CSIR-NET FEB 2022

MATHEMATICAL SCIENCE QUESTION PAPER

https://byjusexamprep.com/

1.A piece in a board game starts at the centre of the board having 5 × 5 squares. The piece can be moved one square horizontally or vertically in one move. If all its moves are random, the chance that it will be in one of the outer squares at the end of 2 move is

A. 1/4

B. 2/3

C. 16/25

D. 1/2

2.A book has 40 pages and each page has x lines. If the number of lines were reduced by 2 in each page, the number of pages would increase by 10 for the identical text. What is the value of x?

A. 7

B. 10

C. 20

D. 30

3.Two windows of a building are exactly one above the other and their lower edges are 2m and 4m above the ground. The angle of elevation of the bird from the lower edge of the lower window is 60° and that from the lower edge of the upper window is 30°. How high is the bird above the ground?

A. 5 m

B. 8 m

C. 10 m

D. 15 m

4.Six indistinguishable balls are to be distributed amongst A, B and C, such that each gets at least one. Then the number of ways to make this distribution is

A. 6

B. 10

C. 18

D. 15

5.Section A has 24 students. If one student of this section is exchanged for another in section B, then the average mark of A is increased by 1.25, while that of B reduced by 1. The number of students in section B is

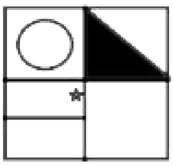
A. 16

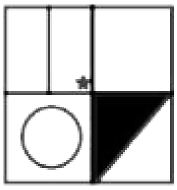
B. 20

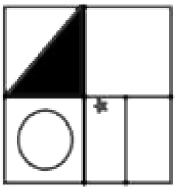
C. 25

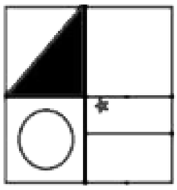
D. 30

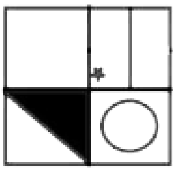
6.Which of the following figures matches exactly the figure below, but for orientation?



A. 

B. 

C. 

D. 

7.Six persons P, Q, R, S, T and U sit around a circular table with equal distance between neighbours. P is to the immediate left of R. T and S do not sit next to each other, and T and R are diametrically opposite to each other. Which of the following is NOT possible?

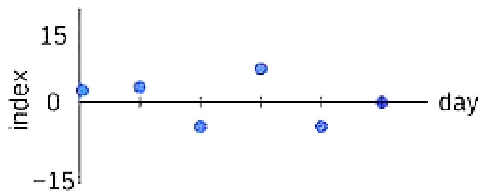
A. Q and P are sitting diametrically opposite to each other

B. P and U are sitting diametrically opposite to each other

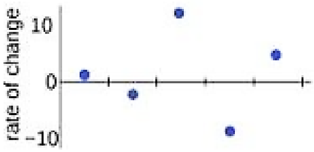
C. S and T are sitting diametrically opposite to each other

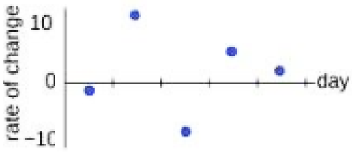
D. S and Q are sitting diametrically opposite to each other

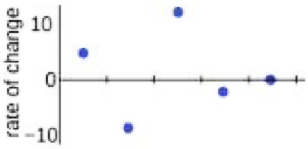
8.The graph shows the day-on-day changes in a certain index.

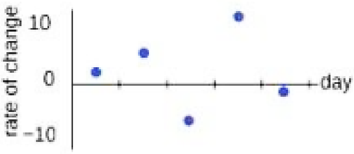


Which of the following is the correct graph of the rate of change of the index?

A. 

B. 

C. 

D. 

9.If all the planets in our solar system were to move in the same plane, it would necessarily imply that

A. The Sun is at the center of the solar system

B. The visible planets will appear aligned along a straight line in the sky as seen from the earth

C. Motion of planets is governed by Newton's law of gravity

D. The radii of the orbits of the planets are in harmonic progression

10.Machine A cuts rods whose mean length is 10 cm with a standard deviation of 3 mm. Machine B cuts rods of the mean length 20 cm with a standard deviation of 4 mm. Rods of 30 cm mean length are made, each by joining one rod from each machine. The standard deviation in the length of the joined rod is

A. 10 mm 10

B. 7 mm

C. 5 mm

D. 3.5 mm

11.At a particular location, a mobile app shows that there are 6, 13, 28 and 50 infected persons with radii of 1, 2, 3 and 4 km, respectively. Within which radius is the density of infected persons the largest?

A. 1 km

B. 2 km

C. 3 km

D. 4 km

12.The square of a two digit number, with non-zero digits, is the number itself preceded by the digit C. Then C is

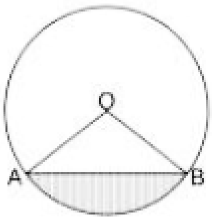
A. 1

B. 2

C. 4

D. 6

13.As shown in the figure, a chord AB of circle of unit radius subtends an angle of 90º at the center, O. The area of the shaded region is



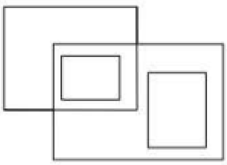
A. $\frac{1}{2}\left(\pi - \frac{1}{2}\right)$

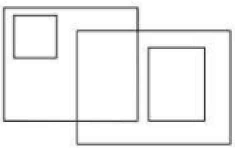
B. $\frac{1}{4}(\pi - 2)$

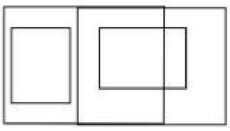
C. $\frac{1}{2}(\pi - 2)$

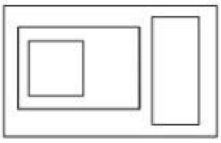
D. $\frac{1}{4}\left(\pi - \frac{1}{2}\right)$

14.An appropriate Venn diagram to represent the relationships between the categories 'inflammable substance', 'water', 'petrol' and 'liquid' is

A. 

B. 

C. 

D. 

15.Water from a completely filled cylindrical jar is poured into smaller cylindrical jars having 1/10th of its diameter but same height. How many smaller jars can be completely filled with water?

A. 10

B. 31

C. 100

D. 314

16.The number of dates between 1st January 2000 and 31st December 2020 when written in the format DDMMYYYY that read the same from left to right and right to left (e.g. 12th February 2021) is

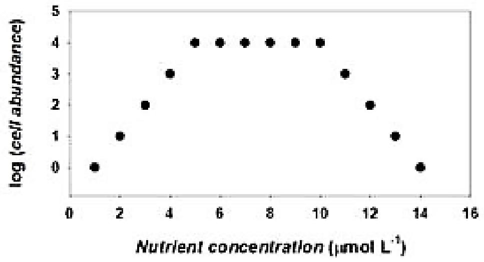
A. 6

B. 7

C. 20

D. 21

17.The figure shows the result of an experiment in which cells of a certain bacterium were grown in different nutrient concentrations while keeping all other parameters constant. Which of the following inferences is correct?



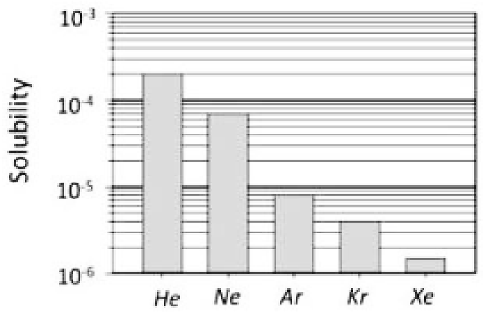
A. Nutrients do not have any role in cell growth.

B. There is a linear increase in cell abundance at low nutrient concentrations.

C. Cell abundance was limited by the availability of nutrients but a high nutrient concentration seems toxic for the bacteria.

D. Cell abundance decreased rapidly once nutrient concentration reached 5µ mol L^{-1}.

18.The bar chart shows the solubility of five species of noble gases in a solvent. The ratio of the solubility of He and Kr is



A. 200/4

B. 100/3

C. 900/7

D. 300/2

19.A and B have coins of Rs.1, Rs.2, Rs.5 and Rs.10, in the ratio 4:3:6:2 and 3:5:7:3, respectively. A has Rs.6/- more than B. Which of the following can be the number of coins with A and B, respectively?

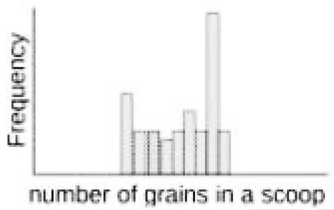
A. 42, 36

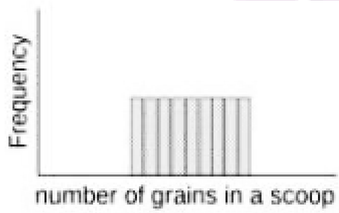
B. 45, 54

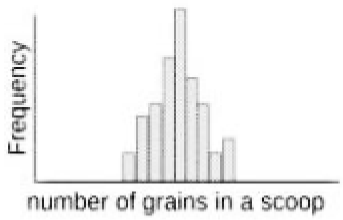
C. 60, 54

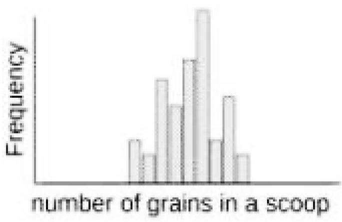
D. 60, 72

20.An experiment is done to count the number of small grains that make up a level scoop, taking adequate precautions. If the measurements are repeated several times, which of the following frequency distributions is the most unlikely to occur?

A. 

B. 

C. 

D. 

21.Let A be a 4 × 4 matrix such that −1, 1, 1, −2 are AS eigenvalues. If B = A^4 − $5A^2$ + 5I then trace (A + B) equals

A. 0

B. −12

C. 3

D. 9

22.Let M = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 2 & -1 \\ 1 & 1 & 3 \end{bmatrix}$. Given that 1 is an eigenvalue of M, which of the following statements is true?

A. −2 is an eigenvalue of M.

B. 3 is an eigenvalue of M

C. The eigen spare of each eigen value has dimension 1

D. M is diagonalizable

23.Let $S_1 = \frac{1}{3} - \frac{1}{2} \times \frac{1}{3^2} + \frac{1}{3} \times \frac{1}{3^3} - \frac{1}{4} \times \frac{1}{3^4} + \ldots$ and $S_2 = \frac{1}{4} + \frac{1}{2} \times \frac{1}{4^2} + \frac{1}{3} \times \frac{1}{4^3} + \frac{1}{4} \times \frac{1}{4^4}$ + …

Which of the following identities is true?

A. $3S_1 = 4S_2$

B. $4S_1 = 3S_2$

C. $S_1 + S_2 = 0$

D. $S_1 = S_2$

24.Consider the two statements given below:

I. There exists a matrix N $\in \mathbb{M}_4(\mathbb{R})$ such that {(1, 1, 1, −1), (1, −1, 1, 1)} is a basis of Row(N) and (1, 2, 1, 4) $\in$ Null(N)

II. There exists a matrix M $\in \mathbb{M}_4(\mathbb{R})$ such that $\{(1, 1, 1, 0)^T, (1, 0, 1, 1)^T\}$ $\in$ is a basis of Col(M) and $(1, 1, 1, 1)^T$, $(1, 0, 1, 0)^T \in$ Null(M)

Which of the following statements is true?

A. Statement I is FALSE and Statement II is TRUE

B. Statement I is TRUE and Statement II is FALSE

C. Both Statement I and Statement II are FALSE

D. Both Statement I and Statement II are TRUE

25.Let v = {A ∈ $M_{3\times3}(\mathbb{R})$At + A ∈ℝ . I}, where I is the identity matrix. Consider the quadratic form defined as q(A) = Trace$(A)^2$ −Trace(A^2). What is the signature of this quadratic form?

A. (+ + ++)

B. (+ 0 0 0)

C. (+ − − −)

D. (− − − 0)

26.Consider the sequence $\{a_n\}_{n>1}$, where $a_n = 3 + 5\left(-\frac{1}{2}\right)^n + (-1)^n\left(\frac{1}{4} + (-1)^n\frac{2}{n}\right)$

. Then the interval $\left(\lim_{n\to\infty} a_n, \lim_{n\to\infty} a_n\right)$is given by

A. (−2, 8)

B. $\left(\frac{11}{4}, \frac{13}{4}\right)$

C. (3, 5)

D. $\left(\frac{1}{4}, \frac{7}{4}\right)$

27.Let A and B be n × n matrices. Suppose the sum of the elements in any row of A is 2 and the sum of the elements in any column of B is 2 . Which of the following matrices is necessarily singular?

A. $I - \frac{1}{2}BA^T$

B. $I - \frac{1}{2}AB$

C. $I - \frac{1}{4}AB$

D. $I - \frac{1}{4}BA^T$

28.Which of the following sets are countable?

A. The set of all polynomials with rational coefficients

B. The set of all polynomials with real coefficients having rational roots

C. The set of all 2 × 2 real matrices with rational eigenvalues

D. The set of all real matrices whose row echelon form has rational entries

29.$\lim_{n\to\infty} \frac{1}{n}\left(1 + \sqrt{2} + \sqrt[3]{3} + \ldots + \sqrt[n]{n}\right)$

A. is equal to 0

B. is equal to 1

C. is equal to 2

D. does not exist

30.Let n > 1 be a fixed natural number. Which of the following is an inner product on the vector space of n × n real symmetric matrices?

A. (A, B) = (trace(A))(trace(B))

B. (A, B) = trace(AB)

C. (A, B) = determinant(AB)

D. (A, B) = trace(A) + trace(B)

31.Let f, g: $\mathbb{R} \to \mathbb{R}$ be given by and $f(x) = x^2$ and g(x) = sin x. Which of the following functions is uniformly continuous on $\mathbb{R}$?

A. h(x) = g(f(x))

B. h(x) = g(x)f(x)

C. h(x) = f(g(x))

D. h(x) = f(x) + g(x)

32.Let S = {1, 2, ..., 100} and let A = {1, 2, ..., 10} and B = {41, 42, ..., 50}. What is the total number of subsets of S, which have non-empty intersection with both A and B?

A. $\frac{2^{100}}{2^{20}}$

B. $\frac{100!}{10!10!}$

C. $2^{80}(2^{10} - 1)^2$

D. $2^{100} - 2(2^{10})$

33.Let f(z) be a non-constant. entire function and z = x + iy. Let u(x, y), v(x, y) denote its real and imaginary parts respectively. Which of the following statements is FALSE?

A. $u_x = v_y$ and $u_y = -v_x$

B. $u_y = v_x$ and $u_x = -v_y$

C. $|f'(x + iy)|^2 = u_x(x, y)^2 + v_x(x, y)^2$

D. $|f'(x + iy)|^2 = u_y(x, y)^2 + v_y(x, y)^2$

34.Let $\mathbb{D} \subset \mathbb{C}$ be the open unit disc $\{z \in \mathbb{C} : |z| < 1\}$ and $O(\mathbb{D})$ be the space of all holomorphic functions on $\mathbb{D}$.

Consider the sets A = $\{f \in O(\mathbb{D}): f\left(\frac{1}{n}\right) = \{e^{-n}\ if\ n\ is\ even\ for; n \geq 2\}$, B = $\{f \in O(\mathbb{D}): f(1/n) = (n - 2)/(n - 1), n \geq 2\}$. Which of the following statements is true?

A. Both A and B are non-empty,

B. A is empty and B has exactly one element

C. A has exactly one element and B is empty

D. Both A, B are empty

35.How many generators does a cyclic group of order 36 have?

A. 6

B. 12

C. 18

D. 24

36.Let (X, d) be a metric space and let f: X → X be a function such that d(f(x), f(y)) ≤ d(x, y) for x, y ∈ X. Which of the following statements is necessarily true?

A. f is continuous.

B. f is injective

C. f is surjective

D. is injective if and only if f is surjective

37.Let f be a rational function of a complex variable z given by f(z) = $\frac{z^3+2z-4}{z}$

The radius of convergence of the Taylor series of f at z = 1 is

A. 0

B. 1

C. 2

D. ∞

38.Let γ be the positively oriented circle {z ∈ℂ: |z| = 3/2}. Suppose that $\int_\gamma \frac{e^{i\pi z}}{(z-1)(z-2i)^2} dz = 2\pi i C$. Then |C| equals

A. 2

B. 5

C. 1/2

D. 1/5

39.Let S = {n: 1 ≤ n ≤ 999; 3|n or 37|n}. How many integers are there in the set S^C = {n: 1 ≤ n ≤ 999; n ∉ S}?

A. 639

B. 648

C. 666

D. 990

40.Which of the following statements is necessarily true for a commutative ring R with unity?

A. R may have no maximal ideals

B. R can have exactly two maximal ideals

C. R can have one or more maximal ideals but no prime ideals

D. R has at least two prime ideals

41.Consider the following two initial value ODEs

(A) $\frac{dx}{dt}$ = x^3, x(0) = 1;

(B) $\frac{dx}{dt}$ = $x\sin x^2$, x(0) = 2

Related to these ODEs, we make the following assertions.

I. The solution to (A) blows up in finite time.

II. The solution to (B) blows up in finite time.

Which of the following statements is true?

A. Both (I) and (II) are true

B. (I) is true but (II) is false

C. Both (I) and (II) are false

D. (I) is false but (II) is true

42.A body moves freely in a uniform gravitational field. Which of the following statements is true?

A. Stable equilibrium of the body is possible

B. Stable equilibrium of the body is not possible

C. Stable equilibrium of the body depends on the strength of the field

D. Equilibrium is metastable

43.Which of the following is an external of the functional J(y) = $\int_{-1}^{1}(y'^2 - 2xy)dx$ that satisfies the boundary conditions y(−1) = −1 and y(1) = 1 ?

A. $-\frac{x^3}{5} + \frac{6x}{5}$

B. $-\frac{x^5}{8} + \frac{9x}{8}$

C. $-\frac{x^3}{6} + \frac{7x}{6}$

D. $-\frac{x^3}{7} + \frac{8x}{7}$

44.Let the solution to the initial value problem y' = y – t^2 + 1, 0 ≤ t ≤ 2, y(0) = 0.5 be computed using the Euler's method with step-length h = 0 .4 . If y(0.8) and w(0 .8) denote the exact and approximate solutions at t = 0 .8, then an error bound for Euler's method is given by

A. $0.2(0.5e^2 - 2)(e^{0.4} - 1)$

B. $0.1(e^{0.4} - 1)$

C. $0.2(0.5e^2 - 2)(e^{0.8} - 1)$

D. $0.1(e^{0.8} - 1)$

45.Let u(x, y) solve the Cauchy problem $\frac{\partial u}{\partial y} + x\frac{\partial u}{\partial x}$ + u – 1 = 0 where $-\infty < x < \infty$, y ≥ 0 and u(x, 0) = sin x. Then u(0, 1) is equal to

A. $1 - \frac{1}{e}$

B. $1 + \frac{1}{e}$

C. $1 - \frac{1-\sin e}{e}$

D. $1 + \frac{1-\sin e}{e}$

46.If y(x) is a solution of the equation 4xy" + 2y' + y = 0

Satisfying y(0) = 1 . Then y"(0) is equal to

A. 1/24

B. 1/12

C. 1/6

D. 1/2

47.Which of the following partial differential equations is NOT PARABOLIC for all x, y $\in \mathbb{R}$?

A. $x^2\frac{\partial^2 u}{\partial x \partial y} - 2xy\frac{\partial u}{\partial y} + y^2 = 0$

B. $x^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$

C. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$

D. $x^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

48.Let a, b, c $\in \mathbb{R}$ be such that the quadrature rule $\int_{-1}^{1} f(x)dx$ = af(−1) + bf'(0) + cf'(1) is exact for all polynomials of degree less than or equal to 2 . Then a + b + c equal to

A. 4

B. 3

C. 2

D. 1

49.Let $X_1, X_2, \ldots, X_{16}$ be a random sample from normal distribution with unknown mean μ and variance 4 . Suppose Z ~ N(0 .1). For the most powerful test for testing H_0: μ = 3 vs H_1: μ = 0, which one of the following is the p-value where the observed sample mean is 2 .5?

A. P(Z > 1)

B. P(Z > −1)

C. P(Z > 0.5)

D. P(Z > −0.5)

50.Let S = {1, 2, 3, 4, 5} Consider a Markov chain on the state space S with transition probability matrix $\begin{pmatrix} 0 & 0.30 & & 0 \\ 0 & 0 & 0.7 & 0 \\ 0 & 0 & 0.30.7 & \\ 0.7 & 0 & 0 & 0.3 \\ 0.30.7 & 0 & 0 & \end{pmatrix}$. Then which of the following is always true?

A. State 1 has period 2

B. State 2 is recurrent

C. State 3 is transient

D. The chain admits at least two stationary distributions

51.Consider a M/M/1 queuing system with traffic intensity p < 1 . The probability of having n customers in the system at the steady state is given by

A. p^n

B. $p(1 - p^n)$

C. $p^{n-1}(1 - p)$

D. $p^n(1 - p)$

52.Suppose Poisson $X|\lambda \sim \text{Poisson}(\lambda)$ where $\lambda > 0$. Consider the exponential distribution with mean 1/4 for the prior on λ. If the observed value of X is 0, then which among the following is the 95% Bayesian credible (confidence) interval for λ of smallest length?

A. (0, c) where c = 0.95

B. (0, c) where c $\frac{log(20)}{5}$

C. (c, exp(c)) where c = $\frac{log(20)}{5}$

D. (0.2 – c, 0 .2 + c) where = $\frac{log(20)}{5}$

53.Suppose that Y has Exponential distribution with mean θ and that the conditional distribution of X given Y = y is Normal with mean 0 and variance y, for all y > 0 Identify the characteristic function of X. (defined as $\phi(t)$ = $[e^{itX}]$ from the following.

A. $e^{-\frac{\theta}{2}t^2}$

B. $e^{-\frac{1}{2\theta}t^2}$

C. $\frac{1}{1+\frac{1}{2}\theta t^2}$

D. $\frac{\theta}{\theta+\frac{1}{2}t^2}$

54.Let X_1, X_2, X_3, X_4 be i.i.d. random variables having Uniform distribution on $(0, \theta)$ where $\theta > 0$ is an unknown parameter, Define $X_{(4)} = \max\{X_1, X_2, X_3, X_4\}$. Consider the confidence intervals I = $[2X_{(4)}, 3X_{(4)}]$ and J = $[X_{(4)}, 1 + X_{(4)}]$ for θ. Which of the following is true?

A. The coverage probabilities of I and J are both independent of θ.

B. The coverage probability of I is independent of θ but the coverage probability of J is NOT independent of θ

C. The coverage probability of J is independent of θ but the coverage probability of I is NOT independent of θ

D. The coverage probabilities of both I and J are NOT independent of θ

55.A newly developed algorithm for random number generation needs to be tested. The first step is to check whether the sequence of numbers generated can be considered a random sample from the uniform distribution on the interval (0, 1). Which of the following is an appropriate nonparametric test?

A. Wilcoxon signed rank test

B. Sign test

C. Paired t test

D. Kolmogorov-Smirnov test

56.Suppose $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ are bivariate measurements where $n > 2$. Assume that all the x_i are distinct and all the y_i are distinct too. Let r_p denote the ordinary (Pearson) correlation coefficient and r_s denote the (Spearman) rank correlation coefficient. Suppose $r_p = 1$. Which of the following is true?

A. $0.5 < r_s < 1$

B. $r_s = 0.5$

C. $r_s = 1$

D. $r_s = -1$

57.A proportion p of a large population has particular disease. A random sample of k people is drawn from the population and their blood samples are combined. An accurate test for the disease applied to the combined blood sample shows a positive result, hence at least one of the k people has the disease. What is the probability that exactly one of the k people has the disease?

A. $\frac{kp(1-p)^{k-1}}{1-(1-p)^k}$

B. $\frac{(1-p)^k+kp(1-p)^{k-1}}{1-(1-p)^k}$

C. $\frac{p}{p+p^2+\ldots+p^k}$

D. $\frac{1}{k}$

58.Let $X_1, X_2, \ldots$ be i.i.d. random variables with uniform distribution on the interval [0, 1]. Let $Y_{n,k}$ denote the , k^{th} order statistic based on the sample $X_1, \ldots, X_n$ (e.g. $Y_{n,1} = \min\{X_1, \ldots, X_n\}$). What is the probability that $Y_{21,7} = Y_{22,7}$?

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{7}{11}$

D. $\frac{15}{22}$

59.Let $X_1, X_2, \dots, X_n$ be a random sample from exponential distribution with mean θ. Which of the following is NOT a sufficient statistic for θ?

A. $\frac{1}{X_1+X_2+\dots+X_n}$

B. $X_1 + X_2 + \dots + X_n$

C. $\frac{X_n}{X_1+X_2+\dots+X_{n-1}}$

D. $(X_n, X_1 + X_2 + \dots + X_{n-1})$

60.Consider a BIBD [Balanced incomplete Block Design) with v treatments in b blocks, each of which has k plots. Let r denote the number of blocks in which each treatment occurs. Let λ be the number of blocks in which each pair of treatment occurs. Which of the following statements is necessarily true?

A. vb = rk

B. vr = bk

C. $r(b - 1) = \lambda (k - 1)$

D. $r(v - 1) = \lambda (b - 1)$

61.Let A be an m × m matrix with real entries and let x be an m × 1 vector of unknowns. Now consider the two statements given below:

I: There exists non-zero vector $b_1 \in \mathbb{R}^m$ such that the linear system $Ax = b_1$ has NO solution.

II: There exist non-zero vectors $b_2, b_3 \in \mathbb{R}^m$ with $b_2 \neq cb_3$ for any $c \in \mathbb{R}$ such that the linear systems $Ax = b_2$ and $Ax = b_3$ have solutions.

Which of the following statements are true?

A. II is TRUE whenever A is singular,

B. I is TRUE whenever A is singular

C. Both I and II can be TRUE simultaneously

D. If m = 2, then at least one of I and II is FALSE

62.Let f: [0 1] → $\mathbb{R}$ be a continuous function such that $\int_0^t f(x)dx = \int_t^1 f(x)dx$ every ∈ [0, 1] Then which of the following are necessarily true?

A. f is differentiable on (0, 1)

B. f is monotonic on [0, 1]

C. $\int_0^1 f(x)dx = 1$

D. f(x) > 0 for all rationale x ∈ [0, 1]

63.Let $A \subseteq \mathbb{R}$ and let f: $\mathbb{R} \to \mathbb{R}$ be continuous. Which of the following statements are true?

A. If A is closed then f(A) is closed.

B. If A is bounded then $f^{-1}(A)$ is bounded

C. If A is closed and bounded then f(A) is closed and bounded

D. If A is bounded then f(A) is bounded.

64.Let Y be a nonempty bounded, open subset of $\mathbb{R}^n$and let $\overline{Y}$ denote its closure. Let $\{U_j\}_{j\geq 1}$ be a collection of open sets in $\mathbb{R}^n$ such that $\overline{Y} \subseteq U_{j\geq 1} U_j$. Which of the following statements are true?

A. There exist finitely many positive integers j1, ..., j_N such that $Y \subseteq U_{k=1}^{N} U_{jk}$

B. There exists a positive integer $j_1, j_2, \ldots$ such that $Y \subseteq U_{j=1}^{N} U_j$

C. For every subsequence $j_1, j_2, \ldots$ we have $Y \subseteq U_{k=1}^{\infty} U_{jk}$

D. There exists a subsequence $j_1, j_2, \ldots$ such that $Y = U_{k=1}^{\infty} U_{jk}$

65.Let M $\in \mathbb{M}_n$ ($\mathbb{R}$) such that M $\neq$ 0 but M^2 = 0. Which of the following statements are true?

A. If n is even then dim(Col(M)) > dim(Null(M))

B. If n is even then dim(Col(M)) < dim(Null(M))

C. If n is odd then dim(Col(M)) < dim(Null(M))

D. If it is odd then dim(Col(M)) > dim(Null(M))

66.Which of the following are inner products on $\mathbb{R}^2$?

A. $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 + 2x_1y_2 + 2x_2y_1 + x_2y_2$

B. $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 + x_1y_2 + x_2y_1 + 2x_2y_2$

C. $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$

D. $\left\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\rangle = x_1y_1 - \frac{1}{2}x_1y_2 - \frac{1}{2}x_2y_1 + x_2y_2$

67.Let f: $\mathbb{R}^2 \to \mathbb{R}$ be a bounded function such that for each t $\in \mathbb{R}$, the functions g_t and h_t given by $g_t(y)$ = f (t, y) and $h_t(x)$ = f(x, t) are non decreasing functions. Which of the following statements are necessarily true?

A. k(x) = f(x, x) is a non-decreasing function

B. Number of discontinuities of f is at most countably infinite

C. $lim_{(x,y)\to(+\infty,+\infty)}$ f(x, y) exists

D. $lim_{(x,y)\to(+\infty,-\infty)}$ f(x, y) exists

68.It is known that $x = X_0 \in M_2(\mathbb{Z})$ is a solution of $AX - XA = A$ for some $A \in \left\{\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\right\}$. Which of following values are NOT possible for the determinant of X_0?

A. $\det(X_0) = 0$

B. $\det(X_0) = 2$

C. $\det(X_0) = 6$

D. $\det(X_0) = 10$

69.In which of the following cases does there exist a continuous and onto function $f: X \to Y$?

A. $X = (0, 1)$ $Y = (0, 1]$

B. $X = [0, 1]$, $Y = (o, 1]$

C. $X = (0, 1)$, $Y = \mathbb{R}$

D. $X = (0, 2)$, $Y = \{0, 1\}$

70.Let $\mathbb{R}^+$ denote the set of all positive real numbers. Suppose that $f: \mathbb{R}+ \to \mathbb{R}$ is a differentiable function. Consider the function $g(x) = e^x f(x)$. Which of the following are true?

A. If $\lim_{x\to\infty} f(x) = 0$ then $\lim_{x\to\infty} f'(x) = 0$

B. If $\lim_{x\to\infty} f(x) + f'(x)) = 0$ then $\lim_{\substack{x\to\infty \\ y\to\infty}} \frac{g(x)-g(y)}{e^x - e^y} = 0$

C. If $\lim_{x\to\infty} f'(x) = 0$ then $\lim_{x\to\infty} f(x) = 0$

D. If $\lim_{x\to\infty} (f(x) + f'(x)) = 0$ then $\lim_{x\to\infty} f(x) = 0$

71.Let A be an n × n matrix. We say that A is diagonalizable if there exists a non-singular matrix p such that PAP^{-1} is a diagonal matrix. Which of the following conditions imply that A is diagonalizable?

A. There exists integer k such that $A^k = 1$

B. There exists integer k such that A^k is nilpotent

C. A^2 is diagonalizable

D. A has n linearly independent eigenvectors

72.Let X be a topological space and E be a subset of X. Which of the following statements are correct?

A. E is connected implies $\overline{E}$ is connected

B. E is connected implies ∂E is connected.

C. E is path connected implies $\overline{E}$ is path connected.

D. E is compact implies $\overline{E}$ is compact

73.Let (a_n) and (b_n) be two sequences of real numbers and E and F be two subsets of $\mathbb{R}$. Let $E + F = \{a + b: a \in E, b \in F\}$. Assume that the right hand side is well defined in each of the following statements. Which of the following statements are true?

A. $\limsup_{n\to\infty} (a_n + b_n) \leq \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n$

B. $\limsup (E + F) \leq \limsup E + \limsup F$

C. $\liminf_{n\to\infty} (a_n + b_n) \leq \liminf_{n\to\infty} a_n + \liminf_{n\to\infty} b_n$

D. $\liminf (E + F) = \liminf E + \limsup F$

74.Let $T: X \to Y$ be a bounded linear operator from a Banach space X to another Banach space Y. Which of the following conditions imply that T has a bounded inverse?

A. $inf_{\|x\| = 1} \|Tx\| = 0$

B. $inf_{\|x\| = 1} \|Tx\| = 0$ and T(X) is dense in Y

C. $inf_{\|x\| = 1} \|Tx\| > 0$

D. $inf_{\|x\| = 1} \|Tx\| > 0$ and T(X) is dense in Y

75.Consider the system

$2x + ky = 2 - k$

$kx + 2y = k$

$ky + kz = k - 1$

in three unknowns and one real parameter k. For which of the following values of k is the system of linear equations consistent?

A. 1

B. 2

C. −1

D. −2

76.Let $f: \mathbb{R}^3 \to \mathbb{R}^2$ be a C_1 function with f(0, 0, 0) = (0, 0). Let A denote the derivative of f at (0, 0, 0). Let $g: \mathbb{R}^3 \to \mathbb{R}$ be the function given by g(x, y, z) = xy + yz + zx + x + y + z. Let $h: \mathbb{R}^3 \to \mathbb{R}^3$ be the function defined by h = (f, g). In which of the following cases, will the function ^ admit a differentiate inverse in some open neighbourhood of (0, 0, 0)?

A. $A = \begin{pmatrix} 121 \\ 000 \end{pmatrix}$

B. $A = \begin{pmatrix} 222 \\ 655 \end{pmatrix}$

C. $A = \begin{pmatrix} 00-1 \\ 01\ 0 \end{pmatrix}$

D. $A = \begin{pmatrix} 424 \\ 032 \end{pmatrix}$

77.For non-negative integers k $\geq$ 1 define $f_k(x) = \frac{x^k}{(1+x)^2} \forall x \geq 0$. Which of the following statements are true?

A. For each k, f_k is a function of bounded variation on compact intervals

B. For every k, $\int_0^\infty f_k(x)dx < \infty$

C. $\lim_{k\to\infty}\int_0^1 f_k(x)dx$ exists

D. The sequence of functions f_k converge uniformly on [0, 1] as k $\to\infty$

78.Let A be an m × n matrix such that the first r rows of A are linearly independent and the first s columns of A are linearly independent, where r < m and s < n. Which of the following statements are true?

A. The rank of A is at feast max{r, s}

B. The sub-matrix formed by the first r rows and the first s columns of A has rank min{r, s}

C. If r < s, then there exists a row among rows r + 1, ..., m which together with the first r rows form a linearly independent set

D. If s < r, then there exists a column among columns s + 1, ..., n which together with the first s columns form a linearly dependent set.

79.Let f =$a_0 + a_1X + \ldots + a_nX^n$ be a polynomial with $a_i \in \mathbb{Z}$ for $0 \leq i \leq n$. Let p be a prime such that $p|a_i$ for all $1 < i \leq n$ and p^2 does not divide a_n. Which of the following statements are true?

A. f is always irreducible

B. f is always reducible

C. f can sometimes be irreducible and can sometimes be reducible.

D. f can have degree 1

80.Let $\mathbb{T}$ denote the unit circle {z $\in\mathbb{C}$: |z| = 1} in the complex plane and let $\mathbb{D}$ be the open unit disc {z $\in\mathbb{C}$: |z| < 1}. Let R denote the set of points z_0 in $\mathbb{T}$ for which there exists a holomorphic function f in an open neighbourhood U_{z_0} of z_0 such that $f(z) = \sum_{n=0}^{\infty} z^{4n}$ in $U_{z_0} \cap\mathbb{D}$. Then R contains

A. All points of $\mathbb{T}$

B. Infinitely many points of $\mathbb{T}$

C. All points of $\mathbb{T}$ except a finite set

D. No points of $\mathbb{T}$

81.Let G be a group of order 24. Which of the following statements are necessarily true?

A. G has a normal subgroup of order 3

B. G is not a simple group

C. There exists an injective group homomorphism from G to S_8

D. G has a subgroup of index 4

82.Let f be an entire function such that |zf{z) − 1 + e^z| $\leq$ 1 + |z| for all z $\in \mathbb{C}$. Then

A. f'(0) = −1

B. f'(0) = −1/2

C. f"(0) = −1/3

D. f"(0) = −1/4

83.For a positive integer n, let $\Omega(n)$ denote the number of prime factors of n, counted with multiplicity. For instance, $\Omega(3)$ = 1, $\Omega(6) = \Omega(9)$ = 2. Let p > 3 be a prime number and let N = p(p + 2)(p + 4). Which of the following statements are true?

A. $\Omega(N) \geq 3$

B. There exist primes p > 3 such that $\Omega(N) = 3$

C. p can never be the smallest prime divisor of N

D. p can be the smallest prime divisor of N

84.Consider the function f(z) = $\frac{(\sin z)^m}{(1-\cos z)^n}$ for 0 < |z| < 1 where m n are positive integers. Then z _ Q Is

A. A removable singularity if m $\geq$ 2n

B. A pole if m < 2n

C. A pole if m $\geq$ 2n

D. An essential singularity for some values of m, n

85.Which of the following statements are true?

A. All finite field extensions of $\mathbb{Q}$ are Galois

B. There exists a Galois extension of $\mathbb{Q}$ of degree 3

C. All finite field extensions of $\mathbb{F}_2$ are Galois

D. There exists a field extension of $\mathbb{Q}$ of degree 2 which is not Galois

86.For any complex valued function f let D_f denote the set on which the function f satisfies Cauchy-Riemann equations. Identify the functions for which D_f is equal to $\mathbb{C}$.

A. f(z) = $\frac{z}{1+|z|}$

B. f(z) = {cos αx – sin αy) + i{sin αx + cos αy), where z = x + iy

C. f(z) = $\begin{cases} e^{-\frac{1}{z^4}} if z \neq 0 \\ 0 if z = 0 \end{cases}$

D. f(z) = $x^2 + iy^2$, where z = x + iy

87.Let p be a prime number and N_p be the number of pairs of positive integers (x, y) such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{p^4}$.

A. 0

B. 4

C. 5

D. 9

88.Which of the following statements are true about subsets of $\mathbb{R}^2$ with the usual topology?

A. A is connected if and only if its closure $\overline{A}$ is connected

B. Intersection of two connected subsets is connected

C. Union of two compact subsets is compact

D. There are exactly two continuous functions from $\mathbb{Q}^2$ to the set {(0,0),(1,1)}

89.Consider A = {1, 1/2, 1/3, ... , 1/n,... |n $\in \mathbb{N}$} and B = A $\cup$ {0}.

Both the sets are endowed with subspace topology from $\mathbb{R}$. Which of the following statements are true?

A. A is a closed subset of $\mathbb{R}$

B. B is a closed subset of $\mathbb{R}$

C. A is homeomorphic to $\mathbb{Z}$, where $\mathbb{Z}$ has subspace topology from $\mathbb{R}$

D. B is homeomorphic to $\mathbb{Z}$, where $\mathbb{Z}$ has subspace topology from $\mathbb{R}$

90.A positive integer n co-prime to 17, is called a primitive root modulo 17 if n^k – 1 is not divisible by 17 for all k with i $\leq$ k < 16 . Let a, b be distinct positive integers between 1 and 16 . Which of the following statements are true?

A. 2 is a primitive root modulo 17

B. If a is a primitive root modulo 17, then a^2 is not necessarily a primitive Root modulo 17

C. If a, b are primitive roots modulo 17, then ab is a primitive root modulo 17

D. Product of primitive roots modulo 17 between 1 and 16 is congruent to 1 modulo 17

91.A mass m with velocity v approaches a stationary mass M along the x-axis. The masses bounce of each other elastically. Assume that the motion takes place in one dimension along the x axis, and v_f and V_f represent the final velocities of masses m and M along the x-axis. Which of the following are true?

A. $v_f = v$, $V_f = v$

B. $v_f = 0$, $V_f = v$

C. $v_f = \frac{(m-M)v}{m+M}$, $V_f = \frac{2mv}{m+M}$

D. $v_f = \frac{mv}{m+M}$, $V_f = \frac{mv}{m+M}$

92.Which of the following expressions for u = u(x, t) are solutions $u^t - e^{-t} u_x + u = 0$ with u(x, 0) = x?

A. $e^t(x + e^t - 1)$

B. $e^{-t}(x - e^{-t} + 1)$

C. $x - e^t + 1$

D. xe^t

93.Let u be a positive eigen function with eigenvalue λ for the boundary value problem $u + 2u + a(t)u = \lambda u$, $u(0) = 0 = u(1)$ where a: [0,1] → (1, ∞) is a continuous function. Which of the following statements are possibly true?

A. $\lambda > 0$

B. $\lambda < 0$

C. $\int_0^1 (u)^2\, dt = 2\int_0^1 uu\, dt + \int_0^1 (a(t) - \lambda)u^2 dt$

D. $\lambda = 0$

94.Let f: $\mathbb{R}^2 \to \mathbb{R}^2$ be a nonzero smooth vector field satisfying div$f \neq 0$. Which of the following are necessarily true for the ODE $x = f(x)$?

A. There are no equilibrium points

B. There are no periodic solutions

C. All the solutions are bounded

D. All the solutions are unbounded

95.Consider the integral equation

$\int_0^x (x-t)u(t)dt$ = x; x $\geq$ 0

for continuous functions u defined on $[0, \infty)$. The equation has

A. A unique bounded solution

B. No solution

C. A unique solution u such that $|u(x)| \leq C(1 + |x|)$ for some constant c

D. More than one solution u such that $|u(x)| \leq C(1 + |x|)$ for some constant c

96.Consider the Euler method for integration of the system of differential equations $\dot{x}$ = −y, $\dot{y}$ = x. Assume that (x_i^n, y_i^n) are the points obtained for i = 0, 1, …, n^2 using a time-step h = 1/n starting at the initial point (x_0, y_0). Which of the following statements are true?

A. The points (x_i^n, y_i^n) lie on a circle of radius 1

B. $lim_{n\to\infty}(x_n^n, y_n^n)$ = (cos(1), sin(1))

C. $lim_{n\to\infty}(x_2^n, y_2^n)$ = (1, 0)

D. $(x_i^n)^2 + (y_i^n)^2 < 1$, for i $\geq$ 1

97.Let u(x, y) solve the partial differential equation (PDE) $x^2\frac{\partial^2 u}{\partial x \partial y} + 3y^2u = 0$ with $u(x, 0) = e^{1/x}$. Which of the following statements are true?

A. The PDE is not linear PDE

B. $u(1, 1) = e^2$

C. $u(1, 1) = e^{-2}$

D. The method of separation of variables can be utilized to compute the solution u(x, y)

98.Let X = {y $\in$ $C^1[0, \pi]$: y(0) = 0 = y(π)] and define J: X → $\mathbb{R}$ by J(y) = $\int_0^\pi y^2(1-y'^2)dx$. Which of the following statements are true?

A. y = o is a local minimum for J with respect to the C^1 norm on X

B. y = 0 is a local maximum for J with respect to the C^1 norm on X

C. y = o is a local minimum for J with respect to the sup norm on X

D. y = 0 is a local maximum for J with respect to the sup norm on X

99.The values of a, b, c, d, e for which the function

f(x) = $\begin{cases} a(x-1)^2 + b(x-2)^3 - \infty < x \le 2 \\ c(x-1)^2 + d 2 \le x \le 3 \\ (x-1)^2 + e(x-3)^3 3 \le x < \infty \end{cases}$

is a cubic spline are

A. a = c = 1, d = 0, b, e are arbitrary

B. a = b = c = 1, d = 0, e is arbitrary

C. a = b = c = d = 1, e is arbitrary

D. a = b = c = d = e = 1

100.Let K(x, y) be a kernel in [0, 1] × [0, 1], defined as K(x, y) = $sin(2\pi x) sin(2\pi y)$. Consider the integral operator K(u)(x) = $\int_0^1 u(u)K(x,y)dy$ where u $\in$ C([0, 1]). Which of the following assertions on K are true?

A. The null space of K is infinite dimensional

B. $\int_0^1 v(x)K(u)(x)dx = \int_0^1 K(v)K(x)u(x)dx$ for all u, v ∈ C([0, 1])

C. K has no negative eigenvalue

D. K has an eigenvalue greater than ¾

101.Let B be the unit ball in $\mathbb{R}^3$ centered at origin, The Euler-Lagrange equation corresponding to the functional I(u) = $\int_B (1 + |\nabla u|^2)^{\frac{1}{2}} dx$ is

A. $div\left(\frac{\nabla u}{(1+|\nabla u|^2)^{\frac{1}{2}}}\right) = 0$

B. $\frac{\nabla u}{(1+|\nabla u|^2)^{\frac{1}{2}}} = 1$

C. $|\nabla u| = 1$

D. $(1 + |\nabla u|^2)\Delta u = \sum_{i,j=1}^{3} u_{xi} u_{xj} u_{xixj}$

102.Consider the 2nd order ODE x+ p(t)x + q(t)x = 0 and let x_1, x_2 be two solutions of this ODE in [a, b]. Which of the following statements are true for the Wronslcian W of x_1, x_2?

A. W $\equiv$ 0 in (a, b) implies that x_1, x_2 are independent

B. W can change sign in (a, b)

C. $W(t_0) = 1$ for some $t_0 \in (a, b)$ implies that W = 1 in (a, b)

D. $W(t_0) = 0$ for some $t_0 \in (a, b)$ implies that W = 0 in (a, b)

103.Let (X_1, Y_2), (X_2, Y_2), ..., (X_n, Y_n) be i.i.d from a continuous bivariate distribution. Let R_i be rank of X_i among the X observations and S_i be rank of Y_i among the Y observations. Let Spearman's statistic for testing independence between X and Y observations be denoted by T. Then, which of the following are true?

A. $T = \frac{12\sum_{i=1}^{n} R_i S_i}{n(n^2-1)} - \frac{3(n+1)}{n-1}$

B. E(T) = 0 when X and Y are independent

C. $Var(T) = \frac{1}{n-1}$ when X and Y are independent

D. $T \geq 0$

104.Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. random variables which are uniformly distributed on the interval $(0,\theta)$ where $\theta > 0$. Let $X_{(1)} \leq X_{(2)} \ldots \leq X_{(n)}$ be the corresponding order statistics. Consider testing the hypotheses H_0: $\theta = 1$ versus H1: $\theta > 1$. Which of the following tests have significance level a for o < α < 0.5?

A. Reject H_0 when $X_1 > 1 - \alpha$

B. Reject H_0 when $X_{(n)} > (1 - \alpha)^{1/n}$

C. Reject H_0 when $X_{(n)} < (1 - \alpha)$

D. Reject H_0 when $X_{(1)} > 1 - \alpha^{1/n}$

105.Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. $N(\theta, 1)$ where $\theta \geq 0$. Let $T = T(X_1, \ldots, X_n)$ be the maximum likelihood estimate of θ. Which of the following statements are true?

A. $E_\theta(T) - \theta \geq 0$ for all $\theta \geq 0$

B. $E_\theta(T) - \theta = 0$ for all $\theta \geq 0$

C. $E_\theta(T) - \theta < 0$ for all $\theta \geq 0$

D. There exists $\theta_0 > 0$ such that $E_\theta(T) - \theta < 0$ for all $0 < \theta < \theta_0$ and $E_\theta(T) - \theta > 0$ for all $\theta \geq \theta_0$

106.Consider the following Linear Programming Problem. Minimise 14x + 9y + 6z subject to

$2x + y + z \geq 8$

$3x + 2y + z \geq 10$

$x \geq 0, y \geq 0, z \geq 0.$

What is the optimal value of the objective function?

A. 52

B. 48

C. 50

D. 56

107.A population of size N is divided into L strata of sizes N_1, N_2, ..., N_L respectively. A stratified random sample of size n is drawn from the population where n_1, n_2, ..., n_L denote the sample size in each of the L strata. Note that within each stratum units are chosen using simple random sampling.

Suppose the sample mean of the j-th stratum is denoted by $\overline{y}_j$ and let

$\overline{y}_{st} = \sum_{j=1}^{L} \frac{N_j \overline{y}_j}{N}$ and $\sigma_{st}^2 = V(\overline{y}_{st})$.

Consider a simple random sample of size n drawn from the population, independently of the first sample. Let $\overline{y}$ and σ^2 denote the sample mean and the variance of the sample mean for this sample. Which of the following statements are correct?

A. $\overline{y}_{st}$ is an unbiased estimator of the population mean

B. $\overline{y}$ is an unbiased estimator of the population mean

C. $\sigma_{st}^2 \leq \sigma^2$

D. If $\frac{n_j}{N_i} = \frac{n}{N}$ for all j, then $\sigma_{st}^2 = \sigma^2$

108.let {(x_i, y_i): i = 1, 2, ..., n} be given data points, where not all x_i s are the same. C decides to fit a linear regression model of y on x with an intercept and D decides to fit a linear regression model of y on x without an intercept. Let $\overline{x}$ and $\overline{y}$ be the sample means of x and y respectively. Which of the following statements regarding the two fitted models are NOT necessarily true?

A. Both the fitted regression lines will pass through the point $(\overline{x}, \overline{y})$

B. C's fitted line will pass through $(\overline{x}, \overline{y})$, but D's fitted line will not pass through $(\overline{x}, \overline{y})$

C. For both the fitted models, the sample correlation coefficient between x_i s and the corresponding residuals is zero

D. The sample correlation coefficient between x_i s and the corresponding residuals is zero for C's fitted model, but not for D's fitted model

109.Let $\{N_t, t \geq 0\}$ be a Poisson Process with intensity parameter 10, Let T be an exponential random variable with mean 6, and independent of the Poisson process. Which of the following are true?

A. $P(N_T = k) = e^{-10T}(10T)^k/k!$

B. $E(N_T) = 60$

C. $(N_{T+t} - N_T; t \geq 0)$ is a Poisson process with intensity parameter4

D. $(N_{T+t} - N_T; t \geq 0)$ is a Poisson process with intensity parameter 10

110.Consider a Markov chain with state space S = {0, 1, 2, ….,} and transition probabilities given as follows:

$p_{0,j} = i/(j!\ e)$ for $j \geq 0$

$p_{i,i-1} = 1$ for i > 0 and i odd; $P_{i,i+1} = 1$ for i > 0 and i even.

Which of the following are true?

A. The chain is irreducible

B. The chain has period 2

C. There are infinitely many recurrent classes

D. Zero is a transient state

Answer |||

111.Let $(x_1, y_1), (x_2, y_2), …., (x_n, y_n)$ be n independent observations from the uniform distribution an $S_\theta = \{(x, y): \theta \leq x^2 + y^2 \leq \theta + 1\}$ 1 where $\theta > 1$ is an unknown parameter. Which of the following statements are correct?

A. $\max_{1\leq i\leq n}\{x_i^2 + y_i^2\} - 1$ is a maximum likelihood estimate of θ

B. $\min_{1\leq i\leq n}\{x_i^2 + y_i^2\} - 1$ is a maximum likelihood estimate of θ

C. Any value between $\max_{1\leq i\leq n}\{x_i^2 + y_i^2\} - 1$ and min $\min_{1\leq i\leq n}\{x_i^2 + y_i^2\} - 1$ is a maximum likelihood estimate of θ

D. Any value between $\max_{1\leq i\leq n}\{x_i^2\} + \max_{1\leq i\leq n}\{y_i^2\} - 1$ and $\min_{1\leq i\leq n}\{x_i^2\} + \min_{1\leq i\leq n}\{y_i^2\} - 1$ is a maximum likelihood estimate of θ

112.Suppose we fit the linear model $Y = X\beta + \in$ using least squares, where $Y = (Y_1, Y_2, …, Y_n\}^T$ and $\in = (\in_1, \in_2, \ldots, \in_n)^T$. Here X is a n × p non-stochastic matrix with full column rank and $\in_i$ s are i.i.d. with mean 0 and variance 1. For I ∈ {1, 2, …, n}. Let $\hat{Y}_i$ be the fitted value of Y_i and let $\hat{\in}_i$ be the corresponding residual. For r, s ∈ {1, 2, …, n} , $r \neq s$, which of the following must be true?

A. The random variables $\in_r$ and $\hat{Y}_s$ are uncorrelated

B. The random variables $\in_s$ and $\hat{Y}_s$ are uncorrelated

C. The random variables $\hat{\in}_r$ and $\hat{Y}_s$ are uncorrelated

D. The random variables $\hat{\in}_s$ and $\hat{Y}_s$ are uncorrelated

113.Let X be a non-negative random variable with E[X] = 1 . Which of the following quantities is necessarily greater than or equal to 1?

A. $E[X^4]$

B. $(E[\cos X])^2 + (E[\sin X])^2$

C. $E\left[\sqrt{x}\right]$

D. E[1/X]

114.Let $X_1, X_2, \ldots, X_n$ random variables whose marginal distributions are N(0, 1). Suppose $E(X_iX_j) = o$ for i, j, i $\neq$ j. Let $Y = X_1 + X_2 + \ldots + X_n$ and V = $X_1^2 + X_2^2 + \ldots + X_n^2$. Which of the following statements follow from the given conditions?

A. Y has normal distribution with mean zero and variance n

B. V has Chi-square distribution with n degrees of freedom

C. $E\left(X_i^3 X_j^3\right)$ = o for all i, j, i $\neq$ j

D. P(|Y| > t} < $\frac{n}{t^2}$ for all t > o

115.Consider a sequence of i.i.d. observations $X_1, X_2, \ldots$ from $N(0,\sigma^2)$. Which of the following are unbiased and consistent estimators of σ?

A. $\sqrt{\frac{\pi}{2}\frac{1}{n}\sum_{i=1}^{n}|X_i|}$

B. $\sqrt{\frac{1}{n}\sum_{i=1}^{n}X_i^2}$

C. $\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(X_i-\overline{X}\right)^2}$

D. $\sqrt{\frac{1}{2\pi}(X_1^2 - X_n^2)}$

116.Suppose X ~ Geometric(l/2) (taking values in {1, 2, 3, ...}) conditional on X the variable Y has Poisson(X) distribution. Similarly suppose U ~ Poisson(1) and conditional on U, the variable V has Geometric(1/(U + 1)) distribution, Then..

A. E[Y] $\geq$ E[V]

B. E[Y] $\leq$ E[V]

C. Var[Y] $\geq$ Var[V]

D. Var[Y] $\leq$ Var[V]

117.Consider a small clinical trial to study the effectiveness of a treatment for a particular illness. 10 patients are enrolled in this experiment. Let θ denote the probability that a randomly chosen patient in the population recovers from this illness due to this treatment. For a standard Bayesian analysis, consider the Beta (0.5, 0.5) prior on θ (with density proportional $(\theta\ (1 - \theta))^{-1/2}$) Suppose exactly 6 out of the 10 patients recover. Which of the following are Bayes estimates of θ under the squared error loss function?

A. $\frac{13}{22}$

B. $\frac{11}{20}$

C. $\frac{1}{2}$

D. E(θ \ 6 out of 10 patients recovered]

118.Let (X_n, n $\geq$ 1) and X be random variables defined on a common probability space, all having finite expectation and having characteristic functions $(\varphi_n, n \geq 1)$ and φ respectively. Which of the following are true?

A. If E(X_n) → E(X) then there is at least one sample point ω such that $X_n(\omega) \to X(\omega)$

B. If $X_n(\omega) \to X(\omega)$ for every sample point ω then E(X_n) → E(X)

C. If $X_n(\omega) \to X(\omega)$ for every sample point ω then $\varphi_n(t) \to \varphi(t)$ for all t

D. If $\varphi_n(t) \to \varphi(t)$ for all t then $X_n(\omega) \to X(\omega)$ for at least one sample point ω

119.If n is a permutation of {1, 2, ..., n}, let $X_n(\pi)$ denote the number of fixed points, that is cardinality of the set {i $\leq$ n: π (i) = i}. If a permutation π is chosen uniformly at random, then X_n is a random variable. Which of the following are correct?

Options:-

A. $E(X_{25}) = 5E(X_5)$

B. $E(X_{25}) = E(X_5)/5$

C. $E(X_{25}) = E(X_5)$

D. $E(X_{25}) = [E(X_5)]^2$

120.Consider an irreducible Markov chain with finite state space S. Let $p = ((p_{ij}))$ be its transition probability matrix and let $p^n = \left(\left(p_{ij}^{(n)}\right)\right)$ denote the n-step transition probability matrix for the chain. Let

$\alpha_{ij} = \lim_{n\to\infty} \frac{1}{n}\sum_{m=1}^{n} p_{ij}^{(m)}$, i, j ∈ S.

Recall that the limit above always exists. Which of the following statements are necessarily true?

A. $\alpha_{ij} = \alpha_{kj} \forall i, j, k \in S$

B. $\sum_j \alpha_{ij} = 1$ for all $i \in S$

C. $\alpha_{ij} > 0$ for all $i, j \in S$

D. For all $i, j \in S$, the sequence $p_{ij}^{(n)}$ converges to α_{ij} as $n \to \infty$

ANSWERS

1.	(A)	41.	(B)	81.	(B)(D)
2.	(B)	42.	(A)	82.	(B)(C)
3.	(A)	43.	(C)	83.	(A)(C)
4.	(B)	44.	(A)	84.	(A)(B)
5.	(D)	45.	(A)	85.	(B)(C)
6.	(B)	46.	(A)	86.	(C)
7.	(C)	47.	(A)	87.	(D)
8.	(A)	48.	(C)	88.	(A)(B)
9.	(B)	49.	(A)	89.	(A)(C)
10.	(C)	50.	(A)	90.	(B)(D)
11.	(A)	51.	(A)	91.	(D)
12.	(D)	52.	(A)	92.	(B)
13.	(B)	53.	(A)	93.	(A)(B)
14.	(A)	54.	(A)	94.	(A)(B)(D)
15.	(C)	55.	(A)	95.	(B)
16.	(A)	56.	(A)	96.	(B)(C)(D)
17.	(C)	57.	(A)	97.	(B)(D)
18.	(A)	58.	(A)	98.	(A)
19.	(C)	59.	(A)	99.	(A)(B)
20.	(B)	60.	(A)	100.	(A)(B)(C)
21.	(C)	61.	(B)(C)(D)	101.	(A)(D)
22.	(C)	62.	(A)(B)	102.	(B)
23.	(D)	63.	(B)(D)	103.	(A)(B)(C)
24.	(A)	64.	(A)(B)	104.	(A)(B)(D)
25.	(A)	65.	(B)(C)	105.	(A)
26.	(B)	66.	(D)	106.	(A)
27.	(D)	67.	(C)(D)	107.	(C)(D)
28.	(A)	68.	(D)	108.	(A)(B)(C)(D)
29.	(B)	69.	(A)(C)	109.	(A)(B)(D)
30.	(B)	70.	(B)(D)	110.	(C)(D)
31.	(C)	71.	(A)(D)	111.	(A)(B)
32.	(C)	72.	(A)	112.	(C)(D)
33.	(B)	73.	(A)	113.	(B)(E)
34.	(B)	74.	(C)	114.	(D)
35.	(B)	75.	(A)(C)	115.	(A)
36.	(A)	76.	(A)(C)	116.	(A)(B)(C)(D)
37.	(B)	77.	(A)(C)	117.	(B)(E)
38.	(D)	78.	(A)(C)	118.	(A)(B)
39.	(B)	79.	(C)	119.	(C)(D)
40.	(A)	80.	(B)(C)	120.	(A)(B)(C)