Mechanical Engineering

Complete Syllabus

FORMULA BOOK

Important Formula (Engineering Mechanics)

CHAPTER-1 FORCE ANALYSIS AND EQUILIBRIUM

INTRODUCTION

Engineering mechanics is the branch of science that considers the motion of bodies and the effects of forces on that motion. It may be divided into two parts

Statics:

Statics deals with the special case of a body at rest or a body that moves with a constant velocity.

Dynamics:

Dynamics is further divided into Kinematics and Kinetics. Kinematics is concerned with rates of change of geometrical quantities in a moving system; it does not involve the concept of force. Kinetics treats the causes and the nature of motion that results from specified forces.

Note:

- A particle has a mass, but a size that can be neglected. For example, the size of the earth is insignificant compared to the size of its orbit and thereafter the earth can be modeled as a particle when studying its orbital motion.
- A rigid body can be considered as a combination of a large number of particles in which particles remain at a fixed distance from one another both before and after applying a load.

FORCE

An agent which produces or tends to produce, destroy or tends to destroy motion.

SI Unit: Newton (N)

System of forces

When two or more than two forces of different magnitude and direction act upon a body, they constitute a system of forces.

Concurrent forces

Two or more forces that act at the same point are called concurrent forces. Concurrent forces need not have the same direction. They simply act at the same point.

Collinear forces

If concurrent forces do have the same direction, they are collinear forces.

Coplanar forces

Two or more forces whose directed arrows lie in the same plane are called coplanar forces.

- Since two concurrent forces always lie in a common plane, they are always coplanar.
- Three or more concurrent forces are not necessarily coplanar.

PRINCIPLE OF TRANSMISSIBILITY

The state of rest or of motion of a rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the applied forces.

RESULTANT OF FORCE

It is possible to find a single force which will have the same effect as that of a number of forces acting on a body. The single force is called resultant force and the process of finding out the resultant force is called composition of forces. The reverse of the composition of forces is called resolution of force.

PARALLELOGRAM LAW OF FORCES

If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram, which passes through their point of intersection, their resultant force is represented, both in magnitude and in direction, by the diagonal of the parallelogram drawn through their point of intersection. Let two forces P and Q are represented by OP and OQ respectively, two sides of the parallelogram. Now the parallelogram OPRQ is completed and the resultant is represented by diagonal OR.
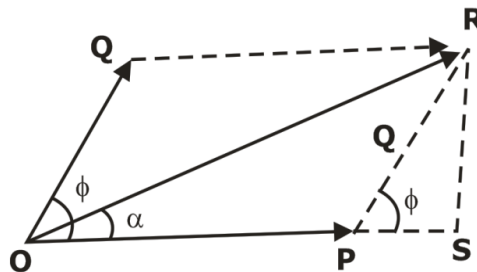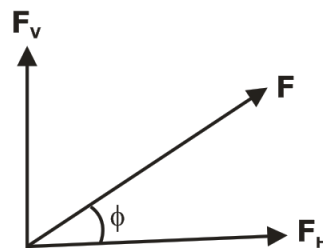

$$R = \sqrt{\left(P^2 + Q^2 + 2PQ\cos\phi\right)}$$

$$\tan\alpha = \frac{Q\sin\phi}{P + Q\cos\phi}$$

ANALYTICAL METHOD

Resolve all forces horizontally and vertically, then determine

ΣH = algebraic sum of all horizontal component.

ΣV = algebraic sum of all vertical component.

$$R^2 = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$\tan\phi = \frac{\Sigma V}{\Sigma H}$$

TRIANGLE LAW OF FORCES

If two forces acting simultaneously on a body are represented in magnitude and direction by two sides of a triangle in order, then the third side will represent the resultant of the two forces in the direction and magnitude taken in opposite order.

POLYGON LAW OF FORCES

When the forces acting on a body are more than two, the triangle law can be extended to polygon law. Polygon Law states that if a number of coplanar concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, then their resultant can be represented by closing side of the polygon in magnitude and direction in the opposite order.

Let forces F_1, F_2, F_3 and F_4 are acting at a point O as shown in figure and can be represent by sides of polygon OA, AB, BC, CD respectively. Vector OD represents the resultant force $\mathbf{R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} + \mathbf{F_4}$ in magnitude and direction.
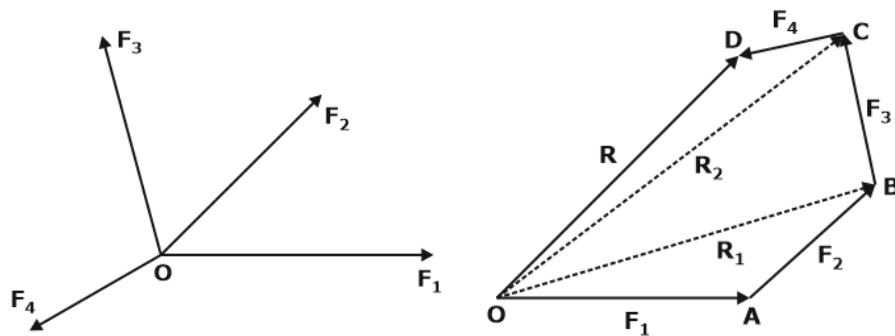
RESOLUTION AND RESULTANT OF FORCE IN SPACE

Consider a force $\mathbf{F}$ acting at the origin O of the system of rectangular coordinates x, y, z as shown in Fig. The direction of force $\mathbf{F}$ is defined by three angles θ_x, θ_y, and θ_2. Let F_x, F_y, and F_z be the components of force F in x, y, and z directions, respectively.
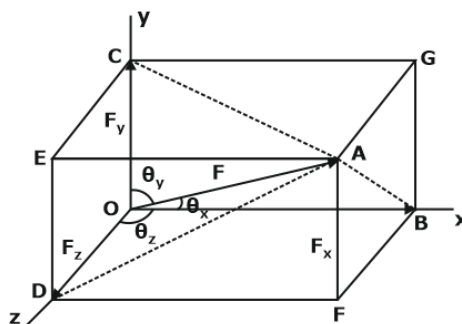
From the triangle OAB $\cos\theta_x = \dfrac{OB}{OA} = \dfrac{F_x}{F}$, component of $\mathbf{F}$ along x-direction $F_4 = F\cos\theta_x$

From the triangle OAC $\cos\theta_y = \dfrac{OC}{OA} = \dfrac{F_y}{F}$, component of $\mathbf{F}$ along y-direction $F_y = F\cos\theta_x$

From the triangle OAD $\cos\theta_x = \dfrac{OC}{OA} = \dfrac{F_z}{F}$, component of $\mathbf{F}$ along z-direction $F_z = F\cos\theta_z$

The cosines of θ_x, θ_y and θ_z are known as the direction cosine of the force **F**.

The angles θ_x, θ_y and θ_z are not independent, they are related as $\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$.

We may express **F** in the vector form $\mathbf{F} = F_x i + F_y j + F_x k$

When the components F_x, F_y and F_z given, the magnitude of force **F** is $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

and direction of force $\theta_x = \cos^{-1}\left(\dfrac{F_x}{F}\right), \theta_y = \cos^{-1}\left(\dfrac{F_y}{F}\right)$, and $\theta_z = \cos^{-1}\left(\dfrac{F_z}{F}\right)$

MOMENTS

The turning effect produced by a force, on the body, on which it acts.

SI Unit: Nm

Moment M = F × L, Where F = Force acting on the body, and L = Perpendicular distance of the point, about which the moment is determined and the line of action of the force.

Force F and position vector r in rectangular components may be written as

$$\mathbf{F} = F_x i + F_y j + F_z k$$

$$\mathbf{r} = xi + yj + zk$$

Thus,

$$\mathbf{M}_o = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$M_o = i\left(yF_z - zF_y\right) - \left(xF_z - zF_x\right)j + \left(xF_y - yF_x\right)k$$

In case of problems involving only two dimensions, the force F may be assumed to lie in the xy-plane. Carrying $z = 0$ and $F_z = 0$, we obtain

$$\mathbf{M}_o = \left(xF_y - yF_x\right)k$$

Note.

Just as force has a tendency to translate the body, moment has a tendency to rotate the body about the point.

DETERMINATION OF DIRECTIONS

Assume perpendicular distance as hand of clock. Keep the point (one end of perpendicular distance), about which the moment is to be determined, hinged like hinged end of hands of a clock. Move the other end of perpendicular distance in the direction of action of force.
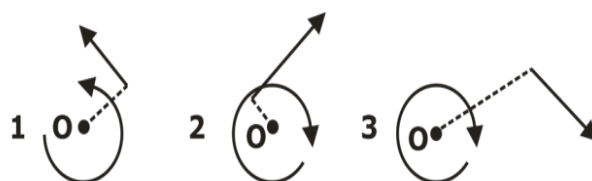

1. Anticlockwise
2. Clockwise

3. Clockwise

Note.

Clockwise whose effect is to turn, in the same direction in which the hands of clock move.

Anticlockwise whose effect is to turn, in the opposite direction in which the hands of clock move.

VARIGNON'S THEOREM

Moment of a force about any point is equal to the algebraic sum of the moments of the components of that force about the same point.
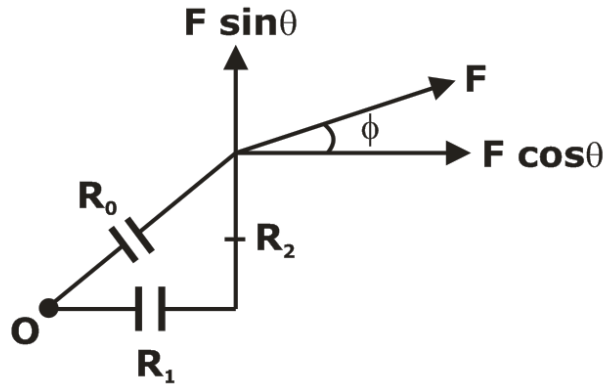
$$M_o = F\cos\theta \times R_2 - F\sin\theta \times R_1$$

Couples

* The moment produced by two equal, opposite and non-collinear forces is called couple.
* It does not produce any translation but produces only rotation.
* The resultant force of a couple is zero.
* The moment of a couple is the product of the magnitude of one of the forces and the perpendicular distance between their lines of action.

CONDITIONS OF EQUILIBRIUM

* The term equilibrium implies that either the body is at rest or it moves with a constant velocity.
* A body is said to be in static equilibrium when the resultant force in it must be zero and the body must have no tendency to rotate.

i.e. $\Sigma F = 0$ means that $\Sigma H = 0$ And $\Sigma V = 0$

and $\Sigma M = 0$ about any point.

FREE BODY DIAGRAM

A free body diagram is a sketch of the body that shows the body (by itself, free of the other part of the system) and all the forces applied to it, that is, all forces acting on the body. Fig shows free body diagrams.

FBD of A   FBD of B   FBD of sphere
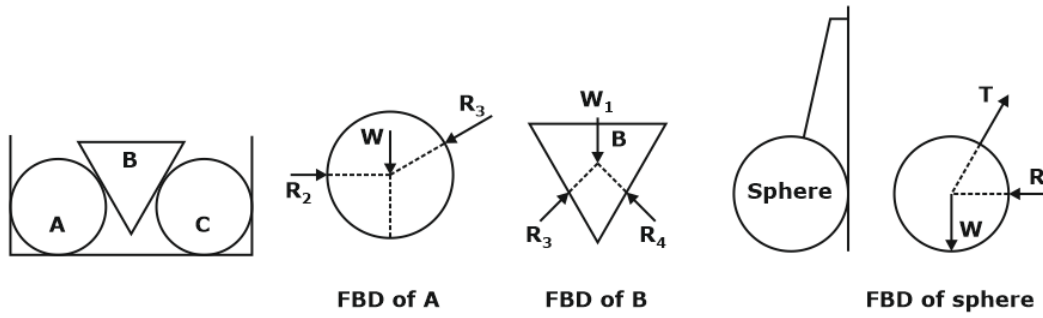
LAMI'S THEOREM

- If three coplanar forces acting at a point be in equilibrium, then each force is proportional to sine of the smaller angle Between other two forces.
- Here P, Q & R are the three coplanar forces and α, β, γ are three angles. According to Lami's theorem $\dfrac{P}{\sin\alpha} = \dfrac{Q}{\sin\beta} = \dfrac{R}{\sin\gamma}$
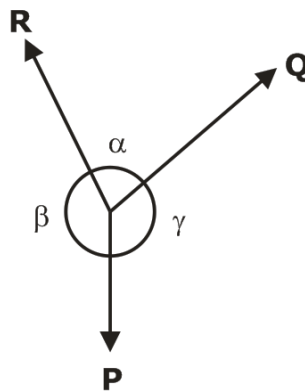
CHAPTER-2 ANALYSIS OF TRUSSES

INTRODUCTION

A structure refers to a system of connected parts used to support a load. Important examples include buildings, towers, ship, aircraft frames, tanks, pressure vessels, mechanical systems and electrical supporting structures.

TRUSSES

- A framework composed of straight members joined at their ends to form a structure is called a truss. Truss is used for supporting moving or stationary load. Bridges, roof supports, and other such structures are common example of trusses.

- When the members of the truss lie essentially in a single plane, the truss is called a plane truss.

- When the members of the truss lie in three-dimension, the truss is called a space truss.

- The simple plane truss is built up from an elementary triangle by adding two new members for each new pin.

CLASSIFICATION OF TRUSS

1) **EFFICIENT OR PERFECT TRUSS**

   - A truss is said to be perfect when the number of members in the truss is just sufficient to prevent distortion of its shape when loaded with an external load.

   - A perfect truss has to satisfy the following equation m = 2j-3, where m is the number of members and j is the number of joints.

2) **DEFICIENT OR COLLASIBLE TRUSS**

   - A deficient truss is an imperfect truss, in which the number of members is NOT sufficient to prevent distortion of its shape when loaded with an external load.

   - It is also known as collapsible truss as under the action of collapsible truss the truss tends to collapse.

   - It satisfies the following equation m < 2j-3

3) **REDUNDANT TRUSS**

   - A redundant truss is an imperfect truss in which the number of members is more than 2j-3 i.e. m > 2j-3

ASSUMPTIONS OF IDEALIZE TRUSS ANALYSIS

- Members are connected at their ends by smooth frictionless pins.

- The weight of members is negligible.

- Loads and reactions are applied to truss at joints only.

- The centroidal axis of each member is straight and coincides with the line connecting the joint centres at the end of the member.

- All members function as two-force members i.e. they are subjected to either tension or compression.

ANALYTICAL METHOD OF TRUSS ANALYSIS

1) METHOD OF JOINTS

- In order to analyze or design a truss, we must obtain the
- Force in each of its members can be obtained by making free body diagram of respective joint and applying equation of equilibrium analysis.
- At each joint the forces in the members meeting at the joint and the loads at the joint, if any, constitute a system of concurrent forces.
- For starting analysis, a joint is selected where there are only two unknown forces.

Note.

- If two non collinear members are in static equilibrium, then these two members must have zero force.
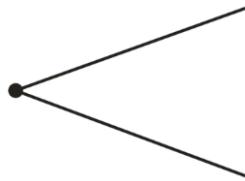
- When three members are meeting at an unloaded joint and out of them two are collinear, then the force in third member will be zero.
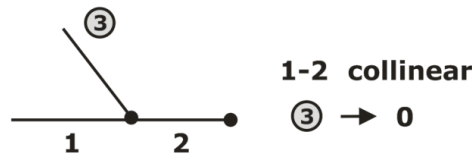
2) METHOD OF SECTIONS

- When we need to find the force in only a few members of a truss, we can analyze the truss using method of sections.
- It is based on the principle that if a body is in equilibrium then any part of the body is also in equilibrium.
- In the method of sections, a section is of the truss is cut, such that not more than three unknown forces are required to be computed. Free body diagram of either of its two parts is drawn and unknown obtain by applying equation of equilibrium analysis.

Note.

- Firstly, the reactions are need to be find.

CHAPTER-3 FRICTION

INTRODUCTION

Friction is the force distribution at the surface of contact between two bodies that prevents or impedes sliding motion of one body relative to the other. It is the frictional force, which causes body to stop.

DRY FRICTION

Dry friction is said to occur when there is relative motion between two completely unlubricated surfaces. It is further divided into two types:

- **Sliding Friction**: When the two surfaces have a sliding motion relative to each other.
- **Rolling Friction**: Friction due to rolling of one body over another.

FILM FRICTION

When two surfaces in contact are completely separated by lubrication, friction will occur due to the shearing of different layers of the lubricant. This is known as film friction or viscous friction.

BOUNDARY FRICTION

When two surfaces in contact have a minute thin layer of lubricant between them, it is known as boundary or skin friction.

Note.

- Friction force is self-adjusting in nature. The friction force cannot increase beyond a limit. Thus, there is a limiting value of friction, known as **limiting friction**.
- When the applied force is less than limiting friction, the body remains at rest and such frictional force is called **static friction**, which may have any value between zero to limiting friction.
- If the value of the applied force exceeds the limiting friction, the body starts moving over the other body and the frictional resistance experienced by the body while moving is known as **Dynamic friction** or **Kinetic friction.**
- It is experimentally found that the magnitude of limiting friction bears a Constant ratio to the normal reaction between the two surfaces and this ratio is called coefficient of Friction

$$\text{Coefficient of friction} = F_s/R$$

where F_s is limiting friction and R, the normal reaction between the contact surfaces.
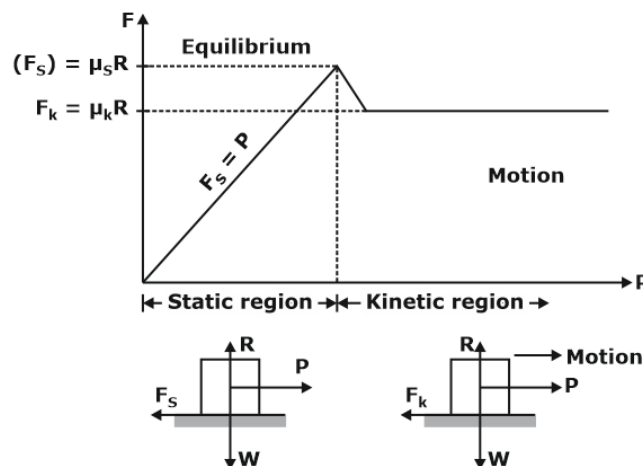
- The graph between the applied load P and the friction force F is linear up to the point when limiting friction is reached.

- The force of static friction between any two surfaces in contact is opposite the applied force and can have values given by $F_s \leq \mu_s R$, where μ_s is called coefficient of static friction.

- When the block is on the verge of slipping, force of static friction is given by
$$F_s = \left(F_s\right)_{max} = \mu_s R$$

- The force of kinetic friction acting on an object is opposite to the direction of motion of the object and is given by $F_k = \mu_k R$, where μ_k is a constant called coefficient of kinetic friction.

- The values of coefficients of friction μ_s and μ_k do not depend upon the area of the surfaces in contact. Both the coefficients, however, depend strongly on the nature of the surface in contact i.e. roughness/smoothness.

- μ_k is generally less than μ_s

SIGNIFICANCE OF FRICTION

Friction is generally undesirable force because additional power is required to overcome the friction. However, it is a desirable feature in certain situation, For example

- friction helps in walking and moving on road
- friction forces are used to stop moving vehicle by applying brakes
- to transmit power by belt drives
- facilitates use of ladder when friction force prevents slipping of the ladder

ANGLE OF FRICTION

- Angle θ which resultant S subtends with the normal to the plane, when the body just starts sliding over the horizontal plane. This is also known as limiting angle of reaction or friction.

- It is sometimes found convenient to replace the normal force R and the friction force F_s by their resultant S. From the geometry of Fig, we note that

$$\tan\phi = \frac{F_s}{R} = \frac{\mu R}{R} = \mu$$
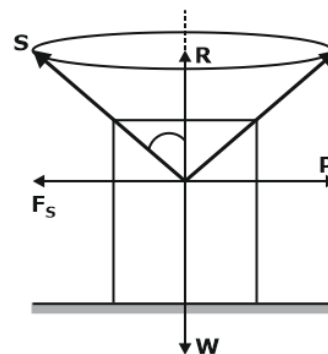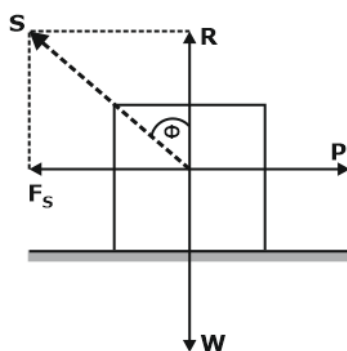
- limiting angle of friction $\phi = \tan^{-1}\mu$

- This indicates that the friction coefficient is tangent of angle of friction.

- We know F_s vary from 0 to limiting value, So if F_s form a circle, then resultant S will describe a right circular cone of apex angle 2ϕ about the line of action R as its axis, then this cone is called a **Cone of friction**.
- The body will be stationary, if resultant is within cone of friction.

ANGLE OF REPOSE

- The maximum inclination of the plane on which a body, free from external forces, experiences repose (sleep) is called Angle of Repose.

- Applying Equilibrium equation to the block shown above, we get $\mu = \tan\theta$

- In terms of angle of friction ϕ, $\theta = \phi$
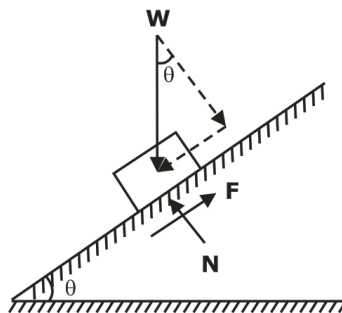
Direction of friction on DRIVING AND DRIVEN WHEELS OF AN AUTOMOBILE

- On a smooth surface driving wheels will rotate about axis of axle but vehicle will not move.
- Frictional torque on driving wheels is overcome by engine torque i.e. frictional torque direction opposite to engine torque. It provides driving force or tractive force to the vehicle.

$$0 \le (F = T / r) \le F_s$$

where F_s is limiting friction, T is engine torque and r is radius of wheel.

- Friction on driven wheels provides motion to driven wheels i.e. friction direction is along the motion of wheel.

CHAPTER-4 DYNAMIC

INTRODUCTION

Dynamics is part of mechanics that deals with the analysis of bodies in motion. Its subparts are

- **Kinematics** is the study of the geometry of motion. It is used to related to displacement, velocity, acceleration and time, without reference to the cause of the motion.

- **Kinetics** is the study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body.

REST

When a body occupies a fixed position for any length of time, it is said to be at rest.

MOTION

A body is said to be in motion when it changes its position with respect to some other fixed object.

- **RECTILINEAR OR TRANSLATORY MOTION**

  Motion in which a particle or point mass body is moving along a straight line.

- **CIRCULAR OR ROTARY MOTION**

  A circular motion is that motion in a particle or a point mass body is moving on a circle.

- **OSCILLATORY OR VIBRATORY MOTION**

  Oscillatory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point (called mean position) in a definite interval of time.

FRAME OF REFERANCE

Frame of reference is a system of coordinate axes attached to an observer having a clock with him, with respect to which, the observer can describe position, displacement, acceleration etc. of a moving object.

- **Inertial frame** of reference is one in which newton's first law of motion holds good. For example, a frame of reference attached to a person in a bus at rest or moving with a uniform velocity along a straight line.

- **Non-Inertial frame** of reference is one in which newton's first law of motion does not hold good. For example, a frame of reference attached to a person in a bus moving with uniform velocity along circular curve or moving with acceleration along a straight line.

DISPLACEMENT

- The change in position of a body in a definite direction is known as displacement and it is measured by straight distance between the initial and the final positions of the moving body.

- The displacement represents both magnitude and direction followed by the body.

- It is a vector quantity.

SPEED

The speed of a moving body is the rate of change of its position.

$$Speed = \frac{Distance \ travelled}{Time \ Taken}$$

UNIFROM SPEED

An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time.

VARIABLE SPEED

An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time.

AVERAGE SPEED

Average speed of the object is that constant speed with which the object covers the same distance in a given time as it does while moving with variable speed during the given time.

$$\text{Average Speed} = \frac{\text{Total Distance travelled}}{\text{Total Time Taken}}$$

INSTANTANEOUS SPEED

The speeds of an object at a given instant of time is called its instantaneous speed.

VELOCITY

The velocity of a moving body is the time rate of change of its displacement.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time Taken}}$$

UNIFORM VELOCITY

Velocity of object with which, it undergoes equal displacements in equal intervals of time. An object is said to be in uniform motion if its velocity is uniform.

VARIABLE VELOCITY

An object is said to moving with a variable velocity, if either its speed (magnitude) or its direction of motion or both change with time.

AVERAGE VELOCITY

Average velocity is that uniform velocity with which the object will cover the same displacement in a given interval of time as it does with its actual velocity during the given interval of time.

INSTANTANEOUS VELOCITY

The velocity of an object at a given instant of time is called its instantaneous velocity.

**GRAPHICAL REPRESENTATION OF VELOCITY, TIME AND DISTANCE TRAVELLED**

- The slope of the position-time graph gives velocity of the object. Straight line parallel to position axis represent infinite velocity, which is not possible.
- The slope of velocity-time graph gives acceleration of object.
- In velocity-time graph, for the given interval of time, the positive area enclosed with the time axis gives positive displacement and the negative area enclosed with the time axis gives negative displacement.
- Distance travelled by a body in a given interval of time is equal to total area of velocity-time graph, without considering sign.
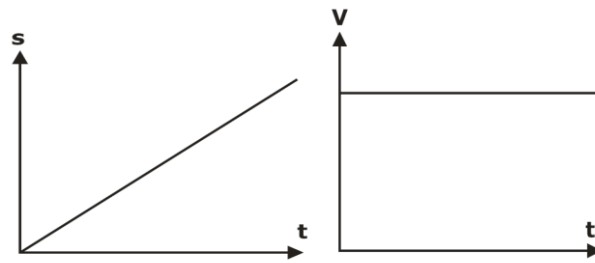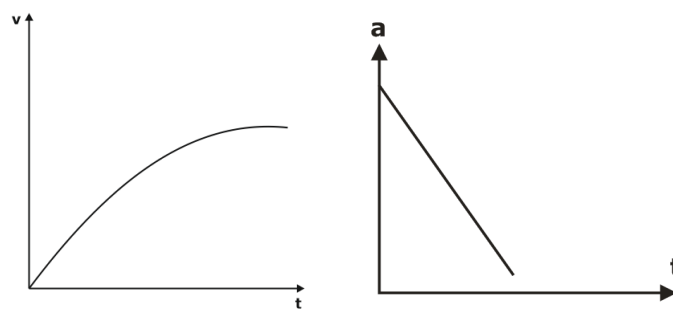
Uniform Motion (Uniform Velocity)

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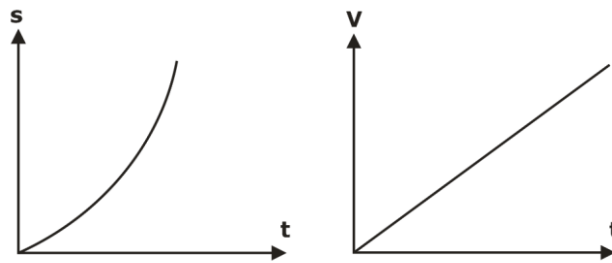
Uniform Acceleration
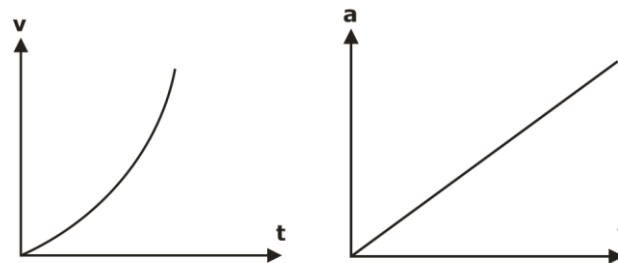
Uniformly Increasing Acceleration

Uniformly Decreasing acceleration

MOTION UNDER VARIABLE ACCELERATION

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{ds}{dt} \times \frac{dv}{ds} = v\frac{dv}{ds}$$

$$ads = vdv$$

SCALAR & VECTORS

- Scalars are those physical quantities which have magnitude only but no direction i.e. mass, length.
- Vectors are those physical quantities which have both magnitude and direction i.e. velocity, acceleration etc.
- A vector can be represented by a straight line with arrow head on it. The length of line drawn on suitable scale represents the magnitude of vector and arrow head tells the direction of the vector.
- The starting point of arrowed line is called tail or origin of vector and the end of the arrowed line is called tip or head of the vector.

VECTOR ADDITION

The general rule of vector addition is used which states that the vectors to be added are arranged in such a way so that the head of first vector coincides with the tail of second vector, whose head coincides with the tail of third vector and so on, then the single vector drawn from the tail of the first vector to the head of the last vector represents their resultant vector.

RELATIVE VELOCITY OF RAIN WRT THE MOVING MAN

The relative velocity of rain with respect to moving man is the velocity of rain which the man will feel.

➢ STEP 1. Draw Lines showing actual velocity of Rain and Man... (OC and OA in the Fig.)

➢ STEP 2. Now draw a Line in opposite direction of actual velocity of Man... (OB in Fig.)

➢ STEP 3. The Resultant velocity CB in triangle COB is relative velocity of the rain and man. It will give the magnitude and direction of rain which a man will feel.
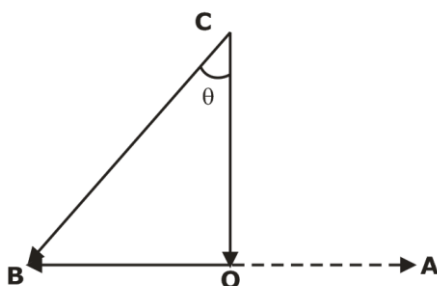
Equation of PLANE MOTION under uniform acceleration

$v = u + at$

$s = ut + \frac{1}{2} at^2$

$s_{nth} = u + \frac{1}{2} a(2n-1)$

$v^2 - u^2 = 2as$

where u = Initial velocity

  v = Final velocity

  t = Time taken by the particle to change its velocity from u to v

  a = Uniform +ve acceleration

  s = distance travelled in t seconds

  n = n is integer number for which second we study

PROJECTILE

A Particle, moving under combined effect of horizontal and vertical forces is called a projectile.

- Horizontal component: remains constant if air resistance is ignored.
- Vertical component: always subjected to gravitational acceleration. A rocket is not a projectile.
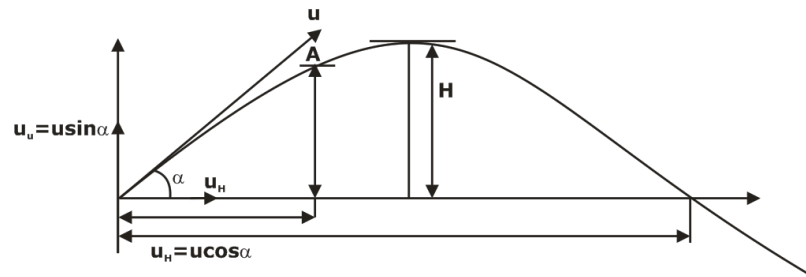
Here u = Velocity of projection

α = Angle of projection with horizontal

Initial Horizontal component of velocity = u_H = u cosα

Initial Vertical Component of velocity = u_v = u sinα

Applying Equation of plane motion along horizontal and vertical direction we get

- Trajectory of projectile $$y = x tan\alpha - \frac{gx^2}{2 u^2 cos^2\alpha}$$

- Time for which the projectile remained in air $$T = \frac{2 u\, sin\alpha}{g}$$

- Horizontal Range of a Projectile $R = \dfrac{u^2 sin2\alpha}{g}$, $R_{max} = \dfrac{u^2}{g}$ at $\alpha = 45°$

- Maximum height of a projectile $H = \dfrac{u^2 \sin^2 \alpha}{2g}$

MOTION OF ROTATION

Motion of rotation which takes place about the geometric axis of the body.

- **ANGULAR VELOCITY**
  - ➢ Rate of change of angular displacement of a body.
  - ➢ Denoted by ω.
  - ➢ Expressed in rad/s.
  - ➢ For N rpm, ω = 2π radian/ minute.
  - ➢ For N rpm, ω = 2π N/60 r.p.m = 2π N/60 rad/sec

- **ANGULAR ACCELERATION**
  - ➢ Rate of change of angular velocity.
  - ➢ Denoted by α.
  - ➢ Expressed in rad/sec^2.

- **ANGULAR DISPLACEMENT**

- Total angle, through which a body has rotated.
- Denoted by θ.
- Expressed in radian.
- $\theta = \omega \times t$ for uniform velocity

- **MOTION UNDER UNIFORM ACCELERATION**

  $\omega = \omega_0 + \alpha t$

  $\theta = \omega_0 t + \alpha t^2/2$

  $$\theta_{nth} = \omega_1 + (2n-1)\frac{\alpha}{2}$$

  $$\omega_0^2 = \omega^2 + 2\alpha\theta$$

  Where ω_0 = initial velocity

  ω = Final velocity

  t = Time taken by the particle to change its velocity from ω_0 to ω

  α = Uniform angular acceleration

  θ = Total angular displacement in t seconds

- **LINEAR (TANGENTIAL) VELOCITY OF A ROTATING BODY**

  ω = Angular velocity of the body in rad/sec

  r = Radius of circular path

  v = Linear velocity of particle on the periphery in m/sec.

  θ = Total angular displacement in t seconds

  l = Total length of arc covered in t seconds = $r\theta$

  Linear Velocity = Length of arc/ time taken

  $$v = \frac{d}{dt}(r\theta) \quad \rightarrow \quad v = \omega r$$

- **LINEAR (TANGENTIAL) ACCELERATION OF A ROTATING BODY**

  α = Uniform angular acceleration

  a = Linear acceleration

  Linear acceleration (a) = dv/dt and v = r*ω

  $$\frac{dv}{dt} = r\frac{d\omega}{dt} \quad \rightarrow \quad a = r\alpha$$

- **RADIAL AND NORMAL VELOCITIES IN CYLINDRICAL COORDINATES**

  Resultant velocity of particle at $P(r,\theta)$ $\qquad v = \sqrt{v_r^2 + v_\theta^2}$

  $$v = \sqrt{\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2}$$

- **RADIAL AND NORMAL ACCELERATIONS IN CYLINDRICAL COORDINATES**

  Resultant acceleration, $a = \sqrt{a_r^2 + a_\theta^2}$

  Where $a_\theta = r\dfrac{d^2\theta}{dt^2} + 2\left(\dfrac{dr}{dt}\right)\left(\dfrac{d\theta}{dt}\right) = \dfrac{1}{r}\dfrac{d}{dt}\left(r^2\dfrac{d\theta}{dt}\right)$

And $a_r = \dfrac{d^2r}{dt^2} - r\left(\dfrac{d\theta}{dt}\right)^2$

Special case: When the path of particle is circular, radius of circle, r, is constant.

Radial acceleration, $a_r = 0 - r\left(\dfrac{d\theta}{dt}\right)^2 = -r\omega^2$, where $\omega = \dfrac{d\theta}{dt}$

Transverse acceleration, $a_\theta = r\left(\dfrac{d^2\theta}{dt^2}\right) = r\alpha$ where $\alpha = \dfrac{d^2\theta}{dt^2}$

$$a_r = -\omega^2 r \text{ and } a_\theta = r\alpha$$

MOTION ALONG A CIRCULAR PATH

A body, moving in a circle or along a circular path, with a constant velocity

- Suffers a continuous change in its direction at every point of motion

- But, magnitude of its speed remains the same.

- Since velocity involves both magnitude and direction, hence velocity is continuously changing.

- Some force must act at right angle to the direction of motion at every point, which should change the direction of the motion of the body.

- The force, which acts along the radius of the circle at every point, and is always directed towards the centre of the circle along which the body moves, is known as Centripetal Force.

- A pseudo force centrifugal force act opposite to centripetal force.

    r = radius of the circular path in m, and

    v = Linear velocity of the particle in m/s

    ω = Angular velocity in rad/s

    a = centripetal or centrifugal acceleration.

    $a = v^2/r = \omega^2 r$

LAWS OF MOTION

- **NEWTON'S FIRST LAW OF MOTION**

    "Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."

- **NEWTON'S SECOND LAW OF MOTION**

    "The rate of change of momentum is directly proportional to the impressed force and takes place, in the same direction in which the force acts."

    m = mass of the body

    u = Initial velocity of the body

    v = Final velocity of the body

    a = Constant acceleration

    t = time required to change velocity from u to v

    Initial momentum = mu

    Final momentum = mv

    Rate of change of momentum = (mv – mu)/t = m (v – u)/t = ma

Acc. To law, Impulse force P α ma $\rightarrow$ P = ma

- **NEWTON'S THIRD LAW OF MOTION**

  The mutual forces of action and reaction between two particles are equal, opposite and collinear or "To every action, there is always an equal and opposite reaction."

D'ALEMBERT'S PRINCIPLE

D'Alembert proposed a method of analysis that would transform a dynamic problem into equivalent problem in statics. The essence of his idea, which is known as D'Alembert's principle is to consider ma as the force acting on particle called inertial force. Newton's second law F-ma=0 can then be treated as an equilibrium condition that involves the applied forces together with inertial force.

MOTION OF A LIFT

Consider a lift carrying some mass m and moving with a uniform acceleration a.

W = Weight carried by Lift

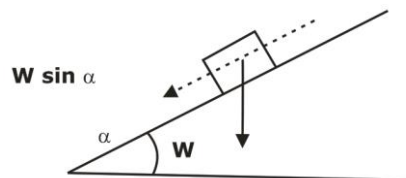R = Reaction of the lift or tension in the cable, supporting the lift.

- **WHEN THE LIFT IS MOVING UPWARDS** Accelerating force F = ma = R – mg
- **WHEN THE LIFT IS MOVING DOWNWARDS** Accelerating force F = ma = mg – R
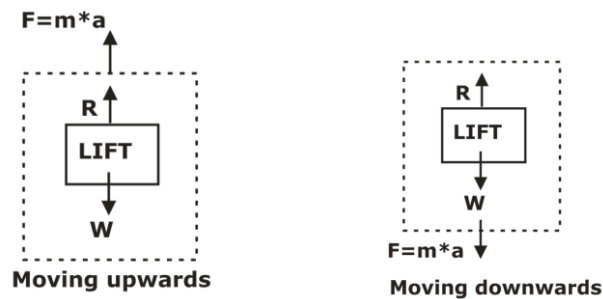

MOTION ON AN INCLINED SMOOTH SURFACE

Consider a body having weight W = mg sliding downwards with acceleration a.

Accelerating Force = ma = mg Sin α i.e.   a = g Sin α

WORK

- Whenever a force acts on a body, and the body undergoes a displacement.

  Work done = Component of the force in the direction of motion x Distance

  W = P Cos θS

  Where P = Applied force

  S = Displacement of the body

  θ = Angle between applied force and direction of motion

- Unit of work: 1 J or Nm

- Work done is equal to Area under curve on Force displacement diagram.
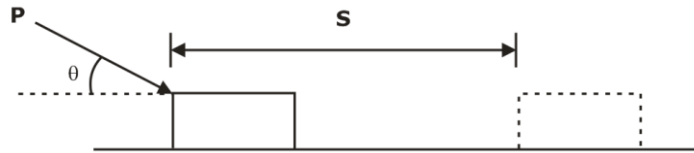
POWER

- Rate of doing work
- Indicated Power: The actual power generated in engine cylinder.
- Brake Power: Power available for useful work.
- Unit: watt (W) on Nm/s.

ENERGY

- Capacity to do work.
- **Potential Energy**. Energy possessed by a body by virtue of its position. P.E. = mgh

    Where m = mass of a body
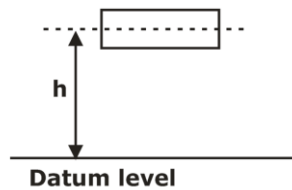
    h = position of body above the datum level

Fig:40

- **Kinetic Energy**: Energy possessed by a body by virtue of its velocity. K.E. = ½ (mv^2)

    Where m = mass of body

    v = velocity of body

- **Law of Conservation of Energy:** "The energy can neither be created nor destroyed, though it can be transformed from one form into any of its forms, in which energy can exist."

Corresponding term used in Linear motion and Rotation motion

Linear Motion	Motion of Rotation
1. Mass(m)	Moment of Inertia (I)
2. Distance travelled (s)	Angular displacement (θ)
3.Linear motion (v)	Angular motion (ω)
4. Linear acceleration (a)	Angular acceleration (α)
5.Linea Momentum = mv	Angular Momentum = Iω
6.Force (F)	Torque (T)
7. Force F = ma	Torque T = Iα
8.Linear K.E. = mv^2/2	Angular K.E. = Iω^2/2
9. Work done = Fs	Work done = Tθ
10. Conservation of momentum	Conservation of angular momentum

CHAPTER-6-CONSERVATION OF MOMENTUM PRINCIPLE

COLLISION

- The bodies after collision come momentarily to rest.

- The two bodies tend to compress each other, so long as they are compressed to the maximum value. The time taken is called time of compression.

- The two bodies attempt to regain its original shape due to their elasticity. This process is known as restitution and time taken during restitution is called the time of restitution.

- The line of impact is the common normal to both the bodies. The plane of impact is the common tangent plane of bodies.

- ***Time of collision = time of compression + time of restitution***

- **Direct collision** is the collision in which the two bodies, before impact are moving along the line of impact.

- **Indirect collision** if the two bodies, before impact are not moving along the line of impact.

LAW OF CONSERVATION OF MOMENTUM

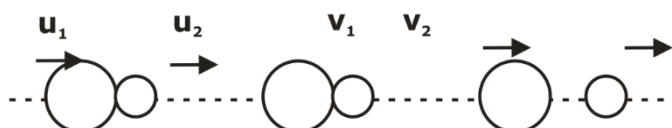"The total momentum of two bodies remains constant after their collision."

m_1 = mass of first body

u_1 = Initial velocity of first body

v_1 = Final velocity of first body

m_2, u_2, v_2 = corresponding values for the second body.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

NEWTON'S LAW OF COLLISION

"When two bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach."

m_1 = mass of first body

u_1 = velocity of first body before impact

v_1 = velocity of first body after impact

m_2, u_2, v_2 = Corresponding values for the second body.

Let us assume that $u_1 > u_2$ and $v_2 > v_1$

$$v_2 - v_1 = e(u_1 - u_2), \text{ Where } e = \text{coefficient of restitution}$$

- e varies between 0 and 1.

- Perfectly elastic collision: When e = 1, velocity of approach = velocity of separation

- Inelastic collision: when e = 0, $v_2 = v_1$

LOSS OF KINETIC ENERGY

- Only in Inelastic collision.
- Difference between kinetic Energies of system before and after collision.

$$E_L = E_f - E_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_2u_2^2$$

CHAPTER-7 CENTROID, CENTRE OF GRAVITY, MOMENT OF INERTIA

INTRODUCTION

- In engineering structures, the members of various cross-sections are used to withstand loads.

- The load carrying capacity depends on type of material used as also cross-section of the members.

CENTROID

- Centroid is the point where the whole area is assumed to be concentrated.

- It is related to shape or the geometry of object.

- Centroid of line $\bar{x} = \dfrac{\int x dL}{L}$, $\bar{y} = \dfrac{\int y dL}{L}$ and $\bar{z} = \dfrac{\int z dL}{L}$. where $\int dL = L$

- Centroid of Area $\bar{x} = \dfrac{\int x dA}{\int dA}$, $\bar{y} = \dfrac{\int y dA}{\int dA}$, $\bar{z} = \dfrac{\int z dA}{\int dA}$, where $\int dA = A$

- Centroid of Volume $\bar{x} = \dfrac{\int x dV}{V}$, $\bar{y} = \dfrac{\int y dV}{y}$ and $\bar{z} = \dfrac{\int z dV}{V}$ where $\int dV = V$

CENTRE OF GRAVITY

- Centre of gravity of the body or the system of particles rigidly connected together is that point where the weight of body is concentrated and gravitational force acts through it.

- $\bar{x} = \dfrac{\int x dm}{\int dm}$, $\bar{y} = \dfrac{\int y dm}{\int dm}$, $\bar{z} = \dfrac{\int z dm}{\int dm}$, where $\int dm = M$

Note.

When density is uniform i.e. the body material is homogenous, centroid and C.G are same but when density is not same, two point will not be some. C.G lies where density is more, i.e. weight more.

SHAPE	FIGURE	$\overline{x}$	$\overline{y}$
Rectangle		$\dfrac{b}{2}$	$\dfrac{d}{2}$

Triangle		$\dfrac{b}{2}$	$\dfrac{h}{3}$
Circle		$\dfrac{r}{2}$	$\dfrac{r}{2}$
Semi-Circle			$\dfrac{4R}{3\pi}$
Quarter of Circle		$\dfrac{4R}{3\pi}$	$\dfrac{4R}{3\pi}$
Sector of a Circle		$\dfrac{2R}{3\alpha}\sin\alpha$	0

AREA MOMENT OF INERTIA

Moment of Inertia for an area: - This is also known as second moment of area.

$I_{xx} = \int dA\, y.y$ = second moment of area about x axis

$$I_{yy} = \int dA \, x.x = \text{second moment of area about y axis.}\backslash$$

THEOREM OF PARALLEL AXIS

The moment of Inertia about any axis parallel to centroidal axis at a distance (h) is equal to sum of moment of Inertia about centroidal axis and product of area and square of distance B/w two axis.

$$I_{parallelAxis} = I_{Centroid} + Ah^2$$

THEOREM OF PERPENDICULAR AXIS

If moment of Inertia about two perpendicular centroidal axis in a plane are given (I_1, I_2) then moment of Inertial about third axis which is perpendicular to both mutually perpendicular axis passing through the point of intersection is equal to sum of moment of Inertia about two centroidal axis.

$$I_{zz} = I_{xx} + I_{yy}$$

SHAPE	AXIS	Moment of Inertia
	x-x AB	$\dfrac{bh^3}{36}$ $\dfrac{bh^3}{12}$
	x-x y-y	$\dfrac{\pi}{64}D^4$ $\dfrac{\pi}{64}D^4$
	x-x y-y	$\dfrac{\pi}{64}(D^4 - d^4)$ $\dfrac{\pi}{64}(D^4 - d^4)$

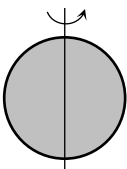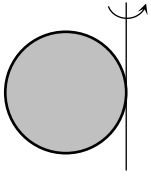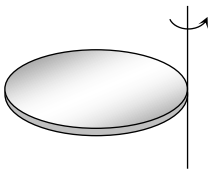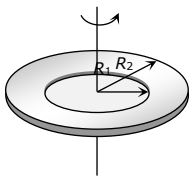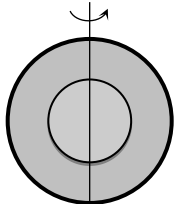| Figure	Axis	Moment of Inertia
	x-x AB	0.0068598 d^4 $\dfrac{\pi d^4}{128}$
	x-x AB	0.00343 d^4 $\dfrac{\pi d^4}{256}$

MASS MOMENT OF INERTIA
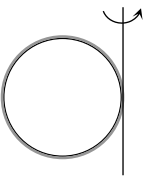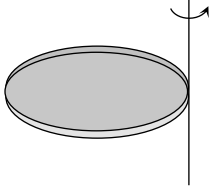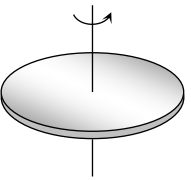
Moment of moment of mass. Also called second moment of mass. Theorem of parallel axis and perpendicular axis also applies in it.
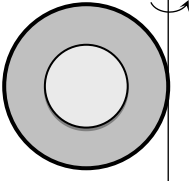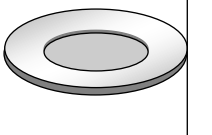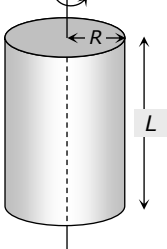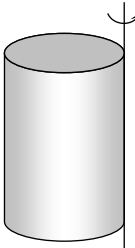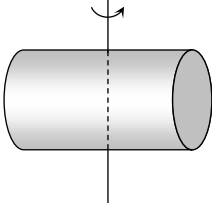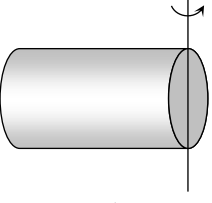
RADIUS OF GYRATION
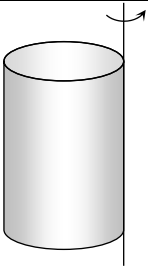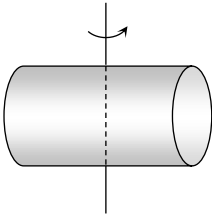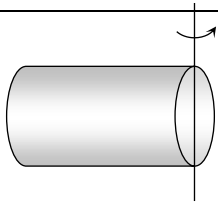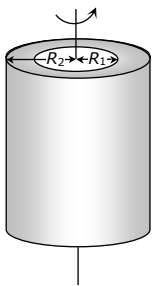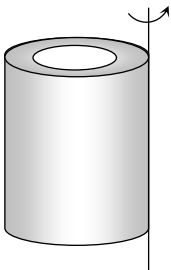
• The radius of gyration k describes the way in which the area of a cross-section is distributed around its centroidal axis.

• If the area is concentrated far from the centroidal axis, it will have a greater value of k and a greater resistance to buckling (or bending).
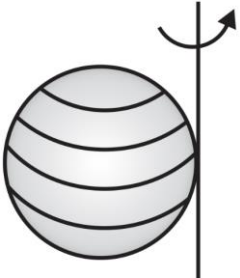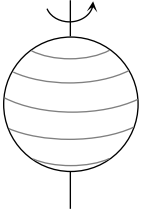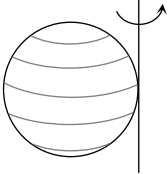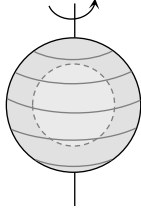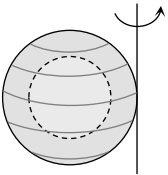
• The radius of gyration is defined as: $k = \sqrt{\dfrac{I}{A}}$, where k is the radius of gyration, I is the moment of inertia, and A is the area of cross-section.
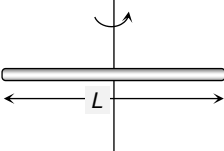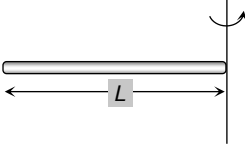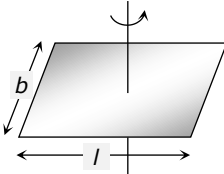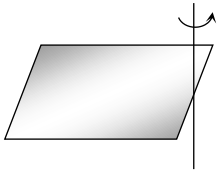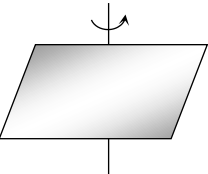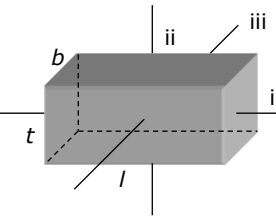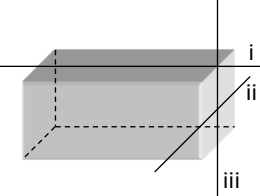
Body	Axis of Rotation	Figure	Moment of inertia	k	k²/R²
Ring	About an axis passing through C.G. and perpendicular to its plane		MR^2	R	1
Ring	About its diameter		$\dfrac{1}{2}MR^2$	$\dfrac{R}{\sqrt{2}}$	$\dfrac{1}{2}$

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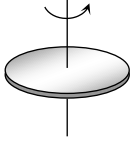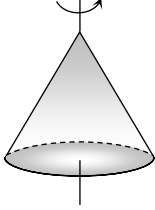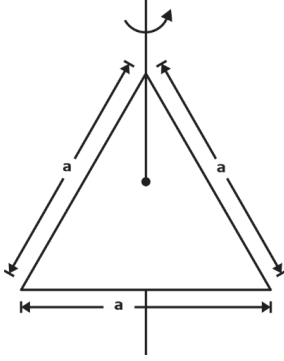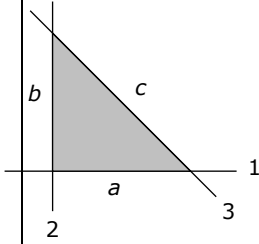
Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Ring	About a tangential axis in its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Ring	About a tangential axis perpendicular to its own plane		$2MR^2$	$\sqrt{2}R$	2
Disc	About an axis passing through C.G. and perpendicular to its plane		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Disc	About its Diameter		$\frac{1}{4}MR^2$	$\frac{R}{2}$	$\frac{1}{4}$
Disc	About a tangential axis in its own plane		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$	$\frac{5}{4}$
Disc	About a tangential axis perpendicular to its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Annular disc inner radius = R_1 and outer radius = R_2	Passing through the centre and perpendicular to the plane		$\frac{M}{2}[R_1^2 + R_2^2]$	–	–
Annular disc	Diameter		$\frac{M}{4}[R_1^2 + R_2^2]$	–	–

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Annular disc	Tangential and Parallel to the diameter		$\frac{M}{4}[5R_1^2 + R_2^2]$	–	–
Annular disc	Tangential and perpendicular to the plane		$\frac{M}{2}[3R_1^2 + R_2^2]$	–	–
Solid cylinder	About its own axis		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Solid cylinder	Tangential (Generator)		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Solid cylinder	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{12} + \frac{R^2}{4}}$	
Solid cylinder	About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3} + \frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{3} + \frac{R^2}{4}}$	
Cylindrical shell	About its own axis		MR^2	R	1

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Cylindrical shell	Tangential (Generator)		$2MR^2$	$\sqrt{2}R$	2
Cylindrical shell	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\dfrac{L^2}{12} + \dfrac{R^2}{2}\right]$	$\sqrt{\dfrac{L^2}{12} + \dfrac{R^2}{2}}$	
Cylindrical shell	About the diameter of one of faces of the cylinder		$M\left[\dfrac{L^2}{3} + \dfrac{R^2}{2}\right]$	$\sqrt{\dfrac{L^2}{3} + \dfrac{R^2}{2}}$	
Hollow cylinder with inner radius = R_1 and outer radius = R_2	Axis of cylinder		$\dfrac{M}{2}(R_1^2 + R_2^2)$		
Hollow cylinder with inner radius = R_1 and outer radius = R_2	Tangential		$\dfrac{M}{2}(R_1^2 + 3R_2^2)$		

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Solid Sphere	About its diametric axis		$\dfrac{2}{5}MR^2$	$\sqrt{\dfrac{2}{5}}R$	$\dfrac{2}{5}$
Solid sphere	About a tangential axis		$\dfrac{7}{5}MR^2$	$\sqrt{\dfrac{7}{5}}R$	$\dfrac{7}{5}$
Spherical shell	About its diametric axis		$\dfrac{2}{3}MR^2$	$\sqrt{\dfrac{2}{3}}R$	$\dfrac{2}{3}$
Spherical shell	About a tangential axis		$\dfrac{5}{3}MR^2$	$\sqrt{\dfrac{5}{3}}R$	$\dfrac{5}{3}$
Hollow sphere of inner radius R_1 and outer radius R_2	About its diametric axis		$\dfrac{2}{5}M\left[\dfrac{R_2{}^5 - R_1{}^5}{R_2{}^3 - R_1{}^3}\right]$		
Hollow sphere	Tangential		$\dfrac{2M[R_2^5 - R_1^5]}{5(R_2^3 - R_1^3)} + MR_2^2$		

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Long thin rod	About on axis passing through its centre of mass and perpendicular to the rod.		$\dfrac{ML^2}{12}$	$\dfrac{L}{\sqrt{12}}$	
Long thin rod	About an axis passing through its edge and perpendicular to the rod		$\dfrac{ML^2}{3}$	$\dfrac{L}{\sqrt{3}}$	
Rectangular lamina of length l and breadth b	Passing through the centre of mass and perpendicular to the plane		$\dfrac{M}{12}[l^2 + b^2]$		
Rectangular lamina	Tangential perpendicular to the plane and at the mid-point of breadth		$\dfrac{M}{12}[4l^2 + b^2]$		
Rectangular lamina	Tangential perpendicular to the plane and at the mid-point of length		$\dfrac{M}{12}[l^2 + 4b^2]$		
Rectangular parallelopiped length l, breadth b, thickness t	Passing through centre of mass and parallel to (i) Length (x) (ii) breadth (z) (iii) thickness (y)		(i) $\dfrac{M[b^2 + t^2]}{12}$ (ii) $\dfrac{M[l^2 + t^2]}{12}$ (iii) $\dfrac{M[b^2 + l^2]}{12}$		
Rectangular parallelepiped length l, breath b, thickness t	Tangential and parallel to (i) length (x) (ii) breadth (y) (iii) thickness(z)		(i) $\dfrac{M}{12}[3l^2 + b^2 + t^2]$ (ii) $\dfrac{M}{12}[l^2 + 3b^2 + t^2]$ (iii) $\dfrac{M}{12}[l^2 + b^2 + 3t^2]$		

Body	Axis of Rotation	Figure	Moment of inertia	k	k²/R²
Elliptical disc of semimajor axis = *a* and semiminor axis = *b*	Passing through CM and perpendicular to the plane		$\dfrac{M}{4}[a^2 + b^2]$		
Solid cone of radius R and height h	Axis joining the vertex and centre of the base		$\dfrac{3}{10}MR^2$		
Equilateral triangular lamina with side a	Passing through CM and perpendicular to the plane		$\dfrac{Ma^2}{6}$		
Right angled triangular lamina of sides *a*, *b*, *c*	Along the edges		(1) $\dfrac{Mb^2}{6}$ (2) $\dfrac{Ma^2}{6}$ (3) $\dfrac{M}{6}\left[\dfrac{a^2 b^2}{a^2 + b^2}\right]$		

****

Important Formula (Heat Transfer)

CHAPTER-1: FUNDAMENTALS OF CONDUCTION

1. Modes of Heat Transfer:

- Conduction
- Convection
- Radiation

2. Conduction:

- Conduction is a mode of heat transfer which generally occurs in solids and static fluids by ***molecular lattice vibration energy transfer (30%) & also by free electron transfer (70%).***

- The highest conductivity (2100 W/m-K) of diamond is due to its perfect crystalline molecular lattice arrangement.

- The thermal conductivity of gases increases with increase of their temperature.

- ***Thermal conductivity of liquids decreases with increasing temperature*** as the liquid expands and the molecules move apart.

- In the case of solids, because of lattice distortions, higher temperatures make it difficult for electrons to flow, hence the **thermal conductivity of metals decreases with increase in temperature.**

  Exception – Aluminium & Uranium

3. Convection:

- Convection is a mode of heat transfer which generally occurs between a solid & a fluid due to temperature difference associated ***with macroscopic bulk displacement of the fluid*** transporting thermal energy (heat energy).

- In case of forced convection heat transfer this macroscopic bulk motion of the fluid is provided by an external agency like far or a blower or a pump.

- In case of free convection heat transfer, this motion is provided by buoyancy forces arising out of density changes of fluid due to its temperature change.

4. Radiation:

- Radiation is a mode of heat transfer which do not require any material medium for its propagation & hence occurs by electromagnetic waves.

- All bodies emit radiation above the 0K.

5. Governing Laws of Heat Transfer:

5.1. Fourier's Law of Conduction:

The law states that "the rate of heat transfer by conduction in a given direction is directly proportional to the temperature gradient along that direction & the area of heat transfer lying perpendicular to the direction of heat transfer".

It is given as

$$Q_x = -kA\left(\frac{dT}{dx}\right) \text{Watts}$$

'k' is thermal conductivity (thermo physical property) of material of slab.

A is the area perpendicular to direction of the heat flow.

Thermal conductivity - It is a thermophysical property of material which tells about its ability to allow the heat energy to get conducted through the material more rapidly.

Here are the list of some material and its thermal conductivity

Materials	Thermal Conductivity
Silver	405 W/mK
Copper	385 W/mK
Gold	319 W/mK
Aluminium	200 W/mK
Steels	17 to 45 W/mK
Asbestos	0.2 W/mK
Refractory bricks	0.9 W/mK
Glass wool	0.075 W/mK
Polyurethane Foam	0.02W/mK
Air	0.026W/mK
Water	0.63 W/mK
Mercury	8.34 W/mK

NOTE- Conductivity of Pure metal is higher than the conductivity of Alloy.

Conductivity in Gases - Conductivity in Gases is due to molecular momentum transfer.

NOTE- Conductivity of gases increases with the increase in temperature.

5.2 Newton's Law of Cooling of Convection:

$$Q = hA\left(T_w - T_\infty\right)$$

Where:

T_∞: Ambient temperature

h is convection heat transfer coefficient.

A is the area of contact.

It is not the property of the material but depends on some of the thermo physical properties of fluid.

5.3. Stefan – Boltzman law of radiation:

The law states that "Radiation Energy emitted from the surface of a black body per unit time & per unit area is directly proportional to the fourth power of the absolute temperature of the body".

$$Q_b = \sigma \varepsilon A T^4$$

Where σ is Stefan-Boltzman constant.

A = Area of Contact

ϵ = emissivity of the material

$\sigma = 5.67 \times 10^{-8}$ W/m²k⁴

For Black Body: $\epsilon = 1$

$$Q_b = \sigma A T^4$$

Where σ is Stefan-Boltzman constant

$\sigma = 5.67 \times 10^{-8}$ W/m²k⁴

6. CONDUCTION EQUATION

6.1. Conduction Heat Transfer Through a Slab:

Generalised conduction equation for 3-Dimensional body in cartesian coordinate:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \left(\frac{\rho C_p}{K} \right) \frac{\partial T}{\partial \tau}$$

Thermal diffusivity:

- It is given by: $\alpha = \dfrac{K}{\rho C_p}$.

- Its unit is m²/sec.

- Thermal diffusivity (α) of a medium signifies the rate at which heat energy can diffuse through the medium.

Case-1: If conditions are steady: $\dfrac{\partial T}{\partial \tau} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = 0$$

$$\nabla^2 T + \frac{q}{K} = 0$$ Poisson equation

Case-2: If there is no heat generation (q = 0):

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \left(\frac{\rho C_p}{K} \right) \frac{\partial T}{\partial \tau}$$

$$\nabla^2 T = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$ Diffusion equation

Case-3: When there is no heat generation and conditions are steady:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

$\boxed{\nabla^2 T = 0}$ Laplace equation

6.2. Radial conduction heat transfer through a cylinder:

Generalised conduction equation:

$$\boxed{\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot q}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

6.3. Radial conduction heat transfer through a sphere:

Generalised conduction equation:

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\psi}\frac{\partial}{\partial \psi}\left(\sin\psi \frac{\partial T}{\partial \psi}\right) + \frac{1}{r^2 \sin^2\phi}\left(\frac{\partial^2 T}{\partial \phi^2}\right) + \frac{\dot q}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

General Equation,

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(kr^2 \frac{\partial T}{\partial r}\right) = \left(\frac{\rho C_P}{K}\right)\frac{\partial T}{\partial \tau}$$

For plane slab, n = 0

For cylinder, n = 1

For sphere, n = 2

Conduction with Heat Generation,1D

Slab

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot q}{K} = 0$$

$$T\left(at\, x\right) = T_0 - \frac{\dot q x^2}{2k} \quad \text{(Parabolic Profile)}$$

Temperature Distribution, $\dfrac{T_0 - T}{T_0 - T_w} = \dfrac{x^2}{L^2}$

Wall temperature: $T_w = \dfrac{\dot q L}{h} + T_\infty$

Maximum temperature: $T_0 = T_w + \dfrac{\dot q}{k}\dfrac{L^2}{2} = \dfrac{\dot q L^2}{2k} + \dfrac{\dot q L}{h} + T_\infty$
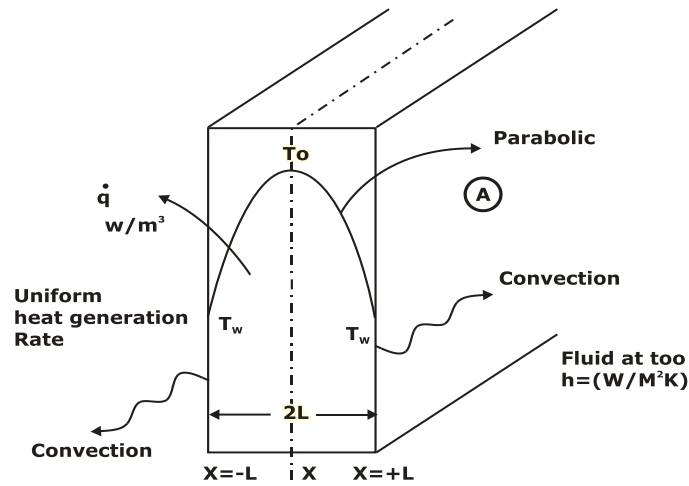
In case, if one side of sab is completely insulated:

Boundary condition: $\left(\dfrac{dT}{dx}\right) = 0$ at the insulated surface.

Cylinder (cable wire)

Generalised conduction equation for radial conduction:

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{\dot q}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

Wall temperature: $T_w = \dfrac{\dot q R}{2h} + T_\infty$

Maximum temperature: $T_{max} = T_w + \dfrac{\dot q R^2}{4K} = \dfrac{\dot q R^2}{4K} + \dfrac{\dot q R}{2h} + T_\infty$

Temperature Distribution, $\dfrac{T_O - T}{T_O - T_w} = \dfrac{r^2}{R^2}$

NOTE – Suppose current 'I' is flowing through a wire having length, 'L' radius 'R" Electrical resistance 'R$_{elec}$' for a time period of 't'

Then, heat generation inside the cylinder per unit Volume is given by $\dot q = \dfrac{I^2 R_{elec}}{\pi R^2 L}$

Sphere

Wall temperature: $T_w = \dfrac{\dot q R}{3h} + T_\infty$

Maximum temperature: $T_{max} = T_w + \dfrac{\dot q R^2}{6K} = \dfrac{\dot q R^2}{6K} + \dfrac{\dot q R}{3h} + T_\infty$

7. Electrical Analogy of Heat Transfer:
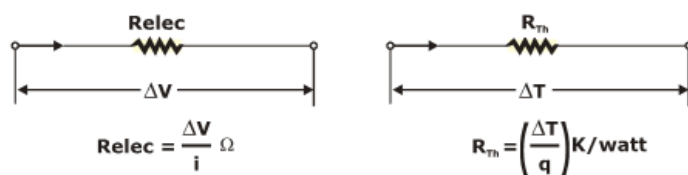
7.1. Plane slab with steady state, no heat generation:

Heat flow rate: $\boxed{Q = \dfrac{KA(T_1 - T_2)}{b}}$

$\boxed{R_{Th} = \dfrac{T_1 - T_2}{Q} = \dfrac{b}{KA}}$ K / watt

Temperature at a distance of 'x' is given as $T = T_1 - \dfrac{(T_1 - T_2)x}{L}$ $T_1 > T_2$

Linear Temperature Profile

7.2. Cylinder with steady state, no heat generation:

Heat flow is given by: $q = \dfrac{2\pi kL(T_1 - T_2)}{\ell n \dfrac{r_2}{r_1}}$

Thermal resistance: $(R_{Th})_{cylinder} = \dfrac{(T_1 - T_2)}{Q} = \dfrac{\ell n\left(\dfrac{r_2}{r_1}\right)}{2\pi KL}$ K / watts

Logarithmic Temperature profile

7.3. Sphere with steady state, no heat generation:

Heat flow: $Q = \dfrac{4\pi k[T_1 - T_2]r_1 r_2}{(r_2 - r_1)}$ watt

Thermal resistance: $(R_{Th})_{spherical} = \dfrac{(r_2 - r_1)}{4\pi kr_1 r_2}$ K / watt

Hyperbolic temperature Profile

7.4. Convection thermal resistance:

- Heat flow: $Q = hA\Delta T$

☐ $\boxed{(R_{Th})_{convection} = \left(\dfrac{1}{hA}\right)}$ K / watt

☐   Higher the convection heat transfer coefficient h or more the convection heat transfer area, smaller the convection thermal resistance.

7.5. Conduction – Convection Heat Transfer Through A Composite Slab:

7.6. Overall Heat Transfer Coefficient (U):

Overall heat transfer coefficient is a parameter which consider all the types of heat transfer into single entity.

Unit – W/m^{2}K

☐ q = UA.ΔT watts

☐ $\boxed{(R_{Th}) = \left(\dfrac{1}{UA}\right)}$ K / watt

8. Critical Radius of Insulation:

It is the radius at which heat transfer rate from the surface is maximum.

For sufficiently thin wires, whose radius is lesser than critical radius of insulation, any insulation wrapped around the wire may result in increase of heat transfer rate (left hand side domain).
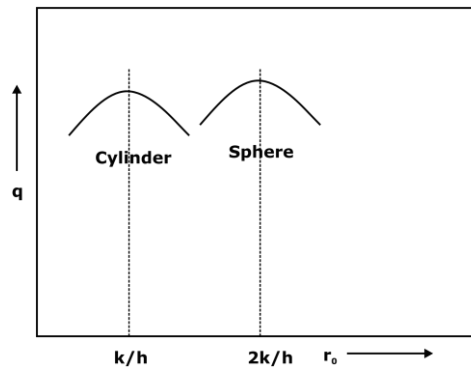
For cylinder: $\boxed{r_o = \dfrac{K_{ins}}{h_{surrouding}}}$

For sphere: $\boxed{(R_{critical})_{sphere} = \dfrac{2K_{ins}}{h_{surrouding}}}$

9. FINS (EXTENDED SURFACES):

Fins are projection protruding from the hot surface into ambient fluid to increase the heat transfer rate from the surface.

Fin efficiency: it is defined as the ratio of actual heat transfer from the fin to maximum possible heat transfer from the fin.

$$\text{Efficiency of fin}(\eta) = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin,max}}$$

Effectiveness of fin: it is defined as the ratio of heat transfer rate when the fin is attached to the surface to the heat transfer rate hen there is no fin.

$$\text{Effectiveness}(\varepsilon_{fin}) = \frac{\dot{Q}_{fin}}{\dot{Q}_{withoutf\,fin}}$$

Case 1: Fin is infinitely long (or) very long fin:

Boundary condition, At x= 0, T = T_O, & At x = ∞, T = $T_∞$

Temperature distribution: $\boxed{\dfrac{\theta}{\theta_o} = \dfrac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}}$

m, $m = \sqrt{\dfrac{hp}{kA_c}}$

where, A_c is the cross-sectional Area

P is the perimeter

Heat transfer through Fin: $\boxed{Q_{fin} = \sqrt{hPKA}\,\theta_0}$ watts

Fin efficiency: $\boxed{\eta_{fin} = \dfrac{1}{mL}}$

Fin effectiveness: $\boxed{\epsilon_{fin} = \dfrac{1}{\sqrt{\dfrac{hA}{kP}}}}$

Case 2: Fins is finite in length & its tip is insulated:

Boundary condition, At x= 0, T = T_O, & At x = L, $\dfrac{dT}{dx} = 0$

Temperature distribution: $\boxed{\dfrac{\theta}{\theta_0} = \dfrac{T - T_\infty}{T_0 - T_\infty} = \dfrac{\cosh m(L - x)}{\cosh mL}}$

Heat transfer through Fin: $\boxed{q_{thro\ fin} = \sqrt{hPkA}\,\theta_0 \tanh(mL)}$ watt

Fin efficiency: $\boxed{\eta_{fin} = \dfrac{\tanh(mL)}{mL}}$

10. Relation between efficiency and effectiveness of the fin:

$\boxed{\text{Effectiveness}(\varepsilon) = \dfrac{\text{Surface area }(A_s)}{\text{Cross} - \sec\text{tion area }(A_c)} \times \text{Efficiency}(\eta)}$

For Fin being effective:

For fin being effective: $\epsilon > 1$

$\boxed{\varepsilon = \dfrac{1}{\sqrt{B_i}}}$

Thus, fin will be more effective if Biot No. is less than 1.

Addition of extended surfaces will not always increase in the heat transfer rate. For effective heat transfer rate h < mk.

CHAPTER-2: TRANSIENT HEAT CONDUCTION

1. Transient Heat Conduction:

T = f (time) or T = f (τ) not the function of space when a body is kept in environment whose temperature is either less or more than body. This type of conduction is called transient heat conduction.

2. Lumped heat capacity analysis:

If in any material, the internal temperature gradient within the body are neglected i.e. the temperature of the body is assumed to be uniform throughout it mass at any instant of time. Such analysis is called **lumped Heat capacity analysis**.

T = f (time) but T ≠ f (space)

$$Bi = \frac{\text{Internal conductive resistance offered by body}}{\text{External convective resistance}} = \frac{ICR}{ECR}$$

- Biot Number: Bi < 0.1

- Biot Number: $Bi = \dfrac{hL_c}{K_{solid}}$

Where L_c = characteristic dimension of body

$$L_c = \left(\frac{V}{A}\right) = \frac{\text{volume}}{\text{Surface Area}}$$

For spherical Body: $L_c = \dfrac{R}{3}$

When a body having an initial temperature T is immersed in a fluid having temperature T∞ then, temperature of body after time "τ" will be T and can be calculated by given relation,

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{hA}{\rho v C_p}\right)\tau}$$
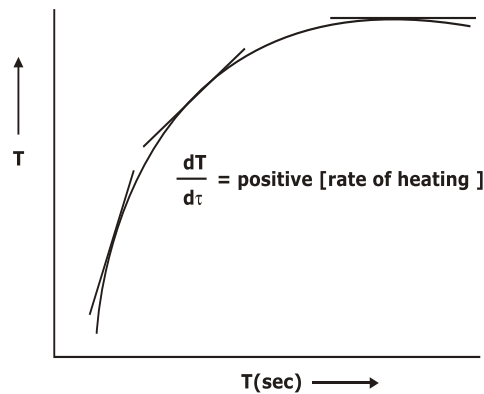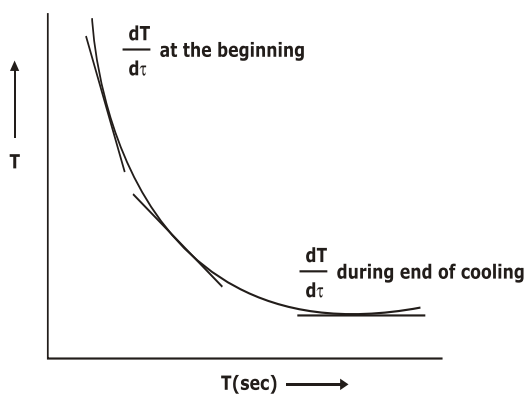
Fourier Number = It is the ratio of amount of heat conduction to the capacity of heat storage in given volume of element.

It signifies the degree of penetration of heating & cooling effect through a solid.

Fourier Number: $F_o = \dfrac{\alpha\tau}{L_c^2} = \dfrac{k}{\rho C}\dfrac{\tau}{L_c^2}$

Product of Biot number and Fourier number $= Bi \times Fo = \dfrac{hL_c}{k} \times \dfrac{k}{\rho C}\dfrac{\tau}{L_c^2} = \dfrac{h\tau}{\rho C L_c}$

Thus: $\dfrac{T - T_\infty}{T_i - T_\infty} = e^{-\left(\frac{hA}{\rho v C_p}\right)\tau} = \dfrac{T - T_\infty}{T_i - T_\infty} = e^{-Bi \times Fo}$ $\boxed{\left[\dfrac{(T - T_\infty)}{(T_i - T_\infty)}\right] = e^{-Bi \times Fo}}$

Time constant (τ) is given by:

$$\boxed{\tau = \dfrac{\rho V C_p}{hA}}$$

It signifies that larger masses ρV and larger heat capacities c_p lead to slower changes in temperature, while larger surface areas As and better heat transfer h lead to faster temperature changes.

CHAPTER-3: HEAT EXCHANGER

1. Introduction

Heat exchanger is a steady flow adiabatic device (open system) in which two flowing fluids exchange or transfer heat between them without using or gaining any heat from the ambient.

Heat exchanger classification by construction:
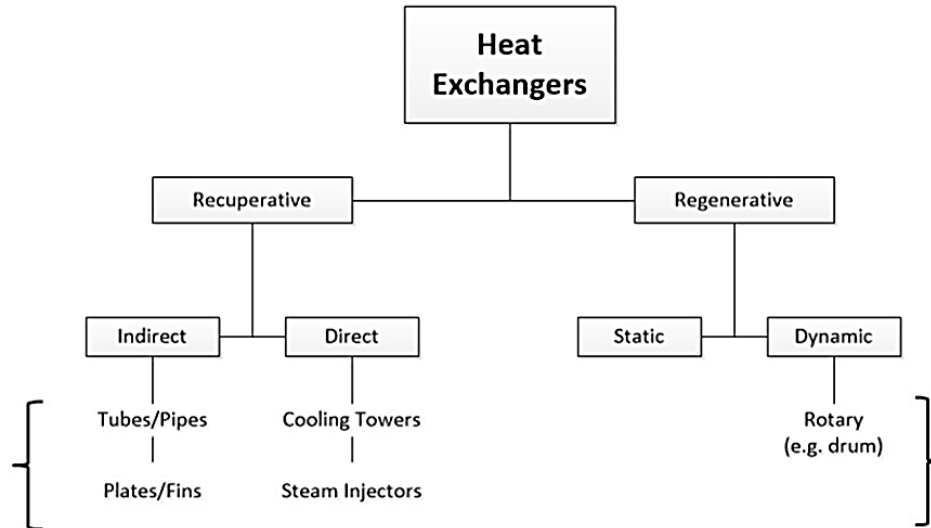
Base of Heat Exchanger

Rate of decrease of Enthalpy of hot fluid = Rate of increase of Enthalpy of cold fluid

$$m_h C_{ph} \left(T_{hi} - T_{he} \right) = m_c C_{pc} \left(T_{ce} - T_{ci} \right)$$

2. **Parallel flow heat exchanger:** Flow of hot and cold fluid are in same direction parallel to each other.
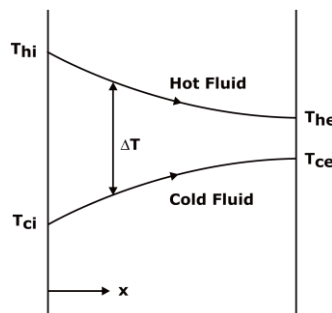
Logarithmic mean temperature difference is given by

$$LMTD = \left(\Delta T_m \right)_{parallel} = \frac{\Delta T_i - \Delta T_e}{\ln \dfrac{\Delta T_i}{\Delta T_e}}$$

Where, $\left(\Delta T_i \right) = T_{hi} - T_{ci}$

$\left(\Delta T_e \right) = T_{he} - T_{ce}$

Heat transfer rate

$$Q = m_h C_{ph} (T_{hi} - T_{he}) = m_c C_{pc} (T_{ce} - T_{ci}) = UA (\Delta T_m)_{parallel}$$

Effectiveness – Effectiveness of a heat exchanger is given as the ratio actual rate of heat transfer to the maximum possible rate of heat transfer.

$$\text{Effectiveness, } \in = \frac{Q_{actual}}{Q_{max.possible}} = \frac{m_h c_{ph}(T_{hi} - T_{he})}{(mc_p)_{small}(T_{hi} - T_{ci})} = \frac{m_c c_{pc}(T_{ce} - T_{ci})}{(mc_p)_{small}(T_{hi} - T_{ci})}$$

Effectiveness in terms of NTU and heat capacity ratio,

$$\text{Effectiveness: } \in_{parallel\ flow} = \left[\frac{1 - e^{-(1+c)NTU}}{1 + C}\right]$$

$$\text{Where, } C\,(\text{Heat Capacity}) = \frac{(mC_p)_{small}}{(mC_p)_{large}} \qquad 0 \le C \le 1$$

$$NTU\,(\text{Number of Transfer unit}) = \frac{UA}{(mC_p)_{small}}$$

3. **Counter flow heat exchanger:** Hot & Cold Fluid travel in the opposite direction parallel to each other.

$$(LMTD)_{counter} = \frac{\Delta T_i - \Delta T_e}{in\,\frac{\Delta T_i}{\Delta T_e}}$$

Where: $(\Delta T_i) = T_{hi} - T_{ce}$

$(\Delta T_e) = T_{he} - T_{ci}$

Heat transfer rate

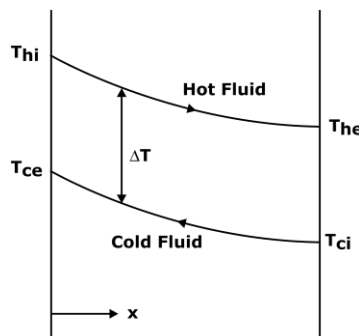$$Q = m_h C_{ph} (T_{hi} - T_{he}) = m_c C_{pc} (T_{ce} - T_{ci}) = UA (\Delta T_m)_{counter}$$

Effectiveness in terms of NTU and heat capacity ratio,

$$\text{Effectiveness: } \boxed{\in_{counterl\ flow} = \left[\frac{1 - e^{-(1-c)NTU}}{1 - Ce^{-(1-c)NTU}}\right]}$$

LMTD FOR CROSS FLOW HEAT EXCHANGER:

LMTD for cross-flow heat exchanger can be obtained as

$$\left(\Delta T_m\right)_{\text{cross flow}} = \left(\Delta T_m\right)_{\text{counter flow}} \times F$$

Where, F = correction factor (To be obtained from databook)

NOTE - For the same inlet & exit temperature of hot & cold fluids:

(ΔT_m) counter flow > (ΔT_m) shell & tube with one fluid condensing (surface condenser) > (ΔT_m) cross flow > (ΔT_m) parallel flow

F= < 1

4. Special cases:

Case-1: when the one of the fluids is changes its phase i.e one of the fluid id having infinite heat capacity.

$$C\,(\text{Heat Capacity}) = \frac{\left(mC_p\right)_{\text{small}}}{\left(m\,C_p\right)_{\text{large}}} = \frac{\left(mC_p\right)_{\text{small}}}{\infty} = 0$$
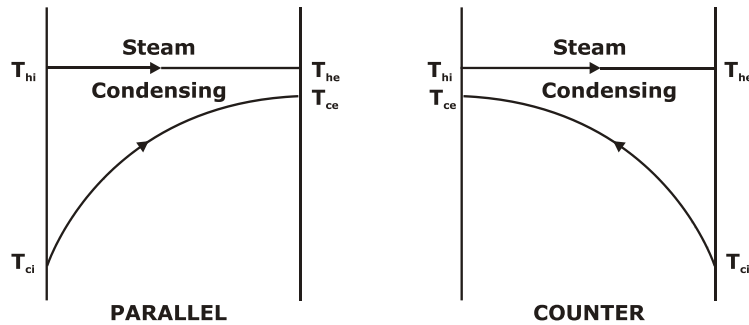
Condenser
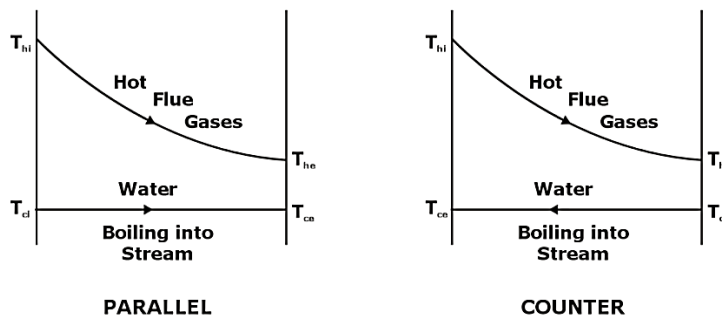
Evaporator

Effectiveness: $\in_{\text{parallel flow}} = \left[\dfrac{1 - e^{-(1+c)NTU}}{1 + C}\right] = 1 - e^{-NTU}$

Effectiveness: $\in_{\text{counterl flow}} = \left[\dfrac{1 - e^{-(1-c)NTU}}{1 - Ce^{-(1-c)NTU}}\right] = 1 - e^{NTU}$

Thus,

$$(\Delta T_m) \text{ parallel flow} = (\Delta T_m) \text{ counter flow}$$

Case-2: when both hot & cold fluids have equal capacity rates ($m_h Cp_h = m_c Cp_c$):

i.e. C = 1

Parallel Flow heat exchanger:

$$\in_{\text{parallel flow}} = \frac{1 - e^{-2NTU}}{2}$$

Counter flow Heat exchanger:

$$\in_{\text{counterl flow}} = \frac{NTU}{1 + NTU}$$
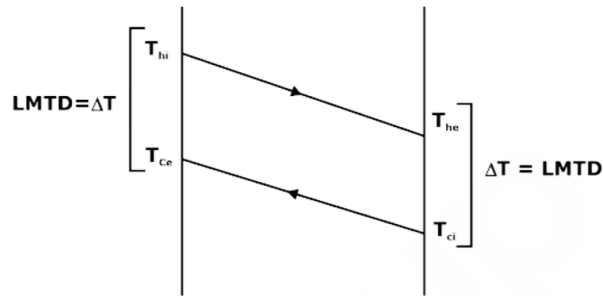
Then the temperature profiles in such a case will be two parallel lines.

Effectiveness in Terms of Temperatures Only –

CASE -1 When $m_h C_{ph} > m_c C_{pc}$

Thus, $(mC_p)_{\text{small}} = m_c C_{pc}$

$$\text{Effectiveness,} \in = \frac{Q_{\text{actual}}}{Q_{\text{max.possible}}} = \frac{m_c c_{pc}(T_{ce} - T_{ci})}{(mc_p)_{\text{small}}(T_{hi} - T_{ci})} = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$$

CASE -2 When $m_c C_{pc} > m_h C_{ph}$

Thus, $(mC_p)_{\text{small}} = m_h C_{ph}$

$$\text{Effectiveness,} \in = \frac{Q_{\text{actual}}}{Q_{\text{max.possible}}} = \frac{m_h c_{ph}(T_{hi} - T_{he})}{(mc_p)_{\text{small}}(T_{hi} - T_{ci})} = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$$

NOTE- Only one of the above two formulae given for effectiveness hold good at a time.

NOTE - In case if both the capacity rates are equal, any one of the above two formulae can be used.

5. Fouling Factor:

Fouling factor 'F' is the factor which takes into account the thermal resistance offered by any scaling or chemical deposit that is formed on the heat transfer surfaces over a period of time. Unit of Fouling Factor is (m^2 K/watt).

$$\frac{1}{U_{\text{with fouling}}} = \frac{1}{h_1} + F_{1 \text{ Hot side}} + \frac{1}{h_2} + F_{2 \text{ Cold side}}$$

6. Industries and Applications of Heat Exchangers by Type:

Type of heat exchanger	Applications
Shell and Tube	Oil refining, Preheating, Steam generation, Boiler blowdown heat recovery, Vapour recovery systems, Industrial paint systems
Double pipe	Industrial cooling processes, Small heat transfer area requirements
Plate type	Cryogenic, food processing, Chemical processing, Furnaces, Closed loop to open loop water cooling
Condensers	Power plants, Refrigeration, HVAC, Chemical processing, Distillation and refinement processes
Evaporators and boilers	distillation and refinement processes, Steam trains, Refrigeration, HVAC
Compact	Limited space requirements (e.g., aircrafts and automobiles), cryogenics, Electronics cooling

CHAPTER-4: RADIATION HEAT TRANSFER

1. Introduction:

☐ Radiation is the heat transfer by electromagnetic waves.

☐ Fastest method of heat transfer.

☐ No medium required.

☐ All bodies above temperatures absolute zero emits thermal radiations.
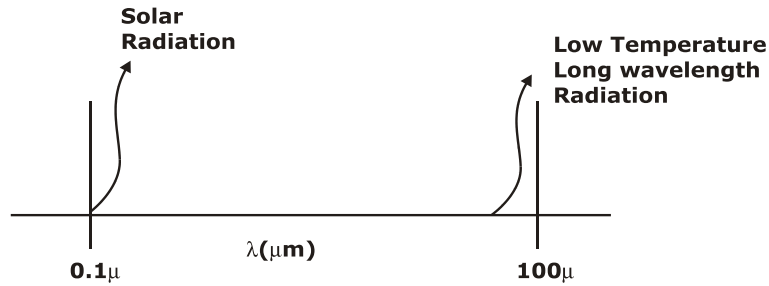
Electromagnetic Spectrum

Diffuse and Specular radiation:

Incident light reflected symmetrically with respect to the normal line is called "specular reflection," while incident light scattered in different directions is called "diffuse reflection."
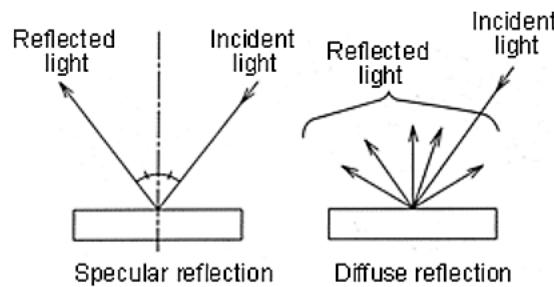
2. Basic definitions:

• Emissive Power is defined as the radiation energy emitted from the surface of a body per unit time & per unit area in all possible hemispherical directions integrated over all the wavelengths (in J/s m^2 or W/m^2).

☐ Black body is the body which absorbs all the thermal radiation incident or falling upon it. It is also the perfect emitter.

☐ **Emissivity (ϵ):**

The total hemispherical emissivity of a surface is simply the average emissivity over all directions and wavelengths:

$$\epsilon = \frac{E(T)}{E_b(T)}$$

☐ Monochromatic Emissive power is emissive power per unit wavelength. Spectral emissivity or monochromatic emissivity is defined in a similar manner:

$$\epsilon_\lambda (T) = \frac{E_\lambda(T)}{E_{b\lambda}(T)}$$

where E_λ (T) is the spectral or monochromatic emissive power of the real surface.

☐ A diffuse surface shall have the same intensity of radiation along al the directions.

- When the emissivity of the material does not change with temperature, it is called a gray body. A Gray body has a constant monochromatic emissivity, i.e. with respect to wavelength.

3. Absorptivity (α), Reflectivity (ρ) & Transmissivity (τ):

Absorptivity (α): It is the fraction of radiation energy incident upon a surface which is absorbed by it.

Reflectivity (ρ) is the fraction of radiation energy incident upon a surface which is reflected by it.

Transmissivity (τ) is the fraction of radiation energy incident upon a surface which is transmitted by it.

$$\boxed{\alpha \ + \ \rho \ + \ \tau \ = \ 1}$$

For opaque surface (τ =0): $\boxed{\alpha + \rho = \ 1}$

Note.

Air at atmospheric pressure and temperature is transparent to thermal radiation for which τ = 1 and α = ρ = 0.

For black body: α = ε = 1

There are some surfaces which absorb nearly all incident radiation, yet do not appear black. *Ice, snow, white-washed walls* have absorptivities greater than 0.95.

For white body: ρ = 1

3. Laws of thermal radiation:

3.1. Kirchhoff's law of radiation

The law states that "Whenever a body is in thermal equilibrium with its surrounding's its emissivity is equal to its absorptivity".

For a body in thermal equilibrium: α = ε

Kirchhoff's law also holds for monochromatic radiation: i.e. $\alpha_\lambda = \epsilon_\lambda$

3.2. Planck's law of thermal radiation
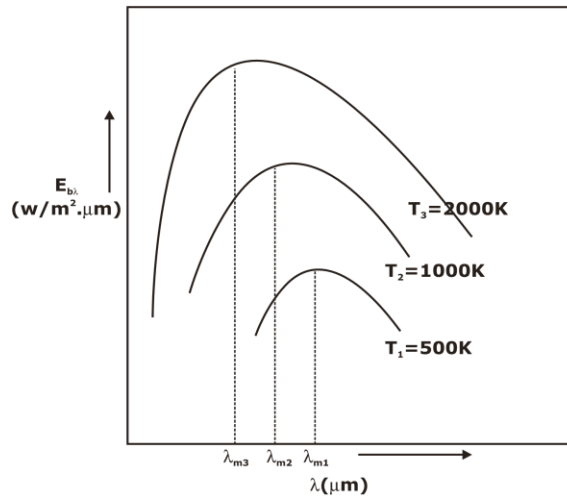
The law states that the monochromatic emissive power of black body is dependent on both absolute temperature of black body & also on wavelength of emission.

Planck deduced by his quantum theory the expression for $E_{b\lambda}$ as a function of temperature and Wavelength:

$$\boxed{E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}}$$

$E_{b\lambda}$ = f(λ,T), T is in kelvin.

Planck's law is the basic law of thermal radiation.

3.3. Rayleigh Jeans's Law ($C_2/\lambda T \lll 1$):

$$E_{b\lambda} = \frac{C_1 T}{C_2 \lambda^4}$$

This equation is known as Rayleigh–Jeans law which is found to be valid for large wavelengths.

3.4. Wein's Law ($C_2/\lambda T \ggg 1$):

$$E_{b\lambda} = \frac{C_1}{\lambda^5} e^{-C_2/\lambda T}$$

This equation is known as Wien's law, which is found to be valid for short wavelengths.

3.5. Wein's displacement law:

This law tells about the wavelength at which $E_{b\lambda}$ will be maximum at a specified temperature

$$\frac{C_2}{\lambda_{max} T} = 4.965$$

$$\lambda_m T = 2898 \ \mu m\text{-}K$$

The value of $(E_{b\lambda})_{max}$ shifts towards the shorter wave lengths with increasing temperature.

$$(E_{b\lambda})_{max} = C_3 T^5$$

The magnitude of $(E_b)_{max}$ is proportional to the fifth power of the absolute temperature of the body.

3.6. Stefan Boltzmann's law:

The law states that the total hemispherical emissive power of a black body is directly proportional to fourth power of the absolute temperature of black body.

$$E = \sigma \varepsilon T^4 \ W/m^2$$

σ = Stefan Boltzmann Constant= 5.67×10^{-8} W/m^2 K^4

For black body's: $\epsilon = 1$

$$\boxed{E_b = \sigma T^4} \; W/m^2$$

3.7. Lambert's cosine law:
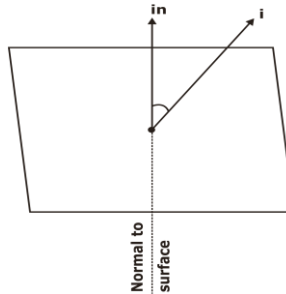
As per this law, Intensity of radiation in a direction θ from the normal to a diffuse emitter is proportional to cosine of the angle θ and given as,

$$i = i_n \cos \theta$$

Where, i_n = normal intensity of radiation

i = Intensity of Radiation along a direction making an angle θ w.r.t normal

3.8. Intensity of radiation:

The intensity of radiation, I is defined as the rate of heat radiation in a given direction from a surface per unit solid angle per unit area of the projection of the surface on a plane normal to the direction of radiation.

$$I = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi}$$

4. Shape factor or view factor or configuration factor:

The radiation heat transfer between bodies depends on their relative orientation. F_{12} (Shape factor of 1 relative to 2) is Fraction of radiation energy leaving surface 1 that reaches surface 2.
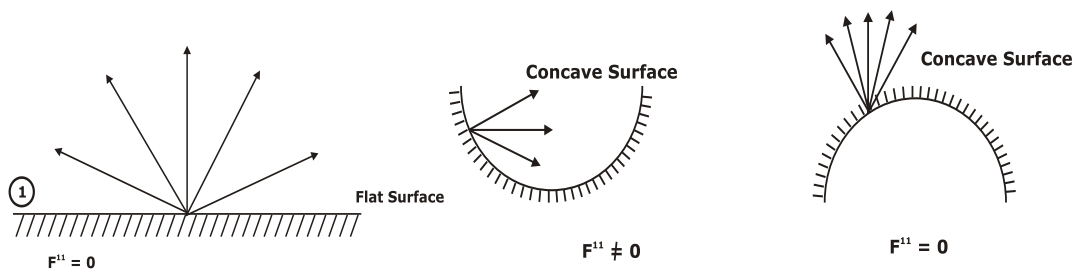
SOLID ANGLE

The relation of an angle and arc length to three dimensions is defined by solid angle.

4.1. SHAPE FACTOR OR VIEW FACTOR OR CONFIGURATION FACTOR

Radiation heat transfer between surfaces depends upon the orientation of the surfaces relative to each other. To account for the effects of orientation on radiation heat transfer

between the surfaces, a new parameter VIEW FACTOR is defined which is purely geometric quantity and is independent of radiation properties and temperatures.

NOTE – Shape factor for concave surface is always zero.

Shape factor for convex surface is always less than one.

Shape factor for plane or Flat surface is always zero.

4.1.1. Reciprocity relation:

Reciprocity Relation is valid between any two surfaces even when there are more than two number of surfaces involved in Radiation Heat Exchange & it is given as,

$$A_1 F_{12} = A_2 F_{21}$$

4.1.2. Summation rule:

If there are n number of surfaces involved in any radiation heat exchange then.

$F_{11} + F_{12} + F_{13} + \ldots\ldots\ldots + F_{1n} = 1$

$F_{21} + F_{22} + F_{23} + \ldots\ldots\ldots + F_{2n} = 1$

$F_{n1} + F_{n2} + F_{n3} + \ldots\ldots\ldots + F_{nn} = 1$

NOTE - If there are Total n surfaces of an enclosure, then total numbers of view factors that need to be evaluated directly $= {}^n C_2 = \dfrac{n(n-1)}{2}$

4.1.3. Symmetry rule:

Two or more surfaces that possess symmetry about a third surface, will have identical view factors from both the surface.

For equilateral triangle: $F_{12} = F_{13}$
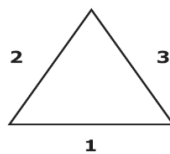
4.1.4. Enclosure:

Total number of shape factors in "n" surface enclosure is **n²**.

5. Radiation Networks:

Irradiation (G) is defined as the total thermal radiation incident upon a surface per unit time per unit area (in W/m²).

The total thermal radiation leaving a surface per unit time & per unit area is called **Radiosity**(J) of the surface.

Suppose a surface having emissivity ϵ & Irradiation is G, then Radiosity will be expresses as

$J = \varepsilon E_b + (1-\varepsilon) G$

• The net radiation heat liberated from the surface:

$q_{net} = (J - G)A$ Watts

$q_{net} = \dfrac{E_b - J}{\left(\dfrac{1-\varepsilon}{\varepsilon A}\right)}$ watts

Comparing it with Current voltage analogy,

$Re\,sis\tan ce = \dfrac{1-\varepsilon}{\varepsilon A}$, It is known as surface resistance.

q
surface - surroundings

E_b ———————⟶————⟩⟩⟩⟩⟩———————— J

$\left[\dfrac{I-\varepsilon}{\varepsilon A}\right]$

surface resistance of body
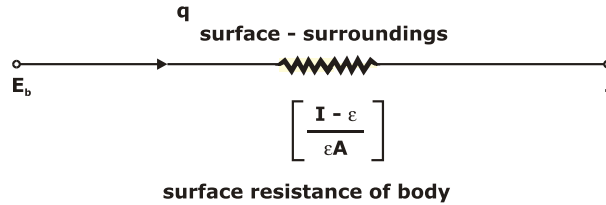
☐ Net radiation heat exchange between two black surface 1 and 2 having radiosity J_1 & J_2

q_{1-2}
net

J_1 ———————⟶————⟩⟩⟩⟩⟩———————— J_2

$\left[\dfrac{1}{A_1F_{12}}\right]$
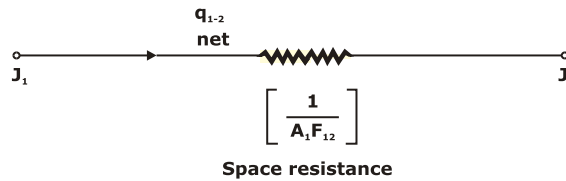
Space resistance

$q_{1-2} = J_1A_1F_{12} - J_2A_1F_{21}$

Since $A_1F_{12} = A_2F_{21}$

$q_{1-2} = \dfrac{J_1 - J_2}{\dfrac{1}{A_1F_{12}}} = \dfrac{J_1 - J_2}{\dfrac{1}{A_2F_{21}}}$

$\dfrac{1}{A_1F_{12}} = \dfrac{1}{A_2F_{21}}$ is called the space resistance.

• Net radiation heat exchange between two gray surface 1 and 2 having radiosity E_{b1} & E_{b2}

$\left(q_{(1-2)}\right)_{net} = \dfrac{E_{b1} - E_{b2}}{\sum R_{Th}} = \dfrac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$ Watts

Thus, two surface exchanging heat will consist two surface resistance (one on each) and one space resistance because of the gap prevails between them.

6. APPLICATIONS OF RADIATIONS NETWORKS

(a). Heat transfer rate between two infinitely large plane surfaces having emissivity ϵ_1 & ϵ_2
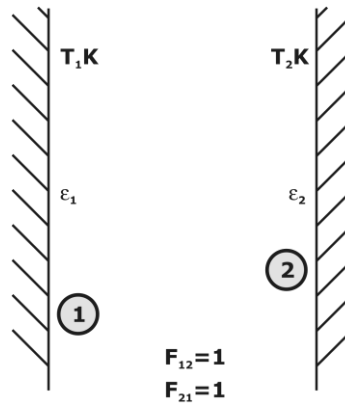
$$\text{Radiation Heat Flux} = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right)} \text{ watt / m}^2$$

(b). Two infinitely long cylinders having emissivity ϵ_1 & ϵ_2

$$\left(q_{1-2}\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right) A_1}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2}\left[\dfrac{1}{\varepsilon_2} - 1\right]} \text{ watts}$$

(c). A very small body is Kept in Very large room, i.e. $A_2 \ggg A_1$

$$A_1 \lll A_2 \Rightarrow \frac{A_1}{A_2} \approx 0$$

$$\left(q_{1-2}\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right) A_1}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2}\left[\dfrac{1}{\varepsilon_2} - 1\right]} \Rightarrow \left(q_{1-2}\right)_{net} = \frac{\sigma\left(T_1^4 - T_2^4\right) A_1}{\dfrac{1}{\varepsilon_1} + 0 \times \left[\dfrac{1}{\varepsilon_2} - 1\right]}$$

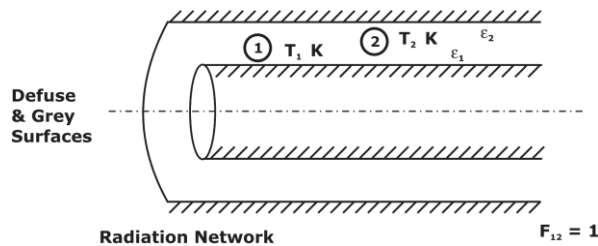$$\left(q_{1-2}\right)_{net} = \sigma\left(T_1^4 - T_2^4\right) A_1 \varepsilon_1$$

7. Radiation shields: It is simply a plane surface, which is place in between two surface exchanging heat such that the heat transfer rate can be reduced.

CASE – 1 if two infinitely long surfaces having emissivity ϵ_1 & ϵ_2 exchanging heat and a third surface having emissivity ϵ_3 is introduced between them then net radiation between them will be given as

$$\left(\left(q / A\right)_{(1-2)}\right)_{net\ with\ screen} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right) + \left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1\right)} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} + \left(\dfrac{2}{\varepsilon_3} - 2\right)}$$

NOTE - In case if all the surfaces including n screens have equal emissivity ϵ then

$$\left(q_{1-2}\right)_{\text{Net with 'n' screen}} = \left(\frac{1}{n+1}\right)q_{\text{without any screen}}$$

NOTE - If there are n number of screens being used between the planes, there will be total (2n + 2) surface resistances in the entire radiation network drawn with the screens thus each surface brings two space resistance and 1 space resistance.

CHAPTER-5: CONVECTION

1. Convection heat transfer: Convection is a mode of heat transfer which generally occurs between a solid & a fluid due to temperature difference associated **with macroscopic bulk displacement of the fluid** transporting thermal energy (heat energy).

The rate of heat transfer is governed by Newton's Law of cooling:

$q = hA(\Delta T)$

Where A = heat transfer surface area

h = heat transfer coefficient

CONVECTION	
Forced	**Free or Natural**
velocity is evident i.e. heat transfer due to bulk motion of the fluid.	No velocity is evident, but flow occurs due to buoyant forces arising out of density changes of fluid).

Various dimensionless number used in Convection

(a). Reynold's number (Re) – it is defined as the ratio of inertia force to viscous force.

$$Re = \frac{Inertia\,force}{Viscous\,force} = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

Where, D = characteristics dimensions, $D = \frac{4 \times cross\,sec\,tional\,Area}{Perimeter} = \frac{4A}{P}$

For circular pipe, D = d = diameter of pipe

For rectangular duct having dimension a×b, $D = \frac{4 \times cross\,sec\,tional\,Area}{Perimeter} = \frac{2ab}{a+b}$

It tells about the behaviour of the fluid whether it is laminar or turbulent.

For Laminar Flow

Through Pipe, Re < 2000

Through plate, Re < 5×10^5

For channel, Re < 1000

For turbulent Flow

Through Pipe, Re < 4000

Through plate, Re < 7×10^5

(b) Prandtl Number (Pr) = Prandtl Number (Pr) is the only dimensionless number which is a properly of the fluid defined as the ratio between kinematic viscosity of fluid & its thermal diffusivity.

$$Pr = \frac{Kinematic\,vis\,cosity}{Thermal\,Diffusivity} = \frac{\upsilon}{\alpha} = \frac{(\mu/\rho)}{(k/\rho C_p)} = \frac{\mu C_p}{K}$$

Prandtl number signifies the relative magnitude of momentum diffusion rate & thermal diffusion rate.

NOTE - (i). Pr of air = 0.65 to 0.71

(ii). Pr or water = 2 to 6

(iii). Pr of Mercury [Liquid Metal] is very Low, since k is very high.

(iv). Pr of Lubricating oils is high [may be up to 100].

(c). Nusselt number - It is the ratio of convective heat transfer to conductive heat transfer.

$$Nu = \frac{conductive\ resis\tan ce\ of\ fluid}{Surface\ convective\ resis\tan ce} = \frac{\dfrac{D}{Ak_f}}{\dfrac{1}{hA}} = \frac{hD}{k_f}$$

$$D = \frac{4 \times cross\ sec tional\ Area}{Perimeter} = \frac{4A}{P}$$

(d). Peclet Number – It is the product of Reynold number and Prandtl number.

$$Pe = Re \times Pr$$

(e). Stanton Number – It is the ratio of heat transfer coefficient to the flow of heat per unit temperature rise due to the velocity of the fluid.

It is also the ratio of Nusselt number and Peclet number.

$$St = \frac{Nu}{Pe} = \frac{Nu}{Re \times Pr} = \frac{h}{\rho VC_p}$$

2. Boundary layer:

2.1. Hydrodynamic boundary layer:

Hydrodynamic Boundary Layer is defined as a thin region formed on the plate inside which velocity gradients are seen in a normal direction to the plate.
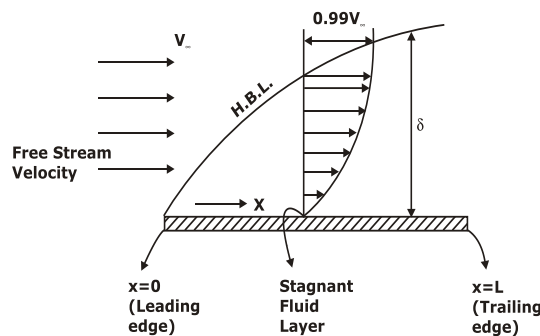
2.2. Thermal boundary layer: Thermal boundary Layer is a region inside which temperature gradients are seen in the normal direction to the plate.

T = f (x,y)

δ_t = Thickness of thermal Boundary Layer

δ_t = f(x)

The Thermal boundary layer thickness is defined as the distance measured form the solid boundary in y direction at which $\dfrac{T_s - T}{T_s - T_\infty} = 0.99$

$T_\infty > T_w$

Flow over

Relation between the Prandtl Number (Pr), thermal boundary layer thickness (δₜ) and velocity boundary layer thickness (δ):

$$\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}$$
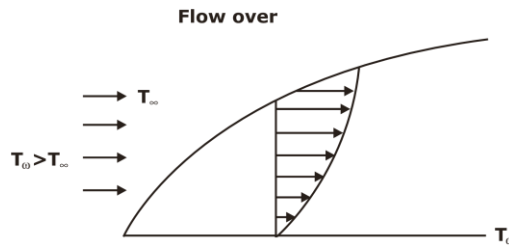
Case-1: If Pr > 1 then $\dfrac{\delta_t}{\delta} < 1$

Case-2: If Pr < 1 then $\dfrac{\delta_t}{\delta} > 1$

Case-3: If Pr ≈ 1 then δ ≈ δₜ

ENERGY BALANCE FOR THERMAL BOUNDARY LAYER-

In a thermal boundary layer, first heat energy gets conducted through plate and then convected through fluid.

$$dq = -K_f\, dA \left(\frac{\partial T}{\partial y}\right)_{at\, y=0} = h_x\, dA \left(T_w - T_\infty\right)$$

FOURIER's Law   NEWTON's LAW

$$h_x = \frac{-K_f \left(\dfrac{\partial T}{\partial y}\right)_{at\, y=0}}{\left(T_w - T_\infty\right)} \ W/m^2K$$

Where, h_x = Local convective heat transfer coefficient at that x
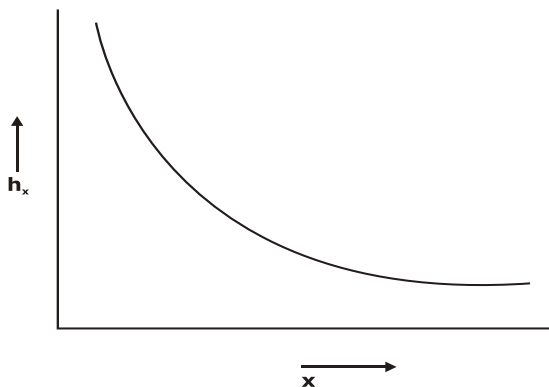
variation of hₓ with x

3. Free convection or natural convection:

Heat transfer takes place due to temperature difference present in the fluid in motion & the motion is fluid occurs due to buoyancy forces arising out of density changes.

Characteristic dimension for various geometry in Free convection:

For vertical plates & for vertical cylinder
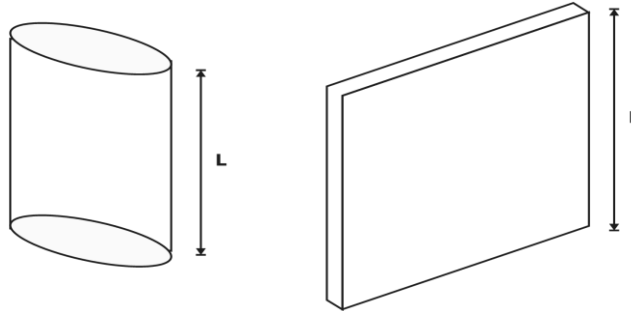
For horizontal cylinder & sphere -
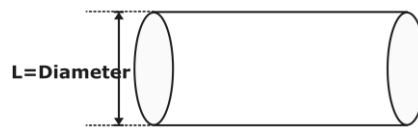
For Sphere
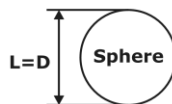
For horizontal plate

$$L = \frac{\text{Area}}{\text{perimeter}} = \frac{ab}{2(a+b)}$$
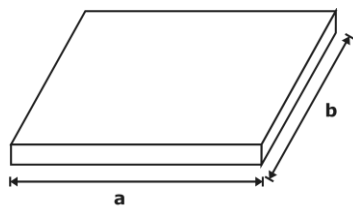
Grashoff number (Gr):

$$Gr = \frac{\text{Inertia Force} \times \text{Buoyancy Force}}{(\text{viscous force})^2}$$

$$Gr = \frac{g\beta\Delta TL^3}{\left[\dfrac{\mu}{\rho}\right]^2} = \frac{g\beta\Delta TL^3}{\upsilon^2}$$

28

Where, β = Isobaric volume expansion coefficient of fluid $\beta = \dfrac{1}{V}\left(\dfrac{\partial V}{\partial T}\right)_{P=C}$ [per kelvin]

In any free convection heat transfer,

Nu = f(Gr Pr)

Rayleigh Number – It is the product of Grashoff Number and Prandtl Number.

Rayleigh number tells weather the flow is laminar or turbulent.

If Ra $< 10^9$ then the flow is laminar

If Ra $> 10^9$ then the flow is turbulent

Relation between Nusselt Number and Grashoff Number-

Nu = C (Gr Pr)^m

C & m are constants which vary from case to case.

m = 1/4 for Laminar Flow

m = 1/3 for Turbulent Flow

****

Important Formula (Machine Design)

CHAPTER-1: STATIC, DYNAMIC & FATIGUE LOADING

Static Load: Static load is defined as a force, which is gradually applied in a mechanical component and which does not change its magnitude or direction with respect to time.

Fatigue Loading – Fatigue load is defined as the load whose magnitude or direction or both changes with respect to time and the same load is applied repeatedly.

Fatigue – Fatigue is the phenomenon of failure under fatigue stress condition.

Factor of Safety: The factor of safety is defined as the ratio of Failure stress to the permissible stress or Allowable stress.

$$FOS = \frac{\text{Failure Stress}}{\text{Allowable Stress}}$$

For ductile materials, the allowable stress, σ is obtained by the following relationship,
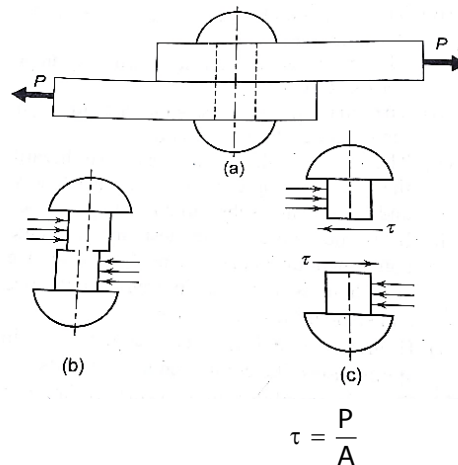
$$\sigma = \frac{S_{yt}}{FOS}$$

For brittle materials, the relationship is $\sigma = \frac{S_{ut}}{FOS}$

Modes of failure

A mechanical component may fail as a result of any one of the following three modes of failure:

• Failure by elastic deflection

• Failure by general yielding

• Failure by fracture

Shear Stress:

$$\tau = \frac{P}{A}$$

where, τ =shear stress (N/mm² or MPa), and A= cross-sectional area under shear of the rivet (mm²)

Stress Concentration: Stress concentration is defined as the localization of high stresses due to the irregularities present in the component and abrupt changes of the cross-section.
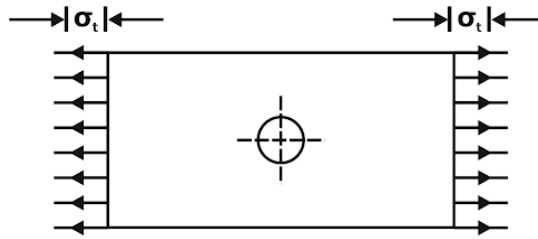
The following figure shows the stress distribution when the effect of stress concentration is taken into account.
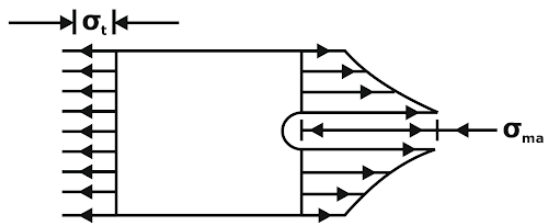
In order to consider the effect of stress concentration and find out localized stresses, a factor called **theoretical stress concentration factor** is used.

It is denoted by K_t and defined as,

$$K_t = \frac{\text{Maximum stress near the discontinuity}}{\text{Nominal stress near the discontinuity}} = \frac{\sigma_{max.}}{\sigma_n} = \frac{\tau_{max.}}{\tau_n}$$

$$\sigma_{max.} = K_t \times \sigma_n$$

NOTE – The value of theoretical stress concentration factor is always greater than or equal to one.

Theoretical stress concentration factor for an elliptical hole in a plate.

If "a" (semi major axis) is the half-length perpendicular to the load and "b" (semi minor axis) is the half-length parallel to the load as shown in figure then theoretical stress concentration factor is given as
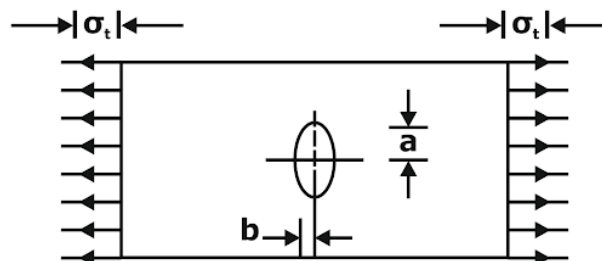
$$K_t = \frac{\sigma_{max}}{\sigma} = 1 + 2\left(\frac{a}{b}\right)$$

Fatigue stress concentration factor – It is the ratio of endurance limit of unnotched specimen to endurance limit of notched specimen.

$$K_f = \frac{\text{Endurance limit of unnotched specimen}}{\text{Endurance limit of notched specimen}}$$

NOTE – The value of Fatigue stress concentration factor is always greater than one.

Notch Sensitivity (q):

$$q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

Relation between Fatigue stress concentration factor and theoretical stress concentration factor,

$$K_f = 1 + q\,(K_t - 1)$$

Where K_f = fatigue stress concentration factor

K_t = theoretical stress concentration factor

NOTE – If q = 0, $K_f = 1$ & If q = 0, $K_f = K_t$

Thus, $1 < K_f < K_t$

Causes of stress concentration are as follows:

(a)Variation In Properties Of Materials:

☐ Internal cracks and flaws like blow holes

☐ Cavities in welds

☐ Air holes in steel components

☐ Non-metallic or foreign inclusions

(b)Load Application:

☐ Contact between the meshing teeth of the driving and the driven gear.

☐ Contact between the cam and the follower

☐ Contact between the balls and the races of ball bearing

REDUCTION OF STRESS CONCENTRATION:

ADDITIONAL NOTCHES AND HOLES IN TENSION MEMBER,

(i) The severity of stress concentration can be reduced by providing multiple notches.
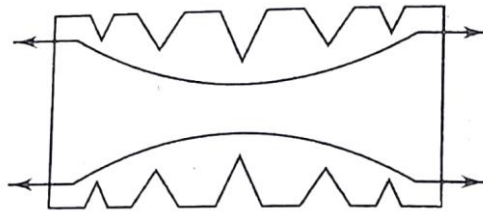
(ii) The severity of stress concentration can be reduced by providing multiple drilled holes along with notches.
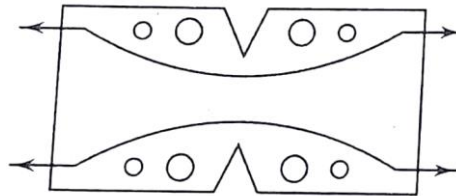
(iii) The severity of stress concentration can be reduced by providing undercut and notches.
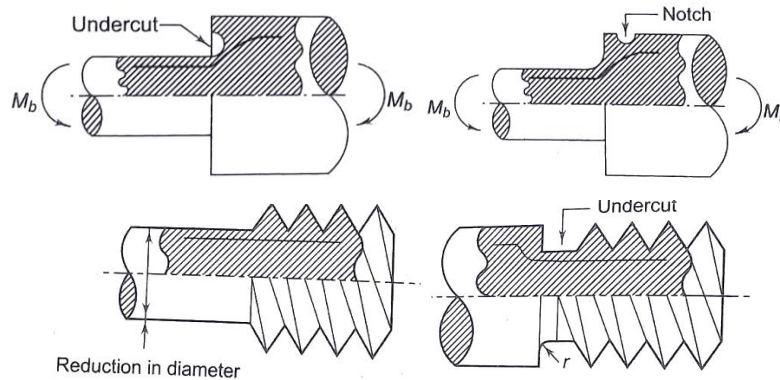
Types of Fatigue Stress -

(i) Completely reversed stress – In this type of stress, magnitude of stresses does not change, only direction changes.

Example = stress changes from +100MPa to -100MPa

(ii) Fluctuating Stress - In this type of stress, magnitude of stresses changes but direction does not change.

Example = stress changes from +100MPa to +200MPa

(iii) Alternating Stress - The stresses which vary from a minimum value to maximum value of the opposite nature (i.e. from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called alternating stresses.

Example = stress changes from +100MPa to -200MPa

(i) Repeated Stress - The stresses which vary from zero to a certain maximum value are called repeated stresses

Example = stress changes from +100MPa to 0MPa or -50MPa to 0MPa.

Important Terms

Stress Cycle: It is the smallest portion of stress time plot which repeats itself periodically and identically.

Maximum Stress: It is the largest algebraic stress in a stress cycle.

Minimum Stress: It is the smallest algebraic stress in a stress cycle.

Mean/ Average Stress: It is the average of maximum Stress & minimum Stress.

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Range of Stress: It is the difference of Maximum Stress & Minimum Stress.

$$Range = \sigma_{max} - \sigma_{min}$$

Variable Stress or Stress Amplitude: It is the difference of maximum Stress & minimum Stress.

$$\sigma_{Variable} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

NOTE – Mean stress mean may positive, negative or zero value but variable stress will always be positive.

Stress Ratio: It is the ratio of minimum stress to maximum stress.

$$Stress\,ratio, R = \frac{\sigma_{min}}{\sigma_{max}}$$

Amplitude Ratio – It is the ratio of variable stress to mean stress.

$$Amplitude\,ratio, A = \frac{\dfrac{\sigma_{max} - \sigma_{min}}{2}}{\dfrac{\sigma_{max} + \sigma_{min}}{2}} = \frac{\sigma_{max} - \sigma_{min}}{\sigma_{max} + \sigma_{min}} = \frac{1 - R}{1 + R}$$

Fatigue failure: Fatigue failure is defined as time delayed fracture under cyclic loading.

• The fatigue failure depends upon a number of factors, such as number of cycles, mean stress, stress amplitude, stress concentration, residual stresses, corrosion, size of the component and creep.

Fatigue Life – Fatigue Life of a component is defined as the number of revolutions it has undergone before evidence of first fatigue crack.

Endurance limit: Endurance limit of a material is defined as the maximum value of completely reversed stress that the standard specimen can sustain for an infinite number of cycles (10^6 cycles) without fatigue failure.

S - N Curve – it is plot between logS (stress) Vs logN (Number of revolutions).
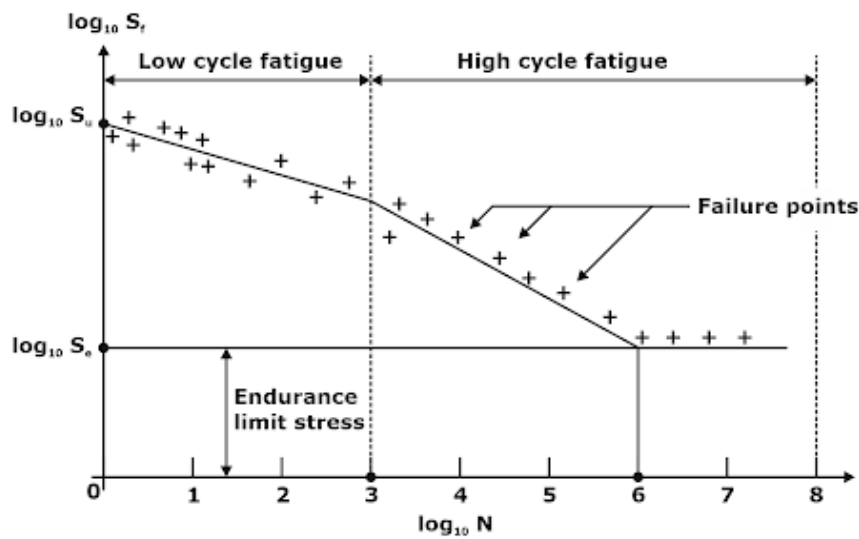
NOTE- If stress induced is above endurance limit, then we will get finite life cycle i.e. in between 10^3 to 10^6 revolution.

If stress induced is below endurance limit, then we will get infinite life cycle i.e. more than 10^6 revolution.

High Cycle Fatigue- If failure takes place above 1000 cycles then it is known as High Cycle Fatigue.

Low Cycle Fatigue- If failure takes place under 1000 cycles then it is known as Low Cycle Fatigue.

Corrected Endurance limit:

S'_e= Endurance limit stress of a rotating beam specimen (standard specimen) subjected to reversed bending stress (N/mm^2)

S_e= Corrected endurance limit stress of a particular mechanical component subjected to reversed bending stress (N/mm^2)

$S_e = K_a K_b K_c K_d S'_e$

K_a= Surface finish factor

K_b= Size factor

K_c= Reliability factor

K_d= Modifying factor to account for stress concentration

12.3 Reversed Stress - Design For Finite and Infinite Life

Case 1: When the component is to be designed for infinite life, the endurance limit becomes the criterion of failure. Such components are designed with the help of the following equations:

$$\sigma_a = \frac{S_e}{FOS} \text{ and } \tau_a = \frac{S_{se}}{FOS}$$

where σ_a and τ_a are stress amplitude in the component and S_e and S_{se} are corrected endurance limits in reversed bending and torsion respectively.

Case 2: When the component is to be designed for finite life, the S-N curve can be used. The curve is valid for steels.
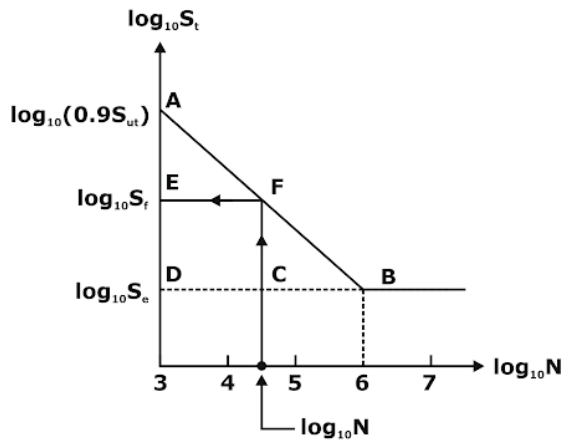
13. Soderberg, Goodman and Gerber lines:

These equations should be used for safe design of a component under fluctuating, alternating and repeated stress loading condition.
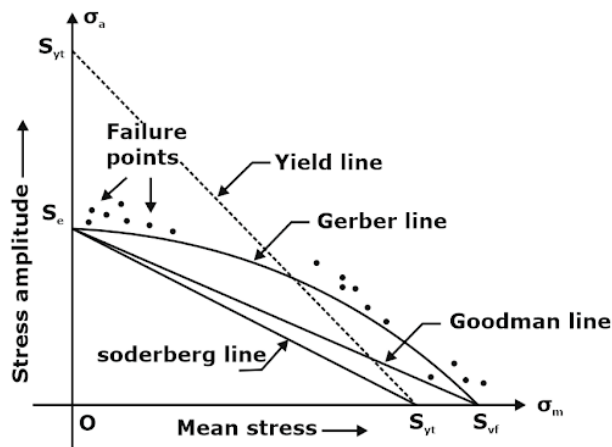
SODERBERG LINE:

A straight line joining S_e on the ordinate to S_{yt} on the abscissa

$$\frac{\sigma_m}{S_{yt}} + k_f \frac{\sigma_a}{S_e} = \frac{1}{FOS}$$

GOODMAN LINE:

A straight line joining S_e on the ordinate to S_{ut} on the abscissa

$$k_t \frac{\sigma_m}{S_{ut}} + k_f \frac{\sigma_a}{S_e} = \frac{1}{FOS}$$

Gerber Parabola:

A parabolic curve joining S_e on the ordinate to S_{ut} on the abscissa,

$$k_t \left(N \frac{\sigma_m}{S_{ut}} \right)^2 + k_f N \frac{\sigma_a}{S_e} = 1 , \text{ Where n= FOS}$$

$$N \times k_t \left(\frac{\sigma_m}{S_{ut}} \right)^2 + k_f \frac{\sigma_a}{S_e} = \frac{1}{N}$$

Modified Goodman Diagrams:
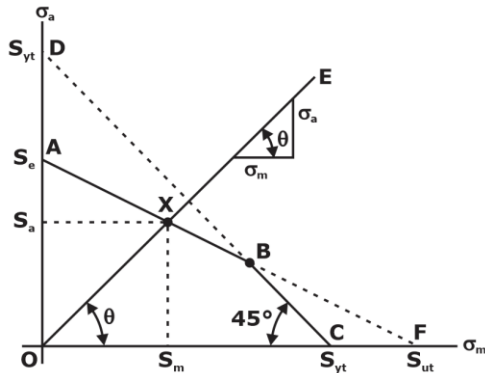
$$\tan \theta = \frac{\sigma_a}{\sigma_m} , \text{ and } \frac{\sigma_a}{\sigma_m} = \frac{(P_a/A)}{(P_b/A)} = \frac{P_a}{P_m} , \text{and} \tan \theta = \frac{P_a}{P_m}$$

The magnitudes of P_a and P_m can be determined from maximum and minimum forces acting on the component.

Similarly, it can be proved that $\tan \theta = \dfrac{(M_b)_a}{(M_b)_m}$

The magnitudes of $(M_b)_a$ and $(M_b)_m$ can be determined from maximum and minimum bending moment acting on the component.

CHAPTER-2: JOINTS

Introduction:
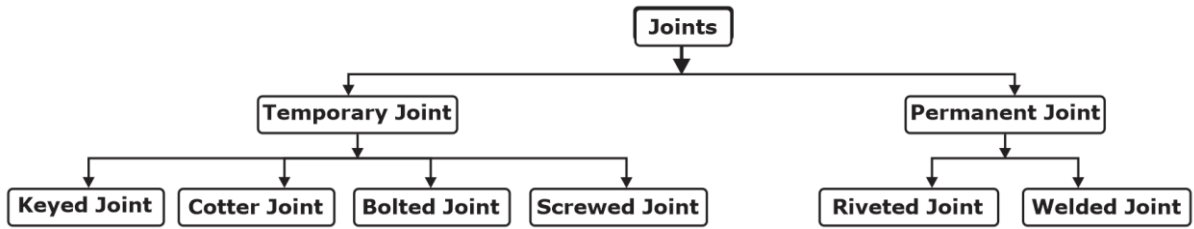
Riveted Joints: A rivet consists of cylindrical shank with a head at one end.

A rivet is specified by the shank diameter of the rivet.

(i)Lap Joint:

Lap joint consists of two overlapping plates, which are held together by one or more rows of rivets Depending upon the number of rows of rivets.

Lap joints are further classified into single-riveted lap joint, double-riveted lap joint.
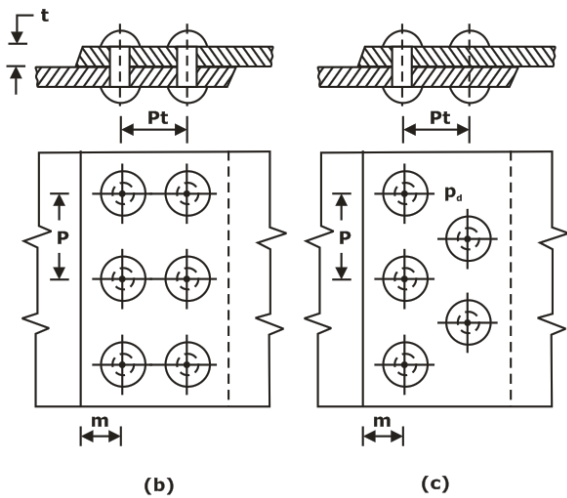
(ii)Butt Joint:

A Butt joint consists of two plates, which are kept in alignment against each other in the same plane and a strap or cover plate is placed over these plates and riveted to each plate.

• Depending upon the number of rows, butt joints are classified into single-row butt joint and double-row butt joint.

• Depending upon the number of straps, the butt joints are classified into single-strap butt joint and double-strap butt joint.

Terminology of Riveted Joints:

Pitch (p): Distance between the centre of one rivet to the centre of the adjacent rivet in the same row. Usually, P=3d

Margin (m): Distance between the edge of the plate to the centreline of the rivets in the nearest row. Usually, m=1.5d

Transverse pitch (p_t): Also called back pitch or row pitch, is the distance between two consecutive rows of rivets in the same plate.

Usually, P_t = 0.8p (for chain riveting)

P_t = 0.6p (for zig-zag riveting)

• **Diagonal pitch (p_d):** Distance between the centre of one rivet to the centre of the adjacent rivet located in the adjacent row.

Types of Failure in Riveted Joints:

(a)   Shear failure of the rivet

(b)   Tensile failure of the plate between two consecutive rivets

(c)   Crushing failure of the plate

(d)   Shear failure of the plate in the margin area

(e)   Tearing of the plate in the margin area

(a)   (b)   (c)   (d)   (d)
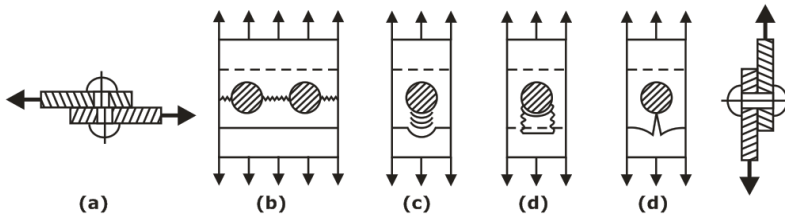
• **Shear strength of Riveted Joint:**
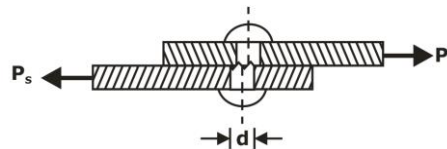
In case of double or triple riveted lap joints

$$P_s = n\frac{\pi}{4}d^2\tau_{per}$$

n= number of rivets per pitch length,

For double-riveted n=2, and for triple-riveted n=3,
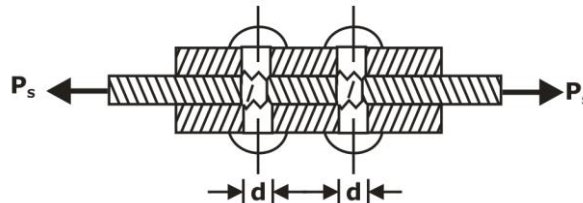
If there more than one shear plane, then

$$P_s = nK\frac{\pi}{4}d^2\tau_{per}$$

Where K is the number of shear plane.

- **Tensile Strength of Plate between Rivets:**
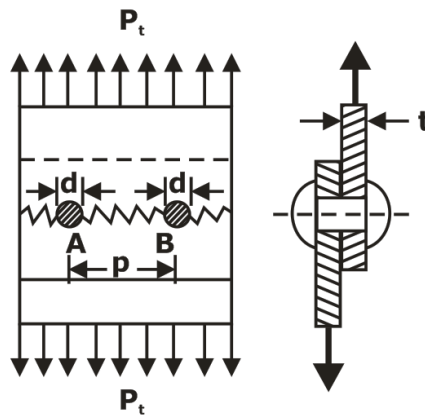
Width, $w = (p - d)$, thickness $= t$

Tensile strength $P_t = (p - d)t\sigma_t$

It is also be given as, $P_t = (b - N_R d_h)t.\sigma_t$

Where, N_R = Number of Rivet in each row.

D_h = diameter of hole

- **Crushing Strength of Riveted joint:**
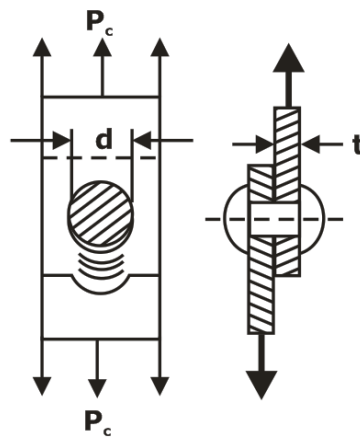
Crushing strength of Riveted Joint $P_c = ndt\sigma_c$,

Where σ_c = Crushing or bearing strength

dt = Crushing Area

NOTE- Strength of any rivet is given by Minimum of Crushing strength and shear strength.

- **Efficiency of Riveted Joint:**

Strength of solid plate $P = pt\sigma_t$

$$\text{Efficiency}(\eta) = \frac{\text{Lowest of } P_s, P_t \text{ and } P_c}{P}$$

• Tearing Efficiency:

Tearing efficiency in terms of width of plate,

$$\eta_{\text{Tearing}} = \frac{P_t}{P_{\text{solid plate}}} = \left(1 - \frac{N_R d_h}{b}\right) \times 100$$

Tearing efficiency in terms of pitch,

$$\eta_{\text{Tearing}} = \frac{P_t}{P_{\text{solid plate}}} = \left(1 - \frac{d_h}{p}\right) \times 100$$

Riveted joints under Eccentric Loading:

An eccentrically loaded riveted joint is shown in figure
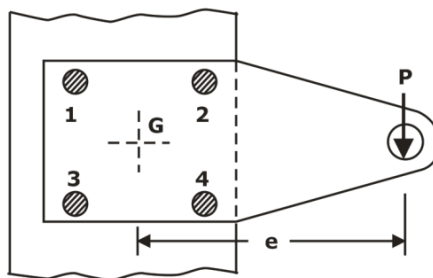
STEPS - Find the centre of gravity of the system.

Primary shear forces P'_1, P'_2 etc. given as

$$P_1' = P_2' = P_3' = P_4' = \frac{P}{(\text{No. of Rivets})}$$

The moment $(P \times e)$ about the centre of gravity results in secondary shear forces $P_1", P_2"$

……. etc.
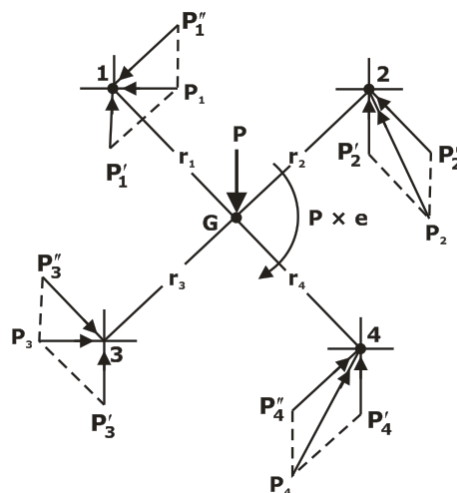
$$P \times e = P_1" r_1 + P_2" r_2 + P_3" r_3 + P_4" r_4$$

$P_1'' = Cr_1, P_2'' = Cr_2 \dots$ etc.

$$C = \frac{P \times e}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$P_1'' = \frac{P \times e \times r_1}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}, \quad P_2'' = \frac{P \times e \times r_2}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

The primary and secondary shear forces are added by vector addition method to get the resultant shear forces P_1, P_2, P_3 and P_4.

$$P_{Resultant} = \sqrt{P_P^2 + P_s^2 + 2P_P P_s \cos\theta}$$

NOTE – (1) If all the rivets are equidistance from CG then the maximum stress (Worst rivet) will be induced in the rivet in which the angle between primary shear force and secondary shear force is minimum.

(2) Secondary shear force is maximum for the rivet which is nearest from the line of action of force.

(3) If all the rivets are arranged in a single column then rivet which is far away from the CG of group is the worst rivet.

WELDED JOINTS:

• Welding can be defined as a process of joining metallic parts by heating to a suitable temperature with or without the application of pressure and with or without the addition of filler metal.

Welding Symbols:

 The basic symbols used to specify the type of weld are shown:

Type of Weld	Symbol
Fillet	◣
Square Butt	⏛
Single V-Butt	▽
Double V-Butt	⨂
Spot	✳
Speam	✕✕✕
Projection	△

(a) Double Fillet Weld

(b) Single Fillet Weld

(c) Single V-Butt Weld

(d) Double V-Butt Weld

(e) Lap joint

(f) Edge joint
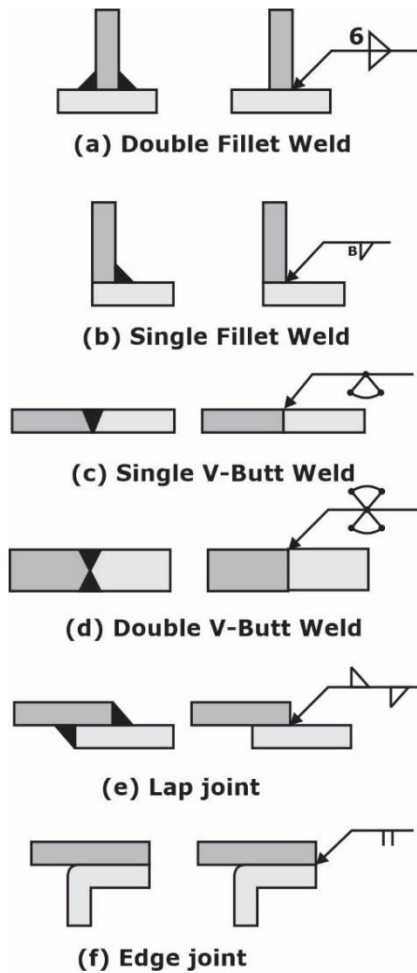
3.1 Strength of Butt Joints:
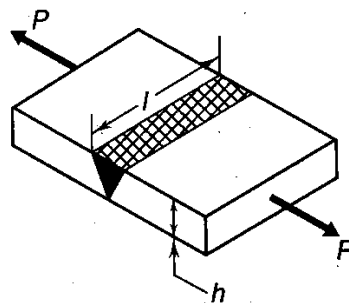
The average tensile stress in the weld is given by, $\sigma_t = \dfrac{\text{Force}}{\text{Throat Area}} = \dfrac{P}{tl}$ …….. (i)

Relation between throat thickness and les of the fillet weld

$t = \dfrac{h}{\cos\theta + \sin\theta}$

The throat is the minimum cross-section of the weld located at 45° to the leg dimension.

Strength of Fillet weld,

$P_S = \sigma_t \times t \times l$

If efficiency of welded joint is also given,

Then, $P_S = \sigma_t \times t \times l \times \eta$

Where η = efficiency of the welded joint (in fraction)

Fillet Joint:

(A) Parallel Fillet Welds: Weld is parallel to the length of the plate & direction of load is parallel to the length of the weld.

The throat is the minimum cross-section of the weld located at 45° to the leg dimension.

$t = h \, Cos45°$ and $t = 0.707h$
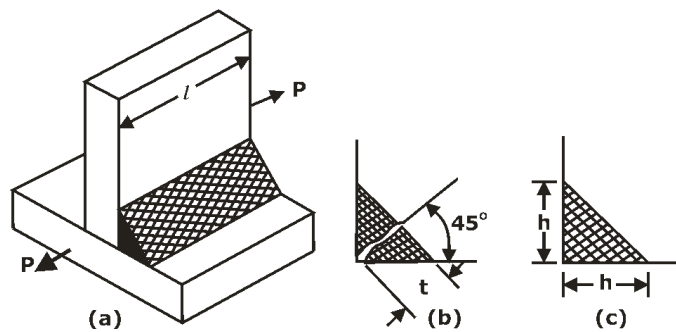
The shear stress in the fillet weld is given by, $\tau = \dfrac{P}{0.707hl}$

For two welds of equal length on two sides of the vertical plate

$P = 2 \, (0.707hl)\tau = 1.414hl\tau$

(B) Transverse Fillet Welds: Weld is perpendicular to the length of the plate & direction of load is perpendicular to the length of the weld.
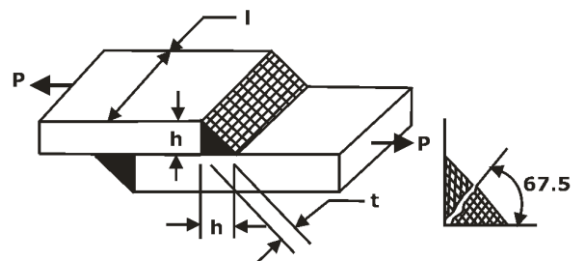
The transverse fillet welds are subjected to both tensile stress and shear stress. The inclination of the plane, where maximum shear stress is induced, is 67.5° to the leg.

Area of the weld =0.765hl

• The tensile stress in the transverse fillet weld is given by $\sigma_t = \dfrac{P}{tl}$

$P = 0.707hl\sigma_t$

• Two welds of equal length on two sides of the vertical plate, $P = 1.414hl\tau$

NOTE- In order to simplify the design of fillet welds, shear failure is used as the failure criterion.

Comparison of Parallel Fillet Weld and Transverse Fillet Weld

Parameters	Parallel Fillet Weld	Transverse Fillet Weld
Direction of load	Parallel to length of Weld	Perpendicular to length of Weld
Shear load	P	$P\sin\theta$
Normal load	zero	$P\cos\theta$
Location of Throat Plane	45°	67.5°
Throat thickness	0.707h	0.765h
Area of Weld	0.707hl	0.765hl
Strength of Weld	$0.707hl\tau_{per}$	$0.832hl\tau_{per}$

Axially Loaded Unsymmetrical Welded Joints:
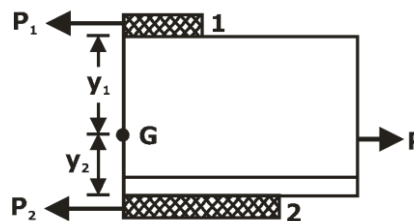
• P_1 and P_2 are the resisting forces set up in the welds 1 and 2 respectively,

• Sum of horizontal forces is equal to zero, $P = P_1 + P_2$

• $P_1 = 0.707hl_1\tau$ and $P_2 = 0.707hl_2\tau$

• moment of forces about the centre of gravity is equal to zero

$P_1y_1 = P_2y_2$

Assuming total length of welds as $l = l_1 + l_2$

Circular Fillet Weld under Pure Torsion- If a circular weld having leg h and throat thickness t is subjected to pure torsion as shown in figure,

Then,

Strength of the Fillet weld be given as, $\tau_{per} = \dfrac{2\sqrt{2}T}{\pi d^2 h}$

Circular Fillet Weld under Pure Bending- If a circular weld having leg h and throat thickness t is subjected to pure bending as shown in figure,

Then,

Strength of the Fillet weld be given as, $\left(\sigma_b\right)_{per} = \dfrac{2\sqrt{2}M}{\pi d^2 h}$

Eccentric Load in the Plane of Welds:

(a)   (b)   (c)   (d)
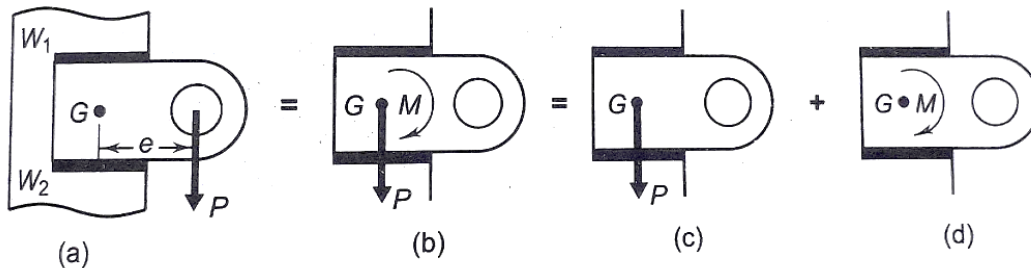
• Suppose G is the centre of gravity of two welds and e is the eccentricity between the centre of gravity and the line of action of force P.

• The stresses in this welded joint are shown in figure,

(a)   (b)   (c)
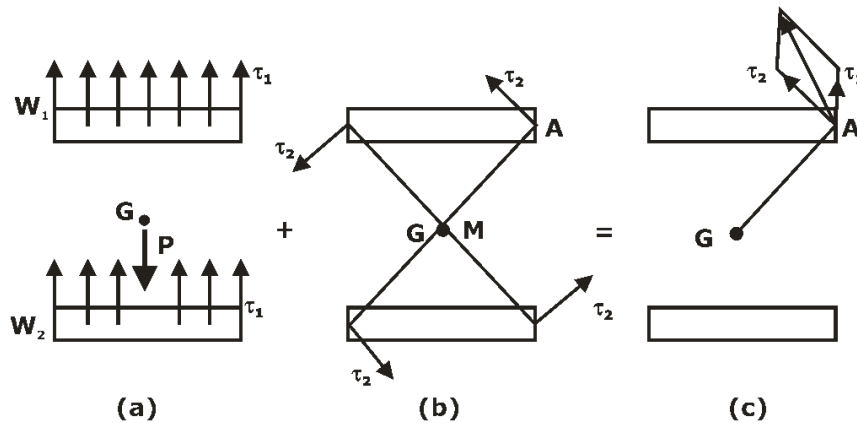
• The force P acting through the centre of gravity causes direct shear stress in the welds. The primary shear stress $\tau = \dfrac{P}{A}$,A is the throat area of all welds.

• The couple M causes torsional shear stresses in the throat area of welds.

Secondary shear stresses (due to couple acting at centre of gravity), $\tau_2 = \dfrac{Mr}{J}$

Where, M = P×e

r = distance of a point in the weld from G

J = polar moment of inertia of all welds about G

Welding Joint subjected to Bending Moment:
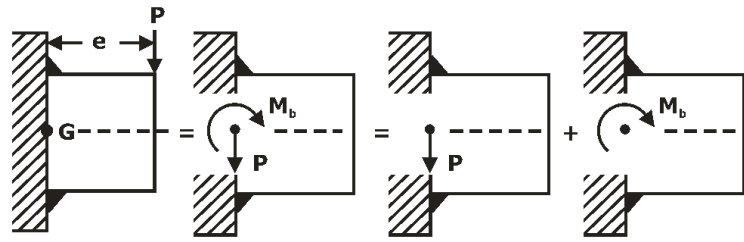
The force P through the plane of welds causes the primary shear stress τ_1, which is given by $\tau_1 = \dfrac{P}{A}$, where A is the throat area of all welds.

The moment M_b causes bending stresses in the welds.

The bending stresses are given by, $\sigma_b = \dfrac{M_b.y}{I}$

The resultant shear stress in the welds is given by,

$$\tau = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + (\tau_1)^2}$$

• It is observed that the allowable load for a transverse fillet weld is more than that of a parallel fillet weld.

• The strength of transverse fillet weld is 1.17 times of the strength of parallel fillet weld.

THREADED JOINTS:

Threaded joint is a separable joint of two or more machine parts which are held together by means of a threaded fastening such as a bolt and a nut.

Terminology of Screw Threads
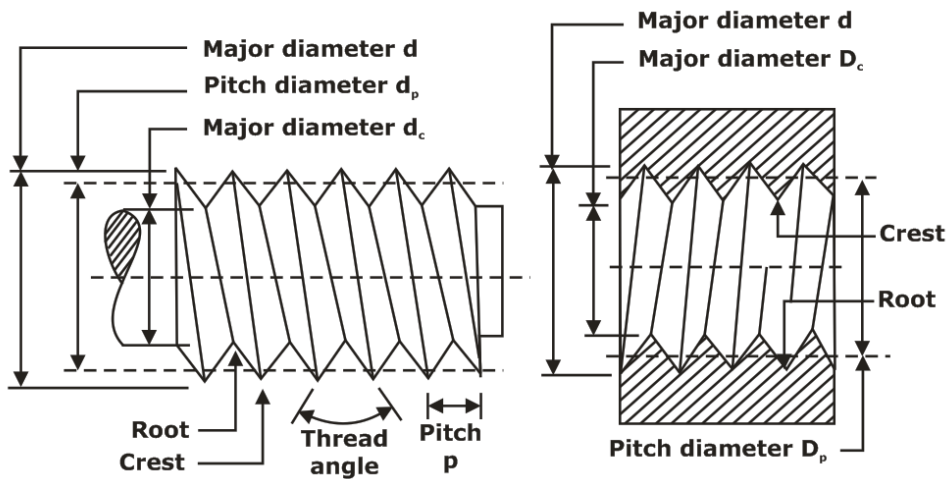
• The major diameter (D) is the largest diameter of the screw thread. It is also called the nominal diameter of the thread.

• The minor diameter (d) is the smallest diameter of the screw thread. It is also called core or root diameter of the thread.

- Pitch diameter (D_P) it is also called the effective diameter of the thread. It is denoted by d_p for external threads and D_p for internal threads.

Designation of ISO metric Screw Threads:

• A screw thread of coarse series is designated by the letter 'M' followed by the value of the value of the nominal diameter (in mm).

• For example, M12, a screw thread of fine series is specified by the letter 'M', followed by the values of the nominal diameter and the pitch in mm and separated by the symbol 'x'. For example, M 12 × 1.25

Bolted Joint Analysis:

• A bolted joint subjected to tensile force P is shown in figure
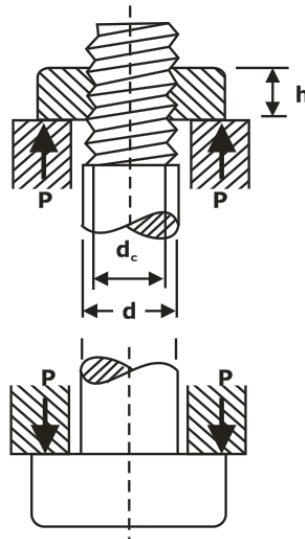
The cross-section at the core diameter d_c is the weakest section.

The maximum tensile stress in the bolt at this cross-section is given by,

$$\sigma_t = \frac{P}{\frac{\pi}{4}d^2_c}$$

$d_c = 0.84d$

Where d is the nominal diameter.

4.5. Eccentrically Loaded Bolted joints in Shear:

An eccentrically loaded bolted connection is shown in figure
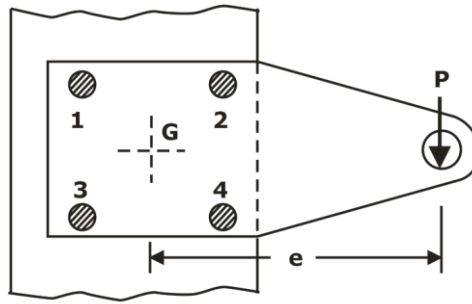
- Find the centre of gravity of the system.

- Primary shear forces P'_1, P'_2 etc. given as

$$P_1' = P_2' = P_3' = P_4' = \frac{P}{(\text{No. of bolts})}$$

- The moment $P \times e$ about the centre of gravity results in secondary shear forces P_1'', P_2''

……. etc.

$$P \times e = P_1'' r_1 + P_2'' r_2 + P_3'' r_3 + P_4'' r_4$$

$$P_1'' = Cr_1, \quad P_2'' = Cr_2 \ldots \text{.etc.}$$
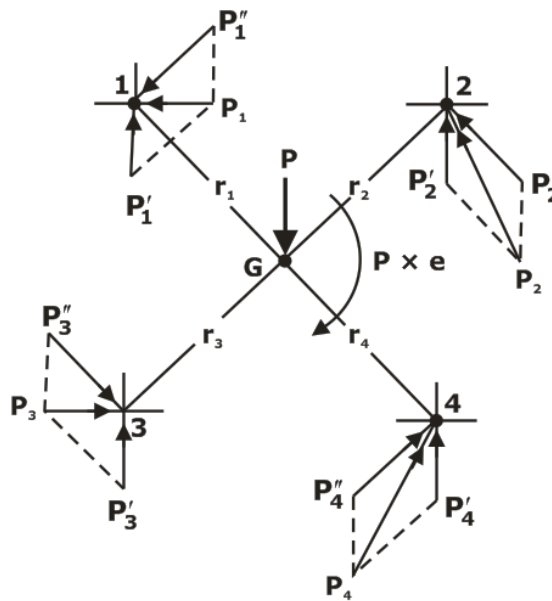
$$C = \frac{Pe}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$P_1'' = \frac{Per_1}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)} \quad \text{and} \quad P_2'' = \frac{Per_2}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

- The primary and secondary shear forces are added by vector addition method to get the resultant shear forces P_1, P_2, P_3 and P_4.

4.6. Eccentric Load perpendicular to Axis of Bolt:

(a)   (b)
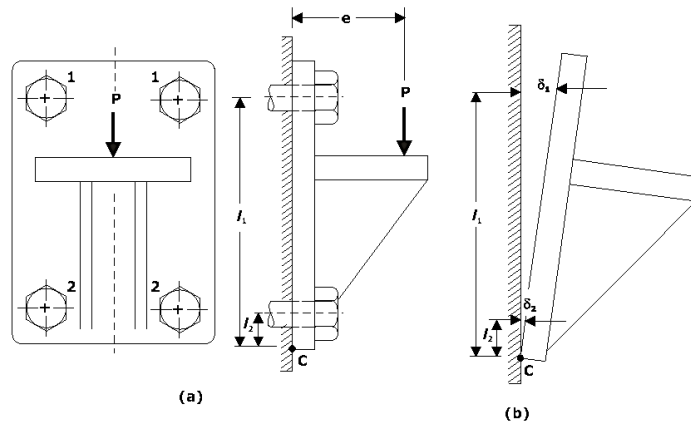
- The shear force on each bolt is given by,

$$P_1{'} = P_2{'} = \frac{P}{(\text{No. of bolts})}$$

- The moment $P \times e$ tends to tilt the bracket about the edge C.

Each bolt is stretched by an amount (δ) which is proportional to its vertical distance from the point C.

$$\delta_1 \alpha\ l_1 \text{ and } \delta_2 \alpha\ l_2$$

If $P_1{''} \alpha\ l_1$ and $P_2{''} \alpha\ l_2$

Hence, $P_1{''} = Cl_1$ and $P_2{''} = Cl_2$

Where C is the constant of proportionality.

$$P_1{''} = \frac{Pel_1}{2(l_1{}^2 + l_2{}^2)} \text{ and } P_2{''} = \frac{Pel_2}{2(l_1{}^2 + l_2{}^2)}$$

The bolts denoted by 1 are subjected to maximum force.

- Direct shear stress in the bolt is given by, $\tau = \dfrac{P_1{'}}{A}$

- Tensile stress in the bolt is given by, $\sigma_t = \dfrac{P_1{''}}{A}$

- The bolts can be designed on the basis of maximum principal stress theory or maximum shear stress theory.

4.7 Elastic Analysis of Bolted Joints:

$$\Delta P = P \left(\frac{k_b{'}}{k_b{'} + k_c{'}} \right)$$

The resultant load on the bolt, i.e., (P_b) is given by,

$$P_b = P_i + \Delta P$$

CHAPTER-3: BEARINGS

Introduction: Bearing is a mechanical element that supports as well as permits relative motion between two machine components such as the shaft and the housing, with minimum friction.

Classification of Bearings:

(A) Depending upon the direction of force direction,

Radial bearings- Force acts perpendicular to the axis of rotation of shaft in the radial direction.
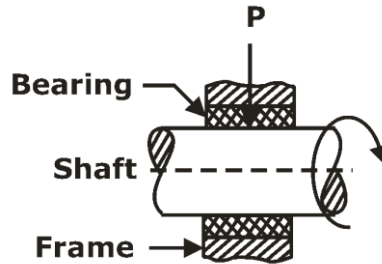
Thrust bearings - Force acts along the axis of rotation of shaft & perpendicular to the cross section.
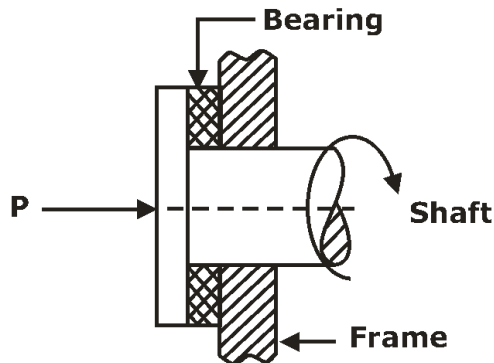
(B) Depending upon the type of friction.

Sliding contact bearing- There is surface contact between
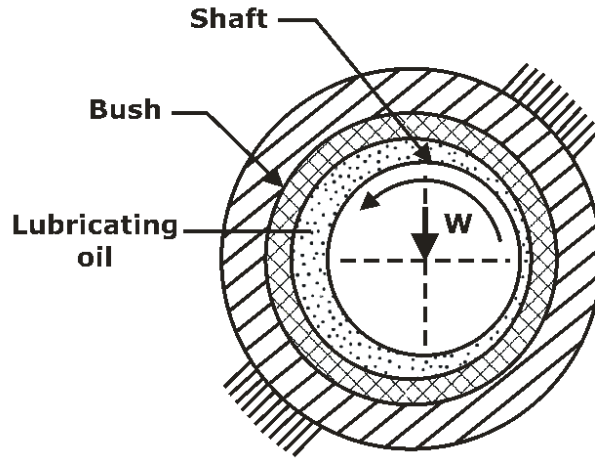
Rolling contact bearings-
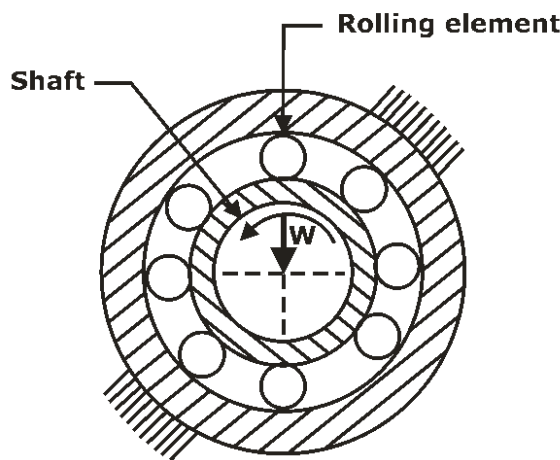
CLASSIFICATION
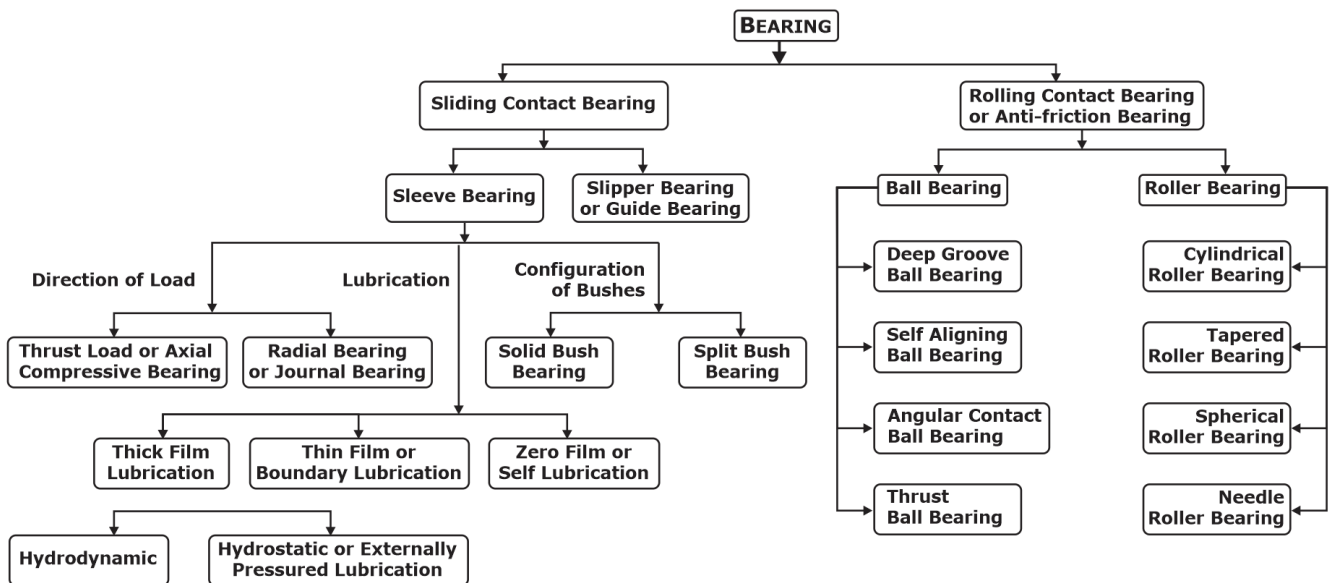
Modes of lubrication in Sliding Contact Bearings:

Thick Film:

Hydrodynamic lubrication:

It is defined as a system of lubrication in which the load–supporting fluid film is created by the shape and relative motion of the sliding surfaces.

A journal bearing is a sliding contact bearing working on hydrodynamic lubrication and which supports the load in radial direction.

There are two types of hydrodynamic journal bearings, namely, **full journal bearing** and **partial bearings.**

• In full journal bearing, the angle of contact of the bushing with the journal is 360°. Full journal bearing can take loads in any radial direction.

• In partial bearings, the angle of contact between the bush and the journal is always less than 180°. Partial bearing can take loads in only one radial direction.

A clearance bearing is a bearing in which the radius of the journal is less than the radius of the bearing.

Hydrostatic lubrication:

It is defined as a system of lubrication in which the load supporting fluid film, separating the two surfaces is created by an external source, like a pump, supplying sufficient fluid under pressure.

Thin Film: Thin film lubrication, which is also called boundary lubrication, is defined as a condition of lubrication where the lubricant film is relatively thin and there is partial metal to metal contact.

Journal- It is the part of the shaft which lies inside the bearing.

Diameter of journal = Diameter of shaft

Length of journal = Length of shaft

O and O' are the axes (centre of rotation) of bearing and journal respectively.

Eccentricity (**OO')** - It is the radial gap between axis of shaft and axis of journal and denoted by the letter e.

Radial clearance, C = It is the radial gap between shaft and bearing in which lubricating oil presents,

$C_1 = R - r$

Diameter clearance = it is twice of the radial clearance & denoted by C.

Diametral clearance – It is the ratio if diameter clearance and diameter of shaft.

Radius of bearing $\Rightarrow R = r + e + h_0 \Rightarrow C = e + h_0$

Eccentricity ratio (ε) = It is defined as the ratio of eccentricity to radial distance.

$$\varepsilon = \frac{e}{C_1} = \frac{2e}{C} = 1 - \frac{2h_0}{C}$$
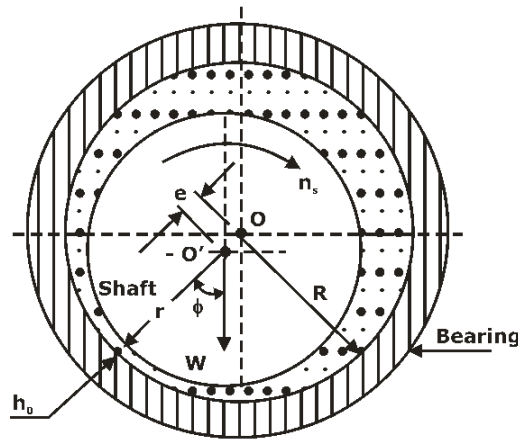
The quantity $\dfrac{h_o}{C}$ is called the minimum film thickness variable.

Bearing Pressure = It is defined as the ratio of the load to the projected area of the bearing.

$$P = \dfrac{W}{LD}$$

Power Loss in Journal Bearing – (Heat generated, H_g)

P = Force× Velocity = fWV = fWrω Watt

Where, $V = \dfrac{\pi DN}{60}$, N = Speed in RPM, ω = angular velocity of shaft in radian/s

μ = Coefficient of friction between lubricant and shaft

Heat Dissipated to the surrounding (H_g) -

$H_d = C_dA (t_b - t_0) = C_d (LD) (t_b - t_0)$

C_d = heat Dissipation Constant

t_b = Bearing surface Temperature, t_0 = Atmospheric temperature

Volume Flow rate of Coolant –

If $H_g > H_d \Rightarrow$ then Artificial colling required.

$H_c = H_g - H_d$

H_c = heat carried by coolant

$H_c = mC_p(t_e - t_i)$

C_p = specific heat

t_e = exit temperature of coolant, t_i = entry temperature of coolant

Sommerfeld number:

$$S = \left(\dfrac{D}{C}\right)^2 \dfrac{\mu N_s}{P}$$

N_s = Rotational speed of shaft in RPS

Attitude angle - The angle ϕ is called the angle of eccentricity or Attitude angle. It locates the position of minimum film thickness with respect to the direction of load.

Coefficient of friction variable (CFV):

$CFV = \left(\dfrac{D}{C}\right)f$, f is the coefficient of friction.

Expression for frictional Torque in terms of Viscosity -

Torque, T_f = Frictional Force (F_f) × Radius, R

$T_f = fWR = \dfrac{\mu\pi^2 D^3 NL}{60C}$

Petroff's Equation:

Petroff's equation is used to determine the coefficient of friction in journal bearings.

$f = 2\left(\dfrac{D}{C}\right)\left(\dfrac{\mu N_s}{P}\right)\pi^2$

Bearing Characteristic Number (BCN)$= \dfrac{\mu N}{P}$, it is dimensionless.

Equivalent Bearing Load:

$P_e = S\left[XVF_r + YF_a\right]$

V= race rotation factor

V is 1 when the inner race rotates while the outer race is held stationary in the housing.

V is 1.2 when the outer race rotates with respect to the load, while the inner ace remains stationary.

S = Service or shock factor

X = Radial Load factor,   Y = Axial Load factor

For Cylindrical roller bearing, X = 1, Y= 0

For Thrust bearing, X = 0 , Y = 1

Design for Cyclic Loads and Speeds:

If there are different loads acting at A different speed for different interval of time,

Then, $P_{eq} = \sqrt[3]{\dfrac{N_1 P_1^3 + N_2 P_2^3 + ...}{N_1 + N_2 + ...}}$

N_1, N_2, N_3…. is the total revolutions corresponding to given loads.

$N_1 = N\times$ time (N is in RPS or RPM depending upon the unit of time)

Life of an Antifriction Bearing – It is defined as the number of revolutions it has undergone before evidence of first fatigue crack either in race or rolling element.

Nominal Life or Rated Life - The rating life of a group of an apparently identical bearings is defined as the number of revolutions that 90% of the bearings will complete or exceed before the first evidence of fatigue crack.

L_{90} = Rated bearing life (in million revolutions),

L_{90} shows the Probability of Survival

L_{10} shows the Probability of Failure

Life at any given reliability(R) - Reliability R is defined as,

$$R = \frac{\text{No. of bearing which have successfully Completed L million revolutions}}{\text{Total number of bearings under test}}$$

$$\frac{L}{L_{90}} = \left[\frac{\ln R}{\ln R_{90}}\right]^{\frac{1}{1.17}}$$

R – is the required reliability

For 50% , R = 0.5

Average Life – Life of bearing at which probability of survival becomes equal to probability of Failure.

It is denoted by L_{50}.

$$L_{50} = 5 L_{90}$$

Static Load Carrying Capacity:

The static load carrying capacity of a bearing is defined as the static load which corresponds to a total permanent deformation of balls and races, at the most heavily stressed point of contact, equal to 0.0001 of the ball diameter.

$C_o = \dfrac{Zkd^2}{5}$ for ball bearing and $C_o = \dfrac{Zkdl}{5}$ for roller bearing

Dynamic Load Carrying Capacity:(C)

The dynamic load carrying capacity of a bearing is defined as the radial load in radial bearings (or thrust load in thrust bearings) that can be carried for a minimum life of one million revolutions. The dynamic load carrying capacity is based on the assumption that the inner race is rotating while the outer race is stationary.

If $P_e < C \Rightarrow L_{90} > 1$ million

If $P_e > C \Rightarrow L_{90} < 1$ million

Bearing Life:

$$L_{90} = \left(\frac{C}{P_e}\right)^k$$

L_{90} = Rated bearing life (in million revolutions),

C= Dynamic load capacity (N),

K = 3 (for ball bearings) and $\frac{10}{3}$ (for roller bearings),

$P_e.$ = Equivalent load,

If Life of bearing is given in hours and speed of the shaft is also given,

$$L_h = \frac{L_{90} \times 10^6}{N \times 60}$$

L_h = Rated bearing life (hours)

N = rotation speed (rpm)

Thrust Bearing – These bearings are used to support the shaft subjected to Thrust load i.e Axial compressive load.
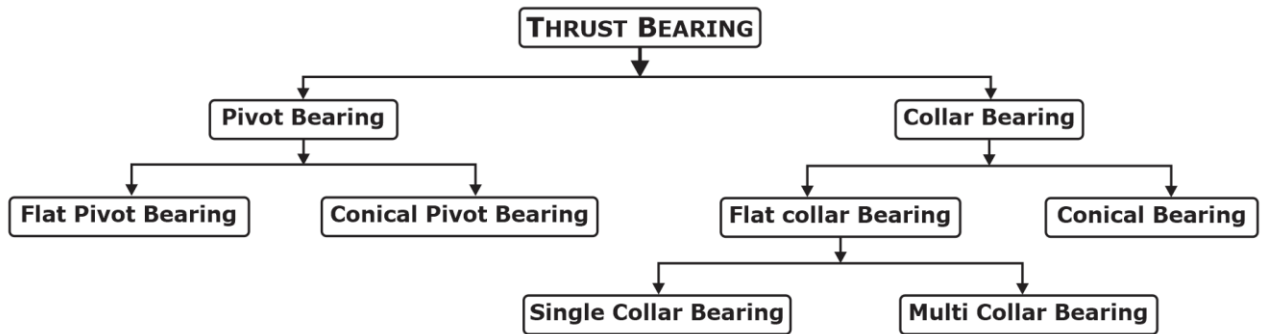
Pivot bearing – It is used to support vertical shaft subjected to Thrust load.

Collar bearing – It is used to support Horizontal shaft subjected to Thrust load.

Flat Collar Bearing –

Load & torque which can be resisted by Flat collar bearing-

(a) By Assuming Uniform Pressure theory – (P = C)

$$P = \frac{W}{\pi(R_o^2 - R_i^2)}$$

$$T = \frac{2}{3}fW\left(\frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)}\right)$$

(b) By Assuming Uniform Wear theory – (Pr = C)

$$P = \frac{W}{2\pi R_i(R_o - R_i)}$$

$$T = \frac{1}{2}fW\frac{(R_o + R_i)}{2}$$
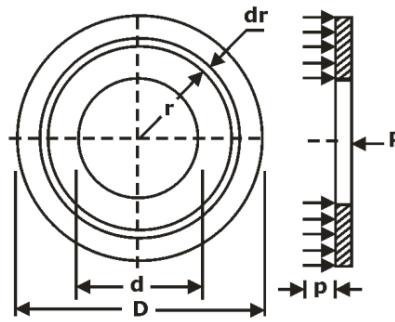
NOTE – For flat pivot bearing, $R_i = 0$, $R_o = R$

NOTE – For safe design of bearing, it is always better to use uniform pressure theory.

Multi Collar Bearing -

Total number of collar, $n = \dfrac{W_{total}}{W_{each}} = \dfrac{W_{total}}{\pi(R_o^2 - R_i^2)P_{per}}$

Some Important bearing & their advantages -

(i) Rolling Contact Bearings: A rolling contact bearing consists of four parts – inner and outer races, a rolling element like ball, roller or needle and a cage which holds the rolling elements together and spaces them evenly around the periphery of the shaft

(ii) Deep Groove Ball Bearing:

The radius of the ball is slightly less than the radii of curvature of the grooves in the races.

Advantages:

• They have high load carrying capacity

• They take load in the radial as well as axial direction

• Generates less noise due to point contact

Disadvantages:

• They are not self – aligning

• They have poor rigidity

(iii)Cylindrical Roller Bearing:

It consists of relatively short rollers that are positioned and guided by the cage.

Advantages:

• Due to line contact, the radial load carrying capacity is very high

• It is more rigid than ball bearing

• The coefficient of friction is low and frictional loss is less in high – speed applications

Disadvantages:

• It cannot the thrust load

• It is not self – aligning. It cannot tolerate misalignment.

(iv) Angular Contact Bearing:

The grooves in inner and outer races are so shaped that the line of reaction at the contact between the balls and races makes an angle with the axis of the bearing. Two components – radial and axial. Therefore, angular contact bearing can take radial and thrust loads. Specific magnitude of pre – load required.

Advantages:

• Load carrying capacity is more than that of deep groove ball bearing

Disadvantages:

• Two bearings are required to take thrust load in both directions

• The angular contact bearing must be mounted without axial play

(v) Self–aligning Bearings:

• Two types – self-aligning ball bearing and spherical roller bearing

• The external surface of the bearing bush is made spherical.

• In spherical roller bearing, balls are replaced by two rows of spherical rollers.

  ➢ Compared with the self-aligning ball bearing, the spherical roller bearing can carry relatively high radial and thrust loads

(vi)Taper Roller Bearing:

Rolling elements in the form of frustum of cone. In taper roller bearing, the line of resultant reaction through the rolling elements makes an angle with the axis of the bearing. Therefore, taper roller bearing can carry both radial and axial loads

Advantages:

• They have more rigidity.

• They can be easily assembled and disassembled due to separable parts.

Disadvantages:

• Use of two taper roller bearings is necessary to balance the axial force

• They cannot tolerate misalignment.

(viii)Thrust Ball Bearing:

It consists of a row of balls running between two rings – the shaft ring and the housing ring Thrust ball bearing carries thrust load in only one direction and cannot take radial load

Advantages

• They have high thrust load carrying capacity due to large number of balls

Disadvantage

• It is not self-aligning and cannot tolerate misalignment

• They do not operate as well on horizontal shafts as they do on vertical shafts

Only for low and medium speed.

CHAPTER-4: DESIGN OF SHAFTS & SPRINGS SHAFTS

Transmission Shafts: These are the Rotating machine elements, circular in cross-section, which supports transmission elements like gears, pulleys and sprockets and transmits power.

Design of Shaft: The design of the shaft can be done on the basis of two criteria which are explained below.

Strength Criteria: The strength criteria uses the first two terms of torsion equation and design is done on basis that stress induced in the shaft must not exceed the strength of material of shaft.

$$\frac{\tau}{R} = \frac{T}{J} \text{ Or } \tau = \frac{16T}{\pi d^3}$$

Rigidity Criteria: The rigidity criteria uses the last two terms of torsion equation and design is done on the basis that maximum angular twist must exceed a certain value.

$$\frac{T}{J} = \frac{G\theta}{L} \text{ Or } \theta = \frac{TL}{GJ}$$

The shaft is subjected to a combination of axial force, bending moment and torsional moment,

$\sigma_x = \sigma_t + \sigma_b$

The Principal stress, $\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$

The maximum shear stress; $\tau_{max.} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$

Taking the case when the shaft is subjected to combined bending and torsional moments,

$\sigma_x = \sigma_b = \frac{32M_b}{\pi d^3}, \tau = \frac{16M_t}{\pi d^3}$

• **Maximum Principal Stress Theory:**

$\sigma_1 = \frac{16}{\pi d^3}\left[M_b + \sqrt{\left(M_b\right)^2 + \left(M_t\right)^2}\right]$

The permissible value of maximum principal stress is given by,

$\sigma_1 = \frac{S_{yt}}{FOS}$

• **Maximum Shear Stress Theory:**

$\tau_{max.} = \frac{16}{\pi d^3}\left[\sqrt{\left(M_b\right)^2 + \left(M_t\right)^2}\right]$

The permissible value of maximum shear stress is given by,

$\tau_{max.} = \frac{S_{sy}}{FOS} = \frac{0.5S_{yt}}{FOS}$

The shafts are made of ductile materials, hence maximum shear stress theory is applied for shaft design.

• **Equivalent Moments:**

$$\text{Equivalent Torsional Moment} = T_{eq} = \sqrt{\left(M_b\right)^2 + \left(M_t\right)^2}$$

$$\text{Equivalent Bending Moment} = M_{eq} = \frac{\left[M_b + \sqrt{\left(M_b\right)^2 + \left(M_t\right)^2}\right]}{2}$$

SPRINGS

Introduction - A spring is defined as an elastic machine element, which deflects under the action of the load and returns to its original shape when the load is removed.

Types of Springs:

• The spring can be a helical coil of a wire, a piece of stamping or a flat wound-up strip.

• There are two types of helical springs – compression spring and extension spring.
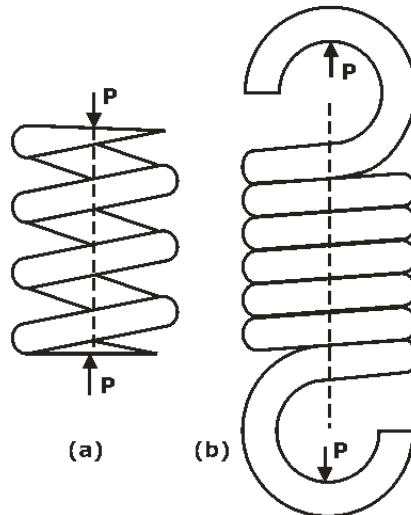
(a) Helical Compression spring, (b) Helical Extension spring

Terminology of Helical Spring:

d = wire diameter of spring (mm) & D_i = inside diameter of spring coil (mm)

D_o=outside diameter of spring coil (mm) & D = mean coil diameter (mm)

Therefore, $D = \dfrac{D_i + D_o}{2}$ & Spring index C= ratio of mean coil diameter to wire diameter Spring index, $C = \dfrac{D}{d}$

Spring compliance – it is the reciprocal of spring index.
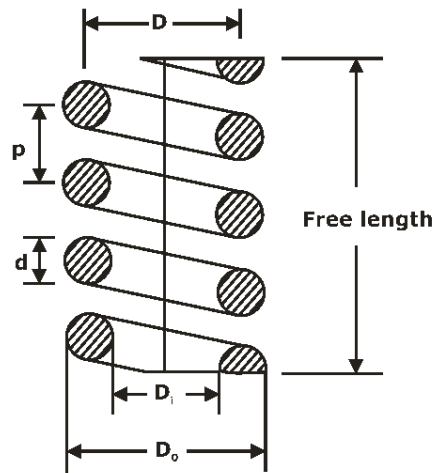
Solid length- Solid length = N_t d ,Where N_t = Total number of coils

Compressed length: Total gap = $(N_t - 1) \times$ gap between adjacent coils

Free length: Free length = Compressed length + δ

Free length = Solid length + total axial gap + δ

(a)   **(b)**
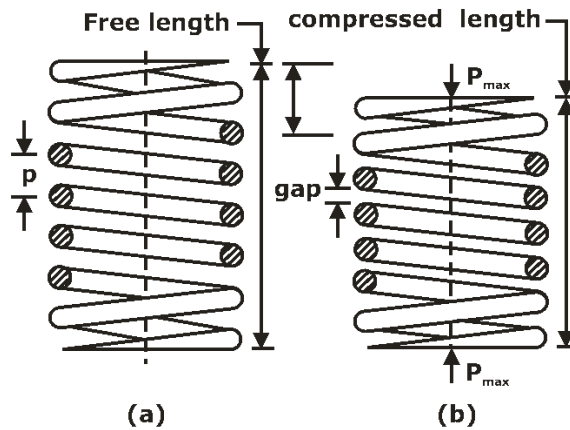
Stress and Deflection equations:

Stress is independent of spring length.

$\tau_{max.} = \tau_s + \tau_d$

$\tau_{max.} = K_w \dfrac{8PD}{\pi d^3}$

Where, K_w is Wahl's correction factor = $K_c \times K_s$

K_c=Curvature factor & K_s=Shear stress factor

Wahl's correction factor $K_w = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$

Deflection of a spring under load W is given as $\text{Deflection}(\delta) = \dfrac{8PD^3 n}{Gd^4}$

Stiffness of the spring: It is defined as the force required to produce unit deflection.

$$k = \frac{P}{\delta} \Rightarrow k = \frac{Gd^4}{8D^3n} = \frac{Gd}{8C^3n}$$

connections of Springs:

Series connection: if two spring having stiffness k_1 & k_2 connected in series, then equivalent stiffness is given as

$$\frac{1}{k_{equi}} = \frac{1}{k_1} + \frac{1}{k_2}$$

In Series connection of spring,

$P_1 = P_2 = P$ & $\delta = \delta_1 + \delta_2$

Parallel connection: : if two spring having stiffness k_1 & k_2 connected in parallel , then equivalent stiffness is given as

$k = k_1 + k_2$, k is equivalent stiffness

In Parallel connection of spring,

$P = P_1 + P_2$ & $\delta_1 = \delta_2 = \delta$

CHAPTER-5: BRAKES & CLUTCHES

BRAKES

Introduction: A Brake is a mechanical device which absorbs the energy possessed by the moving system.

Classification of Brakes:

(i) Mechanical Brakes

(ii) Hydraulics and Pneumatics Brakes

(iii) Electric Brakes

Energy Balance for Brakes:

• For a mechanical system of mass m moving with Velocity V, K.E. $= \frac{1}{2} m(V_1^2 - V_2^2)$

• The kinetic energy of Rotating body K.E. $= \frac{1}{2} I(\omega_1^2 - \omega_2^2)$

Block Brake with short shoe: Angle is less than 45-degree, pressure on shoe is uniform.
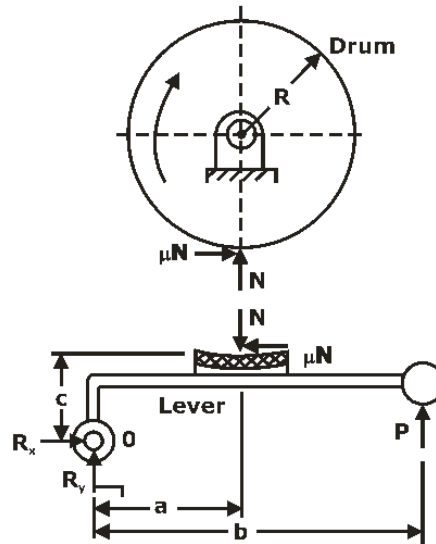
M_t = Braking -torque (N-m)

Considering the equilibrium of forces in vertical and horizontal directions,

$R_x = \mu N$ and $R_y = (N - P)$

Taking moment of forces acting on the lever about the hinge point O,

$P \times b - N \times a + \mu N \times c = 0$

$P = \frac{(a - \mu c)}{b} \times N$

Case I: $a > \mu c$

The friction force μN helps to reduce the magnitude of the actuating force P.

Moment due to braking effort $P \times b$, and moment due to friction force $\mu N \times c$ are both anticlockwise. Such a brake is called a partially **'self – energizing' brake**

Case II: $a = \mu c$, Such a brake is called a **'self – locking' brake**

Case III: $a < \mu c$, The actuating force, P becomes negative. uncontrolled braking.

Block Brake with Long Shoe:

Braking -torque $M_t = \mu' NR$

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin2\theta}$$

θ is the semi – cone angle.

Band Brakes:
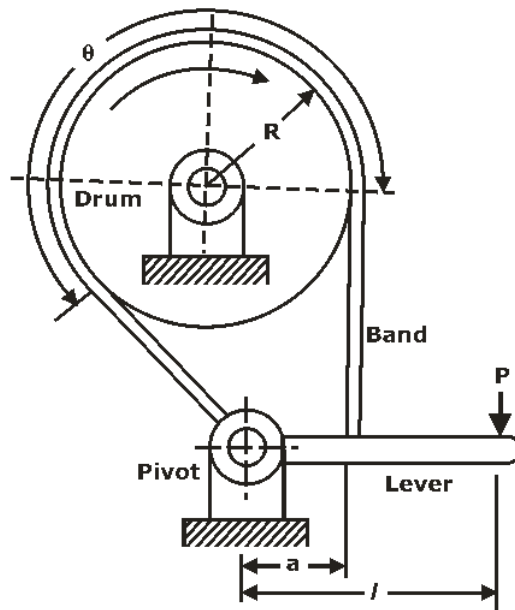
The ratio of bend tensions is given by,

$$\frac{P_1}{P_2} = e^{\mu\theta}$$

Torque $M_t = (P_1 - P_2)R$
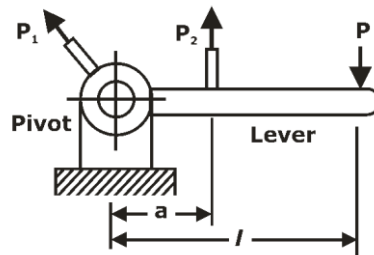
Taking moments about the pivot,

$$P_2 \times a = P \times l$$
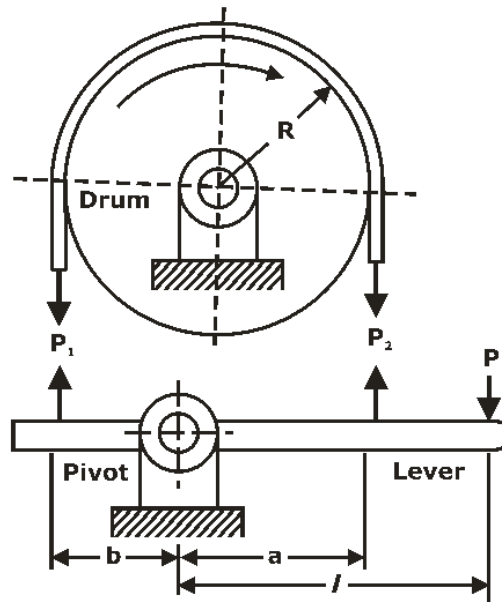
Considering forces acting on the lever and taking moments about the pivot,

$P \times l + P_1 \times b - P_2 \times a = 0$ gives $P = \dfrac{P_2 \times a - P_1 \times b}{l}$

• Substituting the expression of $\dfrac{P_1}{P_2}$ in the above equation,

$P = \dfrac{P_2(a - b \times e^{\mu\theta})}{l}$

• **For the self – locking condition,**

$P \le 0$ and $a \le b \times e^{\mu\theta}$

Therefore, the condition of self-locking is given by $\dfrac{a}{b} \le e^{\mu\theta}$

Disk Brake- A disk brake is similar to a plate clutch, except that one of the shafts is replaced by a fixed member.
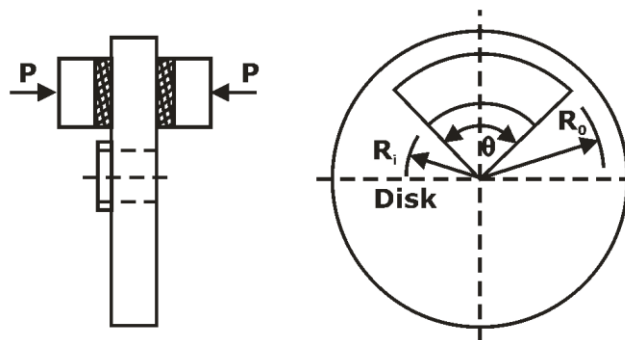
The torque transmitting capacity of the disk is given by, $M_t = \mu P R_f$

The area of the pad is given by, $A = \left(\dfrac{\theta}{2\pi}\right)\pi(R_o{}^2 - R_i{}^2)$

CLUTCHES

Introduction- Connect and disconnect driving and driven shaft at the will of operator without stopping prime mover.

Types of Clutches:

(i) Positive contact clutches

(ii)Friction clutches

(iii)Electromagnetic clutches

(iv)Fluid clutches

Friction Clutches: The force of friction is used to start the driven shaft from rest and gradually brings it up to proper speed.

Advantages:

(i)The engagement is smooth

(ii)Slip occurs only during engaging operation and once the clutch is engaged, there is no slip between the contacting surfaces.

Torque transmitting capacity of single-plate clutch:
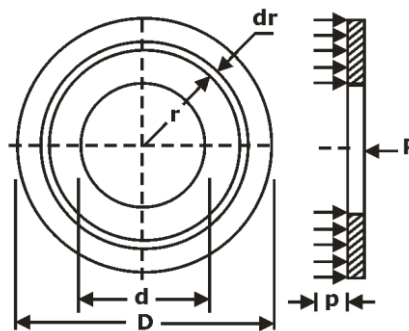
$$M_t = 2\pi\mu \int_{\frac{d}{2}}^{\frac{D}{2}} Pr^2 \, dr$$

Uniform Pressure Theory- In case of new clutches employing a number of springs, the pressure remains constant over the entire surface area of the friction disk.

With this assumption, P is assumed to be constant.

$$M_t = \frac{\mu W}{3} \frac{(D^3 - d^3)}{(D^2 - d^2)}$$

3.1.2. Uniform Wear Theory:

• This assumption is used for worn-out clutches.

• When the wear is uniform,

$Pr = Cons \tan t$ and $M_t = \mu W \frac{(R_O + R_i)}{2}$

The above equation can be rewritten as,

$M_t = \mu W R_f$ where, R_f is called the friction radius.

For uniform wear theory or worn – out clutches:

$$R_f = \frac{(R_O + R_i)}{2} = \frac{(D + d)}{4}$$

For uniform pressure theory or new clutches:

$$R_f = \frac{2}{3}\left(\frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)}\right) = \frac{1}{3}\frac{(D^3 - d^3)}{(D^2 - d^2)}$$

Conclusions:

• The friction radius for new clutches is slightly greater than that of worn – out clutches.

• The torque transmitting capacity of new clutches is slightly more than that of worn – out clutches.

• A major portion of the life of friction lining comes under the uniform wear criterion. Hence, it is more logical and safer to use uniform wear theory for design of clutches.

Multi – disk clutches:

• For uniform pressure criterion,

$$M_t = \frac{z\mu W}{3}\frac{(D^3 - d^3)}{(D^2 - d^2)}$$

• For uniform wear criterion,

$$M_t = \frac{z\mu W}{4}(D + d)$$

Where, z is the number of pairs of contacting surfaces,

Note:

Number of disks = number of pairs of contacting surfaces+1= $z+1$

Suppose,

z_1= number of disks on driving shaft

z_2= number of disks on driven shaft,

$z_1 + z_2$ = number of pairs of contacting surfaces +1

or number of pairs of contacting surfaces= $z_1 + z_2$ -1

Cone clutches:
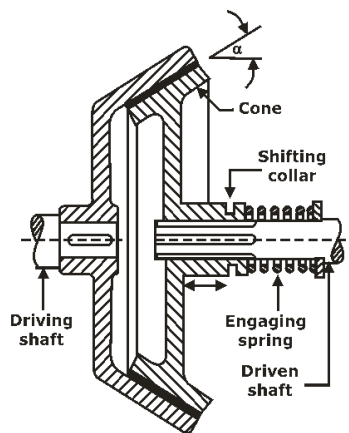
• In case of uniform pressure theory,

$$M_t = \frac{\mu W}{3 Sin\alpha} \frac{(D^3 - d^3)}{(D^2 - d^2)}$$

• In case of uniform wear theory,

$$M_t = \frac{\mu W}{Sin\alpha} \frac{(D + d)}{4}$$

where, α is the semi cone angle,

Centrifugal Clutch:
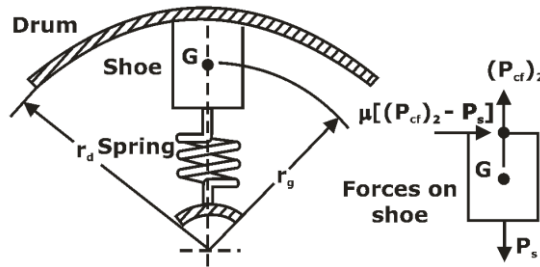
If the number of shoes if z,

The torque transmitting capacity of the clutch is given by,

$$M_t = \frac{\mu m r_g r_d z(\omega_2^2 - \omega_1^2)}{1000}$$

CHAPTER-6: GEARS

Gear Drives- Gears are defined as toothed wheels or toothed wheels or multilobed cams, which transmit power and motion from one shaft to another by means of successive engagement of teeth.

Classification of Gears:

• Spur Gears

• Helical Gears

• Bevel Gears

• Worm Gears

Force Analysis In Gears:
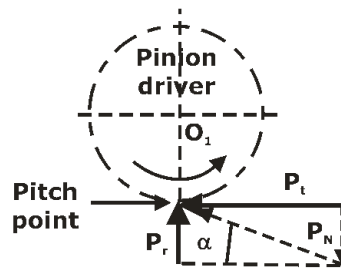
• P_N always acts along the pressure line. This force can be resolved into two components – tangential component P_t and radial component P_r at the pitch point.

$$M_t = P_t \times \frac{d'}{2} \ , \ P_N = \frac{P_t}{Cos\alpha} \ , \ P_r = P_t \tan\alpha \ \text{ and } M_t = \frac{60 \times 10^6 \times P}{2\pi N}$$

Beam Strength of Gear Design:
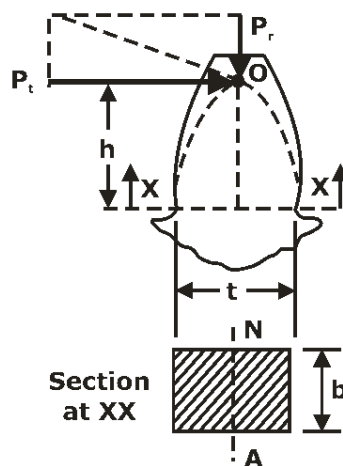
• The beam strength S_b is the maximum value of the tangential force that the tooth can transmit without bending failure.

$$S_b = \sigma_b bmY \ , \text{ Lewis equation}$$

For safe design $P_{eff.} \leq S_b$

- Y is called the Lewis form factor.

- When the different materials are used, the product of $\sigma_b \times Y$ decides the weaker between the pinion and gear.

- The Lewis form factor Y is always less for a pinion compared with gear.

- Effective tangential load on tooth, $P_{eff.} = (FOS)\dfrac{C_s}{C_v}P_t$

- Service Factor $C_s = \dfrac{Maximum\ torque}{Rated\ torque}$ & Velocity Factor C_v

- For ordinary and commercially cut gears made with form cutters and with $0 < V < 10$ m/s

$$C_v = \dfrac{3}{3+V}$$

- For accurately hobbed and generated gears with $10 \le V \le 20$ m/s

$$C_v = \dfrac{6}{6+V}$$

For $20 < V$ and $C_v = \dfrac{5.6}{5.6+\sqrt{V}}$

- According to wear strength of tooth (S_w)

$S_w = d_p b \theta K$ Practically $P_{eff.} \le S_b \le S_w$

- $\theta = \dfrac{2d'_g}{d'_g \pm d'_p}$

- load-stress factor K;

$$K = \dfrac{\sigma_e^2 Sin\varphi Cos\varphi}{1.4}\left[\dfrac{1}{E_p}+\dfrac{1}{E_g}\right]$$

σ_e =surface endurance strength of the material (N/mm^2)

For $\phi = 20^o$ K$=0.16\left(\dfrac{BHN}{100}\right)^2$

****

Important Formula (Fluid Mechanics)

CHAPTER 1: FLUID PROPERTIES

Introduction: Fluid is a substance that deforms continuously under the application of shear (tangential) stress no matter how small the stress may be.

Fluid properties:

Mass density (ρ): It is defined as mass per unit volume.

$$\rho = \frac{M}{V} \; \text{kg} / \text{m}^3$$

Weight density or specific weight (w): It is defined as weight per unit volume.

$$w = \frac{\text{weight}}{\text{volume}} = \frac{m \times g}{V} = (\rho g)$$

Specific gravity or (relative density S): It is defined as ratio between density of fluid and density of standard fluid i.e., water

$$S = \frac{\rho}{\rho_w}$$

Specific gravity of Hg is 13.6.

Specific volume (v): It is defined as volume per unit mass.

$$v = \frac{V}{m} = \frac{1}{\rho} \; \left(\text{m}^3 / \text{kg} \right)$$

Viscosity (μ): The viscosity of a <u>fluid</u> is a measure of its <u>resistance</u> to deformation at a given rate. Viscosity can be conceptualized as quantifying the <u>frictional force</u> that arises between adjacent layers of fluid that are in relative motion.

Causes of viscosity

- intermolecular force of cohesion.
- molecular momentum exchange.

Kinematic viscosity (v):

$$v = \frac{\mu}{\rho}$$

Newton's Law of Viscosity:

$$\tau = \mu \frac{du}{dy}$$

μ = dynamic viscosity of fluid

Fluid Flow Behaviour:
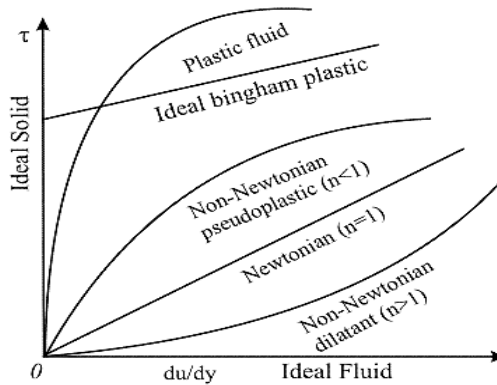
- General Relationship between shear stress and velocity gradient is given by:

$$\tau = \mu \left(\frac{du}{dy} \right)^n + B$$

- The fluids in which the apparent viscosity increases with increases with velocity gradient(n>1) the fluid is termed as dilatant (shear thickening) fluid.
- The fluids in which the apparent viscosity decreases with increases with velocity gradient(n<1) the fluid is termed as pseudoplastic (shear thinning) fluid.

Surface Tension (σ): Surface tension is the apparent interfacial tensile stress (force per unit length of interface) that acts whenever a liquid has a density interface, such as when the liquid contacts a gas, vapour, second liquid, or a solid. Surface tension is due to intermolecular force.

Gauge pressure inside a water droplet:

$$P_{gauge} = \frac{2\sigma}{R} \; N/m^2$$

Gauge pressure for soap bubble:

$$P_{gauge} = \frac{4\sigma}{R} \; N/m^2$$

Gauge pressure inside a jet of water:

$$P_{gauge} = \frac{\sigma}{R} \; N/m^2$$

Capillarity: This phenomenon is due to interplay of terms of cohesion and adhesion.

Capillarity rise $\boxed{h = \dfrac{4\sigma \cos\theta}{\rho g d}}$

For water: θ = 0°, capillary rise: h=+ve.

For Hg: θ = 130°, capillary fall: h= −ve.

Vapour pressure and Cavitation: The vapour molecules execute a partial pressure in the space above liquid, known as vapour pressure. Cavitation is formation of vapor bubbles in the liquid flowing through any Hydraulic Turbine.

Compressibility and Bulk Modulus:

- Bulk modulus of elasticity:

$$\boxed{\left(k\right)=-\frac{\Delta P}{\left(\Delta V / V\right)}=\rho\left(\frac{dp}{d\rho}\right)}$$

- Compressibility $\left(C\right)$ of fluid $=\dfrac{1}{\text{Bulk modulus of elasticity}}$

$$\boxed{\begin{aligned} C &=\frac{1}{K}=\frac{1}{\rho}\left(\frac{d\rho}{dp}\right) \\ C &=-\frac{1}{V}\left(\frac{\Delta V}{\Delta P}\right) \end{aligned}}$$

Isothermal Bulk Modulus: $\boxed{K_T =\rho RT}$

Adiabatic Bulk Modulus: $\boxed{K_A =\gamma\left(\rho RT\right)}$

CHAPTER 2: FLUID STATICS

Introduction: Fluid statics deals with fluids at rest. The fluid can be either gaseous or liquid.

Pressure: Pressure is defined as normal force per unit area and its SI unit is N/m^2 or Pascal (Pa).

$$\Pr essure = \frac{Normal\ force}{Area}$$

Hydrostatic law: The variation of pressure in a fluid in vertical direction is directly proportional to specific weight.

$$\frac{dP}{dh} = \rho g$$

$$P_{Gauge} = \rho gh\ N/m^2 or\ Pascal$$

- As we move vertically down is a fluid the pressure increase as +ρgh. As we move vertically up in a fluid the pressure decreases as −ρgh.
- There is no charge g pressure in horizontally same level.

Pascal's Law: According to Pascal's Law in a static fluid, the pressures at a point is equally distributed in all the direction. Ex: hydraulic lift, hydraulic break etc**.**

Pressure measurement:
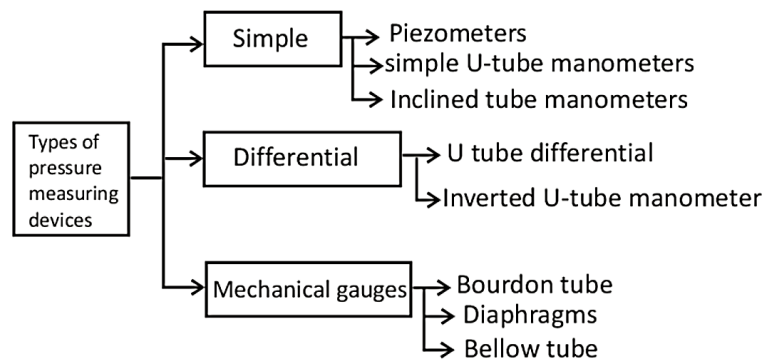
Piezometers: A Piezometer is a simple glass tube that is open at both the ends. Piezometers can't be used to measured very high pressure and gas pressures.

$$P = \rho gH$$
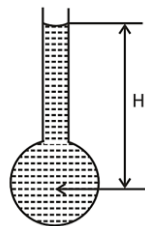
Simple U-tube manometer: It consists of a glass tube with one end open to the atmosphere and other end connected to a point at which pressure is to be measured.
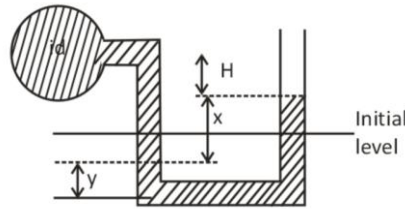
$$P_A + \rho_o g (H + x) + \rho_m gy - \rho_m gy - \rho_m gx = 0$$

Differential Manometer: A differential manometer is used to measure the difference in pressures in two pipes or two points in the pipeline.

Upright U-tube differential manometer:
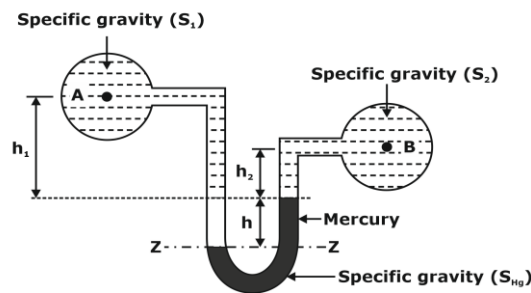
$$P_A - P_B = \rho_2 gh_2 + (\rho_{Hg} - \rho_1)gh - \rho_1 gh_1$$

Inverted U-tube differential manometer:
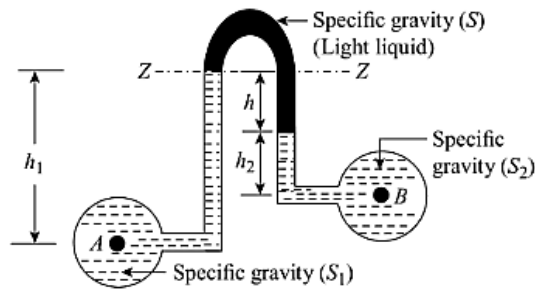
$$P_A - P_B = \rho_1 gh_1 - \rho_2 gh_2 - \rho gh$$

When both the pipes are at the same level i.e. $h_1 = h_2$:

$$P_A - P_B = (\rho_1 - \rho_2)gh_1 - \rho gh$$

Inverted U-tube manometers are used when the pipelines are underground and in these manometers the density of manometric fluid is less than the density of flowing fluid ($\rho_m < \rho$).

The Hydrostatic Paradox:

The pressure at any point depends only upon the depth below the free surface and unit weight of the liquid. It is independent form the size and shape of the container.
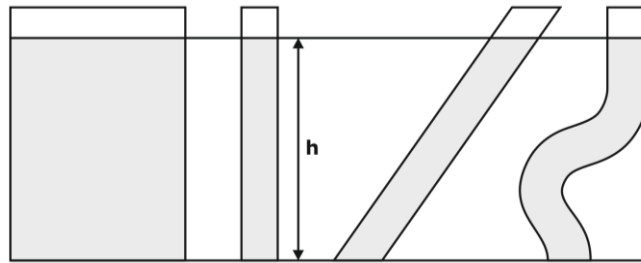
Hydrostatic forces: When a fluid is in contact with a surface is exerts a normal force on the surface which is termed as the hydrostatic force.

Hydrostatic forces on submerged plane surfaces:

Plane inclined surface at angle θ:

Force on the surface is given by: $\boxed{F = \rho g A \bar{h}}$

Centre of pressure for the vertical submerged surface is given by:

$$\boxed{h^* = \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h}}$$

h = vertical distance of centre of Pressure from free surface

I_G = moment of inertia at centroidal axis.

Plane vertical surface (θ = 90°):

$$\boxed{\begin{aligned} F &= \rho g A \bar{h} \\ h^* &= \frac{I_G \sin^2 \theta}{A\bar{h}} + \bar{h} \end{aligned}}$$

Plane horizontal surface (θ = 0°):

$$\boxed{\begin{aligned} F &= \rho g A \bar{h} \\ h^* &= \bar{h} \end{aligned}}$$

Hydrostatic forces on curved surfaces:
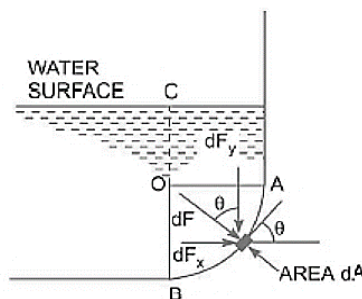
Horizontal component of force on curved surface: Hydrostatic force on the vertical projection area and this force will act at center of pressure of the corresponding area.

Horizontal force: $\boxed{F_X = \gamma A \bar{h}}$

Where,

A = Projected Area

$\bar{h}$ = depth of centroid of an area.

Resultant Force: $\boxed{F_R = \sqrt{F_X^2 + F_Y^2}}$

The angle from the horizontal at which this force will act:

$$\boxed{\tan\alpha = \frac{F_Y}{F_H}}$$

Vertical component of force on curved surface: The vertical component of force on a curved surface is equal to weight of the fluid contained by the curved surface till the free surface and this force will act at the center of gravity of the corresponding weight.

Buoyancy & floatation:

Center of buoyancy (B): Center of buoyancy is defined as the point of application of buoyancy force and this force will act at the centroid of volume of fluid displaced.

Principal of floatation:

Weight of Body = Buoyant Force (F_B)

$\rho_{body}.g.A.h = \rho_f.g.A.x$

(a). If $\rho_{body} > \rho_f$, then body will be submerged totally in the fluid and will rest at the body of the container.

(b). If $\rho_{body} = \rho_f$, then body will be submerged totally in the fluid. In this body will float in the liquid and remains at rest at any point in the fluid if given slight displacement from original position.

(c). If $\rho_{body} < \rho_f$, then body will be partially submerged in the fluid and will float.

For floating bodies:

$F_B = W$

$\dfrac{V_{sub}}{V_{total}} = \dfrac{\rho_{ave,body}}{\rho_f}$

Stability conditions of a completely submerged bodies under angular deflection:

(i). Stable Equilibrium-B above G.

(ii). Unstable Equilibrium-B below G.

(iii). Neutral Equilibrium-B and G at same point.

(a) STABLE EQUILIBRIUM   (b) UNSTABLE EQUILIBRIUM   (c) NEUTRAL EQUILIBRIUM
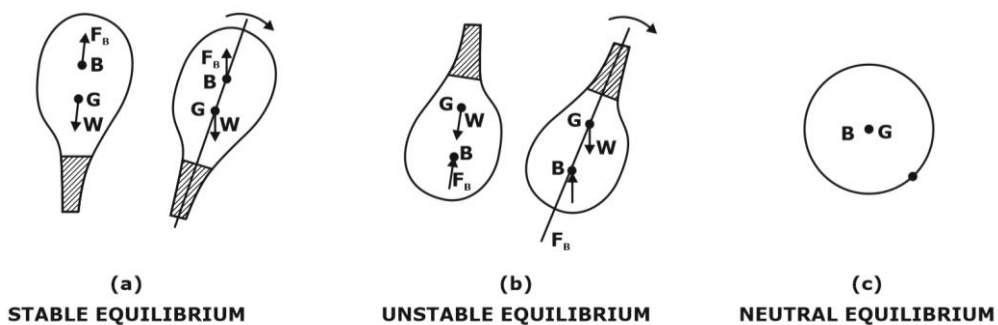
Stability conditions of partially submerged bodies under angular deflection:

Metacenter (M) it is defined as the point of intersection of normal Axis and new line of Action of buoyancy force.

(i). Stable Equilibrium-M above G.

(ii). Unstable Equilibrium-M below G**.**

(iii). Neutral Equilibrium-Mand G at same point.

Metacentric height (GM):

Metacentric Height (GM): it is defined as the vertical distance between center of gravity "G" and metacenter "M" which is the intersection point of the lines of action of the buoyant force through the body before and after rotation.

$$\left(BM\right)_{xx} = \frac{I_{xx}}{V_{disp}} \ and \left(BM\right)_{y-y} = \frac{I_{yy}}{V_{disp}}$$

$$\left(BM\right)_{xx} > \left(BM\right)_{yy}$$

$$\left(BM\right)_{pitching} > \left(BM\right)_{rolling}$$

Stability decrease ➝
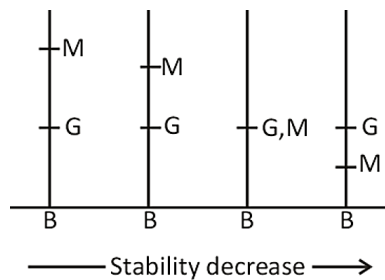
Determination of the metacentric height:

Experimental Method:

Consider a ship is floating in water, let w be a movable weight placed centrally on the deck of the ship and W be the total weight of the ship including w.

$$\overline{GM} = \frac{wx}{W} \times \frac{l}{d}$$

If l is the length of the pendulum and d is the distance moved by it on the horizontal scale.

Theoretical Method:

$$\overline{GM} = \pm(\overline{BM} - \overline{BG})$$

In the above expression:

(i). Positive (+Ve) sign is used when the metacentre M lies above the G.

(ii). Negative (- Ve) sign is used when the metacentre M lies below the G.

Where: $\overline{BM} = \dfrac{I_{body,freesurafce}}{V_{disp}}$

Time period of Oscillation of the floating body:

The time period of the oscillation is given by the following expression:

$$T = 2\pi \sqrt{\frac{K_G{}^2}{g\overline{GM}}}$$

Since $\left(\overline{GM}\right)_{pitching} > \left(\overline{GM}\right)_{rolling}$

Increasing the metacentric height gives the greater stability but reduces the time period of the roll so the ship will be less comfortable for the passengers.

CHAPTER 3: FLUID KINEMATICS

Fluid kinematics: It describes the motion of fluid and its consequences without any consideration of the forces invaded.

Flow field: It is the region in which the flow parameters i.e. pressure, velocity, etc. are defined at each and every point at any instant of time.

Approaches to study fluid motion:

Lagrangian approach: In the Lagrangian description of fluid flow, individual fluid particles are "marked," and their positions, velocities, etc. are described as a function of time.

Eulerian approach: In the Eulerian description of fluid flow, individual fluid particles are not identified. Instead, a control volume is defined**.**

Types of fluid flow:

Steady & Unsteady flow:

For steady flow: $\boxed{\left(\dfrac{\partial V}{\partial t}\right)_{x_0,y_0,z_0} = 0, \left(\dfrac{\partial P}{\partial t}\right)_{x_0,y_0,z_0} = 0, \left(\dfrac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} = 0}$

For unsteady flow: $\boxed{\left(\dfrac{\partial V}{\partial t}\right)_{x_0,y_0,z_0} \neq 0, \left(\dfrac{\partial P}{\partial t}\right)_{x_0,y_0,z_0} \neq 0, \left(\dfrac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} \neq 0}$

Uniform & Non-uniform flow:

For uniform flow: $\boxed{\vec{V} = V(t)}$ and $\boxed{\left(\dfrac{dV}{ds}\right)_{t=constant} = 0}$

For non-uniform flow: $\boxed{\left(\dfrac{dV}{ds}\right)_{t=constant} \neq 0}$

Laminar & Turbulent flows:

Laminar flow is the flow in which the fluid particles move along well-defined paths or steam line & all the streamlines are straight & parallel.

Turbulent floe is the flow in which the fluid particles move in a zig zag which results in eddies formation which results in energy loss.

Compressible & incompressible flow:

Compressible flow: $\boxed{\rho \neq constant}$

Incompressible flow: $\boxed{\rho = constant}$

Rotational & Irrotational flows: Rotational flow is the flow in which the fluid particles while flowing along streamlines also rotate about their own axis.

Irrotational flow is the flow in which the fluid particles while flowing along the streamlines do not rotate about their own axis.

Flow pattern description:

Streamlines: It is defined as an imaginary curve or line in the flow field so that the tangent to the curve at any point represents the direction of the instantaneous velocity at that point.

$$\boxed{\dfrac{dx}{u} = \dfrac{dy}{v} = \dfrac{dz}{w}}$$ equation of streamline in 3D

$$\begin{vmatrix} i & j & k \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0$$

Streak line: It is the focus of all fluid particles which at same instant of time all of which had passed through a common point

Path line: It represents the path traced by an inert tracer fluid particle over a period of time.

Note:

For steady flow, Streamlines, Streak lines & path lines are identical.

Timeline: A timeline is a set of adjacent fluid particles that were marked at the same (earlier) instant in time

Continuity equation:

By continuity equation:

Mass flow rate at inlet = mass flow rate at outlet

Fluid in compressible: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

If fluid is Incompressible i.e. ρ = constant

$A_1 V_1 = A_2 V_2$

Continuity Equation for Steady and 3 – D Incompressible Flow:

$$\boxed{\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0}$$

Continuity Equation for Steady and 2 – D Incompressible Flow:

$$\boxed{\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0}$$

Acceleration of fluid flow:

$$a_x = \dfrac{Du}{Dt} = u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} + \dfrac{\partial u}{\partial t}$$

Similarly, acceleration in y and z directions is given by:

$$a_y = \dfrac{Dv}{Dt} = u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial v}{\partial z} + \dfrac{\partial v}{\partial t}$$

$$a_z = \dfrac{Dw}{Dt} = u\dfrac{\partial w}{\partial x} + v\dfrac{\partial w}{\partial y} + w\dfrac{\partial w}{\partial z} + \dfrac{\partial w}{\partial t}$$

$$\underbrace{\dfrac{D(\ )}{Dt}}_{\substack{\text{Total}\\\text{acceleration}}} = \underbrace{\dfrac{\partial(\ )}{\partial t}}_{\substack{\text{because of}\\\text{unsteadiness}\\\text{(local / temporal}\\\text{acceleration)}}} + \underbrace{u\dfrac{\partial(\ )}{\partial x} + v\dfrac{\partial(\ )}{\partial y} + w\dfrac{\partial(\ )}{\partial z}}_{\substack{\text{because of non} - \text{uniformity}\\\text{(convective acceleration)}}} = \dfrac{\partial(\ )}{\partial t} + (\vec{V}.\nabla)$$

(i). The total differential D/Dt is known as the **material or substantial derivative** with respect to time.

(ii). The first term $\dfrac{\partial}{\partial t}$ in the right-hand side of is known as **temporal or local derivative** which expresses the rate of change with time, at a fixed position.

(iii). The last three terms in the right-hand side of the equation, are together known as **convective derivative** which represents the time rate of change due to change in position in the field.

Total acceleration: The total acceleration is given by the following vector:

$$\boxed{\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$$

Thus, magnitude of the total acceleration is given by:

$$A = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$$

$$\boxed{\vec{A} = \frac{D\vec{V}}{Dt} = \frac{\partial\vec{V}}{\partial t} + \vec{V}.\nabla\vec{V}}$$

Angular velocity: Angular velocity is given by:

$$\boxed{\vec{\omega} = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}}$$

$$\boxed{\vec{\omega} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ u & v & w \end{vmatrix}}$$

Where $\omega_x = \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y} - \dfrac{\partial v}{\partial z}\right)$, $\omega_y = \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z} - \dfrac{\partial w}{\partial x}\right)$ and $\omega_z = \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right)$.

Vorticity (Ω): Vorticity (ξ) in the simplest form is defined as a vector which is equal to two times the rotation vector. It is given by:

$$\boxed{\vec{\Omega} = 2\vec{\omega}}$$

Circulation (Γ): Circulation is defined as the line integral of tangential component of velocity vector along a closed curve.

$$\boxed{\text{Circulation}(\Gamma) = \text{Vorticity} \times \text{Area}}$$

Velocity potential(ϕ): It is a scalar function of space & time such that its negative derivate with respect to any direction gives the fluid velocity in that direction.

$$\boxed{u = -\frac{\partial\phi}{\partial x} \text{ and } v = -\frac{\partial\phi}{\partial y}}$$

The continuity equation is given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi = 0$$

$$\nabla^2 \phi = 0$$

- For flow to be possible velocity potential function must satisfy Laplace equation and velocity potential function only exists for irrotational flow.

- Slope for equipotential line: $\left|\left(\frac{dy}{dx}\right)_{\phi=c} = -\frac{u}{v}\right|$.

Stream function(Ψ): It is defined as the scalar function of space and time, such that its patrial derivative with respect to any direction gives the velocity component at right angles to that direction.

$$\left.u = \frac{\partial \Psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}\right\} \text{where, } \psi \rightarrow \text{streamfunction}.$$

In cylindrical polar co-ordinates: $\left|u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \text{and} \quad u_\theta = -\frac{\partial \psi}{\partial r}\right|$

- For flow to be possible stream function must exist.
- For irrotational flow stream function must satisfy Laplace equation.

- Slope of equistream line: $\left|\left(\frac{dy}{dx}\right)_{\psi=c} = \frac{v}{u}\right|$.

For 2 – D compressible steady flow: Discharge per unit width between two points in a flow = absolute difference between values of stream function through those two points, i.e.

$$\left|\frac{Q_2 - Q_1}{b} = \left|\Psi_2 - \Psi_1\right|\right|$$

Relation between equipotential line and streamline:

$$\left|\left(\frac{dy}{dx}\right)_{\phi=c} \times \left(\frac{dy}{dx}\right)_{\psi=c} = \frac{v}{u} \times \left(-\frac{u}{v}\right) = -1\right|$$

Hence, equipotential lines and streamlines are orthogonal to each other everywhere in flow field except at stagnation point where $V = 0$.

CHAPTER 4: FLUID DYNAMICS

Introduction: The dynamic behaviour of fluid is analysed by Newton's Second law of motion, which relates acceleration with the forces.

Equation of motion for fluid flow:

According to newton's second law:

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

For uniform flow: $\dfrac{d\vec{v}}{dt}$

$$\vec{F} = \vec{v}\frac{dm}{dt}$$

- Net force $F_x = F_g + F_p + F_v + F_T + F_C$
- Reynold's equation of motion: $F_x = F_g + F_p + F_v + F_T$
- Navier-Stokes equation of motion: $F_x = F_g + F_p + F_v$,
- Euler equation of motion: $F_x = F_g + F_p$,

Euler's Equation: This is the equation of motion in which the forces due to gravity and pressure are taken into consideration.

Assumptions:

(a). Flow is non-viscous, $F_v = 0$ i.e., viscous forces are zero.

(b). Flow is ideal.

$$\frac{dp}{\rho} + vdv + gdz = 0$$

This equation is known as Euler's equation of motion.

Bernoulli's Theorem:

Assumptions:

(i). Fluid is Non-viscous.

(ii). Steady flow.

(iii). incompressible fluid.

(iv). ideal fluid.

(v). irrotational flow.

$$\frac{P}{\rho} + \frac{v^2}{2} + gz = const.$$

Where,

$\dfrac{P}{\rho g}$ = Pressure energy of fluid per unit weight of fluid.

$\dfrac{v^2}{2g}$ = kinetic energy per unit weight or kinetic head.

Z = potential energy per unit weight or potential head

Classical Bernoulli's equation is also known as 'mechanical energy conservation Bernoulli's theorem state that for a non-viscous, steady and incompressible flow along a streamline the summation of kinetic energy, potential energy and Pressure energy is a constant.

$$\frac{P}{\rho g} + z = Piezometric\ head$$

Applications of Bernoulli's equation:

Venturi meter: It is a converging-diverging device that is used to measure discharge.
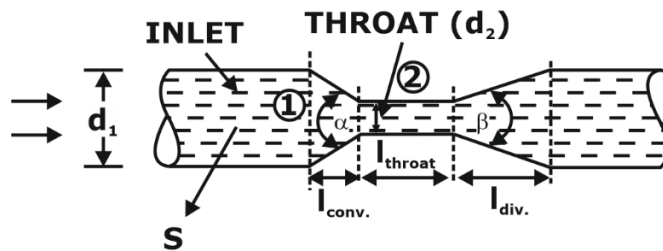
Geometric details: $d_2 = \left(\frac{1}{3} to \frac{1}{2}\right) d_1$ [To avoid cavitation]

Angle of convergence (α) = 20 — 22°

Angle of divergence (β) = must be below 7° (To avoid Boundary layer separation).

$$Q_{actual} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$

$$C_d = \frac{Q_{actual}}{Q_{theoretical}}$$

Calculation of h: The value of h is calculated with the help of the deflection of the manometric column.
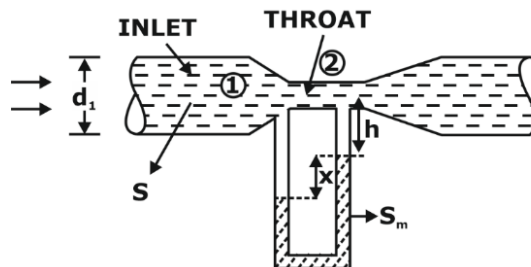
Case 1: S_m > S (flowing fluid), then: $\boxed{h = x\left(\frac{S_m}{S} - 1\right)}$

Case 2: S_m < S (flowing fluid), then: $\boxed{h = x\left(1 - \frac{S_m}{S}\right)}$

Orifice-meter: An orifice meter is essentially a thin circular plate with a sharp edged concentric circular hole in it.
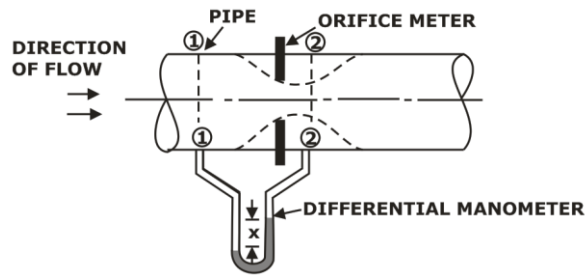
Coefficient of contraction (C$_c$): $C_c = \dfrac{\text{Actual Area at vena-contracta}}{\text{Theoretical Area at vena-contracta}} = \dfrac{A_2}{A_o}$

Discharge: $Q = C_d \cdot \dfrac{A_1 A_o}{\sqrt{A_1^2 - A_o^2}} \times \sqrt{2gh}$

Pitot tube: A right-angled glass tube, large enough for capillary effects to be negligible, is used for the purpose. One end of the tube faces the flow while the other end is open to the atmosphere
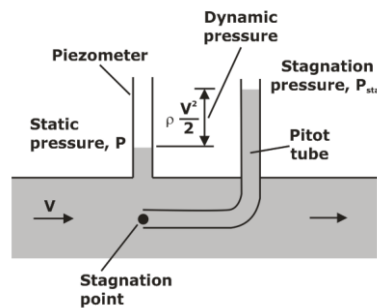
$$V_1 = \sqrt{2gh} = \sqrt{2g\left(stagnation\,head - static\,head\right)}$$

$$h = x\left[\frac{S_m}{S} - 1\right]$$

Momentum equation:

F.dt = d(mv)

which is known as the impulse-momentum equation and states that impulse of a force F acting on a fluid of mass m in a short interval of time dt is equal to the change of momentum d(mv) in the direction of force.

(a) **(b)**
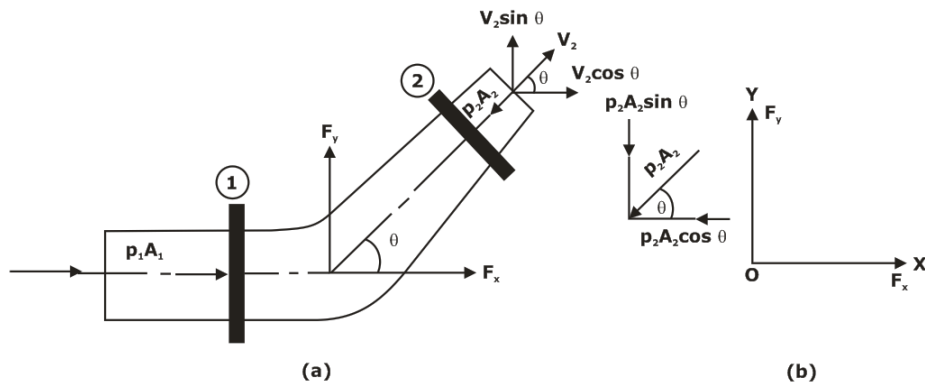
$\therefore F_x = \rho Q (V_1 - V_2 \cos \theta) + p_1 A_1 - p_2 A_2 \cos \theta$

$\therefore F_y = \rho Q (- V_2 \sin \theta) - p_2 A_2 \sin \theta$

Resultant force: $\boxed{F_R = \sqrt{F_x^2 + F_y^2}}$

The angle made by the resultant force with horizontal direction is given by:

$$\boxed{\tan \theta = \frac{F_y}{F_x}}$$

Moment of momentum equation: It states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

According to moment of momentum principle:

Resultant torque = rate of change of moment of momentum

$T = \rho Q [V_2 r_2 - V_1 r_1]$

Applications:

1. For analysis of flow problems in turbines and centrifugal pumps.

2. For finding torque exerted by water on sprinkler

Vortex flow: Vortex flow is defined as the flow of fluid along a curved path or the flow of a rotating mass of fluid is known as vortex flow.

Equation of Motion for Vortex Flow:

$$\boxed{dp = \rho \frac{v^2}{r} dr - \rho g dz}$$

Plane vortex flows: It is the flow where streamlines are concentric circles.

$V_r = 0$ and $V_\theta \neq 0$ and $\frac{\partial V_\theta}{\partial \theta} = 0$

Forced Vortex Flow: In this type of flow, external torque is required to rotate the fluid mass.

$$\boxed{v = w \times r \Rightarrow \omega = \frac{v}{r} = \text{constant}}$$

$$P_2 - P_1 = \frac{\rho}{2} \left[v_2^2 - v_1^2 \right] - \rho g \left[z_2 - z_1 \right] \qquad \left\{ \begin{array}{c} \because \quad v_2 = \omega r_2 \\ v_1 = \omega r_1 \end{array} \right\}$$

where r = Radius of fluid particle from the axis of rotation.

For free surface of the liquid, then $P_1 = P_2$:

$$\left[z_2 - z_1 \right] = \frac{1}{2g} \left[v_2^2 - v_1^2 \right]$$

Note: In case of forced vortex, the rise of liquid level at the ends is equal to the fall of liquid level at the axis of rotation.

$$\text{volume of paraboloid} = \frac{1}{2} \left[\text{Area of cross} - \text{section} \times \text{Height of parabola} \right]$$

Examples: Flow of liquid inside the impeller of a centrifugal pump, Flow of water through the runner of a turbine

Free Vortex Flow: Total mechanical energy remains constant in the entire flow field i.e. does not vary from streamline to streamline. There is neither any addition nor any destruction of energy in the flow field.

$$vr = \frac{\text{Constant}}{m} = \text{Constant}$$

Examples: Flow of liquid through a hole provided at the bottom of a container, Flow of liquid around a circular bend in a pipe, A whirlpool in a river, Flow of fluid in a centrifugal pump casing.

CHAPTER 5: LAMINAR FLOW THROUGH PIPES & PLATES

Introduction: Laminar flow (streamline flow) occurs when a fluid flows in parallel layers, with no disruption between the layers.

Reynold's number: $R_e = \dfrac{\rho VD}{\mu} = \dfrac{VD}{\upsilon}$

Where, V=mean velocity of flow in pipe.

μ = dynamic viscosity of the liquid (N – s /m²)

v = Kinematic viscosity of the liquid (m²/s)

D= diameter → for pipe, D = length → for plate

Hydraulic Diameter: For the flow through non – circular pipes characteristic dimension is given by:

$D_h = \dfrac{4A_c}{P}$

Where,

A_c = Cross – section area of the pipe

P = Perimeter of the pipe

Pipe	Plate
R_e < 2000 laminar	R_e < 5 × 10⁵ Laminar
2000 < R_e < 4000 Transient	R_e > 5 × 10⁵ turbulent
R_e > 4000 turbulent	[transient is small so neglected]

Hagen-Poiseuille flow [Laminar flow in a pipe]:

Assumptions:

(1). steady flow.

(2). flow is fully developed.

Velocity profile is constant w.r.t length in the fully developed flow.

Entrance length
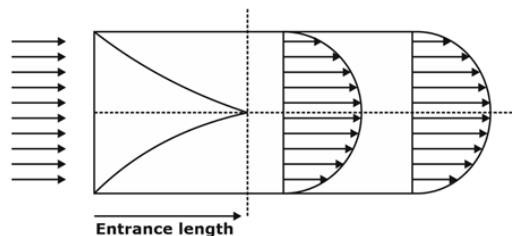
$\dfrac{\partial P}{\partial x} = \text{constant}$

Laminar flow through circular pipe:
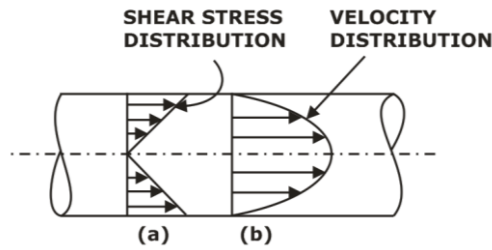
Shear stress: $\tau = \dfrac{-\partial P}{\partial x} \times \dfrac{r}{2}$

It varies linearly along the pipe and it becomes maximum at r=R.

Velocity variation: $u = \dfrac{-R^2}{4\mu}\left(\dfrac{\partial P}{\partial x}\right)\left(1 - \dfrac{r^2}{R^2}\right)$

Velocity varies parabolically along the pipe and it becomes maximum at r = 0.

Maximum velocity: $U_{max} = \dfrac{-R^2}{4\mu}\left(\dfrac{\partial P}{\partial x}\right)$

$u_{avg.} = \dfrac{u_{max}}{2}$

Pressure drop: $P_1 - P_2 = \dfrac{32\mu v L}{D^2} = \dfrac{8\mu v L}{R^2}$

Friction factor: $f = \dfrac{64}{R_e}$

coefficient of friction: $f' = \dfrac{f}{4} = \dfrac{16}{Re}$

Laminar flow between two fixed parallel plates [with unit width]:
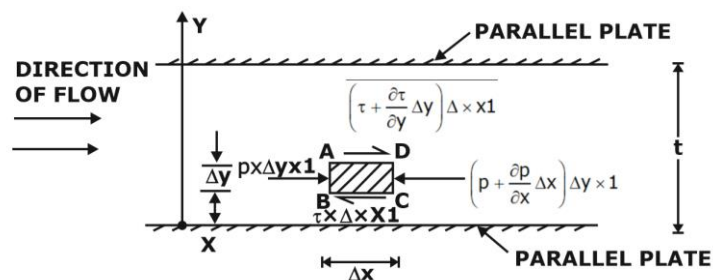
Laminar flow between parallel plates when one of the plates is moving and other one is stationary is known as **"Couette"** flow.

$\dfrac{\partial P}{\partial x} = \dfrac{\partial \tau}{\partial y}$

Velocity Distribution: $u = \dfrac{1}{2\mu}\left(\dfrac{-\partial P}{\partial x}\right)\left(By - y^2\right)$

Maximum velocity: $u_{max} = -\dfrac{t^2}{8\mu}\left(\dfrac{\partial P}{\partial x}\right)$

$V_{avg.} = \dfrac{2}{3}U_{max} = Average\ velocity$

Shear Stress distribution: $\tau = \left(\dfrac{-\partial P}{\partial x}\right)\left(\dfrac{t}{2} - y\right)$
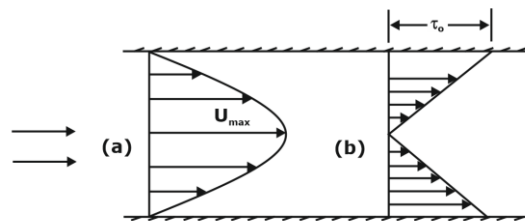
Discharge per unit width: $Q = \dfrac{-1}{12\mu}\left(\dfrac{\partial P}{\partial x}\right)t^3$

Pressure difference: $P_1 - P_2 = \dfrac{12\mu V_{avg.}L}{t^2}$

Correction factors:

Momentum correction factor (β): It is defined as the ratio of momentum per second based on actual velocity to the momentum per second based on average velocity across a section. It is denoted by β.

$\beta = \dfrac{1}{AV^2}\displaystyle\int_0^R u^2 dA$

Kinetic energy correction factor (α): It is defined as the ratio of kinetic energy of flow per second based on actual velocity to the kinetic energy of the flow per second based on average velocity across a same section.

$\alpha = \dfrac{1}{AV^3}\displaystyle\int_0^R u^3 dA$

Correction-factor	Laminar	Turbulent
α	2	1.33
β	1.33	1.2

CHAPTER 6: TURBULENT FLOW

Introduction: In case of turbulent flow there is huge order intermission fluid particles and due to this, various properties of the fluid are going to change with space and time.
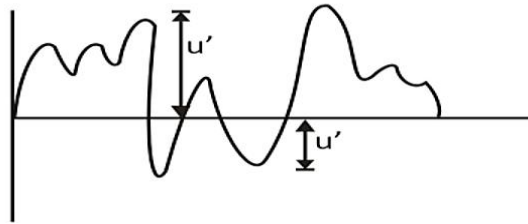
$\overline{u}$ = mean or average component of velocity

Any parameter = Average component + fluctuating component

$$u = u' + \overline{u}$$

Shear stress in turbulent flow:

Shear stress due to turbulence: $\tau_t = \eta \dfrac{\overline{du}}{dy}$ (Boussinesq Hypothesis).

Kinematic eddy viscosity: $\varepsilon = \dfrac{\eta}{\rho}$

Total shear stress: $\tau = \tau_v + \tau_t = \mu \dfrac{du}{dy} + \eta \dfrac{\overline{du}}{dy}$

Reynold's hypothesis: $T_{turbulent.} = \rho\, u'\, v'$

According to Prandtl Mixing length hypothesis: $u' = v' = l\dfrac{du}{dy}$

l = Prandtl's mixing length

$$\tau_{turb.} = \rho l^2 \left(\dfrac{du}{dy} \right)^2$$

$\tau_{total} = \boxed{\tau_{laminar}} + \tau_{turb.} \xrightarrow{\text{Negligible}}$

Relation of Shear Stress with Coefficient of friction: $f = \dfrac{2\tau_0}{\rho V^2}$

Velocity distribution in turbulent flow:

Shear or Fictitious velocity: $u^* = \sqrt{\dfrac{\tau_0}{\rho}}$

$$u = u_{max} + 2.5\, u^* \log_e \left(\dfrac{y}{R} \right)$$

The above equation is known as 'Prandtl's universal velocity distribution equation for turbulent flow in pipes.

$$\frac{u_{max} - u}{u^*} = 5.75 \log_e(R / y)$$

The difference between the maximum velocity u_{max}, and local velocity u at any point i.e. $(u_{max} - u)$ is known as 'velocity defect'.
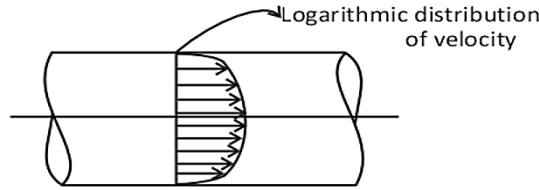
Hydrodynamically smooth & rough pipes:
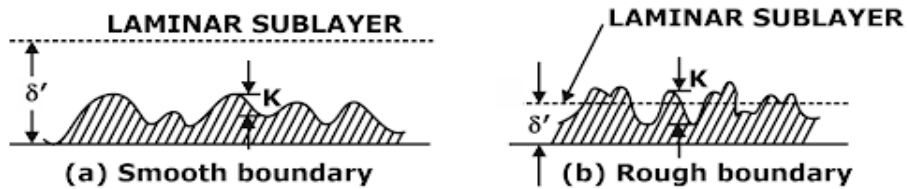
K = average height of roughness

δ' = height of Laminar sublayer

Nikuradse's conditions for smooth & rough boundary.

Nikuradse's conditions	In terms of roughness Reynolds number $\left(\dfrac{u^*k}{\nu}\right)$
$\dfrac{K}{\delta'} < 0.25 \rightarrow smooth$	If $\dfrac{u^*k}{\nu} < 4$, boundary is considered smooth.
$0.25 < \dfrac{K}{\delta'} < 6 \rightarrow Transition$	If $4 < \dfrac{u^*k}{\nu} < 100$, boundary is in transition stage
$\dfrac{K}{\delta'} > 6 \rightarrow Rough$	If $\dfrac{u^*k}{\nu} > 100$, the boundary is rough

Laminar sublayer thickness: $\boxed{\delta' = \dfrac{11.6\upsilon}{V_s}}$

shear friction velocity or hypothetical velocity: $\boxed{V_s = \sqrt{\dfrac{\tau}{\rho}} = V.\sqrt{\dfrac{f}{8}}}$ =

and V is the average velocity.

Velocity distribution for turbulent flow:

Velocity distribution in rough pipes: $\boxed{\dfrac{u}{u^*} = 5.75\log_{10}\left(\dfrac{y}{k}\right) + 8.5}$

Average velocity in Rough pipes: $\boxed{\dfrac{\bar{U}}{u^*} = 5.75\log_{10}\left(\dfrac{R}{K}\right) + 4.75}$

Velocity distribution in rough pipes: $\boxed{\dfrac{u}{u^*} = 5.75\log_{10}\left(\dfrac{u^*y}{v}\right) + 5.5}$

Average velocity in smooth pipes: $\boxed{\dfrac{\bar{U}}{u^*} = 5.75\log_{10}\left(\dfrac{u^*R}{v}\right) + 1.75}$

Now: $\boxed{\dfrac{u - \bar{U}}{u^*} = 5.75\log_{10}(y/R) + 3.75}$

Velocity distribution for turbulent flow with average velocity in smooth pipes for power law:

$\boxed{\dfrac{u}{u_{max}} = \left(\dfrac{y}{R}\right)^{1/n}}$ and $\boxed{n = \dfrac{1}{7} \Rightarrow \dfrac{u}{u_{max}} = \left(\dfrac{y}{R}\right)^{1/7}}$

Resistance of smooth and rough pipes:

The term of the right-hand side is called co-efficient of friction f. Thus:

$\boxed{f = \phi\left[R_e, \dfrac{k}{D}\right]}$

This equation shows that friction co-efficient is a function of Reynolds number and k/D ratio, where k is the average height of pipe wall roughness protrusions.

Friction factor in turbulent flow:

(a). For Smooth pipes:

The value of 'f' for smooth pipe for Reynolds number varying from 4000 to 100000 is given by the relation:

$\boxed{f = \dfrac{.0791}{(R_e)^{1/4}}}$

When, Re $= 10^5$ to $4 \times 10^7 \Rightarrow \boxed{f = 0.0032 + \dfrac{0.221}{Re^{0.232}}}$

(b). For rough pipes:

$\boxed{\dfrac{1}{\sqrt{4f}} = 2\log_{10}\left(\dfrac{R}{K}\right) + 1.74}$ When, Re $> 6 \times 10^8$

CHAPTER 7: LOSSES THROUGH PIPES

Introduction: Major loss is due to friction and calculated by Darcy Weisbach equation and Chezy's formula**.**

Minor loss into five types of losses i.e., sudden expansion loss, sudden contraction loss, exit loss, entry loss and bend losses.

Major losses:

Darcy Weisbach equation: $h_f = \dfrac{fLV^2}{2gD} = \dfrac{4f'LV^2}{2gD}$

Where, f is friction factor

f = 4f',

where f' is friction coefficient

V is mean velocity of flow in the pipe, D is diameter of the pipe, L is length of the pipe

Chezy's formula:
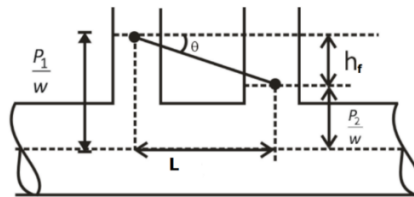
$V = C\sqrt{mi}$

Where, C = Chezy's constant

V = Average velocity

i = hydraulic slope

Hydraulic mean depth: $m = \dfrac{Area\ of\ cross-\sec tion}{wetted\ perimeter} = \dfrac{\frac{\pi}{4}D^2}{\pi D} = \dfrac{D}{4}$

Minor losses:

Sudden expansion loss (h$_l$)$_{se}$: $h_e = \dfrac{(V_1 - V_2)^2}{2g}$

Sudden contraction loss: $(h_l)_{S.C.} = \left(\dfrac{1}{C_C} - 1\right)^2 \dfrac{V_2^2}{2g} = k\dfrac{V_2^2}{2g}$

Entry losses: $(h_l)_{entry} = 0.5\dfrac{V^2}{2g}$

Exit losses: $(h_l)_{exit} = \dfrac{V^2}{2g}$

Bend losses: $(h_l)_{Bend} = K\dfrac{V^2}{2g}$

Hydraulic gradient line (HGL) & total energy line (TEL):

(i). The line that represents the sum of the static pressure and the elevation heads $\dfrac{p}{\rho g} + z$ is called the hydraulic grade line.

(ii). The line that represents the total head of the fluid $\dfrac{p}{\rho g} + \dfrac{V^2}{2g} + z$ is called the energy grade line.

(iii). The difference between the heights of EGL and HGL is equal to the dynamic head $\dfrac{V^2}{2g}$

.

(iv). For *stationary bodies* such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid. The elevation of the free surface *z* in such cases represents both the EGL and the HGL since the velocity is zero and the static pressure (gage) is zero.

(v). *A steep jump or droop* occurs in EGL and HGL whenever mechanical energy is added to the fluid (by a pump or mechanical energy is removed from the fluid (by a turbine) respectively.
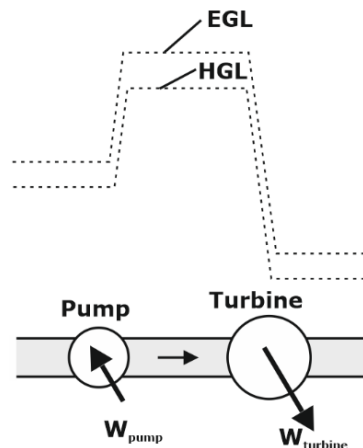
Pipes in series & parallel
(a). Pipes in series:

$$Q_1 = Q_2 + Q_3 + Q_4 \ldots\ldots\ldots Q_n = Q = const.$$

$$h_l = h_{l_1} + h_{l_2} + h_{l_3} \ldots\ldots h_{l_n}$$

Equivalent length is given by(*Dupit's formula*):

$$\frac{L_e}{D_e^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots\ldots\ldots$$

(b). Pipes in Parallel:

$Q = Q_1 + Q_2 + Q_3 \ldots\ldots Q_n$

$h_{l_1} = h_{l_2} = h_{l_3} = \ldots\ldots h_{l_n}$

For "n" pipes → *Discharge through each pipe* $= \dfrac{Q}{n}$

$h_{l_e} = \left(h_l\right)$

$$\frac{fL_e Q^2}{12 D_e^5} = \frac{fLQ^2}{n^2 12 D^5} \Rightarrow \frac{L_e}{D_e^5} = \frac{L}{n^2 D^5}$$

(When all pipes are similar).

$$\sqrt{\frac{D_e^5}{L_e}} = \sqrt{\frac{D_1^5}{L_1}} + \sqrt{\frac{D_2^5}{L_2}} + \sqrt{\frac{D_3^5}{L_3}} + \ldots\ldots$$

(When pipes are different & head loss is same).

Power transmission through pipes:

$$P_{th.} = \rho g Q H$$
$$P_{act} = \rho g Q \left(H - h_f \right)$$

Efficiency is given by:

$$\eta = \frac{P_{act}}{P_{th.}} = \frac{H - h_f}{H}$$

Condition for maximum power transmission:

$$h_f = \frac{H}{3}$$ and $$\eta_{max} = \frac{H - h_f}{H} = 66.67\%$$

CHAPTER 8: BOUNDARY LAYER THEORY

Introduction: It is a narrow thin region near the solid boundary where velocity gradient exists.

Growth of boundary layer over a flat plate:
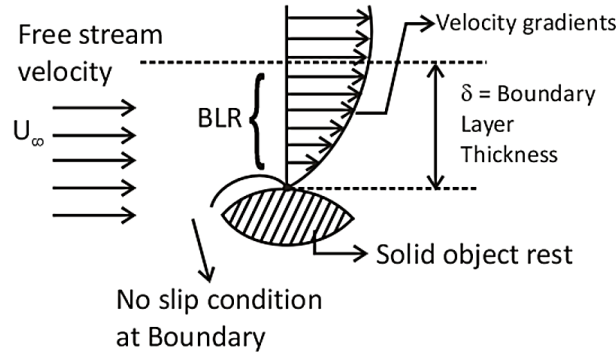
The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the **boundary layer.**
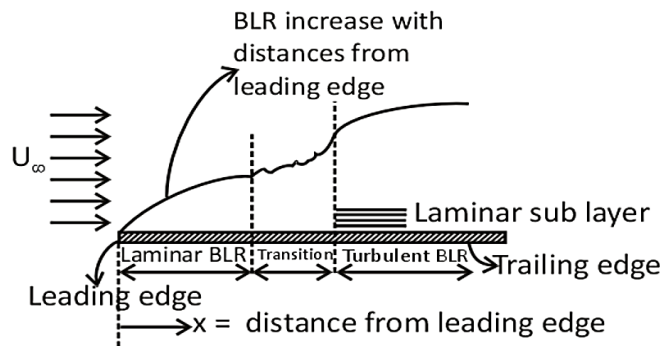
Growth of boundary layer over a flat plate:
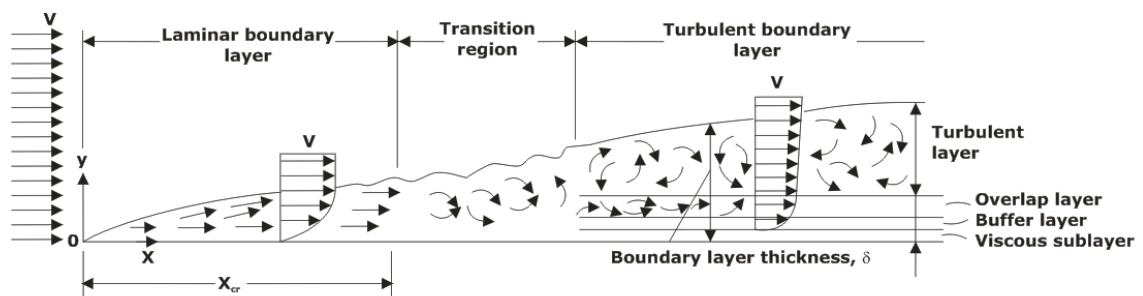
Boundary conditions:

(i). at y = 0, u = 0

(ii). y = δ, u = u∞

(iii). y = δ, $\dfrac{du}{dy} = 0$

(iv). x = 0, δ = 0

Boundary layer thickness (δ): It is defined as the vertical distance from boundary up to the point where velocity becomes 99% of free stream velocity.

For numerical, at y = δ; u = u∞

Displacement thickness (δ*): It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation. It is denoted by δ*.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$$

Momentum thickness (θ): It is defined as "the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation".

$$\theta = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$$

Energy thickness (δₑ): It is defined as "the distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation".

$$\delta_e = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u^2}{u_\infty^2}\right) dy$$

Shape Factor(H): It is defined as the ratio of displacement thickness to momentum thickness.

$$H = \frac{\delta^*}{\theta}$$

Flow over a flat plate: Von-Karman momentum integral equation:

$$\frac{\tau_0}{\rho v_\infty^2} = \frac{d\theta}{dx}$$

Reynolds number is given by: $Re = \dfrac{\rho u_\infty L}{\mu}$

If, Re < 5 × 10⁵ → Laminar flow.

Re > 5 × 10⁵ → turbulent flow.

Coefficient of Drag (C_D)

$$F_D = \frac{C_D^*}{2} \rho A U^2$$

Where, F_D is the drag force experienced by the motion of fluid over flat plate C_D* = local skin friction coefficient.

Analysis of turbulent boundary layer:

(a). If Reynold number is more than 5×10^5 and less than 10^7 the thickness of boundary layer and drag co-efficient are given as:

$$\delta = \frac{0.37x}{\left(R_{e_x}\right)^{1/5}} \quad \text{and} \quad C_D = \frac{0.072}{\left(R_{e_L}\right)^{1/5}}$$

where x = Distance from the leading edge

R_{e_x} = Reynold number for length x

R_{e_L} = Reynold number at the end of the plate.

(b). If Reynold number is more than 10^7 but less than 10^9, Schlichting gave the empirical equation as:

$$C_D = \frac{0.455}{\left(\log_{10} R_{e_L}\right)^{2.58}}$$

Boundary layer separation:
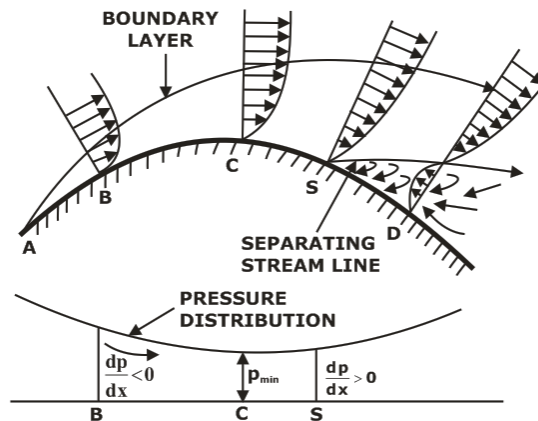
The separation point S is determined from the condition: $\left(\dfrac{\partial u}{\partial y}\right)_{y=0} = 0$

1. If $\left(\dfrac{\partial u}{\partial y}\right)_{y=0} > 0$, the flow has separated, and it is the necessary condition for flow separation.

2. If $\left(\dfrac{\partial u}{\partial y}\right)_{y=0} = 0$, the flow is on the verge of separation. This is the sufficient condition for the flow separation to occur.

3. If $\left(\dfrac{\partial u}{\partial y}\right)_{y=0} < 0$, the flow will not separate, or flow will remain attached with the surface.

CHAPTER 9: DIMENSIONLESS ANALYSIS

Introduction: length(L), Mass(M), Time(T) dimensions are called fundamental dimension.

Some important dimensions:

Area	A	L^2
Volume	V	L^3
Velocity	v	LT^{-1}
Angular velocity	ω	T^{-1}
Acceleration	a	LT^{-2}
Angular Acceleration	α	T^{-2}
Discharge	Q	L^3T^{-1}
Acceleration due to Gravity	g	LT^{-2}
Force	F	MLT^{-2}
Weight	W	MLT^{-2}
Dynamic viscosity	μ	$ML^{-1}T^{-1}$
Kinematic viscosity	ϑ	L^2T^{-1}
Pressure	p	$ML^{-1}T^{-2}$
Surface Tension	σ	MT^{-2}
Work/Energy	E	ML^2T^{-2}
Power	P	ML^2T^{-3}

Methods of dimensional analysis:

(i). Rayleigh method:

It is used for determining the expression for a variable which depends maximum on three or four variables.

Let X be a variable which depends on X_1, X_2, X_3 then, according to Rayleigh method

X=f (X_1, X_2, X_3)

$$X = KX_1^{a}X_2^{b}X_3^{c}$$

(**ii). Buckingham π-theorem:**

Buckingham's π theorem states that "If there are n variables in a problem and these variables contain m primary dimensions (for example M, L, T) the equation relating all the variables will have (n-m) dimensionless groups".

Buckingham referred these groups as π groups. The final equation obtained is in the form of π_l = f (π_2, π_3 .... π_{n-m}).

Method of selecting repeating variables: The number of repeating variables is equal to the number of fundamental dimensions of the problem. The choice of repeating variables is governed by following considerations:

(a). Dependent variable should not be repeating variable

(b). Repeating variable should be selected in such a way that one of the variables should be geometric property, other should be flow property and third variable should be fluid property.

Various forces in fluid mechanics:

Inertia force (F$_i$): $\boxed{F_i = \rho l^2 v^2}$

Surface tension force (F$_s$): It is equal to the product of surface tension and length of surface of the flowing fluid.

$F_s = \sigma \times l$

Gravity force (F$_g$): It is equal to the product of mass and acceleration due to gravity of the flowing fluid.

$F_g = \rho l^3 g$

Pressure force (F$_P$): It is equal to the product of pressure intensity and cross-sectional area of the flowing fluid

$F_P = Pl^2$

Viscous force (F$_v$): It is equal to the product of shear stress (τ) due to viscosity and surface area of the flow.

$F_v = \mu l v$

Elastic force (F$_e$): It is equal to the product of elastic stress and area of the flowing fluid.

$F_e = Kl^2$

Dimensionless numbers:

Dimensionless Number with Symbol	Group Variables	Significance	Field of Use
1. Reynolds number (R$_e$)	$\dfrac{\rho VL}{\mu}$	$\dfrac{\text{inertia force}}{\text{viscous force}}$	Laminar viscous flow in confined passages where viscous effects are predominant.
2. Froude number (F$_r$)	$\dfrac{V}{\sqrt{lg}}$	$\dfrac{\text{inertia force}}{\text{gravitational force}}$	Free surface flows where effects of gravity are important.
3. Mach number (M)	$\dfrac{V}{\sqrt{K/\rho}}$	$\dfrac{\text{inertia force}}{\text{elastic force}}$	High speed flow where compressible effects are important.
4. Weber number (W)	$\dfrac{\rho l V^2}{\sigma}$	$\dfrac{\text{inertia force}}{\text{surface tension force}}$	Small surface waves, capillary and sheet flow where surface tension is important.
5. Euler number (E)	$\dfrac{p}{\rho V^2}$	$\dfrac{\text{inertia force}}{\text{pressure force}}$	Conduit flow where pressure variations are significant.

Similitude & modeling:

Geometric similarity:

Length ratio: $\dfrac{l_m}{l_p} = \dfrac{b_m}{b_p} = \dfrac{t_m}{t_p} = L_r$

Area ratio: $\dfrac{A_m}{A_P} = \dfrac{l_m b_m}{l_p b_p} = L_r^{2}$

Volume ratio: $\dfrac{V_m}{V_p} = L_r^{3}$

Kinematic similarity: Kinematic similarity means the similarity of motion between the model and prototype.

$$\boxed{\dfrac{V_{1m}}{V_{1P}} = \dfrac{V_{2m}}{V_{2p}} = \dfrac{V_{3m}}{V_{3p}}}$$

Dynamic Similarity: A model and prototype are said to be in dynamic similarity, if the ratio of forces at corresponding points in model and prototype are same.

$$\dfrac{(F_v)_{1m}}{(F_V)_{1p}} = \dfrac{(F_i)_{1m}}{(F_i)_{1p}} = \dfrac{(F_g)_{1m}}{(F_g)_{1p}} = \dfrac{(F_p)_{1m}}{(F_P)_{1p}}$$

$$\dfrac{(F_i)_{1m}}{(F_v)_{1m}} = \dfrac{(F_i)_{1p}}{(F_v)_{1p}} \Rightarrow (R_e)_{1m} = (R_e)_{1p} \rightarrow \text{Reynolds model law}$$

Model laws: The laws on which the models are designed for dynamic similarity are called model laws or laws of similarity. The followings are the model laws:

Reynold's Model Law: Reynold's model law is the law in which models are based on Reynold's number.

$$[R_e]_m = [R_e]_P \quad \text{or} \quad \dfrac{\rho_m V_m L_m}{\mu_m} = \dfrac{\rho_P V_P L_P}{\mu_P}$$

Froude's Model Law:

$$(F_e)_{model} = (F_e)_{prototype} \Rightarrow \dfrac{V_m}{\sqrt{g_m L_m}} = \dfrac{V_P}{\sqrt{g_P L_P}}$$

$$\boxed{\dfrac{V_P}{V_m} = \sqrt{\dfrac{L_P}{L_m}} = \sqrt{L_r}}$$

$$\boxed{a_r = 1}$$

$$\boxed{Q_r = L_r^{2.5}}$$

Distorted Models: A model is said to be distorted if it is not geometrically similar to its prototype. For a distorted model different scale ratio for the linear dimensions are adopted. For example, in case of rivers, harbours, reservoirs etc., two different scale ratios, one for horizontal dimensions and other for vertical dimensions are taken.

CHAPTER 10: FLOW THROUGH JET

Introduction: A jet is a stream of fluid that is projected into a surrounding medium, usually from some kind of a nozzle, aperture or orifice.

Force exerted by fluid jet:

(a). Force exerted on a stationary flat plate held normal to jet:

$F_x = \rho a V^2$

V = velocity of jet,

d = diameter of the jet

a = area of cross section of the jet

(b). Force of Jet Impinging on An Inclined Fixed Plate:
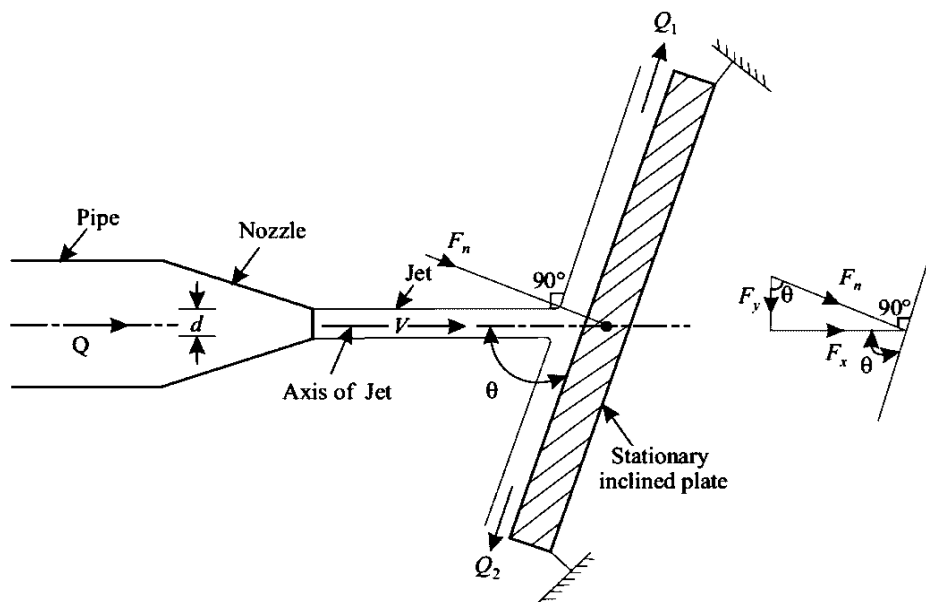
Force normal to plate: $F_n = \rho a V (V \sin\theta - 0) = \rho a V^2 \sin\theta$

Horizontal component in x direction:

$$\boxed{F_x = F_n \sin\theta = \rho a V^2 \sin^2\theta}$$

Horizontal component in x direction:

$$\boxed{F_x = F_n \cos\theta = \rho a V^2 \sin\theta \cos\theta}$$

$$\boxed{Q_1 = \frac{Q}{2}(1 + \cos\theta)} \quad \text{and} \quad \boxed{Q_2 = \frac{Q}{2}(1 - \cos\theta)}$$

Force exerted on stationary curved plate:

(a). Jet strikes the curved plate at the centre:
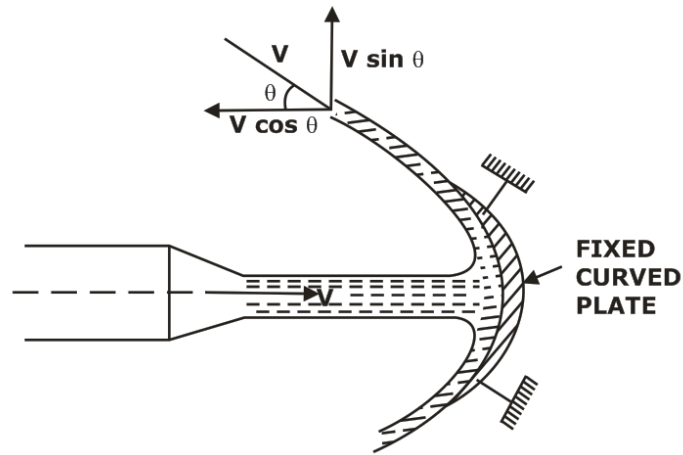
Force in x-direction is given by:

$$F_x = \rho AV(V_{1x} - V_{2x})$$

$$F_x = \rho AV(V - (-V\cos\theta)) = \rho aV^2(1 + \cos\theta)$$

Force in y-direction is given by:

$$F_y = \rho AV(V_{1y} - V_{2y})$$

$$F_y = \rho AV[0 - V\sin\theta] = -\rho AV^2 \sin\theta$$

Negative sign indicates that force is in downward direction.

(b). Jet strikes the stationary curved plate at one end tangentially when the plate is symmetrical:

Force in horizontal direction is given by:

$$F_x = \rho AV(V_{1x} - V_{2x})$$

$$F_x = \rho AV(V\cos\theta - (-V\cos\theta)) = 2\rho aV^2 c\,os\theta$$

Force in y – direction is given by:

$$\boxed{F_y = \rho AV(V_{1y} - V_{2y})}$$

$$\boxed{F_y = \rho AV(V\sin\theta - V\sin\theta) = 0}$$

(c). Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical:

$$F_x = \rho AV(V_{1x} - V_{2x})$$

$$\boxed{F_x = \rho AV(V\cos\theta - (-V\cos\phi)) = \rho aV^2(\cos\theta + \cos\phi)}$$

Force in y direction is given by:
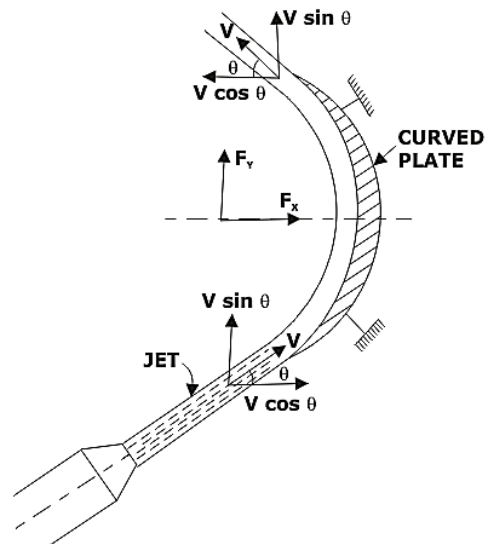
$$F_y = \rho AV(V_{1y} - V_{2y})$$

$$\boxed{F_y = \rho AV(V\sin\theta - (-V\sin\phi)) = \rho aV^2(\sin\theta + \sin\phi)}$$

Force exerted on moving plate:

(a). Force exerted on a moving flat plate held normal to jet:
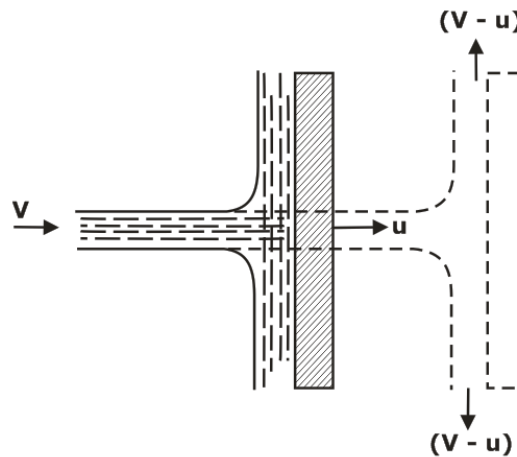
Horizontal component of force in x – direction:

$$\boxed{F_x = \rho a(V - u)[(v - u) - 0] = \rho a(V - u)^2}$$

$$\boxed{\text{Work done} = F_x.u = \rho a(V - u)^2 u}$$

(b). Force of Jet Impinging on An Inclined moving Plate:
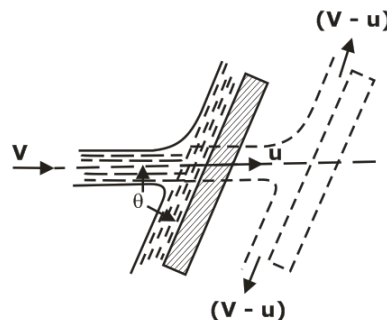
$$\boxed{F_n = \rho a(V - u)[(v - u)\sin\theta - 0] = \rho a(V - u)^2 \sin\theta}$$

$F_x = F_n\sin\theta$ and $F_y = F_n\cos\theta$

$$\boxed{F_x = \rho a(V - u)\sin\theta \times \sin\theta = \rho a(V - u)^2 \sin^2\theta}$$

$$F_y = \rho a(V-u)\sin\theta \times \cos\theta = \rho a(V-u)^2 \sin\theta\cos\theta$$

$$\text{Work done} = F_x \times u = \rho a(V-u)^2 u \sin^2\theta$$

Force exerted by jet on curved plate(vane) when the plate/vane is moving in the direction of jet

(a). When single curved vane is used:

Horizontal component of force:

$$F_x = \rho a(V-u)^2(1+\cos\theta)$$

Work done one the vane per sec:

Work done/sec = $\rho a(V-u)^2(1+\cos\theta)$ u

Kinetic energy of issuing jet:

$$KE = \frac{1}{2}mV^2 = \frac{1}{2}\rho AV \times V^2 = \frac{1}{2}\rho AV^3$$

Efficiency is given as:

$$\eta = \frac{\text{Work done / sec}}{\text{Kinetic energy}} = \frac{\rho a(v-u)^2(1+\cos\theta)u}{\frac{1}{2}\rho AV^3}$$

At u = V/3 maximum efficiency will be obtained.

(b). When series of vane is used:

Horizontal component is given by:

$F_x = \rho aV(V-u)(1+\cos\theta)$

Work done one the vane per sec = $\rho aV(V-u) \times (1+\cos\theta)u$

Kinetic energy of issuing jet:

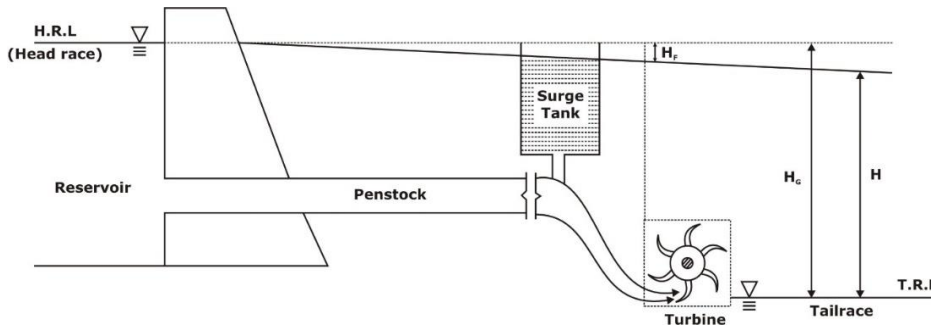$$KE = \frac{1}{2}mV^2 = \frac{1}{2}\rho AV \times V^2 = \frac{1}{2}\rho AV^3$$

$$\eta = \frac{\text{Work done / sec}}{\text{Kinetic energy}} = \frac{\rho aV(V-u)(1+\cos\theta)u}{\frac{1}{2}\rho AV^3}$$

At u=V/2, maximum efficiency will be obtained

Maximum efficiency is given by: $\eta = \dfrac{1+\cos\theta}{2}$

CHAPTER 11: HYDRAULIC MACHINERY

Hydraulic turbine:

Classification of turbines:

(a). On the basis of energy available at inlet:

(i). Kinetic energy only: Impulse turbine

Ex: Pelton Wheel (tangential flow)

(ii). Kinetic energy + Pressure energy = Impulse Reaction Turbine.

Ex: Francis (mixed flow) and Kaplan (Axial flow)

(b). On the basis of head, discharge and specific speed:

Turbine	Head	Discharge	Specific speed
Pelton	High (H > 250 m)	Low	Low
Francis	Medium (50 < H < 250)	Medium	Medium
Kaplan	Low (H < 50 m)	High	High

Type of Heads:

Gross Head (H_G): It is elevation difference between tail race level (TRL) and head race level (HRL). It is the total head under which power plant is working.

Net head (H): It is calculated by subtracting the heat loss in penstock from gross head. It is the net head under which turbine is working.

$H_{net} = H_G - h_f$

$$H_{net} = H_G - \left(\frac{fLV^2}{2gD} \right)$$

Types of Power:

Waterpower/Hydraulic (power power at inlet to turbine):

$$\text{Water power} = \dot{m}gh = \rho QgH$$

Power available at exit of nozzle (KE/sec):

Power = $\frac{1}{2}\dot{m}v_1^2$,   (v_1= velocity of jet at the exit of nozzle)

Thus: $\rho QgH = \frac{1}{2}\dot{m}v_1^2$ (if there is no head loss in nozzle)

$$gH = \frac{V_1^2}{2} \Rightarrow V_1 = \sqrt{2gH}$$

Practically : $v_1 = C_v\sqrt{2gH}$

$$\text{Runner power} = \rho Q\left(V_{w1}u_1 \pm V_{w2}u_2\right)$$

Final power available on the shaft:

Shaft power (SP) = (RP) – (Mechanical frictional losses) = Final power output

Efficiencies

(a). Volumetric efficiency $\left[\eta_{vol}\right]$

$$\eta_{vol} = \frac{\text{Discharge striking the runner}}{\text{Discharge supplied by penstock}} = \frac{Q - \Delta Q}{Q}$$

$\Delta Q \Rightarrow$ leakage loss (some amount of water directly goes to tailrace without striking the runner).

(b). Hydraulic efficiency $\left(\eta_n\right)$:

$$\eta_n = \frac{\text{Runner Power}}{\text{Water Power}} = \frac{R.P.}{W.P.}$$

(c). Mechanical efficiency $\left(\eta_m\right)$:

$$\eta_m = \frac{S.P.}{R.P.}$$

(d). Overall efficiency $\left(\eta_o\right)$:

$$\eta_o = \frac{S.P.}{R.P.} \times \frac{R.P.}{W.P.} = \eta_m \times \eta_H \qquad \left[\text{when } \eta_{vol} \text{ is not given}\right]$$

$$\eta_o = \eta_{vol} \cdot \eta_m \cdot \eta_h \qquad \left[\text{when } \eta_{vol} \text{ is given}\right]$$

Pelton wheel (Tangential flow runner):

Bucket velocity is given by: $u = \frac{\pi DN}{60}$

and $u = u_1 = u_2$

Waterpower is given by: $W.P. = \rho QgH$

Runner Power: $P = \text{Work Done / sec} = F.u. = \rho \cdot aV_1 Vr_1\left(1 + k\cos\phi\right) \cdot u$

Hydraulic efficiency is given by:

$$\eta_h = \frac{\rho \cdot a \cdot V_1 Vr_1\left(1 + k\cos\phi\right) \cdot u}{\frac{1}{2}\dot{m}V_1^2}$$

Condition for maximum hydraulic efficiency: $\boxed{u = \dfrac{V_1}{2}}$

$$\boxed{(\eta_h)_{max} = \frac{1 + k\cos\phi}{2}}$$

Design Data for Pelton turbine

(a). $\phi = 10 - 20° = $ exit blade angle

$\delta = $ angle of deflection $= 180 - \phi$ (Generally 160 – 170°).

(b). Jet ratio (m) $= \dfrac{D}{d} = \dfrac{\text{Runner dia}}{\text{Jet dia}}$

(c). No. of vanes $(\text{Bucket}) = \left[\dfrac{m}{2} + 15\right] = \left[\dfrac{D}{2d} + 15\right]$

Francis turbine: Francis turbine is an inward flow reaction turbine. To maximise runner power, V_2 should be minimum. It can be done by converting absolute velocity direction into radial direction to the runner at exit.

Runner Power per unit weight: $\boxed{\dfrac{RP}{\dot{m}g} = \dfrac{\rho Q V_{w_1} \cdot u_1}{\rho Q g} = \dfrac{V_{w_1} \cdot u_1}{g}}$

$\boxed{\text{Runner head} = \dfrac{RP}{\dot{m}g} = \dfrac{V_{w1} \cdot u_1}{g}}$

Hydraulic efficiency: $\boxed{\eta_h = \dfrac{RP}{HP} = \dfrac{\rho Q \cdot V_{w_1} u_1}{\rho Q g H} = \dfrac{V_{w_1} \cdot u_1}{gH}}$
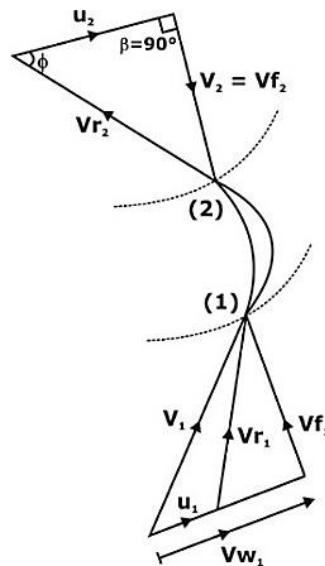
Axial flow reaction turbine: Kaplan and propeller turbines are the example of axial flow reaction turbine.

Performance of turbine:

Unit speed (Nu): $\boxed{Nu = \dfrac{N}{\sqrt{H}}}$

Unit discharge (Q_u): $\boxed{Q_u = \dfrac{Q}{\sqrt{H}}}$

Unit power (P_u): $\boxed{Pu = \dfrac{P}{H^{3/2}}}$

Specific speed of Turbine (N_s):

It is defined as the speed of a turbine which is identical in shape, geometrical dimensions, blade angles, gate opening etc., with the actual turbine but of such a size that it will develop unit power when working under unit head.

$$\boxed{N_S = \dfrac{N\sqrt{P}}{H^{5/4}}}$$

Degree of Reaction (R): Degree of reaction is defined as the ratio of pressure energy change inside a runner to the total energy change inside the runner. It is represented by 'R'.

$$\boxed{R = \dfrac{\text{Change of pressure energy inside the runner}}{\text{Change of total energy inside the runner}}}$$

$$\boxed{R = 1 - \dfrac{\left(V_1^2 - V_2^2\right)}{2g\,H_e}}$$

(i) For a Pelton turbine**:** $u_1 = u_2$ and $V_{r2} = V_{r1}$

Degree of reaction : $\boxed{R = 0}$

(ii). For a Francis turbine: $\boxed{R = 1 - \dfrac{\cot\alpha}{2\left(\cot\alpha - \cot\theta\right)}}$

Draft tube: It is a diverging tube fitted at the exit of runner which connect exit of the runner to the tailrace. It's diverging section helps in converging of velocity head into pressure head, but the angle of divergence should not be greater than 6-8°.
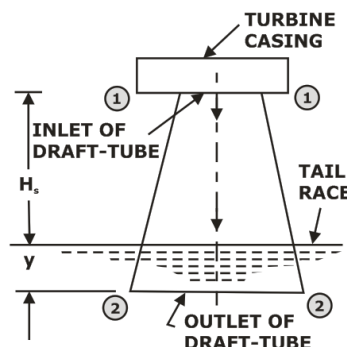
Efficiency of Draft tube: $\eta_d = \dfrac{\left(\dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g}\right) - h_f}{\left(\dfrac{V_1^2}{2g}\right)}$

Hydraulic pumps: The device which converts Mechanical energy into hydraulic energy or pressure energy is called hydraulic pumps.

Types of pumps: The pumps are classified as follows:

(1). Positive displacement Pumps: Positive displacement pumps are further classified into two categories:

(a). Rotary Pumps: Rotary pumps include Gear Pump, Lobe pump, Vane Pump, Screw Pump.

(b). Reciprocating pumps: Reciprocating pumps include piston and diaphragm pumps.

(2). Dynamic Pumps: Dynamic pumps include centrifugal and axial pumps.

Centrifugal pump: When certain mass of fluid is rotated about centre of rotation, increase in head takes place.

Component of Centrifugal Pump:

(i). Casing **(ii).** Impeller
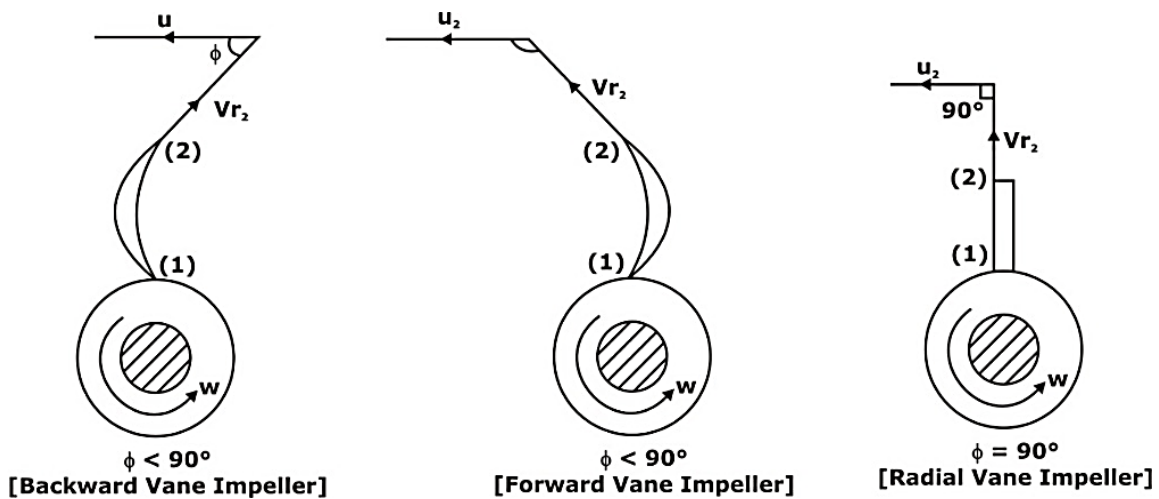
Types of Head:

Static Head (H_S): $H_S = h_s + h_d$

Friction head loss (H_F): $H_F = h_{fs} + h_{fd}$

Manometric head (H_m): Actual head produced by the pump. It is given by:

For ideal pump: $H_m = \dfrac{V_{w2} u_2}{g}$

Power: Power requirement of pump = shaft power (SP)

Impeller power (IP) = SP − (mechanical frictional losses)

$$IP = \rho Q V_{w2} \cdot u_2$$

Output power:

Waterpower (Manometric power) = $\boxed{\rho Q g H_m}$

Note:

$$SP > IP > MP$$

Efficiencies:

(a). Volumetric Efficiency (η_v) : $\boxed{\eta_v = \dfrac{Q}{Q + \Delta Q}}$

ΔQ = amount of discharge not delivered

(b). Manometric efficiency (η_{mano}) :

$$\eta_{man} = \frac{\text{Mechanical.Power.}}{\text{I.P.}} = \frac{\rho Q \cdot g H_m}{\rho Q v_{w2} u_2} = \frac{g \cdot H_m}{V_{w2} \cdot u_2}$$

(c). Mechanical Efficiency (η_{mech}) : $\boxed{\eta_{mech} = \dfrac{\text{I.P.}}{\text{S.P.}}}$

(d). Overall Efficiency (η_o) : $\boxed{\eta_o = \eta_{mano} \cdot \eta_{mech} = \dfrac{\rho Q g H_m}{\text{S.P.}}}$

Minimum starting speed of pump: $\boxed{N \geq \dfrac{60}{2\pi} \sqrt{\dfrac{2 g H_m}{r_2^2 - r_1^2}}}$

Specific speed of pump (N$_s$): The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver one cubic metre of liquid per second against a head of one metre. It is denoted by 'N$_s$'.

$$N_S = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

Non-dimensional form of specific speed $\Rightarrow$ It is called the Shape number of the pump.

$$\text{Shape no} = \frac{N\sqrt{Q}}{(g H_m)^{3/4}}$$

Model-Prototype relationship

1. $\boxed{\left.\dfrac{H_m}{D^2 N^2}\right|_m = \left.\dfrac{H_m}{D^2 N^2}\right|_p}$

2. $\boxed{\left.\dfrac{Q}{D^3 N}\right|_m = \left.\dfrac{Q}{D^3 N}\right|_p}$

3. $\boxed{\left.\dfrac{P}{D^5 N^3}\right|_m = \left.\dfrac{Q}{D^5 N^3}\right|_p}$

Priming of pump: Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe up to the delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump.

Cavitation: Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure and causing damage to the components.

Thomas cavitation parameter:

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H}$$

where H_b = Barometric pressure head in m of water,

H_{atm} = Atmospheric pressure head in m of water,

H_v = Vapour pressure head in in of water.

H_s = Suction pressure at the outlet of reaction turbine in m of water or height of turbine runner above the tail water surface,

H = Net head on the turbine in m

If the value of σ is greater than σ_c the cavitation will not occur in that turbine or pump.

Net positive suction head (NPSH): Net positive suction head is defined as the sum of absolute pressure head and kinetic head at the inlet to the pump minus absolute vapour pressure head.

$$NPSH = \frac{P_1}{\rho g} - \frac{P_v}{\rho g} + \frac{V_s^2}{2g}$$

$$\boxed{NPSH = \frac{P_{atm}}{\rho g} - h_s - h_{fs} - \frac{P_{vap}}{\rho g}}$$

Minimum NPSH is given by: $\boxed{\sigma = \dfrac{NPSH}{H_m}}$

Where σ is the Thomas cavitation parameter.

To avoid cavitation ⇒ $\boxed{\sigma > \sigma_c}$

σ_c = critical cavitation factor

Multi-stage Pumps

(a). Pumps in series:

Q = constant

Total head = n·H$_m$

Where n = no. of stages/no. of impellers/no. of pumps in series

(b). Pumps in parallel:

H$_m$ = constant

Total discharge = n · Q

Where n = no. of pumps in parallel

Reciprocating Pump: In the reciprocating pumps the mechanical energy is converted to hydraulic energy (or pressure energy) by sucking the liquid into a cylinder in which a piston is reciprocating (moving backwards and forward) which exerts the thrust on the fluid and increases the pressure energy.

Single acting Reciprocating Pump:

Theoretical discharge: Q$_{th}$ = Volume/sec

Volume per cycle = A · L

Cycle per second = $\dfrac{N}{60}$

Volume per second = $AL \cdot \left(\dfrac{N}{60} \right)$

Discharge from a pump:

For single acting pump: $\boxed{Q_{th} = \dfrac{ALN}{60}}$

For double acting pump: $\boxed{Q_{th} = 2\left(\dfrac{ALN}{60} \right)}$

Work done/sec= ρgQ(h$_s$+h$_d$)

Slip of reciprocating pump: Slip of a pump is defined as the difference between the theoretical discharge and actual discharge of the pump.

$\boxed{Slip = Q_{th} - Q_{act}}$

Percentage slip is given by:

$\boxed{Percentage\ slip = \dfrac{Q_{th} - Q_{act}}{Q_{th}} \times 100}$
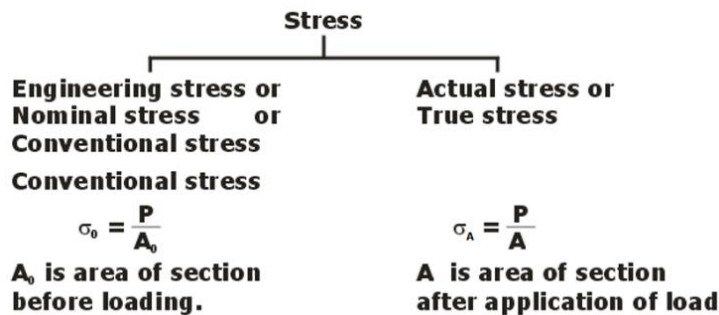
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Important Formula (Strength of Materials)

CHAPTER 1: STRESS AND STRAIN

Stress: The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the load of force.

Units of stress: N/m^2 or Pa.

- 1 Pa = 1 N/m^2
- 1 MPa = 10^6 Pa and 1 GPa = 10^9 Pa

Normal Stress: Either tensile or compressive in nature. Its magnitude is given by:

$$\sigma = \frac{P}{A}$$

The tensile forces are termed as (+ve) while the compressive forces are termed as negative (-ve).
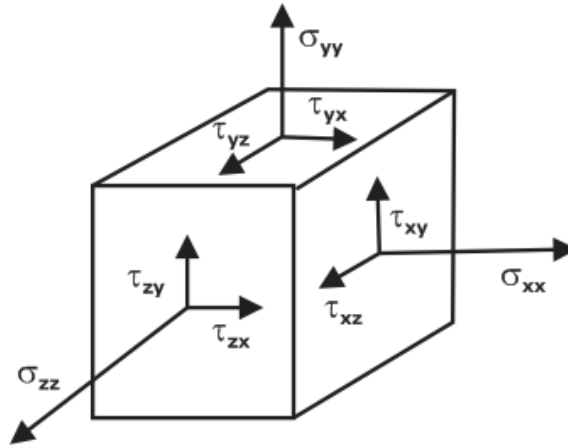
Shear Stress: Stress produced due to forces which are parallel to the area concerned, is called shear stress.

$$\tau = \frac{Shear\ resistance}{Shear\ area} = \frac{P_s}{A}$$

True stress: It is the applied load divided by the instantaneous cross-sectional area (the changing area with respect to time) of the specimen at that load.

$$True\ stress(\sigma_T) = \frac{Load\ (P)}{Instantenous\ area(A)}$$

Cartesian co-ordinate system:

Stress tensor is given by:

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

σ_{xx}, σ_{yy} and σ_{zz} are the tensile stresses on the x, y and z faces while various shear stress component in a 3-dimensional body is as follows $\tau_{xy}, \tau_{yx}, \tau_{yz}, \tau_{zy}, \tau_{zx}, \tau_{xz}$. Here, first subscript indicates the direction of the normal to the surface while second subscript indicates the direction of the stress.

For conditions of equilibrium:

$$\tau_{xy} = \tau_{yx} , \tau_{yz} = \tau_{zy} , \tau_{zx} = \tau_{xy}$$

Strain: It is the ratio of change of dimension of the body to the original dimension is known as strain. It is dimensionless.

Strain may be of following types:

Normal strain: $\boxed{\text{strain(e)} = \dfrac{\text{change in length}}{\text{Original lenth}} = \dfrac{\Delta L}{L}}$
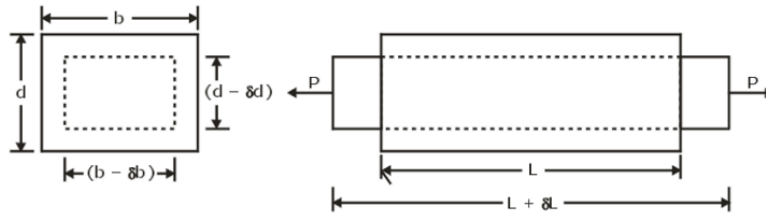
True strain: It equals to the natural log of the quotient of instantaneous length over the original length.

$$\boxed{\varepsilon_T = \ln\left(\frac{L}{L_i}\right) = \ln\left(1 + e\right)}$$

Shear strain: $\boxed{\gamma = \dfrac{\tau}{G}}$

Where γ is the shear strain and G is the modulus of rigidity.

Lateral strain:

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}$$

Poisson's ratio: The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio and it is generally denoted by μ. Hence mathematically:

Poisson's ratio: $\mu = -\dfrac{\text{Lateral strain}}{\text{Longitudinal strain}}$

Material	μ
Cork	Zero
Concrete	0.1 to 0.2
Metals	$\dfrac{1}{4}$ to $\dfrac{1}{3}$
Rubber, Clay, Paraffin	0.5 → Behaves like perfect plastic material

Three-Dimensional Stress System:
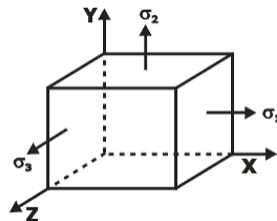
Total strain in the direction of x due to stresses σ_1, σ_2 and σ_3 is:

$$e_1 = \frac{\sigma_1}{E} - \mu\left(\frac{\sigma_2}{E} + \frac{\sigma_3}{E}\right)$$

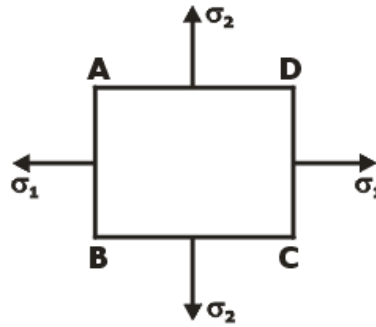Similarly, total strain in the direction of y due to stresses σ_1, σ_2 and σ_3 is:

$$e_2 = \frac{\sigma_2}{E} - \mu\left(\frac{\sigma_1}{E} + \frac{\sigma_3}{E}\right)$$

and total strain in the direction of z due to stress σ_1, σ_2 and σ_3 is:

$$e_3 = \frac{\sigma_3}{E} - \mu\left(\frac{\sigma_1}{E} + \frac{\sigma_2}{E}\right)$$

Relationship between stress and strain:

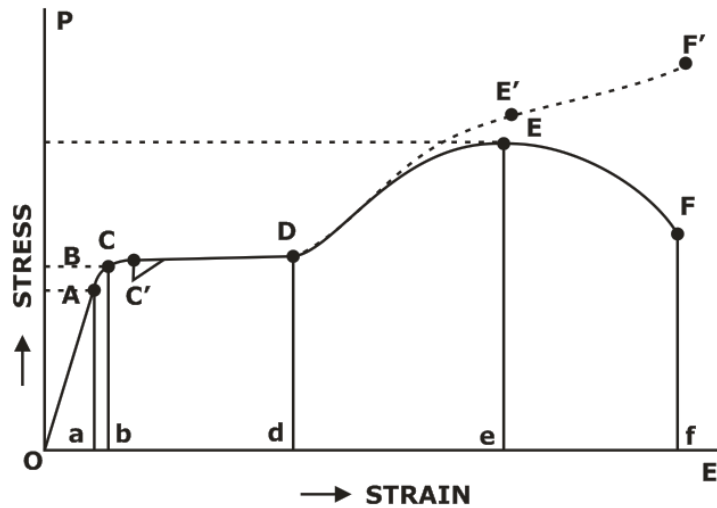Consider a two-dimensional figure ABCD, subjected to two mutually perpendicular stresses σ_1 and σ_2.

Let, e_1 = Total strain in x-direction and e_2 = Total strain in y-direction.

Thus: $\boxed{e_1 = \dfrac{\sigma_1}{E} - \mu\dfrac{\sigma_2}{E}}$ and $\boxed{e_2 = \dfrac{\sigma_2}{E} - \mu\dfrac{\sigma_1}{E}}$

Stress-strain curve: The stress strain curve for mild steel is shown in figure.

A = Proportional Limit   Oa = Linear Deformation

B = Elastic Limit   Ob = Elastic Deformation

C = Yield Point   bd = Perfect Plastic Yielding

C' = Lower Yield Point   de = Strain hardening

E = Ultimate Strength   ef = Necking

F = Rupture Strength/ Fracture strength

The slope of stress-strain curve is called the young's modulus of elasticity (E).

Slope of stress-strain curve: $\boxed{E = \dfrac{\sigma}{\varepsilon}}$

i.e., $\sigma = \varepsilon.E$

This equation is known as Hooke's law. Thus, the modulus of elasticity (E) is the constant of proportionality which is defined as the intensity of stress that causes unit strain.

Stress strain curve for different materials:

Stress strain curve for various grades of steel in tension

Stress strain curve of different material
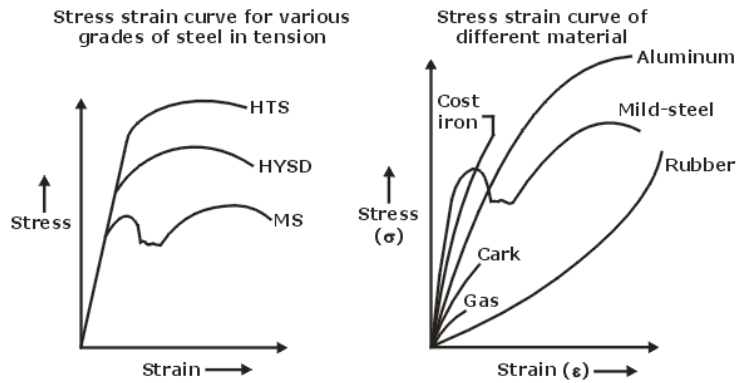
$$E_{cast\ iron} = \frac{1}{2} E_{Steel} \quad \text{and} \quad E_{Eluminium} = \frac{1}{3} E_{Steel}$$

$$E_{concrete} = \left(\frac{1}{10} \text{ to } \frac{1}{20}\right) E_{steel} \quad \text{and} \quad E_{wood} = \left(\frac{1}{8} \text{ to } \frac{1}{20}\right) E_{Steel}$$

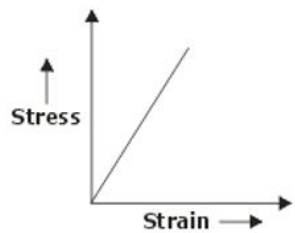**Approximate stress-strain curves for certain materials:**
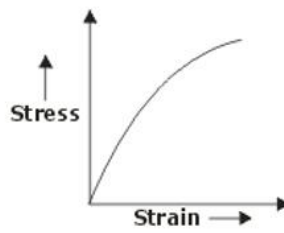

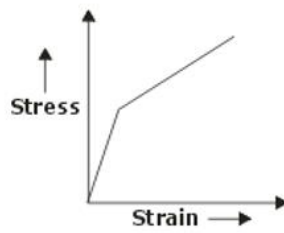Fig. Linear-elastic curve

Fig. non-linear curve

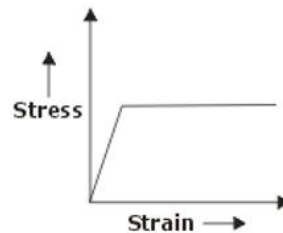Fig. Elasto-plastic with strain hardening

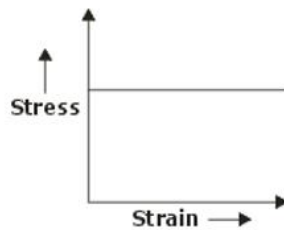Fig. Elasto-plastic or visco-plastic
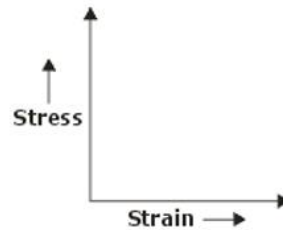
Fig. Perfectly-plastic

Fig. Perfectly Rigid (Ideal rigid))

Properties of Metals:

Ductility: It is that properly of material due to which a metal piece can be drawn into wires of thin section under tensioning effect.

Brittleness: It is the lack of the ductility. Such metal doesn't show necking before fracture.

Malleability: It is that property of metal due to which a metal can be drawn into a thin sheet of negligible section by pressing/forging through the compression process.

Proof stress: A proof stress is a level of stress at which a material undergoes plastic deformation. It is often defined as the point when the material undergoes an amount of plastic deformation equal to 0.2 percent.
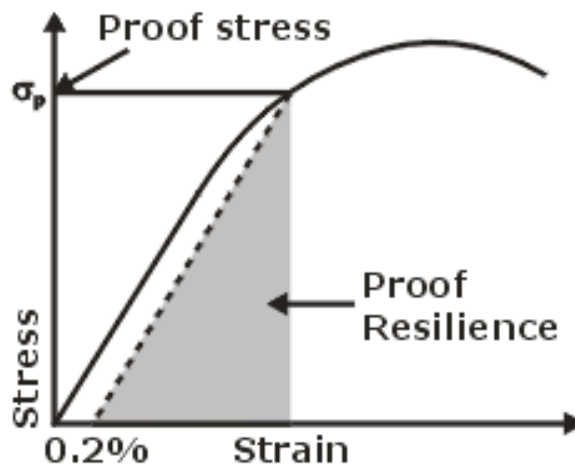
Resilience: Resilience is the ability of a material to absorb energy when it is deformed elastically and release that energy upon unloading. It is defined as the maximum energy that can be absorbed up to the elastic limit, without creating a permanent distortion. The modulus of resilience is defined as the maximum energy that can be absorbed per unit volume without creating a permanent distortion.
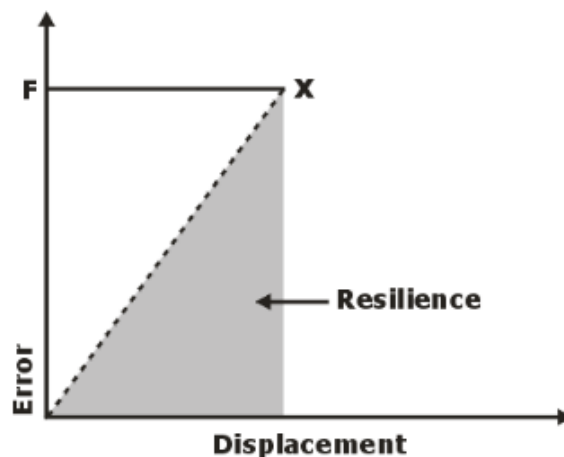
Toughness: It is the ability of a material to absorb energy and plastically deform without fracturing. One definition of material toughness is the amount of energy per unit volume that a material can absorb before rupturing.

Modulus of toughness: The modulus of toughness is the amount of strain energy per unit volume (i.e., strain energy density) that a material can absorb just before it fractures. The modulus of toughness is calculated as the area under the stress-strain curve up to the fracture point.
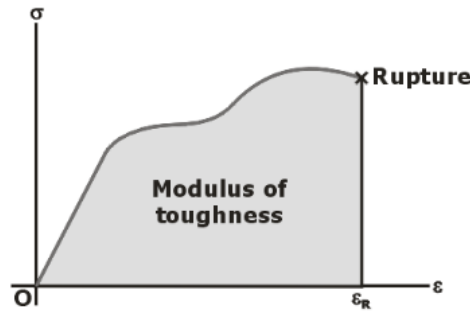
Hardness: Hardness is defined as the resistance of a material to local plastic deformation achieved from indentation of a predetermined geometry indenter onto a flat surface of metal under a predetermined load.

Creep: Creep (sometimes called cold flow) is the tendency of a solid material to move slowly or deform permanently under the influence of persistent mechanical stresses.

Fatigue: Fatigue strength is the highest stress that a material can withstand for a given number of cycles without breaking.

Endurance limit: An endurance or fatigue limit which is defined as the maximum stress below which the steel could presumably endure an infinite number of cycles without failure.
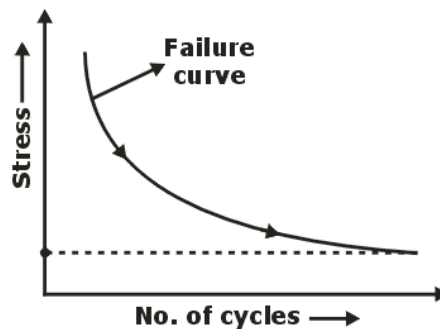
Elasticity: It is that property of metals due to which original dimensions will be recovered offer loading within elastic limits the stress-strain curve may be linear or non-linear.
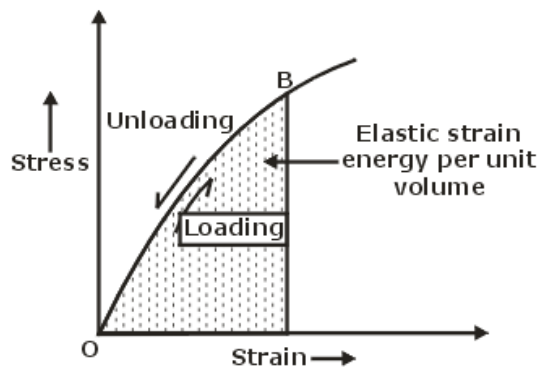
Bulk Modulus: When a body is subjected to three mutually perpendicular like and equal direct stresses, then the ratio of direct stress to the volumetric strain is termed as bulk modulus.

$$K = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\Delta P}{\Delta V / V}$$

Inter relationship of Elastic Constants:

$$E = 3K(1 - 2\mu)$$

$$E = 2G(1 + \mu)$$

$$E = \frac{9KG}{3K + G}$$

$$\mu = \frac{3K - 2G}{6K + 2G} = \frac{1}{m}$$

Here, μ =Poisson's ratio and m = 1/μ.

	μ	G	K
Min limit	0	$\frac{E}{2}$	$\frac{E}{3}$
Max limit	$\frac{1}{2}$	$\frac{E}{3}$	∞

For metals:

μ	G	K
$\frac{1}{4}$	0.4E	0.67E
$\frac{1}{3}$	0.375E	E

Materials based on elastic properties:

Homogeneous Material: When a material exhibits Same elastic properties at any point in a given directions than the material is known as homogenous material i.e., elastic properties are independent of location.
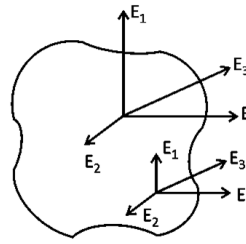
Isotropic Material: When a material exhibits Same elastic properties at any direction at a given point than the material is known as Isotropic Material i.e., elastic properties of material are independent of direction.
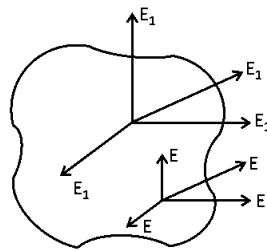
Homogenous & isotropic material: When a material exhibits Same elastic properties at any direction at a every point than the material is known as homogeneous Isotropic Material.
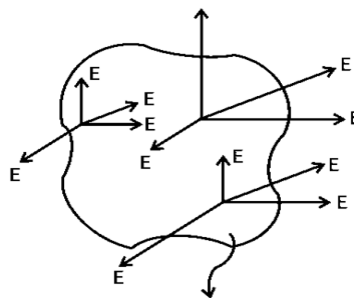
Anisotropic Material: When a material exhibits different elastic properties at every direction at a every point than the material is known as Isotropic Material i.e., they exhibit direction dependent elastic property.
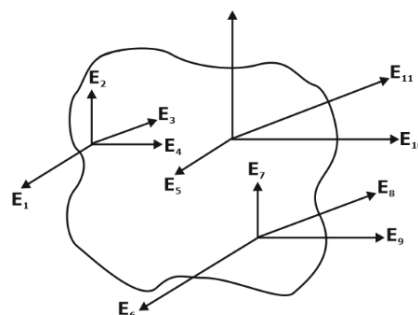
Orthotropic Material: When a material exhibits Same elastic properties at only orthogonal direction at a given point than the material is known as Orthotropic Material.

Material	No. of independent elastic constants
Isotropic	2
Orthotropic	9
Anisotropic	21

Analysis of stresses in different bars

(a). Elongation in Bars of Varying Sections:
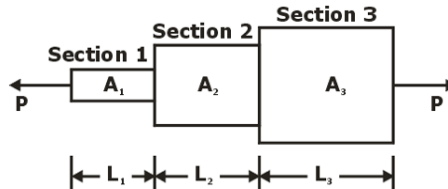
Total change in the length of the bar:

$$dL = dL_1 + dL_2 + dL_3 = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E} = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

(b). Elongation of Tapering Circular Rod:
A bar uniformly tapering from a diameter D_1 at one end to a diameter D_2 at the other and

L = Total length of the bar and E = Young's modulus.
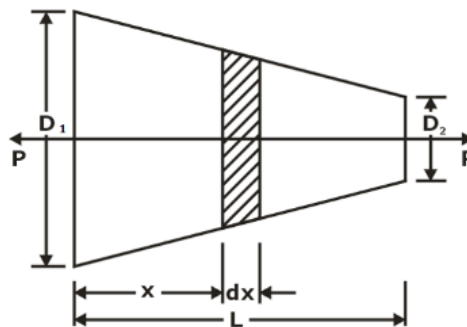
$$\Delta L = \frac{4PL}{\pi E D_1 D_2}$$

(c). Elongation of Tapering Rectangular Bar
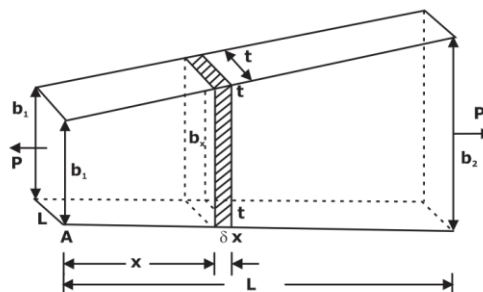
∴ Total extension: $\boxed{\Delta = \dfrac{PL}{(b_2 - b_1)\,t\,E} \log_e \dfrac{b_2}{b_1}}$

(d). Elongation of uniform section bar due to self-weight:
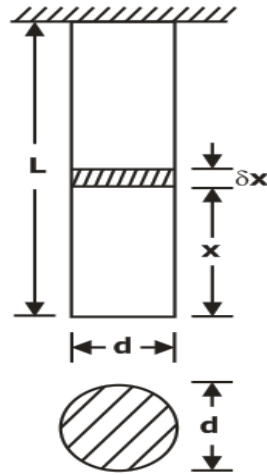
$$\boxed{\Delta = \frac{\lambda L^2}{2E} = \frac{\rho g L^2}{2E}}$$

Coefficient of Thermal Expansion (or contraction): The coefficient (α) is a property of the material and has a unit reciprocal of temperature change.

In SI units, α has the dimensions of either 1/K (the reciprocal of kelvins) or 1/° C (the reciprocal of degree Celsius), because the change in temperature is numerically the same in both kelvins and degrees Celsius.

Common values of α are:

Material	Coefficient of Thermal Expansion
Steel	10×10^{-6} to 18×10^{-6} / °C
copper	17×10^{-6} / °C
aluminium and aluminium alloys	23×10^{-6} / °C

Thermal stresses:

(a). When bar is constrained (supports unyielding):

$$\boxed{\sigma_{Th} = \frac{\Delta_{Th} E}{L} = L\alpha T \cdot \frac{E}{L} = E\alpha T} \text{ and } \boxed{(\Delta L)_{Total} = 0}$$

(b). When Supports yield: If the support yields by an amount a. In this case, the total amount of expansion checked will be $(\Delta_t - a)$. Hence the resulting temperature stress in:

$$\boxed{\sigma_{Th} = (\Delta_{Th} - a)\frac{E}{L} = (L\alpha T - a)\frac{E}{L}}$$

BYJU'S EXAM PREP

CHAPTER 2: SHEAR FORCE AND BENDING MOMENT DIAGRAM

Introduction: The algebraic sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force. The algebraic sum of the moments of all the forces acting to the right of left of the section is known as bending moment.

A shear force diagram is one which shows the variation of the shear force along the length of the beam. And a bending moment diagram is one which shows the variation of the bending moment along the length of the beam.

Types of supports:

(1). Simple Supports

(a). Roller Support:

Number of restricted motions by support = Number of reactions at any support

Hence, number of rection in any roller support is 1.
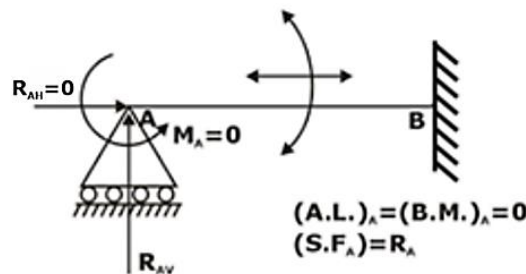
(b). Hinge Support (or) Pin Support:

Number of restricted motions support = Number of reactions at any support

Hence, number of rection is any hinged support is 2.
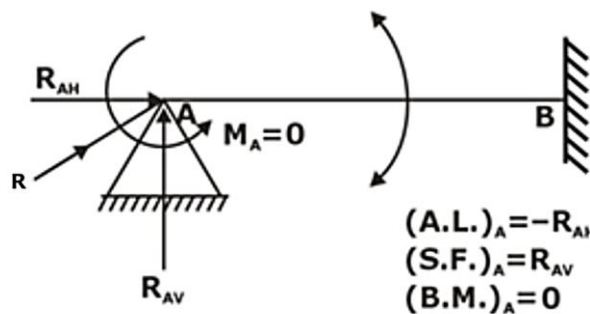
$$R = \sqrt{(R_{AH})^2 + (R_{AV})^2}$$

$$\tan\theta = \frac{R_{AV}}{R_{AH}}$$

Fixed Supports:

Clamped Supports (or) Built-in Supports:
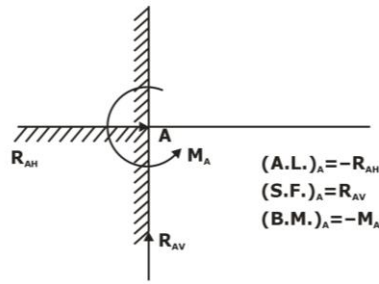
Axial Load at any support = $\pm R_H$ at that point

Shear force at any support = $\pm R_V$ at that point

Moment reaction at any support = $\pm$ Moment reaction at that support

Sign Convention for Shear Force and Bending Moment:

Shear force: If moving from left to right, then take all upward forces as positive and downward as negative.

Left to right	Right to left
↑ (+ve)	↓ (+ve)
↓ (−ve)	↑ (−ve)

Bending moment: If moving from left to right, take clockwise moment as positive and anticlockwise as negative.

Left to right	Right to left
↷ (+ve)	↶ (+ve)
↶ (−ve)	↷ (−ve)

Points of Concern for SFD and BMD (Critical Points):

(i). Starting and end points of beam.

(ii). Point where concentrated point load or concentrated moment is acting.

(iii). Starting and end point of distributed load (UDL or UVL).

(iv). Point where SFD changes sign.

Relationship between SF, BM and load:

(i). Slope of sfd=intensity of distributed load.

$$\boxed{\frac{dV}{dx} = W_x}$$

(ii). Slope of BMD = Shear force at that section.

$$\boxed{\frac{dM}{dx} = V}$$

Loading	Shape of SFD	Shape of BMD
No load	Straight line	Inclined straight line
UDL	Inclined straight line	2° Curve
UVL	2° curve	3° curve

Loading	Shear Force diagram $\frac{dV}{dx} = W$	Bending Moment diagram, $\frac{dM}{dx} = V$
	 Constant slope	 Positive decreasing slope

- Loading is negative and constant ⇒ SF slope is negative and constant

- SF is positive decreasing ⇒ Bending moment slope is decreasing

- If load intensity is UDL ⇒ SFD is learner → BMD is parabolic

- Slope of BMD at any section is equal to SFD ordinate at that section

- Slope of SFD at any section is equal to load intensity at that section

- If load intensity is n-degree curve, SFD will be (n + 1) degree curve and BMD will be (n + 2) degree curve.

	 Negative decreasing slope	 Positive decreasing slope

- Loading is negative and increasing ⇒ SFD slope is negative and increasing.

- SF is positive and decreasing ⇒ BMD slope is positive and decreasing.

- If load intensity is UVL (uniformly varied load) SFD is parabolic and BMD is cubic.

	 Negative decreasing slope	 Positive decreasing slope

- SF is positive and decreasing ⇒ BMD slope is positive and decreasing

- Loading is negative and decreasing ⇒ SFD slope is positive and decreasing

Bending Theory: The following are the important assumptions in the theory of bending

1. The material of the beam is homogeneous and isotropic.
2. The value of Young's modulus of elasticity is the same in tension and compression.
3. The transverse sections which were plane before bending, remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bending into circular arcs with a common centre of curvature.
5. The radius of curvature is large compared with the dimensions of the cross-section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

$$\boxed{\frac{M}{I} = \frac{\sigma}{R} = \frac{E}{R}}$$

The equation is known as bending equation.

2. Section Modulus:

Section modulus is defined as the ratio of moment inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It is denoted by the symbol Z.

$$\boxed{Z = \frac{I}{y_{max}}}$$

I = M.O.I. about neutral axis

y_{max} = Distance of the outermost layer from the neutral axis.

$$\boxed{M = \sigma_{max} \cdot Z}$$

In the above equation, M is the maximum bending moment (or moment of resistance offered by the section). Hence moment of resistance offered the section is maximum when section modulus Z is maximum. Hence section modulus represents the strength of the section.

Section Modulus for Various Shapes or Beam Sections:

Rectangular Section:

Moment of inertia of a rectangular section about an axis passing through its C.G. (or through N.A. is given by).

$$\boxed{I = \frac{bd^3}{12}}$$

Distance of outermost layer from N.A. is given by,

$$\boxed{y_{max} = \frac{d}{2}}$$
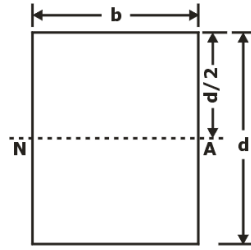
$\therefore$ Section modulus is given by:

$$Z = \frac{I}{y_{max}} = \frac{bd^3}{12 \times \left(\frac{d}{2}\right)} = \frac{bd^2}{6}$$

Hollow Rectangular Section: $Z = \frac{1}{6D}[BD^3 - bd^3]$

Circular Section: $Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}d^4}{\left(\frac{d}{2}\right)} = \frac{\pi}{32}d^3$

Hollow Circular Section: $Z = \frac{\pi}{32D}[D^4 - d^4]$

Strength of a section: The strength of a section means the moment of resistance offered by the section and moment of resistance is given by:

Maximum stress induced $\leq$ permissible Stress.

$$\frac{M}{Z_{N.A}} \leq \sigma_{per}$$

$$M \leq \{(\sigma_{per} \cdot Z_{N.A}) = M_R\}$$

$$M_R = \sigma_{per} \cdot Z_{N.A}$$

For unit radius of bend:

$$EI_{N.A} \uparrow = M_R (\uparrow) = (\theta \ \& \ \delta) \downarrow = \text{chances of beams failures } (\downarrow).$$

Ratio of depth of width of the strongest beam that can be cut from a circular log:
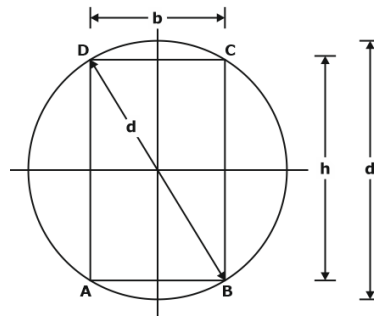
$$d = \sqrt{3}\,b$$

$$\frac{h}{b} = \sqrt{2} = 1.414$$

Beam of uniform strength: It is the beam of uniform moment of resistance.

Constant width		Constant depth	
Simply supported $d_x = \sqrt{\dfrac{3W}{fb}}\sqrt{x} = c\sqrt{x}$		**Simple Supported** $b_x = \dfrac{3W}{\sigma d^2}.x = c'x$	
Cantilever beam **M=constant** $\left[(\sigma_b)_{max}\right]_{x-x} = \pm\dfrac{6M}{bd^2}$ **M=Varying** $d_x = d\sqrt{\dfrac{x}{L}}$		Cantilever beam $b_x = b\dfrac{x}{L}$	

CHAPTER 4: SHEAR STRESS IN BEAMS

Shear stress at a section:

The shear stress is given by:

$$\tau = \frac{F}{Iz}.A\bar{y}$$

Assumptions: The above analysis is based on the following assumptions:

1. For all values of y, τ is uniform across the width of the cross-section, irrespective of its shape.

2. $F = \dfrac{dm}{dx}$ is derived from the assumption that bending stress varies linearly across the section and is zero at the centroid.

3. The material is homogeneous and isotropic, and the value of E is the same for tension as well as compression.

Assumption No. 1 is not strictly correct because the tangential value must be zero at the boundaries of the section. Hence it is understood that τ is the average value across the section. Hence it is understood that q is the average value across the action. Regarding assumption No 2, the stress curve is not a straight line passing through the centroid of the section.

Shear stress distribution and relations for different sections:

(a). Rectangular section: Shear stress: $\tau = \dfrac{F}{2I}\left(\dfrac{d^2}{2} - y^2\right)$ $\therefore \dfrac{\tau_{max}}{\tau_{mean}} = \dfrac{3}{2} = 1.5$	
(b). Circular section: $\tau = \dfrac{F}{12I}\{4(r^2 - y^2)\} = \dfrac{F}{3I}(r^2 - y^2)$ $\tau_{max} = \dfrac{4}{3}\tau_{avg}$	
(c). Triangular section: $\tau = \dfrac{F}{3I}(xh - x^2)$ $\therefore \tau_{N.A.} = \dfrac{8}{3}\cdot\dfrac{F}{bh} = \dfrac{4}{3}\dfrac{F}{\left(\frac{1}{2}bh\right)} = \dfrac{4}{3}\tau_{avg}$ $\tau_{max} = \dfrac{3F}{bh} = \dfrac{3}{2}\dfrac{F}{\left(\frac{1}{2}bh\right)} = \dfrac{3}{2}\tau_{avg}$	
(d). Square Diagonal Section: $\tau = \dfrac{4Fy}{h^4}\left[3h - 4y\right]$ $\tau_{N.A.} = \dfrac{2F}{(\sqrt{2}\,a)^2} = \dfrac{F}{a^2} = \tau_{avg}$	

$$\tau_{max} = \frac{9F}{2 \times (\sqrt{2}a)^2} = \frac{9F}{8a^2} = \frac{9}{8}\tau_{avg}$$	
(e). I beam: $$\tau_{flange} = \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right)$$ $$\tau = \frac{F}{8I}\frac{B}{t_w}(D^2 - d^2) + \frac{F}{8I}(d^2 - 4y^2)$$ $$F_{web} = \frac{t_w \cdot d}{3}(2\tau_{max} + \tau_{min})$$	

Shear stress distribution over other sections:

(a) Rectangle   (b) Solid Circle   (c) Hollow Circle

(d) I-Section   (e) T-Section   (f) L-Section

(g) Cross   (h) Built-up Section

(i) Square   (j) Triangle   (k) Rolled Section
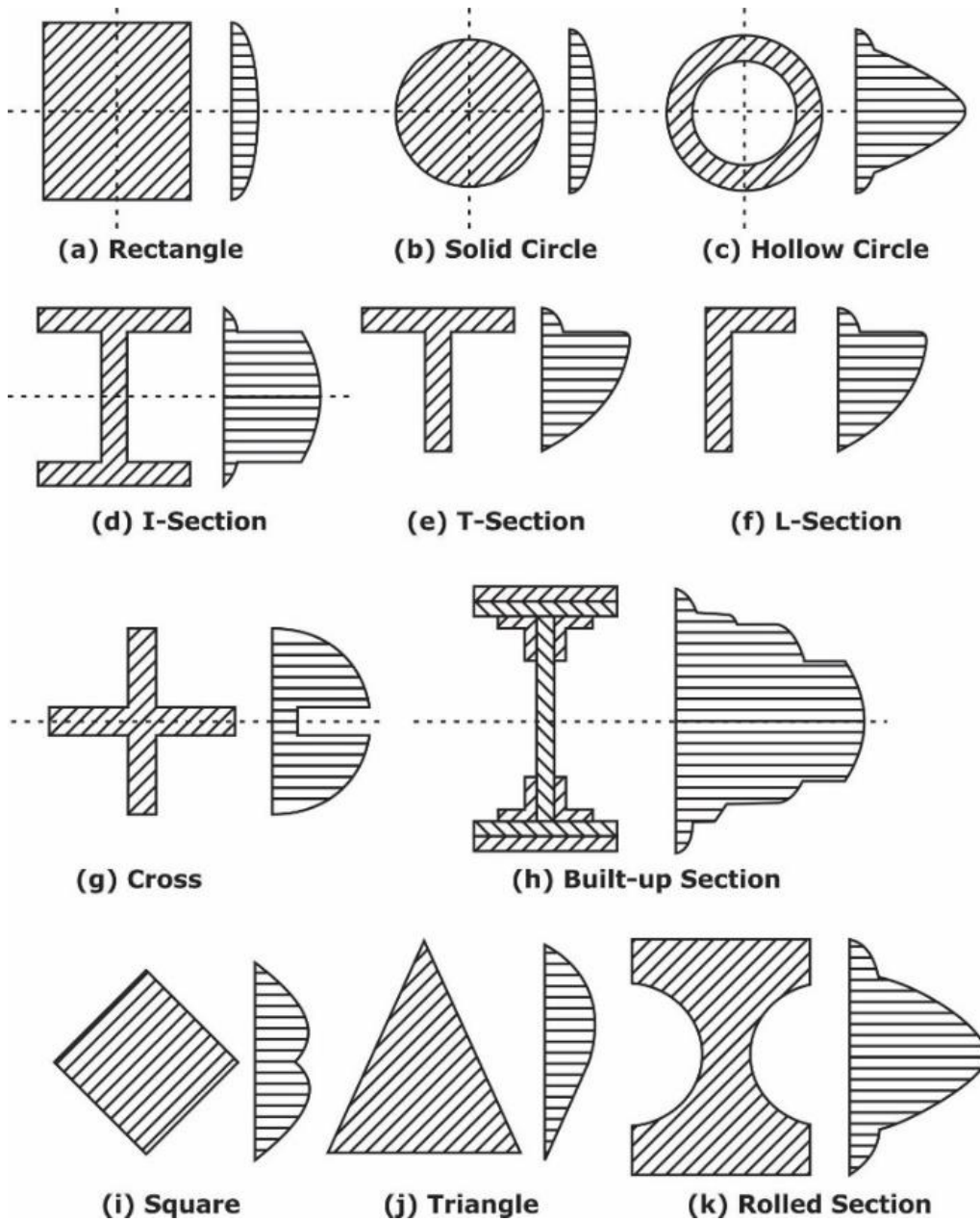
CHAPTER 5: PRINCIPAL STRESSES

Principal stresses:

Three mutually perpendicular planes, on each of which the resultant stress is wholly normal. These are known as principal planes and the normal stress across these planes, as principal stresses.

$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y}$$

Which provides two values of 2θ differing by 180° or two values of θ differing by 90°. Thus, the two principal planes are perpendicular to each other.
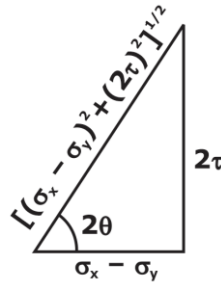
Principal stresses are given by:

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

Maximum (principal) shear stresses:

For maximum value of τ_θ: $\boxed{\tan 2\theta = -\dfrac{\sigma_x - \sigma_y}{2\tau}}$

Maximum shear stress: $\boxed{\tau_{max} = \dfrac{1}{2}(\sigma_1 - \sigma_2) = \dfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$

Principal planes are given by:

$$\tan 2\theta_p = \frac{2\tau}{\sigma_x - \sigma_y}$$

and planes of maximum shear stress:

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau}$$

Multiplying the two:

$\tan 2\theta_p . \tan 2\theta_s = -1$

which means $2\theta_s = 2\theta_p + 90°$

i.e. $\theta_s = \theta_p + 45°$.

This indicates that the planes of maximum shear stress lie at 45° to the planes of principal axes.

The maximum value of shear stress lies in the planes at 45° to the principal planes:

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

Normal stress on the planes of maximum shear stress:

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y)$$

Plane Stress Condition: It is used by the designer for designing thin sheets. In this condition, stress in one of the directions is neglected but strain is assumed to exist in that direction.

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Plane strain condition: In plane strain condition designer assumes strain in one of the directions is zero but for preventing this deformation stress is required in that direction. Designing of thick plate, thick pressure vessel designers use this method.

Strain Rosette: A strain gauge rosette is a term for an arrangement of two or more strain gauge that are positioned closely to measure strains along different directions of the component under evaluation.

Mohr's circle:

Let CR and CS represent two perpendicular planes BD and AB respectively so that OL= σ_x, OM= σ_y and LR and MS each equal to τ in the clockwise and counter-clockwise directions respectively (Fig.9). Now if it is desired to find stresses on an inclined plane at angle θ clockwise with plane BD, a radial line CP may be drawn at angle 2θ in the clockwise direction with CR. Then ON and NP will represent the direct and shear components respectively on the plane AD and the resultant is given by OP.

Thus, the procedure may be summarised as follows:

☐ Take OL and OM as the direct components of the two perpendicular stresses σ_x and σ_y.

☐ At L and M draw ⊥ LR and MS on the x-axis each equal to τ using the same scale as for the direct stresses. For the stress system shown in Fig.8, LR is taken upwards as the direction on plane BD is clockwise and MS downwards as the direction on plane AB is counter clockwise.

☐ Bisect LM at C and draw a circle with C as centre and radius equal to CR (= CS).

☐ Rotate the radial line CR through angle 2θ in the clockwise direction if θ is taken clockwise and let it take the position CP.

☐ Draw NP ⊥ on x-axis. Join OP.

It can be proved that ON and NP represent the normal and the shear stress components on the inclined plane AD.
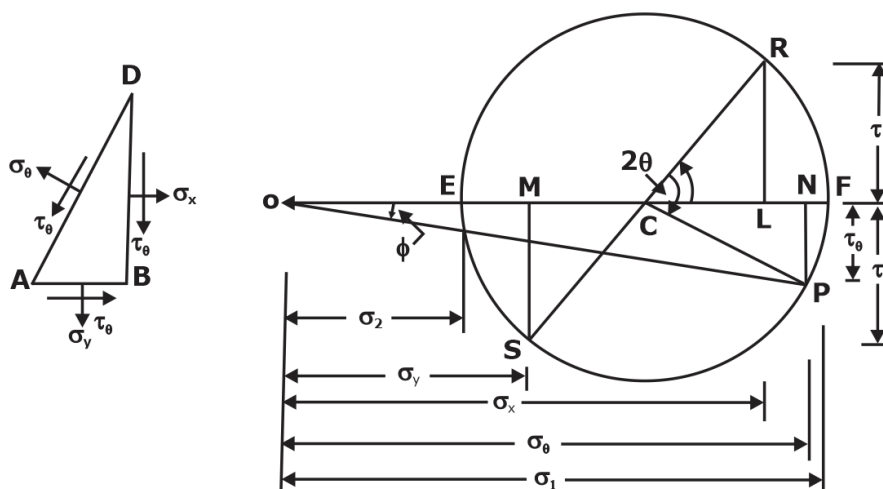
$$\text{Major principal stress} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\text{Minor principal stress} = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

Radius of the circle: $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$

Centre of Mohr's circle: $C = \left(\dfrac{\sigma_x + \sigma_y}{2}, 0\right)$

Mohr's circle different cases:

(a). When both the stresses are tensile in nature and no shear stress:
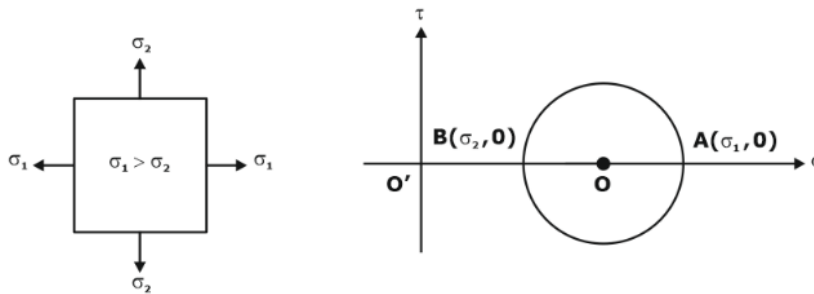
Thin cylinder stress case.

Hoop stress : $\sigma_h = \dfrac{PD}{2t}$ (tensile)

Longitudinal stress : $\sigma_l = \dfrac{PD}{4t}$ (tensile)

(b). When one is tensile and other is compressive in nature and no shear stress:
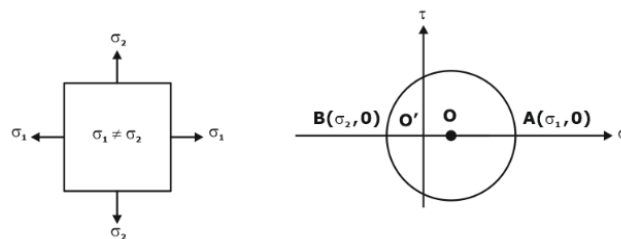
(c). When both the stresses are tensile in nature and are equal: Spherical pressure vessel case.
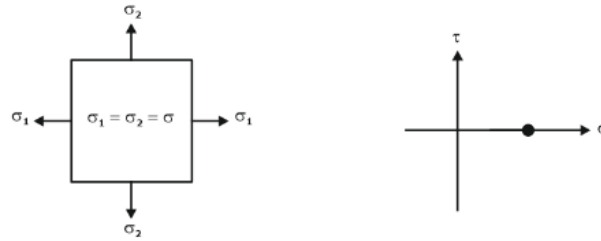
$$\boxed{\sigma_h = \sigma_l = \frac{PD}{4t} \quad \text{(tensile)}}$$

Strain analysis:
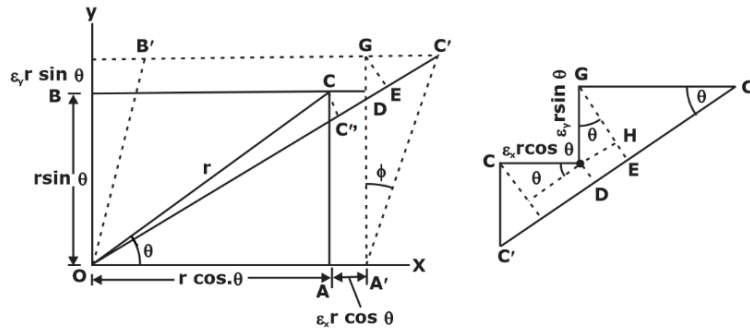
$$\boxed{\varepsilon_\theta = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_y - \varepsilon_y)\cos 2\theta + \frac{1}{2}\varphi \sin 2\theta}$$

$$\boxed{\gamma = -\frac{1}{2}(\varepsilon_x - \varepsilon_y)\sin 2\theta + \varphi \sin^2 \theta}$$

Compare the results with bi-axial and shear stresses conditions:

☐ In a linear system: $\varepsilon_\theta = \varepsilon_x . \cos^2 \theta$ or $\varepsilon_x \left(\dfrac{1 + \cos 2\theta}{2} \right)$.

☐ In a pure shear system and for θ = 45°, $\varepsilon_{45°} = \varphi/2$

Principal strains:

The maximum and the minimum values of strains on any plane at a point are known as the principal strains and the corresponding planes as the principal planes for strains.

$$\boxed{\tan 2\theta = \frac{\varphi}{\sigma_x - \sigma_y}}$$

For shear strain to be maximum or minimum:

$$\boxed{\tan 2\theta = -\frac{\varepsilon_x - \varepsilon_y}{\varphi}}$$

The planes of maximum shear strain are inclined at 45° to the planes of maximum shear strain as in case of maximum shear stress.

Sum of direct strains on two mutually perpendicular planes:

$$\boxed{\varepsilon_\theta + \varepsilon_{\theta+90°} = \varepsilon_x + \varepsilon_y}$$

Relation between principal stress and strain:

$$\boxed{\sigma_2 = \frac{E(\mu\varepsilon_1 + \varepsilon_2)}{1-\mu^2}} \text{ and } \boxed{\sigma_1 = \frac{E(\mu\varepsilon_2 + \varepsilon_1)}{1-\mu^2}}$$

Types of strain rosette:

Let ϵ_x and ϵ_y be the linear strains in x and y directions and ϕ be the shear strain at the point under consideration. Then linear strains in any three arbitrary chosen directions at angles θ_1, θ_2 and θ_3 made with the x-axis will be:

$$\varepsilon_{\theta_1} = \varepsilon_x.\cos^2\theta_1 + \varepsilon_y.\sin^2\theta_1 + \varphi.\sin\theta_1.\cos\theta_1$$

$$\varepsilon_{\theta_2} = \varepsilon_x.\cos^2\theta_2 + \varepsilon_y.\sin^2\theta_2 + \varphi.\sin\theta_2.\cos\theta_2$$

$$\varepsilon_{\theta_3} = \varepsilon_x.\cos^2\theta_3 + \varepsilon_y.\sin^2\theta_3 + \varphi.\sin\theta_3.\cos\theta_3$$

Rectangular strain Rosette:
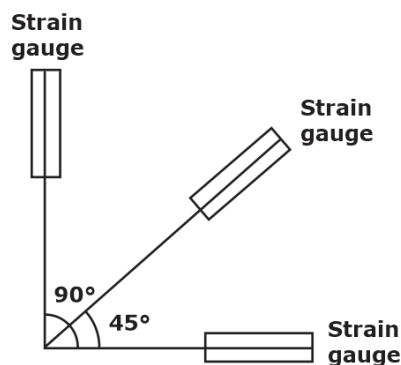
$\theta_1 = \theta\,°$, $\theta_2 = 45°$ and $\theta_2 = 90°$

The above equations can be written:

$\epsilon_{0°} = \epsilon_x$ and $\epsilon_{90°} = \epsilon_y$

$$\varepsilon_{45°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y + \varphi)$$

From which:

$\epsilon_x = \epsilon_{0°}$, $\epsilon_y = \epsilon_{90°}$ and $\Phi = 2\epsilon_{45°} - (\epsilon_x + \epsilon_y)$

Delta strain rosette:
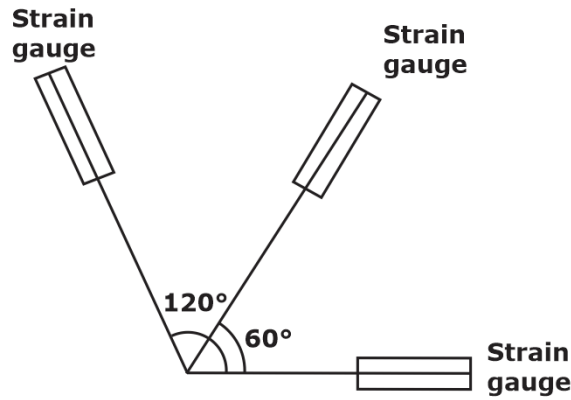
$\epsilon_x = \epsilon_{0°}$ and $\boxed{\varepsilon_y = \dfrac{1}{3}(2\varepsilon_{60°} + 2\varepsilon_{120°} - \varepsilon_{0°})}$

and $\boxed{\varphi = \dfrac{2}{\sqrt{3}}(\varepsilon_{60°} - \varepsilon_{120°})}$

Strain tensor: Strain tensor is used to define the state of strain at a point (i.e., different strains developed on three mutual perpendicular planes, passing through a point.

$\epsilon \rightarrow$ [Normal Strain]

$\tau \rightarrow$ [Shear strain]

$$[\varepsilon]_{3D} = \begin{bmatrix} \varepsilon_{xx} & \dfrac{\phi_{xy}}{2} & \dfrac{\phi_{XZ}}{2} \\ \dfrac{\phi_{xy}}{2} & \varepsilon_{yy} & \dfrac{\phi_{YZ}}{2} \\ \dfrac{\phi_{xz}}{2} & \dfrac{\phi_{YZ}}{2} & \varepsilon_{Zz} \end{bmatrix}_{3 \times 3}$$

$[\varepsilon]_{3D}$ → Strain Tensor at a point in 3D.

For the 2-D:

$$[\varepsilon]_{2D} = \begin{bmatrix} \varepsilon_{xx} & \dfrac{\phi_{xy}}{2} \\ \dfrac{\phi_{xy}}{2} & \varepsilon_{yy} \end{bmatrix}$$

Let ϵ, ϕ are the normal & shear strain or an oblique plane passing through a point under bi-axial state of strain.

CHAPTER 6: TORSION AND SPRINGS

Torsion: A shaft is said to be under pure torsion when it is subjected to two equal & opposite couples in a plane perpendicular to the longitudinal axis of the shaft.
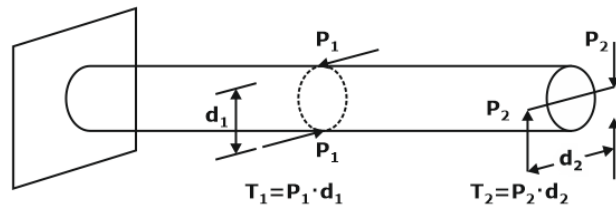
Sign convention:

Right hand thumb rule → If right hand fingers represent direction of torque applied and thumb is pointing towards the section, the torque is taken positive.

Assumptions:

The following assumptions have been made in developing the equations for stresses and deformations in a bar subjected to pure torsion.

1. Shaft is loaded with twisting couples in planes that are perpendicular to the axis of the shaft.
2. Torsion is uniform along the length i.e., all normal cross-section which are the same axial distance suffer equal relative rotations.
3. Circular sections remain circular. Thus, radii remain straight after torsion.
4. Plane normal sections of shaft remain plane after twisting, i.e., no warping or distortion of parallel planes normal to the axis of the shaft takes place.
5. Stress is proportional to strain, i.e., stresses do not exceed proportional limit.
6. Material is homogenous and isotropic.

Torsion formula:
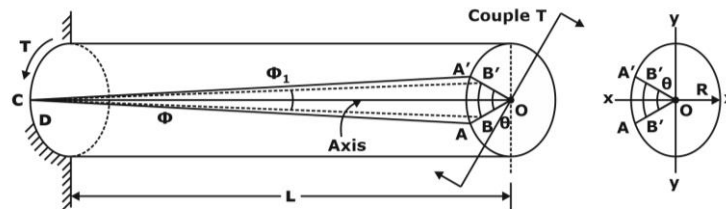
$$\frac{\tau_r}{r} = \frac{\tau}{R} = \frac{T_R}{J} = \frac{G\theta}{L}$$

Where:

θ = twist angle in radians

τ = shear stress developed in the material

T = torque applied

G = modulus of rigidity
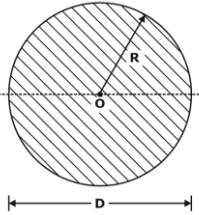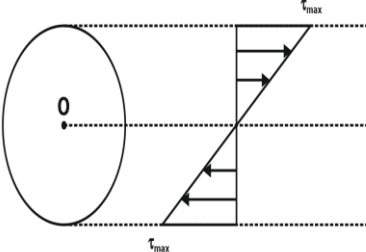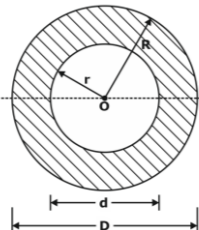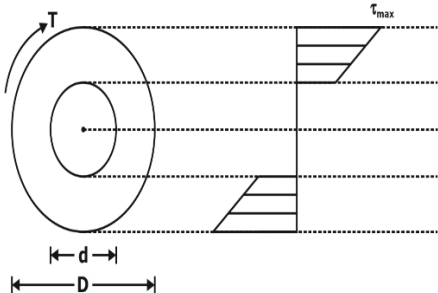
I_P = Polar moment of inertia
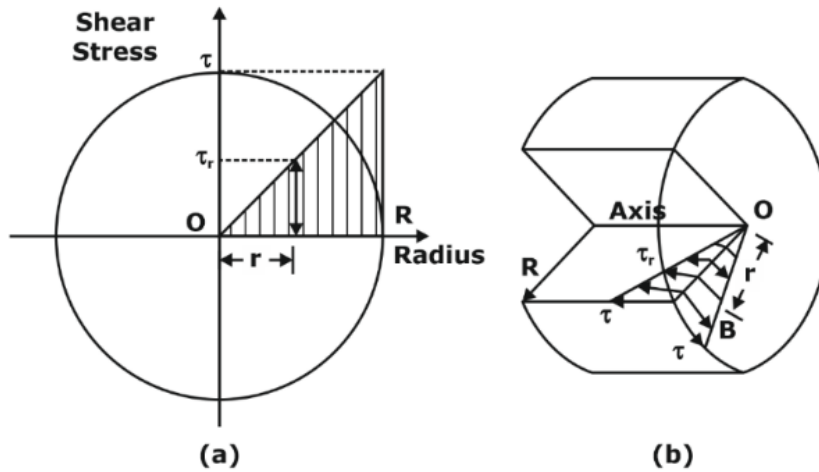
(a)   (b)

Sign convention:

Right hand thumb rule → If right hand fingers represent direction of torque applied and thumb is pointing towards the section, the torque is taken positive.

Polar moment of inertia:

Cross section	Polar moment of Inertia and maximum torque	Shear stress variation
Solid circular shaft	$$I_{zz} = \frac{\pi}{32}D^4$$ $$Z_P = \frac{J}{R} = \frac{J}{D/2} = \frac{\pi}{16}D^3$$ $$T = \tau \times \frac{\pi}{2} \times \frac{D^3}{8} = \frac{\pi}{16}\tau D^3$$	
Hollow circular shaft	$$I_{zz} = \frac{\pi}{32}\left(D^4 - d^4\right)$$ $$Z_p = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)$$ $$T = \tau\frac{\pi}{16}\left[\frac{D_0^4 - D_i^4}{D_0}\right]$$	

Connection of shafts:

(i). Series connection:

Here,

$$\boxed{\begin{array}{l} T_1 = T_2 = T \\ \theta_{AC} = \theta_{AB} + \theta_{BC} \end{array}}$$
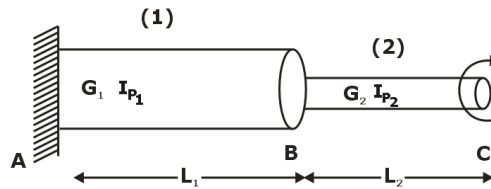
(ii). Parallel connection:
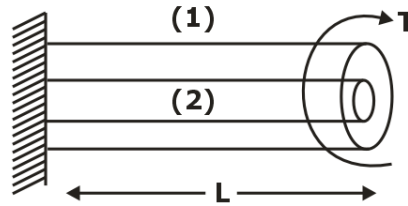
Here, $\boxed{\begin{array}{l} T_1 + T_2 = T \\ \theta_1 = \theta_2 \Rightarrow \dfrac{T_1 L_1}{G_1 J_1} = \dfrac{T_2 L_2}{G_2 J_2} \end{array}}$

Shaft of two materials: composite shaft:
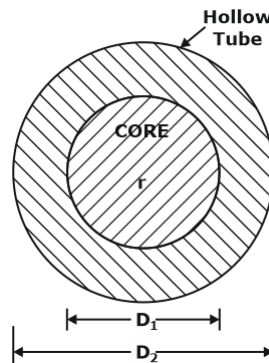
Torques: $\boxed{T_1 = T\left(\dfrac{G_1 J_1}{G_1 J_1 + G_2 J_2}\right)}$ and $\boxed{T_2 = T\left(\dfrac{G_2 J_2}{G_1 J_1 + G_2 J_2}\right)}$

Twist angle: $\boxed{\theta = \dfrac{TL}{G_1 J_1 + G_2 J_2}}$

Ratio of shear stresses: $\boxed{\dfrac{\tau_1}{\tau_2} = \dfrac{T_1 D_1}{T_2 D_2} \cdot \dfrac{J_2}{J_1} = \dfrac{G_1}{G_2} \cdot \dfrac{D_1}{D_2}}$

Strain energy due to torsion:

Total strain energy:

$$U = \frac{1}{2}T.\theta = \text{Area under } T - \theta \text{ diagram}$$

$$\boxed{U = \frac{\tau^2}{4G} \times \text{Volume of the shaft}}$$

For hollow shaft:

$$\boxed{U = \frac{\tau^2}{4G}\left(\frac{D^2 + d^2}{D^2}\right) \times \text{Volume of the shaft}}$$

For the thin tube: $D \approx d$

$$\boxed{U \approx \frac{\tau^2}{2G} \times \text{Volume of the shaft}}$$

Torsional stiffness (K_t): Torsional stiffness is defined as the amount of torque or twisting couple required to produce a twist of unit radian. And it represented by 'K'. $\boxed{K_t = \frac{T}{\theta}}$

For a given twisting couple 'T' $K_t = \frac{GJ}{L} \uparrow \Rightarrow \theta \downarrow \Rightarrow \varphi \downarrow \Rightarrow \gamma \downarrow \tau_{induced} \downarrow \Rightarrow$ Chances of torsional failure $\downarrow$

Torsional Stiffness is used to compare angular twist of two different shafts which are subjected to same twisting moment whereas Torsional Rigidity is used to compare angular twist of two different shafts which are having same length and subjected to same torque.

Combined bending and torsion: Consider any point on the cross-section of as shaft.

Let, T = Torque at the section

D = Diameter of the shaft

M = B.M. at the section

Major principal stress: $\boxed{\sigma_1 = \frac{16\,T}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)}$

Minor principal stress: $\boxed{\sigma_2 = \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)}$

Maximum shear stress

$$\tau_{max} = \frac{\text{Major principal stress} - \text{Minor principal stress}}{2}$$

$$\boxed{\tau_{max} = \frac{16}{\pi D^3}\left(\sqrt{M^2 + T^2}\right)}$$

For a hollow shaft:

Major principal stress: $\sigma_1 = \dfrac{16D_0}{\pi\left[D_0^4 - D_i^4\right]}\left(M + \sqrt{M^2 + T^2}\right)$

Minor principal stress: $\sigma_2 = \dfrac{16D_0}{\pi\left[D_0^4 - D_i^4\right]}\left(M - \sqrt{M^2 + T^2}\right)$

Maximum shear stress: $\tau_{max} = \dfrac{16D_0}{\pi\left[D_0^4 - D_i^4\right]}\left(\sqrt{M^2 + T^2}\right)$

SPRINGS

A spring is defined as an elastic machine element, which deflects under the action of the load and returns to its original shape when the load is removed.

Springs are classified according to their shape.

- The spring can be a helical coil of a wire, a piece of stamping or a flat wound-up strip.
- The most popular type of spring is helical spring.
- The helical spring is made from a wire, usually of usually of circular cross-section, which is bent in the form of a helix.
- There are two types of helical springs – compression spring and extension spring.

Helical spring:
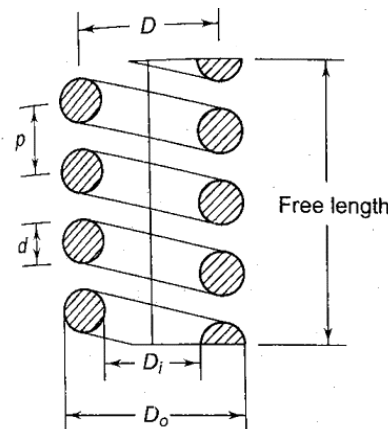
$\boxed{\text{Solid length } = N_t d}$

Where

N_t = total number of coils

$\boxed{\text{Total gap} = (N_t - 1) \times \text{Gap between adjacent coils}}$

Spring deflection:

Assumptions:

(1) The Bending & shear effects may be neglected.

(2) For the purpose of derivation of formula, the helix angle is considered to be so small that it may be neglected.

Spring deflection: $\delta = \dfrac{8W.D^3n}{G.d^4}$

Where, W = axial load

D = mean coil diameter

d = diameter of spring wire

n = number of active coils

spring index : $C = \dfrac{D}{d}$ (For circular wires)

l = length of spring wire

G = modulus of rigidity

Spring stiffness: $k = \dfrac{G.d^4}{8.D^3n} = \dfrac{G.d}{8.C^3n}$

Shear stress: $\tau_{max} = \dfrac{8WD}{\pi d^3}\left(1 + \dfrac{d}{2D}\right) = \dfrac{8WD}{\pi d^3}\left(1 + \dfrac{1}{2C}\right)$

$k_{sh} = \left(1 + \dfrac{d}{2D}\right) = $ shear stress correction factor
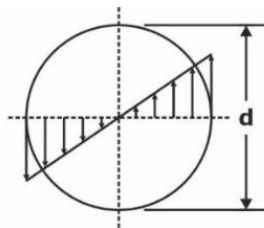
(a) Torsional shear stress diagram.
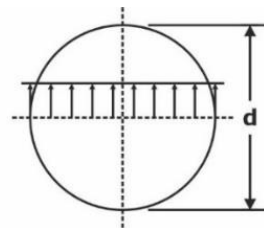
(b) Direct shear stress diagram.
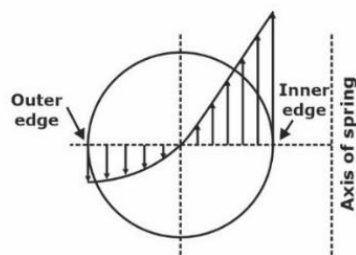
(c) Resultant torsional shear and direct shear stress diagram.

Wahl's correction factor:

$$K_w = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

If we take into account, the Wahl's factor than the formula for the shear stress becomes

$$\tau_{max} = \frac{8WD}{\pi d^3} K_w$$

The Wahl's stress factor (K) may be considered as composed of two sub-factors, K_{sh} and K_C, such that

$$K_w = K_{sh} \times K_c$$

where K_{sh} = Stress factor due to shear, and

K_c = Stress concentration factor due to curvature.

Strain Energy: The strain energy is defined as the energy which is stored within a material when the work has been done on the material.

$$U = \frac{T^2 L}{2GJ} \quad \text{and } L = \pi Dn$$

So, after substitution:

$$U = \frac{16T^2 Dn}{G.d^4}$$

Connection of springs:
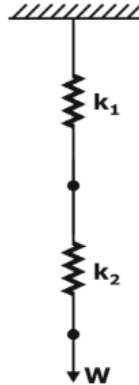
Series connection:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

For n number of springs:

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \dots \frac{1}{K_n}$$

Spring in parallel:
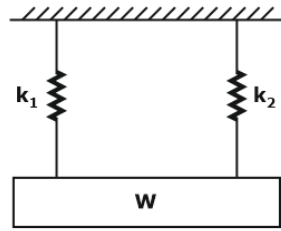
$$k = k_1 + k_2$$

∴ For n number of springs

$$K_{eq} = K_1 + K_2 + K_3 + \ldots + K_n$$

equivalent stiffness of the spring in parallel connection is the sum of the stiffness of the individual spring.

CHAPTER 7: THEORIES OF FAILURE

Maximum principal stress theory: According to this theory, the failure of a material will occur when the maximum principal tensile stress ($σ_1$) reaches the value of the maximum stress at the elastic limit in simple tension or the minimum principal stress reaches the value of the maximum stress at the elastic limit in simple compression.

This is the simplest and oldest theory of failure and is known as Rankine's theory. If the maximum principal stress ($σ_1$) is the design criterion, then maximum principal stress must not exceed the permissible stress ($σ_t$) for the given material.

Hence, $σ_1 = σ_t$

Where, $σ_t$ = permissible stress and is given by:

$$σ_t = σ_t^* / \text{safety factor}$$

(a). Maximum Principal Stress Theory [M.P.S.T.] or Rankine's Theory:

Condition for failure: $σ_1 > S_{yt}$ or S_{ut}

Condition for Safe Design: $σ_1 \leq S_{yt}$ or S_{ut}
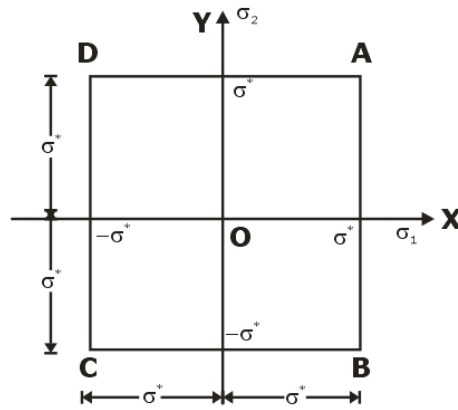
If Factor of safety is taken into consideration, then: $\sigma_1 \leq \dfrac{S_{yt}}{N}$ or $\dfrac{S_{ut}}{N}$

Where N is the Factor of safety.

- Best theory of failure for brittle materials, under any state of stress condition.
- It is suitable for ductile materials under uniaxial state of stress condition, Biaxial state of stress condition when σ_1, σ_2 are like in nature and hydrostatic state of stress.

(b). Maximum Shear Stress Theory [M.S.S.T.]:
Under Triaxial State of Stress

Absolute $\tau_{max} = $ max of $\left| \dfrac{\sigma_1 - \sigma_2}{2}, \dfrac{\sigma_2 - \sigma_3}{2}, \dfrac{\sigma_3 - \sigma_1}{2} \right|$

max of $\left| \dfrac{\sigma_1 - \sigma_2}{2}, \dfrac{\sigma_2 - \sigma_3}{2}, \dfrac{\sigma_3 - \sigma_1}{2} \right| \leq \dfrac{S_{yt}}{2N}$

For Biaxial State of Stress: $\sigma_3 = 0$

$$\text{max of } \left| \dfrac{\sigma_1 - \sigma_2}{2}, \dfrac{\sigma_2}{2}, \dfrac{\sigma_1}{2} \right| \leq \dfrac{S_{yt}}{2N}$$

Since $\sigma_1 > \sigma_2$, thus we can remove σ_2 from the above expression. So, for biaxial state of stress:

max of $\left| \dfrac{\sigma_1 - \sigma_2}{2}, \dfrac{\sigma_1}{2} \right| \leq \dfrac{S_{yt}}{2N}$

Absolute $\tau_{max} > \dfrac{S_{ys}}{N}$ or $\dfrac{S_{yt}}{2N}$

Since for brittle material: $S_{yt} \approx S_{ut}$

Absolute $\tau_{max} > \dfrac{S_{yt}}{2N}$ or $\dfrac{S_{ut}}{2N}$

It gives over safe design for ductile materials [Since it gives safe and uneconomic design].

(c). Maximum Principal Strain Theory [M.P.St.T.]: According to this theory, when Principal strain developed at a critical point under combined loading conditions exceeds yield strain under tension test, failure is likely to occur.
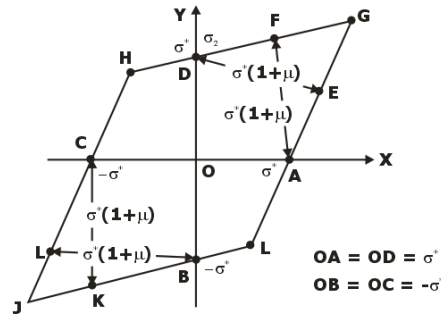
Maximum normal Strain > strain at yield point in tension test

$$\varepsilon_1 > \varepsilon_{YP}$$

Condition for Safe design

$$\varepsilon_1 \le \varepsilon_{YP} = \frac{S_{yt}}{E}$$

$$\boxed{\sigma_1 - \mu(\sigma_2 + \sigma_3) \le \frac{S_{yt}}{N}}$$

Under biaxial state of stress: $\sigma_3 = 0$

$$\boxed{\sigma_1 - \mu\sigma_2 \le \frac{S_{yt}}{N}}$$

(d). Total strain energy theory:

According to this theory, when total strain energy per unit volume at a critical point under combined loading conditions exceeds yield strength in tension test, failure is likely to occur.

Condition for failure

$$\boxed{\text{Total Strain energy / Volume} > \frac{\text{Strain Energy at yield point}}{\text{Volume}}}$$

Condition for Safe Design

$$\boxed{\text{Total Strain energy / Volume} \le \frac{\text{Strain Energy at yield point}}{\text{Volume}}}$$

$$\boxed{\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right) - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right) \le \left(\frac{S_{yt}}{N}\right)^2}$$

Under biaxial state of stress, $\sigma_3 = 0$

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \leq \left(\frac{S_{yt}}{N}\right)^2$$

It represents an equation of ellipse

whose semi-major axis = $\dfrac{S_{yt}}{\sqrt{1-\mu}}$ and semi-minor axis = $\dfrac{S_{yt}}{\sqrt{1+\mu}}$

(e). Maximum distortion energy theory: According to this theory, when maximum distortion energy developed at a critical point under combined loading conditions exceeds the yield strength in tension test, failure is likely to occur.

Deformation Energy / volume $=$ Total strain energy / Volume

$-$ maximum distortion energy / Volume

$$\text{Defomation Energy / volume} = \frac{1+\mu}{6E}\left((\sigma_2 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)$$

Under biaxial state of stress condition, $\sigma_3 = 0$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \leq \left(\frac{S_{yt}}{N}\right)^2$$

Which is an equation of an ellipse. Whose semi major axis = $\sqrt{2}S_{yt}$ and semi minor axis = $\sqrt{\dfrac{2}{3}}S_{yt}$.

It is the best theory of failure for Ductile materials [Since it gives safe and economic design].

CHAPTER 8: COLUMNS

Introduction:

Column:

☐ These are the vertical slender members subjected to an axial compressive load is called a column and used to carry loads of beams, slabs etc. stanchions are steel columns made of rolled steel sections, commonly used in buildings.

The failure of columns is due to the buckling at loads considerably less than those required to cause failure by crushing.

Struts: The term strut is commonly used for compression member in a roof truss, it may either be in vertical position or in inclined position.

Note.1: A compression member is generally considered to be a column when its unsupported length is more than 10 times its least lateral dimension.

Types of Columns

i. Short Columns: Short columns fail under direct compression, also called as crushing (at ultimate strength).

ii. Long Columns: Long columns fail at loads considerably lower than those required to cause crushing due to elastic instability, also called as buckling. The ***buckling behaviour is explained using Euler's theory.***

iii. Intermediate Columns: Intermediate columns **fail by a combination of crushing and buckling**. This behaviour is complex, and several semi-empirical formulations are used for ***analysis such as Rankine's formula***.

Slenderness ratio: The buckling tendency of a column varies with the ratio of the length to least lateral dimension. The ratio is known is known slenderness ratio. It is given by the following relation:

$$\text{Slenderness ratio } (S) = \frac{L_e}{K} = \frac{\text{Effective length of member}}{\text{Least radius of gyration}}$$

Its numerical value indicates whether the member falls into the class of columns or struts.

S.No.	Type of column	Slenderness ratio
1	Short Columns	0-40
2	Intermediate columns	40-125
3	Long Columns	>125

Radius of gyration: It is defined as the distance from the axis of rotation to a point where the total mass of the body is supposed to be concentrated so that the moment of inertia about the axis may remain the same.

The radius of gyration of a section is given by:

$$K = \sqrt{\frac{I}{A}}$$

Euler's theory: The struts which fails by buckling can be analysed by Euler's theory.

Assumptions of Euler's theory: The Euler's theory is based on the following assumptions:

(i). Axis of the column is perfectly straight when unloaded.

(ii). The line of thrust coincides exactly with the unstrained axis of the column.

(iii). Flexural rigidity El is uniform.

(iv) Material is isotropic and homogeneous.

(v). The buckling value of P = PE is assumed to obtain for all degrees of flexure.

Usually, the two assumptions are not really realised in practice. The column may have initial curvature, or crookedness. The theory, therefore, refers to an ideal column and not to a real one.

Equivalent length (L_e): The Effective length L_e for any column is the length of the equivalent pinned end column, i.e., it is the pinned end column having a deflection curve that exactly matches all part of deflection curve of an original column.

Condition and effective length	Diagram
a. Strut with pinned ends: $$P_e = \frac{\pi^2 EI}{L^2} \text{ and } L_e = L$$	
b. One end fixed and the other free: $$P_e = \frac{\pi^2 EI}{4L^2} \text{ and } L_e = 2L$$	
c. Strut with fixed ends: $$P_e = \frac{4\pi^2 EI}{L^2} \text{ and } L_e = L/2$$	
d. One and fixed and other hinged: $$P_e = \frac{2\pi^2 EI}{L^2} \text{ and } L_e = \frac{L}{\sqrt{2}}$$	

Rankine's Formula: Rankine proposed an empirical formula for columns which coven all Lasts ranging from very short to very long struts.

$$\frac{1}{P_r} = \frac{1}{P_c} + \frac{1}{P_E}$$

Where

$P_c = \sigma_C . A$ = ultimate load for a strut.

$P_E = \frac{\pi^2 EI}{L^2}$ = Eulerian crippling load for the standard case.

In the above relation, $\frac{1}{P_c}$ is constant for a material.

$$P_r = \frac{\sigma_c . A}{1 + \left(\frac{\sigma_c}{\pi^2 E}\right)\left(\frac{L}{k}\right)^2} = \frac{\sigma_c . A}{1 + a\left(\frac{L}{k}\right)^2}$$

Above equation is the Rankine's formula for the standard case of column hinged at ends.

Where Rankine's constant: $a = \frac{\sigma_c}{\pi^2 E}$

Values of σ_c, and a for the materials commonly used for columns and struts:

Material	σ_c		a
	Kg/cm²	N/mm²	(For hinged ends)
1. Wrought Iron.	2550	255	1/9000
2. Cast Iron	5670	567	1/1400
3.Mild steel	3300	330	1/7500
4.Strong timber	500	50	1/750

Combined bending and direct stresses (Eccentric Loading):

A column subjected by a compressive load P whose line of action is at a distance of 'e' from the axis of the column. Here 'e' is known as eccentricity of the load. The eccentric load will cause direct stress and bending stress.

CHAPTER 9: THIN AND THICK SHELLS

Introduction: Thin pressure vessel is defined as a closed cylindrical or spherical container designed to hold or store fluids at a pressure substantially different from ambient pressure.

Shape of Shell

Pressure Vessels

Spherical Pressure Vessels

Cylindrical Pressure Vessels

Thin Pressure Vessels $[^D/_t \geq 20]$

Thick Pressure Vessels $[^D/_t < 20]$
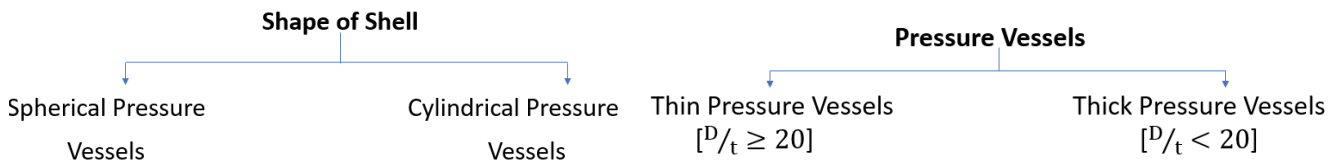
Common examples of pressure vessels are steam boilers, reservoirs, tanks, working chambers of engines, gas cylinders etc.

Thin cylindrical shell subject to internal pressure:

Assumptions:

(i). Stresses are assumed to be distributed uniformly.

(ii). Area is calculated considering the pressure vessel as thin.

(iii). Radial stresses are neglected.

(iv). Biaxial state of stress is assumed to be applicable.

Circumferential stress or Hoop stress: $\boxed{\sigma_H = \dfrac{Pd}{2t}}$

Longitudinal stress (or axial stress) σL: $\boxed{\sigma_L = \dfrac{Pd}{4t}}$

From here, we can say that

$$\boxed{\sigma_L = \dfrac{\sigma_H}{2}}$$

Thus, **the magnitude of the longitudinal stress is one half of the circumferential stress**, both the stresses being of tensile nature.

Note:

The radial stress is negligible as compare to the axial stress and hoop stress. Hence the third stress is neglected.

Maximum shear stress in the plane of σ_H and σ_L :

$$\tau_{max} = \frac{\sigma_H - \sigma_L}{2} = \frac{1}{2}\left(\frac{Pd}{2t} - \frac{Pd}{4t}\right) = \frac{Pd}{8t}$$

$$\text{Absolute } \tau_{max} = \frac{Pd}{4t}$$

Strain:

Hoop strain or Circumferential strain: $\boxed{\varepsilon_c = \frac{\Delta d}{d} = \frac{Pd}{4tE}[2 - \mu]}$

Longitudinal Strain or axial strain: $\boxed{\varepsilon_L = \frac{\Delta L}{L} = \frac{Pd}{4tE}[1 - 2\mu]}$

Note:

The ratio of circumferential strain and longitudinal strain under given loading condition is

$$\boxed{\frac{\text{circumeferential strain}(\varepsilon_c)}{\text{longitudinal strain}(\varepsilon_L)} = \frac{\frac{Pd}{4tE}(2 - \mu)}{\frac{Pd}{4tE}(1 - 2\mu)} = \frac{(2 - \mu)}{(1 - 2\mu)}}$$

Volumetric Strain or Change in the Internal Volume:

$$\boxed{\text{volumetric strain} = \text{longitudinal strain} + 2 \times \text{circumferential strain}}$$

Volumetric Strain: $\boxed{\varepsilon_v = \frac{\Delta V}{V} = \frac{Pd}{4tE}[5 - 4\mu]}$

Thin spherical shells under internal pressure:

Hoop stress: $\boxed{\sigma_h = \frac{Pd}{4t'}}$

And longitudinal stress: $\boxed{\sigma_L = \frac{Pd}{4t'}}$

In plane shear stress in a spherical pressure vessel:

$$\boxed{\text{Maximum shear stress} = \frac{\sigma_1 - \sigma_2}{2} = \frac{P \times d}{4t} - \frac{P \times d}{4t} = 0}$$

$$\boxed{\text{Absolute } \tau_{max} = \frac{Pd}{8t}}$$

$$\boxed{\varepsilon_c = \varepsilon_L = \frac{Pd}{4tE}(1-\mu)}$$

Volumetric strain: $\boxed{\varepsilon_v = \dfrac{dV}{V} = \dfrac{3Pd}{4tE}(1-\mu)}$.

Volumetric strain in spherical shell is thrice of the longitudinal strain or hoop strain.

Thin spherical shells under external pressure:

$$\boxed{\varepsilon_v = \frac{dV}{V} = -\frac{3Pd}{4tE}(1-\mu)}.$$

Thin cylinder with hemispherical ends:

If there is no distortion of the junction of hemispherical ends with cylindrical ends under pressure, then:

$$\boxed{\frac{t'}{t} = \frac{1-\mu}{2-\mu}}$$

Thus, the thickness of cylindrical wall must be greater than thickness of hemispherical ends.

Strengthening of cylindrical pressure vessel:

Wire Winding: A tube can be strengthened against the internal pressure by winding it with wire under tension and putting the tube wall in compression. Since the vessel is in compression, as the pressure is applied, the resultant hoop stress produced is much less as it would have been in the absence of wire.

CHAPTER 10: DEFLECTION OF BEAMS

Introduction: The effect of bending results in the deflection of the beam. This is the stiffness aspect of the beam. For design purpose, a beam should be so designed that it has adequate stiffness so that the deflections are within the permissible limits.

Differential equation of the deflection curve of beam:

$$\boxed{\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d\theta}{dx} = -\frac{M}{EI}}$$

This equation can be integrated in each particular case to find the angle of rotation θ (usually called the slope) or the deflection y provided the bending moment M is known.

Sign Conventions: The following sign conventions are adopted:
(i) x is positive when measured towards the right.
(ii) y is positive when measured downwards.

(iii) θ is positive when the rotation is clockwise from the x-axis.

(iv) M is positive when sagging, i.e., when it produces compression in the upper portion of the beam.

Methods of determining deflection of beams:

(a). Double integration method: $\boxed{EI\dfrac{dy}{dx} = EI.\theta = -\int M}$

(b). Area moment method (Mohr's method): This method utilizes the properties of the area of the bending moment diagram and the moment of that area.

Mohr's first theorem: 1 The angle θ_B^A between the tangent of the deflection curve of two points A and B is equal to the negative area of M/EI diagram between the points.

Mohr's Second Theorem: The deviation of B from tangent at A is equal to the negative of the statical moment (or the first moment) with respect to B, of the $\dfrac{M}{EI}$ diagram area between A and B.

Note:

(i). Non- zero slope cross-section should be a cross-section where slope & deflection are to be determined.

(ii). $\bar{X}$ - should be measured from origin (i.e., Non-zero slope cross-section).

(c). Conjugate beam method (method of elastic weights): A conjugate beam is an imaginary secondary beam, which when loaded with the M/EI diagram of the real beam, yield directly the slope and deflection of the real beam in the form of shear force and bending moment of the conjugate beam.

Preposition 1: Similarity between S.F. of beam and slope of real beam.

$\boxed{F_x = \int \dfrac{d^2y}{dx^2}dx = \dfrac{dy}{dx} = slope}$

Preposition 2: Similarity between B.M. of conjugate beam and deflection of real Shear force.

$\boxed{M_x{}^{'} = \int_0^x \dfrac{dy}{dx}dx = y = deflection}$

Rules	Existing support condition of actual beam	Corresponding support condition for the conjugate beam
Rule -1	Fixed end	Free end
Rule -2	Free end	Fixed end
Rule -3	Simple support at the end	Simple support at the end
Rule -4	Simple support not at the end	Unsupported hinge
Rule -5	Unsupported hinge	Simple support

(d). Castigliano's first theorem (deflection from strain energy):

It may be observed that though the differentiation with respect to load may be carried out before or after the integration, the calculations are simplified if the differentiation is carried out before the integration.

In case of a beam, $U = \int_0^l \frac{M^2 .dx}{2EI}$

Deflection: $\boxed{\delta_i = \frac{\partial U}{\partial W_i} = \frac{1}{EI}\int_0^l M.\frac{\partial M}{\partial W_i}dx}$

The slope θ_i of a beam at a point can also be obtained by applying a virtual couple M_i at the point and putting the same to zero before integration.

$$\boxed{\theta_i = \frac{\partial U}{\partial M_i} = \int_0^l \frac{M}{EI}\frac{M}{\partial M_i}dx = \frac{1}{EI}\int_0^l M.\frac{\partial M}{\partial W_i}dx}$$

(e). Maxwell's reciprocal deflection theorem: "The deflection of any point P resulting from application of a load at any other point Q is the same as the deflection of Q resulting from the application of the same load at P".

Deflections and slopes of different beams under different loadings:

Case (1): Deflection and slope under constant bending moment M:

$\boxed{\theta_{max} = \left[\frac{1}{K_1}\right]\left[\frac{ML}{EI_{N.A.}}\right]}$, and $\boxed{y_{max} = \left[\frac{1}{K_2}\right]\left[\frac{ML^2}{EI_{N.A.}}\right]}$

Where K_1 and K_2 are constants.

Case (2): In presence of concentrated point loads:
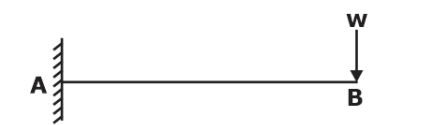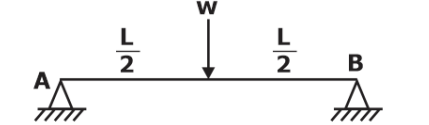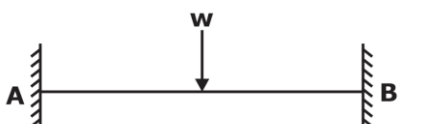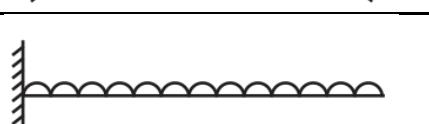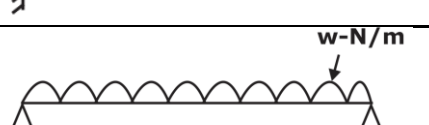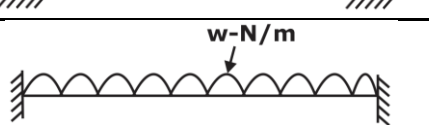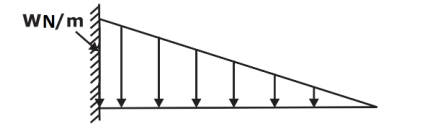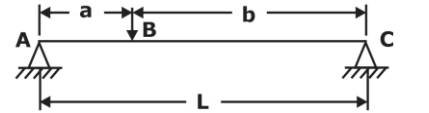
$\boxed{\theta_{max} = \left[\frac{1}{K_1}\right]\left[\frac{WL^2}{EI_{N.A.}}\right]}$ and $\boxed{y_{max} = \left[\frac{1}{K_2}\right]\left[\frac{WL^3}{EI_{N.A.}}\right]}$

Case (3): In presence of distributed loads:

$\boxed{Q_{max} = \left[\frac{1}{K_1}\right]\left[\frac{wL^3}{EI_{N.A.}}\right]}$, and $\boxed{y_{max} = \left[\frac{1}{K_2}\right]\left[\frac{wL^4}{EI_{N.A.}}\right]}$

$\boxed{y_{max} = \theta_{max}\left[\frac{k_1}{k_2}\right]L}$

S. No.	Type of beam	K_1	K_2
1		1	2
2		2	8

3		2	3
4		16	48
5		-	192
6		6	8
7		24	$\dfrac{384}{5}$
8		-	384
9		24	30
10		$\theta_B = \dfrac{Wb}{3EI_{N.A.}}[a^2 - ab]$ (Not maximum)	$Y_B = \dfrac{wa^2b^2}{3EI_{N.A.}L}$

****

Important Formula (Theory of Machines)

CHAPTER-1: SIMPLE MECHANISMS

INTRODUCTION

If a number of bodies are connected & the motion of one body causes the predictable motion to the others, it is known as a mechanism.

Kinematic Link or Element

- Every part of machine which is having some relative motion w.r.t. some other part.
- It is the smallest unit of a mechanism.
- Link required to be a resistant body, so it is capable of transmitting power & motion.

Rigid and Resistant Bodies

- Rigid body does not deform under the action of the load and the distance between any two points does not change.
- Resistant bodies deform upto some extent under the action of external load.

Link Classification on the basis of Property

- **Rigid Links**: Those links in which deformations are negligible.
- **Flexible Links**: in which deformations occur but within permissible limits.
- **Fluid Links**: When power is transmitted because of fluid pressure.

Link Classification on the basis of Connection

- **Binary Links**: which are connected at two places.
- **Ternary Links**: which are connected at three places.
- **Quaternary Links**: which are connected at four places.

Binary Links   Ternary Links   Quaternary Links
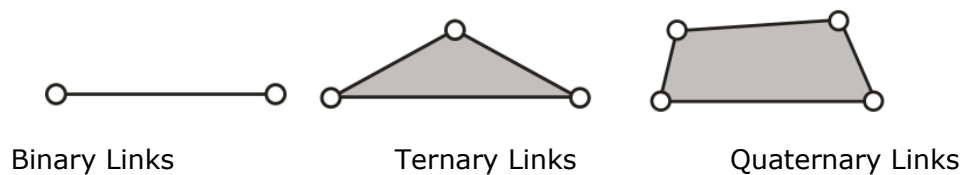
JOINT- Connection of two links is always a joint.

Different Types of Joints

- **Binary Joint:** Where two links are connected.
- **Ternary Joint:** Where three links are connected. 1 Ternary Joint $\equiv$ 2 Binary Joint
- **Quaternary Joint:** here four links are connected. 1 Quaternary Joint $\equiv$ 3 Binary Joint

Binary joint   Ternary joint   Quaternary joint
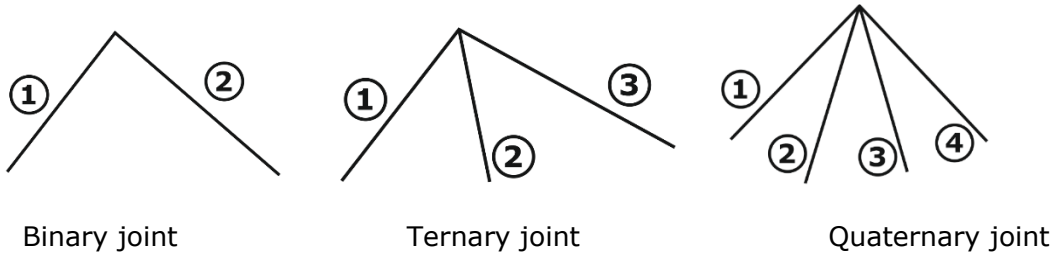
Types of Relative Motion

- Constrained Motion: It is the desired motion which we want from the system.
- Completely Constrained Motion: When motion is constrained by the system property.
- Successfully Constrained Motion: When motion is constrained by surrounding property.
- Incompletely Constrained Motion (Unconstrained Motion): When for a given input more than one output is present.

KINEMATIC PAIR

Any connection of two links is always a joint or a pair but this pair will only be a kinematic pair when the relative motion between the links is a constrained motion.

Classification according to type of Relative Motion

- **Turning pair** (Revolute pair/ Pin Joint): When the relative motion is pure turning.
- **Sliding Pair** (Prismatic Pair): When the relative motion is pure sliding.
- **Rolling Pair**: When the relative motion is pure rolling (Rolling without slipping).
- **Screw Pair** (Helical Pair): When relative motion is over the threads [Nut and Bolt]
- **Spherical Pair** [Ball and Socket Joint]: When the relative motion is 3-dimensional rotation (Spherical Motion)

Turning Pair   Sliding Pair   Rolling Pair   Screw Pair   Spherical Pair
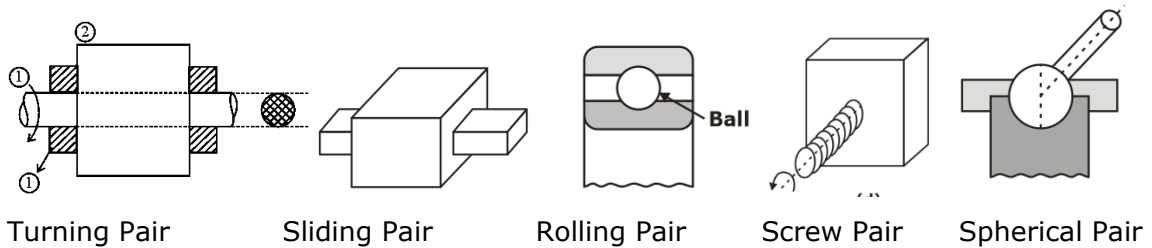
Classification according to type of Contact between the links

- **Lower Pair:** Surface or Area Contact between links.
- **Higher Pair:** Point/Line Contact between links.
- **Wrapping Pair:** When one link is wrapped over other link [Multiple point contacts]

Classification according to type of Closure between the links

- **Self-Closed Pair [Closed Pair]:** Permanent Contact without any external support.
- **Forced Closed Pair [Open Pair]:** not permanent but it is made to contact by applying some external load. Also called Forceful Contact.

KINEMATIC CHAIN

If all the links are connected in such a way that first link is connected to the last link in order to get the closed chain and the relative motions in this closed chain are constrained.

Kinematic Chain [Constrained Chain]

To use this chain one link must be fixed

Mechanism [Which can give desired output with respect to given input]

Utilize

Machine [which is giving desired output]
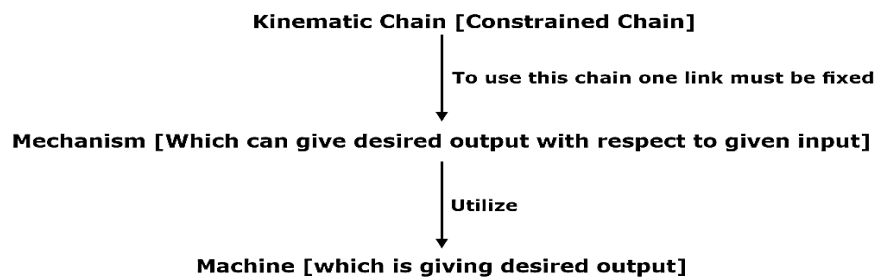
Degrees of Freedom [Mobility]

The minimum number of independent variables required to define the position or motion of the system is known as degrees of freedom of the system.
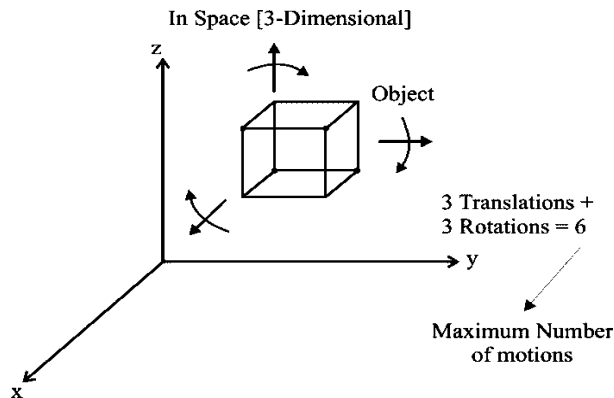
- (DOF) = 6 – Restraints (number of those motions which are not possible/ restricted).
- In a plane $F = 3(l - 1) - 2j - h - F_r$ {where F_r is number of redundant motions which are not the part of a mechanism}. This equation is **KUTZBACK'S EQUATION.**
- The **Redundant link** is the additional link which is added to original mechanism which do not affect the motion of original mechanism.
- Degree of freedom is calculated by removing the redundant link.

Linkage Mechanism and Structure

Structure is a mechanism; whose degree of freedom is zero. No relative motion is possible in case of structure/frame.

- Super Structure is mechanism whose degree of freedom is less than zero. It is also known as indeterminate structure.
- Kinematics Chain is the mechanism whose degree of freedom is equal to one.
- Unconstrained Chain is mechanism whose degree of freedom is more than one.
- A spring can be considered equivalent to two links connected by a binary joint.

GRUBLER'S EQUATION is derived form of kutzback's equation when F = 1 and h = 0

$$3l - 2j - 4 = 0$$

FOUR BAR MECHANISM

- Simple mechanism which consists of four links connected by four turning pairs.
- Fixed position is the best position as it governs both input and output link.
    - a. if any link makes COMPLETE ROTATION → **CRANK**
    - b. if any link makes INCOMPLETE ROTATION (oscillation) → **ROCKER OR LEVER**
    - c. the link opposite to the fixed link → **Coupler**

[Coupler or Coupling Rod]
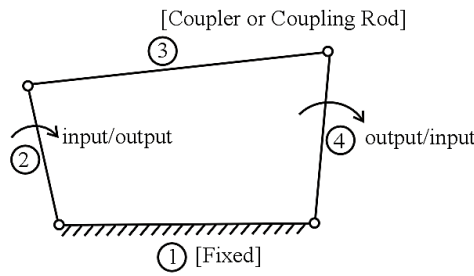
- When different links are fixed, then we obtain different mechanisms and these different mechanisms are called **inversions**. If no. of links = l, then no. of inversions ≤ l

GRASHOF'S LAW

- For the continuous relative motion OR For complete rotation of at least one link following condition must be satisfied.   **(s + l) ≤ (p + q)**

- **CASE I** (s + l) < (p + q)
  - o If shortest link is fixed: DOUBLE CRANK MECHANISM
  - o If shortest link is adjacent to the fixed link: CRANK ROCKER MECHANISM
  - o If shortest link is made coupler or coupling rod: DOUBLE ROCKER MECHANISM

- **Case II** (s + l) = (p + q)
  - o Not having pair of equal links (e.g. 3, 6, 5, 4), Inversions will be same as Case I.
  - o Having pair of equal links (e.g. 3, 3, 5, 5)
    - **a.** If similar links are opposite to each other [PARALLELOGRAM LINKAGE].
      - i.   If longest (or shortest) link is fixed: DOUBLE CRANK MECHANISM
    - **b.** If similar links are adjacent to each other [DELTOID LINKAGE]
      - i.   If shortest link is fixed: DOUBLE CRANK MECHANISM
      - ii. If longest link is fixed: CRANK ROCKER MECHANISM

- **Case III** (s + l) > (p + q)
  - o Grashoff's law is not satisfied. No link will completely rotate.
  - o In all such arrangements, DOUBLE ROCKER MECHANISM will be obtained.

Inversions of Four-Bar Mechanism

- BEAM ENGINE- crank rocker mechanism
- COUPLING ROD OF LOCOMOTIVE- double crank mechanism

TRANSMISSION ANGLE

- The angle between the output link and the coupler is known as transmission angle.
- It is designated by μ.
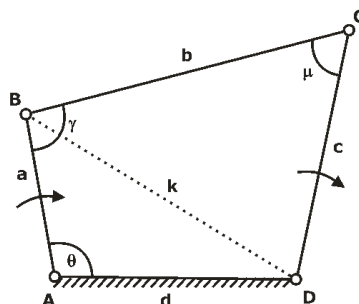
- If some input torque is given to link AB then the torque transmitted to the output link is maximum when the transmission angle μ is 90°.
- If links BC and DC become coincident, the transmission angle is zero and the mechanism would lock or jam.
- From above figure we can see that,

    μ will be maximum when θ is 180°

    μ will be minimum when θ is 0°

SINGLE SLIDER CRANK MECHANISM

Mechanism consists of four links connected by three turning pairs and one sliding pair.
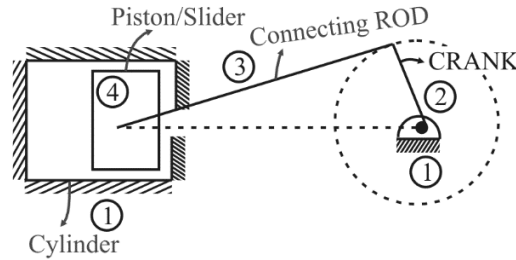
Inversions of single slider crank mechanism

- **Crank is fixed**
    - o Whitworth quick return motion mechanism
    - o rotary internal combustion engine (gnome engine)

- **Connecting rod is fixed**
    - o Crank and slotted lever quick return motion mechanism
    - o Oscillating cylinder engine mechanism

- **Piston/Slider is fixed**
    - o Hand pump (pendulum pump/bull engine)

Whitworth Quick Return Motion Mechanism
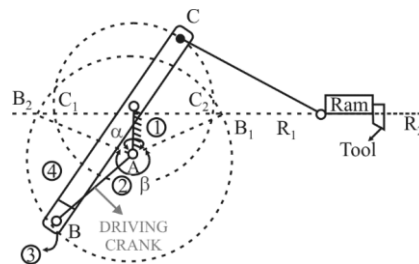
β = cutting stroke angle

α = return stroke angle

α + β = 360°

$$\text{QRR (Quick return ratio)} = \frac{T_{forward}}{T_{Return}} = \frac{T_{Cutting}}{T_{ideal/return}} = \frac{\beta}{\alpha} = \frac{360° - \alpha}{\alpha} > 1$$

$$\text{stroke length} = R_1 R_2 = 2C_1 C_2 = 2 \times OC = 2 \times \text{Radius of smaller circle}$$

Where, OC = length of extension link on which tool is pivoted.

Limitation: Variations in stroke length are limited

Crank and slotted lever quick return motion mechanism
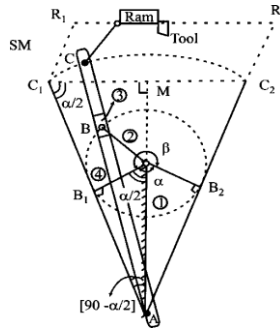
β = cutting stroke angle

α = return stroke angle

α + β = 360°

$$QRR = \frac{T_{forward}}{T_{Return}} = \frac{T_{Cutting}}{T_{ideal/return}} = \frac{\beta}{\alpha} = \frac{360° - \alpha}{\alpha} > 1$$

$$\text{stroke length} = R_1R_2 = 2C_1C_2 = \frac{2 \times \text{crank length} \times \text{length of slotted lever}}{\text{connecting rod length}}$$

Thus, stroke length of crank & slotted lever mechanism depends upon 3 parameters whereas in whitworth quick return motion mechanism stroke length depends upon single parameter.

Oscillating cylinder engine mechanism

Here, connecting rod is fixed. It converts rotary motion into oscillating motion.
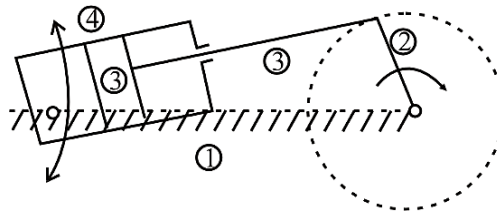
Hand pump mechanism

If the link 4 of the slider-crank mechanism is fixed, then inversion obtained is the fourth inversion of slider crank mechanism.
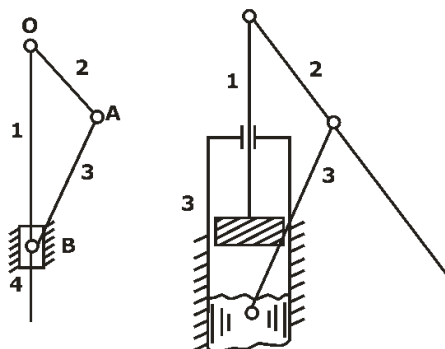
DOUBLE SLIDER CRANK CHAIN

Chain which consists four links connected by two turning pairs and two sliding pairs.
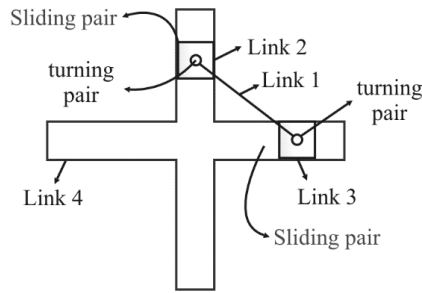
Inversions of double slider crank chain

- **Slotted bar is fixed:** Elliptical trammels
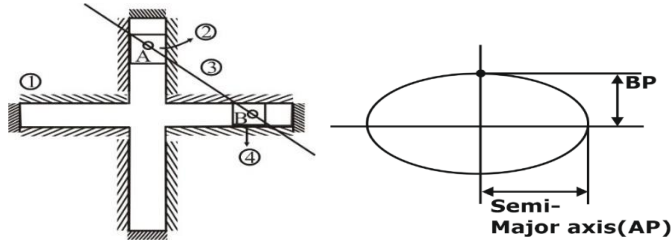
Every point on 'link 3' except mid-point will trace out an ellipse. The mid-point will trace a circle.

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = 1$$

        AP – Semi-major axis

        BP – Semi-minor Axis

- **One of the sliders is fixed:** Scotch-yoke mechanism

It converts rotary motion into reciprocating motion and being used in power hacksaws.
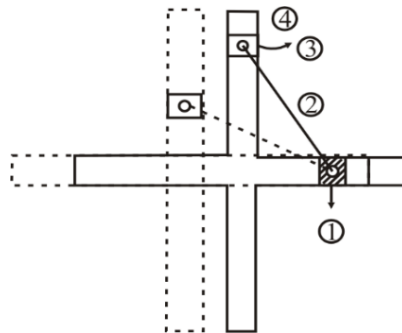
- **Link connecting sliders is fixed:** Oldham's coupling

It is used to connect the shafts having lateral misalignment.
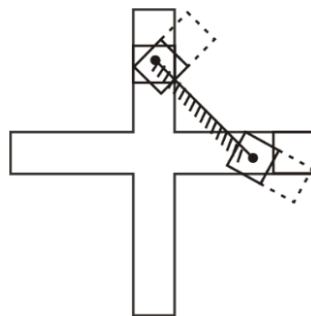
WATT'S INDICATOR MECHANISM

This mechanism was invented by Sir James Watt to measure the pressure of steam inside the cylinder used in the steam engine.
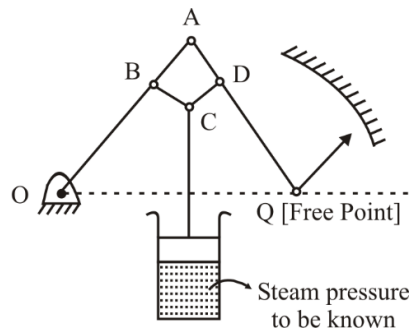
- Point C & point Q both move in approximate straight-line motion. Hence, this mechanism is also known as approximate straight-line motion mechanism.
- There is no relative motion between BC & CD. Hence, BCD behaves as one link.
- Both links BCD and AQ behaves as levers. Hence, this mechanism is a double lever/double rocker mechanism.

MISCELLANEOUS MECHANISM

Geneva mechanism, also called Geneva Stop, one of the most commonly used devices for producing intermittent rotary motion, characterized by alternate periods of motion and rest with no reversal in direction. It is also used for indexing (i.e., rotating a shaft through a prescribed angle).
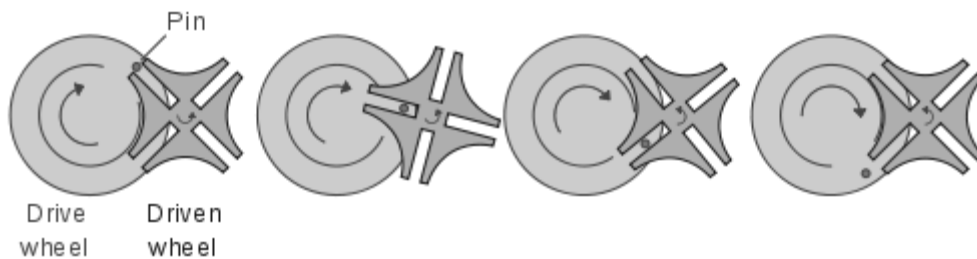
Rack and Pinion is a type of linear actuator that comprises a circular gear (the pinion) engaging a linear gear (the rack), which operate to translate rotational motion into linear motion. Driving the pinion into rotation causes the rack to be driven linearly.
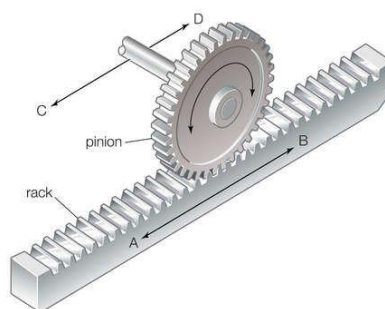
Ratchet mechanism- A wheel with suitably shaped teeth, receiving an intermittent circular motion from an oscillating member, is a ratchet wheel. The figure below shows a simple ratchet mechanism.
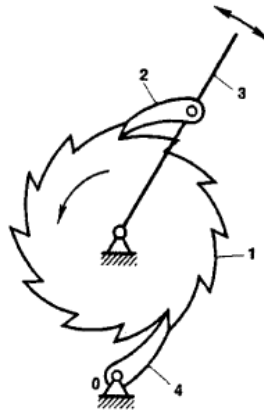
CHAPTER-2: VELOCITY & ACCELERATION ANALYSIS

VELOCITY ANALYSIS (I-centre)

- For the velocity analysis the knowledge of position of all the links is required which is available from the position analysis or from configuration diagram.
- I centre is the point about which motion can be assumed pure rolling.
- Locus of instantaneous centre for a link in its motion → **Centrode**
- Locus of instantaneous axis of rotation for a link → **Axode**

Motion	Centrode	Axode
General Motion	Curve	Curved surface
Pure Translation	Straight line	Plane Surface
Pure rotation	Point	Line

- Number of I-Centres in n links mechanism $^nC_2 = \dfrac{n(n-1)}{2}$

LOCATION OF I CENTRE IN MECHANISM
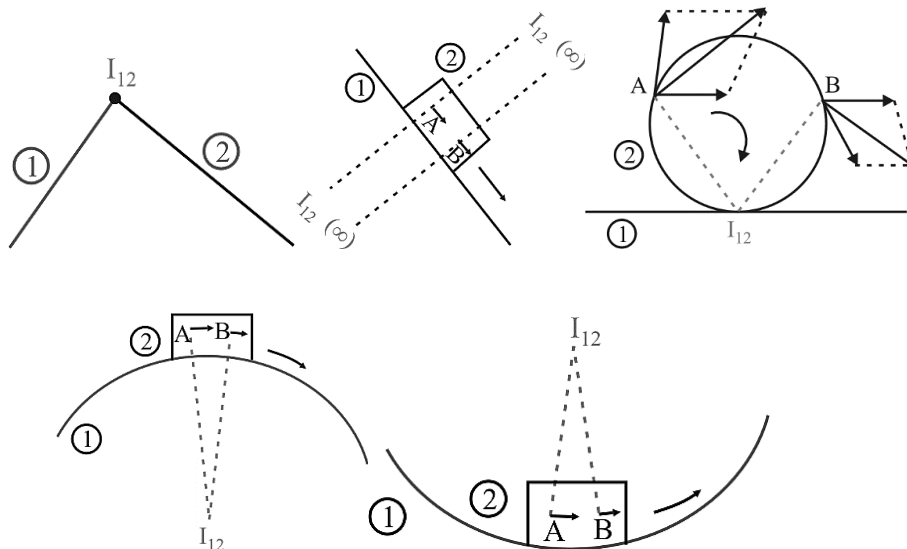
KENNEDY'S THEOREM

This theorem states that "For the relative motion between the number of links in a mechanism, any three links and their three instantaneous centres must lie on a straight line".

Theorem of Angular Velocities

- $V_m = \omega_m(I_{mn} - I_{1m}) = \omega_n(I_{mn} - I_{1n}) = V_n$

- If I_{1m} and I_{1n} lie at the same side of I_{mn}, then direction of rotation of ω_m and ω_n will be same.

- If I_{1m} and I_{1n} lie on the opposite side of I_{mn}, then direction of rotation of ω_m and ω_n will be opposite.
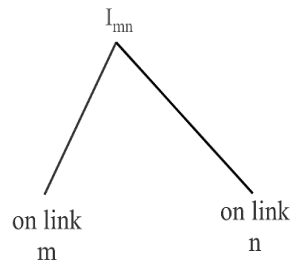
Relative Velocity Method

- For a rigid link the relative velocity between any two points on same link will always be perpendicular to line joining these two points.
- The motion of link can be completely described by line present on link.
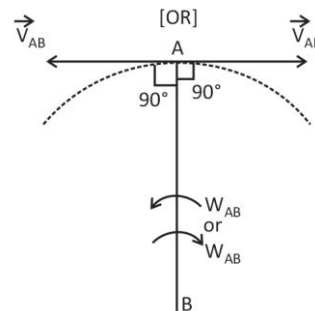
ACCELERATION ANALYSIS

Velocity of a moving body is a vector quantity having magnitude and direction. A change in the velocity requires any of the following conditions to be fulfilled:

- Magnitude change only
- Direction change only
- Both magnitude and direction change

Let a link AB is rotating about point B with angular velocity ω_{AB} & Angular Acceleration α_{AB}. Now, there will be two components of acceleration of this link AB.

- Radial component of acceleration, a^r_{AB} which will be always directed towards the centre of rotation of link.
- Tangential component of acceleration, a^t_{AB} which will act in a direction perpendicular to link.

a^r_{AB} (Due to change in direction of velocity) $= \dfrac{v^2_{AB}}{AB}$

(Direction from A to B)

a^t_{AB} (Change in magnitude of velocity) $= AB(\alpha_{AB})$

(Direction perpendicular to radial acceleration)

Configuration Diagram
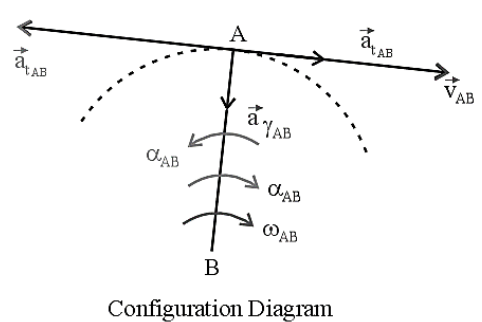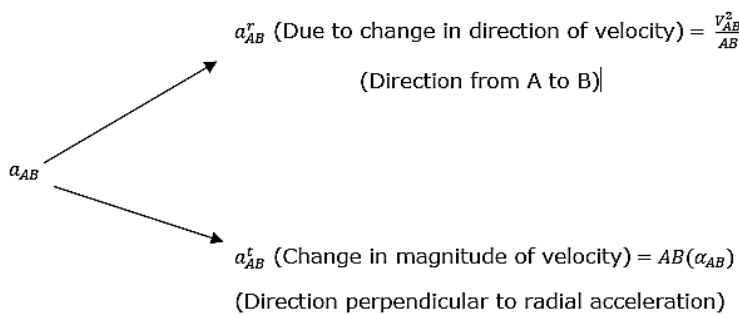
Resultant acceleration is given by $a = \sqrt{a^r_{AB} + a^t_{AB}}$

Coriolis Acceleration (a^c)

- This acceleration will always be associated with the slider when slider is sliding on the rotating object.   a^c = 2vω

- For direction of Coriolis acceleration, take the sense of direction of ω, Rotate the velocity vector in that direction of ω by an angle of 90 degrees.

- This extra acceleration component a^c = 2vω can be understand by below figure using separate motion of translation and rotation of slider crank mechanism. And after combining both motion and looking at their effect.
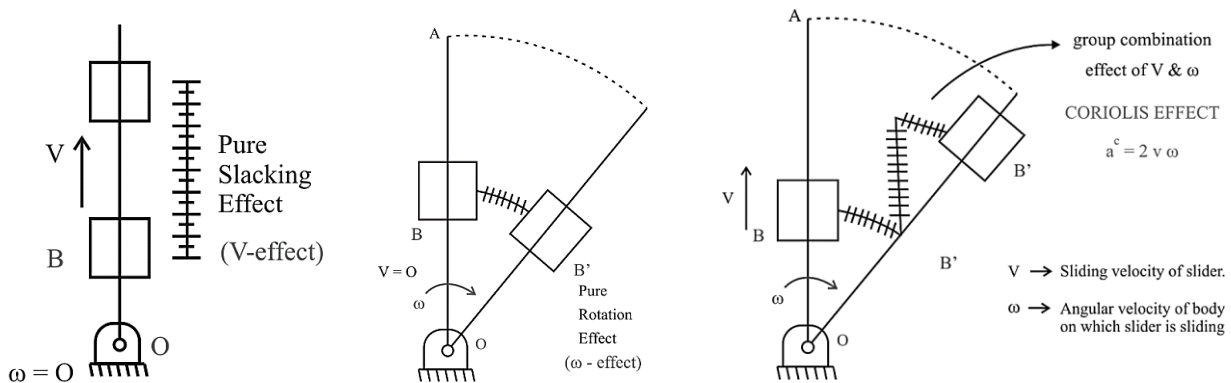
KLEIN'S CONSTRUCTION

- In Klein's construction, the velocity and the acceleration diagrams are made on the configuration diagram itself.

- The line that represents crank in configuration diagram also represents the velocity and the acceleration of its moving end in the velocity and the acceleration diagrams respectively.

- This can only be applied in Basic Single Slider Crank Mechanism when ω$_{crank}$ is given and α$_{crank}$ is zero.

        Here, OAP = velocity diagram of single slider crank mechanism

          OARS = acceleration quadrilateral
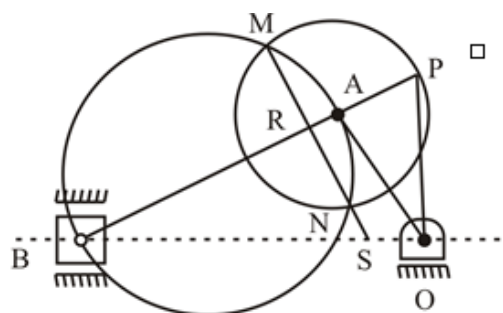
- Both Velocity and acceleration diagram can be summarized by following equations:

$$\frac{V_A}{OA} = \frac{V_B}{OP} = \frac{V_{AB}}{AP} = \omega_{crank}$$

$$\frac{a_A}{OA} = \frac{a_{BA}^r}{AR} = \frac{a_{BA}^t}{RS} = \frac{a_B}{OS} = \omega_{crank}^2$$

MECHANICAL ADVANTAGE (M.A.) OF THE MECHANISM

- A form of calculation that measures the amplified force taken by the mechanical system.
- It gives the ratio between the force applied to the load and the force needed to overcome the given force. It is a unit less expression since the two ratio quantities are the force.
- also defined as the force created by the machine to the for applied on it.
- The formula of mechanical advantage is given as It is defined as the ratio of output force or torque to the input force or torque.

$$\text{Mechanical advantage} = \frac{F_{output}}{F_{input}} = \frac{T_{output}}{T_{input}}$$

$$\text{Efficiency, } \eta = \frac{P_{output}}{P_{input}} = \frac{T_{output} \times \omega_{output}}{T_{input} \times \omega_{input}}$$

$$\text{Mechanical advantage} = \eta \times \frac{\omega_{output}}{\omega_{input}} = \eta \times \frac{V_{output}}{V_{input}}$$

TOGGLE POSITION

- Simple toggle consists of two links which tend to line-up in a straight line at one point in their motion. The mechanical advantage of the simple toggle above is the velocity ratio of the input point A to the output point B
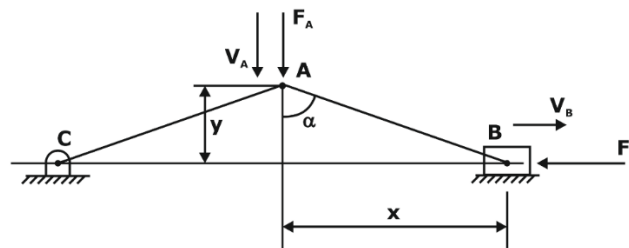
- Mechanical advantage $\dfrac{F_B}{F_A} = \dfrac{x}{y} = \tan \alpha = \dfrac{v_A}{v_B}$

- As angle α approaches 90°, CA & AB come in to toggle. And mechanical advantage becomes **infinite**.

- This type of mechanism is used in punch presses, riveting machines, stone crusher, etc.

- In 4 bar mechanism toggle position is defined as that position when the angle between the input link and coupler link is 180°. In such a situation, the velocity of the output link becomes **zero** and hence, the mechanical advantage becomes **infinite**.

- **Stone Crusher**

  The stone crusher shown uses two toggle linkages in series to obtain high mechanical advantage. This configuration results in a very large crushing force.
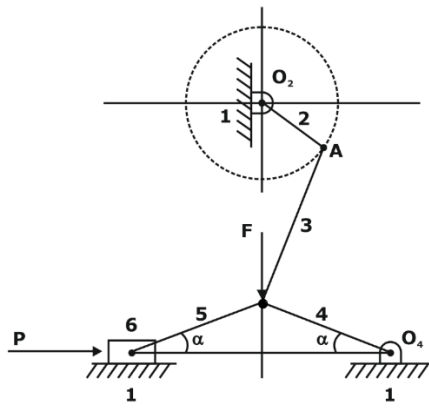
CHAPTER-3: GEARS & GEAR TRAINS

INTRODUCTION

- Gears are used to transmit motion from one shaft to the another by direct contact.
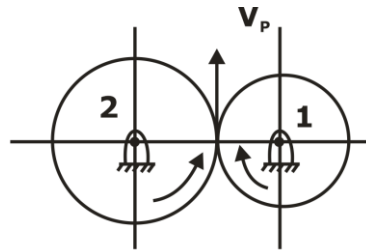
$$V_P = \omega_1 r_1 = \omega_2 r_2 \qquad \frac{\omega_1}{\omega_1} = \frac{N_1}{N_2} = \frac{r_2}{r_1}$$

Where: N = Angular velocity (rpm), ω = Angular velocity (rad/s), r = radius of the disc

- Slipping is not present in gear drives [Positive Drives]

- When the power transmitted increases, slip may occur between the discs and now the motion is no longer definite.

- It is to be noted that if the gear 1 rotates in the clockwise direction, then gear 2 rotates in the counter clockwise direction and vice-versa.

- Practically possible velocities ratio for gears: 6 for spur gears and 10 for helical and herringbone gears. To obtain large reductions, two or more pairs of gears are used.

CLASSIFICATION OF GEARS

- **SPUR GEARS**
  o Teeth are straight and parallel to the axis of rotation. (Shafts are Parallel)
  o Power transmissions at very low speed.
  o Instantaneous engagement and disengagement is present which causes impact stresses in the gears and noise.

- **HELICAL GEARS**
  o Teeth are straight but inclined to the axis of the rotation. (Shafts are Parallel)
  o Gradual Engagement: NO IMPACT STRESSES.
  o To minimize the Axial Thrust, double helical gears [herringbone gears] are used.

- **BEVEL GEARS**
  o Shafts are non-parallel but intersecting.
  o STRAIGHT BEVEL GEARS
    ▪ Teeth are straight and parallel to the axis of rotation.
    ▪ Very less used as IMPACT STRESSES are induced in it
  o HELICAL BEVEL GEARS
    ▪ Teeth are straight and inclined to the axis of rotation
    ▪ Widely used as IMPACT STRESSES are not induced in it

- **SPIRAL GEAR [Skew – Bevel Gear]**
  o Axes are neither parallel nor interesting.

- **WORM and WORM WHEEL**
  - o Used for transmitting rotary motion into linear motion.
  - o Axes are neither parallel nor interesting.

GEAR TERMINOLOGY

- **Pitch Circle-** Imaginary circle in the gears where pure rolling motion is observed when the mating gears are transmitting power.
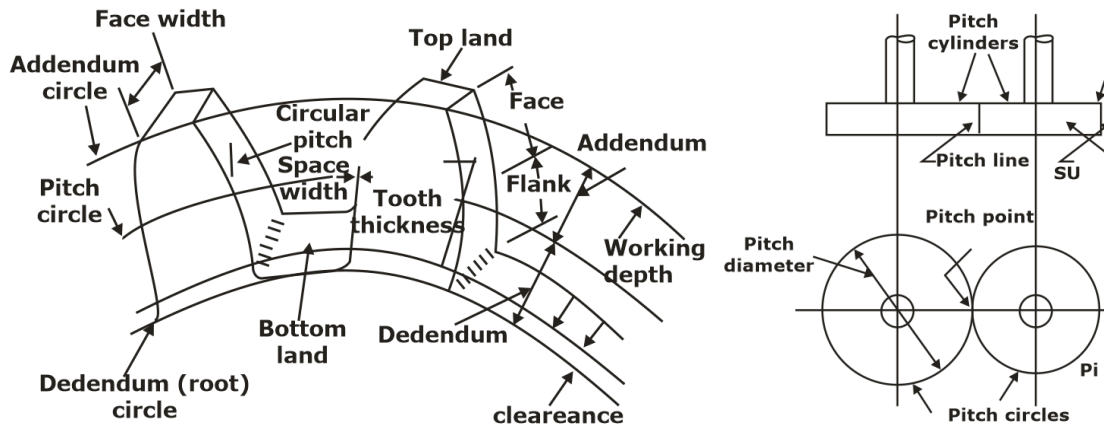
- **Circular Pitch (P_c)-** Then, circular pitch $P_c = \dfrac{\pi d}{T}$, For two mating gears, $P_{c1} = P_{c2}$

- **Module (m)** $m = \dfrac{d}{T}$

- **Diametral Pitch (P_d)** $P_d = \dfrac{T}{d}$

- **Gear Ratio (G)**: It is the ratio of the number of teeth on the gear to that on the pinion.

  $G = \dfrac{T}{t}$, where T = number of teeth on the gear, t = number of teeth on the pinion.

- **Velocity Ratio (VR):** The velocity ratio is defined as the ratio of the angular of the follower to the angular velocity of the driving gear.

  $$VR = \dfrac{\text{angular velocity of follower}}{\text{Angular velocity of driver}} = \dfrac{\omega_2}{\omega_1} = \dfrac{N_2}{N_1}$$

Gear Geometry terminologies:

- **Addendum Circle:** It is a circle passing through the tips of teeth.
- **Addendum:** radial height of a tooth above the pitch circle. Its standard value is one module.
- **Dedendum or Root Circle:** It is a circle passing through the roots of the teeth.
- **Dedendum:** radial depth of a tooth below the pitch circle. Its standard value is 1.157 m.
- **Clearance:** Radial difference between the addendum and the dedendum of a tooth. Thus,

  Addendum circle diameter = d + 2m

  Dedendum circle diameter = d − 2 × 1.157 m

  Clearance = 1.157 m − m = 0.157m

- **Full Depth of Teeth**: It is the total radial depth of the tooth space.

  Full depth = Addendum + Dedendum

- **Working Depth of Teeth:** The maximum depth to which a tooth penetrates the tooth space of the mating gear is the working depth of teeth.

  Working depth = Sum of addendums of the two gears.

- **Space Width:** It is the width of the tooth space along the pitch circle.

- **Tooth Thickness**: It is the thickness of the tooth measured along the pitch circle.

- **Backlash:** difference between the space width and the tooth thickness along the pitch circle.

  Backlash = Space width – Tooth thickness

- **Face Width:** The length of the tooth parallel to the gear axis is the face width.

OTHER TERMINOLOGIES

- **Line of Action or Pressure Line:**
  - The force, which the driving tooth exerts on the driven tooth, is along a line from the pitch point to the point of contact of the two teeth.
  - This line is known as the line of action or the pressure line and it is also the common normal at the point of contact of the mating gears.

- **Pressure Angle or Angle of Obliquity (φ):**
  - The angle between the pressure line and the common tangent to the pitch circles is known as the pressure angle or the angle of obliquity.
  - For more power transmission and lesser pressure on the hearings. the pressure angle must be kept small. Standard pressure angles are 20° and 25°. Gears with 14.5° pressure angles have become almost obsolete.

- **Path of Contact or Contact Length:**
  - The locus of the point of contact of two mating teeth from the beginning of engagement to the end of engagement is known as the path of contact or the contact length.
  - The pitch point P is always one point on path of contact. It can be subdivided as follows:
  - ➢ **Path of Approach:** Portion of the path of contact from the beginning of engagement to the pitch point i.e. the length NP.
  - ➢ **Path of Recess:** Portion of the path of contact from the pitch point to the end of engagement i.e. length PM.
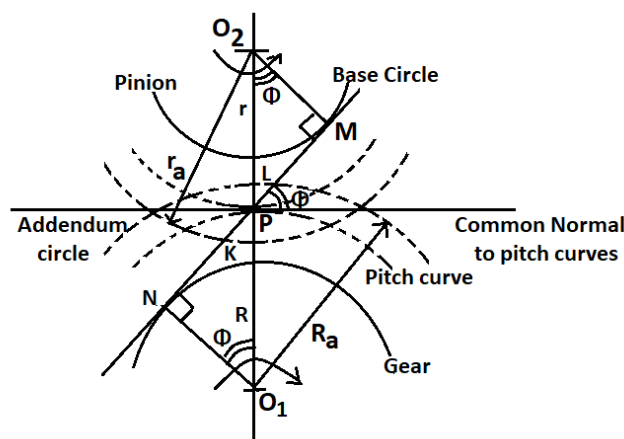
- **Arc of Contact:**

- o The locus of a point on the pitch circle from the beginning to the end of engagement of two mating gears is known as the arc of contact.
  - ➢ **Arc of Approach:** It is the portion of the arc of contact from the beginning of engagement to the pitch point.
  - ➢ **Arc of Recess**: The portion of the arc of contact from the pitch point to the end of engagement is the arc of recess.
- **Angle of Action (δ):**
  - o It is angle turned by a gear from the beginning of engagement to the end of engagement of a pair of teeth, i.e. the angle turned by arcs of contact of respective gear wheels.
  - o Similarly angle of approach (α) & angle of recess (β) can be defined. δ=α+β
  - o The angle will have different values for the driving and the driven gears.
- **Contact Ratio:** It is the angle of action divided by the pitch angle i.e.

$$\text{Contact ratio} = \frac{\delta}{\gamma} = \frac{\alpha + \beta}{\gamma}$$

  - o As the angle of action is the angle subtended by arc of contact and the pitch angle is the angle subtended by the circular pitch at the centre of the pitch circle.
  - o Thus, contact ratio is also the ratio of the arc of contact to the circular pitch. i.e.

$$\text{Contact ratio} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

  - o No. of pairs engaged in one engagement period. Minimum > 1, & Generally: 1.2 to 1.8

LAW OF GEARING

This law states that for the constant velocity ratio between the mating gears, line of action must always pass through the fixed point on the line joining the centres of rotation of gears. This fixed point is known as pitch point.

P = pitch point & Q =actual point of contact
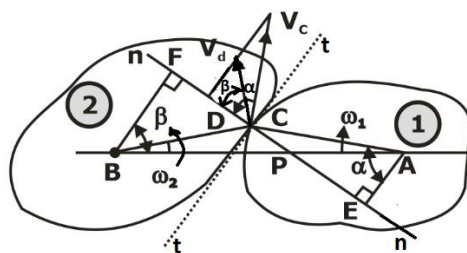
- Velocity of Sliding between mating gears (in case of external gearing),

$$V_{\text{sliding}} = |(\omega_1 + \omega_2)QP|$$

TOOTH PROFILE

- Two curves of any shape that fulfil the law of gearing can be used as the profiles of teeth. Such gear is said to have conjugate teeth.
- However, it will be very difficult to manufacture such gears and the cost will be high.

- Common forms of teeth that also satisfy the law of gearing are:
    1. Cycloidal profile teeth
    2. Involute profile teeth

INVOLUTE PROFILE

Involute is the Locus of a point on a line which rolls without slipping on the fixed circle.
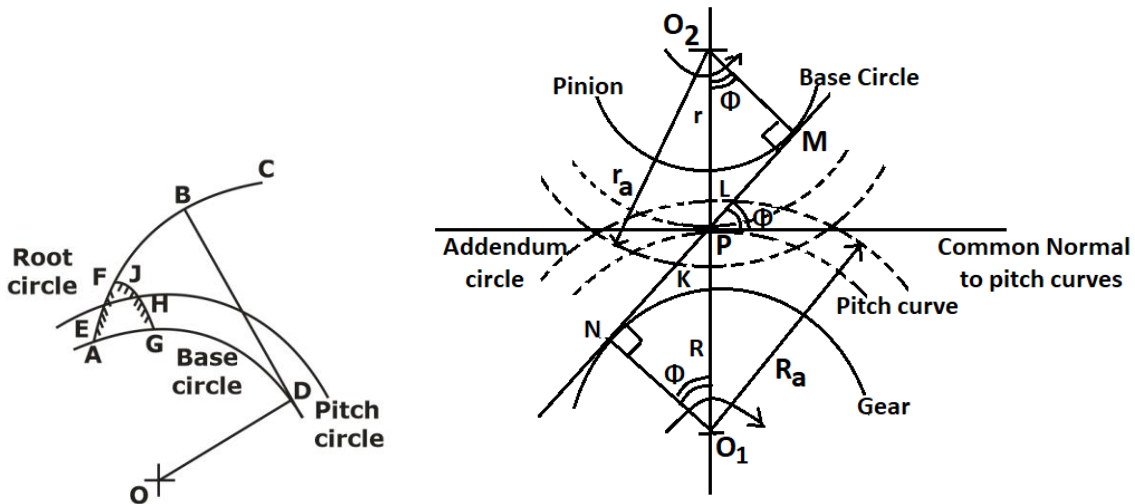
- Points of contact lies on line of action which is the common tangent to the two base circles.

- When the tip of a tooth of the driven wheel touches the flank of a tooth of the driving wheel, the contact is made and when the tip of the driving wheel touches the flank of the driven wheel, the contact is broken.

- If the direction of angular movement of the wheels is reversed, the points of contact will lie on the other common tangent to the base circles.

- Initial contact occurs where the addendum circle of the driven Wheel intersects the line of action. Final contact occurs at a point where the addendum circle of the driver intersects the line of action.

$$KP = \sqrt{R_A^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$PL = \sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi$$

- Path of contact (KL) = KP + PL
- When pinion is driver KP is path of approach and PL is path of recess.
- When gear is driver PL is path of approach and KP is path of recess.
- Arc of approach $= \frac{\text{Path of approach}}{\cos\emptyset}$

- Arc of recess $= \frac{\text{Path of recess}}{\cos\emptyset}$

- Arc of contact $= \frac{\text{Path of contact}}{\cos\emptyset}$

- Contact Ratio $= \frac{\text{Arc of contact}}{\text{Circular Pitch}}$

CYCLOIDAL PROFILE

- Cycloidal is the Locus of a point on the circumference of a circle which rolls without slipping on a fixed straight line.

- Face of cycloidal profile is epicycloid, and flank is hypocycloid.

- Pressure angle is not constant, maximum at start and end of engagement and zero at the pitch point.

- If centre distance between two pitch circles varies, point P is shifted, and speed of the driven gear would vary. Thus, to achieve uniform rotary motion centre distance should not vary.

- Since the cycloidal teeth are made up of two curves, it is very difficult to produce accurate profiles. Thus, this system has been obsolete.

INTERFERENCE

Critical points are M &N. Involute tip of pinion will remove some material from non-Involute flank portion of gear and some of material of pinion will also be removed which will disturb the profile of pinion also. This process of removal of material is called undercutting and whole phenomenon is called interference.

METHODS TO PREVENT INTERFERENCE

- Undercut gears

- Increasing the pressure angle [∅]

- By stubbing the teeth

- Increasing the number of teeth

$$t_{min} = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2 \varphi} - 1}$$ minimum no. of teeth required on pinion

$$T_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \varphi} - 1} = \text{minimum no. of teeth required on gear}$$

INTERCHANGEABLE GEARS

The gears are interchangeable if they are standard ones. It is always a matter of convenience to have gears of standard dimensions which can be replaced easily when they are worn out. The gears are said to be interchangeable if they have:

- The same module.
- The same pressure angles.
- The same addendums and dedendum.
- The same thickness.

TERMINOLOGY OF HELICAL GEARS

- **Helix Angle (φ):** It is the angle at which the teeth are inclined to the axis of a gear. It is also known as spiral angle.

- **Circular Pitch (P_C):** It is the distance between the corresponding points on adjacent teeth measured on the pitch circle. It is also known as transverse circular pitch.

- **Normal Circular Pitch (P_n):** Normal circular pitch or simply normal pitch is the shortest distance measured along the normal to the helix between corresponding points on the adjacent teeth.

GEAR TRAIN

- It's a mechanical system in which more than one gear engage with each other to transmit torque and thus motion transfer from one shaft to another.
- Gear trains are used for power transmission when input and output shafts are at some distance, then driving and driven shafts are connected by some intermediate gears which are known as Idler gears.

Why Gears trains?

- Gear trains are used to speed up or stepped down the speed of driven shaft so that required speed can be achieved for specific application.

Simple Gear Train

- Only one gear on one shaft. and it is used where there is a large distance to be covered between the input shaft and the output shaft.
- In it, all gear axes remain fixed relative to the frame and each gear is on a separate shaft.
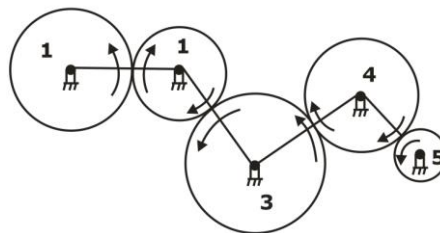
- All odd-number gears move in one direction & all even-number gears in opposite direction.

$$\text{Train value} = \frac{N_5}{N_1} = \frac{T_1}{T_5} = \frac{\text{Number of theeth on driving gear}}{\text{Number of teeth on driven gear}} = \frac{1}{\text{Speed Ratio}}$$

- Thus, it is seen that the intermediate gears have no effect on the ratio and, therefore, they are known as idlers.

Compound Gear Train

- When a series of gears are connected in such a way that two or more gears are mounted on the same shaft and rotate with the same angular velocity.
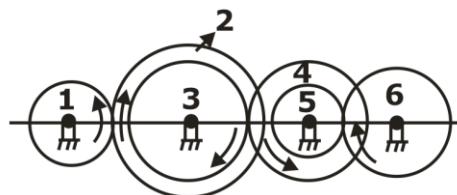
$$\text{Train value} = \frac{\text{Product of number of teeth on driving gears}}{\text{Product of number of teeth on driven gears}}$$

Reverted Gear Train

- It is a special case of compound gear train in which input and output shafts are coaxial.
- Such an arrangement is used in clocks and in simple lathes where back gear is used to give a slow speed to the chuck.
- Also, if r is the pitch radius of a gear. $r_1 + r_2 = r_3 + r_4$

$$\text{Velocity Ratio} = \frac{\text{Product of number of teeth on driven gears}}{\text{Product of number of teeth on driving gears}}$$
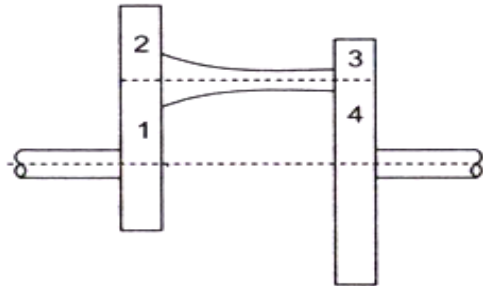
Planetary or Epicyclic Gear Train

- A gear train having a relative motion of axes is called a planetary or an epicyclic gear train.
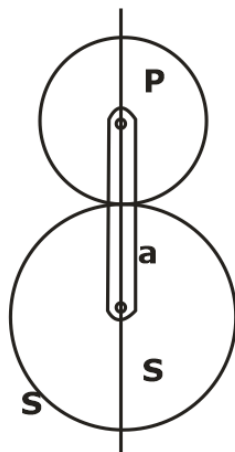
Operation	Revolution of elements (N)		
	Arm C	Gear A	Gear B
Fix the arm and give +1 revolution to gear A clockwise.	0	1	$\dfrac{N_B}{N_A} = \dfrac{-t_A}{t_B}$ $N_B = \dfrac{-t_A}{t_B} \times 1$
Now, multiplying by m (It means m rotation is given to gear A in clockwise.)	0	+m	$-m\dfrac{t_A}{t_B}$
Add n (It means + n rotations are given to all elements of gear train in clockwise)	n	n	n
Total motion	n	m+n	$-m\dfrac{t_A}{t_B} + n$

- On the basis of given condition, we can determine the values m and n.

TORQUES IN EPICYCLIC TRAINS

- Torques are transmitted from one element to another when a geared system transmits power. Assume that all the wheels of a gear train rotate at uniform speeds, i.e. accelerations are not involved. Also, each wheel is in equilibrium under the action of torques acting on it.
- Let N_s, N_a, N_p and N_A be the speeds and T_s, T_a, T_p, and T_A the torques transmitted by S, a, P and A respectively.

$T_s + T_a + T_p + T_A = 0$

$T_s N_s + T_a N_a + T_a N_a = 0$
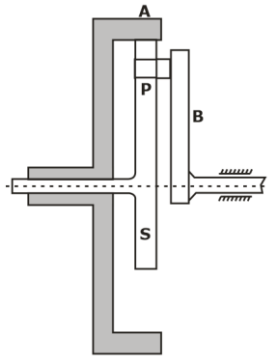
CHAPTER-4: KINEMATIC & DYNAMIC ANALYSIS

Kinematic analysis of single-slider crank mechanism

- Figure shows a slider-crank mechanism in which the crank OA rotates in the clockwise direction. ℓ and r are the lengths of the connecting rod and the crank respectively.

- In kinematic analysis, inertia of connecting rod is not considered in this analysis
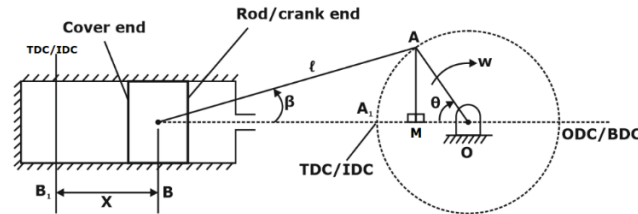
- Here

    m — mass of reciprocating parts

    r — crank radius

    ℓ — length of connecting rod

    $n = \dfrac{\ell}{r}$ = obliquity ratio (very large)

    ω — angular velocity of crank

    θ — angle turned by crank from IDC/TDC

    β → corresponding angle turned of connecting rod

- $\omega = \dfrac{d\theta}{dt}$

- $\sin\beta = \dfrac{1}{n}\sin\theta$

- $\cos\beta = \sqrt{1 - \sin^2\beta} = \sqrt{1 - \dfrac{\sin^2\theta}{n^2}}$

- **Displacement Analysis (x) of piston**

    x = displacement of piston from inner-dead centre at the moment when the crank has turned through angle θ from the inner-dead centre

    $x = r\left[(1 - \cos\theta) + \left(n - \sqrt{n^2 - \sin^2\theta}\right)\right]$

- **Velocity Analysis (V) of piston**

    $V = \dfrac{dx}{dt} = \left(\dfrac{dx}{d\theta}\right)\left(\dfrac{d\theta}{dt}\right) \qquad \Rightarrow V = \left(\dfrac{dx}{d\theta}\right)\omega$

    $\Rightarrow V = r\omega\left[\sin\theta + \dfrac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$

    If connecting rod is very large as compared to the crank, n^2 will be large compared to $\sin^2\theta$

    $\Rightarrow v = r\omega\left[\sin\theta + \dfrac{\sin 2\theta}{2n}\right]$

- **Acceleration Analysis (f) of piston**

$$f = \frac{dv}{dt} = \frac{dv\,d\theta}{d\theta\,dt} = \frac{d}{d\theta}\left[r\omega\left(\sin\theta + \frac{\sin2\theta}{2n}\right)\right]\omega = r\omega\left(\cos\theta + \frac{2\cos2\theta}{2n}\right)\omega$$

$$\Rightarrow f = r\omega^2\left(\cos\theta + \frac{\cos2\theta}{n}\right) \quad \leftarrow \text{Acceleration of Reciprocating mass}$$

When $\theta = 0^\circ$, i.e., at IDC, $f = r\omega^2\left(1 + \frac{1}{n}\right)$

When $\theta = 180^\circ$, i.e., at ODC, $f = r\omega^2\left(1 - \frac{1}{n}\right)$

- **Angular velocity of connecting rod**

We know that angular velocity of connecting rod is given by $\omega_{CR} = \left(\frac{d\beta}{dt}\right)$

$$\omega_{CR} = \frac{\omega\cos\theta}{n\dfrac{\sqrt{n^2 - \sin^2\theta}}{n}}$$

$$\omega_{CR} = \frac{\omega\cos\theta}{n} \quad \text{(n is large)}$$

- **Angular Acceleration of connecting rod**

Let $\propto_{CR}$ = angular acceleration of the connecting Rod

$$\alpha_{CR} = \left(\frac{d\omega_{CR}}{dt}\right) = \left(\frac{d\omega_{CR}}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$$

$$\propto_{CR} = \omega^2\sin\theta\left[\frac{\cos^2\theta - (n^2 - \sin^2\theta)}{(n^2 - \sin^2\theta)^{3/2}}\right]$$

$$\alpha_{CR} = -\left(\frac{\omega^2}{n}\right)\sin\theta \quad \text{(n is large)}$$

The negative sign indicates that the sense of angular acceleration of the rod is such that it tends to reduce the angle β. Thus, in the given case, the angular acceleration of the connecting rod is clockwise.

DYNAMIC-ANALYSIS OF SINGLE-SLIDER CRANK MECHANISM

- The piston effort is termed as the net or effective force applied on the piston.
- In reciprocating engines, the reciprocating masses accelerate during the first half of the stroke and inertia force tends to resist the same. Thus, net force on the piston is decreased.
- During the latter half of the stroke, the reciprocating masses decelerate and the inertia force opposes this deceleration or acts in the direction of the applied gas pressure and thus, increases the effective force on the piston.
- In a vertical engine, the weight of the reciprocating masses assists the piston during the out-stroke (down stroke), thus, increasing the piston efforts by an amount equal to the weight of the piston. During the in-stroke (up stroke), the piston effort is decreased by the same amount.
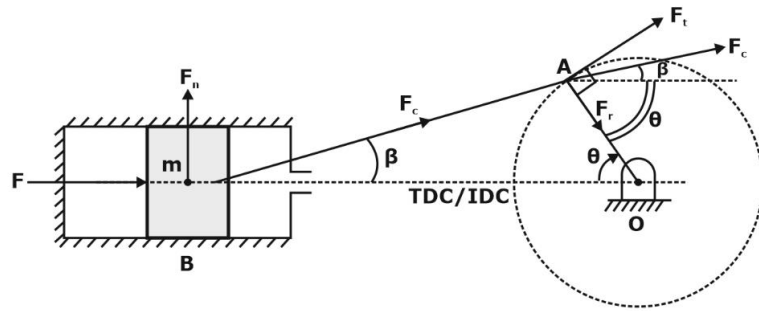
- We assume direction of effective force from cover end to crank end.

- Here

  A_1, A_2 → Cross-sectional area of piston at cover end and rod ends.

  D → Piston dia.

  d → Piston Rod dia.

  $$A_1 = \frac{\pi}{4}D^2 = \text{Area of Piston}$$
  $$A_2 = \frac{\pi}{4}\left(D^2 - d^2\right) = \text{Net effective area of Piston}$$

  P_1, P_2 → Pressure of combustion gases at cover end and crank rod end side

  m = mass of the reciprocating parts

- Force on the piston due to gas pressure, ∴ $F_{gas} = P_1A_1 - P_2A_2$

- Inertia force, F_i or $f_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

- Which is in the opposite direction to that of the acceleration of the piston.

- Kinetic friction $= \pm F \begin{bmatrix} \text{we have to decide sign} \\ \text{Acc. to position of crank, } \theta \end{bmatrix}$

- Hence, Total Piston Effort (F)

  $$F = \left(F_{gas} - F_i \pm f\right) \pm \boxed{mg}, \quad (\text{mg is used for vertical engines})$$
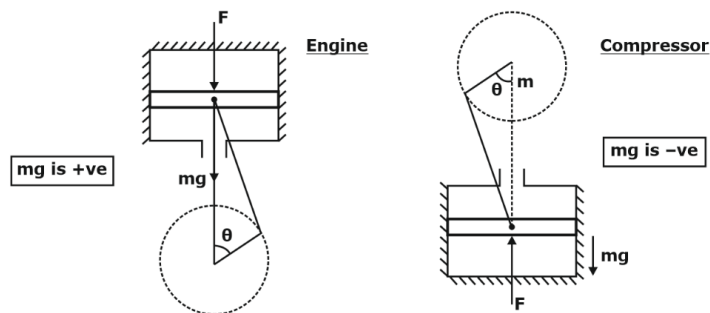
- **Force/thrust along connecting Rod (F_c)**

  $$F_c \cos\beta = F \qquad \Rightarrow F_c = \frac{F}{\cos\beta}$$

- **Normal thrust to cylinder walls (F_n)-** It is the normal reaction on the cylinder walls.

  $F_n = F_c \sin\beta$ $\Rightarrow F_n = F\tan\beta$

- **Radial thrust on crank shaft Bearings (F_r)-** The component of FC along the crank (in the radial direction) produces a thrust on the crankshaft bearings.

  $F_r = F_c \cos(\theta + \beta)$ $\Rightarrow F_r = \dfrac{F}{\cos\beta}.\cos(\theta + \beta)$

- **Crank Effort (F_t):** Force is exerted on the crankpin as a result of the force on the piston. Crank effort is the net effort (force) applied at the crankpin perpendicular to the crank which gives the required turning moment on the crankshaft. It is the force needed to drive crank.

  $F_t = F_c \sin(\theta + \beta)$ $\Rightarrow F_t = \dfrac{F}{\cos\beta}.\sin(\theta + \beta)$

- **Turning moment on crank-Shaft:** $T = F_t.r = \dfrac{F}{\cos\beta}.\sin(\theta + \beta).r$

  $\Rightarrow \boxed{T = f(\theta)} \leftarrow$ Turning moment Diagram

  And, $\theta = f$ (time) $\Rightarrow T = f$ (time)

  we know, for anybody, I = constant $\Rightarrow a = F$(time)

  Hence, the acceleration on crank shaft will continuously vary with time, which will produce jerk and to control these fluctuations, **flywheel** should have to be used. (Without flywheel, Engine can't be started due to jerks.)

- The net torque or turning moment on the crankshaft will be the algebraic sum of-
    i.   Turning moment due to the force of gas pressure.
    ii.   Inertia torque due to the inertia force at the piston as a result of inertia of the reciprocating mass including the mass of the portion of the connecting rod.
    iii. Inertia torque due to the weight (force) of the mass at the crank pin which is the portion of the mass of the connecting rod taken at the crank pin.
    iv.   Inertia torque due to the correction couple.
    v.   Turning moment due to the weight (force) of the piston in case of vertical engines.

CHAPTER-5: FLYWHEEL

INTRODUCTION

- A flywheel acts as an energy reservoir, which stores energy during the period when the supply of energy is more than the requirement and releases energy during the period when the requirement is more than the supply.

- In case a variable torque is supplied where demand is a constant torque or demand is variable torque whereas constant torque is supplied. In both these cases there is mismatch between the supply and demand. This results in **speed variation**.

- To smoothen these variations in torque, flywheel is used which works as an energy storage.

- During one revolution of the crankshaft of a steam engine or IC engine, the torque on it varies and is given by

$$T = F_t \times r = Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$ (Refer chapter Kinematic & Dynamic Analysis)

where F is the net piston effort.

- A plot of T vs. θ is known as the turning-moment diagram. The inertia effect of the connecting rod is, usually ignored while drawing these diagrams, but can be taken into account if desired.
- As T=F_t × r, a plot of F_t vs. θ known as crank effort diagram is identical to a turning-moment diagram.

Turning moment diagram for a single cylinder double acting steam engine
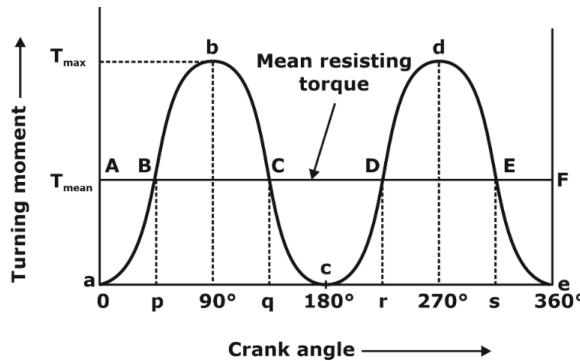
- The mean resisting torque against which the engine works is given by $T_{mean} = \dfrac{W_{cycle}}{2\pi}$

- If T = Torque on the crankshaft at any instant, and T_{mean} = Mean resisting torque. Then accelerating torque on the rotating parts of the engine = T – T_{mean}

- If (T – T_{mean}) is positive, there will be excess energy. This energy will be stored in the flywheel in form of kinetic energy and the flywheel accelerates.

- If (T – T_{mean}) is negative, there will be shortage of energy. This energy will be provided by the flywheel to the engine in form of kinetic energy and hence the flywheel retards.

Turning moment diagram for a four-stroke internal combustion engine
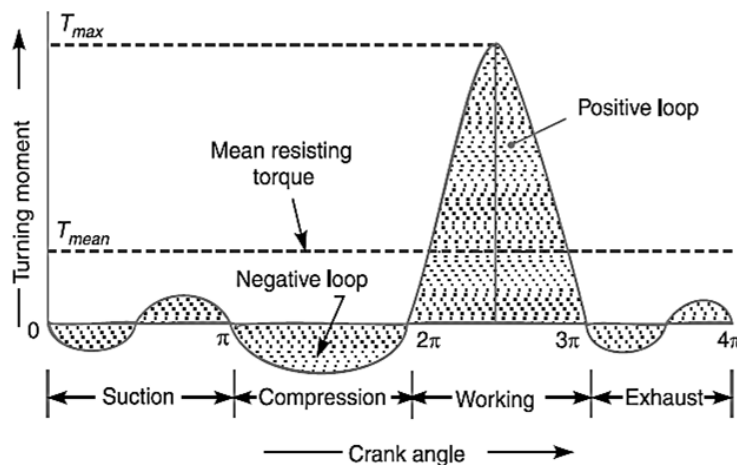
- The mean resisting torque against which the engine works is given by $T_{mean} = \dfrac{W_{cycle}}{4\pi}$

Turning moment diagram for multi-cylinder engines
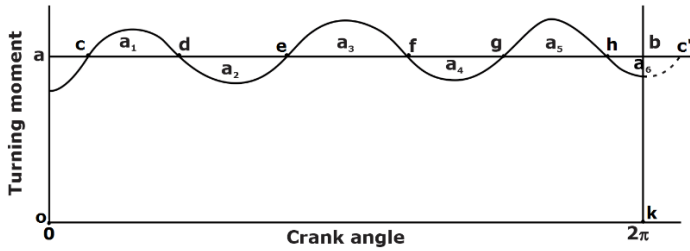
Crank Position	Flywheel energy
c	E
d	$E + a_1$
e	$E + a_1 - a_2$
f	$E + a_1 - a_2 + a_3$
g	$E + a_1 - a_2 + a_3 - a_4$
h	$E + a_1 - a_2 + a_3 - a_4 + a_5$
c'	$E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

- The greatest of these energy is the maximum kinetic energy of the flywheel E_{max} and for the corresponding crank position, the speed is maximum N_{max}.
- The least of these energy is the least kinetic energy of the flywheel E_{min} and for the corresponding crank position, the speed is minimum N_{min}.

FLUCTUATION OF ENERGY

- The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. The variations of energy above and below the mean resisting torque line are called fluctuations of energy.
- Even after installing the flywheel, some fluctuations (2-3%) are still present in the engine.
- The difference between the greatest speed and the least speed is known as the maximum fluctuation of speed and the ratio of the maximum fluctuation of speed to mean speed is define as the coefficient of fluctuation of speed.

  Maximum fluctuation of speed = $N_{max} - N_{min}$

  Coefficient of fluctuation of Speed, $C_S = \dfrac{N_{max} - N_{min}}{N_{mean}}$ (N_{mean} = mean speed of engine)

  $$N_{mean} = \dfrac{N_{max} + N_{min}}{2}$$

- The difference between the maximum and minimum kinetic engines of the flywheel is known as the maximum fluctuation of energy whereas the ratio of this maximum fluctuation of energy to the work done per cycle is defined as the coefficient of fluctuation of energy.

  Maximum Fluctuations in energy (ΔE) = $E_{max} - E_{min}$

  $$\Delta E = \frac{1}{2}I\omega_{max}^2 - \frac{1}{2}I\omega_{min}^2 \qquad (I = \text{moment of inertia of flywheel})$$

  $$\Delta E = \frac{1}{2}I(\omega_{max} + \omega_{min})(\omega_{max} - \omega_{min})$$

  $$\Delta E = I(\omega_{max} - \omega_{min})\frac{\omega \times \omega}{\omega} \qquad (\text{writing } \omega_{mean} \text{ as } \omega)$$

$\Delta E = I\omega^2 C_s$

Coefficient of fluctuation of energy, $C_E = \dfrac{E_{max} - E_{min}}{W_{cycle}}$

MOMENT OF INERTIA OF FLYWHEEL

- The inertia of a flywheel is provided by the hub, spokes and the rim. However, as the inertia due to the hub and the spokes is very small, usually it is ignored. In case it is known, it can be taken into account.

- If   m= mass of the flywheel,

    R = radius of flywheel

    Then, for disc type flywheel, I = $\dfrac{mR^2}{2}$

    For Ring type flywheel, I = mR^2

REQUIREMENT OF FLYWHEEL IN POWER PRESSES
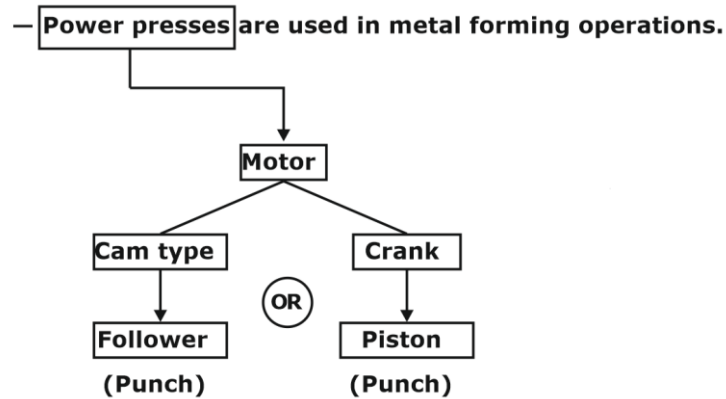
- Cycle time is the total time required for producing a hole using punching press. But actually, only for small part of total time punching is being done.

- Motor provides energy at uniform rate such that total energy required for punching is given in one revolution. Hence excess energy during actual punching is supplied by flywheel.

DESIGNING OF FLYWHEEL RIM

- **Rim type Flywheel** (practical not usable)

  Let, A → cross section area of the rim
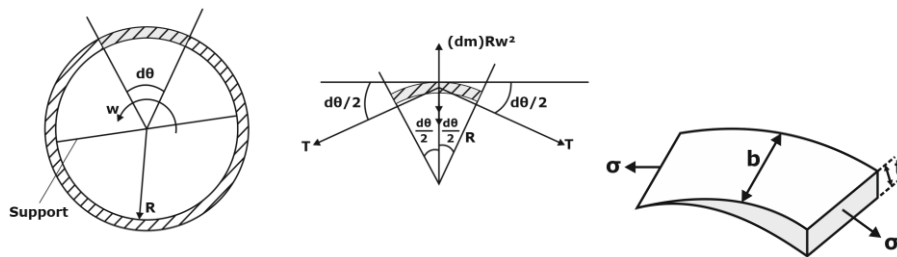
  Equating forces along vertical direction for a very small element

  $$2T\sin\left(\frac{d\theta}{2}\right) = (dm)R\omega^2 \qquad \text{(Taking angle very small } \sin\theta \sim \theta)$$

  $$\left(\frac{T}{A}\right) = \sigma = \text{hoop spess developed in Rim}$$

  $$\therefore V = R\omega = \sqrt{\frac{\sigma}{\rho}}$$

  For safe designing, $\qquad \sigma \leq \sigma_b$ (Bearing strength)

  $$\therefore \boxed{V_{max} = (R\omega)_{max} = \sqrt{\frac{\sigma_b}{\rho}}}$$

  These flywheels can be used for **small speed** only [∵ σ ∝ ω]

- **Flywheel with spokes**

  For medium speeds flywheel with spokes are used.
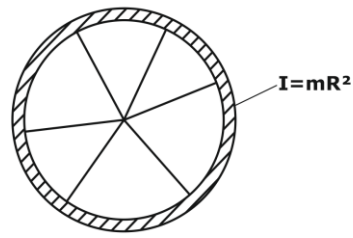
$I = mR^2$

- **Disc shaped flywheels**

  For high speeds Disc shaped flywheels are used.

  MOI of solid disc, $I = mR^2/2$
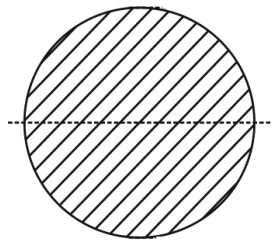
- **Flywheel rim with non-uniform thickness:**

  This is the best flywheel because, it has highest energy storage capacity.

  ∵ Energy stored ∝ speed of rotation

Thicker at periphery and thinner at centre to increase (MOI)

$$\frac{1}{2}mR^2 < I < mR^2$$
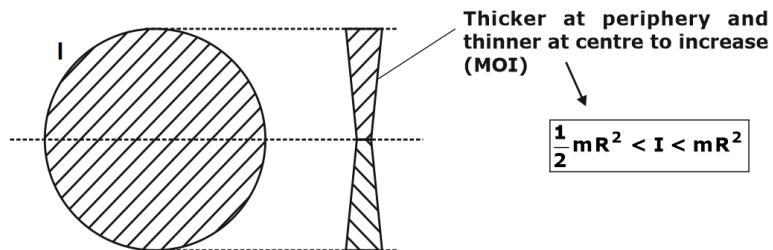
CHAPTER-6: GOVERNORS

INTRODUCTION
- The function of a governor is to maintain the speed of an engine within specified limits.
- If the load on the shaft increases, the speed of the engine decreases unless the supply of fuel is increased by opening the throttle valve.
- On the other hand, if the load on the shaft decreases, the speed of the engine increases unless the fuel supply is decreased by closing the valve sufficiently to slow the engine to its original speed.
- The throttle valve is operated by the governor through a mechanism for the purpose

DIFFERENCE BETWEEN FLYWHEEL AND GOVERNOR
- In general, the speed of an engine varies in two ways—during each revolution (cyclic variation) and over a number of revolutions.
- In the former case, it is due to variation in the output torque of the engine during a cycle and can be regulated by mounting a suitable flywheel on the shaft.
- In the latter case, it is due to variation of load upon the engine and requires governor to maintain the speed.
- The operation of a flywheel is continuous whereas that of a governor is more or less intermittent.
- A flywheel may not be used if there is no undesirable cyclic fluctuation of the energy output, but a governor is essential for all types of engines as it adjusts the supply according to the demand.

TYPES OF GOVERNORS- Governors can broadly be classified into two types.
- **Centrifugal governor**
    - o This is the more common type. Its action depends on the change of speed.
    - o It has a pair of masses, known as governor balls, which rotate with a spindle.
    - o The spindle is driven by an engine through bevel gears (Fig.1).
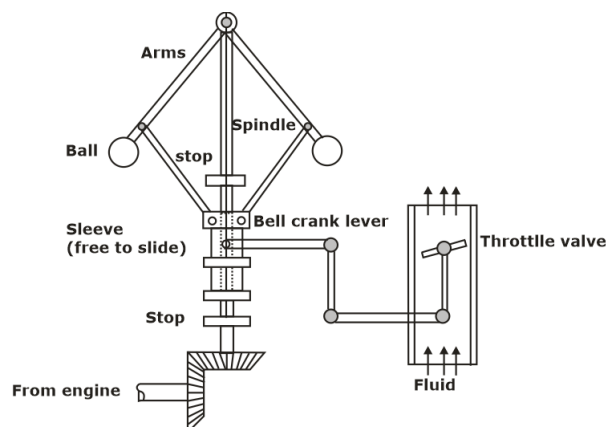
Further classification of Centrifugal Governor is as follows
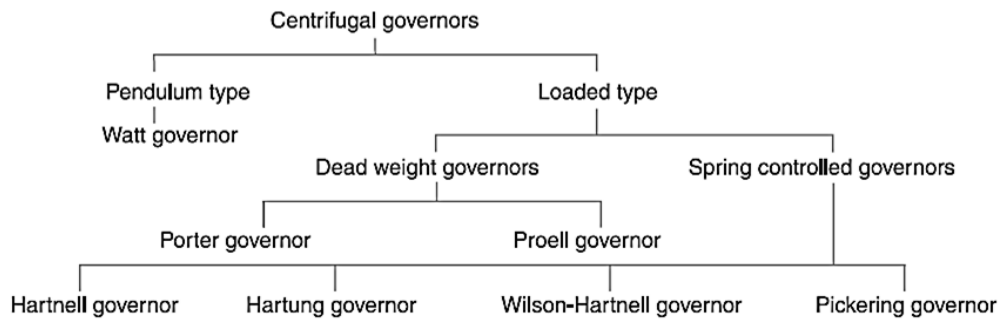
- **Inertia Governor**
  - In this type, positions of balls are affected by forces set up by an angular acceleration or deceleration of the given spindle in addition to centrifugal forces on the balls.
  - Using suitable linkages and springs, the change in position of the balls is made to open or close the throttle valve.

Centrifugal governor vs Inertia Governor

- The balls are operated by the actual change of engine speed in the case of centrifugal governors, it is by the rate of change of speed in case of inertia governors.
- Response of inertia governors is faster than that of centrifugal types.

GOVERNORS

- The function of a governor is to regulate the mean speed of an engine, when there are variations in the load.
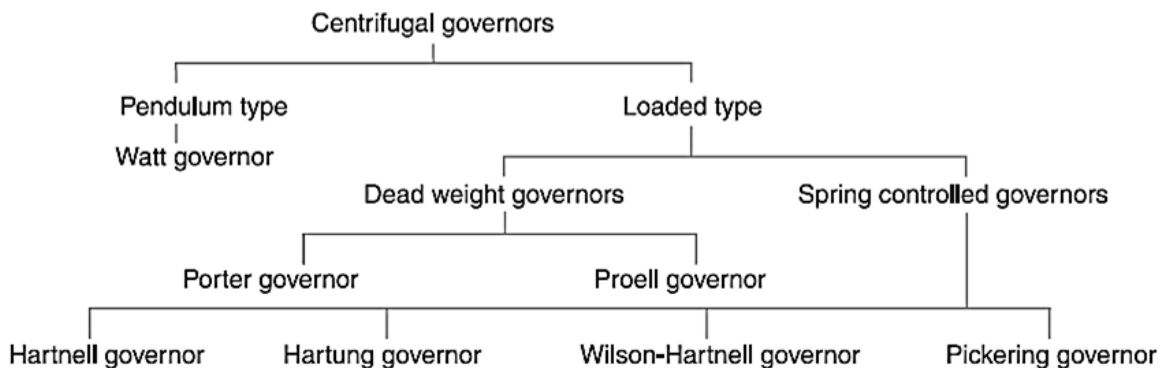
- Height of a governor is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis.

TERMS USED IN GOVERNORS

- **Height of a governor-** It is the vertical distance from centre of ball to a point where axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by "h".
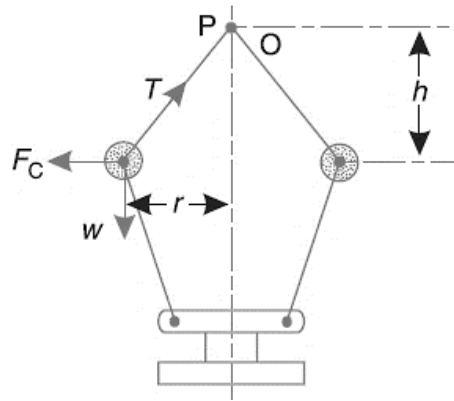
- **Equilibrium speed-** It is the speed at which governor balls, arms, dead weight, spring force etc. are in complete equilibrium and sleeve does not tend to move upwards or downwards.
- **Mean equilibrium speed-** It is the speed at the mean position of the balls or the sleeve.
- **Maximum and minimum equilibrium speeds-** The speeds at the maximum and minimum radius of rotation of the balls without tending to move either way, are known as maximum and minimum equilibrium speeds respectively.
- **Sleeve lift-** It is vertical distance which sleeve travels due to change in equilibrium speed.

Watt Governor

$$h = \frac{g}{\omega^2} = \frac{895}{N^2}$$ (N is in rpm)
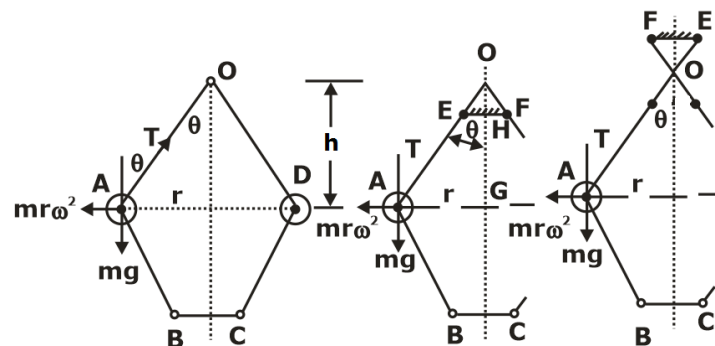
Porter Governor

The Porter governor is a modification of the Watt's governor, with central dead load attached to the sleeve.

$$N^2 = \frac{895}{h}\left(\frac{2mg + (Mg \pm F)(1+K)}{2mg}\right), \text{ here } k = \frac{\tan\beta}{\tan\theta}$$

Case 1 – When $\beta = \theta \rightarrow k = 1$

$$N^2 = \frac{895}{h}\left(\frac{mg + (Mg \pm f)}{mg}\right)$$

Case 2 – f = 0

$$N^2 = \frac{895}{h}\left(\frac{2m + M(1+k)}{2m}\right)$$

Case 3 – When β = θ & f = 0

$$N^2 = \frac{895}{h}\left(\frac{m+M}{m}\right)$$

(a) **(b)**

$$\frac{Mg \pm f}{2}$$
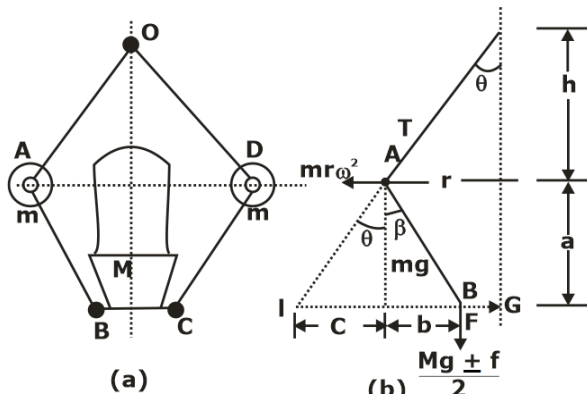
Proell Governor

$$N^2 = \frac{895}{h}\frac{a}{e}\left(\frac{2mg+(Mg\pm f)(1+k)}{2mg}\right) \text{, here } k = \frac{\tan\beta}{\tan\theta}$$

- **Special cases:**

  If k = 1,
  $$N_2 = \frac{895}{h}\frac{a}{e}\left(\frac{mg+(Mg\pm f)}{mg}\right)$$

  If f = 0,
  $$N_2 = \frac{895}{h}\frac{a}{e}\left(\frac{2m+M(1+k)}{2m}\right)$$

  If k = 1, f = 0
  $$N_2 = \frac{895}{h}\frac{a}{e}\left(\frac{m+M}{m}\right)$$

$$(Mg + f)/2$$
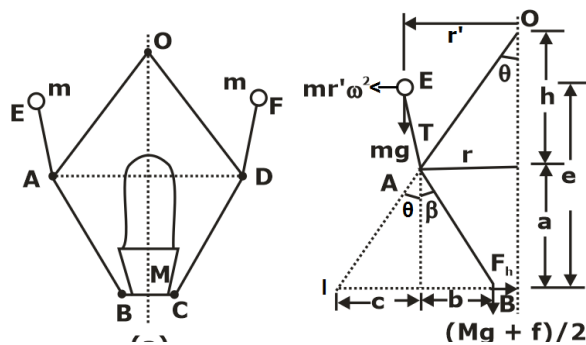
Hartnell Governor

$$F_{s2} - F_{s1} = \frac{2a}{b}(F_2 - F_1) \text{, here } \frac{h}{b} = \frac{r_2 - r_1}{a}$$

(a)
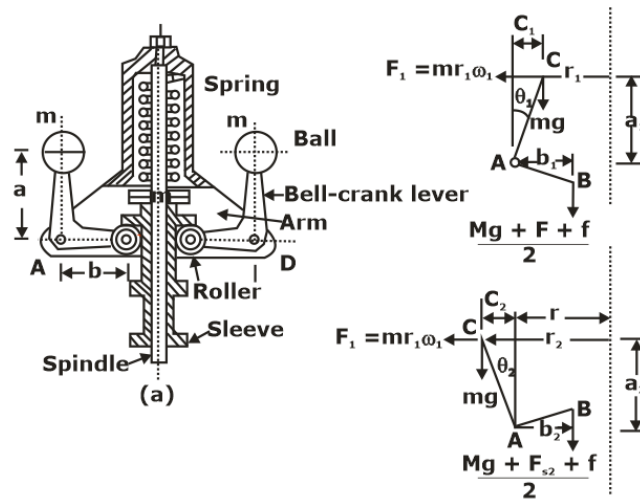
PICKERING GOVERNOR

- A Pickering governor consists of three leaf springs which are arranged at equal angular intervals around the governor spindle.

- The upper end of each spring is fixed by a screw to a hexagonal nut attached to the spindle. The lower end is fastened to the sleeve which can move up and down the governor spindle.

- Each spring has a fly mass m attached at its centre. As the spindle rotates, a centrifugal force is exerted on the leaf spring at the centre which causes it to deflect. This deflection makes the sleeve move up. A stop is also provided to limit the movement of the sleeve.

- Pickering governor is used in **gramophones** to adjust the speed of the turn table.

GOVERNOR TERMINOLOGY

- **Stability-** A governor is said to be stable if it brings the speed of the engine to the required value and there is not much hunting.

- **Hunting-** Hunting is the violent up and down movement of sleeve due to high sensitivity.

- **Sensitiveness of a Governor-** A governor is said to be sensitive when there is larger displacement of the sleeve due to a fractional change in speed.

$$\text{Sensitiveness} = \frac{N_1 - N_2}{N_{mean}} = \frac{2(N_2 - N_1)}{(N_2 + N_1)}$$

- **Isochronous Governor-** A governor is said to be *isochronous* when the equilibrium speed is constant (*i.e.* range of speed is zero) for all radii of rotation of the balls.

- **Coefficient of insensitiveness due to friction:**

for watt governor
$$(C_f)_{Porter} = \frac{f}{mg}$$

for porter governor
$$(C_f)_{Porter} = \frac{f}{(m+M)g}$$

for Hartnell governor
$$(C_f)_{HG} = \frac{f}{F_s}$$

- **Power of a Governor**

$$\text{Power} = \frac{E}{2} \times \text{sleeve displacement}$$

$$\text{Power} = \left[m + \frac{M}{2}(1+k) \right] gh \left(\frac{4c^2}{1+2c} \right)$$

- **Effort of a Governor-** It is the amount of average force acting on the sleeve to raise or lower it for a given change of speed.

- **Controlling force**
  - Controlling force is equal and opposite to the centrifugal force and act readily inward.
  - A graph showing controlling force with radius of rotation is called controlling curve.
  - A governor is said to be stable when as speed increases radius also increases.
  - A governor is said to be unstable when radius decreases as speed increases

Stable governor   **Unstable governor**
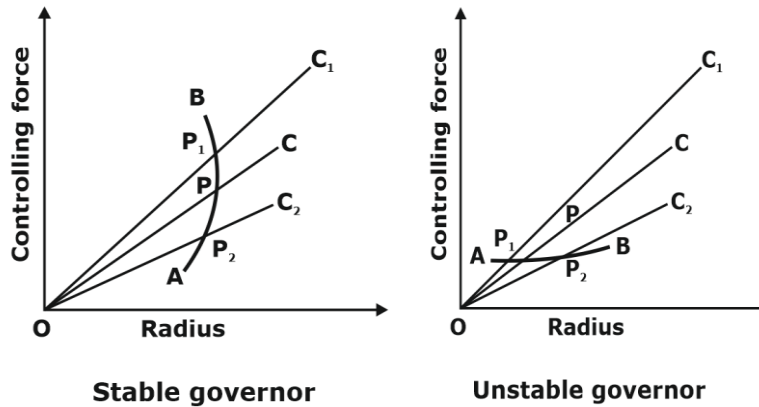
- **Spring-controlled governor Controlling force diagram**
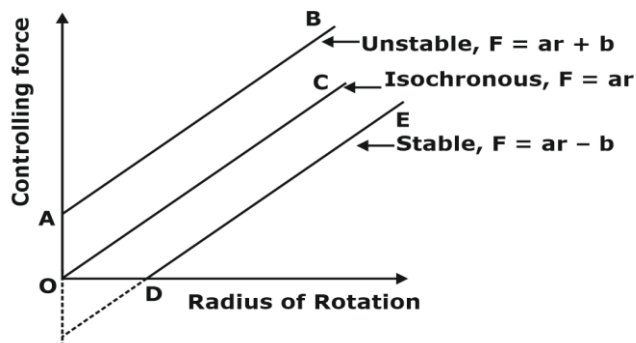
CHAPTER-7: BALANCING

INTRODUCTION

- Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level.

(a)   (b)
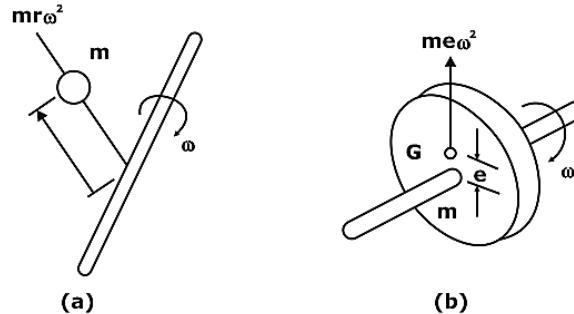
BALANCING OF ROTATING MASSES

- In any rotating system, having one or more rotating masses, if the centre of mass of the system does not lie on the axis of rotation the system is unbalanced.

- When an unbalanced mass rotating about an axis it experiences a centrifugal force in a radially outward direction. This force is called as disturbing force of the system.

- The magnitude of disturbing force is $F_c = mr\omega^2$

    where, F_c = centrifugal force or disturbing force, N

               m = mass of the rotating body, kg.

               r = distance of centre of mass (C.G) from the axis of rotation, m

               ω = angular speed of rotation, rad/s.

Balancing of Single Rotating Mass
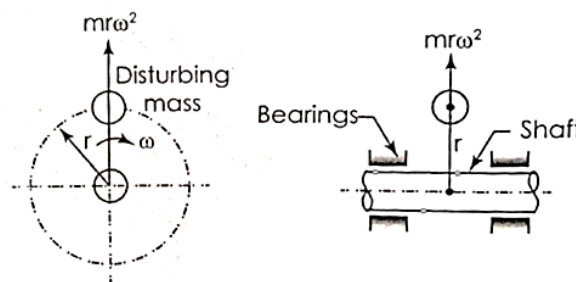
- During the rotation of shaft, a dynamic force (centrifugal force) equal to $mr\omega^2$ acts in a radially outward direction as shown in the above figure.

- This unbalanced force results in increase in load on the bearings, increased bending moment on the shaft and vibrations of the system.

    ❖ **By single mass rotating in same plane (Internal Balancing)**   ➔ $mr = m_b r_b$
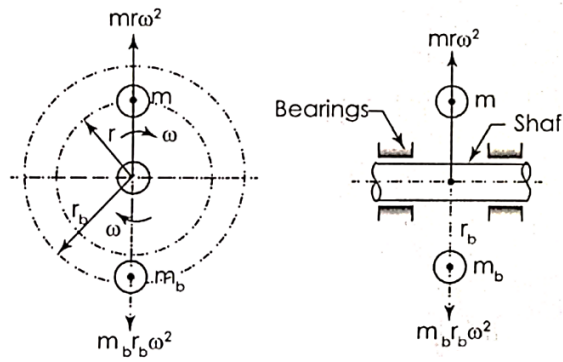
❖ **Balancing by two masses rotating in different planes (External Balancing)**

Balancing masses are placed in two different planes on opposite side of the plane of rotation of disturbing mass:
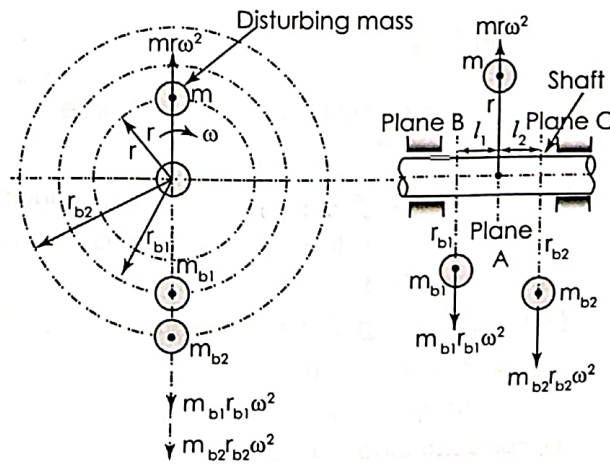
$$mr = m_{b1}r_{b1} + m_{b2}r_{b2}$$

Taking moments about plane A $\qquad m_{b1}r_{b1}l_1 = m_{b2}r_{b2}l_2$

Several masses rotating in same plane
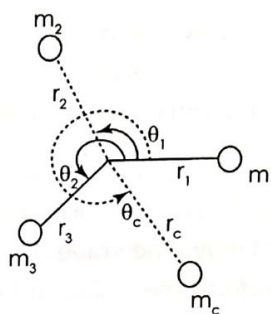
- **Static balancing**

The resultant dynamic force acting on the shaft must be equal to zero. For this, the line of action of three dynamic forces must be the same. This is the condition for **static balancing.**

$$\Rightarrow m_c r_c = \sqrt{\left(\Sigma mr\cos\theta\right)^2 + \left(\Sigma mr\sin\theta\right)^2}$$

$$\tan\theta_c = \frac{-\Sigma mr\sin\theta}{-\Sigma mr\cos\theta}$$

- **Dynamic Balancing**

The resultant couple due to dynamic forces acting on the shaft must be equal to zero. In other words, the algebraic sum of the moments due to dynamic forces about any point in the plane must be zero. This is the condition for **dynamic balancing.**

$$m_{c2}r_{c2}l_{c2}=\sqrt{(\sum mrl \cos \theta)^2 + (\sum mrl \sin \theta)^2}$$

$$\tan\theta_{c2}=\frac{-\sum mrl \sin \theta}{-\sum mrl \cos \theta}$$

RECIPROCATING MASS BALANCING

- Inertia force produced due to this acceleration $F_i = mr\omega^2 \cos \theta + mr\omega^2 \dfrac{\cos 2\theta}{n}$, where $\left(n = \dfrac{\ell}{r}\right)$

- $F_P = mr\omega^2\cos\theta$ is primary unbalanced force acting along line of a stroke of the cylinder. It is due to SHM of parts

- $F_s = \dfrac{mr\omega^2 \cos 2\theta}{n}$ is secondary unbalanced force. It is due to obliquity of arrangement.

  ❖ **Complete balancing**

  Mount a counter balancing mass 'm' on the diagonally opposite to the crank, with crank radius.
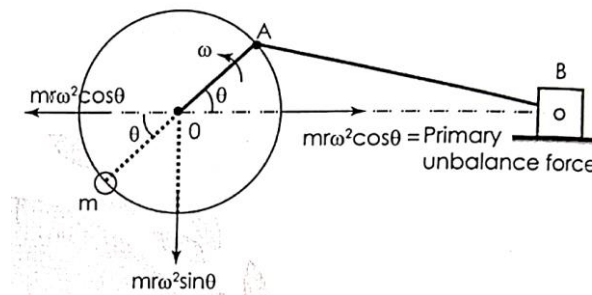
  But there is unbalanced Vertical force which is large. $mr\omega^2 \sin \theta = 0$ at $\theta = 0°$ or 180°. Maximum ($mr\omega^2$) at $\theta = 90°$ or 270°. The piston experiences up and down jumps

  ❖ **Partial balancing**

  Primary force balanced = $cmr\omega^2 \cos\theta$

  Unbalanced horizontal forcer = $(1 – C) mr\omega^2\cos\theta$

  Unbalanced vertical force = $cmr\omega^2 \sin\theta$

  Resultant unbalanced force $R = \sqrt{\left[(1 - c)mr\omega^2 \cos \theta\right]^2 + \left[cmr\omega^2 \sin \theta\right]^2}$

BALANCING OF LOCOMOTIVES

- **Effect of partial primary balancing of locomotives**
  o Unbalanced force along the line of stroke F_H.
  o Unbalanced force perpendicular to the line of stoke F_v.

  ❖ **Variation of tractive force (F_T)**

  Crank for the first cylinder and second cylinder are inclined at an angle θ° and (90°+ θ°) respectively with the line of stroke.

$F_{HU1} = (1 - c) m\omega^2 r\cos\theta$

$F_{HU2} = - (1 - c) m\omega^2 r\sin\theta$

$F_T = F_{HU1} + F_{HU2}$

$F_T = (1 - c) m\omega^2 r [\cos\theta - \sin\theta]$

$F_T = \pm\sqrt{2}(1 - c)m\omega^2 r$ At $\theta = 135°$ minimum   At $\theta = 315°$ maximum

❖ **Swaying Couple (C_s)**

The two unbalanced force acting along the line of stroke of two cylinders constitute a couple in the horizontal direction between the center plane of two cylinders known as swaying couple.

$$C_S = (1 - c)m\omega^2 r(\cos\theta + \sin\theta) \times \frac{\ell}{2}$$

$C_s = \pm\dfrac{\ell}{\sqrt{2}}(1 - c)m\omega^2 r$, Minimum value of swaying couple is given by when $\theta = 225°$. Maximum value of swaying couple is given by when $\theta = 45°$.

❖ **Hammer Blow**

The unbalanced force perpendicular to line of stroke produces the variation of pressure on the rails which causes hammering action on the rails which is called as hammer blow.

Net pressure between the wheel and the rail is $= W \pm m_b\omega^2 r_b$

- **Secondary balancing**

  If the secondary force is not neglected, secondary force. $F_s = mr\omega^2\dfrac{\cos 2\theta}{n} = mr(2\omega)^2\dfrac{\cos 2\theta}{4n}$

Complete balancing

1. Primary force balancing (closed force polygon)
2. Primary couple balancing (closed couple polygon)
3. Secondary force balancing
4. Secondary couple balancing

Dynamically equivalent link

- $m_B l_B = m_C l_C$
- $m_B + m_C = m_2$
- $m_B l_B^2 + m_C l_C^2 = I_{G2}$

Balancing of radial engines

- A radial engine is a multi-cylinder engine in which all the connecting rods are connected to a common crank.
- The analysis of forces in such type of engines is much simplified by using the method of *direct* and *reverse* cranks. As all the forces are in the same plane, no unbalance couples exist.
- The crank rotating in the direction of engine rotation is known as the **direct crank** and the imaginary crank rotating in the opposite direction is known as the **reverse crank**

OA — Primary direct crank   **OC — Secondary direct crank**
OA'— Primary reverse crank   **OC'— Secondary reverse crank**
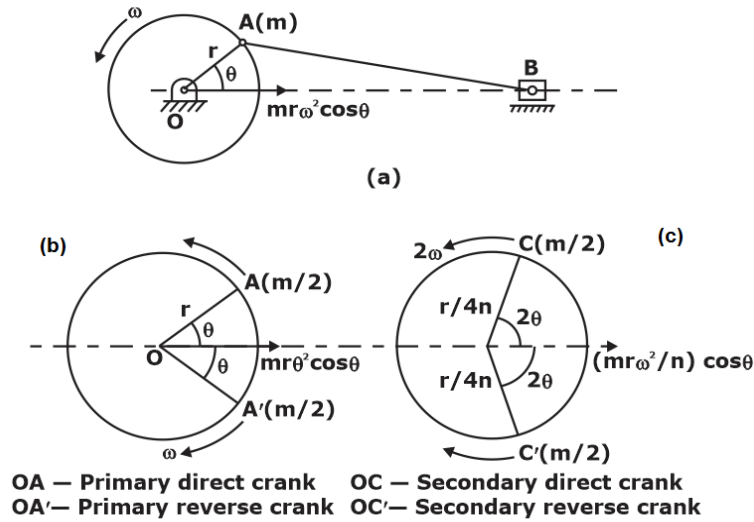
- **Primary accelerating force:**

Primary force = $mr\omega^2\cos\theta$ (along line of stroke)

In the method of direct and reverse cranks, a force identical to this force is generated by two masses in the following way:

   i. A mass m/2, placed at the crank pin A and rotating at an angular velocity ω in the given direction.

   ii. A mass m/2, placed at the crank pin of an imaginary crank OA' at the same angular position as the real crank but in the opposite direction of the line of stroke. This imaginary crank is assumed to rotate at the same angular velocity ω in the opposite direction to that of the real crank.

- **Secondary accelerating force**

Secondary Force $= mr\omega^2 \dfrac{\cos 2\theta}{n} = mr(2\omega)^2 \dfrac{\cos 2\theta}{4n} = m\dfrac{r}{4n}(2\omega)^2 \cos 2\theta$ (along line of stroke)

This force can also be generated by two masses in a similar way as follows.

   i. A mass m/2, placed at the end of direct secondary crank of length $\dfrac{r}{4n}$ at angle 2θ and rotating at an angular velocity 2ω in the given direction.

   ii. A mass m/2, placed at the end of reverse secondary crank of length $\dfrac{r}{4n}$ at angle −2θ rotating at an angular velocity 2ω in the opposite direction.

3-cylinder radial engine equally spaced
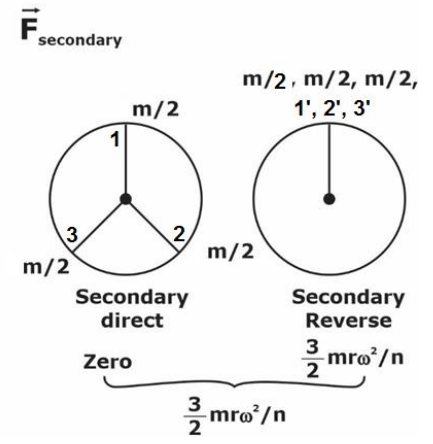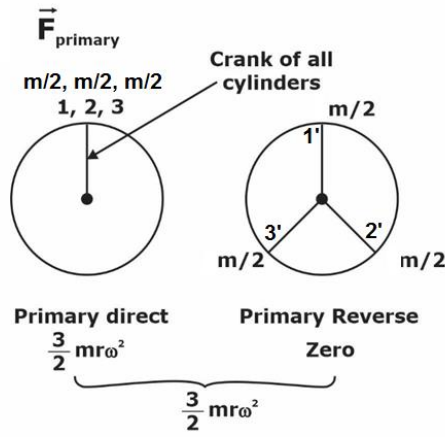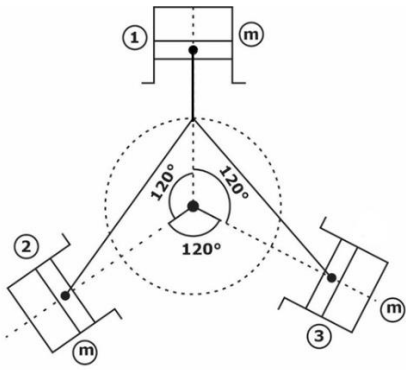
CHAPTER-8: CAMS

CAMS

- A cam is a rotating machine element which gives reciprocating or oscillating motion to another element known as follower.
- Plate cam/disc cam or radial cam, Wedge cam, Cylindrical cam, Face cam, Conical cam, Knife edge follower, Roller follower, Flat face follower.

Definitions and cam terminologies
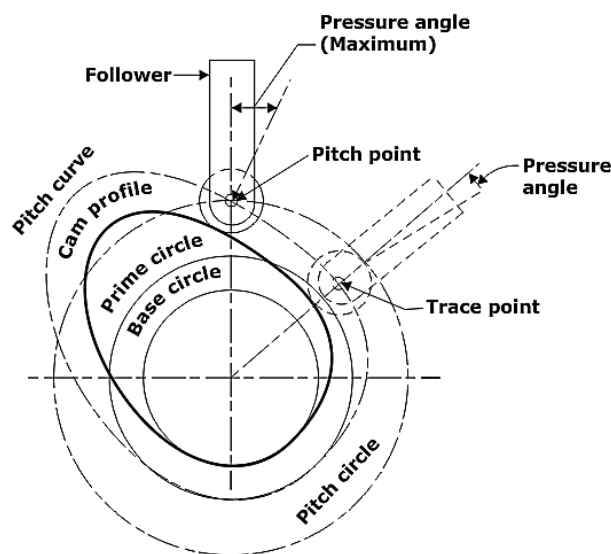
- **Base circle-** It is the smallest circle that can be drawn tangential to the cam profile (contour) from the centre of rotation of a radial cam. The base circle decides the overall size of a cam and is, therefore, a fundamental feature of the cam.
- **Trace point-** It is a reference point on the follower and is used to generate the pitch curve. In case of knife edge follower, the knife edge represents the trace point and in a roller follower, the centre of the roller represents the trace point.
- **Pitch curve-** It is the curve generated by the trace point assuming that the cam is fixed, and the trace point of the follower rotates around the cam i.e. the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.
- **Pressure angle-** The pressure angle, representing the steepness of the cam profile, is the angle between the normal to the pitch curve at a point and the direction of the follower motion. During a complete rotation, the pressure angle varies from its maximum to its minimum value. The greater the pressure angle, the higher will be the thrust, and consequently chances of jamming of the translating follower in its guide will increase. In an offset follower, the pressure angle reduces during the follower rise but at the cost of increased pressure angle during the return stroke.
- **Pitch point-** It is a point on the pitch curve having the maximum pressure angle.

- **Pitch circle-** It is the circle passing through the pitch point and concentric with the base circle.
- **Prime circle-** It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.
- **Lift or stroke-** It is the maximum travel of the follower from its lowest position to the topmost position.
- **Common Normal-** Line joining cam centre to the corresponding point of contact with cam denotes common normal to the roller surface and cam surface.
- **Angle of Ascent** (ϕ_a) **-** It is the angle through which the cam turns during the time the follower rises.
- **Angle of Dwell** (δ)**-** It is the angle through which the cam turns while the follower remains stationary at the highest or the lowest position.
- **Angle of descent** (ϕ_d) **-** It is the angle through which the cam turns during time the follower returns to the initial position.

DERIVATIVES OF FOLLOWER MOTION

- The derivatives of follower motion can be kinematic (with respect to θ) which relate to the geometry of the cam system or physical (with respect to time) which relate to the motion of the follower of the cam system.
- **Kinematic derivatives-** A displacement diagram of the follower motion is plotted with the cam angle θ as the abscissa and the linear or angular motion of follower as the ordinate. Thus, it is a graph that relates the input and output of the cam system.
- If s is the displacement of the follower. Then s = s (θ) and $\theta = \theta(t)$

- **First derivative:** $\dot{s} = \dfrac{ds}{dt} = \dfrac{ds}{d\theta}\dfrac{d\theta}{dt} = \omega\dfrac{dx}{d\theta}$

- **Second derivative:** $\ddot{s} = \dfrac{d^2s}{dt^2} = \omega^2\dfrac{d^2s}{d\theta^2}$

- **Third derivative:** $\dddot{s} = \dfrac{d^3s}{dt^3} = \omega^3\dfrac{d^3s}{d\theta^3}$

Motion of the Follower

- ❖ **Uniform velocity**

$$V = \left(\dfrac{h}{\theta_0}\right)\omega$$
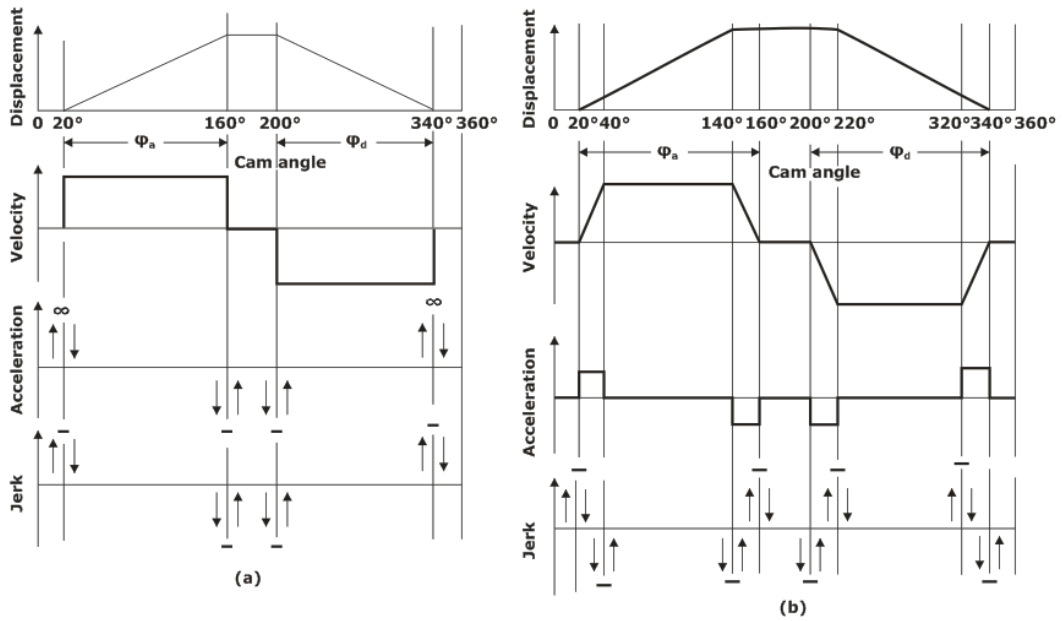
(a): Programme when the follower moves with uniform velocity

(b): Modified Programme when the follower moves with uniform velocity

❖ **Uniform Acceleration and Retardation**

$$V_{max} = \frac{2h\omega}{\phi}$$
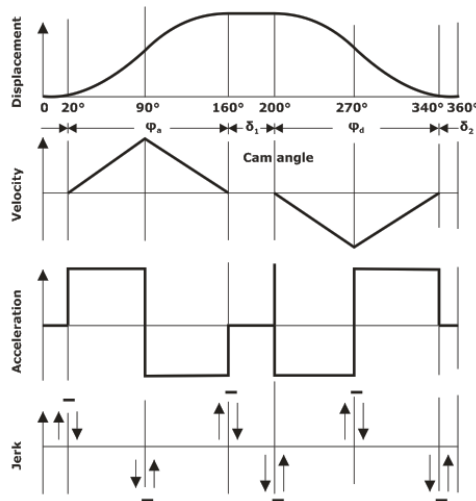
❖ **Simple harmonic motion**

$$V_{max} = \frac{h}{2}\frac{\pi\omega}{\phi}, \text{ at } \theta = \frac{\phi}{2}$$

$$\alpha = \frac{dv}{dt} = \frac{h}{2}\left(\frac{\pi\omega}{\phi}\right)^2 \cos\frac{\pi\theta}{\phi}$$

$$\alpha_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi}\right)^2, \text{ at } \theta = 0$$

Fig. 7.16
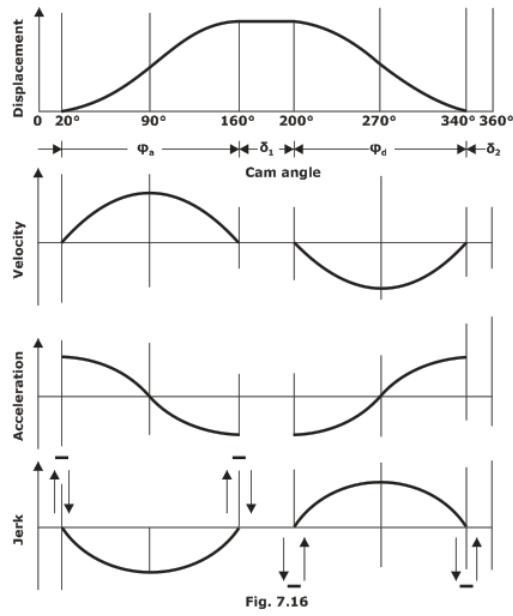
❖ **Cycloidal Motion**

$$a_{max} = \frac{2h\pi\omega^2}{\varphi^2}, \text{at } \theta = \frac{\varphi}{4}$$
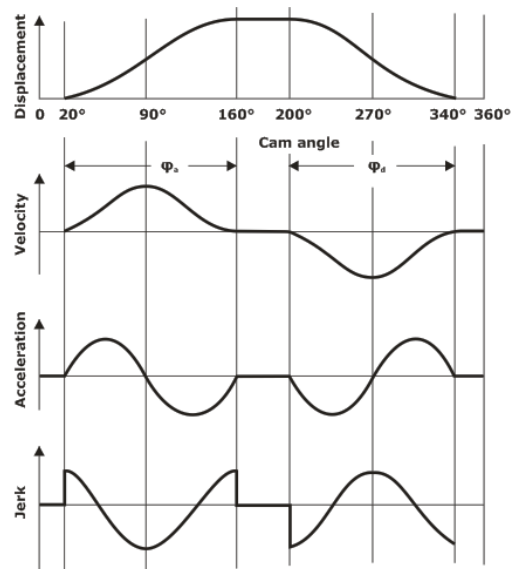
Gyroscopic Effect

- A body rotating about an axis of symmetry offers resistance to change in the direction of axis of spin. This tendency to oppose precessional motion is called gyroscopic effect.

- $T = I_p \alpha = I_p \omega_s \omega_p = - (\text{Re active gyroscope couple})$ Where I_p is polar moment of inertia about spinning axis.

- Axis of spin, axis of gyroscopic couple and axis of precessional motion are mutually perpendicular to each other.

Gyroscopic effect on aeroplanes

- **Propeller rotating clockwise when viewed from rear**
  - o For left turn nose will go up and tail will go down.
  - o For right turn nose will go down and tail will go up.

- **Propeller rotating Anti-clockwise when viewed from rear**
  - o For left turn nose will go down and tail will go up.
  - o For right turn nose will go up and tail will go down.

- **Propeller rotating clockwise when viewed from front**
  - o For left turn nose will go down and tail will go up.
  - o For right turn nose will go up and tail will go down.

- **Propeller rotating Anti-clockwise when viewed from front**
  - o For left turn nose will go up and tail will go down.
  - o For right turn nose will go down and tail will go up.

Gyroscopic effect on Naval ships

Terminology of Naval ships - Bow is the fore or the front end. Stern or aft is the rear end. Starboard is the right-hand side when looking from the stern. Port is the left-hand side when looking from the stern.

- **Gyroscopic effect during Steering**

  Steering is turning on the side when viewing from the top.

  - ❖ **Propeller rotating clockwise when viewed from stern**
    - ▪ For left turn bow will go up and stern will go down.
    - ▪ For right turn bow will go down and stern will go up.

  - ❖ **Propeller rotating Anti-clockwise when viewed from stern**
    - ▪ For left turn bow will go down and stern will go up.
    - ▪ For right turn bow will go up and stern will go down.

  - ❖ **Propeller rotating clockwise when viewed from bow**
    - ▪ For left turn bow will go down and stern will go up.
    - ▪ For right turn bow will go up and stern will go down.

  - ❖ **Propeller rotating Anti-clockwise when viewed from bow**
    - ▪ For left turn bow will go up and stern will go down.

- For right turn bow will go down and stern will go up.

- **Gyroscopic effect on Pitching**

  Pitching is limited angular motion of the ship about the transverse axis. $\theta = \Phi \sin \omega_0 t$

  Φ = Amplitude (angular) of swing or the maximum angle turned from the mean position in radians

  $\omega_0 = $ angular velocity of SHM $== \dfrac{2\pi}{\text{time period}}$

  Angular velocity of precession, $\dfrac{d\theta}{dt} = \Phi \omega_0 \cos \omega_0 t$

  This is maximum when $\cos \omega_0 t = 1$

  Maximum angular velocity of precession, $\omega_P = \Phi \omega_0$

  Gyroscopic couple, $T = I \omega \omega_p = I \omega \left(\Phi \times \dfrac{2\pi}{\text{time period}} \right)$

  ❖ **Propeller is rotating anticlockwise when viewed from bow**
    - When bow is rising, the gyroscopic effect will be to turn ship towards starboard.
    - When bow is falling, the gyroscopic effect will be to turn ship towards port side.

- **Gyroscopic Effect on Rolling**
  o Rolling is limited angular motion of the ship about the longitudinal axis.
  o As the axes of the rolling of the ship and of the rotor are parallel, there is no precession of the axis of spin and thus, there is no gyroscopic effect.

INTRODUCTION

- To and fro motion of a body about its mean position is termed as mechanical vibrations or harmonic vibrations or oscillations.

- For a system to have vibration, it must have mass and restoring device.

Simple Harmonic Motion

$$x = A\sin\omega t$$

$$\dot{x} = A\omega\cos\omega t$$

$$\ddot{x} = -A\omega^2\sin\omega t = -\omega^2 x$$

- $\ddot{x} + \omega^2 x = 0$ This is the equation of single degree of freedom system having free vibration with natural frequency ω.

Single degree of freedom system free undamped vibration

- Vibrations without any restriction and without under the influence of external force is called free undamped vibration.

Methods to Calculate Natural Frequency

- **D-Alembert's Principle**
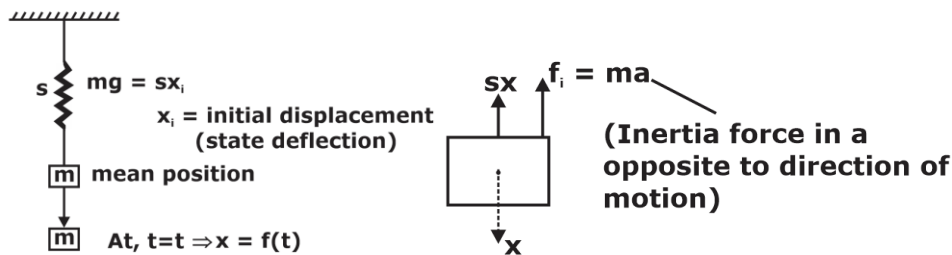
m $\ddot{x}$ + sx = 0

$\ddot{x} + \left(\dfrac{s}{m}\right)x = 0 \leftarrow$ Eqauation of natural system .

By comparing with equation of SHM

$\omega_n = \sqrt{\dfrac{s}{m}}$ Rad / s

- **Energy Method**

  Total energy = constant

  For a spring mass system E= K.E. + P.E. = $\dfrac{1}{2}Sx^2 + \dfrac{1}{2}mv^2$

  So, $\dfrac{dE}{dt} = 0$ $\Rightarrow \ddot{x} + \left(\dfrac{s}{m}\right)x = 0 \leftarrow$ Eqauation of natural system

  $\omega_n = \sqrt{\dfrac{S}{m}}$

- **Rayleigh's Method to calculate natural frequency**

  Max kinetic Energy = Max Potential energy

  $x = A\sin\omega t$

$\dot{x} = A\omega Cos\omega t$

Maximum Potential Energy $= \dfrac{1}{2}SA^2$

Maximum Kinetic Energy $= \dfrac{1}{2}m\omega^2 A^2$

$\omega_n = \sqrt{\dfrac{S}{m}}$

- **Effect of mass of spring**

$\omega_n = \sqrt{\dfrac{S}{\left(M + \dfrac{M_s}{3}\right)}}$

- **Analogy Between Linear and Torsional Vibration**

For Spring mass system $\omega_n = \sqrt{\dfrac{S}{m}}$, For torsional system $\omega_n = \sqrt{\dfrac{k_T}{I_{Rotor}}}$ (k_T=Torsional stiffness of

shaft).
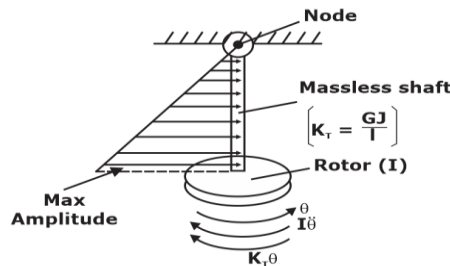
For Spring mass system $S = \dfrac{W}{\delta}$, For Torsional System $K_T = \dfrac{T}{\theta}$

For Spring mass system $\omega_n = \sqrt{\dfrac{S}{\left(M + \dfrac{M_s}{3}\right)}}$, For Torsional System $\omega_n = \sqrt{\dfrac{K_T}{\left(I_{rotor} + \dfrac{I_s}{3}\right)}}$

SINGLE Degree of Freedom System Free Damped Vibration

- Damping is an effect which causes a Reduction in the amplitude of an oscillation as a Result of energy being drained from the system to overcame frictional/other Resistive forces.
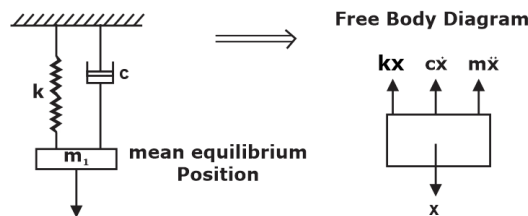
$m\ddot{x} + c\dot{x} + kx = 0$

Roots of above linear differential equation $\alpha_{1,2} = -\left(\dfrac{c}{2m}\right) \pm \sqrt{\left(\dfrac{c}{2m}\right)^2 - \omega_n^2}$

$\zeta(\text{Damping Factor}) = \dfrac{c}{2m\omega_n} = \dfrac{c}{2\sqrt{mk}}$

- **Overdamped System** $(\zeta > 1)$

$$\alpha_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n$$

x = $Ae^{(-\zeta + \sqrt{\zeta^2 - 1})w_n t} + Be^{(-\zeta - \sqrt{\zeta^2 - 1})w_n t}$

No vibrations will be present in an over-damped system
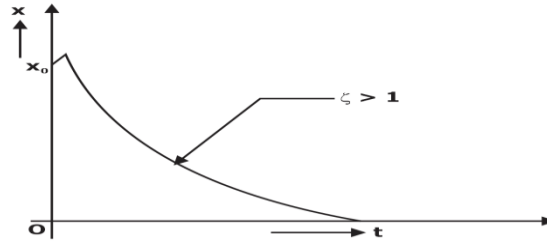
- **Critically Damped Systems** $(\xi = 1)$

  $\alpha_1 = \alpha_2 = -w_n$

  x = $(A + Bt)e^{-W_n t}$

  System Returns to its position in shortest time.

  $$\xi = \frac{C}{C_c} = \frac{\text{Actual Damping Coeff}}{\text{Critical Damping Coeff}}$$
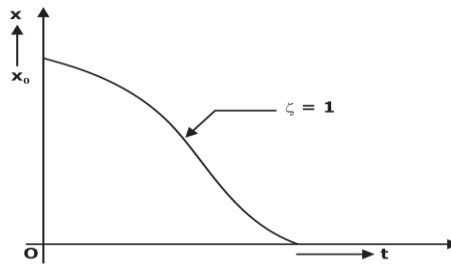
- **Under Damped Systems** $(\xi < 1)$

  $\alpha_{1,2} = -\xi\omega_n \pm i\,\omega_d$

  $\omega_d(\text{Damped Natural Frequency}) = \sqrt{1 - \xi^2}\,\omega_n$

  x = $e^{-\xi w_n t}\ A\left[\sin(w_d t + \phi)\right]$

  Here

$Ae^{-\xi\omega_n t}$	$\left[\sin(\omega_d t + \phi)\right]$
Amplitude [Decreasing with time]	Vibrations with Damped frequency $\omega_d = \left(\sqrt{1 - \xi^2}\right)\omega_n$
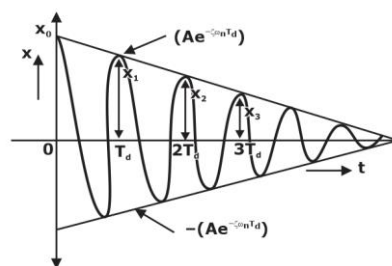
  o   Periodic motion.

- o Decrement Ratio $= \dfrac{x_0}{x_1} = \dfrac{x_1}{x_2} = \dfrac{x_2}{x_3} = \ldots \dfrac{x_n}{x_{n+1}} =$ Constant

- o Logarithmic Decrement (δ) is natural logarithm of ratio of two consecutive oscillation.

$$\delta = \ln(\text{D.R.})$$
$$\delta = \xi \omega_n T_d$$
$$\delta = \dfrac{2\pi\xi}{(1-\xi^2)^{1/2}}$$

Single Degree of Freedom System Forced Damped Vibration
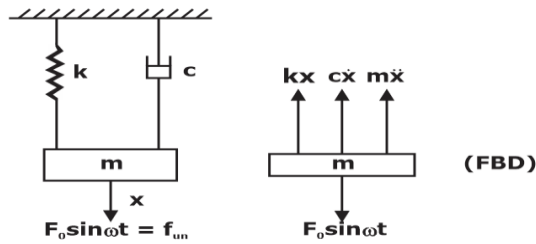
$$m\ddot{x} + c\dot{x} + kx = f_0 \sin \omega t$$

Total Solution =Complimentary Function + Particular Integral

After some time, CF = 0, $x = PI$

$$x = A\sin(\omega t - \phi)$$

$$A(\text{Amplitude}) = \dfrac{(f_0/k)}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left[\dfrac{2\xi\omega}{\omega_n}\right]^2}}$$

$$MF = \dfrac{A}{(f_0/k)} = \dfrac{1}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left(\dfrac{2\xi\omega}{\omega_n}\right)^2}}$$
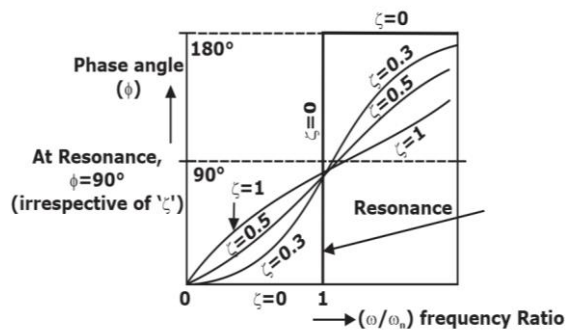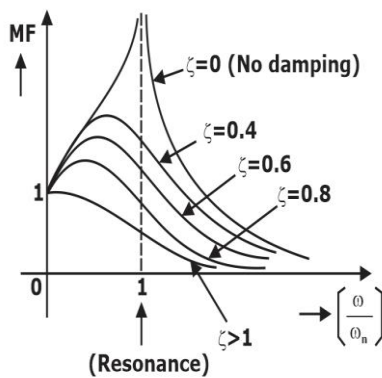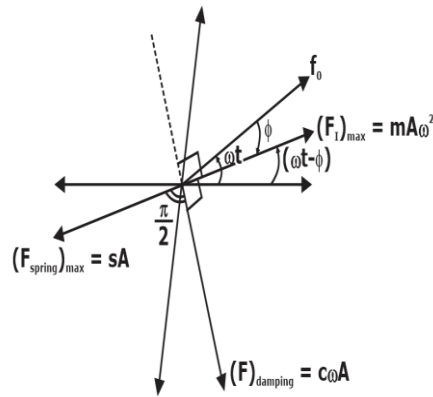
From Phase Diagram Spring and damping force's max values are perpendicular to each other. Inertia force lies exactly opposite (At 180°) To the spring force's max value.

$$\tan \varphi = \dfrac{\left(\dfrac{2\xi\omega}{\omega_n}\right)}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}$$

Vibration Isolations

- It is used to isolate the ground from the vibrations of the Running machine so as to save other stationary m/cs from these vibration effects.
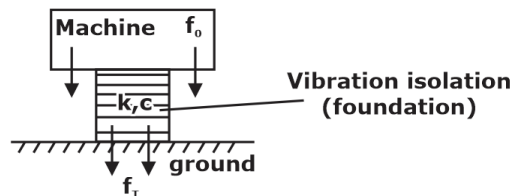
$$\in (\text{Transmissibility}) = \dfrac{f_T}{f_0} = \dfrac{\sqrt{1 + \left(\dfrac{2\xi\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \left(\dfrac{\omega}{\omega_n}\right)^2\right)^2 + \left(\dfrac{2\xi\omega}{\omega_n}\right)^2}}$$
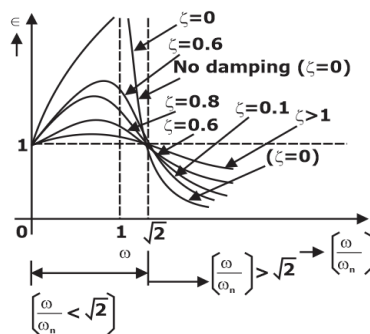
Whirling of Shafts
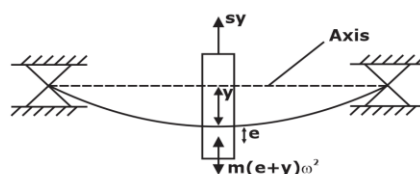
$$y = \left[\frac{e}{\left(\dfrac{\omega_n}{\omega}\right)^2 - 1}\right]$$

At, $\omega = \omega_n$: $y \to \infty$ and hence very dangerous Vibrations will be induced in the shaft and causing whipping of shafts.

Torsional Equivalent Shaft

$$\frac{L_E}{J_E} = \frac{L_1}{J_1} + \frac{L_2}{J_2} + \cdots \frac{L_N}{J_N}$$

- **Two-Rotor System**
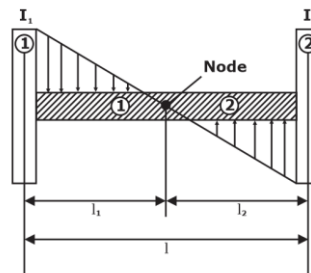
$$I_1 \ell_1 = I_2 \ell_2$$

Node divides the distance in inverse ray.

****

Important Formula (IC Engine)

CHAPTER-1 AIR STANDARD CYCLE

NOMANCLATURE OF AN INTERNAL COMBUSTION ENGINE

➢ **Cylinder Bore(d)-** Nominal inner diameter of the working cylinder.

➢ **Piston Area (A)-** Cross-sectional area of cylinder $A = \dfrac{\pi}{4}d^2$

➢ **Stroke (L)-** Nominal distance b/w Top Dead centre & Bottom Dead centre, piston reciprocates

➢ **Stroke to Bore Ratio (L/d)**- important parameter in classifying the size of the engine.

If $\dfrac{L}{d} > 1 \rightarrow$ under-square engine | If $\dfrac{L}{d} = 1 \rightarrow$ square engine | If $\dfrac{L}{d} < 1 \rightarrow$ over-square engine

➢ **Dead Centre-** Position of piston when direction of piston motion is reversed at either end

   o **Top Dead Centre (TDC) or Inner Dead Centre (IDC) -** when piston is farthest from the crankshaft or nearest to cylinder head.

   o **Bottom Dead Centre (BDC) or Outer Dead Centre (ODC) -** when piston is nearest to the crankshaft or farthest from the cylinder head.
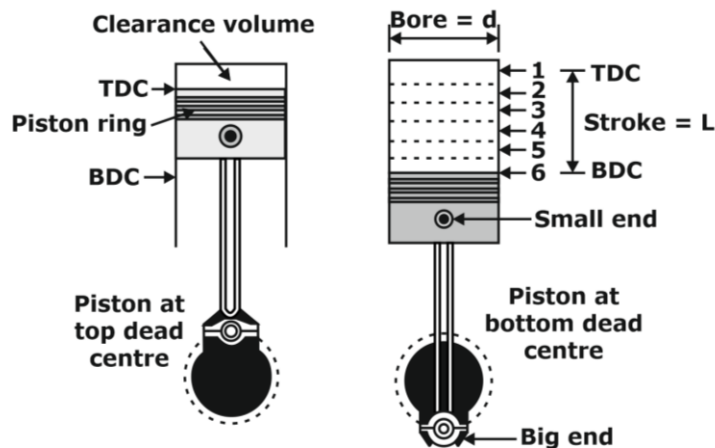
➢ **Displacement or Swept Volume (Vₛ)-** Volume swept by piston when travelling b/w TDC and BDC. $Swept \; \text{Volume} = V_s = A \times L = \dfrac{\pi}{4}d^2 L$

➢ **Engine Capacity**- Displacement volume of a cylinder multiplied by number of cylinders in an engine will give the engine capacity. It is also known as cubic capacity.

➢ **Clearance Volume (V_c)**- Gap b/w piston (when piston at TDC) & cylinder head.

➢ **Compression Ratio (r)-** Ratio of the total cylinder volume (when piston at BDC) to clearance volume. $r = \dfrac{V_{Total}}{V_C} = \dfrac{V_C + V_S}{V_C} = 1 + \dfrac{V_S}{V_C}$

CONSTANT VOLUME OR OTTO CYCLE

- Nicolaus Otto (1876), proposed a constant-volume heat addition cycle

- theoretical cycle for the spark-ignition engine or petrol engine or gasoline engine.

- mixture of fuel and air is used as charge at inlet

- Working of otto cycle –

  o **Processes 0→1 and 1→0** Suction and exhaust processes (nullified effect on full throttle)

  o **Process 1→2** isentropic compression of charge

  o **Process 2→3** heat is supplied reversibly at constant volume (corresponds to spark-ignition)

  o **Processes 3→4** isentropic expansion

  o **Process 4→1** constant volume heat rejection
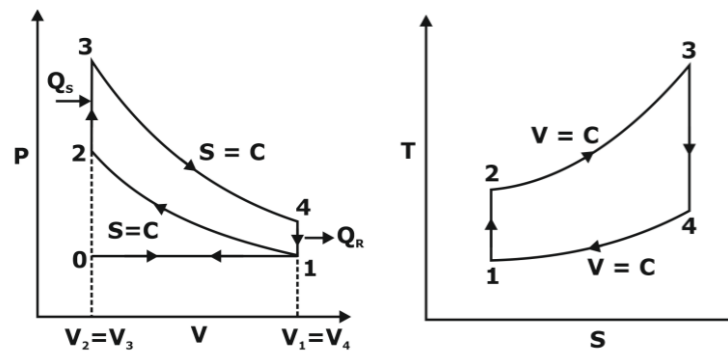
Swept volume $= (V_1 - V_2) = V_2\left(\dfrac{V_1}{V_2} - 1\right) = V_2(r - 1)$

Compression ratio $r = \dfrac{V_S + V_C}{V_C} = \dfrac{V_1}{V_2}, \Rightarrow \dfrac{T_2}{T_1} = \dfrac{T_3}{T_4} = \left(\dfrac{V_1}{V_2}\right)^{(\gamma-1)} = (r)^{(\gamma-1)}$

Expansion ratio $r_e = \dfrac{V_4}{V_3}, \Rightarrow r = r_e$

- **Efficiency** $\eta_{otto} = 1 - \dfrac{Q_R}{Q_S} = 1 - \dfrac{C_v(T_4 - T_1)}{C_v(T_3 - T_2)} = 1 - \dfrac{T_1}{T_2} = 1 - \dfrac{1}{r^{(\gamma-1)}}$

It is function of compression ratio r and the ratio of specific heats, γ & independent of heat supplied and pressure ratio. The normal range of compression ratio for spark-ignition engines is 6 to 10.
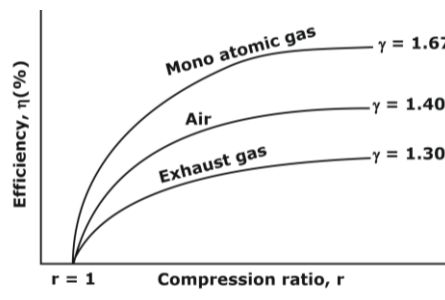
- **Work Output** $W_{net} = W_{1\to2} + W_{3\to4} = \dfrac{P_1V_1}{\gamma - 1}(r_P - 1)(r^{\gamma-1} - 1)$

directly proportional to pressure ratio, r_p.

- **Mean effective pressure** $P_{MEP} = \dfrac{\text{Work output}}{\text{Swept volume}} = \dfrac{P_1 r(r_p - 1)(r^{\gamma-1} - 1)}{(\gamma - 1)(r - 1)}$

increase in compression ratio leads to increase in MEP as well as the thermal efficiency.

CONSTANT PRESSURE OR DIESEL CYCLE

- To overcome upper limit of compression ratio in SI engine, air and fuel are compressed separately and brought together at the time of combustion. It is working principal of CI engine.

- Working of Diesel cycle-

  o **Process 1→2** isentropic compression of the air

  o **Process 2→3** heat is supplied reversibly at constant pressure (corresponds to injection of fuel)

  o **Processes 3→4** isentropic expansion

  o **Process 4→1** constant volume heat rejection
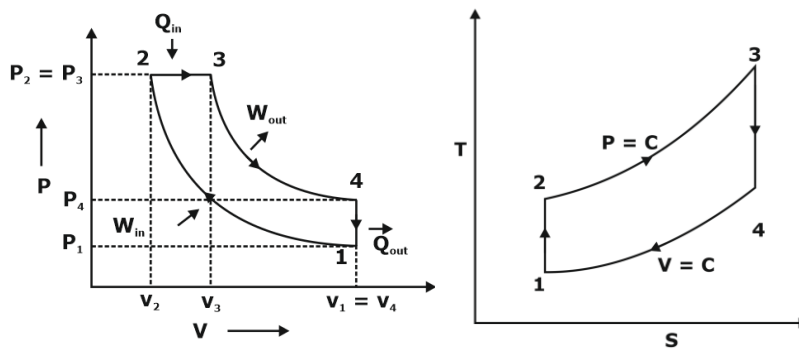
$\text{Swept volume} = (V_1 - V_2) = V_2\left(\dfrac{V_1}{V_2} - 1\right) = V_2(r - 1)$

$\text{Compression ratio } r = \dfrac{V_S + V_C}{V_C} = \dfrac{V_1}{V_2}, \Rightarrow \dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{(\gamma-1)} = (r)^{(\gamma-1)}$

$\text{Expansion ratio } r_e = \dfrac{V_4}{V_3}$

$\text{Cut off ratio, } \rho = \dfrac{\text{volume after heat addition}}{\text{Volume before heat addition}} = \dfrac{V_3}{V_2}, \Rightarrow r = \rho \times r_e$

- **Efficiency** $\eta_{Diesel} = 1 - \dfrac{Q_R}{Q_S} = 1 - \dfrac{C_v(T_4 - T_1)}{C_P(T_3 - T_2)} = 1 - \dfrac{1}{r^{\gamma-1}}\left[\dfrac{\rho^\gamma - 1}{\gamma(\rho - 1)}\right]$

Value bracketed factor is always greater than unity.

Fuel cut-off ratio ρ depends on output, thus for maximum output ρ is maximum.

Unlike the Otto cycle the air-standard efficiency of the Diesel cycle depends on output.

The normal range of compression ratio for diesel engine is 16 to 20.

- **Work Output** $W_{net} = W_{1\to2} + W_{2\to3} + W_{3\to4} = \dfrac{P_1 V_1 r^{(\gamma-1)}\left[\gamma(\rho - 1) - r^{(1-\gamma)}(\rho^\gamma - 1)\right]}{\gamma - 1}$

- **Mean effective pressure** $P_{MEP} = \dfrac{\text{Work output}}{\text{Swept volume}} = \dfrac{P_1[\gamma r^\gamma(\rho - 1) - r(\rho^\gamma - 1)]}{(\gamma - 1)(r - 1)}$

DUAL COMBUSTION OR MIXED OR LIMITED PRESSURE CYCLE

- In this cycle, part of the heat addition is at constant volume and remaining at constant pressure
- Heat addition at constant volume tends to increase the efficiency of the cycle whereas switching over to constant pressure heat addition limits the maximum pressure. Hence, this cycle is also called limited pressure cycle.
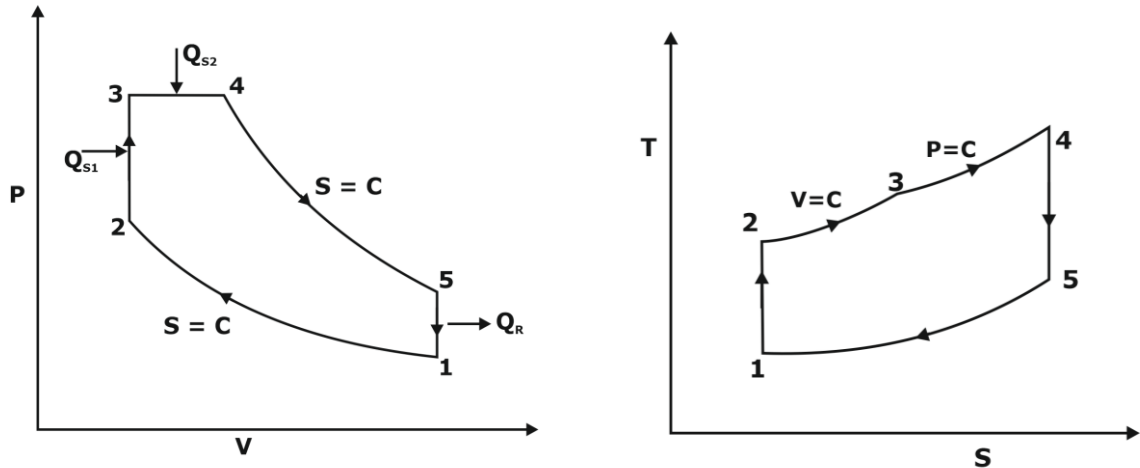
$$\text{Swept volume} = (V_1 - V_2) = V_2\left(\frac{V_1}{V_2} - 1\right) = V_2(r-1)$$

$$\text{Compression ratio } r = \frac{V_S + V_c}{V_C} = \frac{V_1}{V_2}, \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = (r)^{(\gamma-1)}$$

$$\text{Expansion ratio } r_e = \frac{V_5}{V_4}$$

$$\text{Cut off ratio, } \rho = \frac{\text{volume after heat addition}}{\text{Volume before heat addition}} = \frac{V_4}{V_3}, \Rightarrow r = \rho \times r_e$$

$$\text{Pressure ratio } \alpha = \frac{P_3}{P_2}$$

- **Efficiency** $\eta_{Dual} = 1 - \frac{Q_R}{Q_S} = 1 - \frac{C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_P(T_3 - T_3)} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\alpha\rho^\gamma - 1}{(\alpha-1) + \alpha\gamma(\rho-1)}\right]$

Value of $\alpha > 1$ results in an increased efficiency for a given value of ρ and γ.

Thus, efficiency of dual cycle lies between that of Otto cycle and diesel cycle.

With $\rho = 1$, it becomes an Otto cycle. With $\alpha = 1$, it becomes a Diesel cycle.

- **Work Output** $W_{net} = W_{1\to2} + W_{3\to4} + W_{4\to5} = \frac{P_1V_1}{\gamma-1}\left[\gamma\alpha r^{\gamma-1}(\rho-1) + r^{\gamma-1}(\alpha-1) - (\alpha\rho^\gamma - 1)\right]$

- **Mean effective pressure** $P_{MEP} = \frac{Work\ output}{Swept\ volume} = P_1\frac{[\gamma\alpha r^\gamma(\rho-1) + r^\gamma(\alpha-1) - r(\alpha\rho^\gamma-1)]}{(\gamma-1)(r-1)}$

COMPARISON OF OTTO, DIESEL, AND DUAL CYCLES

➢ **For same compression ratio and same heat input**

Otto cycle allows working medium to expand more (to lower pressure) compare to Diesel cycle.

So, heat energy converted into work is more in otto cycle due to more expansion.

Otto cycle 1→2→3→4→1,

Diesel cycle1→2→3'→4'→1

Dual cycle 1→2→2'→3" →4" →1

$\Rightarrow Q_{R_{otto}} < Q_{R_{dual}} < Q_{R_{diesel}}$
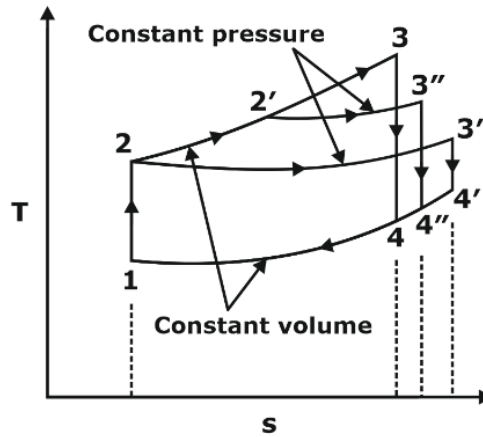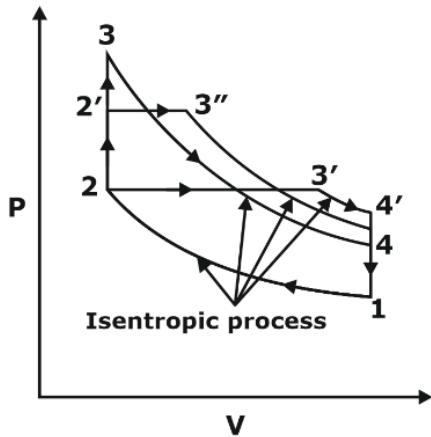
$\Rightarrow \eta_{otto} > \eta_{dual} > \eta_{diesel}$

➢ **Same Compression Ratio and Heat Rejection**

Otto cycle 1→2→3→4→1,

Diesel cycle1→2→3'→4→1

$\Rightarrow Q_{S_{otto}} > Q_{S_{dual}} > Q_{S_{diesel}}$
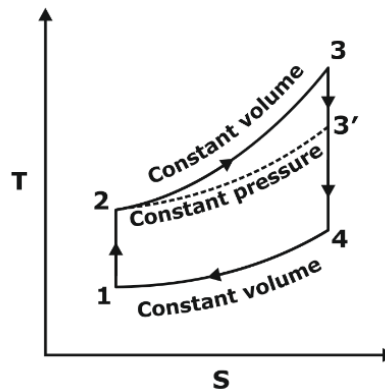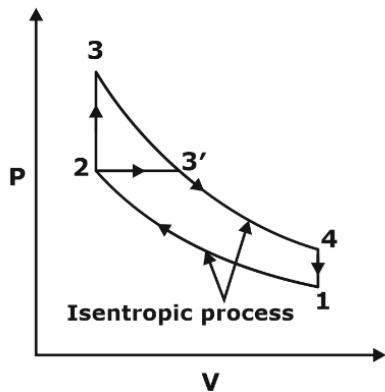
$\Rightarrow \eta_{otto} > \eta_{dual} > \eta_{diesel}$

➢ **For constant maximum pressure and same heat input**

Otto cycle 1→2→3→4→1

Diesel cycle 1→2'→3'→4'→1

$\Rightarrow Q_{R_{otto}} > Q_{R_{dual}} > Q_{R_{diesel}}$
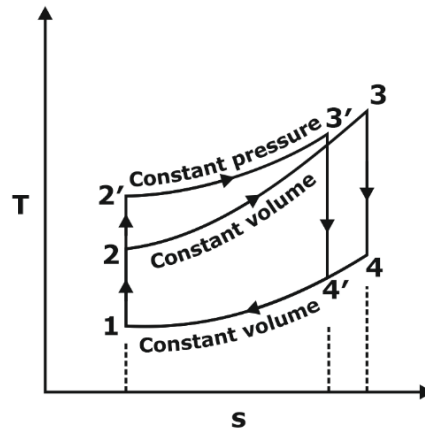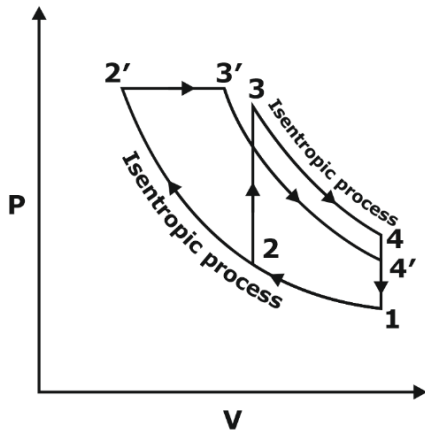
$\Rightarrow \eta_{otto} < \eta_{dual} < \eta_{diesel}$

> **Same Peak Pressure, Peak Temperature and Heat Rejection**

   Otto cycle 1→2→3→4

   Diesel cycle 1→2→2'→3→4

$\Rightarrow Q_{S_{otto}} < Q_{S_{dual}} < Q_{S_{diesel}}$
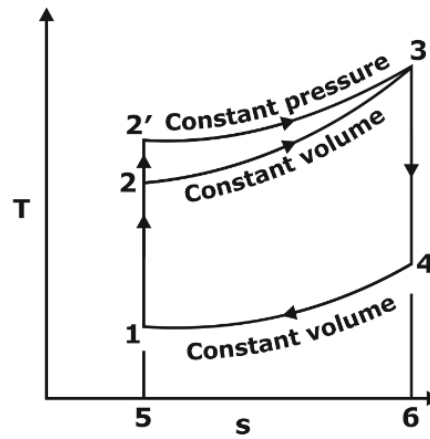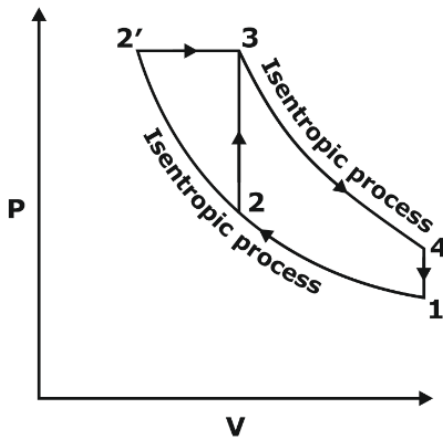
$\Rightarrow \eta_{otto} < \eta_{dual} < \eta_{diesel}$

> **For same maximum pressure and work output**

$\Rightarrow \eta_{otto} < \eta_{dual} < \eta_{diesel}$
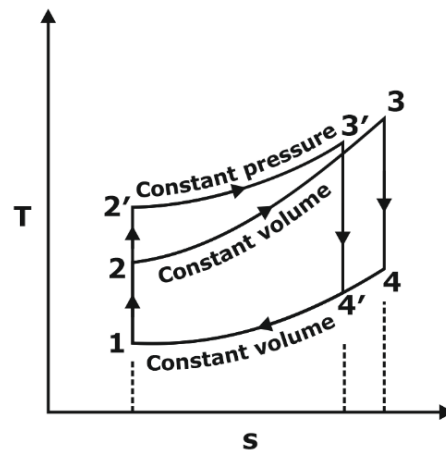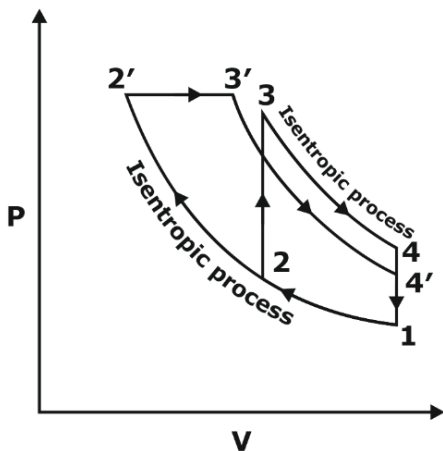
CHAPTER-2 IC ENGINE PERFORMANCE PARAMETERS AND TESTING

INTRODUCTION
- To compare develop engine with other engines in terms of its output & efficiency, we need some parameters that reflect the performance of the engine.
- The performance of engine depends on inter-relationship b/w power developed, speed & specific fuel consumption at each operating condition within the useful range of speed and load.

Indicated Power (IP)
- It tells about the health of the engine. It is conversion of chemical energy of fuel into heat energy.
- Area of the indicator diagram will represent the indicated power of the engine. IP = (GP – PP)

- Indicated power also given by $IP = P_{imep}\dot{V}_s = P_{imep} \times \dfrac{V_S nk}{60}$

    Here P_{imep} = Indicated mean effective pressure, N/m^2

    $V_s = A \times L = \dfrac{\pi}{4}d^2 L$ Swept Volume

    L = Length of stroke, (m)
    d = inner diameter of piston, (m)
    k = number of cylinders
    n = number of power strokes per minute
    (n = N/2 for a four-stroke engine, n = N for a two-stroke engine)
    N= engine speed in revolutions per minute

Brake Power (BP) or shaft power or delivered power
- It is power delivered by engine at output shaft (engine crankshaft) and calculated using torque and angular speed of output shaft.
- Dynamometer is used to measure power output of an engine either by dissipating absorbed power as heat or by transmitting power to the load coupled to the engine.

    $BP = T \times \omega = T \times \dfrac{2\pi N}{60}$

    Here N= engine speed in revolutions per minute
       T = Torque at output shaft in N-m

- Brake power also given by $BP = P_{bmep}\dot{V}_s = P_{bmep} \times \dfrac{V_S nk}{60}$

    Here P_{bmep} = brake mean effective pressure, N/m^2

    $V_s = A \times L = \dfrac{\pi}{4}d^2 L$ Swept Volume

    L = Length of stroke, (m)
    d = inner diameter of piston, (m)
    k = number of cylinders
    n = number of power strokes per minute
    (n = N/2 for a four-stroke engine, n = N for a two-stroke engine)

N= engine speed in revolutions per minute

Frictional power (FP)

• The mechanical losses occur while transmitting work from piston to crank-shaft.

FP = IP − BP

Indicated thermal efficiency (η_{ith})

• ratio of indicated power (IP) and heat addition by fuel per second

$$\eta_{ith} = \frac{IP}{HA_{/sec}}$$

Brake thermal efficiency (η_{bth})

• ratio of brake power (BP) to heat addition by fuel per second

• It is also called actual efficiency of engine.

$$\eta_{bth} = \frac{BP}{HA_{/sec}}$$

Mechanical efficiency

• Ratio of BP to IP

• It takes into account mechanical losses in an engine.

• In general, mechanical efficiency of engines varies from 65 to 85%

$$\eta_{mech} = \frac{BP}{IP} = \frac{P_{bmep}}{P_{imep}} = \frac{\eta_{bth}}{\eta_{ith}}$$

Relative Efficiency

• ratio of actual efficiency obtained from an engine to the theoretical efficiency of air-standard cycle.

$$\text{Re}lative\ efficiency,\ \eta_{rel} = \frac{Actual\ brake\ thermal\ efficiency}{Air-s\tan dard\ efficiency}$$

• Relative efficiency for most of the engines varies from 75 to 95% with theoretical air and decreases rapidly with insufficient air to about 75% with 90% air.

Specific Fuel Consumption

• It is the mass of fuel consumed in kg/hour per kW of power developed by engine.

$$bsfc = \frac{\dot{m}_f (in\ \text{kg/hr})}{BP(in\ \text{kW})}$$

Equivalence ratio

• Ratio of actual fuel-air ratio to stoichiometric fuel-air ratio

$$\text{Equivalence ratio} = \frac{actual\ fuel-air\ ratio}{stoichiometric\ fuel-air\ ratio}$$

Volumetric Efficiency

• it indicates the breathing capacity of the engine.

• ratio of actual mass of air drawn into engine during a given time period to the theoretical mass.

$$\eta_v = \frac{m_{act}}{m_{th}},$$

Where m_{act} = a measured quantity

$$m_{th} = \rho_a n V_s,$$

n = number of power strokes per minute

(n = N/2 for a four-stroke engine, n = N for a two-stroke engine)

N = engine speed in revolutions per minute

ρ_a = density of the surrounding atmosphere.

$$V_s = A \times L = \frac{\pi}{4} d^2 L k \quad \text{Swept Volume}$$

L = Length of stroke, (m)

d = inner diameter of piston, (m)

k = number of cylinders

Mean Effective Pressure (P_{mep})

- It is average pressure at which, if engine operates then area under this horizontal line in between TDC and BDC will be equal to the net-work of the system.

- It is a mean value expressed in N/m², which, when multiplied by the swept volume V_s gives the same net-work as actually produced with the varying pressures.

$$P_{mep} = \frac{Network\ output}{Swept\ volume}$$

Indicated Mean Effective Pressure (P_{imep})

- It is a mean value expressed in N/m², which, when multiplied by the swept volume V_s gives the same indicated net-work as actually produced with the varying pressures.

$$P_{imep} = \frac{Net\ \text{indicated}\ work\ output}{Swept\ volume}$$

BRAKE MEAN EFFECTIVE PRESSURE (P_{bmep})

- P_{imep} may be considered to consist of P_{fmep} and P_{bmep}, two **hypothetical** pressures.

- Fiction mean effective pressure is that portion of P_{imep} which is required to overcome friction losses

- Brake mean effective pressure is the portion which produces useful power delivered by the engine.

$$P_{imep} = P_{fmep} + P_{bmep}$$

MEASUREMENT OF INDICATED POWER- Morse Test

- Let total brake power is B of 4-cylinder engine

    $I_1 = B - B_1$, when engine 1 is cut-off

    $I_2 = B - B_2$, when engine 2 is cut-off

    $I_3 = B - B_3$, when engine 3 is cut-off

    $I_4 = B - B_4$, when engine 4 is cut-off

    Then the total indicated power of engine will be equal to $I = I_1 + I_2 + I_3 + I_4$

    And Frictional Power, FP = IP- BP

STIRLING CYCLE

- It consists of two isothermal and two constant volume processes.
- The heat rejection and addition take place at constant temperature.
- from Figure, amount of heat addition and rejection during constant volume processes is same.
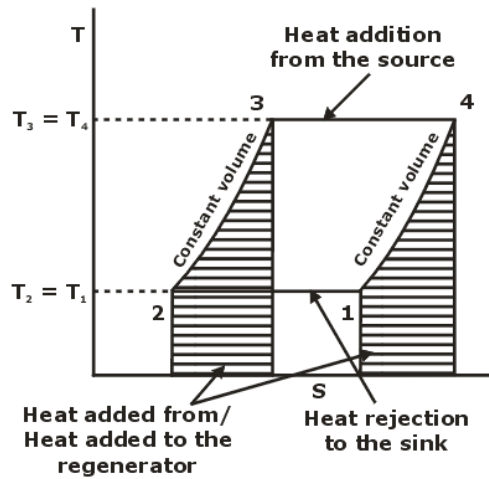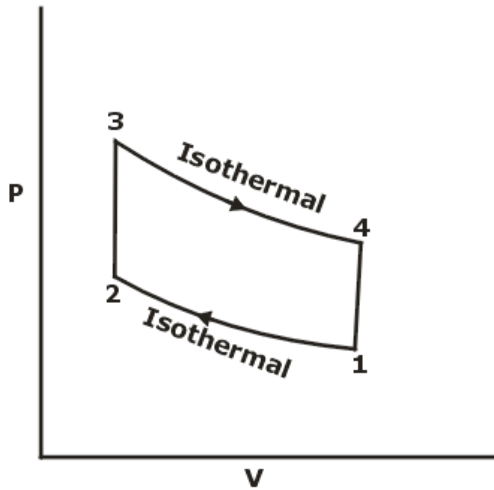
- **Thermal Efficiency** $\eta_{\text{Stirling}} = 1 - \dfrac{Q_R}{Q_S} = 1 - \dfrac{T_1}{T_3}$

It is same as Carnot efficiency but work output is more in case of Stirling cycle.

ERICSSION CYCLE

- The Ericsson cycle consists of two isothermal and two constant pressure processes.
- The heat addition and rejection take place at constant pressure as well as isothermal processes.
- from Figure, amount of heat addition and rejection during constant pressure processes is same.
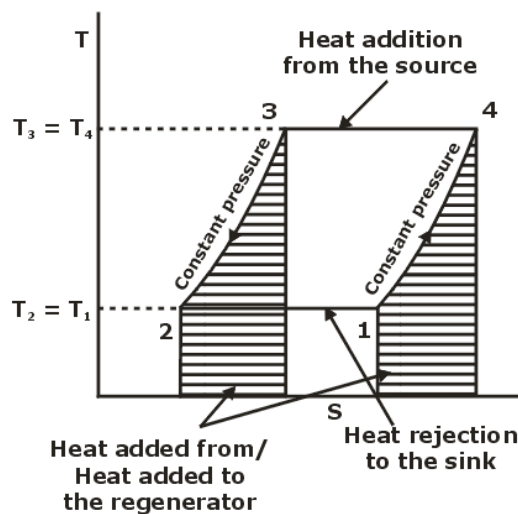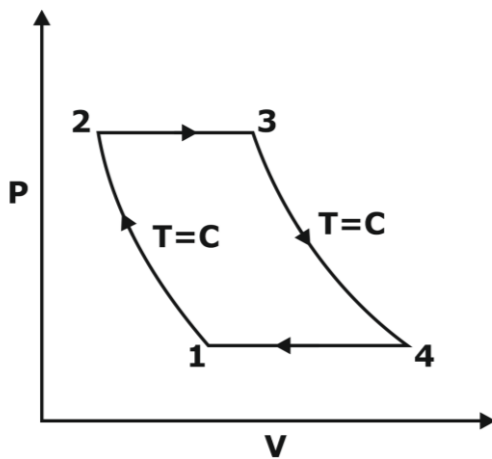
LENOIR CYCLE

- The Lenoir cycle consists of the 3 processes to complete its cycle.

    Process 1→2 - Constant volume heat addition

    Process 2→3 - isentropic expansion

    Process 3→1 - constant pressure heat rejection
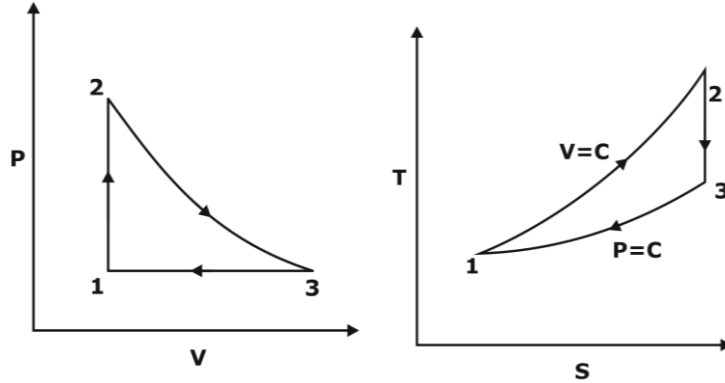
$$\frac{P_2}{P_1} = \frac{P_2}{P_3} = r_p = \frac{T_2}{T_1} \Rightarrow T_2 = r_p T_1$$

- **Thermal Efficiency** $\eta_{Lenoir} = 1 - \dfrac{Q_R}{Q_S} = 1 - \gamma\left(\dfrac{T_3 - T_1}{T_2 - T_1}\right) = 1 - \gamma\left(\dfrac{T_1\,\alpha^{1/\gamma} - T_1}{T_1\,\alpha - T_1}\right) = 1 - \gamma\left(\dfrac{\alpha^{1/\gamma} - 1}{\alpha - 1}\right)$

efficiency of Lenoir cycle depends upon the pressure ratio as well as the ratio of specific heats, γ.

ATKINSON CYCLE

- Atkinson cycle is an ideal cycle for Otto engine exhausting to a gas turbine.
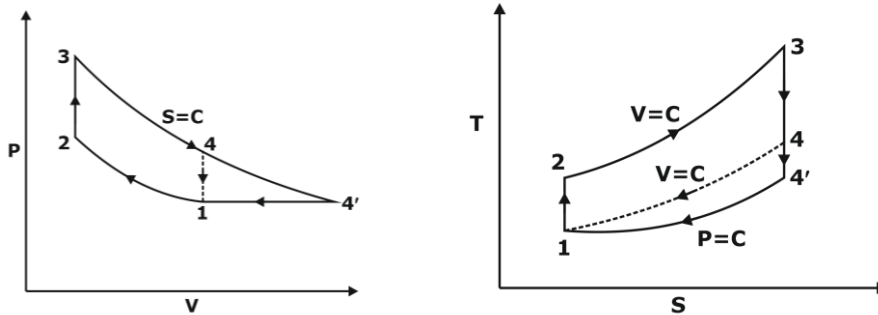
Compression ratio, $r = \dfrac{V_1}{V_2}$

Expansion ratio $r_e = \dfrac{V_{4'}}{V_3}$

Thermal Efficiency $\eta_{Atkinson} = 1 - \dfrac{Q_R}{Q_S} = 1 - \gamma\left(\dfrac{T_{4'} - T_1}{T_3 - T_2}\right) = 1 - \gamma\left[\dfrac{r_e - r}{r_e^{\gamma} - r^{\gamma}}\right]$

****

Important Formula (Industrial Engineering)

CHAPTER 1: FORECASTING

Forecasting: Forecasting can be termed as prediction of future sales or demand of a particular product it is a projection based upon past data and art of human judgement.

Categorized by time period:

Short term: Usually for a period of time less than a year. It avoids underproduction and over production, inventory, cost on variable factor, sales target and appropriate pricing.

Long term: time horizon of 5-7 years,10- 20 years. It deals with manpower planning, long term capital requirement, investment decisions, interdependence of industry.

Forecasting based on analysis methods:
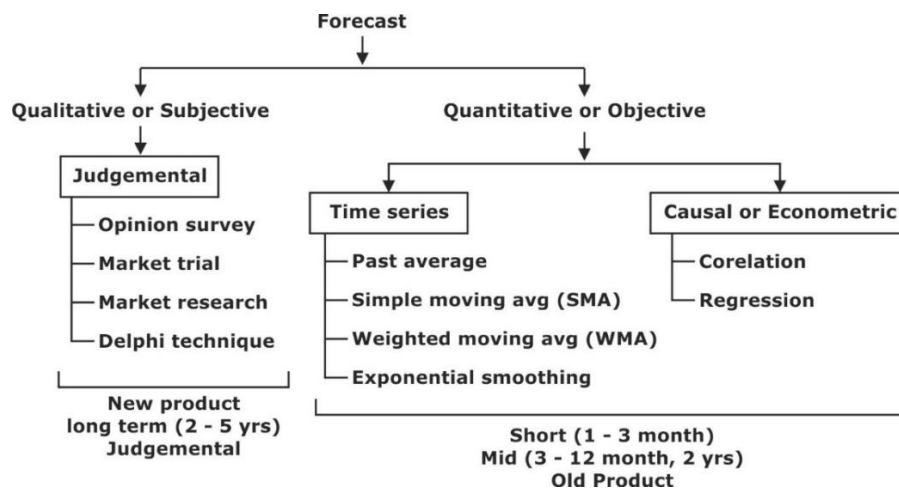
Time Series:

a. Past Average: In this method forecast is given by average or mean of the actual demand data of the previous period.

b. Simple moving Avg. or Simple rolling avg. (SMA):

$$\text{Forecast} = \frac{\text{Sum of demands for given period}}{\text{Number of Period}}$$

c. Weighed moving avg. (WMA): WMA method uses past data similar to simple moving average. The difference being it gives unequal weight to each demand data in such a manner that summation of all weight always equal to one. The most recent data is given the highest weight and the weight assigned to the oldest data will be the least.

n = no. of period of WMA

$$\sum n = \frac{n(n+1)}{2}$$

Arrange then in decreasing order of weight as: $\dfrac{n}{\sum n}, \dfrac{n-1}{\sum n}, \dfrac{n-2}{\sum n}, ..., \dfrac{1}{\sum n}$

d. Exponential smoothing: This method required only the current demand and forecasted value for the current period to give the next forecast.

This method is the modified form of weighted moving avg. which gives weight to all the previous data and the weight assigned are in exponentially decreasing order.

General form is:

$f_t = \alpha D_{t-1} + (1 - \alpha) [\alpha D_{t-2} + \alpha(1 - \alpha) D_{t-3} + \alpha(1-\alpha)^2 D_{t-4} .... \infty]$

$$\boxed{f_t = \alpha D_{t-1} + (1 - \alpha)f_{t-1}}$$

$f_t = f_{t-1} + \alpha (D_{t-1} - f_{t-1})$

$$\boxed{f_t = f_{t-1} + \alpha e_{t-1}}$$

α is known as smoothing constant and is equivalent to n period of SMA.

$$\boxed{\alpha = \dfrac{2}{n+1}}, \text{ n = No. of SMA}$$

Practically, $\alpha \to 0.1$ to 0.2.

Responsiveness & stability:
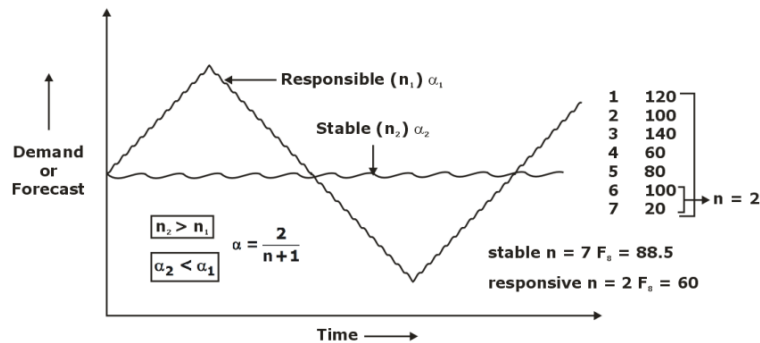
Responsiveness: It indicates that forecast have fluctuating or swinging pattern. It is preferred for new product and for that no. of period is kept small.

Stability: It means the forecast pattern is flat, smooth or has less fluctuation. It is preferred for old existing product and for that no. of period is kept large.

$$\boxed{f_t = f_{t-1} + \alpha \left(D_{t-1} - f_{t-1}\right)}$$

$$\boxed{\alpha = \dfrac{2}{n+1}}$$

(i). if $\alpha = 0$, $n \to \infty$ (limit of stability)

$$\boxed{f_t = f_{t-1}}$$

(ii). if $\alpha = 1$, $n \to 1$ (limit of responsiveness)

$$\boxed{f_t = D_{t-1}}$$

Forecast error: $\boxed{e_i = D_i - f_i}$

Running Sum Forecast Error : $\boxed{RSFE = \sum_{i=1}^{n}(D_i - f_i)}$

Mean absolute deviation: $\boxed{MAD = \dfrac{\sum_{i=1}^{n}|D_i - f_i|}{n}}$

Bias /Mean Forecast Error: $\boxed{BIAS = \dfrac{\sum_{i=1}^{n}(D_i - f_i)}{n}}$

$\boxed{Bias = \dfrac{RSFE}{n}}$

Mean square error (MSE): $\boxed{MSE = \dfrac{\sum_{i=1}^{n}(D_i - f_i)^2}{n}}$

Standard deviation: $\boxed{\sigma = \sqrt{MSE}}$

MSE is used to compare Forecast error which is utilized to plot control chart for forecast error.

Mean absolute Percentage error (MAPE): $\boxed{MAPE = \dfrac{\sum_{i=1}^{n}\left|\dfrac{D_i - f_i}{D_i} \times 100\right|}{n}}$

Tracking signal (TS): It tells how well forecaster is predicting the actual value.

$\boxed{TS = \dfrac{RSFE}{MAD}}$

Causal or Econometric Forecasting:

Linear Regression analysis:
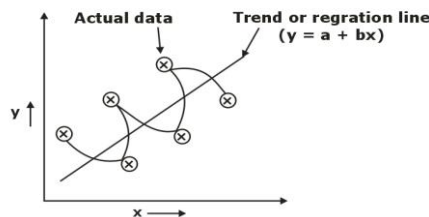
$\boxed{a = \dfrac{\sum y - b\sum x}{n}}$ and $\boxed{b = \dfrac{n\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}}$

Least square method: $\Sigma x = 0$

$\boxed{a = \dfrac{\sum y}{n}}$ and $\boxed{b = \dfrac{\sum x.y}{\sum x^2}}$

Correlation coefficient method: The co-relation coefficient b/w the two variable x & y is given by,

4

$$r = \frac{\Sigma (x - \bar{x})(y - \bar{y})}{\sqrt{\Sigma (x - \bar{x})^2 \, \Sigma (y - \bar{y})^2}}$$

where $\bar{x}$ and $\bar{y}$ are average values of x and y respectively.

Sequencing: In sequencing our aim is to find the order in which different jobs are to be processed on different machines so that the idle time is minimized, and utilization is optimized.

Sequencing rules for N jobs on one Machine:

Shortest processing time (SPT) Rule: In this rule jobs are sequenced in increasing order of their processing time i.e. minimum time first and max time last. SPT is the best technique for minimizing job flow and average number of jobs in the system.

b. Earliest due date (EDD) Rule: Jobs are sequenced in increasing order of their due date. EDD minimizes the maximum (or average) Tardiness.

c. Critical Ratio rule (CR) Rule: $$CR = \frac{Due\ date}{Processing\ time}$$

Jobs are sequenced in increasing order of their critical ratio (minimum at 1st and maximum at last) CR.

d. Slack time remaining (STR) Rule: $$STR = due\ date - processing\ time$$

Jobs are sequenced in increasing order of their slack time remaining (minimum at 1st and maximum at last).

N jobs on two machines:

Let, A_i = processing time of ith job on machine 1

B_i = processing time of ith job on machine 2

These problems are solved by *Johnson's rule* and steps involved are:

(i). Find out the minimum of Ai and Bi.

(ii). If the minimum is for a particular job on machine A then, perform that job at the start or beginning.

(iii). If the minimum is for a particular job on machine B then, perform that job in the last.

(iv) Strike-off the job which is assigned so that it cannot be considered again.

(v). Continue in the similar manner until all the jobs are assigned.

Example:

Job	A	B	C	D	E	F
Machine I	4	3	9	6	4	9
Machine II	8	5	7	2	3	1

B	A	C	E	D	F

↑
has least time of processing
and it is on machine-II

Sequence:

N jobs on 3 machines (Jackson Rule):

i. Check $(\text{Minimum})_{i1} \geq (\text{maximum})_{i2}$ **and** $(\text{Minimum})_{i3} \geq (\text{Maximum})_{i2}$

At least one must satisfy.

ii. Convert to two machines problem by adding machine 1 and 2, then machine 2 and 3.

iii. Solve using Johnson's rule.

Job Flow time: It is the time from some starting time until that particular job is completed.

Make span time (MST): It is time from when processing begins on the first job in set until the last job is completed.

Tardiness or Lateness: It is the amount of time by which a job is delayed beyond its due date.

Average no. of jobs in system: It is a term used to represent avg. no. of jobs present all the time within the system until one set of job is completed. It is the ratio of total job flow time over make span time

Machine Utilization:

$$\%\text{Utilization} = \frac{\text{Working time}}{\text{Make span time(MST)}} \times 100\%$$

$$\text{Idle time} = \text{Make span time (MST)} - \text{working time (WT)}$$

Line balancing: Assembly line balancing is to know how tasks are to be assigned to workstations, so that the predetermined goal is achieved.

Minimization of the number of workstations and maximization of the production rate are the most common goals.

Efficiency of the system is given by:

$$\eta_{\text{system}} = \frac{\text{Actual output}}{\text{Installed output}}$$

Terms related:

a. Work Element: Every job is completed by a set of operation and each operation which is performed on the job is called work element.

b. Task time (T_1): It is the slandered time required to complete work-element.
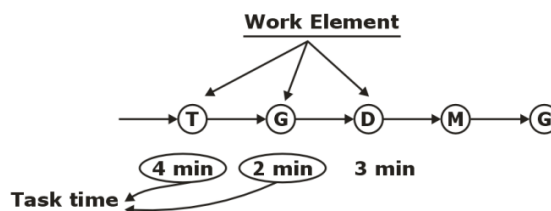
c. Workstation: It is the specific location on the assembly line where the given amount of work elements is completed within a fixed period of time.

$$T_c = 16$$

WS-I ④   WS-II ①   WS-III ③

③ ④   ② ⑤   ② ④
② ③   ⑧   ⑥ ①

TS-I = 12 min   TS-II = 15 min   TS-III = 13 min
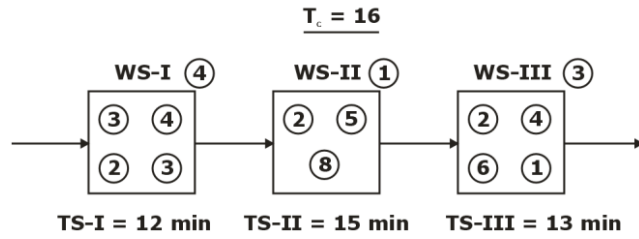
d. Station time (T_{s1}): It is a time required to complete work element assigned in a workstation.

e. Total work content (TWC): It is time required to complete one set of job. It is given by either the summation of all the station time or the summation of all the elemental task time.

$\Sigma T_{si} = \Sigma T_i = 40$ min (in above eq.)

f. Cycle time (T_c): $T_c \geq (T_{si})_{max}$

It is an amount of time for which a job to be assembled remains in a workstation. It is a time gap between two successive products coming out from the assembly line.

$$BD\% = \left(\frac{nT_c - T_{wc}}{nT_c} \right) \times 100$$

Where n: no. of workstation

T_c: cycle time

T_{wc}: total work content

g. Line efficiency (η_L):

$$\eta_L = \frac{T_{wc}}{nT_c} \times 100$$
$$\eta_L = 100 - BD\%$$

h. Smoothness index (SI): It is a term used to represent the load distribution between the different workstations. Compare to a station consuming max. time.

$$S.I. = \sqrt{\sum_{i=1}^{n} \left((T_{si})_{max} - T_{si} \right)^2}$$

i. Minimum of workstation required: $n_{min} = \dfrac{T_{wc}}{T_c}$

CHAPTER-3: PRODUCTION PLANNING AND CONTROL

Master Production Schedule (MPS): It is the complete timetable of our scheduled production in future. It gives information about which product is to be produced when it is to be produced and in what quantity.

Inventory record file: This file gives complete and up to date information about on hand inventory, transit inventory, planned order and scheduled receipt.

Bill of Material (BOM): It gives information about how our product is manufactured, specifying all sub-component item and their sequence of build-up in the final product.
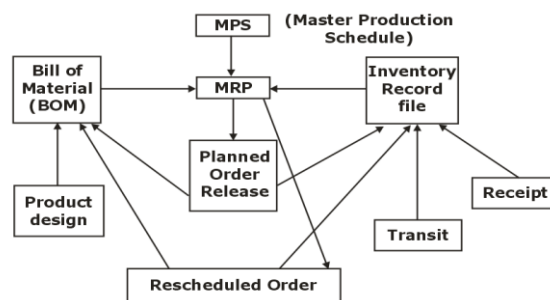
MRP Processing: MRP processing takes the end item requirements specified by the master schedule and "explodes" them into time-phased requirements for assemblies, parts, and raw material using the bill of materials offset by lead times.

Net requirements in Period (t) = Gross requirements in period (t) – Projected inventory in period (t) + Safety stock (SS)

Product structure:
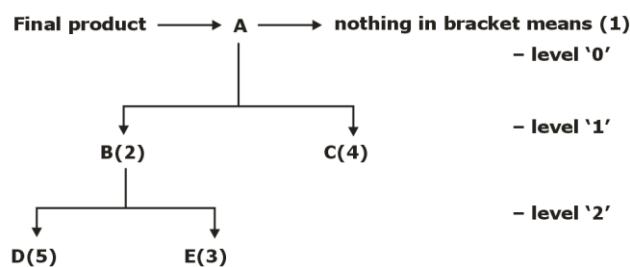
Net Requirement = Gross requirement – (Inventory on hand + Scheduled receipt)

Scheduled receipts: Open orders scheduled to arrive from vendors or elsewhere in the pipeline by the beginning of a period.

Planned-order receipts: The quantity expected to be received by the beginning of the period in which it is shown. Under lot-for-lot ordering, this quantity will equal net requirements. Under lot-size ordering, this quantity may exceed net requirements.

Updating the system: The two basic systems used to update MRP records are regenerative and net change. A regenerative system is updated periodically; a net-change system is continuously updated.

MRP Outputs:

Primary Reports: Production and inventory planning and control are part of primary reports.

Secondary Reports: Performance control, planning, and exceptions belong to secondary reports.

Material resource Planning-II (MRP-II): MRP II is a computer-based system that can create detailed production schedules using real-time data to coordinate the arrival of component materials with machine and labour availability.

Enterprise resource Planning (ERP): ERP stands for Enterprise Resource Planning and refers to software and systems used to plan and manage all the core supply chain, manufacturing, services, financial and other processes of an organization.

Lean Manufacturing: Lean is a methodology to reduce waste in a manufacturing system without sacrificing productivity.

Three types of wastes:

Mura: Unevenness, or waste due to fluctuations in demand.

Muri: Overburden, or waste due to trying to do too much at once.

Muda: Non-value-adding work, or process waste.

Time study:
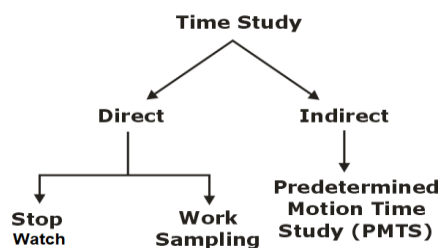
Stopwatch method:

Observed time (OT): It is the time measured or observed by an observer using some measuring device like stopwatch.

Normal time:

N.T. = O.T. × Performance rating factor

N.T. = O.T. × RF

For machines: NT = OT

Standard time: $\boxed{S.T. = N.T. + \text{allowances}}$

$$\text{Allowance} = 20\%(NT)$$

** If nothing is mentioned then NT

Kanban Cards: A Kanban card is small card containing information about a specific part used in production. It is a signal that tells someone upstream to move, purchase, or build more of a component for production. Kanban cards do not reduce inventory. They manage it.

The number of Kanban you need to have in the circuit between downstream and upstream is a simple calculation. It works both for "production" and "move" Kanban.

$$\text{Number of Kanban} = \frac{DD \times LT}{Q}(1 + \%SS)$$

DD = Daily Demand

LT = Lead Time (in Days)

SS = Safety Stock

Q = Quantity in a Container

Production planning & control:
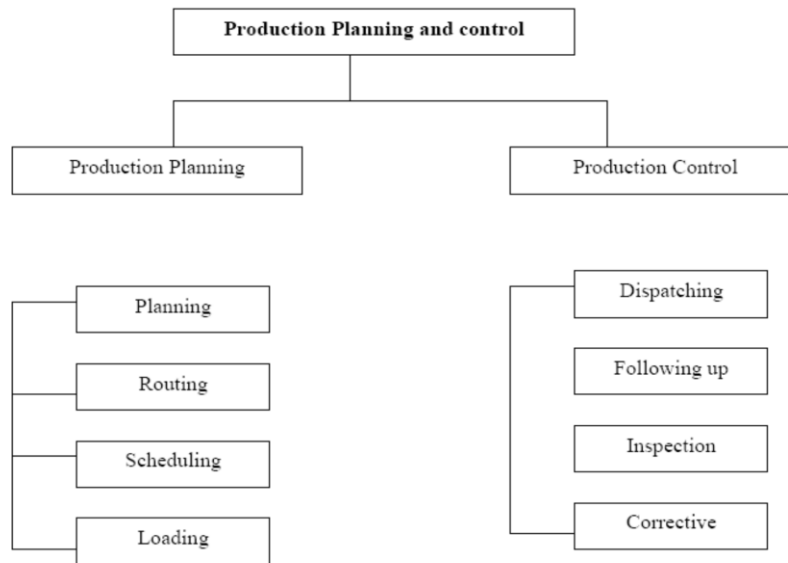
Routing: Various operations, their path and sequence are established. To perform these operations the proper class of machines and personnel required are also worked out. The main aim of routing is to determine the best and cheapest sequence of operations and to ensure that this sequence is strictly followed.

Scheduling: It means working out of time that should be required to perform each operation and the time necessary to perform the entire series as routed, making allowances for all factors concerned. It mainly concerns with time element and priorities of a job.

Loading: The next step is the execution of the scheduled plan as per the route chalked out it includes the assignment of the work to the operators at their machines or workplaces. So, loading determines who will do the work as routing determines where and scheduling determines when it shall be done.

Dispatching: Dispatching involves issue of production orders for starting the operations

Follow up: Every production programming involves determination of the progress of work, removing bottlenecks in the flow of work and ensuring that the productive operations are taking place in accordance with the plans. It spots delays or deviations from the production plans. It helps to reveal detects in routing and scheduling, misunderstanding of orders and instruction,

under loading or overloading of work etc. All problems or deviations are investigated, and remedial measures are undertaken to ensure the completion of work by the planned date.

Inspection: This is mainly to ensure the quality of goods. It can be required as an effective agency of production control.

Corrective measures: Corrective action may involve any of those activities of adjusting the route, rescheduling of work changing the workloads, repairs and maintenance of machinery or equipment, control over inventories of the cause of deviation is the poor performance of the employees. Certain personnel decisions like training, transfer, demotion etc. may have to be taken.

CHAPTER-4: INVENTORY CONTROL

Break-even point analysis: B.E.P. is the volume of production where total cost is equal to total sales and organization neither gains profit nor suffers loss it is also known as no profit no loss point.

Total Cost: $\boxed{TC = (F + vx)}$
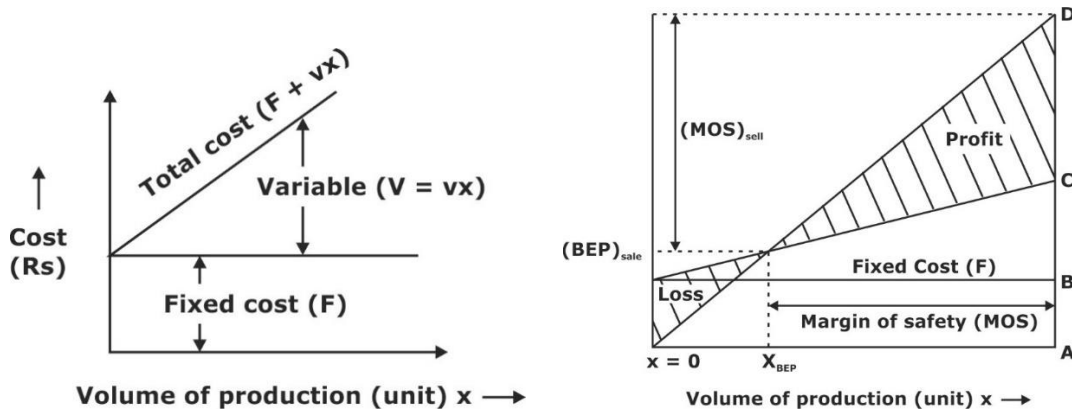
Selling cost: $\boxed{S = F + V + P}$

$\boxed{sx = F + vx + P}$

$sx - vx = F + P$

Break Even point: $\boxed{x = \dfrac{F + P}{(s - v)}}$

At BEP: Profit (P) = 0

$\boxed{x_{BEP} = \dfrac{F}{s - v}}$ units

Break Even sales: $\boxed{(BEP)_{sale} = x_{BEP \cdot s} \times \dfrac{F}{(s - v)} \times s}$

$\boxed{contribution = (s - v) = F + P}$

Profit volume Ratio (P/V ratio): $\boxed{\left(\dfrac{P}{V}\right)_{ratio} = \dfrac{CM}{S} = \dfrac{S - V}{S} = \dfrac{s - v}{s}}$

Margin of safety: $\boxed{(MOS)_{sale} = S_x - S_{BEP}}$

$\boxed{MOS\% = \left(\dfrac{S_x - S_{BEP}}{S_x}\right) \times 100}$

Profit volume graph: It is a term used to represent profitability related to sells. It is the ratio of contribution margin to the volume of sales. This ratio always remains const. For a particular product and is normally preferred when the organization deals in multiproduct.
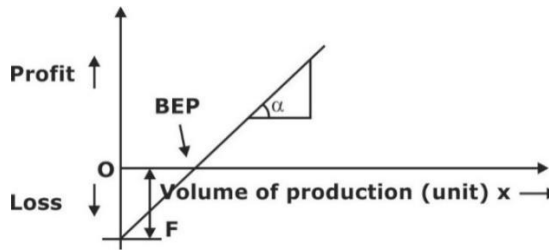
$$\left(\frac{P}{V}\right)_{ratio} = \frac{CM}{S} = \frac{S-V}{S} = \frac{s-v}{s}$$

$$\left(\frac{P}{V}\right)_{ratio} = \frac{F+P\uparrow}{S\uparrow} = \frac{s-v}{s}$$

$$\frac{F+P_1}{S_1} = \frac{F+P_2}{S_2} = \frac{F}{S_{BEP}} = \frac{s-v}{s}$$

Inventory control: Inventory can be termed as stock in-hand at a given point of time which may be held for the purpose of later use or sell. It has an economic value, and it may include raw material, work in process, semi-finished or sub assembly and final product.

Dependent: The demand for these items is directly related or linked to any other item, usually of a higher level of which it becomes a part.

Independent: The demand for this item is not directly related or linked to any other item it is difficult to compute and it is projected with the help of forecasting.

Inventory review system:
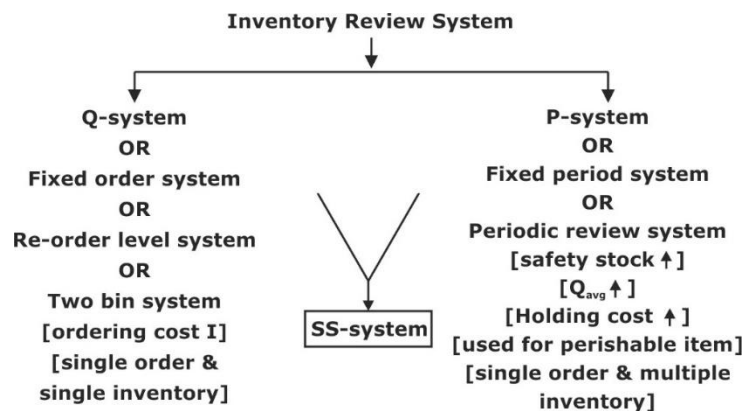
Fixed order system: In these system as inventory decreases to reorder level a fresh order for fixed quantity is placed at that point. In these system sizes of order is fixed but the time of order is variable.

Fixed period system: In this system, inventory is reviewed after fixed period and a fresh order for variable quantity is placed at that point. In these system sizes of order is variable but the time of order is fixed.

Type of inventory costs:

Inventory Cost
- Purchase cost
- Ordering OR Set-up cost
- Holding OR Carring cost
- Stortage OR Stockout cost
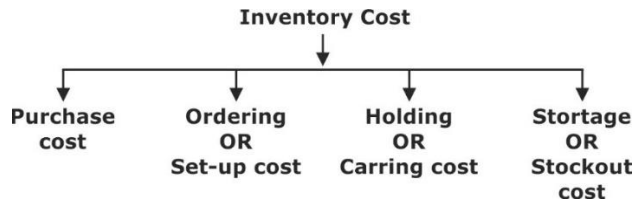
Purchase Cost: $P.C. = \text{no. of unit} \times \dfrac{cost}{unit}$

Ordering Cost: $O.C. = \text{no. of order} \times \dfrac{cost}{order}$

Set up cost: $S.C. = \text{no. of set up} \times \dfrac{cost}{set\ up}$

Holding or Carrying Cost: $H.C. = \text{Average invectonry for a period} \times \dfrac{Holding\ cost}{unit \times time}$

Shortage or Stock out cost: $S.C. = \text{Average number of unit short} \times \dfrac{Stortage\ cost}{unit \times time}$

Deterministic Inventory Models:

Economic Order Quantity for Harris Wilson model or Infinite Rate of Replenishment
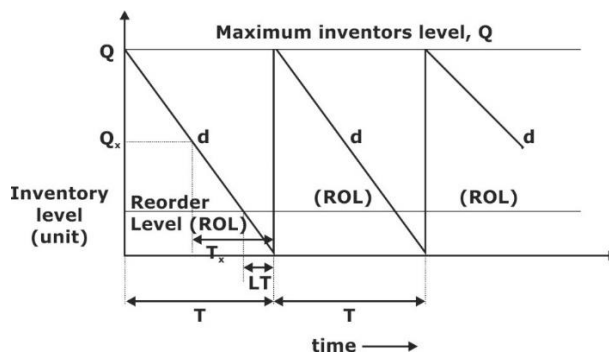
$$\text{Total annual cost } (TAC) = D.C. \text{ (constant)} + \left(\dfrac{D}{Q}\right) \cdot C_0 + \left(\dfrac{Q}{2}\right) C_h$$

$$Q^* = \sqrt{\dfrac{2DC_0}{C_h}}$$

$TVC^* = \sqrt{2DC_0C_h}$ only for EOQ

Model sensitivity or model Robustness:

$$\frac{TIC(Q)}{TIC(Q^*)} = \frac{1}{2}\left[k + \frac{1}{k}\right]$$

Production or Build up model:
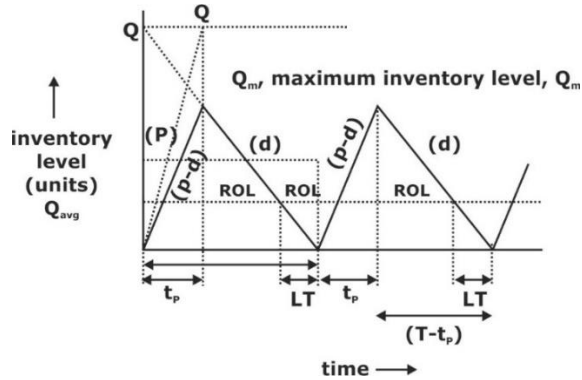
Economic order quantity: $\boxed{Q^* = \sqrt{\frac{2DC_0}{C_h}}\sqrt{\frac{P}{(P-d)}}}$ Production factor > 1

Maximum inventory: $\boxed{Q_m = Q\left(\frac{P-d}{P}\right)}$ $Q_m < Q$

Total inventory cost: $\boxed{TIC^* = \sqrt{2DC_0C_h}\sqrt{\frac{(P-d)}{P}}}$

Shortage or Stockout or Back order:
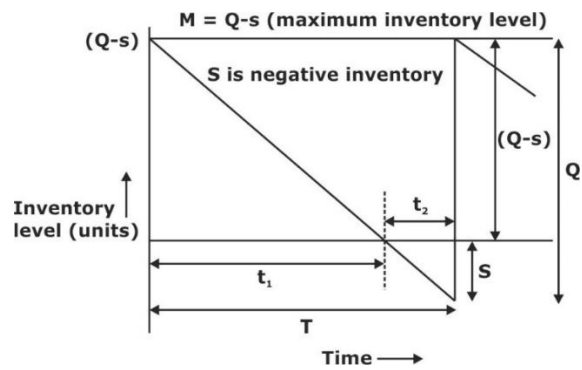
Economic order quantity: $\boxed{Q^* = \sqrt{\frac{2DC_0}{C_h}}\sqrt{\frac{C_b + C_h}{C_b}}}$

Total inventory cost: $\boxed{TIC^* = \sqrt{2D\,C_0C_h}\sqrt{\frac{C_b}{C_b + C_h}}}$

Stock out quantity: $\boxed{S^* = Q^*\left[\frac{C_h}{C_b + C_h}\right]}$

Maximum inventory level: $M^* = Q^* \left(\dfrac{C_b}{C_b + C_h} \right)$

Production and Shortage:

Economic order quantity: $Q^* = \sqrt{\dfrac{2DC_0}{C_h}} \sqrt{\dfrac{P}{(P - d)}} \sqrt{\dfrac{C_b + C_h}{C_b}}$

Total inventory cost: $TIC^* = \sqrt{2DC_0C_h} \sqrt{\dfrac{(P - d)}{P}} \sqrt{\dfrac{C_b}{C_b + C_h}}$

Probabilistic Models:
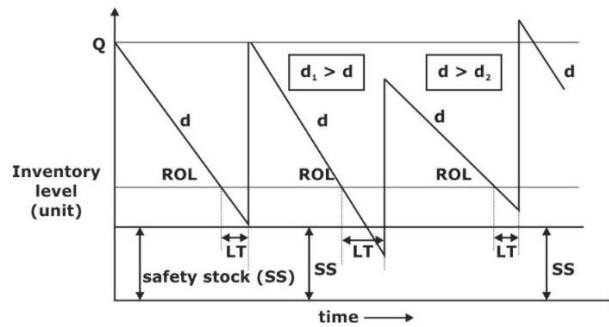
$\boxed{Re - order\ level = Average\ demand\ during\ LT\ (ADDLT) + Safety\ Stock\ (SS)}$

$ADDLT = \bar{x}$ or $\mu = LT \times d$

$\Rightarrow \boxed{ROL = LT \times d + SS}$

$Q_{max} = Q + SS$

$Q_{min} = SS$

$\boxed{Q_{avg} = \dfrac{Q}{2} + SS}$

Demand Profit or Static Inventory Model:

D – demand, S – supply, P – profit/unit **and** l – loss/unit

If $\underset{300}{D} > \underset{200}{S}$, (D – S) P = loss $\rightarrow$ potential profit loss.

If D < S (S – D) l = loss

$$\boxed{\begin{array}{c} P = S_P - C + C_b \\ \hline l = C - C_s + C_n \end{array}}$$

P: potential profit loss per unit for not meeting the demand.

S_P: selling price per unit

C: purchasing cost per unit

C_b: back order / shortage cost / good will loss per unit

l: unsold item loss per unit

C_s: Salvage / scrap value per unit

C_h: holding cost per unit

Service Level Model: 95% service level is the standard value and it mean that 95% of the customer demand on and avg. is fulfilled during lead time and only 5% of the customer's order are rejected because of stock out during lead time.

$$\text{Service level} = \frac{\text{Number of units supplied without delay}}{\text{total number of units demanded}}$$

$$\boxed{ROL = \bar{x} + z\sigma}$$

$\bar{x}$ avg. demand during lead time = LT × d

σ: std deviation for the demand variation during lead time.

z: standard normal variate whose value depends upon the service level required.

Z	Service level (%)
0.84	80%
1.28	90%
1.645	95%
2.33	99%
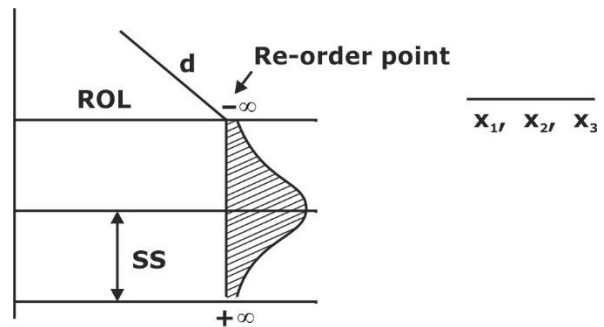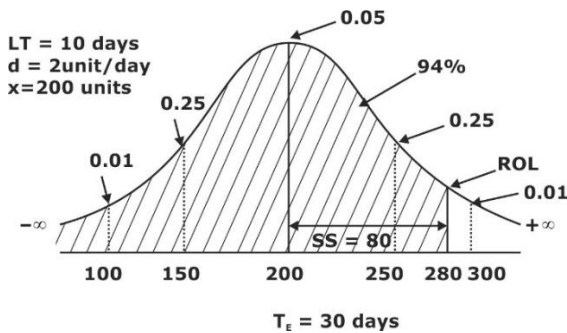
Standard deviation: $\sigma = \sqrt{\dfrac{(\bar{x} - x_1)^2 + (\bar{x} - x_2)^2 + (\bar{x} - x_3)^2}{3}}$

$$\boxed{\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}}$$

Newspaper boy problem: G = gross profit for each unit sold.

L = Loss for each unit not sold.

$$P(s) = \frac{G}{G+L}$$
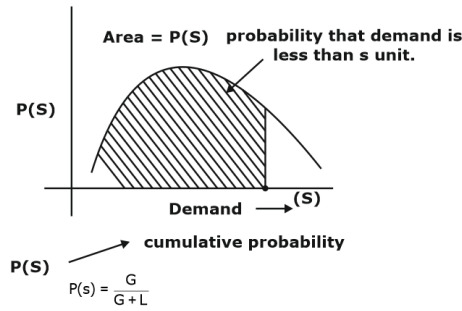
The conditions: $P(S-1) < \left(\dfrac{G}{G+L}\right) \le P(S)$

Inventory classification and control: ABC Control (Always better control): In ABC control, inventory items are classified into A, B & C category depending upon their usage value. It is based on Pareto law or 80 – 20 law. A category inventory items have almost 50-60 % of total usage value and stoke is kept just 10-20 % of such items.

	Usage %	Item %	
A	50 – 60%	10 – 20%	Vital few trivial many
B	30 – 40%	30 – 40%	
C	10 – 20%	50 – 60%	

VED (Vital essential & desirable): Inventory is classified based on importance of inventory for the production system.

HML (High Medium & Law): Inventory are classified based on unit price of inventory.

SDE (Source difficult & Easy): Inventory are classified based on availability of inventory for the production system e.g. Nuclear power plant.

CHAPTER 5: OPERATION RESEARCH

Linear programming:

L.P is used for optimization of our limited resources when there are no of alternate solution possible for the problem.

Max. $Z = C_1x_1 + C_2x_2 + C_3x_3 + C_4x_4 + \ldots\ldots\ldots\ldots + C_nx_n$ – objective fu^n

$$\text{Constaints or Condition}\begin{cases} a_{11}\,x_1 + a_{12}\,x_2 + \ldots\ldots\ldots + a_{1n}x_n\,x \le b_1 \\ a_{21}\,x_1 + a_{22}\,x_2 + \ldots\ldots\ldots + a_{2n}x_n\,x \le b_2 \\ \quad\vdots\qquad\quad\vdots\qquad\qquad\quad\vdots \\ a_{m1}\,x_1 + a_{m2}\,x_2 + \ldots\ldots\ldots + a_{mn}x_n\,x \le b_m \end{cases}$$

Non-negative condition $\rightarrow x_1, x_2 \ldots\ldots x_n \ge 0$

Where a_{ij}, b_i, c_j are constant & x_j is variable

$i = 1, 2, 3 \ldots\ldots\ldots n$

$j = 1, 2, \ldots\ldots\ldots\ldots n$

a_{ij}: technological constant or substitution

b_i: resource value

c_j: Profit coefficient

x_j: decision or choice variable

Graphical Method:

- Identify the problem, define decision variables, objective function, and constraints.

- Draw a graph that include all the constraints and identify the common feasible region.

- Find out the point within the feasible region which optimizes the objective function. These point gives the final optimum sol^n.

Special cases:

a. Infinite or multi optimal solution: We get infinite solutions if same value of objective function is obtained at two or more points and these points lie on same straight line.

b. No solution or infeasibility:
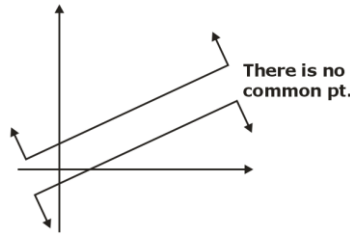
c. Unbounded solution:
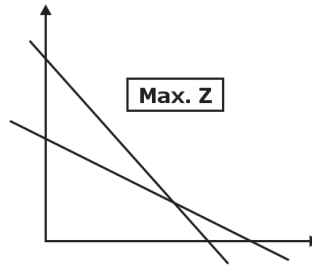
Simplex method:

- All the resource value for the given constraint should be non-negative ($b_j > 0$).

- All the inequalities of the given constant should be converted into equalities.

    $3x_1 + x_2 \leq 15$

    $5x_1 + 2x_3 \geq 80$

- Each of the decision variable for the objective fu^n and constraint should be linear and non-negative.

    $x_j \geq 0$

    If there are m equality constraint & n no. of variable, then we need to put (n-m) variable equal 'o' known as non-basic variable and solve the remaining (m) basic variable to give basic solution.

- Calculate Δ_j value as a difference of c_j and z_j and it is termed as **"Net evaluation row"** or **"Net opportunity cost row".** The value of Δ_j row give the amount of increase or

decrease in the objective fu^n if one unit represented by the column head is brought into the current solution.

Negative or zero when LP is for maximization

Positive or zero when LP is for minimization.

- In case of maximization, we select the Highest +ve in the Δ_j row and the selected row is called key column & variable in column head as incoming variable.

- Now divide bi values by the corresponding elements of key column to get replacement ratio column in these columns we select the min positive value selected row is called key row with the variable in the row as outgoing variable.

- The element at the intersection of key column & key row is termed as key element.

- Key element is converted to unity by multiplying or dividing key row by a common multiplying or dividing key row by a common multiplying.

- All the element in the key column are made zero except key element which will be unity these is done by adding or subtracting the proper multiples of key row from other row.

- In the new table outgoing variable is replaced by incoming variable.

Special cases:

a. Infinite or multi optimum solution: When a non-basic variable in an optimum solution has zero value for Δ_j row then the solution is not unique, and it indicates that the problem has infinite no. of solution.

b. Unbounded solution: If in a case all the value in the replacement ratio column are either −ve or infinite then the solution terminates, and it indicates that the problem has unbounded solution.

c. No-solution infeasibility: When in the final solution artificial variable remains in the basis then there is no feasible solution to the problem.

d. Degenerate solution: When one or more of the basic variables becomes equal to zero during calculation then the solution is called degenerate & the condition is known as degeneracy. In a degenerate solution the no. of basic variables becomes less than equality constraint.

Duality:

Primal   **Dual**

$\text{maximum} \longleftrightarrow \text{Minimum}$

$n \longleftrightarrow m$

$m \longleftrightarrow n$

$b_i \longleftrightarrow c_j$

$c_j \longleftrightarrow b_i$

$\leq$ type constraint ↔ nonnegative variable

– type constraint ↔ Unremitted sign variable.

TRANSPORTATION

• The aim of transportation problem is to reduce the cost of transporting commodities from different suppliers to different destinations.

• The number of sources is m and number of destinations are n.

• Number of variables are m × n.

• Total number of constraints are m+n.

$$\text{min. } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} ci_j.xi_j$$

• If total supply = total demand → Balanced

• Otherwise→ Unbalanced then balance it by adding dummy source or destination.

Basic feasible solution methods:

(i). North -West Corner Method

(ii). Row-Minima Method

(iii). Column-Minima Method

(iv). Least Cost Method

(v). Vogel's Approximation method (VAM)

OPTIMALITY:

Modified Distribution (MODI-Method) or U-V Method to check optimality:

(a). Develop cost matrix for allocated cell only.

(b). Computing U_i & V_j value by taking $V_1 = 0$.

(c). Develop $U_i + V_j$ matrix for unallocated cells.

(d). Subtract the cell value of $U_i + V_j$ matrix for unallocated cell from original cost matrix to get cell evaluation matrix.

(e). In the cell evaluation matrix if any of the cell value is negative then the current solution is not optimum.

→ In the cell evaluation matrix identify the cell with the most negative value mark it and it is called identified cell.

→ Trace a path in matrix such that it starts from the identified cell and corner of the path should already have allocation make identified cell as +ve and each other cell at the corner of the path alternatively −ve, +ve, −ve and so on.

ASSIGNMENT MODEL:

Assignment problem is a special case of transportation, where matrix must be a square matrix and, in each row and each column, only one allocation is possible.

$m = n$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ and } \sum_{i=1}^{n} x_{ij} = 1$$

$x_{ij} = 0 \text{ or } 1$

All $a_i = 1$ and $b_j = 1$

Hungarian Method for Solving Assignment Problem:

The Hungarian method of assignment provides us with an efficient method of finding the optimal solution without having to make a-direct comparison of every solution. It works on the principle of reducing the given cost matrix to a matrix of opportunity costs.

Opportunity cost show the relative penalties associated with assigning resources to an activity as opposed to making the best or least cost assignment. If we can reduce the cost matrix to the extent of having at least one zero in each row and column, it will be possible to make optimal assignment.

CHAPTER-6: QUEUING THEORY

Queuing theory: The aim of queuing theory is achievement of economic balance between cost of providing service.

Characteristics of queuing model:

Arrival rate / Arrival Pattern (λ)**:** It is assumed to follow Poisson distribution.

Service Rate or Service Pattern (μ): It is assumed to follow exponential distribution (negative exponential).

Service Rule or Service Order: (1) FIFO or FCFS (2) UFO or LCFS (3) SIRO (4) Priority treatment (G-D).

Customer Attitude:

(i). Jockeying: Customer keep on changing queue in hope to get service faster.

Balking: Customer does not join the queue & leave the system without joining as queue is very long.

(iii). Reneging: Customer joins the queue for short duration & then leave the system as queue is very slow.

(iv). Cheater: Customer takes illegal means like fighting, bribing, etc. in hope to get service faster.

Representation of queuing model:
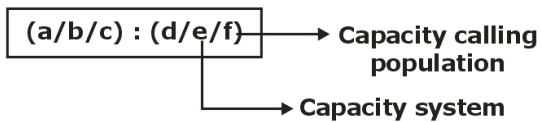
a: probability distribution for arrival pattern

b: probability distribution for service pattern

c: no of server within the system

d: service rule or service order

e: size or capacity of system

f: size or capacity of calling population or input source.

(M/M/1): (FIFO/∞/∞) Model:

Arrival Rate: λ

Inter arrival rate (exponential) or time $= \dfrac{1}{\lambda}$

Service rate: μ

If λ $\leq$ μ → System works

λ < μ → prefer

Probability that the server is busy:

Traffic intensity: $\rho = \dfrac{\text{Arrival rate}}{\text{Service rate}} = \dfrac{\lambda}{\mu}$

It also indicates the probability that a new customer will have to wait.

Probability that the system is idle or Prob. of zero customer in the system.

$P_0 = 1 - \rho$

Probability of having exactly 'n' customer in the system: $P_n = \rho^n \cdot P_0$

Avg. no. of customers in the system: $L_s = \dfrac{\rho}{1-\rho}$

Avg. no. of customers in the queue: $L_q = \dfrac{\rho^2}{1-\rho} = L_s - \rho = L_s \cdot \rho$

Avg. length of non-empty queue or Avg. length of queue containing at least one

Customer:

$L_q = \dfrac{1}{1-\rho}$

Probability of 'n' arrival in the system during period T:

$P(n, T) = \dfrac{e^{-\lambda T} \cdot (\lambda T)^n}{n_1}$

Probability that more than T time period is required to service a customer

$P = e^{-\mu T}$

Probability that the waiting time in the queue is greater than 'T'

$P(w_q > T) = \rho\, e^{-\left(\frac{T}{w_s}\right)}$

Probability that the waiting time in the system is greater than T

$P(w_s > T) = e^{-\left(\frac{T}{w_s}\right)}$

CHAPTER-7: PERT – CPM

PERT stands for Program Evolution and Review Technique while CPM – Critical Path Method.

Project: It is a group or combination of interrelated activities that must be executed in a certain fixed order before the entire task is completed.

Event: It denotes the point of time or accomplishment occurring at a moment and is used to denote the starting and end point of an activity.

Activity: It is the recognizable part of a project which consume time and resources for its completion.

Network diagram: It is a graphical representation of the logical sequence in which different activities are interrelated to each other white completing.

PERT (β-distribution): 3-time estimate.

(i). Optimistic time (t_0 or q): It is the minimum time required to complete an activity when everything goes acc. to the plan.

(ii). Pessimistic time (t_P or b): It is the maximum time required to complete an activity when everything goes against the plan.

(iii). Most likely time (t_m or m): It is a time required to complete an activity when executed under normal working condition.
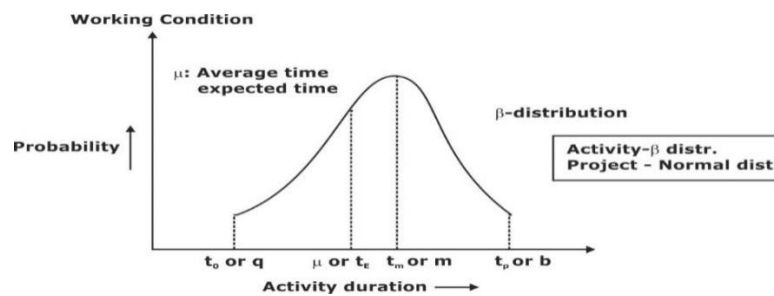
$$\mu \text{ or } t_E = \frac{a + 4m + b}{6} = \frac{t_0 + 4t_m + t_P}{6}$$

$$\text{Variance} = \sigma^2 = \left(\frac{b-a}{6}\right)^2 = \left(\frac{t_P - t_0}{6}\right)^2$$

Critical Path: It is the maximum time-consuming path from the 1st event to last event in a network diagram the time taken along a critical path is termed as expected project completion time (T_E). The activity along the critical path are termed as critical activity and are represented by double line arrow ($\Rightarrow$)

Probability of completing project within scheduled time:

$$Z = \frac{T_S - T_E}{\sigma}$$
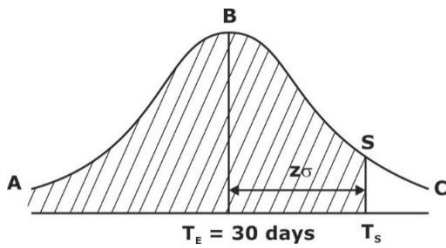
$$\boxed{P(T_S) = \frac{\text{Area (ABS)}}{\text{Area (ABC)}}}$$

$\text{ROL} = \bar{X} + Z\sigma$

$T_S = T_E + Z\sigma$

$Z = 1.645$

Prob = 95%

$\underbrace{\sigma_1}$, σ_2, $\underbrace{\sigma_3}$, σ_4, σ_5, $\underbrace{\sigma_6}$, σ_7, $\underbrace{\sigma_8}$

Standard deviation along critical path:

$$\sigma = \sqrt{\sigma_1^2 + \sigma_3^2 + \sigma_6^2 + \sigma_8^2}$$

Critical Path Method:

(i). Forward Pass Computation: In these we compute the time by which the event is expected to be completed at the earliest.
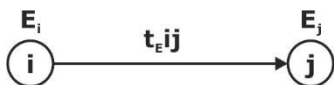
$$\boxed{E_j = \text{max. of all } [E_i + t_{E,ij}]}$$

where E_i: Earlist expected time for event i

E_j: Earliest expected time for event j

t_{Eij} : expected time for activity iJ

(ii). Backward Pass Computation:
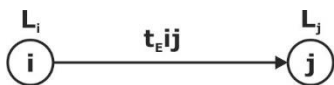
In these we compute the time by which an event must be completed at the latest.

$$\boxed{L_i = \text{minimum of all } [L_j - t_{E\,ij}]}$$

where L_i: Latest allowable time for event i

L_j: Latest allowable time for event j

$t_{E\,ij}$: Expected time for activity i – j

For any activity to be critical following 3 condition must be satisfied.
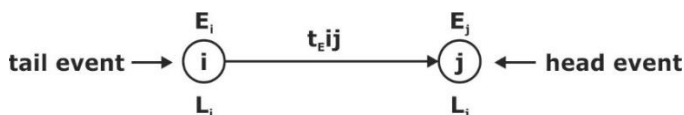
(i) Head event slack = 0

$$L_j - E_j = 0$$

(ii). Tail event slack = 0

$$Li - Ei = 0$$

(iii). $$L_j = L_i = E_j - E_i = t_{E,iJ}$$

Critical path is termed as critical because if any activity along this path is delayed by certain amount of time the whole project is delayed by the same amount of time.

SLACK & FLOAT:
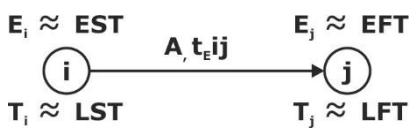
$$E_i + t_{E,ij} = E_j$$
$$EST + t_{E,ij} = EFT$$

EFT: Earliest finish time for activity A

LST: Latest start time for activity A

Generally, Slack $\rightarrow$ PERT (event)

Float $\rightarrow$ CPM (activity)

$$S_j = L_j - E_j$$ and $$S_i = L_i - E_i$$

(i) Slack: Amount of time by which particular event can be delayed without delaying the project schedule.

(ii) Float

(a). Total Float (TF): It denotes the amount of time by which an activity can be delayed without delaying the project completion time.
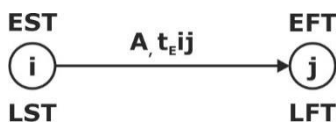
$$TF = LFT - IFT = LST - EST$$

(b) Free Float (FF): It is extra time by which an activity can be delayed so that the succeeding activity can be started at their earliest start time.

$$FF = TF - \text{Head event slack}$$

(c) Independent float (IF): It is the amount of time which can be used without affecting either the head or the tail event.

$$IF = FF - \text{tail event slack}$$

For any activity: $\boxed{TF \geq FF \geq IF}$

For critical path: $\boxed{TF = FF = IF = 0}$

Crashing or Time Cost Model:
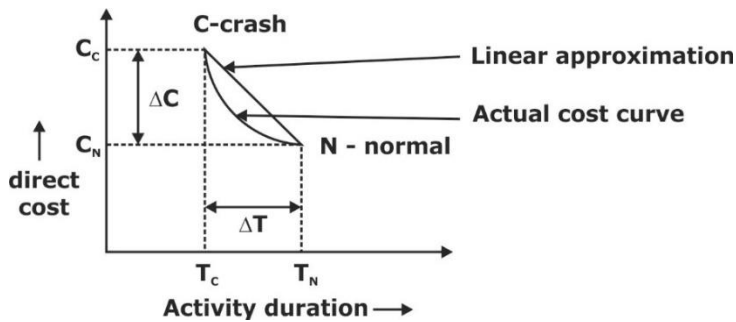
Crash time is minimum activity duration to which a activity can be compressed by increasing the resources and hence by increasing direct cost.

The slope of the line gives the amount of increase in the direct cost per unit time for crashing an activity.

$$\boxed{\text{Cost time slope} = \frac{\Delta C}{\Delta T} = \frac{C_C - C_N}{T_N - T_C}}$$

The objective of crashing a network is to determine optimum project duration corresponding to minimum cost of project and the steps involved are:

(i). In the critical path select the critical activity having minimum cost slope.

(ii). Reduce the duration of this activity by one-time unit.

(iii). Revise the network diagram by adjusting the time & cost of crashed activity again find critical path, project duration and total cost of project.

(iv). If the optimum project duration is obtained, then stop otherwise repeat from step 1.

****

Important Formula (Thermodynamics)

CHAPTER-1 BASICS OF THERMODYNAMICS

INTRODUCTION

Thermodynamics is the branch of science which deals with energy interaction between one body to another body and the effect of the energy transfer on them.

BASIC TERMS

1. SYSTEM

A system is defined as a quantity of matter (control mass) or a region (control volume) in space chosen for study.

2. SURROUNDINGS

Everything external to the system is called the surroundings or the environment.

3. BOUNDARY

- A real or imaginary surface that separates the system from its surroundings
- Mathematically, the boundary has zero thickness, and thus it can neither contain any mass nor occupy any volume in space.
- The boundary may be either fixed or moving.

4. Universe

- A system and its surroundings together comprise a universe.
- Universe consist of everything, so everything happening whether energy transfer or transformation or losses remain inside universe.

TYPES OF SYSTEM

A. Depending upon the mass and energy interaction.

   a. Open System

- when there is mass as well as energy can transfer across the boundary.
- Example - air compressor, boiler, pump, IC engine with valve open etc.

   b. Closed system

- if its mass remains fixed but there may be energy transfer into or out of the system
- Example – a certain quantity of fluid in a completely enclosed cylinder, Tea in kettle etc.

   c. Isolated system

- when there is no mass and energy interaction between the system and the surroundings.
- Example – thermo-flask, Universe (since all energy interaction taking place between systems and surrounding which is the part of universe)
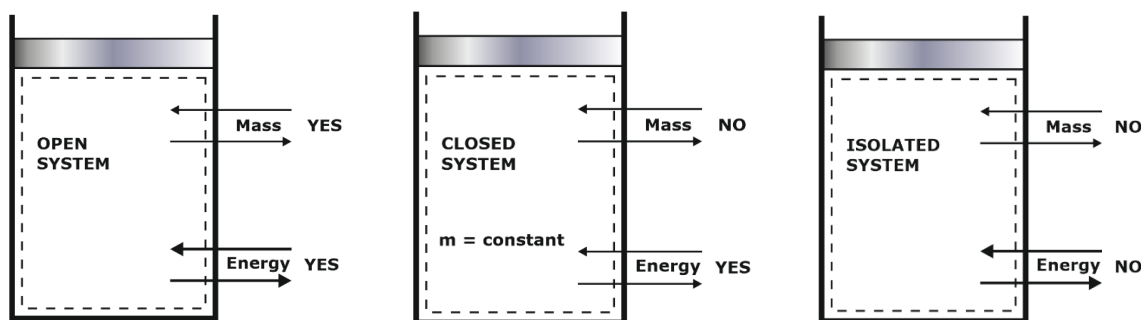
B. Depending upon the Chemical Composition

a. Homogeneous System

- A quantity of matter homogeneously distributed throughout system with respect to its chemical composition and physical structure is called homogenous system.
- A system consisting of a single phase is called a homogeneous system.

b. Heterogeneous System

- A system consisting of more than one phase is known as a heterogeneous system.
- Heterogenous system is the combination of two or more homogeneous system.

Macroscopic Approach

- In the macroscopic approach, a certain quantity of matter is considered, without molecular level being taken into account.
- It is also known as **classical approach**. All the properties will be the average of the properties of each molecule passing through that space.

Microscopic Approach

- From the microscopic point of view, matter is composed of numbers of molecules and study is made on molecular level.
- It is also known as **statistical approach**. In this approach, we sit on a particular molecules and whatever changes occurs is recorded and average of this will be our required properties.

CONCEPT OF CONTINUUM

- The concept of continuum is an idealization of the continuous description of matter where the properties of the matter are considered as continuous functions of space i.e. the space between the molecules (mean free path) is almost zero or very small as compare to the size of the system.
- At very low pressure (i.e. space outside earth atmosphere) gas is studied by **Gas rarefied theory** in which microscopic approach is used.
- Example: Density is calculated assuming continuum concept, so we assumed same uniform density all over the material.

Extensive Properties

- The properties which dependent on mass is known as extensive properties.
- It is also known as Extrinsic properties.
- Example – Volume, Enthalpy, Weight, All forms of energy etc.

Intensive Properties

- extensive properties per unit mass, are intensive properties.

- These properties do not dependent on mass of the system.

- It is system property independent of quantity.

- It is also known as intrinsic properties.

- Example– Temperature, Pressure, Density, Viscosity, Ratio of two extensive properties, specific energy, specific enthalpy etc.

THERMODYNAMIC EQUILIBRIUM

- A system is said to exist in a state of thermodynamic equilibrium when no change in any macroscopic property is registered.

- Above condition satisfy, if the following three types of equilibrium are satisfied:

  a. Mechanical Equilibrium

  b. Chemical equilibrium

  c. Thermal equilibrium

- When the conditions for any one of the three types of equilibrium are not satisfied, a system is said to be in a **non-equilibrium** state.

- Thermodynamic properties defined only for thermodynamic equilibrium states.

State

- When all the properties of a system have definite values, then it is said to exist at a definite state.

- The number of properties required to fix the state of a system is given by the state postulate.

Change of state

Any operation in which one or more of the properties of a system changes is called a change of state.

Process

Any change that a system undergoes from one equilibrium state to another equilibrium state.

Path

The succession of states passed through during a change of state is called the path of the change of state. Or curve by joining the two equilibrium states of the system is known as Path.

Quasi-Static Process

- 'Quasi' meaning 'almost' and 'Static' means 'at rest'.

- Infinite slowness is the characteristic feature of a quasi-static process. Such a process, which is a locus of all the equilibrium states is known as a quasi-static process.

Cycle

A thermodynamic cycle is defined as a series of state changes such that the final state is identical with the initial state.

Reversible and Irreversible Process

- Reversible process is subset of Quasi static process which is also frictionless.

- A reversible process is defined as a process that can be reversed without leaving any trace on the surroundings. Processes that are not reversible are called irreversible processes.

- While plotting the curve, reversible processes are shown by continuous line or curve whereas irreversible processes are shown by dotted line or curve.

- Reversible processes actually do not occur in nature. They are merely idealizations of actual processes.

Point function and Path function

- If $\int_{1}^{2} d\varphi = \varphi_2 - \varphi_1$, hence φ depend only on Initial and final states. φ is called Point Function

- All thermodynamic Properties are point function.

- If $\int_{1}^{2} d\varphi \neq \varphi_2 - \varphi_1$, hence it depends on Path followed by the process irrespective of their end point, is known as path function.

- Path function are inexact differential, so we use δφ instead of dφ

IDEAL GAS EQUATION

- Ideal (perfect) gas equation is a special equation of state, which is applicable to ideal gases.

- The molecular forces of attraction between gas molecules are zero, a gas is called a **perfect gas**.

- In addition, the volume of the molecules should be negligible compared to total volume for a perfect gas.

- The perfect or ideal gas equation of state is given by: $PV = n\bar{R}T$ or $PV = mRT$

  Where   P = Absolute Pressure = atmospheric pressure + Gauge pressure (in pascal)

  V = Volume in m^3

  $\bar{R}$ = Universal Gas constant = 8.314KJ/Kmol-K

  T = Absolute temperature in kelvin

  n = number of moles (in k-mol)

  R = characteristic gas constant $R = \dfrac{\bar{R}}{\text{Molecular weight}} = \dfrac{\bar{R}}{M}$

  m = weight (in kg)

- Deviations from ideal behaviour are observed particularly at high pressures or low temperatures.

- A gas which does not obey the ideal gas equation, PV = nRT strictly at all temperatures and pressures is said to be a real gas.

Boyle's Law

Boyle's law states that when the temperature is kept constant, the volume of a given mass of gas varies inversely with the pressure, $P \propto \dfrac{1}{V}$

Charles Law

Charles's law, if the pressure remains constant then volume occupied by a fixed amount of gas is directly proportional to its absolute temperature, $V \propto T$

Some important values for air

Specific heat at constant pressure = C_P = 1.005KJ/ kg – K

Specific heat at constant Volume = C_v = 0.718KJ/ kg – K

Specific heat ratio = $\gamma = \dfrac{C_P}{C_V} = 1.4$

Characteristic gas constant, $R = \dfrac{8.314}{29} = 0.287 \dfrac{kJ}{kg - K}$

Mayer's equation $= C_P - C_V = R$

by using Mayer's equation and specific heat ratio, $C_P = \dfrac{R\gamma}{\gamma - 1}$, $\quad C_V = \dfrac{R}{\gamma - 1}$

PURE SUBSTANCE

- one that is homogeneous and invariable in chemical composition throughout its mass.
- The relative proportions of the chemical elements constituting the substance are also constant.
- Examples: Atmospheric air, steam-water mixture and combustion products of a fuel etc.
- But the mixture of air and liquid air is not a pure substance, since the relative proportions of oxygen and nitrogen differ in gas and liquid phases in equilibrium.

GIBBS PHASE RULE

It is an empirical formula to calculate the degree of freedom when number of component and number of phases are known in a given substance. $P + F = C + 2$

Where, P = number of phases, F = degree of freedom, C = number of components

DEGREE OF FREEDOM

- It is defined as the minimum number of independent intensive variables required to fix the position of any system.
- NOTE – degree of freedom of triple point is ZERO
- NOTE – degree of freedom of Critical point is ZERO

ZEROTH LAW OF THERMODYNAMICS

- It is the basic law of thermodynamics which define the temperature.
- Zeroth law states that: "When a body A is in thermal equilibrium with a body B, and also separately with a body C, then B and C will be in thermal equilibrium with each other".
- It is the basis of temperature measurement. Zeroth law comes after 1st law of thermodynamic.

THERMOMETRIC PROPERTIES

- The selected physical characteristic of the body which changes with temperature, is called the thermometric property.
- the reference body which is used in the determination of temperature is called the thermometer.
- For linearly varying thermometer property **T = a*N** where **a** is random constant & **N** is the thermometric property

$$\frac{T_1}{T_2} = \frac{N_1}{N_2}$$

- Five different kinds of thermometer, each with its own thermometric property

Thermometer	Thermometric property	Symbol
1 Constant volume gas thermometer	Pressure	p
2. Constant pressure gas thermometer	Volume	V
3. Electrical resistance thermometer	Resistance	R
4. Thermocouple	Voltage/ EMF.	e
5. Mercury-in-glass thermometer	Length/ Volume	L

TEMPERATURE SCALE

   a. **Method Used Before 1954**

   - Two fixed points were chosen and intermediate point are interpolated as straight line.

   - Two points are, ice point and steam point selected at one atmosphere pressure, and

   b. **Method in Use After 1954**

   - only one fixed point has been in use, viz. the triple point of water, the state at which ice, liquid water and water vapour co-exist in equilibrium.

   - The temperature at which this state exists is arbitrarily assigned the value of 273.16 degrees Kelvin, or 273.16 K

ENERGY

A closed System and its Surroundings can interact by work transfer and Heat transfer, whereas open system can interact by work, Heat and mass transfer (because mass also carries energy).

REPRESENT OF ALL PROCESSES IN SINGLE P-V GRAPH

Any process can be represented as $PV^K = C$
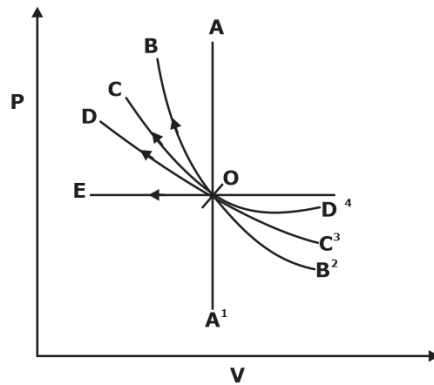
   OA → Constant Volume → $k = \infty$

   OB→ Reversible adiabatic → $k = \gamma$

   OC→ Reversible Polytropic → $k = n$

   OD→ Isothermal → $k = 1$

   OE→ Constant Process → $k = 0$

WORK TRANSFER

- Work can be defined as force multiplied by distance moved by object in the direction of applied load; we call it as mechanical work.

- "Work is said to be done by a system if the sole effect on things external to the system can be used to raising of a weight".

- **Sign Convention**- Work done by system is taken as Positive and work done on system is negative.

NON-FLOW OR CLOSED SYSTEM WORK FOR VARIOUS PROCESS

- $W = \int P\, dV \Rightarrow$ area under the P -V graph projected on volume axis.

- Work is a Path function. It is written as δW not dW. $\int_{1}^{2} \delta W \neq W_2 - W_1$

- For closed system $dV = \dfrac{1}{P} \delta W$, Here $\dfrac{1}{P}$ is integrating factor. Integrating factor is defined as the factor which when multiplied by inexact differential gives exact differential.

  ➢ **Reversible Constant Volume or Isochoric Process** (V = Constant)

  $W = \displaystyle\int_{1}^{2} P\, dV = 0$, since $dV = 0$

  ➢ **Reversible Constant Pressure or Isobaric Process or Isopistic Process** (P = Constant)

  $W = \displaystyle\int_{1}^{2} P\, dV = P\,(V_2 - V_1)$

  ➢ **Reversible Isothermal Process** (T = Constant)

  Using Ideal Gas Equation PV = mRT = Constant ⇒ This is equation of Rectangular Hyperbola.

  $W = P_1 V_1 \ln\left(\dfrac{V_2}{V_1}\right) = P_1 V_1 \ln\left(\dfrac{P_1}{P_1}\right) == mRT_1 \ln\left(\dfrac{V_2}{V_1}\right) = mRT_1 \ln\left(\dfrac{P_1}{P_1}\right)$

  $\dfrac{dP}{dV} = \dfrac{-P}{V} \Rightarrow$ Slope of reversible Isothermal process.

  ➢ **Reversible Adiabatic Process** (Q = 0, No Heat transfer, PV^{γ} = Constant)

  Using Ideal Gas Equation $\dfrac{T_2}{T_1} = \left(\dfrac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\dfrac{v_1}{v_2}\right)^{\gamma-1}$, where γ =adiabatic Index and greater than 1

  $W = \dfrac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \dfrac{mR\left(T_1 - T_2\right)}{\gamma - 1}$

  $\dfrac{dP}{dV} = -\gamma.\dfrac{P}{V} \Rightarrow$ Slope of reversible adiabatic process.

  Hence slope of Reversible adiabatic process γ times more than slope of Reversible Isothermal process. Hence slope will be steeper.

  ➢ **Reversible Polytrophic Process** (PV^{n} = Constant)

  Using Ideal Gas Equation $\dfrac{T_2}{T_1} = \left(\dfrac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\dfrac{V_1}{V_2}\right)^{n-1}$, Generally $1 < n < \gamma$

  $W = \dfrac{P_1 V_1 - P_2 V_2}{n - 1} = \dfrac{mR\left(T_1 - T_2\right)}{n - 1}$

FLOW OR OPEN SYSTEM WORK FOR VARIOUS PROCESS

- $W = -\displaystyle\int V\, dP \Rightarrow$ area under the P -V graph projected on pressure axis.

  ➢ **Reversible Constant Volume or Isochoric Process** (V = Constant)

  $W = -\displaystyle\int_{1}^{2} V\, dP = -V(P_2 - P_1) = V(P_1 - P_2)$

  ➢ **Reversible Constant Pressure or Isobaric Process or Isopistic Process** (P = Constant)

$W = \int_{1}^{2} -VdP = 0$. since $dP = 0$

➢ **Reversible Isothermal Process** (T = Constant)

Using Ideal Gas Equation PV = mRT = Constant ⇒ This is equation of Rectangular Hyperbola.

$$W = P_1V_1 \ln\left(\frac{V_2}{V_1}\right) = P_1V_1 \ln\left(\frac{P_1}{P_1}\right) == mRT_1 \ln\left(\frac{V_2}{V_1}\right) = mRT_1 \ln\left(\frac{P_1}{P_1}\right)$$

Thus, work flow process and non-flow process is same for isothermal process.

➢ **Reversible Adiabatic Process** (Q = 0, No Heat transfer, Pv^{γ} = Constant)

Using Ideal Gas Equation $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$, where γ =adiabatic Index and greater than 1

$$W = \gamma\left(\frac{P_1V_1 - P_2V_2}{\gamma - 1}\right)$$

$W_{open} = \gamma \times W_{close}$

➢ **Reversible Polytrophic Process** (Pv^n = Constant)

Using Ideal Gas Equation $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{v_1}{v_2}\right)^{n-1}$, Generally 1 < n < γ

$$W = n\left(\frac{P_1V_1 - P_2V_2}{n - 1}\right)$$
$W_{open} = n \times W_{close}$

OTHER TYPES OF WORK TRANSFER

➢ **ELECTRICAL WORK**

Work $W = \int_{1}^{2} VI \, dt$, where V is Voltage, I is Current and t is time

Power P = VI Watt

This is the rate at which work is transferred.

➢ **SHAFT WORK**

Work $W = \int_{1}^{2} T d\theta$, Where T is the torque on shaft and dθ is the angular displacement of shaft

Power P $= \int_{1}^{2} T \frac{d\theta}{d\tau} = T\omega$, where ω is the angular velocity

➢ **PADDLE-WHEEL WORK OR STIRRING WORK**

As the weight is lowered, and the paddle wheel turns, and there is work transfer into the fluid system which gets stirred.

➢ **WORK DONE IN STRETCHING A WIRE**

$W = -\int_{1}^{2} F \, dL$, where F is force, dL means an expansion of the wire, Minus sign is used for negative work

➢ **WORK DONE IN CHANGING THE AREA OF A SURFACE FILM**

The work done on a homogeneous liquid film in changing its surface area by an infinitesimal amount dA is $W = -\int_1^2 \sigma \, dA$ where σ is the surface tension (N/m).

HEAT TRANSFER

- "The energy transfer because of the temperature difference is known as Heat"

- If mass of the body B is m and specific heat of the body is C, then heat gain by the body B or heat transfer between both the body A & B will be given by $Q = mC(T_A - T_B)$

- **"Specific heat** is the amount of heat required to raise the temperature of unit mass by unit degree centigrade (or Kelvin)"

$$C = \frac{Q}{m(T_A - T_B)} = \frac{Q}{m.\Delta T}, \quad \text{Unit -} \quad C = \frac{Q}{m.\Delta T} = \frac{kJ}{kg - K} \text{ or } \frac{kJ}{kg - °C}$$

- **NOTE –** value of specific heat varies with the process for gases whereas in case of solid and liquid it remains constant irrespective of process because solid and liquid both can be assumed to be incompressible.

- Values of specific heat for air $C_p = 1.005 \frac{kJ}{kg - K}$, $C_v = 0.718 \frac{kJ}{kg - K}$ and water $C_{water} = 4.18 \frac{kJ}{kg - K}$

- The specific Heat at Constant Pressure greater than specific Heat at constant volume because in constant pressure Process, the Heat is utilized to raise the temperature and to do PdV work.

$$C_P > C_v, \quad \gamma = \frac{C_p}{C_v}$$

- **Sensible Heat-** Heat transfer because of the temperature difference between two bodies is known as sensible heat. $Q = mC\Delta T$

- **Latent Heat-** Energy released or absorbed by a substance when it is subjected to change in phase.
  latent heat of fusion - Amount of energy absorbed during melting of a substance is called the latent heat of fusion and is equivalent to the amount of energy released during freezing.
  latent heat of vaporization - Amount of energy absorbed during vaporization is called the latent heat of vaporization and is equivalent to the energy released during condensation.

- **Sign Convention for Heat Transfer**
  Heat supplied to the system is taken as positive. Heat rejected from the system is taken as negative.

    1. **Enthalpy, H**: It represents the total heat content of the system.

    2. **Specific Enthalpy, h**: Summation of specific internal energy and flow work. $\Rightarrow h = u + P\upsilon$

    3. **Flow work:** It is amount of work which is required to displace a infinitely small fluid element to enter or exit from the control volume known as flow work.

Process	Work (Non0flow)	Heat

Reversible Constant Volume or Isochoric Process	$\delta W = 0$	$Q = dU = mC_v \Delta T$
Reversible Constant Pressure or Isobaric Process or Isopistic Process	$\delta W = \int\limits_{1}^{2} P\,dV = P(V_2 - V_1)$	$Q = dH = mC_p \Delta T$
Reversible Isothermal Process	$\delta W = P_1 V_1 \ln\left(\dfrac{V_2}{V_1}\right) = mRT_1 \ln\left(\dfrac{V_2}{V_1}\right)$	$Q = W$
Reversible Adiabatic Process	$W = \dfrac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \dfrac{mR(T_1 - T_2)}{\gamma - 1}$	$Q = 0$
Reversible Polytrophic Process	$W = \dfrac{P_1 V_1 - P_2 V_2}{n - 1} = \dfrac{mR(T_1 - T_2)}{n - 1}$	$Q = \left(\dfrac{\gamma - n}{\gamma - 1}\right) \times W_{polytropic}$

CHAPTER-2 : FIRST LAW OF THERMODYNAMICS

FIRST LAW OF THERMODYNAMICS

- First law of thermodynamics is to relate the various form of energy and energy interaction.

- Its states that "Energy can be neither created nor destroyed during a process; it can only change forms". It is also known as "conservation of energy principle"

- It represents by E,

  E = Mechanical Energy + Thermal Energy + Internal Energy + potential Energy + Kinetic Energy + Chemical Energy ETC.

FIRST LAW FOR A CLOSED SYSTEM UNDERGOING A CYCLE

- Net heat interaction in a cycle is equal to the net Work interaction. $(\Sigma\ W)_{cycle} = J(\Sigma\ Q)_{cycle}$ where J is the Joule's equivalent.

- This is also expressed in the form $\oint \delta W = \oint \delta Q$, where the symbol $\oint$ denotes the cyclic integral for the closed path.

FIRST LAW FOR A CLOSED SYSTEM UNDERGOING A CHANGE OF STATE (for a process)

- If Q is the amount of heat transferred to the system and W is the amount of work transferred from the system during the process, then, net energy transfer (Q – W) will be stored in the system.

- Energy in storage is neither heat nor work, and is given the name internal energy or simply, the energy of the system. Therefore, $\delta Q = dE + \delta W$, where dE is the increase in the energy of the system

CONSEQUENCES OF 1ST LAW OF THERMODYNAMICS

- Heat is a path function; hence, heat transfer does not depend only end point but also depends upon the followed by the process.

- Energy has a definite value for every state of the system. Hence, it is a point function and a properly of the system. The energy E is an extensive property. So $\oint dE = 0$

- As we know that, for an isolated system, Heat and work interaction from the surrounding is zero. Energy of isolated system is Constant. So, Q =0, W = 0 → ΔE=0 ⇒ E_2-E_1 =0 → E_2=E_1

- Perpetual motion machine of first kind-PMM1 is not possible. Thus, $\oint \delta W \neq \oint \delta Q$

PERPETUAL MOTION MACHINE

- Any device which violate any of the thermodynamic laws is called a perpetual motion machine.

- A device which violate the first low of thermodynamic is called perpetual motion machine of first kind. (PMM-1)

STEADY FLOW ENERGY EQUATION (SFEE)

If the rates of flow of mass and energy through the control surface does change with time, then it is said to be steady flow.
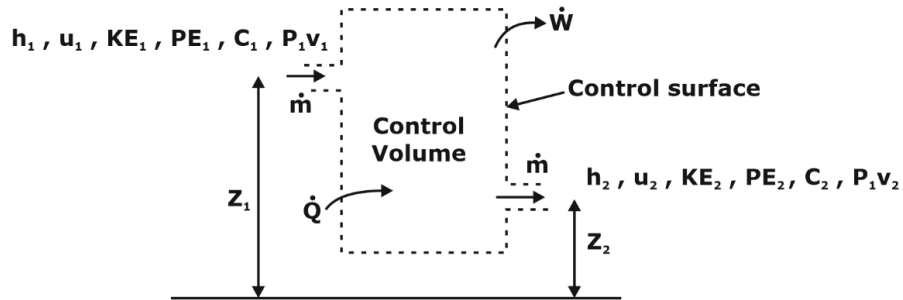
Here   i and e subscripts used for inlet and exit condition respectively.

A_i, A_e – cross-section of stream, m²

$\dot{m}_i$, $\dot{m}_e$ – mass flow rate, kg/s

P_i, P_e – pressure, absolute, N/m²

V_i, V_e – volume, m³

υ_i, υ_e – Specific volume, m³/kg

u_i, u_e – Specific internal energy, J/kg

C_i, C_e – Velocity, m/s

z_i, z_e - Elevation above an arbitrary datum, m

$\dfrac{dQ}{dt} = \dot{Q}$ = Net rate of heat transfer through the control surface, J/s

$\dfrac{dW}{dt} = \dot{W}$ = Net rate of work transfer through the control surface, J/s

- **Continuity Equation** or **Conservation of mass**, $\dot{m}_i = \dot{m}_e \Rightarrow (\rho AC)_i = (\rho AC)_e \Rightarrow \left(\dfrac{AC}{\upsilon}\right)_i = \left(\dfrac{AC}{\upsilon}\right)_e$

- **Energy balance** $E_{in} = E_{out} \Rightarrow E_i = E_e$

$$U_i + (KE)_i + (PE)_i + (PV)_i + \dot{Q} = U_e + (KE)_e + (PE)_e + (PV)_e + \dot{Q}$$

$$\dot{m}_i\left(u + P\upsilon + \frac{1}{2}C^2 + gz\right)_i + \dot{Q} = \dot{m}_e\left(u + P\upsilon + \frac{1}{2}C^2 + gz\right)_e + \dot{W}$$

$$\dot{m}_i\left(h + \frac{1}{2}C^2 + gz\right)_i + \dot{Q} = \dot{m}_e\left(h + \frac{1}{2}C^2 + gz\right)_e + \dot{W}$$

UNSTEADY FLOW ENERGY EQUATION

In this type of flow, the flow rate is changing with respect to time, so these are also known as variable flow process.

Here   1 and 2 subscripts used for initial and final condition inside control volume respectively.

m_1, m_2 = initial and final mass inside the control volume

Others notation are same as previously explained

- **Continuity Equation** or **Conservation of mass -** $\dfrac{d}{dt}(m_{cv}) = \dfrac{d}{dt}(m_2 - m_1) = \dot{m}_i - \dot{m}_e$

- **Energy balance –** $\dfrac{d}{dt}(E_{cv}) = \dfrac{d}{dt}(E_2 - E_1) = \dot{E}_i - \dot{E}_e$

$$\frac{d}{dt}(E_{cv}) = \frac{d}{dt}\left[m_i\left(h + \frac{1}{2}C^2 + gz\right)_i + Q\right] - \frac{d}{dt}\left[m_e\left(h + \frac{1}{2}C^2 + gz\right)_e + W\right]$$

Assumptions- Neglecting KE and PE changes $\Rightarrow E = U + KE + PE \Rightarrow E = U$

$$\frac{d}{dt}(U_{cv}) = \frac{d}{dt}(m_i h_i + Q) - \frac{d}{dt}(m_e h_e + W)$$

Neglecting the variation of enthalpy at inlet and outlet with respect to time $\dfrac{dh_i}{dt} = \dfrac{dh_e}{dt} = 0$

$$\frac{d(U_{cv})}{dt} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}$$

APPLICATIONS OF STEADY FLOW PROCESSES

➢ **Nozzle and Diffuser**

- A nozzle is a device of variable cross section which increases the velocity or Kinetic Energy of a fluid at the expense of its pressure.

- A Diffuser is a device of variable cross section which increases the Pressure Energy of a fluid at the expense of its kinetic energy.

- Both devices are work **transforming device** in which one form of energy is transformed into another form so no work interaction will be there.
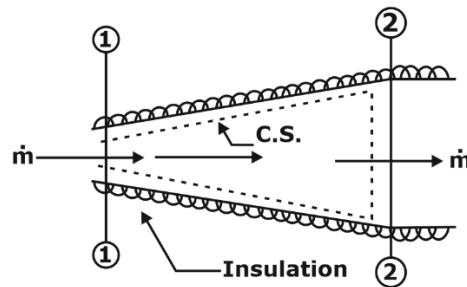
If the inlet is 1-1 then it will work as diffuser

If the inlet is 2-2 then it will work as nozzle

Assumption –

1. It is completely insulated so, Q=0

2. No work interaction, so W = 0

3. It is assumed to be horizontal so, $\Delta PE \sim 0$

- Thus, from SFEE $h_1 + \dfrac{1}{2}C_1^2 = h_2 + \dfrac{1}{2}C_2^2$

- In case of nozzle, generally, $C_1 \lll C_2$, Then $C_1 \approx 0 \Rightarrow C_2 = \sqrt{2(h_1 - h_2)}$

- In case of diffuser, generally, $C_2 \lll C_1$, Then $C_2 \approx 0 \Rightarrow C_1 = \sqrt{2(h_2 - h_1)}$

- ➢ **Turbine and Compressor**
  - Turbines are power producing device in which the fluid imparts its energy to the rotor and due to which mechanical energy generates
  - Compressor or pump are power consuming device in which the mechanical energy of the rotor is given to the fluid which results in increase in temperature and pressure of the fluid.
  - Both devices are work **transferring device** in which work input is required from the outside in case of compressor whereas work is produced in case of turbine.
  - **Assumption –**
    1. It is completely insulated so no heat transfer, Q = 0
    2. It is assumed to be horizontal thus PE can be neglected
    3. Kinetic energy is very low as compare to enthalpy so KE of the fluid is neglected.
  - From SFEE, For turbine $\dot{W} = \dot{m}(h_1 - h_2)$, For compressor, $\dot{W} = \dot{m}(h_2 - h_1)$

- ➢ **Boiler and Condenser**
  - Both are a type of heat exachanger in which energy from high energy fluid is imparted to low energy fluid.
  - In boiler, heat is being absorbed by working fluid at constant pressure,
  - In condenser heat is being rejected by working fluid at constant pressure.
  - **Assumption** –
    1. No work interaction, so W = 0
    2. It is assumed to be horizontal thus PE can be neglected.
    3. There is no change in kinetic energy.
  - From SFEE, For Boiler $\dot{Q} = \dot{m}(h_2 - h_1)$, For Condenser $\dot{Q} = \dot{m}(h_1 - h_2)$

- ➢ **Throttling Device -**
  - When a fluid flows through a restricted passage, like a partially opened valve, an orifice, or a porous plug, there is an appreciable drop in pressure, and the flow is said to be throttled.
  - **Assumption –**
    1. It is completely insulated so no heat transfer, Q = 0
    2. No work interaction, so W = 0
    3. It is assumed to be horizontal thus potential energy can be neglected
    4. There is no change in kinetic energy,
  - From SFEE $\quad \dot{m}h_1 = \dot{m}h_2 \qquad \Rightarrow h_1 = h_2$
  - During the throttling process enthalpy of fluid remains constant whereas pressure of fluid decrease.
  - It is an irreversible process.

APPLICATIONS OF UNSTEADY FLOW PROCESSES

- ➢ **Bottle filling process or Charging of tank**
  - Assumption –
    1. In this case, mass is not leaving the system boundary, so $m_e = 0$
    2. Bottle is initially vacuum and work against Vacuum is zero, W = 0

3. System is insulated, Q = 0

4. Initially mass in the bottle is zero, so $m_1 = 0$, thus $m_i = m_2$

- With above assumption, if the working fluid is ideal gas then $u_2 = h_i$ i.e. Enthalpy is converted to internal energy and $T_2 = \gamma T_i$ i.e. the final temperature inside the tank is γ times the temperature of supply line.

- Note: notation has usual meaning

➢ **Discharging of tank**

- Assumption –

1. Mass is not entering the system boundary, so $m_i = 0$

2. Pipeline is initially vacuum and work against Vacuum is zero, W =0

3. System is insulated, Q = 0

4. Neglecting the variation of enthalpy at inlet $\dfrac{dh_i}{dt} = 0$

5. The pressure in the supply line remains constant

- With above assumption, discharging from a tank is adiabatic process.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

- Note: notation has usual meaning

FREE EXPANSION

- It is also known as unstrained expansion or Expansion against vacuum

- It is a case of irreversible process.

- Since the system is insulated and gas is expanding against the vacuum thus, dQ=0, dW=0

- From the 1st law of thermodynamics $dQ = dE + dW \implies dE = 0$

CHAPTER-3 SECOND LAW OF THERMODYNAMICS

INTRODUCTION

- Satisfying the first law alone does not ensure that process will actually take place.
- Thus, second law of thermodynamic asserts that.
    1. A process can occur in certain direction.
    2. Energy has quality as well quantity.
- A process if to occur must satisfy both 1st law and 2nd law.

THERMAL ENERGY RESERVOIR (TER)

- A TER is a Hypothetical body which have relatively very large thermal heat capacity, that can supply or absorb finite amount of heat without undergoing any change in temperature. ΔT = 0
- We know $\Delta T = \dfrac{Q}{mC}$, So mC has to be large. It is possible under 2 cases-
    o Any physical body whose specific capacity is large relative to amount of energy can be TER.
    o Large water bodies such as oceans, lakes, rivers, atmospheric air can be modeled as TER.
- A two-phase system can also be modeled as reservoir since it can absorb and release large quantities while remaining of constant temperature.
- **Source** ⇒ A thermal energy reservoir that supplies energy in form of heat is called source.
- **Sink** ⇒ A thermal energy reservoir that absorbs energy in the form of heat is called sink.

HEAT ENGINE

- Heat engine is a device which converts the part of heat energy into work and rejects remaining to sink or surrounding.
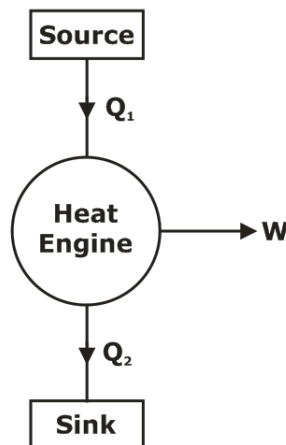
- Heat engines are characterized by following
    1. They receive heat from high temperature source = Q_1
    2. They convert part of this heat into work = W
    3. They reject the remaining heat to low temperature sink = Q_2
    4. They operate on cycle.
- Heat engines and other cyclic devices usually involve a fluid to and from which heat is transferred while undergoing a cycle this fluid is called **working fluid.**
- **Thermal Efficiency -** The fraction of the heat input that is converted to net-work output is known as thermal efficiency and is the measure of the performance of heat engine.

$$\eta = \frac{\text{Net work output}}{\text{Total Heat input}} = \frac{W_{net}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

REFRIGERATOR

- A refrigerator is a device which maintains lower temperature as compared to surrounding. As lower temperature is to be maintained continuously, refrigerator must operate on cycle.

- The efficiency of a refrigerator is expressed as coefficient of performance. For refrigerator, Desired effect is extracting Q_2.

$$(COP)_{Ref.} = \frac{\text{Desire effect}}{\text{Work Input}} = \frac{Q_2}{W_{net\ in}} = \frac{Q_2}{Q_1 - Q_2}$$

- This equation is valid for reversible as well as irreversible cycle.

- The COP may or may not be greater than 1, that's why a new term coefficient of performance is used.
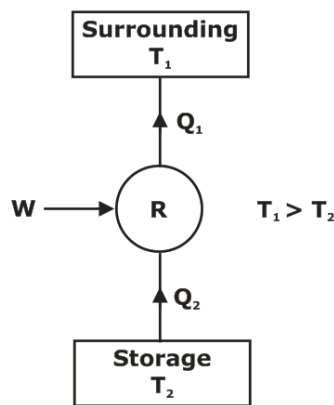
HEAT PUMP

- Heat pump is a device which maintains higher temperature compared to surroundings.

- The thermodynamic cycle of refrigerator and heat pump are same, but they differ in their objective and the temperature limits under which they operate.

$$(COP)_{HP} = \frac{\text{Desire effect}}{\text{Work Input}} = \frac{Q_1}{W_{net\ in}} = \frac{Q_1}{Q_1 - Q_2}$$

- This equation is valid for reversible as well as irreversible heat pump.

- Since $Q_1 > W_{net\ in}$, the COP of heat pump is always greater than 1.

- Relationship between cop of a heat pump and refrigerator operating between same temperature limits $(COP)_{HP} = 1 + (COP)_{Ref.}$
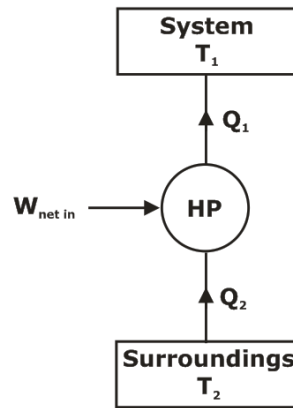
TWO STATEMENT OF SECOND LAW OF THERMODYNAMIC

- **KELVIN PLANCK STATEMENT-**
  - It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work.
  - Hence a heat engine can not 100% efficiency this limit to the efficiency is not due friction or other dissipative effect but applies to both idealized and actual heat engines.
- **CLAUSIUS STATEMENT-**
  - It is impossible to construct a device that operates in a cycle and produces no effect other than the heat transfer of heat from a lower temperature body to a higher temperature body.
- **Note:** Both the Kelvin-Planck and Clausius statements of second law are negative statements and a negative statement cannot be proved.
- The Kelvin-Planck and the Clausius statements are equivalent in their consequences and either statements can be used as expression of second law of thermodynamic.

PERPETUAL MOTION MACHINE

- Any device which violate any of the thermodynamic laws is called a perpetual motion machine.
- A device which violates the second law of the thermodynamic is called perpetual motion machine of 2nd kind (PMM-2)

IRREVERSIBILITIES

- The factor that causes a process to be irreversible are called irreversibilities.
- The irreversibilities of a process may be due to either one or both of the following.
  1. Lack of equilibrium (e.g. mechanical, thermal, chemical equilibrium)
  2. Involvement of dissipative effect (e.g. Friction)

CARNOT CYCLE (REVERSIBLE CYCLE)

- Carnot cycle is the best-known reversible cycle which is both internally and externally reversible.
- The theoretical heat engine that operates on Carnot cycle is called Carnot heat engine.
- The four reversible process that make up the Carnot cycle are as follows

  1-2 ⇒ Reversible Isothermal expansion process ⇒ slow

  2-3 ⇒ Reversible adiabatic expansion process ⇒ fast

  3-4 ⇒ Reversible isothermal compression process ⇒ slow

  4-1 ⇒ Reversible adiabatic compression process ⇒ fast.
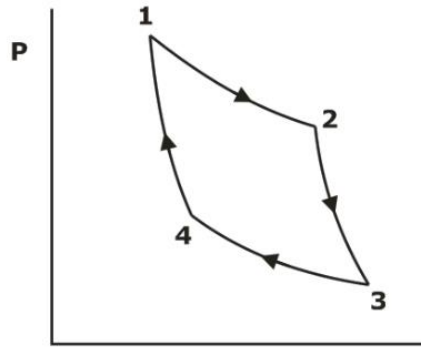
CARNOT THEOREM OR PRINCIPLES

1. The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between same two reservoirs. $\eta_{rev} > \eta_{irrev}$

2. The efficiency of all reversible heat engine operating between same two reservoir are same.

3. Efficiency of a reversible cycle depends only on temperature limit.

• A temperature scale that is independent of the properties of the substance that are used to measure temperature is called **thermodynamic temperature scale.**

• As per Kelvin Experiment taking simplest function $\dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2}$ ⇒ valid for reversible cycle.

• We know for heat engine $\eta = 1 - \dfrac{Q_2}{Q_1}$, **Efficiency of reversible heat engine** $\eta_{rev} = 1 - \dfrac{T_2}{T_1}$

• for refrigerator $(COP)_R = \dfrac{Q_2}{Q_1 - Q_2}$, **Efficiency of reversible refrigerator** $(COP)_R = \dfrac{T_2}{T_1 - T_2}$

• for heat pump $(COP)_{HP} = \dfrac{Q_1}{Q_1 - Q_2}$, **Efficiency of reversible heat pump** $(COP)_{HP} = \dfrac{T_1}{T_1 - T_2}$

DRAWBACKS OF CARNOT CYCLE

• It consists of mainly isothermal & adiabatic process to achieve isothermal condition the process has to be extremely slow whereas to achieve adiabatic condition the process has to be extremely fast, so both the process is not possible in a single cycle.

• The area under P-V curve is having very narrow region therefore the net workout of Carnot cycle is having low value, moreover to obtain the same workout the swept volume is very large.

Note:

• The max possible efficiency of reversible engine is given by Carnot

• the expression of Carnot efficient is $\eta_{Carnot} = 1 - \dfrac{T_L}{T_H}$, T_L and T_H are low and high temperature limit.

• Carnot efficient is function of temperature limits only.

• If there are n number of reversible engines operative between same temperature limits with different working fluid then the value of Carnot efficiency or ideal efficiency or maximum possible efficiency is having same value. Carnot efficiency is independent of working fluid.

COMBINATION OF HEAT ENGINE

- If two reversible heat engines operating in series and heat rejected by engine A directly supplied to engine B.

- **Combined efficiency of the engine**

  We know that $\eta = \dfrac{W_{max}}{Q_{supplied}}$, So $\eta_1 = \dfrac{W_1}{Q_1}$ & $\eta_2 = \dfrac{W_2}{Q_2}$

  $\eta_{Overall} = \dfrac{W_1 + W_2}{Q_1} = \eta_1 + \eta_2 (1 - \eta_1) = \eta_1 + \eta_2 - \eta_1\eta_2$ (As per Kelvin Experiment $Q \propto T$)

- **Expression of intermediate temperature if both engines are equally efficient**

  We know $\eta_1 = 1 - \dfrac{T_2}{T_1}$ and $\eta_2 = 1 - \dfrac{T_3}{T_2}$, Since, $\eta_1 = \eta_2 \Rightarrow T_2 = \sqrt{T_1 T_3}$

  Thus, when efficiency of both engines is same then intermediate temperature is geometric mean of higher and lower temperature.

- **Expression of intermediate temperature if both engines produces same amount of work**

  Since $W_1 = W_2 \quad \Rightarrow Q_2 = \dfrac{Q_1 + Q_3}{2} \quad \Rightarrow T_2 = \dfrac{T_1 + T_3}{2}$

  Thus, when work produced by both engines are same then intermediate temperature is arithmetic mean of higher and lower temperature.
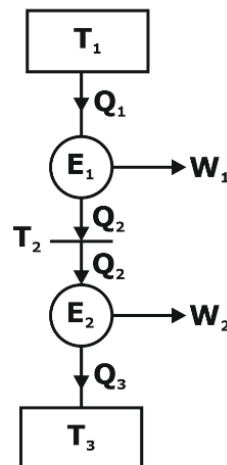
CLAUSIUS INEQUALITY

- The cyclic integral of $\dfrac{\delta Q}{T}$ is less than or equal to zero. $\oint \dfrac{\delta Q}{T} \leq 0$

  $\oint \dfrac{\delta Q}{T} = 0 \Rightarrow$ Reversible cycle

  $\oint \dfrac{\delta Q}{T} < 0 \Rightarrow$ Cycle is Irreversible

CHAPTER-4 ENTROPY

INTRODUCTION

- From Clausius inequality, cyclic integral of $\frac{\delta Q}{T}$ is zero for a reversible process & if cyclic integral of any variable is zero then it must be property of system, Thus $\frac{\delta Q}{T}$ is a property of the system.

- So, a new property Entropy is defined.

- "S" is an extensive property named as entropy, sometimes is referred to as total entropy. Entropy per unit mass, designated "s", is an intensive property.

- **Units –** unit of entropy is $\Delta S = \int_1^2 \frac{\delta Q}{T} = \frac{kJ}{K}$, unit of Specific entropy is $\Delta s = \frac{\Delta S}{mass} == \frac{kJ}{kg-K}$

- the integral of $\frac{\delta Q}{T}$ gives the value of entropy change only if the integration is carried out along an internally reversible path between the two states.

- The integral of $\frac{\delta Q}{T}$ along an irreversible path is not a property, and in general, different values will be obtained when the integration is carried out along different irreversible paths.

- For a reversible process, $\frac{\delta Q}{T} = dS_{rev.process}$, For an irreversible process, $\frac{\delta Q}{T} + S_{gen} = dS_{rev.process}$

- For a reversible process, $S_2 - S_1 = \int_1^2 \frac{dQ}{T}$, For an irreversible process, $S_2 - S_1 = \int_1^2 \frac{dQ}{T} + S_{gen}$

Note:

- The word 'entropy' was first used by Clausius, taken from the Greek word `tropee' meaning `transformation'.

- Entropy is a thermodynamic property that measures the degree of randomization or disorder at the microscopic level.

- A macroscopic feature which is associated with entropy production is a loss of ability to do useful work. Energy is degraded to a less useful form, and it is sometimes said that there is a decrease in the availability of energy.

- The notion that entropy can be produced, but never destroyed, is the second law of thermodynamics.

- **Area under the T -S plot** on S axis will give the heat transfer in a reversible process.

$$Q_{rev} = \int_i^f TdS$$

- **Slope of Constant Volume Process in T-S diagram** $\left(\frac{dT}{ds}\right)_{v=c} = \frac{T}{C_v}$

- **Slope of Constant Pressure Process on T-S diagram** $\left(\frac{dT}{ds}\right)_{P=c} = \frac{T}{C_P}$

- As we know that, $C_P > C_v$, thus, $\dfrac{T}{C_P} < \dfrac{T}{C_v}$

- So, slope of constant volume line on T-S Curve is always greater than the slope of constant pressure line.
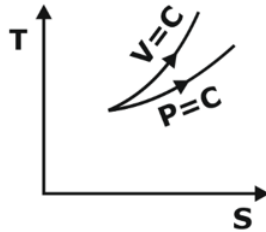
ENTROPY GENERATION

- Entropy charge of the system is summation entropy change due to internal irreversibility S_{gen} and entropy change due to external interaction $\dfrac{\delta Q}{T}$.

- Entropy generation S_{gen} is always a positive quantity or zero $S_{gen} \geq 0$. Its value depends on the process, and thus it is not a property of the system. $S_{gen} < 0$ (impossible Process)

- Some entropy is generated or created during an irreversible process, and this generation is due entirely to the presence of **irreversibilities**.

- For an isolated system dQ = 0, Therefore, for an isolated system $dS_{iso} \geq 0$

- Entropy of an isolated system can never decrease. It always increases and remains constant only when the process is reversible. This is known as **entropy principle.**

- If the system is reversible then entropy change of the system is given by $S_{gen} = 0$

- If the system is irreversible then entropy change of the system is given by $S_{gen} \neq 0$

- If the system is reversible and adiabatic it must be isentropic.

- Entropy change on the system may be positive, Negative or zero but entropy change of the universe can never have negative value.

CONCLUSIONS

1. Processes can occur in a certain direction only, not in any direction. A process must proceed in the direction that complies with the increase of entropy principle, that is, $S_{gen} \geq 0$.

2. A process that violates this principle is impossible.

3. Entropy is a non-conserved property, and there is no such thing as the conservation of entropy principle. Entropy is only conserved during the idealized reversible processes only.

4. Performance of engineering systems is degraded by the presence of irreversibilities, and entropy generation is a measure of the magnitudes of the irreversibilities present during that process.

5. The greater the extent of irreversibilities, the greater the entropy generation. Therefore, entropy generation can be used as a quantitative measure of irreversibilities associated with a process.

EXPRESSION OF ENTROPY FOR VARIOUS PROCESSES (of Ideal gas)

> **For Reversible process** (Using 1st Law equations, T-dS equations and Ideal gas equations)

$$s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right) = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{p_1}\right) = C_p \ln\left(\frac{V_2}{V_1}\right) + C_V \ln\left(\frac{P_2}{p_1}\right)$$

Notations have usual meanings.

> **Adiabatic Process**

- For an adiabatic process, dQ_{rev}, = 0, → dS = 0, → S = constant
- A reversible adiabatic process is, therefore, an isentropic (constant entropy) process.

> **Isothermal process**

- The system is taken from i to f reversibly.

- For reversible isothermal heat transfer $\therefore Q_{rev} = T\int_i^f dS = T(S_f - S_i)$

$$s_2 - s_1 = R \ln\left(\frac{V_2}{V_1}\right) = -R \ln\left(\frac{P_2}{p_1}\right) = C_p \ln\left(\frac{V_2}{V_1}\right) + C_V \ln\left(\frac{P_2}{p_1}\right)$$

> **Constant Volume Process –**

$$s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{p_1}\right) = C_V \ln\left(\frac{P_2}{p_1}\right)$$

> **Constant Volume & constant Pressure Process –**

$$s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right) = C_p \ln\left(\frac{T_2}{T_1}\right) = C_p \ln\left(\frac{V_2}{V_1}\right)$$

APPLICATIONS OF ENTROPY PRINCIPLE (ΔS$_{univ}$ ≥ 0)

> **Transfer of Heat through a Finite Temperature Difference Reservoir**

- Q be the rate of heat transfer from reservoir A at T_1 to reservoir B at T_2
- For reservoir A, $\Delta S_A = -Q/T_1$. It is negative because heat Q flows out of the reservoir.
- For reservoir B, $\Delta S_A = +Q/T_2$. It is positive because heat Q flows into the reservoir.
- The rod connecting the reservoirs suffers no entropy change, as it is in the steady state.

$$\Delta S_{univ} = \Delta S_A + \Delta S_B = -\frac{Q}{T_1} + \frac{Q}{T_2} = Q\cdot\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$
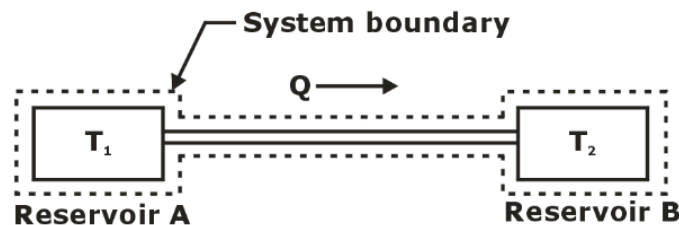
1. If $T_1 > T_2$, ΔS$_{univ}$ is positive.

2. If $T_1 = T_2$, ΔS$_{univ}$ is zero, and the process is reversible.

3. If $T_1 < T_2$, ΔS$_{univ}$ is negative and the process is impossible.

> **Mixing of two fluids (excluding gas) or Transfer of Heat through a Finite body at Temperatures T$_1$ and T$_2$ brought together into thermal contact**

- Subsystem-1 (mass m_1, specific heat C_1, temperature T_1) and Subsystem-2 (mass m_2, specific heat C_2, temperature T_2) comprise a composite system in an adiabatic enclosure.

- After partition is removed, the two fluids mix and T_F be the final equilibrium temperature.

- If the two bodies are merely brought together into thermal contact, delivering no work, the final temperature T_F reached would be the maximum.

- From Energy conservation $T_F = \dfrac{m_1 C_1 T_1 + m_2 C_2 T_2}{m_1 C_1 + m_2 C_2}$

- Entropy change for subsystem 1 and 2, $\Delta S_1 = m_1 C_1 \ln \dfrac{T_f}{T_1}$, $\Delta S_2 = m_2 C_2 \ln \dfrac{T_F}{T_2}$

$$\Delta S_{uni} = \Delta S_1 + \Delta S_2 = m_1 C_1 \ln \dfrac{T_F}{T_1} + m_2 C_2 \ln \dfrac{T_F}{T_2}$$

- ΔS_{univ} will be positive definite, and the mixing process is irreversible.
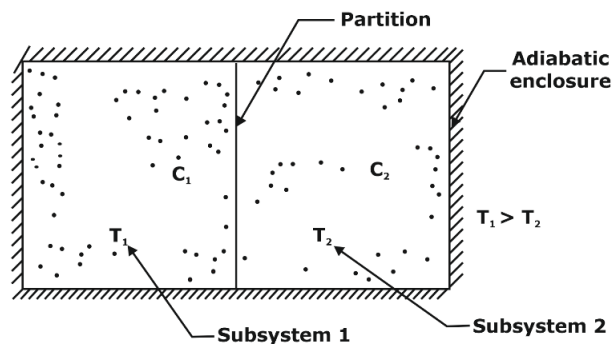
- **SPECIAL CASE** – If $m_1 = m_2 = m$ & $C_1 = C_2 = C$. Then, $T_F = \dfrac{T_1 + T_2}{2}$

$$\Delta S_{univ} = 2mC \ln \dfrac{T_F}{\sqrt{T_1 \cdot T_2}} = 2mC \ln \left(\dfrac{AM}{GM} \right),$$ Where AM & GM are the Arithmetic mean &

Geometric mean of temperatures. This is always positive, since the AM of any two numbers is always greater than their GM.

> **Transfer of Heat through a Finite body at Temperatures T_1 and T_2 interacting through reversible Heat Engine**

- Consider two finite bodies of constant heat capacity at temperatures T_1 and T_2, ($T_1 > T_2$).

- Via Heat Engine, part of the heat from body 1 converted to work W, and the remainder is rejected to body 2. HE will stop operating when bodies attain the final temperature T_F.

- Total heat withdrawn and rejected from body 1 to 2, $Q_1 = m_1 C_1 (T_1 - T_F)$, $Q_2 = m_2 C_2 (T_F - T_2)$

- Amount of total work delivered by the heat engine, $W = Q_1 - Q_1 = m_1 C_1 (T_1 - T_F) - m_2 C_2 (T_F - T_2)$

- Entropy change for body 1 and 2, $\Delta S_1 = m_1 C_1 \ln \dfrac{T_f}{T_1}$, $\Delta S_2 = m_2 C_2 \ln \dfrac{T_F}{T_2}$

$$\Delta S_{uni} = \Delta S_1 + \Delta S_2 = m_1 C_1 \ln \dfrac{T_F}{T_1} + m_2 C_2 \ln \dfrac{T_F}{T_2}$$

- For given values of mC, T_1 and T_2, the magnitude of work W depends on T_F. Work obtainable will be maximum when **T_F is minimum**.

- For **maximum work** in the heat engine cycle, entropy change should be **ΔS$_{univ}$ ≥ 0**
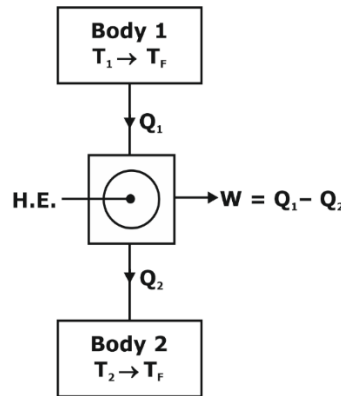
- **SPECIAL CASE** – If $m_1 = m_2 = m$ & $C_1 = C_2 = C$. Then, $T_F = \sqrt{T_1 . T_2}$ → The lowest attainable final temperature T_F corresponds to $ΔS_{univ} = 0$. T_F will be GM of the initial temperatures.

   The largest possible amount of work $W_{max} = mC(T_1 + T_2 - 2\sqrt{T_1 T_2}) = mC\left(\sqrt{T_1} - \sqrt{T_2}\right)^2$

- ➢ **Transfer of Heat from a Finite body at Temperatures T and TER at T$_o$ interacting through reversible Heat Engine**
  - Let finite body of constant heat capacity at temperatures T_1 and TER is at temperature T_o
  - Via Heat Engine, part of the heat from body 1 converted to work W, and the remainder is rejected to TER. The engine stop working, when the temperature of the body reaches T_o.
  - Total heat withdrawn and rejected from body 1 to TER, $Q_1 = Q = mC(T - T_o)$, $Q_2 = Q - W$

  - Entropy change for body 1 and 2, $\Delta S_{Body} = mC \ln \dfrac{T_0}{T}$, $\Delta S_{TER} = \dfrac{Q - W}{T_0}$

$$\Delta S_{univ} = \Delta S_{Body} + \Delta S_{TER} = mC \ln \dfrac{T_0}{T} + \dfrac{Q - W}{T_0}$$
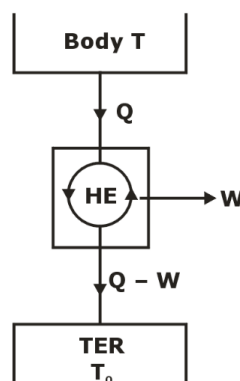
- For **maximum work** in the heat engine cycle, entropy change should be **ΔS$_{univ}$ ≥ 0**

$$\Rightarrow W_{max} = Q + T_0 mC \ln \dfrac{T_0}{T} = mC\left[(T - T_0) - T_0 \ln \dfrac{T}{T_0}\right]$$

- For **max efficiency** $\eta = \dfrac{W_{max}}{Q_{supplied}} = 1 - T_o\left[\dfrac{\ln(T/T_o)}{T - T_o}\right]$

CHAPTER-5 AVAILABILITY & IRREVERSIBILITY

INTRODUCTION

- Available energy or Exergy is the energy which is used for doing useful work. It is the amount of low-grade energy that can be completely converted into useful work.

- In exergy analysis the initial state of system is specified, the work output would be maximum when the process is ideal or reversible. Finally, the system must be at dead state to obtain maximum useful work. → Exergy = f (Initial state, Reversible process, dead state)

- **Dead state** is the state of the system in equilibrium with the environment. At the dead state, a system is at environment temperature and pressure, it has no KE & PE relative to environment.
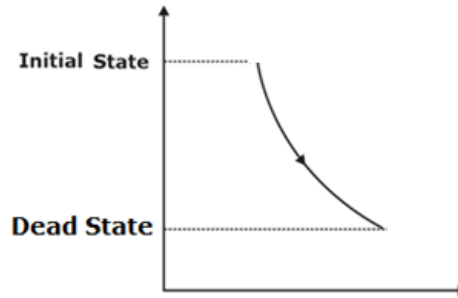
- A system delivers the maximum useful work as it undergoes a reversible process from specified initial state to dead state. This maximum useful work is called **exergy** or **availability**.

EXERGY OR AVAILABLE ENERGY OF HEAT

- Heat is a low-grade energy. For abstracting the max. useful work out of particular amount of heat available at high temperature T_1, we fix a reversible heat engine between T_1 and surrounding T_0.

- Available energy = $W = Q_1 - Q_2$

- $(W)_{max} = \eta_{reversible} \times Q_1 = \left(1 - \dfrac{T_0}{T_1}\right) Q_1$

- The energy which cannot be utilised for doing useful work is called **unavailable energy**.
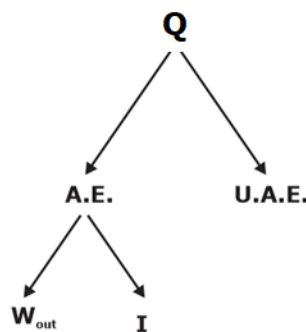
- **Irreversibility I** is equivalent to energy destroyed, hence also known as energy destruction.

  Work Producing device $(W_{Rev})_{out} > (W_{act})_{out} \Rightarrow I = (W_{Rev})_{out} - (W_{act})_{out}$

  Work Consuming device $(W_{Rev})_{in} < (W_{act})_{in} \Rightarrow I = (W_{act})_{in} - (W_{Rev})_{in}$

EXERGY ASSOCIATED WITH KINETIC AND POTENTIAL ENERGY

- Kinetic energy is a form of mechanical energy (high grade energy) and thus it can be converted to work completely. Hence the exergy of KE of a system is equal to KE itself.

- Potential energy is also a form of mechanical energy (high grade energy) and thus it can be converted to work completely. Hence exergy of PE of a system is equal to PE itself.

SURROUNDING WORK

- When a gas in Piston cylinder device expands, part of work done is used to overcome atmospheric pressure. This work which cannot be recovered and utilized for any useful purpose is called surrounding work. $(W)_{surr} = P_0 (V_2 - V_1)$

- Surrounding work is equal to irreversibility associated with above process.

DECREASE IN AVAILABILITY or INCREAE IN IRREVERIBILITY WHEN HEAT TRANSFER THROUGH FINITE TEMPERATURE DIFFERENCE

- Consider a reversible heat engine operating between T_1 and T_0 and Heat transfer is Q_1

- Another reversible heat engine operating between T_1' and T_0 and Heat transfer is Q_1

- Available energy and entropy for 1st case $W = AE = Q_1\left(1 - \dfrac{T_0}{T_1}\right)$ and $\Delta s = \dfrac{Q_1}{T_1}$

- Available energy and entropy for 2nd case $W' = AE' = Q_1\left(1 - \dfrac{T_0}{T_1'}\right)$ and $\Delta s' = \dfrac{Q_1}{T_1'}$

- We know $\Rightarrow T_1 > T_1'$, So $\Rightarrow \Delta s' > \Delta s$ and $\Rightarrow W > W'$

- Decrease in AE or Increase in UAE = $W - W' = T_0\left(\dfrac{Q_1}{T_1'} - \dfrac{Q_1}{T_1}\right) = T_o\left(\Delta s' - \Delta s\right)$

- Hence increase in UAE and the shaded portion represent increase in UAE.
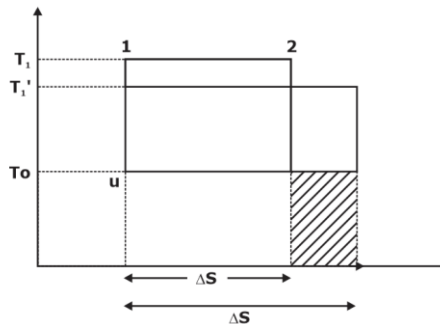
SECOND LAW EFFICIENCY

- Second law efficiency is the ratio of the exergy recovered to the exergy spent.

- It is the ratio of actual work produced to the max work produced under reversible condition.

$$\eta_{II} = \frac{\text{Exergy Re cov ered}}{\text{Exergy spent}} = \frac{W_{act}}{W_{max}}$$

- The second law efficiency is measure of the performance of a device relative to its performance under reversible conditions.

EXERGY OF A CLOSED SYSTEM

- Consider a piston cylinder device that contains a fluid of mass m at temperature T and pressure P. The system is then allowed to undergo a differential change of state.

- For change of exergy from state 1 to state 2

Exergy at state 1 $\Phi_1 = (E_1 - E_0) + P_0(V_1 - V_0) - T_0(S_1 - S_0)$

Exergy at state 2 $\Phi_2 = (E_2 - E_0) + P_0(V_2 - V_0) - T_0(S_2 - S_0)$

Change in Exergy $\Phi_1 - \Phi_2 = (E_1 - E_2) + P_0(V_1 - V_2) - T_0(S_1 - S_2)$

EXERGY OF OPEN SYSTEM

- A flowing fluid has an additional form of energy, called flow energy, which is the energy needed to maintain flow in Pipe. $W_{flow} = PV$

- The flow work essentially is the boundary work done by a fluid on fluid downstream and thus energy associated with flow work is equivalent to energy associated with boundary work.

- For change of exergy from state 1 to state 2 $\Phi_{flowing\ fluid} = \phi_{non\ flow\ fluid} + \phi_{flow}$

   Exergy at state 1 $\Phi_{flowing\ fluid} = (E_1 - E_0) + P_0(V_1 - V_0) - T_0(S_1 - S_0) + P_1V_1 - P_0V_0$

   $$= U_1 - U_0 + (KE)_1 - (KE)_0 + (PE)_1 - (PE)_0 + P_1V_1 - P_0V_0 - T_0(S_1 - S_0)$$

   $$= (U_1 + P_1V_1) - (U_0 + P_0V_0) - T_0(S_1 - S_0) + (KE)_1 - (KE)_0 + (PE)_1 - (PE)_0$$

   $$= (H_1 - H_0) - T_0(S_1 - S_0) + (KE)_1 - (KE)_0 + (PE)_1 - (PE)_2$$

EXERGY BALANCE

- The exergy transfer can be by heat, work and mass.

- Consider an **Open System**, the exergy balance is as follows

   (Total Exergy Entering – Total Exergy leaving)= (Change in total exergy of system + Irreversibility)

   $$m_{in}\phi_{in} + Q_{in}\left(1 - \frac{T_0}{T}\right) + W_{in} - m_{out}\phi_{out} - Q_{out}\left(1 - \frac{T_0}{T}\right) - W_{out} = m_2\phi_2 - m_1\phi_1 + I$$

- For **Closed system** $m_{in} = 0$, $m_{out} = 0$, and $m_1 = m_2 = m$

   The exergy balance gives $m(\phi_2 - \phi_1) + I = Q_{in}\left(1 - \frac{T_0}{T}\right) + W_{in} - Q_{out}\left(1 - \frac{T_0}{T}\right) - W_{out}$

CHAPTER-6 PROPERTIES OF PURE SUBSTANCES

INTRODUCTION

- A pure substance is a substance of constant chemical composition throughout its mass.
- It is one component system and may exist in one or more phases.

SOME IMPORTANT DEFINITIONS

➢ **Sensible heating & cooling:**
- It refers to the heating of substance in single phase. It causes rise in temperature of substance.
- In case of cooling in above conditions it shall be called sensible cooling.

➢ **Latent heating & cooling:**
- It is the heating of substance for causing its phase change without any change in temperature.
- If heat is extracted for causing phase change without any change in its temperature it will be called latent cooling.

➢ **Normal boiling point:**
- It is the temperature at which vapour pressure equals to atmospheric pressure and at this temperature, phase change from liquid to gas begins.
- Examples: Boiling point of water 100 °C, Boiling point of nitrogen −195.8 °C

➢ **Melting point:**
- It is the temperature at which phase change from solid to liquid takes place upon supplying latent heat.

➢ **Saturation states:**
- Saturation state of a substance refers to the state at which its phase transformation takes place without any change in pressure and temperature.
- These can be saturated solid state, saturated liquid state and saturated vapour state.

➢ **Saturation pressure:**
- It is the pressure at which substance changes its phase for any given temperature.
- Example: For water at 100°C the saturation pressure is 1 atm pressure.

➢ **Saturation temperature:**
- It refers to the temperature at which substance changes its phase for any given pressure.
- Example: For water at 1 atm pressure the saturation temperature is 100°C.

➢ **Triple point:**
- Triple point of a substance refers to the slate at which substance can coexist in all three phases (in solid, liquid and gaseous phase) in equilibrium.
- For water it is P_{tp} = 0.611 KPa, T_{tp} = 0.01 °C i.e. at this temperature ice, water and steam can coexist in equilibrium.

➢ **Critical states:**

• "Critical state refers to that state of substance at which liquid and vapour coexist in equilibrium."

• In case of water at 22.12 MPa (221.2 bar), and 374.15°C the water and vapour coexist in equilibrium. Thus, it is the highest pressure and temperature at which distinguishable water and vapour exist together. Specific volume at critical point for water is 0.00317 m^3/kg.

➢ **Dryness fraction:**

• It is the mass fraction of vapour in a mixture of liquid and vapour at any point in liquid-vapour mixture region. It is generally denoted by 'x'. It is also **called quality of steam.**

• $x = \dfrac{m_v}{m_L + m_v}$, where m_v = mass of water vapour and m_L = mass of the liquid

➢ **Compressed liquid or subcooled liquid:**

• Liquid at temperature less than saturation temperature corresponding to a given pressure is called compressed liquid or subcooled liquid.

• Degree of subcooling is given by the temperature difference between liquid temperature and saturation temperature of liquid at given pressure.

➢ **Superheated steam:**

• Steam having temperature more than the saturation temperature corresponding to given pressure is called superheated steam.

• Degree of superheating is given by difference between temperature of steam and saturation temperature at given pressure.

➢ **Vapor pressure:**

• When a liquid or solid is in equilibrium with its vapour at a given temperature, the vapour exerts a pressure that depends only on the temperature which is known as **Vapor pressure.**

• In general, the greater the temperature, the higher is the vapour pressure.

• The temperature at which the vapour pressure is equal to 760 mm Hg is called the **normal boiling point**.

• If v_f is the specific volume of the saturated liquid at a given pressure, and v_g the specific volume of the saturated vapour, then ($v_g - v_f$) or v_{fg} is the change in specific volume during phase transition (boiling or condensation) at given pressure.

• As pressure increases, v_{fg} decreases, and at the critical point v_{fg} becomes zero.

P-V DIAGRAM FOR A PURE SUBSTANCE

• The saturated liquid line with respect to vaporization and the saturated vapour line with respect to vaporization incline towards each other and form what is known as the saturation or vapour dome. The two lines meet at the critical state.

• The liquid-vapour mixture region (L + V) exists within the vapour dome between the saturated liquid and saturated vapour lines known as Wet region.

• To the right of the saturated vapour line is the vapour region.
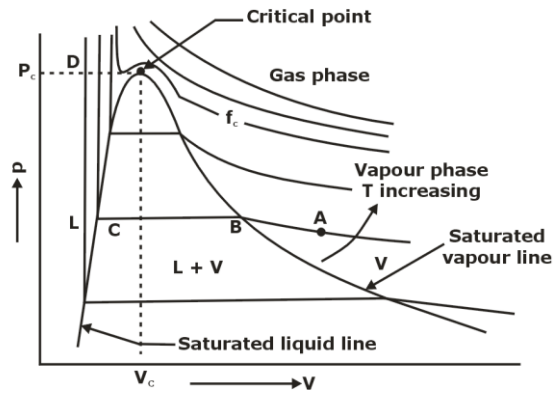
- ABCD is a typical isotherm of a pure substance.

- If the vapour at state A is compressed slowly and iso-thermally, the pressure will rise until there is saturated vapour at point B. If the compression is continued, condensation takes place, the pressure remaining constant so long as the temperature remains constant. At any point between B and C, the liquid and vapour are in equilibrium. Since a very large increase in pressure is needed to compress the liquid, line CD is almost vertical.

- As the temperature increases, the liquid- vapour transition, as represented by BC, decreases, and becomes zero at the critical point. Below the critical point only, there is a liquid-vapour transition zone, where a saturated liquid, on heating, absorbs the latent heat of vaporization, and becomes saturated Vapour at a constant pressure and temperature.

- Above, the critical point, a liquid upon heating suddenly flashes into vapour, or a vapour upon cooling suddenly condenses into liquid. There is no distinct transition zone from liquid to vapour and vice versa.

- The isotherm passing through the critical point is called the **critical isotherm**, and the corresponding temperature is known as the **critical temperature (t_c)**. The pressure and volume at the critical point are known as the **critical pressure** (p_c) and critical **volume** (v_c) respectively.

- Above the critical point, the isotherms are continuous curves that at large volumes and low pressures approach equilateral hyperbolas.

P–T DIAGRAM FOR A PURE SUBSTANCE

- The state changes of a pure substance, upon slow heating at different constant pressures, are plotted on P–T coordinates.

- There are 3 curves on P-T diagram, **fusion curve, vaporization curve, sublimation curve** which divide entire region in 3 phases- **solid, liquid, gas.**

- The fusion curve, the vaporization curve, and the sublimation curve meet at the triple point.

- The slopes of the sublimation and vaporization curves for all substances **are positive.** The slope of the fusion curve for most substances is positive, but **for water, it is negative.**
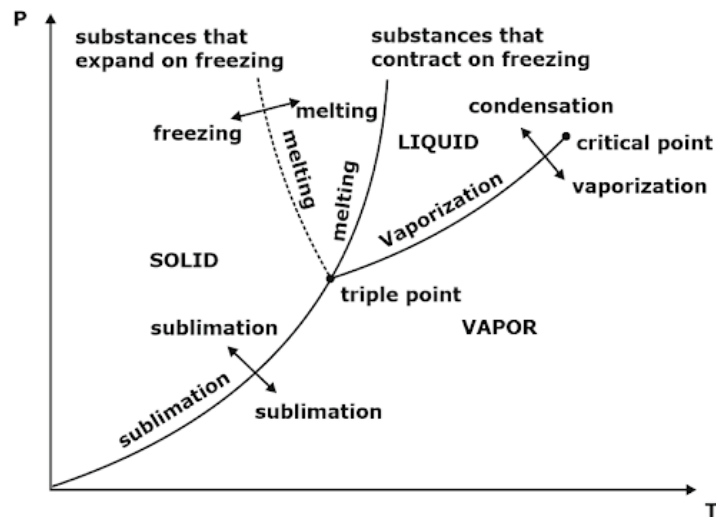
Note:

- The triple point line when projected to the p–T plane becomes a point.
- The critical isotherm has a point of inflection at the critical point.
- It has been found that on a 'P-T' diagram the triple point is represented by **a point.**
- On a 'P-V' diagram and "T-S' diagram triple point is **a line**.
- On a 'u-v' diagram triple point is **a triangle**.

T–S DIAGRAM FOR A PURE SUBSTANCE

- At particular pressure if s_f is the specific entropy of saturated water, and s_g is that of saturated vapour. The entropy change of the system during the phase change from liquid to vapour at that pressure is s_{fg} (s_g – s_f).
- The value of s_{fg} decreases as the pressure increases and becomes zero at the critical point
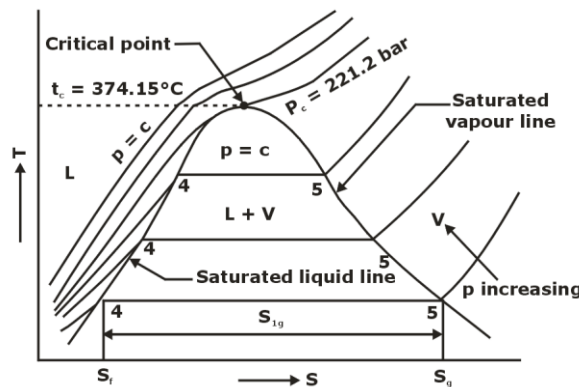
h—s DIAGRAM OR MOLLIER DIAGRAM FOR A PURE SUBSTANCE

- From the first and second laws of thermodynamics, the following property relation was obtained. This equation forms the basis of the h–s diagram of a pure substance, also called Mollier diagram.

$$\left(\frac{\partial h}{\partial s}\right)_P = T$$

- The slope of an isobar on the h–s coordinates is equal to the absolute saturation temperature, T (t_{sat} + 273) at that pressure. If the temperature remains constant the slope will remain constant. If the temperature increases, the slope of the isobar will increase.

- Below figure is the h –s or Mollier diagram indicating only the liquid and vapour phases.
- Constant pressure lines diverge from one another, and the critical isobar is a tangent at the critical point. In the vapour region, the states of equal slopes at various pressures are joined by lines, as shown, which are the constant temperature lines.
- Although the slope of an isobar remains, continuous beyond the saturated vapour line, **the isotherm bends towards the right and its slope decreases asymptotically to zero**, because in the ideal gas region it, becomes horizontal and the constant enthalpy implies constant temperature.
- At particular pressure, h_f is the specific enthalpy of saturated water, h_g is that of saturated vapour, and h_{fg} (h_g — h_f) is the latent heat of vaporization at that pressure, As the pressure increases. h_{fg} decreases, and at the critical pressure, h_{fg} becomes zero.
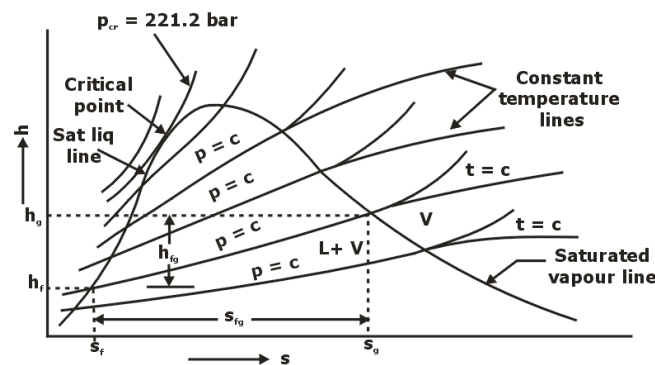
QUALITY OR DRYNESS FRACTION

- If in 1 kg of liquid-vapour mixture, x kg is the mass of vapour and (1 – x) kg is the mass of liquid. Then x is known as the quality or dryness fraction of the liquid-vapour mixture. Therefore, quality indicates the mass fraction of vapour in a liquid vapour mixture,

$$x = \frac{m_v}{m_v + m_L}$$, where m_v and m_L are the masses of vapour and liquid respectively in the mixture.

- The value of x varies between 0 and 1.
- For saturated water (when water just starts boiling): x = 0.
- For saturated vapour (when vaporization is complete): x = 1
- Also $s = (1 - x) s_f + x s_g$

  $h = (1 - x) h_f + x h_g$

  $v = (1 - x) v_f + x v_g$

  $u = (1 - x) u_f + x u_g$, where s, h, v and u refer to specific entropy, specific enthalpy and specific internal energy of the mixture of quality x, the suffix f and suffix g indicate the conditions of saturated liquid and saturated vapour, respectively.

MEASUREMENT OF STEAM QUALITY

- The state of a pure substance gets fixed if two independent properties are given. A pure substance is thus said to have **two degrees of freedom**.
- Of all thermodynamic properties, it is easiest go measure the pressure and temperature of a substance. Therefore, whenever pressure and temperature are independent properties, it is the practice to measure them to determine that state of the substance.

- This is done in the compressed liquid region or the superheated vapour region, where the measured values of pressure and temperature would fix up the state.
- But when the substance is in the two-phase region, the measured values of pressure and temperature could apply equally well to saturated liquid point f, saturated vapour point g, or to mixtures of any quality, points x_1, x_2 or x_3.
- Now, of the two properties, P and T, only one is independent; the other is a dependent property. If pressure is given, the saturation temperature gets automatically fixed for the substance. In order to fix up the state of the mixture, apart from either pressure or temperature, one more property, such as specific volume, enthalpy or composition of the mixture (quality) is required to be known.
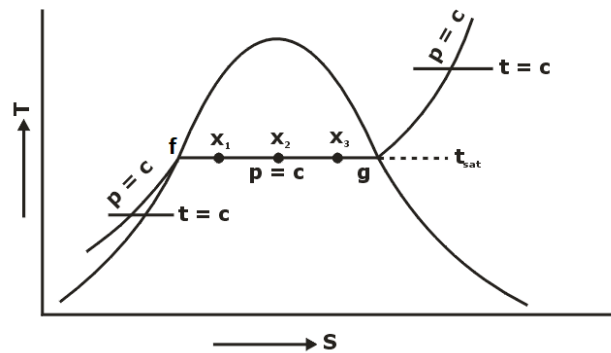
- Since it is relatively difficult to measure the specific volume of a mixture, devices such as calorimeters are used for determining the quality or the enthalpy of the mixture.
- In the measurement of quality, the object is always to bring the state of the substance from the two-phase region to the single-phase or superheated region, where both pressure and temperature are independent, and measured to fix the state, either by **Adiabatic Throttling** or **Electric heating**.

CHAPTER-7 THERMODYNAMIC RELATIONS

INTRODUCTION

- Properties such as temperature, pressure, volume and mass can be calculated directly. Other properties such as density and specific volume can be determined from these properties using some simple relations.

- However, energy, enthalpy and entropy are not so easy to determine because they cannot be measure or related to easily measurable properties through some simple relations.

- Therefore, thermodynamic relations are fundamental relations between commonly encountered thermodynamics properties and express the properties that cannot be measured properties.

PARTIAL DIFFERENTIAL RELATION

• **Theorem: 1**	• **Theorem: 2**	• **Theorem: 3**
$z = f(x,y)$	$z = f(x,y)$	$f = \phi(x, y, z)$
$dz = \left(\dfrac{\partial z}{\partial x}\right)_y dx + \left(\dfrac{\partial z}{\partial y}\right)_x dy$ $\qquad$ M $\qquad$ N	$\left(\dfrac{\partial z}{\partial x}\right)_y \left(\dfrac{\partial x}{\partial y}\right)_z \left(\dfrac{\partial y}{\partial z}\right)_x = -1$	$\left(\dfrac{\partial x}{\partial y}\right)_f \left(\dfrac{\partial y}{\partial x}\right)_f \left(\dfrac{\partial z}{\partial x}\right)_f = 1$
$dz = Mdx + Ndy$ A function is said to exact, if	This relation is known as cyclic relation.	
$\left(\dfrac{\partial M}{\partial y}\right)_x = \left(\dfrac{\partial N}{\partial x}\right)_y \Rightarrow$ exact		

MAXWELL'S 4 EQUATIONS

- The equations that relate the partial derivative of properties P, V, T and S of a simple compressible system to each other are called Maxwell relations. They are obtained from the **four Gibbs equations** by exploring the exactness of the differentials of thermodynamic properties.

- Two of them are taken from TdS relations.

$$dU = TdS - PdV$$
$$dH = TdS + VdP$$

- The other two are based on Helmholtz and Gibbs function.

   Helmholtz function   F = U – TS (availability of closed system)

   Gibb's function   G = H – TS (availability of open system)

- For all real processes the value of Helmholtz function & Gibbs function decreases & attains a min value at equilibrium. These four relations will be used to derive Maxwell relations by two important theorems of partial derivative.

MAXWELL 4 RELATIONS

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V \qquad \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P \qquad \left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \qquad \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

COEFFICIENT OF VOLUME EXPANSION (β)

$\beta = \dfrac{1}{v}\left(\dfrac{\partial V}{\partial T}\right)_P$, It is rate of change of volume with respect to temperature at constant pressure.

ISOTHERMAL COMPRESSIBILITY (K_T)

$K_T = -\dfrac{1}{v}\left(\dfrac{\partial V}{\partial P}\right)_T$, It shows the rate of change of volume with respect to pressure at constant

temperature or at isothermal conditions.

T-dS EQUATIONS

Using S = f(T, V) & Maxwell relations $\Rightarrow TdS = C_v dT + T\left(\dfrac{\partial P}{\partial T}\right)_v dV$ → **first Tds equation**

Using S = f(T, P) & Maxwell relations $\Rightarrow TdS = C_p dT - T\left(\dfrac{\partial v}{\partial T}\right)_T dP$ → **second TdS equation**

MAYER'S FORMULA

Using both TdS equation and using $T = \phi(P, V)$, we have $\qquad \Rightarrow C_p - C_v = \dfrac{-Tv\beta^2}{k_T}$

ENERGY EQUATIONS

Using TdS relation & U= f (T, P, V) & Ideal gas equations $\qquad \Rightarrow \left(\dfrac{\partial U}{\partial V}\right)_T = 0$ & $\left(\dfrac{\partial U}{\partial P}\right)_T = 0$

→ This shows that the internal energy of an ideal gas does not vary with respect to pressure under isothermal conditions. Therefore, **Internal energy of an ideal gas is a function of temperature only.**

JOULE THOMSON COEFFICIENT (μ)

- When a fluid passes through porous plug, capillary tube or valve, its pressure decreases.
- Throttling process is isenthalpic in nature. The temperature behaviour of the fluid during throttling is described by joule Thompson coefficient (μ).

$$\Rightarrow \mu = \left(\dfrac{\partial T}{\partial P}\right)_h = \dfrac{1}{C_P}\left\{T\left(\dfrac{\partial v}{\partial T}\right)_P - v\right\}$$

- Hence, for an ideal gas, μ = 0, if enthalpy is constant then on changing or reducing pressure, temperature will not change. Therefore, when we use air as refrigerant in aircraft refrigerant throttling is not used instead, we use an isentropic expander or turbine.
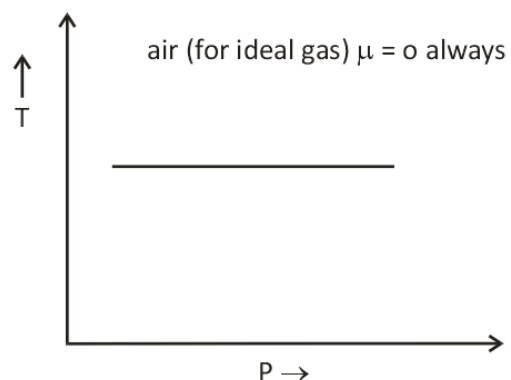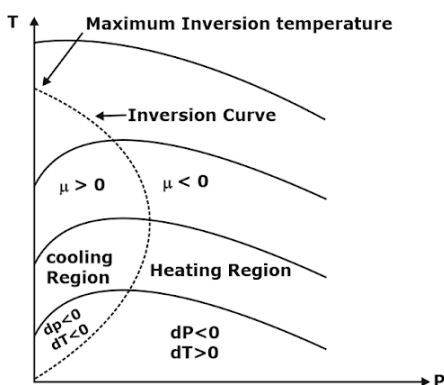
- μ is +ve in cooling region i.e. slope of isenthalpic curve on T – P diagram is +ve in cooling region
- μ is −ve in heating region i.e. slope of isenthalpic curve on T – P diagram is −ve in heating region
- There is nothing as heating or cooling region for an ideal gas & value of μ is zero everywhere.

CLAUSIUS CLAPEYRON EQUATION

- Clausius Clapeyron equations is a relationship between saturation pressure, temperature and enthalpy of vaporisation and the specific volume of two phases involved.
- This equation helps in calculations of properties in two phase regions.
- The Clausius Clapeyron equation can be derived using the Maxwell equation during phase change

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T = \frac{ds}{dv} = \frac{s_{fg}}{v_{fg}} = \frac{h_{fg}}{T_{sat}v_{fg}} = \frac{h_{fg}}{T_{sat}(v_g - v_f)} \rightarrow \text{Clapeyron Equation}$$

- If $v_g >>> v_f$ and behaviour of vapour is like ideal gas $v_g = \frac{RT_{sat}}{P_{sat}}$

$$\frac{dP}{dT} = \frac{P_{sat} \times h_{fg}}{RT_{sat}^2} \rightarrow \text{Clausius Clapeyron equation}$$

- Integrating above equation $\quad \ln\left(\frac{P_1}{P_2}\right) = \frac{h_{fg}}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right) \rightarrow$ It helps to determine enthalpy change associated with phase change by measuring pressure, temperature and volume.

CHAPTER-8 BEHAVIOUR OF IDEAL AND REAL GAS

IDEAL GAS EQUATION

- Deviations from ideal behaviour are observed particularly at high pressures or low temperatures.
- A gas which does not obey the ideal gas equation, PV = nRT strictly at all temperatures and pressures is said to be a real gas.

COMPRESSIBILITY FACTOR (Z)

- The deviation from ideal behaviour is expressed by introducing a factor Z known as compressibility factor in the ideal gas equation.
- Z may be expressed as, Z = PV / nRT
    - In case of ideal gas, PV = nRT ∴ Z = 1,   In case of real gas, PV ≠ nRT ∴ Z ≠ 1
    - Also, $z = \dfrac{v}{v_i} = \dfrac{\text{volume at an instant}}{\text{volume predicted by ideal gas}}$

- Compressibility factor (z) can be viewed as the ratio of actual volume of a real gas to the volume predicted by ideal gas equation.
- In case of real gases, Z can be less than 1 or greater than 1. Farther the value of z from unity, the more will the behaviour of gas deviates from ideal gas.
- When Z < 1, it is a negative deviation. It shows that the gas is more compressible than expected from ideal behaviour.
- When Z > 1, it is a positive deviation. It shows that the gas is less compressible than expected from ideal behaviour.

CAUSES OF DEVIATION FROM IDEAL BEHAVIOUR

- The causes of deviations from ideal behaviour may be due to the following two assumptions of kinetic theory of gases.
    1. Volume occupied by gas molecules is negligibly small as compared to the volume occupied by the gas. The first assumption is valid only at low pressures and high temperature.
    2. The forces of attraction between gas molecules are negligible. The second assumption is also valid at low pressures and high temperature. Otherwise, forces of attraction become appreciable and cannot be ignored.

CRITICAL PROPERTIES

- **Critical Temperature (T$_c$)-** It is the maximum temperature at which a gas can be liquefied i.e. the temperature above which a gas can't exist as liquid.
- **Critical Pressure (P$_c$)-** It is the minimum pressure required to cause liquefaction at T$_c$.
- **Critical Volume (V$_c$)-** It is the volume occupied by one mol of a gas at T$_c$ and P$_c$

REDUCED PROPERTIES

- The gases follow the ideal-gas equation closely at low pressure and high temperature. But what exactly constitutes low pressure or high temperature?
- pressure or temperature of a substance is high or low relative to its critical temperature or pressure

- Gases behave differently at a given temperature and pressure, but they behave very much the same at temperature and pressure normalized with respect to their critical temperature and pressure. The normalization is done as

  Reduced pressure $P_R = \dfrac{P}{P_{cr}}$

  Reduced temperature $T_R = \dfrac{T}{T_{cr}}$

  Reduced volume $V_R = \dfrac{V}{V_{cr}}$

- The ratios of pressure, temperature and specific volume of a real gas to the corresponding critical values are called the reduced properties.

COMPRESSIBILITY CHART

- The compressibility chart is drawn between compressibility factor & reduced pressure.

- Behaviour of different real gases can be predicted approximately by use of compressibility chart.
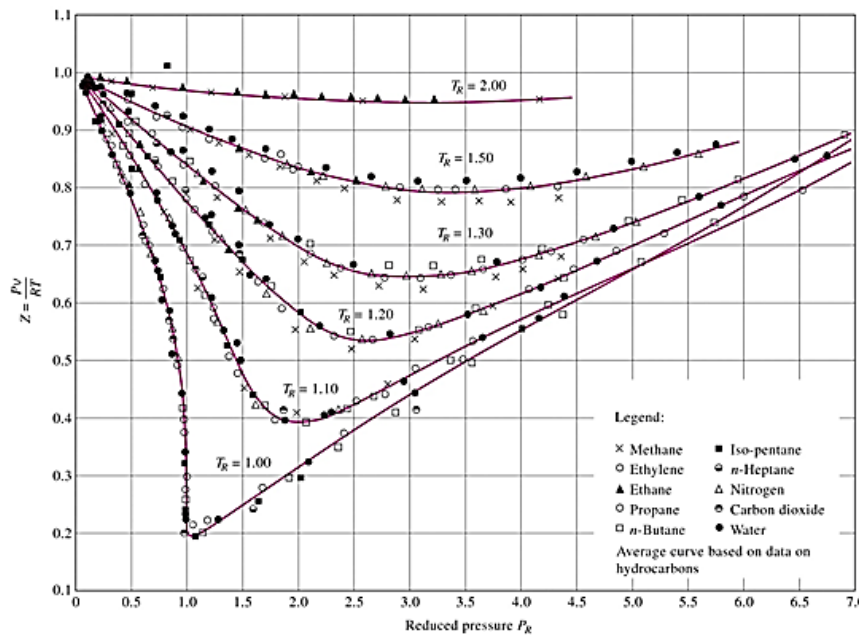
- The following observations can be made from the generalized compressibility chart
  1. At very low pressure, gases behave as an ideal gas regardless of temperature.
  2. At high temperature, ideal gas behavior can be assumed with good accuracy regardless of pressure.
  3. The deviation of a gas from ideal gas behavior is greatest in the vicinity of the critical point.

VAN DER WAALS EQUATION OF REAL GASES

- Van der Waals equation explains quantitatively observed behaviour of real gases, so it is an improvement over the ideal gas equation.

- Vander wall equation is given as, $\left(P + \dfrac{a}{v^2}\right)(v - b) = \bar{R}T$ per mole

  a → correction factor to account for intermolecular attractions

b → correction factor to account for the finite size of the molecules

- The determination of the two constants appearing in this equation is based on the observation that the critical isotherm on a P-V diagram has a horizontal **inflection point** at the critical point. Thus, the first and the second derivative of P with respect to V at the critical point must be zero.

i.e. $\left(\dfrac{\partial P}{\partial v}\right)_{T=T_c} = 0$ and $\left(\dfrac{\partial^2 P}{\partial v^2}\right)_{T=T_c} = 0$
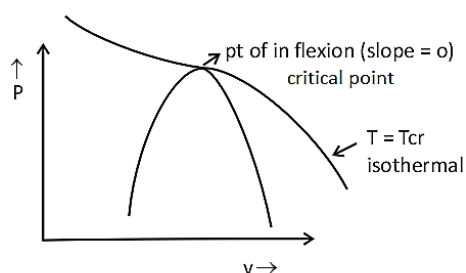
Using Van der Waals equation and above derivatives,

we get

$$T_c = \dfrac{8a}{27\bar{R}b}, \qquad P_c = \dfrac{a}{27b^2}, \qquad V_c = 3b$$

$$a = \dfrac{27\bar{R}^2 T_c^2}{64 P_c}, \qquad b = \dfrac{\bar{R}T_c}{8 P_c}$$

Compressibility factor at critical point $Z_c = \dfrac{P_c V_c}{\bar{R}T_c} = \dfrac{3}{8}$

****

Important Formula (Power Plant)

CHAPTER 1: RANKINE CYCLE

1. **RANKINE CYCLE: -** Rankine cycle has 4 major components.

   (i) Heat addition in boiler,

   (ii) Steam expansion in turbine,

   (iii) Condensation of saturated steam in condenser,

   (iv) Compression of saturated liquid by pump.
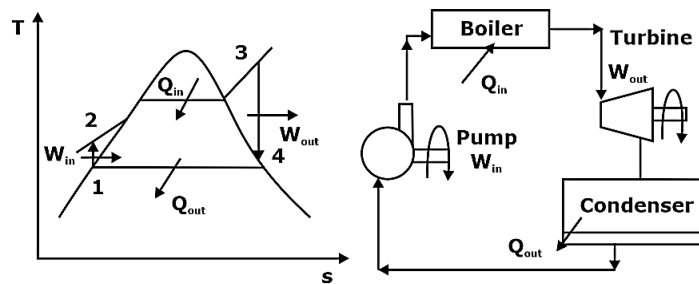
The ideal Rankine cycle

Pump	Q=0	$W_{pump,in} = h_2 - h_1$
Boiler	W=0	$Q_{in} = h_3 - h_1$
Turbine	Q=0	$W_{turbine,out} = h_3 - h_4$
Condenser	W=0	$Q_{out} = h_4 - h_1$

- **Efficiency:** $\eta_R = \dfrac{\text{net work output}}{\text{heat supplied to the boiler}} = \dfrac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)}$

- **Specific steam consumption (SSC):** It is the mass flow rate of steam, required to produce 1kW of power.

$$\text{ssc} = \frac{\dot{m}_f}{P} = \frac{3600}{W_{net}} \text{in kg / kWhr} \quad \text{where, } W_{net} = (h_3 - h_4) - (h_2 - h_1) \text{ in kJ / kg.}$$

- **Heat rate (H.R.):** It is the amount of heat required to produce 1kW of power output.

$$\text{H.R.} = \frac{\text{heat supplied(kW)}}{\text{power developed(kW)}} = \frac{1}{\eta}$$

- A steam power plant system consists of many components like, boiler, turbine, different auxiliaries, generator etc. So, the overall efficiency of steam power plant can be written as:

$$\eta_{overall} = \eta_{boiler} \times \eta_{cycle} \times \eta_{mechnical} \times \eta_{generator} \times \eta_{auxiliaries}$$

- Due to irreversibility in the turbine and pump, process in them is not isentropic, so to find actual turbine and pump work isentropic efficiency is used.
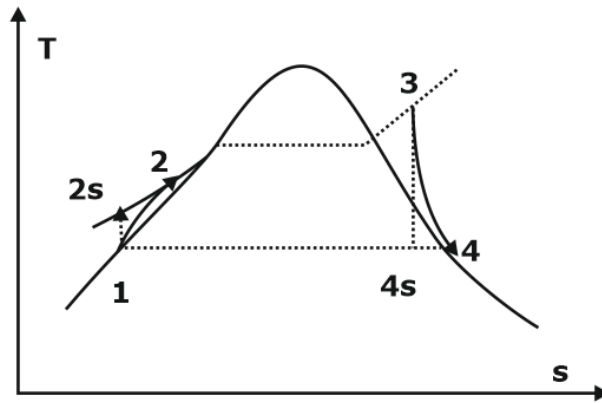
Isentropic efficiency of pump, $\eta_{pump} = \dfrac{\text{isentropic work}}{\text{actual work}} = \dfrac{(h_{2s} - h_1)}{(h_2 - h_1)}$

Isentropic efficiency of turbine, $\eta_{turbine} = \dfrac{\text{actual work}}{\text{isentropic work}} = \dfrac{(h_3 - h_4)}{(h_3 - h_{4s})}$

1.1. Methods of increasing the efficiency of Rankine cycle:

1.1.1. Decreasing the Condenser Pressure

By decreasing the condenser pressure, mean temperature of heat rejection in the condenser decreases. Thus, the thermal efficiency of the cycle will be increased.

$$\eta \propto 1 - \dfrac{T_L}{T_H}$$
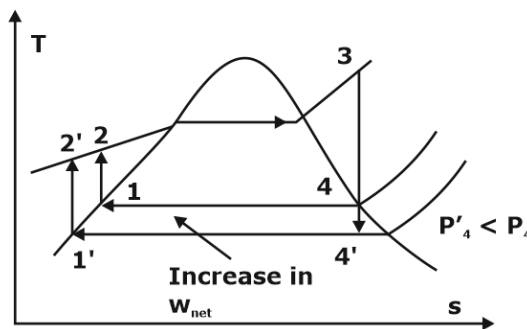
Effect of lowering condenser pressure on ideal Rankine cycle

1.1.2. Superheating the Steam to High Temperature

Superheating the steam will increase the network output and the efficiency of the cycle. It also decreases the moisture contents of the steam at the turbine exit.
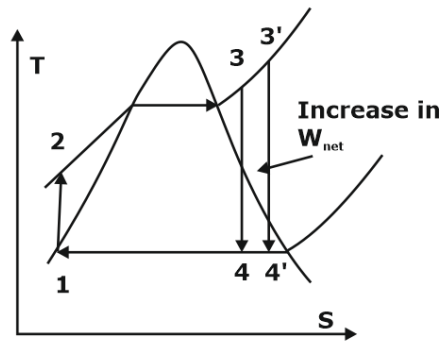
The effect of superheating on the ideal Rankine cycle.

1.1.3. Increasing the Boiler Pressure

On increasing the boiler pressure, the mean temperature of heat addition increases hence thermal efficiency increases.
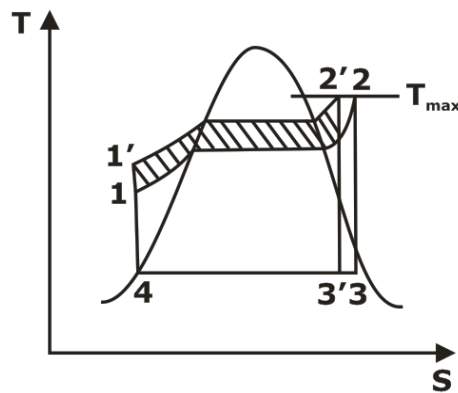
Increasing the boiler Pressure

1.2. IDEAL REHEAT RANKINE CYCLE

To take advantage of the increased efficiencies at higher boiler pressure without facing the excessive moisture content at final stages of the turbine, reheating is used. In this process steam is expended in more than one turbine by providing heat between two turbines.
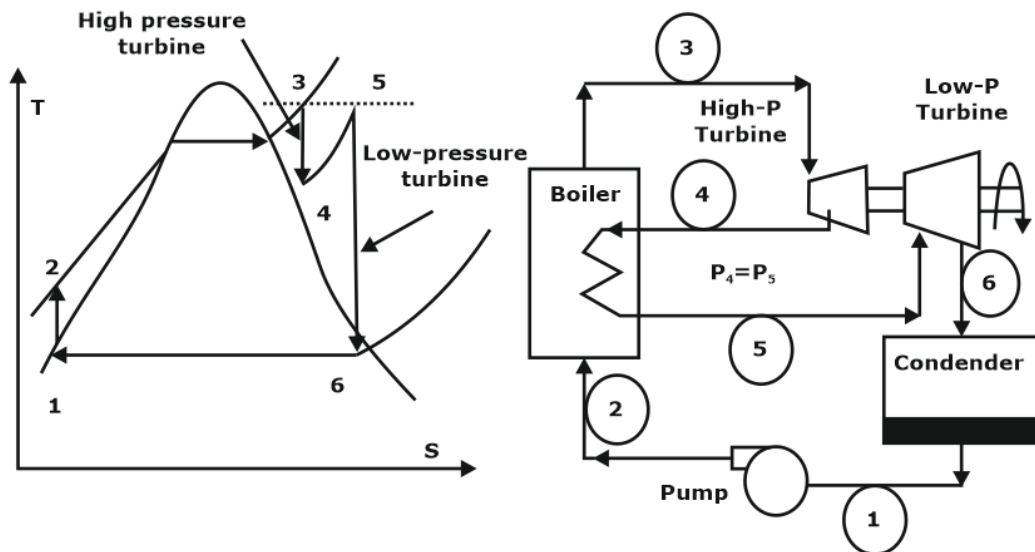
The ideal reheat Rankine cycle

- Optimum reheat pressure for the most of the modern power plant is 0.20 to 0.25 of the initial steam pressure.

$$\text{Heat input} = \text{Primary heat} + \text{Reheat} = (h_3 - h_2) + (h_5 - h_4)$$

$$W_{\text{turbine}} = W_{\text{H-P turbine}} + W_{\text{L-P turbine}} = (h_3 - h_4) + (h_5 - h_6)$$

1.3. IDEAL REGENERATIVE RANKINE CYCLE

The mean temperature of heat addition can also be increased if heat addition at low temperature is avoided such that feed water enters the boiler at saturated liquid condition. The feed water can be brought to saturated condition by internal heating using extracted steam. This concept is known as regeneration.
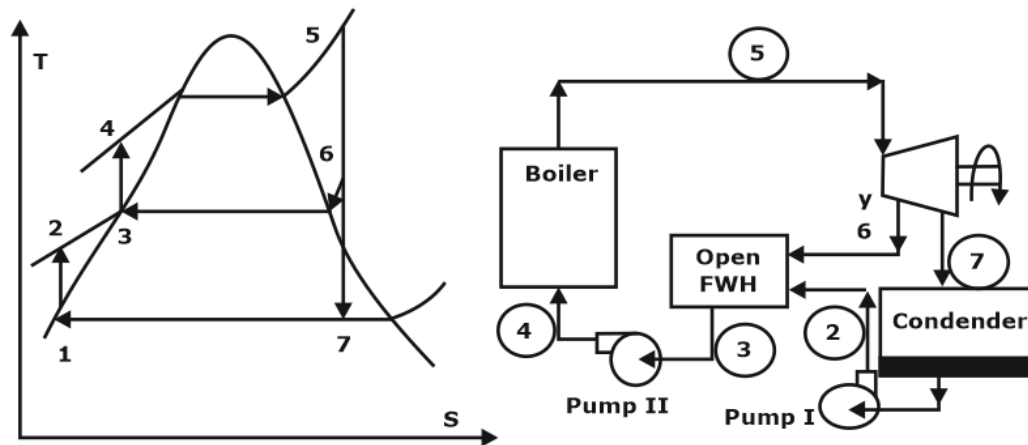
The ideal regenerative Rankine cycle with an open FWH

y = amount of bleed from turbine

Q_{in}= h_5-h_4

Q_{out}= $(1-y)$ (h_7-h_1)

$W_{\text{turbine out}}$= (h_5-h_6) +$(1-y)$ (h_6-h_7)

W_{pump}=$(1-y)$ W_{pump1}+ W_{pump2}

W_{pump1}= $v_1(P_2-P_1)$ and W_{pump2}= $v_3(P_4-P_3)$,

(Because change in specific volume of liquid is very small)

Where v is specific volume at pump inlets.

- **Canonization of Rankine Cycle**: If infinite number of feed water heater is used, Rankine cycle can be converted into a Carnot cycle.

- **Law of diminishing returns**: It states that the greatest increment in efficiency is brought by the 1st heater, the increment for each additional heater thereafter successively diminish. This law is used to determine the optimum number of regenerator.

- Regeneration is done with the help of water heaters. Rankine cycle uses mainly two type of water heaters:

(i) **Open feed water heater**: In this type of water heater direct mixing of two fluids occurs i.e. both fluid come in physical contact with each other. Example : Deaerator .

(ii) **Close feed water heater**: In this type of water heater no physical mixing of fluid is allowed and heat exchange takes place by the non-contact type heat exchangers.

1.4 Binary fluid cycles:- This type of cycle is used to get the advantage of two fluids in one system. In this cycle heat rejected by one cycle works as source for other cycle.

$$\eta_{binary} = 1 - (1 - \eta_{I})(1 - \eta_{II})$$

Where η_{I} = efficiency of 1st cycle, η_{II} =efficiency of 2nd cycle.

CHAPTER 2: GAS TURBINE

2.1 Brayton Cycle: - It consists of 2 isentropic and 2 isobaric processes. It can be used as open Brayton cycle and close Brayton cycle.

- In open cycle, fuel is burn inside combustion chamber and burnt gases are used as the working fluid.

- In close cycle, fuel burn outside the cycle and heat is exchanged with the help of heat exchanger. They are larger in size because of the use of two heat exchangers.
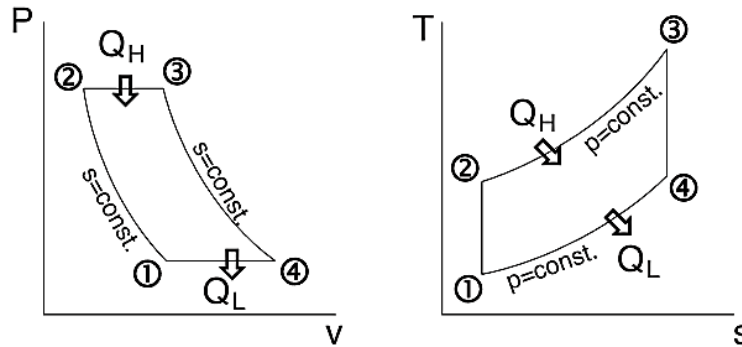
- Work done by compressor (1-2): $W_c = h_2 - h_1 = C_p(T_2 - T_1)$

- Heat addition (2-3): $\qquad$ $Q_{addition} = h_3 - h_2 = C_p(T_3 - T_2)$

- Work done by turbine (3-4): $\qquad$ $W_T = h_3 - h_4 = C_p(T_3 - T_4)$

- Heat rejection (4-1): $\qquad$ $Q_{rejection} = h_4 - h_1 = C_p(T_4 - T_1)$

- Important relationship: $\qquad$ $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_4} = r_p^{\frac{\gamma-1}{\gamma}}$

- Efficiency: $\eta_{brayton} = 1 - \dfrac{1}{r_p^{\frac{\gamma-1}{\gamma}}}$ $\qquad$, where $r_p = \dfrac{P_2}{P_1} = \dfrac{P_3}{P_4}$ (pressure ratio)

- Efficiency of Brayton cycle increases with increasing the value of pressure ration and γ.

- **Work ratio (r_w):** It is the ratio of net work and turbine work.

$$r_w = \frac{W_{net}}{W_T}$$

- **Back work ratio (r_{bw}):** It is the ratio between compressor work and turbine work.

$$r_{bw} = \frac{W_c}{W_T} \quad , \quad r_w + r_{bw} = 1$$

- Back Work Ratio for gas power cycles: 40% - 60%

- **Air rate (A.R.)**: It is the mass flow rate to air used to produce 1kW of net power.

$$\text{A.R.} = \frac{\dot{m}_{air}}{P} = \frac{3600}{W_{net}} \text{ in kg/kW-hr} \text{ , where } W_{net} \text{ is in kJ/kg.}$$

- **Power (P):** $P = \dot{m}_{air} \times W_{net}$ in kW

2.1.1. Maximum Pressure Ratio

- $(r_p)_{max} = \left(\dfrac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{\gamma-1}}$

- At maximum pressure ratio the efficiency of brayton cycle is equal to Carnot cycle operating between same temperature limit.

- At maximum pressure ratio the net-work is zero.

2.1.2. Optimum Pressure Ratio

- $(r_p)_{optimum} = \left(\dfrac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}} = \sqrt{(r_p)_{max}}$

- Optimum pressure ratio gives maximum net-work.

- In this condition: $T_2 = T_4 = \sqrt{T_1 \times T_3}$ and $(W_{net})_{max} = C_p\left(\sqrt{T_3} - \sqrt{T_1}\right)^2$.

- Efficiency can be written as: $\eta_{optimum} = 1 - \sqrt{\dfrac{T_1}{T_3}}$

2.1.3. Actual Brayton Cycle: In actual brayton cycle compression and expansion are not isentropic and pressure drop occurs due to friction in the heat exchangers /combustion chamber.
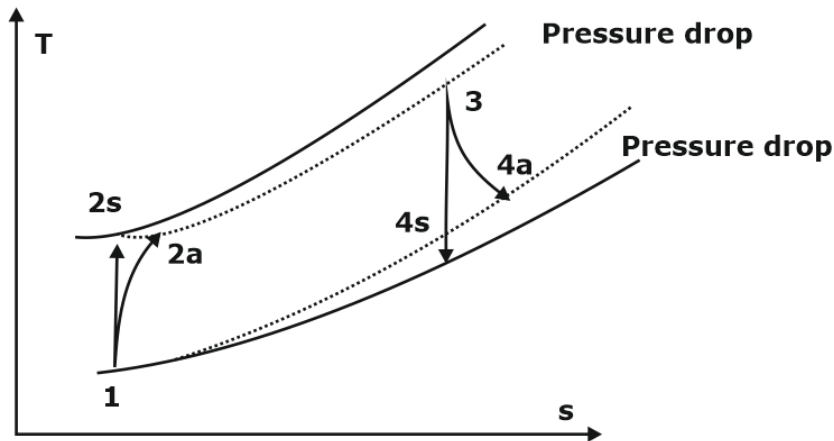
$$\text{Isentropic efficiency of compressor, } \eta_{compressor} = \frac{\text{isentropic work}}{\text{actual work}} = \frac{(h_{2S} - h_1)}{(h_{2a} - h_1)}$$

$$\text{Isentropic efficiency of turbine, } \eta_{turbine} = \frac{\text{actual work}}{\text{isentropic work}} = \frac{(h_3 - h_{4a})}{(h_3 - h_{4s})}$$

2.1.4. Brayton Cycle with Reheat

- This uses more than one gas turbine that results in increase in net work output. But the efficiency decrease.
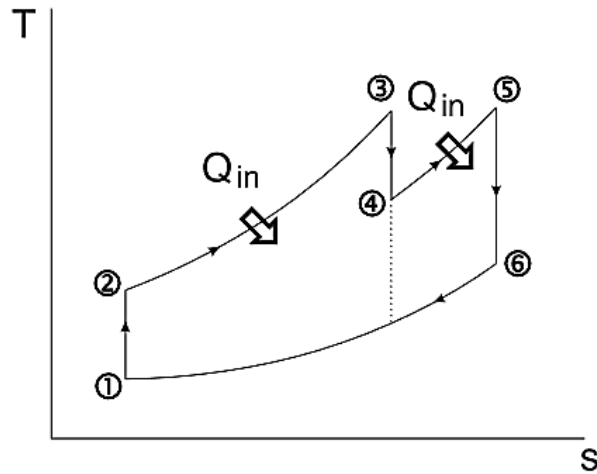
- For perfect reheating:
    - $T_3 = T_5$
    - Intermediate pressure will be, $P_i = \sqrt{P_1 \times P_2}$
    - Work done by both the turbine will be same, $W_{T_1} = W_{T_2}$

2.1.5. Brayton Cycle with Intercooling

- It uses more than one compressor to compresses the air. By this modification, compressor work reduces and network done increases. But due to this efficiency of cycle decreases.
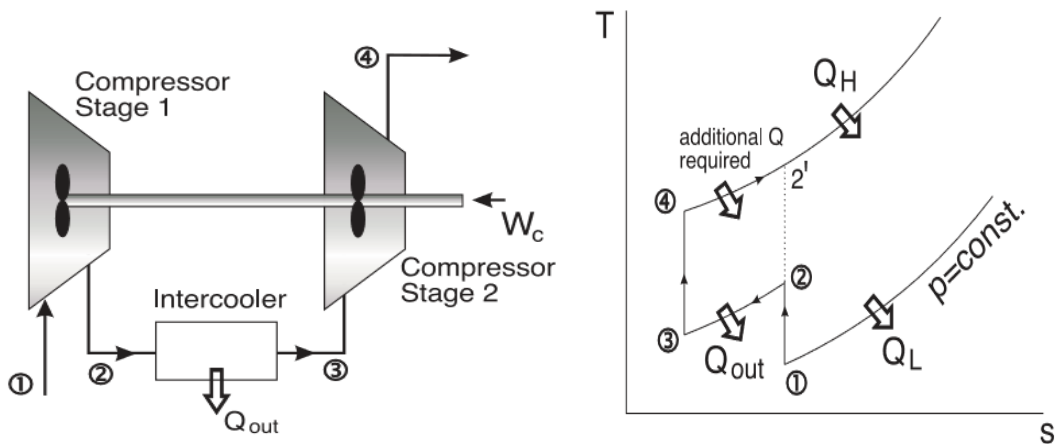
- For perfect intercooling:
    - $T_3 = T_1$
    - Intermediate pressure will be: $P_i = \sqrt{P_1 \times P_4}$
    - Work done of both compressor will be same: $W_{c_1} = W_{c_2}$

2.1.6. Brayton Cycle with Regeneration

- In this exhaust gas from turbine is used to exchange heat with the air coming to the combustion chamber.

- This will increase the mean heat addition temperature and decrease the mean heat rejection temperature hence increases the efficiency of cycle.
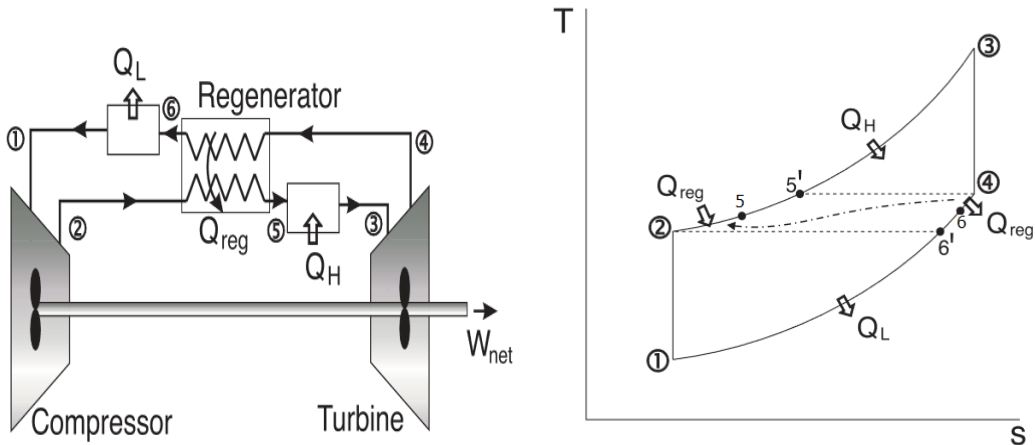
- Effectiveness in this type of system can be defined as the ratio of actual temperature rise and maximum possible rise in temperature due to regeneration.

$$Q_{regeneration,actual} = h_5 - h_2, \ Q_{regeneration,max} = h_4 - h_2$$

$$\text{Effectivenss}(\varepsilon) = \frac{Q_{regeneration,actual}}{Q_{regeneration,max}} = \frac{h_5 - h_2}{h_4 - h_2} = \frac{T_5 - T_2}{T_4 - T_2}$$

- For ideal regeneration:

  - $T_5 = T_4 = T_{5'}$ and $T_2 = T_6 = T_{6'}$

  - Efficiency: $\eta_{\text{ideal regenerative}} = 1 - \frac{T_1}{T_3}\left(r_p^{\frac{\gamma-1}{\gamma}}\right)$

  - For ideal regenerative cycle efficiency will decrease on increasing the pressure ratio and γ.

2.1.7. COMBUSTION CHAMBER:

Combustion chamber is used in open brayton cycle. Generally, A very lean mixture is used in brayton cycle i.e. A/F=50:1 to 80:1. For open cycle equation of combustion chamber can be written as:

$$\dot{m}_f \times (CV)_{fuel} \times \eta_{combustion} = (\dot{m}_f + \dot{m}_a) \times C_{p_{gas}} \times T_3 - \dot{m}_a \times C_{p_{air}} \times T_2$$

Where $\dot{m}_f$ = mass flow rate of fuel, $\dot{m}_a$ =mass flow rate of air, $\eta_{combustion}$ = combustion efficiency, $C_{p_{gas}}$ = specific heat of burnt gases, $C_{p_{air}}$ =specific hear of air, T_2, T_3 are temperature at the inlet and exit of combustion chamber.

CHAPTER 3: COMPRESSOR

Compressor is a device which is used to compress the gas or vapour from lower to higher pressure and for that work input is required from outside. In process of compressing, temperature of fluid also increases.
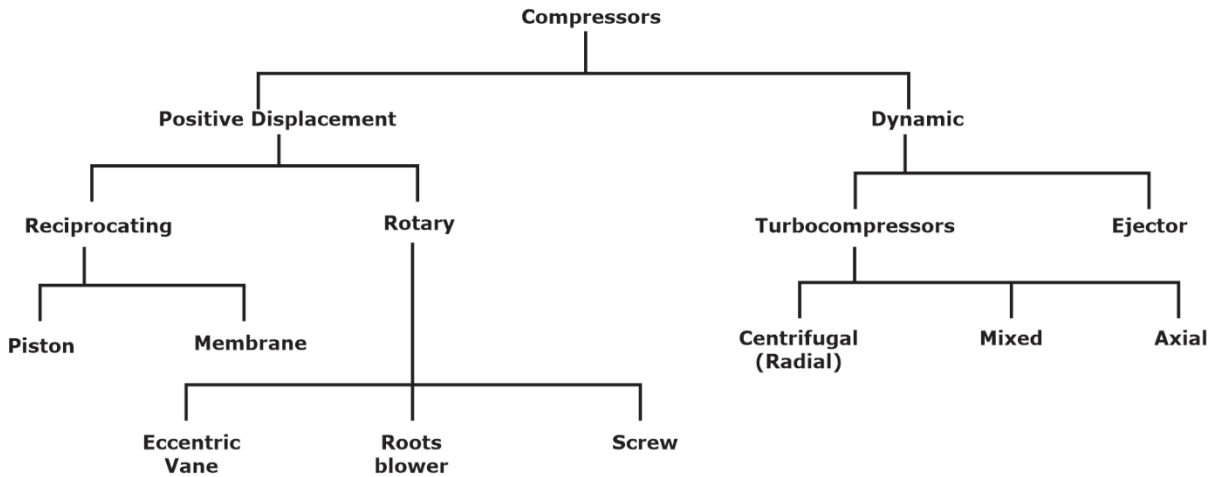
3.1. Reciprocating compressor

3.1.1. Work input required (without clearance volume)
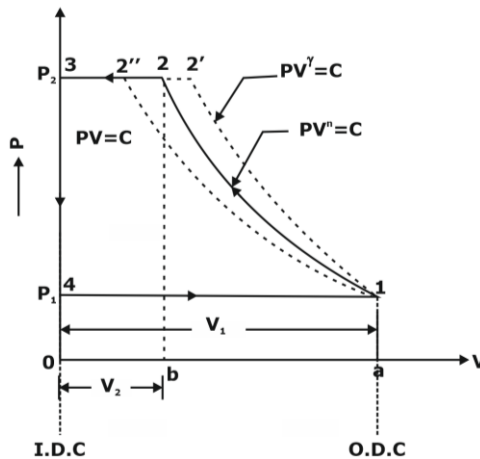
Fig.: Representation of compression Process on P-V diagram.
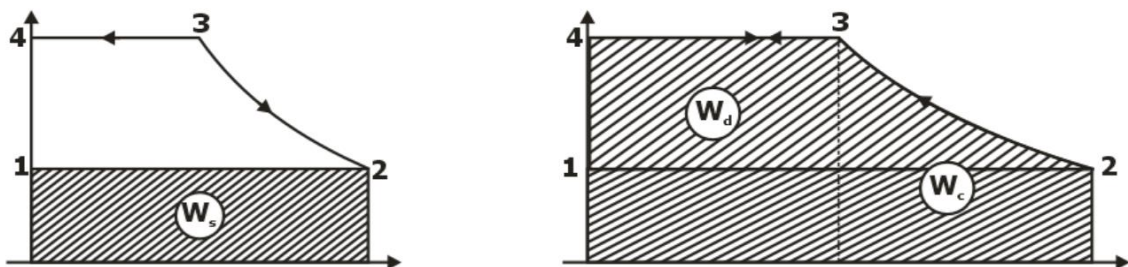
$$W / cycle = \left(\frac{n}{n-1} \right) \left[p_2 v_2 - p_1 v_1 \right]$$

$$W / cycle = \left(\frac{n}{n-1} \right) p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

3.1.2. Compressor work input (with clearance volume)
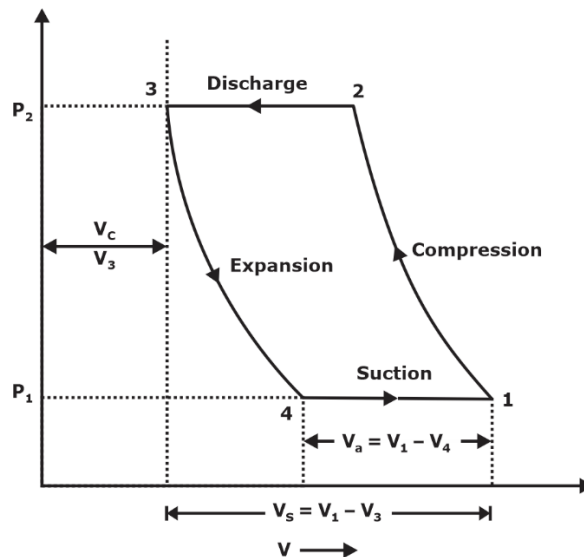
Fig.: P-V diagram with clearance

$$W / cycle = \frac{n}{n-1} p_1 \left[v_1 - v_4 \right] \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

- For compressor minimum work input is required if the compression is isothermal. So isothermal efficiency is used for reciprocating compressor.
- Isothermal efficiency is the ratio of isothermal work and actual work of compressor. Isothermal work to compress air from pressure P_1 to P_2 is

$$W_{isothermal} = P_1 V_1 \times \ln \left(\frac{P_2}{P_1} \right)$$

$$\eta_{isothermal} = \frac{\text{isothermal work}}{\text{actual work}}$$

3.1.3. Volumetric efficiency: It is ratio of actual volume intake and swept volume.

$$\eta_{vol} = \frac{\text{Actual volume}}{\text{Swept volume}}$$

$$\eta_v = 1 + C - C \left(\frac{p_2}{p_1} \right)^{1/n}$$

Where C= clearance ratio.

- Volumetric efficiency decreases with increase to pressure ratio. So to get higher pressure ratio multi-stage compression is used.

- Volumetric efficiency becomes zero at pressure ratio of:

$$\frac{P_2}{P_1} = \left(\frac{1}{C} + 1\right)^n$$

- FAD (free air delivered) is the volume of air compressed at atmospheric conditions i.e. P= 1 bar, and T=288 K. FAD is used to compare different compressors capacity.

3.1.4. Multi-stage compression:
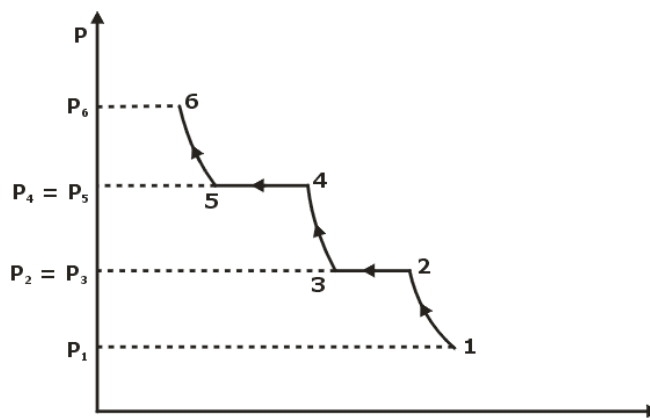
P-V diagram of Multiple stage compression

- The compression is done with multi-stage with intercooling between stages which results in less total compressor work.
- The optimum pressure ratio per stage in case of n stage compression with intercooling

$$(r_P)_{opt} = \left[\frac{P_F}{P_i}\right]^{1/n}$$

- The work done in each stage is same.
- For same efficiency and perfect intercooling, dimensions of cylinder can be related as:

$$P_1 D_1^2 = P_2 D_2^2 = \text{constant}$$

4. CENTRIFUGAL COMPRESSOR:-

- The principal components of centrifugal compressor are impeller and diffuser.
- Theoretical Power consumed by compressor is:

$$P_{theoritical} = \dot{m} \times \left(V_{w_2} u_2 - V_{w_1} u_1\right)$$

Where $\dot{m}$ = mass flow rate of air, V_{w_2}, V_{w_2} are the whirl velocity at exit and entry respectively, u_2, u_1 are the rotor velocity at exit and inlet respectively.

4.1 FOR RADIAL COMPRESSOR:-

- Actual power produced by turbine (for radial blade with slipping):

$$P_{actual} = \dot{m} \times \phi_s \times \phi_w \times u_2^2$$

  Where, $\phi_s = \text{slip factor} (\phi_s < 1)$, $\phi_w = \text{work factor} (\phi_w > 1)$.

- In terms of enthalpy and stagnation temperature actual power consumed will be:

$$P_{actual} = \dot{m} \times \left(h_{o_2} - h_{o_1}\right) = \dot{m} \times C_p \left(T_{o_2} - T_{o_1}\right)$$

  If the velocity at inlet and exit to be same. i.e. $v_1 = v_2$

$$P_{actual} = \dot{m} \times C_p \left(T_2 - T_1\right)$$

  Where, h_{o_2}, h_{o_1} are the stagnation enthalpy at entry and exit of compressor,

  T_{o_2}, T_{o_1} are the stagnation temperature at entry and exit of compressor,

  T_2, T_1 are the temperature at the entry and exit of compressor.

4.2 FOR AXIAL COMPRESSOR:

- Power consumed by the compressor is:

$$W_{net} / \text{stage} = \dot{m} \times u \times v_f \times \left(\tan\beta - \tan\alpha\right) = \dot{m} \times u \times v_f \times \left(\tan\theta - \tan\phi\right)$$

  Where u= velocity of blade at mean diameter, v_f=flow velocity of fluid,

  α, β are the flow angles at entry and exit, θ, ϕ are the blade angle at entry and exit

- Forward curved blade consumes maximum power.

- Backward curved blade gives best efficiency and are stable for wide operating range.

- **Surging** is the complete breakdown of steady flow through compressor, due to periodic flow reversal. This reversal of flow causes abnormal sound, vibration, decrease in efficiency, and increase in temperature and if the intensity is more it will lead to mechanical damage.

- The maximum mass flow rate possible through compressor is termed as **choking**. It occurs when the Mach number corresponding to relative velocity at inlet become sonic. Choking means fixed mass flow rate irrespective of pressure ratio.

- **Stalling** is an aerodynamic flow separation form the blade surface due to improper design of blade. It is a local phenomenon and chances of flow separation are more at low mass flow rate, non-uniform surface, improper design of blades and higher no. of diffuser blades.

CHAPTER 4: STEAM TURBINE

A steam turbine converts the energy of high-pressure, high temperature steam produced by a steam generator into shaft work.

- On the basis of working principal steam turbine can be classified into Impulse type turbine and reaction type turbine.
- In **impulse turbine** pressure drop take place only in the nozzle and there is no drop of pressure in moving blade. This turbine converts the only kinetic energy into electrical energy.
- In **reaction turbine** pressure drop takes place both in fixed as well as moving blade. This turbine converts pressure and kinetic energy into electric energy.
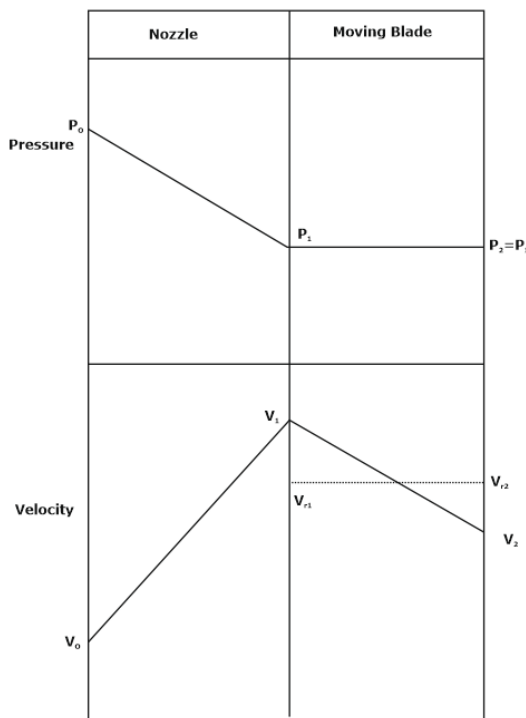
Impulse turbine
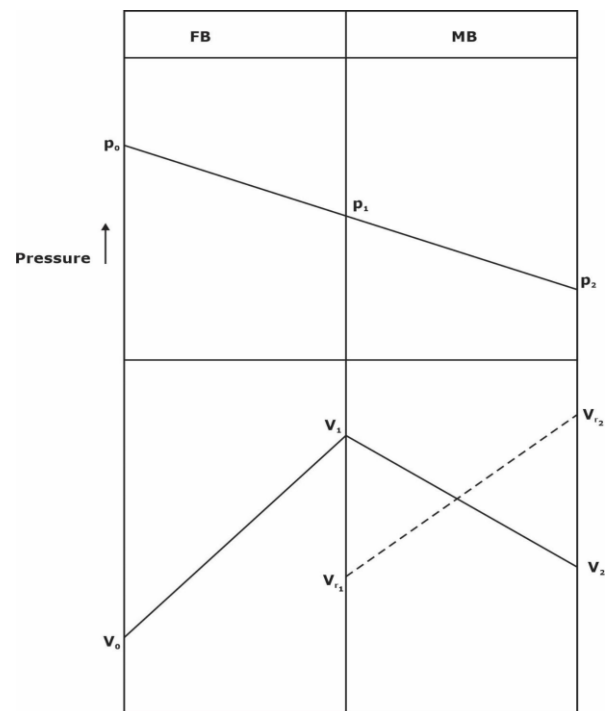
Reaction turbine

- **Degree of reaction:** This can be defined as the ratio of enthalpy drop in the rotor to the total enthalpy drop (in stator +rotor blade).

$$R = \frac{\text{enthalpy drop in rotor}}{\text{enthalpy drop in stator+enthalpy drop in rotor}} = \frac{\Delta h_{rotor}}{\Delta h_{stator} + \Delta h_{rotor}}$$

Degree of reaction for impulse turbine is always zero.

- In Impulse steam turbine, two type of compounding is possible.

- **Pressure compounding:** In this pressure drop takes place in more than one row of nozzles and the increase in kinetic energy after each nozzle is held within limits. Example: Rateau turbine.
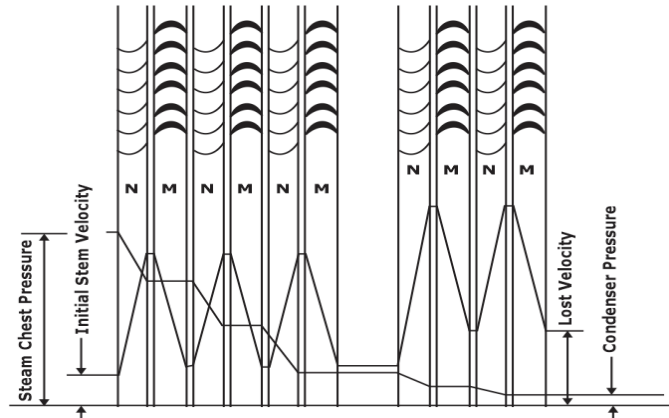
- **Velocity compounding:** In this type of compounding first all the pressure energy is converted into kinetic energy with the use of single nozzle, then this kinetic energy of steam is extracted by the row of moving and fixed blades. Example: Curtis turbine.
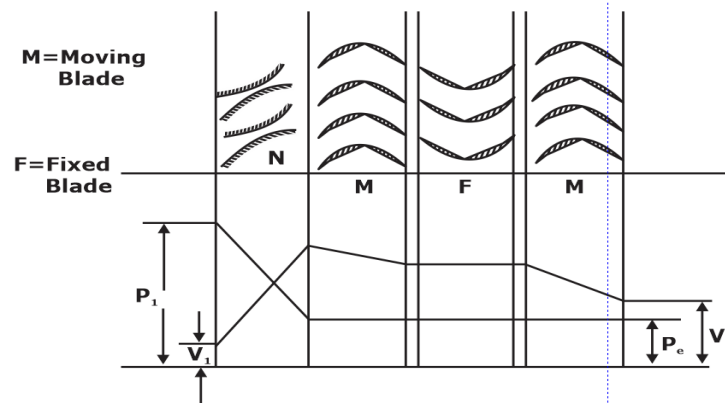
DIFFERENT STEAM TURBINES:-

1. **De Laval turbine:-** A single-stage impulse turbine is called the de Laval turbine. It consists of a single rotor with impulse blades attached to it. In this turbine all the pressure energy converted into kinetic energy with the help of nozzle and then kinetic energy is converted into shaft work with the help of impulse blade attached to rotor.
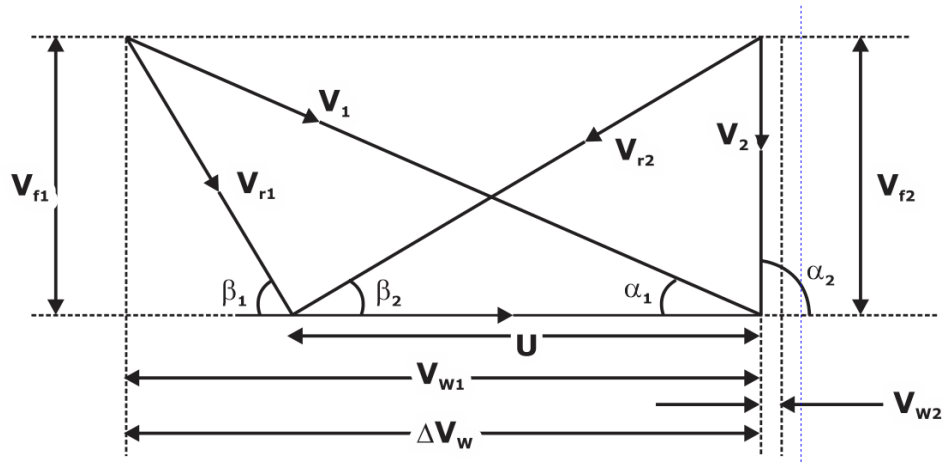
Fig.: Velocity Triangle of Impulse Turbine

Power developed: $\qquad P = \dot{m} \times u \times \left(V_{w_2} - V_{w_1} \right)$

Where, V_{w_2}, V_{w_1} are the whirl velocity at inlet and exit respectively, u= velocity of blade/rotor, $\dot{m}$ = mass flow rate of steam.

Nozzle efficiency: It is defined as the ratio of kinetic energy at nozzle exit and isentropic enthalpy change within nozzle.

$$\eta_{nozzle} = \frac{\text{Kinetic energy at exit of nozzel}}{\text{isentropic enthalpy change in nozzle}} = \frac{\frac{1}{2}\dot{m} \times V_1^2}{\dot{m} \times \Delta h_{isentropic}}$$

Blade efficiency: It is the ratio of power developed and the kinetic energy provided by the nozzle.

$$\eta_{blade} = \frac{\text{work by turbine blade}}{\text{Kinetic energy supplied by nozzel}} = \frac{\dot{m} \times u \times \left(V_{w_2} - V_{w_1} \right)}{\frac{1}{2}\dot{m} \times V_1^2}$$

It can be written as:

$$\left(\eta_b\right) = 2\left(\rho\cos\alpha - \rho^2\right)\left(1 + k_b\right)$$

Where, $\rho = \dfrac{V_b}{V_1}$

For maximum efficiency,

$$\rho_{optimum} = \frac{V_b}{V_1} = \frac{\cos\alpha}{2}$$

$$\left(\eta_b\right)_{max} = \left(\frac{1 + k_b}{2}\right)\cos^2\alpha$$

If friction is neglects (k_b=1), maximum efficiency will be,

17

$$(\eta_b)_{max} = \cos^2 \alpha$$

Where, V_b is the blade velocity, V_1= fluid velocity at inlet, α =nozzle angle at inlet, k_b=constant for friction loss.

2. **Rateau turbine :-**To resolve the problem of very high speed of rotor in case of single-stage impulse turbine, the total enthalpy is divided among many single-stage impulse turbine in series. And such a turbine is known as Rateau turbine. In this turbine :

$$(\Delta h)_{stage} = \frac{\Delta h_{total}}{n}$$

3. **Curtis turbine: -** This is an impulse turbine with velocity compounding. In this turbine pressure and enthalpy drop of steam take place in single stage/row of nozzle and the kinetic energy of steam is absorb by the number of moving blades. For n-row Curtis turbine condition of maximum efficiency will be (for no friction, symmetric blades and axial discharge):

$$\rho = \frac{v_b}{v_1} = \frac{\cos \alpha}{2n}$$

NOTE: Pearson turbine is a reaction turbine with degree of reaction 0.5. Hero's turbine is a reaction turbine with degree of reaction 1.

4. **Reaction turbine:**
   - A reaction turbine is one that is constructed of rows of fixed and rows of moving blades. The fixed blades act as nozzles. The moving blades move as a result of the Impulse of steam received (caused by change in momentum) and also as a result of expansion and acceleration of the steam relative to them.
   - The moving blades of a reaction turbine are easily distinguishable from those of an impulse turbine In that they are not symmetrical and, because they act partly as nozzles, have a shape similar to that of the fixed blades, although curved in the opposite direction.
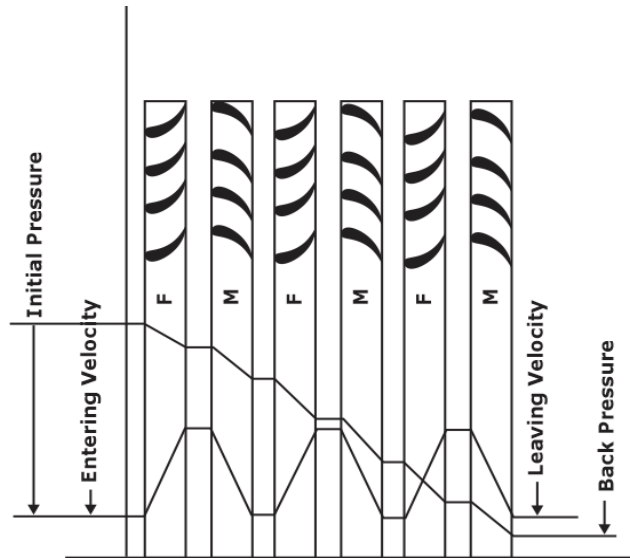
Reaction turbine indicating pressure and velocity distribution

- Condition for maximum efficiency in reaction turbine:

$$\rho_{optimum} = \frac{V_b}{V_1} = \cos\alpha$$

And maximum efficiency will be: $\eta_{max} = \dfrac{2\cos^2\alpha}{1+\cos^2\alpha}$
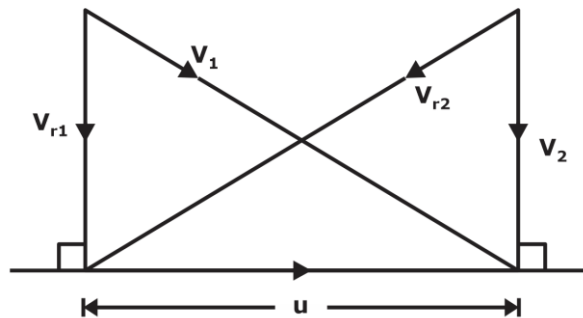
Velocity diagram for maximum efficiency

Generally $\qquad \eta_{reaction} > \eta_{rateau} > \eta_{curtis}$

Reheat factor and Stage efficiency: - The Thermodynamic effect on the turbine efficiency can be best understood by considering a number of stages between two stages, 1 and 2 as shown in Figure. (4 stages in between).
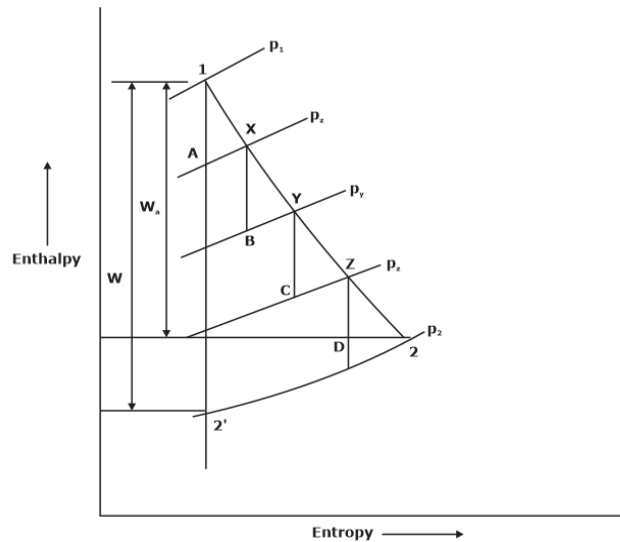
Fig: Different stage of a steam turbine

The total expansion is divided into four stages of the same efficiency (η_s) and pressure ratio.

$$\frac{P_1}{P_x} = \frac{P_x}{P_y} = \frac{P_y}{P_z} = \frac{P_z}{P_2}$$

So, Overall efficiency $(\eta_0) = \dfrac{W_a}{W} = \dfrac{\text{actual enthalpy drop}\,(1-2)}{\text{isentropic heat drop}\,(1-2')}$

Reheat factor is defined as:

$$\text{Reheat factor(RF)} = \frac{\text{Cumulative enthalpy drop (isentropic)}}{\text{Isentropic enthalpy drop (overall)}} = \frac{\Delta h_{1A} + \Delta h_{XB} + \Delta h_{YC} + \Delta h_{ZD}}{\Delta h_{12}}$$

Generally R.F is varied from 1.03 to 1.04

Stage efficiency $\left(\eta_s\right)$ is defined as the ratio of actual enthalpy drop and isentropic enthalpy

drop for a particular stage. If η_s remains same for all the stages or η_s is the mean stage efficiency.

$$\eta_s = \frac{\Delta h_{1x}}{\Delta h_{1A}} = \frac{\Delta h_{xy}}{\Delta h_{xB}} = \frac{\Delta h_{yz}}{\Delta h_{yc}} = \frac{\Delta h_{z2}}{\Delta h_{zD}}$$

So, overall efficiency can be written as

$$\eta_o = \eta_s \times R.F.$$

This makes the overall efficiency of the turbine greater than the individual stage efficiency.

Steam Turbine Governing and Control:

The objective of governing is to keep the turbine speed fairly constant irrespective of load. The principal methods of steam turbine governing arc as follows:

(i). Throttle governing

(ii). Nozzle governing

(iii). By-pass governing

Comparison of Throttle Governing and Nozzle Control Governing

S. No.	Aspects	Throttle Control	Nozzle Control
1.	Throttling losses	Severe	No throttling losses (Actually there are a little throttling losses in nozzles valves which are partially open).
2.	Partial admission losses	Low	High
3.	Heat drop available	Lesser	Larger
4.	Use	Used in impulse and reaction turbines both.	Used in impulse and also in reaction (if initial stage impulse) turbines.
5.	Suitability	Small turbines	Medium and larger turbines

CHAPTER 5: COMPRESSIBLE FLOW

A flow is said to be compressible flow when there is considerable change in the density of fluid during process. Mach number is an important non-dimensional number used in study of compressible flow.

- Mach number is the ratio of velocity of object and velocity of sound in fluid.

$$M = \frac{\text{velocity of object}}{\text{velocity of sound}} = \frac{v}{c}$$

For M<1 : flow is called **subsonic**.

For M=1 : flow is called **sonic.**

For M>1 : flow is called **supersonic.**

For M>5 : flow is called **hypersonic.**

- **Stagnation state:** when a fluid is decelerated isentropically to a zero velocity state than the final zero velocity state is known as stagnation state. All properties corresponding to the

stagnation state is known as stagnation properties. Example: stagnation enthalpy, stagnation temperature, stagnation pressure etc.

- For compressible flow stagnation temperature and pressure can be written as:

$$T_o = T \times \left(1 + \left(\frac{\gamma - 1}{2} \right) \times M^2 \right)$$

$$\frac{P_o}{P} = \left(\frac{T_o}{T} \right)^{\frac{\gamma}{\gamma - 1}}$$

For isentropic process: stagnation pressure and stagnation temperature remains constant.

i.e. $T_{o_1} = T_{o_2}$ and $P_{o_1} = P_{o_2}$.

For adiabatic process: only stagnation temperature remains constant. i.e. $T_{o_1} = T_{o_2}$ but

$P_{o_1} \neq P_{o_2}$.

- A state is said to be critical state when Mach number becomes one. i.e. M=1 (v=c). And the properties at critical state are known as **critical properties**.

- Relation between critical and stagnation properties:

$$\frac{T^*}{T_o} = \left(\frac{2}{\gamma + 1} \right) \text{ and } \frac{P^*}{P_o} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}}$$

- Stagnation velocity of sound: $c_o = \sqrt{\gamma R T_o}$

- Relation between maximum velocity and stagnation velocity of sound:

$$\frac{v_{max}}{c_o} = \sqrt{\frac{2}{\gamma - 1}} \text{ , where } v_{max} = \sqrt{2 C_p T_o}$$

- **Fanno flow**: Under idealized condition this type of flow may be assumed in gas duct of aircraft, air conditioning system etc. where there is friction but no heat transfer. Fanno flow is the adiabatic flow through a constant area duct where the effect of friction is taken into consideration. The Fanno line defines the possible states for a gas when the mass flow rate and total enthalpy are held constant, but the momentum varies.

  The Fanno flow model is often used in the design and analysis of nozzles.
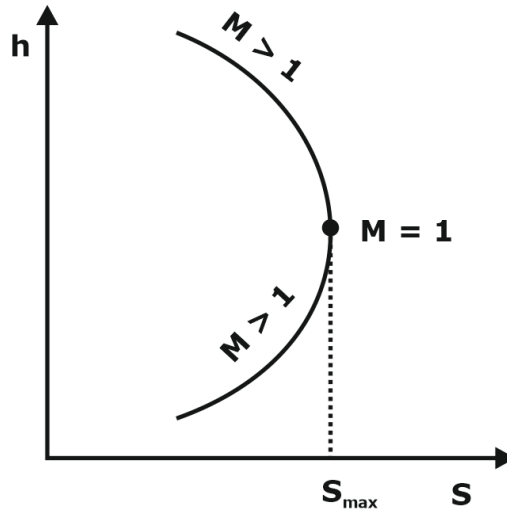
- **Rayleigh flow**: Flow in a constant area duct with heat transfer and without friction can be assumed as Rayleigh flow.

  The Mach number at maximum temperature is:

  $$M = \frac{1}{\sqrt{\gamma}}$$
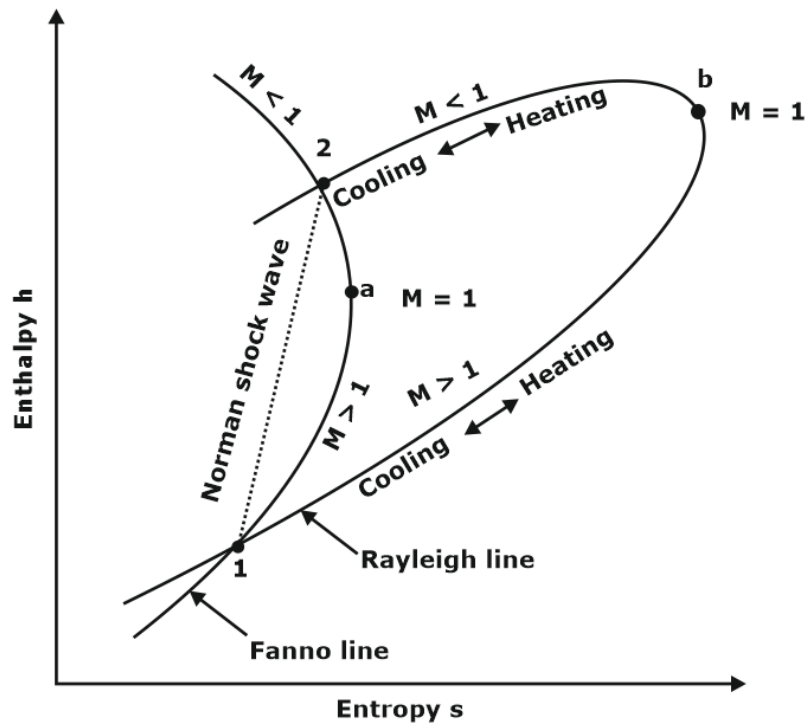
- **Nozzle:** nozzle is a device used to increase the velocity of fluid and decrease the pressure of fluid and nozzle efficiency can be define as:

  $$\text{nozzle efficiency}(\eta_n) = \frac{\text{actual enthaply drop}}{\text{ideal enthalpy drop}}$$

  Types of Nozzles:

(i) Convergent Nozzle:
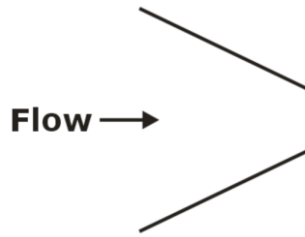
Cross section of the nozzle decreases continuously from entrance to exit

(ii) Divergent Nozzle:
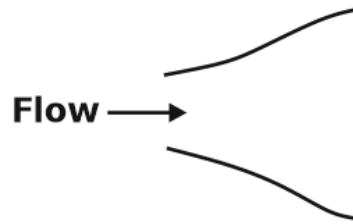
Cross section of the nozzle increases continuously Cross section of the nozzle increases continuously from entrance to exit

(iii) Convergent –Divergent Nozzle
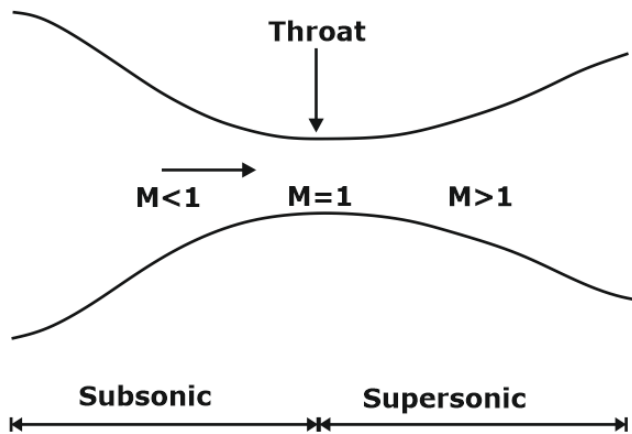
Fig.: Cross section of the nozzle first decreases and then increases

- **Diffuser:** Diffuser is a device used to increase the pressure of fluid and decrease the velocity of fluid and diffuser efficiency can be define as:

$$\text{diffuser efficiency}(\eta_d) = \frac{\text{ideal enthaply rise}}{\text{actual enthalpy rise}}$$

- For nozzle relation between area and velocity:

$$\frac{dA}{A} = \frac{dV}{V}(M^2 - 1)$$

Where A= Areas, V= Velocity of fluid.

****

Important Formula (Refrigeration & Air Conditioning)

CHAPTER-1 INTRODUCTION

- **Refrigeration** is the process of maintaining a space at a lower temperature than that of surrounding.

- **Heat pump** works in the same principle as that of refrigerator. In case of Heat Pump, our objective is to maintain a space at a temperature higher than surrounding.

- **Refrigeration effect** is the amount of the heat that has to be removed from the space to maintain it at a temperature lower than the surrounding per kg of refrigerant. Its **unit** is kJ/kg

- **Refrigerating Capacity** is the product of refrigeration effect to the mass flow rate of refrigerant. Its **unit** is kW.

$$\text{Refrigeration Capacity} = \dot{m} \times \text{Refrigeration Effect}$$

- **Unit of Refrigeration (Tons of refrigeration)**

  It is defined as the amount of heat which is required to extract from 1 ton of water at 0°C in order to convert it into ice at 0°C in a day or 24 hours.

  $1\text{TR} = 3.5\text{kW} = 210\text{kJ/min} = 50\text{kcal/min} = 4.71$ HP (1 horse power = 746W = 0.746kW)

- **Coefficient of performance COP / Energy performance Ratio EPR**

  It is defined as the ratio of desired effect to the work input.

  $$\text{COP} = \text{ER} = \text{EPR} = \frac{\text{Desired Effect}}{\text{Work Input}}$$

  Refrigerator – Desire effect is cooling at lower temperature, Thus $\text{COP} = \dfrac{Q_L}{W_{input}} = \dfrac{Q_L}{Q_H - Q_L}$

  If the refrigerator is reversible, Then, $Q \propto T \Rightarrow \text{COP} = \dfrac{T_L}{T_H - T_L}$

  Heat pump – Desire effect is Heating at higher temperature, Thus, $\text{COP} = \dfrac{Q_H}{W_{input}} = \dfrac{Q_H}{Q_H - Q_L}$

  If the refrigerator is reversible, Then, $Q \propto T \Rightarrow \text{COP} = \dfrac{T_H}{T_H - T_L}$
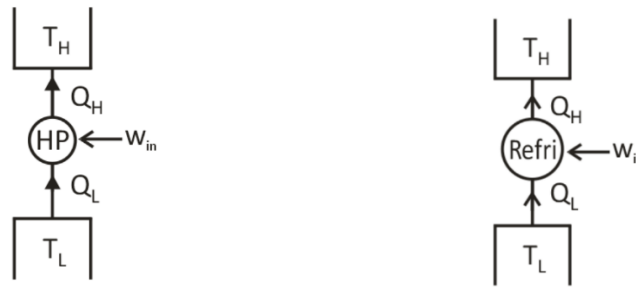
- **Relationship** between COP of a heat pump and COP of refrigerator and efficiency of heat engine operating between same temperature limits $(COP)_{HP} = 1 + (COP)_{Ref.} = \dfrac{1}{\eta_{HE}}$

- In a refrigerating machine, the heat exchanger that absorbs heat is connected to the conditioned space whereas in a heat pump, the heat exchanger that rejects heat is connected to the conditioned space.

- We can conclude that the same machine can be used either for cooling or for heating. When used for cooling, it is called a Refrigerating Machine and when used for heating it is called a Heat Pump.

- **Heat Rejection Ratio-** ratio of heat rejected in the condenser to the refrigeration effect

$$HRR = \dfrac{Q_C}{R.E} = 1 + \dfrac{1}{COP} \qquad \text{Also } HRR > 1$$

REVERSED CARNOT CYCLE

- A reversible heat engine can be reversed in operation to work as a refrigerating. The cycle consists of two isothermal and two isentropic.
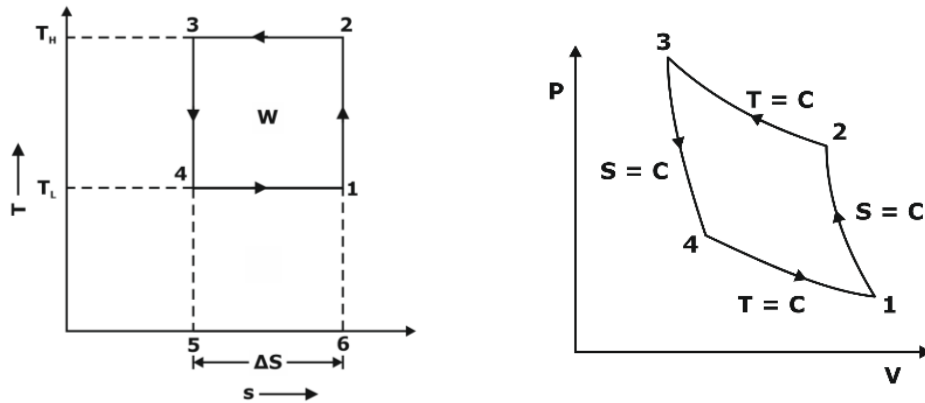
- A reversed Carnot cycle or reversible refrigeration cycle would measure maximum possible COP operating between two temperatures T_H of heat rejection and T_L of refrigeration.

- The Carnot COP depends on the operating temperatures T_H and T_L only. It does not depend on the working substance (refrigerant) used.

- for cooling, T_L is the refrigeration space temperature and its lowest possible value is '0' (absolute zero temp.) & maximum value T_H (ambient). Thus, **COP for Refrigeration varies between 0 and ∞**

- for heating, T_L is heat absorption temp. from the surroundings and its lowest possible value is '0' (absolute zero temp.) & maximum value T_H. So, **COP for Heat Pump varies between 1 and ∞**

- It may, therefore, be noted that to obtain maximum possible COP in any application,
    - the cold body temperature T_L should be as high as possible, and
    - the hot body temperature T_H should be as low as possible.

- For the purpose of comparison between the actual and Carnot cycle, a new efficiency is defined known as second law efficiency which is also known as exergy efficiency.

- Second law effieciency, $\eta_{II} = \dfrac{COP_{actual}}{COP_{ideal\,or\,carnot}}$

- **CASCADE REFRIGERATION SYSTEM** $(COP)_{cascade} = \dfrac{(COP)_1 \times (COP)_2}{1 + (COP)_1 + (COP)_2}$

CHAPTER-2 VAPOUR COMPRESSION REFRIGERATION SYSTEM

- VCRS is modified cycle over reversed Carnot cycle (practically not possible) with vapour as a refrigerant. it is most widely used in commercial refrigeration systems. Consequently, the theoretical COP of the vapour compression cycle is lower than that of the reversed Carnot cycle.

- VCRS consists of four basic components: Evaporator, Compressor, Condenser, Expansion device.

- **Refer below modifications**-

  ➢ The isothermal processes of heat rejection and heat absorption in reverse Carnot cycle, replaced by **condensation** and **evaporation** respectively.

  ➢ **Wet compression** advantages and disadvantages

   ▪ liquid refrigerant may be trapped in the head of the cylinder and may damage the compressor valves and the cylinder itself.

   ▪ liquid-refrigerant droplets may wash away the lubricating oil from the walls of the compressor cylinder, thus increasing wear.

   ▪ With a reciprocating compressor, wet compression is not found suitable

  ➢ **Dry compression** advantages and disadvantages

   ▪ In dry compression refrigeration effect is more compare to wet compression.

   ▪ Refrigerant leaves the compressor superheated due to which compressor work will increase.

  ➢ The isentropic expansion process of the Carnot cycle replaced by simple **throttling process** as the positive work of isentropic expansion is not large enough to justify cost of an expander.
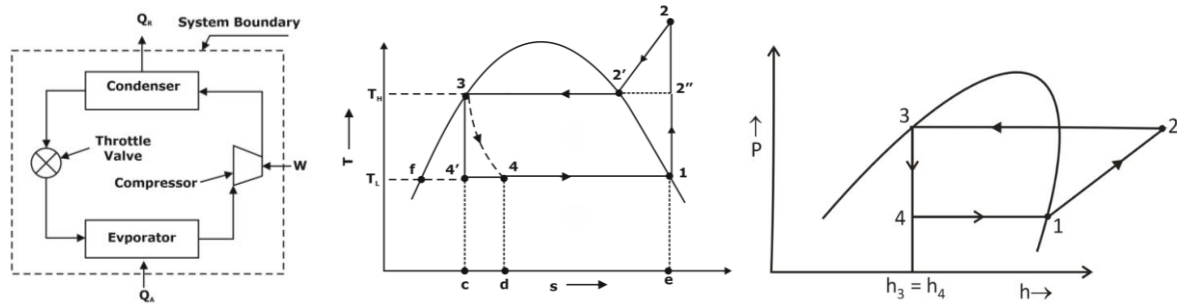
- The thermodynamic processes are as follows:

Process 1-2	Reversible Adiabatic compression	$S_2 = S_1$, $Q = 0$ Compression work, $W = h_2 - h_1$
Process 2-3	Constant Pressure Heat rejection (condensation)	P_H = const., W =0 Heat rejected $Q_R = (h_2 - h_3)$
Process 3-4	Isenthalpic expansion	$h_3 = h_4 = h_{f4} + x(h_1 - h_{f4})$ Expansion Work, W = 0
Process 4-1	Constant pressure heat absorption (Evaporation)	P_L = const., W =0 Heat absorbed $Q_A = (h_1 - h_4)$

- Refrigerating effect, RE = area 1-4-d-e = $h_1 - h_4$

- Refrigeration Capacity = $\dot{m}(h_1 - h_4)$

- Heat rejected: Q_R = area 2-2'-3-c-e $= \dot{m}(h_2 - h_3)$

- Work done: W = $Q_A - Q_R$ = area 1-2-2'-3-c-d -4-1 $= \dot{m}(h_2 - h_1)$

- COP for cooling $COP = \dfrac{\text{Desired effect}}{\text{Work Input}} = \dfrac{R.E}{w_{in}} = \dfrac{h_1 - h_4}{h_2 - h_1}$

- The isentropic discharge temperature T_2 may be found by using saturation properties and the specific heat of vapour

$$s_1 = s_2 = s_2' + C_p \ln \frac{T_2}{T_2'} \quad \text{where } s_2' = s_{g2} \text{ and } T_2' = T_H$$

EFFECT OF OPERATING CONDITIONS ON THE PERFORMANCE OF VCRS

- ➢ **Decrease in evaporator pressure/temperature**
    - ○ Decrease in refrigeration effect
    - ○ Increase in work input
    - ○ Decrease in COP
    - ○ Decrease in η_{vol} due to the increase in pressure ratio
    - ○ Increase in the specific volume of suction vapour

- ➢ **Increase in condenser pressure/temperature**
    - ○ Decrease in refrigeration effect
    - ○ Increase in work input
    - ○ Decrease in COP
    - ○ Decrease in η_{vol} due to the increase in pressure ratio

- ➢ **Super heating (within evaporator)**
    - ○ Increase in Refrigeration effect.
    - ○ Increase in w_{in} to the compressor as w_{in} is a function of inlet temp. to the compressor
    - ○ COP may increase or decrease depending on the type of refrigerant used
    - ○ In case of R−12 refrigerant, superheating would result an increase in COP whereas in case of NH_3 superheating would result in decrease in COP.

- ➢ **Liquid Subcooling (within condenser)**
    - ○ Increase in refrigeration effect
    - ○ No change in work input
    - ○ Increase in COP

- ➢ **Liquid-Vapour Regenerative Heat Exchanger**
    - ○ Keeping mass flow rate of the liquid and vapour same, from energy balance of the heat exchanger Q= $h_1' - h_1 = h_3 - h_3'$ but $T_3 - T_3' \neq T_1' - T_1$
    - ○ both refrigeration effect and work input to compressor increases, COP may increase or decrease depending on the type of refrigerant used.

FLASH CHAMBER- The vapour is separated from the liquid refrigerant in the flash chamber & therefore only liquid enters the evaporator.

CHAPTER 3- VAPOUR ABSORPTION REFRIGERATION SYSTEM

- In VCRS, main cost of the refrigeration unit is compressor due to vapour phase compression. So, in VARS, compressor is replaced with:
  - o **Absorption unit** to convert refrigerant vapour into liquid state.
  - o **Pump** to compress it from evaporator pressure to compressor pressure in liquid state only
  - o **Generator unit** to convert compressed liquid refrigerant into vapour form.
  - o **Analyser and Rectifier** is optional. It is used remove water particles from the refrigerant vapour. The complete elimination of water particles is taken place in rectifier.
- In VARS heat rejection occurs in condenser & absorber and heat absorption occurs in evaporator & generator as shown in schematic diagram.
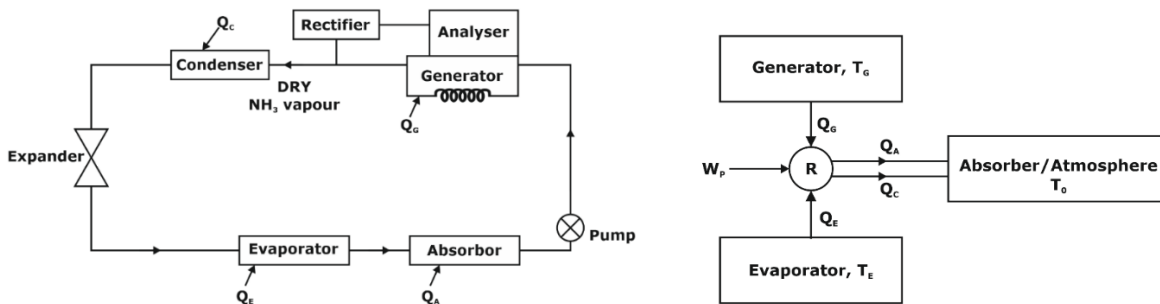
Here Q_E and $Q_G \rightarrow$ heat absorbed by the refrigerants across evaporator and generator

Q_c and $Q_A \rightarrow$ heat rejected by the refrigerant across condenser and absorber

$T_o \rightarrow$ atmospheric temperature

- Using first law of thermodynamics and Clausius inequality $(COP)_{actual} = \dfrac{Q_E}{W_p + Q_G}$

- Pump work $W_{pump} = -\int V_f dP$, specific volume of solution V_f very small. $W_p \rightarrow 0$ Neglected

- $(COP)_{VARS} = \dfrac{Q_E}{Q_G} = \dfrac{(T_G - T_O)}{T_G} \times \dfrac{T_E}{T_O - T_E} = \left(1 - \dfrac{T_O}{T_G}\right) \times \left(\dfrac{T_E}{T_O - T_E}\right) = \eta_{carnot} \times (COP)_{carnot}$

  here Carnot Engine operating between T_G & T_o and Carnot Refrigerator operating between T_0 & T_E.

COMMONLY USED ABSORBER REFRIGERANT PAIR AND THEIR WORKING

- **Ammonia - Water**
  - In this Ammonia is used as a refrigerant and water is used as absorber.
  - Ammonia absorb heat from the **evaporator**, then rejects heat in **absorber** and convert into liquid form. Due to special property of NH_3, ammonia absorb in water at lower temperature.

- Then ammonia water mixture is compressed to condenser pressure by **pump**. After this water get separated at higher temperature in **generator.** However, some water particle gets removed in analyser and rectifier assembly.

- **Li Br and water**
  - In this water is used as a refrigerant and lithium bromide is used as absorber.
  - The above pair is not preferable below 0°C (the freezing point of water is 0°C).

DIFFERENCE BETWEEN VCRS AND VARS

VCRS	VARS
1. Compressor is used.	1. Compressor is replaced with absorber, pump and generator.
2. COP of VCRS system is having higher value and lies b/w (3 to 5).	2. Relatively lower COP (0.3 to 0.5)
3. VCRS is work operated unit and runs on high grade energy.	3. It is heat operated unit or runs low grade energy.
4. Moisture related problem is more severe in VCRS.	4. Relatively lesser problem.
5. Chances for leakage of refrigerant are higher.	5. Relatively lesser
6. Creates more noise	6. Relatively lesser
7. Heat rejection occurs only in condenser.	7. Heat rejection occurs in condenser and absorber

ELECTROLUX REFRIGERATOR (R)

- Main aim of using this refrigerator system is to create noiseless operation. i.e., no use of pump.
- It is also called as **triple fluid** vaporization absorption system. Three-fluid are:
  - $NH_3 \rightarrow$ Refrigerant
  - Water (H_2O) $\rightarrow$ absorber
  - Hydrogen (H_2) $\rightarrow$ create low partial vapour pressure of NH_3 vapours (NH_3 Evaporates in H_2).
- In condenser pure NH_3 vapour pressure = Total pressure
- In Evaporator pure NH_3 vapour pressure = Total pressure – Partial pressure of H_2.

CHAPTER-4 GAS REFRIGERATION CYCLE

INTRODUCTION

- Gas is used as the working fluid which does not undergo any phase change during the cycle.
- Consequently, all internal heat transfer processes are sensible heat transfer.
- It finds applications in aircraft cabin cooling and also in liquefaction of various gases.
- Air is used as a refrigerant in aircraft system because of low weight per ton of refrigeration

STANDARD AIR REFRIGERATION CYCLE ASSUMPTION

- The working fluid is a fixed mass of air that behaves as an ideal gas.
- The cycle is assumed to be a close loop cycle heat transfer processes to or from surrounding.
- All the processes within the cycle are reversible, i.e. the cycle is internally reversible.
- The specific heat of air remains constant throughout the cycle.

BELL COLEMAN OR REVERSED BRAYTON CYCLE

- Modification over reversed Carnot cycle (2 isothermal processes of reverse Carnot cycle replaced by two isobaric processes)
- The thermodynamic processes are as follows:

    1 – 2: Reversible adiabatic compression or isentropic compression

    2 – 3: Constant pressure heat rejection

    3 – 4: Reversible adiabatic expansion

    4 - 1: Constant pressure heat addition

- **Why not isenthalpic expansion as in VCRS-** Air is an ideal gas & for an ideal gas h = f(T) alone & hence if we are using isenthalpic expansion then, there would be no change in temperature i.e. ($T_3 = T_4$) hence instead of heat absorbing it will reject heat.
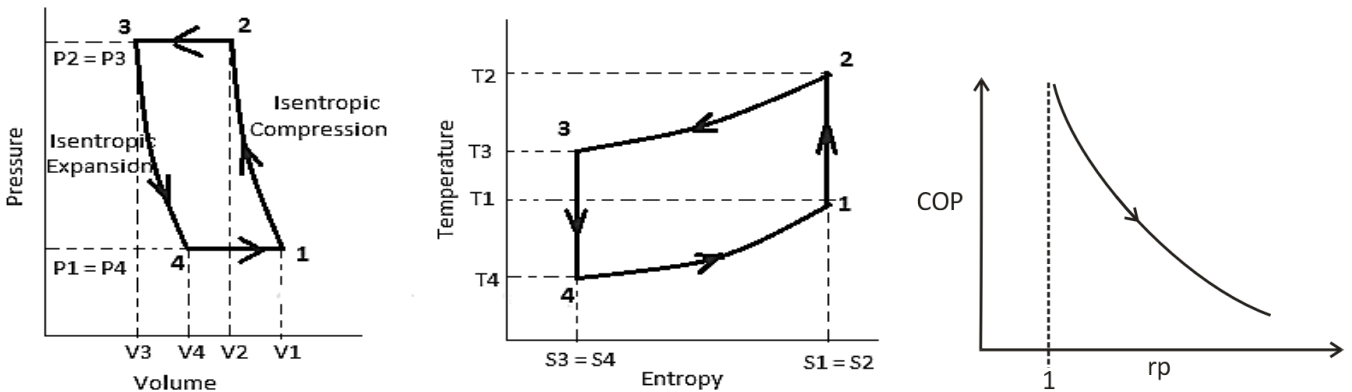
- $$COP = \frac{RE}{w_{in}(=w_c - w_t)} = \frac{h_1 - h_4}{(h_2 - h_1) - (h_3 - h_4)} = \frac{1}{(r_p)^{(\gamma-1)/\gamma} - 1}, \text{ where } r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4} \text{ (Pressure Ratio)}$$

ACTUAL CYCLE

- The actual reverse Brayton cycle differs from the ideal cycle due to:
    - Non-isentropic compression and expansion processes.

o Pressure drops in cold and hot heat exchangers.
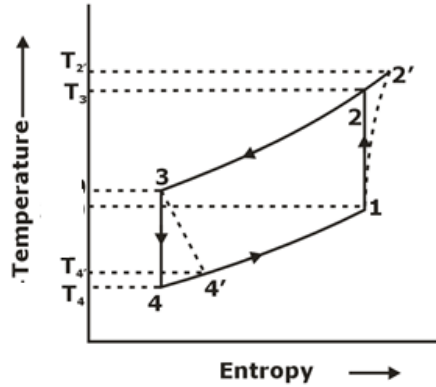
- Isentropic efficiency of compressor $\eta_{is,compressor} = \dfrac{h_2 - h_1}{h_2{'} - h_1} = \dfrac{\text{Ideal work}}{\text{Actual work}}$

- Isentropic efficiency of turbine $\eta_{is,turbine} = \dfrac{h_3 - h_4{'}}{h_3 - h_4} = \dfrac{\text{Actual work}}{\text{Ideal work}}$

- Due to above irreversibilities net-work input increases. (compressor work $\uparrow$ & turbine work $\downarrow$)

- The refrigeration effect also reduces. COP of actual reverse Brayton cycles will decrease.

OPEN & CLOSE CYCLE

OPEN-AIR REFRIGERATION CYCLE	CLOSED AIR REFRIGERATION CYCLE
1. directly come in contact with space to be cooled	1. not come in contact with space to be cooled
2. suction pressure at atmospheric pressure	2. suction pressure higher than that of atmospheric pressure
3. volume of air handled by the compressor and expander is large	3. volume of air handled by the compressor and expander are smaller

CHAPTER-5 REFRIGERANTS

- Refrigerants are those working substance in a refrigerant cycle.
- **Primary Refrigerants-** those working refrigerants which directly flows into refrigerating machine and produce cooling effect. Ex: R – 11, R – 12, R – 22, R – 134
- **Secondary Refrigerants-** those working fluids which are first cooled by primary refrigerants & then used for cooling at the desired place. Ex: H_2O, Brine solution, sometimes ammonia.
- Almost all refrigerators are having similar value of COP when operating between same temp. limits.

NOMENCLATURE OF REFRIGERANTS

- If the refrigerant is **saturated HC** $C_mH_nF_pCl_q$ i.e. $n + p + q = 2m + 2$ $\Rightarrow R - (m-1)(n+1)p$

- If the refrigerant is **unsaturated HC** $C_mH_nF_pCl_q$ i.e. $n + p + q = 2m$

$$\Rightarrow R - 1(m-1)(n+1)p$$

- If the refrigerant is **inorganic** compound $\Rightarrow R - [700 + Molecular\ weight]$

- **Azeotropes** are the mixture of refrigerants which behaves like a pure substance. Their designation is started with $\Rightarrow R - 500$

SELECTION OF REFRIGERANT (Desirable properties of refrigerant)

- **THERMODYNAMIC PROPERTIES**
  - **Critical temperature** should be as high as possible above the condensing temperature (normally ambient temperature).
  - More the **enthalpy of vaporization**, lee will be the mass flow rate of refrigerant for same amount of refrigeration capacity, then size of evaporator will be less.
  - Low **freezing point** is desirable to avoid freezing of refrigerant in the evaporator coils and thus choking of coils.

    Note: Above 3 properties of some refrigerant are given below.

Critical Temperature	Enthalpy of Vaporization	Freezing Point
$H_2O \Rightarrow 374°C$	$H_2O \Rightarrow 2261$ kJ/kg	$H_2O \Rightarrow 0°C$
$SO_2 \Rightarrow 156.5°C$	$NH_3 \Rightarrow 1369$ kJ/kg	$R–22 \Rightarrow - 160.5°C$
$NH_3 \Rightarrow 132.4°C$	$R–22 \Rightarrow 234.7$ kJ/kg	$R–12 \Rightarrow - 157.4°C$
$R–12 \Rightarrow 111.5°C$	$R–12 \Rightarrow 165.7$ kJ/kg	$R–13 \Rightarrow - 77°C$
$R–22 \Rightarrow 96.5°C$	$R–134 \Rightarrow 197.3$ kJ/kg	$R–134(a) \Rightarrow - 103.2°C$
$R–134 \Rightarrow 101.21°C$		
$CO_2 \Rightarrow - 31°C$		
Ethylene $\Rightarrow - 10.2°C$		

- **Specific heat of the vapour** should be high in order to limit the degree of superheat.

Specific heat of liquid of refrigerant should be low in order to reduce irreversibility.

- High value of **thermal conductivity** is desirable as it help in reducing size of evaporator & condenser.

- **Evaporator pressure** and **condenser pressure** both should be positive (more than or equal to atmospheric pressure) to avoid possibility for leakage of air.

- Low **compression ratio** is desirable because high compression ratio results increase in work input to the compression and decrease in volumetric efficiency.

- High **Compressor discharge temperature** not desirable because more temperature → more pressure ratio → reduced volumetric efficiency. Also, at high temperature compressor's material of construction may react with refrigerant due to change in its properties. So, compressor need to be cool down.

Ammonia compressors	water cooled (due to high compressor discharge temperature)
R – 11/R – 12 Compressors	air cooled

- Lower **COP** represent higher running cost of equipment.

- **Volume at the inlet** of the compressor should be low as possible to avoid higher work input.

Type of Compressor	Pressure	Volume	Ex:
Reciprocating Comp.	High	Low	NH_3, CO_2
Centrifugal or Rotary comp.	Low	High	R – 11, R – 113

- **CHEMICAL PROPERTIES**
  - **Toxicity**: The refrigerant should be Non-toxic in nature. E.g. Ammonia is toxic in nature.
  - **Flammability:** The refrigerants should be non-inflammable in nature.
    Note: Ammonia (NH_3) is both toxic as well as flammable in nature.
  - **Action with oil:**
    o At condenser pressure and temperature refrigerants which are **fully miscible** or **fully immiscible** with oil does not create any problem.
    o Refrigerants which are **partially miscible** with oil are very dangerous because they can wash away all the lubricants present in the compressor. To avoid this an **oil separator** is installed between compressor and condenser. It will separate lubricants from refrigerant and bring back to the compressor.
    R + Fully Miscible with oil → R – 11, R – 12
    R + Fully immiscible with oil → NH_3, CO_2
    R + Partially Miscible with oil → R – 22
    o Presence of lubricating oil (liquid) in evaporation will decrease the heat transfer coefficient as if oil accumulates, the thickening oil film will act as an insulating layer.
  - **Action with material of construction:**

o Copper is not usually used in ammonia system because with the presence of little water only, ammonia becomes ammonium hydroxide, the OH^{-1} combined with Cu making cupric hydroxide of blue colour. Besides moist ammonia will not react with iron or steel. Therefore, only steel or ductile iron should be used for ammonia systems, ammonia containers.

o At high temperatures aluminium may act as catalysts for thermal breakdown of Freon refrigerant. Hence freons refrigerants are not used with aluminium construction.

Refrigerant	Attack	Favourable
NH_3	Copper	Wrought iron
Freon or Halocarbon compounds	Aluminium	Copper

- **PHYSICAL PROPERTIES**
  - **Cost:** It should be low.
  - **Viscosity:** viscosity of the refrigerant should be low for the easy flow of refrigerants.
  - **Leak detection:** there should not be leakage of refrigerant at any cost but if it leaks out then its detection should be as fastest as possible and by the simplest method.

Refrigerant used	Leak detection Test	In case of refrigerant leak…
Freon or Halo carbon compounds	Halide torch test	colour of light changes from blue to bluish green
NH_3	sulphur stick method	white fumes of Ammonium sulphite form
SO_2	Ammonia swab test	A dense white smoke form

- **Environmental Factor**
  - The chlorine element present in refrigerator, attacks the ozone layer which is situated in stratosphere. Therefore, refrigerants with lower ozone Depletion potential (ODP) preferred.
  - Eco-friendly refrigerants should use in which there is absence of chlorine elements.
  - E.g. Hydrocarbon,
    Flouro carbon,
    R-290 Propane (C_3H_8),
    R-134a Tetra-fluoroethane ($C_2H_2F_4$)
    R-600a Isobutane (C_4H_{10})

REFRIGERANT AND THEIR APPLICATIONS
- R – 11 : Large central AC plants.
- R – 12 : Domestic refrigerator, water cooler.
- R – 22 : Window AC
- NH_3: Cold storage plants.
- Brine : Milk chilling plants.
- CO_2 : Direct contact freezing of foods.
- AIR : Gas Liquification.
- Azeotropes: Aircraft refrigeration system.

CHAPTER-6 PSYCHROMETRY

It is the branch of science which deals with the study of properties of moist air

Moist air = Dry air + water vapour

Specific Humidity (w) $\quad \omega = \dfrac{m_v}{m_a} = 0.622 \left(\dfrac{Pv}{P-Pv} \right)$

Relative Humidity (ϕ) $\quad \phi = \dfrac{m_v}{m_{v_s}} = \dfrac{\dfrac{P_v V}{R_v T}}{\dfrac{P_{v_s} V}{R_v T}} = \dfrac{P_v}{P_{v_s}}$

Degree of saturation $\quad \boxed{\mu = \dfrac{\omega}{\omega_s}} = \phi \left[\dfrac{P-P_{vs}}{P-P_v} \right]$

Enthalpy of Moist Air

h_{moist} = 1.005t + w[2500 + 1.88t] kJ/kg

t is dry bulb temperature in °C.

w is specific humidity.

PSYCHROMETRIC CHART
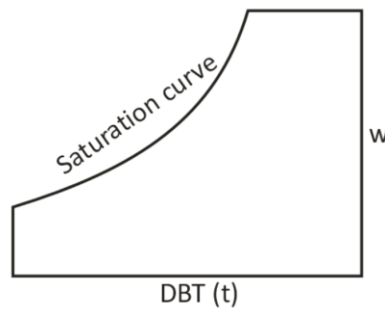
- W is specific humidity.
- DBT is dry bulb temperature.
- Saturation curve corresponds to 100 % relative humidity.

PSYCHROMETRIC PROCESSES
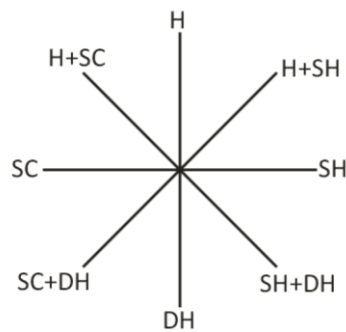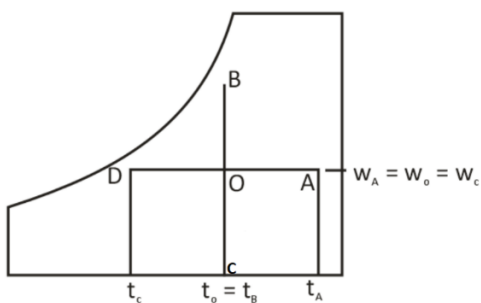
- H-Humidification
- DH-Dehumidification
- H+SC-Humidification and sensible cooling.
- H+SH-Humidification and sensible heating.
- SC- Sensible cooling
- SH-Sensible heating
- SC+DH-Sensible cooling and dehumidification.
- SH+DH-Sensible heating and dehumidification
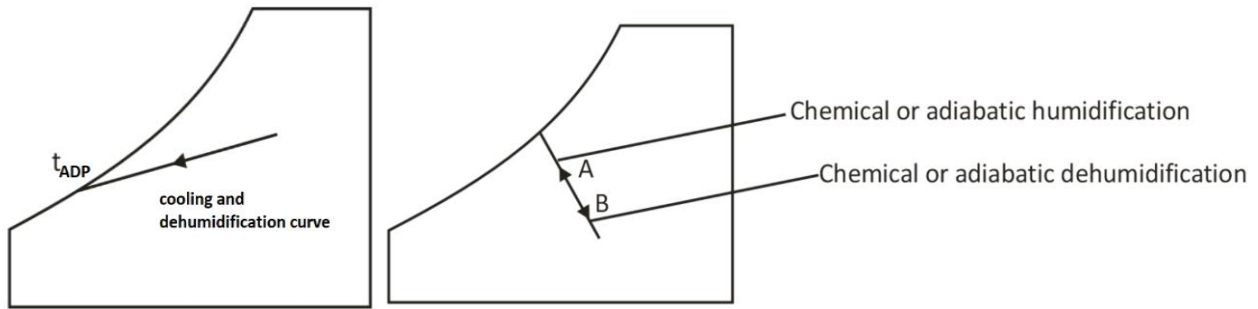
Sensible Heat Factor $SHF = \dfrac{SH}{TH} = \dfrac{SH}{SH + LH}$
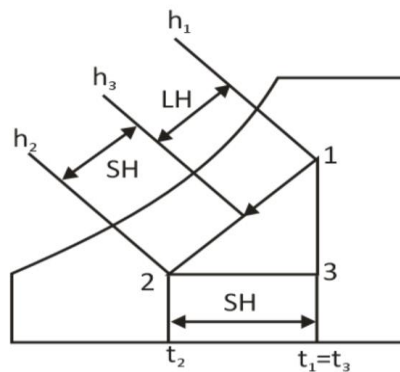
Bypass factor of a heating coil

$t_3 > t_2 > t_1$

$\left(BPF\right)_{H.C.} = \dfrac{t_3 - t_2}{t_3 - t_1}$

$\eta_{H.C.} = 1 - \left(BPF\right)_{H.C.}$

Bypass factor of cooling coil

$t_1 > t_2 > t_3$

$\left(BPF\right)_{CC} = \dfrac{t_2 - t_3}{t_1 - t_3} \quad \because t_2 > t_3$

$$\boxed{\eta_{cc} = 1 - (BPF)_{CC}}$$

Bypass factor of a coil with more than one row of coils:

B.P.F = X (for each coil)
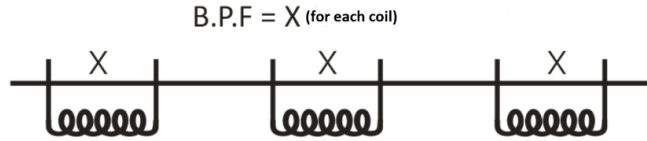
Combined BPF = X^N N → No. of coils

Where X = BPF of each coil

MIXING OF AIR STREAMS

t → DBT in °C

m → mass flow rate

h_2 → enthalpy

w → sp. Humidity

(a) $m_1 + m_2 = m_3$

(b) $m_1h_1 + m_2h_2 = m_3h_3$

(c) $m_1w_1 + m_2w_2 = m_3w_3$

(d) $m_1t_1 + m_2t_2 = m_3t_3$

Notes:

(1) Cooling and Dehumidification is possible when the cooling coil temperature is less than DPT of entering air.

(2) During heating and dehumidification, the dry bulb temperature increases and specific humidity decreases.

(3) Example of heating and humidification → steam spray in air

(4) In case of desert coolers, we achieve cooling and humidification

(5) Desert coolers are more effective when the value of **"Wet Bulb depression = DBT – WBT is high"**

****

Important Formula (Production Engineering)

CHAPTER 1: METAL CASTING

CASTING

Casting is a process in which the liquid molten metal is poured into the mould cavity whose shape is same as that of the casting to be produced, allow it to solidify and after solidification the casting can be taken out by breaking the mould.

Steps involved in casting:

(i). Preparation of pattern.

(ii). Preparation of mould.

(iii). The Gating system (Design)

(iv). Design of Riser

(v). Melt the Metal

(vi). Pouring of molten metal

(vii). Solidification

(ix). Fettling, Finishing, Testing.
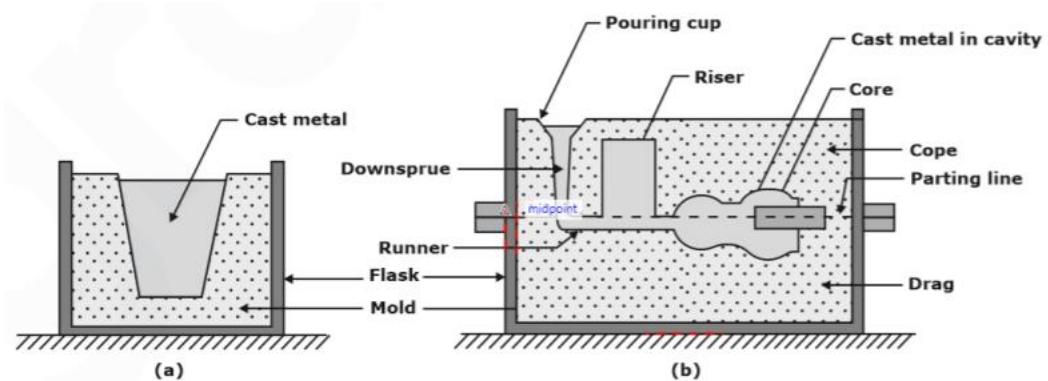
Pattern and mould:

A Pattern is the replica of the part to be cast (to be produced) and is used to prepare the mould cavity. Patterns are made of either wood or metal.

A Mould is an assembly of two or more metal blocks, or bonded refractory particles (sand) consisting of a primary cavity.

Pattern making: Pattern size = casting size ± allowances

Allowances:

Shrinkage Allowance:

Shrinkage allowance is a positive allowance which is provided to take care of the contractions of a casting. The total contraction of a casting takes place in three stages:

Liquid Shrinkage: It is the shrinkage of molten metal when it cools from pouring temperature to freezing temperature and phase of molten metal remains liquid.

Solidification Shrinkage: It is the shrinkage of molten metal when the phase of the molten metal changes from liquid to solid.

Solid Shrinkage: It is the Shrinkage associated when the temperature of solid casting changes from freezing temperature to room temperature.
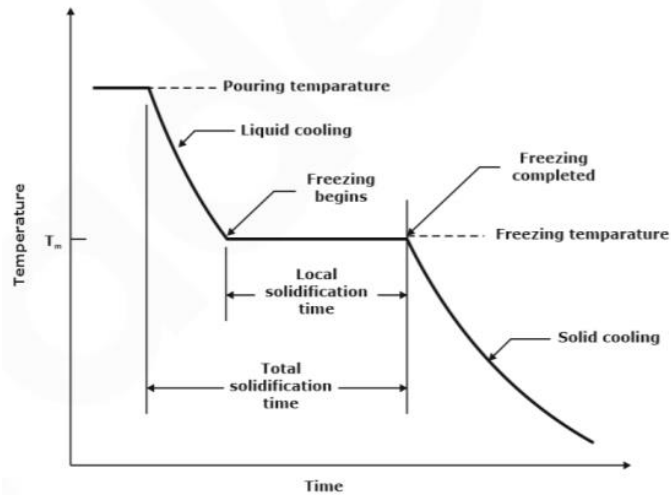
Note:

(i). The first two will be taken care by providing riser during casting. But the third will be provided as a shrinkage allowance in the pattern (taking place during the cooling of the material from freezing temp to room temp as a solid).

Solid shrinkage allowance for different materials:

Metals	Solid shrinkage in mm (per 1000mm)
Invar, Bismuth	0
White Metal	5
Cast iron	10
Aluminium	13
Copper	17
Steel	20
Brass	23

Machining Allowance: It is provided to take care of the machining to produce good surface finish is called the machining allowance.

Draft Allowance: It is provided to withdraw the pattern from the cavity without the damage. In general, 5° to 8° draft is given for internal surfaces and ½° and 2° is given for external surfaces.

3

Rapping allowance or Shake allowance: It is rapped all around the vertical faces to enlarge the mould cavity slightly which facilitates its *easy removal*. It is *a negative allowance* and is to be applied only to those dimensions which are parallel to the parting plane.

Note: If the pattern is made by using the materials like wax, mercury, polystyrene as pattern material, no shake allowance is provided.

Distortion Allowance: "To avoid the distortion, the shape of pattern itself should be given a distortion of equal amount in the opposite direction of the likely distortion direction so that final product will come in true shape known as distortion allowance"

Types of patterns:

Solid or single piece pattern:

solid pattern placed in the drag position and it is used for making a flat surface like as gear blanks, square blocks etc.
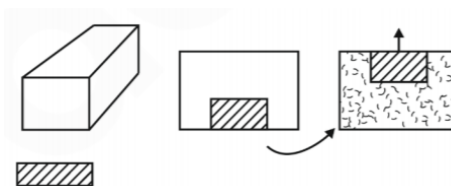
Split pattern: It is widely used type of pattern for intricate castings.
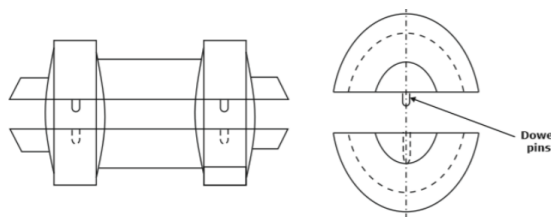
Match plate patterns: The match plate is accurately placed between the cope and the drag flasks by means of locating pins. Production efficiency and dimensional accuracy is improved by this method.

Gated pattern: The parting line should he chosen so as the smallest portion of the pattern in the cope.

Sweep pattern: Sweep patten is used to generate surfaces of revolution in large castings which are axi-symmetrical or prismatic in nature such as bell shaped or cylindrical.

Skeleton pattern: This type of pattern is useful generally for very large castings required in small quantities.

Loose piece patterns: In it overhanging parts are fastened loosely to the main part of the pattern by wires or wooden pins and pattern cannot be removed in any direction.

Follow board pattern: It is used for those castings where there are some portions which are structurally weak and if not supported properly are likely to break under the force of ramming.

Mould making:

Types of sands:

Green sand: a mixture of sand, clay, water, and some organic additives, e.g., wood flour, dextrin, and sea coal.

Composition: 70-85% sand, 10-20% clay, 3 – 6% water, and 1-6% additives.

Dry sand: Silica sand + clay +sodium silicate.

Loam sand: Green or dry sand with at least 50% clay and dries hard. It also contains fire clay. It has 18 to 20% moisture and produces good surface finish.

Baking sand: It consists of refractory material and it is made of used sand or burnt sand.

Facing sand: Carboneous material sprinkled on the inner surfaces of the moulding cavity for obtaining better surface finish

CO_2 sand: In this sand in place of clay if sodium silicate is used, called as CO_2 sand.

Additives (up to 2% each):

(i). Cereal binder up to 2% increases the strength.

(ii). By-product in coke making up to 3% would improve the hot strength.

(iii). Saw dust: up to 2% improves the collapsibility by slowly burning and increase the permeability.

(iv). Starch or dextrin: Used for increasing strength and resistance for deformation of moulding sand.

(v). Coal Dust: It is basically used for providing better surface finish to the castings.

Properties of moulding sand:

Permeability and Permeability number: "Permeability is the ability of moulding sand to allow the air to escape". Permeability test is used for determining the Porosity property of moulding sand is denoted by:

$$P_n = \frac{V.H}{P.A.T}$$

As per the American foundry society (AFS) standard or ASTM standard, the standard test conditions are

D = H = 5.08 cm = 2inch

$P_n = 50.127/T$

Green strength: The optimum moisture content in the moulding sand is 7-8% and strength are such a condition is called the green strength of the sand.

Refractoriness: The ability of withstanding higher temperature of the molten metal without losing its strength and hardness is called refractoriness.

Collapsibility: It is the property of material due to which, it does not provide any resistance during the contraction of the solidified casting.

Flowability: The ability of flowing of moulding sand into each and every corner of the mould is called flowability.
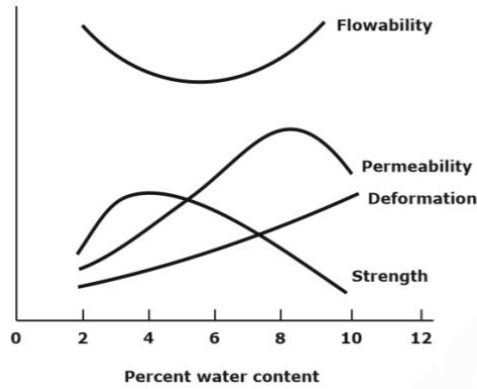
Effect of moisture content on different properties

Fluidity: It depends on the Viscosity, solidification pattern of the alloy, Degree of super heat.
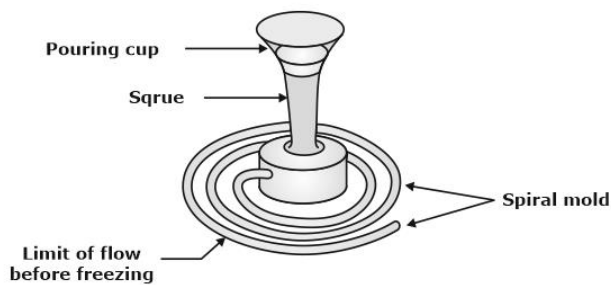
Spiral fluidity test

Properties increases ↑	Effect on Fluidity
Pouring temperature	↑
Surface finish of the mould	↑
Viscosity	↓
Density	↓
Surface tension	↓
Moisture content	↓

Machine moulding:

Jolting: Sand filled mould is raised to certain amount of height and it is allowed to fall freely on to the ground so that the reaction load produced by the ground will be used for ramming. The resulting impact forces the *sand to get compacted uniformly* into the mould. This type of ramming is suitable for horizontal surfaces.

Squeezing: A plate *slightly smaller than the inside dimensions of the moulding flask* is fitted into the flask already filled with the moulding sand and a uniform pressure is applied on the plate by either moving it down or by moving the flask upwards. The resulting force compacts the sand uniformly. *The sand next to the plate rams hardest* while the sand below is progressively less hard. This type is *suitable for small castings and is generally suitable for shallow flasks.*

Sand slinging (combination of jolt squeezing machines) is done by throwing sand into the flask rapidly and with great force. This process develops uniformly high mould hardness. The process is very fast and gives high uniform sand ramming.

Core making: Core is used for making cavities and hollow projections and cores are normally made by CO_2 moulding.

Recess provided in the mould *for locating, positioning, and supporting of cores* is called core print.

Buoyancy force: $F_b = Vg\rho$

Net buoyancy force acting on the core = Weight of liquid displaced due to projected portion — total weight of core

$$F = \rho_m Vg - \rho Vg = Vg(\rho_m - \rho)$$ (For Horizontal cores)

Where:

ρ_m = density of molten metal

ρ = density of core material

V = volume of core

$$F = \frac{\pi}{4}(D_1^2 - D^2)H\rho_m - V\rho$$ (For Vertical cores)

Where, V= total volume of the core in the mould.

The basic *function of chaplet is to act as an additional support* for supporting the unsupported length of the core.

Chill: The use of chill is used to get the *directional solidification.* The paddings are used to avoid the sand erosion taking place during sharp edged casting.

Gating system:

Elements: Pouring basin, Sprue, Runner, Ingate
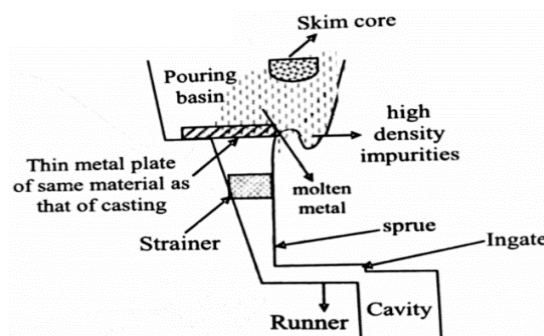
Strainer: acting as filter for separating the impurities present in molten metal.

Skim Bob: a semi-circular cut in a horizontal gate to prevent heavier and lighter impurities from entering the mould.

Splash core: reduces the eroding force of the liquid metal stream and also ensures settle down of high-density impurities.

Runner: Always horizontal with uniform trapezoidal cross-section and mainly used for minimizing the sand erosion in casting process.
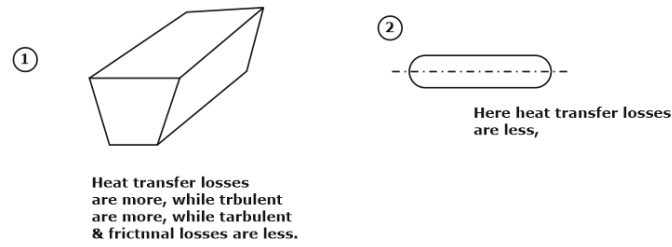
Gating ratio: The gating ratio refer to the proportion of the cross-sectional areas between the sprue, runner, and in-gates, and is generally denoted as sprue area, runner area, and ingate area.

Gating ratio = $A_S : A_R : A_G$

Where: A_S = sprue area, A_R = runner area and A_G = ingate area.

Types:

Non pressurized gating system: If pressure above molten metal in gating system is equal to atmospheric pressure, it is called as non-pressurized gating system, i.e. P = P_{atm}.

Choke area is at the sprue base i.e. sprue base area is minimum in un-pressurized gating system i.e. 1:2:2, 1:4:4, 1:2:4, 0.5:1.5:1.

Due to low turbulence oxides formation will not takes place thus we can *cast Non-Ferrous alloys such as aluminium and magnesium alloys.*

Pressurized gating system: If top of the pouring basin is closed and the pressure above molten metal in pouring basin is maintained greater than atmospheric pressure is called as pressurized gating system, i.e. P > P_{atm}.

Choke area is at the ingate i.e. ingate area is minimum in pressurized gating system i.e. 4:2:1, 2:2:1, 2:1:0.5.

It is **used for Ferrous castings.**

Top Gating System:

$$\text{Pouring time} = \frac{\text{Volume}}{\text{Flow rate}} = \frac{\text{Volume}}{A_c \times V_{max}}$$

$$\boxed{V_{max} = \sqrt{2gh_t}}$$

A_c = Choke area = $Min\left(A_s, A_r, A_g\right)$

Volume = Volume of the casting or pattern
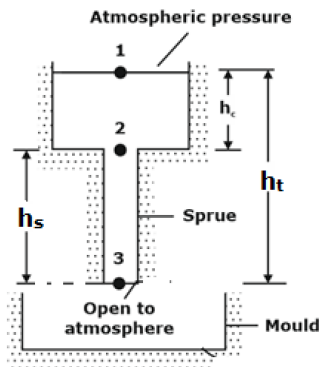
Mould Filling time: $t_f = \dfrac{A_m \times h_m}{A_g \times \sqrt{2gh_t}}$

To avoid air Aspiration:

$\Rightarrow \boxed{\dfrac{A_2}{A_3} = \dfrac{\sqrt{h_t}}{\sqrt{h_c}}}$

$A \propto \dfrac{1}{\sqrt{h}}$

Thus, ideally, the sprue profile parabolic as shown by the solid lines in the Fig. but it is difficult to design thus, a straight tapered sprue (shown by the dashed lines) is preferred.
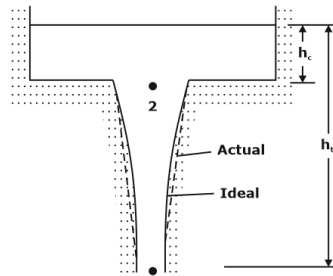
Bottom Gating System:
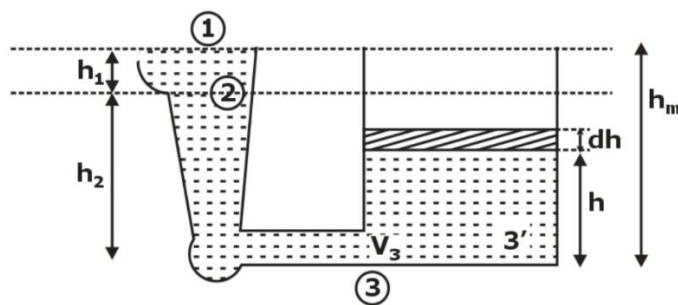
Mould Filling time: $t_f = 2\dfrac{A_m}{A_g}\dfrac{1}{\sqrt{2g}}\left[\sqrt{h_t} - \sqrt{h_t - h_m}\right]$

If height of the mould(h_m) is equal to total height (h_t):

$t_f = 2\dfrac{V_m}{A_g\sqrt{2gh_t}}$

$$\boxed{\left(t_f\right)_{Bottom} = 2 \times \left(t_f\right)_{Top}}$$

Cooling and solidification time:

Pure metals: A pure metal solidifies at a constant temperature equal to its freezing point, which is the same as its melting point.
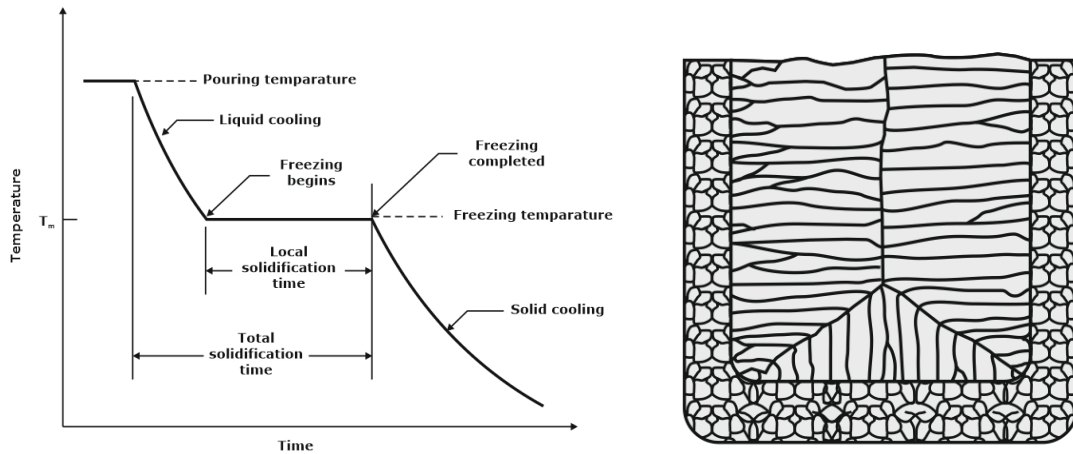
As the solidification starts on the mould wall and is rapidly cooled by the extraction of heat through the mould wall resulting in the fine and randomly oriented grains while coarse, columnar grains aligned toward the centre of the casting.

Alloys:

(b)
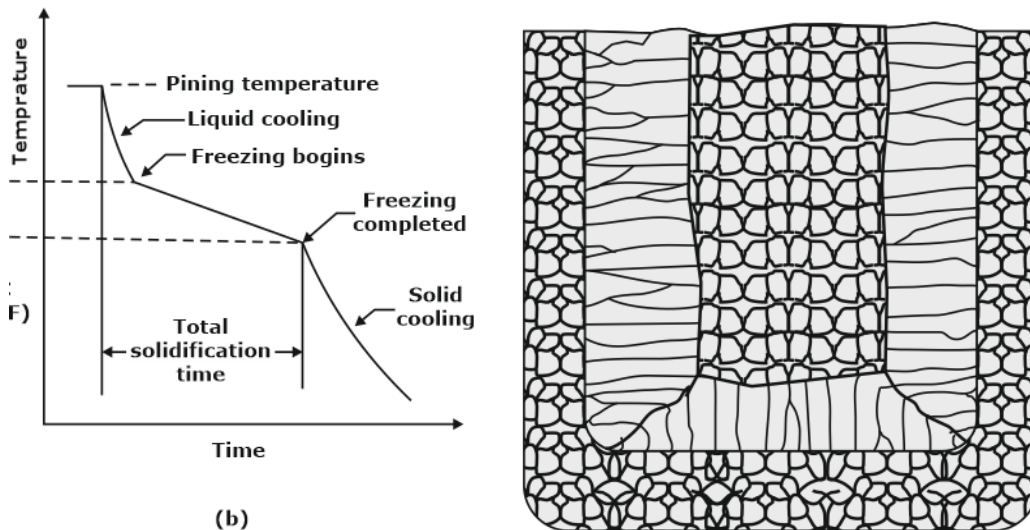
The start of freezing is like that of the pure metal. The solid portions are the dendrite structures that have formed sufficiently to trap small islands of liquid metal in the matrix and a mushy zone at the centre. As freezing continues and the dendrites grow, there develops an ***imbalance in composition between the metal that has solidified and the remaining molten metal.*** This composition imbalance is finally manifested in the completed casting in the form of ***segregation of the elements.***

Solidification time: "Solidification time is the time required for the casting to solidify after pouring". This time is dependent on the size and shape of the casting by an empirical relationship known as Chvorinov's rule, which states:

$$t_S = k \left(\frac{V}{SA} \right)^n$$

Where,

t_s = total solidification time

k = mould constant (or) solidification factor

V = volume of the casting,

SA = surface area of the casting,

n is an exponent usually taken to have a value = 2

$$t_S = k \left(\frac{V}{SA} \right)^2$$

$$\text{Modulus (M)} = \frac{V}{SA} = \frac{\text{Volume of casting}}{\text{Surface area}}$$

$$t_S = k (M)^2$$

$$T_S \propto \left(\frac{V}{SA} \right)^2$$

Riser design:

Condition to Design the Riser

1. $V_R \geq 3V_{Sc}$ ... [Necessary condition] i.e. Volume of riser should be at least 3 times the shrinkage volume of castings.

2. $T_{S_{Riser}} \geq T_{S_{cavity}}$ [Sufficient condition] The solidification time of molten metal in the riser must be at least equal to the solidification time of molten metal in the casting cavity.

Types of Riser:

Side Riser: $\boxed{SA = 2 \left(\frac{\pi}{4} d^2 \right) + \pi dh}$

Top Riser: $\boxed{SA = \left(\frac{\pi}{4} d^2 \right) + \pi dh}$

Optimum condition to get minimum surface area or **maximum solidification time in case of cylindrical riser:**

Side Riser	h = d	$\left(\frac{A}{V} \right) = \frac{6}{d}$
Top Riser	h = d/2	$\left(\frac{A}{V} \right) = \frac{6}{d}$

Location of risers:

One central riser is satisfactory if the maximum feeding distance is less than 4.5 times the plate thickness. The feeding distance should be measured from the edge of the riser.
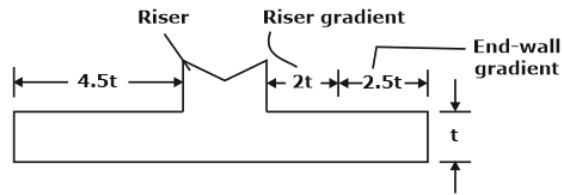
(a) Plate with one central riser

It should be noted that, of the total distance 4.5t the riser gradient prevails up to a distance 2t, whereas the end wall gradient prevails in the remaining distance 2.5t. Thus, the maximum distance between the edges of two consecutive risers is 4t and not 9t.
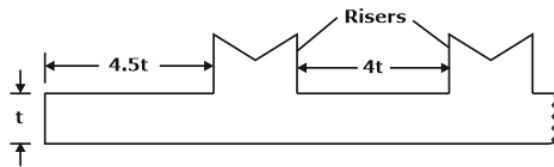
(b) Maximum distance between
two consecutive risers

Caine's Method

$$\text{Freezing ratio} : X = \frac{\left(V/A_s\right)_{riser}}{\left(V/A_s\right)_{casting}} = \frac{M_{riser}}{M_{casting}}$$

Freezing ratio, $X = \dfrac{a}{y-b} - c$

y = volumetric ratio $= \dfrac{V_r}{V_c}$

$$\frac{M_r}{M_c} = \frac{a}{\left(\dfrac{V_r}{V_c}\right) - b}$$

Modulus value for different type of risers:

Type of Riser	Modulus value
Spherical Riser	$M = \dfrac{R}{3} = \dfrac{D}{6}$
Top Cylindrical riser	$M = \dfrac{Dh}{D + 4h} = \dfrac{D}{6}$

side cylindrical riser	$M = \dfrac{Dh}{2D + 4h} = \dfrac{D}{6}$

Modulus Method:

Riser Solidification time: $\boxed{t_r \geq t_C}$

and $\boxed{M_r = 1.2 M_C}$

Chvorinov's Law: $\boxed{t_s = k\left(\dfrac{V}{A_s}\right)^2}$

Shape factor (SF):

$\boxed{shape\ factor(S.F) = \dfrac{L+W}{t}}$

For Plate (L × w × t): $\boxed{SF = \dfrac{L+w}{t}}$

For Cube (a × a × a): $\boxed{SF = \dfrac{a+a}{a} = 2}$

For Sphere (of Diameter D): $\boxed{SF = \dfrac{D+D}{D} = 2}$

For Solid cylinder (Diameter D and Height H): $\boxed{SF = \dfrac{D+H}{D}}$

For thin cylinder: $\boxed{SF = \dfrac{L + \dfrac{\pi}{2}(D_o + D_i)}{\dfrac{(D_o - D_i)}{2}}}$

Special casting processes:

Expendable mould casting processes:

1. Shell moulding: The mould is a thin shell (typically 9 mm) made of sand held together by a **_thermosetting resin binder_** and the thickness of the shell can be determined accurately by controlling the time that the pattern is in contact with the mould.

It can be mechanized for mass production and is very economical for large quantities.

Applications: high precision parts such as gear housings, cylinder heads, and connecting rods.

2. Vacuum Moulding: It is also called the V-process uses a sand mould held together by vacuum pressure.

Applications: It is suitable particularly for thin walled (0.75 mm) complex shapes with uniform properties. Typical parts made are superalloy gas-turbine components with walls as thin as 0.5 mm.

3. Expanded Polystyrene process: also known as Evaporative-pattern and investment casting.

Applications: Typical applications are cylinder heads, engine blocks, crankshafts, brake components, manifolds, and machine bases. It is applied to mass produce castings for automobiles engines.

4. Investment casting: a pattern made of wax is coated with a refractory material to make the mould. The parts of great complexity and intricacy can be cast and close dimensional control—tolerances of ±0.075 mm, are possible.

Applications: All types of metals including steels, stainless steels, and other high temperature alloys, can be investment cast. Examples of parts include complex machinery parts, blades. and other components for turbine engines, Jewellery, and dental fixtures.

5. Plaster mould casting:

Mould is made of plaster of Paris (gypsum: $CaSO_4.2H_2O$) instead of sand.

Additives such as talc and silica flour arc mixed with the plaster to control contraction and setting time, reduce cracking, and increase strength. It is the casting of lower-melting-point alloys such as aluminium, magnesium. and some copper-base alloys.

6. Ceramic-mould casting:

Mould is made of refractory ceramic materials that can withstand higher temperatures than plaster.

It is used to cast steels, cast irons, and other high-temperature alloys. Its applications (relatively intricate parts) are like those of plaster mould casting except for the metals cast.

Permanent-Mould Casting Processes:

It uses reusable metal moulds.

Metals commonly cast in permanent moulds include aluminium, magnesium, copper-base alloys, and cast iron. The very high pouring temperatures of steel make permanent moulds unsuitable for this metal unless the mould is made of refractory material.

Permanent-mould casting include good surface finish and close dimensional control, as previously indicated.

More rapid solidification caused by the metal mould results in a finer grain structure, so stronger castings are produced.

Typical parts include automotive pistons. pump bodies, and certain castings for aircraft and missiles.

Variations of Permanent mould casting:

1. Slush casting: The flow of metal into the mould cavity is caused by gravity. Slush casting is used to make hollow parts such as statues, lamp pedestals, and toys out of low-melting-point metals such as zinc and tin.

2. Low pressure casting:

The liquid metal is forced into the cavity under low pressure— approximately 0.1 MPa from beneath so that the flow is upward.

3. Vacuum Permanent-Mould Casting:

Low-pressure casting operation where reduced air pressure from the vacuum in the mould is used to draw the liquid metal into the cavity, rather than forcing it.

4. Die Casting:

Die casting is a permanent-mould casting process in which the molten metal is injected into the mould cavity under high pressure ranging from 7 to 350 MPa.

There are two main types of die-casting machines:

(a). Hot chambers die casting:

Typical injection pressures: 7 to 35 MPa.

Limited in its applications **to low-melting-point metals** that do not chemically attack the plunger and other mechanical components. The metals include **zinc, tin, lead, and sometimes magnesium.**

(b). Cold chambers die casting:

Injection pressures: typically, 14 to 140 MPa.

Typically used for casting aluminium, brass, and magnesium alloys.

Centrifugal casting:

True Centrifugal Casting:

The parts made by this process include pipes, tubes, bushings, and rings.

G-factor GF is the ratio of centrifugal force divided by weight:

$$GF = \frac{F}{W} = \frac{mv^2}{Rmg} = \frac{v^2}{Rg}$$

$$N = \frac{30}{\pi}\sqrt{\frac{gGF}{R}} = \frac{30}{\pi}\sqrt{\frac{2gGF}{D}}$$

GF = 60 to 80 are found to be appropriates for horizontal centrifugal casting.

For Vertical casting:

$$N = \frac{30}{\pi}\sqrt{\frac{2gL}{R_t^2 - R_b^2}}$$

Where L = vertical length of the casting (in m), R_t = inside radius at the top of the casting (in m) and R_b = inside radius at the bottom of the casting (in m).

Semi centrifugal castings:

It is used to produce solid castings rather than hollow castings.

G factors of around 15 are obtained.

Wheels and pulleys are examples of castings that can be made by this process.

Centrifuge casting:

The mould is designed with part cavities located away from the axis of rotation.

CHAPTER 2: ENGINEERING MATERIALS

CRYSTAL STRUCTURES

- A crystalline material is one in which atoms are arranged in a regular pattern over large atomic distances.

- **Space lattice:** Infinite array of points in 3-D space in which each point located with respect to other.

- **Unit Cell** are the smallest unit of a structure which when repeated in all 3-demensions produces the crystal structure.

| Space lattice + Basis = Unit cell |

- **Primitive cell:**

  This may be defined as a geometrical shape which, when repeated indefinitely in 3-dimensions, will fill all space and is equivalent of one lattice point i.e. the unit cell that contains one lattice point only at the corners.
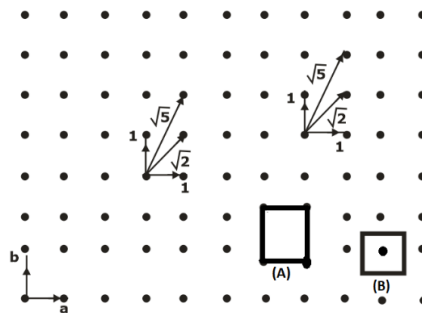
Crystal families and crystal class:

If all the atoms at the lattice points are identical, the lattice is said to be Bravais lattice. There are four systems and five possible Bravais lattices in two dimensions as shown in Fig. The four crystal systems of two-dimensional space are oblique, rectangular, square and hexagonal.

(i) Square
a = b, γ = 90°

(ii) Rectangular (primitive)
a = b, γ = 90°

(ii) Rectangular (centered)
a = b, γ = 90°

(iii) Hexagonal
a = b, γ = 90°

(iv) Oblique
a = b, γ = 90°
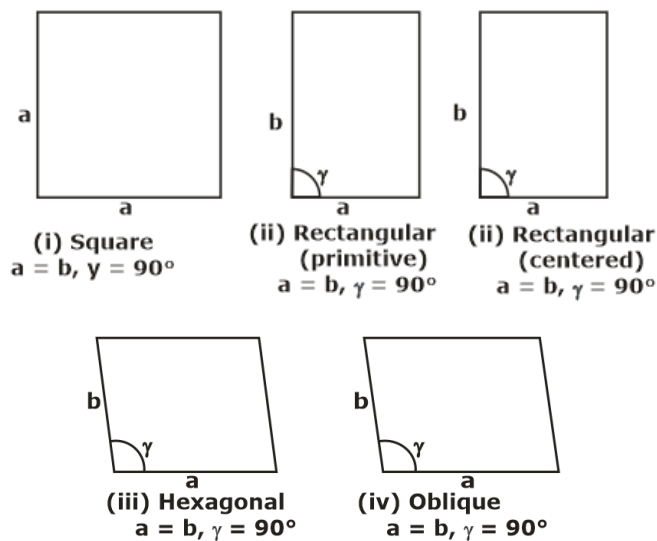
Bravais Lattices in two dimensions

Seven crystal systems:

Crystal family	Crystal system	Axial relationships
Isometric	Cubic	a = b = c and α = β = γ = 90°
Tetragonal	Tetragonal	a = b ≠ c and α = β = γ = 90°
Orthorhombic	Orthorhombic	a ≠ b ≠ c and α = β = γ = 90°
Monoclinic	Monoclinic	a ≠ b ≠ c and α = γ = 90°; β = 90°
Anorthic	Triclinic	a ≠ b ≠ c and α ≠ 90°; β ≠ 90°; γ ≠ 90°
Hexagonal	Hexagonal	a = b ≠ c and α = β ≠ 90°; γ = 90°;
	Trigonal	A = b = c, α = β = γ; or
	Rhombohedral	a' = b' ≠ c' and α' =β' = 90°, γ' = 120° (Hexagonal axes)

Effective number of lattice points in the unit cell of the three cubic space lattices:

Space lattice	Abbreviation	Effective number of lattice points in unit cell
Simple cubic	SC	1
Body centred cubic	BCC	2
Face centred cubic	FCC	4

The Bravais lattices in three dimensions (the seven basic crystal systems)

Crystal system	Lattice type	No. of lattices	Examples
Cubic	P, F, C	3	Au, NaCl, CaF_2, C_rCl, CaO (I)
Monoclinic	P, B	2	$2H_2O$, $NaSO_4$, $CaSO_4$, $FeSO_4$
Triclinic	P	1	$K_2Cr_2O_7$, $CuSO_4$, $K_2S_2O_8$
Tetragonal	P, C	2	$NiSO_4$, Sn, TiO_3, and SnO_2
Orthogonal	P, B, F, C	4	$MgSO_4$, KNO_3, and $BaSO_4$
Rhombohedral (Trigonal or orthorhombic)	P or R	1	SiO_2, $CaSO_4$, and $CaCO_3$
Hexagonal	P	1	$AgCl_2$, Sio_2, Zn and Graphite

Representation of symbols:

P → primitive, B → base centred

C → body centred and

F → face centred

Atomic Packing Factor (APF): This is defined as the ratio of total volume of atoms in a unit cell to the total volume of the unit cell. This is also called relative density of packing (RDP). Thus:

$$APF = \frac{No.\ of\ atoms \times Volume\ of\ one\ atom}{Volume\ of\ unit\ cell} = \frac{v}{V}$$

Simple Cubic Crystal System: (a = b = c, α = β = γ = 90):
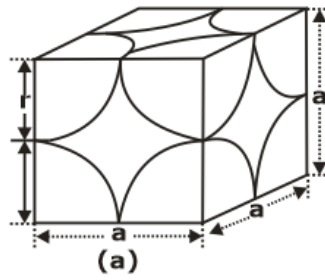
No. of atoms in all corners: $n = \dfrac{1}{8} \times 8 = 1$

From the diagram: $a = 2r$

volume of cubic cell $= a^3 = (2r)^3$

$$\therefore \boxed{APF = \dfrac{1 \times \dfrac{4}{3}\pi r^3}{(2r)^3} = \dfrac{\pi}{6}(= 0.52) = 52\%}$$

Coordination number of BCC crystal structure is 6.

Body-centred Cubic Structure (BCC):

$\therefore$ Total no. of atoms for corners $= \dfrac{1}{8} \times 8 = 1$ atom

BCC crystal has one atom at the centre = 1 atom

$\therefore$ Total atoms in BCC unit cell = 1 + 1 = 2 atoms
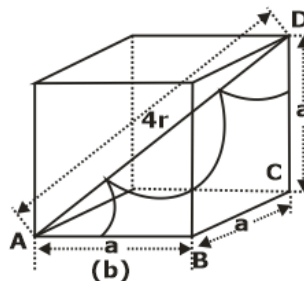
$$\boxed{a = \sqrt{\dfrac{16r^2}{3}} = \dfrac{4r}{\sqrt{3}}}$$

$$\boxed{APF = \dfrac{2 \times \dfrac{4}{3}\pi r^3}{\left(4r/\sqrt{3}\right)^3} = \dfrac{\sqrt{3}}{8}\pi = 0.68}$$

Coordination number of BCC crystal structure is 8.

Examples: α-Fe (below 910°C), Mo, V, Mn, Nb, W, α -Cr, δ -iron (1400°C to 1539°C), etc.

Face-centred Cubic Structure (FCC):

(c)
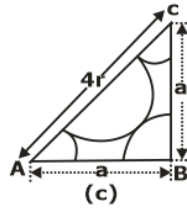
No. of atoms in all six faces = $\frac{1}{2} \times 6 = 3$

No. of atoms in all corners = $\frac{1}{8} \times 8 = 1$

∴ Total atoms in FCC unit cell = 1 + 3 = 4 atoms

$$a = \sqrt{8}r = 2\sqrt{2}r$$

$$\therefore APF = \frac{4 \times \frac{4}{3}\pi r^3}{\left(2\sqrt{2}r\right)^3} = \frac{\pi}{3\sqrt{2}} = 0.74$$

Coordination number of BCC crystal structure is 12.

Examples: Cu, Al, Ag, Au, α-Fe, Ca, β-Co, γ-Iron (910°C to 1400°C).

Hexagonal closed packing (HCP) Structure:

The coordination number of HCP crystal is 12.

$$APF = \frac{\pi\sqrt{2}}{6} = 0.74$$

Examples of HCP structure are Be, Mg, Zn, Cd, Ti, Co, Hf, Se, Te, etc.

CYRSTAL DENSITY:

$$Density = \frac{nM}{VNo}$$

n = no. of atoms per unit cell.

M = At weight

V = Volume of unit cell

N_o = Avogadro's number (6.023×10^{23} atoms/mol).

Linear Densities:

$$Linear\ desnity\ (LD) = \frac{No.\ of\ atoms\ centered\ on\ unit\ length\ of\ direction\ vector}{Length\ of\ the\ direction\ vector}$$

Planer Densities:

$$PD = \frac{No.\ of\ atoms\ centered\ on\ a\ plane}{Area\ of\ the\ plane}$$

Miller Indices: Miller indices are used to specify directions and planes.
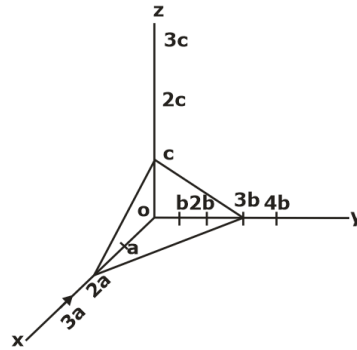
Procedure:

(a). Identify the plane intercepts on the x, y and z-axes.

(b). Specify intercepts in fractional coordinates.

(c). Take the reciprocals of the fractional intercepts.

Notation summary:

(a). (h, k, l) represents a point – note the exclusive use of commas.

(b). Negative numbers/directions are denoted with a bar on top of the number.

(c). [h k l] represents a direction.

(d). <h k l> represents a family of directions.

(e). (h k l) represents a plane.

(f). {h k l} represents a family of planes.

Interplanar spacings:

$$d_{hkl} = \frac{1}{\sqrt{\dfrac{h^2}{a^2} + \dfrac{k^2}{b^2} + \dfrac{l^2}{c^2}}}$$

d_{hkl} gives the distance between two successive (h k l) places.

For a cubic system: a = b = c

$$\therefore \quad d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Angle between two planes or directions:

$$\cos\theta = \frac{h_1 h_2 + k_1 k_2 + l_1 l_2}{\sqrt{h_1^2 + k_1^2 + l_1^2} \times \sqrt{h_2^2 + k_2^2 + l_2^2}}$$

The angle θ between the two directions or planes having Miller indices (h_1, k_1, l_1) and (h_2, k_2, l_2) respectively.

Defects and imperfections: Crystalline defects can be classified on the basis of their geometry as follows:

(i). Point imperfections

(ii). Line imperfections

(iii). Surface and grain boundary imperfections

(iv). Volume imperfections

Point Defects:

Vacancies: The simplest point defect is a vacancy i.e. an empty site of a crystal lattice which arise either from imperfect packing during original crystallization or from thermal vibrations of the atoms at higher temperature.

Interstitial Imperfections: In a closed packed structure of atoms in a crystal if the atomic packing factor is low, an extra atom may be lodged within the crystal structure.

Frenkel defect: Frenkel defect is a combination of vacancy and interstitial defects. It is ***more common in ionic crystals,*** because the positive ions, being smaller in size, get lodged easily in the interstitial positions.

Schottky defect: It is caused, whenever a pair of positive and negative ions is missing from a crystal. ***It maintains a charge neutrality.***

Linear Detects:

a. Edge Dislocation: An extra half plane of atoms is introduced into the crystal structure. The dislocation line is perpendicular to burger vector.

The elastic strain energy E per unit length of a dislocation of Burgers vector b can be expressed approximately by

$$E = \frac{\mu b^2}{2}$$

b. Screw Dislocation: Dislocation is formed by shear stress; the upper front region of the crystal is shifted one atomic distance to the right relative to the bottom portion.

The dislocation line is parallel to burger vector.

Surface and grain boundaries:

a. High angle and low angle grain boundaries: When the orientation difference between neighbouring grains is more than 10°-15°, boundaries are called high angle grain boundaries. While in case of low angle grain boundaries this orientation mismatch is not more than of 5°.

b. Twin boundaries: Here, the atomic arrangement on one side of a twin boundary is a mirror reflection of the arrangement on the other side.
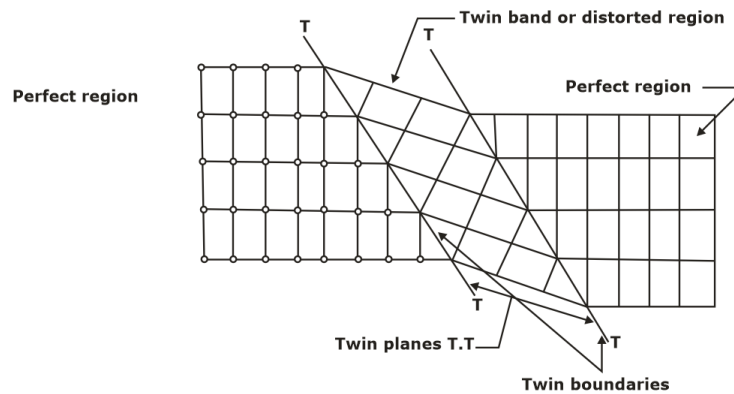
Strengthening Mechanisms and phase diagrams:

Strengthening Mechanisms:

Any restriction to the movement of the dislocations are said to increase the strength of the material.
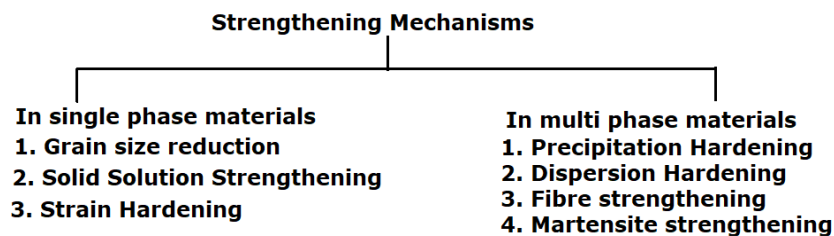
Solid solution strengthening:

Hume-Rothary rule: It gives Conditions for unlimited solubility.

1. Difference in atomic radius of both the materials should be less than 15 %.

2. Crystal Structure: The materials must have the same crystal structure.

3. Valency of both the materials should be same.

4. Electronegativity and electron affinity of both the materials should be comparative. Alloying the element either produces interstitial or substitutional impurity.

Grain size strengthening: Yield strength is related to grain size (diameter, d) as Hall Petch relation:

$$\boxed{\sigma_y = \sigma_o + Kd^{-1/2}}$$

Strain hardening:

Dislocation density (ρ) and shear stress (τ) are related as follows:

$$\boxed{\tau = \tau_o + A\sqrt{\rho}}$$

Percentage cold work is given by:

$$\boxed{\%CW = \frac{A_o - A_d}{A_o} \times 100\%}$$

Where:

A_o = Original Area of specimen

A_d = area after deformation

PHASE:

A phase is a physically distinct, chemically homogeneous, and mechanically separable region in a system in equilibrium.

According to Gibb's Phase Rule:

$$\boxed{F + P = C + 2}$$

where, F = No. of degrees of freedom.

P = No of phases present.

C = No. a component

2 is for temperature and pressure.

When pressure is held constant

F+ P = C+1, 1 is only for temperature.

Type of Phase Diagram

Unary phase diagram (single component): Example: Water, graphite, metallic carbon, diamond.

Binary phase diagram (two components):

Type 1 (Binary Isomorphous Systems): The materials which are completely soluble in liquid as well as solid state.

Type-II: Two Components are Completely Soluble in Liquid State and are Completely insoluble in each Other in Solid State.

Example: Bismuth-cadmium system

Type III: Two Components are Completely Soluble in Liquid State and are Partially Soluble in Solid State.

Examples: Pb-Sn, Cu-Ag, Pb-Sb, Cd-Zn, Sn-Bi, etc.

Type IV: When two metals are completely soluble in liquid state, show partial solubility in the solid state, and it their melting points are vastly different from each other, then, a peritectic phase diagram may result.

LEVER RULE:

Draw a horizontal line until it intersects the curve on both sides. This line is called tie line.

Composition, weight percent B
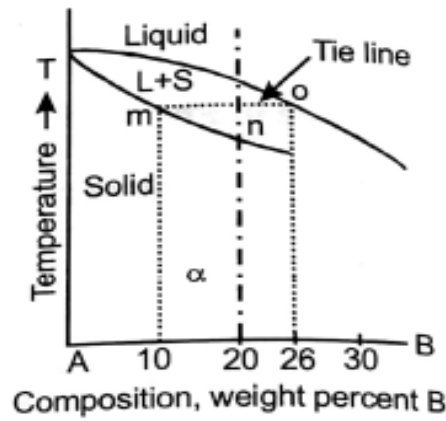
$$\boxed{\text{Liquid}(\%) = \frac{mn}{mo} \times 100}$$

$$\boxed{\text{Solid}(\%) = \frac{no}{mo} \times 100}$$

Various types of phase Diagram Reaction

Reaction	Symbolic equation	Schematic presentation	Example
Eutectic	$L \leftrightarrow \alpha + \beta$		Fe-C, 4.27C%, 1147°C
Eutectoid	$\alpha \leftrightarrow \beta + \gamma$		Fe-C, 0.80C%, 723°C
Peritectic	$L + \alpha \leftrightarrow \beta$		Fe-C, 0.16C%, 1495°C
Peritectoid	$\alpha + \beta \leftrightarrow \gamma$		
Monotectic	$L_1 \leftrightarrow L_2 + \alpha$		Fe-C, 0.51C%, 1495°C

Iron-carbon phase diagram:
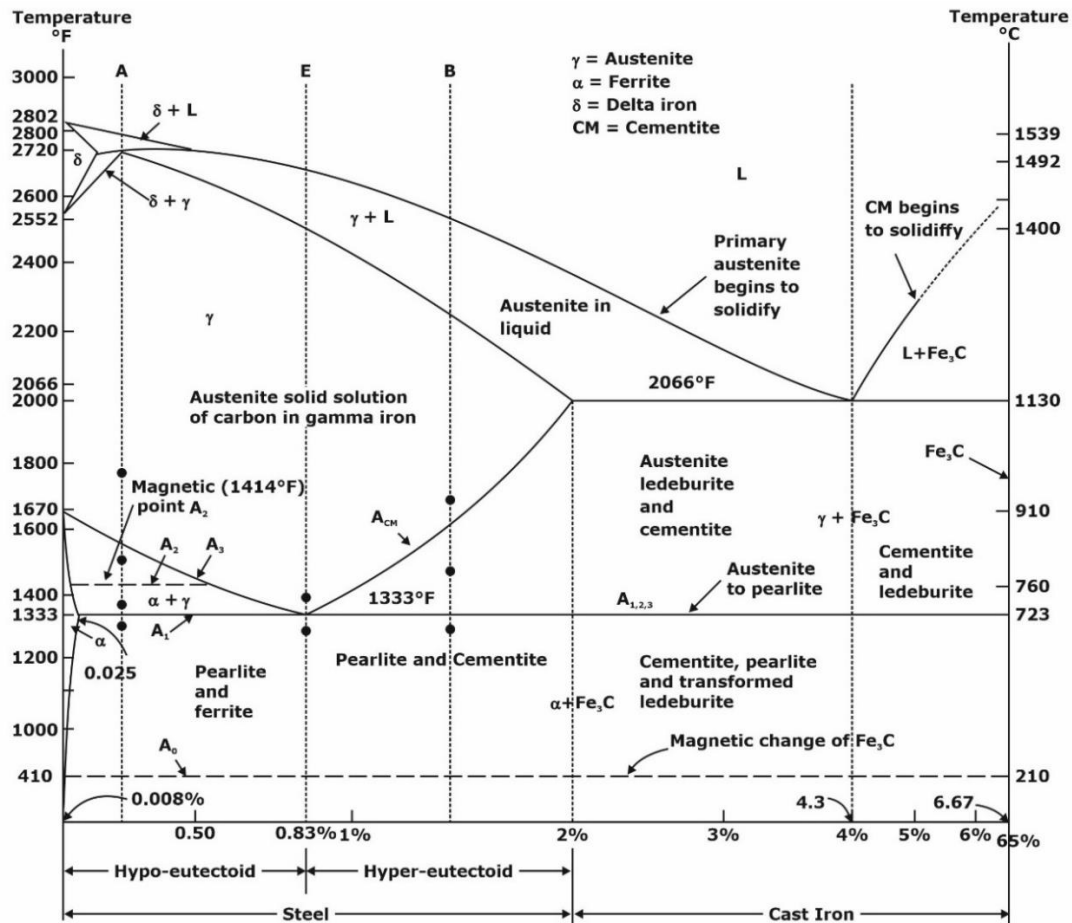
α-ferrite: It is a solid solution of **BCC iron** and carbon with maximum solubility of carbon is 0.022% at 727°C. It is softest phase and act as matrix of a composite material.

γ-iron or austenite: It is a solid solution of FCC iron and carbon with maximum solubility of 2.11% C at 1148°C. It is present between temperature limits from (910°C to 1395°C).

Cementite or Fe₃C: It is 100% iron carbide having maximum solid solubility of 6.67% C. It is *very hard and brittle intermetallic compound* and has significant influence on the properties of steel.

Peritectic reaction at 1495°C and 0.16%C.

δ-ferrite + L ↔ γ-iron (austenite)
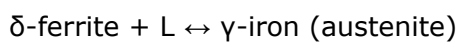
Monotectic reaction 1495°C and 0.51%C.

L ↔ L+ γ-iron (austenite)
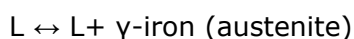
Eutectic reaction: at 1147°C and 4.3%C.

L ↔ γ-iron + Fe₃C (cementite) [ledeburite].

Eutectoid reaction: at 723°C and 0.8%C.

γ-iron L ↔ α-ferrite + Fe₃C (cementite) [pearlite]

Types of steels:

Low Carbon steels: C < 0.25 wt%. These are unresponsive to heat treatments intended to form martensite; strengthening is accomplished by cold work.

Medium carbon steels: 0.25 wt% < C < 0.60 wt%. These alloys may be heat treated by austenitizing, quenching, and then tempering to improve their mechanical properties.

High-carbon steels: 0.60 wt% < C < 1.4 wt%, are the hardest, strongest, and yet least ductile of the carbon steels.

Cast Irons: Generically, cast irons are a class of ferrous alloys with carbon contents above 2.14 wt%; in practice, however, most cast irons contain between 3.0 and 4.5 wt% C and with other alloying elements.

Gray Cast Iron: C varies from 2.5 to 4.0 wt% and Si varies from 1.0 to 3.0 wt%, respectively. The graphite exists in the form of flakes (similar to corn flakes).

Ductile (or Nodular Iron): It is obtained by adding a small amount of magnesium and/or cerium to the gray iron. Graphite still forms, but as nodules or sphere-like particles.

White Cast Iron: Most of the carbon exists as cementite instead of graphite. Due to large amounts of the cementite phase, white iron is extremely hard and brittle (unmachinable).

Malleable cast iron: Heating white iron at temperatures between 800°C and 900°C for a prolonged time period and in a neutral atmosphere (to prevent oxidation) causes a decomposition of the cementite, forming graphite, which exists in the form of clusters or rosettes surrounded by a ferrite or pearlite matrix.

TTT DIAGRAMS: While quenching austenite below 725° C, it was observed that for some time there was no change in the microstructure this period is called incubation time.

- Incubation time decreases as the temperature of quenching decreases.
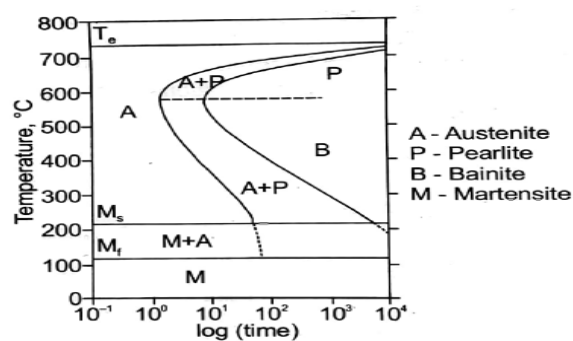
Quenching medias and their severity:

"Severity of quench" is a term often used to indicate the rate of cooling, the more rapid the quench, the more severe the quench. Quenching media with severity:

Brine > water > oil > air

Brine produces the most severe quench, followed by water and then oil, which is more effective than air.

Martensite:

Martensite is a ***body-centered tetragonal (BCT)*** form of iron in which some carbon is dissolved. The transformation to martensite does not involve atom diffusion, but rather occurs by a sudden diffusion-less shear process.

Bainite:

Bainite is formed at cooling rates slower than that for martensite formation and faster than that for ferrite and pearlite formation. Upper bainite generally forms at temperatures between 550 and 400°C and Lower bainite generally forms at temperatures between 400 and 250°C.

HEAT TREATMENT:

a. Annealing:

Process Annealing: used to negate the effects of cold work—that is, to soften and increase the ductility of a previously strain-hardened metal.

Stress Relief annealing: It is used to relieve internal residual stresses developed due to nonuniform distortion and warpage.

Full Annealing:

For hypoeutectoid steels: at about 50°C above the A_3 line (to form austenite).

For hypereutectoid steels: 50° C above the A_1 line.

The microstructural product of this anneal is coarse pearlite that is relatively soft and ductile.

Spheroidizing: A coalescence of the Fe_3C to form the spheroid particles.

b. NORMALIZING:

For hypoeutectoid steels: at about 55°C above the A_3 line (Upper critical temperature).

For hypereutectoid steels: 55° C above the A_{cm} line.

The sample is cooled in air. It is used to refine the grains (i.e., to decrease the average grain size) and produce a more uniform and desirable size distribution.

c. TEMPERING: used to relive residual stress, improve ductility and Toughness.

Surface Hardening Mechanisms:

CASE HARDENING:

(i). In Pack Carburizing Piece is surrounded by a carburizing mixture and packed in a steel box.

Carburizing mixture: 50% charcoal + 20% $BaCO_3$ + 5% $CaCO_3$ + 5-12% Na_2CO_3

(ii). In Gas Carburizing work-part is treated in an atmosphere of gases containing carbon and hydrocarbon gases such as CH4, butane etc.

(iii). In Liquid Carburizing Workpiece is heated to 950°C and dipped in a molten salt bath containing 20% NaCN, which provides Carbon & Nitrogen.

Nitriding is done by heating steel in the atmosphere of NH_3 gas. It produces the hardest surface of steel.

Cyaniding: work part is immersed in molten salt bath containing sodium cyanide (NaCN) which is heated to 820-860°C.Mixture is 20-30% NaCN + 25-50% NaCl + 25-50% Na_2CO_3

Flame Hardening Process consists of heating the surface of high carbon steels by a high temperature gas flame at 2400-3500°C, followed by immediate cooling is air or water.

Induction hardening: Workpieces is heated in an induction furnace surrounded by copper coils which are water cooled. High frequency AC current is passed through Cu coils and thus alternate magnetic field set up which induce eddy currents on the surfaces.

Effects of alloying elements:

Element	Effect(s)
Mn	Increase hardenability Increase resistance Reduces ductility and weldability Increase hardenability significantly
Mo	Increase strength, toughness, red hardness, and hot When used with Cr, Mn, and V Enhances corrosion and abrasion resistance
Ni	Increases toughness and impact resistance
P	Increases strength in low carbon steel Improves corrosion resistance
Si	Strengthens low alloy steels Increases hardenability Acts as deoxidizer Improves magnetic properties when present in large percentage.
S	Improves machinability of very low carbon steels
Ti	Increases austenitic hardenability Reduces martensitic hardness in Cr steels
V	Increases strength while retaining ductility Produces fine grain size Increases hardenability
W	Imparts hardness and wear resistance Significantly improves red hardness Imparts strength temperature

CHAPTER 3: FORMING

Introduction: Metal Forming is the manufacturing process in which the parts are produced by plastic deformation.

Types of metal forming:

Cold working: plastic deformation of metals and alloys at a temperature below their recrystallization temperature. Parts produced from it have better dimensional accuracy, better surface finish, residual stresses, large power requirement.

$T < 0.3\ T_m$

Warm Forming: Metal deformation carried out at temperatures intermediate to hot and cold forming.

$0.3\ T_m < T < 0.5\ T_m$

Hot working: Plastic deformation of metals and alloys at such a temperature above recrystallization temperature at which recovery and recrystallization take place simultaneously with the strain hardening.

$T > 0.6\ T_m$

Parts produced from hot working have poor surface finish, poor dimensional accuracy, less power requirement.

Typical values for different type metalworking:

| Category	Temperature range	Strain rate sensitivity exponent	Coefficient of friction
Cold working	$\leq 0.3 T_m$	$0 \leq m \leq 0.05$	0.1
Warm working	$0.3 T_m - 0.5\ T_m$	$0.05 \leq m \leq 0.1$	0.2
Hot working	$0.5 T_m - 0.75\ T_m$	$0.05 \leq m \leq 0.4$	0.4 – 0.5

Classification of metalworking processes:

1. Direct compression type: Forging and Rolling

2. Indirect compression type: Wire drawing, Extrusion, Deep drawing

3. Tension type: Stretch Forming

4. Bending type: Bending of sheets

5. Shearing processes: Blanking, Coining, Joggling, Twisting.

Forging: Forging is a basic process in which the work piece is shaped by compressive forces applied through various dies and tooling.

Open die forging: In this, the work piece is compressed between two platens. There is no constraint to material flow in lateral direction.

- In edging the cross-section area is increased.
- In fullering cross section area is decreased and length is increased.

- In Upsetting forging major length of part is in die and some part is out of die. Force is applied parallel to axis to increase area of part outside the die. It is used to produce bolt heads.
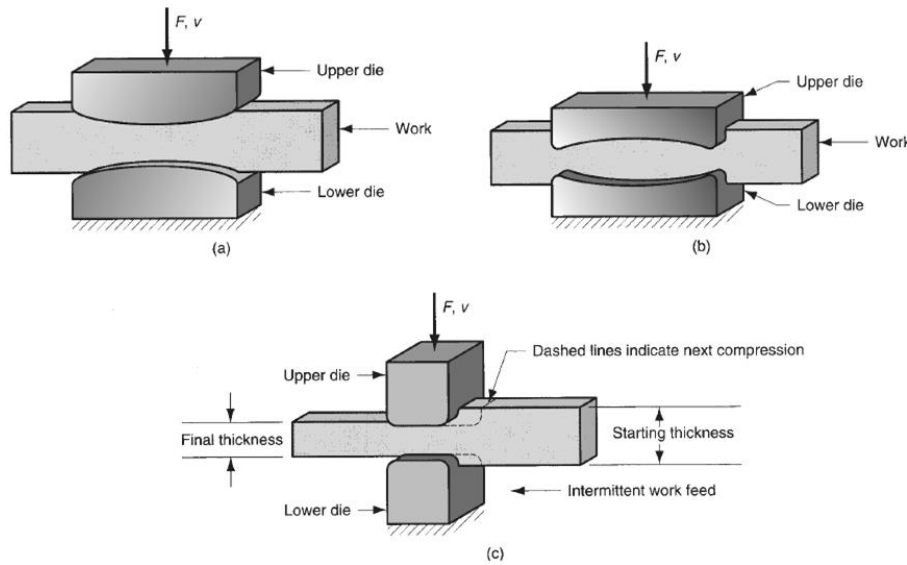
Open die forging operations: (a) fullering, (b) edging, and (c) cogging

Impression die forging: In impression die forging, the work piece is pressed between the dies. As the metal spreads to fill up the cavities sunk in the dies, the requisite shape is formed between the closing dies.

Some material which is forced out of the dies is called "flash".

Closed die forging: It is very similar to impression die forging, but in true closed die forging, the amount of material initially taken is very carefully controlled, so that no flash is formed.

Drop forging: It utilizes a closed impression die to obtain the desired shape of the component. The shaping is done by the repeated hammering given to the material in the die cavity. The equipment used for delivering the blows are called drop hammers.

Press forging: In press forging the metal is shaped not by means of a series of blows as in drop forging, but by means of a single continuous squeezing action.

Swaging: Swaging is a special variation of impact forging where the repeated blows are obtained by a radial movement of shaped dies.

Analysis of forging:

Volume before forging = Volume after forging

$$\frac{\pi}{4} d_0^2 \times h_0 = \frac{\pi}{4} \times d_1^2 \times h_1$$

$$\boxed{d_1 = d_0 \sqrt{\frac{h_0}{h_1}}}$$

Forging force calculation is done based on final dimensions.

$$\boxed{F_{act} = \sigma_y \times A_f \left[1 + \frac{2\mu r_f}{3h_f} \right]}$$

Where A_f = final cross section area, r_f = final radius, h_f = final height.

True stress:

$$(\sigma_T) = \frac{\text{load}}{\text{Instantaneous area}} = \sigma(1 + \varepsilon)$$

Where σ = engineering stress

ϵ = engineering strain

True strain is given by:

$$(\varepsilon_T) = \int_{L_o}^{L} \frac{dx}{x} = \ln\left[\frac{L}{L_o}\right] = \ln(1 + \varepsilon) = \ln\left[\frac{A_o}{A}\right] = 2\ln\left[\frac{d_o}{d}\right]$$

Flow stress is given by the power law:

$$\sigma_T = K(\epsilon_T)^n$$

Thus, mean flow stress is given by:

$$\sigma_0 = \frac{K\epsilon_T^n}{n+1}$$

Forging defects:

(i). Barrelling is the defect when material near plates does not flow and material in middle flow.

(ii). Die shift

(iii). Cold shut

Rolling: In this process, metals and alloys are plastically deformed into semi-finished or finished products by being pressed between two rolls which are rotating.
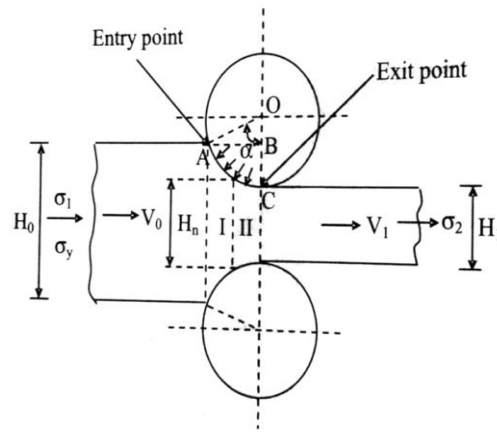
$V_{\text{top roller}} = V_{\text{bottom roller}}$

$$\left(\frac{\pi DN}{60}\right)_{\text{top}} = \left(\frac{\pi DN}{60}\right)_{\text{bottom}}$$

$D_t N_t = D_b N_b$

$H_0 B_0 V_0 = H_1 B_1 V_1$

$$\frac{H_0}{H_1} = \frac{V_1}{V_0} \left(\because H_0 > H_1\right)$$

$\Rightarrow V_1 > V_0$

ΔH = reduction in thickness = $(H_0 - H_1)$

α = Deformation angle (or) Bite angle (or) Angle of bite.

$$\Delta H = D (1 - \cos \alpha)$$

Length of Contact : $L = \sqrt{R\Delta H}$

Maximum reduction possible per pass: $\Delta H_{max} = \mu^2 R$

The neutral point defined in the deformation zone is dividing the deformation zone into two zones:

(i). The zone between the entry and neutral points is called "lagging zone".

(ii). The zone between neutral point and exit is called "leading zone".

At the entry, the velocity of the strip is much less than the velocity of the roller, the relative velocity between rollers and the strip is maximum.

(a). When we are moving along the deformation zone because of increase of velocity of strip their relative velocity is reducing.

(b). At the neutral point the relative velocity becomes equal to zero.

(c). Beyond the neutral point the relative velocity again increasing in the opposite direction and becomes maximum at the exit. But the maximum relative velocities at the entry and at exit are not equal.

From the above in the deformation zone the relative velocity is reducing first and then increasing, whereas in lagging zone relative velocity is reducing and in leading zone relative velocity is increasing.

As slip $\propto$ Relative velocity

$$\text{Backward slip} = \frac{V - V_0}{V} = 1 - \frac{V_0}{V}$$

The maximum % slip taking place in the leading zone is called as "forward slip".

$$\text{Forward slip} = \frac{V_1 - V}{V} = \frac{V_1}{V} - 1$$

- In the deformation zone the pressure is increasing first and then decreasing:

$n = \dfrac{2\,\mu L}{\Delta H}$

μ = coefficient of friction

$(P_x)_{lag} = \left(\dfrac{\sigma_y}{n}\right)\left[(n-1)\left(\dfrac{H_0}{H_x}\right)^n + 1\right] - \left(\dfrac{H_0}{H_x}\right)^n \sigma_1$

$(P_x)_{lead} = \left(\dfrac{\sigma_y}{n}\right)\left[(n+1)\left(\dfrac{H_0}{H_x}\right)^n - 1\right] - \left(\dfrac{H_0}{H_x}\right)^n \sigma_1$

H_x = Thickness of stirp in leading zone at a distance of 'x'.

$P_{x,\,lag}$, $P_{x,\,lead}$ is pressure in lag and lead zones at a distance x respectively.

σ_2 = front tension (not compulsory)

At the neutral point the pressure is equal:

$(P_n)_{lagging} = (P_n)_{leading}$

Rolls Power: $P = 2T\omega$ (As two power rollers considered)

T = Torque required per single roller

Angular velocity: $\boxed{\omega = \dfrac{2\pi N}{60}}$

N = rpm of rollers

T = F_{avg} × a

Moment arm: a = λL

T = F_{avg} × λL

λ (arm factor) = 0.3 to 0.5

$\boxed{P_{avg} = \dfrac{2}{\sqrt{3}}\,\sigma_y \times \left(1 + \dfrac{\mu L}{4H}\right)}$

Where $H = \dfrac{H_0 + H_1}{2}$

Defects in Rolling:

- Alligatoring
- Wavy edges

Extrusion: Extrusion is a process in which the metal is subjected to plastic flow by enclosing the metal in a closed chamber in which the only opening provided is through a die.

Types of Extrusion Processes:

These are mainly Forward extrusion or backward extrusion. These two can be in both the hot and cold conditions:

Forward or Direct extrusion: A metal billet is loaded into a container, and a ram compresses the material, forcing it to flow through one or more openings in a die at the opposite end of the container.
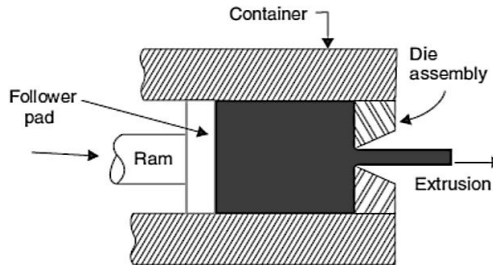
Forward or direct extrusion

Backward or Indirect extrusion: The die is mounted to the ram rather than at the opposite end of the container. As the ram penetrates the work, the metal is forced to flow through the clearance in a direction opposite to the motion of the ram. Since the billet is not forced to move relative to the container, there is no friction at the container walls, and the ram force is therefore lower than in direct extrusion.
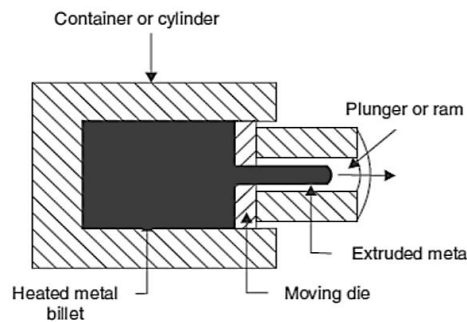

Backward or Indirect extrusion

Hot extrusion vs cold extrusion: Metals that are typically extruded hot include aluminium, copper, magnesium, zinc, tin, and their alloys. These same metals are sometimes extruded cold. Steel alloys are usually extruded hot, although the softer, more ductile grades are sometimes cold extruded (e.g., low carbon steels and stainless steel).

Cold Extrusion Processes:

(a). Hydrostatic extrusion: This is a direct extrusion process. But the pressure is applied to the metal blank on all sides through a fluid medium.

(b). Impact extrusion: Collapsible bottles are made by impact extrusion.
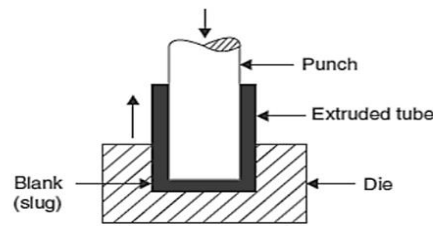
Impact extrusion

Calculations in Extrusion:

Extrusion or reduction ratio: $R = \dfrac{A_0}{A_f}$

Where A_o and A_f are the original and final areas, respectively.

Johnson's equation:

$$\sigma = \sigma_0[a + b\ln R]$$

Where a and b are Johnson's constants, σ_o is the nominal stress and R is the extrusion ratio.

$$\sigma_0 = \dfrac{K\varepsilon_T^n}{n+1}$$

Slab Method: $\sigma_E = \sigma_y \left(\dfrac{1+B}{B} \right) \left[1 - \left(\dfrac{A_f}{A_0} \right)^B \right]$

Where B = μ.cot α

μ = coefficient of friction

α = semi die angle

For Plane strain: $\sigma_y = \dfrac{2\sigma_o}{\sqrt{3}}$

For Plane stress: $\sigma_y = \sigma_o$

For Hot Extrusion:

$$\sigma_d = k_1 \ln R$$

K_1 = Extrusion constant which depends upon the temperature

Extrusion defects:

Surface Cracking: Sometimes the surface of extruded metal/products develops surface cracks. If extrusion temperature, friction, or speed is too high, surface temperatures can rise significantly, which may cause surface cracking and tearing.

Piping: The type of metal-flow pattern in extrusion tends to draw surface oxides and impurities toward the centre of the billet-much like a funnel. This defect is known as pipe defect, tailpipe, or fishtailing.

Wire drawing: Wire drawing process is a cold working process used to produce wires from solid rods by pulling through a stationary die.

Analysis of wire drawing operation
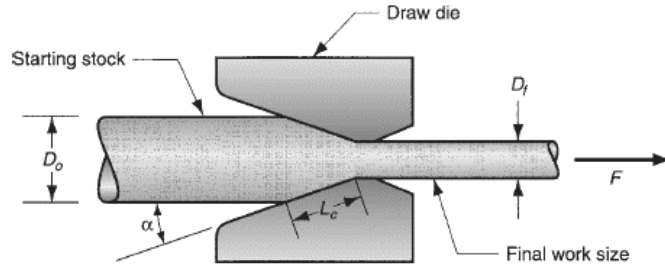
Original area: $A_0 = \dfrac{\pi}{4}D_0^2$

Final Area: $A_f = \dfrac{\pi}{4}D_f^2$

The draft is simply the difference between original and final stock diameters:

$$d = D_o - D_f$$

In a drawing operation, the change in size of the work is usually given by the area reduction, defi ned as follows:

% Reduction in area: $r = \dfrac{A_0 - A_f}{A_0}$

Contact length (L_c) of the work with the draw die: $L_c = \dfrac{D_0 - D_f}{2\sin\alpha}$

Mechanics of drawing: $\varepsilon = \ln\dfrac{A_o}{A_f} = \ln\dfrac{1}{1-r}$

Drawing stress (σ_d) is given by:

$$\sigma_d = \sigma_y\left(\dfrac{1+B}{B}\right)\left[1 - \left(\dfrac{A_f}{A_0}\right)^B\right] + \sigma_b\left(\dfrac{A_f}{A_0}\right)^B$$

Where σ_b = Back pull stress

B = μcot α

μ = coefficient of friction

α = semi die angle

Drawing Force: F = σ_d × A_f

When σ_b = 0, Back pull stress

$$\sigma_d = \sigma_y\left(\dfrac{1+B}{B}\right)\left[1 - \left(\dfrac{A_f}{A_0}\right)^B\right]$$

For maximum reduction case: **$\sigma_d = \sigma_y$**

$$\Rightarrow \dfrac{\sigma_d}{\sigma_y} = 1$$

But, $\dfrac{\sigma_d}{\sigma_y} = \left(\dfrac{1+B}{B}\right)\left[1 - \left(\dfrac{A_f}{A_0}\right)^B\right]$

$\therefore \left(\dfrac{1+B}{B}\right)\left[1 - \left(\dfrac{A_1}{A_0}\right)^B\right] = 1$

Under ideal conditions of wire drawing operation, the coefficient of friction is assumed to be zero.

Therefore, μ = 0 ⇒ B = 0

$$\sigma_{d\,(ideal)} = \sigma_y \ln\dfrac{A_0}{A_f}$$

Sheet metal working operations: The basic cutting operations which come under Sheet Metal Operations are:

(a). Punching Operation

(b). Blanking Operation

(a). Punching Operation: When the force is applied by using the punch on to the sheet, the cutting or shearing action will be taking place in the sheet producing piece/blank leaving a hole in the sheet.

In punch and die working, if the sheet with the hole is useful, it is called Punching or Piercing operation.

$$\text{Punch Size} < \text{Die Size (Basic Requirement)}$$
$$\text{Punch Size} = \text{Hole Size (Needed)}$$

Clearance → Die.

Shear → Punch.

(b). Blanking Operation: In punch and die working, if the Piece/blank produced in the sheet is useful, it is called as Blanking operation.

In blanking Operation, the die size is made equal to blank size and clearance is provided only on the Punch.

$$\text{Blank Size} = \text{Die Size (Basic Requirement)}$$
$$\text{Punch Size} = \text{Die size} - 2C\,(\text{Needed})$$

- Clearance → Punch.
- Shear → Die.

Analysis of punching and blanking

- Optimum Radial clearance: $C = 0.0032t\sqrt{\tau_u}$

- $\boxed{F_{max} = A_s \times \tau_u}$

A_s = shearing area = $p \times t$

Where p is the perimeter of the hole.

τ_u = ultimate shear stress

- For rectangular blanks with length L and width b

$$\boxed{F_{max} = 2(L + b)\,t.\tau_u}$$

- Work done = Force × distance

$$\boxed{\text{Work Done} = F_{max} \times Kt}$$

K = % penetration required for completing the shearing action.

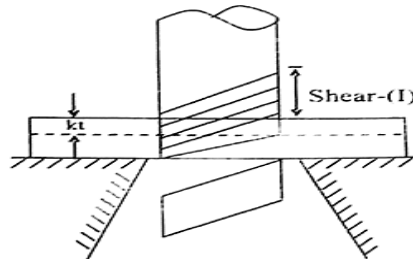$$\boxed{d_{min}(\text{Smallest Diameter}) = \frac{4t\tau_u}{\sigma_{c,allowable}}}$$

Methods of reducing punch force: It is done by providing shear on punch. Energy required if shear is provided

$$\boxed{\text{Energy required} = F \times (Kt + I)} \qquad (\text{always } F < F_{max}).$$

The energy required for punching or blanking is remains same with and without provided shear:

$$\boxed{\begin{aligned} F_{max}.\,K.t &= F\,(K.t + I) \\ F &= \left[\frac{F_{max}Kt}{(Kt + 1)}\right] \end{aligned}}$$

Slotting is the term sometimes used for a punching operation that cuts out an elongated or rectangular hole.

Perforating involves the simultaneous punching of a pattern of holes in sheet metal.

Notching involves cutting out a portion of metal from the side of the sheet or strip. Seminotching removes a portion of metal from the interior of the sheet.

Lancing: It is creating a tab on the edge without removal of material.

Nibbling: The process of creating a profile in the sheet is called nibbling.

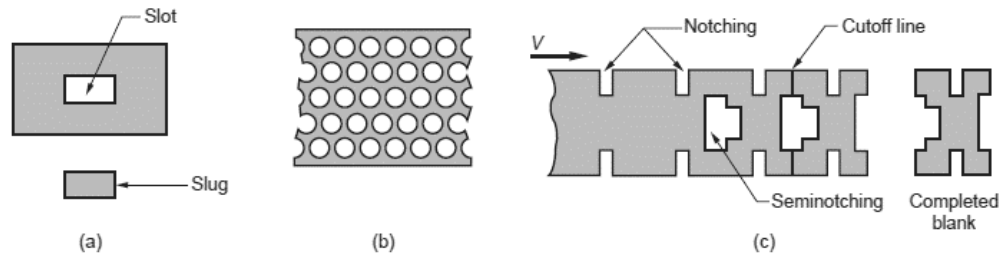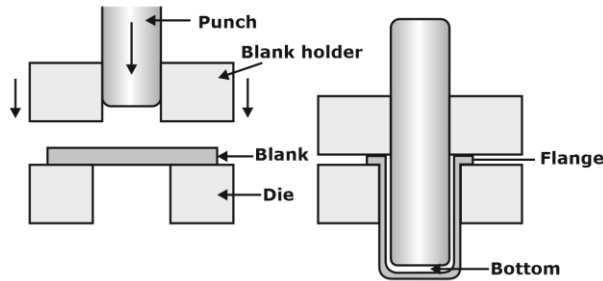**Parting:** Shearing the plates into two parts.

Figure: a) Slotting, (b) perforating, (c) notching and semi notching.

Deep drawing: It is a Sheet Metal forming process in which a sheet metal blank is radially drawn into a forming die by the mechanical action of a punch.

If $\dfrac{h}{d} \geq 0.5$ it is called deep drawing.

If $\dfrac{h}{d} < 0.5$ is called shallow drawing.

Blank diameter is given by:

$$\boxed{D = \sqrt{d^2 + 4dh}} \text{ if } \dfrac{d}{r} \geq 20$$

$$\boxed{D = \sqrt{d^2 + 4dh} - \dfrac{r}{2}} \quad \text{If } \dfrac{d}{r} = 15 \text{ to } 20$$

$$\boxed{D = \sqrt{d^2 + 4dh} - r} \quad \text{If } \dfrac{d}{r} = 10 \text{ to } 15$$

Draw ratio : $\boxed{DR = \dfrac{\text{Blank Diameter}}{\text{Punch Diameter}}}$

An approximate upper limit on the drawing ratio is a value of 2.0.

$$DR_1 = \dfrac{D}{d_1}$$

$$DR_2 = \dfrac{d_1}{d_2}$$

$$DR_3 = \dfrac{d_2}{d_3}$$

Deep Drawing force is given by: $\boxed{F = \pi D_p t (TS) \left(\dfrac{D_b}{D_p} - 0.7 \right)}$

Where t original blank thickness, mm; TS = tensile strength, MPa and D_b and D_p are the starting blank diameter and punch diameter, respectively.

Drawing Without a Blank holder: One of the primary functions of the blank holder is to prevent wrinkling of the flange while the cup is being drawn. The tendency for wrinkling is reduced as the thickness-to-diameter ratio of the blank increases. If the t/D_b ratio is large enough, drawing can be accomplished without a blank holder. The limiting condition for drawing without a blank holder can be estimated from the following:

$$\boxed{D_b - D_p < 5t}$$

Defects in Drawing:

(a). Wrinkling in the flange: Wrinkling in a drawn part consists of a series of ridges that form radially in the undrawn flange of the work part due to compressive buckling.

(b). Wrinkling in the wall: If and when the wrinkled flange is drawn into the cup, these ridges appear in the vertical wall.

(c). Tearing: Tearing is an open crack in the vertical wall, usually near the base of the drawn cup, due to high tensile stresses that cause thinning and failure of the metal at this location. This type of failure can also occur as the metal is pulled over a sharp die corner.

(d). Earing: This is the formation of irregularities (called ears) in the upper edge of a deep drawn cup, caused by anisotropy in the sheet metal. If the material is perfectly isotropic, ears do not form.

(e). Surface scratches: Surface scratches can occur on the drawn part if the punch and die are not smooth or if lubrication is insufficient.

Bending operation:

Bend allowance: $L_b = r_n \times \phi$

r = inside radius

Neutral plane radius: $r_n = r + Kt$

Where K = Stretch factor or bend factor

Thus: $\boxed{L_b = \phi[r + Kt]}$

Where ϕ is in radians.

$K = \dfrac{1}{3}$ if $r < 2t$ and $K = \dfrac{1}{2}$ if $r \geq 2t$

Spring back: It is the elastic recovery partially toward its original shape. In overbending, the punch angle and radius are fabricated slightly smaller than the specified angle on the final part so that the sheet metal springs back to the desired value.

Bending force: $F = \dfrac{K(TS)wt^2}{D}$

where F = bending force, N; TS = tensile strength of the sheet metal, MPa; w = width of part in the direction of the bend axis, mm; t = stock thickness, mm; and D = die opening dimension.

Spinning: In the spinning process, an object with surface of revolution is produced from a sheet metal.
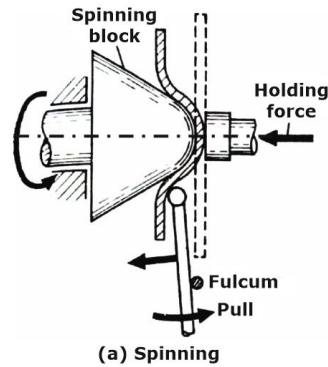
(a) Spinning

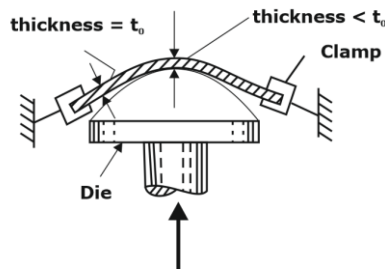The thickness of the sheet after the spinning operation is given by:

$t_c = t_b \sin\alpha$

where α is the semi die angle.

t_c=Thickness of sheet after spinning.

t_b=Thickness of sheet before spinning.

Stretch forming: a metal forming process in which a piece of sheet metal is stretched and bent simultaneously over a die in order to form large, contoured parts.

For biaxial stretching of sheets:

$$\varepsilon_1 = \ln\left(\dfrac{L_{i1}}{L_{o1}}\right), \varepsilon_2 = \ln\left(\dfrac{L_{i2}}{L_{o2}}\right)$$

Where ε_1 is the true strain for the one part of sheet and ε_2 is the true strain for the other part of the sheet.

$$\text{Final thickness}(t_f) = \dfrac{\text{Initial thickness}(t_i)}{e^{\varepsilon_1} \times e^{\varepsilon_2}}$$

Ironing Force: The objective is only to reduce the wall thickness of the cup and hence, no blank holding is required because the punch is fitted closely inside the cup.

Ironing force: $\boxed{P = \pi d_1 t_1 s_{av} \log_e \dfrac{t_0}{t_1}}$

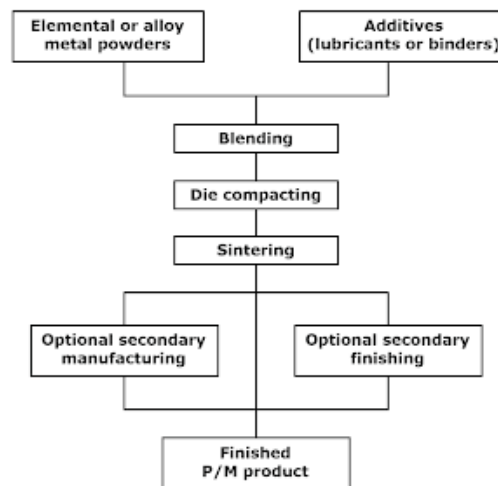Where

F = Ironing force, N

d_1 = Mean diameter of the shell after ironing,

t_1 = Thickness of shell after ironing,

t_0 = Thickness of the shell before ironing, and

S_{av} = Average of tensile strength before and after ironing.

Powder metallurgy:

Blending: Blending or mixing operations can be done either dry or wet. Lubricants such as graphite or stearic acid improve the flow characteristics and   compressibility at the expense of reduced strength.

Compacting: Powder is pressed into a "green compact" and still very porous, ~70% density.

Sintering: Controlled atmosphere: no oxygen, Heat to $0.75T_m$, Particles bind together and density increase, up to 95%.

Infiltration: The liquid would flow into the voids simply by capillary action, thereby decreasing the porosity and improving the strength of the component.

Impregnation: It is used for self-lubrication under action conditions. The liquid would flow into the voids simply by capillary action.

CHAPTER 4: JOINING

Introduction: Welding is the process of joining together two pieces of metal with the application of heat or pressure or both is applied and with or without added metal for formation of metallic bond.

Types of welding:
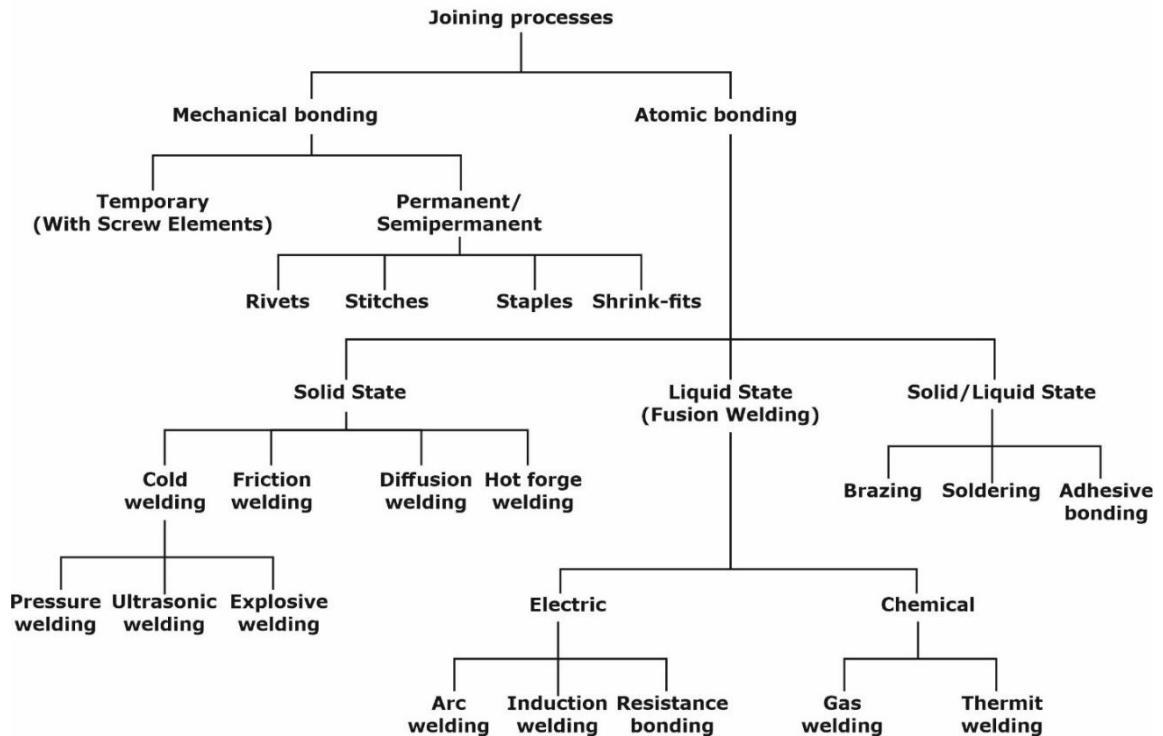
Arc Welding: Electric arc welding is one of the fusion welding processes in which coalescence of the metal is achieved by the heat from an electric arc between an electrode and workpiece.

- Electric arc is generated when electrode is brought into contact with the work and is then quickly separated by a short distance approximately 2 mm.

- In order to produce the arc, potential difference between the two electrodes should be sufficient to allow them to move across the air gap.
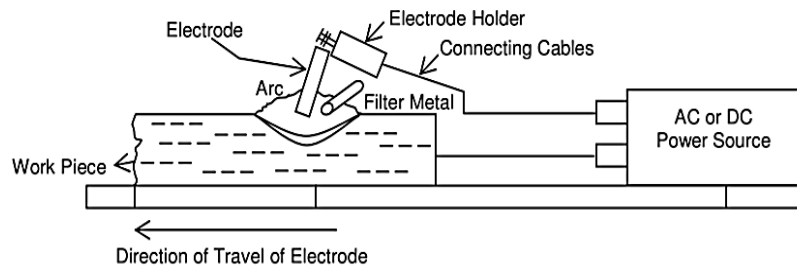

- The non-consumable electrodes made of tungsten or carbon. These do not melt in the process of welding and so called non-consumable electrodes.

- Consumable electrodes are consumed in the welding operation.

- In case of use of non-consumable electrode separate filler metal is used to improve properties of weldment. Selection of a filler metal depends on the metal to be welded.

- Flux gives alloys to bead materials, bead becomes stronger than parent metal. It protects the bead from atmospheric gases.

V-I characteristics of arc welding:

$V = V_0 - V_{Drop}$

$$\boxed{V = V_0 - \left(\frac{V_0}{I_S}\right)I}$$
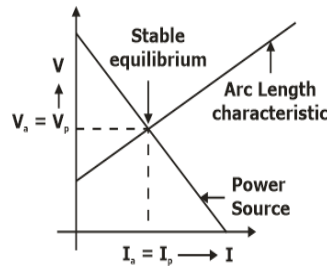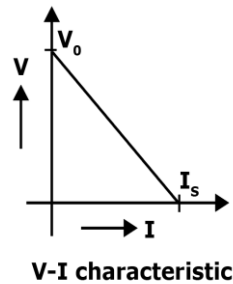
V-I characteristic

$\boxed{V = a + bL}$ → arc length characteristics

Where L = Arc Length

At stable equilibrium condition:

$V_a = V$

Power P = VI

$$P = (a + bL)I = \left(a + bL\right)\left[\frac{I_s}{V_o}(V_0 - a - bL)\right]$$

For maximum power output:

$$\frac{dP}{dL} = 0 \rightarrow L = ...\text{optimum arc length.}$$

$V_{opt} < V_o$

$I_{opt} < I_S$

Duty cycle: Duty cycle is the percentage of time that a machine will safely operate (or weld), within a certain time period, at a given amperage.

$$\boxed{\text{Duty Cycle} = \frac{\text{Arc on Time (AOT)}}{\text{Total welding Time}}}$$

Total welding time = Arc on time + Rest

Time For a welding transformer

$$\boxed{I_d^2 D_d = I_r^2 D_r}$$

Where

I_d = Desired output current in Amp

I_r = Rated output current in Amp

D_d = Desired duty Cycle %

D_r = Rated duty Cycle%

Hear flow characteristics in Arc Welding:

Heat input rate: $Q = KVI$

$$\text{No of electrodes required / pass} = \frac{\text{Length of weld head}}{x}$$

$$\text{Number of passes} = \frac{\text{Total number of electrodes required}}{\text{Number of electrodes/pass}}$$
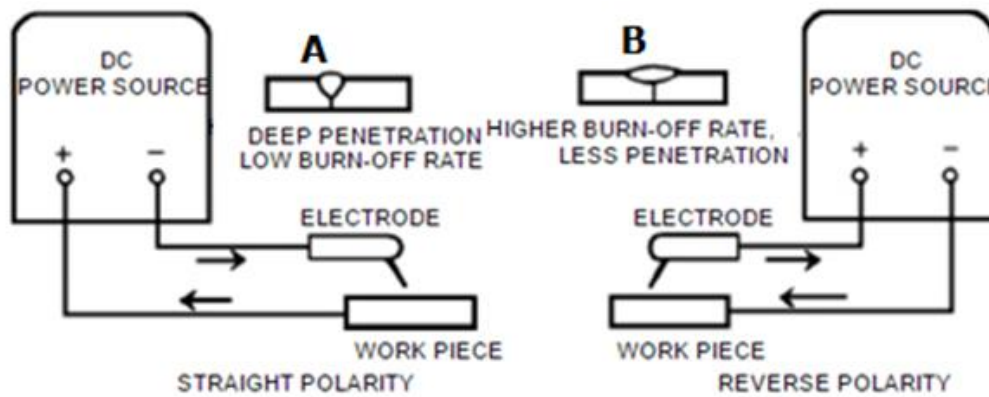
$$\text{Arc on time / pass} = \frac{\text{Length of weld head}}{\text{welding speed}}$$

Total Arc on Time = A.O.T/pass × No of passes

$$\text{Total welding time} = \frac{\text{Time Arc on Time}}{\text{Duty cycle}}$$

Comparison of Different Electrode polarities:

DCRP and DCSP:

	Direct Current Straight Polarity **(DCSP)**	Direct Current Reverse Polarity **(DCRP)**	**ACHF**
1. Penetration	shallow	Deep	Intermediate
2. Heat generation	$2/3^{rd}$ at electrode, $1/3^{rd}$ at workpiece	$1/3^{rd}$ at electrode, 2/3 at workpiece	50% on both
3. Metal deposition rate	High	Low	Intermediate
4. Thickness of work to be welded	Thin sheets	Thick sheets	Intermediate
5. Stable smaller arc	Easier	Easier	Difficult
6. Arc blow	Severe	Severe	Insignificant

(a). Gas tungsten arc welding (GTAW) or TIG Welding:

- In a TIG welding process, *a high intense arc is produced between non consumable tungsten electrode* and work piece. Typical shielding gases include *argon, helium, or a mixture of these gas elements*.

- A separate filler metal rod is used to deposit the material

- This was primarily invented to weld alloys of Aluminium and Magnesium. Aluminium is very difficult to weld because as soon as it is exposed to atmosphere it forms a layer over it. To weld these materials work piece should be given negative polarity and electrode positive polarity.

(b). Gas Metal arc welding (GMAW) or MIG Welding: The electrode is a *consumable bare metal wire*, and shielding is accomplished by flooding the arc with Gases such as *argon and helium, and active gases such as carbon dioxide*. The bare wire is fed continuously and automatically from a spool through the welding gun.

Inert gases are used for welding aluminium alloys and stainless steels, while CO_2 is commonly used for welding low and medium carbon steels. GMAW is widely used in fabrication operations in factories for welding a variety of ferrous and nonferrous metals

(c). Shielded Metal arc welding (SMAW) or stick welding: It uses a consumable electrode consisting of a filler metal rod coated with chemicals that provide flux and shielding. Currents typically used in SMAW range between 30 and 300 A at voltages from 15 to 45 V.

Shielded metal arc welding is usually performed manually. Common applications include construction, pipelines, machinery structures, shipbuilding, job shop fabrication, and repair work.

(d). Submerged Arc Welding (SAW): an arcwelding process that uses a continuous, consumable bare wire electrode, and arc shielding is provided by a cover of granular flux. The Submerged Arc Welding can be used to weld pressure vessels like boilers.
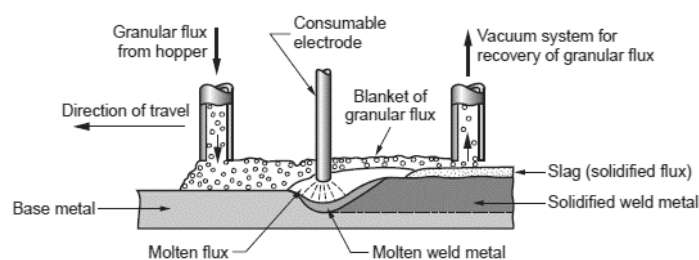
Submerged arc welding is widely used in steel fabrication for structural shapes (e.g., welded I-beams); longitudinal and circumferential seams for large diameter pipes, tanks, and pressure vessels; and welded components for heavy machinery.

Low-carbon, low-alloy, and stainless steels can be readily welded by SAW; but not high-carbon steels, tool steels, and most nonferrous metals. Because of the gravity feed of the granular flux, the *parts must always be in a horizontal orientation*.

(e). Electro gas welding: an AW process that uses a continuous consumable electrode (either flux-cored wire or bare wire with externally supplied shielding gases) and molding shoes to contain the molten metal. The process is primarily applied to vertical butt welding.

Principal applications of electro gas welding are steels (low-and medium-carbon, low-alloy, and certain stainless steels) in the construction of large storage tanks and in ship building.

(f). Plasma Arc Welding (PAW): A special form of gas tungsten arc welding in which a constricted plasma arc is directed at the weld area.

Argon, argon–hydrogen, and helium are also used as the arc-shielding gases. Plasma is the pool of ionized gas. Temperature is about $17000\,^{\circ}C$.

The process can be used to weld almost any metal, including tungsten. Difficult-to-weld metals with PAW include bronze, cast irons, lead, and magnesium.

(g). Carbon arc welding (CAW): is an arc-welding process in which a non-consumable carbon (graphite) electrode is used. The carbon arc process is used as a heat source for brazing and for repairing iron castings. It can also be used in some applications for depositing wear resistant materials on surfaces.

(h). Stud welding (SW) is a specialized AW process for joining studs or similar components to base parts. SW applications include threaded fasteners for attaching handles to cookware, heat radiation fi ns on machinery, and similar assembly situations.

Resistance Welding: The resistance welding is produced by means of electrical resistance across the two components to be joined.

Heat generated: $\boxed{H_g = I^2 R t}$

Where I = current passing through circuit
R = Electrical Resistance at the interface
t = time during which current is passing
Heat required for melting:

Heat Required: $\boxed{H_m = mL + mC_P\left(T_m - T_a\right)}$

Where: L = latent heat of fusion
T_m = Melting point of the material
T_a = ambient temperature
Melting efficiency (η_m) is given by:

$$\boxed{\eta_m = \frac{H_m}{H_g}}$$

For MIG welding:
Wire melting rate = filling rate of weld bead

$$\boxed{\frac{\pi}{4}d^2 \times f = A \times v}$$

Where f = feed rate of wire

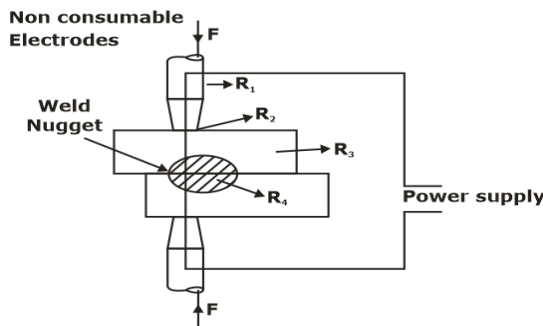d = diameter of the wire

A = area of the weld

v = welding speed

Heat transfer efficiency (η_t) is given by:

$$H_t = \frac{VI}{Av} \times \eta_t \; J/mm^3$$

(a). Resistance Spot Welding: An RW process in which fusion of the faying surfaces of a lap joint is achieved at one location by opposing electrodes. The process is used to join sheet-metal parts of thickness 3 mm or less, using a series of spot welds, in situations where an airtight assembly is not required.

Materials used for RSW electrodes consist of two main groups: (1) copper-based alloys and (2) refractory metal compositions.


Volume of nugget: $V = \frac{\pi}{4}D^2h$

Diameter of the nugget: $D = 6\sqrt{t}$ mm

Press-type spot welders are intended for larger work than rocker arm type guns.

It is widely used in mass production of automobiles, appliances, metal furniture, and other products made of sheet metal.

(b). Seam Welding: Electrodes are in the form of rotating wheels and the welding is the Continuous spot welding. The current is applied through the heavy copper electrodes in a series of pulses at proper intervals. The process is capable of producing air-tight joints, and its industrial applications include the production of gasoline tanks, automobile mufflers.
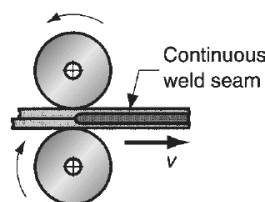
(c). Projection Welding: Resistance projection welding (RPW) is an RW process in which coalescence occurs at one or more relatively small contact points on the parts. The welding of two sheets without any indentation is done. One sheet has some projections.
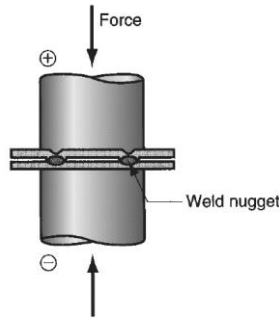
(d). Other Resistance-Welding Operations:

(i). Flash butt Welding: In flash welding (FW), normally used for butt joints, the two surfaces to be joined are brought into contact or near contact and electric current is applied to heat the surfaces to the melting point, after which the surfaces are forced together to form the weld.
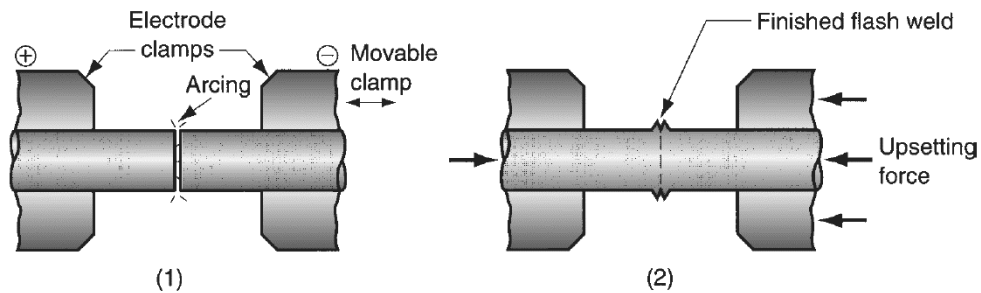

(ii). Upset welding (UW) is similar to flash welding except that in UW the faying surfaces are pressed together during heating and upsetting.

(iii). Percussion welding (PEW) is also similar to flash welding, except that the duration of the weld cycle is extremely short, typically lasting only 1 to 10 ms.

(iv). High-frequency resistance welding (HFRW) is a resistance-welding process in which a high-frequency (10 to 500 kHz) alternating current is used for heating, followed by the rapid application of an upsetting force to cause coalescence.

(v). Ultrasonic Welding (USW): It is a welding technique which uses ultrasonic vibration of high frequency to weld the two pieces together. It is most used to weld thermoplastic materials and dissimilar materials. Metal with thin section can also be welded with USW.
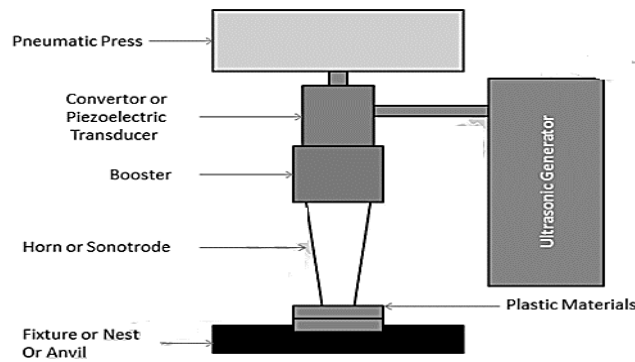
- The frequency applied is 20-60 KHz.
- It is a case of solid-state welding.

Electron beam welding (EBW): In this high velocity electron beam is made to strike at workpiece. ***High Vacuum is required*** and Entire set up is lead lines to avoid getting out of X rays. The electron beam gun operates at high voltage to accelerate the electrons (e.g., 10–150 kV typical), and beam currents are low (measured in milliamps). Welding speeds are high compared to other continuous welding operations. No filler metal is used, and no flux or shielding gases are needed. Minimum heat affected zone.

Any metals that can be arc welded can be welded by EBW, as well as certain refractory and difficult-to-weld metals that are not suited to AW. Work sizes range from thin foil to thick plate. EBW is applied mostly in the automotive, aerospace, and nuclear industries.

Laser beam welding (LBW): A fusion-welding process in which coalescence is achieved by the energy of a highly concentrated, coherent light beam focused on the joint to be welded. It is used to weld copper and aluminium alloys in electronics industry.

LBW produces welds of high quality, deep penetration, and narrow heat-affected zone. here are several advantages of LBW over EBW: no vacuum chamber is required, no X-rays are emitted, and laser beams can be focused and directed by optical lenses and mirrors.

Explosive welding: Explosion welding is solid state welding process where welding is accomplished by accelerating one of the components at extremely high velocity through the use of chemical explosives.

This process is most commonly utilized to clad carbon steel plate with a thin layer of corrosion resistant material.

Friction Welding: Friction welding (FRW) is a solid-state welding process in which coalescence is achieved by frictional heat combined with pressure. It is used for welding of dissimilar metals.

When properly carried out, no melting occurs at the faying surfaces. No filler metal, flux, or shielding gases are normally used.
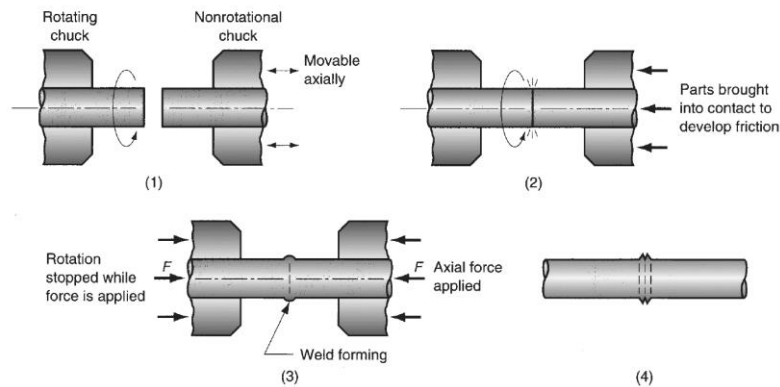
Friction stir welding (FSW): Friction stir welding (FSW) is a solid-state joining process that uses frictional heat generated by a rotating tool to join materials. The process is primarily used in industry to join aluminium alloys of all grades, whether cast, rolled or extruded. FSW has been used for a variety of applications across industries ranging from aerospace to shipbuilding and rail to electronics, including EV battery trays.
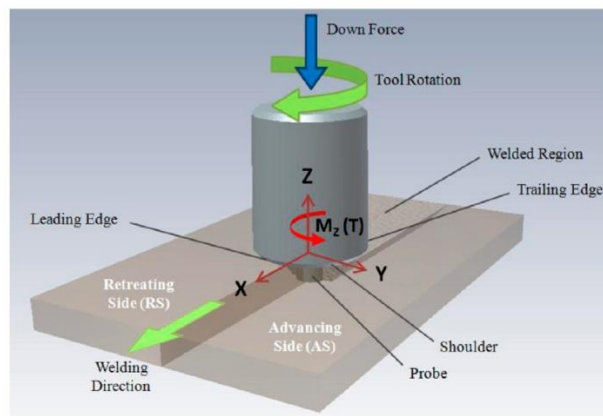
Thermit Welding: Mixture of aluminium powder and iron oxide that produces an exothermic reaction when ignited. It is used in incendiary bombs and for welding. Mixture is placed in a crucible and ignited by means of a firecracker. It is used for repair of railway track.

Finely mixed powders of aluminium and iron oxide (in a 1:3 mixture), when ignited at a temperature of around 1300°C, produce the following chemical reaction:

$8\ Al + 3\ Fe_3O_4 \longrightarrow 9\ Fe + 4\ Al_2O_3 + Heat$

The temperature from the reaction is around 2500°C (4500°F), resulting in superheated molten iron plus aluminium oxide that floats to the top as a slag and protects the iron from the atmosphere.

GAS WELDING: Oxyfuel gas welding (OFW) is the term used to describe the group of FW operations that burn various fuels mixed with oxygen to perform welding. Oxyfuel gas is also commonly used in cutting torches to cut and separate metal plates and other parts. The most important OFW process is oxyacetylene welding.

Carburizing Flame: It has silent flame. This flame is obtained when excess of acetylene is supplied than which is theoretically required. It is used for cast iron. The inner cone has a feathery edge extending beyond it. This white feather is called the acetylene feather. Maximum temperature obtained is 3040°C.
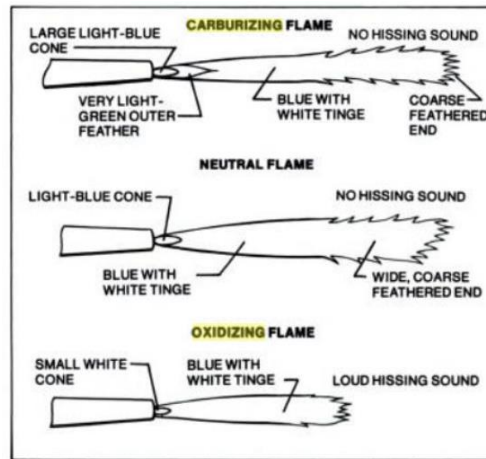
Neutral Flame: This consists of nearly one to one ratio of acetylene and oxygen by volume. It is used for mild steel. Maximum temperature obtained is 3200°C.

Oxidizing Flame: Because of an excess amount of oxygen is supplied from the oxygen cylinder, distance to be travelled by the flame for complete combustion will be reducing, therefore, the length of inner cone is reduced. It produces roaring sound. It is used for copper zinc-based alloys. Maximum temperature obtained is 3380°C.

Soldering, Brazing:

Soldering: A joining process in which a filler metal with melting point (liquidus) not exceeding 450°C is melted and distributed by capillary action between the faying surfaces of the metal parts being joined

The flux is zinc chloride.

The filler material is lead with tin.

Brazing: Brazing joints forms at temperatures higher than 450°C & the non-ferrous filler metal is drawn into and fills the closely fitted mating surface by capillary action. The joints obtained in Brazing are stronger than soldered joints.

Common ingredients for brazing fluxes include borax, borates, fluorides, and chlorides.
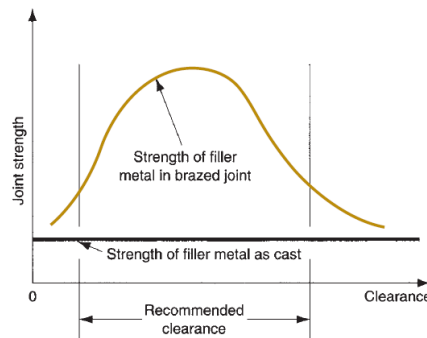
Braze welding is different from Brazing & here the filler metal is deposited in groves & capillary attraction is not the factor in distributing the filler metal. Braze welding is frequently used to repair cracked or broken cast Iron parts.

CHAPTER 5: MACHINING AND MACHINABILITY

MACHINING: Machining is the process of removing unwanted material from workpiece. The important elements are workpiece, cutting tool, chips. Culling tools are classified into two major groups:
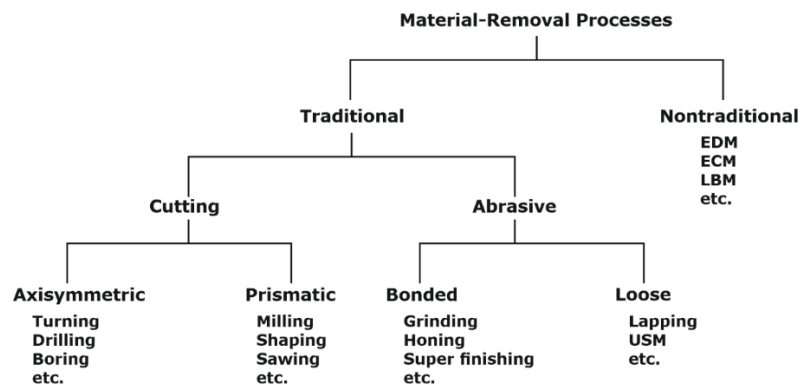

Single point cutting tools: In a single-point tool, there is one tool point from which the name of this cutting tool is derived. The point is usually rounded to a certain radius, called the nose radius.

Example: Turning tool, parting tool, Shaping tool etc.

Multipoint cutting tools: They have more than one cutting edge to remove excess material from the work piece.

Examples: Milling cutters, drills, reamers, broaches and grinding wheels are multi point cutting tools.

Single point cutting tool:
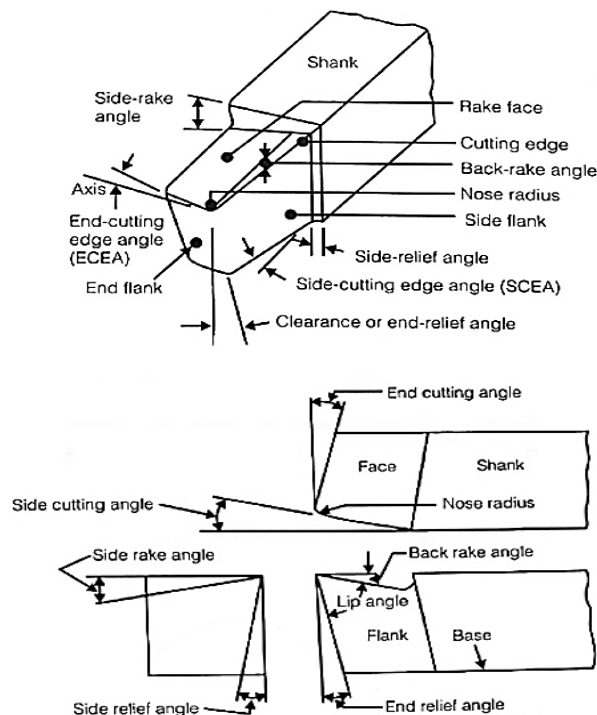
ASA Tool Signature:

Back rake angle - Side rake angle - End relief angle - Side relief angle - End cutting edge angle - Side cutting edge angle- Nose radius.

Orthogonal rake system (ORS):

Inclination angle-Normal Rake Angle - side relief angle- end relief angle - end cutting edge angle - approach angle λ – Nose Radius R.

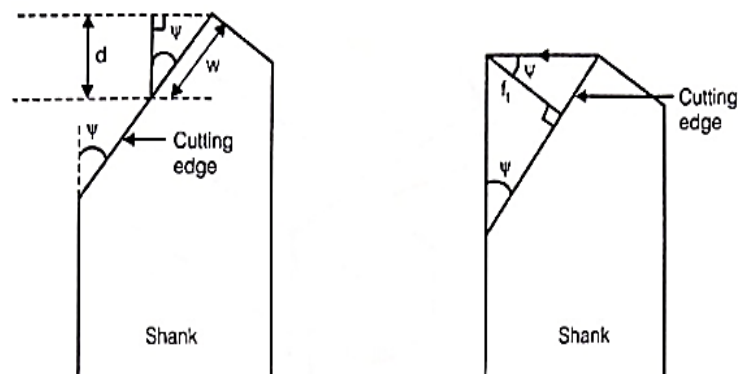**Conversion formulas from one system to other:**

$$\tan I = \cos \Psi \tan \alpha_{ab} - \sin \Psi \tan \alpha_{as}$$
$$\tan \alpha_n = \cos \Psi \tan \alpha_{as} + \sin \Psi \tan \alpha_{ab}$$

Where Ψ = side cutting edge angle

Back rake angle: For Machining brass, zero-degree rake angles are chosen.

Side cutting edge angle (Ψ):

$$w = \frac{d}{\cos \Psi}$$
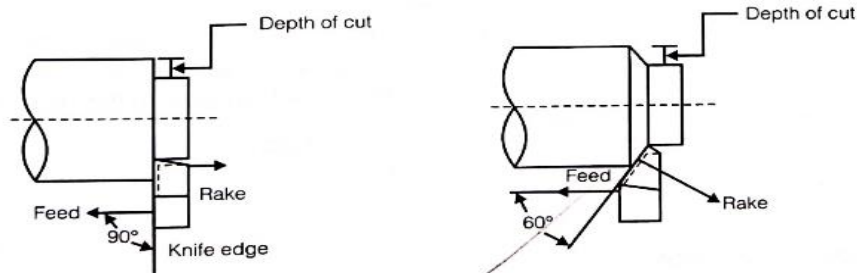
d = depth of cut

w = width of cut

$$\frac{f_t}{f} = \cos \Psi$$

True feed : $f_t = f \cos \Psi$

t_1 = uncut chip thickness.

TYPES OF METAL CUTTING PROCESS:

Orthogonal cutting (Two-dimensional cutting). Cutting edge is at right angle to tool feed.

Oblique cutting (Three-dimensional cutting): Cutting edge is at acute angle to tool feed.
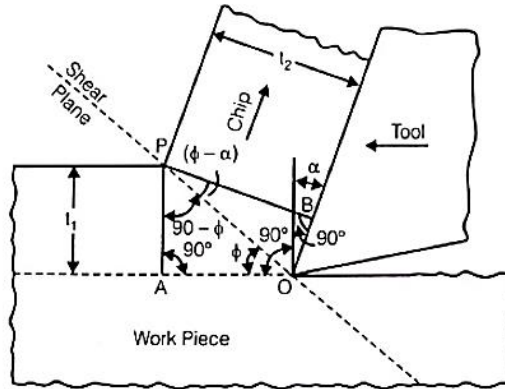
Types of chips:

Continuous Chips: Ductile materials, High speed, Low feed and depth of cut and High back rake angle.

Discontinuous Chips: Brittle materials, Low speed, High feed and depth of cut and Low back rake angle.

Chips with built-up edge: Ductile material, Low speed and High feed and depth of cut.
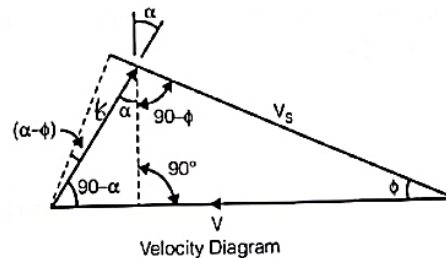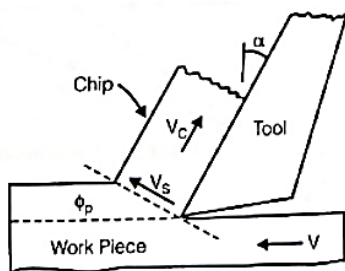
MERCHANT'S ANALYSIS:

$$\tan\phi = \frac{\cos\alpha}{\dfrac{t_2}{t_1} - \sin\alpha}$$

chip thickness ratio: $r = \dfrac{t_1}{t_2} < 1$

$$r = \frac{t}{t_c} = \frac{l_c}{l} = \frac{V_C}{V} = \frac{\sin\phi}{\cos(\phi - \alpha)}$$

Chip reduction coefficient: $\zeta = \dfrac{1}{r} = \dfrac{t_2}{t_1} > 1$
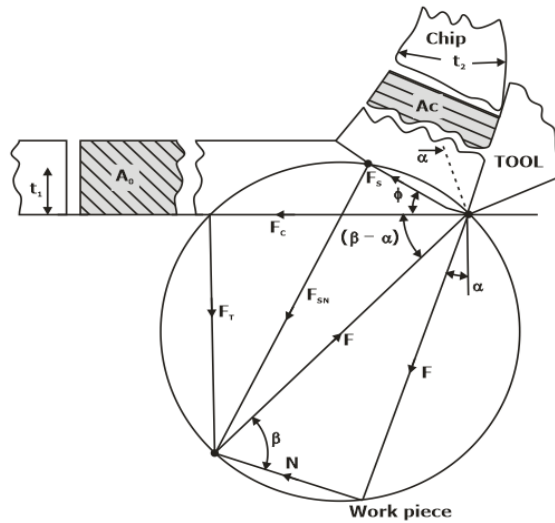
Shear strain: $\gamma = \tan(\phi - \alpha) + \cot\phi$

$$\frac{V}{\sin(90° - \phi + \alpha)} = \frac{V_C}{\sin\phi} = \frac{V_S}{\sin(90° - \alpha)}$$

$$\frac{V}{\cos(\phi - \alpha)} = \frac{V_C}{\sin\phi} = \frac{V_S}{\cos\alpha}$$

$$V_s = \frac{V\cos\alpha}{\cos(\phi - \alpha)} \text{ and } V_c = \frac{V\sin\phi}{\cos(\phi - \alpha)}$$

Resultant force: $R = \sqrt{F_C^2 + F_T^2}$ = Diameter of Merchant circle

$$R = \frac{F}{\sin\beta} = \frac{N}{\cos\beta} = \frac{F_S}{\cos(\phi + \beta - \alpha)}$$

Friction angle: $\beta = \alpha + \tan^{-1}\left(\dfrac{F_T}{F_C}\right)$

In general $F_C > F_T$, but in some cases $F_C < F_T$ like face turning operation, broaching, grinding etc

$$\frac{F_T}{F_C} = 2.5\text{(Grinding)}$$

In that case According to Classical frictional theorem:

$$\mu = \frac{\ln\left(\dfrac{1}{r}\right)}{\dfrac{\pi}{2} - \alpha}$$

Shear plane area: $A_s = AB \times b$

Where AB is the shear plane length.

$$A_s = \frac{t_1}{\sin\phi} \times b$$

$$W.D = \text{Energy required} = F_C \times V_C$$

Merchant's 1st angle relation (Minimum Power Requirement):

$$2\phi + \beta - \alpha = 90°$$

Lee and Shaffer relation:

$$\theta + \beta - \alpha = 45°$$

Machining constant or Merchants constant (C_m):

$$C_m = 2\phi + \beta - \alpha$$

SPECIFIC CUTTING ENERGY: $SCE = \dfrac{\text{Work done}}{\text{material removal rate}} \dfrac{J}{mm^3}$

Specific cutting pressure or specific cutting energy:

$$SCE = \frac{F_C \times V_C}{t_1 \times b \times V_C} = \frac{F_C}{A_0} \quad N/mm^2$$

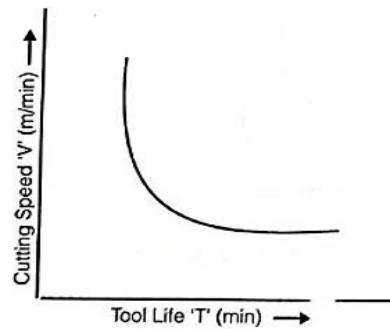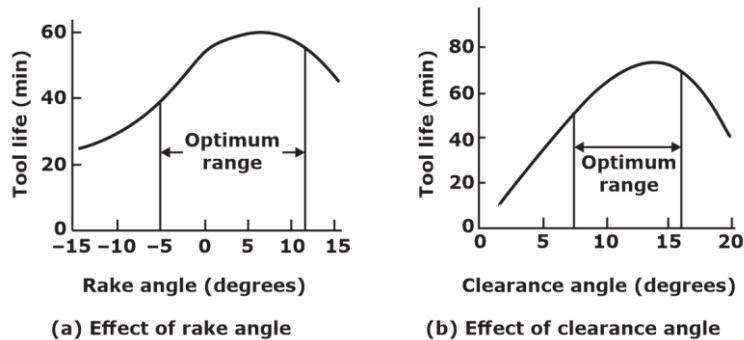**TAYLOR'S TOOL LIFE EQUATION:**

$$VT^n = C$$

V = cutting speed

T = tool life.

C = machining constant.

n = Tool life exponent (depends only on tool material)

Effect of Back Rake angle on tool life:

(a) Effect of rake angle   (b) Effect of clearance angle

When the rake angle increases, Life starts improving because the cutting force reduces. A further increase in the rake angle results in a larger temperature since the tool becomes thinner and the area available for heat conduction reduces.

Similarly, when the clearance angle increases, the tool life increases at first. This is due to for the same volume of flank wear, h_f reduces. However, with a further increase in the clearance angle, the tool becomes thinner and the tool life decreases due to the higher temperature.

Effect of parameters:

Higher the cutting speed tool life will decrease. By increasing the feed and depth of cut, tool life will decrease.

$$\text{Machinability index} = \frac{V_t}{V_s} \times 100$$

V_S = Cutting speed of standard free-cutting steel for 1 min tool life.

V_t = Cutting speed of metal for 1 min tool life.

Economics of machining:

Minimum Cost Criteria:

$$T_{opt} = \left[\left(\frac{1-n}{n} \right) \frac{C_t}{C_m} \right]$$

Maximum production rate criteria:

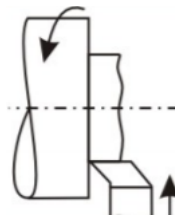$$T_{opt} = \left[\left(\frac{1-n}{n} \right) T_c \right]$$

Velocity and Tool life order:

$$\left(V_{opt} \right)_{min\ cost} < \left(V_{opt} \right)_{max\ profit} < \left(V_{opt} \right)_{max.\ prod.\ rate}$$

$$\left(T_{opt} \right)_{max\ prod.\ rate} < \left(T_{opt} \right)_{max\ profit} < \left(T_{opt} \right)_{min\ cost}$$

Various Operation of Metal cutting:

Facing Operation:

$$\text{Time per cut} = \frac{\text{length of tool travel}}{\text{feed velocity}} = \frac{L}{fN}$$

$$L = \frac{D}{2} + AP + OR$$

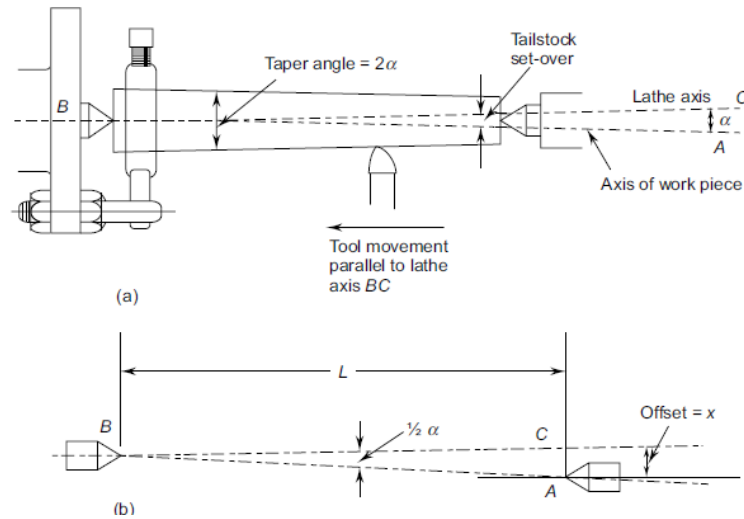**Turning operation:**

$$\text{Time per cut} = \frac{\text{length of tool travel}}{\text{feed velocity}} = \frac{L}{fN}$$

The empirical formula used for calculating the taper is:

$$\tan\theta = \frac{D_1 - D_2}{2L}$$

Where θ is half of the included angle, D_1 and D_2 are the major and minor diameters of the workpiece and L is the length of the tapered portion.

Offsetting the tailstock:

(a)

(b)

If α is very small, then we can approximate

$$\sin\alpha = \tan\alpha = \frac{D-d}{2L}$$

$\therefore$ Offset: $\quad S = L\dfrac{(D-d)}{2l} = \dfrac{(D-d)}{2} \times \dfrac{\text{Total length of workpiece}}{\text{Taper length}}$

This is the most general situation where the taper is to be obtained over a small portion of the length (l) of the job while the actual length of the work piece, L could be long. However, when they are equal i.e. L = l, then:

Offset: $\quad S = \dfrac{(D-d)}{2}$

In turning:

As depth of cut: $d = \dfrac{D_i - D_f}{2}$ and $V = \pi D_{avg} N \text{ mm / min}$

$D_{avg} = \dfrac{D_i + D_f}{2} \Rightarrow V = \pi\dfrac{D_i + D_f}{2}N$

Thus, $\quad MRR = \pi\left(\dfrac{D_i^2 - D_f^2}{4}\right)fN \quad$ mm³/min

Thread cutting operation

Time/cut : $\quad t_m = \dfrac{L}{fN}$

L = Length of the component + AP + OR

f = pitch → single start = lead → multi-starts

Lead = pitch × number of starts

Gear ratio = Train value = speed of follower/speed of driver

$$\boxed{\text{Gear ratio} = \frac{\text{number of teeth on driver gear}}{\text{number of teeth on driven gear}} = \frac{\text{pitch to be cut on job}}{\text{pitch on lead screw}}}$$

$$\boxed{\text{Gear ratio} = \frac{\text{Lead of job threads}}{\text{lead of lead screw threads}}}$$

Drilling:

Time/hole : $\boxed{t_m = \dfrac{L}{fN}}$

L = Length of tool travel

L = t + (AP$_1$) + AP + OR

Break through distance: $\boxed{A = \dfrac{D}{2\tan\alpha}}$

MRR in drilling: $\boxed{MRR = \dfrac{\pi}{4}D^2fN}$

Broaching: Broaching is a method of removing metal by a tool that has successively higher cutting edges in a fixed path.

Knurling Process: Knurling is a manufacturing process, typically conducted on a lathe, whereby a pattern of straight, angled or crossed lines is rolled into the material to make a grip on the surface.

Thread Rolling: A work blank is pressed between either two flat dies or three circular die process and threads are produced plastic deformation process.

Boring: It is the process of enlarging already existing hole to bring it to the required size.

Reaming: It is the process of finishing the hole.

Counter Boring: Counterboring provides a stepped hole, in which a larger diameter follows a smaller diameter partially into the hole.

Counter sinking: This is similar to counterboring, except that the step in the hole is cone-shaped for flat head screws and bolts.

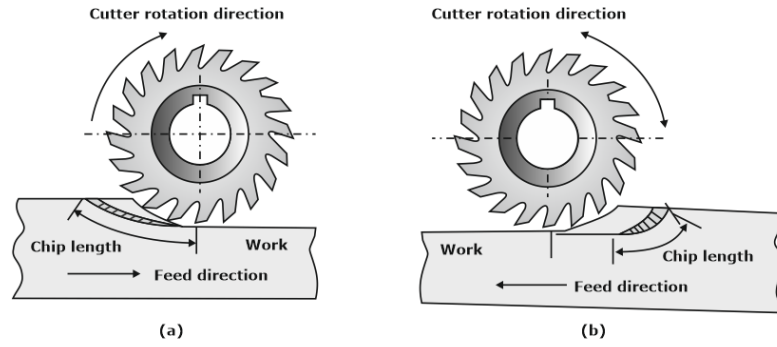**MILLING:**

The material removal rate is:

$$\boxed{MRR = w \times d \times f_m}$$

Peripheral or slab milling operation: In peripheral milling, also called plain milling, the axis of the tool is parallel to the surface being machined.

Two forms of peripheral milling: In peripheral milling, the direction of cutter rotation distinguishes two forms of milling: up milling and down milling.

(a)   (b)

Two forms of peripheral milling operation Up milling or conventional milling

Up milling (conventional milling): the direction of motion of the cutter teeth is opposite the feed direction when the teeth cut into the work. the chip formed by each cutter tooth starts out very thin and increases in thickness during the sweep of the cutter.

Down Milling (Climb Milling): The direction of cutter motion is the same as the feed direction when the teeth cut the work. Each chip starts out thick and reduces in thickness throughout the cut.

$AP = O_1O_2$

$$AP = \sqrt{Dd - d^2} = \sqrt{d(D - d)}$$

Maximum chip thickness: $t_{1max} = \dfrac{2f_m}{NZ}\sqrt{\dfrac{d}{D}}$

Average chip thickness: $t_{1avg} = \dfrac{f_m}{NZ}\sqrt{\dfrac{d}{D}}$

f_m = Table speed or feed in mm/min

$f \times N = f_t \times Z \times N$

Feed per tooth: $f_t = \dfrac{f_m}{NZ}$

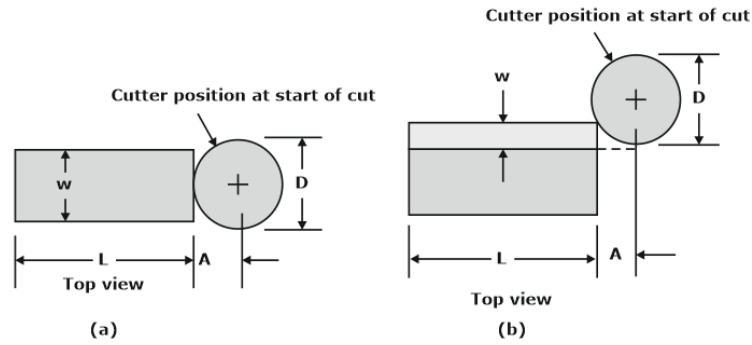f = table feed in mm/rev

f_t = table feed in mm/tooth

N = rpm of cutter

Z = Number of teeth

d = depth of cut

D = diameter of milling cutter

Face Milling: In face milling, the axis of the cutter is perpendicular to the surface being milled.

(a). When cutter is centered over the work piece and (b). When cutter is offset to one side over the work.

Symmetric milling: $A = 0.5(D - \sqrt{D^2 - w^2})$

If D = w, $A = 0.5D$

If D < w, then a slot is cut into the work and it = 0.5D.

When Cutter is offset: $A = \sqrt{w(D - w)}$

Grinding: Grinding is a chip-removal process that uses an individual abrasive grain as the cutting tool, and it is accomplished by abrasive particles that are contained in a bonded grinding wheel rotating at very high surface speeds.

Grinding Wheel Specification:

The preceding parameters can be concisely designated in a standard grinding wheel marking system defined by the American National Standards Institute (ANSI). This marking system uses numbers and letters to specify abrasive type, grit size, grade, structure, and bond material.

The undeformed chip length (l) in surface grinding is approximated by the equation

$l = \sqrt{Dd}$

The undeformed chip thickness, t, by the equation:

$$t = \sqrt{\left(\frac{4V}{VCr}\right)}\sqrt{\left(\frac{d}{D}\right)}$$

Grinding Ratio: Grinding-wheel wear is generally correlated with the amount of workpiece material ground by a parameter called the grinding ratio, G, defined as:

$$G = \frac{\text{volume of material removed}}{\text{Volume of wheel wear}}$$

External Centreless grinding: The following equation can be used to predict through feed rate, based on inclination angle and other parameters of the process:

$$f_r = \pi D_r N_r \sin I$$

Where:

f_r: through feed rate, mm/min

D_r: diameter of the regulating wheel, mm

N_r: rotational speed of the regulating wheel, rev/min

I: inclination angle of the regulating wheel

Honing: Honing is an abrasive process performed by a set of bonded abrasive sticks. A common application is to finish the bores of internal combustion engines. The motion of the honing tool is a combination of rotation and linear reciprocation, regulated in such a way that a given point on the abrasive stick does not trace the same path repeatedly. *Honing speeds are 15 to 150 m/min. Hone pressures of 1 to 3 MPa are typical.*

Lapping: Lapping is an abrasive process used to produce surface finishes of extreme accuracy and smoothness. Common abrasives are aluminium oxide and silicon carbide with typical grit sizes between 300 and 600.

NON -TRADITIONAL MACHINING:

Requirements: When material is very hard and strong which is difficult to machine by traditional process. When job is very complex.

(a). Electric Discharge Machining: The shape of the finished work surface is produced by a formed electrode tool.

(i). High voltage, low current process.

(ii). Mechanism of metal removal: Erosion, melting, vaporisation.

(iii). Dielectric is kerosene.

(iv). Energy released/spark: $E = \frac{1}{2}CV_d^2$ J

Cycle time : $t_c = RC \ln\left[\frac{V_0}{V_0 - V_d}\right]$ sec

Avg. power input: $P_{avg} = \frac{E}{t_c}$

The discharge voltage (V_d) and dc source voltage (V_o) relation is given by:

$$V_d = V_o \left(1 - e^{-t_c/RC}\right)$$

For maximum power delivery:

$$\left(\frac{V_d}{V_o}\right)_{opt} = 0.72$$

For a purely inductive discharging circuit, the critical value of resistance is:

$$R_{min} = \sqrt{\frac{L}{C}}$$

Where,

L = inductance of discharge circuit,

If R falls below this critical value, arcing, instead of sparking, will take place.
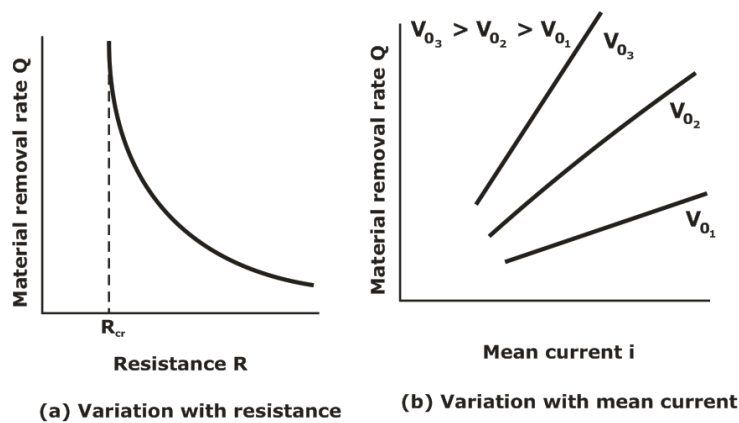
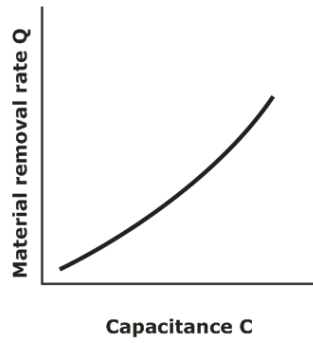As in EDM, an overcut exists in wire EDM that makes the kerf larger than the wire diameter.

$$\boxed{MRR = \text{Cross section area of cut} \times \text{wire feed}} \left(mm^3 / sec\right)$$
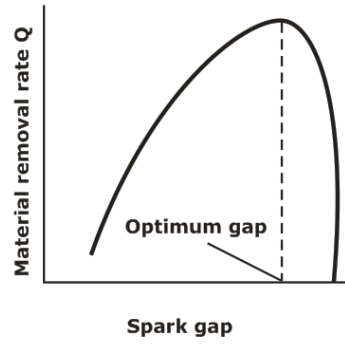
CSA of cut = width of cut × thickness of W/P

Width of cut = Wire diameter + Spark gap around the wire

MRR in the RC circuit:

(a) Variation with resistance   (b) Variation with mean current

MRR characteristics in EDM using RC relaxation circuit

(b). Electro Chemical Machining (ECM):

- High current low voltage.

- Mechanics of MRR: Ion displacement

- Medium: Conducting electrolyte

- Tool Materials: Cu, Brass, Steel

$$MRR = \left(\frac{AI}{ZF}\right)\frac{g}{sec} = \left(\frac{AI}{\rho ZF}\right)\frac{cm^3}{sec}$$

F = Faraday's constant = 96,500 coleuses

I = current flowing in amperes

Z = Valences of metal dissolved

A = atomic wt of material in gms.

ρ = density of work piece, gm/cm^3

MRR for an alloy:

$$Q = \frac{0.1035 \times 10^{-2}}{\rho}\left(\frac{1}{\sum (x_i Z_i / A_i)}\right) cm^3 / amp - sec$$

If the total overvoltage at the anode and the cathode be ΔV and the applied Voltage is V, the current I is given by:

$$I = \frac{V - \Delta V}{R}$$

Where R is ohmic resistance of the electrolyte.

Kinematics of ECM:

Current density = VK /y= ρ.f/Z

Where,

y = gap between tool and work,

V = applied voltage,

K = conductivity of electrolyte (mho/mm)

ρ = density of work material kg/mm 9

f = tool feed rate (mm/sec)

(c). Ultrasonic Machining (USM):

- **Mechanics of MRR:** Brittle fracture caused by impact of abrasive grains due to tool vibrating at high frequency.

- **Medium:** Slurry (abrasives mixed with water, paraffin etc.)

- **Abrasives:** Al_2O_3, B_4C (Boron Carbide), SiC, diamond (Usually B_4C with water as slurry and SiC with paraffin as slurry) and 100 — 800 grit size.

- Vibration frequency: 15 to 30 KHz

- Vibration Amplitude: 25 to 100 gm

- **Tool:** Material soft steel, cu or brass.

- For a given work material, the removal rate in USM:

$$\boxed{Q \propto vZv}$$

Where:

Q = Volume of work material removal rate

v = frequency

Z = Number of particles making impact/cycle

v = volume of work material dislodged/impact

Shape Application: Round and irregular holes, impressions.

Limitations: Very low MRR, tool wear, depth of holes and small cavities

(d). Abrasive jet machining:

• Mechanism of material removal is due to brittle fracture by impinging abrasive grains at high speed.

• This process is more suitable when the work material is brittle and fragile.

• Media for flow of abrasives is air or CO_2, abrasive material is Al_2O_3 or SiC.

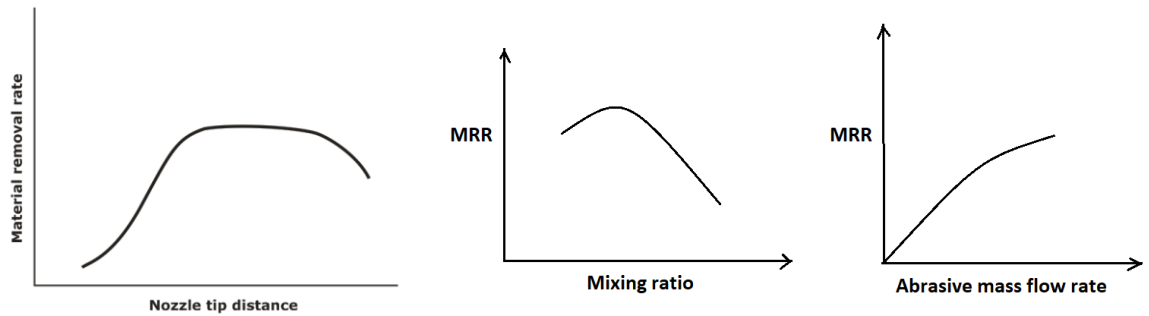**MRR in AJM:**

Metal removal Rate in AJM is given by:

$$\boxed{MRR = \chi Z d^3 V^{1.5} \left(\frac{\rho}{12 H_w} \right)^{3/4}}$$ Where,

χ = constant,

Z = no. of abrasive particles impinging per unit time

d = mean diameter of abrasive grains

V = velocity of abrasive grains

ρ = density of abrasive materials

H_W = hardness of work material

MRR Variation in AJM with different parameters

Shape Applications: Deburring, trimming and deflashing, cleaning, and polishing.

Materials Application: Cutting is accomplished successfully on hard, brittle materials (e.g., glass, silicon, mica, and ceramics) that are in the form of thin flat stock.

(e). Electron beam machining (EBM): It uses a high velocity stream of electrons focused on the workpiece surface to remove material by melting and vaporization. EBM must be carried out in a vacuum chamber to eliminate collision of the electrons with gas molecules.

The Total range to which electron can penetrate (δ) depends on the kinetic energy i.e. on the accelerating voltage V. It is given by:

$$\delta = 2.6 \times 10^{-17} \frac{V^2}{\rho}$$

Where:

δ = range in mm

V = the accelerating voltage in volts

ρ = density of the material in kg/mm^3

(f). Laser Beam Machining (LBM): It uses the light energy from a laser to remove **material by vaporization and ablation**. Ideal properties of a material for LBM include high light energy absorption, poor reflectivity, good thermal conductivity, low specific heat, low heat of fusion, and low heat of vaporization.

The time required to rise the surface to melting temperature is

$$t_m = \frac{\pi}{\alpha} \left[\frac{\theta_m K}{2H} \right]^2$$

α = thermal diffusivity = $\dfrac{K}{\rho C}$

K = thermal conductivity, J/m°C

H = heat flux = heat absorbed

θ_m = melting point temperature of work.

The critical value of 'H' is given by: $\boxed{H_{cr}\ \dfrac{2K\theta_m}{d}}$

Where:

d = focused diameter of incident beam.

If H = H_{cr}

Power intensity is the minimum value.

(g). Plasma Arc cutting (PAC): A plasma is a superheated, electrically ionized gas. Plasma arc cutting (PAC) uses a plasma stream operating at temperatures in the range 10,000°C to 14,000°C to cut metal by melting.

CHAPTER 6: METROLOGY AND INSPECTION

Introduction:

- **Fits:** Assembly condition between "Hole" & "Shaft".
- **Hole** – A feature engulfing a component
- **Shaft** – A feature being engulfed by a component.
- **Tolerance** is the difference between the upper limit (UL) and lower limit (LL).
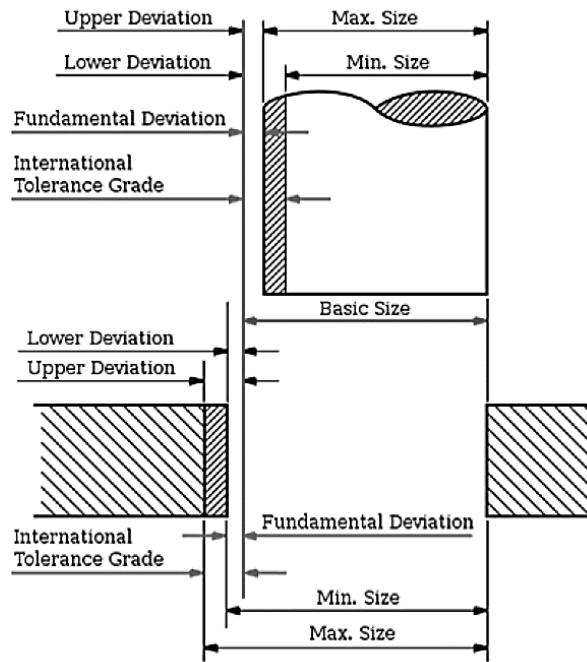
  $\boxed{\text{Tolerance} = UL - LL}$

- Limits of sizes are two extreme permissible sizes for a dimension of the part.

  Unilateral Limits i.e. only on one side of basic limit e.g. $\varnothing 25^{+0.18}_{+0.10}$

  Bilateral Limits i.e. on both sides of the basic size e.g. $\varnothing 25^{\pm 0.04}$

- **Upper deviation** is the algebraic difference between the maximum size and the basic size. The upper deviation of a hole is represented by a symbol ES (Ecart Superior) and of a shaft, it is represented by es.
- **Lower deviation** is the algebraic difference between the minimum size and the basic size. The lower deviation of a hole is represented by a symbol EI (Ecart Inferior) and of a shaft, it is represented by ei.
- Mean deviation is the arithmetical mean of upper and lower deviations.
- **Fundamental deviation** is the deviation, either the upper or the lower deviation, which is nearest one to zero line for a hole.

BASIS OF FITS:

Hole Basis: If the system of assembly of shaft and hole is consisting of basic hole, then that type of system is known as Hole Basis System.

Hole Basis Fits
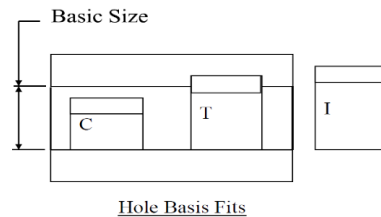
Legends:
Hole	C - Clearance
Shaft	T - Transition
Tolerance	I - Interference

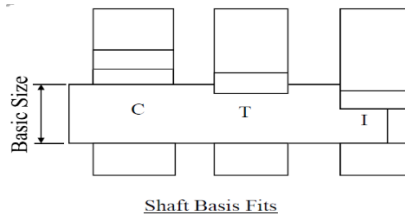**Shaft Basis:** If the system of assembly of shaft and hole consisting of basic shaft, then that type of system is known as Shaft Basis System.
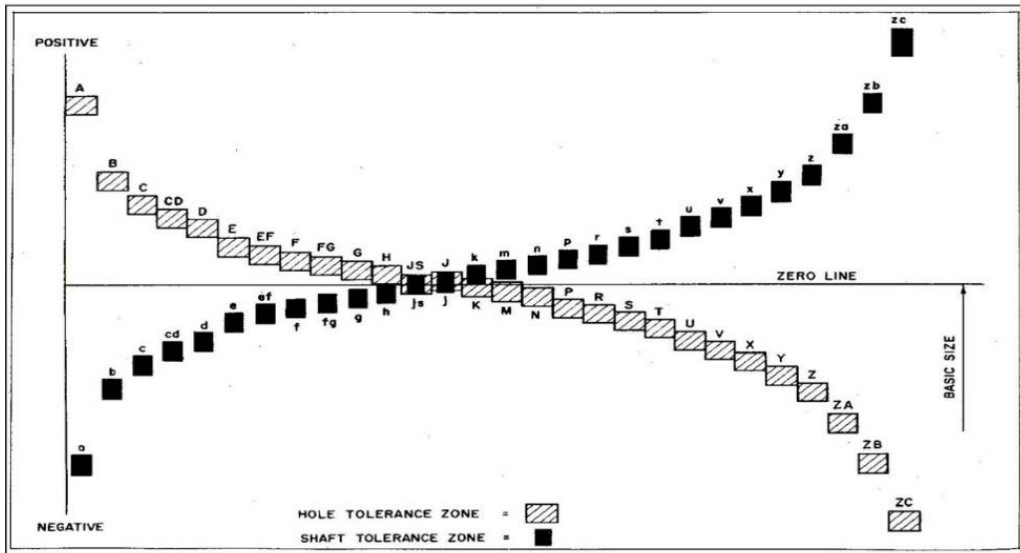

Shaft Basis Fits

Legends:
Hole	C - Clearance
Shaft	T - Transition
Tolerance	I - Interference

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IS: LIMITS AND FITS

• Limits and fits comprise 18 grades of fundamental tolerances

• There are 25 types of fundamental deviations:

$$i(\text{microns}) = 0.45\sqrt[3]{D} + 0.001\,D$$

D is the size or geometric mean diameter in mm.

Tolerance grade	IT5	IT6	IT7	IT8	IT9	IT10	IT11	IT12	IT13	IT14	IT15	IT16
Magnitude	7 i	10 i	16 i	25 i	40 i	64 i	100 i	160 I	250 i	400 i	640 i	1000 i

Fits: The condition which denotes the relationship between two mating parts with respect to the degree of clearance or interference appearing on the assembly is known as fit.
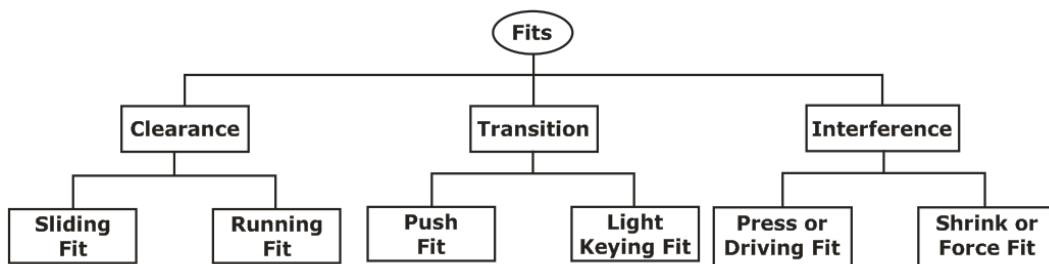
HOLE BASE & SHAFT BASE SYSTEM

(a). Clearance Fit: When lower limit of hole is greater than upper limit of shaft. Such fits give loose joint.
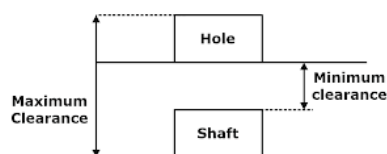
(i). Loose Fit: It is used between those mating parts where no precision is required. It provides minimum allowance and is used on loose pulleys, agricultural machineries etc.

(ii). Running Fit: For a running fit, the dimension of shaft should be smaller. For a running fit, the dimension of shaft should be smaller enough to maintain a film of oil for lubrication. It is used in bearing pair etc.

(iii). Slide Fit or Medium Fit: It is used on those mating parts where great precision is required. It provides medium allowance and is used in tool slides, slide valve, automobile parts, etc

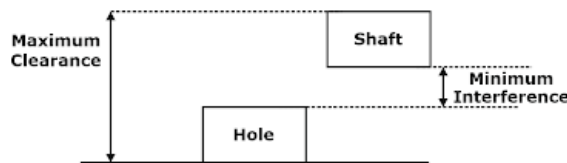**(b). Interference fits:** When lower limit of shaft is greater than upper limit of hole.

There are three types of interference fits namely: Shrink Fit or Heavy Force Fit, Medium Force Fit and Tight Fit or Force Fit.

(c). Transition fit: When a part is selected randomly from whole lot and randomly from shaft lot, some of the assembly have clearance fit, some are having interference fit. This is called as transition fit.

There are three types of transition fits namely: Push Fit or Snug Fit, Force Fit or Shrink Fit and Wringing Fit.

Allowance: It is the difference between the basic dimensions of the mating parts. When the shaft size is less than the hole size, then the allowance is positive and when the shaft size is greater than the hole size, then the allowance is negative.

Unilateral system: In this system, the dimension of a part is allowed to vary only on one side of the basic size, i.e. tolerance lies wholly on one side of the basic size either above or below it.

Bilateral system: In this system, the dimension of the part is allowed to vary on both the sides of the basic size, i.e. the limits of tolerance lie on either side of the basic size.
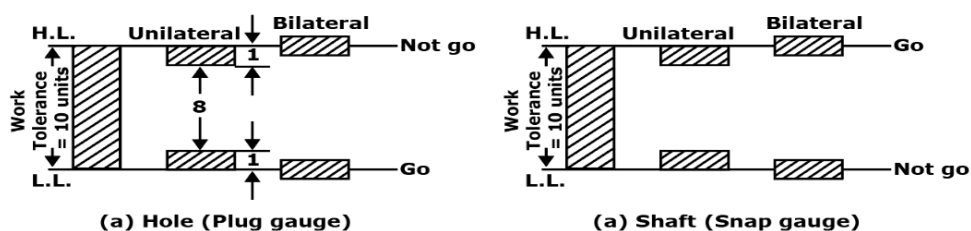
Allocation of Manufacturing Tolerance

LIMIT GAUGES:

- **Plug gauge:** used to check the holes.

- **Snap, Gap or Ring gauge**: used for gauging the shaft and male components.

Wear allowance:

- GO gauges which constantly rub against the surface of the parts in the inspection are subjected to wear and loose their initial size.

- The size of go plug gauge is reduced while that of go snap gauge increases.

Gauge tolerance (GT) = $\frac{1}{10}$ (work tolerance)

Wear allowance = $\frac{1}{10}$ (gauge tolerance)= $\frac{1}{100}$ (work tolerance)

Wear tolerance is only provided where W.T ≥ 0.1 mm

Slip gauges:

They are in the form of rectangular prisms, very accurately made in varying lengths. They are made of hardened steel having flat parallel surfaces. They are also called Gauge blocks.

Slip-gauge size of range, mm	Increment, mm	Number of pieces
1.005	-	1
1.001 to 1.009	0.001	9
1.010 to 1.490	0.010	49
0.500 to 9.500	0.500	19
10 to 100	10.000	10

One of the principles to be remembered is that the number of blocks used should always be the smallest.

Vernier Scale: A caliper is a device used to measure the distance between two opposing sides of an object. It can be as simple as a compass with inward or outward-facing points.
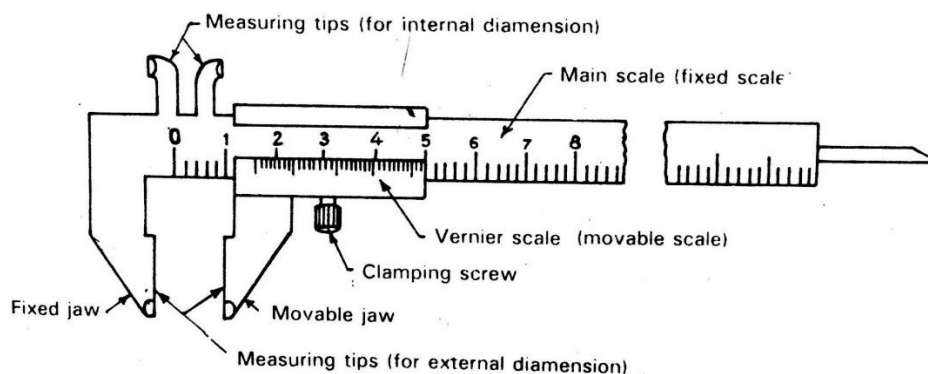
Least Count: The least count or the smallest reading which you can get with the instrument can be calculated as:

LC = 1 MSD − 1 VSD

$$LC = \frac{1\ MSD}{Number\ of\ divisions\ on\ vernier\ scale}$$

If the zero of the vernier scale lies ahead of the Nth division of the main scale, then the main scale reading (MSR) is:

MSR = N

If nth division of Vernier scale coincides with any division of the main scale, then the Vernier scale reading (VSR) is:

VSR = n × LC

Total reading: $TR = MSR + VSR = N + n \times LC$

Micrometre: A micrometre, sometimes known as a micrometre screw gauge, is a device incorporating a calibrated screw widely used for accurate measurement of components. Micrometre screw gauge is an instrument used to measure the diameter of thin wires, the thickness of small sheets of glass, plastic, etc.
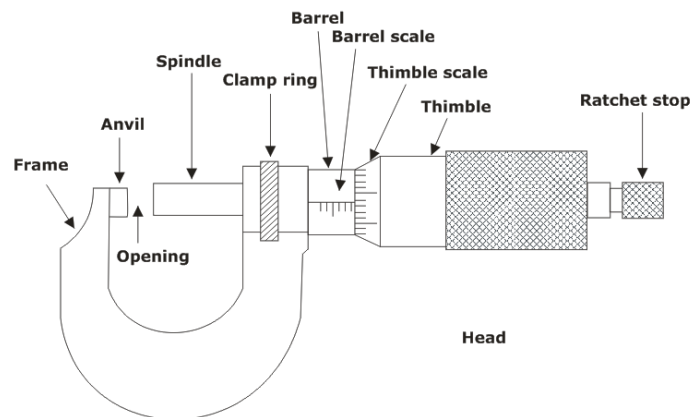
Least count of micrometer is given by:

$$LC = \frac{Pitch\ of\ screw\ gauge}{Total\ number\ of\ divisions\ on\ circular\ scale}$$

Dial indicator: It Converts a linear displacement into a radial movement to measure over a small range of movement for the plunger.

Its application is direct measurement to be measured by the actual dimensions, and comparative measurement to read the amount of displacement from the phrase reference dimension.

Angular measurement devices:

(a). Sine bar: It is a simple instrument which can be easily used for setting and measuring angles. Fairly high accuracy can be expected when measuring smaller angles, that is less than 45°.
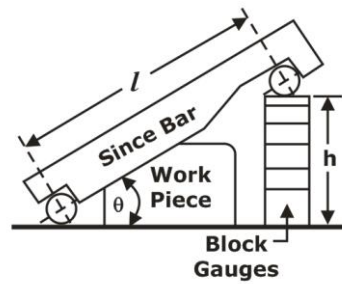
$$\sin\theta = \frac{h}{l} \Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{h}{l}\right)}$$

(b). Bevel Protractor: It is part of the machinist's combination square. The flat base of the protractor helps in setting it firmly on the workpiece and then by rotating the rule, it is possible to measure the angle.

(c). Clinometer: A clinometer is a tool that is used to measure the angle of elevation, or angle from the ground, in a right - angled triangle. A Clinometer basically consists of a precision level mounted in a holder which is attached to a rotatable member.

(d). Autocollimator: An autocollimator is an optical instrument that is used to measure small angles with very high sensitivity. As such, the autocollimator has a wide variety of applications including precision alignment, detection of angular movement, verification of angle standards, and angular monitoring over long periods.

Straightness, Flatness and Squareness:

Straightness: It is defined as the deviation of surface from ideal straight line. This straightness can be measured in 3 ways.

Spirit level: Surface under examination is divided into number of segments equal to the size of spirit level. Spirit level is there kept from one segment to another and position of bubble in it is noted down. The deviations of bubble form the center position are recorded.

Flatness: Flatness is defined as the departure of surface from ideal flat surface.

Interferometry:

Optical flat as comparator: Using optical flat difference is the size of slip gauge can be calculated from a master reference. Suppose the difference Δh has to be calculated:

From the similar triangles

Optical flat
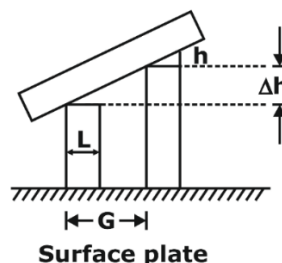
$$\frac{h}{l} = \frac{\Delta h}{G}$$

$$\Rightarrow \Delta h = \left(\frac{h\lambda}{2}\right)\left(\frac{G}{L}\right)$$

Surface Finish:

Roughness height: This is the parameter with which generally the surface finish is indicated. It is specified either as arithmetic average value or the root-mean-square value.

Roughness width: It is distance parallel to the nominal part surface within which lie the peaks and valleys, which constitute the predominant pattern of the roughness.

Roughness width cut-off: This is the maximum width of the surface that is included in the calculation of the roughness height.

Waviness: Waviness refers to those surface irregularities that have a greater spacing than that of roughness width.

Standard terminology and symbols to describe surface finish
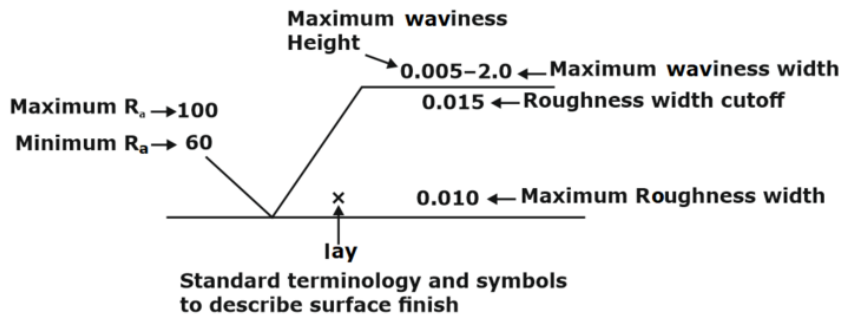
Example:

LAY DIRECTION: It is the direction of the predominant surface pattern produced on the workpiece by the tool marks. The different types of lays are as under

Symbol	Diagram	Description
=		**Parallel lay:** Surface is produced
⊥		**Perpendicular lay:** Surface is produced by shaping and planning
X		**Crossed lay:** Such surface can be produced by knurling operation
M		**Multidirectional lay:** Such surfaces are produced by grinding operation
C		**Circular lay:** The surfaces are produced by facing operation

R	⊕	Radial lay

Types of lays

CENTER LINE AVERAGE VALUE (CLA, R$_A$):

If y=(f(x) is the characteristic equation of the roughness, R$_a$ value can also be expressed as

$$R_a = \frac{1}{L} \int_0^L |y(x)|\, dx \cong \frac{1}{N} \sum |y_i|$$
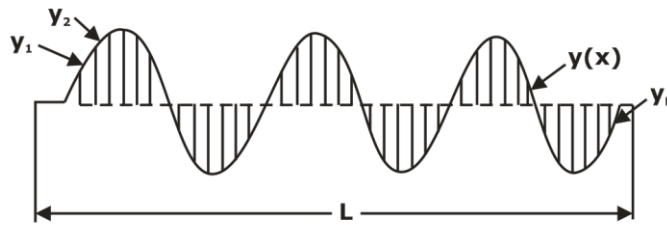
Cutoff length is one in which measurement of roughness is being carried out (or length of travel of stylus). So, R$_a$ value can also be represented as

$$R_a = \frac{\sum a + \sum b}{L}$$

Where $\sum a$ = area above the line

$\sum b$ = area below the line

L= cutoff length

Root mean square roughness: $R_{rms} \cong \sqrt{\frac{1}{N} \sum y_i^2}$

PEAK TO VALLEY HEIGHT (R$_T$ OR R$_{MAX}$): It is the difference between highest peak and deepest valley.

$$R_a = \frac{H_{max}}{4}$$

In case of turning operation when the nose radius (R) and the feed rate (f) is given Maximum height of unevenness can also be expressed as:

$$H_{max} = \frac{f^2}{8R}$$

If complete tool signature is given, the peak to valley height can also be calculated as

$$H_{max} = \frac{f}{\tan\psi + \cot\psi_1}$$

Where

f= feed rate

ψ =side cutting edge angle

ψ₁ =end cutting edge angle

CHAPTER 7: COMPUTER INTEGRATED MANUFACTURING

CIM: A computer is a machine that can be instructed to carry out sequence of arithmetic or logical operation automatically via computer programming.

CIM is the technique of using computers to control an entire production process. It's commonly used by factories to automate functions such as analysis, cost accounting, design, distribution, inventory control, planning and purchasing.

These functions are often linked to a central, computer-controlled station to enable efficient materials handling and management, while delivering direct control and monitoring of all operations simultaneously.

Methodology CAD/CAM: To use technical data from a database in the design and production stages. Information on parts, materials, tools, and machines are integrated.

CAD (Computer Aided Design): Allows the design in a computer environment.

CAM (Computer Aided Manufacturing): To manage programs and production stages on a computer.

Evolution of Numerical Control:

(i). Numerical Control (NC): Data on paper or received in serial port**,** NC machine unable to perform computations **and** Hardware interpolation.

(ii). Direct Numerical Control (DNC): Central computer control a number of machines DNC or CNC**.**

(iii). Computer Numerical control (CNC): A computer is on the core of each machine tool**,** Computation and interpolation algorithms run on the machine.

(iv). Distributive numerical control: scheduling**,** Quality control **and** Remote monitoring.

Solid modelling:

Wire frame Geometric modelling:

- 2-D Two-dimensional representation is used for a flat object.

- $2\frac{1}{2}D$: It allows somewhat beyond 2D capability by permitting a 3-D object to be represented if it has no side wall details.

- 3-D allows for full three-dimensional modelling of a more complex geometry.

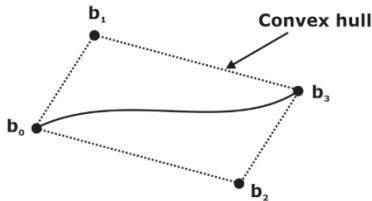- The most advanced method of Geometric modelling in 3-D is solid modeling.

Bezier Curve:

- A parameter Bezier curve segment is a weighted sum of (n + 1) control points P_0, P_1, P_2, ..., P_N

$$P(t) = \sum_{i=0}^{n} P_i \, B_{i,n}(t)$$

Where, $B_{i,n}(t)$ = Bezier blending function

- The Blending function is always a polynomial one degree less than the no. of control point.

- Bezier as convex hull of its control points.

- A Bezier curve of order (n+1) [degree n] has (n+1) control points.

- Bezier curves are widely used in computer graphics to model smooth curves.

Coon's Patch:

- A Coons patch is a bicubic parametric surface formed by four corner points, eight tangent vectors (two vectors in the u and w directions, respectively, at each of the four corners), and four twister vectors at the respective four corner points.

NURBS:

- It stands for the Non uniform rational basis spline. It is a mathematical model commonly used in computer graphics for generating and representing curves and surfaces.

- It offers a greater flexibility NURBS and precision for handling both the analytical and modeled shapes.

- NURBS is flexible for designers, designing a large variety of shapes by manipulating the control points and the weightages.

- Weights in the NURBS data structure determine the amount of surface deflection towards or away from its control points.

Transformation of geometry:

1. Translation:

Any graphical entity can be translated or moved in X or Y direction by using this routine. Basic equations used in this subroutine are:

$X' = X + T_x$

$Y' = Y + T_y$

(X', Y') are the new coordinates after translation and (X, Y) are the old coordinates before translation. T_x and T_y are the distance to be translated in x and y direction respectively.

The equations can be written in matrix form as

$$\begin{bmatrix} X' & Y' & 1 \end{bmatrix} = \begin{bmatrix} X & Y & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix}$$

Here matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix}$ is called the translation matrix.

In 3D transformations, the x, y and z coordinates of a point are considered. For translation, the transformation matrix $[R_T]$ is given by:

$$[R_T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

T_x, T_y, T_z being translation in x, y and z. directions respectively.

2. Scaling: This routine is used to enlarge the object or make it small. The basic equations are:

$X' = X. S_x$

$Y' = Y. S_y$

where S_x and S_y are the scaling factor in x and y direction, respectively.

$$\begin{bmatrix} X' & Y' & 1 \end{bmatrix} = \begin{bmatrix} X & Y & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For 3-D scaling, the transformation matrix is given by: $[R_s] = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

where S_x, S_y and S_z are scaling factors in x, y and z direction respectively.

3. Rotation: Rotation of any point is effective with respect to some fixed point. We assume anticlockwise rotation as positive and clockwise rotation as negative.
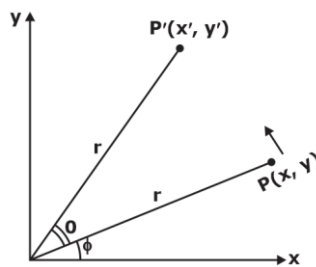
From the figure:

$X = r \cos \phi$ and $Y = r \sin \phi$

$X' = r \cos (\theta + \phi)$ and $Y' = r \sin (\theta + \phi)$

$$\begin{bmatrix} X' & Y' & 1 \end{bmatrix} = \begin{bmatrix} X & Y & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation in 3-D can be about x, y or z axis. These equations of rotation of a point about z-axis is:

Rotation about Z-axis: $[R_z] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Rotation about X-axis: $[R_x] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Rotation about Y-axis: $[R_y] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4. Reflection:

(i). Reflection of X-axis (Y = 0 axis): Reflection matrix is given by:

$[R_x] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

After reflection, coordinates are (x', y') then:

$\begin{bmatrix} X' & Y' & 1 \end{bmatrix} = \begin{bmatrix} X & Y & 1 \end{bmatrix} \times \begin{bmatrix} R_x \end{bmatrix}$

(ii). Reflection of Y-axis (X = 0 axis): Reflection matrix is given by:

$[R_Y] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence to find coordinates of point P' i.e. (x', y') after reflection about y-axis:

$\begin{bmatrix} X' & Y' & 1 \end{bmatrix} = \begin{bmatrix} X & Y & 1 \end{bmatrix} \times \begin{bmatrix} R_Y \end{bmatrix}$

(iii). To Find Reflection Matrix when the Axis of Reflection is the Line Passing Through origin (Y = X):

$[R_{x=Y}] = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence to find coordinates of point P' i.e. (x', y') after reflection about X= Y-axis:

$\begin{bmatrix} X' & Y' & 1 \end{bmatrix} = \begin{bmatrix} X & Y & 1 \end{bmatrix} \times \begin{bmatrix} R_{X=Y} \end{bmatrix}$

(iv). To Find Reflection Matrix when the Axis of Reflection is the Line Passing Through origin (Y = - X):

$$\left[R_{X=-Y}\right] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence to find coordinates of point P' i.e. (x', y') after reflection about X= - Y axis:

$$\left[X' \quad Y' \quad 1\right] = \left[X \quad Y \quad 1\right] \times \left[R_{X=-Y}\right]$$

Numerical control:

Open systems: The open loop system means the output of the system is free from their input due to being the non-feedback system. The open loop system is more stable as compared to a closed loop system. The open loop system gives the fast response.

Examples: The traffic light, automatic washing machine, etc. are the examples of the output system.
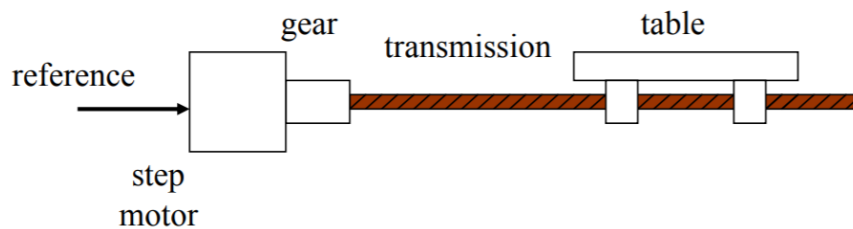
Closed system: In the closed-loop system, the desired output depends on their input due to being the feedback system. The construction of the closed-loop system is quite difficult. closed-loop system is reliable, and the accuracy of the system is more as compared to open loop system.

Example: The temperature controller, toaster etc. are the examples of the closed-loop system.
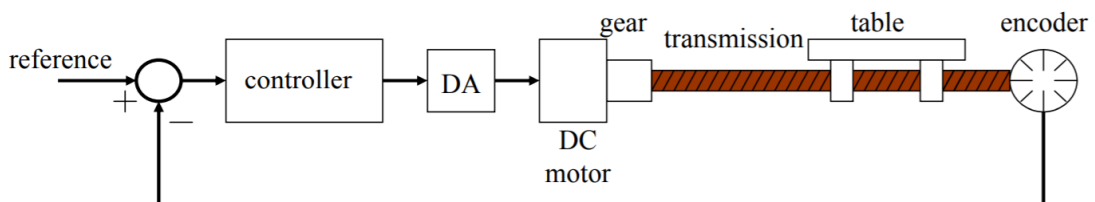
BASIC LENGTH UNIT(BLU): BLU is the distance moved by table corresponding to single pulse.
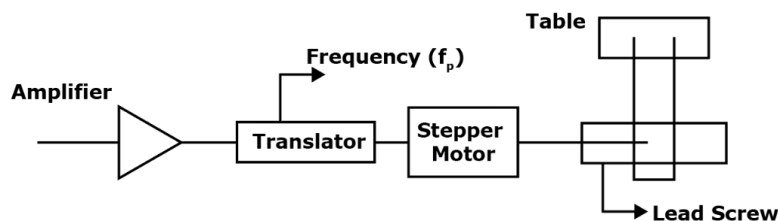
n_S = No. of steps

$$\text{Step angle} = \frac{360°}{n_S}$$

$$BLU = \frac{\text{Lead screw pitch(mm)}}{\text{Steps per revolution of stepper motor}}$$

t = time period of pulse coming to stepper motor

f_P = frequency

Total pulse = $f_P \times t$

$$\text{Total angle} = \frac{360°}{n_S} \times f_p t$$

So, $\boxed{\text{No. of revolutions} = \frac{360° \times f_p t}{n_S \times 360°} = \frac{f_p t}{n_S} \text{rps}}$ $\boxed{N = \frac{60 f_p t}{n_S}}$

Also, linear velocity of stepper motor, V is given by the expression:

$$\boxed{V = \text{pulse frequency x BLU x 60 mm / min}}$$

PART PROGRAMING CODES:

G and M Codes:

G codes	Interpretation	M codes	Interpretation
G00	rapid traverse	M00	Programs stop
G01	linear interpolation	M01	Planned stop
G02	circular interpolation (CW)	M02	end of program
G03	circular interpolation (CCW)	M03	CW spindle rotation
G04	Dwell	M04	CCW spindle rotation
G05	Hold	M05	spindle off
G08	For acceleration	M06	Tool change
G09	For Retardation	M07, 08	Coolant on
G17	XY plane selection	M09	Coolant off
G18	YZ plane selection	M10	Clamp
G19	ZX plane selection	M11	unclamp
G33	thread cutting constant lead	M17	end of sub program
G34	thread cutting with increasing lead	M30	end of sub of main program
G35	thread cutting with decreasing lead		
G41	Tool radius compensation left		
G42	Tool radius compensation Right		
G63	Tapping		
G70	English programming		
G71	Metric programming		
G90	Absolute Positioning		

G91	Incremental positioning		

Other important codes for programming:

Symbol	Interpretation
N	Sequence Number or block number
G	Preparatory Function
X	X Axis Command
Y	Y Axis Command
Z	Z Axis Command
R	Radius from specified center
A	Angle ccw from +X vector
I	X axis arc center offset
J	Y axis arc center offset
K	Z axis arc center offset
F	Feed rate
S	Spindle speed
T	Tool Number
M	Miscellaneous function

NC Words order:

N _ _ G _ _ X _ _ Y _ _ Z _ _ f _ _ S _ _ T _ _ M _ _

It is the sequence of the words which will be followed while writing a block of the program.

Note:

- It is generally possible to include more than one G- address in a block, provided these functions are mutually exclusive.
- G02 and G03 together in one block are not permissible. If they are given, the latter i.e. G03 will become operational overriding the earlier of same category.
- Model G-codes behave as settings to the control.
- T01: tool designation represents the first tool used in the CNC program. T02: tool designation represents any tool in the CNC program between the first and last tool. T03: Tool designation represents the last tool used in the CNC program.
- T99: Tool designation represents empty tool (dummy tool) as an empty tool pocket identification.

Interpolator:

- Interpolator in a CNC machine coordinates axes movements.
- An interpolator determines the velocities of individual axis to drive the tool along the programmed path at given feed rate.
- It also provides intermediate coordinate positions along the programmed path.

Advanced CNC programming languages

- Automatically Program Tool (APT) Developed at MIT in 1954. Derived from APT:

(i). ADAPT (IBM)

(ii). IFAPT (France)

(iii). MINIAPT (Germany)

APT Language: APT is not only an NC language, it is an computer program that performs the calculations to generate cutter positions based on the APT statements.

Four type of statements in APT Language:

(a). Geometry statements

(b). Motion statements:

Point to point motions: Only two basic PTP motion commands: GOTO and GODLTA. The GOTO statement instructs the tool to go to a particular location specified in the descriptive data. The GODLTA command specifies an incremental move for the tool.

(c). Postprocessor statements

(d). Auxiliary statements

• Compact II

• Autospot

• SPLIT

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