Forced Vibration

Forced vibration occurs when an external force is applied to a system, causing it to vibrate at a specific frequency. The force can be a periodic force, such as a sinusoidal or random force. The response of the system to the applied force depends on the frequency of the force, the natural frequency of the system, and the damping of the system. If the frequency of the force is close to the natural frequency of the system, the system will respond with large amplitude vibrations, a phenomenon known as resonance.

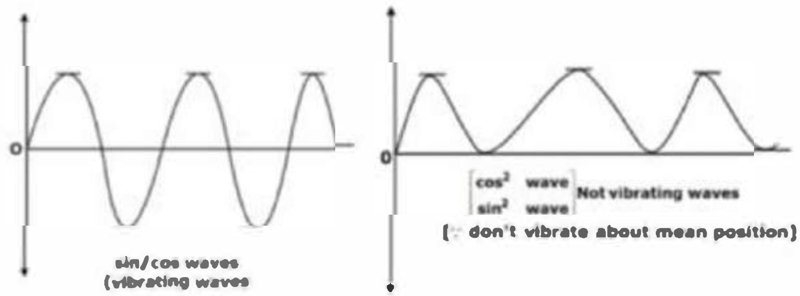

Fig.1: Type of oscillating waves

Any vibrating system is a combination of the following:

- E storing device (i.e., mass = m)
- E storing device (having stiffness = s)
- Kinetic friction
- Unbalanced forces

Forced Vibration Formula

The mathematical derivation of forced vibrations can be complex and depends on the system being analyzed. In general, however, the process involves solving a set of differential equations that describe the system's motion. These equations are based on the system's mass, stiffness, and damping properties, as well as the characteristics of the applied force. Damped vibration is somehow similar to forced vibration. A simple example of forced vibration can be a mass-spring-damper system that is excited by a sinusoidal force. The equation of motion for this system can be written as follows:

$mx'' + cx' + kx = F(t) \cos(wt)$

Where:

- m = mass of the system,
- c = damping coefficient,
- k = spring constant,
- x = displacement of the mass,

- F(t) = amplitude of the applied force,
- w = angular frequency of the applied force

This is a second-order differential equation, which can be solved using various methods. One common method is to use the method of undetermined coefficients, which involves assuming a particular solution for x and then determining the coefficients of the solution using the initial conditions and the equation of motion. The solution to this differential equation represents the displacement of the mass over time, and it can be used to determine the amplitude and phase of the system's response to the applied force.

In general, the solution to the differential equation of motion would be complex numbers (complex amplitude) represented in amplitude and phase shift. The equation of motion for more complex systems, such as multi-degree-of-freedom systems or systems with distributed mass and stiffness, can be more challenging to solve. In such cases, numerical methods, such as the finite element method, may be used to determine the system's response to the applied force. Solving the equation of motion can be complex and require knowledge of mathematical methods such as the Laplace Transform, Fourier series, etc. But overall, it is the key step to understanding and predicting the behaviour of mechanical systems under forced vibrations.

Types of Forced Vibration

There are several different types of forced vibrations, each with its own unique characteristics and uses. Some common types of forced vibrations include:

1. **Harmonic excitation:** This type of excitation involves a sinusoidal force with a constant amplitude and frequency. Harmonic excitation is often used in the testing and analysis of mechanical systems, as the sinusoidal nature of the excitation makes it relatively easy to predict and measure the system's response.
2. **Random excitation:** This type of excitation involves a force that varies randomly in amplitude and frequency. Random excitation is often used to simulate real-world loading conditions, as many natural forces, such as wind and waves, are random in nature.
3. **Impact excitation:** This type involves a sudden, short-duration force, such as a hammer strike. Impact excitation is often used to test a system's dynamic response and determine its natural frequency.
4. **Transient excitation:** This type of excitation involves a force that varies over time, such as a force that starts at one amplitude and frequency and then gradually changes. Transient excitation is often used to test the dynamic response of a system to a step change in loading conditions.
5. **Periodic excitation:** This type of excitation involves a repetitive force but is not necessarily sinusoidal in nature. Periodic excitation can be found in many applications, such as in Rotating Machineries.
6. **Continuous excitation:** This type of excitation is continuous and can be seen in everyday life, such as people are walking and vehicles pass by.

The choice of excitation type depends on the test or analysis's specific goals and the system's excited characteristics. Engineers may choose one type of excitation or a combination to achieve the desired results.

Applications of Forced Vibration

There are several applications of forced vibration in various fields, as we have seen in free vibration, such as engineering, physics, and materials science. Some examples include:

1. **Structural dynamics:** Engineers use forced vibration analysis to study the response of structures such as bridges, buildings, and towers to wind and earthquake forces. This helps ensure that the structures can withstand these forces without damaging or collapsing.
2. **Vibration testing:** Engineers use forced vibration to test the structural integrity of products such as aircraft, automobiles, and electronic devices. By subjecting the products to various types of vibrations, engineers can identify design weaknesses and improve the robustness of the products.
3. **Manufacturing:** Forced vibration is used to improve the quality and consistency of manufactured products. For example, it can shake loose any loose particles or contaminants on a surface and ensure that a surface or material is level or homogeneous.
4. **Energy Harvesting:** Vibration energy harvesting devices use forced vibration of various sources to generate electrical energy. Piezoelectric and electromagnetic materials are used to convert mechanical energy into electrical energy.
5. **Medical:** Vibration therapy is used to help treat conditions such as osteoarthritis and improve muscle strength and flexibility. This therapy is delivered through vibration plates or devices and applies forced vibrations to specific body areas.
6. **Seismology and Geophysics:** Forced vibration caused by earthquakes, volcanic activity or human activities such as construction and blasting can be used to study the subsurface geology of an area. The resulting vibrations are measured and analyzed to learn about the subsurface layers' composition, structure and properties.

Advantages of Forced Vibration

Forced vibration, also known as excitation vibrations, has several advantages in engineering and industrial applications. These include:

1. **Improved performance:** Forced vibrations can be used to improve the performance of certain mechanical systems, such as increasing the amplitude of vibration in a machine to increase its efficiency.

2. **Testing and analysis:** Forced vibrations can be used to test and analyze the behaviour of mechanical systems and structures under different loading conditions.
3. **Detection of defects:** Forced vibrations can detect defects in mechanical systems and structures, such as cracks or looseness in bolts, by analyzing the system's response to the excitation.
4. **Control of vibrations:** Forced vibrations can control unwanted vibrations in a system. By introducing a controlled amount of excitation, it can be possible to cancel out or reduce the amplitude of unwanted vibrations.
5. **Dynamic balancing:** Forced vibrations can be used to balance the rotor of the machine dynamically by adjusting the position of the mass of the rotor
6. **Maintenance schedule:** Forced vibrations can be used for scheduling maintenance on mechanical systems by monitoring changes in vibration levels over time, allowing for early detection of potential problems.

Overall, Forced vibrations can be a powerful tool for improving mechanical systems' performance, testing, and maintenance.

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Disadvantages of Forced Vibration

Forced vibrations, also known as excitation vibrations, have several disadvantages in certain engineering and industrial applications. These include:

1. **Damage to the system:** Forced vibrations can cause damage to the mechanical system or structure being excited. High amplitude vibrations can lead to fatigue failures and other types of damage.
2. **Noise and vibration:** Forced vibrations can generate noise and vibration that can disrupt people and equipment. This can be a problem in industrial and residential areas.
3. **Extra cost:** Introducing the excitation source and measuring the response of a system to the excitation may incur an extra cost, both in terms of equipment and labour.
4. **The complexity of the system:** The complexity of the systems and their behaviour under different loading conditions can make the testing and analysis of the system more complex and difficult to interpret.
5. **Inaccurate results:** Forced vibrations may give inaccurate results for the system's behaviour, as it may not simulate the real loading conditions of the system.
6. **Safety hazards:** Forced vibrations can create safety hazards, particularly if the system is not properly secured or the excitation source is not controlled properly.

Overall, Forced vibrations can be a powerful tool for testing and analyzing the behaviour of mechanical systems but can also damage the system, be disruptive and costly, and create hazards. Careful planning and execution are necessary to minimize these disadvantages and use forced vibrations effectively.

Torsional Vibrations

Torsional vibrations, also known as torsional oscillations, refer to rotational motion about an axis perpendicular to the shaft's axis. These vibrations occur in systems with a shaft or rod subjected to torque, such as a crankshaft in an engine or a propeller shaft in a vehicle. Torsional vibrations can be caused by system imbalances or external forces such as torque or bending loads.

Torsional vibrations can significantly affect a system's performance and durability. If the force vibrations are not properly damped, they can lead to fatigue and failure of the shaft or other components in the system. Torsional vibrations can also cause unwanted noise and vibration in the system, which can be a problem in some applications. To mitigate torsional vibrations, it is important to design the system with the appropriate stiffness and damping properties and to carefully balance the system to minimize imbalances. In some cases, torsional vibration dampers or other devices may be used to reduce the amplitude of the vibrations.

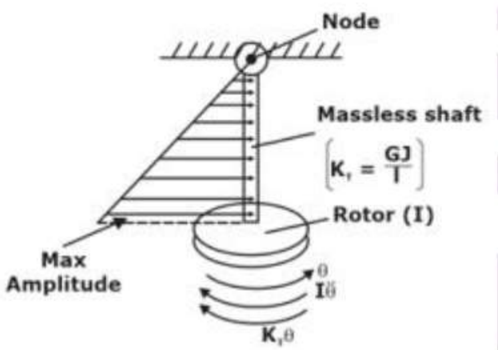

Fig.3: shaft-rotor system

$$\omega_n = \sqrt{\frac{k_T}{I_{Rotor}}}$$

(k_T=Torsional stiffness of shaft)

Note: If shaft mass MOI is also considered ($=I_s$)

$$\therefore \omega_n = \sqrt{\frac{k_t}{I + \left(\frac{I_{shaft}}{3}\right)}}$$

1.3.1. Two-Rotor System

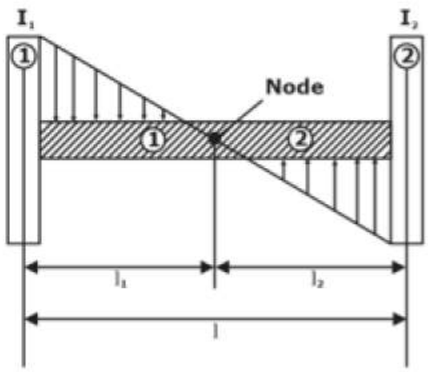

Fig.4: 2-rotor system

At Node Point:

$\omega_{n1}=\omega_{n2}$ (i.e. net vibrations of Node Point=0)

$$\Rightarrow \quad \sqrt{\frac{k_{t_1}}{I_1}} = \sqrt{\frac{k_{t_2}}{I_1}}$$

$$\Rightarrow \quad \sqrt{\frac{G_1 J_1}{\ell_1 I_1}} = \sqrt{\frac{G_2 J_2}{\ell_2 I_2}}$$

$$\Rightarrow \quad \frac{G_1 J_1}{\ell_1 I_1} = \frac{G_2 J_2}{\ell_2 I_2} \qquad (1)$$

for this same shaft, $G_1J_1=G_2J_2$ $\quad [\because G_1 = G_2, J_1 = J_2]$

$$(1) \Rightarrow \quad I_1 \ell_1 = I_2 \ell_2 \qquad (2)$$

Also, $\quad \ell_1 + \ell_2 = \ell \qquad (3)$

1.4. Rayleigh's Method to calculate natural frequency (Method of Static Deflection of Mass)

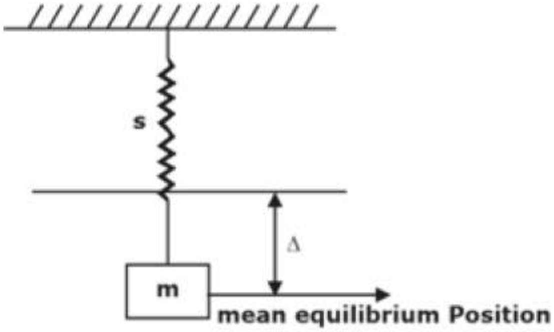

Fig.5: Basic spring Mass System

∇ = static deflection of mass 'm'

$$= \left(\frac{mg}{s}\right) \qquad [\because mg = s \cdot \nabla]$$

$$\therefore \omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{s}{m}}$$

Longitudinal Vibrations of Beams

Longitudinal vibrations of beams refer to vibrational motion along the length of a beam. These forced vibrations occur when a beam is subjected to external loads or forces that cause it to deflect from its equilibrium position. A beam's natural frequencies and vibrational modes depend on its length, cross-sectional shape, material properties, and the distribution of mass and stiffness along its length.

Longitudinal vibrations of beams can be analyzed using various mathematical models, such as the Euler-Bernoulli beam theory or the Timoshenko beam theory. These models allow engineers to calculate the natural frequencies and mode shapes of a beam, as well as the response of the beam to external loads. It is important to understand a beam's longitudinal vibrations to design stable structures capable of withstanding the loads they will be subjected to. In some cases, dampers or other devices may be used to reduce the amplitude of the vibrations and improve the stability of the beam.

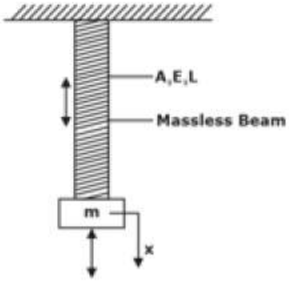

Fig.6: longitudinal vibrations of a beam

Axial or longitudinal stiffness of beam,

$$s = \left(\frac{AE}{L}\right)$$

$$\omega_n = \sqrt{\frac{s}{m}}$$

Transverse Vibrations of Beams

Transverse vibrations of beams refer to vibrational motion perpendicular to the length of a beam. This type of vibration occurs when a beam is subjected to external loads or forces that cause it to deflect from its equilibrium position in a direction perpendicular to its length. A beam's natural frequencies and vibrational modes depend on its length, cross-sectional shape, material properties, and the distribution of mass and stiffness along its length.

Transverse vibrations of beams can be analyzed using various mathematical models, such as the Euler-Bernoulli beam theory or the Timoshenko beam theory. These models allow engineers to calculate the natural frequencies and mode shapes of a beam, as well as the response of the beam to external loads. It is important to understand the transverse vibrations of a beam to design stable structures capable of withstanding the loads they will be subjected to. In some cases, dampers or other devices may be used to reduce the amplitude of the vibrations and improve the stability of the beam.

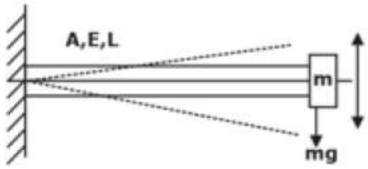

Fig.7: Transverse vibrations of a beam