BYJU'S EXAM PREP

GATE/ESE

Civil Engineering

Engineering Hydrology

Important Formula Notes

IMPORTANT FORMULAS ON ENGINEERING HYDROLOGY

CHAPTER-1- PRECIPITATION

i. INDEX OF WETNESS

$$Index\ of\ wetness = \frac{Rainfall\ in\ a\ year}{Average\ annual\ rainfall} \times 100$$

If the index of wetness is 100%, it indicates a normal year. If it is greater than 100%, it is called a good year, and if it is less than 100%, it is called a bad year.

ii. Aridity index

Aridity index $= \frac{PET - AET}{PET} \times 100$

Here, PET = potential evapotranspiration.

AET = actual evapotranspiration.

Aridity index %)	Condition
0 – 25	Mild
25 – 50	Moderate
>50	Severe

iii. Optimum Number of rain gauges

$$\boxed{N = \left(\frac{C_v}{\epsilon}\right)^2}$$

C_v = coefficient of variation.

ϵ = allowable percentage error.

For a given number of rain gauge standard error ϵ_s

$$\boxed{\epsilon_s = \frac{C_v}{\sqrt{n}}}$$

iv. ESTIMATION OF MISSING RAINFALL DATA

Let N_1, N_2, N_3, … and N_x be the normal precipitation values for station' 1 to m', and 'x' Normal precipitation is an average rainfall value for a day.

Let P_1, P_2, P_3, … and P_x be the rainfall for station '1 to m', and 'P_X' is the rainfall of station x.

Case 1: when N_1, N_2, … N_m differs from N_x by less than 10%, the Value of P_x is given as follows

$$\boxed{P_x = \frac{P_1 + P_2 + P_3 + P_3 + \ldots + P_m}{m}}$$

Case 2: when one or more of N_1, N_2,…N_m differs from N_x by 10% or more, the Value of P_x is calculated by:

$$\boxed{P_x = \frac{N_x}{m}\left[\frac{P_1}{N_1} + \frac{P_2}{N_2} + \ldots\ldots + \frac{P_m}{N_m}\right]}$$

v. AVERAGE PRECIPITATION/RAINFALL

a. ArithmeticMean/Average Method

This method is suitable if rainfall is uniformly distributed and the area is not very large.

$$\boxed{P_{avg} = \frac{P_1 + P_2 + P_3 + \ldots\ldots + P_n}{n}}$$

This method does not give very good results and hence is not used very frequently. Any station outside the area of consideration is not taken into account in this method.

b. Thiessen polygon/Weighted Area Method

In this method, the rainfall recorded at each station is given a weightage based on an area closest to the station. That is why this method is also known as the **weightage area method**.

$$P_{avg} = \frac{P_1A_1 + P_2A_2 + \ldots + P_nA_n}{A_1 + A_3 + \ldots + A_n}$$

$$\Rightarrow P_{avg} = \frac{\sum_{i=1}^{n} P_iA_i}{A}$$

The ratio A_i/A is called the weightage factor for each station.

This method of finding average rainfall is suitable when the area is large, and rainfall is non-uniformly distributed. This method is superior to the arithmetical mean method.

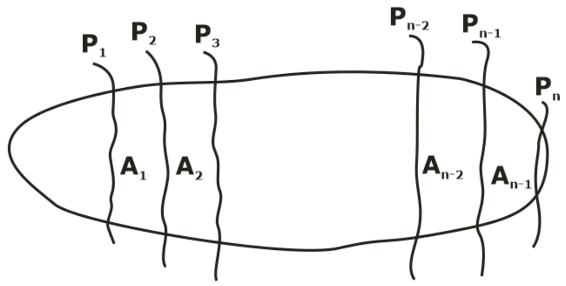
c. Isohyetal Method

An isohyet is a line joining all the points having the same value of rainfall, and isohyetal maps are the one which shows contours of equal rainfall magnitude.

In an isohyetal method, it is assumed that the precipitation in areas between isohyetal lines is equal to the mean of the precipitation of at isohyetal lines.

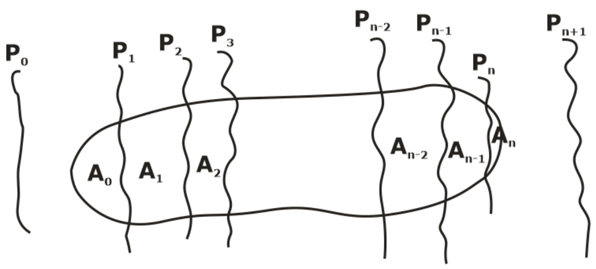
Mathematically, various following cases are possible.

Case 1:



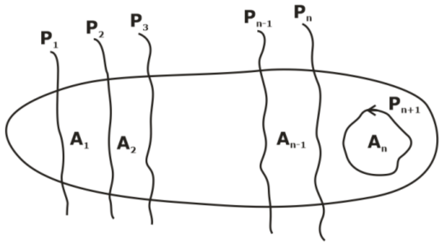
$$P_{avg} = \frac{\left(\frac{P_1 + P_2}{2}\right)A_1 + \left(\frac{P_2 + P_2}{2}\right)A_2 + \ldots + \left(\frac{P_{n-1} + P_n}{2}\right)A_{n-1}}{A_1 + A_2 + A_3 + \ldots + A_{n-1}}$$

Case 2: If isohyets outside the considered area are given.



$$P_{avg} = \frac{\left(\frac{P_0 + P_1}{2}\right) A_0 + \left(\frac{P_1 + P_2}{2}\right) A_1 + \left(\frac{P_2 + P_3}{2}\right) A_3 + \left(\frac{P_{n-1} + P_n}{2}\right) A_{n-1} + \left(\frac{P_n + P_{n+1}}{2}\right) A}{A_0 + A_1 + A_2 \ldots\ldots + A_{n-1} + A_n}$$

Case 3:



$$P_{avg} = \frac{\left(\frac{P_1 + P_2}{2}\right) A_1 + \left(\frac{P_2 + P_3}{2}\right) A_2 + \ldots + \left(\frac{P_{n-1} + P_n}{2}\right) A_{n-1} + P_{n+1} A_n}{A_1 + A_2 + A_3 + \ldots + A_{n-1} + A_n}$$

CHAPTER-2- EVAPORATION

i. Measurement of evaporation

a. Experimental method

$$\boxed{Lake\ Evaporation\ = C_P \times Pan\ evaporation}$$

S.No.	Type of Pan	C_P
1.	Class A	0.7
2.	Indian standard	0.8
3.	Colorado	0.78

b. Empirical Method

$$E = k \times m \times (e_w - e_a) \times \left(1 + \frac{V_9}{16}\right)$$

E = Rate of evaporation per day.

K_m = constant, which depends on the size of the water body.

e_w = Saturated vapour pressure in mm of mercury.

e_a = vapour pressure of air (e_a) in mm of mercury.

V_9 = Mean monthly wind velocity in km/hr at the height of about 9m from the ground surface.

ii. Transpiration

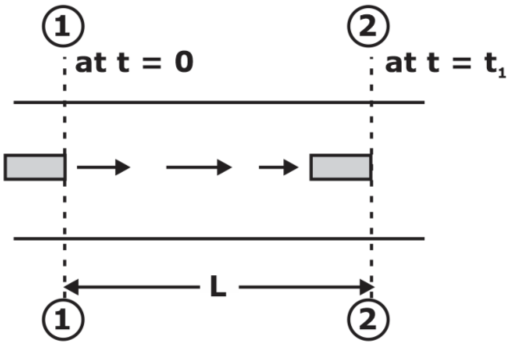
- It is the process by which water leaves the body of a living plant and reaches the atmosphere as water vapour.
- It can be measured by an instrument called as phytometer
- Following are the methods to find out the evapotranspiration.
  - Lysimeter
  - Field plot
  - Penman's equation
  - Blaney Criddle equation.

CHAPTER-3- STREAMFLOW MEASUREMENT

i. MEASUREMENT OF VELOCITY

a. Float

In this method, a very simple float device is used, which flows along the river surface.



Mathematically, $V = \frac{L}{t_1}$

b. Current meter

$$V = aN_s + b$$

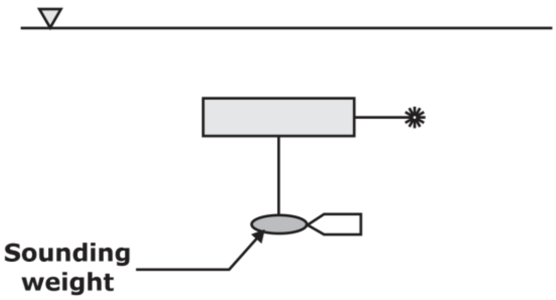
N_s = Number of revolutions per sec.

Here, a and b are called characteristic constants.

Note:

Calibration of the current meter is done by using a 'Towing tank'.

Sounding weight: It is a standard weight attached to a current meter to keep it at a fixed location.



To reduce the drag force, these are streamlined in shape. The value of this sounding weight is given as

$$W = 50\,\bar{v}\,y$$

Here, w = weight in 'N'

$\bar{v}$ = Average velocity in 'm/s'

Y = depth in 'cm'.

Average velocity can be obtained as follows

Case 1: For deep water bodies, k/a two-point formula.

$$\boxed{V_{avg} = \frac{V_{0.2y} + V_{0.8y}}{2}}$$

Case 2: For shallow k/a one point formula

$$\boxed{V_{avg} = V_{0.6y}}$$

Case 3: For flashy river and flood-like situation.

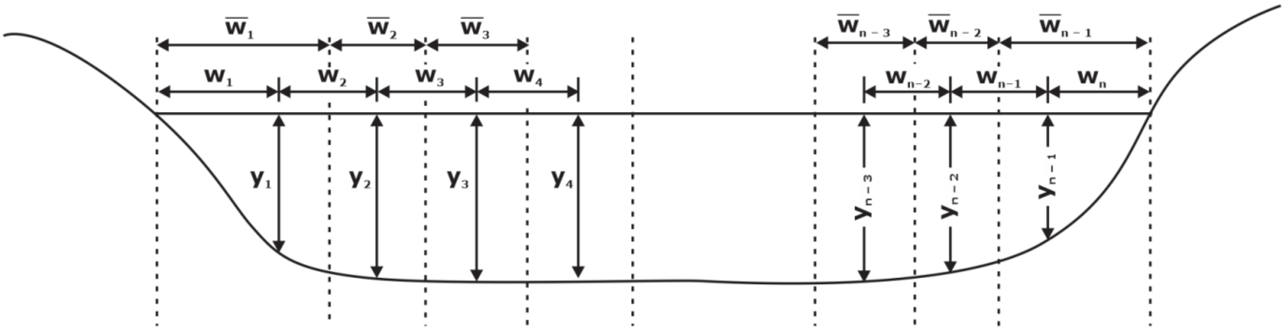
$$\boxed{V_{avg} = kv_s}$$

Here, k = 0.85 – 0.95

V_s = Surface velocity

ii. MEASUREMENT OF DISCHARGE

a. Area velocity method



$$\boxed{Q = \sum_{i=1}^{n} \Delta Qi}$$

$$\Delta Q_1 = A_1V_1$$

$$\Delta Q_2 = A_2V_2$$

$$\Delta Q_n = A_nV_n$$

$$\text{Here, } A_1 = \bar{w}_1 y_1$$

$$A_2 = \bar{w}_2 y_2$$

$$A_n = \bar{w}_n y_2$$

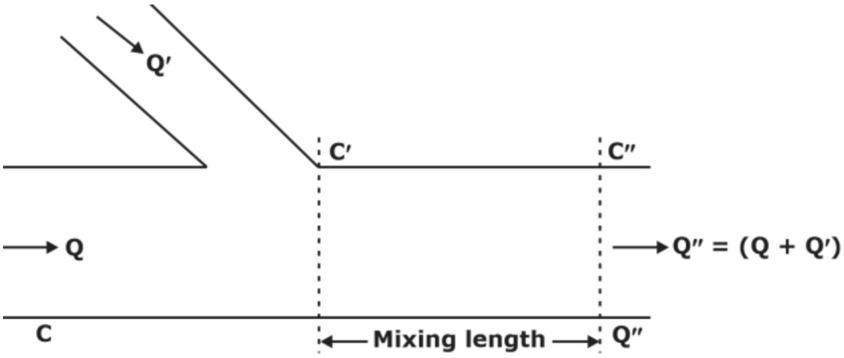
Here,

$$\bar{w}_1 = \frac{\left(w_1 + \frac{w_2}{2}\right)^2}{2w_1}$$

$$\bar{w}_3 = \frac{w_3 + w_4}{2}$$

$$\text{Similarly, } \bar{w}_{n-1} = \frac{\left(w_n + \frac{w_{n-1}}{2}\right)^2}{2w_n}$$

b. Dilution method:

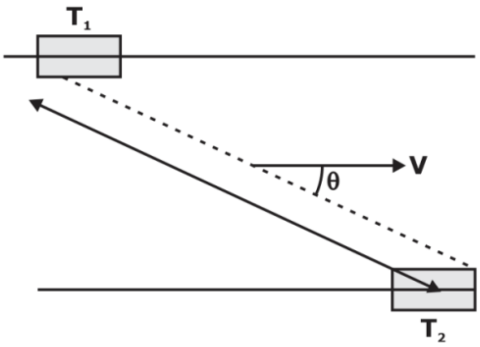


Here C = background concentration.

Mathematically,

$$\boxed{CQ + c'Q' = C''(Q + Q')}$$

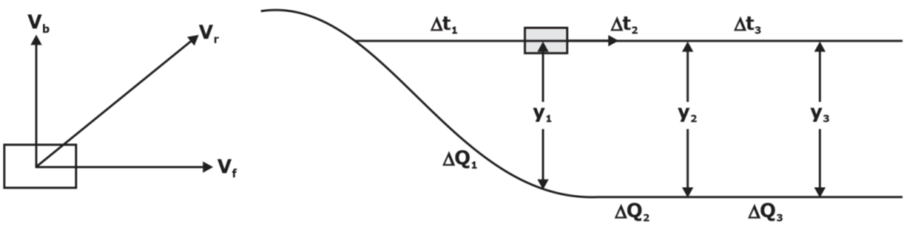
c. Ultrasonic Method:



$$v = \frac{l}{2\cos\theta}\left(\frac{1}{t_1} - \frac{1}{t}\right)$$

$$Q = A \times V$$

d. Moving Boat Method:



$$\Delta Q_1 = A_1 V_f = \left[V_b \Delta t_1 \left(\frac{0 + y_1}{2}\right)\right] v_f = v_{b0} v_f \Delta t_1 \frac{y_1}{2}$$

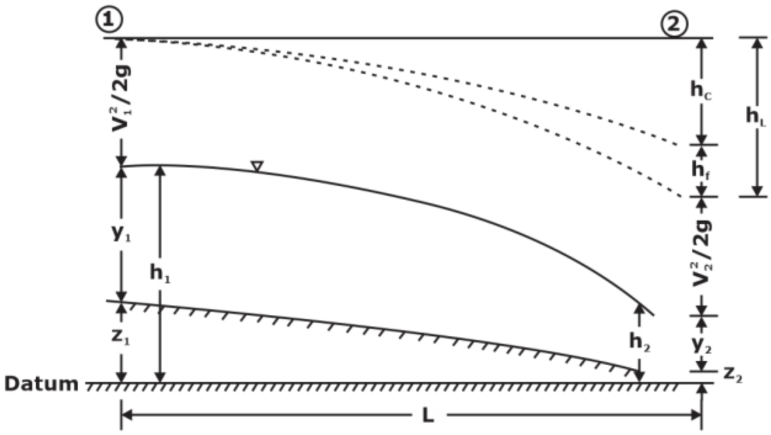
$$= V_r^2 \sin\theta \cos\theta \Delta t_1 \frac{y_1}{2}$$

Similarly,

$$\Delta Q_2 = V_r^2 \sin\theta \, \Delta t_2 \left(\frac{y_1 + y_2}{2}\right)$$

$$\Delta Q_3 = V_r^2 \sin\theta \, \Delta t_3 \left(\frac{y_2 + y_3}{2}\right)$$

e. **Slope Area Method :**



$$h_f = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2g}\right) - h_e$$

$$\begin{bmatrix} h_f = \text{frictional} \\ h_c = \text{eddies} \end{bmatrix}$$

Through experiments, it has been found that eddy head loss

$$h_e = K_e \left|\frac{V_1^2 - V_2^2}{2g}\right|$$

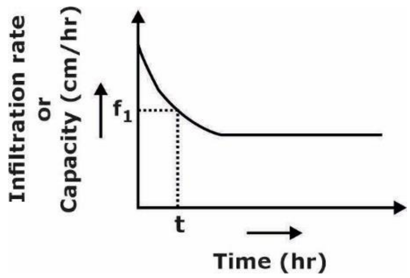
Where K_e = eddy head loss coefficient.

- $Q = k\sqrt{\frac{h_f}{L}}$

$$K = (K_1 \times K_2 \times K_3 \times K_4 \times K_5 ...... \times K_n)^{1/n}$$

CHAPTER-4- INFILTRATION

i. INFILTRATION CAPACITY



$$F_t = F_f + (F_i - F_f)e^{(-kt)}$$

F_t = Infiltration rate or capacity at a time 't'.

F_f = Final infiltration rate or capacity.

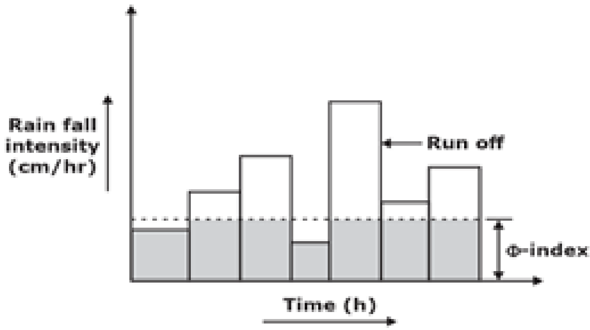
F_i = Initial infiltration rate or capacity.

K = Decay constant (T^{-1} or /s or / hr)

ii. INFILTRATION INDICES

a. ϕ - index

The ϕ - index is the average rainfall above which the rainfall volume is equal to runoff volume. The ϕ *Index* is derived from the rainfall hyetograph with the knowledge of the resulting runoff volume. The initial losses are also considered infiltration



b. w-index

This is the average infiltration rate during the entire duration of rainfall. In calculatingthe w-index, the initial losses are separated from total abstractions to refine the ϕ *Index*.

Mathematically it is defined as

$$w - \text{index} = \frac{P - R - t_a}{T_c}$$

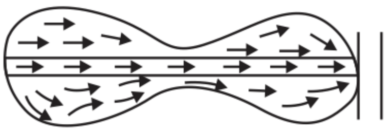
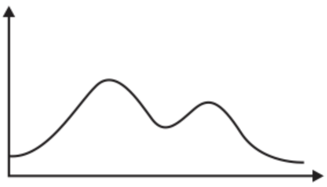
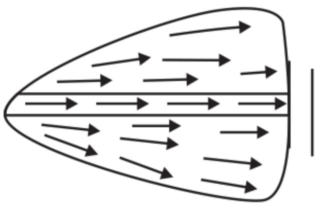
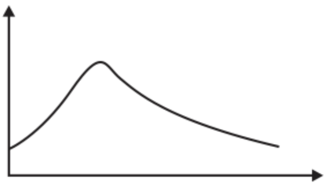
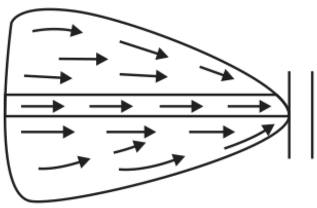
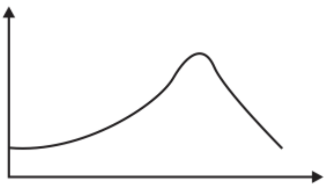
Here, P = total storm precipitation (cm)

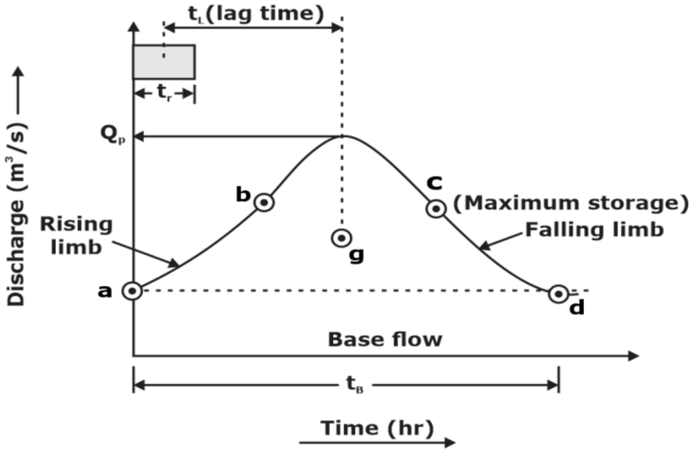
R = total run off (cm)

t_a = minor loss in total duration

T_c = duration of the rainfall excess i.e. total.

CHAPTER-5- RUN-OFF & HYDROGRAPH

1. Shape of Catchment	Hydrograph
	
	
	

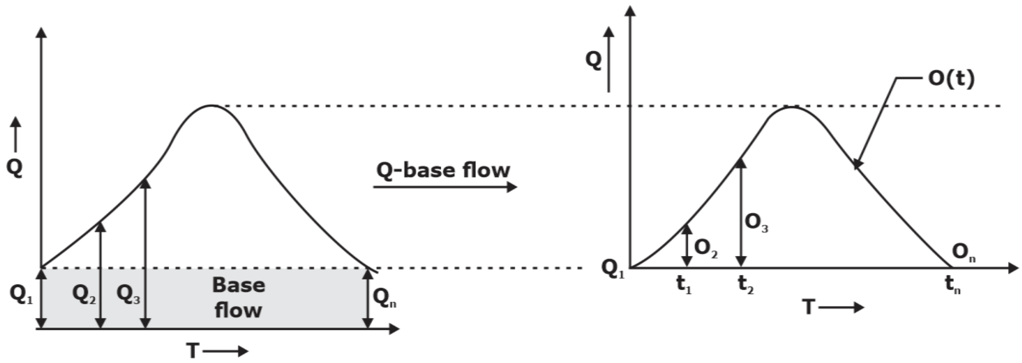


i. DETERMINATION OF DIRECT RUNOFF HYDROGRAPH (DRH)

Hydrograph having base flow is known as flood hydrograph or storm hydrograph.

Flood

hydrograph $\xrightarrow{-\text{Base flow}}$ Direct runoff hydrograph



Direct runoff = depth

n =(Direct runoff volume)/(catchment area)

So, Mathematically it can express as

$$n = \frac{0.36\sum o\ t}{A}$$

Here, n is in cm

A is in km^2

t is in hr

O is in m^3/sec

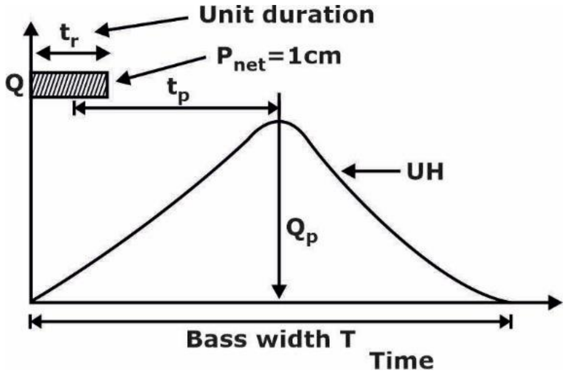
ii. SYNTHETIC UNIT HYDROGRAPH

Snyder selected three parameters for the development of SUH. These parameters relate to the catchment characteristics

(i) Basin time width T

(ii) Peak discharge QP

(iii) Lag time,i.e. basin lag time tp. (Snyder defined lag time as the time interval from mid pt. of rainfall to the peak of UH (instead of centroid).



He proposed the following three equations for these three parameters

Lag time, $t_p = C_t(LL_{ca})0.3$

Basin time width $T = (72 + 3t_p)$

Peak discharge $Q_P = \frac{2.78C_pA}{t_p}$

Where tp is in hr

Ct is a coefficient reflecting slope, land use, and associated storage characteristics of the basin. Its value varies between 1.35 to 1.65, the average being 1.5

L = basin length measured along the watercourse from the basin divide to the gauging station in km.

L_{ca} = Distance of centroid of the catchment from the gauging point (in km)

T is in hr

Q_p is in m^3/s

A = Catchment area in km^2

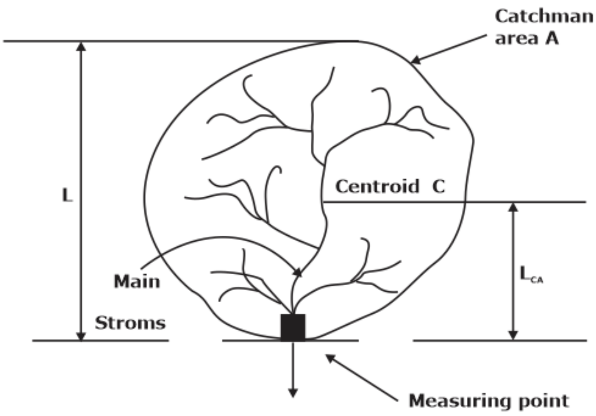


Fig. 6.15

CP = a regional constant having value between 0.56 to 0.69

Synder used the standard duration tr (or D-hr) in hr for unit hydrograph

$$t_r = D_{hr} = \frac{t_p}{5.5}$$

If a synthetic unit hydrograph of other duration then D'_{hr} is required, then lag time, t_{pr}' is given by

$$t_{pr} = t_p = \frac{D' - t_r}{4}$$

To plot the smooth synthetic unit hydrograph, US army crops of engineering gave the width of SUH as

$$W_{50} = \frac{5.87}{\left(\frac{Q_p}{A}\right)^{1.08}}$$

$$W_{75} = \frac{3.35}{\left(\frac{Q_p}{A}\right)^{1.08}} = \frac{W_{50}}{1.75}$$

W_{50} and W_{75} are the widths of synthetic unit hydrograph in hr at 50% and 75% of Q_p, respectively. Where Q_p is in m^3/s and A is an area of the catchment in km^2.

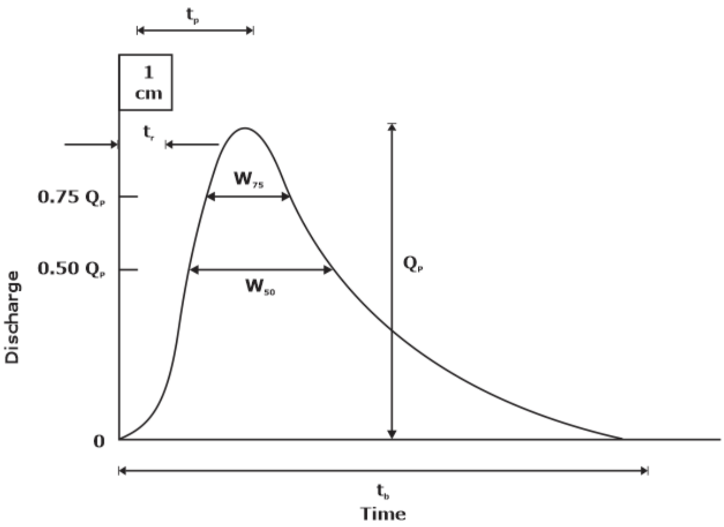
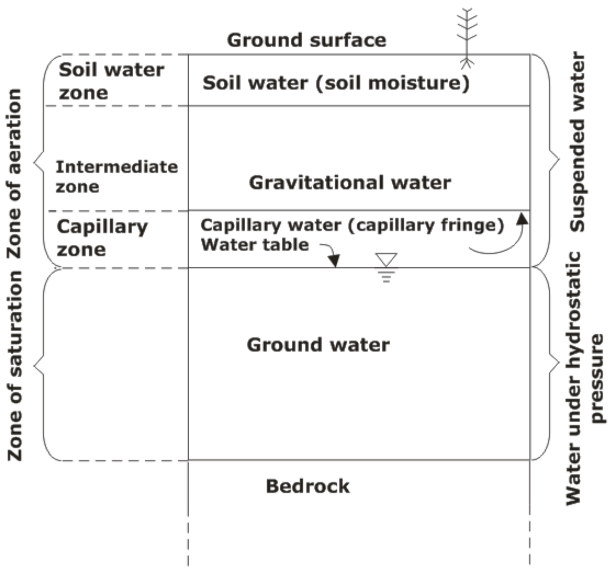
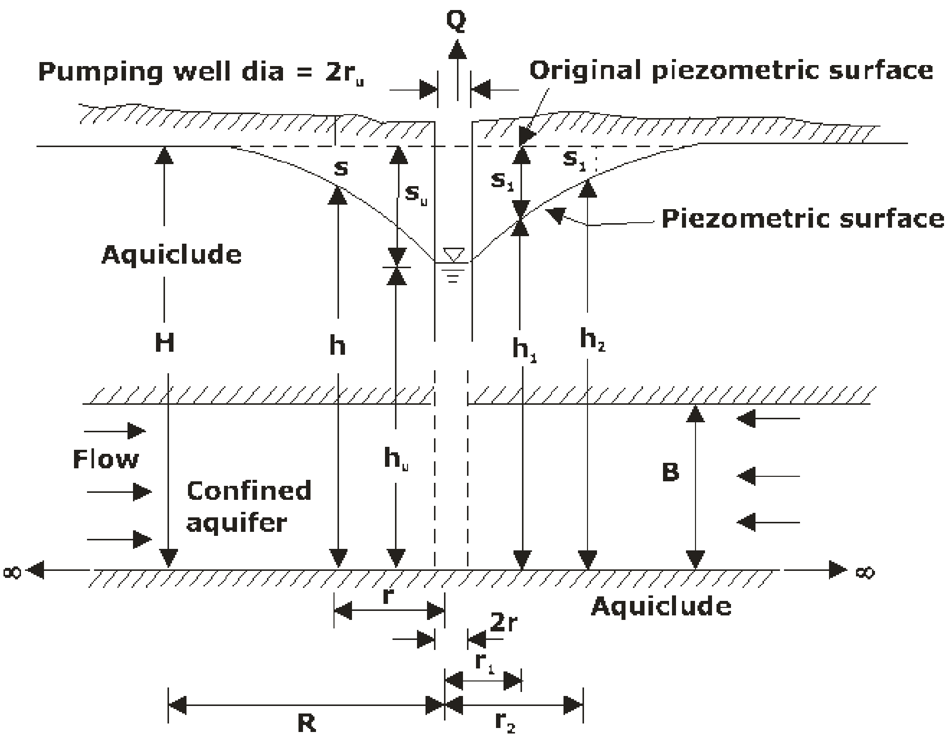


Fig. 6.17: Elements of a synthetic unit hydrograph

CHAPTER-6- GROUNDWATER HYDROLOGY



i. STEADY CONFINED FLOW (FULLY PENETRATING WELL)



A. Theims Theory

$$Q = \frac{2\pi KB\left(h_2 - h_1\right)}{\ln\frac{r_2}{r_1}}$$

$$h_1 + s_1 = h_2 + s_2$$

Q= Rate of flow

h_1 =height of water table in 1st well

h_2 =height of water table in 2nd well

S_1 = drawdown in 1st well

S_2 = drawdown in 2nd well

r_2, r_1 = radius

B. Duipit's theory

Further, at the edge of the zone of influence, s =0, r_2 = R and h_2= H;at the well wall r_1 = r_w, h_1 = h_w and s_1 = s_w. Hence

$$Q = \frac{2\pi KBS_w}{\ln\frac{R}{r_w}}$$

This is called the Dupit's formula.

ii. STEADY UNCONFINED FLOW

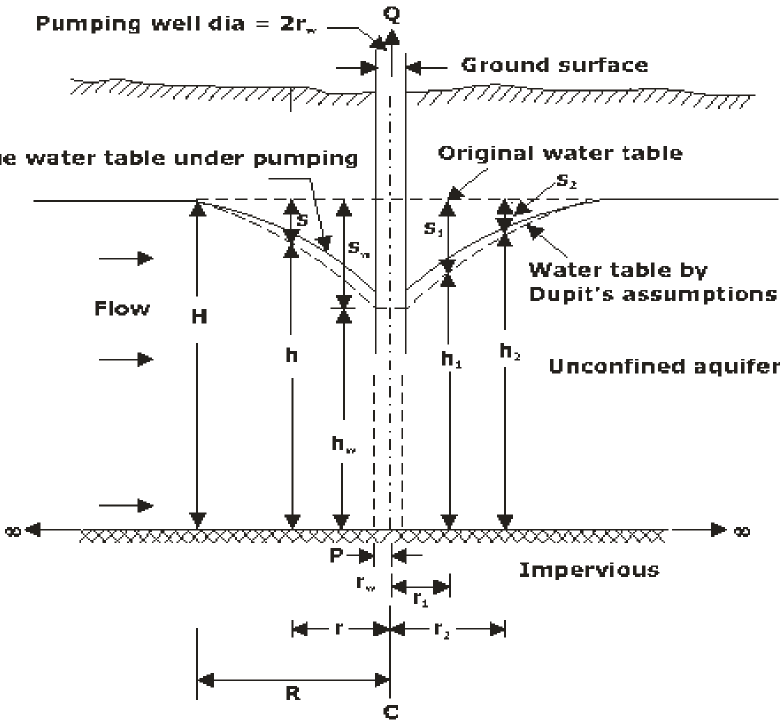


Fig. 10.15. Radial flow to well in an unconfined aquifer

A. Theim's Theory

$$Q = \frac{\pi k\left(h_2^2 - h_1^2\right)}{\ln\left(\frac{r_2}{r_1}\right)}$$

B. Duipit's theory

$$Q = \frac{\pi k\left(H^2 - h_w^2\right)}{\ln\frac{R}{r_w}}$$

$$R = 3000 \times S \times \sqrt{K}$$

h_w = depth of water in the pumping well of radius r_w.

CHAPTER-7- RESERVOIR CAPACITY & FLOOD ROUTING

i. RATIONAL METHOD

This method is suitable for small size catchments whose area is less than 50 km^2. Mathematically it is expressed as

$$Q_P = \frac{1}{36} k p_c A$$

Here, p_c = critical rainfall intensity in 'cm/hr'

k = runoff coefficient

A = Area in hectares

It is applicable only if rainfall duration is greater than or equal to the time of concentration.

ii. EMPIRICAL FORMULAE

The empirical formulae used for estimating the flood peak are essentially regional formulae based on statistical correlation of the observed peak and important catchment properties. These equations are given as follows.

(a) Dicken's Equation

This equation is applicable for North and central India. Mathematically, it is given as

$$\boxed{Q_P = C_D A^{3/4}}$$

Here,

Q_P = m^3/s

A = Area in km^2

C_D =dicken's constant (6 to 30)

(b) Ryve's Equation

This equation is applicable for southern parts of India. Mathematically it is given as

$$\boxed{Q_P = C_R A^{2/3}}$$

Here, C_R =ryve's constant.

(c) Inglis Equation

This equation is applicable for WesternGhats and Maharashtra region. Mathematically it is given as,

$$\boxed{Q_P = \frac{124A}{\sqrt{A + 10.4}}} \approx 123\sqrt{A}$$

iii. STATICAL PROBABILITY METHOD

This method is suitable if a large number of data is given, and it is required to find the peak value of discharge for any given period or return period.

Return Period

The return period is calculated for each event using Weibull's formula.

$$T_v = \frac{n+1}{m}$$

T_r = return period in yr.

m = order no.

n = no. for yr of records

return period represents the average no. of years within which a given event will be equalled or exceeded.

Probability of exceedance = $\frac{1}{T_r} = P$

RISK AND RELIABILITY

The probability of a particular event happen exactly 'r' times out of 'n' trials is given as

$${}^n c_r p^r (1-p)^{n-r}$$

p = probability of exceedance.

Reliability

This is the probability that a particular flood or rainfall is never equalled or exceeded (r = 0) in a period of 'n' years, Mathematically

$\Rightarrow$ Reliability $= {}^{n}c_0 p^0 (1-p)^{n-0}$

$\Rightarrow \boxed{\text{Reliability} = (1-p)^n}$

Risk

This is a probability that a particular flood or rainfall is equalled or exceeded at least once in a period of 'n' years.

P (at least once) = 1 – p(never happen)= 1- Reliability

The most commonly used distributions are :

(a) Gumbel's distribution

(b) Log Pearson Type III distribution.

Gumbel's Method:

As per Gumbel's method

$X_T = \bar{X} + K.\sigma_{n-1}$

where X_T = value of variate (i.e. flood) with a return period of T

$\bar{X}$ = Mean Value for variate $\frac{\Sigma X}{n} = \bar{X}$ (from annual series)

n = no. of yrs of record

σ_{n-1} = Standard deviation of the sample of size n

$$\sigma_{n-1} = \sqrt{\frac{\sum\left(X-\bar{X}\right)^2}{n-1}}$$

K = Frequency factor

$$K = \frac{y_T - \bar{y}_n}{S_n}$$

y_T = reduced variate

$$\frac{1}{t} = (-)\left[\ln \ln \frac{T}{T-1}\right]$$

$\bar{y}_n$ = mean of reduced variate

S_n = Standard deviation of reduced variate.

$\bar{y}_n$ And S_n function of n (no. of yr. of record).

However if, n is large (generally >200)

$y_n \rightarrow 0.577$

$S_n \rightarrow 1.2825$

[Normal for n> 50 also some time we use y_n = 0.577. S_n= 1.2825 without much error]

iv. CONFIDENCE LIMIT

Confidence interval indicates the limits about the calculated value between which the true value can be said to lie with a specific probability based on sampling errors only.

For confidence probability 'α', the confidence interval of variate X_T is bounded by values X_1 and X_2 given by

$$\boxed{X_{1/2} = X_T \pm f(\alpha).S_c}$$

where f(α) = function of confidence probability 'α'. It can be found using the following table

A is percent	50	68	80	90	95	99
f(α)	0.674	1.0	1.282	1.645	1.96	2.58

S_c = Probable error = $b\,\frac{\sigma_{N-1}}{\sqrt{n}}$

$$b = \sqrt{1 + 1.3K + 1.1\,K^2}$$

$$K = \frac{y_T - y_n}{S_n}$$

$$y_T = -\ln\ln\frac{T}{T-1}$$

n = Sample size

T = Return period

σ_{n-1} = Standard deviation of sample

v. Rainfall run off correlation

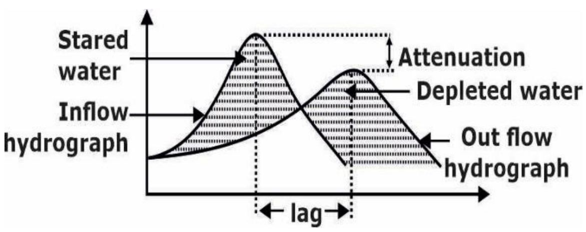
$$\boxed{R = ap + b}$$

Here,

$$a = \frac{n\sum PR - \sum P \sum R}{n\sum P^2 - (\sum P)^2}$$

$$b = \frac{\sum R - a\sum P}{n}$$

vi. Flood Routing



This reduction in peak is called attenuation.

The time difference between the two peaks is called lag.

a. Prism storage

It is the volume that would exist if the uniform flow occurred at the downstream flow

b. Wedge storage

It is the wedge-like volume formed between the actual surface profile and the top surface of prism storage.

c. Muskingum method

$\Delta S = (I - Q)\Delta t$

ΔS = Change in storage in time Δt

I= Avg. inflow rate over the time Δt

Q= Avg outflow rate over the time Δt

$$\Delta S = \left(\frac{I_2 + I_1}{2}\right)\Delta t - \left(\frac{Q_2 + Q_1}{2}\right)\Delta t$$

$$S_1 = K\left[XI_1 + (1 - x)Q_1\right]$$

$$S_2 = K\left[XI_2 + (1 - x)Q_2\right]$$

$S_2 - S_1 = K [x (I_2 - I_1) + (1 - x) (Q_2 - Q_1)]$

$Q_1 = I_1$

$Q_2 = C_0I_2 + C_1I_1 + C_2Q_1$

$Q_n = C_0I_n + C_1I_{n-1} + C_2Q_{n-1}$

$$C_0 = \frac{-Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_1 = \frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_2 = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$C_0 + C_1 + C_2 = 1.0$

For the results routing interval, Δt should be so chosen that $K > \Delta t > 2Kx$

The following steps are used for channel routing using the Muskingum method.

(i) Knowing K and x, select an appropriate value of Δt [$K > \Delta t > 2Kx$]

(ii) Calculate C_0, C_1 and C_2

(iii) Starting from the initial conditions I_1. Q_1 and known I_2 at the end of the first time step Δt, calculate Q_2 by eq. (C)

(iv) The outflow calculated in step (iii) becomes the known initial outflow for the next step. Repeat the calculations for the entire-inflow hydrograph.

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