Turbulent Flow

Turbulent flow is the type of flow that can occur in a closed pipe or in an open channel. Flow categorization depends on the Reynolds number of the flow. It can be laminar flow, turbulent flow or the transition between laminar and turbulent state. This type of flow is essential for the GATE CE exam. Turbulent flow is a flow regime characterized by the following points as given below.

Shear stress in the turbulent flow

$$T = \tau_v + \tau_t = \mu\ (du/dy) + \eta\ (du/dy)$$

where

- τ_v and τ_t = shear stress due to viscosity and turbulence.
- η = eddy viscosity coefficient.

Turbulent Shear Stress by Reynolds

$$\tau = \rho u'v'$$

u' and v' fluctuating components of velocity.

Shear Stress in Turbulent Flow

Shear stress is the value of the force applied on the pipe wall per unit cross-section area. In the turbulent flow, the fluid particles will move in a random direction that will cause a shearing force on the pipe wall. Hence shearing stress will be generated on the wall. Shear flow in turbulent flow can be explained below.

$$T = \rho l^2\ (du/dy)^2$$

where, l = Mixing length

The expression gives the velocity distribution in the turbulent flow for pipes.

$$u = u_{max} + 2.5\ u^*\ \log_e(y/R)$$

$$U_{max} = \text{center velocity}$$

where,

- y = Distance from the pipe wall,
- R = radius of the pipe

$$u^* = \text{Shear velocity} = (\tau_0/\rho)^{0.5}$$

Velocity defect is the difference between the maximum velocity (u_{max}) and local velocity (u) at any point given by

$$u_{max} - u = 5.75\, u^* \log_{10}(R/y)$$

Karman Prandtl Velocity Distribution Equation in Turbulent Flow

The velocity distribution equation in turbulent flow follows logarithmic distributions, while in laminar flow, velocity distribution follows the parabolic distribution. Karman Prandtl gives a velocity distribution equation for the turbulent flow through a pipe, the hydrodynamically smooth pipe and rough pipes. It will explain further:

Hydrodynamically pipe

$$\frac{u}{u^*} = 5.75 \log_{10}\left(\frac{u^* y}{v}\right) + 5.5 \ (for\ smooth\ pipe)$$

$$= 5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5 \ (for\ rough\ pipe)$$

where

- u = velocity at any point in the turbulent flow
- u* = shear velocity = $(\tau_0/\rho)^{0.5}$
- v = Kinematic viscosity of the fluid
- y = Distance from the wall of the pipe
- k = Roughness factor

Velocity distribution in terms of average velocity

$$\frac{\bar{u}}{u^*} = 5.75 \log_{10} \frac{u^* R}{v} + 5.5 \ (for\ smooth\ pipe)$$

$$= 5.75 \log_{10} \frac{R}{k} + 4.75 (for\ rough\ pipe)$$

Common Mean Velocity Distribution Equation

Common mean velocity is the velocity of the fluid for which the same discharge and energy will be found as that of the actual flow. Actual velocity distribution for the turbulent flow will follow logerthic variation explained as below.

$$\mathbf{[(u-\bar{u})/ u^*]= 5.75 \log_{10} (y/R) + 3.75}$$

(This is valid for both rough and smooth pipes, that's why it is referred to as Common Mean Velocity Distribution Equation)

Coefficient of friction

$$f = 16/R_e \text{ (for laminar flow)}$$

$$f = 0.0791/(Re)^{0.25} \text{ ; } 4000 \le R_e < 10^5$$

$$f = 0.0008 + 0.05525/(R_e)^{0.257} \text{ , } 4\times10^7 \le R_e \le 10^5$$

(for smooth pipe)

$$1/(4f)^{0.25} = 2\log_{10}(R/k) + 1.74 \text{ (for rough pipe)}$$

Difference Between Laminar Flow and Turbulent Flow

Laminar and turbulent flow are both conditions of a fluid flow. These conditions depend on the Reynolds number of the flow and are very important for the GATE CE question paper. Here a few points explain the difference between laminar and turbulent flow.

- In the laminar flow, fluid particles move in a layered form, while in the turbulent flow, the fluid particle moves haphazardly.
- Velocity distribution in laminar flow follows parabolic distribution, while in turbulent flow, it varies in a logarithmic way.
- The magnitude and direction both of velocities vary for the turbulent flow but for the laminar flow, only the magnitude of the velocity varies.
- Conditions for Reynolds number for both types of flow are different, and it also depends on the type of conduit, whether it is open or closed.