}) \mathrm{t}}+\mathrm{e}^{-(5-7 \mathrm{j}) \mathrm{t}}}{2}$
$\Rightarrow \mathrm{x}_{\mathrm{cs}}(\mathrm{t})=\frac{\mathrm{e}^{5 \mathrm{t}}(\cos (7 \mathrm{t})+\mathrm{j} \sin (7 \mathrm{t}))+\mathrm{e}^{-5 \mathrm{t}}(\cos (7 \mathrm{t})+\mathrm{j} \sin (7 \mathrm{t}))}{2}$
$\Rightarrow \mathrm{x}_{\mathrm{cs}}(\mathrm{t})=\frac{(\cos (7 \mathrm{t})+\mathrm{j} \sin (7 \mathrm{t}))\left(\mathrm{e}^{5 \mathrm{t}}+\mathrm{e}^{-5 \mathrm{t}}\right)}{2}$
The real part of conjugate symmetric signal is:
$\operatorname{Re}\left(\mathrm{x}_{\mathrm{cs}}(\mathrm{t})\right)=\frac{\left(\mathrm{e}^{5 \mathrm{t}}+\mathrm{e}^{-5 \mathrm{t}}\right)}{2} \cos (7 \mathrm{t})$
$\Rightarrow \operatorname{Re}\left(\mathrm{x}_{\mathrm{cs}}(\mathrm{t})\right)=\cosh (5 \mathrm{t}) \cos (7 \mathrm{t})$
73. Let $\mathrm{h}[\mathrm{n}]$ be the impulse response of a discrete-time linear time invariant (LTI) filter. The impulse response is given by
$\mathrm{h}[0]=\frac{1}{3} ; \mathrm{h}[1]=\frac{1}{3} ; \mathrm{h}[2]=\frac{1}{3}$; and $\mathrm{h}[\mathrm{n}]=0$ for $\mathrm{n}<0$ and $\mathrm{n}>2$
Let $\mathrm{H}(\omega)$ be the discrete-time Fourier system transform (DTFT) of $\mathrm{h}[\mathrm{n}$ ], where $\omega$ is the normalized angular frequency in radians. Given that $\mathrm{H}(\omega)=0$ and $0<\omega_{0}<\pi$, the value of $\omega_{0}$ (in radians) is equal to $\qquad$ .
Ans. 2.094
Sol. It is given that,
$\mathrm{h}[0]=\frac{1}{3} ; \mathrm{h}[1]=\frac{1}{3} ; \mathrm{h}[2]=\frac{1}{3}$
$\mathrm{h}[\mathrm{n}]=0$ for $\mathrm{n}<0$ and $\mathrm{n}>2$.
$\therefore \mathrm{h}[\mathrm{n}]=\mathrm{h}[0] \delta[\mathrm{n}]+\mathrm{h}[1] \delta[\mathrm{n}-1]+\mathrm{h}[2] \delta[\mathrm{n}-2]$
$\frac{1}{3}[\delta[n]+\delta[n-1]+\delta[n-2]]$
Apply DTFT on both sides,
$\therefore \mathrm{H}(\omega)=\frac{1}{3}\left[1+\mathrm{e}^{-\mathrm{j} \omega}+\mathrm{e}^{-2 \mathrm{j}_{\omega}}\right]$
Given that $\mathrm{H}\left(\omega_{0}\right)=0 \& 0<\omega_{0}<\pi$
$\frac{1}{3}\left[1+\mathrm{e}^{-\mathrm{j} \omega}+\mathrm{e}^{-2 \omega_{\omega}}\right]=0$
$1+e^{-j \omega}\left(1+e^{-j \omega}\right)=0$
$1+\mathrm{e}^{-\mathrm{j} \omega}=-\mathrm{e}^{\mathrm{j} \omega}$
$e^{-j \omega}+e^{j \omega}=-1$
$\cos \omega=-\frac{1}{2}$
$\therefore \omega_{0}=\frac{2 \pi}{3}$
$\omega_{0}=2.094$
74. A continuous time signal $x(t)=4 \cos (200 \pi t)+8 \cos (400 \pi t)$, where $t$ is in seconds, is the input to a linear time invariant (LTI) filter with the impulse response
$h(t)=\left\{\begin{array}{cc}\frac{2 \sin (300 \pi t)}{\pi t} & t \neq 0 \\ 600 & t=0\end{array}\right\}$
Let $y(t)$ be the output of this filter. The maximum value of $|y(t)|$ is $\qquad$ -.
Ans. [7.9 to 8.1]
Sol. Given: $(t)=\left\{\begin{array}{cc}\frac{2 \sin (300 \pi t)}{\pi t} & t \neq 0 \\ 600 & t=0\end{array}\right\}$
Thus $h(t)=600 \sin c(300 t)$
$\therefore H(f)=2 \operatorname{rect}\left(\frac{f}{300}\right)$
Given $x(t)=4 \cos 200 n t+800 \cos 400 n t$
In f-domain

$$
X(f)=2[\delta(f-100)+\delta(f+100)]+4[\delta(f-200)+\delta(f+200)]
$$




For the whole signal $=4 \times 2=8$
75. An optical fiber is kept along the $\hat{z}$ direction. The refractive indices for the electric fields along $\hat{x}$ and $\hat{y}$ directions in the fiber are $n_{x}=1.5000$ and $n_{y}=1.5001$, respectively ( $n_{x} \neq n_{y}$ due to the imperfection in the fiber cross-section). The free space wavelength of a light wave propagating in the fiber is $1.5 \mu \mathrm{~m}$. If the light wave is circularly polarized at the input of the fiber, the minimum propagation distance after which it becomes linearly polarized, in centimeter, is $\qquad$ -.

Ans. [0.36 to 0.38]
Sol. For circular polarization, the phase difference between $E_{x}$ and $E_{y}$ is $\pi / 2$
The phase difference for linear polarization should be $\pi$
$\Rightarrow$ So the wave must travel a minimum distance such that the extra phase difference of $\pi / 2$ must occur.
$\beta_{y I} I_{\text {min }}-\beta_{x} I_{\text {min }}=\frac{\pi}{2}$
$\Rightarrow I_{\text {min }}$
$\frac{\omega}{\mathrm{c}}\left[\mathrm{n}_{\mathrm{y}}-\mathrm{n}_{\mathrm{x}}\right]=\frac{\pi}{2} \Rightarrow \frac{2 \pi \mathrm{I}_{\min }}{\lambda_{0}}\left[\mathrm{n}_{\mathrm{y}}-\mathrm{n}_{\mathrm{x}}\right]=\frac{\pi}{2}$
$\Rightarrow I_{\text {min }}=$
$\frac{\lambda_{0}}{4\left[\mathrm{n}_{\mathrm{y}}-\mathrm{n}_{\mathrm{x}}\right]}=\frac{1.5 \times 10^{-5}}{4[0.0001]}=\frac{1.5}{4} \times 10^{-2}$
$=0.375 \times 10^{-2} \mathrm{~m}=0.375 \mathrm{~cm}$
76. The band diagram of a p-type semiconductor with a band-gap of $1 e \mathrm{~V}$ is shown. Using this semiconductor, a MOS capacitor having $\mathrm{V}_{\text {TH }}$ of $-0.16 \mathrm{~V}, \mathrm{C}^{\prime}{ }_{\text {ox }}$ of $100 \mathrm{nF} / \mathrm{cm}^{2}$ and a metal work function of 3.87 eV is fabricated. There is no charge within the oxide. If the voltage across the capacitor is $\mathrm{V}_{\mathrm{TH}}$, the magnitude of depletion charge per unit area (in $\mathrm{C} / \mathrm{cm}^{2}$ ) is

0.2 ev
A. $1.41 \times 10^{-8}$
B. $1.70 \times 10^{-8}$
C. $0.93 \times 10^{-8}$
D. $0.52 \times 10^{-8}$

## Ans. B

Sol. From the figure, $\mathrm{E}_{\mathrm{C}}-\mathrm{E} v_{\mathrm{v}}=1 \mathrm{eV}=0.5 \mathrm{Ev}+\mathrm{q} \varphi_{\mathrm{B}}+0.2 \mathrm{eV}$
$\Rightarrow \mathrm{q} \varphi_{\mathrm{B}}=0.3 \mathrm{eV}$
$\Rightarrow \varphi B=0.3 \mathrm{~V}$, where $\mathrm{q} \varphi \mathrm{B}=\mathrm{E}_{\mathrm{i}}-\mathrm{E}_{\mathrm{FS}}$
The magnitude of depletion charge density
$\rho_{\mathrm{s}}=\sqrt{2 \in_{\mathrm{S}} \mathrm{N}_{\mathrm{A}} \psi_{\mathrm{S}}}$ $\qquad$
Where, $\psi_{\mathrm{s}}=2 \varphi_{\mathrm{B}}=2 \times 0.3 \mathrm{~V}=0.6 \mathrm{~V}$ $\qquad$
Voltage across capacitor,
$V_{T H}=V_{F B}+\frac{\sqrt{2 \epsilon_{S} N_{A} \psi_{S}}}{C_{o x}}+\psi_{S}$ $\qquad$
Where, $\mathrm{V}_{\mathrm{FB}}=\varphi_{\mathrm{ms}}=\varphi_{\mathrm{m}}-\varphi \mathrm{S}$
$=3.87-4.8$
$V_{\mathrm{FB}}=\varphi_{\mathrm{ms}}=-0.93 \mathrm{~V}$ $\qquad$
From (i), (ii), (iii) and (iv),
$\rho_{\mathrm{s}}=1.7 \times 10^{-8} \mathrm{C} / \mathrm{cm}^{2}$
77. Four points $P(0,1), Q(0,-3), R(-2,-1)$, and $S(2,-1)$ represent the vertices of a quadrilateral. What is the area enclosed by the quadrilateral?
A. 4
B. $4 \sqrt{2}$
C. 8
D. $8 \sqrt{2}$

Ans. B
Sol.


Length of $P S=\sqrt{(2-0)^{2}+(-1-1)^{2}}=\sqrt{8}$
Length of $\mathrm{SQ}=\sqrt{4+4}=\sqrt{8}$
Length of $\mathrm{QR}=\sqrt{4+4}=\sqrt{8}$
Length of RP $=\sqrt{4+4}=\sqrt{8}$
Length of $R S=\sqrt{16+0}=4$
Length of $\mathrm{PQ}=\sqrt{0+16}=4$
Here, Length of $\mathrm{PQ}=$ Length of RS
Hence, PQRS is square
Area under PQRS $=(\sqrt{8})^{2}$
Area under $\mathrm{PQRS}=8$
78. If the op-amp is ideal, then the current in amperes through the $90 \Omega$ resistor is


Ans. [0.02 to 0.02]
Sol. Since DC 4V is given and there are no transients, we can do steady state analysis.
Under DC: C becomes open circuited and $L$ becomes short circuited
The circuit then will be as below:

From superposition theorem:
$\mathrm{V}_{\mathrm{o}}=-\frac{90}{100}(4)+\left[1+\frac{90}{100}\right]\left[\frac{4(2 \mathrm{~K})}{2 \mathrm{~K}+2 \mathrm{~K}}\right]$
$\Rightarrow \mathrm{V}_{\mathrm{o}}=-3.6+3.8=0.2 \mathrm{~V}$
From the circuit we can write as below:
$V_{2}=\frac{4(2 K)}{2 K+2 K}=V_{1}$
$\Rightarrow V_{1}=2 \mathrm{~V}$
$\Rightarrow \mathrm{I}_{90 \Omega}=\frac{\mathrm{V}_{1}-\mathrm{V}_{\mathrm{o}}}{90}=\frac{2-0.2}{90}=0.02 \mathrm{~A}$
79. For a class $B$ amplifier providing a 20 V peak signal to a 160 hm load speaker and a power supply of $V_{c c}=30 \mathrm{~V} .$. The efficiency is
A. 76.3\%
B. $52.3 \%$
C. $48.37 \%$
D. $81.25 \%$

Ans. B
Sol. A 20 V peak signal across a $16 \Omega$ load providing a peak load current of:
$\mathrm{I}_{\mathrm{m}}=\frac{\mathrm{V}_{\mathrm{m}}}{\mathrm{R}_{\mathrm{L}}}=\frac{20}{16}=1.25 \mathrm{~A}$
$\mathrm{I}_{\mathrm{dc}}=\frac{2 \mathrm{I}_{\mathrm{m}}}{\pi}=\frac{2}{\pi}(1.25)=0.796 \mathrm{~A}$
The input power delivered by the supply voltage is:
$\mathrm{P}_{\mathrm{i}_{\mathrm{DC}}}=\mathrm{V}_{\mathrm{cc}} \mathrm{I}_{\mathrm{dc}}=30 * 0.796=23.9 \mathrm{~W}$
The output AC power is given by:
$\mathrm{P}_{\mathrm{oAC}}=\frac{\mathrm{V}_{\mathrm{m}}^{2}}{2 \mathrm{R}_{\mathrm{L}}}=\frac{20^{2}}{2 * 16}=12.5 \mathrm{~W}$
Therefore, efficiency is given by:
$\eta=\frac{P_{o A C}}{P_{i_{D C}}} * 100 \%=\frac{12.5}{23.9} * 100 \%=52.3 \%$
80. An analog signal of $B W 20 \mathrm{KHz}$ is sampled at a rate of 40 KHz and quantized into 16 levels resultant signal is transmitted using M-PSK with raised cosine pulse of roll off factor 0.3 . Find minimum value of $M$ if channel with a 110 KHz BW is available to transmit data?
A. 2
B. 4
C. 8
D. 16

Ans. B
Sol. Number of bits required for 16 levels is:
$n=\log _{2} L=\log _{2} 16=4$
Bit rate is given by:
$R_{b}=n f_{s}=4 \times 40 \mathrm{~K}=160 \mathrm{~K}$
BW for M-ary PSK is given by:
$B W=\frac{R_{b}}{\log _{2} M}(1+\alpha)$
$\Rightarrow 110 K=\frac{160 K}{\log _{2} M}(1+0.3)$
$\Rightarrow M=2^{1.89}=3.7$
$\Rightarrow M=4$
Since, $M$ is always an integer.
81. A HWR with capacitor filter is to supply 30 V to a $500 \Omega$ load. If the ripple factor is 0.02 and frequency of $A C$ supply is 50 Hz , then the value of peak diode current is
A. 2.12 A
B. 1.2 A
C. 1.48 A
D. 2.36 A

Ans. A
Sol. Given values are:
$V_{D C}=30 \mathrm{~V} \quad \mathrm{R}_{\mathrm{c}}=500 \Omega \quad \mathrm{r}=0.02 \quad \mathrm{f}=50 \mathrm{~Hz}$
$\Rightarrow \mathrm{I}_{\mathrm{DC}}=\frac{\mathrm{V}_{\mathrm{DC}}}{\mathrm{R}_{\mathrm{c}}}=\frac{30}{500}=0.06 \mathrm{~A}$
Ripple factor of HWR is given by:
$r=\frac{1}{2 \sqrt{3} R_{L} f C}$
$\Rightarrow \mathrm{C}=\frac{1}{2 \sqrt{3} \mathrm{R}_{\mathrm{L}} \mathrm{fr}}=\frac{1}{2 \sqrt{3} * 50 * 0.02 * 500}=0.58 \mathrm{mF}$
$\mathrm{V}_{\mathrm{r}}=\frac{\mathrm{I}_{\mathrm{DC}}}{\mathrm{fC}}=\frac{0.06}{50 * 0.58 * 10^{-3}}=2.07 \mathrm{~V}$
$\Rightarrow \mathrm{V}_{\mathrm{m}}=\mathrm{V}_{\mathrm{DC}}+\frac{\mathrm{V}_{\mathrm{r}}}{2}=30+\frac{2.07}{2}=31.035 \mathrm{~V}$
The value of peak diode current is given by:
$\Rightarrow i_{D \text { peak }}=I_{D C} \sqrt{1+2 \pi \sqrt{\frac{2 V_{m}}{V_{r}}}}$
$\Rightarrow \mathrm{i}_{\text {D peak }}=0.06 \sqrt{1+2 \pi \sqrt{2 * \frac{31.035}{2.07}}}=2.12 \mathrm{~A}$
82. $A$ and $B$ play a game where each is asked to select a number from 1 to 25 . If the two numbers match both of them win a prize, and then probability that they will not win a prize in single trail is $\qquad$ —.
Ans. [0.95 to 0.97]
Sol. No. of ways in which either player can choose a number from 1 to 25 is 25
Total number of ways of choosing numbers is $25 \times 25=625$
There are 25 ways in which the number chosen by both players is the same.

The probability that they will win a prize in single trail is:
$P=\frac{25}{625}=\frac{1}{25}$
The probability that they will not win a prize in single trail is:
$\Rightarrow P^{c}=1-\frac{1}{25}=\frac{24}{25}=0.96$
83. If the differential equation $\frac{d y}{d x}=\sqrt{x^{2}+y^{2}}, y(1)=2$ is solved using the Eulers method with step size $\mathrm{h}=0.1$, then $\mathrm{y}(1.2)=$ $\qquad$ .
Ans. [2.3 to 2.5]
Sol. Let, $f(x, y)=\frac{d y}{d x}=\sqrt{x^{2}+y^{2}}$
From Euler's method:
$\Rightarrow y_{1}=y(1.1)=y_{0}+h f\left(x_{0}, y_{0}\right)$
$\Rightarrow y_{1}=2+(0.1) f(1,2)$
$\Rightarrow y_{1}=2+(0.1)(\sqrt{1+4})$
$\Rightarrow y_{1}=2.2236$
Again from Euler's method:
$\Rightarrow y_{2}=y(1.2)=y_{1}+h f\left(x_{1}, y_{1}\right)$
$\Rightarrow y_{2}=2.2236+(0.1) f(1.1,2.2236)$
$\Rightarrow y_{1}=2+(0.1)\left(\sqrt{(1.1)^{2}+(2.2236)^{2}}\right)$
$\Rightarrow y_{1}=2.4717$
84. A base band signal with $f=4 \mathrm{MHz}$ is uniformly distributed in the range from -4 V to 4 V . This signal is sent on a channel whose maximum capacity is 32 Mbps using PCM. The best SNR (in dB) that could be achieved is $\qquad$
Ans. [23.5 to 24.5]
Sol. Signal power is Mean Square Value (MSV) of Uniform random variable
$\Rightarrow S=\int_{-4}^{4} x^{2} f(x) d x=\int_{-4}^{4} x^{2} \times \frac{1}{8} d x=\frac{16}{3} W$
Ideal sampling frequency is given by
$f_{s}=2 f_{m}$
$\Rightarrow f_{s}=2 \times 4 \mathrm{M}=8 \mathrm{MHz}$
For error free transmission through a channel, using PCM:
$C \geq B W=n f_{s}$
$\Rightarrow 32 \mathrm{Mbps} \geq n \times 8 \mathrm{MHz}$
$\Rightarrow n \leq 4$

Hence maximum value of $n=4$ bits/sample
For best SNR , Noise power $\mathrm{N}=\frac{\Delta^{2}}{12}$ should be small. So, step size $\Delta=\frac{\mathrm{V}_{\max }-\mathrm{V}_{\min }}{2^{\mathrm{n}}}$ should be small.
$\Rightarrow$ ' $n$ ' Should be considered the maximum value
$\Rightarrow \Delta=\frac{4-(-4)}{2^{4}}=\frac{8}{16}=\frac{1}{2}$
$N=\frac{1}{4 \times 12}=\frac{1}{48} W$
$S N R=\frac{S}{N}=\frac{16}{3} \times 48=256$
$\Rightarrow S N R($ in $d B)=10 \log (256)=24.08 d B$
85. The transmitting and receiving antennas are separated by a distance of $200 \lambda$ and having directive gain of 25 and 18 dB respectively. If 5 mW of power is to be received, the minimum transmitted power in watts is $\qquad$
Ans. [1.57 to 1.59]
Sol. Given that $G_{t}(d B)=25 d B=10 \log \left(G_{t}\right)$
$\Rightarrow G_{t}=10^{2.5}=316.23$
Also given that $G_{r}(d B)=18 d B=10 \log \left(G_{r}\right)$
$\Rightarrow G_{r}=10^{1.8}=63.1$
Using the Friis equation, we have:
$P_{r}=G_{t} G_{r}\left[\frac{\lambda}{4 \pi R}\right]^{2} P_{t}$
$\Rightarrow P_{t}=P_{r}\left[\frac{4 \pi R}{\lambda}\right]^{2} \frac{1}{G_{t} G_{r}}$
$\Rightarrow P_{t}=5 * 10^{-3} \times\left[\frac{4 \pi * 200 \lambda}{\lambda}\right]^{2} \times \frac{1}{316.23 * 63.1}$
$\Rightarrow P_{t}=1.583 \mathrm{~W}$
86. A finite state machine (FSM) is implemented using the D flip-flops A and B, and logic gates, as shown in the figure below. The four possible states of the $F S M$ are $Q_{A} Q_{B}=00,01,10$ and 11 .


Assume that $X_{1 N}$ is is held at a constant logic level throughout the operation of the FSM. When the FSM is initialized to the state $\mathrm{Q}_{A} \mathrm{Q}_{\mathrm{B}}=00$ and clocked, after a few clock cycles, it starts cycling through
A. all of the four possible states if $X_{1 N}=1$
B. three of the four possible states if $X_{1 N}=0$
C. only two of the four possible states if $X_{1 N}=1$
D. only two of the four possible states if $X_{\text {IN }}=0$

Ans. D
Sol. In given diagram

| Present <br> State | $\mathbf{D}_{\mathrm{A}}$ | $\mathbf{D}_{\mathrm{B}}$ | $\mathbf{X}_{\mathrm{a}}$ | $\mathbf{X}_{\mathrm{a}}$ | $\mathbf{X}_{\mathrm{a}}=0$ Next State |  |  | $\mathbf{X}_{\mathrm{a}}=1$ Next State |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |  |  |
| 01 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |  |  |
| 11 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |  |  |
| 01 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |  |  |

When $X_{\text {in }}=02$ States
When $\mathrm{Xin}_{\text {in }} 13$ States
87. Consider the $D$-Latch shown in the figure, which is transparent when its clock input $C K$ is high and has zero propagation delay. In the figure, the clock signal CLK1 has a $50 \%$ duty cycle and CLK2 is a one-fifth period delayed version of CLK1. The duty cycle at the output latch in percentage is $\qquad$ .


Ans. [29.9 to 30.1]

## Sol.


$\Rightarrow$ Duty-cycle of output $=\frac{\frac{T_{C L K}}{2}-\frac{T_{C L K}}{5}}{T_{C L K}} \times 100=30 \%$
88. In a non-degenerate bulk semiconductor with electron density $n=10^{16} \mathrm{~cm}^{-3}$, the value of $\mathrm{E}_{\mathrm{c}}-$ $\mathrm{E}_{\mathrm{Fn}}=200 \mathrm{meV}$, where $\mathrm{E}_{\mathrm{c}}$ and $\mathrm{E}_{\mathrm{Fn}}$ denote the bottom of the conduction band energy and electron Fermi level energy, respectively. Assume thermal voltage as 26 mV and the intrinsic carrier concentration is $10^{10} \mathrm{~cm}^{-3}$. For $\mathrm{n}=0.5 \times 10^{16} \mathrm{~cm}^{-3}$, the closest approximation of the value of ( $\mathrm{E}_{\mathrm{c}}-\mathrm{E}_{\mathrm{Fn}}$ ), among the given options, is $\qquad$ .
A. 226 meV
B. 174 meV
C. 218 meV
D. 182 meV

Ans. C

## Sol.


n-type
Given, $E_{c}-E_{f n}=200 \mathrm{meV}=0.2 \mathrm{eV}$
We know that, $E_{C}-E_{F}=K T \ln \frac{N_{C}}{N_{D}}$

## Case 1:

$\mathrm{n} \cong \mathrm{N}_{\mathrm{D} 1}=10^{16} / \mathrm{cm}^{3}$
$E_{C}-E_{F n 1}=K T \ln \frac{N_{C}}{N_{D_{1}}}=0.2 \mathrm{eV}=200 \mathrm{meV}$
$\frac{\mathrm{N}_{\mathrm{C}}}{\mathrm{N}_{\mathrm{D}_{1}}}=2191.43 \Rightarrow \mathrm{~N}_{\mathrm{C}}=2191.43 \times 10^{16}$

## Case 2:

$\mathrm{n} \cong \mathrm{N}_{\mathrm{D} 2}=0.5 \times 10^{16} / \mathrm{cm}^{3}$
$\mathrm{E}_{\mathrm{C}}-\mathrm{E}_{\mathrm{Fn} 2}=\mathrm{kT} \ln \frac{\mathrm{N}_{\mathrm{C}}}{\mathrm{N}_{\mathrm{D}_{2}}}$
$=k T \ln \frac{2191.43 \times 10^{16}}{0.5 \times 10^{16}}$
$=26 \ln \left(\frac{2191.43}{0.5}\right)$
$=218 \mathrm{meV}$
Other Method:
Let, $E_{C}-E_{F n 1}=k T \ln \frac{N_{C}}{N_{D_{1}}}$
$\mathrm{E}_{\mathrm{C}}-\mathrm{E}_{\mathrm{Fn}_{2}}=\mathrm{kT} \ln \frac{\mathrm{N}_{\mathrm{C}}}{\mathrm{N}_{\mathrm{D}_{2}}}$
(ii) - (i)
$\left(E_{C}-E_{F_{n_{2}}}\right)-\left(E_{C}-E_{F_{n_{1}}}\right)=k T \ln \frac{N_{C}}{N_{D_{1}}} \cdot \frac{N_{D_{1}}}{N_{C}}$
$E_{C}-E_{F_{n_{2}}}-200=k T \ln \frac{10^{16}}{0.5 \times 10^{16}}$
$\mathrm{E}_{\mathrm{C}}-\mathrm{E}_{\mathrm{Fn}_{2}}=200+\mathrm{kT} \ln 2$
$=200+26 \times \ln (2)$
$\cong 218 \mathrm{meV}$
89. Consider the homogeneous ordinary differential equation $x^{2} \frac{d^{2} y}{d x^{2}}-3 x \frac{d y}{d x}+3 y=0, x>0$ with $y(x)$ as a general solution. Given that $y(1)=1$ and $y(2)=14$ the value of $y(1.5)$, rounded off to two decimal places, is $\qquad$ .
Ans. [5.24 to 5.26]
Sol. Given differential equation is of Cauchy - Euler differential equation type.
So let $\mathrm{x}=\mathrm{e}^{\mathrm{z}} . \mathrm{z}=\ln \mathrm{x}$
The differential equation can be written as,
$D(D-1)-3 D+3=0$
$\therefore \mathrm{D}^{2}-4 \mathrm{D}+3=0$
$\therefore \mathrm{D}=1,3$
$\therefore \mathrm{y}=\mathrm{C}_{1} \mathrm{e}^{\mathrm{z}}+\mathrm{C}_{2} \mathrm{e}^{3 \mathrm{z}}$
$\therefore \mathrm{y}=\mathrm{C}_{1} \mathrm{x}+\mathrm{C}_{2} \mathrm{x}^{3}$
Now y (1) $=1$
$\therefore \mathrm{C}_{1}+\mathrm{C}_{2}=1 \ldots$ (i)
And $y(2)=14$
$\therefore 2 C_{1}+8 C_{2}=14$
From (i) and (ii)
$C_{1}=-1, C_{2}=2$
$\therefore y=-x+2 x^{3}$
$\therefore y(1.5)=-1.5+2(1.5)^{3}$
$\therefore y(1.5)=5.25$
90. Consider the DE given below with $\mathrm{y}(0)=1$. Then $\mathrm{y}(3)=$ ?
$\left(x^{2}+1\right) \frac{d y}{d x}+4 x y=\frac{1}{x^{2}+1}$
A. 0.01
B. 0.02
C. 0.03
D. 0.04

Ans. D
Sol. Given DE can be rearranged as below:
$\frac{d y}{d x}+\frac{4 x}{1+x^{2}} \cdot y=\frac{1}{\left(1+x^{2}\right)^{2}}$
This is a standard Bernoulli's equation, whose integrating factor is given by:
$I F=e^{\int \frac{4 x}{1+x^{2}} d x}$
$\Rightarrow I F=e^{\int 2 \frac{2 x}{1+x^{2}} d x}=e^{2 \ln \left(1+x^{2}\right)}=\left(1+x^{2}\right)^{2}$
The general solution is given by:
$y(I F)=\int(I F) \cdot \frac{1}{\left(1+x^{2}\right)^{2}} d x$
$\Rightarrow y\left(1+x^{2}\right)^{2}=\int\left(1+x^{2}\right)^{2} \cdot \frac{1}{\left(1+x^{2}\right)^{2}} d x$
$\Rightarrow y\left(1+x^{2}\right)^{2}=x+c$
$\Rightarrow y(x)=\frac{x}{\left(1+x^{2}\right)^{2}}+\frac{c}{\left(1+x^{2}\right)^{2}}$
Given, $y(0)=1$,
$\Rightarrow y(0)=1=0+c$
$\Rightarrow c=1$
$\Rightarrow y(x)=\frac{x+1}{\left(1+x^{2}\right)^{2}}$
$\Rightarrow y(3)=\frac{4}{100}=0.04$
91. If $f(z)=x-2 a y+i(b x-c y)$ is an analytic function, then $\left(\frac{a c}{b}\right)=$ ?
A. -0.5
B. -1
C. 0.5
D. 1

## Ans. A

Sol. Let, $f(z)=x-2 a y+i(b x-c y)=u+i v$
$\Rightarrow u=x-2 a y \& v=b x-c y$
$\Rightarrow u_{x}=1 \& u_{y}=-2 a$
$\Rightarrow v_{x}=b \& v_{y}=-c$
For a function to be analytic, it should satisfy CR equations
$\Rightarrow u_{x}=v_{y} \& u_{y}=-v_{x}$
$\Rightarrow 1=-c \&-2 a=-b$
$\Rightarrow c=-1$
$\Rightarrow b=2 a$
$\Rightarrow\left(\frac{a c}{b}\right)=\frac{a(-1)}{2 a}=-0.5$
92. For a MOD-28 ripple counter, each flip flop has a propagation delay of 50 ns and the NAND gate has a propagation delay of 30 ns . The maximum frequency of clock that can be applied to the counter is
A. 9.2 MHz
B. 2.5 MHz
C. 3.57 MHz
D. 4.34 MHz

Ans. C
Sol. We know that:
$M O D \leq 2^{n}$
$\Rightarrow 28 \leq 2^{n}$
$\Rightarrow \mathrm{n}=5$
Total delay in the flip flops is given by:
$T_{D}=n t_{p d}$
$\mathrm{T}_{\mathrm{D}}=5 \times 50 \mathrm{~ns}=250 \mathrm{~ns}$
So, the total propagation delay is:
$\Rightarrow \mathrm{T}=250+30=280 \mathrm{~ns}$
$\Rightarrow \mathrm{f}_{\max }=\frac{1}{\mathrm{~T}}=\frac{1}{280 \mathrm{~ns}}=3.57 \mathrm{MHz}$
93. The logic levels used in a 6-bit $R-2 R$ ladder type DAC are $1=5 \mathrm{~V}$ and $0=0 \mathrm{~V}$. Find the output voltage for input 101011 . Consider $R=1 \mathrm{k} \Omega$.
A. $\frac{55}{32} \mathrm{~V}$
B. $\frac{13}{8} \mathrm{~V}$
C. $\frac{215}{64} \mathrm{~V}$
D. $\frac{97}{4} \mathrm{~V}$

Ans. C
Sol. Output voltage of $\mathrm{R}-2 \mathrm{R}$ ladder is given by:
$V_{o}=-I_{F} R$
$\Rightarrow V_{o}=-\left[\frac{5}{2 R}+0+\frac{5}{8 R}+0+\frac{5}{32 R}+\frac{5}{64 R}\right]$
$\Rightarrow V_{o}=-\left[\frac{160+40+10+5}{64 R}\right]$
$V_{o}=-\frac{215}{64} V$
Here, negative sign only shows that input is given to inverting pin of op-amp or output is $180^{\circ}$ phase reversed input.
$\Rightarrow \mathrm{V}_{0}=\frac{215}{60} \mathrm{~V}$
94. The following program is executed on 8085 microprocessor. After execution the contents of stack pointer is

LXI H, 0100H
DCX H
SPHL
PUSH H
PUSH PSW
CALL SUBI
POP H
RET
A. 0100 H
B. 00FD H
C. 00FF H
D. 00FB H

Ans. C
Sol. LXI H, 0100H HL $=0100 \mathrm{H}$
DCX H HL $=0100 \mathrm{H}-1=00 F F \mathrm{H}$
SPHL SP $=\mathrm{HL}=00 F F \mathrm{H}$
PUSH H SP $=00 F F \mathrm{H}-2=00 F D \mathrm{H}$
PUSH PSW SP $=00 F D \mathrm{H}-2=00 F B \mathrm{H}$
CALL SUBI SP $=00 F B \mathrm{H}-2+2=00 \mathrm{FB} \mathrm{H}$
POP H SP $=00 F B H+2=00 F D H$
RET SP $=00 F D H+2=00 F F H$
So, the contents of SP is 00FF H
95. The approximate delay in LOOP2 that the below 8085 microprocessor program with clock frequency of 2 MHz produces in milliseconds is?

MVI B, 38H
LOOP2: MVI C, FFH
LOOP1: DCR C
JNZ LOOP1
DCR B
JNZ LOOP2
Options:
A. 50
B. 75
C. 100
D. 125

Ans. C
Sol. Instruction with corresponding T-states is given as:
MVI C, FFH 7 T-states
DCR C 4 T-states
JNZ LOOP1 10/7 T-states
DCR B 4 T-states
JNZ LOOP2 10/7 T-states
So, the delay in LOOP1 is given by:
$T_{D 1}=\left(T_{\text {clk }} \times\right.$ Loop $T$ states $\left.\times N_{10}\right)-($ correction $)$
$T_{D 1}=(0.5 \times 14 \times 255)-(0.5 \times 3)=1783.5 \mu s$
And the delay in LOOP2 is given by:
$T_{D 2}=\left(0.5 \times 21+T_{D 1}\right) \times 56-(0.5 \times 3)$
$\Rightarrow T_{D 2} \approx 100 \mathrm{~ms}$
96. A parallel plate capacitor with plate area of $5 \mathrm{~cm}^{2}$ and plate separation of 3 mm has a voltage $50 \sin \left(10^{3} \mathrm{t}\right) \vee$ applied to its plates. The displacement current assuming $\varepsilon=2 \varepsilon_{0}$ is?
A. $147.4 \sin \left(10^{3} t\right) n A$
B. $274.1 \cos \left(10^{3} \mathrm{t}\right) \mathrm{nA}$
C. $147.4 \cos \left(10^{3} \mathrm{t}\right) \mathrm{nA}$
D. $274.1 \sin \left(10^{3} t\right) n A$

Ans. C
Sol. From the below equations:
$\mathrm{D}=\varepsilon \mathrm{E}=\frac{\varepsilon \mathrm{V}}{\mathrm{d}}$
$\Rightarrow J_{d}=\frac{\partial D}{\partial t}=\frac{\varepsilon}{d} \frac{d V}{d t}$
Displacement current is thus given by:
$I_{d}=J_{d} \cdot S=\frac{\varepsilon S}{d} \frac{d V}{d t}=C \frac{d V}{d t}$
$\Rightarrow I_{d}=\frac{2 * 10^{-9}}{36 \pi} \times \frac{5 * 10^{-4}}{3 * 10^{-3}} \times 10^{3} \times 50 \cos \left(10^{3} t\right) A$
$\Rightarrow I_{d}=147.4 \cos \left(10^{3} t\right) n A$
97. Consider a super heterodyne receiver tuned to 600 kHz . If the local oscillator feeds a 1000 kHz signal to the mixer, the image frequency (in integer) is $\qquad$ kHz .

Ans. 1400

## Sol.

$\mathrm{F}_{\mathrm{L}_{0}}>\mathrm{F}_{\mathrm{R}_{\mathrm{F}}}$
$\mathrm{F}_{\mathrm{L}_{0}}=\mathrm{F}_{\mathrm{R}_{\mathrm{F}}}+\mathrm{F}_{\mathrm{I}_{\mathrm{F}}}$
$1000=600+\mathrm{F}_{\mathrm{I}_{\mathrm{F}}} \Rightarrow \mathrm{F}_{\mathrm{I}_{\mathrm{F}}}=400 \mathrm{kHz}$
$F_{\text {Image }}=F_{f s}+2^{F_{I_{F}}}$
$F_{\text {Image }}=600+2(400)=1400 \mathrm{kHz}$
98. For a unit step input $u[n]$, a discrete-time LTI system produces an output signal $(2 \delta[n+1]+$ $\delta[n]+\delta[n-1])$. Let $y[n]$ be the output of the system for an input $\left(\left(\frac{1}{2}\right)^{n} u[n]\right)$. The value of $y[0]$ is $\qquad$ .
Ans. 0
Sol. $u[n] \longrightarrow \mathbf{h}[\mathrm{n}] \longrightarrow \mathbf{2}[\mathbf{n}+1]+\delta[\mathrm{n}]+\delta[\mathrm{n}-1]$
$\mathrm{u}[\mathrm{n}] \stackrel{\text { Z.T. }}{\longleftrightarrow} \frac{\mathrm{z}}{\mathrm{z-1}}$
$2 \delta[n+1]+\delta[n]+\delta[n-1] \stackrel{\text { z.T. }}{\longleftrightarrow} 2 z+1+z^{-1}$
$=2 z+1+\frac{1}{z}$
$=\left(\frac{2 z^{2}+z+1}{z}\right)$
Now,
$H(z)=\left(\frac{2 z^{2}+z+1}{z}\right) /\left(\frac{z}{z-1}\right)$
$H(z)=\frac{(z-1)\left(2 z^{2}+z+1\right)}{z^{2}}$
$H(z)=\frac{2 z^{3}-z^{2}-1}{z^{2}}$
$H(z)=2 z-1-z^{-2}$
$h[n]=2 \delta[n+1]-\delta[n]-\delta[n-2]$
$x[n] \longrightarrow h[n] \longrightarrow y[n]$
$x[n]=\left(\frac{1}{2}\right)^{n} \cdot u[n], h[n]=2 \delta[n+1]-\delta[n]-\delta[n-2]$
$\mathrm{y}[\mathrm{n}]=\mathrm{x}[\mathrm{n}] * \mathrm{~h}[\mathrm{n}]$
$y[n]=x[n] *[2 \delta[n+1]-\delta[n]-\delta[n-1]]$
$y[n]=2 x[n+1]-x[n]-x[n-2]$
$y[n]=2\left(\frac{1}{2}\right)^{n+1} \cdot u[n+1]-\left(\frac{1}{2}\right)^{n} \cdot u[n]-\left(\frac{1}{2}\right)^{n-2} \cdot u[n-2]$
$\mathrm{y}[0]=2\left(\frac{1}{2}\right)^{1} \cdot \mathrm{u}[1]-\left(\frac{1}{2}\right)^{0} \cdot \mathrm{u}[0]-\left(\frac{1}{2}\right)^{-2} \cdot \mathrm{u}[-2]$
$y[0]=1-1-0$
$y[0]=0$
99. The transfer function of a linear time invariant system is given by $H(s)=2 s^{4}-5 s^{3}+5 s-2$. The number of zeros in the right half of the s-plane is $\qquad$ -.

## Ans. 3

Sol. Since, Routh Hurwitz gives information about location of roots about imaginary axis.
So, the roots i.e. zeroes of $\mathrm{H}(\mathrm{s})$ can be found by Routh Hurwitz.
We can proceed here by taking this polynomial as characteristic equation and conclusion can be draw by using RH criterion. As we are interested to know how many roots are lying on right half of s plane.

| $\mathrm{S}^{4}$ | 2 | 0 | -2 |
| :--- | :--- | :--- | :--- |
|  | -5 | +5 | 0 |
|  |  |  | $\left\{\begin{array}{l}\text { Since row of zero } \\ \text { occursthe } \\ \text { awciliaryequation } \\ \text { is } \\ \text { A. } \varepsilon: 2 s^{2}-2 \\ d \\ \hline\end{array}\right.$ |
|  | -2 |  |  |
| $s$ | -2 | 0 |  |

$\rightarrow$ The number of roots i.e. the number of zeros in this case in right half of plane is number of sign changes
$\rightarrow$ Number of sign changes $=3$
100. The open-loop transfer function of a unity-feedback control system is

$$
G(s)=\frac{K}{s^{2}+5 s+5}
$$

The positive value of $K$ at the breakaway point of the feedback control system's root-locus plot is

Ans. 1.25
Sol. In this first we need to find the break point by finding the root of $\frac{d k}{d s}=0$ and then by using magnitude condition value of $k$ can be obtained.
$G(s)=\frac{K}{s^{2}+5 s+5}$
$k=\left(s^{2}+5 s+5\right)$
$\frac{\mathrm{dk}}{\mathrm{ds}}=0$
$\Rightarrow 2 s+5=0 \Rightarrow s=-2.5$
Applying magnitude condition $|G(s)|=1$
$\left|\frac{K}{s^{2}+5 s+5}\right|_{s=-2.5}=1$
$\Rightarrow\left[\frac{\mathrm{k}}{\left(-2.5^{2}\right)[5 \times(-2.5)]+5}\right]=1$
$\Rightarrow\left[\frac{\mathrm{k}}{6.25-12.5+5}\right]=1$
$\Rightarrow\left|\frac{\mathrm{k}}{-1.25}\right|=1$
$\Rightarrow \mathrm{k}=1.25$

