CSIR-NET NOV. 2020 MATHEMATICAL SCIENCE QUESTION PAPER

1.Find the value of f (0) if $f(x + 2) = (x + 1)^{34} - (x + 1)^{33} + 5$

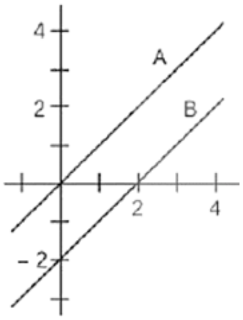
A. 5

B. 7

C. 6

D. 72

2.The shortest distance between the parallel lines A and B in the following figure is



A. $\sqrt{2}$

B. 2

C. $2\sqrt{2}$

D. $2\sqrt{3}$

3.Two verities A and B of rice cost Rs. 30 and Rs . 90 per kg whereas two varities C and D of pulses, Rs 100 and Rs 120 per kg, respectively. If at least one kg each of A and B and at least half a kg each of C and D have to be purchased, then the minimum and maximum costs of a total of 5 kg of these provision are, respectively

A. Rs 150 and Rs 600

B. Rs 260 and Rs 530

C. Rs 290 and Rs 470

D. Rs. 370 and Rs 460

4.One of four suspects A, B, C and D has committed a crime. A and D are always truthful, and B and C are always untruthful. C and D are identical twins and the interrogator does not know who is who. If A says, "D is innocent", B says, "A is guilty" and among C and D one says, "A is innocent" and the other says, "B is guilty", then which of the following is FALSE?

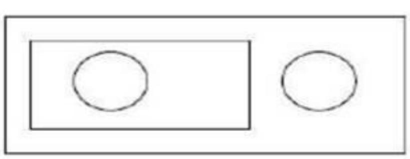
A. D said "A is innocent"

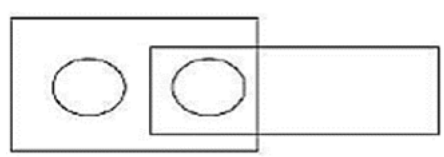
B. D is innocent

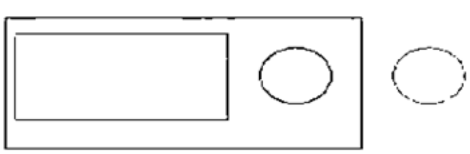
C. B is innocent

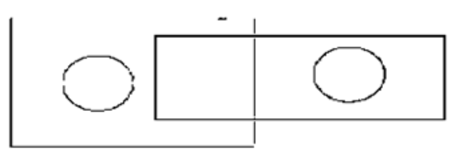
D. C is innocent

5.Which is an appropriate diagram to represent the relation between the following categories: quadruped, mammal, whale house lizard?

A. 

B. 

C. 

D. 

6.A 7m long tube having inner diameter of 2 cm is filled with water. The water is then poured into a cylindrical bucket having inner base area of 200cm^2. What will be the approximate height (in cm) of water in the bucket?

A. 22

B. 44

C. 9

D. 11

7.Water is being filled in a cone from the top at a constant volumetric rate. The rate of increase of the height of the water column

A. is linearly dependent on time

B. Depends on the apex angle of the cone.

C. Increases as cube-root of the volumetric rate.

D. Increases as square-root of the volumetric rate

8.A square board is divided into 9 smaller identical squares by drawing lines. Three bullets are shot at the board randomly. The probability that at least 2 bullets hit the same small square is,

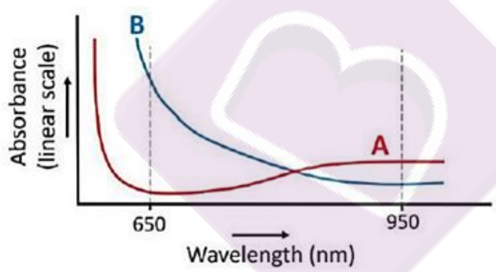
A. 1/3

B. 56/81

C. 25/81

D. 2/3

9.The wavelength dependent absorbance of two compounds, A and B , is shown. Absorbance of mixture is a linear function of the concentration of the compounds. R is defined as a ratio of absorbance at 650nm to the absorbance at 950 nm.



If the mixture contains 95% of compound A and R must be

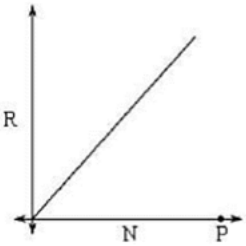
A. 95

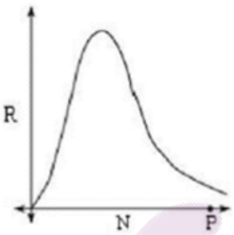
B. 5

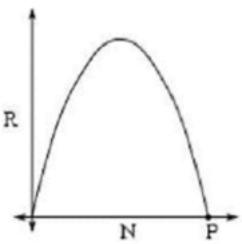
C. 1

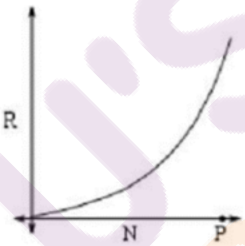
D. Less than 1

10.An epidemic is spreading in a population of size P. The rate of spread R of the disease at a given time is proportional to both the number of people affected by the disease (N), and the number of people not yet affected by the disease. Which of the following graphs of R vs N is correct?

A. 

B. 

C. 

D. 

11.A and B complete a work in 30 days. B and C complete the same work in 24 days whereas C and A complete the same work in 28 days. Based on this statement which of the following conclusions is correct?

A. C is the most efficient and B is the least efficient

B. B is the most efficient but the least efficient one cannot be determined

C. C is the most efficient but the least efficient one cannot be determined

D. C is the most efficient and A is the least efficient

12.Clock A loses 4 minutes every hour, clock B always shows the correct time and clock B shows the correct time and clock C gains 3 minutes every hour. On a Monday, all the three clocks showed the same time 8 pm. On the following Wednesday, when the clock C shows 2pm, what time will clock A show?

A. 7:20 am

B. 8:40 am

C. 9:20 are

D. 10: 40 am

13.In a class, there is one pencil for every two students, one pencil for every two student, one eraser for every student, and one ruler for every four student's. If the total number of these stationery items required is 65, how many students are present in the class?

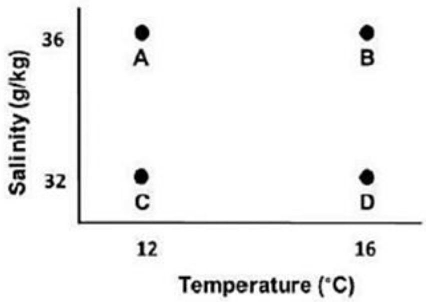
A. 55

B. 60

C. 65

D. 70

14.The figure shows temperature and salinity of four samples of water. Which one of the samples has the highest density?



A. A

B. B

C. C

D. D

15.The given tables show the number of active and recovered cases of a certain disease. Assuming that the linear trend for both continues, on which day will recovered cases be twice that of the active cases?

Day	0	1	4	7	10
Active cases	990	1000	1030	1060	1090
Recovered cases	760	800	920	1040	1160

A. 61

B. 62

C. 63

D. 64

16.A boat weighs 60 kg, and oarsmen A and B weigh 80 and 90 kg, respectively. Rowing at a constant power, the time required to complete a course is proportional to the total weight. Rowing alone, A and B complete the course in 1 and 1 ½ hours, respectively. Assuming that their power add up, how long they take to complete the course if they row together?

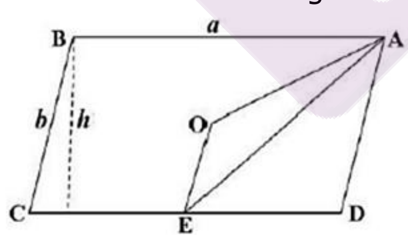
A. 49.4 min

B. 57.5 min

C. 62.6 min

D. 72.5 min

17.Consider a parallelogram ABCD with center O and E as the midpoint of side CD. The area of the Triangle OAE, is



A. $\frac{1}{5}$ah

B. $\frac{1}{6}$ah

C. $\frac{1}{8}$ah

D. $\frac{1}{7}$ah

18.The sum of the first n even numbers is

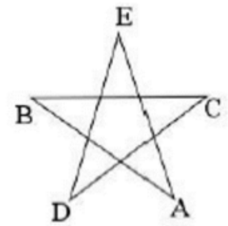
A. divisible by and not by (n+1)

B. Divisible by (n+1) and not by n

C. divisible by both n and (n+1)

D. Neither divisible by n nor by (n+1)

19.A, B, C, D and E are the vertices of a regular pentagon as shown in the figure



The angle ΔABC is

A. 48º

B. 72º

C. 54º

D. 36º

20.On a 200 m long straight road, maximum numbers of poles are fixed at 20 m interval. How many of these poles should be removed in order to have maximum number of poles at an interval of 40 m on the road?

A. 8

B. 6

C. 5

D. 4

21.Let $\{E_n\}$ be a sequence of subsets of R Define

$$\limsup_n E_n = \bigcap_{k=1}^{\infty} \bigcup_{n=k}^{\infty} E_n$$

$$\liminf_n E_n = \bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} E_n$$

Which of the following statements is true?

A. $\limsup_n E_n = \liminf_n E_n$

B. $\limsup_n E_n = \{x : x \in E_n$ for all but finitely many $n\}$

C. $\liminf_n E_n = \{x : x \in E_n \; for \; infinitely \; many \; n\}$

D. $\limsup_n E_n = \{x : x \in E_n$ for some $n\}$

22.$f : N \rightarrow N$ Be a bounded function. Which of the following statements is NOT true?

A. $f\ (n) \in N$

B. $f\ (n) \in N$

C. $(f(n) + n) \ \in N$

D. $(f(n) + n) \ \notin N$

23.Which of the following statements is true?

A. There are at most countably many continuous maps from R^2 to R

B. There are at most finitely many continuous surjective maps from R^2 to R

C. There are infinitely many continuous injective maps from R^2 to R

D. There are no continuous bijective maps from R^2 to R

24.The series

$$\sum_{n=1}^{\infty} \frac{(-1)^n . \sin nx}{n \log_e n}, x \in \mathbb{R}$$

A. Only for x = 0

B. Uniformly only for $x \in [-\pi, \pi]$

C. Uniformly only for $x \in R - \{n\pi : n \in Z\}$

D. Uniformly for all $x \in R$

25.Given $(a_n)_{n\geq1}$ a sequence of real numbers, which of the following statements is true?

A. $\sum_{n\geq1} \quad (-1)^n \frac{a_n}{1+|a_n|}$ *converges*

B. There is a subsequence $(a_{nk})_{k\geq1}$ such that $\sum_{k\geq1} \quad \frac{a_{nk}}{1+|a_{nk}|}$ converges

C. There is a number b and a subsequence $(a_{nk})_{k\geq1}$ such that

D. $\sum_{k\geq1} \quad \left|b - \frac{a_{nk}}{1+|a_{nk}|}\right|$ converges

26.Given f, g are continuous functions on [0, 1] such that f(0) = f(1) = 0; g(0) = g(1) = 1 and f(1/2) > g(1/2). Which of the following statements is true?

A. There is no t ∈ [0, 1] such that f(t) = g(t)

B. There is exactly one t ∈ [0, 1] such that f(t) = g(t)

C. There are at least two t ∈ [0, 1] such that f(t) = g(t)

D. There are always infinite many t ∈ [0, 1] such that f(t) = g(t)

27.Let A be an n×n matrix such that the set of all its nonzero eigenvalues has exactly r elements. Which of the following statements is true?

A. Rank A ≤ r

B. if r = 0, then rank A < n - 1

C. Rank A ≥ r

D. A^2 has r distinct nonzero eigenvalues

28.Let A and B be 2X2 matrices. Then which of the following is true?

A. det (A + B) + det (A – B) = det A + det B

B. det (A + B) + det (A – B) = 2det A - 2detB

C. det (A + B) + det (A – B) = 2det A + 2detB

D. det (A + B) - det (A – B) = 2det A - 2detB

29.If A = $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$, then A^{20} equals

A. $\begin{pmatrix} 41 & 40 \\ -40 & -39 \end{pmatrix}$

B. $\begin{pmatrix} 41 & -40 \\ 40 & -39 \end{pmatrix}$

C. $\begin{pmatrix} 41 & -40 \\ -40 & -39 \end{pmatrix}$

D. $\begin{pmatrix} 41 & 40 \\ 40 & -39 \end{pmatrix}$

30.Let A be a 2X2 real matrix with det A = 1 and trace A = 3 . What is the value of trace A^2?

A. 2

B. 10

C. 9

D. 7

31.For $a, b \in R$, let $p(x,y) = a^2x_1y_1 + abx_2y_1 + abx_1y_2 + b^2x_2y_2, x = (x_1, x_2), y = (y_1, y_2) \in R^2$

for what values of a & b does $p: R^2 \times R^2 \to R$ define an Inner product?

A. a > 0, b > 0

B. ab > 0

C. a = 0, b = 0

D. for no values of a, b

32.Which of the following real quadratic form on R_2 is positive definite?

A. $Q(x,y) = xy$

B. $Q(x,y) = x^2 - xy + y^2$

C. $Q(x,y) = x^2 + 2xy + y^2$

D. $Q(x,y) = x^2 + xy$

33.Let be the positively oriented circle in the complex plane given by Then equals

A. 3

B. 1/3

C. 2

D. 1/2

34.For a positive integer p, consider the holomorphic function $f(z) = \frac{\sin z}{z^p}$ for z ∈ $C - \{0\}$. For which values of p does there exist a holomorphic function such that f(z)=g'(z) for z ∈ $C - \{0\}$?

A. All even integers

B. All odd integers

C. All multiple of 3

D. All multiple of 4

35.Let γ be the positively oriented circle in the complex plane given by $\left\{Z \in C: |z-1| = \frac{1}{2}\right\}$ the line integral $\int_\gamma \frac{z.e^{1/z}}{z^2-1} dz$ equals

A. $i\pi e$

B. $-i\pi e$

C. πe

D. $-\pi e$

36.Let p be a positive integer. Consider the closed curve $\gamma(t) = e^{it}$. $0 \leq t < 2\pi$. Let f be the function holomorphic in {z: |z| < R}. Where R>1. If f has zero only at z_0. $0 < |z_0| < R$, and it is of multiplicity q. then $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)} z^p . dz$ equals

A. qz_0^P

B. $z_0 q^P$

C. pz_0^q

D. $z_0 p^q$

37.Which of the following statements is true?

A. Every even integer $n \geq 16$ divides (n – 1)! +3

B. Every odd integer $n \geq 16$ divides (n-1)!

C. Every even integer $n \geq 16$ divides (n-1)!

D. For every integer $n \geq 16, n^2$ divide n! +1

38.Let X be a non-empty set & P(X) be the set of all subsets of X, On P(X), define two operations * & Δ as follows: for

$A, B \in P(X), A * B = A \cap B; A\Delta B = (A \cup B) - (A \cap B)$

Which of the following statements is true?

A. P(X) is a group under * as well as under Δ

B. P(X) is group under *, but not under Δ

C. P(X) is a group under Δ but not under*

D. P(X) is neither a group under * nor under Δ

39.Let $\varphi(n)$ be the cardinality of the set $\{a | 1 \leq a \leq n, (a, n) = 1\}$ where (a, n) denotes the gcd of a and n.

Which of the following is NOT true?

A. There exist infinitely many n such that $\varphi(n) > \varphi(n+1)$.

B. There exist infinitely many n such that $\varphi(n) < \varphi(n+1)$.

C. There exists $N \in N$such that N > 2 and for all $n > N,\ \varphi(N) < \varphi(n)$

D. The set $\left\{\frac{\varphi(n)}{n} : n \in N\right\}$ has finitely many limit points.

40.For any two metric spaces (X, d_x), (Y, d_y) a map $f: X \to Y$is said to be a closed map if whenever F is closed in X, then f(F) is closed in Y. For any subset B of a metric space, B is given the induced metric. The metric on X × Y is given by d((x, y), (x', y') = max {d_x(x,x'), d_y(y, y')} which of the following are true?

A. For any subset $A \subseteq$ X the inclusion map i: $A \to X$ is closed

B. The projection map $p_1: X \times Y \to X$ given by p_1(x, y) = x is closed

C. Suppose that $f: X \to Y,\ g: Y \to X$ are continuous maps if g. $f; X \to Z$is a closed map then $g|_{f(x)}: f(X) \to Z$ is closed. Here $g|_{f(X)}$means the map restricted to f(X)

D. If $f: X \to Y$ takes closed balls into closed sets then f is closed

41.Let K be a positive integer. Consider the differential equations $\{\ \frac{dy}{dt} = \frac{5k}{y^{5k+2}} for\ t > 0\ y(0) = 0$

Which of the following statements is true?

A. It has a unique solution which is continuously differentiable on $(0, \infty)$

B. It has at most two solutions which are continuously differentiable on $(0, \infty)$

C. It has infinitely many solutions which are continuously different on $(0, \infty)$

D. It has no continuously differentiable solution on $(0, \infty)$

42.Let $y_0 > 0,\ z_0 > 0$ and $\alpha > 1$consider the following two differential equations:

$$(*)\{\frac{dy}{dt} = y^{\alpha}\ for\ t > 0, y(0) = y_0$$

$$(**)\{\frac{dz}{dt} = -z^{\alpha}\ \ fot\ t > 0, z(0) = z_0$$

We say that the solution to a different equation exists globally if it exists for all t > 0 Which of the following statements is true?

A. Both (*) and (**) have global solutions

B. None of (*) and (**) have global solution

C. There exists a global solution for (*) and there exists a $T < \infty$ such that $|\ z(t)| = +\infty$

D. There exists a global solution for (*) and there exists a $T < \infty$ such that $|\ y(t)| = +\infty$

43.The general solution of the surface which are perpendicular to the family of surface $z^2 = kxy,\ k \in R$ is

A. $\phi(x^2 - y^2, xz) = 0\ \phi \in C^1(R^2)$

B. $\phi(x^2 - y^2, x^2 + z^2) = 0\ \phi\ \in C^1(R^2)$

C. $\phi(x^2 - y^2,\ 2x^2 + z^2) = 0\ \phi \in C^1(R^2)$

D. $\phi(x^2 + y^2,\ 3x^2 - z^2) = 0\ \phi\ \in C^1(R^2)$

44.The general solution of the equation $\frac{x\partial z}{\partial x} + y\frac{\partial z}{dy} = 0$

A. $z = \phi\left(\frac{|x|}{|y|}\right), \phi \in C^1(R)$

B. $z = \phi\left(\frac{x-1}{y}\right)\phi \in C^1(R)$

C. $z = \phi\left(\frac{x+1}{y}\right)\phi \in C^1(R)$

D. $z = \phi(|x| + |y|), \phi \in C^1(R)$

45.Let f be an infinitely differentiable real-valued function on a bounded interval I take $n \geq 1$ interpolation points $\{x_0, x_1, \ldots, x_{n-1}\}$ for the function f. In the $\varepsilon \to 0$limit the quantity

$$|\ p_{2n-1}(x) - q_{2n-1}(x)|$$

A. Does not necessarily converge

B. Converges to $\frac{1}{2n}$

C. Converges to 0

D. Converge to $\frac{1}{2n+1}$

46.The extremal of the functional

$J(y) = \int_0^1 \quad [2(y')^2 + xy]dx,\ y(0) = 0, y(1) = 1, y \in C^2[0,1]$is

A. $y = \frac{x^2}{12} + \frac{11x}{3}$

B. $y = \frac{x^3}{3} + \frac{2x^3}{3}$

C. $y = \frac{x^2}{7} + \frac{6x}{7}$

D. $y = \frac{x^3}{24} + \frac{23x}{24}$

47.The solution of the Fredholm integral equation $y(s) = s + \int_0^1 \quad (st^2 + s^2t)y\ (t)\ dt$is

A. y(s) = -(50s + $40s^2$)

B. y(s) = (30s + $15s^2$)

C. y(s) = -(30s + $40s^2$)

D. y(s) = (60s + $70s^2$)

48.Consider the solid S made of material of constant density in the shape of hemisphere of unit radius:

$S = \{(x, y, z) | x^2 + y^2 + z^2 \leq 1\ z \geq 0\}$ Which of the following statements is true?

A. The centre of mass of S is at the origin

B. the x-axis is a principal axis for S

C. The moment of inertia tensor of S is not a diagonal matrix

D. The z-axis is principal axis for S

49.In an examination involving multiple choice questions, a student works out the solution in 50% of the questions. In the remaining question the student guesses the answer. However, when the answer is guessed the probability that it is correct is 0.30. When the student works out the solutions it may be wrong with probability 0.10. If the answer to a particular question is correct, what is the probability that the student guessed the answer?

A. 0.25

B. 0.50

C. 0.90

D. 0.30

50.Let X_1, X_2, … be i.i.d random variable having a x^2 – distribution with 5 degrees of freedom. Let $a \in R$be constant. Then the limiting distribution of $a\left(\frac{X_1 + .. + X_n - 5n}{\sqrt{n}}\right)$is

A. Gamma distribution for an appropriate value of a

B. X^2 – Distribution for an appropriate value of a

C. Standard normal distribution for an appropriate value of a

D. A degenerate distribution for an appropriate value of a

51.Consider a Markov chain $X_0, X_1, X_2, \ldots$ with state space S. suppose $i, j \in S$ are two states which communicate with each other. Which of the following statements is NOT correct?

A. Period of i = period of j

B. i is recurrent if and only if j is recurrent

C. $\lim_{n\to\infty} P[X_n = i \mid X_0 = k] \neq \lim_{n\to\infty} P[X_n = j \mid X_0 = k]$

D. $\lim_{n\to\infty} P[X_n = j \mid X_0 = i] = \lim_{n\to\infty} P[X_n = j \mid X_0 = j]$

52.Suppose that X has uniform distribution on the interval [0,100]. Let Y denote the greatest integer smaller than or equal to X. which of the following is true?

A. $P(Y \leq 25) = 1/4$

B. $P(Y \leq 25) = 26/100$

C. $E(Y) = 50$

D. $E(y) = 101/2$

53.Let $X_1, X_2, .., X_n$ be i.i.d random variable with common pdf $f(x|\theta) = \frac{(\log \theta)\theta^x}{\theta-1}$ for $0<x<1$ where $\theta > 1$ is an unknown parameter. Then the statistic $T = \sum_{i=1}^{n} X_i$ is

A. Sufficient, but not complete

B. Sufficient but not minimal sufficient

C. complete sufficient

D. Neither complete, nor sufficient

54.Consider the pdf given $f(x|\theta) = \frac{e^{(x-\theta)}}{\left[1+e^{(x-\theta)}\right]^2} - \infty < x < \infty, -\infty < \theta < \infty$ based on one observation X with the above pdf, a UMP test of size α for testing $H_0: \theta \leq \theta_0$ is

A. X > k for k such that $\alpha = P\theta_0[X > k]$

B. X < k for k such that $\alpha = P\theta_0[X > k]$

C. X > k for such that $\alpha = P\theta_0[X < k]$

D. X < k for such that $\alpha = P\theta_0[X < k]$

55.Consider 35 i.i.d observations $X_1, X_2, .., X_{15}$ and $Y_1, Y_2, \ldots Y_{20}$ Let R be the Wilcoxon's rank sum statistic based on the ranks of the X' s in the combined sample. Then the expected value of R is

A. 270

B. 300

C. 360.5

D. 330.5

56.Let I, J > 5. Consider two-way ANOVA where the observation satisfy the linear model $y_{ij} = \alpha + \beta_i + y_j + \varepsilon_{ij}, 1 \le i \le I, 1 \le j \le J$ $E(\varepsilon_{ij}) = 0, V ar\ (\varepsilon_{ij}) = \sigma^2, \sum_{i=1}^{I} \beta_i = \sum_{j=1}^{s} \gamma_j = 0$ In this set-up

A. β_1 is estimable

B. γ_1 is estimable

C. $\beta_1 - \beta_2$ is estimable

D. $\beta_1 + \gamma_2$ is estimable

57.Let X_1 and X_2 be two i.i.d pX1 multivariate normal random vector with mean μ and positive definite dispersion matrix Σ. Then which of the following random variable always has a central chi-square distribution

A. $\frac{1}{2}(X_1 - X_2)^T(X_1 - X_2)$

B. $2(X_1 - X_2)^T(X_1 - X_2)$

C. $2(X_1 - X_2)^T\Sigma^{-1}(X_1 - X_2)$

D. $\frac{1}{2}(X_1 - X_2)^T\Sigma^{-1}(X_1 - X_2)$

58.10 units are chosen by simple random sampling without replacement from a population of size 100. Consider the sample variance $\frac{1}{10}\sum_{i=1}^{10} (y_i - \underline{y})^2 = s^2$. An unbiased estimate of population variance $\sigma^2 = \frac{1}{100}\sum_{i=1}^{100} (y_i - \underline{y})^2$. Is

A. $\frac{11}{10}s^2$

B. $\frac{10}{11}s^2$

C. $\frac{100}{99}s^2$

D. $\frac{100}{111}s^2$

59.Consider a Randomized Block Design with b blocks and k treatments. Let the observation corresponding to the i^{th} treatment and the j^{th} block be $y_{ij}, 1 \le i \le k,\ 1 \le j \le b$, which satisfies the usual linear model. Which of the following is true?

A. The estimated of any to treatment contrasts are uncorrelated

B. The error sum of square ha bk-1 degrees of freedom

C. The estimate of any treatment contrast is uncorrelated with estimated of any contrast

D. The correlation between the estimated of two treatment contrast is always negative

60.The maximum and the minimum values of 5x + 7y, when $|x| + |y| \le 1$ are

A. 5 and -5

B. 5 and -7

C. 7 and -5

D. 7 and -7

61.Which of the following sets are in bijection with $\mathbb{R}$?

A. Set of all maps from {0, 1} to $\mathbb{N}$

B. Set of all maps from $\mathbb{N}$ to {0, 1}

C. Set of all subsets of $\mathbb{N}$

D. Set of all subsets of $\mathbb{R}$

62.Which of the following statements are true?

A. The series $\sum_{n\geq 1}\frac{(-1)^n}{\sqrt{n}}$ is convergent

B. The series $\sum_{n\geq 1}\frac{(-1)^n}{\sqrt{n}+n}$ is absolutely convergent

C. The series $\sum_{n\geq 1}\frac{(1+(-1)^n)\sqrt{n}+\log n}{n^{\frac{3}{2}}}$ is convergent

D. The series $\sum_{n\geq 1}\frac{(-1)^n\sqrt{n}+1}{n^{\frac{3}{2}}}$ is convergent

63.Let $f:\mathbb{R}^2\to\mathbb{R}$ be defined by $f(x,y)=\begin{cases}\frac{2xy}{x^2+y^2} & \text{for } (x,y)\neq(0,0)\\ 0 & \text{for } (x,y)=(0,0)\end{cases}$

$$g(x,y)=\sum_{n=1}^{\infty}\frac{f((x-n),(y-n))}{2n}$$

Which of the following statements are true?

A. The function $h(y)=g(c,y)$ is continuous on $\mathbb{R}$ for all c

B. g is continuous form $\mathbb{R}^2$ into $\mathbb{R}$

C. g is not well-defined function

D. g is continuous on $\mathbb{R}^2\setminus\{(k,k):k\in\mathbb{R}\}$

64.Consider the two series $A(x)=\sum_{n=1}^{\infty}x^n(1-x)$ and $B(x)=\sum_{n=1}^{\infty}(-1)^n x^n(1-x)$

Where $x\in[0,1]$.Which of the following statements are true?

A. Both A(x) and B (x) converge pointwise

B. Both A(x) ad B (x) converge uniformly

C. A(x) converge uniformly but B(x) does not

D. B(x) converge uniformly but A(x) does not

65.For $p > 1$,consider the improper integral

$$I_p = \int_0^1 t^p \sin t\, dt$$

Which of the following statements are true?

A. I_p is convergent for p = -1/2

B. I_p is divergent for p = -3/2

C. I_p is convergent for p = 4/3

D. I_p is divergent for p = -4/3

66.Suppose that $\{f_n\}$ is a sequence of real-valued function on R. suppose it converges to a continuous function f uniformly on each closed and bounded subset $\mathbb{R}$. Which of the following statements are true?

A. The sequence $\{f_n\}$ converges to f uniformly on $\mathbb{R}$

B. The sequence $\{f_n\}$ converges to f pointwise on $\mathbb{R}$

C. For all sufficiently large n, the function f_n is bounded

D. For all sufficiently large n the function f_n is continuous

67.Let $f(x) = e^{-x}$ and $g(x) = e^{-x^2}$, which of the following statements are true?

A. Both f and g are uniformly continuous on $\mathbb{R}$

B. f is uniformly continuous on every interval of the interval of the form $[a, +\infty), a \in \mathbb{R}$

C. g is uniformly continuous on $\mathbb{R}$

D. f(x).g(x) is uniformly continuous on $\mathbb{R}$

68.Define

$$f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2}, (x,y) \neq (0,0) \\ 0,\ (x,y) = (0,0) \end{cases}$$

Which of the following statements are true?

A. f is discontinuous at (0, 0)

B. f is continuous at (0, 0)

C. f is not differentiable at (0, 0)

D. All directional derivatives of f at (0, 0) exist.

69.Define

$$f(x,y)=\begin{cases}\frac{x^2-y^2}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Which of the following statements are true?

A. f is continuous at (0, 0)

B. f is bounded in a neighbourhood of (0, 0)

C. f is not bounded in any neighbourhood of (0, 0)

D. f has all directional derivatives at (0, 0)

70.Let $p: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$p(x,y) = \begin{cases}|x| if\ x \neq 0\\ |y| if\ x = 0\end{cases}$$

Which of the following statements are true?

A. p(x, y) = 0 if and only if x = y = 0

B. p(x, y) ≥ 0 for all x,y

C. p(ax, ay) = $|\alpha|p(x,y)$ for all $\alpha \in \mathbb{R}$ and for all x, y

D. $p(x_1 + x_2, y_1 + y_2) \leq p(x_1, y_1) + p(x_2, y_2)$ for all (x_1, y_1), (x_2, y_2)

71.Let P be a square matrix such that $P^2 = P$ Which of the following statement are true?

A. Trace of P is an irrational number

B. Trace of P = rank of P

C. Trace P is an integer

D. Trace of P is an imaginary complex number

72.Let A and B be n × n real matrices and let $C=\begin{pmatrix}A & B\\ B & A\end{pmatrix}$ which of the following statements are true?

A. If λ is an eigenvalue of A + B then λ is an eigenvalue of C

B. If λ is an eigenvalue of A – B then λ is an eigenvalue of C

C. If λ is an eigenvalue of A or B then λ is an eigenvalue of C

D. All eigenvalues of C are real

73.Let A be an n×n real matrix. Let b be a n×1 vector. Suppose Ax=b has no solution. Which of the following statements are true?

A. There exists a n×1 vector c such that Ax=c has a unique solution

B. There exist infinitely many vectors c such that Ax=c has no solution

C. If y is the first column of Ax=y has a unique solution

D. det A=0

74.Let A be a n × n matrix such that the first 3 rows of A are linearly independent and the first 5 column of A are linearly independent. Which of the following statement are true?

A. A has at least 5 linearly independent rows

B. $3 \le \rho(A) \le 5$

C. $\rho(A) \ge 5$

D. $\rho(A^2) \ge 5$

75.Let n be a positive integer and F be an non-empty proper subset of {1, 2,…,n} Define $\langle x, y\rangle_F = \sum_{K\in F} x_k y_{k,} x = (x_1, \ldots, x_n), y = (y_1, \ldots, y_n) \in \mathbb{R}^2$ Let $T = \{x \in \mathbb{R}^2 : \langle x, y\rangle_F = 0\}$ Which of the following statement are true? For $y \in \mathbb{R}^n y \neq 0$

A. $inf_{x\in T}\langle x+y, x+y\rangle_F = \langle y, y\rangle_F$

B. $suf_{x\in T}\langle x+y, x+y\rangle_F = \langle y, y\rangle_F$

C. $inf_{x\in T}\langle x+y, x+y\rangle_F < \langle y, y\rangle_F$

D. $sup_{x\in T}\langle x+y, x+y\rangle_F > \langle y, y\rangle_F$

76.Let $v \in \mathbb{R}^3$ be a non-zero vector. Define a linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ by $T(x) = x - 2\frac{x.v}{v.v}v$ where x.v denotes the standard inner product $\mathbb{R}^{3.}$ Which of the following statements are true?

A. The eigenvalues of T are +1, -1

B. The determinant of T is -1

C. The trace of T is + 1

D. T is distance preserving

77.A quadratic form Q(x, y, z) over $\mathbb{R}$ represent 0 non trivially if there exists $(a,b,c) \in \mathbb{R}^3 - \{(0,0,0)\}$ such that Q(a, b, c) = 0 Which of the following quadratic forms Q(x, y, z) over $\mathbb{R}$ represent 0 non trivially?

A. $Q(x, y, z) = xy + z^2$

B. $Q(x, y, z) = x^2 + 3y^2 - 2z^2$

C. $Q(x, y, z) = x^2 - xy + y^2 - z^2$

D. $Q(x,y,z)=x^2+xy+z^2$

78.Let Q(x, y, z) be a real quadratic form. Which of the following statements are true?

A. $Q(x_1 + x_2, y, z) = Q(x_1, y, z) + Q(x_2, y, z)$f or all x_1,x_2,y,z

B. $Q(x_1 + x_2, y_1 + y_2, 0) + Q(x_1 - x_2, y_1 - y_2, 0) = 2Q(x_1, y_1, 0) + 2Q(x_2, y_2, 0)$ for all x_1, x_2, y_1, y_2

C. $Q(x_1 + x_2, y_1 + y_2, z_1 + z_2) = Q(x_1, y_1, z_1) + Q(x_2, y_2, z_2)$ for at least one choice of $x_1, x_2, y_1, y_2, z_1, z_2$

D. $2Q(x_1 + x_2, y_1 + y_2, 0) + 2Q(x_1 - x_2, y_1 - y_2, 0) = Q(x_1, y_1, 0) + Q(x_2, y_2, 0)$f or all x_1, x_2, y_1, y_2

79.For $z \neq -i$, let $f(z) = \exp\left(\frac{1}{z+i}\right) - 1$. Which of the following are true?

A. f has finitely many zeros

B. f has a sequence of zeros that converges to a removable singularity of f

C. f has a sequence of zero that converge to a pole of f

D. f has a sequence of zeros that converge to an essential singularity of f

80.Let f be a holomorphic function on the open units disc $D = \{z \in \mathbb{C} : |z| < 1\}$ suppose that $|f| \geq 1$ on $\mathbb{D}$ and f(0) = i which of the following are possible values of $f\left(\frac{1}{2}\right)$?

A. -i

B. i

C. 1

D. -1

81.Let $D = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc and f: $\mathbb{D} \to \mathbb{D}$ be a holomorphic function. Suppose that f(0)=0 and f′(0) = 0 which of the following possible values are of $f\left(\frac{1}{2}\right)$?

A. 1/4

B. -1/4

C. 1/3

D. -1/3

82.Let n be a positive integer. For a real number R > 1 let $z(\theta) = Re^{i\theta}, 0 \leq \theta < 2\pi$ the set $\{\theta \in [0,2\pi): |z(\theta)^n + 1| = |z(\theta)|^n - 1\}$ contains which of the following sets?

A. $\{\theta \in [0,2\pi): \cos n\theta = 1\}$

B. $\{\theta \in [0,2\pi): \sin n\theta = 1\}$

C. $\{\theta \in [0,2\pi): \cos n\theta = -1\}$

D. $\{\theta \in [0,2\pi): \sin n\theta = -1\}$

83.Which of the following statements are true?

A. $\mathbb{Q}$ has countably many subgroups

B. $\mathbb{Q}$ has uncountably many subsets

C. Every finitely generated subgroup of $\mathbb{Q}$ is cyclic

D. $\mathbb{Q}$ is isomorphic to $\mathbb{Q}$X$\mathbb{Q}$ as groups

84.Let $SL_2(\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(\mathbb{Z}) \mid ad - bc = 1 \right\}$ and for any prime p, let

$$\Gamma(P) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in (SL_2(Z)) \mid \begin{matrix} a \equiv 1(\text{mod} p), d \equiv 1(\text{mod} p) \\ c \equiv 0(\text{mod} p), b \equiv 0(\text{mod} p) \end{matrix} \right\}$$

A. Γ(p) is a subgroup of $SL_2(\mathbb{Z})$

B. Γ(p) is not a normal subgroup of $SL_2(\mathbb{Z})$

C. Γ(p) has atleast two element

D. Γ(p) is uncountable

85.Let G be a finite group. Which of the following are true?

A. If $g \in G$ has order m and if $n \geq 1$ divides m then G has a subgroup of order n.

B. If for any two subgroup A and B of G either $A \subset B$ or $B \subset A$ then G is cyclic

C. If G is cyclic, then for any two subgroups A and B of G either $A \subset B$ or $B \subset A$

D. If for every positive integer m dividing |G|, G has subgroup of order m, then G is abelian

86.Let R, S be commutative rings with unity, $f: R \to S$ be surjective ring homomorphism, $Q \subseteq S$ be a non-zero prime ideal. Which of the following statements are true?

A. $f^{-1}(Q)$ is a non-zero prime ideal in R

B. $f^{-1}(Q)$ is a maximal ideal in R if R is a PID

C. f^{-1} (Q) is a maximal ideal in R if R is a finite commutative ring with unity

D. $f^{-1}(Q)$ is a maximal ideal in R if $x^5 = x$ for all $x \in R$

87.Consider the polynomial $f(x) = x^2 + 3x - 1$. Which of the following statement are true?

A. f is irreducible over $\mathbb{Z}[\sqrt{13}]$

B. f is irreducible over $\mathbb{Q}$

C. f is reducible over $\mathbb{Q}[\sqrt{13}]$

D. $\mathbb{Z}[\sqrt{13}]$ is a unique factorization domain.

88.Let p be an odd prime such that $p \equiv 2$ (mode 3). Let $\mathbb{F}_p$ be the field with p elements. Consider the subset E of $\mathbb{F}_p \times \mathbb{F}_p$ given by $E = \{(x, y) \in \mathbb{F}_p \times \mathbb{F}_p : y^2 = x^3 + 1\}$ which of the following are true?

A. E has at least two elements

B. E has at most 2p elements

C. E has at most p^2 elements

D. E has at least 2p elements

89.Consider the subset of $\mathbb{R}^2$ defined as follows $A = \{(x, y) \in \mathbb{R} \times \mathbb{R}: (x-1)(x-2)(y-3)(y+4) = 0\}$ which of the following statements are true?

A. A is connected

B. A is compact

C. A is closed

D. A is dense

90.Let X be a non-empty set. Suppose that τ_1 and τ_2 are two topologies over X such that $\tau_2 \subset \tau_1$ which of the following statements imply that $\tau_1 = \tau_2$?

A. (X, τ_1) is compact and τ_1 is T_2 (Hausdorff)

B. (X, τ_1) is compact and τ_2 is T_2 (Hausdorff)

C. The connected component of both (X, τ_1) and (X, τ_2) is same

D. For any subset $A \subset X$ the closure of A in (X, τ_2) is contained in the closure of A in (X, τ_1)

91.The following two-point boundary value problem $y''(x) + \lambda y(x) = 0$ $x \in (0, \pi)$, $y(0) = 0$ and $y(\pi) = 0$ has a trivial solution y = 0, it also has non-trival solution for

A. No values of $\lambda \in \mathbb{R}$

B. $\lambda = 1$

C. $\lambda = n^2$ for all $n \in \mathbb{N},\ n > 1$

D. $\lambda \le 0$

92.Let A be a n × n matrix with distinct eigenvalues $\{\lambda_1 \ldots \lambda_n\}$ with corresponding linearly independent eigenvectors $\{v_1 \ldots v_n\}$.

Then, the non-homogenous differential equation

$x'(t) = Ax(t) + e^{\lambda_1 t} \cdot v_1$

A. does not have a solution of the form $e^{\lambda_1 t}$a for any vector a $\in \mathbb{R}^n$

B. has a solution of the form $e^{\lambda_1 t}$a for vector a $\in \mathbb{R}^n$

C. has a solution of the form $e^{\lambda_1 t}a + te^{\lambda_1 t}b$ for any vector a, b $\in \mathbb{R}^n$

D. does not have a solution of the form $e^{\lambda_1 t}a + te^{\lambda_1 t}b$ for any vector $a, b \in \mathbb{R}^n$

93.Consider the solution $y_1 := \begin{pmatrix} e^{-3t} \\ e^{3t} \\ 0 \end{pmatrix}, y_2 := \begin{pmatrix} 0 \\ e^{-5t} \\ e^{-5t} \end{pmatrix}$

To the homogeneous linear system of differential equation

$$y'(t) = \begin{bmatrix} -5 & -2 & -2 \\ 1 & -4 & -1 \\ -1 & 1 & -6 \end{bmatrix} y(t)$$

Which of the following statements are true?

A. y_1 and y_2 form a basis for the set of all solutions to (*)

B. y_1 and y_2 are linearly independent but do not form a basis for the set of all solution to (*)

C. There exists another solution y_3 such that {y_1, y_2, y_3} form a basis for the set of all solutions to (*)

D. y_1 and y_2 are linearly dependent

94.Consider the partial differential equations:

(i) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x^2 y} + (1 - \mathrm{Sgn}(y))\frac{\partial^2 u}{\partial^2 y} = 0$

(ii) $y\frac{\partial^2 y}{\partial r^2} + x\frac{\partial^2 y}{\partial y^2} = 0$

Which of the following statements are true?

A. Equation (i) is parabolic for y > 0 and elliptic for y < 0

B. Equation (i) is hyperbolic for y > 0 and elliptic for y < 0

C. Equation (ii) is elliptic in I and III quadrant and hyperbolic in II and IV quadrant

D. Equation (ii) is hyperbolic in I and III quadrant and elliptic in II and IV quadrant

95.Consider the Cauchy problem $\frac{\partial^2 u}{\partial x \partial y} = 0, \ |x| < 1, \ 0 < u < 1$

$u(x, x^2) = 0, \ \frac{\partial u}{\partial y}(x, x^2) = g(x), |x| < 1$

Which of the following statements are true?

A. A necessary condition for a solution to exist is that g is an odd function

B. A necessary condition for a solution to exist is that g is even function

C. The solution (if it exists) is given by u(x,y) $= 2\int_x^{\sqrt{y}} zg(z)dz$

D. The solution (if it exists) is given by u (x, y) $= 2\int_{\sqrt{y}}^{x^2} zg(z)dz$

96.Fix a $\alpha \in (0,1)$. Consider the iteration defined by

$$(*) x_{k+1} = \frac{1}{2}(x_k^2 + \alpha), k = 0,1,2\ldots$$

The above iteration has two distinct fixed points ξ_1 and ξ_2 such that $0 < \xi_1 < 1 < \xi_2$.

Which of the following statements are true?

A. The iteration (*) is equivalent to the recurrence relation $x_{k+1} - \xi_1 = \frac{1}{2}(x_k + \xi_1)(x_k - \xi_1), k = 0,1,2\ldots$

B. The iteration (*) is equivalent to the recurrence relation $x_{k+1} - \xi_1 = \frac{1}{2}(x_k + \xi_2)(x_k - \xi_1), k = 0,1,2,\ldots$

C. If $0 \le x_0 < \xi_2$ then $lim_{k\to\infty} x_k = \xi_1$

D. If $-\xi_2 < x_0 \le 0$ then $lim_{k\to\infty} x_k = \xi_1$

97.Consider the function $f: [0,1] \to \mathbb{R}$ defined by

$$f(x) := \begin{cases} 2^{-\left\{1+(log_2\left(\frac{1}{x}\right)^{\frac{1}{\beta}}\right\}^{\beta}} & for\ x \in (0,q] \\ 0 for\ x = 0 \end{cases}$$

Where $\beta \in (0,\infty)$ is parameter? Consider the iterations

$$x_{k+1} = f(x_k), k = 0,1,\ldots; x_0 > 0$$

Which of the following statement are true about the iteration?

A. For $\beta = 1$, the sequence $\{x_k\}$ converges to 0 linearly with rate of convergence $\log_{10}2$

B. For $\beta > 1$, the sequence $\{x_k\}$ does not converge to 0

C. For $\beta \in (0,1)$, the sequence $\{x_k\}$ converges to 0 sublinearly

D. For $\beta \in (0,1)$, the sequence $\{x_k\}$ converges to 0 superlinearly

98.The extremal of the functional

$f(x,y,y')=e^x\cdot\sqrt{1+y'^2}, y\in C^2[0,1]$

Is of the form

A. y = $\sec^{-1}\left(\frac{x}{c_1}\right)+c_2$, where c_1 and c_2 are arbitrary constant

B. y = $\sec^{-1}\left(\frac{x}{c_1}\right)+c_2$, where $|c_1| < 1$ and c_2 is an arbitrary constants

C. y = $\tan^{-1}\left(\frac{x}{c_1}\right)+c_2$, where c_1 and c_2 are arbitrary constants

D. y = $\tan^{-1}\left(\frac{x}{c_1}\right)+c_2$,, where $|c_1| > 1$ and c_2 is an arbitrary constant

99.Consider the functional

$J(y)=\int_0^\pi (y'^2-ky^2)dx$ with boundary condition $y(0)=0, y(\pi)=0$

Which of the following statements are true?

A. It has a unique extremal for all $k \in \mathbb{R}$

B. It has at most one extremal if $\sqrt{k}$ is not an integer

C. It has at most many extremal if $\sqrt{k}$is not an integer

D. It has a unique extremal if $\sqrt{k}$is an integer

100.For the Fredholm integral equation

$y(s)=\lambda\int_0^1 e^x e^t y(t)dt$

Which of the following statements are true?

A. It has a non-trivial solution satisfying $\int_0^1 e^t y(t)dt = 0$

B. Only the trivial solution satisfies $\int_0^1 e^t y(t)dt = 0$

C. It has non-trivial solution for all $\lambda \neq 0$

D. It has non-trivial solutions only if $\lambda = \frac{2}{e^2-1}$ and $\int_0^1 e^t y(t)dt \neq 0$

101.Consider the particle differential equation:

$$z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + \frac{\partial z}{\partial x}\frac{\partial z}{\partial y}$$

Which of the following statements are true?

A. The complete integral is $z = xa + yb + ab, a, b$ arbitrary constants

B. The complete integral $z = xa + yb + \sqrt{a^2 + b^2}, a, b$ arbitrary constants

C. The particular solution passing through x = 0 and $z = y^2$ is $\left(\frac{x}{4} - y\right)^2$

D. The particular solution passing through x = 0 and $z = y^2$ is $\left(\frac{x}{4} + y\right)^2$

102.Consider a dynamical system with the Lagrangian function L(q,q) = T – U, where the kinetic

$$T = a(q)q^2 \geq 0$$

And the potential energy $U := U(q)$ ad $a(q) > 0$. Which of the following statements are true?

A. The associated Lagrange's equation is $\frac{d}{dt}\frac{\partial L}{a\dot{q}} = \frac{\partial L}{aq}$

B. The associated Lagrange's equation is $\frac{d}{dt}\frac{\partial L}{aq} = \frac{\partial L}{a\dot{q}}$

C. The point $(q_0, a\dot{q})$ is an equilibrium position of the dynamical system if and only if

$$\dot{q}_0 = 0 \left.\frac{\partial U}{\partial q}\right|_{q=q_0} = 0$$

D. The point $(q_0, \dot{q}_0)$ is an equilibrium position of the dynamical system if and only

$$\dot{q}_0 = 0 \left.\frac{\partial U}{\partial q}\right|_{q=q_0} > 0$$

103.Let X and Y be independent random variable with E(X)=E(Y)=0 and Var(x)=Var(Y)=1 . Let Z=X+Y. which of the following statements are correct?

A. $P(|Z| > \varepsilon) \leq 2/\varepsilon^2$

B. $E(|Z|) \leq \sqrt{2}$

C. $E(Z^2) = 2$

D. $P(Z \leq 0) = P(Z \geq 0)$

104.Fr n > 1, let X_1, X_2 …, X_n be random variables such that $E(X_i) = 0$ and $E(X_i^2) = 1$ for all I and $E(X_iX_j) = \rho$ for all $i \neq j$. Which of the following statements are true?

A. $\rho = 0$ if and only if $X_1, X_2 \dots, X_n$ are independent

B. Var $(X_1 + X_2 + \dots + X_n)$ if and only if $X_1, X_2 \dots, X_n$ are independent

C. Var $(X_1 + X_2 + \dots + X_n) = n$ if and only if $X_1, X_2 \dots, X_n$ are pairwise independent

D. Var $(X_1 + X_2 + \dots + X_n) = n$ if and only if $\rho = 0$

105.Consider a Markov chain with a countable state space S. Identify the correct statements.

A. If the Markov chain is aperiodic and irreducible then three exists a stationary distribution

B. If the Markov chain is aperiodic and irreducible then three is at most one stationary distribution

C. If S is finite then there exists a stationary distribution

D. If S is finite then three is exactly one stationary distribution

106.Consider a Markov chain with transition probability matrix

$$P = \begin{pmatrix} 0\ 0\ \frac{1}{2}\ 0\ \frac{1}{2} \\ 0\ \frac{1}{2}\ 0\ \frac{1}{2}\ 0 \\ \frac{1}{2}\ 0\ 0\ 0\ \frac{1}{2} \\ 0\ \frac{1}{2}\ 0\ \frac{1}{2}\ 0 \\ \frac{1}{2}\ 0\ \frac{1}{2}\ 0\ 0 \end{pmatrix}$$

Let $\pi = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$

A. π is a stationary distribution

B. If η is a stationary distribution, then $\eta = \pi$

C. The Markov chain in periodic

D. The Markov chain is irreducible

107.Suppose $X_1, X_2, \dots, X_n$ are i.i.d. random variable with characteristic function $\phi(t;\theta) = E[e^{itX_t}]$, where $\underline{\theta} \in \mathbb{R}^k$ the parameter of the distribution is. Let $Z = X_1 + X_2 + \dots + X_n$. Then for which of the following distributions of X_1 would the characteristic function Z be of the form $\phi(t;\underline{\alpha})$ for some $\underline{\alpha} \in \mathbb{R}^k$?

A. Negative Binomial

B. Geometric

C. Hypergeometric

D. Discrete Uniform

108.Let $X_1, X_2,\dots, X_n$ be i.i.d. with the common pdf $f(x|\theta) = \frac{\theta}{x^{\theta+1}}$ for x > 1 where $\theta > 1$ is an unknown parameter. Which of the following estimators of θ are consistent?

A. $\frac{1}{n}\sum_{i=1}^{n} X_i$

B. $\frac{1}{n}\sum_{i=1}^{n} log(X_i)$

C. $\frac{n}{\sum_{i=1}^{n} X_i}$

D. $\frac{n}{\sum_{i=1}^{n} log(X_i)}$

109.Let $X_1,X_2\dots,X_n$ be i.i.d. with the common probability mass function $p(x|\theta) = \theta^x(1-\theta)^{1-x}, x = 0$ or 1, and $0 \le \theta \le \frac{1}{2}$ Then

A. The method of moment's estimator of θ is $\frac{1}{2n}\sum_{i=1}^{n} X_i$

B. The MLE of θ is $min_{1\le i\le n} X_i$

C. The method of moment's estimator of θ is $min_{1\le i\le n} X_i$

D. The MLE of $\theta$is $min\{\frac{1}{n}\sum_{i=1}^{n} X_i , \frac{1}{2}\}$

110.Let the pdf of X be $f(x|\theta) = \frac{2x}{\theta^2}$, for $0 < x < \theta$, where $\theta > 0$ is unknown parameter. Which of the following are $100(1-\alpha)\%$ confidence intervals for θ?

A. $\left[x, \frac{x}{\sqrt{\alpha}}\right]$

B. [X, 2X]

C. $\left[\frac{\sqrt{2}}{\sqrt{2-\alpha}}X, \frac{\sqrt{2}}{\sqrt{\alpha}}X\right]$

D. [0, X]

111.X has binomial distribution with parameter n and p. suppose that n is given and the unknown parameter p has prior distribution, which is uniform on the interval [0, 1]. Consider the squared error loss function and the observation X = n. Identify the correct statement.

A. The Bayes estimate of p is $\left(\frac{n+1}{n+2}\right)$

B. The Bayes estimate of p is $2^{-1/(n+1)}$

C. The median of the posterior distribution of p is $2^{-1/(n+1)}$

D. The median of the posterior distribution of p is $\left(\frac{n+1}{n+2}\right)$

112.Let X_1,X_2,X_3 be a random sample from the uniform distribution on the interval $(0,\theta)$. Suppose the prior distribution of θ is uniform on the interval (0, 1). Let $X_{(3)} = \max\{X_1,X_2,X_3\}$. Consider the squared error loss function. Which of the following statements are necessarily true?

A. Bayes estimator of θ is unique

B. $\frac{1}{x_{(3)}}$ Is a Bayes estimator of θ

C. $X_{(3)}$ Is a Bayes estimator of θ

D. $\frac{1-x_{(3)}}{x_{(3)}}$ Is a Bayes estimator of θ

113.Consider the Gauss-Markov model $Y_{n\times 1} = X_{n\times p}\beta_{p\times 1} + \varepsilon_{n\times 1}$, where $E(\varepsilon) = 0$ and Dispersion $(\varepsilon) = \sigma^2 I_{n\times n}$. Suppose that p < n. Which of the following are correct?

A. Least-squares estimate of $\beta$is unique

B. Least-square estimate of an estimable linear function of β is unique

C. Least-squares estimate $X\beta$ is unique

D. Determinant $(X^TX) > 0$

114.Let p > 1 and $1 > \rho \geq 0$. Consider a multiple linear regression problem with p independent variables $X_1,X_2,\ldots X_p$ and a dependent variable Y. Suppose that the correlation between Y and X_1 is $\rho$and the correlation between X_i and X_j is also ρ for all $1 \leq i \leq j \leq p$. Which of the following a correct?

A. The multiple correlation between Y and $(X_1, \ldots, X_p)$ is larger than or equal to ρ

B. The multiple correlation between Y and $(X_1, \ldots, X_p)$ will be ρif $\rho = 0$

C. The multiple correlation between Y and $(X_1, \ldots, X_p)$ will be ρ only if $\rho = 0$

D. The multiple correlation between Y and $(X_1, \ldots, X_p)$ tends to 1 as $p \to \infty$

115.Let n > 2 and $0 < \theta \frac{\pi}{2}$ be fixed. Let $X_1 \ldots X_n$ be i.i.d. normal random variable with mean zero and variance $\sigma^2 > 0$. For i = 1, …, n define

$Y_{2i-1} = X_i \cos\theta$ and $y_{2i} = X_i \sin\theta$. Further, let $Z^T = (Y_1, Y_2, \ldots Y_{2n})$ and $V^T = (X_1,Y_1,Y_2,X_2,Y_3,Y_4, \ldots,X_n, Y_{2n-1}, Y_{2n})$

which of the following statements are correct?

A. Z^T has multivariate normal distribution

B. There exists a constant C, such that CZ^TZ has a chi-square distribution

C. V^T has a multivariate normal distribution

D. $E\left(\frac{1}{V^TV}\right) < \infty$

116.For circular systematic sampling, which of the following are correct?

A. Sample mean is an unbiased estimate for population mean but sample variance is not an unbiased estimate for population variance

B. Sample mean and sample variance are unbiased estimate for population mean and population variance respectively

C. Sample mean is not an unbiased estimate for population mean but sample variance is an unbiased estimate for population variance

D. Neither sample mean nor sample variance is an unbiased estimate for their population counterparts

117.In a Randomized Block Design with one observation per cell, and date satisfying the standard linear model, which of the following are correct?

A. Mean treatment effects are estimable

B. Mean block effects are estimable

C. Treatment-Block interactions are NOT estimable

D. Treatment and block effects as well as treatment-block interactions are estimable

118.Suppose $\lambda(t)$ for $t \geq 0$ is a continuous hazard function of non-negative random variable X, where $\lambda(t) \geq 1$. Which of the following statements are always true?

A. $\frac{1}{\lambda(t)}$ is a hazard function

B. $\lambda^2(t)$ is also a hazard function

C. $c\lambda(t)$ for $c \geq 0$ is also a hazard function

D. $log\, \lambda(t)$ is a hazard function

119.Let $X_i = \theta + \varepsilon_i,\ 1 \leq i \leq n$ where $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ are i.i.d. with pdf $g(\varepsilon) = |\varepsilon|, -1 < \varepsilon < 1$. Let $T_1 = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $T_2 = X_{([\frac{3n}{4}]+1)}$, the sample 75th percentile. Which of the following are correct?

A. T_1 is a consistent and asymptotically normal estimator of θ

B. $T_2 - \frac{1}{\sqrt{2}}$ is consistent and asymptotically normal estimation of θ

C. The asymptotic variance of T_1 is $\frac{1}{2n}$

D. The asymptotic variance of T_2 is $\frac{3}{8n}$

120.Consider a M/M/1 queue with arrival rate λ and service rate μ. Let $Q_0 = 0$ and Q_t denote the queue length at time t. which of the following statements are true?

A. (Q_t) admits a stationary distribution if and only if $\lambda \leq \mu$

B. The stationary distribution of the process (Q_t) is geometric, when it exists

C. $lim_{t\to\infty} P\,(Q_t > k) = 1$ for all $k < \infty$ if $\lambda > \mu$

D. $lim\, e_{t\to\infty} P(Q_t > k) = 2^{-(k+1)}$ for all $k < \infty$ if $\lambda = \frac{\mu}{2}$

ANSWERS

1. Ans. B.
2. Ans. C.
3. Ans. C.
4. Ans. D.
5. Ans. D.
6. Ans. D.
7. Ans. B.
8. Ans. C.
9. Ans. D.
10. Ans. D.
11. Ans. D.
12. Ans. C.
13. Ans. B.
14. Ans. A.
15. Ans. A.
16. Ans. B.
17. Ans. C.
18. Ans. C.
19. Ans. D.
20. Ans. C.
21. Ans. A.
22. Ans. C.
23. Ans. A.
24. Ans. D.
25. Ans. D.
26. Ans. C.
27. Ans. C.
28. Ans. C.
29. Ans. B.
30. Ans. D.
31. Ans. D.
32. Ans. B.
33. Ans. B.
34. Ans. B.
35. Ans. A.
36. Ans. A.
37. Ans. C.
38. Ans. C.
39. Ans. D.
40. Ans. A.
41. Ans. C.
42. Ans. D.
43. Ans. C.
44. Ans. A.
45. Ans. A.
46. Ans. D.
47. Ans. C.
48. Ans. A.
49. Ans. A.
50. Ans. C.
51. Ans. C.
52. Ans. B.
53. Ans. C.
54. Ans. A.
55. Ans. A.
56. Ans. C.
57. Ans. D.
58. Ans. A.
59. Ans. A.
60. Ans. D.
61. Ans. B.
62. Ans. A. D.
63. Ans. A.
64. Ans. A. D.
65. Ans. C. D.
66. Ans. B.
67. Ans. B. C. D.
68. Ans. B. D.
69. Ans. B.
70. Ans. A.
71. Ans. B. C.
72. Ans. A. B.
73. Ans. B. D.
74. Ans. A. C.
75. Ans. A.
76. Ans. A.B.C. D.
77. Ans. A. B. D.
78. Ans. B. C.
79. Ans. D.
80. Ans. B.
81. Ans. A. B.
82. Ans. C.
83. Ans. B. C.
84. Ans. A. C.
85. Ans. A. D.
86. Ans. A. B. C. D.
87. Ans. A. B. C.
88. Ans. A. C.
89. Ans. A. C.
90. Ans. A.
91. Ans. B. C.
92. Ans. A. C.
93. Ans. B. C.
94. Ans. B. C.
95. Ans. B. C.
96. Ans. A.
97. Ans. A.
98. Ans. B.
99. Ans. B. C.
100. Ans. B. D.
101. Ans. A. C.
102. Ans. A.
103. Ans. A. B. C.
104. Ans. D.
105. Ans. A.
106. Ans. A.
107. Ans. D.
108. Ans. D.
109. Ans. D.
110. Ans. B. D.
111. Ans. B. D.
112. Ans. A.
113. Ans. B.
114. Ans. A.
115. Ans. A. B. C. D.
116. Ans. A.
117. Ans. A. B. C.
118. Ans. B. C.
119. Ans. A. B. C. D.
120. Ans. B. C. D.