## CIVIL ENGINEERING

## Complete Formulas and Short Notes

## IMPORTANT FORMULAS ON BUILDING MATERIALS

## CHAPTER-1-CEMENT

## INTRODUCTION

- Cement is material having adhesive and cohesive properties in presence of water which provide a binding medium for the discrete ingredients.
- Standard bulk density of cement is $1440 \mathrm{~kg} / \mathrm{m}^{3}$ \& Sp. Gravity is 3.15
- Volume of cement of 1 bag of 50 kg is 35 litres.


## MANUFACTURING OF CEMENT:

- Raw materials required for manufacture of Portland cement are calcareous and argillaceous material. Calcareous Material-Compounds of calcium such as limestone or chalk Argillaceous Material-Compounds of Silica, Alumina \& oxides of iron such as clay and shale.
- Process consists of grinding the raw materials, mixing them in certain proportions depending upon their purity and composition and burning them in a kiln at 1300 to 1500 C.
- At this temperature, material sinters and fuses to form what is called as clinker (a solid solution)
- Clinker is cooled \& grounded to fine powder, while grinding Gypsum (3-5\%) is added in order to prevent flash-setting of the cement, this results in formation of OPC.
- There are two processes known as "wet" and "dry" processes depending upon whether the mixing and grinding of raw materials is done in wet or dry conditions
- Previously wet process was used because control in the mixing of raw materials in powder form was not available then.
- Today dry process is used because as technique of dry mixing of powdered materials is available and it is more energy efficient (requires much less fuel).
- In dry process as materials are already in a dry state, whereas in wet process material are in slurry form having $35 \%$ to $50 \%$ of water.
- To dry the slurry we require more fuel.


## CHEMICAL COMPOSITION OF RAW MATERIAL:

- Three basic constituents of hydraulic cement are lime, silica and alumina.
- Relative composition and their respective functions are listed below.

| Ingredient | Function | \% | Av. \% |
| :---: | :---: | :---: | :---: |
| Lime (Cao) | - It controls strength and soundness. <br> - Its deficiency reduces strength and setting action(time required to change from plastic to solid state of paste) and excess of it cause unsoundness (cement to expand and disintegrate). | 60-65 | 62 |
| Silica ( $\mathrm{SiO}_{2}$ ) | - It imparts strength. <br> - Excess of it increase the strength but setting action is prolonged. | 17-25 | 22 |
| Alumina ( $\mathrm{Al}_{2} \mathrm{O}_{3}$ ) | - Responsible for quick setting. <br> - Strength decreases as alumina in excess amount. | 3-8 | 5 |
| Calcium sulphate $\left(\mathrm{CaSO}_{4}\right)$ | - It increases initial setting time (time period during paste remains in plastic state) of cement. | 3-4 | 4 |
| $\begin{gathered} \text { Iron oxide } \\ \left(\mathrm{Fe}_{2} \mathrm{O}_{3}\right) \end{gathered}$ | - Gives colour and helps in fusion of different ingredients. <br> - Excess of it produce a hard clinker which is difficult to grind. | 0.5-6 | 3 |
| Magnesia (MgO) | - it imparts colour and hardness (rigidity of paste) <br> - Excess of it makes cement unsound. | 0.5-4 | 2 |
| Sulphur trioxide $\left(\mathrm{SO}_{3}\right)$ | - Excess of it makes cement unsound | 1-3 | 1 |
| Alkalies \{Soda and potash\} | - These are residue. <br> - Excess of it cause efflorescence and cracking | 0.5-1.3 | 1 |

## BOGUE'S COMPOUNDS

- Raw material when subjected to high clinkering temperature combines with each other to form complex compounds known as Bogue's Compounds.
- Bogues compounds are formed during clinkering process.
- Properties of Portland cement varies significantly with the proportions of these four compounds, as substantial difference is observed in their individual behaviour.

| Bogue's compound | Functions |
| :---: | :---: |
| Tri Calcium <br> Silicate <br> ( $\mathrm{C}_{3} \mathrm{~S}$ ) <br> 25 to 50\% <br> normally 40\% <br> $3 \mathrm{CaO} . \mathrm{SiO}_{2}$ <br> Alite | - Best cementing material among all four Bogues Compounds. <br> - Helps clinkers easy to grind. <br> - Increase resistance to freezing and thawing (melting) <br> - It hydrates rapidly generating high heat and develops an early hardness and strength (mainly 7 Days). <br> - Raising of $\mathrm{C}_{3} \mathrm{~S}$ content beyond the specified limits increases heat of hydration and solubility of cement in water, but free lime will be more and cement will be unsound. Main cause of hardness and Party strength of cement paste <br> - Heat of hydration is $500 \mathrm{~J} / \mathrm{gm}$ |
| Di Calcium <br> Silicate <br> ( $\mathrm{C}_{2} \mathrm{~S}$ ) <br> 25 to 40\% <br> normally $32 \%$ <br> $2 \mathrm{CaO} . \mathrm{SiO}_{2}$ <br> Belite | - It hydrates and harden slowly (>1 year or more) hence responsible for ultimate strength. <br> - helps resistance to chemical attack. <br> - raising of $\mathrm{C}_{2} \mathrm{~S}$ content result in harder to grind clinker, reduces early strength and decrease resistance to freezing and throwing at early ages and decreases heat of hydration. <br> - At early ages ( $<1$ month) $C_{2} S$ has little influence on strength and hardness but after one year it is almost equal to $\mathrm{C}_{3} \mathrm{~S}$. <br> - Heat of hydration is $260 \mathrm{~J} / \mathrm{gm}$. |
| TriCalcium Aluminate ( $\mathrm{C}_{3} \mathrm{~A}$ ) <br> 8 to 12\% normally $10 \%$ $3 \mathrm{CaO} . \mathrm{Al}_{2} \mathrm{O}_{3}$ Celite | - rapidly reacts with water and responsible for flash set (stiffening without strength development). <br> - rapidity of action is controlled by addition of 2 to $3 \%$ of gypsum (calcium sulphate) at the time of grinding of cement. |


|  | - most responsible for initial setting, high heat of hydration and has greatest tendency to volume change causing cracking. <br> - Raising of $\mathrm{C}_{3} \mathrm{~A}$ contents reduces the setting time, resistance to sulphate attack declines and lowers the ultimate strength <br> - Heat of hydration is $865 \mathrm{~J} / \mathrm{gm}$. |
| :---: | :---: |
| ```Tetra calcium alumina ferrite (C4AF) 6 to 10% normally 8% 4CaO.Al2O3.Fe2O3``` | - it is responsible for flash set but generates less heat. <br> - it has poor cementing value because hydrated product due to $\mathrm{C}_{4} \mathrm{AF}$ also does not contribute anything to strength <br> - if $\mathrm{C}_{4} \mathrm{AF}$ is increased it slightly reduces the strength <br> - heat of hydration is $420 \mathrm{~J} / \mathrm{gm}$. <br> - It is also called Felite. |

## HYDRATION OF CEMENT

- Chemical reaction between cement and water is known as hydration of cement. it is an exothermic reaction.
- This setting (the change of cement paste from plastic to stiff solid state) and hardening (gain of strength with hydration) is a chemical reaction, wherein water plays an important role and it's not just a matter of drying out, in fact setting and hardening stops as soon as the concrete becomes dry.
- $\mathrm{C}_{3} \mathrm{~S}$ produces less $\mathrm{C}-\mathrm{S}-\mathrm{H}$ gel and more $\mathrm{Ca}(\mathrm{OH})_{2}$ as compared to $\mathrm{C}_{2} \mathrm{~S}$.
- $\mathrm{Ca}(\mathrm{OH})_{2}$ is not desirable product in the concrete Mass because it is soluble in water and gets leached out making the concrete porous particularly in hydraulic structure. That's why cement with smaller percentage of $\mathrm{C}_{3} \mathrm{~S}$ and more $\mathrm{C}_{2} \mathrm{~S}$ is recommended for using in hydraulic structures.
- Alternative way to overcome this difficulty is to grind some pozzolanic material (fly ash combined with lime)with cement. pozzolana is a siliceous material which reacts with lime in presence of moisture to give a relatively strength producing calcium silicate.
- the only advantage of $\mathrm{Ca}(\mathrm{OH})_{\text {z }}$ is that being alkaline in nature it maintains pH of 13 in concrete which resist the corrosion of reinforcement.
- Rate of hydration of the bogue's compounds will be in following descending order $\mathrm{C}_{4} \mathrm{AF}>\mathrm{C}_{3} \mathrm{~A}>\mathrm{C}_{3} \mathrm{~S}>\mathrm{C}_{2} \mathrm{~S}$
- Heat of hydration both books combined will be in following descending order
$\mathrm{C}_{3} \mathrm{~A}>\mathrm{C}_{3} \mathrm{~S}>\mathrm{C}_{4} \mathrm{AF}>\mathrm{C}_{2} \mathrm{~S}$
- Higher is the temperature rapid is the hydration, hence in cold weather aggregates are heated before they are used for making concrete. Hydration of concrete ceases at $11^{\circ} \mathrm{C}$.
- Finer is the cement rapid will be the hydration, because finer cement has larger surface area. Although total heat evolved will be same both in case of fine cement or coarser cement. However, a very fine ground cement is susceptible to air set and deteriorates early.
- Water requirement for hydration: About an average 23\% (24\% for $\mathrm{C}_{3} \mathrm{~S}$ and $21 \%$ for $\mathrm{C}_{2} \mathrm{~S}$ ) water by weight of cement is required for complete hydration of Portland cement. It is further observed that $15 \%$ of water by weight of cement is required to fill the gel pores. A total of $38 \%$ of water by weight of cement is required to complete the chemical reaction and to occupy the space within the gel pores.
- Flash setting of cement: flash setting is defined as immediate stiffening of Portland cement paste, mortar or concrete. It is due to very fast reaction of $C_{3} A$ with water. This rigidity cannot be overcome and also plasticity cannot be regained by further mixing without addition of water. causes of flash setting are (i) presence of high tricalcium aluminate (ii) less gypsum added to the cement and (iii) presence of alkalis.
- false set: Rapid stiffening or hardening in freshly mixed Portland cement concrete motor or concrete (with no appreciable evolution of heat). Plasticity of paste can be regained by only remixing of cement paste without addition of water. Reason for false set are (i) grinding too hot clinkers with gypsum dihydrate and formation of gypsum Semi-hydrate (ii) low $\mathrm{C}_{3} \mathrm{~A}+$ high gypsum will result in false set (iii) some concrete admixtures such as lignin sulfonate (water reducer) negatively affect the solubility of gypsum and increases the tendency towards false set (iv) alkalis in the cement.


## FIELD TESTS FOR CEMENT

- Colour: Grey colour with a light greenish shade.
- Physical Properties: Cement should feel smooth when touched between fingers.
- If a hand is inserted in a bag or heap of cement, it should feel cool.
- If a small quantity of cement is thrown in a bucket of water, it should float for some time before sinking.
- A thin paste between fingers should feel sticky.
- Presence of lumps: Cement should be free from lumps.


## LABORATORY TESTS FOR CEMENT

## 1. Chemical Composition Test

- Ratio of percentage of lime to percentage of silica, alumina and iron oxide known as Lime Saturation Factor (LSF), when calculated by the formula shall not be greater than 1.02 and not less than 0.66 .

$$
\frac{\mathrm{CaO}-07 \mathrm{SO}_{3}}{\left(28 \mathrm{SiO}_{2}+12 \mathrm{Al}_{2} \mathrm{O}_{3}+0.65 \mathrm{Fe}_{2} \mathrm{O}_{3}\right)}
$$

- Ratio of percentage of alumina $\left(\mathrm{Al}_{2} \mathrm{O}_{3}\right)$ to that of iron oxide $\left(\mathrm{Fe}_{2} \mathrm{O}_{3}\right)$ shall not be less than 0.66
- Weight of insoluble residue shall not be more than 4 per cent.
- Weight of Magnesia Shall not be more than 6 per cent
- Total loss on ignition shall not be more than 5 per cent.
- Total sulphur content calculated as sulphuric anhydride shall not be more than $2.5 \%$ when $C_{3} A$ is $5 \%$ or less and shall not be more than $3 \%$ when $C_{3} A$ is more than $5 \%$


## 2. Normal Consistency Test

- The normal (standard) consistency of a cement paste is defined as that consistency which will permit a Vicat plunger having 10 mm diameter and 50 mm length to penetrate a depth of 33 to 35 mm from the top (or 5 to 7 mm from the bottom) of the mould.
- Vicat Apparatus: Vicat apparatus assembly consists of a plunger 300 gm in weight with a length of 50 mm and diameter of 10 mm and a mould which is 40 mm deep and 80 mm in diameter.


## 3. Initial Setting Time Test

When water is added to cement, the resulting paste starts to stiffen and gain strength and loose the consistency simultaneously. The term setting implies solidification of the plastic cement paste. Initial and final setting times may be regarded as the two stiffening states of the cement. The beginning of solidification, called the initial set, marks the point in time when the paste has become unworkable

- Initial setting time should not be less than 30 minutes for OPC and 60 minutes for low heat cement.
- A cement paste is prepared by gauging cement with 0.85 times the water required to prepare a paste of standard consistency.
- In vicat's apparatus a square needle of area $1 \mathrm{~mm}^{2}$ is used and when the penetration is not more than 33 to 35 mm from the top it is assumed that initial setting has started.


## 4. Final Setting Time Test

- The cement paste is prepared same as the initial setting time test, here the needle with 5 mm diameter annual collar is attached in place of $1 \mathrm{~mm}^{2}$ for determination of final setting time.
- if the needle makes an impression not more than point 0.5 mm but the edge of the collar fails to do so the paste is assumed to be finally set.
- The final setting time should not be more than 10 hours.


## 5. Soundness Test

- The purpose of soundness test is to determine change in volume of cement after setting.
- Unsoundness of cement is due to the presence of free lime and magnesia which slake slowly causing change in volume of cement. Therefore, freshly grinded cement is allowed to aerate for two to three weeks allowing the lime to hydrate and overcome unsoundness. sometimes excess gypsum added to cement to retard the setting time can cause unsoundness by formation of calcium-sulpho-eliminate hence very strict control is kept over quantity of gypsum added to clinker.
- unsound cement causes cracks distortion and disintegration (due to expansion), ultimately leading to failure of the structure.
- Le Chatelier's Methodcan only indicate unsold next due to free line it does not indicate presents and after effect of excess of magnesium.
- Le Chatelier's method 100 grams cement is mixed with 0.78 times the water required to give a paste of standard consistency.
- Autoclave Testheat sensitive to both free lime and magnesia.


## 6. Strength Test

## (a) Compressive Strength Test

- For compressive strength test 200 grams of cement is mixed with 600 grams standard sand (Ennore sand) and ( $\mathrm{P} / 4+3$ )\% off water is added until the mixture is of uniform color. where P is the percentage of water required to produce a paste of normal consistency.
- Three cubes having size 70.6 mm or face area of $5000 \mathrm{~mm}^{2}$ are tested for compressive strength, at 1 day, 3 day, 7 days and 28 day where the period of testing is reckoned from the completion of vibration.
- Load is applied starting from zero at a rate of $35 \mathrm{~N} / \mathrm{mm}^{2} / \mathrm{min}$. The compressive strength shall be the average of the strengths of the three cubes for each period respectively.


## (b) Tensile Strength Test

- Motor is prepared by $1: 3$ (cement: sand) by weight, water is to be used is $(P / 5+2.5) \%$ where $P$ is the standard consistency of cement.
- Six briquettes are tested, and average tensile strength is calculated.
- A load is applied steadily and uniformly, starting from zero and increasing at the rate of $0.7 \mathrm{~N} / \mathrm{mm}^{2}$ in 12 seconds.
- Strength increases when the loading rate is increased from the specified rate.
- OPC should have a tensile strength of not less than 2 MPa after 3 days and 2.5 MPa after 7 days respectively.
- Generally, tensile strength is 10-15\% of compressive strength.


## 7. Fineness Test:

- Fineness of cement is measure of mean size of grains i.e. the degree to which it is grinded. Finner is the cement more is the surface area of a given volume of cement higher will be the rate of hydration because more surface area is available for cement water reaction.
- Cement become finer with age.
- Setting time of the cement decreases as the fineness of the cement increases.
- Rate of gain of strength is higher but ultimate strength is not affected.
- Fine cement bleeds less than the course cement because more surface area is available for water to be attached.
- Finer cement leads to a stronger reaction with alkali reactive aggregates and increases the chances to shrinkage and cracking of cement paste.
- Fineness of cement is expressed as $\mathrm{cm}^{2} / \mathrm{gm}$.

There are three methods for testing fineness viz.
(a) Sieve Method

- 100 gm of cement sample is taken and air set lumps, if any, in the sample are broken with fingers.
- The sample is placed on a 90 micron sieve and continuously sieved for 15 minutes.
- The residue should not exceed the limits specified below:

|  | Type of cement | \% Residue by <br> weight |
| :--- | :--- | :---: |
| (i) | Ordinary Portland cement | 10 |
| (ii) | Rapid hardening cement | 5 |
| (iii) | Portland Pozzolana <br> cement | 5 |

## (b) Air Permeability Method

(c) Wagner Turbidimeter Test

## 8. Heat of hydration Test

- The apparatus used to determine the heat of hydration of cement is known as calorimeter.
- The heat of hydration for low heat Portland cement should not be more than 66 and $75 \mathrm{cal} / \mathrm{gm}$ for 7 and 28 days respectively.


## 9. Specific Gravity Test

- The specific gravity of cement is obtained by using Le Chatelier's flask.
- Generally specific gravity of OPC is equal to 3.15.
- cement with higher content of iron oxides have higher specific gravity.


## 10. Loss on ignition

- 1 gm of cement is heated for 15 min in a weighed and covered platinum Crucible of 20 to 25 ml capacity by placing it in a muffle furnace at any temp between 900-1000 degree Celsius.
- The percentage loss on ignition should not be more than $4 \%$.


## TYPES OF CEMENT

## 1. Ordinary Portland cement

- Higher is the strength of Portland cement higher will be the rate of heat development during hydration of cement.
- OPC can be classified in three grades: 33 grade 43grade53 grade. where 33, 43, 53 are characteristic strength of cement at 28 days.


## 2. Rapid Hardening Cement (IS: 8041)

- This cement is obtained from OPC but grinded more.
- More $\mathrm{C}_{3} \mathrm{~S}$ (up to $50 \%$ ) and less $\mathrm{C}_{2} \mathrm{~S}$.
- 1 day strength of RHC 3-day strength of OPC with the same W/C ratio.
- 3-day strength of RHC 7-day strength of OPC with the same W/C ratio.
- Used in situations where a rapist development of strength is desired (Ex when form work is to be removed early for reuse).
- This cement must not be used for mass concrete because due to large quantity of heat of hydration the inside temperature in concrete increases and causes cracking.
- Large shrinkage and water requirement for workability is more Concrete made with RHC can be safely exposed to frost as a matures more quickly.
- Cost of rapid hardening cement is about $10 \%$ more than the ordinary cement.


## 3. High alumina cement (IS:6452)

- Raw material used are $40 \%$ bauxite, $40 \%$ lime, $15 \%$ iron oxide with little $\%$ of ferric oxide, silica and magnesia etc by fusing at very high temperature $1550-1500^{\circ} \mathrm{C}$ )
- As the heat required for the manufacture of this cement is more the manufacturing cost of this cement is high
- Allows more time for mixing and placing operation.
- $\quad$ Since $C_{3} A$ is not present the cement is good resistance against attack by sulphate.
- it evolves great heat during setting It is therefore not affected by frost but cannot be used in mass concreting works
- Due the property of rapid hardening (high heat of hydration), high early strength and good chemical attack resistance, widely used in marine construction \& sewer infrastructure.
- It has high resistance against fire, used in refractory concrete where it requires more strength at very high temp.


## 4. Portland slag cement (IS:455)

- It is obtained by grinding OPC clinker, granulated blast furnace slag (25-65\%) and gypsum. Slag is waste product in manufacture of pig iron which contains silica around 25-40\% and lime 35-50\%.
- Fineness, setting time, soundness and strength are same as that for 33 grade OPC
- Low heat of hydration hence can be used for mass concrete.
- Bate of hardening an OPC during the first 28 days but there after increases and close to 12 months strength becomes equal to or more than OPC.
- They have lower lime high silica and alumina
- Better resistance to chemical attack (especially chlorides \& sulphates), hence can be used for Marine Works and Sewer Works.


## 5. sulphate resistant Portland cement (IS: 12330)

- It is similar to OPC except that it contains very low $\mathrm{C}_{3} \mathrm{~A}$ content ( $<5 \%$ ), ground finer than OPC and high silicate content
- This cement is sulphate resistant because the disintegration of harden concrete, caused by the chemical reaction of $\mathrm{C}_{3} \mathrm{~A}$ with soluble sulphate like $\mathrm{MgSO}_{4}, \mathrm{CaSO}_{4}$, and $\mathrm{Na}_{2} \mathrm{SO}_{4}$ is inhibited.
- It can be used in Structures in sea water, coastal area, marshy lands, sewage and canal linings
- Note: OPC subjected to sulphate attack specially $\mathrm{MgSO}_{4}$, Sulphates react with free calcium hydroxide to form calcium sulphate and hydrate of calcium aluminate to form calcium sulpho aluminate volume of which is approximately $227 \%$ of the volume of the original volume causes cracks in cement paste and concrete due to expansion.


## 6. Low heat Portland cement (IS:12600)

- It is a Portland cement with relatively lower contents of the more hydrating compounds $\mathrm{C}_{3} \mathrm{~S}$ and $\mathrm{C}_{3} \mathrm{~A}$ and more contents of $\mathrm{C}_{2} \mathrm{~S}$.
- Rate of development of strength is slow but ultimate strength is same as that of OPC
- Low heat evolved, preventing shrinkage at high temperature. Used for mass concreting of dam and bridges etc.


## 7. Quick setting cement

- In the manufacturing of this cement gypsum content is reduced who get the quick setting property, also small amount of alumina sulphate.
- It is grounded much finer than OPC.
- it sets quickly but does not harden quickly.
- initial setting time is 5 minutes and final setting time is around 30 minutes.
- it is used for concreting under water or in running water.


## 8. White and colored Portland cement

- Manufactured from pure white chalk and clay free from iron oxide.
- Gray color of cement is due to iron oxide, so for white cement iron oxide should be less than $1 \%$. colored cements are made by adding 5 to $10 \%$ coloring pigments before grinding.
- Compressive strength of this cement is $10 \%$ of that 33 grade of OPC.
- hunter scale or ISI scale is used to measure the whiteness of white cement.


## 9. Expensive cement

- this cement does not shrink or suffer any overall change in volume on drying while hardening and thereafter expand slightly with time.
- Commonly used for grouting anchor bolts or grouting machine foundation or prestressed concrete ducts where in drying shrinkage may otherwise defeat the purpose of grouting.
- expensive cement is obtained by mixing 8 to 20 parts of sulpho -aluminate clinkers with 100 parts of OPC and 15 parts of stabilizer.


## 10. Hydrophobic cement or water repellent cement

- stearic acid, boric acid, oleic acid and pentachlorophenol are added to OPC (0.1\%$0.5 \%$ of weight of cement) during grinding of cement clinker. These acids form a film around the cement particles which prevent the entry of atmospheric moisture and this film breaks down when the concrete is mixed and then the normal hydration take place.
- Note: hydrophobic cement has small strength gain initially because of the hydrophobic films on cement grains which prevent the interaction with water but it's 28 day strength is equal to OPC.


## 11. Air entraining cement

- It is manufactured by adding small amount off air entraining agent with OPC clinker at the time of grinding. This improves workability and water-cement ratio can be reduced which reduces shrinkage.
- Also minute voids increases resistance against freezing and scaling action of salts.
- Air entraining cements are used for the same purpose as that of OPC but it has higher initial setting time and longer final setting time than OPC.
- Various air entraining agents are aluminum powder, zinc powder, hydrogen peroxide, natural wood resigns and calcium ligno-sulphates.


## 12. Portland pozzolana cement

- It is manufactured by grinding Portland cement clinker and pozzolana (15 to 35\%) or by uniformly blending OPC and with fine pozzolana.
- Pozzolana's a siliceous and aluminous material which has no cementing property itself but has the property of combining with lime to produce a stable compound which has definite cementitious property.
- Pozzolana can be Fly ash, blast furnace slag, rice husk etc. calcium hydroxide.
- Pozzolanic action is very slow hence it has low heat of hydration and low rate of gain of strength, but ultimate strength is comparable to OPC.
- as low heat is involved so it can be used for mass concreting such as dams, it is low post because mostly clinkers are replaced by cheaper pozzolanic material.
- As free line present in the cement is removed and hence resistance to chemical attack increases, making it suitable for marine works.


## CHAPTER-2-CONCRETE

## 1. PERMISSIBLE LIMITS FOR IMPURITIES IN WATER

| Impurity | Permissible Limits |
| :--- | :--- |
| Organic | $200 \mathrm{mg} / \mathrm{L}$ |
| Inorganic | $3000 \mathrm{mg} / \mathrm{L}$ |
| Sulphates <br> $\left(\mathrm{SO}_{3}\right)$ | $400 \mathrm{mg} / \mathrm{L}$ |
| Chlorides (CI) | $2000 \mathrm{mg} / \mathrm{L}$ for plain concrete work, 500 <br> $\mathrm{mg} / \mathrm{L}$ for reinforced concrete work |
| Suspended <br> matter | $2000 \mathrm{mg} / \mathrm{L}$ |

## 2. WHY NOT TO USE SEA WATER

- Seawater generally contains 3 to $5 \%$ of dissolved salts of which $78 \%$ is NaCl and $15 \%$ chloride/sulphate of $\mathrm{Mg}^{+2}$, this will cause alkali aggregate reactions.
- As per IS456 seawater cannot be used for RCC and PSC but can we use four PCC in unavoidable condition.
- Seawater cannot be use for PCC also if aggregates are all alkali reactive.
- salt in seawater may cause efflorescence and persistence dampness hence it should be avoided when appearance is important.
- although seawater slightly accelerates the setting time but reduces the 28 days strength by 10 to $20 \%$.


## 3. COMPRESSIVE STRENGTH OF CONCRETE

- 3specimens of a sample are taken to report the strength and compressive strength is average of 3 specimens.
$0.85\left(\mathrm{f}_{\mathrm{c}}\right)_{\text {sample }} \leq\left(\mathrm{f}_{\mathrm{c}}\right)_{\text {specimen }} \leq 1.15\left(\mathrm{f}_{\mathrm{c}}\right)_{\text {sample }}$
- standard cube sizes $150 \times 150 \mathrm{~mm}$, standard cylinder sizes 150 mm diameter and 300 mm height. The faces in touch with molds are in touch with the plates of the machine.
- Compressive strength of a cylindrically specimen is it 0.8 times the compressive strength of a cubic specimen it is due to restraining effect of plates (because of friction) of the testing machine extends over the entire height of a cube but leaves unaffected a part of a test slender.
- Strength decreases with increase in size, however after a certain value it is almost constant. Compressive strength of concrete in structure is it 0.85 times strength in cylindrical specimen and 0.67 times strength in cubical specimen. IS code takes cubic strength as standard strength.

Note: (a) Specimen tested in dry condition shows about 15\% decrease in elastic modulus as compared to wet specimens this is explained by the fact that drying produces more micro cracks in the transition zone which affects behavior of curve for stress strain.
(b) This is opposite to effect of moisture on the compressive strength which decreased by $15 \%$ when tested wet as compared to dry. Because moisture content in concrete produces lubrication effect and reduces the strength.
4. FACTORS AFFECTING STRENGTH OF CONCRETE

- Water cement ratio: Water content required to react completely with cement is $23 \%$, rest of the water used for preparing paste will result in voids in the concrete matrix. But the water cement paste also work as lubricant for compaction, if the mixture is not fully compact it will not develop the strength fully. Strength of concrete is inversely proportional to $\mathrm{w} / \mathrm{c}$ ratio, but when $\mathrm{w} / \mathrm{c}$ ratio is very small then compaction become difficult, and strength gained is not equal to theoretically observed.

- Size of specimen: Smaller is the size of specimen better is its strength. Strength of cube is greater than strength of cylinder is greater than strength of building unit.
- Size of aggregate: Bigger is the aggregate lesser are the voids better should be the strength, but after a particular size further increase in size will decrease the strength because proper bonding between the aggregate is not possible because of the lesser surface area to the volume ratio.
- Shape of aggregate: round aggregate requires lesser paste to be bounded because have least voids but angular aggregate shows better interlocking so increases strength of concrete.
- texture of aggregate: more rough texture of aggregate better bonding between them.
- Rate of loading: faster is the loading more will be the strength because in slow loading there is more time for creep to occur as failure is governed by limiting strain not by stress.
- Compaction/air voids: strength of concrete decreases with increase in air voids in concrete which is due to improper compaction. Approximately $1 \%$ air voids will result in 5\% strength reduction.

5. CREEP OF CONCRETE

- Under sustained compression (loading can be tensile also) deformation in concrete increases with time even though applied stress level is not changed, this time dependent component of strain is called creep.
- Creep depends on applied stress level, higher is the applied stress higher will be the creep, it is only due to dead load + permanent live load.
- creep occurs due to,(i) internal movement to adsorb water (ii) sliding between gel particles (iii) moisture loss and (iv) growth in microcracks.
- effect of creep are, (i) increase in deflection of beam, column and slabs, (ii) loss of prestress (iii) gradual transfer of load from concrete to the reinforcement in compression member. Due to create compressive strain in compression steel increases and tensile strain in tensile steel increases.
- Presence of compression reinforcement reduces creep, this effects sometime are beneficial like reduction of stress due to support movement.
- In general creep increases (i) when cement content is high (ii) aggregate content is Iow (iii) W/C ratio is high (iv) higher volume of voids (v) relative humidity is low (vi) temperature is high (vii) size or thickness is small (viii) loading occurs at early age (ix) load in sustained over long period of time.

6. SHRINKAGE OF CONCRETE

- It results from volume change in concrete, similar to creep it also introduces time dependent strain but shrinkage strain are independent of applied stress.Also shrinkage is reversible to great extent, alternate dry and wet processes will results in alternatevolume change in concrete.All factors related to material properties, composition of mix, curing, environmental condition, member size, that effect creep also effects in shrinkage.
- Types of shrinkage:
chemical shrinkage:this is caused due to chemical reaction (hydration of cement), as you know absolute volume of unhydrated material is more than absolute volume of hydrated material.At early stage it results in volume reduction and at later stages with causes void formation.
plastic shrinkage:it occurs due to loss of moisture from top surface, cracks genrated are generally on the top surface and it is a short-term process.
autogenous shrinkage:it results in volume reduction with no moisture transfer from outside, this is mainly due to self-desiccation of cement (extreme state of dryness).This results in rise of capillary pressure and occurs in early days. It increases with increasing grade of concrete.
drying shrinkage: contraction of harden concrete due to loss of water from the pores.It is a long-term process it increases with decreasing grade of concrete.
- For normal concrete, generally drying shrinkage dominates.
- half of the totalshrinkage occurs in first half month and $3 / 4^{\text {th }}$ in first six month, in absence of data shrinkage strain for design shall be taken as $3 \times 10^{-4}$.


## 7. MATURITY OF CONNCRETE

Strength of concrete depends on time period of curing end temperature maintained during Curing for early period of hydration.

$$
\text { Maturity }=\sum(\text { time } \times \text { temperature })
$$

temperature is taken starting room $-11^{\circ} \mathrm{C}$, because experimentally hydration of concrete stops below $-11^{\circ} \mathrm{C}$.

$$
\% \text { strength of concrete }=A+B \log _{10}\left(\text { maturity } \times 10^{-3}\right)
$$

## 8. WORKABILITY OF CONCRETE

- Workability is the amount of work required to produce full compaction.
- Consistency indicates the fluidity or mobility. it must be noted that workability is different from consistency optimum workability of concrete will be dependent on the type of job situation.
- The important factors affecting workability are:
(i) water content: higher is the water content better is fluidity of the mix, water act as a lubricant. For better workability more water is to be added but adding only water will increase water cement ratio which reduces strength so both water as well as cement must be added in the same ratio as that of the original mix.
(ii) Aggregate sites: for larger size aggregates total surface area to be wetted per unit volume of the aggregate is lesser, so additional paste is available to help lubrication.
(iii) shape of aggregate: rounded aggregates give better workability. Angular,elongated or flaky aggregates make concrete harsh.
(iv) mix proportion and grading of aggregate: better is the grading lesser will be the voids to be filled by the paste so more paste is available to work as lubricant
(v) surface texture of aggregate: smoother is the surface of aggregate better is its work ability.


## 9. Mass concreting

- During massconcreting, concrete is placed in layers of $350-450 \mathrm{~mm}$, before placing the concrete on next layer previous surface must be properly cleared with waterjet and scrubbing by airbrush.
- Concrete subjected to lateral thrust, bond bars and bond stones are provided to form a key between different layers.
- In case of mass concreting due to large heat of hydration trapped within the concrete mass, it tries to expandand hence tensile stresses develops on outside and compressive in interior which causes cracks on the surface.
- To avoid these cracks, we use low heat of hydration cements, external ice bags are placed to lower the surface temperature.


## 10.VARIOUS TESTS FOR WORKABILITY ARE:

## Slump Test

- Slump test is the most commonly used method of measuring consistency of concrete which can be employed either in laboratory or at site of work.
- It is not a suitable method for very wet or very dry concrete and stiff mix.
- Maximum size of aggregate is limited 38 mm .



## Compaction Factor Test

- In the compaction factor test the degree of workability is measured in terms of internal energy required to compact the concrete thoroughly.
- A compaction factor increase is workability also increases.
- The compacting factor test is designed primarily for use in the laboratory, but it can also be used in the field.
- The degree of compaction called the compacting factor is measured by the density ratio i.e., the ratio of the density actually achieved in the test to the density of the same concrete fully compacted.


## Vee Bee Consistometer

- This test consists of a vibrating table, metal pot, sheet metal concrete and a standard iron rod.
- The time required for the shape of concrete to change from slump concrete shape to cylindrical shape in seconds is known as Vee Bee Degree.
- This method is very suitable for very dry concrete whose slump value cannot be measured by slump test but the vibration is too vigorous for concrete with a slump greater than about 50 mm .
- In this test, more is the time lesser will be the workability.


## Flow Test

- This test is appropriate for concrete having very high work ability including flowing concrete.
- the flow of concrete is the percentage increase in diameter hope spread concrete due to falling off table from 12.5 mm @ of 60 RPM for 15 times. The spread or the flow of the concrete is measured, and this flow is related to workability.
- This is a laboratory test which gives an indication of the quality of concrete with respect to consistency,cohesiveness and the proneness to segregation.

Uses of concrete having different consistency:

| Degree of <br> workability | Slump <br> $(\mathrm{mm})$ | Compacting <br> factor |  | Use for which concrete is suitable |
| :---: | :---: | :---: | :---: | :--- |
| Very low | - | 0.78 | 0.90 | Roads vibrated by power-operated machines. At the <br> more workable end of this group, concrete may be <br> compacted in certain cases with hand-operated <br> machines. |
| Low | $24-75$ | 0.85 | 0.87 | Roads vibrated by hand-operated machines. At the <br> more workable end of this group, concrete may be <br> manually compacted in roads using aggregate of <br> rounded or irregular shape. Mass concrete foundations <br> without vibration or flighty reinforced sections with <br> vibration. |


| Medium | $50-100$ | 0.92 | 0.935 | At the less workable end of this group, manually <br> competed flat stables using crushed aggregates. <br> Normal reinforced concrete manually compacted and <br> heavily reinforced sections with vibrations. |
| :---: | :---: | :---: | :---: | :--- | :--- |
| High | $100-$ <br> 150 | 0.95 | 0.96 | For sections with congested reinforcement, Not <br> normally suitable for vibration, for pumping and <br> tremble placing. |
| Very high | - | - | - | Flow table test is more suitable. |

## 11.COMPACTION

- Compaction is a process of removal of entrapped air and uniform placement of concrete to form a homogeneous dense mass. Density strength durability impermeability obse concrete depends on compaction.
- compaction by mechanical vibration reduces internal friction between the different particles of concrete by imparting oscillations to particles and dust compacts the concrete.
different types of mechanical vibrators are:
- internal vibrator/needle/poker/immersion vibrator: it is generally best suited for construction work as all the energy is directly transferred to concrete.A steel tube with eccentric loading inside is rotated with external motor and this tube is inserted in concrete itself.Do not try to move concrete with vibrator, it will cause segregation of concrete mix.
- Form or external vibrators: clamped rigidly do the form work at the predetermined points so that both concrete and form work both are vibrated. it is used where needle vibrator cannot be inserted like dense RCC or narrow walls or columns. you can compact up to 450 mm from surface. This has to be moved from surface to surface, it requires more power to give same degree of compaction effect as given by the internal vibrator.
- vibrating tables: it is very efficient in compacting stiff and harsh concrete mixes, required for precasting elements, it is also used for laboratory compaction of test specimens.
- surface vibrator or screed board vibrator: it is placed directly on the surface of concrete mass shallow depth.It is mainly used in case of road construction, it should not be used when depth of the concrete is more than 150 mm .


## 12. NONDESTRUCTIVE TEST

- Nondestructive tests do not cause any damage to the structure or specimen and hence save a lot of time and money.They are more or less done at same location to know variation of property with time.
- rebound hammer test: it is done to find out the compressive strength of concrete.Its principle is, rebound of an elastic mass depends on hardness of surface against which it strikes. The compressive strength can be read directly from the graph provided on the body of hammer.Surface of specimens should be smooth.For old concrete surface hardness will be more as compared to strength of interior because in old concrete due to carbonation at surface hardness increases and it will overestimate the strength of concrete.
- penetration test: it is done to determine resistance of concrete to penetration by a steel Rod, driven by a fixed amount of energy.It is used to assess the compressive strength of concrete; depth of the penetration is inversely proportional to strength.
- ultrasonic pulse velocity test: principal of this test is velocity of sound in a solid material is a function of modulus of elasticity and density of material.The instruments consist of a pulse transmitter and receiver which are held on opposite faces.velocity of a wave is calculated which is the correlated to the strength of concrete, type and amount of aggregate affects the pulse velocity, paste has lesser velocity as compared to that ofaggregate. Saturated sample has more velocity as compared to dry sample.If reinforcement is present in path, then proper adjustment is to be made for corrections.It can estimate strength and homogeneity of concrete.thisdetermines the dynamic modulus of elasticity.
- pull out test: this test measures the force required to pull out a precast metal entered earlier with enlarged end.Pull out forces related to the compressive strength of concrete; it actually measures the shear strength of concrete.This test has to be preplanned and damage caused to the location has to be maintained.


## 13.DEFECTS IN CONCRETE

## Bleeding of Concrete

- Bleeding is also known as water gain it is a type of segregation in which some of the water in the mix tends to rise to the surface of freshly prepared or placed concrete. it is caused due to inability of solid constituents to hold all the mixing waters when settles downward, water being lightest hence comes to the top.
- Due to bleeding water comes up and accumulates at the surface, along with water some cement will also come out.This is observed in case of rich mix and where excessive vibration is done.
- Bleeding leads to creation of pores inside concrete hence causes decrease in strength.
- Bleeding can be reduced by using finer cement, by use of uniformly graded aggregates, air entraining agents and pozzolana, by breaking continuous water channel.


## Segregation

- It is the separation of the constituent material of concrete. In a good concrete all the constituents are distributed uniformly to make a homogeneous mixture.
- Cohesive and fatty characteristic of matrix does not allow aggregate to fall apart.
- Causes of segregation are excessive water, dropping concrete from height, badly designed mix, poor aggregate grading, concrete carried away for long distances.
- If the mix is too wet then excessive vibration will make aggregate to settle down, it can be avoided by air entrainment using dispersing agent, finer coarse aggregates.
- Do not use vibrator to spread the mixture.


## Crazing

- Development of fine random cracks on the surface of concrete due to difference in shrinkage between surface and interior. These are not harmful apart from appearance. Their depth is less than 12 mm .


## Cracks

- Cracks cannot be completely prevented but can be minimized, use of unsound material high w/c ratio, bad jointing, freezing, thermal effects etc. leads to cracks.This makes the concrete less durable.
- when more water is used,coarse aggregates will settle down and upper layer will have more pores as the water quantity is high.This porous concrete could not withstand sinkage stresses and will lead to cracks.
- shrinkage cracks due to early Loss of water, water is absorbed by dry aggregates, dry shuttering or hot sunny day. Which leads to shrinkage cracks.
- alkali aggregates reactions, when expensive compound is formed by reaction of silica from aggregate and alkali from cement in presence of moisture in concrete will lead to cracks and disruption of concrete.
- freezing and thawing, if $W / C$ ratio was high concrete would be more porous and rainwater can percolate easily inside and when temperature goes below freezing point these voids tried to expand to accommodate the volume change.


## 14.Underwater concreting

- Bottom dump truck: In this method a bucket of concrete is taken through water in a watertight box or bucket and on reaching the final place of deposition the bottom is made open by some mechanism. Some amount of cement is washed off and concrete masses is full of voids.
- Tremie pipe: This is the best method for underwater concreting. The pipe of 200 mm diameter is used with funnel at top. Firstly, pipe is filled with concrete and then a small jerk is given after placing the pipe at desired location, so that bottom lock isunlocked, and concrete is placed, now keeping the end of pipe submerged in concrete we fill the pipe again and again till desired depth is reached. Concrete in inner layer is not affected by water except the outer layers. concrete automatically get compacted by water pressure. Here concrete is used having very high slump about 150 to 200.
15.High performance concrete:This is not any special type of concrete but are created by using one or more cementitious materials such as Flyash, silica flumes or granulated blast furnace slag and usually a superplasticizer.


## their basic properties are

1. it has very less permeability, high toughness and good workability.
2. high durability and energy adsorption capacity for earthquake resistant structure.
16.Ferrocement: the term ferrocement implies the combination of ferrous product with cement, generally this combination is in the form of steel wires meshes embedded in the Portland cement motor.

## its properties are

1. it has high strength per unit mass
2. capacity of to resist shockload
3. it is impervious
4. these can be constructed without using formwork.

## CHAPTER-3-STEEL \& MISCELLANEOUS MATERIALS

## 1. Steel

The carbon content in steel lies between cast iron (2-4 percent) and wrought iron (0.15 percent maximum). Usually, it is between two percent. Carbon in excess of percent does not combine with iron and exists as free graphite. This makes the dividing line between cast iron and steel where former possesses free graphite whereas later does not. As the carbon content increases steel becomes harder and tougher. Steel possesses sufficient tensile and compressive strength unlike cast iron (good compressive strength but poor tensile strength) and wrought iron (good tensile strength by virtue of fibrous nature).

## - Manufacturing of steel

The processes commonly used for manufacturing of steel are
(1) Bessemer process
(2) Cementation process
(3) Crucible steel process
(4) Duplex process
(5) Open hearth process
(6) Electric process

- Defects in steel: The commonly occurring defects in steel are
(i) Cavities or blow holes: These are formed by trapping of gases in the molten mass which produce cavities on solidification.
(i) Cold shortness: Higher percentage of phosphorus causes this defect in steel. The steel cracks when worked in cold state
(ii) Red shortness: Excess amount of sulphur in steel causes cracking of a steel when worked in hot state.
(iii) Segregation: This defect refers to the separation of some constituents of steel solidifying at an early stage.
- Properties of mild steel: Mild steel possesses the following properties.
(1) It can beEasily forged and welded
(ii) It cannot be easily hardened and tempered
(iii) It has a fibrous structure
(iv) It is malleable and ductile
(v) It is susceptible to rusting
(vi) it has specific gravity of $7.70-7.80$


## 2. PAINTS

The paints are coatings of fluid materials, and they are applied over surfaces of timber and metal to protect the surface from weathering effects. Also, it gives a good appearance and a smooth surface for easy cleaning.

## - Ingredients of an oil paint:

An oil paint essentially consists of the following ingredients:
(i) Base: A base is a solid substance in a line state of division, and it forms the bulk of a paint. It determines the character of the paint and imparts durability to the surface which is painted. It reduces shrinkage cracks formed on drying and it also forms an opaque layer to obscure the surface of material to be painted. Common bases are white lead, red lead, oxide of zinc and zinc white, Lithophone etc.
(ii) Vehicles: The vehicles are the liquid substances which hold the ingredients of a paint in liquid suspension. They help in even spreading of paint and acts as a binder material so that the paint may stick to the surface properly. Common vehicles used in the paint are Linseed oil, Tung oil, Poppy oil, Nut oil etc.
(iii) Driers: These substances accelerate the process of drying. A drier absorbs oxygen from the air and transfers it to the linseed oil, which in turn gets hardened. Litharge is the most commonly used drier.
(iv) Colouring pigments: Pigments are added to the paint to impart desired colour. They might be natural or artificially made.
(v) Solvents: The function of a solvent is to make the paint thin so that it can be easily applied on the surface. It also helps the paint in penetrating the porous surfaces. The most commonly used solvent is the spirit of turpentine.

- Different Varieties of Paint:

Following are the different types of paints available in the market.
(i) Aluminium paint: The very finely ground aluminium is suspended in either quick drying spirit varnish or slow-drying oil varnish as per requirement. The oil evaporates and a thin metallic film of aluminium is formed on the surface.
(ii) Anticorrosive paint: This paint essentially consists of oil and a strong drier. A pigment such as chromium oxide or lead or red lead or zinc chrome is taken and after mixing it with some quantity of very fine sand, it is added to the paint.
(iii) Asbestos Paint: This type of paints are applied on the surfaces which are exposed to the acidic gases and steam.
(iv) Bituminous paint: This paint is prepared by dissolving asphalt in any type of oil or petroleum. The paint presents a black appearance, and it is used for painting ironwork under water.
(v) Cellulose paint: This paint is prepared from nitro-cotton. Celluloid sheets, photographic films, etc. It hardens by evaporation of a thinning agent. It gives a flexible, hard and smooth surface. Also, the painted surfaces with cellulose paint can be washed and easily cleaned.
(vi) Cement paint: This paint consists of white cement, pigment, accelerator and other additives. It is available in dry powder form. The cement paint is available in a variety of shades, and it exhibits excellent decorative appearance. It is waterproof and durable.
(vii) Colloidal paint: No inert material is mixed in this type of paint. It requires more time to settle and in the process of settlement, it penetrates through the face. It may be used for interior as well as exterior walls.
(viii) Emulsion paint: A variety of emulsion paints is available. It contains binding materials such as polyvinyl acetate, synthetic resins, etc. This paint is easy to apply and it dries quickly in about 1 to 2 hours.
(ix) Enamel paint: This paint is available in different colours. It contains white lead or zinc white, oil, petroleum spirit and resinous matter. It dries slowly and forms a hard and durable surface.
(x) Graphite paint: The paint presents a black colour and it is applied on iron surfaces which come in contact with ammonia, sulphur gases, chlorine etc.
(xi) Luminous paint: This paint contains calcium sulphide with varnish. The surface on which luminous paint is applied shines like radium dials of watches after the source of light has been cut off.
(xii) Oil paint: This is ordinary paint and it is generally applied in three coats of varying composition. They are respectively termed as primes, undercoats and finishing coats. This paint is cheap and easy to apply and it possesses good opacity and low gloss.
(xiii) Plastic paint: This paint contains the necessary variety of plastics. The paint possesses a pleasing appearance and it is attractive in colour. It is widely used in showrooms and auditoriums.

## 3. VARNISHES

The term varnish is used to indicate the solution of resins or resinous substances prepared either in alcohol, oil or turpentine.

## - Following are the characteristics of an ideal varnish

(i) It should render the surface glossy.
(ii) It should dry rapidly and present a finished surface which is uniform in nature and pleasing in appearance.
(iii) The colour of varnish should not fade away when the surface is exposed to the atmospheric actions.
(iv) The protecting film developed by varnish should be tough, hard and durable.
(v) It should not shrink after drying.

## - Ingredients of a Varnish:

Following are the ingredients of a varnish:
(i) Resins or resinous substances: The commonly used resins are copal, lac, shellac and rosin.
(ii) Driers: The function of a drier in varnish is to accelerate the process of drying. The common dryers used in varnishes are litharge, white copper and lead acetate.
(iii) Solvents: Depending upon the nature of resin, the type of solvent is decided. For Copal boiled linseed solvent is used. For lac and shellac, the Methylated spirit of wine is used as solvent. While for rosin, turpentine is used as solvent.

## - Types of Varnishes:

Varnishes are classified on the basis of the type of solvent used. They are basically of four types:
(i) Oil varnishes: Linseed oil is used as solvent.
(ii) Spirit varnishes: The methylated spirit of wine is used as a solvent.
(iii) Turpentine varnishes: Turpentine is used as solvent.
(iv) Water varnishes: Hot water is used as solvent.

## 4. ADMIXTURES:

- Admixtures are chemical compounds other than water, aggregates, hydraulic cement and additives like pozzolana, fiber $\mathrm{R} / \mathrm{f}$ used as ingredient of concrete.
- it is added before or during mixing to modify one or more property of concrete in wet and hardened state.
- admixtures are liquid generally because liquids are better to mix.
- admixtures do never replace the good workmanship or good material to be used.
- reasons for adding admixture to the concrete are, to increase workability, to reduce water content, to accelerate or retard setting of concrete and increase resistance to the chemical attack.
- plasticizers(water reducers): plasticizers mainly fluidify the mix and improve the workability for a given water content or help decreasing the water content for a given workability.Ex are ligno-sulphonic acid,hydroxylated carboxylic acid. when plasticizers are added they get adsorbed on the cement particles, adsorption of charged polymer on cement creates repulsive force between particles and flocculated structures changes to dispersed and water entrapped in voids of cement structure is free to help in improve fluidity.
- Superplasticizers: they are chemically distinct from normal plasticizers, but the action is basically same. use of superplasticizers can reduce water requirement by $30 \%$ as compared to plasticizers which can help reduce water quantity by $15 \%$.Finer is the cement more will be the plasticizer required. examples are sulphonated melamine formaldehyde.It is used in case of flowing concrete, self-compacting concrete and production of high-performance concrete.
- Airentrainers: air entraining agent introduces air in the form of minute air bubbles distributed uniformly throughout the cement paste, to improve resistance against freezing and thawing of hardened concrete, to improve workability in case of harsh mix or whereas fines are absent.It also helps prevent bleeding. Air entrainer reduces the strength, $1 \%$ air bubble will reduce $5 \%$ compressive strength.
- Accelerators:These admixtures speed up the chemical reaction of cement and water hence accelerate the rate of setting or early gain of strength.also, it increases initial
heat of hydration.Ex. of accelerator are $\mathbf{N a C l a n d ~} \mathbf{N a}_{2} \mathbf{S O}_{4}$ and $\mathbf{N a O H}$ etc.Used where early setting or high initial strength is required. concreting is taking place in very cold region.
- Note: chloride-based accelerator promote corrosion of R/f steel, so avoid using this for RCC.
- Retarders: These slow down the chemical process of hydration so concrete remain plastic and workable for a longer time. examples are gypsum, sugar.Overdosing candelay setting for days.Used when concreting is done in hot weather, to prevent cold jointingdue to duration of placing, to transport concrete from one place to another.Note: $\mathrm{CaCl}_{2}$ is added up to $2 \%$ it acts as it accelerator but on increasing the proportions it acts as retarder.


## GATE/ESE

## Civil Engineering

Project Management

## IMPORTANT FORMULAS ON PROJECT MANAGEMENT

## CHAPTER-1-PROJECT MANAGEMENT

Project Management deals with both 'material' as well as 'human factors' to increase productivity.

Objectives of a Project:

- It should be completed in minimum time with minimum capital investment
- It should use available manpower and other resources optimally.

Phases of Project Management
Planning
Planning involves:

1. Defining objectives of the project.
2. Listing of jobs that have to be performed.
3. Determining gross requirements for materials, equipment and manpower and preparing estimates of costs and duration for various jobs.
4. To bring about the satisfactory completion of the project.

## Scheduling

Scheduling is the allocation of resources such as time, material, space, equipment and human and technological effort.

It involves:

1. Finalizing the planned functions mechanically.
2. Assigning starting and completion dates to each activity to proceed in a logical sequence and in a systematic manner.

## Controlling

Controlling involvesdetermination of deviations from basic plan and their effects on the project.

- It should be noted that planning and scheduling are accomplished before the actual protection starts while controlling is operative during execution of the project.


## CHAPTER-2-FUNDAMENTALS OF NETWORK

## Event

- An event is either the start or completion of an activity.
- Events are significant points in a project which act as control points of the project.
- An event is an instant of time and it does not require time or resources.

Following are examples of an event:

1. All parts assembled
2. A budget prepared
3. Construction completed

Following taskscannot be termed as events:

1. Prepare budget
2. Assemble pals

3 Excavate trench

- Events are represented by nodes in a network. It may have any of the following shapes
(I) Circular

(iii) Rectangular
(ii) Square

(iv) Oval



## Different Shapes for Events

Most commonly adopted shape for events is circular shape.

- Tail event or Start event:

It is the beginning of an activity.

If it is the first event of the project then known as 'initial as start event". It has only an outgoing arrow. e.g., Event C is a tail event.


## - Head event or Final event:

The event which marks the completion of an activity is known as 'head event". If this event represents completion of the entire project, then it is called "Finish event'.

It has only incoming arrows.
E, g.: Event C is a head event.


Head Event
(i) When a tail event represents the beginning of more than one activity, then the event is said to occur when the first activity starts from it.
(ii) Similarly, when a head event occurs at the end of more than one activity, the event is said to have occurred only when all the activities leading to it are completed.

- Dual role events: All events except the first and the last event of a project are dual role events. They have both incoming and outgoing arrows.


Dual Role Events
e.g.: Events 2and 6 are dual role events.

- Successor events: The event or events that follow another event are called successor events to that event.


Successor Events
e g.: Event 2 and 3 are successor events of event 1

- Predecessor events: The event or events that occur before another event are called predecessor oven' to that event.
In the above figure, events 2 and 3 are predecessor to event 5.
Activity
Activity is the actual performance of a job. It requires lime and resources for its completion.
Following are examples elan activity:

1. Excavate trench
2. Mix concrete
3. Prepare budget

In A-0-A system (Activity on Arrow network system), activity is represented by arrows between events while in A-O-N (Activity on Node system), activities are represented by nodes. In A-O-N system, events have no place.


Here A \& B activities are represented in two different systems.

- The activities which can be performed simultaneously are independent of each other called parallel activities. In the above figure, activities A \& Bare parallel activities.
- Activities or activities that can be performed after performance of other activities are known as successor activities to that activity. Activity F is the successor activity to activity C in above figure.

- Similarly, activities that are required to be performed before next activity can begin are called predecessor activities to that activity. Activity (A) is predecessor activity to activity D.


## Dummy

- A dummy is a type of operation which neither requires time nor any resource, but it denotes dependency among the activities.
- It is represented by a dashed arrow.

In the figure shown below, a dummy activity is shown.


- Dummy is used to serve following purposes:

1. Grammatical purpose:

To prevent two arrows having a common beginning and common end.


B
(a) Ambiguous Representation

(b) Grammatically Clean Representation
2. Logical purpose:

To show relationships with other activities.
Here dummy is required to show that activity $D$ can start after completion activities of A \& B both


- Unnecessary dummies should be avoided.
- Dummies are used to show predecessor relations but if that relation is already established in the network, then that dummy is redundant and has to be removed.
- If dummy is only an incoming/outgoing arrow to/from a node then it can be removed provided there is no logical or grammatical error.


## Rules of a Network

1. There can be only one initial and one final event.
2. An event cannot occur unless all preceding activities are completed.
3. An event cannot occur twice.
4. Number of arrows should be equal to the number of activities.
5. Time should always flow from left to right.
6. Length of the arrow does not show any magnitude. Straight arrows should be taken as far as possible.

7. Arrows should normally not cross each other. If it is necessary to cross, one should be bridged over the other.
8. No activity can start until its tail event has occurred.

## Fulkerson's rule for numbering the events:

1. The single initial event is numbered as $0,1,10$ etc.
2. All arrows emerging out of the initial event are neglected. Doing so, the created one or more new initial events are numbered as $2,3,4$ or $20,30,40$ etc.
3. Step - 2 is repeated unless all events are numbered.

## Errors in Network

1. Looping error: Loops should not be formed.


## 2. Dangling error:

Project is complete only when all its activities are complete but the duration of activity 'R has no effect on the project time as shown in figure.


To avoid dangling error, the network must be examined in such a manner that all events except initial and final events must have at least one activity entering and one activity leaving them.
3. Wagon wheel error: As shown in figure below, each of the activities $P, 0$ and $R$ cannot start until all the three activities $A, B$ and $C$ are completed. But in reality, this may not be the situation. There is no error visible in the construction of the diagram but logical error has crept into it.


## CHAPTER-3

## PROGRAMME EVALUATION \& REVIEW TECHNIQUE (PERT)

## INTRODUCTION

- For PERT. Employs Beta-distribution for the time - expectation for an activity.
- The limits within which the duration will lie, is estimated.
- Pert follows the probabilistic approach and absorbs the uncertainties into the time estimates for activity and project durations.
- Therefore, PERT is well suited for those projects where there is insufficient or no background information for estimation of time duration
- PERT is used in R\&D type projects such as space industry, defense industry etc. As such projects are of non-repetitive type or once-through type for which correct time estimates cannot be made.
- Further a PERT analysis is event oriented $i$ e in this analysis interest is more focussed on the PERT (start or completion of activity) rather than the activities


## TIME ESTIMATES

In order to take into account, the uncertainties involved in the activity times three kinds of time estimates are made for each activity in PERT.
(i) Optimistic time ( $\mathbf{t}_{\mathbf{o}}$ ): If everything in the project goes well, it is the minimum time required for an activity if everything goes perfectly well without any problems or adverse conditions developed during the execution of the activity. In this time estimate, no provisions are made for delays or setbacks and better than normal conditions are assumed to prevail during the execution of the activity.
(ii) Pessimistic Time ( $\mathbf{t}_{\mathbf{p}}$ ): It is the time for completing an activity that is best.It is the maximum time required for an activity if everything goes wrong and abnormal situations prevail This time estimate does not include the possible effects of major catastrophes such as flood earthquakes, fire, labor-strikes etc.
(iii) Most Likely Time ( $\mathbf{t}_{\mathbf{m}}$ ): It is the time required to complete the activity if normal conditions prevail. This time estimate lies between pessimistic and optimistic time estimates.

- In PERT activity time is probabilistic but in CPM activity time is deterministic.
- The other difference: PERT is Event - Oriented. While the CPM is Activity - Oriented (in CPM we actually know the Activity time)


## (i) Expected completion time of an Activity: ( $\mathrm{t}_{\mathrm{E}}$ )

$$
t_{E}=\frac{t_{0}+4 t_{m}+t_{p}}{6}
$$

Where, $\mathrm{t}_{0}=$ Optimistic time
$\mathrm{t}_{\mathrm{p}}=$ Pessimistic time
$\mathrm{t}_{\mathrm{m}}=$ Most likely time
(ii) Standard deviation of an Activity ( $\sigma$ )

$$
\sigma=\frac{t_{p}-t_{0}}{6}
$$

(iii) Variance of an activity: ( $\boldsymbol{\sigma}^{\mathbf{2}}$ )

$$
\sigma^{2}=\left(\frac{t_{p}-t_{0}}{6}\right)^{2}
$$

## (iv) Central limit theorem:

(a) The mean time of the project as a whole is $t_{E}=t_{E 1}+t_{E 2}+\ldots \ldots$. along the critical path. Probability of completion of project in time $t_{E}$ is $50 \%$.
(b) The standard deviation of the project as a whole is
$\sigma=\sqrt{\sigma_{1}^{2}+\sigma_{2}^{2}+\sigma_{3}^{2}+\ldots \ldots . .}$ along the critical path.

Critical Path: The time wise longest path is called critical path. In this path any type of delay in any event will cause delay to the project. These are shown by double lines or dark lines in a network.

An event is critical if its slack is zero.

## Event Time:

## (i) Earliest expected event occurring time ( $\mathrm{T}_{\mathrm{E}}$ )

$T_{E}^{j}=T_{E}^{i}+t_{\varepsilon}^{i j}$ When there is only one path.


Where, $t_{\varepsilon}^{i j}=$ Expected completion time of an activity $\mathrm{I}-\mathrm{j}$
$T_{E}^{j}=\left(T_{E}^{i}+t_{\varepsilon}^{i j}\right)_{\max }$ $\qquad$ when there is more than one path.
Where $T_{E}^{i}=$ Earliest expected time of event i . $T_{E}^{j}=$ Earliest expected time of event j .
(ii) Latest allowable occurrence time ( $\mathrm{T}_{\mathrm{L}}$ ):
$T_{L}^{i}=T_{L}^{i}-T_{\varepsilon}^{i j}$ When there is only one path.
$T_{L}^{i}=\left(T_{L}^{i}-T_{\varepsilon}^{i j}\right)_{\min }$ When there is more than one path.
(iii) Slack (s): This is the time by which an event may be delayed without affecting the completion time of the project.


## - Probability Factor (z)

$$
z=\frac{T_{S}-T_{E}}{\sigma}
$$

Where, $\mathrm{T}_{\mathrm{s}}=$ Given scheduled completion time of the project

| $z$ | $P$ |
| :--- | :--- |
| 0 | $50 \%$ |
| +1 | $84.13 \%$ |
| +2 | $97.72 \%$ |
| +3 | $99.87 \%$ |


| $Z$ | $P$ |
| :--- | :--- |
| 0 | $50 \%$ |
| -1 | $15.87 \%$ |
| -2 | $2.28 \%$ |
| -3 | $0.13 \%$ |

$T_{E}=$ Expected completion time of the project.
$\sigma=$ Standard deviation

## Frequency Distribution Curve for PERT

It is assumed to be a $\beta$-distribution curve with a unimodal point occurring at tm and its end points occurring at $\mathbf{t}_{\mathbf{o}}$ and $\mathbf{t}_{\mathbf{p}}$. The most likely time need not be the midpoint of $\mathbf{t}_{\boldsymbol{o}}$ and $\mathbf{t}_{\boldsymbol{p}}$ and hence the frequency distribution curve may be skewed to the left, skewed to the right or symmetric.

symmetric

skewed to Left
$\beta$-Distribution curve


Skewed to Right

CHAPTER-4-CRITICAL PATH METHOD (CPM)

## Critical Path Method (CPM)

1. A network diagram in CPM is activity oriented.
2. Cost is the most important criteria. Minimum is found corresponding to optimum time.
3. There is only a single time estimate for each activity.
4. The probability of completion of activity in this estimated duration is $100 \%$.
5. It is based on a deterministic approach.
6. Suitable for repetitive types of work.
7. Normal distribution is followed.

## Activity times:


(i) Earliest Start Time (EST)

$$
E S T=T_{E}^{i}
$$

(ii) Earliest FinishTime (EFT) = EST + Activity time

$$
E S T=T_{E}^{i}+t_{i j}
$$

(iii) Latest Finish Time (LFT) $=$ TL of head event

$$
L F T=T_{L}^{j}
$$

(iv) Latest StartTime (LST) $=\mathbf{L F T}-\mathrm{t}_{\mathrm{ij}}$

$$
L S T=T_{L}^{j}-t_{i j}
$$

## Float:

Float denotes the range within which activity time or its finish time may fluctuate without affecting the completion of the project.

## (i) Total Float ( $\mathrm{F}_{\mathrm{T}}$ :

$\mathrm{F}_{\mathrm{T}}=\mathrm{LST}-\mathrm{EST}$ or $\mathrm{F}_{\mathrm{T}}=$ LFT $-\mathbf{E F T}$

$$
F_{T}=T_{L}^{j}-T_{E}^{i}-t_{\varepsilon}^{i j}
$$

(ii) Free Total (FF):

$$
\begin{gathered}
F_{T}=T_{E}^{i}-T_{E}^{i}-t_{e}^{i j} \text { Or } F_{F}=F_{T}-S_{j} \\
\text { Where } \mathrm{S}_{\mathrm{j}}=\text { Head event slack }
\end{gathered}
$$

(iii) Independent Float (Fid):

$$
\begin{gathered}
F_{I D}=T_{E}^{j}-T_{L}^{i}-t_{e}^{i j} \\
F_{I D}=F_{F}-S_{i} \\
F_{I D}=F_{T}-S_{i}-S_{i}
\end{gathered}
$$

Where $\mathrm{S}_{\mathrm{i}}=$ Tail event slack
$\mathrm{F}_{\mathrm{T}}=0$ - for critical path $\mathrm{F}_{\mathrm{T}}>0$-for subcritical path
$\mathrm{F}_{\mathrm{T}}<0$ - for Supercritical path
(iv) Interfering float (Fin)

It is another name for head event slack.

$$
F_{D N}=S_{j}=F_{T}-F_{F}
$$

## CHAPTER-5-CRASHING

## Cost Model Analysis

- In CPM, time is related to cost and the object is to develop an optimum time-cost relationship.
- The overall project duration can be reduced by reducing the duration of only the critical activities in the project network. The durations of such activities may be reduced in two ways.
(a) By deploying more resources for the early completion of such activities.
(b) By relaxing the technical specifications for such activities.
- In the CPM Cost model, we will be assuming that project duration is reduced by deploying more resources on critical activities.
- In CPM, there are two time and cost estimates for each activity: 'normal estimate' and 'crash estimate' in the normal estimate, the emphasis is on cost with time being associated with minimum cost. The 'crash' estimate involves the absolute minimum time required for the job and the cost necessary to achieve it. Here the emphasis is on 'time'.


## Project Cost

Total project cost is the sum of two separate costs:
(a) The direct cost for accomplishing the work, and
(b) The indirect cost related to the control or direction of that work, financial overhead, lost production, and the hike etc.

The components of the total cost are shown in figure below.



## Components of Project Cost

## Indirect Project Cost

- Indirect costs on a project are the expenditures which cannot be apportioned or clearly allocated to the individual activities of a project, but are assessed as a whole. The indirect cost includes the expenditures related to administrative and establishment charges, overhead, supervision, expenditure on a central store organization, loss of revenue, lost profit, penalty etc.
- Indirect cost rises with increased duration, considering only overhead and supervision. It is represented by a straight line, with a slope equal to daily overhead.
- But when there is a loss in profits, due to inability to meet demand or due to some penalty due to delay, a corresponding cost increase must be added to the cost of overheads producing the curve. Such a loss is called the outage loss.
- The total indirect cost curve will thus be curved.




## Direct Project Cost

- It is a cost which is directly dependent on the amount of resources involved for completion of activities.
- It includes labour materials, plants and machining etc.
- To get the same work done in less time, we have to increase the amount of labour, equipment and time saving materials, plants and machining etc.
- To get the same work done in less time, we have to increase the amount of labour, equipment and time saving material that takes extra charges which simply means increases in direct cost.
- The project has the highest cost corresponding to the crash duration, and has normal cost corresponding to the normal duration.
- Normal time ( $\mathbf{t}_{\mathbf{n}}$ ): Normal time is the standard time that an estimator would usually allow for an activity.

- Crash Time ( $\mathbf{t}_{\mathbf{c}}$ ): Crash times is the minimum possible time in which activity can be completed, by employing extra resources. Crash time is that time, beyond which the activity cannot be shortened by any amount of increase in the resources.
- Normal Cost ( $\mathbf{C n}_{\mathbf{n}}$ ): This is the direct cost required to complete the activity in normal time duration.
- Crash Cost $\left(\mathbf{C}_{\mathbf{c}}\right)$ ): This is a direct cost corresponding to the completion of the activity within crash time.
- Crash Slope: Cost Slope $=\frac{\text { Crash Cost }- \text { Normal Cost }}{\text { Normal Time }- \text { Crash Time }}$


## Note- Activities having minimum cost slope are crashed first.

## CHAPTER-6-COST ESTIMATION

## Unit of MEASUREMENT:

| S No. | Particulars of Item | Unit of measurement |
| :---: | :---: | :---: |
| 1. | Earthwork <br> (i) Earthwork in excavation <br> (ii) Earthwork in filling in foundation trenches <br> (iii) Earthwork in filling in plinth | Cum <br> Cum <br> cum |
| 2. | Concrete <br> (i) Concrete in foundation <br> (ii) Cement concrete in beam, slab <br> (iii) Lime concrete in roof terracing (Thickness specified) <br> (iv) Cement concrete in Lintels | Cum <br> Cum <br> Sqm <br> cum |
| 3. | Damp proof course (Thickness specified) | sqm |
| 4. | Brick work <br> (i) Brickwork in foundation <br> (ii) Brickwork in superstructure <br> (iii) Thin partition walls <br> (iv) reinforced brick work <br> (v) One brick thick wall (Thickness specified) | Cum <br> Cum <br> Cum <br> Cum <br> sqm |
| 5. | Stone work | cum |
| 6. | Wood work <br> (i) doors and windows frames and chowkhats <br> (ii) Shutters of doors and windows (thickness specified) | $\begin{aligned} & \text { Cum } \\ & \text { Sqm } \end{aligned}$ |


|  | (iii) doors and windows fitting (Hinges, handles etc.) | Number |
| :---: | :---: | :---: |
| 7. | Steel work <br> (i) Steel reinforcement bars <br> (ii) rivet, bolt and nuts <br> (iii) Binding of steel reinforcement <br> (iv) Iron hold fast <br> (v) Iron railing <br> (vi) iron grills, iron gate and shutters | Quintal <br> Quintal <br> Quintal <br> Quintal <br> Quintal <br> sqm |
| 8. | Roofing <br> (i) RCC slab roof <br> (ii) LC roofs over bricks or stone slab (Thickness specified) <br> (iii) Centering and shuttering form work <br> (iv) $A C$ sheet roofing | Cum <br> Sqm <br> Sqm <br> sqm |
| 9. | Plastering, pointing and finishing <br> (i) Plastering, cement or lime mortar (Thickness specified) <br> (ii) Pointing <br> (iii) White washing, colour washing (Number of coats specified) <br> (iv) Distempering, painting and varnishing <br> (Number of coats specified) | Sqm Sqm Sqm sqm |
| 10. | Flooring <br> (i) 25 mm or 40 mm CC floor <br> (ii) Doors and window sills | $\begin{aligned} & \text { Sqm } \\ & \text { Sqm } \end{aligned}$ |


|  | (iii) 25 mm CC over 75 mm LC floor | Sqm |
| :---: | :--- | :---: |
| 11. | Steel wooden trusses | No |
| 12. | Glass panels | sqm |
| 13. | Fixing of Glass panels | No. |

Sinking Fund: A fund which is kept aside annually to reconstruct the property after the expiry of the period of utility is known as sinking Fund.

The sinking fund is calculated using the following formula:

$$
I=\frac{S i}{(1+i)^{n}-1}
$$

Where,
$I=$ Amount of Sinking fund
$N=$ Life of the property
$\mathrm{i}=$ Rate of interest expressed in decimal
$S=$ Money required to buy the property
Depreciation: The loss in the value of a property due to constant wear and tear is termed as depreciation.

The depreciation can be calculated using the following methods:
(a) Straight Line Method: In this method, it is assumed that the property loses its value by the same amount every year. A fixed amount of the original cost is deducted every year, so that at the end of the utility period, only the scrap value is left.

$$
\text { Annual Depreciation }=\frac{C-S}{n}
$$

(b) Constant Percentage Method: In this method, it is assumed that the property will lose its value by a constant percentage of its value at the beginning of every year.

$$
\text { Annual Depreciation }=1-\left(\frac{S}{C}\right)^{1 / n}
$$

Where,

S = Scrap value
C = Original cost
$\mathrm{n}=$ life of property in years
The value of the property at the end of first year $=C-D C$.
(c) Sinking Fund Method: In this method the depreciation of a property is assumed to be equal to annual sinking fund plus interest on the fund for that year.

If $A$ is the annual sinking fund and $b, c, d$ etc. represent interest on the sinking fund for subsequent years, then the depreciation at the end of various years can be calculated as:

| year | Depreciation | Total Depreciation | Book Value |
| :---: | :---: | :---: | :---: |
| $1^{\text {st }}$ year | A | A | $\mathrm{C}-\mathrm{A}$ |
| $2^{\text {nd }}$ year | $\mathrm{A}+\mathrm{b}$ | $2 \mathrm{~A}+\mathrm{b}$ | $\mathrm{C}-(2 \mathrm{~A}+\mathrm{b})$ |
| $3^{\text {rd }}$ year | $\mathrm{A}+\mathrm{c}$ | $3 \mathrm{~A}+\mathrm{b}+\mathrm{c}$ | $\mathrm{C}-(3 \mathrm{~A}+\mathrm{b}+\mathrm{c})$ |

(d) Sum of Year Digit Method: In this method, the digits corresponding to the number of each year of life are listed in reverse order. The general expression for the annual depreciation for any year ( $q$ ) when the life is $p$ years is expressed as:

$$
D_{q}=(C-S)\left[\frac{(p-q+1)}{\frac{p(p+1)}{2}}\right]
$$

Where,
$\mathrm{q}=$ number of years in which depreciation is to be calculated
$p=$ life of property in years

## GATE/ESE

## Civil Engineering

## Design of Concrete Structures

Important Formula Notes

## IMPORTANT FORMULAS ON DESIGN OF CONCRETE STRUCTURES

## CHAPTER-1-INTRODUCTION

## 1. Important Codes considered in the design

| IS 456:2000 | RCC |
| :---: | :---: |
| IS 1893 | Earthquake |
| IS 13920 | Ductile <br> Detailing |
| IS 1343 | Prestress |
| IS 3370 (Part- <br> I/II/III/IV) | Water tank |
| IS 800:2007 | Steel |
| IS 1905 | Masonry work |

2. Permissible Limit for Impurities in the water as per IS 456:2000

| Impurity | Maximum permissible limit <br> $\mathbf{( ~ m g / l )}$ |
| :---: | :---: |
| Organic | $\mathbf{2 0 0}$ |
| Inorganic | $\mathbf{3 0 0}$ |
| Sulphates (as $\mathrm{SO}_{3}$ ) | $\mathbf{4 0 0}$ |
| Suspended matter | $\mathbf{2 0 0 0}$ |
| Chloride as Cl | 2000- plain concrete <br> 500- Reinforce concrete work |

## 3. IMPORTANT TESTS ON CEMENT

i. Consistency- Vicat apparatus
ii. Initial and final setting time- Vicat apparatus
iii. Soundness test- Autoclave test
iv. Specific gravity- Le chatelier flask
v. Fineness by specific surface- Blaine air permeability test

Note: Water cement ratio $\propto \frac{1}{\text { Compressive strength }} \quad$ (Abram's law)
4. IMPORTANT CRITERIA USED IN RCC
i. Comparison of workability by various methods

| Degree of workability | Slump <br> (mm) | Vee-Bee <br> (sec) | Compacting |
| :---: | :---: | :---: | :---: |
| Very low | Nil | $20-10$ | $0.70-0.75$ |
| Low | $0-25$ | $10-5$ | $0.75-0.80$ |
| Medium | $25-75$ | $5-3$ | $0.80-0.85$ |
| High | $75-150$ | $3-0$ | $0.85-0.92$ |
| Very high | $>150$ | - | $>0.92$ |

ii. Minimum cement contents and maximum w/c ratio for durability

|  | Plain cement concrete <br> PCC |  | Reinforcement cement <br> concrete RCC |  | Minimum <br> grade of <br> concrete |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Exposure | Minimum <br> cement <br> $\mathbf{k g} / \mathbf{m}^{\mathbf{3}}$ | Maximum <br> Free w/c | Minimum <br> cement <br> $\mathbf{k g / \mathbf { m } ^ { \mathbf { 3 } }}$ | Maximum <br> free w/c | PCC | RCC |
| Mild | 220 | 0.60 | 300 | 0.55 | - | M20 |
| Moderate | 240 | 0.60 | 300 | 0.50 | M15 | M25 |
| Severe | 250 | 0.50 | 320 | 0.45 | M20 | M30 |
| Very <br> Severe | 260 | 0.45 | 340 | 0.45 | M20 | M35 |
| Extreme | 280 | 0.40 | 360 | 0.40 | M25 | M40 |

iii. Exposure conditions

| Environment | Exposure condition |
| :---: | :--- |
| Mild | Concrete surfaces protected against weather or aggressive conditions, <br> except those situated in the coastal area |
| Moderate | Concrete surfaces sheltered from rain or freezing whilst wet <br> Concrete exposed to condensation and rain <br> Concrete continuously underwater Concrete in contact or buried under <br> non-aggressive soil/groundwater <br> Concrete surfaces sheltered from saturated salt air in the coastal area |


| Severe | Concrete surface exposed to severe rain, alternate wetting and drying or <br> occasional freezing whilst wet or severe condensation. <br> Concrete completely immersed in seawater. Concrete exposed to the <br> coastal environment |
| :---: | :--- |
| Very severe | Concrete surfaces exposed to seawater spray. corrosive fumes or severe <br> freezing conditions whilst wet concrete in contact or buried under <br> aggressive subsoil groundwater |
| Extreme | The surface of members in the tidal zone. Members in direct contact with <br> liquid/solid aggressive chemicals |

## 5. NOMINAL COVER

It is minimum clear cover required for the outermost layer of steel reinforcement.
Minimum Nominal cover

| Member | Mild (mm) | Moderate (mm) | Severe (mm) | Very severe (mm) | Extreme <br> $\mathbf{( m m )}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Slab | 20 | 30 | 45 | 50 | 75 |
| Beam | 25 | 30 | 45 | 50 | 75 |
| Column | 40 | 40 | 45 | 50 | 75 |
| Foundation | 40 | 50 | 50 | 50 | 75 |

## 6. CHARACTERISTIC STRENGTH OF CONCRETE (fck)

i. The strength below which $5 \%$ of test results are expected to fail is called characteristic strength of concrete.
ii. fckvalue gives compressive strength at 28 days after casting.
iii. If $f_{m}$ is targeted mean strength and $f_{c k}$ is characteristics strength, then

$$
\begin{gathered}
f_{m}=f_{c k}+1.65 \times \propto \\
\propto=\text { standard deviation }
\end{gathered}
$$

Value of standard deviation

| Grade | standard deviation ( $\boldsymbol{\alpha}$ ) $\mathrm{N} / \mathrm{mm}^{2}$ ) |
| :---: | :---: |
| M10- M15 | 3.5 |
| M 20-M25 | 4 |
| M 30 \& Higher | 5 |

## 7. ACCEPTANCE CRITERIA FOR CONCRETE

As per IS code 456:2000
The avg. strength of four non-overlapping consecutive tests should not be
$>f_{a v} \geq\left(f_{c k}+0.825 \times \sigma\right)$ orfavg $\geq\left(f_{c k}+3\right)$ \{whichever is more\}
> For individual test results ITR $\geq\left(f_{\mathrm{cK}}-3\right)$
> Test results should be obtained after testing on atleast three cubes. The difference in each test block strength and average strength should not be more than $15 \%$

## 8. MODULUS OF ELASTICITY OF CONCRETE

i. Initial Tangent modulus ( $\mathbf{E}_{\mathbf{T}}$ ) - Tangent's slope at any point on the curve is called initial tangent modulus. It gives the instant value of modulus of elasticity.
ii. Secant modulus/ Static modulus (Es) - The slope of the line joining any point of the curve to origin is called the secant modulus of elasticity.
iii. Initial tangent of elasticity/dynamic modulus of elasticity (Ec)- It is the modulus of elasticity of concrete at the origin.

$$
\mathbf{E}_{T}=\mathbf{E}_{\mathrm{s}}=\mathbf{E}_{\mathrm{c}}=5000 \sqrt{\boldsymbol{f}_{c k}}
$$



The above formula holds only for the short term. For long term elastic coefficient (EL)
$\mathrm{E}_{\mathrm{L}}=\frac{5000 \sqrt{\mathrm{f}_{\mathrm{ck}}}}{1+\theta}$
$\theta=$ creep coefficient
Table for creep coefficient

| Age of loading | Creep coefficient |
| :---: | :---: |
| 7 days | 2.2 |
| 28 days | 1.6 |


| 1 year | 1.1 |
| :---: | :---: |

9. STRIPPING TIME

| Type of formwork | Minimum period before <br> removing formwork |
| :---: | :---: |
| a. Vertical formwork to <br> columns, walls, beams | $16-24$ hours |
| b. Soffit formwork to slabs <br> (Props to be refixed immediately after <br> removal of formwork) | 3 days |
| c. Soffit formwork to beams <br> (Props to be refixed immediately after <br> removal of formwork) | 7 days |
| d. Props to slabs <br> i) Spanning upto 4.5 m <br> ii) Spanning over 4.5 m | 7 days <br> 14 days |
| e. Props to beams <br> i) Spanning upto 6 m <br> ii) Spanning over 6 m | 14 days <br> 21 days |

## 10. DESIGN METHODS

i) Working Stress Method or Elastic Theory
ii) Limit State Method
iii) Ultimate Load Method or Whitney's Theory

## CHAPTER-2-LIMIT STATE METHOD OF DESIGN

## 1. Characteristic strength of materials

> The term 'characteristic strength 'means that the value of the strength of material below, which is not more than the minimum acceptable percentage of test results, are expected to fall.
> IS 456:2000 have accepted the minimum acceptable percentage as $5 \%$ for reinforced concrete structures.

> Characteristic strength $=$ Mean strength $-\mathrm{K} \times$ standard deviation or
$f_{k}=f_{m}-K \times S_{d}$
where, $\mathrm{f}_{\mathrm{k}}=$ characteristic strength of the material
$\mathrm{f}_{\mathrm{m}}=$ mean strength
$\mathrm{K}=$ constant $=1.65$
$S_{d}=$ standard deviation for a set of test results.
The value of standard deviation ( $\mathrm{s}_{\mathrm{d}}$ ) is given by
$S_{d}=\sqrt{\frac{\sum \delta^{2}}{n-1}}$
Where $\delta=$ deviation of the individual test strength from the average or mean strength of $n$ samples.
$\mathrm{n}=$ number of test results
IS 456:2000 has recommended minimum value of $n=30$

## 2. Partial safety factor for loads

The partial safety for loads, as per IS 456:2000 are given in the table below

| Load <br> combination | Limit state of collapse |  |  | Limit state of Serviceability |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
|  | DL | LL | WL/EL | DL | LL | WL/EL |  |
| DL+IL | 1.5 | 1.5 | - | 1.0 | 1.0 | - |  |


| DL+WL | $1.50 r$ <br> $0.9 *$ | - | 1.5 | 1.0 | - | 1.0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| DL+IL+WL | 1.2 | 1.2 | 1.2 | 1.0 | 0.8 | 0.8 |

> (* This value is to be considered when stability against overturning or stress reversal is critical)

## LIMIT STATE OF COLLAPSE IN FLEXURE

> In bending, the maximum compressive strain in concrete (at the outermost fibre) $\varepsilon_{\mathrm{cu}}$ shall be taken as 0.0035 .
> For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factorof 1.5 shall be applied in addition to this.


Figure: Stress-strain curve for concrete
> For the design purpose of reinforcement, the partial safety factor $\gamma_{\mathrm{m}}$ equal to 1.15 shall be applied.
> The maximum strain in the tension reinforcement in the section at failure shall not be less than :

$$
\frac{f_{\mathrm{y}}}{1.15 \mathrm{E}_{\mathrm{s}}}+0.002
$$

## 3. Singly Reinforced Beam



$$
\left(\frac{x_{u}}{\boldsymbol{d}}\right) \lim =\frac{0.0035}{\frac{0.87 f_{y}}{E_{S}}+0.0055}
$$

Table: Limiting depth of neutral axis for different grades of steel

| Steel Grade | Fe 250 | Fe 415 | Fe 500 |
| :--- | :--- | :--- | :--- |
| Xu,lim/d | 0.531 | 0.478 | 0.46 |

d= effective width
Note: Limiting depth of neutral axis depends only on the grade of steel and is independent of
the grade of concrete.
i. Depth of neutral Axis
$\mathrm{x}_{\mathrm{u}}=\frac{0.87 \mathrm{f}_{\mathrm{y}} \mathrm{A}_{\text {st }}}{0.36 \mathrm{f}_{\mathrm{ck}} \mathrm{b}}$
ii. Lever Arm $=d-0.42 x_{u}$ ( $d=$ effective width and $x_{u}$ is depth of neutral axis)
iii. Ultimate Moment of resistance
$M_{u}=0.36 f_{c k} \mathrm{bxu}_{u}\left(\mathrm{~d}-0.42 \mathrm{x}_{\mathrm{u}}\right) \quad$; for all $\mathrm{x}_{u}$
Alternatively, in terms of the steel tensile stress,
$M_{u}=0.87 f_{y} \times A_{\text {st }}(d-0.42 x u) ;$ for all $x_{u}$
Case $-1: \frac{x_{u}}{d}$ equal to the limiting value $\frac{x_{u, \text { max }}}{d}$ : Balanced section
Case $-2: \frac{\mathrm{x}_{\mathrm{u}}}{\mathrm{d}}$ less than limiting value: under-reinforced section
Case $-3: \frac{\mathrm{X}_{\mathrm{u}}}{\mathrm{d}}$ more than limiting value: over-reinforced section.

## iv. Computation of $M_{u}$

a. $\mathrm{X}_{u}<\mathrm{X}_{\mathrm{u}, \text { max }}$
> In this case,steel yields before the crushing of concrete and the failure is ductile. In construction, under-reinforced sections are preferred as it gives warning before the collapse.
> $M_{u}=0.87 f_{y} A_{s t}(d-0.42 \mathrm{xu})$ (calculated from tension side)
b. $X_{u}=X_{u, \max }$
> In this case,the yielding of steel and crushing of concrete takes place simultaneously.

$$
M_{u, \text { lim }}=0.36 \frac{x_{u, \max }}{d}\left(1-0.42 \frac{x_{u, \max }}{d}\right) f_{c k} b^{2}
$$

C. $X_{u}>X_{u, \max }$

In this case, crushing of concrete occurs before yielding steel, and sudden failure occurs.
> On the other hand, when steel reaches $\frac{0.87 f_{y}}{E_{s}}+0.002$, the strain of concrete far exceeds 0.0035 . Hence, it is not possible. Therefore, such a design is avoided, and the section should be redesigned.
> The moment of resistance $M_{u}$ for such an existing beam is calculated by restricting $X_{u}$ to $X_{u, m a x}$ only, and the corresponding $M_{u}$ will be as per the case when $X_{u}=X_{u}$, max .

Table: Limiting value of the moment of resistance for different grades of steel

| Steel Grade | Fe 250 | Fe 415 | Fe 500 |
| :---: | :---: | :---: | :---: |
| MORlim $^{2} 0.148 \mathrm{f}_{\mathrm{ck}} \mathrm{bd}^{2}$ | $0.138 \mathrm{f}_{\mathrm{ck}} \mathrm{bd}^{2}$ | $0.133 \mathrm{f}_{\mathrm{ck}} \mathrm{bd}^{2}$ |  |

## 2. DOUBLY REINFORCED SECTION



Fig. Doubly reinforced beam
Analysis of doubly reinforced beam


$M_{u}=M_{1}+M_{2}$

$$
\mathrm{M}_{1}=\mathrm{MOR}_{\lim } \text { (corresponding to limiting moment of resistance) }
$$

$\Rightarrow M_{2}=M_{u}-M_{1}=0.87 f_{y} A_{s t 2}\left(d-d^{\prime}\right)=A_{s c}\left(f_{s c}-f_{c c}\right)\left(d-d^{\prime}\right)$
Where $A_{\text {st2 }}=$ Area of additional tensile reinforcement
$A_{s c}=$ Area of compression reinforcement
$\mathrm{f}_{\mathrm{sc}}=$ stress in compression reinforcement
$\mathrm{f}_{\mathrm{cc}}=$ Compressive stress in concrete at the level of compression reinforcement
Since the addition of reinforcement is balanced by the additional compressive force.
$A_{s c}\left(f_{s c}-f_{c c}\right)=0.87 f^{\prime} A_{s t 2}$
$>$ The strain at the level of compression reinforcement is $0.0035\left(1-\frac{\mathrm{x}_{\mathrm{u}}}{\mathrm{x}_{\mathrm{u}, \max }}\right)$
> The total area of reinforcement shall be obtained by
$A_{s t}=A_{s t 1}+A_{s t 2}$
$A_{s t 1}=$ Area of reinforcement for a singly reinforced section for $\mathrm{M}_{\mathrm{u}, \mathrm{lim}}$
$A_{s t 2}=\frac{A_{s c}\left(f_{s c}-f_{c c}\right)}{0.87 f_{y}}$

## 3. T BEAMS AND L BEAMS



Beam-supported floor slab system


## i. Effective width of Flange

## a. For beam casted monolithically with slab

effective width of flange $b_{f}$ (Cl. 23.1.2 of Code) are given as follows:

$$
\mathrm{b}_{\mathrm{f}}=\left\{\begin{array}{l}
\mathrm{l}_{0} / 6+\mathrm{b}_{\mathrm{w}}+6 \mathrm{D}_{\mathrm{f}} \text { for } \mathrm{T}-\text { Beam } \\
\mathrm{l}_{0} / 12+\mathrm{b}_{\mathrm{w}}+3 \mathrm{D}_{\mathrm{f}} \text { for } \mathrm{L} \text { - Beam }
\end{array}\right. \text { [Eq. 1] }
$$

- $b_{w}$ is the breadth of the web,
- $D_{f}$ is the thickness of the Flange
- $I_{0}$ is the "distance between points of zero moments in the beam" (which may be assumed as 0.7 times the effective span in continuous beams and frames).
b. For Isolated Beams

$$
\mathrm{b}_{\mathrm{f}}=\left\{\begin{array}{l}
\frac{\mathrm{I}_{0}}{\mathrm{I}_{0} / \mathrm{b}+4}+\mathrm{b}_{\mathrm{w}} \text { for isolated } T \text { - Beam } \\
\frac{0.5 I_{0}}{\mathrm{I}_{0} / \mathrm{b}+4}+\mathrm{b}_{\mathrm{w}} \text { for isolated } L-\text { Beam }
\end{array}\right.
$$

## ii. Analysis of Singly Reinforced Flanged Sections

Case A: If the neutral axis lies in the Flange area (i.e., $X_{u}<D_{f}$ )
It will behave as rectangular section with width equal to that of flange.
$\mathrm{x}_{\mathrm{u}}=\frac{0.87 \times \mathrm{f}_{\mathrm{v}} \times \mathrm{A}_{\text {st }}}{0.36 \times \mathrm{f}_{\text {ck }} \times \mathrm{b}_{\mathrm{f}}}$
$b_{f}=$ width of flange
CASE B: If theneutral axis lies in the web region (i.e., $x_{u}>D_{f}$ )
I. When $\mathrm{x}_{u}>$ Dfand $\mathrm{x}_{u}<\frac{7}{3} D_{f}$

$y_{f}=0.15 X_{u}+0.65 D_{f}$
a. For calculation of NA

$$
0.36 \times f_{c k} \times b_{w} \times X_{u}+0.45 \times f_{c k} \times\left(b_{f}-b_{w}\right) \times y_{f}=0.87 \times f_{y} \times A_{s t}
$$

b. For Moment of Resistance
$>M_{u}=0.36 \times f_{c k} \times b_{w} \times X_{u} \times\left(d-0.42 \times X_{u}\right)+0.45 \times f_{c k} \times\left(b_{f}-b_{w}\right) \times y_{f}\left(d-\frac{y_{f}}{2}\right)$
$\Rightarrow M_{u}=0.87 \times f_{y} \times A_{s t 1} \times\left(d-0.42 X_{u}\right)+0.87 \times f_{y} \times A_{s t 2} \times\left(d-\frac{y_{f}}{2}\right)$
$>A_{\text {st1 }}=\frac{0.36 \times f_{c k} \times b_{w} \times X_{u}}{0.87 \times f_{y}}$
$>A_{\text {st2 }}=\frac{0.45 \times f_{c k} \times\left(b_{f}-b_{w}\right) \times y_{f}}{0.87 \times f_{y}}$
II. When $\mathrm{x}_{\mathrm{u}}>\mathrm{D}_{\mathrm{f}}$ and $\mathrm{x}_{\mathrm{u}}>\frac{7}{3} D_{f}$

Variation of stress and strain will be same as that of case I except that $y_{f}=D_{f}$
a. For calculation of NA

$$
0.36 \times f_{c k} \times b_{w} \times X_{u}+0.45 \times f_{c k} \times\left(b_{f}-b_{w}\right) \times D_{f}=0.87 \times f_{y} \times A_{s t}
$$

b. Ultimate moment of resistance

$$
\begin{aligned}
& >M_{u}=0.36 \times f_{c k} \times b_{w} \times X_{u} \times\left(d-0.42 \times X_{u}\right)+0.45 \times f_{c k} \times\left(b_{f}-b_{w}\right) \times D_{f}\left(d-\frac{D_{f}}{2}\right) \\
& >M_{u}=0.87 \times f_{y} \times A_{s t 1} \times\left(d-0.42 X_{u}\right)+0.87 \times f_{y} \times A_{\text {st2 }} \times\left(d-\frac{D_{f}}{2}\right) \\
& >A_{s t 1}=\frac{0.36 \times f_{c k} \times b_{w} \times X_{u}}{0.87 \times f_{y}}
\end{aligned}
$$

$>A_{s t 2}=\frac{0.45 \times f_{c k} \times\left(b_{f}-b_{w}\right) \times D_{f}}{0.87 \times f_{y}}$

## CHAPTER-3-WORKING STRESS METHOD OF DESIGN

## 1. WORKING STRESS METHOD / MODULAR RATIO METHOD

### 1.1. Factor of safety

- In WSM, the structure's design is based on actual stresses developed in concrete and steel due to actual loads.
- These stresses are always kept within maximum permissible stress.
(ii) Assumptions
(a) At any section, a plain section before bending remains plain after bending.
- The strain diagram of the section is linear.
- The stress diagram is also linear in WSM.
- Within the elastic limit, strain is directly proportional to stress.
(b) All tensile stresses are taken by steel only and none by concrete.


### 1.2. Value of modular ratio, (m)

- $\mathrm{m}=\frac{280}{3 \sigma_{c b c}}$
- $\mathrm{m}=\frac{E_{s}}{E_{c}}$
- The value mentioned above of ' $m$ ' is a value with the only partial effect of creep.


## 2. ANALYSIS OF A RCC BEAM FOR FLEXURE


(i) Stress relationship
$\frac{C_{a}}{x_{a}}=\frac{t_{a} / m}{d-x_{a}}$
(ii) Actual depth by NA
$\frac{B x_{a}^{2}}{2}=m \cdot A_{s t}\left(d-x_{a}\right)$
(iii) Total resultant compressive force
$\mathrm{C}=\mathrm{B} \times \mathrm{X}_{\mathrm{a}} \times\left(\frac{\mathrm{C}_{\mathrm{a}}}{2}\right)$
(iv) Total tensile force
$T=t_{a} \times A_{s t}$
(v) Also, $\mathrm{C}=\mathrm{T}$

Therefore, $B \times x_{a} \times \frac{C_{a}}{2}=t_{a} \times$ Ast
(vi) for Bending moment
$B M=B \times x_{a} \times \frac{C_{a}}{2}\left(d-\frac{x_{a}}{3}\right)$ (Compression approach)
$B M=t_{a} \times A_{\text {st }}\left(d-\frac{x_{a}}{3}\right)$ (tension approach)

### 2.1. Stress behaviour

There are three types of sections.
(i) Under reinforced section

- The actual depth of neutral axis < critical depth
- At the maximum bending moment that can be allowed.
$\rightarrow$ stress in concrete $\Rightarrow C_{a}<\sigma_{c b c}$
$\rightarrow$ stress in steel $\Rightarrow t_{a}=\sigma_{\text {st }}$
- Steel reaches its maximum permissible stress first (prior to concrete)
- Under reinforced sections are always preferred to use.
(ii) Over reinforced section
- $X_{a}>X_{c}$
- At moment of resistance,
$\rightarrow$ stress in concrete $\Rightarrow C_{a}=\sigma_{c b c}$
$\rightarrow$ stress in steel $\Rightarrow \mathrm{t}_{\mathrm{a}}<\sigma_{\text {st }}$
- Concrete reaches maximum permissible stress prior to steel.
(iii) Balanced section
- $X_{a}=x_{c}$
- At the moment of resistance,
$\rightarrow$ stress in concrete $\Rightarrow C_{a}=\sigma_{c b c}$
$\rightarrow$ stress in steel $\Rightarrow t_{a}=\sigma_{\text {st }}$
- For balanced section.
- $\mathrm{x}_{\mathrm{c}}=\mathrm{k} \times \mathrm{d}=\left(\frac{\mathrm{m} \sigma_{\mathrm{cbc}}}{\sigma_{\mathrm{st}}+\mathrm{m} \sigma_{\mathrm{cbc}}}\right) \times \mathrm{d}=\left(\frac{280}{3 \sigma_{\mathrm{st}}+280}\right) \times \mathrm{d} \quad\left(\right.$ Since, $\left.\mathrm{m}=\frac{280}{3 \sigma_{c b c}}\right)$
- Value of $x_{c}$ for different grades
$x_{c}=0.40 \mathrm{~d} \rightarrow \mathrm{Fe} 250$
$\mathrm{X}_{\mathrm{c}}=0.289 \mathrm{~d} \rightarrow \mathrm{Fe} 145$
$x_{c}=0.253 \mathrm{~d} \rightarrow \mathrm{Fe} 500$


## CHAPTER-4-DESIGN FOR SHEAR

## 1. NOMINAL SHEAR STRESS

> The average shear stress can be calculated using the following formula:

$$
\tau_{v}=\frac{V_{u}}{b d}
$$

Where,
$\mathrm{V}_{\mathrm{u}}=$ ultimate shear stress at the section
$\mathrm{b}=$ width of the section
$d=$ effective depth of the section
> For beams with varying depth
$\tau_{\mathrm{v}}=\left[\frac{\mathrm{V}_{\mathrm{u}} \pm\left(\mathrm{M}_{\mathrm{u}} / \mathrm{d}\right) \tan \beta}{\mathrm{bd}}\right]$
Where,
$\beta=$ inclination of flexural tensile force to the horizontal.
$M_{u}=$ factored bending moment at the section.

+ sign is used when bending moment increases as the depth increases, and -ve sign is used
when bending moment decreases as the depth increases.


## 2. DESIGN SHEAR STRENGTH

The design shear strength of concrete depends upon two factors:
(i) Grade of concrete
(ii) Percentage tensile reinforcement

## 3. MINIMUM SHEAR REINFORCEMENT

The minimum amount of shear reinforcement should always be provided in the RCC section to
> To prevent bursting of concrete cover.
> Avoid sudden shear failure.
> To hold the reinforcing bars together
> To prevent cracks in the concrete due to shrinkage, thermal stresses etc.
> IS 456 specifies the following formula for the calculation of minimum shear reinforcement.
$\left(\frac{A_{s v}}{b S_{v}}\right) \geq\left(\frac{0.4}{0.87 f_{y}}\right)$
Where $A_{s v}=$ total cross-sectional area of stirrup legs effective in shear.
$\mathrm{S}_{\mathrm{v}}=$ spacing of stirrups.
$\mathrm{b}=$ breadth of the beam or breadth of the web of flanged beams.
$f_{y}=$ characteristic strength of stirrup reinforcement in $\mathrm{N} / \mathrm{mm}^{2}$, which shall not be taken greater than $415 \mathrm{~N} / \mathrm{mm}^{2}$.

## 4. DESIGN OF SHEAR REINFORCEMENT

> When the nominal shear stress exceeds the design shear strength, extra shear reinforcement is provided in the form of

- Vertical/Inclined stirrup

- Bent up bars

> The following formula gives the design shear stress:

$$
\mathrm{V}_{\mathrm{us}}=\left(\mathrm{V}_{\mathrm{u}}-\mathrm{V}_{\mathrm{c}}\right)=\left(\tau_{\mathrm{v}}-\tau_{\mathrm{c}}\right) \mathrm{bd}
$$

Where,
$V_{u}=$ factorshear force.
$\mathrm{V}_{\mathrm{c}}=$ shear resisted by concrete
$\mathrm{V}_{\mathrm{us}}=$ shear resisted by reinforcements (Links or bent up bars)
$\tau_{\mathrm{v}}=$ nominal shear stress.
$\tau_{c}=$ shear stress resisted by concrete

## 5. Vertical/Inclined Stirrup

> The spacing of the vertical stirrup can be calculated by using the following formula:

$$
\mathrm{S}_{\mathrm{v}}=\left[\frac{0.87 \mathrm{f}_{\mathrm{y}} \cdot \mathrm{~A}_{\mathrm{sv}} \cdot \mathrm{~d}}{\mathrm{~V}_{\mathrm{us}}}\right]
$$

Where,
$A_{s v}=$ total area of the legs of shear reinforcement.
$S_{v}=$ spacing of the links.
$d=$ effective depth of the section.
7. For inclined stirrup:

$$
\mathrm{V}_{\mathrm{us}}=\frac{0.87 \mathrm{f}_{\mathrm{y}} \mathrm{~A}_{\mathrm{st}} \mathrm{~d}}{\mathrm{~S}_{\mathrm{v}}}(\sin \alpha+\cos \alpha)
$$

Where,
$a=$ Angle of inclination of stirrup
8. The spacing between two stirrups shall be a minimum of the following values:
(i) $S_{v} \ngtr\left(S_{v}\right)_{\text {min shear rft }}$
(ii) $S_{v} \ngtr 0.75 d$ (for vertical stirrups) $\ngtr d$ (for inclined stirrups)
(iii) $S_{v} \ngtr 300 \mathrm{~mm}$

## 9. Bent Up Bars

$V_{u s}=0.87 \times f_{y} \times A_{s b} \times \sin \alpha$ Where, $\alpha=$ Angle of inclination of the bar with horizontal
$A_{\text {sb }}=$ area of bent up bar

## CHAPTER-5-DESIGN FOR BOND, ANCHORAGE AND LAP LENGTH

## 1. BOND AND ANCHORAGE

> Development Length

- For LSM $L_{d}=\frac{0.87 f_{y}}{4 \tau_{\text {bd }}} \phi$
- For WSM $L_{d}=\frac{\phi \times \sigma_{\text {st }}}{4 \tau_{\text {bd }}}$



Probable variation of anchorage bond stress


Assumed uniform average bond stress
> Permissible bond stress in tension Tbd , $\left(\mathrm{N} / \mathrm{mm}^{2}\right)$

| Grade of concrete | M15 | M20 | M25 | M30 | M35 | M40 and above |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Design bond stress <br> (LSM) | - | 1.2 | 1.4 | 1.5 | 1.7 | 1.9 |
| Design bond stress <br> (WSM) | 0.6 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |

- For deformed bars above value must be increased by $60 \%$
- For a bar in compression, the above value must be increased by $25 \%$


## CHAPTER-6-DESIGN FOR TORSION

## 1. Equivalent Shear

The following formula calculates the equivalent shear:

$$
V_{e}=V_{u}+1.6 \frac{T_{u}}{B}
$$

Where,
$\mathrm{V}_{\mathrm{e}}=$ Equivalent shear force
$V_{u}=$ Shear force
$\mathrm{T}_{\mathrm{u}}=$ Torsional moment
$B=$ Width of the section

## 2. Longitudinal reinforcement

The longitudinal tension reinforcement should be designed to carry equivalent bending moment of

$$
M_{e 1}=M_{u}+M_{t}
$$

Where $M_{u}=$ Flexural moment

$$
\mathrm{M}_{\mathrm{t}}=T_{u}\left(\frac{1+\frac{D}{b}}{1.7}\right)
$$

$\mathrm{T}_{u}=$ Torsional moment
D = Overall depth of the section

## 3. Transverse Reinforcement

As per Is 456, transverse reinforcement is provided in the form of two-legged closed hoops. The following formula obtains the area of transverse reinforcement:

$$
A_{s v}=\frac{T_{u} s_{v}}{b_{1} d_{1}\left(0.87 f_{y}\right)}+\frac{V_{u} s_{v}}{2.5 d_{1}\left(0.87 f_{y}\right)}
$$

Subjected to a maximum value of $\frac{\left(\tau_{v e}-\tau_{c}\right) b s_{v}}{0.87 f_{y}}$.
Where,
$\mathrm{T}_{\mathrm{u}}=$ Torsional moment
$\mathrm{V}_{u}=$ Shear force
$\mathrm{S}_{\mathrm{v}}=$ Spacing of shear reinforcement
$\mathrm{b}_{1}=$ centre to centre distance between corner bar in the direction of width
$d_{1}=$ centre to centre distance between corner bar in the direction of depth
$\mathrm{b}=$ width of the member
$\mathrm{f}_{\mathrm{y}}=$ Characteristics strength of stirrup reinforcement
Tve $=$ equivalent nominal shear stress
$\mathrm{T}_{\mathrm{c}}=$ shear strength of concrete

Note: The distribution of transverse reinforcement should be such that the spacing should be a minimum value of $\mathrm{x}_{1}, \frac{x_{1}+y_{1}}{4}$ or 300 mm where $\mathrm{x}_{1}$ and $\mathrm{y}_{1}$ are the short and long dimensions of the stirrup.

$\mathrm{x}_{1}=\mathrm{b}_{1}+$ Diameter of longitudinal bar + Diameter of stirrup $y_{1}=d_{1}+$ Diameter of longitudinal bar + Diameter of stirrup

## CHAPTER-7-DESIGN OF SLABS

## 1. ONE WAY SLAB

$>$ If $\frac{L_{y}}{L_{x}}>2$, slab is designed as a one-way slab.



Main Reinforcement distribution bars
Ly = longer span
Lx = shorter span
Main reinforcement is always provided along with the supports

## 2. TWO WAY SLAB

$>\frac{L y}{L x} \leq 2$


## 3. GENERAL CONSIDERATIONS FOR DESIGN OF SLABS

Lc = clear span
$\mathrm{w}=$ width of support
d= depth of support

## i. Effective length-

> For slabs that not built integrally with their supports
Leff $=$ Minimum of $\{(L c+d),(L c+w)\}$
Lc = clear span
> For Continuous Slabs

- If width of support $w \leq \frac{L_{c}}{12}$
- $L_{\text {eff }}=\operatorname{minimum}\{(L c+d),(L c+w)\}$
- If the width of the support $w \geq \min \left(\frac{L_{C}}{12}, 600 \mathrm{~mm}\right)$
- One end is fixed other is continuous, or both ends continuous Leff $\left(L_{e}\right)=L c$
- One end is continuous, and the other end is simply supported

$$
L_{e f f}\left(L_{e}\right)=\min \left(\left[L_{C}+\frac{W}{2}\right],\left[L_{C}+\frac{d}{2}\right]\right)
$$


> For cantilevers

- $L e=L c+\frac{d}{2}$ for fixed ends
- $L e=L c+\frac{W}{2}$ for continuous supports


## ii. Deflection

As per clause 23.2 of IS-456:2000,
> Final deflection due to all loads including the effect of temperature, creep and shrinkage should not exceed $\frac{\text { span }}{\mathbf{2 5 0}}$
> Deflection including effect of creep, temperature and shrinkage occurring after creation of partition and application of finishes should not exceed $\frac{\mathbf{s p a n}}{\mathbf{3 5 0}}$ or 20 mm whichever is less.

## iii. Span to depth ratio

> For span < 10m

| Type of support | Span/depth |
| :--- | :---: |
| a. Cantilever | 7 |
| b. Simply supported | 20 |
| c. Continuous | 26 |

> For span $>10 \mathrm{~m}$
$\frac{\text { span }}{\text { depth }} \mathrm{A} \times \frac{10}{\text { span in } m}$
iv. Concrete cover
> The cover at each end of the reinforcement bar should be not less than 25 mm or twice the diameter of the bar

## v. Reinforcement

The reinforcement for a slab spanning in one direction consists of main bars.
> The minimum reinforcement in either direction shall be $0.15 \%$ of the total crosssection area.
> This value is reduced to $0.12 \%$ when high strength deformed bars are used.
> Distribution Reinforcement

- These are reinforcement provided running at right angles to the main steel to distribute the load and the temperature and shrinkage stresses.
> Diameter of bars
$\begin{aligned} & \text { diameter } \\ & \text { of } \text { main bar }\end{aligned}=\frac{\text { thickness of slab }}{8}$
- Diameter of distribution bars $=8 \mathrm{~mm}$


## vi. Spacing between bars

The maximum spacing of main reinforcement should not exceed min(3d, 300 mm ) and for distribution reinforcement it should not exceed min (5d, 300 mm ).

## CHAPTER-8-DESIGN OF COLUMNS

> If leff> 3Least Lateral Dimension, it is called a column
$>$ If $l_{\text {eff }}<3$ LLD, it is called a pedestal
Note: LLD is the least lateral dimension


1. IS RECOMMENDATION REGARDING LONGITUDINAL REINFORCEMENT
> The minimum percentage of longitudinal reinforcement should not be less than $\mathbf{0 . 8 \%}$ to prevent buckling of the column.
> The maximum percentage of longitudinal reinforcement shall not be more than $\mathbf{6 \%}$ to avoid congestion of reinforcements, making it very difficult to place the concrete and consolidate it.
> The minimum number of longitudinal bars provided in a column shall be four in rectangular columns and sixfor circular columns.
> The bars shall not be less than $\mathbf{1 2 ~ m m}$ in diameter
> Maximum spacing of longitudinal bars $\mathbf{=} \mathbf{3 0 0} \mathbf{~ m m}$
> Minimum cover to the column reinforcement equals $\mathbf{4 0} \mathbf{~ m m}$ or diameter of the bar, whichever is greater.

## 2. TRANSVERSE REINFORCEMENT


> Diameter shall not be less than the maximum of these values
$\left\{\begin{array}{c}\frac{\text { Diameter of longitudinal bar }}{4} \\ 6 \mathrm{~mm}\end{array}\right.$
> Suppose the longitudinal bars are not spaced more than $\mathbf{7 5 m}$ mm either side. In that case, transverse reinforcement needs only to go around the corner and alternate bars to provide effective lateral supports.

> The diameter of transverse reinforcement need not exceed $\mathbf{2 0} \mathbf{~ m m}$.
> Spacing of transverse reinforcement shall not exceed the least of the following

- Least lateral dimension
- Sixteen times the diameter of the smallest longitudinal reinforcing rod.
- 48 times the diameter of transverse reinforcement


## 3. EFFECTIVE LENGTH

| Case <br> No. | End condition | Theoretical Leff | Recommended Leff |
| :---: | :---: | :---: | :---: |
| 1 |  | 0.5 Lo | 0.65 Lo |
| 2 | $\begin{gathered} \hline \text { HIP } \\ \text { NRAR } \end{gathered}$ | 0.7 Lo | 0.80 Lo |


|  | HIP RAR |  |  |
| :---: | :---: | :---: | :---: |
| 3 |  | 1.0 Lo | 1.0 Lo |
| 4 |  | 1.0 Lo | 1.20 Lo |
| 5. |  | - | 1.5 Lo |


| 6. | NHIP <br> RAR | 2.0 Lo | 2.0 Lo |
| :---: | :---: | :---: | :---: |
| 7. |  | 2.0 Lo | 2.0 Lo |

## HIP: Held in position

## NHIP: Not held in position

## RAR: Restrained against rotation

## NRAR: Not restrained against rotation

4. SHORT COLUMNS AND LONG COLUMNS
$>$ If $\frac{L_{\text {eff }}}{L L D}<12$, short column
$>$ If $\frac{L_{\text {eff }}}{L L D}>12$, long column
5. SLENDERNESS RATIO (SR ${ }_{x x}$ )
> It is the ratio of the length of a column and the least radius of gyration of its crosssection.
$>\mathrm{SR}_{\mathrm{xx}}=\frac{\mathrm{L}_{\mathrm{ex}}}{\text { Lateral dimension perpendicular to } \mathrm{x}-\mathrm{x}}$


$$
\begin{aligned}
\mathbf{S r}_{\mathrm{xx}} & =\frac{L e_{\mathrm{x}}}{\text { lateral dimension }} \\
& =\frac{L e_{\mathrm{x}}}{\mathrm{D}}
\end{aligned}
$$

> Maximum slenderness ratio for column $=60$
> If one end is restrained, unsupported length

$$
\mathrm{L}>\frac{100 \mathrm{~B}^{2}}{\mathrm{D}}
$$

$>$ For a short column, $\left[\left(\frac{L_{e x}}{D}\right) \text { and }\left(\frac{L_{e y}}{B}\right)\right]_{\text {both }}<12$
$>$ For a long column, $\left[\left(\frac{L_{\text {ex }}}{D}\right) \text { and }\left(\frac{L_{\mathrm{ey}}}{B}\right)\right]_{\text {both }}>12$

## 6. MINIMUM ECCENTRICITY

$$
\begin{aligned}
& \mathrm{e}_{\mathrm{x}, \min }=\max \quad\left\{\frac{l_{e x}}{500}+\frac{D_{x}}{30}, 20 \mathrm{~mm}\right\} \\
& \mathrm{e}_{\mathrm{y}, \min }=\max \left\{\frac{l_{e y}}{500}+\frac{D_{y}}{30}, 20 \mathrm{~mm}\right\}
\end{aligned}
$$

## 7. DESIGN OF COLUMNS

All columns shall be designed for
> Axial load $=P_{0}$
> Moment about $x-x=M_{u x} \quad M_{u x}>M_{u x m i n}$
> Moment about $\mathrm{y}-\mathrm{y}=$ Muy

$$
M_{u y} \nless M_{u y \min }
$$

(a) IS code method - WSM method
> The safe load on a short column is given by
$\mathrm{W}_{\mathrm{c}}=\left[\begin{array}{l}\text { area of } \\ \text { concrete }\end{array}\right] \times\left[\begin{array}{l}\text { safe stress } \\ \text { in concrete }\end{array}\right] \times\left[\begin{array}{l}\text { Area of } \\ \text { steel }\end{array}\right] \times\left[\begin{array}{l}\text { Safe stress } \\ \text { in steel }\end{array}\right]$ $W_{c}=\sigma_{c c} \times A_{c}+\sigma_{s c} \times A_{s}$ $A_{c}=\left(B D-A_{s c}\right)=$ net area of concrete
> Safe stresses in concrete

|  | M20 | M25 | M30 | M35 | M40 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\sigma_{c c}$ | 5 | 6 | 8 | 9 | 10 |

> Safe stresses in concrete

|  | Fe 250 | Fe350 | Fe415 | Fe500 |
| :---: | :---: | :---: | :---: | :---: |
| $\sigma_{\text {sc }}$ | $130 \mathrm{~N} / \mathrm{mm}^{2}$ | $130 \mathrm{~N} / \mathrm{mm}^{2}$ | $190 \mathrm{~N} / \mathrm{mm}^{2}$ | $190 \mathrm{~N} / \mathrm{mm}^{2}$ |

> The load-carrying capacity of a long column

$$
P=C_{r}\left(\sigma_{c c} A_{c}+\sigma_{s c} A_{s c}\right)
$$

$C_{r}=$ reduction coefficient
> For Rectangular or square column

$$
C_{r}=1.25-\frac{L_{e f f}}{48 B}
$$

$B=$ Least lateral dimension
> For irregular shape
$C_{r}=1.25-\frac{L_{\text {eff }}}{160 \times i_{\text {min }}}$
$\mathrm{i}_{\text {min }}=$ Minimum radius of gyration
$i_{\text {min }}=\sqrt{\frac{I}{A}}$
> Load carrying capacity of composite column
$P=C_{r}\left[\sigma_{c c} A_{c}+\sigma_{s c} A_{s c}+\sigma_{m c} A_{m c}\right)$
Where $A_{m c}=$ area of metal core (if provided) $\ngtr 0.2 \times B D$
$\sigma_{\mathrm{mc}}=$ stresses of metal core
$\sigma_{\mathrm{mc}}=125 \mathrm{~N} / \mathrm{mm}^{2}$ for structural steel
$=70 \mathrm{~N} / \mathrm{mm}^{2}$ for cast Iron

## (b) LSM METHOD

i. When column is subjected to only axial load
$P_{u}=0.45 f_{c k} A_{c}+0.75 f_{y} A_{s c}$
Note -The column section should be designed for combined axial load and bending moment due to the minimum specified eccentricity. If minimum eccentricity (as per the above specification) is less than or equal to 0.05 D , the column section can be designed as per the equation

$$
P_{u}=0.4 f_{c k} \times A c+0.67 f_{y} \times A s c
$$

## 8. DESIGN OF CIRCULAR COLUMN

For a column with helical reinforcement
$>$ If $\mathrm{e}_{\min } \leq 0.05 \mathrm{D}$
Load-carrying capacity is increased by $5 \%$
soPu $=1.05\left[0.4 \mathrm{f}_{\mathrm{ck}} \mathrm{Ac}_{\mathrm{c}}+0.67 \mathrm{fy}_{\mathrm{y}} \mathrm{Asc}\right.$ ]
$>$ For Helical reinforcement
$\frac{0.36 f_{c k}}{f_{y}}\left(\frac{A_{g}}{A_{c}}-1\right) \leq \frac{V_{h}}{V_{c}}$
Where,
$A_{g}=$ Gross area of the section
Ac $=$ Area of the core of the helically reinforced column


$$
\begin{aligned}
& \mathrm{Ag}=\frac{\pi}{4} \times \mathrm{D}^{2} \\
& \mathrm{AC}=\frac{\pi}{4} \times \mathrm{D}^{2} \\
& \mathrm{DC}=\mathrm{D}-2 \times \text { clear cover }
\end{aligned}
$$

$A_{C}=\frac{\pi}{4} \times D_{C}^{2}$
$A_{g}=\frac{\pi}{4} \times D^{2}$
Dc $=$ D $-2 \times$ Clear cover
$\mathrm{V}_{\mathrm{c}}=$ Volume of core portion in unit length of column $=1000 \mathrm{Acmm}{ }^{3}$
$\mathrm{V}_{\mathrm{h}}=$ Volume of helical reinforcement in a unit length of the column
$\mathrm{V}_{\mathrm{h}}=$ No. of turns $\times$ length of one turn $\times \mathrm{c} / \mathrm{s}$ are of helical reinforcement $=\frac{1000}{\text { pitch }} *$ $\left(\Pi D_{h}\right) * \frac{\Pi}{4} \varphi_{n}{ }^{2}$

Where, $D_{n}=D_{c}-\varphi_{\mathrm{n}}, \varphi_{\mathrm{n}}$ is the diameter of helical reinforcement
> Pitch ( p )
For helical reinforcement
(i). $p \ngtr 75 \mathrm{~mm}$
(ii). $p \ngtr\left(\frac{D_{C}}{6}\right)$
(iii). $p \nless 25 \mathrm{~mm}$
(iv). $p \nless 3 \phi_{n}$

## CHAPTER-9-DESIGN OF PRESTRESSED CONCRETE

> Prestressed concrete is a block of concrete in which internal stress of suitable magnitude and distribution are introduced to counteract the stresses resulting from external load to the desired degree.
> Prestressed concrete is different from a conventional RCC structure due to the application of an initial load on the structure prior to its use.
> Important points:

\#Pretesion [Hoyer method/Long line method]

- Tensioning $\rightarrow$ casting $\rightarrow$ distress $\rightarrow$ cutting of wires.
- Pre-stressing force is applied by bond action
- Straight wires or bent wire can also be used.
- Transmission length : length required to transmit full prestressing force to concrete.
[Example:- Railway sleepers, electric poles]
\#Post-tension:- Prestress principal tensile stresses $=0.24$ bulk uncrushed reaction.
- Casting with ducts - wire inserted \& than tensioned from
(1) one end jacking (2) Both end jacking.
- Straight, bend, parabolic /suitable for castin site
- Force is transformed by bearing action at ends, force transfer $[r / f \rightarrow$ wedge $\rightarrow$ plate $\rightarrow$ concrete]
- Anchorage zone : bursting and spalling forces occurs at end due to concentration of load [compression forces], after this zone only longitudinal stresses occurs
(a) Freyssinet system
- Conical steel wedges
- Multiple wires at a line.
(b) Magnel Blaton,
- Flat sandwich wedges
- 2 wires on either side of wedge.
- Normally total 8 wires
(c) LeMcall system
- High strength ruts are used on threaded r/f.
(b) Gifford Udall
- One wire with one set of half split cones
$\Rightarrow$ High strength steel is used as losses are high (10-20\%) of initial prestress forces
$\Rightarrow$ Min. Grade of concrete $(\mathrm{M}-40) \rightarrow$ pretension $\quad(\mathrm{M}-30) \rightarrow$ port tension.
$\Rightarrow$ Height [fck] concrete means less creep, more tensile strength but more brittle, more shrinkage ,less ductile as high cement content[prestressed concrete remains uncracked \& reduction in steel corrosion]
$\Rightarrow$ Min. clear cover between cable or tendon

1. 40 mm .
2. $5 \mathrm{~mm}+$ largest size of aggregate.
$\Rightarrow$ Maximum stress in tendon just behind the anchorage zone is $\Rightarrow 76 \%$ of ultimate strength of steel.
$\Rightarrow$ Types of section (as per cracking)

| Class-I <br> No tension <br> No cracking <br> Only compression <br> Section uncracked | Class-II <br> Tension allowed <br> No cracking <br> Tensile stress < $3 \mathrm{~N} / \mathrm{m}^{2}$ <br> Section uncracked | Class-III <br> - Tension allowed <br> - Cracking also allowed <br> - Tensile stress <fcr. <br> - Section uncracked |
| :---: | :---: | :---: |

$\Rightarrow$ Some properties

- $\quad P($ force $)=\frac{\left(f_{1}+f_{2}\right) A}{2}$ (only for symmetrical section)
- $C \Rightarrow(B \times D) \frac{\left(f_{1}+b_{2}\right) A}{2}$

c-force $\Rightarrow \mathrm{P}$-force at every section
$B M M x \Rightarrow P \bar{x}$ or $C \bar{x} \quad(\bar{x}=$ lever arm)
- Location of c-force is line of pressure \& location of P-force is line of cable.


## 1. Stress concept

2. Strength concept $\rightarrow$ check $M_{x}\left(\right.$ moment at section)' ${ }^{\prime}$ ' or ${ }^{\prime}-$ - at section
$+\Rightarrow C$ is above $P$

- $\Rightarrow C$ is below $P$
$\left(\bar{x} \Rightarrow \frac{M_{p}}{p}\right)$ now relate \& find eccentricity $\left(e_{c}\right)$ for $c$
$\mathrm{f}_{1} / \mathrm{f}_{2} \rightarrow \frac{\mathrm{C}}{\mathrm{A}} \pm \frac{\mathrm{Ce}_{\mathrm{c}}}{\mathrm{I}} \mathrm{y}$


## 3. Load balancing :-



Now find $M_{x}$ net \& find stress as with this condition for

- Bend wire $W \Rightarrow 2 P \sin \theta$
- Parabolic $==\frac{8 \operatorname{Pces} \theta h\left(=\left(e_{1}+e_{2}\right)\right)}{L^{2}} \quad y=\frac{4 h}{L^{2}}\left(L x-x^{2}\right)$
$\Rightarrow$ Cracking moment : Moment at which tensile stresses $\rightarrow \mathrm{f}_{\mathrm{cr}}$
All PSC beams are designed as uncracked section

$$
\mathrm{FOS} \Rightarrow \frac{\mathrm{M}_{\mathrm{cr}}}{\mathrm{BM}_{\text {warking }}} \Rightarrow \frac{\mathrm{W}_{\mathrm{cr}}}{\mathrm{~W}_{\text {warking }}}
$$

$\Rightarrow$ Load balancing profile of cable
$\left[e=\frac{M_{x}}{p}\right] \quad$ (profile is mirror image (scale) of BMD)

## $\Rightarrow$ Concordant profile of cable

To avoid (secondary moments, secondary stress, change in support reaction)
[In continuous beam when simple $P$ is applied on straight cable support reaction change so this concordant profile is used as mirror image of BMD]

possible pressure line (c-force)

Kern Points \& kern distance [No tension in section]
D/3 for $\square$ section, > D/3 for $\square$ section

## A. LOSSES IN PRESTRESS

| Pre-tensioning | Post-tensioning |
| :--- | :--- |
| 1. Elastic deformation of concrete | 1. No loss due to elastic shortening when all bars <br> are simultaneously tensioned. If, however, wires <br> are successively tensioned, there would be loss of <br> prestress due to elastic deformation of concrete |
| 2. Relaxation of stress in steel | 2. Relaxation of stress in steel |
| 3. Shrinkage of concrete | 3. Shrinkage of Concrete |
| 4. Creep of concrete | 4. Creep of concrete |
|  | 5. Frictional losses |
|  | 6. Anchorage slip |



Various losses in prestress

## 1. LOSS OF PRESTRESS DUE TO FRICTION

> The friction generated at the interface of concrete and steel during the stretching of a curved tendon in a post-tensioned member leads to a drop in the prestress along with the member from the stretching end.
> The loss due to friction does not occur in pre-tensioned members because there is no concrete during the stretching of the tendons.
> Force in the cable at a distance x from jacking end, after a frictional loss $-\mathrm{P}_{\mathrm{x}}$


$$
P_{x}=P_{0} e^{-(\mu a+k x)}
$$

Where $P_{x}=$ Prestressing force at a distance x from jacking end.
$P_{0}=$ Prestressing force at jacking end.
$\mathrm{k}=$ coefficient called wobble correction factor
$\mu=$ Coefficient for friction in the curve
$\alpha=$ Cumulative Angle in radian through which the tangent to the cable profile turned between any two points under consideration.
> For small $(\mu+k x)$ values, the Taylor series expansion can simplify the above expression.

$$
P_{x}=P_{o}[1-(\mu \alpha+k x)]
$$



For parabolic profile, Jacking at one end, $a=2 \theta=\frac{8 e}{L}$

Jacking from both ends $a=\theta=\frac{4 e}{L}$
For trapezoidal profile, Jacking at one end $a=2 \theta=\frac{2 e}{a}$

Jacking from both ends $a=\theta=\frac{e}{a}$

2. LOSS OF PRESTRESS DUE TO ANCHORAGE SLIP
> In a post-tensioned member, when the prestress is transferred to the concrete, the wedges slip through a little distance before they get properly seated in the conical space.
$>$ This loss due to anchorage slip $=\frac{\mathrm{E}_{\mathrm{s}} \Delta}{\mathrm{L}}$
$=\left(\frac{\Delta}{L}\right) E_{s}$
$E_{s}=$ Young modulus of steel in $\mathrm{N} / \mathrm{mm}^{2}$
$\Delta=$ Anchorage slip in mm
$\mathrm{L}=$ Length of cable in mm

## Table:- Typical values of anchorage slip

| Anchorage system | Anchorage slip ( $\Delta$ ) |
| :--- | :--- |
| Freyssinet | 4 mm |
| $12-5 \mathrm{~mm} \phi$ strands | 6 mm |
| $12-8 \mathrm{~mm} \phi$ strands | 8 mm |
| Magnet | 1 mm |

## 3. LOSS OF PRESTRESS DUE TO CREEP OF CONCRETE

> Creep is the property of concrete by which it continues to deform with time under sustained loading.
> Creep coefficient is defined as

$$
\phi=\frac{\text { creep strain }}{\text { elastic strain }}=\frac{\varepsilon_{\mathrm{cp}}}{\varepsilon_{\mathrm{c}}}
$$

> Loss of stress $=m \phi f_{c}$
> Note that elastic shorting loss multiplied by creep co-efficient is equal to loss due to creep.

| Age at loading | Creep co-efficient |
| :--- | :--- |
| 7 days | 2.2 |
| 28 days | 1.6 |
| 1 year | 1.1 |

## 4. LOSS DUE TO SHRINKAGE OF CONCRETE

> The loss of stress in steel due to the shrinkage of concrete is estimated as loss of stress $=\varepsilon_{c s} \times E_{s}$
Where $E_{s}=$ modulus of elasticity of steel.
$>\varepsilon_{c s}=$ total residual shrinkage strain having values of $3 \times 10^{-4}$ for pre tensioning and $\varepsilon_{c s}=\left[\left(2 \times 10^{-4}\right) / \log _{10}(t+2)\right]$ for post-tensioning Where, $\mathrm{t}=$ age of concrete at transfer in days.

## 5. LOSS OF PRESTRESS DUE TO RELAXATION OF STEEL

| Initial Stress (1) | Relaxation Loss N/mm ${ }^{2}$ |
| :--- | :--- |
| $0.5 f_{p}$ | 0 |
| $0.6 f_{p}$ | 35 |
| $0.7 f_{p}$ | 70 |
| $0.8 f_{p}$ | 90 |

## Note:

$f_{p}$ is the characteristic strength of prestressing steel.
The conclusion of the above discussions:

| Sr. No. | Type of loss | Equation |
| :--- | :--- | :--- |
| 1 | Wobble \& curvature effect | $(\mu \mathrm{a}+\mathrm{kx}) \mathrm{P}_{0}$ |
| 2 | Anchorage slip | $\mathrm{E}_{s} \Delta / \mathrm{L}$ |
| 3 | Shrinkage loss | $\varepsilon_{s c} \mathrm{E}_{s}$ |
| 4 | Creep of concrete | $\mathrm{m} \phi \mathrm{f}_{\mathrm{c}}$ |
| 5 | Elastic shortening of concrete | $\mathrm{mf}_{\mathrm{c}}$ |


| 6 | Relaxation in steel | 2 to $5 \%$ for initial stress in steel |
| :--- | :--- | :--- |


| Type of loss | Pretensioned (\%) | Post tensioned (\%) |
| :--- | :--- | :--- |
| Elastic shorting of concrete | 3 | 1 |
| Shrinkage | 7 | 6 |
| Creep | 6 | 5 |
| Relaxation | 2 | 3 |
| Total Loss | $18 \%$ | $15 \%$ |


| Losses | Pretensioning | Post-tensioning |
| :--- | :--- | :--- |
| Length effect | No | Yes |
| Curvature effect | No | Yes |
| Anchorage slip | No | Yes |
| Shrinkage of concrete | Yes | Yes |
| Creep of concrete | Yes | Yes |
| Elastic deformation or <br> shortening of concrete | Yes | No (If all wires are simultaneously tensioned) <br> Yes (If wires are successively tensioned) |

## 6. Deflection of Pre-stressed Beam

Short term deflection under uncracked condition can be computed using elastic theory by using area moment method (Mohr's method). Concrete beam deflects upwards on the application or transfer of prestress.

Bending moment at any section is the product of prestressing force and eccentricity at that section.


$$
\Delta=\frac{\mathrm{PeL}^{2}}{8 \mathrm{EI}}
$$



$$
\Delta=\frac{-\mathrm{Pe}}{6 \mathrm{EI}}\left(21_{1}^{2}+61_{1} 1_{2}+3 I_{2}^{2}\right)
$$



$$
\Delta=\frac{-5}{48} \frac{P e L^{2}}{E I}
$$



Note:
Downward deflection due to self wt or imposed load is $\frac{5 w}{384} \frac{L^{4}}{E I}$

## GATE/ESE

## Civil Engineering

Design of Steel Structures

## Important Formula Notes



## IMPORTANT FORMULAS ON DESIGN OF STEEL STRUCTURES

## CHAPTER-1- PLASTIC ANALYSIS

In plastic analysis, we use the reserve of strength beyond the point of the first yield and mostly used for indeterminate structures.

Assumptions:
i. Material must possess ductility so that it could be deformed to the plastic state.
ii. Strain distribution should be linear, i.e. plane section should remain plane after bending (shear deflections are neglected)
iii. Stress-strain curve is assumed to be idealised elastoplastic.
iv. Relation between tensile stress-strain is the same in both tension and compression.
v. Joints should be sufficiently rigid to transfer the moment.

## 1. SAFE AND YIELD MOMENT OF RESISTANCE

### 1.1. Safe moment of resistance: $M=f \times z$

Where, $f=$ stress in extreme fibre and $z=$ section modulus

### 1.2. Yield Moment of Resistance ( $M_{y}$ ): $M_{y}=f_{y} \times z$

Where, $f_{y}=$ yield stress and $z=$ section modulus

## 2. PLASTIC MOMENT CAPACITY

Plastic moment $=M_{p}=C \times$ lever arm $=T \times$ lever arm.
$M_{p}=C \times$ lever arm $=f_{y} \times \frac{A}{2} \times\left(\bar{y}_{1}+\bar{y}_{2}\right)$
$M_{p}=f_{y}\left[\frac{A}{2}\left(\bar{y}_{1}+\bar{y}_{2}\right)\right]=f_{y} z_{p}$
Here, $z_{p}=\frac{A}{2}\left(\bar{y}_{1}+\bar{y}_{2}\right)=$ Plastic modulus
$\bar{y}_{1}, \bar{y}_{2}=$ Centroid distances of compression area and tension area from the neutral axis.

## 3. IMPORTANT FACTORS IN PLASTIC DESIGN

3.1. Shape factor (SF) $=\frac{M_{p}}{M_{y}}=\frac{f_{y} z_{p}}{f_{y} z} \Rightarrow S F=\frac{z_{p}}{z}$
3.2. Load factor (LF) $=\frac{M_{p}}{M}=\frac{f_{y} \cdot z_{p}}{f z}$
$\frac{f_{y}}{f} \times \frac{z_{p}}{z}=F o S \times S F=$ load factor
$\therefore$ Load factor $=$ Factor of Safety $\times$ Shape factor

### 3.3. Factor of Safety (FOS)

Factor of safety for ductile material, FOS $=\frac{f_{y}}{f}$
Factor of safety for brittle material, FOS $=\frac{\text { Ultimate stress }}{\text { Working stress }}$
Shape factors for various cross-sections:

| S.No | CROSS-SECTION | SHAPE FACTOR |
| :---: | :---: | :---: |
| 1. | Rectangle | 1.5 |
| 2. | Circle | 1.7 |
| 3. | Rolled steel I section | 1.12 to 1.14 |
| 4. | H section | 1.5 |
| 5. | Diamond section | 2 |
| 6. | Triangle | 2.34 |

### 3.4. Calculation of Collapse Load

To form a mechanism, the number of plastic hinges required $=\mathrm{n}=\mathrm{D}_{\mathrm{s}}+1$.
In the plastic analysis, the following conditions must be satisfied.
i) Equilibrium condition: the equation of equilibrium should be satisfied.
ii) Mechanism condition: sufficient number of plastic hinges must develop so that a part or entire structure must transform into a mechanism leading to collapse.
iii) Yield condition: Bending moment at any section should not exceed plastic moment capacity ( $M_{p}$ ) of the cross-section.
On the basis of the above 3 conditions, there are two methods:
i) Kinematic method or Kinematic theorem or upper bound theorem
a) It is the combination of equilibrium and mechanism conditions.
b) It also states that the collapse load formed by assuming a mechanism will always be greater than or equal to the true collapse load.
ii) Static method or Static theorem or lower bound theorem
a) It satisfies equilibrium and yield conditions.
b) It states that the collapse load formed based on any collapsed bending moment will always be less or equal to the true collapse load.

## 4. LOCATIONS WHERE PLASTIC HINGES CAN FORM

i) At maximum bending moment locations.
ii) At fixed supports and rigid joints.
iii) Wherever the cross-section changes.
iv) Under point loads in supported spans but not at a free end.
v) Whenever the material changes.
4.1. Principle of Virtual Work: "when a body is in equilibrium, the total virtual work done by all forces is zero".

### 4.2. Collapse Load for Various Cases

i) Simply Supported beam
a) Point load at mid-length: Collapse load $=\frac{4 M_{P}}{\ell} \mathrm{KN}$
b) Uniformly distributed load: Collapse load $=\frac{8 M_{P}}{\ell^{2}} \mathrm{KN} / \mathrm{m}$
c) Eccentric loaded point load: Collapse load $=\frac{M_{p} \ell}{a b}\{\because a+b=\ell\}$
(ii) Fixed-beam
a) Point load at mid-length: Collapse load $=\frac{8 M_{P}}{\ell} \mathrm{KN}$
b) Uniformly distributed load:
$\Rightarrow$ When two hinges are formed simultaneously at the ends
$\Rightarrow \mathrm{W}_{1}=\frac{12 \mathrm{M}_{\mathrm{P}}}{\ell^{2}} \mathrm{KN} / \mathrm{m}$
$\Rightarrow$ When the third hinge is formed at mid-span after the hinges form at the ends
Collapse load $=\frac{16 \mathrm{M}_{\mathrm{P}}}{\ell^{2}} \mathrm{KN} / \mathrm{m}$
c) Eccentric point load: Collapse load $=\frac{2 M_{p} \ell}{a b} k N$

## (iii) Propped cantilever beam

a) Point load at mid-length: Collapse load $=\frac{6 M_{P}}{\ell} \mathrm{kN}$
b) Uniformly distributed load: Collapse Load $=\frac{11.656 M_{p}}{\ell^{2}} \mathrm{kN}$
c) Eccentric point load: Collapse load $=\frac{M_{p}(\ell+b)}{a b} k N$

### 4.3. Plastic Analysis of Frame

4.3.1. Beam Mechanism: The mechanism is formed only in one member of the portal frame.

4.3.2. Sway Mechanism: If the frame sways to the left or to the right.

4.3.3. Combined Mechanism: It is formed by the combination of sway and beam mechanism.


## CHAPTER-2-DESIGN OF CONNECTIONS

1. INTRODUCTION: Different types of fasteners available are rivets, bolts, pins and welds.

## 2. BOLTED CONNECTIONS



Bolts are classified as:
(a) Unfinished or Black Bolts, (b) Finished (turned) bolts, (c) High strength friction grip (HSFG) bolts
2.1. Black Bolts: Black bolts are also referred to as ordinary, unfinished, rough, or common bolts. They are the least expensive bolts.


Fig. Hexagonal head black bolt and nut.
Figures in bracket are for high-strength bolts and nut
In steel construction, generally, bolts of property class 4.6 are used. In property class 4.6, the number4 indicates $1 / 100^{\text {th }}$ of the nominal ultimate tensile strength in $\mathrm{N} / \mathrm{mm}^{2}$, and the number 6 indicates the ratio of yield stress to ultimate stress expressed as a percentage. Thus, the ultimate tensile strength of a class 4.6 bolt is $400 \mathrm{~N} / \mathrm{mm}^{2}$, and yield strength is 0.6 times 400 , which is $240 \mathrm{~N} / \mathrm{mm}^{2}$.
2.2. Turned Bolts (Close Tolerance Bolts): These are similar to unfinished bolts, with the difference that the shanks of these bolts are formed from a hexagonal
rod. They are mainly used in special jobs (in some machines and where there are dynamic loads).
2.3. High Strength Friction Grip bolts (HSFG): The tension in the bolt ensures that no slip takes place under working conditions, and so the load transmission from plate to the bolt is through friction and not by bearing. HSFG bolts are made from quenched and tempered alloy steels with grades from 8.8 to 10.9.

## 3. CLASSIFICATION OF BOLTS BASED ON METHOD OF LOAD TRANSFER

Based on load transfer in the connection, bolts may be classified as:
(a) Bearing type: Unfinished (black) bolts and finished (turned) bolts are bearing types since they transfer shear force from one member to other members by bearing.
(b) Friction grip type: HSFG bolts belongs to the friction grip type since they transfer shear by friction.

(a) Bearing connection

## 4. TERMINOLOGY IN BOLTED CONNECTION

1. Pitch of the Bolts (P): It is the centre to centre spacing of bolts in a row, measured in the direction of load.
2. Guage (g): It is the distance between the two consecutive bolts of adjacent rows and is measured at a right angle to the direction of the load.
3. Staggered Pitched ( $\mathbf{P}_{\mathbf{s}}$ ): It is the centre to centre distance of staggered bolts measured in the direction of load.

4. Diameter of Bolt Hole: The diameter of the bolt hole is larger than the nominal diameter (shank diameter) of the bolt
5. Area of Bolt at Root (Anb): Area of Bolt at the root of the thread is less than at the shank of the bolt. For some common bolt sizes, $A_{n b}=0.78 \times A_{s b} w h e r e, A_{s b}=$ Area of bolt at the shank.

## 5. IS 800- 2007SPECIFICATIONS FOR SPACING AND EDGE DISTANCE OF BOLT HOLES

(i) Pitch shall not be less than 2.5 d , where ' d ' is the nominal diameter of the bolt.
(ii) Pitch shall not be more than
(a) 16t or 200 mm , whichever is less, in case of tension members.
(b) 12 t or 200 mm , whichever is less, in the case of compression members where t is the thickness of the thinnest member.
(c) In the case of staggered pitch, the pitch may be increased by 50 per cent of the specified value provided the gauge distance is less than 75 mm .
(iii) In the case of butt joints, the maximum pitch is to be restricted to 4.5d for a distance of 1.5 times the width of the plate from the butting surface.
(iv) The gauge length ' $g$ ' should not be more than $100+4$ t or 200 mm , whichever is less.
(v) Minimum edge distance shall be
(a) Less than $1.7 \times$ hole diameter in case of sheared or hand flame cut edge.
(b) Less than $1.5 \times$ hole diameter in case of rolled, machine flame cut, sawn and planed edges.
(vi) Maximum edge distance should not exceed.
(a) $12 t \in$, where $\in=\sqrt{\frac{250}{f_{y}}}$ and $t$ is the thickness of the thinner outer plate.
(b) $40+4 t$, where $t$ is the thickness of thinner connected plate if exposed to the corrosive environment.
(vii) Apart from the required bolt from the consideration of design forces, additional bolts called tacking fasteners should be provided as specified below.
(a)If the value of gauge length exceeds after providing design fasteners at maximum edge distances tacking rivets should be provided.

- At 32 t or 300 mm , whichever is less, if plates are not exposed to the weather.
- At 16 t or 200 mm , whichever is less if plates are exposed to the weather.
(viii) In the case of a member made of up two flats, or angles or tees or channels, tacking rivets are to be provided along the length to connect its components as specified below.
(a) Not exceeding 1000 mm , if it is a tension member.
(b) Not exceeding 600 mm , if it is a compression member.


## 6. TYPES OF BOLTED CONNECTION

Bolted joints may be grouped into the following types.
6.1. Lap Joint: It is the simplest type of joint. In this, the plate to be connected overlaps one another.

$\mathrm{I}_{\mathrm{g}} \ngtr 5 \mathrm{~d}, \mathrm{~d}=$ diameter of the bolt.


Lap Joint
6.2. Single Cover Butt Joint: Bolts in single cover butt joints are subjected to single shear and bearing.


$$
t_{\text {cover }} \geq t_{\text {main }}
$$

6.3. Double cover butt joint: Bolts in double cover butt joint are subjected to double shear and bearing.


$$
\text { sum of } \mathrm{t}_{\text {cover }} \geq \mathrm{t}_{\text {main }}
$$

7. ASSUMPTIONS IN DESIGN OF BEARING BOLTS: The following assumptions are made in the design of bearing (finished or unfinished) bolted connections:
8. The friction between the plates is negligible.
9. The shear is uniform over the cross-section of the bolt.
10. The distribution of stress on the plates between the bolt holes is uniform
11. Bolts in a group subjected to direct loads share the load equally
12. Bending stresses developed in the bolts in neglected.
13. DESIGN TENSILE STRENGTH OF PLATES IN A JOINT

## (i) Failure of bolts

(a) Shear failure of the bolt, the bolt will get cut into 2 pieces.
(b) Bearing failure of the bolt, the bolt will go out of shape.
(ii) Failure of plates
(a) Shear of plates, the cracks are developed parallel to the direction of the force.


Shear stress at (2) - (2)
Tensile stress at (1) - (1)
(b) Splitting failure: It occurs due to diagonal tension in the plate at bolt level.

(c) Bearing failure: Bolt will push the plate forward. It occurs when the bearing strength of the plate is less.

(d)Tension failure or tearing failure of plate: Cracks are developed perpendicular to the direction of the force.


Design strength of bolted joint: It is taken as the least value of

- $\quad$ Shear strength of all bolts in the joint ( $\mathrm{P}_{\mathrm{s}}$ )
- $\quad$ Bearing strength of all bolts in the joint $\left(\mathrm{P}_{\mathrm{b}}\right)$
- $\quad$ Tensile strength of the plate $\left(\mathrm{P}_{\mathrm{t}}\right)$


## (i) Lap joint

Case (a): For the entire width of the plate.
$P_{s}=$ shear strength of all bolts in the joint.
$P_{s}=n \times \frac{\pi}{4} \times d^{2} \times f_{s}$
$\mathrm{n}=$ Total number of bolts in the joint.
$\mathrm{d}=$ diameter of the bolt.
$\mathrm{f}_{\mathrm{s}}=$ permissible shear stress in the bolt.
$f_{s}=\frac{f_{u}}{\sqrt{3} \times 1.25}$ (for LSM)
$\mathbf{P}_{\mathbf{b}}=$ bearing strength of all bolts in the joint.
$P_{b}=n \times d \times t \times f_{b}$
$\mathrm{n}=$ number of bolts in the joint
$\mathrm{d} \times \mathrm{t}=$ projected area of the bolt against the plate.
$\mathrm{t}=$ thickness of the thinner plate.
$f_{b}=$ permissible bearing stress in the bolt.
$f_{b}=2.5 \times k_{b} \times \frac{f_{u}}{1.25}$
$\mathbf{P}_{\mathrm{T}}=$ Tensile strength of the plate.
$P_{T}=A g \times \frac{f_{y}}{1.1}$ (Based on gross area yielding at $x . x$ )
$=A_{\text {net }} \times \frac{0.9 \mathrm{f}_{\mathrm{u}}}{1.25} \quad$ (based on net area)
Least value of $P_{s}, P_{b}$ and $P_{t}$ is taken as the strength of the joint.

## Case (b): Strength of joint / gauge length

$P_{S_{1}}=$ Shear strength of all bolts in shaded gauge length.
$P_{S_{1}}=n_{1} \times \frac{\pi}{4} \times d^{2} \times f_{s}$

$\mathrm{n}_{1}=$ total number of bolts in shaded gauge length.
$\mathrm{n}_{1}=3$
$\mathrm{P}_{\mathrm{b}_{1}}=$ bearing strength of all bolts in shaded gauge length.
$P_{b_{1}}=n_{1} \times d \times t \times f_{b}$
$P_{t_{1}}=$ tensile strength of the plate/gauge length
$=A_{g} \times \frac{f_{y}}{1.1}$
$=A_{\text {net }} \times \frac{0.9 f_{u}}{1.25}$

## (ii) Double Cover butt joint

$P_{s}=$ shear strength of all bolts in the joint
$P_{s}=n \times 2 \times \frac{\pi}{4} \times d^{2} \times f_{s}$
$P_{b}=$ Bearing strength of all bolts in the joint.
$P_{b}=n \times d \times t \times f_{b}$
$\mathrm{n}=\mathrm{g}$
$\mathrm{t}=$ sum of cover plate thickness or thickness of thinner main plate, whichever is less.
$P_{t}=$ tensile strength of the plate.
$=A g \times \frac{f_{y}}{1.1}$
$=A_{\text {net }} \times \frac{0.9 f_{y}}{1.25}$

(iii) Rivet value ( $\mathbf{R}_{\mathrm{v}}$ ): It is the strength of a single bolt. It is taken as the least value of $P_{s}$ and $P_{b}$ of a single bolt.
(a) When bolt is in single shear
$\left.\begin{array}{l}P_{s}=\frac{\pi}{4} \times d^{2} \times f_{s} \\ P_{b}=d \times t \times f_{b}\end{array}\right\}$ whichever is less is $R_{v}$
(b) If bolt is in double shear
$\left.\begin{array}{l}P_{s}=2 \times \frac{\pi}{4} \times d^{2} \times f_{s} \\ P_{b}=d \times t \times f_{b}\end{array}\right\}$ whichever is less

Number of bolts required at a joint $=\mathrm{n}=\frac{\text { Factored load }}{\text { Rivet value }}$
$\mathrm{n}=\frac{\mathrm{F}}{\mathrm{R}_{\mathrm{v}}}$
9. ARRANGEMENT OF BOLT

## (i) Chain bolting:



Here, net area $=A_{\text {net }}=(B-3 d)$ t at $1-1$
$\mathrm{d}=$ diameter of bolt hole
$B=$ width of plate
(ii) Diamond bolting

here, net area $=A_{\text {net }}=(B-d) t$ at $1-1$
So, Diamond riveting/bolting is more efficient than chain bolting.
(iii) Mixed Bolting


In this arrangement, both cover plates and the main plate can take the maximum load.
10. ECCENTRIC BOLTED CONNECTION: If the centre of gravity of the bolt group does not lie on the line of action of the load, then it is called an eccentric connection.
10.1. In-plane eccentricity: In this case, the bolt group and load are on the same plane, but the CG of the bolt group does not lie on the line of action of the load.


There are two effects when the load is applied, as shown above $\rightarrow$ direct load ( P )
$\rightarrow$ twisting moment $(T)=P e$.
Direct shear force on bolt due to load $P, F_{1}=\frac{P}{\Sigma A_{i}} \times A_{i}$
where $A_{i}=$ cross-section area of each bolt.
If the cross-sectional area is the same for all bolts, then, $F_{1}=\frac{P}{n A_{i}} \times A_{i}=\frac{P}{n}$ where $\mathrm{n}=$ number of bolts.
Shear force in the bolt due to twisting moment $(T), F_{2}=\frac{P e}{\Sigma A_{i} r_{i}^{2}} \times r_{i} A_{i}$ If all the bolts have the same area, $F_{2}=\frac{P e}{\Sigma r_{i}^{2}} \times r_{i}$
where $r_{i}=$ radial distance of each bolt from CG of bolt group.
Resultant shear force in the bolt, $F_{R}=\sqrt{F_{1}^{2}+F_{2}^{2}+2 F_{1} F_{2} \cos \theta}$
$\theta=$ the angle between $F_{1}$ and $F_{2}$
$\mathrm{F}_{\mathrm{R}} \ngtr \mathrm{R}_{\mathrm{V}}$ for safe bolting, $\mathrm{R}_{\mathrm{V}}=$ rivet/bolt value .


If the bolts are of the same diameter, the most critically stressed bolt is the one for which ' $r$ ' is maximum and $\theta$ is minimum.

### 10.2. Out of plane eccentricity

Section is subjected to a direct shear force $P$ and moment $M=P e$.


On the tension side, only the bolt resists load but on the compression side entire zone between the columns and the connecting angle resist the load. It is assumed that the neutral axis (NA) lies at a height of $\frac{H}{7}$ of the depth of bracket, measured from the bottom edge of the angle.
Since bolts above NA are subjected to tension and shearing, the governing criteria to prevent failure of the bolt is given by
$\left(\frac{\mathrm{P}_{\mathrm{T}, \text { cal }}}{\mathrm{P}_{\mathrm{T}}}\right)^{2}+\left(\frac{\mathrm{P}_{\mathrm{S}, \text { cal }}}{\mathrm{P}_{\mathrm{S}}}\right)^{2} \leq 1$
Here,
$\mathrm{P}_{\mathrm{T}, \text { cal }}=$ calculated factored tensile force in the bolt.
$\mathrm{P}_{\mathrm{T}}=$ tension capacity of the bolt
$P_{s}=$ shear capacity of the bolt
Interaction curve between $\mathrm{P}_{\mathrm{t}}$ and $\mathrm{Ps}_{\text {s }}$.

11. DESIGN OF WELDED JOINT: There are three types of welded joints
(i) Butt weld
(ii) Fillet weld
(iii)Slot weld or plug weld
11.1. Butt weld: It is also called a groove weld. Depending upon the shape of the groove made for welding, butt welds are classified as square butt weld, single $v$ butt weld, double $v$ butt weld, sing $U$ butt joint, single J butt joint, single bevel butt joint.
11.2. Fillet Weld: It is a weld of approximately a triangular cross-section joining two surfaces approximately at right angles to each other in a lap joint or corner joint. It is assumed that fillet welds always offer resistance in the form of shear only.
(i)Size of fillet weld:
(a) Size of the normal fillet weld shall be taken as the minimum weld leg size.
(b)For deep penetration welds with penetration not less than 2.4 mm , the size of the weld is (minimum leg size +2.4 mm ).
(c)For fillet welds made by semi-automatic or automatic process with deep penetration of more than 2.4 mm ,
$\mathrm{S}=$ minimum leg size + actual penetration
(ii) Throat: It is the minimum dimension in fillet weld. Throat thickness $=t_{t}=$ $\mathrm{k} \times$ size of weld.

Where k is a constant depends on the angle between fusion faces.

| Angle between fusion faces | $k$ |
| :---: | :---: |
| $60^{\circ}-90^{\circ}$ | 0.7 |
| $91^{\circ}-100^{\circ}$ | 0.65 |
| $101^{\circ}-106^{\circ}$ | 0.6 |
| $107^{\circ}-113^{\circ}$ | 0.55 |
| $114^{\circ}-90^{\circ}$ | 0.5 |

Fillet weld should not be used if the angle between fusion faces is less than $60^{\circ}$ or more than $120^{\circ}$.

## (iii) Minimum size of weld

The minimum size of weld specified is 3 mm . To avoid the risk of cracking in the absence of preheating, the minimum size specified are :

| Thickness of thicker plate | Minimum size of weld |
| :---: | :---: |
| $<10 \mathrm{~mm}$ | 3 mm |
| 10 mm | 5 mm |
| $20 \mathrm{~mm}-32 \mathrm{~mm}$ | 6 mm |
| $32 \mathrm{~mm}-50 \mathrm{~mm}$ | 8 mm |

(iv)Maximum size of weld
(a)At a square edge, Maximum size of weld = thickness of plate - 1.5 m
(b)At the round edge, Maximum size of weld $=\frac{3}{4} \times$ thickness of the plate
(v) Effective length of weld: Welding length made is equal to effective length plus twice the size of the weld. Effective length should not be less than four times the size of the weld.
(vi) Lap joint: The minimum lap should be four times the thickness of the thinner part joined or 40 mm , whichever is more. The length of weld along either edge should not be less than the transverse spacing of welds.
(vii) Strength of weld: Load carrying capacity of weld
$P=$ permissible shear stress in weld $\times$ effective area of the weld.
$P=f_{s} \times l_{\text {eff }} \times t_{t}$
Where $f_{s}=$ permissible shear stress in the weld $=\frac{f_{u}}{\sqrt{3} \times 1.25}$
11.3. Slot weld or plug weld: Slot welding is alone to increase the length of the weld.

Minimum width of slot $=3 \mathrm{t}$ or 25 mm , whichever is less.
Here, $\mathrm{t}=$ thickness of the plate in which slot is made.
12. ECCENTRIC WELDED CONNECTION: Plane of the moment and the eccentric load, $P$ is equivalent to a direct load $P$ at the centre of gravity of the group of weld and a twisting moment, $\mathrm{P} \times \mathrm{e}$.


Due to direct load $P$, shear stress $f_{1}$, is developed and due to twisting moment, torsion shear stress $f_{2}$ is developed at $A$ (this torsional shear stress is maximum if radial distance is maximum). Resultant due to $f_{1}$ and $f_{2}$ can be calculated as follows:
$f_{R}=\sqrt{f_{1}^{2}+f_{2}^{2}+2 f_{1} f_{2} \cos \theta}$
$\theta=$ angle between $f_{1}$ and $f_{2}$

## Out of plate eccentricity:

The effect of eccentric load is equivalent to a direct load acting at centre of gravity of weld group and bending moment, $\mathrm{P} \times \mathrm{e}$.


Neutral axis for the weld group lies at the centre of gravity of weld group because bending tensile stress and bending compressive stress are resisted by weld only.
Due to direct load, $P$ vertical shear stress $f_{1}$ is developed and due to bending moment, bending tensile stress is developed.

Equivalent stress is calculated as follows
$f_{E}=\sqrt{f_{1}^{2}+3 f_{2}^{2}} \leq f_{s}=\frac{f_{u}}{\sqrt{3} \times 1.25}$
If $f_{1}+f_{2}<f_{s}$ than
$\mathrm{f}_{\mathrm{r}}=\sqrt{\mathrm{f}_{1}^{2}+\mathrm{f}_{2}^{2}} \leq \mathrm{f}_{\mathrm{s}}$

## CHAPTER-3-TENSION MEMBERS

1. LUG ANGLE: Length of the end connection of a heavily loaded tension member may be reduced by using lug angles. By using the lug angle, there will be a saving in the greatest plate.
(i) The effective connection of the lug angle shall as far as possible terminate at the end of the member.
(ii) The connection of the lug angle to the main angle shall preferably start in advance of the member to the gusset plate.
(iii) Minimum of two bolts rivets or equivalent welds is used for attaching lug angle to the gusset plate.
(iv) The purpose of lug angle is to reduce the shear lag effect and to reduce the length of the connection to the gusset plate.
(v) Shear lag factor, which takes care of loss of efficiency due to shear lag, should not be less than 0.7.
2. LONG JOINTS: If the length of the joint is more than 15 d or $150 \mathrm{t}_{\mathrm{t}}$, where d is the diameter of the bolt hole, and $t_{t}$ is the throat thickness.
3. GRIP LENGTH: grip length should not be more than 5d, where $d$ is the diameter of the bolt hole. If the value of grip length is in the range of 5 d to 8 d , then $\mathrm{P}_{\mathrm{s}}$ is multiplied by a reduction factor to take care of the additional stresses. But in case the value of grip length is more than 8 d , then, in that case, the section must be redesigned.

## 4. SLENDERNESS RATIO LIMITS

| Type of member | Maximum value of slenderness <br> ratio |
| :--- | :---: |
| (i) Pure tension member | 400 |
| (ii) Hanger bars | 160 |
| (iii) Pure compression member | 180 |
| (iv) Lacing member | 145 |
| (v) Tension member subjected to reversal of stresses due to live <br> load | 180 |


| (vi) Pure tension member subjected to reversal of stress due to <br> action of wind. | 350 |
| :--- | :---: |
| (vii)Reversal of load by wind/E.Q. the deformation of such <br> members doesn't affect the other part of the structure | 250 |

## 5. NET SECTIONAL AREA:

Case 1: Chain Bolting: Net area along the section $A B C D, A_{n e t}=(B-n d) t$
Where $\mathrm{n}=$ number of bolt holes, $\mathrm{d}=$ diameter of bolt holes, $\mathrm{t}=$ thickness of the plate.


## Case 2: Staggered Bolting



Net Area along the section 1-2-3-4-5 is given by, $A_{\text {net }}=\left[B-n d+\frac{s_{1}^{2}}{4 g_{1}}+\frac{s_{2}^{2}}{4 g_{2}}\right] \times t$
If $\mathrm{s}_{1}=\mathrm{s}_{2}$, and $\mathrm{g}_{1}=\mathrm{g}_{2}, A_{\text {net }}=\left[B-n d+\frac{n^{\prime} \mathrm{s}^{2}}{4 g}\right] \times t$
Where, $s=$ staggered pitch, $g=$ gauge distance, $\mathrm{n}^{\prime}=$ number of staggered pitches, n
= number of holes in zig-zag line

## 6. LOAD CARRYING CAPACITY OF A TENSION MEMBER

A tension member may fail in any of the following modes:
6.1. Gross Section Yielding: $T_{d g}=\frac{A_{g} f_{y}}{\gamma_{m 0}}$

Here, $\quad A_{g}=$ gross area of plate,
$f_{y}=$ yield stress,
$Y_{m o}=$ Partial factor of safety governed by yielding $=1.1$
6.2. Net section Rupture: $T_{d n}=0.9 \frac{A_{n} f_{u}}{\gamma_{m 1}}$

Here, $A_{n}=$ Net area of plate,
$f_{u}=$ Ultimate strength of plate
$Y_{m 1}=$ Partial factor of safety governed by ultimate strength $=1.25$

### 6.3. Block shear failure:



Shear failure occurs along 1-2 and 3-4, whereas tension failure occurs along 2-3.

For shear yield and tension failure: $T_{d b 1}=\frac{A_{v g} f_{y}}{\sqrt{3} \gamma_{m 0}}+0.9 \frac{A_{t n} f_{u}}{\gamma_{m 1}}$

For tension yield and shear fracture: $T_{d b 2}=\frac{A_{t g} f_{y}}{\gamma_{m 0}}+0.9 \frac{A_{v n} f_{u}}{\sqrt{3} \gamma_{m 1}}$

Where,
$A_{v g}=$ Gross area in shear along the line of action of force
$A_{v n}=$ Net area in shear along the line of action of force
$A_{t g}=$ Gross area in tension across the line of action of force

Atn $=$ Net area in tension across the line of action of force

## CHAPTER-4-COMPRESSION MEMBERS

## 1. EFFECTIVE LENGTH

(a) If end conditions can be assessed: Where the boundary conditions in the plane of buckling can be assessed, the effective length (KL) can be calculated as follows.

| Boundary Condition |  |  |  | Diagram | Effective Length |
| :---: | :---: | :---: | :---: | :---: | :---: |
| At one end |  | At another end |  |  |  |
| Translation | Rotation | Translation | Rotation | $1 \% \uparrow$ |  |
| Restrained | Restrained | Free | Free |  | 2.0 L |
| Restrained | Free | Free | Restrained |  | L |
| Restrained | Free | Restrained | Free |  | L |
| Restrained | Restrained | Free | Restrained |  | 1.2 L |
| Restrained | Restrained | Restrained | Free |  | 0.8 L |



## (b) Compression member in truss

- In the case of bolted, riveted or welded trusses and braced frames, the effective length, $K L$, shall be taken as 0.7 to 1.0 times the actual length, depending upon the degree of end restraints provided.
- For buckling in the plane perpendicular to the plane of the truss, the effective length may be taken as the actual length.
(c) In frames: In frame analyses, if the deformed shape is not considered, the effective length depends upon the stiffness of the members meeting at the joint.

2. EULER'S COLUMN FORMULA: Critical load on column, $P_{C r}=\frac{\pi^{2} E I_{\text {min }}}{L^{2}}$ Where, $L=$ effective length, $I_{\text {min }}=$ minimum moment of inertia $=A r_{\text {min }}^{2}$ $r_{\text {min }}=$ minimum radius of gyration .
As per IS 800: 2007. The factored compressive load carrying capacity of the column, $P_{c}=f_{c d} \times A_{g}$
Here, $f_{c d}=$ design compressive stress, $A_{g}=$ gross cross sectional area of the column.
3. ANALYSIS OF STRUT: To prevent the buckling of strut components between tack bolts, $\left(\frac{\ell_{\mathrm{t}}}{r_{\text {min }}}\right)_{\text {comp }} \ngtr 40$
4. DESIGN OF LACING: If lacing members are used, then the effective length of the column is increased by $5 \%$ in the calculations. Lacing members are designed as truss elements. The maximum slenderness ratio for lacing members is 145 .
The angle of lacing with respect to vertical should not be less than $40^{\circ}$ and more than $70^{\circ}$.
If $\theta>70^{\circ}$, the force in the lacing member will be very high and it may buckle. If $\theta<40^{\circ}$, the length of the lacing member will be more and it may buckle.

To prevent buckling of the column component between lacing connection,
$\left(\frac{C}{r_{\text {min }}}\right) \ngtr 50 \ngtr 0.7 \times$ slenderness ratio of the overall section
Where $C=$ overlap length $k 4 t, r_{\text {min }}=$ minimum radius of gyration
According to IS 800 - 2007, the dimensions of the lacing bar are specified as follows
(i) Length of lacing bar ( $\mathbf{I}_{\mathbf{1}}$ ): It is taken as the distance between inner welds or bolts.
(ii) Effective length of lacing bar (leff):
$l_{\text {eff }}=I_{1}$, for single lacing (with one or two bolts)
$l_{\text {eff }}=0.7 I_{1}$, for double lacing.
$l_{\text {eff }}=0.7 \mathrm{I}_{1}$, for welded lacing.

## (iii) Minimum thickness ( $\mathbf{t}_{\text {min }}$ ):

$t_{\text {min }} \nless \frac{l_{\text {eff }}}{40}$, for single lacing.
$t_{\text {min }} \varangle \frac{I_{\text {eff }}}{60}$, for double lacing.
(iv) Minimum width $\left(b_{\text {min }}\right): b_{\text {min }} \nless 3 \phi$

Where $\phi=$ nominal diameter of bolt.
At the top and bottom, tie plates or batten plates are provided. They prevent distortion of built-up column cross-section due to twisting moment at the connection of the base plate with the column.

## Force in the lacing system:

- The lacing shall be designed to resist transverse shear force, $\mathrm{V}=2.5 \%$ of column load (to take care of the eccentricity of axial loads and moments arising due to accidental lateral loads).
- The shear force V is shared by parallel planes of the lacing system equally. So transverse shear force on each lacing system is $\frac{V}{n}$ where $n$ is the number of parallel planes of lacing.
- If the column is subjected to bending also, $\mathrm{V}_{\mathrm{t}}=$ bending shear $+2.5 \%$ column force.
- Laced members should not be subjected to eccentric loading because additional transverse shear force is developed in the lacing system due to moment.

5. DESIGN OF BATTENED COLUMN: IS 800-2007 specifies the following rules for the design of battened column.
(i) Batten plates should be placed symmetrically.
(ii) At both ends, batten plates should be provided.
(iii) The number of battens should be such that the member is divided into less than three bays. As far as possible, they should be spaced and proportioned uniformly throughout.
(iv) Battens shall be of plates, angles, channels, or I-sections and at their ends shall be riveted, bolted or welded.
(v) By providing battens distance between the members of columns is so maintained that radius of gyration about the axis perpendicular to the plane of battens is not less than the radius of gyration about the axis parallel to the plane of the batten.
(vi) The effective slenderness ratio of battened columns shall be taken as 1.1 times the maximum actual slenderness ratio of the column to account for shear deformation. (vii) The vertical spacing of battens, measured as the centre to centre of its end fastening, shall be such that the slenderness ratio of any component of the column over that distance shall be neither greater than 50 nor greater than 0.7 times the slenderness ratio of the member as a whole about its axis.
(viii) Battens shall be designed to carry the bending moment and shear forces arising from transverse shear force $V_{t}$ equal to $2.5 \%$ of the total axial force.
(ix) In case columns are subjected to moment also, the resulting shear force should be found, and then the design shear is the sum of this shear and $2.5 \%$ of axial load. $(x)$ The design shear and moments for battens plates is given by
$V_{b}=\frac{V_{t} C}{N S}$ and $M=\frac{V_{t} C}{2 N}$ at each connection here, $V_{t}=$ transverse shear force as defined in 8 and 9.

Here,
$\mathrm{C}=$ distance between centre to centre of battens longitudinally.
$\mathrm{N}=$ number of parallel planes.
$\mathrm{S}=$ minimum transverse distance between the centroid of the fasteners connecting batten to main
(xi) The effective depth of end battens (longitudinally), shall not be less than the distance between the centroids of main members.
(xii) The effective depth of intermediate battens shall not be less than $\frac{3}{4}$ th of above distance.
(xiii) In no case, the width of battens shall be less than twice the width of one member in the plane of the batten. It is to be noted that the effective depth of a batten shall be taken as the longitudinal distance between the outermost fasteners.
(xiv) The thickness of battens shall not be less than $\frac{1}{50}^{\text {th }}$ of the distance between the innermost connecting lines of rivets, bolts or welds.
(xv) The length of the weld connecting the batten plate to the member shall not be less than half the depth of the batten plate. At least one-third of the weld shall be placed at each end of this edge.

## CHAPTER-5-BEAMS

## 1. COMMON TYPES OF BEAMS

- Floor beam: A major beam of a floor system usually supporting joints in a building.
- Girder: In buildings, girders are the same, as floor beams are also a major beam in a structure.
- Grit: A horizontal member fastened to and spanning between peripheral columns of an industrial building to support wall cladding.
- Joist: A beam supporting floor construction but not major beams.
- Lintel: Beam members used to carry wall loads over wall openings for doors, windows etc.
- Purlin: A roof beam, usually supported by roof trusses.
- Rafter: A roof beam, usually supporting purlins.
- Spandrels: Exterior beams at floor level of buildings, which carry part of floor load and the exterior wall
- Stringers: Beam supporting stair steps (in case of buildings).
- Header: A beam at stairwell openings.


## 2. TYPES OF BEAMS BASED ON THE LATERAL SUPPORTS TO COMPRESSION FLANGES

(i) Laterally supported beam: If the compression flange of the beam is completely restrained against lateral movement, then it is called a laterally supported beam.


Since the moment of inertia about the $y$-axis increases due to the slab, there is no possibility of the buckling of the compression flange. So, design bending compressive stress, $f_{b d}$ is taken $f_{y}$ (in LSM) or $0.66 f_{y}$ (in WSM).
(ii) Laterally unsupported beam: If the compression flange of the beam is not restrained against lateral movement, then it is called a laterally unsupported beam.


Since there is a possibility of buckling of the compression flange, design bending compressive stress is taken as $f_{b d}=X_{L T}\left(\frac{f_{y}}{1.1}\right)$.

Where, $X_{L T}=$ reduction factor to take care of lateral-torsional buckling of the beam.

## 3. CLASSIFICATION OF CROSS-SECTIONS:

(i) Plastic section: It has the capacity to develop the plastic hinge and collapse mechanism
$M_{P}=$ plastic moment capacity of cross section $=f_{y} \times Z_{p}$
$M_{y}=$ Yield moment carrying capacity $=\frac{f_{y}}{1.1} z_{P}$
$M_{o}=f_{b d} \cdot Z_{p}$
Where $f_{b d}=\frac{f_{y}}{1.1}$ design stress
(ii) Compact section: It has the capacity to develop a plastic hinge but does not have the capacity to form a collapse mechanism.
$M_{p}=f_{y} \cdot z_{p}$
$M_{u}=f_{b d} \cdot z_{p}$
Here, $\quad f_{b d}=\frac{f_{y}}{1.1}$
(iii) Semi-compact section: It has the capacity to develop yield moment only.
$M_{y}=f_{y} \cdot z$
$M_{o}=f_{b d} \cdot z$
(iv) Slender Section: These sections fail by buckling even before reaching yield stress.
$M_{u}=f_{b d} \cdot z$
4. BENDING (FLEXURAL) STRENGTH: Bending strength design of laterally supported beams is governed by yield stress, and lateral or torsional buckling controls the design of laterally unsupported beams.
(a) Laterally supported beams:(For beam supported laterally against lateral torsional buckling)

- Factored design moment at any section (M) $\leq$ (Design bending strength of section $M_{d}$ )
- When $\mathrm{d} / \mathrm{t}_{w}<67 \epsilon$ (No shear buckling in web)
- Nominal shear capacity ( $\mathrm{V}_{\mathrm{n}}$ ) - Plastic shear strength of beam ( $\mathrm{V}_{\mathrm{p}}$ )
- Design shear strength $\mathrm{V}_{\mathrm{d}}=\mathrm{V}_{\mathrm{n}} / \mathrm{Y}_{\text {mo }}$
- When $\mathrm{d} / \mathrm{t}_{\mathrm{w}}>67 \epsilon$ (web of beam susceptible to shear buckling)

Case I: Low shear case (Factored design shear force $\mathrm{V} \leq 0.6 \mathrm{~V}_{\mathrm{d}}$ )
Design bending strength $M_{d}=\beta_{p} Z_{p} f_{y} / Y_{m o}$
$\leq 1.2 Z_{\text {efy }} /$ Ymo (For simply supported beams)
$\leq 1.5 Z_{e} f_{y} /$ Ymo $_{\text {mor }}$ (Fantilever beams)
Where,
$\beta_{b}=1.0$ (for plastic and compact sections)
$\beta_{b}=Z_{e} / Z_{p}$ (for semi compact sections)
$\beta_{b}$ and $Z_{p}$ elastic and plastic section modulus of the cross section
For slender sections:
$M_{d}=Z_{\mathrm{ef}} \mathrm{f}_{\mathrm{y}}\left(\mathrm{f}_{y}^{\prime}\right.$ Reduced design strength)
Case II: High shear case - (Factored design shear force $\mathrm{V}>0.6 \mathrm{~V}_{\mathrm{d}}$ )
Design bending strength
$M_{d}=M_{d v}\left(M_{d v}-d e s i g n\right.$ bending strength under high shear)
For plastic or compact section
$M_{d v}=M_{d} \beta\left(M_{d}-M_{f d}\right) \leq \frac{1.2 Z_{e} f_{v}}{\gamma_{m o}}$ where $\beta=\left(\frac{2 V}{V_{d}}-1\right)^{2}$
$M_{d}=$ Plastic design moment of the whole section neglecting high shear case and
considering web buckling effect
$\mathrm{V}=$ Factored applied shear force
$V_{d}=$ Design shear strength as governed by web yielding fort web buckling.
$M_{\mathrm{fd}}=$ Plastic design strength of the area of cross-section excluding shear area.

## For Semi compact section

$M_{d v}=Z_{e} f_{y} / Y_{m o}$
(b) Laterally unsupported beams (For beam unsupported laterally against lateral torsional buckling)

- Beam with major axis bending and compression flange not restrained against lateral bending fail by lateral. Torsional buckling before attaining their bending strength
- The effect of lateral-torsional buckling need not be considered when $\lambda_{L T} \leq 0.4$ (where
$\lambda_{\text {LT- }}$ non-dimensional effective slenderness ratio for lateral-torsional buckling)
The bending strength of laterally unsupported beam is given by,
$M_{d}=\beta_{b} . Z_{p} . f_{c d}$
$\beta_{b}=1.0$ (For plastic and compact sections)
$=Z_{e} / Z_{p}$ (for semi compact sections)
$\mathrm{Z}_{\mathrm{e}}=$ Elastic section modulus
$Z_{p}=$ Plastic section modulus
$f_{c d}=$ Design bending compressive stress $=X_{L T} \cdot \frac{f_{y}}{\gamma_{\text {mo }}}$
$X_{L T}=$ Bending stress reduction factor to account for lateral-torsional buckling
$X=\frac{1}{\phi_{L T}+\left(\left(\phi_{L T}^{2}-\lambda_{L T}^{2}\right)\right)^{0.5}} \leq 1.0$
$\phi_{L T}=0.5\left[1+\alpha_{L T}\left(\lambda_{L T}-0.2\right)+\lambda_{L T}^{2}\right]$
$\mathrm{a}_{\mathrm{Lt}}=$ Imperfection factor
$=0.21$ (For rolled sections)
$=0.49$ (Welded section)
$\lambda_{L T}=$ Non dimensional slenderness ratio
$=\sqrt{\beta_{b} Z_{p} \cdot f_{y} / M_{c r}} \leq \sqrt{1.2 Z_{e} \cdot f_{y} / M_{c r}}=\sqrt{\frac{f_{y}}{f_{c r} \cdot b}}$
$M_{c r}=\sqrt{\left(\frac{\pi^{2} E I}{\lambda_{L T}^{2}}\right)\left[G I_{t}+\frac{\pi^{2} E I_{w}}{\lambda_{L T}^{2}}\right]}$
$=\beta_{b} \cdot Z_{p} \cdot f_{\text {cr. }}$ b
$M_{c r}=$ The moment at which a beam fail by lateral buckling when subjected to uniform moment is called elastic critical moment.


## 5. SHEAR STRENGTH OF LATERALLY SUPPORTED BEAM

Design shear strength $V_{d}=\frac{A_{v} f_{y v}}{\sqrt{3} \gamma_{m o}}$
Where,
$\mathrm{A}_{\mathrm{v}}=$ Shear area
$f_{y v}=$ Yield strength of the web
The shear area may be calculated as given below :

## For I and Channel Sections

(i) Major axis bending

Hot rolled - htw.
Welded - $\mathrm{dt}_{\mathrm{w}}$
(ii) Minor axis bending

Hot rolled or welded - $2 b_{f}$
For Rectangular Hollow Sections of Uniform Thickness
(i) Loaded parallel to depth (h) : $\frac{\mathrm{Ah}}{(\mathrm{b}+\mathrm{h})}$
(ii) Loaded parallel width (b) : $\frac{\mathrm{Ab}}{(\mathrm{b}+\mathrm{h})}$
(iii) Circular hollow tubes of uniform thickness $-\frac{2 A}{\Pi}$
(iv) Plates and solid bars - A

Where,
A Cross-section area:
$b=$ Overall breadth of the tubular section, breadth of I section flanges
d = Clear depth of web between flanges
$h=$ Overall depth of the section
$\mathrm{t}_{\mathrm{r}}=$ thickness of the flange and
$\mathrm{t}_{\mathrm{w}}=$ thickness of the web.

## 6. DEFLECTION LIMIT

- Vertical deflection for a simply supported span

Elastic cladding - span/240
Brittle cladding - span /300

- Vertical deflection for cantilever span

Elastic cladding - span/120
Brittle cladding - span / 240

- Vertical deflection for purlins and grit

Elastic cladding - span/ 150
Brittle cladding - span/180

## 7. WEB CRIPPLING



The crippling occurs at the root of the radius. IS 800-2007 has accepted the following formula to find crippling of the web: $F_{w}=\left(b_{1}+n_{c}\right) t_{w} \times \frac{f_{y w}}{\gamma_{m o}}$ where,
$\mathrm{b}_{1}=$ Stiff bearing length
$\mathrm{n}_{\mathrm{c}}=$ length obtained by dispersion through the flange to the web function at a slope
1: 2.5 to the plane of flange .
$\Rightarrow \mathrm{n}_{\mathrm{c}}=2.5 \mathrm{t}_{\mathrm{f}}$
$f_{y w}=$ Yield stress of the web

## 8. WEB BUCKLING

The web buckling strength of support will be
$F_{w b}=B . t_{w} . f_{c d}$
$\mathrm{f}_{\mathrm{cd}}=$ Allowable compressive stress
$f_{c d}=$ Allowable compressive stress corresponding to the announced web strut according to buckling curve 'C
$B=$ length of left portion of the bearing plus additional length is given dispersion $45^{\circ}$ to the level of NA.


Hence as per IS 800-2007, effective web buckling is to be found based on the crosssection of the web.

At support, $A=\left(b_{1}+\frac{h}{2}\right) t_{w}$
Web buckling strength $=F_{c d w}=\left(b_{1}+\frac{h}{2}\right) t_{w} f_{c}$
And $f_{c}$ is the allowable compressive stress corresponding to the assumed web column of effective length $=0.7 \mathrm{~d}$, where d is the web height.
At concentrated load

$$
\begin{aligned}
& F_{\mathrm{cdw}}=\left(\mathrm{b}_{1}+2+\frac{h}{2}\right) \mathrm{t}_{\mathrm{w}} \mathrm{f}_{\mathrm{c}} \\
& \mathrm{~F}_{\mathrm{cdw}}=\left(\mathrm{b}_{1}+\mathrm{h}\right) \mathrm{t}_{\mathrm{w}} \times \mathrm{f}_{\mathrm{c}}
\end{aligned}
$$

9. DESIGN OF BUILT-UP BEAMS: If a single beam section could not withstand the applied load, then we use built-up beams. The factored moment carrying capacity can be expressed as
$M_{u}=\left(\frac{f_{y}}{1.1}\right) \cdot z_{p}$
$\left(Z_{p}\right)_{\text {required }}=\frac{M_{u}}{\left(\frac{f_{v}}{1.1}\right)}$
where, $M_{u}=$ factored bending moment.
If cover plates are provided to increase moment carrying capacity, then
$\left(Z_{p}\right)_{\text {req }}=\left(Z_{p}\right)_{\text {beam }}+\left(Z_{p}\right)_{\text {plates }}$
$Z_{p}=A_{p} \cdot d$
Here, $A_{p}=$ area of plate required on each side
$A_{p}=\frac{\left(Z_{p}\right)_{\text {req }}-\left(Z_{p}\right)_{\text {beam }}}{d}$ (in LSM), $\quad A_{p}=\frac{Z_{\text {req }}-Z_{\text {beam }}}{d}$ (in WSM)

## 10. STEPS FOR BEAM DESIGN (LIMIT STATE DESIGN):

## A. Design of laterally supported beams

1. Calculate the factored load and the maximum bending moment and shear force
2. Obtain the plastic section modulus required
$Z_{\text {req }}=\frac{\left(M \times \gamma_{\text {mo }}\right)}{f_{y}}$

Select a suitable section for the beam-ISLB, ISMB, ISWB or suitable built-up sections (doubly symmetric only). (Doubly symmetric, singly symmetric and asymmetricprocedures are different)
3. Check for section classification such as plastic, compact, semi-compact or slender. Most of the sections are either plastic or compact. Flange and web criteria.

$$
\frac{\mathrm{d}}{\mathrm{t}_{\mathrm{w}}}, \frac{\mathrm{~b}}{\mathrm{t}_{\mathrm{f}}} \varepsilon=\sqrt{\frac{250}{\mathrm{fy}}=1}
$$

4. Calculate the design shear for the web and is given by

$$
=V_{d p}=\frac{(A v \times f y)}{\sqrt{3} \times \gamma_{m o}}>V_{d} \text { and } V<0.6 V_{d}
$$

5. Calculate the design bending moment or moment resisted by the section (for plastic and compact) $M_{d}=\beta_{p} Z_{p f y} / Y_{\text {mo }}$
6. Check for buckling
7. Check for crippling or bearing
8. Check for deflection
B. Design of laterally unsupported beams
9. Calculate the factored load and the maximum bending moment, and shear force.
10. Design of ISB is by trial and error method. The design bending stress is significantly less, which is to be assumed to start with. Assume slenderness ratio and Wu and get the corresponding critical bending stress and hence the corresponding design bending stress.
11. Determine the required plastic section modulus and select the section.
12. Determine the actual design bending stress of this selected section knowing in slenderness ratio, which should be greater than that assumed previously. Otherwise, revise the section.
13. Check for shear, buckling, crippling, and deflection should be done. Design bending strength can be calculated as per IS 800:2007.
14. Gantry Girder:A typical arrangement of a crane system is shown in Figure.


Different forms of gantry girder are as follows:

(a)

(d)

(c)

(d)
11.1. LOADS ON GANTRY GIRDER: These are subjected to three forces:
(i) The reaction from the crane girder, acting vertically downwards.
(ii) The longitudinal thrust, due to starting or stopping of crane, acting in the longitudinal direction.
(iii) The lateral thrust, due to starting or stopping of the crab acting horizontally, normal to the gantry girder.


Additional load on the structure due to Impact load:

| Type of Load | Additional load |
| :--- | :--- |
| (a) Vertical forces transferred to the rails <br> (i) For electrically overhead cranes <br> (ii) For hand-operated cranes | $25 \%$ of maximum static wheel load <br> $10 \%$ of maximum static wheel load |
| (b)Horizontal forces transverse to the rails <br> (i) For electrically overhead cranes | $10 \%$ of the weight of the crab and the <br> weight lifted on the crane <br> $5 \%$ of the weight of the crab and the <br> weight lifted on the crane |
| (ii) For hand-operated cranes | $5 \%$ of the static wheel load |
| (c)Horizontal forces along the rails |  |

11.2. PERMISSIBLE DEFLECTION: The vertical deflection of a gantry girder should not exceed the values specified below:
(i) Where the cranes are manually operated $-\frac{L}{500}$
(ii) Where the cranes are travelling overhead and operated electrically up to 500 kN $\frac{L}{750}$
(iii) Where the cranes are travelling overhead and operated electrically over 500 kN $\frac{L}{1000}$
(iv) Other moving loads, such as charging cars, etc.- $\frac{L}{600}$

Where $L=$ span of gantry girder
12. PLATE GIRDER: It is a flexural member designed for bending, which is used when I section are not sufficient to support the anticipated load.

### 12.1. Elements of plate girder

(i) Web plate
(ii) Flange angles with or without flange cover plate
(iii) Stiffeners
(iv) Splices


### 12.2. General Considerations

- The flange angles must be unequal angles with longer legs connected to flange plates to get more bearing area.
- The size of the flange angle should be such that they should form at least $\frac{1}{3}$ of total flange area.
- The stiffeners are designed on the basis of the following condition.
(i) If $\frac{d_{1}}{t_{\omega}}>90$, vertical stiffeners are provided to prevent buckling of the web due to diagonal compression. Vertical stiffeners are provided under point loads. Vertical stiffeners are provided under point loads. These stiffeners are called load-bearing stiffeners. They prevent \& web crippling or web crimpling, or local buckling of the web.
(ii) If $\frac{d_{1}}{t_{\omega}}>200$, horizontal stiffeners are longitudinal, stiffeners are provided above NA at a distance of $0.2 \mathrm{~d}_{\mathrm{w}}$ (depth of web plate) from the compression flange. They prevent buckling of the web due to bending compressive stress.
(iii) If $\frac{d_{1}}{t_{w}}>250$, additional horizontal stiffeners are provided at the neutral axis. These stiffeners prevent buckling of web between vertical stiffeners due to shear force.
(iv) If $\frac{d_{1}}{t_{w}}>400$, then section must be redesigned.
- At the supports, to prevent bending of flange plate and buckling of web plate due to support reaction and bearing stiffeners are provided. If the bearing
stiffeners are the only means of providing torsional restraint, then they are also called torsional stiffeners. The other means of providing torsional restraint are extending the plate girder into the wall or using a small piece of angle (web cleats).


## - Design of end bearing stiffeners

End bearing stiffeners are designed as an imaginary column with both ends fixed whose effective length is $0.71_{1}$.

Here,
$I_{1}=$ length of bearing stiffener between flange angles.


Since bearing stiffeners are designed as columns, they should be vertical and should not be jogged (jogging means bending, i.e., they should not be bent to touch the web plate), and the gap between bearing stiffeners and web plate must be filled by filler plate. Since they act as a column, the end bearing stiffener should have a tight bearing between the flange angles.

## - Design of vertical stiffeners

Vertical stiffeners are used to prevent buckling of the web due to diagonal compression. These stiffeners are not designed as columns. So, the ends of stiffeners need not have a tight bearing with the flange angle.


Since the vertical stiffeners are not designed as column they can be joggled (i.e. they can be bent to touch the web plate so that filler plates need not be provided) Dimensions of vertical stiffeners are as follows:
(i) Spacing of vertical stiffeners (s)

$$
\mathrm{s} \nless 0.33 \mathrm{~d}_{1} \text { and } \mathrm{S} \nless 1.5 \mathrm{~d}_{1}
$$

here, $\quad d_{1}=$ clear depth of web between toes of flange angles.
(ii) Lesser clear panel dimension $\ngtr 180 \mathrm{t}_{\mathrm{w}}$
greater clear panel dimension $\ngtr 270 \mathrm{t}_{\mathrm{w}}$
(iii) So, the spacing of vertical stiffeners increases, permissible shear stress in the web decreases due to buckling possibility of the web.

## GATE/ESE

Civil Engineering
Engineering Hydrology

## Important Formula Notes

## IMPORTANT FORMULAS ON ENGINEERING HYDROLOGY

## CHAPTER-1-PRECIPITATION

## i. INDEX OF WETNESS

$$
\text { Index of wetness }=\frac{\text { Rainfall in a year }}{\text { Average annual rainfall }} \times 100
$$

If the index of wetness is $100 \%$, it indicates a normal year. If it is greater than $100 \%$, it is called a good year, and if it is less than $100 \%$, it is called a bad year.

## ii. Aridity index

Aridity index $=\frac{P E T-A E T}{P E T} \times 100$
Here, PET = potential evapotranspiration.
AET = actual evapotranspiration.

| Aridity index \%) | Condition |
| :---: | :---: |
| $0-25$ | Mild |
| $25-50$ | Moderate |
| $>50$ | Severe |

## iii. Optimum Number of rain gauges

$$
N=\left(\frac{C_{v}}{\epsilon}\right)^{2}
$$

$C_{v}=$ coefficient of variation.
$\epsilon=$ allowable percentage error.
For a given number of rain gauge standard error $\epsilon_{s}$

$$
\epsilon_{s}=\frac{C_{v}}{\sqrt{n}}
$$

## iv. ESTIMATION OF MISSING RAINFALL DATA

Let $N_{1}, N_{2}, N_{3}, \ldots$ and $N_{x}$ be the normal precipitation values for station' 1 to $\mathrm{m}^{\prime}$, and ' $x$ ' Normal precipitation is an average rainfall value for a day. Let $P_{1}, P_{2}, P_{3}, \ldots$ and $P_{x}$ be the rainfall for station '1 to $m$ ', and ' $\mathrm{Px}^{\prime}$ ' is the rainfall of station x .

Case 1: when $N_{1}, N_{2}, \ldots N_{m}$ differs from $N_{x}$ by less than $10 \%$, the Value of $P_{x}$ is given as follows

$$
P_{x}=\frac{P_{1}+P_{2}+P_{3}+P_{3}+\ldots+P_{m}}{m}
$$

Case 2: when one or more of $N_{1}, N_{2}, \ldots N_{m}$ differs from $N_{x}$ by $10 \%$ or more, the Value of $P_{x}$ is calculated by:

$$
P_{x}=\frac{N_{x}}{m}\left[\frac{P_{1}}{N_{1}}+\frac{P_{2}}{N_{2}}+\ldots \ldots+\frac{P_{m}}{N_{m}}\right]
$$

## v. AVERAGE PRECIPITATION/RAINFALL

## a. ArithmeticMean/Average Method

This method is suitable if rainfall is uniformly distributed and the area is not very large.

$$
P_{\text {avg }}=\frac{P_{1}+P_{2}+P_{3}+\ldots .+P_{n}}{n}
$$

This method does not give very good results and hence is not used very frequently. Any station outside the area of consideration is not taken into account in this method.

## b. Thiessen polygon/Weighted Area Method

In this method, the rainfall recorded at each station is given a weightage based on an area closest to the station. That is why this method is also known as the weightage area method.

$$
\begin{aligned}
P_{\text {avg }}= & \frac{P_{1} A_{1}+P_{2} A_{2}+\ldots+P_{n} A_{n}}{A_{1}+A_{3}+\ldots+A_{n}} \\
& \Rightarrow P_{\text {avg }}=\frac{\sum_{i=1}^{n} P_{i} A_{i}}{A}
\end{aligned}
$$

The ratio $A_{i} / A$ is called the weightage factor for each station.
This method of finding average rainfall is suitable when the area is large, and rainfall is non-uniformly distributed. This method is superior to the arithmetical mean method.

## c. Isohyetal Method

An isohyet is a line joining all the points having the same value of rainfall, and isohyetal maps are the one which shows contours of equal rainfall magnitude.
In an isohyetal method, it is assumed that the precipitation in areas between isohyetal lines is equal to the mean of the precipitation of at isohyetal lines.

Mathematically, various following cases are possible.

## Case 1:



Case 2: If isohyets outside the considered area are given.


$$
P_{\text {avg }}=\frac{\left(\frac{P_{0}+P_{1}}{2}\right) A_{0}+\left(\frac{P_{1}+P_{2}}{2}\right) A_{1}+\left(\frac{P_{2}+P_{3}}{2}\right) A_{3}+\left(\frac{P_{n-1}+P_{n}}{2}\right) A_{n-1}+\left(\frac{P_{n}+P_{n+1}}{2}\right) A}{A_{0}+A_{1}+A_{2} \ldots .+A_{n-1}+A_{n}}
$$

Case 3:


$$
P_{\text {avg }}=\frac{\left(\frac{P_{1}+P_{2}}{2}\right) A_{1}+\left(\frac{P_{2}+P_{3}}{2}\right) A_{2}+\ldots+\left(\frac{P_{n-1}+P_{n}}{2}\right) A_{n-1}+P_{n+1} A_{n}}{A_{1}+A_{2}+A_{3}+\ldots+A_{n-1}+A_{n}}
$$

## CHAPTER-2-EVAPORATION

i. Measurement of evaporation

## a. Experimental method

Lake Evaporation $=C_{P} \times$ Pan evaporation

| S.No. | Type of Pan | $C_{P}$ |
| :---: | :---: | :--- |
| 1. | Class A | 0.7 |


| 2. | Indian standard | 0.8 |
| :---: | :---: | :---: |
| 3. | Colorado | 0.78 |

b. Empirical Method

$$
E=k \times m \times\left(e_{w}-e_{a}\right) \times\left(1+\frac{V_{9}}{16}\right)
$$

$E=$ Rate of evaporation per day.
$\mathrm{K}_{\mathrm{m}}=$ constant, which depends on the size of the water body.
$\mathrm{e}_{\mathrm{w}}=$ Saturated vapour pressure in mm of mercury.
$e_{a}=$ vapour pressure of air $\left(e_{a}\right)$ in mm of mercury.
$\mathrm{V}_{9}=$ Mean monthly wind velocity in $\mathrm{km} / \mathrm{hr}$ at the height of about 9 m from the ground surface.

## ii. Transpiration

> It is the process by which water leaves the body of a living plant and reaches the atmosphere as water vapour.
> It can be measured by an instrument called as phytometer
> Following are the methods to find out the evapotranspiration.

- Lysimeter
- Field plot
- Penman's equation
- Blaney Criddle equation.


## CHAPTER-3-STREAMFLOW MEASUREMENT

## i. MEASUREMENT OF VELOCITY

## a. Float

In this method, a very simple float device is used, which flows along the river surface.


Mathematically, $V=\frac{L}{t_{1}}$

## b. Current meter

$$
\mathrm{V}=\mathrm{aN}+\mathrm{b}
$$

$\mathrm{N}_{\mathrm{s}}=$ Number of revolutions per sec.
Here, a and b are called characteristic constants.
Note:
Calibration of the current meter is done by using a 'Towing tank'.
Sounding weight: It is a standard weight attached to a current meter to keep it at a fixed location.
$\qquad$


To reduce the drag force, these are streamlined in shape. The value of this sounding weight is given as

$$
W=50 \bar{v} y
$$

Here, w = weight in ' N '
$\bar{v}=$ Average velocity in ' $\mathrm{m} / \mathrm{s}^{\prime}$
$Y=$ depth in 'cm'.
Average velocity can be obtained as follows
Case 1: For deep water bodies, k/a two-point formula.

$$
\mathrm{V}_{\text {avg }}=\frac{\mathrm{V}_{0.2 \mathrm{y}}+\mathrm{V}_{0.8 y}}{2}
$$

Case 2: For shallow k/a one point formula

$$
\mathrm{V}_{\mathrm{avg}}=\mathrm{V}_{0.6 \mathrm{y}}
$$

Case 3: For flashy river and flood-like situation.

$$
\mathrm{V}_{\mathrm{avg}}=\mathrm{k} \mathrm{v}_{\mathrm{s}}
$$

Here, $\mathrm{k}=0.85-0.95$
$\mathrm{V}_{\mathrm{s}}=$ Surface velocity
ii. MEASUREMENT OF DISCHARGE
a. Area velocity method


Here, $A_{1}=\bar{W}_{1} y_{1}$

$$
A_{2}=\bar{w}_{2} y_{2}
$$

$$
A_{n}=\bar{W}_{n} y_{2}
$$

Here,

$$
\begin{gathered}
\bar{w}_{1}=\frac{\left(w_{1}+\frac{w_{2}}{2}\right)^{2}}{2 w_{1}} \\
\bar{w}_{3}=\frac{w_{3}+w_{4}}{2}
\end{gathered}
$$

$$
\text { Similarly, } \bar{w}_{n-1}=\frac{\left(\mathrm{w}_{\mathrm{n}}+\frac{\mathrm{w}_{\mathrm{n}-1}}{2}\right)^{2}}{2 \mathrm{w}_{\mathrm{n}}}
$$

b. Dilution method:


Here C = background concentration.
Mathematically,

$$
\mathrm{CQ}+\mathrm{C}^{\prime} \mathrm{Q}^{\prime}=\mathrm{C}^{\prime}\left(\mathrm{Q}+\mathrm{Q}^{\prime}\right)
$$

c. Ultrasonic Method:


$$
\begin{gathered}
v=\frac{\mathrm{l}}{2 \cos \theta}\left(\frac{1}{\mathrm{t}_{1}}-\frac{1}{\mathrm{t}}\right) \\
\mathrm{Q}=\mathrm{A} \times \mathrm{V}
\end{gathered}
$$

d. Moving Boat Method:

$\Delta Q_{1}=A_{1} v_{f}=\left[v_{b} \Delta t_{1}\left(\frac{0+y_{1}}{2}\right)\right] v_{f}=v_{b 0} v_{f} \Delta t_{1} \frac{y_{1}}{2}$
$=V_{r}^{2} \sin \theta \cos \theta \Delta t_{1} \frac{y_{1}}{2}$
Similarly,
$\Delta Q_{2}=V_{r}^{2} \sin \theta \Delta t_{2}\left(\frac{\mathrm{y}_{1}+\mathrm{y}_{2}}{2}\right)$
$\Delta Q_{3}=V_{r}^{2} \sin \theta \Delta t_{3}\left(\frac{y_{2}+y_{3}}{2}\right)$

## e. Slope Area Method :


$h_{f}=\left(h_{1}-h_{2}\right)+\left(\frac{V_{1}^{2}-V_{2}^{2}}{2 g}\right)-h_{e}$
$\left[\begin{array}{l}h_{f}=\text { frictional } \\ h_{c}=\text { eddies }\end{array}\right]$
Through experiments, it has been found that eddy head loss

$$
h_{e}=K_{e}\left|\frac{V_{1}^{2}-V_{2}^{2}}{2 g}\right|
$$

Where $\mathrm{K}_{\mathrm{e}}=$ eddy head loss coefficient.

$$
\begin{aligned}
& -\quad Q=k \sqrt{\frac{h_{f}}{L}} \\
& K=\left(K_{1} \times K_{2} \times K_{3} \times K_{4} \times K_{5} \ldots \ldots \times K_{n}\right)^{1 / n}
\end{aligned}
$$

## CHAPTER-4- INFILTRATION

## i. INFILTRATION CAPACITY


$F_{t}=F_{f}+\left(F_{i}-F_{f}\right) e^{(-k t)}$
$\mathrm{F}_{\mathrm{t}}=$ Infiltration rate or capacity at a time ' t '.
$F_{f}=$ Final infiltration rate or capacity.
$\mathrm{F}_{\mathrm{i}}=$ Initial infiltration rate or capacity.
$\mathrm{K}=$ Decay constant ( $\mathrm{T}^{-1}$ or /s or / hr)

## ii. INFILTRATION INDICES

a. $\phi$ - index

The $\phi$ - index is the average rainfall above which the rainfall volume is equal to runoff volume. The $\phi$ Index is derived from the rainfall hyetograph with the knowledge of the resulting runoff volume. The initial losses are also considered infiltration


## b. w-index

This is the average infiltration rate during the entire duration of rainfall. In calculatingthe w-index, the initial losses are separated from total abstractions to refine the $\phi$ Index.

Mathematically it is defined as
$w-$ index $=\frac{P-R-t_{a}}{T_{c}}$
Here, $\mathrm{P}=$ total storm precipitation (cm)

$$
R=\text { total run off }(\mathrm{cm})
$$

$t_{a}=$ minor loss in total duration
$T_{c}=$ duration of the rainfall excess i.e. total.

## CHAPTER-5-RUN-OFF \& HYDROGRAPH

| 1. Shape of Catchment | Hydrograph |
| :---: | :---: |



## i. DETERMINATION OF DIRECT RUNOFF HYDROGRAPH (DRH)

Hydrograph having base flow is known as flood hydrograph or storm hydrograph.
Flood
hydrograph $\xrightarrow{\text {-Baseflow }}$ Direct runoff hydrograph


Direct runoff $=$ depth

$$
\mathrm{n}=(\text { Direct runoff volume) } /(\text { catchment area })
$$

So, Mathematically it can express as

$$
\mathrm{n}=\frac{0.36 \sum \mathrm{ot}}{\mathrm{~A}}
$$

Here, n is in cm
A is in km ${ }^{2}$
t is in hr
O is in $\mathrm{m}^{3} / \mathrm{sec}$

## ii. SYNTHETIC UNIT HYDROGRAPH

Snyder selected three parameters for the development of SUH. These parameters relate to the catchment characteristics
(i) Basin time width T
(ii) Peak discharge QP
(iii) Lag time,i.e. basin lag time tp. (Snyder defined lag time as the time interval from mid pt. of rainfall to the peak of UH (instead of centroid).


He proposed the following three equations for these three parameters
Lag time, $\mathrm{t}_{\mathrm{p}}=\mathrm{C}_{\mathrm{t}}\left(\mathrm{LL}_{\text {ca }}\right) 0.3$
Basin time width $T=\left(72+3 t_{p}\right)$

Peak discharge $Q_{p}=\frac{2.78 C_{p} A}{t_{p}}$
Where tp is in hr
Ct is a coefficient reflecting slope, land use, and associated storage characteristics of the basin. Its value varies between 1.35 to 1.65 , the average being 1.5
$\mathrm{L}=$ basin length measured along the watercourse from the basin divide to the gauging station in km.
Lca $=$ Distance of centroid of the catchment from the gauging point (in km)
T is in hr
$Q_{p}$ is in $\mathrm{m}^{3} / \mathrm{s}$
A = Catchment area in $\mathrm{km}^{2}$


Fig. 6.15
$\mathrm{CP}=$ a regional constant having value between 0.56 to 0.69
Synder used the standard duration $\operatorname{tr}$ (or D-hr) in hr for unit hydrograph
$t_{r}=D_{h r}=\frac{t_{p}}{5.5}$
If a synthetic unit hydrograph of other duration then $D^{\prime} h r$ is required, then lag time, $t_{p r}$ ' is given by
$\mathrm{t}_{\mathrm{pr}}=\mathrm{t}_{\mathrm{p}}=\frac{\mathrm{D}^{\prime}-\mathrm{t}_{\mathrm{r}}}{4}$
To plot the smooth synthetic unit hydrograph, US army crops of engineering gave the width of SUH as
$W_{50}=\frac{5.87}{\left(\frac{Q_{p}}{A}\right)^{1.08}}$
$W_{75}=\frac{3.35}{\left(\frac{Q_{p}}{A}\right)^{1.08}}=\frac{W_{50}}{1.75}$
$W_{50}$ and $W_{75}$ are the widths of synthetic unit hydrograph in hr at $50 \%$ and $75 \%$ of $Q_{p}$, respectively. Where $Q_{p}$ is in $\mathrm{m}^{3} / \mathrm{s}$ and $A$ is an area of the catchment in $\mathrm{km}^{2}$.


Fig. 6.17: Elements of a synthetic unit hydrograph

## CHAPTER-6- GROUNDWATER HYDROLOGY



## i. STEADY CONFINED FLOW (FULLY PENETRATING WELL)


A. Theims Theory
$\mathrm{Q}=\frac{2 \pi \mathrm{~KB}\left(\mathrm{~h}_{2}-\mathrm{h}_{1}\right)}{\ln \frac{\mathrm{r}_{2}}{\mathrm{r}_{1}}}$
$h_{1}+s_{1}=h_{2}+s_{2}$
$Q=$ Rate of flow
$h_{1}=$ height of water table in $1^{\text {st }}$ well
$h_{2}=$ height of water table in $2^{\text {nd }}$ well
$S_{1}=$ drawdown in $1^{\text {st }}$ well
$\mathrm{S}_{2}=$ drawdown in $2^{\text {nd }}$ well
$r_{2}, r_{1}=$ radius

## B. Duipit's theory

Further, at the edge of the zone of influence, $s=0, r_{2}=R$ and $h_{2}=H$;at the well wall $r_{1}=r_{w}, h_{1}=h_{w}$ and $s_{1}=s_{w}$. Hence
$\mathrm{Q}=\frac{2 \pi \mathrm{KBS}_{\mathrm{w}}}{\ln \frac{\mathrm{R}}{\mathrm{r}_{\mathrm{w}}}}$
This is called the Dupit's formula.

## ii. STEADY UNCONFINED FLOW



Fig. 10.15. Radial flow to well in an unconfined aquifer
A. Theim's Theory
$\mathrm{Q}=\frac{\pi \mathrm{k}\left(\mathrm{h}_{2}^{2}-\mathrm{h}_{1}^{2}\right)}{\ln \left(\frac{\mathrm{r}_{2}}{\mathrm{r}_{1}}\right)}$
B. Duipit's theory
$\mathrm{Q}=\frac{\pi \mathrm{k}\left(\mathrm{H}^{2}-\mathrm{h}_{\mathrm{w}}^{2}\right)}{\ln \frac{\mathrm{R}}{\mathrm{r}_{\mathrm{w}}}}$
$R=3000 \times S \times \sqrt{K}$
$h_{w}=$ depth of water in the pumping well of radius $r_{w}$.

## CHAPTER-7-RESERVOIR CAPACITY \& FLOOD ROUTING

i. RATIONAL METHOD

This method is suitable for small size catchments whose area is less than $50 \mathrm{~km}^{2}$. Mathematically it is expressed as
$\mathrm{Q}_{\mathrm{P}}=\frac{1}{36} \mathrm{kp}_{\mathrm{c}} \mathrm{A}$
Here, $\mathrm{p}_{\mathrm{c}}=$ critical rainfall intensity in 'cm/hr'
$k=$ runoff coefficient
A = Area in hectares
It is applicable only if rainfall duration is greater than or equal to the time of concentration.

## ii. EMPIRICAL FORMULAE

The empirical formulae used for estimating the flood peak are essentially regional formulae based on statistical correlation of the observed peak and important catchment properties. These equations are given as follows.

## (a) Dicken's Equation

This equation is applicable for North and central India. Mathematically, it is given as

$$
Q_{P}=C_{D} A^{3 / 4}
$$

Here,
$Q_{p}=m^{3} / \mathrm{s}$
A = Area in km ${ }^{2}$
$C_{D}=$ dicken's constant (6 to 30)

## (b) Ryve's Equation

This equation is applicable for southern parts of India. Mathematically it is given as

$$
Q_{P}=C_{R} A^{2 / 3}
$$

Here, $C_{R}=$ ryve's constant.

## (c) Inglis Equation

This equation is applicable for WesternGhats and Maharashtra region. Mathematically it is given as,

$$
\mathrm{Q}_{\mathrm{P}}=\frac{124 \mathrm{~A}}{\sqrt{\mathrm{~A}+10.4}} \approx 123 \sqrt{\mathrm{~A}}
$$

## iii. STATICAL PROBABILITY METHOD

This method is suitable if a large number of data is given, and it is required to find the peak value of discharge for any given period or return period.

## Return Period

The return period is calculated for each event using Weibull's formula.
$\mathrm{T}_{\mathrm{v}}=\frac{\mathrm{n}+1}{\mathrm{~m}}$
$\mathrm{T}_{\mathrm{r}}=$ return period in yr .
$\mathrm{m}=$ order no.
$\mathrm{n}=\mathrm{no}$. for yr of records
return period represents the average no. of years within which a given event will be equalled or exceeded.

Probability of exceedance $=\frac{1}{T_{r}}=P$

## RISK AND RELIABILITY

The probability of a particular event happen exactly 'r' times out of ' $n$ ' trials is given as
${ }^{n} c_{r} p^{r}(1-p)^{n-r}$
$\mathrm{p}=$ probability of exceedance.

## Reliability

This is the probability that a particular flood or rainfall is never equalled or exceeded ( $r=0$ ) in a period of ' $n$ ' years, Mathematically
$\Rightarrow$ Reliability $={ }^{n} c_{0} p^{0}(1-p)^{n-0}$
$\Rightarrow$ Reliability $=(1-p)^{n}$

## Risk

This is a probability that a particular flood or rainfall is equalled or exceeded at least once in a period of ' $n$ ' years.
$P$ (at least once) $=1-p($ never happen $)=1$ - Reliability
The most commonly used distributions are :
(a) Gumbel's distribution
(b) Log Pearson Type III distribution.

## Gumbel's Method:

As per Gumbel's method
$X_{T}=\bar{X}+K \cdot \sigma_{n-1}$
where $\mathrm{X}_{\mathrm{T}}=$ value of variate (i.e. flood) with a return period of T
$\bar{X}=$ Mean Value for variate $\frac{\Sigma X}{n}=\bar{X}$ (from annual series)
$n=$ no. of $y r s$ of record
$\sigma_{n-1}=$ Standard deviation of the sample of size $n$
$\sigma_{n-1}=\sqrt{\frac{\sum(x-\bar{x})^{2}}{n-1}}$
$\mathrm{K}=$ Frequency factor
$K=\frac{\mathrm{Y}_{\mathrm{T}}-\overline{\mathrm{y}}_{\mathrm{n}}}{\mathrm{S}_{\mathrm{n}}}$
$\mathrm{y}_{\mathrm{T}}=$ reduced variate
$\frac{1}{\mathrm{t}}=(-)\left[\ln \ln \frac{\mathrm{T}}{\mathrm{T}-1}\right]$
$\bar{y}_{\mathrm{n}}=$ mean of reduced variate
$\mathrm{S}_{\mathrm{n}}=$ Standard deviation of reduced variate.
$\bar{y}_{\mathrm{n}}$ And $\mathrm{S}_{\mathrm{n}}$ function of n (no. of yr . of record).
However if, $n$ is large (generally >200)
$y_{n} \rightarrow 0.577$
$\mathrm{S}_{\mathrm{n}} \rightarrow 1.2825$
[Normal for $\mathrm{n}>50$ also some time we use $\mathrm{y}_{\mathrm{n}}=0.577$. $\mathrm{S}_{\mathrm{n}}=1.2825$ without much error]
iv. CONFIDENCE LIMIT

Confidence interval indicates the limits about the calculated value between which the true value can be said to lie with a specific probability based on sampling errors only. For confidence probability ' $\alpha$ ', the confidence interval of variate $X_{T}$ is bounded by values $X_{1}$ and $X_{2}$ given by

$$
X_{1 / 2}=X_{T} \pm f(\alpha) \cdot S_{C}
$$

where $\mathrm{f}(\alpha)=$ function of confidence probability ' a '. It can be found using the following table

| A is percent | 50 | 68 | 80 | 90 | 95 | 99 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(\alpha)$ | 0.674 | 1.0 | 1.282 | 1.645 | 1.96 | 2.58 |

$\mathrm{S}_{\mathrm{c}}=$ Probable error $=\mathrm{b} \frac{\sigma_{\mathrm{N}-1}}{\sqrt{\mathrm{n}}}$
$\mathrm{b}=\sqrt{1+1.3 \mathrm{~K}+1.1 \mathrm{~K}^{2}}$
$K=\frac{\mathrm{Y}_{\mathrm{T}}-\mathrm{y}_{\mathrm{n}}}{\mathrm{S}_{\mathrm{n}}}$
$y_{T}=-\ln \ln \frac{T}{T-1}$
$\mathrm{n}=$ Sample size
T = Return period
$\sigma_{n-1}=$ Standard deviation of sample

## v. Rainfall run off correlation

$\mathrm{R}=\mathrm{ap}+\mathrm{b}$
Here,
$a=\frac{n \sum P R-\sum P \sum R}{n \sum P^{2}-\left(\sum P\right)^{2}}$
$\mathrm{b}=\frac{\sum \mathrm{R}-\mathrm{a} \sum \mathrm{P}}{\mathrm{n}}$

## vi. Flood Routing



This reduction in peak is called attenuation.
The time difference between the two peaks is called lag.

## a. Prism storage

It is the volume that would exist if the uniform flow occurred at the downstream flow

## b. Wedge storage

It is the wedge-like volume formed between the actual surface profile and the top surface of prism storage.
c. Muskingum method
$\Delta \mathrm{S}=(\mathrm{I}-\mathrm{Q}) \Delta \mathrm{t}$
$\Delta S=$ Change in storage in time $\Delta t$
$\mathrm{I}=$ Avg. inflow rate over the time $\Delta \mathrm{t}$
$\mathrm{Q}=\mathrm{Avg}$ outflow rate over the time $\Delta \mathrm{t}$
$\Delta \mathrm{S}=\left(\frac{\mathrm{I}_{2}+\mathrm{I}_{1}}{2}\right) \Delta \mathrm{t}-\left(\frac{\mathrm{Q}_{2}+\mathrm{Q}_{1}}{2}\right) \Delta \mathrm{t}$
$\mathrm{S}_{1}=\mathrm{K}\left[\mathrm{XI}_{1}+(1-\mathrm{x}) \mathrm{Q}_{1}\right]$
$S_{2}=K\left[X_{2}+(1-x) Q_{2}\right]$
$\mathrm{S}_{2}-\mathrm{S}_{1}=\mathrm{K}\left[\mathrm{x}\left(\mathrm{I}_{2}-\mathrm{I}_{1}\right)+(1-\mathrm{x})\left(\mathrm{Q}_{2}-\mathrm{Q}_{1}\right)\right]$
$\mathrm{Q}_{1}=\mathrm{I}_{1}$
$\mathrm{Q}_{2}=\mathrm{C}_{0} \mathrm{I}_{2}+\mathrm{C}_{1} \mathrm{I}_{1}+\mathrm{C}_{2} \mathrm{Q}_{1}$
$Q_{n}=C_{0} I_{n}+C_{1} I_{n-1}+C_{2} Q_{n-1}$
$C_{0}=\frac{-K x+0.5 \Delta t}{K-K x+0.5 \Delta t}$
$C_{1}=\frac{K x+0.5 \Delta t}{K-K x+0.5 \Delta t}$
$C_{2}=\frac{K-K x-0.5 \Delta t}{K-K x+0.5 \Delta t}$
$C_{0}+C_{1}+C_{2}=1.0$
For the results routing interval, $\Delta t$ should be so chosen that $K>\Delta t>2 K x$
The following steps are used for channel routing using the Muskingum method.
(i) Knowing $K$ and $x$, select an appropriate value of $\Delta t[K>\Delta t>2 K x]$
(ii) Calculate $\mathrm{C}_{0}, \mathrm{C}_{1}$ and $\mathrm{C}_{2}$
(iii) Starting from the initial conditions $\mathrm{I}_{1}, \mathrm{Q}_{1}$ and known $\mathrm{I}_{2}$ at the end of the first time step $\Delta t$, calculate $Q_{2}$ by eq. (C)
(iv) The outflow calculated in step (iii) becomes the known initial outflow for the next step. Repeat the calculations for the entire-inflow hydrograph.

## GATE/ESE

Civil Engineering
Engineering Mechanics

## Important Formula Notes

## IMPORTANT FORMULAS ON ENGINEERING MECHANICS

## CHAPTER-1- INTRODUCTION \& SYSTEM OF FORCES

1. FORCE

An agent which produces or tends to produce, destroy or tends to destroy motion. It is a push or a pull.

SI Unit: Newton (N)

## a) System of forces

When two or more than two forces of different magnitude and direction act upon a body, they constitute a system of forces.

## i. Concurrent forces

Two or more forces that act at the same point are called concurrent forces. Concurrent forces need not have the same direction. They act at the same point.
ii. Collinear forces

If concurrent forces have the same direction, they are collinear forces.

## iii. Coplanar forces

Two or more forces whose directed arrows lie in the same plane are called coplanar forces.

- Since two concurrent forces always lie in a common plane, they are always coplanar.
- Three or more concurrent forces are not necessarily coplanar.


## 2. PRINCIPLE OF TRANSMISSIBILITY

> The state of rest or motion of a rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the applied forces.


## 3. RESULTANT OF FORCE

> It is possible to find a single force that produces the same effect as many forces acting on a body.
> The single force is called resultant force, and the process of finding out the resultant force is called the composition of forces.
> The reverse of the composition of forces is called the resolution of force.

## 4. PARALLELOGRAM LAW OF FORCES

> If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram, which passes through their point of intersection, their resultant force is represented, both in magnitude and in direction, by the diagonal of the parallelogram drawn through their point of intersection.
> Let two forces, P and Q , be represented by OP and OQ , respectively, on two sides of the parallelogram.
> Now the parallelogram OPRQ is completed, and diagonal OR represents the resultant.

a. ANALYTICAL METHOD
$\Sigma \mathrm{H}=$ algebraic sum of all horizontal components.
$\Sigma \mathrm{V}=$ algebraic sum of all vertical components.


$$
\mathrm{R}^{2}=\sqrt{\sum \mathrm{H}^{2}+\sum \mathrm{V}^{2}}
$$

$$
\tan \phi=\frac{\sum \mathrm{V}}{\sum \mathrm{H}}
$$

## 5. TRIANGLE LAW OF FORCES

Suppose two forces acting simultaneously on a body are represented in magnitude and direction by two sides of a triangle in order. In that case, the third side will represent
the results of the two forces in the direction and magnitude taken in the opposite order.

## 6. POLYGON LAW OF FORCES

$>$ When the forces acting on a body are more than two, the triangle law can be extended to polygon law. Polygon law states that if many coplanar concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, their resultant can be represented by the closing side of the polygon in magnitude and direction in the opposite order.
$>$ Let forces $F_{1}, F_{2}, F_{3}$ and $F_{4}$ act at a point $O$ as shown in the figure and can be represented by sides of polygon $O A, A B, B C$, and $C D$, respectively.
$>$ Vector $O D$ represents the resultant force $\mathbf{R}=\mathbf{F}_{1}+\mathbf{F}_{2}+\mathbf{F}_{3}+\mathbf{F}_{4}$ in magnitude and direction.


## RESOLUTION AND RESULTANT OF FORCE IN SPACE

> Consider a force $\mathbf{F}$ acting at the origin O of the System of rectangular coordinates x , $y, z$ as shown in Fig. Three angles define the direction of force $F \theta_{x}, \theta_{y}$, and $\theta_{z}$.
$>$ Let $\mathrm{F}_{\mathrm{x}}, \mathrm{F}_{\mathrm{y}}$, and $\mathrm{F}_{\mathrm{z}}$ be the components of force F in $\mathrm{x}, \mathrm{y}$, and zdirections, respectively.

> From the triangle $O A B \cos \theta_{x}=\frac{O B}{O A}=\frac{F_{x}}{F}$, component of $F$ along the $x$-direction $\mathrm{F}_{\mathrm{x}}=\mathrm{F} \cos \theta_{\mathrm{x}}$
> From the triangle $O A C \cos \theta_{y}=\frac{O C}{O A}=\frac{F_{y}}{F}$, component of $\mathbf{F}$ along the $y$-direction $\mathrm{F}_{\mathrm{y}}=\mathrm{F} \cos \theta_{\mathrm{x}}$
$>$ From the triangle $O A D \cos \theta_{x}=\frac{O C}{O A}=\frac{F_{z}}{F}$,component of $F$ along the z-direction $\mathrm{F}_{\mathrm{z}}=\mathrm{F} \cos \theta_{\mathrm{z}}$
> The cosines of $\theta_{x}, \theta_{y}$ and $\theta_{z}$ are known as the direction cosine of the force $\mathbf{F}$.
> The angles $\theta_{x} \theta_{y}$ and $\theta_{z}$ are not independent. They are related as
$\cos ^{2} \theta_{x}+\cos ^{2} \theta_{y}+\cos ^{2} \theta_{z}=1$.
$\Rightarrow F$ in the vector form $\mathbf{F}=F_{x} i+F_{y} j+F_{x} k$
> The magnitude of force $\mathbf{F}$ is $F=\sqrt{F_{x}^{2}+F_{y}^{2}+F_{z}^{2}}$
$>$ Direction of force $\theta_{x}=\cos ^{-1}\left(\frac{F_{x}}{F}\right), \theta_{y}=\cos ^{-1}\left(\frac{F_{y}}{F}\right)$, and $\theta_{z}=\cos ^{-1}\left(\frac{F_{z}}{F}\right)$

## 7. MOMENTS

> It is the turning effect produced by force on the body on which it acts.
> SI Unit: Nm
> Moment $\mathrm{M}=\mathrm{F} \times \mathrm{L}$

- Where $\mathrm{F}=$ Force acting on the body
- $L=$ Perpendicular distance of the point, about which the moment is determined and the line of action of the force.

Force $F$ and position vector $r$ in rectangular components may be written as

$$
\begin{aligned}
\mathbf{F} & =F_{x} \mathrm{i}+\mathrm{F}_{\mathrm{y}} \mathrm{j}+\mathrm{F}_{\mathrm{z}} \mathrm{~h} \\
\mathbf{r} & =\mathrm{xi}+\mathrm{yj}+\mathrm{zk}
\end{aligned}
$$

Thus,

$$
\begin{gathered}
\mathbf{M}_{o}=\mathbf{r} \times \mathbf{F}=\left|\begin{array}{ccc}
i & j & k \\
x & y & z \\
F_{x} & F_{y} & F_{z}
\end{array}\right| \\
M_{o}=i\left(y F_{z}-z F_{y}\right)-\left(x F_{z}-z F_{x}\right) j+\left(x F_{y}-y F_{x}\right) k
\end{gathered}
$$

> In case of problems involving only two dimensions, the force F may be assumed to lie in the xy-plane. Carrying $\mathrm{z}=0$ and $\mathrm{F}_{\mathrm{z}}=0$, we obtain

$$
\mathbf{M}_{o}=\left(x F_{y}-y F_{x}\right) k
$$

## Note.

8. Just as the force tends to translate the body, the moment tends to rotate the body about the point.
9. DETERMINATION OF DIRECTIONS
> Assume perpendicular distance as the hand of a clock.
> Keep the point (one end of perpendicular distance), about which the moment is to be determined, hinged like the hinged end of hands of a clock.
> Move the other end of perpendicular distance in the direction of action of force.

10. Anticlockwise
11. Clockwise
12. Clockwise

Note.
> Clockwise,whose effect is to turn, in the same direction in which the clock's hands move.
> Anticlockwise, whose effect is to turn, in the opposite direction in which the clock's hands move.

## 10. VARIGNON'S THEOREM

> The moment of a force about any point is equal to the algebraic sum of the moments of the components of that force about the same point.

$M_{o}=F \cos \theta \times R_{2}-F \sin \theta \times R_{1}$

## 11. Couples

> The moment produced by two equal, opposite and non-collinear forces is called a couple.
> It does not produce any translation but produces only rotation.
> The resultant force of a couple is zero.
> The moment of a couple is the product of the magnitude of one of the forces and the perpendicular distance between their lines of action.

## CHAPTER-2-EQUILIBRIUM OF RIGID BODY \& FBD

## 1. CONDITIONS OF EQUILIBRIUM

> The term equilibrium implies that either the body is at rest or it moves with a constant velocity.
$>$ A body is said to be in static equilibrium when the resultant force must be zero, and the body must have no tendency to rotate.

$$
\begin{aligned}
& \text { i.e. } \Sigma \mathrm{F}=0 \text { means that } \Sigma \mathrm{H}=0 \text { and } \Sigma \mathrm{V}=0 \\
& \text { and } \Sigma \mathrm{M}=0 \text { about any point. }
\end{aligned}
$$

## 2. FREE BODY DIAGRAM

> A free-body diagram is a sketch of the body that shows the body (by itself, free of the other part of the system) and all the forces applied to it, i.e., all forces acting on the body. Fig shows free body diagrams.



FBD of A


FBD of $B$


FBD of sphere

## 3. LAMI'S THEOREM

> If three coplanar forces acting at a point are in equilibrium, then each force is proportional to the smaller angle between the other two forces.
$>$ Here $\mathrm{P}, \mathrm{Q} \& \mathrm{R}$ are the three coplanar forces and $\alpha, \beta, \gamma$ are three angles. According to Lami's theorem $\frac{\mathrm{P}}{\sin \alpha}=\frac{\mathrm{Q}}{\sin \beta}=\frac{\mathrm{R}}{\sin \gamma}$


## CHAPTER-3-INTERNAL FORCES IN A STRUCTURE

## TRUSSES

$>$ A framework composed of straight members joined at their ends to form a structure is called a truss.

## CLASSIFICATION OF TRUSS

1) EFFICIENT OR PERFECT TRUSS, $\mathrm{m}=2 \mathrm{j}-3$
> DEFICIENT OR COLLAPSIBLE TRUSS, $\mathrm{m}<2 \mathrm{j}-3$
2) REDUNDANT TRUSS , m> m -3
> Where m is the number of members and j is the number of joints.

## ZERO FORCE MEMBERS

> Zero force members in a truss are members which do not have any force in them.
Case 1: At a two-member joint that is not parallel and there are no other external loads or reactions at the joint, both members are zero force members.


Case 2: At a three-member joint, if two of those members are parallel and there are no other external loads (or reaction) at the joint, then the not parallel member is a zero force member.


Methods of analysis of statically determinate trusses
i. Method of joint
$\Sigma F_{x}=0$ and $\Sigma F_{y}=0$
Analysis should start at joint having at least one known force and at most two unknown forces.

## ii. Method of section

A section is cut such that it cuts maximum of those members in which forces are Unknown and subsequently force equilibrium and moment equilibrium equations are used to obtain the unknown forces.

## CHAPTER-4-CENTRE OF MASS \& MOMENT OF INERTIA

## 1. INTRODUCTION

> In engineering structures, the members of various cross-sections are used to withstand loads.
> The load-carrying capacity depends on the type of material used as also the crosssection of the members.

## 2. CENTROID

> Centroid is the point where the whole area is assumed to be concentrated.
> It is related to the shape or the geometry of the object.
> The centroid of line

$$
\bar{x}=\frac{\int x d L}{L}, \bar{y}=\frac{\int y d L}{L} \text { and } \bar{z}=\frac{\int z d L}{L} \text {. where } \int d L=L
$$

> The centroid of the area

$$
\bar{x}=\frac{\int x d A}{\int d A}, \quad \bar{y}=\frac{\int y d A}{\int d A}, \quad \bar{z}=\frac{\int z d A}{\int d A}, \text { where } \int d A=A
$$

> The centroid of Volume

$$
\bar{x}=\frac{\int x d V}{V}, \bar{y}=\frac{\int y d V}{V} \text { and } \bar{z}=\frac{\int z d V}{V} \text { where } \int d V=V
$$

## 3. CENTRE OF GRAVITY

> The centre of gravity of the body or the system of particles rigidly connected together is that point where the body's weight is concentrated, and gravitational force acts through it.

$$
\overline{\mathrm{x}}=\frac{\int x d m}{\int \mathrm{dm}}, \quad \overline{\mathrm{y}}=\frac{\int y d m}{\int \mathrm{dm}}, \quad \bar{z}=\frac{\int z d m}{\int d m}, \text { where } \int \mathrm{dm}=M
$$

Note.
> When density is uniform, i.e. the body material is homogenous, centroid and C.G are the same, but two points will not be the same when density is not the same. C.G lies where density is more, i.e. weight is more.

| SHAPE | FIGURE | $\overline{\mathrm{x}}$ | $\overline{\mathrm{y}}$ |
| :---: | :---: | :---: | :---: |
| Rectangle |  | $\frac{\mathrm{b}}{2}$ | $\frac{\mathrm{d}}{2}$ |
| Triangle |  | $\frac{\mathrm{b}}{2}$ | $\frac{\mathrm{h}}{3}$ |
| Circle |  | $r$ | $r$ |
| Semi-Circle |  | 0 (taking centre as origin) | $\frac{4 \mathrm{R}}{3 \pi}$ |


| Quarter of Circle |  | $\frac{4 \mathrm{R}}{3 \pi}$ | $\frac{4 \mathrm{R}}{3 \pi}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\frac{2 \mathrm{R}}{3 \alpha} \sin \alpha$ | 0 |

## 4. AREA MOMENT OF INERTIA

> Moment of Inertia for an area: - This is also known as the second moment of area.
$\mathrm{I}_{\mathrm{xx}}=\int y^{2} d A=$ second moment of area about x -axis
$I_{y y}=\int x^{2} d A=$ second moment of area about the $y$-axis.

## 5. THEOREM OF PARALLEL AXIS

> The moment of inertia about any axis parallel to the centroidal axis at a distance ( h ) is equal to the sum of moment of Inertia about the centroidal axis and product of area and square of distance $B / w$ two axis.

$$
I_{\text {parallelAxis }}=I_{\text {Centroid }}+A h^{2}
$$

## 6. THEOREM OF PERPENDICULAR AXIS

> If the moment of Inertia about two perpendicular centroidal axis in a plane are given ( $I_{1}, I_{2}$ ), then the moment of Inertial about the third axis, which is perpendicular to both mutually perpendicular axis passing through the point of intersection, is equal to the sum of moment of Inertia about two centroidal axis.

$$
\mathrm{I}_{z z}=\mathrm{I}_{x x}+\mathrm{I}_{y y}
$$

| SHAPE | AXIS | Moment of Inertia |
| :---: | :---: | :---: |
|  |  |  |



## 7. MASS MOMENT OF INERTIA

> Moment of the moment of mass is called the mass moment of inertia.
> It is also called the second moment of mass.
> The theorem of parallel axis and perpendicular axis also applies in it.
8. RADIUS OF GYRATION
> The radius of gyration k describes how the area of a cross-section is distributed around its centroidal axis.
> If the area is concentrated far from the centroidal axis, it will have a greater value of k and a greater resistance to buckling (or bending).
> The radius of gyration is defined as: $\mathrm{k}=\sqrt{\frac{\mathrm{I}}{\mathrm{A}}}$, where k is the radius of gyration, I is the Moment of Inertia, and Ais the cross-section area.

| Body | Axis of Rotation | Figure | Moment of inertia | k | $k^{2} / R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Ring | About an axis passing through C.G. and perpendicular to its plane |  | $M R^{2}$ | R | 1 |
| Ring | About its diameter |  | $\frac{1}{2} M R^{2}$ | $\frac{\mathrm{R}}{\sqrt{2}}$ | $\frac{1}{2}$ |


| Body | About a <br> tangential axis <br> in its own <br> plane |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Ring |  |
| Ring |  |


| Body | Axis of Rotation | Figure | Moment of inertia | $k$ | $k^{2} / R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Annular disc | Diameter |  | $\frac{M}{4}\left[R_{1}^{2}+R_{2}^{2}\right]$ | - | - |
| Annular disc | Tangential and Parallel to the diameter |  | $\frac{M}{4}\left[5 R_{1}^{2}+R_{2}^{2}\right]$ | - | - |
| Annular disc | Tangential and perpendicular to the plane |  | $\frac{M}{2}\left[3 R_{1}^{2}+R_{2}^{2}\right]$ | - | - |
| Solid cylinder | About its own axis |  | $\frac{1}{2} M R^{2}$ | $\frac{\mathrm{R}}{\sqrt{2}}$ | $\frac{1}{2}$ |
| Solid cylinder | Tangential (Generator) |  | $\frac{3}{2} M R^{2}$ | $\sqrt{\frac{3}{2}} \mathrm{R}$ | $\frac{3}{2}$ |
| Solid cylinder | About an axis passing through its C.G. and perpendicular to its own axis |  | $M\left[\frac{L^{2}}{12}+\frac{\mathrm{R}^{2}}{4}\right]$ | $\sqrt{\frac{L^{2}}{12}+\frac{R^{2}}{4}}$ | - |
| Solid cylinder | About the diameter of one of faces of the cylinder |  | $M\left[\frac{L^{2}}{3}+\frac{R^{2}}{4}\right]$ | $\sqrt{\frac{L^{2}}{3}+\frac{R^{2}}{4}}$ | - |


| Body | Axis of Rotation | Figure | Moment of inertia | $k$ | $k^{2} / R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Cylindrical shell | About its own axis |  | $M R^{2}$ | R | 1 |
| Cylindrical shell | Tangential (Generator) |  | $2 M^{2}$ | $\sqrt{2} \mathrm{R}$ | 2 |
| Cylindrical shell | About an axis passing through its C.G. and perpendicular to its own axis |  | $\mathrm{M}\left[\frac{\mathrm{L}^{2}}{12}+\frac{\mathrm{R}^{2}}{2}\right]$ | $\sqrt{\frac{L^{2}}{12}+\frac{R^{2}}{2}}$ | - |
| Cylindrical shell | About the diameter of one of faces of the cylinder |  | $M\left[\frac{L^{2}}{3}+\frac{\mathrm{R}^{2}}{2}\right]$ | $\sqrt{\frac{L^{2}}{3}+\frac{R^{2}}{2}}$ |  |
| Hollow cylinder with inner radius $=R_{1}$ and outer radius $=R_{2}$ | Axis of cylinder |  | $\frac{M}{2}\left(R_{1}^{2}+R_{2}^{2}\right)$ | - | - |



| Body | Axis of Rotation | Figure | Moment of inertia | k | $k^{2} / R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Long thin rod | About on axis passing through its center of mass and perpendicular to the rod. |  | $\frac{M L^{2}}{12}$ | $\frac{\mathrm{L}}{\sqrt{12}}$ | - |
| Long thin rod | About an axis passing through its edge and perpendicular to the rod |  | $\frac{M L^{2}}{3}$ | $\frac{L}{\sqrt{3}}$ | - |
| Rectangular lamina of length I and breadth $b$ | Passing through the center of mass and perpendicular to the plane |  | $\frac{M}{12}\left[I^{2}+b^{2}\right]$ | - | - |
| Rectangular lamina | Tangential perpendicular to the plane and at the mid-point of breadth |  | $\frac{M}{12}\left[\left.4\right\|^{2}+b^{2}\right]$ | - | - |
| Rectangular lamina | Tangential perpendicular to the plane and at the mid-point of length |  | $\frac{M}{12}\left[1^{2}+4 b^{2}\right]$ | - | - |
| Rectangular parallelepiped length I, breadth $b$, thickness $t$ | Passing through center of mass and parallel to <br> (i) Length ( x ) <br> (ii) breadth (z) <br> (iii) thickness <br> (y) |  | (i) $\frac{\mathrm{M}\left[\mathrm{b}^{2}+\mathrm{t}^{2}\right]}{12}$ <br> (ii) $\frac{M\left[\left.\right\|^{2}+t^{2}\right]}{12}$ <br> (iii) $\frac{M\left[b^{2}+I^{2}\right]}{12}$ | - | - |


| Body | Axis of Rotation | Figure | Moment of inertia | k | $k^{2} / R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Rectangular parallelepiped length /, breath $b$, thickness $t$ | Tangential and parallel to <br> (i) length ( $x$ ) <br> (ii) breadth ( y ) <br> (iii) thickness(z) |  | (i) $\frac{M}{12}\left[\left.3\right\|^{2}+b^{2}+t^{2}\right]$ <br> (ii) $\frac{M}{12}\left[I^{2}+3 b^{2}+t^{2}\right]$ <br> (iii) $\frac{M}{12}\left[1^{2}+b^{2}+3 t^{2}\right]$ | - | - |
| Elliptical disc of semimajor axis $=a$ and semi minor axis $=b$ | Passing through CM and perpendicular to the plane |  | $\frac{M}{4}\left[a^{2}+b^{2}\right]$ | - | - |
| Solid cone of radius $R$ and height h | Axis joining the vertex and center of the base |  | $\frac{3}{10} M^{2}$ | $\sqrt{\frac{3}{10}} \mathrm{R}$ | $\frac{3}{10}$ |
| Equilateral triangular Iamina with side a | Passing through CM and perpendicular to the plane |  | $\frac{M a^{2}}{6}$ | $\frac{\mathrm{a}}{\sqrt{6}}$ | - |
| Right angled triangular lamina of sides $a, b, c$ | Along the edges |  | (1) $\frac{\mathrm{Mb}^{2}}{6}$ <br> (2) $\frac{\mathrm{Ma}^{2}}{6}$ <br> (3) $\frac{M}{6}\left[\frac{a^{2} b^{2}}{a^{2}+b^{2}}\right]$ | - | - |


> The graph between the applied load P and the friction force F is linear until limiting friction is reached.
> The force of static friction between any two surfaces in contact is opposite the applied force and can have values given by $F_{s} \leq \mu_{s} R$, where $\mu_{s}$ is called the coefficient of static friction.
> When the block is on the verge of slipping, the force of static friction is given by $F_{s}=\left(F_{s}\right)_{\text {max }}=\mu_{s} R$
> The force of kinetic friction acting on an object is opposite to the object's direction of motion and is given by $F_{k}=\mu_{k} R$, where $\mu_{k}$ is a constantly called coefficient of kinetic friction.
$>$ The values of coefficients of friction $\mu_{\mathrm{s}}$ and $\mu_{\mathrm{k}}$ do not depend upon the area of the surfaces in contact.
> However, both the coefficients depend strongly on the nature of the surface in contact,i.e. roughness/smoothness.
> $\mu_{\mathrm{k}}$ is generally less than $\mu_{\mathrm{s}}$

## ANGLE OF FRICTION

> Angle $\theta$, which resultant S subtends with the normal plane when the body just starts sliding over the horizontal plane.
> This is also known as the limiting angle of reaction or friction.
> It is sometimes found convenient to replace the normal force R and the friction force $F_{s}$ by their resultant S. From the geometry of Fig, and we note that

$$
\tan \phi=\frac{F_{s}}{R}=\frac{\mu R}{R}=\mu
$$


$>$ limiting angle of friction $\phi=\tan ^{-1} \mu$
$>$ This indicates that the friction coefficient is tangent tothe angle of friction.
$>$ We know $F_{s}$ vary from 0 to limiting value, $S o$ if $F_{s}$ form a circle, then resultant $S$ will describe a right circular cone of apex angle $2 \phi$ about the line of action Ras its axis, then this cone is called a Cone of friction.
$>$ The body will be stationary if the resultant is within the cone of friction.

## ANGLE OF REPOSE

> The maximum inclination of the plane on which a body, free from external forces, experiences repose (sleep) is called the Angle of Repose.
$>$ Applying the Equilibrium equation to the block shown above, we get $\mu=\tan \theta$
$>$ In terms of angle of friction $\phi, \quad \theta=\phi$


The direction of friction on DRIVING AND DRIVEN WHEELS OF AN AUTOMOBILE
$>$ On a smooth surface driving wheels will rotate about the axis of the axle, but the vehicle will not move.
$>$ Frictional torque on driving wheels is overcome by engine torque,i.e. frictional torque direction opposite to engine torque. It provides driving force or tractive force to the vehicle.

$$
0 \leq(F=T / r) \leq F_{s}
$$

Where $F_{s}$ is limiting friction, $T$ is engine torque, and $r$ is the wheel's radius.
> Friction on driven wheels provides motion to driven wheels,i.e. friction direction is along with the wheel's motion.

## FRICTION IN WHEELS

## WHEELS BEING PULLED BY A FORCE

> Say a wheel of radius R is being pulled by a force Power a rough surface, as shown in
Fig. If the coefficient of friction between wheel and surface is $\mu$, the frictional force $F=\mu N$, $\mathrm{N}=$ normal reaction

Turning moment, $\mathrm{T}=\mathrm{FR}=\mu \mathrm{WR}$


Fig:13
> If the surface is smooth $\mu=0, \mathrm{~T}=0$, the wheel will not rotate.
> The wheel moves in the clockwise direction, and the wheel moves towards the right.
> The magnitude of frictional force $F$ will be maximum equal to $\mu \mathrm{N}$.
> Force P applied at axis O cannot provide a turning moment about axis O. Frictional force provides a clockwise turning moment.

## DRIVING AND DRIVEN WHEELS OF AN AUTOMOBILE

> Consider a rear-wheel-drive vehicle as shown in Fig travelling towards the right as shown.
> Driving torque on rear wheels is T . The engine provides this torque through the propeller shaft.


However, the road is generally rough, and a frictional force


$>F_{1}$ acts on the outer surface of the wheel, if $R$ is the radius of the wheel, then frictional torque $T_{f}=F_{1} \times R$ (where $F_{1}$ becomes the propelling force on the vehicle)
$>$ In the direction of motion. Friction force $F_{1}$ at contact point $A$ anticlockwise couple replace a case $F_{1} \times R$ and a force $F_{1}$ at axle axis $O$ as shown. So this is the frictional force on the driving wheel responsible for producing motion in the vehicle.
$>F_{2}<F_{1}$ The following points must be remembered in an engine-powered vehicle:
(a) On a smooth surface, driving wheels will rotate about the axis of the axle, but the vehicle will not move.
(b) Frictional torque on driving wheels is overcome by engine torque.
(c) Friction on driven wheels provides motion to driven wheels.
(d) Friction on driving wheels provides driving force or tractive force to the vehicle.

### 1.1. Fundamental of vibrations

a) Free vibration - Vibrations due to inherent forces of the system is called free vibration.
b) Force vibration- Vibrations caused due to external forces is called forced vibrations.
c) Degree of freedom- The number of independent coordinates required to solve the vibrating system is the degree of freedom.

### 1.2. Damping

> Damping is the ability of a building to disintegrate the energy of the earthquake ground shaking.

## 2. FUNDAMENTAL PERIOD OF VIBRATIONS

The following formula is used for vibrations
(a) $\mathrm{T}=0.075 \mathrm{H}^{0.75}$
$\mathrm{H}=$ Height of building above ground level
T= Fundamental period of vibration
(b)If the shear wall is provided

$$
T=\frac{0.09 H}{\sqrt{D}}
$$

$\mathrm{D}=$ depth of the building
Note: The following relationship can be used to find the approximate fundamental period of vibrations

$$
\begin{gathered}
\mathrm{T}=0.1 \times \mathrm{N} \\
\mathrm{~N}=\text { number of storeys above ground level }
\end{gathered}
$$

## 3. EQUATION OF MOTION


mü : fictitious inertia force
cu̇: damping force
Ku: spring force
u : displacement at any time t
ú: velocity at any time t
ü: acceleration at any time t
$\mathrm{P}(\mathrm{t})$ : External force
The general equation of motion is,
$\mathrm{P}(\mathrm{t})=\mathrm{mü}+\mathrm{c} \dot{\mathrm{u}}+\mathrm{ku}$
For undamped $(c=0)$ free vibration $(P=0)$ the above equation reduces to
$\mathrm{mü}+\mathrm{ku}=0$
$\Rightarrow \quad \ddot{u}+\frac{k}{m} u=0$
or $\ddot{u}+w_{n}^{2} u=0$
Where, $w_{n}^{2}=k / m$
$w_{n}=\sqrt{k / m}$ : Natural frequency of vibration
The solution of the above equation of motion is
$u(t)=u(0) \cos w_{n} t+\frac{\vec{u}(0)}{w_{n}} \sin w_{n} t$
where $\mathrm{u}(0)$ : initial displacement
ù (0) : initial velocity
This motion repeats itself after every $\frac{2 \pi}{w_{n}}$ seconds
Motion executed is simple harmonic as the restoring force on the particle is directly proportional to the displacement.
a) Undamped natural circular frequency ( $\omega_{\mathrm{n}}$ )

$$
\begin{aligned}
& \omega_{\mathrm{n}}=\sqrt{\frac{k}{m}} \\
& \mathrm{k}=\text { spring constant } \\
& \mathrm{m}=\text { mass }
\end{aligned}
$$

b) Time period ( T )

$$
\mathrm{T}=\frac{2 \pi}{\omega_{\mathrm{n}}}
$$

## c) Frequency

$$
f=1 / T
$$

****

## GATE/ESE

## Civil Engineering

Environmental Engineering

## Important Formula Notes

## IMPORTANT FORMULAS ON ENVIRONMENTAL ENGINEERING

## CHAPTER-1-WATER DEMAND

## A. WATER DEMAND AND POPULATION FORECASTING

## 1. Population forecasting Methods

## (i) Arithmetic increase method

$$
P_{n}=P_{o}+n \times \bar{x}
$$

Where,
$P_{n}=$ Prospective or forecasted Population after $n$ decades from the present (i.e., last known census)
$\mathrm{P}_{\mathrm{o}}=$ Population at present (i.e., last known census)
$\mathrm{n}=$ Number of decades between now \& future.
$\bar{x}=$ Average (arithmetic mean) of population increases in the known decades.
(ii) Geometric Increase Method
$P_{n}=P_{o}\left(1+\frac{r}{100}\right)^{n}$
where,
Po =Initial Population.
$P_{n}=$ Future Population after ' $n$ ' decades.
$r=$ Assumed growth rate (\%).
$r=\sqrt{\frac{P_{2}}{P_{1}}}-1$
where,
$\mathrm{P}_{2}=$ Final known Population
$P_{1}=$ Initial known Population
$t=$ Number of decades (period) between $P_{1}$ and $P_{2}$
$r=\sqrt[t]{r_{1} r_{2} \ldots \ldots r_{t}}$
(iii) Incremental Increases Method

$$
P_{n}=P_{o}+n \bar{x}+\frac{n(n+1)}{2} \bar{y}
$$

Where,
$\bar{x}=$ Average increase of Population of known decades
$\bar{y}=$ Average incremental increases of the known decades.

## (iv) Decreasing rate of growth method

$>$ In this method, the average decrease in the percentage increase is calculated and then subtracted from the latest percentage increase for each successive decade.
> However, this method is applicable only in cases where the rate of growth shows a downward trend.
(v) Logistic Curve Method
$>\log _{e}\left[\frac{P_{s}-P}{P}\right]-\log _{e}\left[\frac{P_{s}-P_{O}}{P_{O}}\right]=k P_{s} t$
Where,
Po= Population of the start point.
$\mathrm{P}_{\mathrm{s}}=$ Saturation population
$\mathrm{P}=$ Population at any time t from the origin.
$\mathrm{K}=$ Constant.
> $P=\frac{P_{S}}{1+m \log _{e}^{-1}(n t)}$
> $\mathrm{P}_{\text {sat }}=\frac{2 \times\left(\mathrm{P}_{0} \mathrm{P}_{1} \mathrm{P}_{2}\right)-\mathrm{P}_{1}^{2}\left(\mathrm{P}_{0}+\mathrm{P}_{2}\right)}{\mathrm{P}_{0} \mathrm{P}_{2}-\mathrm{P}_{1}^{2}}$
$\Rightarrow m=\frac{P_{s}-P_{0}}{P_{0}}$
> $\mathrm{n}=\frac{1}{\mathrm{t}_{1}} \log _{\mathrm{e}} \frac{\mathrm{P}_{0}\left(\mathrm{P}_{\text {sat }}-\mathrm{P}_{1}\right)}{\mathrm{P}_{1}\left(\mathrm{P}_{\text {sat }}-\mathrm{P}_{0}\right)}$
B. Water Demands

1. Fire Demand
> Kutchling's formula

$$
\begin{gathered}
\mathrm{Q}=3182 \sqrt{\mathrm{P}} \\
\mathrm{Q}=\text { Quantity of water in litres per minute } \\
\mathrm{P}=\text { Population in thousands }
\end{gathered}
$$

## > Buston formula:

$$
\begin{gathered}
\mathrm{Q}=5663 \sqrt{\mathrm{P}} \\
\mathrm{Q}=\text { Quantity of water in litres per minute }
\end{gathered}
$$

$$
P=\text { Population in thousands }
$$

> Freeman's formula:

$$
\begin{array}{r}
Q=1136\left(\frac{\mathrm{P}}{5}+10\right) \\
\mathrm{F}=2.8 \sqrt{\mathrm{P}}
\end{array}
$$

$\mathrm{F}=$ Number of simultaneous fire streams.
$\mathrm{Q}=$ Quantity of water in litres per minute
$\mathrm{P}=$ Population in thousands
> National Board of Fire under Writers formula:

$$
\begin{gathered}
\mathrm{Q}=4637 \sqrt{\mathrm{P}}(1-0.01 \sqrt{\mathrm{P}}) \\
\mathrm{Q}=\mathrm{Quantity} \text { of water in litres per minute } \\
\mathrm{P}=\text { Population in thousands }
\end{gathered}
$$

## Per capita Demand:

> Average Daily per Capita Demand $==\frac{\text { Quantity Required in } 12 \text { Months }}{(365 \times \text { Population })}$
> Maximum daily Demand $=1.8 \times$ average daily demand
> Maximum hourly Demand of maximum day, i.e. Peak demand

$$
=2.7 \times \text { annual average hourly demand }
$$

Note:
> We will design a raw water scheme on the Demand, which has maximum fluctuation.
> To be economical, we will design a raw water scheme taking a Maximum of \{Maximum Hourly Demand and Coincident Demand\}.
> Coincident Demand = Maximum Daily Demand + Fire Demand

## Water consumption or demands for Various Purposes:

| S.No | Types of Consumption | Normal Range <br> (lit/capita/day) | Average | $\%$ |
| :---: | :---: | :---: | :---: | :---: |
| 1. | Domestic Consumption | $65-300$ | 160 | $\mathbf{3 5}$ |
| 2. | Industrial and Commercial <br> Demand | $45-450$ | 135 | 30 |
| 3. | Public Uses including Fire <br> Demand | $20-90$ | 45 | 10 |
| 4. | Losses and Waste | $45-150$ | 62 | 25 |

## CHAPTER-2- WATER QUALITY PARAMETERS

1. CHARACTERISTICS OF WATER

### 1.1. Qualities of Raw Water

The parameters which help in assenting qualities and properties of raw water are referred to as water quality parameters
i. Physical Water Quality Parameters
a. Suspended solids
> Dissolved solids (DS) = Total solids (TS) - suspended solids (SS)
> Acceptable limit of total solid $=500 \mathrm{mg} / \mathrm{lit}$
b. Turbidity
> Turbidity is measured by

- Turbidity rod -It is used for measuring the turbidity of water in the field.
- Jacksons Turbidimeter - It is used to measure turbidity when it is more than 100 p.p.m.
- Bali's Turbidimeter- - It is used to measure the turbidity of the sample when it is less than five units.
> Drinking water should not have a turbidity of more than 10 N.T.U
c. Colour and Temperature
> The permissible colour for domestic water is 20 ppm on the platinum cobalt scale.
> The most desirable Temperature for public supply is about $20^{\circ} \mathrm{C}$. The Temperature above $35^{\circ} \mathrm{C}$ is unfit for public supply because it is not palatable.
d. Taste and odour

Threshold Odour Number, T.O.N $=\frac{\text { Final volume at which odour is hardly detectable }}{\text { Sample volume }}$

$$
1 \leq \mathrm{TON} \leq 3
$$

## ii. Chemical characteristics

a. pH value of water
> $\mathrm{pH}=-\log _{10}\left[\mathrm{H}^{+}\right]$
> $\mathrm{pH}+\mathrm{pOH}=14$
> $\mathrm{pH}<7$ : Acidic
> $\mathrm{pH}=7$ :Neutral
$>\mathrm{pH}=>7$ : Basic
$>\mathrm{pH}$ is measured by potentiometer
> $\mathrm{p}(\mathrm{H})=7$ to 8.5 is acceptable limit
$>\mathrm{p}(\mathrm{H})<6.5$ and $\mathrm{p}(\mathrm{H})>9.2$ is cause for rejection.

## iii. Total Dissolved Solids (TDS)

> Electrical conductivity in $\mu \mathrm{Mho} / \mathrm{cm}$ at $\left.25^{\circ} \mathrm{C}\right) \times 0.65=$ dissolved solid content ( $\mathrm{mg} / \mathrm{l}$ ).
> Source of Total dissolved solids

- Major source- $\mathrm{Na}, \mathrm{Ca}, \mathrm{Mg}, \mathrm{HCO}_{3}^{-}, \mathrm{SO}_{4}{ }^{2-} \mathrm{CI}^{-}$
- Minor source- $\mathrm{Fe}, \mathrm{K}, \mathrm{CO}_{3}{ }^{2-}, \mathrm{NO}_{3}{ }^{-}$, Fluoride Boron, silica
> According to the GOI manual, the acceptable limit of TDS ( $\mathrm{mg} / \mathrm{l}$ ) is 500 , and the cause for rejection is 2000.
iv. Alkalinity
> Major sources
$\mathrm{CO}_{3}{ }^{2-}, \mathrm{HCO}_{3}{ }^{-}, \mathrm{OH}^{-}$.
> Minor sources
$\mathrm{H}_{2} \mathrm{BO}_{3}^{-}, \mathrm{HPO}_{4}^{2-}, \mathrm{HS}^{-}$
> Alkalinity measurements are done by titrating the water with acid and determining the hydrogen equivalent of alkalinity, and it is expressed in terms of $\mathrm{mg} / \mathrm{I}$ as $\mathrm{CaCO}_{3}$.


## Note:

> Equivalent weight $=\frac{\text { Molecular weight }}{\text { Valency }}$
> Gram equivalent $=\frac{\text { Weight in gram }}{\text { Equivalent weight }}$
> Equivalent wt of $\mathrm{CaCO}_{3}=\frac{\text { Molecular wt }}{\text { Valency }}=\frac{100}{2}=50$
> 1 ml of $0.02 \mathrm{~N} \mathrm{H}_{2} \mathrm{SO}_{4} \equiv 1 \mathrm{~m}$ of alkalinity expressed as $\mathrm{CaCO}_{3}$

## Relative Quantity of Alkalinity Species are pH-Dependent


> The reaction of alkalinity with hydrogen ions is shown below.

$$
\begin{aligned}
& \mathrm{H}^{+}+\mathrm{OH}^{-}=\mathrm{H}_{2} \mathrm{O} \\
& \mathrm{CO}_{3}^{2-}+\mathrm{H}^{+}=\mathrm{HCO}_{3}^{-} \\
& \mathrm{HCO}_{3}^{-}+\mathrm{H}^{+}=\mathrm{H}_{2} \mathrm{CO}_{3}
\end{aligned}
$$

> If $P=M$ all alkalinity is caustic alkalinity.
> If $P=\frac{M}{2}$, all alkalinity is carbonate alkalinity
$>$ If $P<\frac{M}{2}$, predominant species are carbonate and bicarbonate
$>$ If $P>\frac{M}{2}$, predominant species are carbonate and hydroxide
> If $\mathrm{P}=0$, total alkalinity is bicarbonate alkalinity.

## v. Nitrogen Content

> Free ammonia(should not be more than $0.15 \mathrm{mg} / \mathrm{l}$.) $\rightarrow$ indicates recent pollution
$>$ Organic ammonia (Albuminoid)( should not be more than $0.3 \mathrm{mg} / \mathrm{I}$ )- $\rightarrow$ indicates the Quantity of nitrogen before decomposition has started.
> Nitrite $\rightarrow$ indicates a partly decomposed condition
$>$ Nitrate (should not be more than $45 \mathrm{mg} / \mathrm{l}$ ) $\rightarrow$ indicates old pollution (fully oxidized)
> Organic ammonia Free ammonia + organic ammonia $=$ Kjeldahl Nitrogen Ammonia.
> Nitrite is highly dangerous; hence its permissible limit is zero.
> The colour for nitrite is developed by sulphonic acid + Napthamine
> Nitrate causes blue baby disease or Mathemoglobineming.

## vi. Hardness

, Total Hardness $=\frac{\left[\mathrm{Mg}^{2+}\right] \mathrm{mg} / \mathrm{I}}{\text { eq. wt of mg }} \times$ eq. Wt of $\mathrm{CaO}_{3}$
$+\frac{\left[\mathrm{Ca}^{2+}\right] \mathrm{mg} / \mathrm{I}}{\text { eq. wt of } \mathrm{Ca}} \times$ eq. wt. of $\mathrm{CaCO}_{3}$
> Carbonate Hardness = Minimum of(Alkalinity, Total hardness)
> Non - Carbonate Hardness = Total hardness - Alkalinity
> Acceptable limit of total hardness $=200 \mathrm{mg} / \mathrm{I}$ and cause for rejection $=600 \mathrm{mg} / \mathrm{l}$

### 1.2. WATERBORNE DISEASES

> Bacteria

- Typhoid fever (salmonella bacteria typhi)
- Cholera (Vibrio cholerae bacteria)
- Bacillary Dysentry (Sonnebacilus)
> Virus
- Jaundice (Hepatitis virus)
- Poliomyelitis or Polio (poliovirus)
> Protozoa
- Amoebic dysentery

| Parameter | Desirable- <br> Tolerable | If no alternative source is available, limit extended upto |
| :---: | :---: | :---: |
| Physical |  |  |
| [Turbidity (NTU unit) | < 10 | 25 |
| Colour (Hazen scale) | < 10 | 50 |
| Taste and Odour | Un-objectionable | Un-objectionable |
| Chemical |  |  |
| pH | 7.0-8.5 | 6.5-9.2 |
| Total Dissolved Solids mg/l | 500-1500 | 3000 |
| Total Hardness mg/l ( $\mathrm{as} \mathrm{CaCO}_{3}$ ) | 200-300 | 600 |
| Chlorides mg/l (as CI) | 200-250 | 1000 |
| Sulphates mg/l ( $\mathrm{as} \mathrm{SO}_{4}$ ) | 150-200 | 400 |
| Fluorides mg/l (as F) | 0.6-1.2 | 1.5 |
| Nitrates mg/l (as $\mathrm{NO}_{3}$ ) | 45 | 45 |
| Calcium mg/l (as Ca) | 75 | 200 |
| Iron mg/l (as Fe) | 0.1-0.3 | 1.0 |

## Biological WQP

Testing of Coliforms is done

| Tests/Technique | Remarks |  |  |
| :--- | :--- | :---: | :---: |
| 1. MPN Test | i. Multiple tube Fermentation tests <br> ii. Nutrient used: Lactose broth <br> iii. More is the dilution of sample lesser <br> is the possibility of getting +ye test. |  |  |
| 2. Membrance Filter <br> Technique | i. Nutrient: M-Endo Medium <br> ii. Coliform colonies are counted |  |  |
| 3. Coliform Index Test | Reciprocal of the smallest quantity of <br> sample giving +ve B-coil test |  |  |
| CHTER-3-WATER TREATMENT PLANT |  |  |  |

## 1. Flowchart of treatment



### 1.1. Block Diagram of various Treatment Stages


2. SCREENING
> These are classified as coarse screen and fine screen.
> Coarse screens are in the form of bars spaced at $20-100 \mathrm{~mm} \mathrm{c} / \mathrm{c}$.
> It is kept inclined 3-6 V: 1 H for increasing flow area, reducing velocity.
> The coarse screen is in the form of wire mesh with an opening of less than 10 mm .


## Fixed Bar Type Coarse or Medium Screen

## 3. AERATION

> Water is brought in intimate contact with air.
> It is used to remove undesirable gases dissolved in water like $\mathrm{CO}_{2} \mathrm{H}_{2} \mathrm{~S}$.
> It is also used to add oxygen to oxidize undesirable substances like oils, a decomposing product of algae, etc.
> It removes iron and manganese. Iron and manganese are soluble in $\mathrm{Fe}^{2+}$ and $\mathrm{Mn}^{2+}$ form. They are oxidized to $\mathrm{Fe}(\mathrm{OH})_{3}$ and $\mathrm{MnO}_{2}$, which precipitates.

$$
\begin{gathered}
4 \mathrm{Fe}^{2}+\mathrm{O}_{2}+10 \mathrm{H}_{2} \mathrm{O} \rightarrow 4 \mathrm{Fe}(\mathrm{OH})_{3} \downarrow+8 \mathrm{H}^{+} \\
2 \mathrm{Mn}^{2}+\mathrm{O}_{2}+2 \mathrm{H}_{2} \rightarrow 2 \mathrm{MnO}_{2} \downarrow+4 \mathrm{H}^{+}
\end{gathered}
$$

> This processes increases the acidity of water.

### 3.1. Gravity Aerator or Tray Tower or Trickling Bed

> It is used mostly for Fe and Mn removal.
> This method is one of the most efficient methods for the removal of $\mathrm{CO}_{2}$.

### 3.2. Spray Tower or Nozzle

> Nozzle type aerators are very efficient and are commonly used in the removal of carbon dioxide and iron.
> Removes $90 \% \mathrm{CO}_{2}$ and $99 \% \mathrm{H}_{2} \mathrm{~S}$.


### 3.3. Air Diffuser

> Aeration tanks are commonly about 4 m deep and have a retention time of about 15 minutes.
> Water absorbs oxygen from compressed air, and colour, odour and taste are removed.


### 3.4. Cascade Aerator

> Cascade type aerators depend on the turbulence created in a thin stream of water flowing swiftly down an incline and impinging against fixed upstate.
$>$ Removes $20-45 \% \mathrm{CO}_{2}$ and $35 \% \mathrm{H}_{2} \mathrm{~S}$.
> Cheap compared to others.


## Note:

## Various Water Treatment Aeration Devices


4. Sedimentation
> It is a natural process by which solids with higher density than the fluid in which they are suspended, settling under gravity's action.
> The purpose of sedimentation take is to remove suspended solids.
> Sedimentation is classified into two categories:

- Plain sedimentation, and
- Sedimentation with coagulation
> Settling velocity will be calculated by stoke's law unless and otherwise given.
- $\mathrm{V}_{\mathrm{s}}=\frac{(\mathrm{G}-1) \gamma_{\mathrm{w}} \mathrm{d}^{2}}{18 \mu} \ldots$ applicable for $\operatorname{Re}<1$ (stoke's law)
- $\mathrm{V}_{\mathrm{s}}=418 \times(\mathrm{G}-1) \times \mathrm{d}^{2} \times\left[\frac{3 \mathrm{~T}+70}{100}\right]$ applicable for $\mathrm{Re}<1$ (stoke's law)
$\mathrm{G}=$ specific gravity of the particle
$\mathrm{V}_{\mathrm{s}}=$ Settling velocity
d= diameter of particle
$\mu=$ Dynamic viscosity
$\mathrm{T}=$ Temperature of water in ${ }^{\circ} \mathrm{C}$
- $V_{s}=\left[\frac{\frac{4}{3} g d(G-1)}{C_{D}}\right]^{1 / 2}$
- $C_{D}=0.4$ for $R e \geq 10^{4}$
- $C_{D}=\frac{24}{R_{e}}$ for $R_{e} \leq 0.5$
- $C_{D}=\frac{24}{R_{e}}+\frac{3}{\sqrt{R_{e}}}+0.34$ for $0.5 \leq R_{e} \leq 10^{4}$
- $V_{s}=\left[\frac{\frac{4}{3} g d(G-1)}{C_{D}}\right]^{1 / 2}$
- $V_{s}=1.8 \sqrt{g d(G-1)}$ for $\mathrm{d} \geq 0.1 \mathrm{~mm}$
- $\mathrm{V}_{\mathrm{s}}=418 \times(\mathrm{G}-1) \times \mathrm{d} \times\left[\frac{3 \mathrm{~T}+70}{100}\right]$ for $0.1 \leq \mathrm{d} \leq 1 \mathrm{~mm}$
> Sedimentation Tanks are classified as Quiescent type (fill and draw-type) and continuous type.
> The quiescent type tank will have detention time $=24 \mathrm{hr}$ and period of cleaning $=8-$ 12 hr .
> Three nos of the tank are required in quiescent type. The Tank will be designed for max daily flow and max daily flow $=1.8 \times$ avg. Daily flow.


### 4.1. Continuous Flow Type

a) Horizontal flow tank $\rightarrow$ Rectangular.
b) Vertical flow tank $\rightarrow$ Circular.

### 4.1.1. HORIZONTAL FLOW TANK

> The design is based on the assumption that:

- the Concentration of suspended particles of each size is the same at all points of the vertical cross-section at the inlet end.
- A particle is removed when it reaches the bottom of the settling zone.

> Time of horizontal flow

$$
\text { Time of horizontal flow }=\frac{\text { Lengthof tank }}{\text { Velocity of flow }}=\frac{L}{V_{f}}
$$

$>$ The velocity of flow $\left(\mathrm{V}_{\mathrm{f}}\right)$

$$
V_{f}=\frac{Q}{B H}
$$

> Time of horizontal flow or (Detention time)

Time of horizontal flow or (Detention time) $=\frac{\mathrm{L}}{\mathrm{Q} / \mathrm{BH}}=\frac{\mathrm{LBH}}{\mathrm{Q}}=\frac{\text { Volumeof tank }}{\text { Discharge }}$
> Time of falling through height $=\frac{\mathrm{H}}{\mathrm{V}_{\mathrm{s}}}$
> If a particle of settling velocity $\mathrm{V}_{2}$ is introduced at the topmost inlet point, it will be assumed to be removed if the time of falling through H is detention time.

$$
\Rightarrow \frac{\mathrm{H}}{\mathrm{~V}_{\mathrm{s}}}=\frac{\mathrm{LBH}}{\mathrm{Q}} \Rightarrow \mathrm{~V}_{\mathrm{s}}=\frac{\mathrm{Q}}{\mathrm{BL}}
$$

> This Quantity $\mathrm{V}_{\mathrm{s}}=\frac{\mathrm{O}}{\mathrm{BL}}$ is called overflow rate.
> The overflow rate for plain sedimentation is 12000 to $18000 \mathrm{lit} / \mathrm{m}^{2} /$ day .
> The overflow rate for sedimentation with coagulation is 24000 to $30000 \mathrm{lit} / \mathrm{m}^{2} / \mathrm{day}$.
> Particles with a size greater than the particle for which settling velocity equals the overflow rate are $100 \%$ removed.
> A particle having a settling velocity less than the overflow rate will not get $100 \%$ removed.
> The percentage removal of these particles will be given by $\left(\frac{h}{H}\right)$

$$
\begin{aligned}
\frac{\mathrm{h}}{\mathrm{~V}_{\mathrm{s}}}=\text { Detention time } & =\frac{\mathrm{H}}{\text { over flow rate }}=\frac{\mathrm{H}}{\mathrm{~V}_{\mathrm{s}}} \\
\Rightarrow \frac{\mathrm{~h}}{\mathrm{H}} & =\frac{\mathrm{V}_{\mathrm{s}}}{\mathrm{~V}_{\mathrm{s}}}
\end{aligned}
$$

> The percentage removal of a particle having settling velocity $V_{s}=\frac{V_{s}}{V_{s}} \times 100$

## Note:

> If the overflow rate is increased, the efficiency of the sedimentation tank is decreased and vice versa.
> Theoretically, depth does not have any effect on the efficiency of the Tank.
> For tube settlers detention time is < or = 10 minutes.

## SHORT-CIRCUITING IN SEDIMENTATION TANK

> The degree of short-circuiting is the derivation of the actual flow pattern to the ideal flow pattern. Hence, Displacement efficiency

$$
\left(\mathrm{n}_{\mathrm{d}}\right)=\frac{\text { Flow through period }}{\text { Theoretical Detention Time }} \times 100
$$

> Generally $\left(n_{d}\right)>30 \%$

### 4.1.2. CIRCULAR SEDIMENTATION TANK


> In a circular tank, the horizontal flow velocity of water continuously decreases at a distance from the centre increases.
> Hence particle path will be parabolic as opposed to the straight-line path in the case of the horizontal flow tank.
> Volume of circular tank is given by

$$
\begin{aligned}
V= & D^{2}(0.785 H+0.011 D) \\
& Q \times t_{d}=\text { volume }
\end{aligned}
$$

$$
\text { Over flow rate }=\frac{\mathrm{Q}}{\pi \mathrm{~d}^{2}}
$$


> Weir loading Rate $=\frac{\mathrm{Q}}{\pi \mathrm{D}}$
> weir loading rate affects lighter particle (flocs) clarification.
> Weir loading Rate is normally taken as $300 \mathrm{~m}^{3} / \mathrm{d} / \mathrm{m}$, but when the Tank is property design, its value goes up to $1500 \mathrm{~m}^{3} / \mathrm{d} / \mathrm{m}$

## 5. SEDIMENTATION WITH COAGULATION

The common coagulants added in water:
(a) Alum
> The chemical formula of alum is $\mathrm{Al}_{2}\left(\mathrm{SO}_{4}\right)_{3} .18 \mathrm{~Hz}_{2} \mathrm{O}$


$\mathrm{Al}_{2}\left(\mathrm{SO}_{4}\right)_{3} \cdot 18 \mathrm{H}_{2} \mathrm{O}+3 \mathrm{Na}_{2} \mathrm{CO}_{3} \longrightarrow 3 \mathrm{Na}_{2} \mathrm{SO}_{4}+\underset{\mathrm{ppt}}{2 \mathrm{Al}(\mathrm{OH})_{3} \downarrow+3 \mathrm{CO}_{2}+15 \mathrm{H}_{2} \mathrm{O}, ~}$
> Normal alum dose is $10-30 \mathrm{mg} /$ litre of water and is very effective in the pH range of 6.5 to 8.5.
> Alum coagulant is cheap and the flocs formed are very stable.
> Alum also reduces colour, taste and odour, but the only disadvantage is that it is difficult to dewater the sludge formed.

NOTE: •
> A mole of alum gives 2 moles of $\mathrm{Al}(\mathrm{OH})_{3}$.
> 666 gm of alum gives $2 \times 78 \mathrm{gm}$ of $\mathrm{Al}(\mathrm{OH})_{3}$.

## (b) Copper: Copperas

> Its chemical formula is $\mathrm{FeSO}_{4} .7 \mathrm{H}_{2} \mathrm{O}$.
> When lime is added first, the following reaction takes

$$
\text { (i) } \underset{\text { Copperas }}{\mathrm{FeSO}_{4} .7 \mathrm{H}_{2} \mathrm{O}}+\underset{\text { Hydrated lime }}{\mathrm{Ca}(\mathrm{OH})_{2}} \longrightarrow \underset{\substack{\text { Ferrous } \\ \text { hydroxide }}}{\mathrm{CaSO}}
$$

> Similarly, when copperas is added earlier to lime, the reaction that takes place is

$$
\text { (ii) } \mathrm{FeSO}_{4} \cdot 7 \mathrm{H}_{2} \mathrm{O}+\underset{\substack{\text { Alkalinity present } \\ \text { in raw water }}}{\mathrm{Ca}\left(\mathrm{HCO}_{3}\right)_{2}} \longrightarrow \mathrm{Fe}\left(\mathrm{HCO}_{3}\right)_{2}+\mathrm{CaSO}_{4}+7 \mathrm{H}_{2} \mathrm{O}
$$

> The ferrous hydroxide formed in either case further gets oxidized, forming hydroxide as below:

$$
4 \mathrm{Fe}\left(\mathrm{HCO}_{3}\right)_{2}+\mathrm{O}_{2}+2 \mathrm{H}_{2} \mathrm{O} \longrightarrow 4 \mathrm{Fe}(\mathrm{OH})_{3}
$$

> Copperas is extensively used as a coagulant for raw waters that are not coloured.
$>$ It is generally cheaper than alum and functions effectively in the pH range of 8.5 and above.
> For coloured raw waters, it is, however, not used, as it does not give satisfactory results

## (c) Use of Chlorinated Copperas as Coagulant:

> When chlorine is added to a solution of copperas (i.e. ferrous sulphate), the two react chemically to form ferric sulphate and ferric chloride.
> The chemical equation is as follows:

$$
6\left(\mathrm{FeSO}_{4} \cdot 7 \mathrm{H}_{2} \mathrm{O}\right)+3 \mathrm{Cl}_{2} \longrightarrow \underset{\text { Ferric sulphate }}{2 \mathrm{Fe}_{2}\left(\mathrm{SO}_{4}\right)}+\underset{\text { Ferric chloride }}{2 \mathrm{FeCl}_{3}}+2 \mathrm{H}_{2} \mathrm{O} 2 \mathrm{H}_{20}
$$

> The resultant combination of ferric sulphate and ferric chloride is known as chlorinated copperas. It is a valuable coagulant for removing colours, especially whose raw water has a low pH value.
> Both the chlorinated copperas and lime constituents are effective coagulants, and their combination is often quite effective.
> The chemical reactions that take place are given below:

$$
\mathrm{Fe}\left(\mathrm{SO}_{4}\right)+3 \mathrm{Ca}(\mathrm{OH})_{2} \longrightarrow 3 \mathrm{CaSO}_{4}+\underset{\substack{\text { Ferric hydroxide } \\ \text { ppt }}}{2 \mathrm{Fe}(\mathrm{OH})_{3}} \downarrow
$$

The resulting ferric hydroxide forms the floc and helps in sedimentation.
> Ferric sulphate is quite effective in the pH range of 4 to 7 and above 9, whereas ferric chloride is quite effective in the pH range of 3.5 to 6.5 and above 8.5.

## (d) Use of Sodium Aluminate as a Coagulant:

$>$ Besides alum and iron salts, sodium aluminate $\left(\mathrm{Na}_{2} \mathrm{Al}_{2} \mathrm{O}_{4}\right)$ is also sometimes used as a coagulant.
> The chemical reactions that are involved are

$$
\begin{gathered}
\mathrm{Na}_{2} \mathrm{Al}_{2} \mathrm{O}_{4}+\mathrm{Ca}\left(\mathrm{HCO}_{3}\right)_{2} \longrightarrow \underset{\text { Cal. aluminate }}{\mathrm{CaAl}_{2} \mathrm{O}_{4}}+\mathrm{Na}_{2} \mathrm{CO}_{3}+\mathrm{CO}_{2} \uparrow+\mathrm{H}_{2} \mathrm{O} \\
\mathrm{Na}_{2} \mathrm{Al}_{2} \mathrm{O}_{4}+\mathrm{CaCl}_{2} \longrightarrow \mathrm{CaAl}_{2} \mathrm{O}_{4}+2 \mathrm{NaCl} \\
\mathrm{Na}_{2} \mathrm{Al}_{2} \mathrm{O}_{4}+\mathrm{CaSO}_{4} \longrightarrow \mathrm{CaAL}_{2} \mathrm{O}_{4}+\mathrm{Na}_{2} \mathrm{SO}_{4}
\end{gathered}
$$

## 6. FILTERATION

> Filters are classified as:
(a) Slow sand filter
(b) Rapid sand filter gravity filters
(c) Pressure filter
> The slow sand filter removes a larger percentage of impurities as compared to rapid sand filters.
> Slow sand filters have a very slow rate of filtration about $\frac{1}{30}$ th of the rapid sand filter.

### 6.1. SLOW SAND FILTER


> It utilizes the effluent from a plain sedimentation tank only, which are relatively clearer.
> The depth of the tank is 2.5 to 3.5 m .
> Plan area required is $100-2000 \mathrm{~m}^{2}$.
> Filter medium is sand or anthracite or garnet
> $\mathrm{D}_{10}($ of filter medium $)=0.2-0.3 \mathrm{~mm}$.
> $\frac{D_{60}}{D_{10}}=$ uniformity coefficient $=5$ (as per GOI manual).
> The depth of sand is $90-110 \mathrm{~cm} \approx 1 \mathrm{~m}$
> The depth of water over the sand medium would be approximately the same as the depth of the sand medium.
> The top 15 cm of the sand layer would be finer remaining may be of uniform size.
> The material is gravel provided in 3 layers.

- Top layer size - 3-6 mm
- Middle layer - 20-40 mm
- Bottom layer - 40-65 mm
> The design period of a slow sand filter is ten years.


### 6.2. RAPID GRAVITY FILTER

> Particles more than and less than one $\mu \mathrm{m}$ diameter are efficiently removed.
> Removes suspended \& colloidal matter
> It also removes micro-organisms (i.e. bacteria and helminth etc.)


### 6.3. PRESSURE FILTERS

$>$ The diameter of the tank is $1.5-3.0 \mathrm{~m}$.
$>$ Height or length is 3.5 to 8.0 m .
> It is operated as a rapid gravity filter except that raw water is neither flocculated nor sedimented before it enters the filter.


## 7. DISINFECTION


> Out of various methods, Chlorination is most commonly adopted.

### 7.1. MINOR METHODS

## a) Treatment with excess lime

b) Treatment with ozone
c) Treatment with F and Br
d) Treatment with $\mathrm{KMnO}_{4}$

### 7.2. MAJOR METHODS

### 7.2.1. Chlorination

> At $\mathrm{pH}<5$, chlorine does not react with water and remains as free chlorine.
$>\left(\mathrm{HOCl}+\mathrm{OCl}^{-}\right.$and $\left.\mathrm{Cl}_{2}\right)$ arc combined called freely available chlorine.
> $\mathrm{P}(\mathrm{H})$ of water should be maintained slightly below 7.
> Moreover, the chlorine will immediately react with ammonia present in water to form chloramines.

$$
\begin{gathered}
\mathrm{NH}_{3}+\mathrm{HOCl} \xrightarrow{\mathrm{pH}>7.5} \mathrm{NH}_{2} \mathrm{Cl}+\mathrm{H}_{2} \mathrm{O} \\
\mathrm{NH}_{2} \mathrm{Cl}+\mathrm{HOCl} \xrightarrow{\mathrm{pH}(5-6.5)} \mathrm{NHCl}_{2}+\mathrm{H}_{2} \mathrm{O}, \mathrm{PH}(5-6.5) \\
\mathrm{NHCl}_{2}+\mathrm{HOCl} \xrightarrow{\mathrm{pH}>4.4} \mathrm{NCl}_{3}+\mathrm{H}_{2} \mathrm{O}, \mathrm{PH}<4.5
\end{gathered}
$$

> Chloramines are combined form of chlorine.
> It is less effective than free chlorine ( 25 times lesser), but they are stable and remain in the water for a greater duration.
> Doses of chlorine should be sufficient to leave a residue of 0.2 mg per litre after 10 minutes of the contact period. This dose is called chlorine demand of water.
> The residual chlorine is tested by DPD (Diethyl-Paraphenylene Diamine) test.

### 7.3. FORMS IN WHICH CHLORINE IS ADDED

> Free chlorine (liquid or gaseous form)
> Hypochlorite's (Bleaching powder)
> Chloramines (ammonia + chlorine)
> Chlorine dioxide $\left(\mathrm{ClO}_{2}\right)$

### 7.4. TYPES OF CHLORINATION

## a) Plain Chlorination

> Only Chlorination and no other treatment is given to water.
> It removes bacteria, organic matter and colour.
> It is used for clean water, i.e. turbidity between $20-30 \mathrm{mg} / \mathrm{l}$.
> The dose is $0.5 \mathrm{mg} / \mathrm{I}$.

## b) Pre Chlorination

> In this case, chlorine is added before filtration or rather before sedimentation and coagulation.

## c) Post Chlorination

> Applying chlorine at the end when all treatment is complete is called post chlorination.
> The dose should be such that 0.1 to $0.2 \mathrm{mg} / \mathrm{lit}$ should be left after a contact period of 20 mm .

## d) Double Chlorination

> Prechlorination \& post-chlorination combinedly is called double Chlorination.

## e) Breakpoint Chlorination


> During the disinfection process amount of residual chlorine is less in the beginning (stage I), during which iron, nitrite etc., are oxidized.
> In stage II, chloramines and combined residual chlorine form. Combined residual chlorine will gradually increase as demand for disinfection is satisfied.
> Chlorine residue is tested by the DPD test, which measures both combined and free chlorine.
> At point C, a bad smell started coming out. It is because oxidation of organic matter starts at point C . Hence the residual decreases.
$>$ In stage III, free chlorine breaks down chloramines into nitrogen compounds.

$$
2 \mathrm{NH}_{3}+3 \mathrm{Cl}_{2} \rightarrow \mathrm{~N}_{2}+6 \mathrm{HCl}
$$

> Chloro-organic compounds are also destructed. At this point, D bad smell suddenly disappears. It implies that organic matter oxidation is complete.
> Any further chlorine addition simply appears as free chlorine, i.e. chlorine breaks away from water. Thus point $D$ is called a breakpoint.
> In general practice, chlorine is added beyond breakpoint to ensure a $0.2-0.3 \mathrm{mg} / \mathrm{litre}$ residual as free chorine.
> The difference between applied chlorine and residual chlorine is called the chlorine demand.

Chlorine demand=Applied chlorine -Residual chlorine

## f) Super Chlorination

- When excess chlorine ( 5 to $15 \mathrm{mg} / \mathrm{I}$ ) is added to water during the epidemic, it gives a residual of 1 to $2 \mathrm{mg} / \mathrm{lit}$ beyond breakpoint is called super Chlorination.
Note: The various dechlorinating agents are:
> Sodium thiosulphate $\left(\mathrm{Na}_{2} \mathrm{~S}_{2} \mathrm{O}_{3}\right)$ - cheapest of all
> Activated carbon
> Sulphur dioxide $\left(\mathrm{SO}_{2}\right)$


## 8. WATER SOFTENING

### 8.1. REMOVAL OF TEMPORARY HARDNESS

### 8.1.1. By simple boiling


> The boiling does not remove temporary hardness due to magnesium because $\mathrm{MgCO}_{3}$ is fairly soluble in water. Hence this hardness is removed by the addition of lime.

$$
\begin{gathered}
\mathrm{MgCO}_{3}+\underset{\text { Hydratedline }}{\mathrm{Ca}(\mathrm{OH})_{2} \rightarrow \mathrm{Mg}(\mathrm{OH})_{2} \downarrow+\mathrm{CaCO}_{3} \downarrow} \\
\mathrm{Mg}\left(\mathrm{HCO}_{3}\right)_{2}+\mathrm{Ca}(\mathrm{OH})_{2} \rightarrow \mathrm{Ca}\left(\mathrm{HCO}_{3}\right)_{2}+\mathrm{Mg}(\mathrm{OH})_{2} \downarrow \\
\mathrm{Ca}\left(\mathrm{HCO}_{3}\right)_{2}+\mathrm{Ca}(\mathrm{OH})_{2} \rightarrow 2 \mathrm{CaCO}_{3} \downarrow+2 \mathrm{H}_{2} \mathrm{O}
\end{gathered}
$$

> 1 mole of $\mathrm{MgCO}_{3}$ requires a mole of hydrated lime, whereas a mole of $\mathrm{Mg}\left(\mathrm{HCO}_{3}\right)_{2}$ requires 2 moles of lime.

### 8.2. REMOVAL OF PERMANENT HARDNESS (WATER SOFTENING)

> Lime soda process
> Base exchange process
> Demineralization process

### 8.2.1. LIME SODA PROCES

$$
\begin{gathered}
\mathrm{Ca}\left(\mathrm{HCO}_{3}\right)_{2}+\mathrm{Ca}(\mathrm{OH})_{2} \rightarrow+2 \mathrm{CaCO}_{3} \downarrow+2 \mathrm{H}_{2} \mathrm{O} \\
\mathrm{Mg}\left(\mathrm{HCO}_{3}\right)_{2}+\mathrm{Ca}(\mathrm{OH})_{2} \rightarrow \mathrm{Ca}\left(\mathrm{HCO}_{3}\right)_{2}+\mathrm{Mg}(\mathrm{OH})_{2} \downarrow \\
\mathrm{MgCO}_{3}+\mathrm{Ca}(\mathrm{OH})_{2} \rightarrow \mathrm{Mg}(\mathrm{OH})_{2} \downarrow+\mathrm{CaCO}_{3} \downarrow \\
\mathrm{Mg}^{2+}+\left\{\begin{array}{l}
2 \mathrm{Cl}^{-}+\mathrm{Ca}(\mathrm{OH})_{2} \rightarrow \mathrm{Mg}(\mathrm{OH})_{2} \downarrow+\mathrm{Ca}^{2+}+ \\
\mathrm{SO}_{4}^{2-} \\
2 \mathrm{NO}_{3}^{-}
\end{array}\right\} 2 \mathrm{Cl}^{-}
\end{gathered}
$$

Further

$$
\begin{gathered}
\mathrm{CaCl}_{2}+\mathrm{Na}_{2} \mathrm{CO}_{3} \rightarrow \mathrm{CaCO}_{3} \downarrow+2 \mathrm{NaCl} \\
\mathrm{CaSO}_{4}+\mathrm{Na}_{2} \mathrm{CO}_{3} \rightarrow \mathrm{CaCO}_{3} \downarrow+\mathrm{Na}_{2} \mathrm{SO}_{4} \\
\mathrm{Ca}\left(\mathrm{NO}_{3}\right)_{2}+\mathrm{Na}_{2} \mathrm{CO}_{3} \rightarrow \mathrm{CaCO}_{3} \downarrow+2 \mathrm{NaNO}_{3} \\
\text { Lastly } \mathrm{CO}_{2}+\mathrm{Ca}(\mathrm{OH})_{2} \rightarrow \mathrm{CaCO}_{3} \downarrow+\mathrm{H}_{2} \mathrm{O}
\end{gathered}
$$

> Lime removes entire carbonate hardness.
> The lime reacts with the non-carbonate hardness of magnesium to convert it to the non-carbonate hardness of calcium.
> The non-carbonates hardness of calcium is finally removed by soda ash.

### 8.3. BASE EXCHANGE PROCESS (CATION EXCHANGE PROCESS)

> Zeolite is a natural or synthetic cation or base exchange hydrated silicates of sodium and aluminium. It is called green sand.
$>$ Zeolite $\rightarrow \mathrm{Na}_{2} \mathrm{O}^{2} \mathrm{Al}_{2} \mathrm{O}_{3} \cdot x \mathrm{SiO}_{3} . \mathrm{yH}_{2} \mathrm{O}$ (greensand)

$$
\mathrm{NaZ}+\underset{\mathrm{Mg}}{\mathrm{Ca}}\left\{\begin{array}{l}
\mathrm{HCO}_{3}^{-} \\
\mathrm{SO}_{4}^{-} \mathrm{Cl}^{-}
\end{array} \rightarrow \mathrm{Na}\left\{\begin{array}{l}
\mathrm{HCO}_{3}^{-} \\
\mathrm{SO}_{4}^{-} \mathrm{Cl}^{-} \\
\mathrm{Mg}
\end{array}\right\} \mathrm{C}\right.
$$

> Thus water will have zero hardness. But sodium zeolite can be regenerated from calcium and magnesium zeolites.
> Regeneration from $\mathrm{CaZ}_{2}$ and Na Z is done by using 5-10\% solution (brine solution).

$$
\underset{\mathrm{Na}}{\mathrm{Ca}}\left\{\mathrm{Z}+2 \mathrm{NaCl} \rightarrow \mathrm{Na}_{2} \mathrm{Z}+\underset{\mathrm{Mg}}{\mathrm{Ca}}\right\} \mathrm{Cl}_{2}
$$

## CHAPTER-4- WATER DISTRIBUTION SYSTEM

## 1. TYPES OF DISTRIBUTION SYSTEM

There are three types of distribution systems :
(i) Gravity Distribution System: In this system, water is distributed from high to lower by the simple action of gravity.
(ii) Direct Pumping System: Treated water is directly pumped to the distribution mains in this type of system.
(iii) Combined System: In this type of distribution system, treated water is pumped and transported to an elevated reservoir and then fed to the distribution system under the action of gravity only.

## 2. LAYOUT OF DISTRIBUTION SYSTEM

There are four methods of laying out of distribution system
(a) Dead end system
(b) Grid system
(c) Circular system
(d) Radial system
(a) Dead End System:

(M): Main Pipe
(B): Branch
(s): Sub Mains

- : Cut off Valves
(b) Grid System:



## (c) Circular/ Ring System:


(M): Main Pipe
(B): Branch
(S): Sub Mains

- : Cut off Valves


## (d) Radial System:



## 3. ANALYSIS OF NETWORK OF PIPES

There are two major methods for the analysis of a network of pipes:
(a) Equivalent Pipe Method: In this system, a complex system of pipes is replaced by a single hydraulically equivalent pipe, causing the same head loss and having the same discharge. This method is used for a large network of pipes.
(b) Hardy Cross Method:
> It is a trial and error method in which flow in the system of pipes is assumed, and resulting head losses are balanced.
> Successive corrections are applied until the network is hydraulically balanced. Two conditions must be satisfied in the analysis:
(i) At any junction, the quantity of water entering the junction should equal the quantity of water leaving the junction.
(ii) In any closed loop, the algebraic sum of pressure drops should be equal to zero.

## Steps for Calculation:

(i) Divide the network into several closed loops.
(ii) Assume flow in each pipe satisfying continuity equation. Take clockwise flow as positive and anticlockwise flow as negative.
(iii) calculate the head loss in each pipe using the formula $h_{f}=r Q^{n}$
(iv) The modification in discharge is given by the following formula:

$$
\Delta Q=\frac{-\Sigma r Q^{n}}{\Sigma\left|r n Q^{n-1}\right|}
$$

(v) $\Delta Q$ is added to the assumed discharge algebraically.

## 4. DESIGN OF BALANCING RESERVOIR

A reservoir is designed such that it can meet the fluctuating demand with a constant rate of supply. The storage capacity of the reservoir can be determined using the mass curve method.

### 4.1. Mass Curve Method

> A mass curve is a plot of accumulated supply/demand with time.
$>$ The amount of balancing storage is determined by adding the maximum ordinate between the demand and supply line.


- In region 1-2 - Demand rate < Supply rate (Accumulation of water)
- In region 2-4 - Demand rate> Supply Rate (Depletion of water)
- In region 4-5 - Demand Rate < Supply rate (Accumulation of water)
$>$ Maximum water that can be stored in the reservoir $=A+B$


## CHAPTER-5-DESIGN OF SEWER

## 1. HYDRAULIC DESIGN OF SEWERS

### 1.1. Important Formulas for Determining Flow Velocity

Following formulas used to determine flow velocities in sewers:
(i) Manning's formula: The flow velocity is given by

$$
v=\frac{1}{n} R^{2 / 3} \sqrt{S}
$$

Where,
$\mathrm{R}=$ Hydraulic radius $=\mathrm{A} / \mathrm{P}$
$A=$ Cross sectional area of sewer
P = Wetted Perimeter
$S=$ Ground slope
$\mathrm{n}=$ manning's constant
(ii) Chezy's formula: The flow velocity is given by

$$
v=C \sqrt{R S}
$$

Where,
C = Chezy's constant

### 1.2. Design Data

> Sewage should be designed for maximum hourly discharge, and it should be ensured that flow velocity will always be greater than self-cleansing velocity.
> To avoid clogging, sufficient velocity known as 'Self-cleansing velocity needs to be maintained in the system.
> Maximum hourly discharge $=3 \times$ Average daily discharge
> Maximum daily discharge $=2 \times$ Average daily discharge
$>$ It is assumed that $80 \%$ of the water supply goes to sewers.
> The self-cleansing velocity can be calculated using Shield's formula

$$
v=\frac{1}{n} R^{1 / 6}\left(K(G-1) d_{p}\right)^{1 / 2}
$$

Where,
$G=$ Specific gravity of the particle
$d_{p}=$ Size of particle
$\mathrm{K}=\mathrm{A}$ constant
$\mathrm{R}=$ Hydraulic radius of sewer
$\mathrm{n}=$ manning coefficient

## 3. Circular Sewer running Partially Full



When the sewage is running partially full at depth $d$ such that,
> Proportional depth

$$
d=\frac{D}{2}\left(1-\cos \frac{\alpha}{2}\right)
$$

> Area of flow

$$
\frac{d}{D}=\frac{1}{2}\left(1-\cos \frac{\alpha}{2}\right)
$$

> Proportional area

$$
\begin{aligned}
a=\frac{\pi D^{2}}{4} & \left(\frac{\alpha}{360}-\frac{\sin \alpha}{2 \pi}\right) \\
& \frac{a}{A}=\left(\frac{\alpha}{360}-\frac{\sin \alpha}{2 \pi}\right)
\end{aligned}
$$

> Wetted Perimeter

$$
p=\pi D \frac{\alpha}{360}
$$

> Proportional wetted perimeter

$$
\frac{p}{P}=\frac{\alpha}{360}
$$

> Hydraulic radius

$$
r=\frac{a}{p}=\frac{D}{4}\left(1-\frac{360 \sin \alpha}{2 \pi \alpha}\right)
$$

> Proportional hydraulic radius

$$
\frac{r}{R}=1-\frac{360 \sin \alpha}{2 \pi \alpha}
$$

> Proportional velocity of flow

$$
\frac{v}{V}=\frac{N}{n}\left(\frac{r}{R}\right)^{2 / 3} \text { (Using manning's formula) }
$$

> Proportional discharge

$$
\frac{q}{Q}=\frac{a \times v}{A \times V}
$$

## Note:

$>$ For the constant value of manning's coefficient, the velocity will be maximum when $d$ $=0.81 \mathrm{D}$
$>$ For a constant value of manning's coefficient, the discharge will be maximum when d $=0.95 \mathrm{D}$
4. Equal Degree of Self Cleansing:
$>$ For equal self-cleansing, the drag force under partial flow should be the same as that under full flow.

$$
\begin{gathered}
\gamma_{w} r s=\gamma_{w} R S \\
\Rightarrow r s=R S \\
\Rightarrow \frac{v}{V}=\frac{N}{n}\left(\frac{r}{R}\right)^{\frac{1}{6}}
\end{gathered}
$$

## 1. INTRODUCTION

- Wastewaters are usually classified as industrial wastewater or municipal wastewater.
- Industrial wastewater with characteristics compatible with municipal wastewater is often discharged to the municipal sewers.


### 1.1. Important Waste Water Contaminates

| SI. <br> No | Contaminant | Sources | Environmental Significance |
| :---: | :---: | :---: | :---: |
| 1. | Suspended solids | Domestic use, <br> Industrial wastes | Causes sludge deposits and Anaerobic <br> condition in Aquatic environment |
| 2. | Biodegradable <br> Organic | Domestic use, <br> Industrial wastes | Cause biological degradation |
| 3. | Pathogens | Domestic water | Transmit communicable diseases |
| 4. | Nutrients | Domestic and <br> Industrial waste | Cause eutrophication |
| 5. | Refractory <br> Organics | Industrial waste | Causes taste and odour Problems |

## 2. PHYSICAL CHARACTERISTICS

The most important physical characteristics of water are its turbidity, colour, odour and temperature.

### 2.1. TURBIDITY

Wastewater is normally turbid, containing wastes from baths, faecal matter, pieces of paper, greases, vegetable debris, fruit skins, etc.

### 2.2. COLOUR

- Fresh wastewater is usually grey or light brown. However, as bacteria break down organic compounds, the dissolved oxygen in the wastewater is reduced to zero and colour changes to black. This condition of waste is said to be septic or stale.
- The common method of colour removal is by coagulation followed by sedimentation.


### 2.3. ODOUR

- Odours in wastewater usually are caused by gases produced by the decomposition of organic matter.
- The most characteristic odour of stale or septic wastewater is hydrogen sulphide produced by anaerobic micro-organisms that reduce sulphates to sulphides.


### 2.4. TEMPERATURE

The average Temperature of sewage in India is $20^{\circ} \mathrm{C}$, near the ideal Temperature for biological activities.

## 3. CHEMICAL CHARACTERISTICS

Important chemical characteristics of wastewater are listed below:
i) Total solids, suspended solids and Settleable solids.
ii) pH value.
iii) Chloride content
iv) Nitrogen content
v) Presence of fats, greases, and oils.
vi) Sulphides, sulphates and $\mathrm{H}_{2} \mathrm{~S}$ gas.
vii) Dissolved oxygen.
viii) Chemical oxygen demand (COD).
ix) Bio-chemical oxygen demand (BOD).
$x$ ) Total organic carbon (TOC).
xi) Theoretical oxygen demand (ThOD).

### 3.1. TOTAL SOLIDS, SUSPENDED SOLIDS AND SETTLEABLE SOLIDS

Solids present in wastewater may be in four forms: suspended solids, dissolved solids, colloidal solids and settlement solids.

- Suspended solids are those which remain floating in the water.
- Dissolved solids are those which dissolve in wastewater.
- Colloidal solids are finely divided solids remaining either in suspension or in solution.
- Settleable solid is that portion of solid matter that settle out if the wastewater is allowed to remain undisturbed for 2 hours.
- Inorganic matter consists of sand, gravel, debris, chlorides, sulphates etc., whereas organic matter consists of:
i) Carbohydrates such as cellulose, cotton, fibre, sugar etc.
ii) facts and oils from kitchens, garages, shops etc.
iii) nitrogenous compounds like proteins, urea, fatty acids etc.

The number of various kinds of solids present in wastewater can be determined as follow:
a) Total solids can be determined by evaporating a known value of the wastewater sample and weighing the dry residue left. The mass of residue left divide by the volume of the sample is total solids in $\mathrm{mg} / \mathrm{l}$.
b) The suspended solids, also called non-filterable solids. Thus weighting the dry residue left and dividing by volume of sample filtered will give suspended solids in $\mathrm{mg} / \mathrm{l}$.
c) The quantity of settlement solids can be determined using the Imhoff cone (figure). Wastewater is allowed to stand in the cone for two hours, and the quantity of solids settled down in the bottom is directly readout, which gives an approximate amount of settlement solids.


Imhoff cone

## 3.2. pH VALUE

- The alkalinity of the fresh wastewater samples is alkaline, but as time passes, it becomes acidic because of the bacteria action in anaerobic or nitrification processes.


### 3.3. CHLORIDE CONTENT

- These are derived from kitchen wastes, human faeces, and urinary discharges etc.
- The chloride content can be measured by titrating the waste sample with a standard silver nitrate solution, using potassium chromate as an indicator.


### 3.4. NITROGEN CONTENT

- The presence of nitrogen in wastewater indicates the presence of organic matter, and may be found, in the following forms:
a) Free ammonia or ammonia nitrogen (indicates recent pollution)
b) Albuminoid nitrogen or organic nitrogen (indicates quantity of nitrogen before decomposition has started).
c) Nitrites (indicates partly decomposed condition).
d) Nitrates [indicates old pollution (fully oxidized)]


### 3.5. PRESENCE OF FATS, OILS AND GREASES

- The amount of facts and greases in the wastewater sample can be determined by evaporating and mixing the residual solids with ether (hexane). The solution is then poured off and evaporated, leaving the facts and greases as residue, easily weighted.


### 3.6. SULPHIDES, SULPHATES AND HYDROGEN SULPHIDE GAS

- Sulphides and sulphates are formed due to the decomposition of various sulphurcontaining substances present in the wastewater.
- This decomposition also leads to the evolution of hydrogen sulphide gas, causing bad smells and odours, besides causing corrosion of concrete sewer pipes.
- The initial decompositions is associated with the formation of $\mathrm{H}_{2} \mathrm{~S}$ gas, which also ultimately gets oxidized to form sulphates ions.


### 3.7. DISSOLVED OXYGEN

- Dissolved oxygen is required for the respiration of aerobic micro-organisms and all other aerobic life forms.
- The dissolved oxygen in fresh wastewater depends upon Temperature.

If the Temperature of sewage is more, the D.O. content will be less. Max quantity of D.O. that can remain mixed in water at a particular temperature is called Saturation Dissolved Oxygen.

- Dissolved oxygen less than 4 ppm is detrimental to the survival of fish.
- The D.O content of wastewater is determined by Winkler's Method.


### 3.8. CHEMICAL OXYGEN DEMAND (COD)

- The COD test is used to measure the organic matter content of wastewater, both biodegradable and non-biodegradable.
- The oxygen equivalent of organic matter that can be oxidized is measured using a strong chemical oxidizing agent in an acidic medium.
- Potassium dichromate is excellent for this purpose.
- This test is also sometimes called the dichromate-oxygen demand test.
- $\left(C O D-B O D_{u}\right)=$ Non-biodegradable organics.


### 3.9. THEORETICAL OXYGEN DEMAND (THOD)

> If the chemical formula and the quantity of all organic matter present in the sewage are known, the exact amount of oxygen required to oxidize can be calculated stoichiometrically. It is called theoretical oxygen demand (ThOD).
> For most practical cases, COD $=$ ThOD(taken), However, generally ThOD > COD > BOD > TOC

### 3.10. TOTAL ORGANIC CARBON (TOC)

> It is another method of expressing organic matter in terms of carbon content.

### 3.11. BIOCHEMICAL OXYGEN DEMAND

> Biochemical oxygen demand is used to measure the quantity of oxygen required for oxidation of biodegradable organic matter present in a water sample by aerobic biochemical action.
> Three classes of materials exert oxygen demand of wastewater:
a) Carbonaceous organic materials.
b) Oxidisable nitrogen derived from nitrite, ammonia, and other organic nitrogen compounds serve as food for specific bacteria (Nitrosomonas and Nitrobacter).
c) Chemical reducing compounds, e.g. $\mathrm{Fe}^{2+}, \mathrm{SO}_{3}{ }^{2-}$ (sulphites), $\mathrm{SO}^{2-}$ (sulfide) which are oxidized by dissolved oxygen.

- For domestic sewage, nearly all oxygen demand is due to carbonaceous organic material and is determined by the BOD test.
$\mathrm{BOD}_{5}=$ D.O. consumed in the test by diluted sample $\times \frac{\text { Vol. of the diluted sample }}{\text { Vol. of the undiluted sewage sample }}$
The above factor in the bracket is called the dilution factor.
- The first demand occurs due to oxidation of organic matter and is called carbonaceous demand or first stage demand. Later demand occurs due to biological oxidation of ammonia and is called nitrogenous demand or second stage demand.
- However, the term BOD usually means the first stage BOD.



### 3.11.1. REACTION KINETICS


$L_{t}=$ amount of organic matter present at time $t$

$$
B O D_{t}=L_{0}\left(1-10^{-k_{D} t}\right)
$$

- Unit of $K_{D}$ is in terms of per day, and it is temperature-dependent.

$$
K_{D, T^{\circ} C}=(1.047)^{T-20} K_{D, 20^{\circ} C}
$$

(Vanhoff-Arrhenius equation)

- $K_{D}$ is also sometimes called deoxygenating constant


### 3.11.2. ESTIMATION OF Kd



## 4. POPULATION EQUIVALENT

- Average standard BOD of domestic sewage is 80 gms per person per day.
- The number of people who produce the amount of BOD at the rate of 80 gms per person per day equal to that produced by industrial sewage is called the population equivalent of industrial sewage.


## 5. RELATIVE STABILITY (S)

- Relative stability ( S ) is calculated as

$$
\begin{gathered}
\mathrm{S}=\frac{\mathrm{O}_{2} \text { available in effluent }}{\text { Total } \mathrm{O}_{2} \text { required for 1st stage } \mathrm{BOD}} \\
\mathrm{~S}=100\left[1-(0.794)^{\mathrm{t}_{20}}\right]=100\left[1-(0.63)^{\mathrm{t}_{37}}\right]
\end{gathered}
$$

$T_{20} / \mathrm{t}_{37}=$ time in days for a sewage sample to decolourise a sample of methylene blue solution when incubated at $20^{\circ}$ and $37^{\circ}$, respectively.
Note: Sometimes, seeded water is used for dilution in the BOD test. The seeded water is the water with seeding of mixed bacterial culture.

In that case, for a seeded sample:

$$
\mathrm{BOD}_{5}=\frac{\left(\mathrm{D}_{1}-\mathrm{D}_{2}\right)-\left(\mathrm{B}_{1}-\mathrm{B}_{2}\right)(1-\mathrm{P})}{\mathrm{P}}
$$

Where,
$\mathrm{D}_{1}=$ Do of diluted sample immediately after dilution $\mathrm{mg} / \mathrm{l}$
$\mathrm{D}_{2}=$ Do of diluted sample after 5 days ( 120 hours) $\mathrm{mg} / \mathrm{l}$
$B_{1}=$ Do of seeded control sample before incubation, $\mathrm{mg} / \mathrm{l}$
$\mathrm{B}_{2}=$ Do of seeded control sample after 5 days incubation, $\mathrm{mg} / \mathrm{l}$
$P=$ Decimal volumetric fraction of sample used
= volume of undiluted sample/volume of diluted sample

## 6. VARIOUS NATURAL CYCLES

(1) Aerobic cycle
(2) Anaerobic cycle
(i) Aerobic cycle


Nitrogen cycle (Aerobic cycle)
(ii) Sulphur Cycle


Sulphur Aerobic cycle

## (iii) Carbon Cycle



Carbon Aerobic cycle

## (iv) Anaerobic Cycle



## CHAPTER-7-SEWAGE TREATMENT PLANT

## Treatment Methods :

## Unit Operations

The means of treatment in which the application of physical forces predomination are known as unit operations.

## Unit process

The types of treatment in which the removal of contaminants is brought about by the addition of chemicals or the use of biological mass or microbial activities.


Schematic flow diagram of a typical conventional treatment plant
PST-Primary Settling Tank, SST-Secondary Settling Tank

## Primary Treatments

## 1. Screening:

- The primary purpose of the screen is to protect pumps and other mechanical equipment.
- Head loss through the screen,
$\mathrm{h}_{\mathrm{L}}(\mathrm{m})=0.0729\left(\mathrm{~V}^{2}-\mathrm{v}^{2}\right)$
$\mathrm{V}=$ velocity through the opening of bar screen (in $\mathrm{m} / \mathrm{s}$ )
$\mathrm{v}=$ approach velocity in upstream channel (in $\mathrm{m} / \mathrm{s}$ )


## 2. Comminution and Maceration

They are installed before the fine screen.

## 3. Grit Chamber

> They are located either before or after sewage pumps in sewage treatment plants to prevent clogging of pipelines, channels, etc., due to the settlement of grit.
> Grit chamber should not allow settlement of organic materials.
> Only one section (channel) is required if velocity control devices has been used, e.g., proportional flow weir, or Parshall flume or Sutro weir.

(a) PROPORTIONAL FLOW WEIR

(b) SUTRO WEIR
> Grit chamber removes particles of size $\geq 0.2 \mathrm{~mm}$
$>$ The specific gravity of the grit is usually in the range of 2.4 to 2.65 , but for design adopt $\approx 2.65$.
> For 0.2 mm particle setting velocity $=0.025 \mathrm{~m} / \mathrm{s}$
> Surface over flow rate $(\mathrm{Q} / \mathrm{A})=2160 \mathrm{~m}^{3} / \mathrm{m}^{2} /$ day .
> Stokes law cannot be applied to grit chamber settling because the particle size is $\geq$ 0.2 mm . In this case, $\mathrm{C}_{\mathrm{D}}$ can be approximated as

$$
C_{D}=\frac{18.5}{(\operatorname{Re})^{0.6}}
$$

and $\quad V_{s}^{2}=\frac{4}{3} \frac{\left(\gamma_{s}-\gamma_{w}\right) d}{C_{D} \rho_{w}}$
> Horizontal flow velocity is $0.15-0.3 \mathrm{~m} / \mathrm{s}$
> Horizontal critical flow velocity is given by

$$
\mathrm{V}_{\mathrm{C}}=3 \text { to } 4.5 \sqrt{\mathrm{gd}\left(\mathrm{G}_{\mathrm{s}}-1\right)}
$$

$>$ Detention time is $40-60 \mathrm{sec}$.
> Depth is $1-1.5 \mathrm{~m}$.
> The freeboard is 0.3 m .
> The settling velocity of grit particles in the transition zone is also calculated by Hazen's modified formula.

$$
V_{s}=60.6 d\left(G_{s}-1\right)\left(\frac{3 T+70}{100}\right)
$$



## 4. Detritus Tank

The function of the detritus tank is to remove finer inorganic particles. It is similar to a grit chamber with the only difference that the grit chamber is meant for the removal of larger particles and detritus tank is meant for removing fine particles.

## 5. Skimming Tank

Skimmimg tank is provided for the removal of oil and greases, soaps. It is provided before sedimentation tank
(i) Detention Period $=3$ to 5 minutes.
(ii) Amount of compressed air required $=300$ to $6000 \mathrm{~m}^{3}$ per million litres of sewage.
(iii) Surface Area,
$A=0.00622 \frac{q}{V_{r}}$
Where $\mathrm{q}=$ rate of flow of sewage in $\mathrm{m}^{3} /$ day.
$\mathrm{V}_{\mathrm{r}}=$ Min. rising velocity of greasy material to be removed in $\mathrm{m} / \mathrm{min}$
$=0.25 \mathrm{~m} / \mathrm{min}$ mostly.

## Sedimentation

In primary sedimentation, organic suspended solids are settled.
Types of settling
Type I: Discrete settling
Type II: Flocculent settling
Type III: Hindered or zero settling
Type IV: Compression settling

|  | PST | Overflow rate <br> $\left(\mathbf{m}^{\mathbf{3} / \mathbf{m}^{\mathbf{2}} \mathbf{/ d a y )}}\right.$ |  | Depth | Detention <br> Time |
| :---: | :--- | :---: | :---: | :---: | :---: |
|  |  | Avg. | Peak |  |  |
| 1 | $1^{\circ}$ settling only | $25-30$ | $50-60$ | $2.5-3.5 \mathrm{~m}$ | $2-2.5 \mathrm{hr}$ |
| 2 | $1^{\circ}$ settling followed <br> by sy secondary <br> treatment | $35-50$ | $80-120$ | $2.5-3.5 \mathrm{~m}$ | $2-2.5 \mathrm{hr}$ |
| 3 | $1^{\circ}$ settling with ASP | $25-35$ | $50-60$ | $3.5-4.5$ | $2-2.5 \mathrm{hr}$ |

## Secondary Treatment (Biological Treatment)

Units based on aerobic treatment: Trickling filter, Activated Sludge Process, Oxidation ponds etc.
Units based on anaerobic treatment: Septic tank, Imhoff tank, UASB reactor
Trickling Filter


COMPARISON OF CONVENTIONAL AND HIGH RATE TRICKLING FILTER

|  | Standard | High Rate | Super high rate |
| :--- | :---: | :---: | :---: |
| Hydraulic loading <br> $\left(\mathbf{m}^{\mathbf{3}} / \mathbf{m}^{\mathbf{2}}\right.$ /day) | $1-4$ | $10-40$ (including <br> recirculation) | $40-200$ (including <br> recirculation) |
| Organic loading <br> $\mathbf{( k g ~}$ <br> $\mathbf{B O D}_{\mathbf{5}} / \mathbf{m}^{\mathbf{3}}$ /day) | $0.08-0.32$ | $0.32-1.0$ <br> (excluding <br> recirculation) | $0.6-0.8$ (excluding <br> recirculation) |
| Depth (m) | $1.8-3.0$ | $0.9-2.5$ | $4.5-12$ |
| Recirculation ratio <br> $\left(\mathbf{Q}_{\mathbf{R}} / \mathbf{Q o}_{\mathbf{0}}\right)$ | 0 | $0.5-3.0$ | $1-4$ |

(a) Conventional Trickling Filter or Low Rate Trickling Filter

$$
\mathrm{n}=\frac{100}{1+0.0044 \sqrt{\mathrm{u}}}
$$

Where,
$n=$ The efficiency of the filter and its secondary clarifier, in terms of \% of applied BOD $\mathrm{u}=$ Organic loading in $\mathrm{kg} / \mathrm{ha}-\mathrm{m} /$ day applied to the filter (called unit organic loading)
(b) High Rate Trickling Filter
(i) $F=\frac{1+\frac{\mathrm{R}}{\mathrm{I}}}{\left(1+0.1 \frac{\mathrm{R}}{\mathrm{I}}\right)^{2}}$

Where $F=$ Recirculation factor
$R / I=$ Recirculation ratio
(ii) $\mathrm{n}=\frac{100}{1+0.0044 \sqrt{\frac{\mathrm{Y}}{\mathrm{VF}}}}$

Where,
$\mathrm{Y}=$ Total organic loading in kg/day applied to the filter, i.e. the total BOD in kg .
$\frac{\mathrm{Y}}{\mathrm{VF}}$ Unit organic loading in $\mathrm{kg} / \mathrm{Ha}-\mathrm{m} /$ day
$\mathrm{V}=$ Filter volume in Ha-m.
\% efficiency of single-stage high rate trickling filter.
(iii) $\mathrm{n}=\frac{100}{1+\frac{0.0044}{1-\mathrm{n}^{\prime}} \sqrt{\frac{\mathrm{Y}^{\prime}}{\mathrm{V}^{\prime} \mathrm{F}^{\prime}}}}$

Where,
$n^{\prime}=$ Final efficiency in the two-stage filter.
$Y^{\prime}=$ Total BOD in the effluent from the first stage in $\mathrm{kg} /$ day.
$F^{\prime}=$ Recirculation factor for second stage filter
$\mathrm{V}^{\prime}=$ volume in second stage filter in ha-m.

## Sludge Digestion Tank

> The diameter of the tank is normally 6-38 m
> The depth of the tank is approximately 6-12 m
> The lower slope is 1 to $1: 3$.
> Dia/depth $=1.5-4$.
(i) When the change during digestion is linear.
(a) $V=\left(\frac{V_{1}+V_{2}}{2}\right) t$

Where,
$\mathrm{V}=$ volume of digestor in $\mathrm{m}^{3}$.
$V_{1}=$ Raw sludge added per day ( $m^{3} /$ day )
$\mathrm{V}_{2}=$ Equivalent digested sludge produced per day on completion of digestion, $\mathrm{m}^{3} /$ day . Digestion period in the day.
(b) $V=\left(\frac{V_{1}+V_{2}}{2}\right) t+V_{2} T$ with monsoon storage

Where,
$\mathrm{T}=$ number of days for which digested sludge $\mathrm{V}_{2}$ is stored (monsoon) storage)
(ii) When the change during digestion is parabolic
(a) $V=\left[V_{1}-\frac{2}{3}\left(V_{1}-V_{2}\right)\right]$ t without monsoon storage
(b) $V=\left[V_{1}-\frac{2}{3}\left(V_{1}-V_{2}\right)\right] t+V_{2} T$ with monsoon storage

## Destruction and Removal Efficiency (DRE)

DRE $=\frac{W_{\text {in }}-W_{\text {out }}}{W_{\text {in }}} \times 100$
Where,
$\mathrm{W}_{\text {in }}=$ The mass fill rate of one POHC (Principal organic Hazardous constituent) in the waste stream.

Wout $=$ Mass emission rate of the same POHC present in the exhaust emission before releasing to the atmosphere.

## Activated Sludge Process


(i) Detention period, $\mathrm{t}=\frac{\mathrm{V}}{\mathrm{Q}}$

Where
$\mathrm{V}=$ volume of the Tank in $\mathrm{m}^{3}$.
Q = Quantity of wastewater flow into the aeration tank excluding the quantity of recycled sludge ( $\mathrm{m}^{3} /$ day)
(ii) Volumetric BOD Loading or Organic Loading, (U)
$\mathrm{u}=\frac{\mathrm{QY}_{0}}{\mathrm{~V}}$
Where,
$\mathrm{QY}_{0}=$ Mass of BOD applied per day to the aeration tank through influent sewage in gm.
$\mathrm{V}=$ The volume of the aeration tank in m3.
$\mathrm{Q}=$ sewage flows into the aeration tank in $\mathrm{m}^{3}$.
$Y_{0}=B O D_{5}$ in $\mathrm{mg} /$ lit (or gm/m3) of the influent sewage.
(iii) $\frac{\mathrm{F}}{\mathrm{M}}=\frac{\mathrm{QY}}{\mathrm{V}} \mathrm{XX}_{\mathrm{t}}$

Where,
$\frac{F}{M}=\operatorname{Food}$ (F) to Microorganism (M) ratio
$Q Y_{0}=$ Daily BOD applied to the aeration system in gm.
$Y_{0}=$ five day BOD of the influent sewage in $\mathrm{mg} / \mathrm{lit}$.
$\mathrm{Q}=$ The flow of influent sewage in $\mathrm{m}^{3} /$ day.
$X_{t}=$ MLSS (Mixed liquor suspended solids) in $\mathrm{mg} /$ lit.
$\mathrm{V}=$ The volume of the Aeration Tank (lit).
$\mathrm{M}=\mathrm{V} X_{t}$ Total microbial mass in the system in gm.

## (iv) Sludge Age ( $\boldsymbol{\theta}_{\mathrm{c}}$ )


(b) $\theta_{C}=\frac{V X_{T}}{Q_{w} X_{R}+\left(Q-Q_{w}\right) X_{E}}$

Where,
$\mathrm{X}_{\mathrm{T}}=$ The Concentration of solids in the influent of the Aeration Tank called the MLSS, i.e. mixed liquor suspended solids in $\mathrm{mg} / \mathrm{lit}$.

V = Volume of Aerator
$\mathrm{Qw}=$ The volume of waste sludge per day
$X_{R}=$ The Concentration of solids in the returned sludge or in the wasted sludge (both being equal) in mg/lit.
$\mathrm{Q}=$ Sewage inflow per day.
$X_{E}=$ The Concentration of solids in the effluent in $\mathrm{mg} /$ lit.
(v) Sludge Volume Index (S.V.I)

$$
\mathrm{SVI}=\frac{\mathrm{V}_{\mathrm{ob}}}{\mathrm{X}_{\mathrm{ob}}} \times 1000
$$

Where,
$\mathrm{X}_{\mathrm{ob}}=$ Concentration of suspended solids in the mixed liquor in $\mathrm{mg} / \mathrm{lit}$.
$V_{o b}=$ Settled sludge volume in $\mathrm{ml} / \mathrm{lit}$.
S.V.I=Sludge volume index in $\mathrm{ml} / \mathrm{gm}$.
(vi) Sludge Recycle and Rate of Return Sludge
$Q_{R} \cdot X_{R}=\left(Q+Q_{R}\right) \times t$
$\frac{Q_{R}}{Q}=\frac{X_{t}}{X_{R}-X_{t}}$
Where,
$Q_{R}=$ Sludge recirculation rate in $\mathrm{m}^{3} /$ day .
$X_{t}=$ MLSS in the aeration tank in $\mathrm{mg} / \mathrm{lit}$.
$X_{R}=$ MLSS in the returned or wasted sludge in $\mathrm{mg} /$ lit.
$X_{R}=\frac{10^{6}}{S . V . I}$
S.V.I = Sludge volume index in $\mathrm{ml} / \mathrm{gm}$.

- Specific substrate utilization rate

$$
\mathrm{U}=\frac{\mathrm{Q}\left(\mathrm{Y}_{0}-\mathrm{Y}_{\mathrm{E}}\right)}{\mathrm{V} \cdot \mathrm{X}_{\mathrm{t}}} \quad \frac{1}{\theta_{\mathrm{C}}}=\alpha_{\mathrm{y}} \mathrm{U}-\mathrm{k}_{\mathrm{e}}
$$

$\alpha_{y}=1$ for MLSS and 0.6 for MLVSS, $k_{e}=0.06$

## Oxygen Requirement of the Aeration Tank

$O_{2 \text { ( equired) }}=\left[\frac{Q\left(Y_{0}-Y_{E}\right)}{f}-1.42 Q_{W} \cdot X_{R}\right] \mathrm{gm} /$ day

Where,
$\mathrm{f}=\frac{\mathrm{BOD}_{5}}{\mathrm{BOD}_{\mathrm{u}}}=\frac{5 \text { day BOD }}{\text { Ultimate BOD }}=0.68$

## Oxidation Ponds

$>$ Depth $\rightarrow 1.0$ to 1.8 m .
> Detention period $\rightarrow 2$ to 6 weeks.
$>$ Organic loading $\rightarrow 150$ to $300 \mathrm{~kg} / \mathrm{ha} /$ day.
$>$ Under hot conditions $\rightarrow$, 60 to $90 \mathrm{~kg} / \mathrm{ha} /$ day.
> Under cold conditions.

- -Length to width ratio $=2$
- Sludge Accumulation $=2$ to $5 \mathrm{~cm} /$ year
- Minimum depth to be kept $=0.3 \mathrm{~m}$.

For Inlet Pipe Design
Assume V $=0.9 \mathrm{~m} / \mathrm{s}$
Assume flow for 8 hrs.
For Outlet Pipe Design
Dia of outlet $=1.5$ dia of the inlet pipe

## Septic Tank

> Detention time $=12$ to 36 hr .
> Sludge accumulation rate $=30 \mathrm{lit} / c a p / y e a r$.
> Sewage flow $=90$ to 150 lit/capita/day.
> Cleaning period $=6$ to 12 months
> Length to width ratio $=2$ to 3 m
> Depth $=1.2$ to 1.8 m
$>$ Width $\varangle 0.9 \mathrm{~m}$.
> Free board $=0.3 \mathrm{~m}$
Volume of septic tank $=\left[\begin{array}{l}(\text { Sewage flow } \times \text { Detention time })+ \\ (\text { Sludge accumulation rate }) \times \text { Cleaning rate }\end{array}\right]$

## CHAPTER-8- DISPOSAL OF TREATED EFFLUENT

## Dilution and Dispersion

$$
C=\frac{C_{s} Q_{s}+C_{R} Q_{R}}{Q_{s}+Q_{R}}
$$

Where,
$\mathrm{C}=$ Final concentration of that material in the river
$\mathrm{C}_{\mathrm{s}}=$ Concentration of material in sewage
$C_{R}=$ Concentration of that material in the river
$\mathrm{Q}_{s}=$ Flow rate of sewage
$Q_{R}=$ Flow rate of river

## Zone of Pollution in River Stream


> Oxygen deficit $=$ Saturation DO- Actual DO
Saturation D.O at $20^{\circ} \mathrm{C} \rightarrow 9.2 \mathrm{mg} / \mathrm{lit}$.
Saturation D.O at $30^{\circ} \mathrm{C} \rightarrow 7.6 \mathrm{mg} / \mathrm{lit}$.
Saturation D.O at $0^{\circ} \mathrm{C} \rightarrow 14.6 \mathrm{mg} / \mathrm{lit}$.

> The curve showing the rate of depletion with time is known as the Deoxygenation curve.
> The curve showing the rate of accumulation of oxygen with time is termed as Reoxygenation curve.
> Oxygen Sag curve: In running a polluted stream, deoxygenation and reoxygenation occur simultaneously. The resultant oxygen deficit can be obtained by adding the deoxygenation and reoxygenation curve algebraically. The resultant curve is known as the oxygen sag curve.
$>$ TOD $>\mathrm{COD}>(\mathrm{BOD})_{\mathrm{U}}>(\mathrm{BOD})_{5}$
Where,
TOD= Theoretical oxygen demand
BOD= Biological oxygen demand
COD = Chemical oxygen demand
(BOD) $u=$ Ultimate BOD (Yu)

## Stretcher-PHELPS EQUATION

$D_{t}=\frac{k_{D} L}{k_{R}-k_{D}}\left[(10)^{-k_{D} t}-(10)^{-k_{R} t}\right]+\left[D_{0} \cdot(10)^{-k_{R} t}\right]$
$k_{D\left(T^{\circ} C\right)}=k_{D\left(20^{\circ} \mathrm{C}\right)}[1.047]^{\left(T-20^{\circ} \mathrm{C}\right)}$
$k_{R\left(T^{\circ} C\right)}=k_{R\left(20^{\circ} \mathrm{C}\right)}[1.016]^{\left(T-20^{\circ} \mathrm{C}\right)}$
$f=\frac{k_{R}}{k_{D}}$
$t_{c}=\frac{1}{k_{D}(f-1)} \log _{10}\left[\left\{1-(f-1) \frac{D_{0}}{L}\right\} f\right]$
$D_{C}=\frac{L}{f} \cdot[10]^{-k_{0} \cdot t_{z}}$
Where,
$D_{t}=$ D.O deficit in $\mathrm{mg} / \mathrm{lit}$ after t days.
$\mathrm{L}=$ Ultimate first stage BOD of the mix at a point of waste discharge in $\mathrm{mg} / \mathrm{lit}$.
$D_{o}=$ Initial oxygen deficit of the mix at the mixing point in $\mathrm{mg} / \mathrm{lit}$.
$K_{R}=$ Reoxygenation constant
$K_{D}=$ Deoxygenation constant
$\mathrm{f}=$ Self-purification constant
$t_{c}=$ Critical time at which minimum dissolved oxygen occurs i.e.
$D_{C}=$ Critical maximum oxygen deficit.

## CHAPTER-9-AIR POLLUTION

## 1. COMPOSITION OF AIR

Air is a mixture of gases present in the atmosphere. Major gases are

| Name of the gas | Percentage by volume (\%) |
| :---: | :---: |
| Nitrogen | $98.09 \%$ |
| Oxygen | $20.95 \%$ |
| Argon | $0.90 \%$ |
| Carbon dioxide | $0.032 \%$ |
| Remaining gases | Traces |

## 2. STRUCTURE OF ATMOSPHERE

| Name of the zone | Extended up to (km) |
| :---: | :---: |
| Troposphere | 0 to 12 km |
| Stratosphere | 12 to 52 km |
| Mesosphere | 52 to 92 km |
| Ionosphere or thermosphere | 92 km |

## 3. SOURCES OF AIR POLLUTION

a. Natural sources of air pollution:
b. Man-made sources of air pollution:
(i) Combustion of fuels: $\left(\mathrm{CO}_{2}\right),\left(\mathrm{SO}_{2}\right),\left(\mathrm{NO}_{2}\right)$.etc. accumulate in the atmosphere.
(ii) Industries: Emit undesirable gases $\left(\mathrm{SO}_{2}, \mathrm{CO}_{2}, \mathrm{NO}_{2}, \mathrm{NH}_{3}, \mathrm{CO}\right)$.
(iii) Thermal Power Plants: Mainly, they emit Sulphur dioxide.
(iv) Automobiles: Exhaust contains carbon monoxide (CO), methane, un-burnt carbon.

CO is the main source of air pollution in congested cities.
(v) Agricultural activities: Crop spraying and field burning.
(vi) Nuclear Power Plants: Emit various radioactive substances

Air pollutants are classified according to how they are formed, and this classification is primarily categorized into three distinct types

### 3.1. Based on Origin

1. Primary air pollutants
2. Secondary air pollutants

### 3.2. Based on Nature

1. Organic air pollutants
2. inorganic air pollutants

### 3.3. Based on the State of Matter

1. Particulates (Aerosols)
2. Gasses and vapours

## a. Primary air pollutants

> Particulate matter such as dust and aerosols.
> Pollens
> Sulphur compounds $\left(\mathrm{SO}_{2}, \mathrm{SO}_{3}, \mathrm{H}_{2} \mathrm{~S}\right)$
> Nitrogen compounds ( $\mathrm{NO}, \mathrm{NO}_{2}, \mathrm{NH}_{3}$ )
> Carbon monoxide ( CO ) and carbon dioxide $\left(\mathrm{CO}_{2}\right)$
> Photochemical oxidants

## b. Secondary Air Pollutants:

> Ozone
> PAN (Peroxy Acetyl Nitrate)
> SMOG (smoke+fog)
$\Rightarrow$ Acid rain- Sulphuric acid $\left(\mathrm{H}_{2} \mathrm{SO}_{4}\right)$ is formed by a simple chemical reaction between sulphur dioxide $\left(\mathrm{SO}_{2}\right)$ and water $\left(\mathrm{H}_{2} \mathrm{O}\right)$ vapour. It causes acid rains.

## 4. SMOG

> Smog is a synchronism of two words - i.e., Smoke and Fog.
> SMOG= SMOKE + FOG
> Smog can be of two types, Photochemical or Coal induced.
> Smog is caused by the interaction of some hydrocarbons and oxidants under sunlight, giving rise to dangerous Peroxy acetyl nitrate (PAN).
> Smog reduces visibility, causes eye irritation and damage to vegetation.

## 5. VERTICAL DISPERSION OF POLLUTANTS

> The rate at which dry air cools as it rises is called the dry adiabatic lapse rate. It is independent of the ambient air temperature.
> The dry adiabatic lapse rate can be calculated from the first law of thermodynamics (1.0 per 100m).
> The saturated adiabatic lapse rate is not a constant. Since the amount of moisture that the air can hold before condensation begins is a function of temperature.
> A reasonable average value of the moist adiabatic lapse rate in the troposphere is about $6^{\circ} \mathrm{C} / \mathrm{Km}$.

## 6. ATMOSPHERIC STABILITY

> When the environmental lapse rate and the dry adiabatic lapse rate are the same, Such atmosphere is neutral.
> The atmosphere is super adiabatic when the environmental lapse rate ( $-\mathrm{dT} / \mathrm{dz}$.) is greater than the dry adiabatic lapse rate.
> When the environmental lapse rate is less than the dry adiabatic lapse rate, Such an atmospheric condition is called stable, and the lapse rate is said to be sub adiabatic.
> When the ambient lapse rate and the dry adiabatic lapse rate are the same, the atmosphere has neutral stability.
> Super adiabatic condition prevails when the air temperature drops more than $1^{\circ} \mathrm{C} / 100 \mathrm{~m}$
> Sub adiabatic conditions prevail when the air temperature drops at the rate of less than $1^{\circ} \mathrm{C} / 100 \mathrm{~m}$.

> Temperature profiles to the left of the adiabatic lapse rate correspond to an unstable atmosphere (line) A; profiles to the right are stable (line C). The dry adiabatic lapse rate is line $B$. The speckled area is meant to suggest slopes that correspond to the wet adiabatic lapse rate.

| Vertical temperature gradient: .......... |  |
| :--- | :--- | :--- |
| Normal state ........ |  |
| Almost isothermal state |  |
| A combination of on inverse state (above the state |  |
| ground) and a normal state (from a height slightly |  |
| below the chimney orfice) |  |


| Particulate air pollutants, their sources, and effects |  |  |
| :---: | :---: | :---: |
| Pollutant | Sources | Effects |


| Suspended particulate <br> Smoke from domestic, <br> Depends on the specific <br> composition <br> matter/dust | Smoke from domestic, <br> Depends on the specific <br> composition <br> matter/dust industrial and <br> vehicular soot R | Depends on the specific <br> composition <br> matter/dust industrial and <br> vehicular soot Reduces sunlight <br> and visibility, <br> increases corrosion, <br> Pneumoconiosis, <br> Asthma, cancer, and other lung <br> diseases. |
| :--- | :--- | :--- |
| Fly ash |  | Part of the Smoke released <br> from Settles down on <br> vegetation, houses. Adds <br> chimneys of factories and to <br> the suspended participate <br> matter (SPM) <br> power plants | | Settles down on vegetation, |
| :--- |
| houses. Adds |
| chimneys of factories and to the |
| suspended participate matter |
| (SPM) |
| Power plants in the air. |
| Leachates contain harmful |
| material |

Gaseous air pollutants: their sources and effects
$\left.\begin{array}{|l|l|l|}\hline \text { Pollutant } & \text { Source } & \text { Harmful effect } \\ \left.\hline \begin{array}{l}\text { Carbon compound (CO and } \\ \mathrm{CO}\end{array}\right) & \begin{array}{l}\text { Automobile exhaust } \\ \text { burning of wood and coal }\end{array} & \begin{array}{l}\text { - Respiratory problems } \\ \text { - Green house effect }\end{array} \\ \hline \begin{array}{l}\text { Sulphur compounds ( } \mathrm{SO}_{2} \\ \left.\text { and } \mathrm{H}_{2} \mathrm{~S}\right)\end{array} & \begin{array}{l}\text { Power plants and refineries } \\ \text { volcanic eruptions }\end{array} & \begin{array}{l}\text { - Respiratory problems in } \\ \text { humnans } \\ \text { - Loss of chlorophyll in } \\ \text { plants (chlorosis) } \\ \text { - Acid rain }\end{array} \\ \hline \begin{array}{l}\text { Nitrogen Compound (NO } \\ \left.\text { and } \mathrm{N}_{2} \mathrm{O}\right)\end{array} & \begin{array}{l}\text { Motor vehicle exhaust } \\ \text { atmospheric reaction }\end{array} & \begin{array}{l}\text { - Irritation in eyes and } \\ \text { lungs }\end{array} \\ \text { - Low productively in } \\ \text { plants }\end{array}\right\}$

|  |  | visibility and aggravates <br> asthma in patients. |
| :--- | :--- | :--- |
| Fibres(Cotton, wool) | Textiles and carpet <br> weaving industries | $\bullet$ Lung disorders. |

## Air pollution control \& its principles

## 1. Source Control Technology

- Air quality management sets the tools to control air pollutant emissions.
- Control measurements describe the equipment, processes, or actions used to reduce air pollution.


## 2. Settling Chambers

- Settling chambers use the force of gravity to remove solid particles.


## 3. Cyclones

Cyclones are efficient in removing large particles but are not as efficient with smaller particles. For this reason, they are used with other particulate control devices.

## 4. Venturi Scrubbers

- Venturi scrubbers use a liquid stream to remove solid particles.
- In the venturi scrubber, gas laden with particulate matter passes through a short tube with flared ends and a constricted middle.
- This constriction causes the gas stream to speed up when the pressure is increased.
- The difference in velocity and pressure resulting from the constriction causes the particles and water to mix and combine.


## 5. Electrostatic Precipitators (ESPs)

- An ESP is a particle control device that uses electrical forces to move the particles out of the flowing gas stream and onto collector plates.
- The ESP places electrical charges on the particles, causing them to be attracted to oppositely charged metal plates located in the precipitator.


## Air Quality Standards

- Air quality standards are generally health-based guidelines that seek to establish the concentrations of air pollutants to which the public can be exposed throughout their lifetime without significant adverse effects at a population level.
- The NAAQS (National Air Quality Standards) are health-based, and the EPA sets two types of standards: primary and secondary. The primary standards are designed to protect the health of 'sensitive' populations such as asthmatics, children, and the elderly. The secondary standards are concerned with protecting the environment.
Legislative methods to control air pollution


## a. The Air (Prevention and Control of Pollution) Act, 1981

b. National Air Quality Index

Currently, the air is assessed by measuring 8 pollutants in the atmosphere. They are Particulate Matter 10 (PM 10), PM 2.5, Nitrogen dioxide ( $\mathrm{NO}_{2}$ ), Sulphur dioxide ( $\mathrm{SO}_{2}$ ), Lead, Carbon monoxide (CO) and ammonia $\left(\mathrm{NH}_{3}\right)$. One of the major drawbacks is that benzene and methane are not considered; however, benzene is carcinogenic, and methane is a greenhouse gas.

|  | National Air Quality Index |  |
| :--- | :--- | :--- |
| Air quality Index (AQI) | Colour | Possible Health Impacts |
| Good (0-50) | Green | Minimal Impact |
| Satisfactory (51-100) | Light green | Minor breathing discomfort to sensitive <br> people |
| Moderate <br> $(101-200)$ | Breathing discomfort to the people with <br> lung, heart disease, children and older <br> adults |  |
| Poor (201-300) | Orange | Breathing discomfort to people on <br> prolonged exposure |
| Very Poor (301-400) | Respiratory illness to the people on <br> prolonged exposure |  |
| Severe (>400) | Maroon | Respiratory effects even on healthy people. |

## c. Safar Mobile Application

SAFAR (System of Air Quality Weather Forecasting and Research) was launched in Delhi in 2010 during the Commonwealth Games. The App was launched by the Union Ministry of Earth Sciences (MoES) to provide online Air Quality information service in real-time.
d. Sameer

It is an app that provides hourly updates on the National Air Quality Index (NAQI) published by CPCB. The pollutants measured are $\mathrm{SO}_{2}, \mathrm{NO}_{2}, \mathrm{PM} 10$ and $2.5, \mathrm{CO}, \mathrm{O}_{3}$ etc.

## CHAPTER-10-SOLID WASTE MANAGEMENT

Solid waste refers to all non-liquid wastes. In general, this does not include excreta.
Common ordinary household and commercial waste are called refuse or municipal solid waste (MSW).

## 1. Different categories of solid waste include:

> Organic waste: wastes from the preparation of food, market place etc.
> Combustibles: paper, wood, dried leaves, packaging for relief items, etc.
> Non-combustibles: Metal, tin cans, bottles, stones, etc.
> Ashes/dust: Residue from fires used for cooking.
> Bulky waste: Tree branches, tyres, etc.
> Dead animals: Carcasses of domestic animals and livestock.
> Hazardous waste: Oil, battery acid, medical waste.
> Construction waste: Roofing, rubble, broken concrete, etc.
2. ON-SITE DISPOSAL OPTIONS

### 2.1. Communal pit disposal

> Consumers dispose of waste directly into a communal pit.
> The size of this pit will depend on the number of people it serves.
> The long-term recommended objective is six cubic meters per fifty people.

### 2.2. Family pit disposal

> Family pits may provide a better long-term option where there is adequate space.
> These should be fairly shallow (up to 1 m deep).

### 2.3. Communal bins

> A single 100-litre bin should be provided for every fifty people in domestic areas, one hundred people at feeding centres, and ten market stalls.
> In general, bins should be emptied daily.

## 3. OFF-SITE DISPOSAL OPTIONS

### 3.1. Landfilling

> Once the solid waste is transported off-site, it is normally taken to a landfill site.
> Here the waste is placed in a large excavation (pit or trench) in the ground, back-filled with excavated soil each day waste is tipped.
> Ideally, about 0.5 m of soil should cover the deposited refuse at the end of each day to prevent animals from digging up the waste and flies from breeding.

### 3.2. Incineration

> Burning or incineration is often used for the disposal of combustible waste.

### 3.3. Composting

> Composting is environmentally friendly and beneficial for crops.

### 3.4. Recycling

> It is used to make the waste reuseable.

## GATE/ESE

## Civil Engineering

Fluid Mechanics

## Important Formula Notes

## IMPORTANT FORMULAS ON FLUID MECHANICS

## CHAPTER-1-FLUID PROPERTIES

i. Mass density ( $\rho$ ):
$\rho=\frac{\mathrm{m}}{\mathrm{V}}$
Where,
$\mathrm{m}=$ mass of the material.
$\mathrm{V}=$ volume of the material.
ii. Weight density or specific weight ( $\omega$ ):
$\omega=\frac{\mathrm{m} \times \mathrm{g}}{\mathrm{V}}=(\rho \mathrm{g})$
Where,
$\mathrm{m}=$ mass of the material.
$\mathrm{V}=$ volume of the material.
$\mathrm{g}=$ acceleration due to gravity.
iii. Specific gravity or (relative density) (S):
$S=\frac{\rho}{\rho_{\omega}}$
Where,
$\rho=$ density of the material.
$\rho_{w}=$ density of water.

## iv. Specific volume (v):

$v=\frac{V}{m}=\frac{1}{\rho}$
Where,
$\mathrm{m}=$ mass of the material.
$\mathrm{V}=$ volume of the material.
$\rho=$ density of the material.
Viscosity ( $\boldsymbol{\mu}$ ): The viscosity of a fluid is a measure of its resistance to deformation at a given rate. Viscosity can be conceptualized as quantifying the frictional force that arises between adjacent layers of fluid that are in relative motion.

## Causes of viscosity

- intermolecular force of cohesion (in liquids).
- molecular momentum exchange (in gases).


## v. Newton's Law of Viscosity:

$$
\tau=\mu \frac{d u}{d y}
$$

Where,
$\mu=$ dynamic viscosity of fluid
$\frac{d u}{d y}$
$=$ rate of shear strain
SI unit of dynamic viscosity is $\mathrm{N}-\mathrm{s} / \mathrm{m}^{2}$ or pascal-sec.
Dimensional unit of dynamic viscosity $=\left[\mathrm{ML}^{-1} \mathrm{~T}^{-1}\right]$
CGS unit for dynamic viscosity is poise.
1 poise $=\frac{1}{10} \mathrm{~Pa}-\mathrm{sec}$

## vi. Kinematic viscosity (v):

$v=\frac{\mu}{\rho}$
Where,
$\mu=$ dynamic viscosity of fluid
$\rho=$ density of the material.
Its SI unit is $\mathrm{m}^{2} / \mathrm{s}$.
CGS unit of kinematic viscosity is stoke.
1 stoke $=1 \mathrm{~cm}^{2} / \mathrm{sec}=10^{-4} \mathrm{~m}^{2} / \mathrm{sec}$.

## Fluid Flow Behaviour:



- General Relationship between shear stress and velocity gradient is given by:
$\tau=\mu\left(\frac{d u}{d y}\right)^{n}+B$

| s. <br> no. | Type of fluid | $\mathbf{n}$ | $\mathbf{B}$ | Example |
| :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1 .}$ | Dilatant | $>1$ | $=0$ | Starch in water, butter etc. |
| 2. | Newtonian | $=1$ | $=0$ | Water, air etc |
| 3. | pseudoplastic | $<1$ | $=0$ | polymer solution, blood, milk etc |
| 4. | Rheopectic | $>1$ | non-zero | Gypsum paste, cream etc |
| 5. | Bingham plastic | $=1$ | Non-zero | Clay suspension, toothpaste etc. |
| $\mathbf{6 .}$ | Thixotropic | $<1$ | Non-zer0 | Inks, paints etc. |

## EFFECT OF TEMPERATURE AND PRESSURE ON VISCOSITY:

For liquid:

|  | Dynamics viscosity | Kinematic viscosity |
| :--- | :--- | :--- |
| Temperature | Decreases with increasing <br> temperature. | Decreases with increasing <br> temperature. |
| Pressure | No effect of pressure. | No effect of pressure. |

For Gas:

|  | Dynamics viscosity | Kinematic viscosity |
| :--- | :--- | :--- |
| Temperature | Increases with increasing temperature. | Increases with increasing <br> temperature. |
| Pressure | No effect of pressure. | Decreases with increasing <br> pressure. |

vii. Surface Tension: Surface tension is the apparent interfacial tensile stress (force per unit length of interface) that acts whenever a liquid has an interface such as the liquid-gas, liquid-vapour, liquid-liquid, or a liquid-solid. Surface tension is due to cohesive intermolecular force. The surface layer of fluid is like a stretched rubber member under tension, is called surface Tension ( $\sigma$ ) N/m. It can also be defined as the surface energy per unit area.
$\sigma=\frac{F}{L}=\frac{E}{A}$
Where $F, L$ are the force and length, and $E, A$ are surface energy and surface area.
Contact angle ( $\phi$ ): It is an important surface effect that causes due to the liquid interface intersection with the solid surface. A liquid is said to wet the surface when $\phi$ $<90^{\circ}$ and not to wet the surface when $\phi>90^{\circ}$.


For a curved curve surface with radius $r_{1}$ and $r_{2}$, the pressure difference between curve surface will be:
$\Delta \mathrm{P}=\sigma\left(\frac{1}{r_{1}}+\frac{1}{r_{2}}\right)\left(\mathrm{N} / \mathrm{m}^{2}\right)$

## Gauge pressure inside a water droplet:

$P_{\text {gauge }}=\frac{2 \sigma}{R} \mathrm{~N} / \mathrm{m}^{2}$

## Gauge pressure for soap bubble:

$$
P_{\text {gauge }}=\frac{4 \sigma}{R} \mathrm{~N} / \mathrm{m}^{2}
$$

## Gauge pressure inside a jet of water:

$P_{\text {gauge }}=\frac{\sigma}{R} N / m^{2}$

## viii. Capillarity Rise

This phenomenon of rise or fall of fluid column due to cohesion or adhesion.
Capillary rise in thin tube:
Capillarity rise $\mathrm{h}=\frac{4 \sigma \cos \theta}{\rho g d}$
For water: $\theta=0^{\circ}$, capillary rise: $\mathrm{h}=+\mathrm{ve}$.
For $\mathrm{Hg}: \theta=130^{\circ}$, capillary fall: $\mathrm{h}=-\mathrm{ve}$.
Capillary rise between two parallel plate with small gap between them:

$$
\mathrm{h}=\frac{2 \sigma \cos \theta}{\rho \mathrm{gt}}
$$

Where $\mathrm{t}=$ gap between two plate.

## Capillary rise between two coaxial tube:

$$
\mathrm{h}=\frac{4 \sigma \cos \theta}{\rho \mathrm{~g}\left(\mathrm{~d}_{\mathrm{o}}-\mathrm{d}_{\mathrm{i}}\right)}
$$

Where $d_{o}=$ outer diameter, $d_{i}=$ inner

## Compressibility and Bulk Modulus:

- Bulk modulus of elasticity:

$$
(k)=-\frac{\Delta P}{(\Delta V / V)}=\rho\left(\frac{d p}{d \rho}\right)
$$

- Compressibility $(\beta)$ of fluid $=\frac{1}{\text { Bulk modulus of elasticity }}$

$$
\begin{aligned}
& \beta=\frac{1}{\mathrm{~K}}=\frac{1}{\rho}\left(\frac{\mathrm{~d} \rho}{\mathrm{dp}}\right) \\
& \beta=-\frac{1}{\mathrm{~V}}\left(\frac{\Delta \mathrm{~V}}{\Delta \mathrm{P}}\right)
\end{aligned}
$$

Isothermal Bulk Modulus: $\mathrm{K}_{\mathrm{T}}=\mathrm{P}=\rho \mathrm{RT}$
Adiabatic Bulk Modulus: $\mathrm{K}_{\mathrm{A}}=\gamma \mathrm{P}=\gamma(\rho \mathrm{RT})$

## CHAPTER-2-PRESSURE AND ITS MEASUREMENT

## i. PRESSURE AT A POINT IN A FLUID AT REST

If the fluid is stationary, Pressure at a point in a fluid at rest,

$$
P=\frac{d F}{d A}
$$

If force (F) is uniformly distributed over area (A), then pressure at any point, $\mathrm{P}=\frac{\mathrm{F}}{\mathrm{A}}=\frac{\text { Force }}{\text { Area }}$

## Units of pressure:

- 1 Pascal $=1 \mathrm{~N} / \mathrm{m}^{2}$
- $1 \mathrm{MPa}=1 \mathrm{~N} / \mathrm{mm}^{2}$
- 1 bar $=10^{5}$ Pascal $=0.1 \mathrm{~N} / \mathrm{mm}^{2}$
- $1 \mathrm{~atm}=101.325 \mathrm{kPa}=0.101325 \mathrm{MPa}$
- 1 torr $=1 \mathrm{~mm} \mathrm{Hg}=133.32 \mathrm{~Pa}$
ii. PASCAL'S LAW

It state that the pressure or intensity of pressure at a point in a static fluid is equal in all directions

## iii. DIFFERENT TYPES OF PRESSURE

a. Absolute Pressure: It is the total pressure exerted on a system measured from zero pressure level or absolute vacuum pressure.
b. Atmospheric Pressure: It is the pressure exerted by environmental mass.
c. Gauge Pressure: It is the pressure, which is measured with the help of a pressure measuring instrument in which atmospheric pressure is taken as datum.
d. Vacuum Pressure: It is the value of pressure below to the atmospheric pressure.


Absolute pressure $=$ atmospheric pressure + gauge Pressure

$$
P_{\mathrm{abs}}=P_{\mathrm{atm}}+P_{\text {Gauge }}
$$

## iv. Pressure Variation In A Fluid At Rest:

$$
P=\rho g h
$$

Where,
$P=$ Pressure in fluid
$\rho=$ density of fluid
$\mathrm{g}=$ acceleration due to gravity
$h$ = height of fluid

## v. Measurement of Pressure:



Barometer: It is a pressure measuring device, that is used to measure the atmospheric pressure. It consists of a small diameter tube with mercury as a manometric fluid.


Piezometers: A Piezometer is a simple glass tube that is open at both the ends. Piezometers can't be used to measured very high pressure and gas pressures.
$\mathrm{P}=\rho \mathrm{gH}$


Simple U-tube manometer: It consists of a glass tube with one end open to the atmosphere and other end connected to a point at which pressure is to be measured.

$P_{A}+\rho_{o} g(H+x)+\rho_{m b y}-\rho_{m b y}-\rho_{m} g x=0$
Differential Manometer: A differential manometer is used to measure the difference in pressures between two pipes or two points in the pipeline.

## Upright U-tube differential manometer:


$\mathrm{P}_{\mathrm{A}}-\mathrm{P}_{\mathrm{B}}=\rho_{2} \mathrm{gh}_{2}+\left(\rho_{\mathrm{Hg}}-\rho_{1}\right) \mathrm{gh}-\rho_{1} \mathrm{gh}_{1}$
NOTE: Multiple u-tube manometer is used to measure the high pressure difference.
Inverted U-tube differential manometer:


$$
\mathrm{P}_{\mathrm{A}}-\mathrm{P}_{\mathrm{B}}=\rho_{1} g \mathrm{~h}_{1}-\rho_{2} g h_{2}-\rho g h
$$

When both the pipes are at the same level i.e. $h_{1}=h_{2}$ :

$$
\mathrm{P}_{\mathrm{A}}-\mathrm{P}_{\mathrm{B}}=\left(\rho_{1}-\rho_{2}\right) \mathrm{gh}_{1}-\rho \mathrm{gh}
$$

Inverted U-tube manometers are used when the pipelines are underground and in these manometers the density of manometric fluid is less than the density of flowing fluid ( $\rho_{\mathrm{m}}<\rho$ ).
Hydrostatic forces: When a fluid is in contact with a surface it exerts a normal force on the surface which is termed as the hydrostatic force.
Hydrostatic forces on submerged plane surfaces: Plane inclined surface at angle $\boldsymbol{\theta}$ :
Force on the surface is given by: $F=\rho g A \bar{h}$
Where $\bar{h}=$ distance of centre of gravity from the free surface.
Centre of pressure for the vertical submerged surface is given by:
$\mathrm{h}^{*}=\frac{\mathrm{I}_{\mathrm{G}} \sin ^{2} \theta}{\mathrm{~A} \bar{h}}+\overline{\mathrm{h}}$
h* = vertical distance of centre of Pressure from free surface
IG = moment of inertia at centroidal axis.
Plane vertical surface ( $\boldsymbol{\theta}=\mathbf{9 0 ^ { \circ }}$ ):

$$
\begin{aligned}
& \mathrm{F}=\rho \mathrm{g} \mathrm{~A} \overline{\mathrm{~h}} \\
& \mathrm{~h}^{*}=\frac{\mathrm{I}_{\mathrm{G}}}{\mathrm{~A} \overline{\mathrm{~h}}}+\overline{\mathrm{h}}
\end{aligned}
$$

Plane horizontal surface $\left(\theta=0^{\circ}\right)$ :
$F=\rho g A \bar{h}$
$h^{*}=\bar{h}$

## Hydrostatic forces on curved surfaces:



Horizontal component of force on curved surface: Horizontal component of force will be equal to the force on the vertical projection area and this force will act at center of pressure of the corresponding area.

Horizontal force: $F_{X}=\rho g A \bar{h}$
Where,
A = Projected Area
$\overline{\mathrm{h}}=$ depth of centroid of an area.
Vertical component of force on curved surface( $\mathrm{F}_{\mathrm{Y}}$ ): The vertical component of force on a curved surface is equal to the weight of the fluid contained by the curved surface till the free surface and this force will act at the center of gravity of the corresponding weight.
$F_{Y}=V \rho_{\text {fluid }} g$
Where $\mathrm{V}=$ volume of fluid above curve surface. $\rho_{\text {fluid }}=$ density of fluid.
So, Resultant Force: $F_{R}=\sqrt{F_{X}^{2}+F_{Y}^{2}}$
The angle from the horizontal at which this force will act:
$\tan \alpha=\frac{\mathrm{F}_{\mathrm{Y}}}{\mathrm{F}_{\mathrm{H}}}$
The moments of inertia and other geometric properties of some important plane surfaces:

| Plane surface | C.G. from <br> the base | Area | Moment of inertia <br> about an axis <br> passing through C.G. <br> and parallel to base <br> $\left(I_{\mathbf{G}}\right)$ | Moment <br> of inertia <br> about <br> (I $\mathbf{I}_{\mathbf{o}}$ |
| :--- | :---: | :---: | :---: | :---: |
| 1 Rectangle | $\mathrm{x}=\frac{\mathrm{d}}{2}$ | bd | $\frac{\mathrm{db}^{3}}{12}$ | $\frac{\mathrm{db}^{3}}{3}$ |





## Center of pressure for some standard cases:

| s.no. | Case | Name | Center of pressure |
| :---: | :---: | :---: | :---: |
| 1. |  | rectangle | $\bar{h}=\frac{H}{2}$ |
| 2. |  | Circle | $\overline{\mathrm{h}}=\frac{5 \mathrm{D}}{8}$ |
| 3. |  | Triangle(upwards) | $\overline{\mathrm{h}}=\frac{3 \mathrm{H}}{4}$ |
| 4. |  | Triangle(downwards) | $\overline{\mathrm{h}}=\frac{\mathrm{H}}{2}$ |

## vi. Liquid Containers Subjects To Constant Horizontal Acceleration:

a. For only Horizontal Acceleration:
$\tan \theta=\frac{\mathrm{a}_{\mathrm{x}}}{\mathrm{g}}$
Where,
a = horizontal acceleration
$\mathrm{g}=$ acceleration due to gravity
b. For both Vertical and Horizontal Acceleration:
$\tan \theta=\frac{\mathrm{a}_{\mathrm{x}}}{\mathrm{a}_{\mathrm{z}}+\mathrm{g}}$
Where,
$\mathrm{a}_{\mathrm{x}}=$ horizontal acceleration
$\mathrm{a}_{\mathrm{z}}=$ vertical acceleration

## Acceleration on a Straight Path:

Consider a partially filled container with a liquid, moving on a straight path with a constant acceleration.


$$
\frac{\partial P}{\partial X}=-\rho a_{x}, \frac{\partial P}{\partial y}=0, \text { and } \frac{\partial P}{\partial z}=-\rho\left(g+a_{z}\right)
$$

$$
\text { Slope }=\frac{\mathrm{dz}_{\text {isobar }}}{\mathrm{dx}}=-\frac{\mathrm{a}_{\mathrm{x}}}{\mathrm{~g}+\mathrm{a}_{\mathrm{z}}}=-\tan \theta
$$

## CHAPTER-4-BUOYANCY AND FLOATATION

## i. ARCHIMEDES PRINCIPLE

When a body submerged or immersed (fully or partially) in a static fluid, a net hydrostatic force acts on the body in the vertically upward direction. This force is known as up thrust or Buoyant force and value of this force is the weight of displaced fluid by the body.

## ii. IMPORTANT DEFINITION

Centre of Buoyancy: It is a point, through which the force of buoyancy is supposed to act.

Meta - Centre: It is defined as point about which a body starts oscillating when the body is tilted by a small angle.

Also, it is the point at which the line of action of forces of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

Meta Centre Height: The distance between the meta centre of a floating body and the centre of gravity of the body is called meta centric height.

## iii. EFFECT OF DISTURBANCES

a. Submerged Body: There is no change in the shape of displaced volume during disturbances so centre of buoyancy remains at the same point of vertical axis of the body.
b. Floating Body: The shape of displaced volume changes during disturbances so centre of buoyancy changes its location and shifts from the vertical axis of the body.

iv. STABILITY OF SUBMERGED BODY

## a. Unstable Condition



## b. Neutral condition



## c. Stable Condition



## v. STABILITY OF FLOATING BODY


a. Neutral Equilibrium (M and G coincide with each other)
b. Stable Equilibrium ( M is above Gi.e. $\mathrm{GM}>0$ )
c. Unstable Equilibrium ( M is below G i.e. $\mathrm{GM}<0$ )

Note: Higher the GM of body, higher will be the stability.

## vi. FORCE OF BUOYANCY

$$
\mathrm{F}_{\mathrm{B}}=\rho \quad \mathrm{g} \mathrm{~V}_{\text {displaced }}
$$

vii. METACENTRIC HEIGHT


Where,
$\mathrm{I}=$ moment of inertia of the plane cut by free surfaced level of liquid.
viii. TIME PERIOD OF ROLLING AND PITCHING
$T=2 \pi \sqrt{\frac{K_{G}{ }^{2}}{g \overline{G M}}}$
Where,
$\mathrm{K}=$ radius of gyration.

## CHAPTER-5-FLUID KINEMATICS

Fluid kinematics: It describes the motion of fluid and its consequences without any consideration of the forces involved.
Flow field: It is the region in which the flow parameters i.e. pressure, velocity, etc. are defined at each and every point at any instant of time.

## Approaches to study fluid motion:

Lagrangian approach: In the Lagrangian description of fluid flow, individual fluid particles are "marked," and their positions, velocities, etc. are described as a function of time.
Mathematical expression of the above statement:

$$
\vec{S}=S\left(\vec{S}_{0}, t\right)
$$

Where,
$\vec{S}_{0}$ : initial position at a given time $\mathrm{t}=\mathrm{t}_{\mathrm{t}}$.
Eulerian approach: In the Eulerian description of fluid flow, a finite flow domain or control volume is considered rather than individual fluid particles. The fluid flow enters and exits this control volume. The Eulerian description is more often used for fluid mechanics applications and experimental measurements. The Lagrangian method of description can always be derived from the Eulerian method.

## Types of fluid flow:

## Steady \& Unsteady flow:

For steady flow: $\left(\frac{\partial V}{\partial t}\right)_{x_{0}, y_{0}, z_{0}}=0,\left(\frac{\partial P}{\partial t}\right)_{x_{0}, y_{0}, z_{0}}=0,\left(\frac{\partial \rho}{\partial t}\right)_{x_{0}, y_{0}, z_{0}}=0$
For unsteady flow: $\left(\frac{\partial V}{\partial t}\right)_{x_{0}, y_{0}, z_{0}} \neq 0,\left(\frac{\partial P}{\partial t}\right)_{x_{0}, y_{0}, z_{0}} \neq 0,\left(\frac{\partial \rho}{\partial t}\right)_{x_{0}, y_{0}, z_{0}} \neq 0$

## Uniform \& Non-uniform flow:

For uniform flow: $\overrightarrow{\mathrm{V}}=\mathrm{V}(\mathrm{t})$ and
$\left(\frac{d V}{d s}\right)_{t=\text { constant }}=0$

For non-uniform flow:
$\left(\frac{d V}{d s}\right)_{t=\text { constant }} \neq 0$

## Laminar \& Turbulent flows:

Laminar flow is the flow in which the fluid particles move along well-defined paths or stream line \& all the streamlines are straight \& parallel (flow of blood in veins).

Turbulent flow is the flow in which the fluid particles move in a zig zag path which results in eddies formation and hence energy losses(pipe flow).

The deciding parameter is the Reynolds number (Re) which determines that whether the flow is laminar or turbulent.

## Compressible \& incompressible flow:

Compressible flow: Compressible flow is a type of flow in which the density of the fluid changes from point to point.

```
\rho\not= constant
```

Incompressible flow: While a flow in which the density is constant for fluid flow is called the incompressible flow.
$\rho=$ constant
Rotational \& Irrotational flows: Rotational flow is the flow in which the fluid particles while flowing along streamlines also rotate about their own axis.

Irrotational flow is the flow in which the fluid particles while flowing along the streamlines do not rotate about their own axis.

## 1-D, 2-D and 3-D flow:

For 1 - D flow: $\mathrm{u}=\mathrm{f}(\mathrm{x}), \mathrm{v}=0 \mathrm{andw}=0$
For $2-\mathrm{D}$ flow: $\mathrm{u}=\mathrm{f}_{1}(\mathrm{x}, \mathrm{y}), \mathrm{v}=\mathrm{f}_{2}(\mathrm{x}, \mathrm{y})$ and $\mathrm{w}=0$

For 3 - $D$ flow: $u=f_{1}(x, y, z), v=f_{2}(x, y, z)$ and $w=f_{3}(x, y, z)$

## Flow pattern description:

Streamlines: It is defined as an imaginary curve or line in the flow field so that the tangent to the curve at any point represents the direction of the instantaneous velocity at that point.
$\frac{d x}{u}=\frac{d y}{v}=\frac{d z}{w}$ equation of streamline in 3D
$\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k} \\ d x & d y & d z \\ u & v & w\end{array}\right|=0$
Streak line: A streak line at any instant is the locus of all fluid particles passed sequentially through a prescribed point (fixed point) in the flow field. The streak line equation can be derived using the Lagrangian approach as the focus is on individual particles passing through a point.

Path line: It represents the path traced by an inert tracer fluid particle over a period of time.

For steady flow, Streamlines, Streak lines \& path lines are identical.
Timeline: A timeline is a set of adjacent fluid particles that were marked at the same (earlier) instant in time

## Continuity equation:

By continuity equation:
Mass flow rate at inlet = mass flow rate at outlet

$$
\frac{\mathrm{d} \rho}{\mathrm{dt}}+\nabla \cdot(\rho \overrightarrow{\mathrm{V}})=0
$$

Continuity Equation for Steady and 3 - D Incompressible Flow:
$\nabla \cdot \vec{V}=\operatorname{Div}(\vec{V})=0$
It can also be written as:
$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$
Continuity Equation for Steady and 2 - D Incompressible Flow:
$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$
Continuity equation in polar coordinate:
$\frac{\mathrm{v}_{\mathrm{r}}}{\mathrm{r}}+\frac{\partial \mathrm{v}_{\mathrm{r}}}{\partial \mathrm{r}}+\frac{\partial \mathrm{v}_{\theta}}{\mathrm{r} \partial \theta}=0$
In simplified form:
Fluid in compressible: $\rho_{1} \mathrm{~A}_{1} \mathrm{~V}_{1}=\rho_{2} \mathrm{~A}_{2} \mathrm{~V}_{2}$
If fluid is Incompressible i.e. $\rho=$ constant
$\mathrm{A}_{1} \mathrm{~V}_{1}=\mathrm{A}_{2} \mathrm{~V}_{2}$

## Acceleration of fluid flow:

$$
a_{x}=\frac{d u}{d t}=u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}+w \frac{\partial u}{\partial z}+\frac{\partial u}{\partial t}
$$

Similarly, acceleration in $y$ and $z$ directions is given by:

$$
\begin{aligned}
& a_{y}=\frac{d v}{d t}=u \frac{\partial v}{\partial x}+v \frac{\partial v}{\partial y}+w \frac{\partial v}{\partial z}+\frac{\partial v}{\partial t} \\
& a_{z}=\frac{d w}{d t}=u \frac{\partial w}{\partial x}+v \frac{\partial w}{\partial y}+w \frac{\partial w}{\partial z}+\frac{\partial w}{\partial t}
\end{aligned}
$$

## Material derivative:

(i). The total differential $\frac{d}{d t}$ is known as the material or substantial derivative with respect to time.
(ii). The first term $\frac{\partial}{\partial t}$ in the right-hand side of is known as temporal or local derivative which expresses the rate of change with time, at a fixed position.
(iii). The last three terms in the right-hand side of the equation, are together known as convective derivative which represents the time rate of change due to change in position in the field.

| Type of flow | Material acceleration |  |
| :---: | :---: | :---: |
|  | Temporal | convective |
| Steady and uniform | 0 | 0 |
| Steady and non-uniform | 0 | exists |
| Unsteady and uniform | exists | 0 |


| Unsteady and non- <br> uniform | exists | exists |
| :---: | :---: | :---: |

Total acceleration: The total acceleration is given by the following vector:
$\vec{a}=a_{x} \hat{i}+a_{y} \hat{j}+a_{z} \hat{k}$
$\overrightarrow{\mathrm{a}}=\frac{\mathrm{d} \overrightarrow{\mathrm{V}}}{\mathrm{dt}}=\frac{\partial \overrightarrow{\mathrm{V}}}{\partial \mathrm{t}}+\overrightarrow{\mathrm{V}} \cdot \nabla \overrightarrow{\mathrm{V}}$
Thus, magnitude of the total acceleration is given by:

$$
a=\sqrt{a_{x}^{2}+a_{y}^{2}+a_{z}^{2}}
$$

## Fundamental types of motion:

- Translation

So, Linear strain (linear deformation):

$$
\dot{\varepsilon}_{\mathrm{xx}}=\frac{\partial \mathrm{u}}{\partial \mathrm{x}}, \dot{\varepsilon}_{\mathrm{yy}}=\frac{\partial \mathrm{v}}{\partial \mathrm{y}} \text { and } \dot{\varepsilon}_{\mathrm{zz}}=\frac{\partial \mathrm{w}}{\partial \mathrm{z}} .
$$

- Shear strain (angular deformation):

$$
\gamma_{x y}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), \gamma_{y z}=\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right), \gamma_{z x}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)
$$

- Rotation:

$$
\omega_{x}=\frac{1}{2}\left(\frac{\partial w}{\partial y}-\frac{\partial \mathbf{v}}{\partial z}\right), \omega_{y}=\frac{1}{2}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right) \text { and } \omega_{z}=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) .
$$

In vector form ,

$$
\vec{\omega}=\frac{1}{2}(\nabla \times \vec{V})
$$

When the rotation components at all the points in the flow field are zero, the flow will constitute the irrotational flow.

$$
\nabla \times \vec{V}=0
$$

Angular velocity: Angular velocity is given by:
$\vec{\omega}=\omega_{x} \hat{i}+\omega_{y} \hat{j}+\omega_{z} \hat{k}$
$\vec{\omega}=\frac{1}{2}\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial \mathbf{x}} & \frac{\partial}{\partial \mathbf{y}} & \frac{\partial}{\partial \mathbf{z}} \\ \mathbf{u} & \mathrm{v} & \mathbf{w}\end{array}\right|$

Where $\omega_{\mathrm{x}}=\frac{1}{2}\left(\frac{\partial \mathbf{w}}{\partial \mathbf{y}}-\frac{\partial \mathbf{v}}{\partial \mathrm{z}}\right), \omega_{\mathrm{y}}=\frac{1}{2}\left(\frac{\partial \mathbf{u}}{\partial \mathbf{z}}-\frac{\partial \mathbf{w}}{\partial \mathbf{x}}\right)$ and $\omega_{\mathrm{z}}=\frac{1}{2}\left(\frac{\partial \mathbf{v}}{\partial \mathbf{x}}-\frac{\partial \mathbf{u}}{\partial \mathbf{y}}\right)$.
Vorticity ( $\boldsymbol{\Omega}$ ): Vorticity $(\xi)$ in the simplest form is defined as a vector which is equal to two times the rotation vector. It is given by:
$\vec{\Omega}=2 \vec{\omega}=\operatorname{curl}(\mathrm{V})$
$\Omega_{\theta}=\frac{\partial \mathrm{V}_{\mathrm{r}}}{\partial \mathrm{z}}-\frac{\partial \mathrm{V}_{\mathrm{z}}}{\partial \mathrm{r}}$
$\Omega_{\mathrm{r}}=\frac{1}{\mathrm{r}} \frac{\partial \mathrm{V}_{\mathrm{z}}}{\partial \theta}-\frac{\partial \mathrm{V}_{\theta}}{\partial \mathbf{z}}$
$\Omega_{\mathrm{z}}=\frac{\partial \mathrm{V}_{\theta}}{\partial \mathrm{r}}-\frac{1}{\mathrm{r}} \frac{\partial \mathrm{V}_{\mathrm{r}}}{\partial \theta}+\frac{\mathrm{V}_{\theta}}{\mathrm{r}}$
Where $\mathrm{V}_{\mathrm{r}}, \mathrm{V}_{\theta}$ and $\mathrm{V}_{\mathrm{z}}$ are the velocities in radial, tangential and z directions respectively.
Circulation (Г): Circulation is defined as the line integral of tangential component of velocity vector along a closed curve.
Circulation $=\oint \overrightarrow{\mathrm{V}} . \mathrm{dl}$
It can also be written as:
Circulation $(\Gamma)=$ Vorticity $\times$ Area
Velocity potential $(\phi)$ : It is a scalar function of space \& time such that its negative derivate with respect to any direction gives the fluid velocity in that direction. Velocity potential is defined for irrotational flow only.

$$
\mathrm{u}=-\frac{\partial \phi}{\partial \mathrm{x}} \text { and } \mathrm{v}=-\frac{\partial \phi}{\partial \mathrm{y}}
$$

In polar coordinate:

$$
u_{r}=-\frac{\partial \phi}{\partial r} \text { and } u_{\theta}=-\frac{1}{r} \frac{\partial \phi}{\partial \theta}
$$

The continuity equation is given by:

$$
\begin{aligned}
& \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right) \phi=0 \\
& \nabla^{2} \phi=0
\end{aligned}
$$

Stream function( $\Psi$ ): It is defined as the scalar function of space and time, such that its patrial derivative with respect to any direction gives the velocity component at right angles to that direction. This is only define for incompressible 2D flow.
$u=\frac{\partial \Psi}{\partial \mathrm{y}}$ and $\left.\mathrm{v}=-\frac{\partial \psi}{\partial \mathrm{x}}\right\}$ where, $\psi \rightarrow$ streamfunction.

- For irrotational flow stream function must satisfy Laplace equation.

So, $\nabla^{2} \Psi=0$, (for irrotational flow)
$\nabla^{2} \Psi \neq 0$, (for rotational flow)

- Slope of equi-stream line: $\left(\frac{d y}{d x}\right)_{\psi=c}=\frac{v}{u}$.
- For 2 - D compressible steady flow: Discharge per unit width between two points in a flow $=$ absolute difference between values of stream function through those two points, i.e.
- $\frac{\mathrm{Q}_{2}-\mathrm{Q}_{1}}{\mathrm{~b}}=\left|\Psi_{2}-\Psi_{1}\right|$
- For 2 - D compressible steady flow: Discharge per unit width between two points in a flow = absolute difference between values of stream function through those two points, i.e.
- $\frac{\mathrm{Q}_{2}-\mathrm{Q}_{1}}{\mathrm{~b}}=\left|\Psi_{2}-\Psi_{1}\right|$


## CHAPTER-6-FLUID DYNAMICS

## i. EQUATION OF MOTION FOR FLUID FLOW

According to Newton's Second law of motion, net force acting on a fluid element in given direction is equal to mass $m$ of the fluid multiplied by the acceleration $a_{x}$ in the $x$-direction.
$\mathrm{F}_{\mathrm{x}}=\mathrm{max}$
In the fluid flow, the following forces are present:
$F_{g}=$ gravity force
$\mathrm{F}_{\mathrm{p}}=$ pressure force
$\mathrm{F}_{\mathrm{v}}=$ viscous force
$\mathrm{F}_{\mathrm{T}}=$ Turbulence force
$\mathrm{F}_{\mathrm{c}}=$ Compressibility force
Thus, Net force, $\mathrm{F}_{\mathrm{x}}=\mathrm{Fg}_{\mathrm{g}}+\mathrm{F}_{\mathrm{p}}+\mathrm{F}_{\mathrm{v}}+\mathrm{F}_{\mathrm{T}+} \mathrm{Fc}_{\mathrm{c}}$
a. Reynold's equation of motion: If, Fc force is neglected.
$F_{x}=F_{g}+F_{p}+F_{v}+F_{T}$.
b. Navier-Stokes equation of motion:If, $F_{T}$ force is also neglected.
$F_{x}=F_{g}+F_{p}+F_{v}$
c. Euler equation of motion: If, $F_{v}$ force is also neglected.

$$
F_{x}=F_{g}+F_{p}
$$

## ii. EULER'S EQUATION OF FLUID MOTION

$$
\frac{d p}{\rho}+v d v+g d z=0
$$

## iii. BERNOULLI'S THEOREM

a. Assumptions:

1. Fluid is Non-viscous.
2. Steady flow.
3. Incompressible Fluid.
4. Ideal Fluid.
5. Irrotational Flow.
b. Classical Bernoulli's equation:
$\frac{P}{\rho g}+\frac{v^{2}}{2 g}+z=$ constant
$\frac{P}{\rho g}+z=$ Piezometric head

## c. For Ideal Fluid:

$$
\frac{p_{1}}{\omega}+\frac{v_{1}^{2}}{2 g}+z_{1}=\frac{p_{2}}{\omega}+\frac{v_{2}^{2}}{2 g}+z_{2}
$$

## d. For Real Fluid:

$\frac{p_{1}}{\omega}+\frac{v_{1}^{2}}{2 g}+z_{1}=\frac{p_{2}}{\omega}+\frac{v_{2}^{2}}{2 g}+z_{2}+\operatorname{head} \operatorname{losses}\left(h_{f}\right)$
iv. VENTURI-METER: It is a converging-diverging device that is used to measure discharge.


Geometric details: $d_{2}=\left(\frac{1}{3}\right.$ to $\left.\frac{1}{2}\right) d_{1}$

Angle of convergence ( a ) $=20-22^{\circ}$ (To avoid cavitation)
Angle of divergence $(\beta)=$ must be below $7^{\circ}$ (To avoid Boundary layer separation).
a. Theoretical discharge: $Q_{\text {theo }}=\frac{A_{1} A_{2}}{\sqrt{A_{1}^{2}-A_{2}^{2}}} \times \sqrt{2 g h}$
b. Actual Discharge: $Q_{a c}=c_{d} \frac{A_{1} A_{2}}{\sqrt{A_{1}^{2}-A_{2}^{2}}} \times \sqrt{2 g h}$
c. Coefficient of Discharge: $C_{d}=\frac{Q_{\text {actual }}}{Q_{\text {theoretical }}}$

## d. Calculation of $\mathbf{h}$ :

1. The manometric fluid has higher specific gravity $\left(\mathrm{S}_{\mathrm{m}}\right)$ than the specific gravity of the flowing fluid(S), then,

$$
\mathrm{h}=\mathrm{x}\left(\frac{\mathrm{~S}_{\mathrm{m}}}{\mathrm{~S}}-1\right)
$$

2. The manometric fluid has lower specific gravity $\left(S_{m}\right)$ than the specific gravity of the flowing fluid(S), then

$$
\mathrm{h}=\mathrm{x}\left(1-\frac{\mathrm{S}_{\mathrm{m}}}{\mathrm{~S}}\right)
$$

## v. ORIFICE-METER


a. Coefficient of Contraction, $C_{c}=\frac{\text { Actual Area at vena-contracta }}{\text { Theoretical Area at vena-contracta }}=\frac{A_{2}}{A_{o}}$
b. Discharge, $Q=C_{d} \cdot \frac{A_{1} A_{o}}{\sqrt{A_{1}^{2}-A_{o}^{2}}} \times \sqrt{2 g h}$
c. Coefficient of Discharge, $C_{d}=C_{c} \times \frac{\sqrt{1-\frac{A_{o}^{2}}{A_{1}^{2}}}}{\sqrt{1-\frac{C_{c}^{2} A_{o}^{2}}{A_{1}^{2}}}}$

## vi. PITOT-TUBE


a. Velocity: $V_{1}=\sqrt{2 g h}=\sqrt{2 g(\text { stagnation head }- \text { static head })}$
b. Measurement of velocity in Pipes:

Velocity, $V_{1}=\sqrt{2 g h}$
Where $h=x\left[\frac{S_{m}}{S}-1\right]$
c. Actual Velocity, $\mathrm{V}_{\mathrm{act}}=\mathrm{C}_{\mathrm{v}} \sqrt{2 \mathrm{gh}}$

## Different types of devices:

| Type of <br> flowmeter | Accuracy | Cost | Loss of total <br> head | Typical values of <br> $\mathbf{C}_{\mathbf{d}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Venturi-meter | High | High | Low | 0.95 to 0.98 |
| Orifice-meter | Low | Low | High | 0.60 to 0.65 |
| Flow Nozzle | Intermediate to <br> venturi and <br> orifice | Intermediate <br> to venturi and <br> orifice | Intermediate to <br> venturi and <br> orifice | $0.70 \quad 00.80$ |

Static pitot tube (Prandtl tube):

Prandtl tube measures both stagnation pressure and static pressure using the same
device.

$v=C_{v} \sqrt{2 g h\left(\frac{S_{m}}{S}-1\right)}$
Where $h=$ manometric head difference.

## Momentum equation:

F. $\mathrm{dt}=\mathrm{d}(\mathrm{mv})$
which is known as the impulse-momentum equation and states that impulse of a force $F$ acting on a fluid of mass $m$ in a short interval of time $d t$ is equal to the change of momentum $d(\mathrm{mv})$ in the direction of force.
Change in momentum of fluid $=$ final momentum - initial momentum.
Change in momentum of bend $=$ initial momentum - final momentum.

$\therefore F_{x}=\rho Q\left(V_{1}-V_{2} \cos \theta\right)+p_{1} A_{1}-p_{2} A_{2} \cos \theta$
$\therefore \mathrm{F}_{\mathrm{y}}=\rho \mathrm{Q}\left(-\mathrm{V}_{2} \sin \theta\right)-\mathrm{p}_{2} \mathrm{~A}_{2} \sin \theta$
Resultant force: $\mathrm{F}_{\mathrm{R}}=\sqrt{\mathrm{F}_{\mathrm{x}}^{2}+\mathrm{F}_{\mathrm{y}}^{2}}$
The angle made by the resultant force with horizontal direction is given by:
$\tan \theta=\frac{F_{y}}{F_{x}}$
Moment of momentum equation: It states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.
According to moment of momentum principle:

Resultant torque $=$ rate of change of moment of momentum
$T=r Q\left[V_{2} r_{2} \pm V_{1} r_{1}\right]$, where sign depends on direction

## Applications:

1. For analysis of flow problems in turbines and centrifugal pumps.
2. For finding torque exerted by water on sprinkler

Vortex flow: A vortex is a region in a fluid in which the flow revolves around an axis that may be straight or curved. Such a type of flow is called vortex flow.
Equation of Motion for Vortex Flow:
$d p=\rho \frac{v^{2}}{r} d r-\rho g d z$
Plane vortex flows: Plane circular vortex is characterized by flow in which streamlines are concentric circles. The velocity field can be described as (assuming circle centre as pole or origin):
$V_{r}=0$ and $V_{\theta} \neq 0$.
Forced Vortex Flow: Forced vortex flow is defined as that type of vortex flow in which some external torque is required to rotate the fluid mass. The liquid will move as a rigid body with the container, and every fluid particle assumes the same angular velocity $(\omega)$ as that of the container.
$\omega=\frac{\mathrm{v}}{\mathrm{r}}=$ constnat

## Equation of Forced Vortex Flow.

$$
\left(p_{2}-p_{1}\right)=\frac{\rho}{2}\left[v_{2}^{2}-v_{1}^{2}\right]-\rho g\left[z_{2}-z_{1}\right]
$$



If points 1 and 2 lies on the free surface of the liquid, then $p_{1}=p_{2}$ and hence above equation reduces to:

$$
\left[\mathrm{z}_{2}-\mathrm{z}_{1}\right]=\frac{\left[\mathrm{v}_{2}^{2}-\mathrm{v}_{1}^{2}\right]}{2 \mathrm{~g}}
$$

Also;
$\mathrm{h}=\frac{\mathrm{v}^{2}}{2 \mathrm{~g}}=\frac{\omega^{2} \times \mathrm{r}^{2}}{2 \mathrm{~g}}$
Thus, ' $h$ ' varies with the square of $r$. Above equation is an equation of a parabola. It means the free surface of the liquid is a paraboloid.

Volume of paraboloid formed due to rotation is:-
$V_{p}=\frac{\pi r^{2} h}{2}$; where ' $r$ 'is radius and height of paraboloid formed is given by ' $h$ '
Free Vortex Flow: Free vortex flows are the plane circular vortex flows where the total mechanical energy remains constant in the entire flow field, i.e. does not vary from streamline to streamline. No external torque is required to rotate the fluid mass; fluid is rotating due to the previously imparted rotation.
$\mathrm{vr}=\mathrm{C}$

## CHAPTER-7- LAMINAR FLOW

Introduction: Laminar flow (streamline flow) occurs when a fluid flows in parallel layers, with no disruption between the layers.

Reynold's number: $R_{e}=\frac{\rho V D}{\mu}=\frac{V D}{v}$
Where, $\mathrm{V}=$ mean velocity of flow in pipe.
$\mu=$ dynamic viscosity of the liquid ( $N-s / m^{2}$ )
$v=$ Kinematic viscosity of the liquid ( $\mathrm{m}^{2} / \mathrm{s}$ )
$D=$ diameter $\rightarrow$ for pipe, $D=$ length $\rightarrow$ for plate
Characteristic dimension: For the flow through non - circular pipes characteristic dimension is given by:

$$
\mathrm{D}_{\mathrm{h}}=\frac{4 \mathrm{~A}_{\mathrm{c}}}{\mathrm{P}}
$$

Where,
$\mathrm{A}_{\mathrm{c}}=$ Cross - section area of the pipe
$P=$ Perimeter of the pipe

| Pipe | Plate |
| :---: | :---: |
| $\mathrm{Re}_{\mathrm{e}}<2000$ laminar | $\mathrm{Re}_{\mathrm{e}}<5 \times 10^{5}$ Laminar |
| $2000<\mathrm{Re}_{\mathrm{e}}<4000$ Transient | $\mathrm{Re}_{\mathrm{e}}>5 \times 10^{5}$ turbulent |
| $\mathrm{R}_{\mathrm{e}}>4000$ turbulent | [transient is small so neglected] |

## Hagen-Poiseuille flow [Laminar flow in a pipe]:

## Assumptions:

(1). steady flow.
(2). flow is fully developed.

Velocity profile is constant w.r.t length in the fully developed flow.

$\frac{\partial P}{\partial x}=$ constant
Laminar flow through circular pipe:

(a)
(b)

Shear stress: $\tau=\frac{-\partial P}{\partial x} \times \frac{r}{2}$
It varies linearly along the pipe and it becomes maximum at $r=R$.
Velocity variation: $u=\frac{1}{4 \mu}\left(-\frac{\partial P}{\partial x}\right) R^{2}\left(1-\frac{r^{2}}{R^{2}}\right)$
Velocity varies parabolically along the pipe and it becomes maximum at $\mathrm{r}=0$.
Maximum velocity: $\mathrm{U}_{\max }=\frac{1}{4 \mu}\left(-\frac{\partial \mathrm{P}}{\partial \mathrm{x}}\right) \mathrm{R}^{2}$
$\mathrm{u}_{\mathrm{avg} .}=\frac{\mathrm{u}_{\mathrm{max}}}{2}$
Pressure drop: $P_{1}-P_{2}=\frac{32 \mu \mathrm{VL}}{D^{2}}$
Friction factor: $f=\frac{64}{\mathrm{R}_{\mathrm{e}}}$
coefficient of friction: $f^{\prime}=\frac{f}{4}=\frac{16}{R e}$
Discharge: $\mathrm{Q}=-\frac{\pi \mathrm{R}^{4}}{8 \mu}\left(\frac{\partial \mathrm{P}}{\partial \mathrm{x}}\right)=-\frac{\pi \mathrm{D}^{4}}{128 \mu}\left(\frac{\partial \mathrm{P}}{\partial \mathrm{x}}\right)$

## Laminar flow between two fixed parallel plates [with unit width]:



Laminar flow between parallel plates when one of the plates is moving and other one is stationary is known as "Couette" flow.
$\frac{\partial \mathrm{P}}{\partial \mathbf{x}}=\frac{\partial \tau}{\partial \mathbf{y}}$
Velocity Distribution: $u=\frac{1}{2 \mu}\left(\frac{-\partial P}{\partial x}\right)\left(\mathrm{ty}-\mathrm{y}^{2}\right)$
Maximum velocity: $\mathrm{u}_{\max }=-\frac{\mathrm{t}^{2}}{8 \mu}\left(\frac{\partial \mathrm{P}}{\partial \mathrm{x}}\right)$
$\mathrm{V}_{\text {avg. }}=\frac{2}{3} \mathrm{U}_{\max }=$ Average velocity
Shear Stress distribution: $\tau=\left(\frac{-\partial \mathrm{P}}{\partial \mathrm{x}}\right)\left(\frac{\mathrm{t}}{2}-\mathrm{y}\right)$


Discharge per unit width: $\mathrm{Q}=\frac{-1}{12 \mu}\left(\frac{\partial \mathrm{P}}{\partial \mathrm{x}}\right) \mathrm{t}^{3}$
Pressure difference: $P_{1}-P_{2}=\frac{12 \mu V_{\text {avg. }}}{t^{2}}$
Share stress distribution: $\tau=\left(\frac{-\partial \mathrm{P}}{\partial \mathrm{x}}\right)\left(\frac{\mathrm{t}}{2}-\mathrm{y}\right)$
$\tau_{\max }=\left(\frac{-\partial \mathrm{P}}{\partial \mathrm{x}}\right) \cdot \frac{\mathrm{t}}{2}$

## iv. CORRECTION FACTOR

## a. Momentum Correction Factor:

$\beta=\frac{\text { Momentum per second based on actual velocity }}{\text { Momentum per second based on average velocity }}$
b. Kinetic Energy Correction Factor:
$\alpha=\frac{\text { K.E. / sec based on acutal velocity }}{\text { K.E. / sec based on average velocity }}$

## CHAPTER-8-BOUNDARY LAYER THEORY

i. Boundary layer is a narrow thin region near the solid boundary where velocity gradient exists.

Growth of boundary layer over a flat plate:


The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the boundary layer.


## Growth of boundary layer over a flat plate:



## Boundary conditions:

(i). at $y=0, u=0$
(ii). $y=\delta, u=u_{\infty}$
(iii). $y=\delta, \frac{d u}{d y}=0$
(iv). $x=0, \delta=0$

Boundary layer thickness ( $\mathbf{\delta}$ ): It is defined as the vertical distance from boundary up to the point where velocity becomes $99 \%$ of free stream velocity.

For numerical, at $y=\delta ; u=u_{\infty}$
Displacement thickness ( $\boldsymbol{\delta}^{*}$ ): It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in mass flow rate on account of boundary layer formation. It is denoted by $\delta^{*}$.
$\delta^{*}=\int_{0}^{\delta}\left(1-\frac{\mathrm{u}}{\mathrm{u}_{\infty}}\right) \mathrm{dy}$
Momentum thickness ( $\boldsymbol{\theta}$ ): It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced in addition to displacement thickness to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation.

$$
\theta=\int_{0}^{\delta} \frac{\mathrm{u}}{\mathrm{u}_{\infty}}\left(1-\frac{\mathrm{u}}{\mathrm{u}_{\infty}}\right) \mathrm{dy}
$$

Energy thickness ( $\boldsymbol{\delta}_{\mathrm{e}}$ ): It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced in addition to displacement thickness to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation".

$$
\delta_{\mathrm{e}}=\int_{0}^{\delta} \frac{\mathrm{u}}{\mathrm{u}_{\infty}}\left(1-\frac{\mathrm{u}^{2}}{\mathrm{u}_{\infty}^{2}}\right) \mathrm{dy}
$$

Shape Factor(H): It is defined as the ratio of displacement thickness to momentum thickness.

$$
\mathrm{H}=\frac{\delta^{*}}{\theta}
$$

Flow over a flat plate: Von-Karman momentum integral equation:

$$
\frac{\tau_{0}}{\rho u_{\infty}^{2}}=\frac{d \theta}{d x}
$$

Reynolds number is given by: $\operatorname{Re}=\frac{\rho \mathrm{u}_{\infty} \mathrm{L}}{\mu}$
If, $\operatorname{Re}<5 \times 10^{5} \rightarrow$ Laminar flow.
Re $>5 \times 10^{5} \rightarrow$ turbulent flow.

## Coefficient of Drag ( $\mathrm{C}_{\mathrm{D}}$ )

$F_{D}=\frac{C_{D}{ }^{*}}{2} \rho A U^{2}$
Where, $\mathrm{F}_{\mathrm{D}}$ is the drag force experienced by the motion of fluid over flat plate, $\mathrm{C}_{\mathrm{D}}{ }^{*}=$ skin friction coefficient.

Average skin friction coefficient:
$C_{D}=\frac{F_{D}}{\frac{1}{2} \rho U^{2}}$
Blasius equations: when no velocity distribution is given consider blasius equations for calculations,

| Parameter | Laminar flow | Turbulent flow |
| :---: | :---: | :---: |
| $\delta$ | $\frac{5 x}{\sqrt{\operatorname{Re}_{x}}}$ | $\frac{0.371 x}{\left(\mathrm{Re}_{x}\right)^{1 / 5}}$ |
| $\mathrm{C}_{D}$ | $\frac{1.328}{\sqrt{\operatorname{Re}_{L}}}$ | $\frac{0.074}{\left(\mathrm{Re}_{L}\right)^{1 / 5}}$ |
| $C_{D x}$ | $\frac{0.664}{\sqrt{\operatorname{Re}_{x}}}$ | $\frac{0.038}{\left(\mathrm{Re}_{x}\right)^{1 / 5}}$ |

Boundary layer thickness and coefficient of drag for different velocity distribution:

| Velocity Distribution | $\delta$ | $\mathbf{C}_{\mathbf{D}}$ |
| :--- | :--- | :--- |


| 1. $\frac{u}{U}=2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^{2}$ | $5.48 \times / \sqrt{R_{e_{x}}}$ | $1.46 / \sqrt{R_{e_{L}}}$ |
| :--- | :--- | :--- |
| 2. $\frac{u}{U}=\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^{3}$ | $4.64 \times / \sqrt{R_{e_{x}}}$ | $1.292 / \sqrt{R_{e_{L}}}$ |
| 3. $\frac{u}{U}=2\left(\frac{y}{\delta}\right)-2\left(\frac{y}{\delta}\right)^{3}+\left(\frac{y}{\delta}\right)^{4}$ | $5.84 \times / \sqrt{R_{e_{x}}}$ | $1.36 / \sqrt{R_{e_{L}}}$ |
| 4. $\frac{u}{U}=\sin \left(\frac{\pi}{2} \frac{y}{\delta}\right)$ | $4.79 \times / \sqrt{R_{e_{x}}}$ | $1.31 / \sqrt{R_{e_{L}}}$ |
| 5. Blasius's Solution | $4.91 \times / \sqrt{R_{e_{x}}}$ | $1.328 / \sqrt{R_{e_{L}}}$ |

## Analysis of turbulent boundary layer:

(a). If Reynold number is more than $5 \times 10^{5}$ and less than $10^{7}$ the thickness of boundary layer and drag co-efficient are given as:
$\delta=\frac{0.37 x}{\left(R_{e_{x}}\right)^{1 / 5}}$ and $C_{D}=\frac{0.072}{\left(R_{e_{L}}\right)^{1 / 5}}$
where $\mathrm{x}=$ Distance from the leading edge $\mathrm{R}_{\mathrm{e}_{\mathrm{x}}}=$ Reynold number for length x
$R_{e_{L}}=$ Reynold number at the end of the plate.
(b). If Reynold number is more than $10^{7}$ but less than $10^{9}$, Schlichting gave the empirical equation as:
$C_{D}=\frac{0.455}{\left(\log _{10} R_{e_{\mathrm{L}}}\right)^{2.58}}$
Boundary layer separation: The boundary layer may not keep sticking to the solid body, if it cannot provide kinetic energy to overcome its resistance. Thus, the boundary layer will be separated from the surface, and this phenomenon is called the boundary layer separation.


The separation point $S$ is determined from the condition: $\left(\frac{\partial u}{\partial y}\right)_{y=0}=0$

1. If $\left(\frac{\partial u}{\partial y}\right)_{y=0}<0$, the flow has separated, and it is the necessary condition for flow separation.
2. If $\left(\frac{\partial u}{\partial y}\right)_{y=0}=0$, the flow is on the verge of separation. This is the sufficient condition for the flow separation to occur.
3. If $\left(\frac{\partial u}{\partial y}\right)_{y=0}>0$, the flow will not separate, or flow will remain attached with the surface.

Methods of Preventing the Separation of Boundary Layer: Separation of the boundary layer is undesirable, and attempts should be made to avoid separation by various methods. The following are the methods for preventing the separation of boundary layers:

1. Suction of the slow-moving fluid by a suction slot.
2. Supplying additional energy from a blower.
3. Providing a bypass in the slotted wing.
4. Rotating boundary in the direction of flow.
5. Providing small divergence in a diffuser.
6. Providing guide-blades in a bend.
7. Providing a trip-wire ring in the laminar region for the flow over a sphere.

## CHAPTER-9-DRAG AND LIFT CONCEPT

i. DRAG FORCE
a. Drag: The component of the total force (FR) in the direction of motion,
b. Drag Force: $\mathrm{F}_{\mathrm{D}}=\oint P d A \cos \theta+\oint \tau_{0} d A \sin \theta$

Here, $\begin{aligned} & \oint \mathrm{PdA} \cos \theta=\text { pressure drag or form drag. } \\ & \oint \tau_{0} \mathrm{dA} \sin \theta=\text { skin friction drag. }\end{aligned}$

## ii. LIFT FORCE

a. Lift: The component of total force in the direction perpendicular to the direction of motion is
b. Lift Force: $F_{L}=\oint P d A \sin \theta+\oint \tau_{0} d A \cos \theta$

## iii. IMPORTANT:

a. $F_{D}=C_{D} \times \frac{1}{2} \times \rho \times A \times u_{\infty}^{2}$
b. $F_{L}=C_{L} \times \frac{1}{2} \times \rho \times A \times u_{\infty}^{2}$

Here,
A = projected area.
$C_{D}=$ coefficient of drag
$C_{L}=$ coefficient of lift
Lift ( $\mathrm{FL}_{\mathrm{L}}$ ): The component of the total force ( $\mathrm{F}_{\mathrm{R}}$ ) in the direction perpendicular to the direction of motion is known as 'lift'. Lift force occurs only when the axis of the body is inclined to the direction of fluid flow. If the axis of the body is parallel to the direction of fluid flow, the lift force is zero.
The lift for a body moving in a fluid of density $\rho$, at a uniform velocity ( $U$ ) are calculated mathematically, as:
$\mathrm{F}_{\mathrm{L}}=\mathrm{C}_{\mathrm{L}} \mathrm{A} \frac{\rho \mathrm{U}^{2}}{2}$
Where:
$C_{L}=$ Co-efficient of lift,
$A=$ projected area of the body perpendicular to the direction of flow $=$ Largest projected area of the immersed body.
The resultant force on the body is given as:

$$
\mathrm{F}_{\mathrm{R}}=\sqrt{\mathrm{F}_{\mathrm{D}}^{2}+\mathrm{F}_{\mathrm{L}}^{2}}
$$

## Flow over the flat plate:

Case-1: When the plate is placed parallel to the direction of the flow, $\theta$ is the angle made by pressure with the direction of the motion.

Thus, $\theta=90^{\circ}$
$\int \mathrm{p} \cos \theta \mathrm{dA}=0$
Thus, the total drag will be equal to friction drag (or shear drag).


Case-II: If the plate is placed perpendicular to the flow
Here, $\theta=0^{\circ}$
Hence, .
Thus, the total drag will be due to the pressure difference between the upstream and downstream sides of the plate.


Case-III: If the plate is held at an angle with the flow direction, both the terms $\int \mathrm{p} \cos \theta \mathrm{dA} \& \int \tau_{0} \sin \theta \mathrm{dA}$ will exist, and total drag will be equal to the sum of pressure drag and friction drag.

## Drag on a sphere:

Case-I: If the Reynolds number of the flow is very small: $\operatorname{Re}<0.2$
The viscous forces are much more important than the inertial forces, which may be assumed negligible.
C.G. Stokes developed a mathematical equation for the total drag on a sphere in a flowing fluid such that $\operatorname{Re}<0.2$ :

Total drag: $\quad F_{D}=3 \pi \mu \mathrm{DU}$
Skin friction drag: $F_{D, f}=\frac{2}{3} F_{D}=\frac{2}{3} \times 3 \pi \mu \mathrm{DU}=2 \pi \mu \mathrm{DU}$

Pressure drag: $F_{D, p}=\frac{1}{3} F_{D}=\frac{1}{3} \times 3 \pi \mu D U=\pi \mu D U$
$\mathrm{C}_{\mathrm{D}}=\frac{24}{\mathrm{Re}}$

Case-II: When $0.2<\operatorname{Re}<5$
With the increase of Reynolds number, the inertia forces increase and must be considered.

Oseen improved Stoke's law and gave a relation for the coefficient of drag:

$$
\mathrm{C}_{\mathrm{D}}=\frac{24}{\mathrm{R}_{\mathrm{e}}}\left[1+\frac{3}{16 \mathrm{R}_{\mathrm{e}}}\right]
$$

Case-III: If $5.0<\operatorname{Re}<1000$
The drag coefficient: $\mathrm{C}_{\mathrm{D}}=0.4$
Case-IV: $1000<\operatorname{Re}<100,000$
$\mathrm{C}_{\mathrm{D}}=0.5$
$C_{D}$ is independent of the Reynolds number.
Case-V: For Re $>10^{5}$
$\mathrm{C}_{\mathrm{D}}=0.2$

Terminal velocity of a body: It is the maximum velocity attained by a falling body in a medium.

The forces acting on the body at this stage will be:

$$
\mathrm{W}=\mathrm{F}_{\mathrm{D}}+\mathrm{F}_{\mathrm{B}}
$$

Where,
$\mathrm{W}=$ weight of the body, acting downward,
$F_{D}=$ Drag force, acting vertically upward
$F_{B}=$ Buoyant force, acting vertically up.
Terminal velocity of a sphere:

$$
\mathrm{U}=\frac{\pi \mathrm{D}^{3}}{18 \mu}\left(\rho_{\mathrm{s}}-\rho_{\mathrm{f}}\right) \times \mathrm{g}
$$

Where:
$\mu=$ dynamic viscosity of the fluid
$\rho_{f}=$ density of the fluid
$\rho_{\mathrm{s}}=$ density of the sphere
$\mathrm{U}=$ Terminal velocity

## Streamlined body:

A streamlined body is defined as that body whose surface coincides with the streamlines when placed in a flow. Thus, the body offers the least resistance in terms of Pressure drag.

- The drag experienced is mainly due to Viscous/frictional drag, which depends on the surface area exposed to fluid.



## Bluff body:

A bluff body is defined as that body whose surface does not coincide with the streamlines when placed in a flow. Thus, the body offers the least resistance to viscous drag. Then the flow is separated from the body's surface much ahead of its trailing edge resulting in a very large wake formation zone. Thus, the pressure drag will be very large. Pressure drag is the function of the cross-section area rather than the surface area.


## CHAPTER-10-DIMENSIONAL ANALYSIS

Introduction: length(L), Mass(M), Time(T) dimensions are called fundamental dimension.

## i. DIMENSIONAL HOMOGENEITY

The power of the fundamental dimension ( $\mathrm{L}, \mathrm{M}$ and T ) on both sides of the equation will be identical for the dimensionally homogeneous equation.

## ii. RAYLEIGH'S METHOD

$x \quad=f \underbrace{f\left(x_{1}, x_{2}, x_{3}\right)}_{\downarrow}$
dependent $\begin{gathered}\text { Independent } \\ \text { variable } \quad \text { variabele }\end{gathered}$
$\Rightarrow \mathrm{x}=\mathrm{kx} x_{1}^{\mathrm{a}} \cdot \mathrm{x}_{2}^{\mathrm{b}} \cdot \mathrm{x}_{3}^{\mathrm{c}}$
Here, $a, b$ and $c$ are arbitrary powers.
iii. BUCKINGHAM'S $\boldsymbol{\Pi}$-THEOREM
$f\left(\pi_{1}, \pi_{2}\right.$ _-___-, $\left.\pi_{m-n}\right)=0$
Here,
$\pi$ term $=$ dimensionless terms
$\mathrm{m}=$ total number of variables (dependent variable + independent variable)
$n=$ number of fundamental dimensions involved in the problem, generally $n=3$. ( $M$,
$L$ and T)
Number of $\pi$ terms $=m-n$
$\pi=$ [set of repeating variable]. Variable
Or
$\pi=\left[\begin{array}{lll}\text { geometric } & \text { kinematic } & \text { Dynamic } \\ \text { variable } & \text { variable } & \text { variable }\end{array}\right]$ variable

## Various forces in fluid mechanics:

Inertia force ( $\mathrm{F}_{\mathrm{i}}$ ): $\quad \mathrm{F}_{\mathrm{i}}=\mathrm{\rho l}^{2} \mathrm{v}^{2}$
Surface tension force ( $\mathrm{Fs}_{\mathbf{s}}$ ): It is equal to the product of surface tension and length of surface of the flowing fluid.
$F_{s}=\sigma \times I$
Gravity force ( $\mathbf{F g}_{\mathbf{g}}$ ): It is equal to the product of mass and acceleration due to gravity of the flowing fluid.
$F_{g}=\rho^{3} g$
Pressure force ( $\mathrm{F}_{\mathrm{p}}$ ): It is equal to the product of pressure intensity and cross-sectional area of the flowing fluid
$\mathrm{F}_{\mathrm{P}}=\mathrm{Pl}^{2}$
Viscous force ( $F_{\mathbf{v}}$ ): It is equal to the product of shear stress ( $\tau$ ) due to viscosity and surface area of the flow.
$\mathrm{F}_{\mathrm{v}}=\mu \mathrm{lv}$
Elastic force ( $\mathbf{F e}$ ): It is equal to the product of elastic stress and area of the flowing fluid.
$\mathrm{Fe}=\mathrm{Kl}^{2}$
Dimensionless numbers:

| Dimensionless <br> number with <br> symbol | Group | Significance | Field of use |
| :--- | :---: | :---: | :--- |
| 1. Reynolds number <br> $\left(R_{e}\right)$ | $\frac{\rho V L}{\mu}$ | $\frac{\text { inertia force }}{\text { viscous force }}$ | Laminar viscous flow in <br> confined passages where <br> viscous effects are <br> predominant. |


| 2. Froude number (Fr) | $\frac{\mathrm{V}}{\sqrt{\text { Ig }}}$ | $\frac{\text { inertia force }}{\text { gravitational force }}$ | Free surface flows where <br> effects of gravity are <br> important. |
| :--- | :---: | :---: | :--- |
| 3. Mach number (M) | $\frac{\mathrm{V}}{\sqrt{\mathrm{K} / \mathrm{\rho}}}$ | $\frac{$ inertia force  <br>  elastic force }{} | High speed flow where <br> compressible effects are <br> important. |
| 4. Weber number (W) | $\frac{\rho / \mathrm{V}^{2}}{\sigma}$ | $\frac{\text { inertia force }}{\text { surface tension force }}$ |  | | Small surface waves, |
| :--- |
| capillary and sheet flow |
| where surface tension is |
| important. |

## iv. MODEL AND SIMILITUDE STUDIES

a. Geometric Similarity: It is said to exist between the model and the prototype. The ratio of all corresponding linear dimension in the model and prototype are equal.
$\frac{L_{p}}{L_{m}}=\frac{b_{p}}{b_{m}}=\frac{D_{p}}{D_{m}}=L_{r}$
Here,
$\mathrm{L}_{\mathrm{r}}=$ length scale ratio
$\mathrm{L}_{\mathrm{m}}=$ length of model
$B_{m}=$ breath of model
$\mathrm{D}_{\mathrm{m}}=$ diameter of model
$A_{m}=$ Area of model
$\mathrm{V}_{\mathrm{m}}=$ volume of model
And similarly dimensions of prototype are Lp, bp, $\mathrm{D}_{\mathrm{p}}, \mathrm{Ap}_{\mathrm{p}}$ and $\mathrm{V}_{\mathrm{p}}$.
b. Kinematic Similarity: It means similarity of motion between model and prototype.

1. Velocity Scale Ratio: $\frac{V_{P_{1}}}{V_{m_{1}}}=\frac{V_{P_{2}}}{V_{m_{2}}}=V_{r}$
2. Acceleration Scale Ratio: $\frac{a_{p_{1}}}{a_{m_{1}}}=\frac{a_{p_{2}}}{a_{m_{2}}}=a_{r}$
3. Time Scale Ratio: $\mathrm{V}_{\mathrm{r}}=\frac{\mathrm{L}_{\mathrm{r}}}{\mathrm{T}_{r}}$
c. Dynamic Similarity: It means the similarity of forced between forced between the model and prototype

Inertia Force: $\quad F_{i}=\rho L^{2} V^{2}$
Viscous Force: $\quad F_{v}=\mu V L$
Pressure force: $\quad F_{P}=P L^{2}$
Gravity Force: $\quad F_{g}=\rho L^{3} g$
Surface Tension Force: $F_{g}=\sigma L$
Model laws: The laws on which the models are designed for dynamic similarity are called model laws or laws of similarity. The followings are the model laws:

Reynold's Model Law: Reynold's model law is the law in which models are based on Reynold's number.

$$
\left[R_{e}\right]_{m}=\left[R_{e}\right]_{p} \text { or } \frac{\rho_{m} V_{m} L_{m}}{\mu_{m}}=\frac{\rho_{p} V_{p} L_{p}}{\mu_{p}}
$$

## Froude's Model Law:

$\left(F_{e}\right)_{\text {model }}=\left(F_{e}\right)_{\text {prototype }} \Rightarrow \frac{V_{m}}{\sqrt{g_{m} L_{m}}}=\frac{V_{p}}{\sqrt{g_{p} L_{p}}}$
$\frac{V_{p}}{V_{m}}=\sqrt{\frac{L_{p}}{L_{m}}}=\sqrt{L_{r}}$
$\mathrm{a}_{\mathrm{r}}=1$
$\mathrm{Q}_{\mathrm{r}}=\mathrm{L}_{\mathrm{r}}^{2.5}$

## Euler's model law:

$\left(\mathrm{E}_{\mathrm{u}}\right)_{\text {model }}=\left(\mathrm{E}_{\mathrm{u}}\right)_{\text {prototype }}$
$\frac{V_{m}}{\sqrt{\mathrm{p}_{\mathrm{m}} / \rho_{\mathrm{m}}}}=\frac{V_{p}}{\sqrt{\mathrm{p}_{\mathrm{p}} / \rho_{\mathrm{p}}}}$

## Weber model law:

$(\mathrm{Wb})_{\text {model }}=(\mathrm{Wb})_{\text {prototype }}$
$\frac{V_{m}}{\sqrt{\sigma_{m} / \rho_{m} L_{m}}}=\frac{V_{p}}{\sqrt{\sigma_{p} / \rho_{p} L_{p}}}$

## Mach Model Law:

$(\mathrm{M})_{\text {model }}=(\mathrm{M})_{\text {prototype }}$
$\frac{V_{m}}{\sqrt{K_{m} / \rho_{m}}}=\frac{V_{p}}{\sqrt{K_{p} / \rho_{p}}}$

Classification of models: The hydraulic models are classified as:

1. Undistorted models
2. Distorted models.

Undistorted Models: Undistorted models are those models that are geometrically similar to their prototypes. Thus, the scale ratio for the linear dimensions of the model and its prototype is the same.

Distorted Models: A model is said to be distorted if it is not geometrically similar to its prototype. For a distorted model different scale ratio for the linear dimensions are adopted. For example, in case of rivers, harbours, reservoirs etc., two different scale ratios, one for horizontal dimensions and other for vertical dimensions are taken.

## CHAPTER-11-FLOW THROUGH PIPES

## i. LOSS OF ENERGY IN PIPES

a. Major Energy Losses: This is due to friction and it is calculated by the following formulae.

- Darcy-Weisbach formula.
- Chezy's formula.


## b. Minor Energy Losses

- Sudden expansion of pipe
- Sudden contraction of pipe
- Bend in pipe
- Pipe fitting
- Obstruction in pipe


## ii. HEAD LOSS DUE TO FRICTION

## a. Darcy-Weisbach Formula

Head loss in terms of Velocity: $h_{f}=\frac{4 \cdot f \cdot L \cdot V^{2}}{2 g d}$
Head loss in terms of Discharge: $h_{f}=\frac{8 \cdot Q^{2} \cdot f \cdot L}{\pi^{2} \cdot g \cdot D^{5}}$
Here,
$h_{f}=$ loss of head due to friction
f $\quad=$ co-efficient of friction which is a function of Reynold's number
$=\frac{16}{R_{e}}$ for $\mathrm{Re}_{\mathrm{e}}<2000$ (viscous flow)
$=\frac{0.079}{R_{e}^{1 / 4}}$ for $R_{e}$ varying from 4000 to $10^{6}$
L = Length of pipe
$\mathrm{V}=$ mean velocity of a flow
d = diameter of pipe
b. Chezy's Formula
$h_{f}=\frac{4 L V^{2}}{C^{2} D}$
Here,
$\mathrm{H}_{\mathrm{f}}=$ loss of head due to friction
$\mathrm{V}=$ mean velocity of flow
C = Chezy's constant

## iii. Minor Energy Losses:

a. Loss of Head due to Sudden Enlargement: $h_{L}=\frac{\left(V_{1}-V_{2}\right)^{2}}{2 g}$
b. Loss of Head due to Sudden Contraction: $\mathrm{h}_{\mathrm{c}}=\frac{\mathrm{kV}}{2 \mathrm{~g}}$, where, $\mathrm{k}=\left[\frac{1}{\mathrm{c}_{\mathrm{c}}}-1\right]^{2}$
c. Loss of Head at the Entrance of a Pipe: $h_{i}=0.5 \frac{\mathrm{~V}^{2}}{2 g}$
d. Loss of Head due to Bend in Pipe: $h_{b}=\frac{k V^{2}}{2 g}$

## iv. FLOW THROUGH SYPHON

Condition to avoid vaporisation.
$\frac{P_{s}}{\rho g} \geq \frac{P_{v}}{\rho g}$ (seporation)
$\frac{P_{s}}{\rho g}($ seporation $)=2.4-2.7 \mathrm{~m}$ of $\mathrm{H}_{2} \mathrm{O}$
v. Water hammer in pipes:

If, $\mathrm{T}=$ time taken to closed the value.
a. Gradual closure, T > 2L/C,
$\frac{P}{\rho g}=\frac{V}{g} \cdot \frac{L}{T}$
b. Sudden closure, $\mathrm{T}<2 \mathrm{~L} / \mathrm{C}$

Rigid pipe: $\frac{P}{\rho g}=\frac{V}{g} \sqrt{\frac{k}{\rho}}$
Elastic pipe: $\frac{p}{\rho g}=\frac{V}{g} \sqrt{\frac{k}{\rho\left[1+\frac{\mathrm{Dk}}{\mathrm{Et}}\right]}}$
Here,
$\mathrm{K}=$ bulk modulus of water
$E=$ modulus of elasticity for pipe material

## CHAPTER-12-TURBULENT FLOW

Turbulent flow: In case of turbulent flow there is huge order intermixing of fluid particles and due to this, various properties of the fluid are going to change with space and time.

$\overline{\mathrm{u}}=$ mean or average component of velocity
Any parameter $=$ Average component + fluctuating component
$\mathrm{u}=\overline{\mathrm{u}}+\mathrm{u}^{\prime}$
Shear stress in turbulent flow:
Shear stress due to turbulence: $\tau_{\mathrm{t}}=\eta \frac{\overline{\mathrm{du}}}{\mathrm{dy}} \quad$ (Boussinesq Hypothesis).
Kinematic eddy viscosity: $\varepsilon=\frac{\eta}{\rho}$
Total shear stress: $\tau=\tau_{v}+\tau_{t}=\mu \frac{d u}{d y}+\eta \frac{d u}{d y}$
Reynold's hypothesis: $\bar{\tau}_{\text {turbulent }}=\rho \overline{u^{\prime} v^{\prime}}$
According to Prandtl Mixing length hypothesis: $u^{\prime}=v^{\prime}=I \frac{d \bar{u}}{d y}$
I = Prandtl's mixing length
$\bar{\tau}_{\text {turb. }}=\rho l^{2}\left(\frac{d \bar{u}}{d y}\right)^{2}$


Relation of Shear Stress with Coefficient of friction: $f=\frac{2 \tau_{0}}{\rho V^{2}}$
Where $\tau_{0}=$ wall shear stress, $\mathrm{v}=$ average velocity
Velocity distribution in turbulent flow:
Shear or Fictitious velocity: $\mathrm{u}^{*}=\sqrt{\frac{\tau_{0}}{\rho}}$

$$
\mathrm{u}=\mathrm{u}_{\max }+2.5 \mathrm{u}^{*} \log _{\mathrm{e}}\left(\frac{\mathrm{y}}{\mathrm{R}}\right)
$$

The above equation is known as 'Prandtl's universal velocity distribution equation for turbulent flow in pipes.
$\frac{u_{\max }-u}{u^{*}}=5.75 \log _{e}(R / y)$
The difference between the maximum velocity $u_{\max }$, and local velocity $u$ at any point i.e. $\left(u_{\max }-u\right)$ is known as 'velocity defect'.


## Hydrodynamically smooth \& rough pipes:


(a) Smooth boundary

$\mathrm{K}=$ average height of roughness
$\delta '=$ height of Laminar sublayer
Nikuradse's conditions for smooth \& rough boundary.

| Nikuradse's conditions | In terms of roughness Reynolds number $\left(\frac{\mathrm{u}^{*} \mathrm{k}}{v}\right)$ |
| :---: | :--- |
| $\frac{K}{\delta^{\prime}}<0.25 \rightarrow$ smooth | If $\frac{\mathrm{u}^{*} \mathrm{k}}{v}<4$, boundary is considered smooth. |
| $0.25<\frac{K}{\delta^{\prime}}<6 \rightarrow$ Transition | If $4<\frac{\mathrm{u}^{*} \mathrm{k}}{v}<100$, boundary is in transition stage |
| $\frac{K}{\delta^{\prime}}>6 \rightarrow$ Rough | If $\frac{\mathrm{u}^{*} \mathrm{k}}{v}>100$, the boundary is rough |

Laminar sublayer thickness: $\delta^{\prime}=\frac{11.6 v}{U^{*}}$
shear friction velocity or hypothetical velocity: $U^{*}=\sqrt{\frac{\tau}{\rho}}=V \cdot \sqrt{\frac{f}{8}}$
where $\mathrm{f}=$ friction coefficient and $\mathrm{V}=$ average velocity.

## Velocity distribution for turbulent flow:

Velocity distribution in rough pipes: $\frac{u}{u^{*}}=5.75 \log _{10}\left(\frac{y}{k}\right)+8.5$
Average velocity in Rough pipes: $\frac{\bar{U}}{u^{*}}=5.75 \log _{10}\left(\frac{R}{K}\right)+4.75$
Velocity distribution in smooth pipes: $\frac{u}{u^{*}}=5.75 \log _{10}\left(\frac{u^{*} y}{v}\right)+5.5$
Average velocity in smooth pipes: $\frac{\bar{U}}{u^{*}}=5.75 \log _{10}\left(\frac{u^{*} R}{v}\right)+1.75$
Velocity distribution for turbulent flow with average velocity in smooth pipes for power law:
$\frac{\mathrm{u}}{\mathrm{u}_{\max }}=\left(\frac{\mathrm{y}}{\mathrm{R}}\right)^{1 / \mathrm{n}}$ and $\mathrm{n}=\frac{1}{7} \Rightarrow \frac{\mathrm{u}}{\mathrm{u}_{\max }}=\left(\frac{\mathrm{y}}{\mathrm{R}}\right)^{1 / 7}$

## Resistance of smooth and rough pipes:

The friction co-efficient is a function of Reynolds number and $k / D$ ratio, where $k$ is the average height of pipe wall roughness protrusions.

$$
\mathrm{f}=\phi\left[\mathrm{R}_{\mathrm{e}}, \frac{\mathrm{k}}{\mathrm{D}}\right]
$$

## Friction factor in turbulent flow:

## (a). For Smooth pipes:

The value of 'f' for smooth pipe for Reynolds number varying from 4000 to 100000 is given by the relation:

$$
\mathrm{f}=\frac{.0791}{\left(\mathrm{R}_{\mathrm{e}}\right)^{1 / 4}}
$$

When, $\operatorname{Re}=10^{5}$ to $4 \times 10^{7} \Rightarrow f=0.0032+\frac{0.221}{\operatorname{Re}^{0.232}}$
(b). For rough pipes:

$$
\frac{1}{\sqrt{4 f}}=2 \log _{10}\left(\frac{R}{K}\right)+1.74 \quad \text { When, Re }>6 \times 10^{8}
$$

## GATE/ESE

## Civil Engineering

## Soil Mechanics

## Important Formula Notes

## IMPORTANT FORMULAS ON SOIL MECHANICS

## CHAPTER-1 ORIGIN \& PROPERTIES OF SOIL



1. Water content (w)

$$
w=\frac{\text { weight of water }}{\text { weight of soil solids }}=\frac{w_{w}}{w_{s}}
$$

- The water content can be any value greater than or equal to zero, there is no upper limit.
- Dry weight of solids,

$$
w_{s}=\frac{w}{1+w}=\frac{\text { total weight }}{1+\text { water content }}
$$

2. Void Ratio (e)

$$
e=\frac{\text { volume of voids }}{\text { volume of soil solids }}=\frac{V_{v}}{V_{s}} .
$$

- void ratio can be any value greater than or equal to zero, there is no upper limit.
- Volume of soil solid,

$$
V_{s}=\frac{V}{1+e}=\frac{\text { total volume }}{1+\text { void ratio }}
$$

## 3. Porosity (n)

$$
n=\frac{\text { volume of void }}{\text { total volume }}=\frac{V_{v}}{V}
$$

- Porosity can be any value between 0 to 1 .
- It can be expressed in terms of void ratios.

$$
\Rightarrow \eta=\frac{\mathrm{e}}{1+\mathrm{e}} \text { or } \mathrm{e}=\frac{\eta}{1-\eta}
$$

4. Degree of Saturation (S)

$$
S=\frac{\text { volume of water }}{\text { volume of void }}=\frac{V_{w}}{V_{v}} \times 100
$$

- It can take any value from 0 to 100 .
- If, $S=0$ then dry soil. If $S=100$ then fully saturated state, if $S$ is in between 0 to 100 then partially saturated state.

5. Air Content ( $\mathbf{a c}_{\text {c }}$ )

$$
a_{c}=\frac{\text { volume of air }}{\text { volume of void }}=\frac{V_{a}}{V_{v}}=1-S
$$

- It can take any value between 0 to 100

6. Percentage Air Voids ( $\mathrm{n}_{\mathrm{a}}$ )

$$
n_{a}=\frac{\text { volume of air }}{\text { total volume }}=\frac{V_{a}}{V} \times 100
$$

## 7. Density of Soil

$$
\text { bulk unit weight }=\frac{(G+e S) \gamma_{w}}{1+e}
$$

- For Dry unit weight, Put $\mathrm{S}=0$.
- For saturated unit weight, Put $\mathrm{S}=1$.

8. Density Index or Relative density or Degree of Density ( $\mathrm{I}_{\mathrm{D}}$ )

$$
\begin{gathered}
\mathrm{I}_{\mathrm{D}}=\frac{\mathrm{e}_{\max }-\mathrm{e}}{\mathrm{e}_{\max }-\mathrm{e}_{\min }} \\
I_{D}=\frac{\gamma_{d_{\max }}}{\gamma_{d}}\left[\frac{\gamma_{d}-\gamma_{d_{\text {min }}}}{\gamma_{d_{\max }}-\gamma_{d_{\text {min }}}}\right]
\end{gathered}
$$

| $\mathbf{I}_{\mathbf{D}}$ | Description of soil |
| :---: | :---: |
| $0-15$ | Very loose soil |
| $15-35$ | Loose Soil |
| $36-65$ | Medium dense soil |
| $66-85$ | Dense soil |
| $86-100$ | Very dense soil |

Soil having higher relative density is denser, possesses high shear strength and low compressibility.
9. Relation between $e, w, G$ and $S$

$$
e \mathrm{~S}=\mathrm{G} w
$$

## 10.Relation between $\mathbf{w}, \mathbf{G}, \mathbf{e}, \mathbf{S}, \gamma_{\mathbf{w}}$ we know that

$$
\text { bulk unit weight }=\gamma=\frac{(G+e s) \gamma_{w}}{1+e}
$$

Case 1: Soil is dry
$\gamma=\gamma_{\mathrm{d}} ; \mathbf{S}=\mathbf{O}$

$$
\Rightarrow \gamma_{d}=\frac{G \gamma_{w}}{1+e}
$$

Case 2: Soil is saturated
$\gamma=\gamma_{\text {sat }} ; \mathrm{S}=1$.

$$
\Rightarrow \gamma_{\text {sat }}=\frac{(G+e) \gamma_{w}}{1+e}
$$

Case 3: Soil is submerged
$\gamma=\gamma^{\prime}$

$$
\begin{aligned}
\gamma^{\prime}= & \gamma_{\text {sat }}-\gamma_{\mathrm{w}} \Rightarrow \gamma^{\prime}=\frac{(G+e) \gamma_{w}}{1+e}-\gamma_{w} \\
& \Rightarrow \gamma^{\prime}=\frac{(G-1) \gamma_{w}}{1+e}
\end{aligned}
$$

## 11. Relationship between $\gamma \mathrm{d}, \gamma$ and w

$$
\gamma_{d}=\frac{\gamma}{1+w}
$$

## 12. Relationship between $\gamma_{\mathrm{d}}, \eta_{a r} \mathbf{G}, \gamma_{\mathbf{w}}$, w

$$
\Rightarrow \gamma_{d}=\frac{\mathrm{G}\left(1-\eta_{a}\right) \gamma_{w}}{1+w G}
$$

## 13. Classification of soil water:

- Gravitational water (free water): water free to move through soil under influence of gravity
- Held water: (held against gravity)
> Structural water: chemically combined in crystal structure of soil particle, it cannot be removed without breaking the structure of soil particles. This water is not removed by heating upto 105 to 110 degrees Celsius (used for drying the soil in laboratory condition).
> adsorbed water:The water held by the fine-grained soil particles due to electrochemical force of adhesion it can be completely removed only above 200 degrees Celsius. At 105 to 110 degrees Celsius it cannot be completely removed but part of that can be removed that is called hygroscopic water.
> capillary water: it is the water held in soil Mass due to capillary action. Capillary water can exist on a macroscopics scale as compared to other type of held water which can exist on microscopic scale.


## 14.Pycnometer

Specific Gravity using Pycnometer


Calculation of specific gravity (G),

$$
G=\frac{\left(m_{2}-m_{1}\right)}{\left(m_{4}-m_{1}\right)-\left(m_{3}-m_{2}\right)}=\frac{\left(m_{2}-m_{1}\right)}{\left(m_{4}-m_{3}\right)+\left(m_{2}-m_{1}\right)}
$$

- For calculation ofG, dry soil is taken.
- The value of G is reported at $27^{\circ} \mathrm{C}$ if the temperature during testing is different, than we should convert the $G$ value at $27^{\circ} \mathrm{C}$ to report its value

$$
\left(G \gamma_{w}\right)_{T^{\circ} \mathrm{C}}=\left(G \gamma_{w}\right)_{27^{\circ} \mathrm{C}}
$$

Calculation of water content (w),

$$
w=\left[\frac{\left(m_{2}-m_{1}\right)}{\left(m_{3}-m_{4}\right)} \times\left(\frac{G-1}{G}\right)-1\right]
$$

- For calculation of water content (w), moist soil is taken.


## 15.Calculation of water content

$$
\text { moisture content } \%=w=\frac{w_{2}-w_{1}}{w_{1}} \times 100
$$

where $W_{2}$ is weight of moist soil and $W_{1}$ is weight of dry soil.

## 16. Measurement of unit weight

- By core cutter method: it is a field method and applicable for soil which are soft and fine grained. It cannot be used for Stoney, gravelly and dry soil. Methodis applicable when surface of soil is exposed, and core cutter can be easily pushed. Volume of core cutter is 1000 cc.
- By sand replacement method: it is a field method and suitable for gravelly, sandy and dry soil. Where core cutter method cannot be used.W= weight of soil excavated from the pit.

$$
\begin{aligned}
& \gamma_{t}=\frac{W}{V}=\frac{W \times \gamma_{\text {sand }}}{\text { weight of sand in pit }} \\
& \text { and } \gamma_{\text {sand }}=\frac{\text { weight of sand in cylinder }}{1000 c c}
\end{aligned}
$$

- Water displacement method:

volume of $w a x=\frac{\text { weight of sample after applying } \operatorname{wax}\left(w_{1}\right)-\text { original weight of sample }(w)}{\gamma_{\text {wax }}}$

$$
\operatorname{so}, \boldsymbol{\gamma}_{t}=\frac{\boldsymbol{W}}{\frac{w_{2}}{\gamma_{w}}-\frac{w_{1}-w}{\gamma_{w a x}}}
$$

## 17.Index properties of soil

- Index properties are those properties which helps in assessing the engineering behavior of soil and in classification of the soil into different groups so that a particular group is representative of a particular behavior.
- For coarsegrain soil, property of particle and the relative state of compactness are the most significant properties and for fine grained soil consistency and plasticity are important.
- The soil grain properties can be determined from remolded or disturbedsample, and they depend on the shape and size of grains and their mineralogical composition. It is independent of mode of soil formation.
- soil aggregate properties depend on mode of soil formation, soil history and soil structure. and these properties are determined from undisturbed sample or preferably from insitu tests.


## 18. Grain size analysis:

- Grain size analysis is important mainly for coarse grain soil because in fine grained soil interparticle forces and water content is more important.
- For coarse grain soilgrain size analysis is carried out by sieve analysis and for fine grain soil it is analyzed by sedimentation analysis.
- For wet analysis soil passing 4.75 mm sieve is washed over $75 \mu$ sieve and then oven dried and fine sieve analysis is performed with this oven dried sample.
- Fineness modulus: it is the sieve number at which average size of particles are said to lie when counted from the lower order sieve size to the higher order sieve size.

| s. no. | Sieve | Wt. retained <br> (gm) | Cumulative <br> weight retained <br> $(\mathrm{gm})$ | \% <br> Cumulative <br> weight <br> retained | \% Finer |
| :---: | :---: | :---: | :---: | :---: | :---: |


| $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| ---: | ---: | ---: | ---: | ---: | ---: |

Fineness modulus $=\frac{\sum \% \text { cummulative weight retained }}{100}$

$$
C_{u}=\frac{D_{60}}{D_{10}}=\text { uniformity coefficient }
$$

$$
c_{c}=\frac{D_{30}{ }^{2}}{D_{10} \times D_{60}}=\text { coefficient of curvature }
$$

- $C_{u}$ basically represent the slope of the curve between $D_{60}$ and $D_{10}$ and hence it is related to degree of uniformity of the sample. If $C_{u}$ is 1 , then soil is perfectly uniform graded, and the curve will be vertical. If $C_{u}$ is lessthen 2 or 3 it is assumed that soil is uniformly graded.
- If $\mathrm{C}_{\mathrm{c}}$ is not between 1-3 then it is said that the soil is gap graded, it means certain size of soil are missing.
19.Sedimentation analysis: stokes law is valid for particle od diameter 0.2 mm to 0.0002 mm .

| $V_{S}=\frac{\left(\gamma_{s}-\gamma_{w}\right) \times d^{2}}{18 \mu}$ and $\frac{H_{e}}{t}=\frac{\left(\gamma_{s}-\gamma_{w}\right) \times d^{2}}{18 \mu}$ | here, |
| :---: | :---: |
| $\%$ finer $=$$\gamma_{s}$ <br> concentration of sample at time 't from height $H_{e}$ <br> original concentration of sample | $\gamma_{\mathrm{w}}=$ unit weight of soil particles weight of water <br> $d=$ size of particle <br> $\mu$ |
|  |  <br> $=$ dynamic viscosity of water |

- Deflocculating agent or dispersing agent like sodium hexametaphosphate and sodium oxalate is used.


## 20.Consistency of soil:

- Consistency represents the relative ease with which soil can be deformed.It is related to clay and the water content. Change in water content leads to the change in consistency of the soil.
- Atterberg analysed the consistency of soil in four stages.
(a) Solid,
(b) Semi-solid,
(c) Plastic,
(d) Liquid

21.The liquid limit is determined by plotting a graph between number of blows as abscissa on a logarithmic scale and the corresponding water content as ordinate. Liquid limit is corresponding to 25 number of blows.
- slope of flows curve is known as 'Flow Index' and it represents the rate of loss of shear strength of soil with increase in water content. Mathematically, flow index ( $\mathrm{I}_{\mathrm{f}}$ ) is given as

$$
\tan \theta=I_{\mathrm{f}}=\frac{\mathrm{W}_{1}-\mathrm{W}_{2}}{\log _{10} \frac{N_{2}}{N_{1}}}
$$



Note: At liquid limit all soils will have same shear strength equal to $2.7 \mathrm{KN} / \mathrm{m}^{2}$. Note: if the rubber pad used during Casagrande method is harder than the standard rubber than the liquid limit reported will be smaller.
22.Plastic limit: Plastic limit is defined as the water moisture content at which a thread of soil with 3.0 mm diameter begins to crumble.
23.Shrinkage Limit (Ws): maximum water content, below which change in water content does not change volume of soil.


- Shrinkage Ratio (SR):

Shrinkage ratio,

$$
S R=\frac{\frac{V_{1}-V_{2}}{V_{d}}}{W_{1}-W_{2}} \times 100=\frac{\frac{V_{2}-V_{d}}{V_{d}}}{W_{2}-W_{d}} \times 100=\frac{\text { slope }}{V_{d}} \times 100
$$

Here,
$\mathrm{V}_{1}=$ volume of soil mass at water content $\mathrm{W}_{1}$
$\mathrm{V}_{2}=$ volume of soil mass of water content $\mathrm{W}_{2}$.
$\mathrm{V}_{\mathrm{d}}=$ volume of dry soil mass.

- Volumetric Shrinkage (Vs):

$$
V S=\left(\frac{V_{1}-V_{d}}{V_{d}}\right) \times 100
$$

## 24.Consistency Indices

(i) Plasticity Index ( $\mathrm{I}_{\mathrm{P}}$ ):

$$
I_{P}=W_{L}-W_{P}
$$

Also, the plasticity can be classified based on $I_{p}$ as

| $\mathbf{I}_{\mathbf{p}}$ | Plasticity |
| :---: | :---: |
| 0 | Non plastic |
| $<7$ | Low plastic |
| $7-17$ | Medium plastic |
| $>17$ | High plastic |

(ii) Relative Consistency / Consistency Index:

$$
I_{C}=\frac{W_{L}-W}{I_{P}}=\frac{W_{L}-W}{W_{L}-W_{P}}
$$

(iii) Liquidity Index ( $\mathrm{I}_{\mathrm{L}}$ )/Water plasticity ratio:

$$
I_{C}=\frac{W-W_{P}}{I_{P}}=\frac{W-W_{P}}{W_{L}-W_{P}}
$$

Note: The sum of liquidity index and consistency index will always be equal to 1 .

$$
\mathrm{I}_{\mathrm{L}}=1-\mathrm{I}_{\mathrm{C}}
$$

| Consistency | Description | $\mathbf{I}_{\mathbf{C}}$ | $\mathbf{I}_{\mathbf{L}}$ | $\mathbf{U C S}\left(\mathbf{K g} / \mathbf{c m}^{\mathbf{2}}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| Liquid | Liquid | $<0$ | $>1$ | -1 |
| Plastic | Very soft | $0-0.25$ | $1-0.75$ | $0-0.25$ |
|  | Soft | $0.25-0.5$ | $0.75-0.50$ | $0.25-0.5$ |
|  | Medium stiff | $0.5-0.75$ | $0.5-0.25$ | $0.5-1.0$ |
| Semi-solid | Very stiff to hard | $>1$ | $0.25-0$ | $1-2$ |
| Solid | Hard to very hard | $>1$ | $<0$ | $2-4$ |

## (iv) Toughness Index ( $\mathrm{I}_{\mathrm{T}}$ ):

$$
I_{t}=\frac{I_{p}}{I_{f}}=\frac{\text { plasticity index }}{\text { flow index }}
$$

It represents the shear strength at plastic limit.
For most of the soil it is between $0-3$, if $\mathrm{I}_{\mathrm{t}}<1$ then soil is friable. For most of the clay it is 1-3.

| 25.Sensitivity: degree of disturbance on remoulding is called sensitivity.$S_{t}=\frac{(U C S)_{\text {undisturbed state }}}{(U C S)_{\text {remoulded state }}}$ | $S_{\text {t }}$ | Description |
| :---: | :---: | :---: |
|  | 1 | Intensive Soil |
|  | 2-4 | Normal/Less sensitive soil |
|  | 4-8 | Sensitive soil |
|  | 8-16 | Extra sensitive soil |
|  | >16 | Unstable/Quick soil |

26.Thixotropy: it is the property of soil due to which loss of strength(shear) on remolding can be regained if left undisturbed for some time.

- Increase in strength in due passes of time is due to tendency of clay soil to gain their chemical equilibrium with reorientation of water molecules in the absorbed layer.
- Clay soil will have large thixotropy then single grain soils.


## 27.Activity:

$$
A_{t}=\frac{I_{P}}{\% C}
$$

Here,
$\mathrm{Ip}=$ plasticity index
\%C = percent of clay particles

| At | Description |
| :---: | :---: |
| $<0.75$ | Inactive soil |
| $0.75-1.25$ | Normal active soil |
| $>1.25$ | Active |

Note 1:AsI ${ }_{\mathbf{p} \uparrow}$ then, plasticity $\uparrow$, shrinkage limit $\downarrow$, swelling shrinkage $\uparrow$, permeability $\downarrow$,clay fraction $\uparrow$, creep rate $\uparrow$, organic content $\uparrow$, dry strength $\uparrow$,compressibility $\uparrow$, cohesion $\uparrow$ and smaller compacted density and larger OMC.

Note 2: As $\mathbf{W}_{\mathbf{L} \uparrow}$ then, compress ability $\uparrow$, organic matter $\uparrow$, rate of volume change $\downarrow$
Note 3: As $\mathbf{A c}_{\boldsymbol{c} \uparrow}$ then, compressibility $\uparrow$,swelling shrinkage $\uparrow$.
Note 4: As organic content $\uparrow$ then, moisture content $\uparrow$, density of soil $\downarrow$, plasticity index $\uparrow$ UCS $\downarrow$, compressibility $\uparrow$, secondary compression $\uparrow$, shrinkage $\uparrow$, permeability $\downarrow$.

Note 5:

| characteristics | $W_{\mathrm{L}}$ same, $\mathrm{I}_{\mathrm{P}} \uparrow$ | $\mathrm{I}_{\mathrm{p}}$ same, $\mathrm{W}_{\mathrm{L}} \uparrow$ |
| :---: | :---: | :---: |
| dry strength | $\uparrow$ | $\downarrow$ |
| toughness near plastic limit | $\uparrow$ | $\downarrow$ |
| compressibility | ------ | $\uparrow$ |
| permeability | $\downarrow$ | $\uparrow$ |
| rate of volume change | $\downarrow$ | $\uparrow$ |

- If $W_{p}$ is less, it means that at smaller moisture content also soil is plastic this means that there must be greater clay fraction.
- $W_{P}$ is more it means soil is plastic at higher moisture content only.


## CHAPTER-2 CLASSIFICATION OF SOIL

## PARTICLE SIZE CLASSIFICATION

According to the IS classification:

| Type of soil |  | Size of particles (mm) |
| :---: | :---: | :---: |
| Clay | - | $<0.002$ |
| silt |  | $0.002-0.075$ |
| Sand | Fine | $0.075-0.425$ |
|  | Medium | $0.425-2.0$ |
|  | Coarse | $2.0-4.75$ |
| Gravel | Fine | $4.75-20$ |
|  | Coarse | $20-80$ |
|  | - | $80-300$ |
| Boulder | - | $>300$ |

## INDIAN SOIL CLASSIFICATION (ISC) SYSTEM

- Classification of soil is done the basic of IS1498. Soil isclassified as coarse grained/fine grained/peat.
- Prefix and suffix used in soil classification.

| Soil type | Prefix | subgroup | suffix |
| :---: | :---: | :---: | :---: |
| Gravel | G | Well graded | W |
| Sand | S | Poor graded | P |
| Silt | M | Silty | M |
| Clay | C | Clayey | C |
| organic | O | $\mathrm{W}_{\mathrm{L}}<35$ | L |
| peat | P | $35<\mathrm{W}_{\mathrm{L}}<50$ | I |
|  |  | $\mathrm{W}_{\mathrm{L}}>50$ | H |

- Plasticity chart:
- Equation of A-line:

$$
I_{P}=0.73\left(W_{L}-20\right)
$$

- Equation of U-line:

$$
I_{P}=0.9\left(W_{L}-8\right)
$$

- points which represent different sample from the same soil stratum will define a line that is roughly
 parallel to A-line.
- The soil sample from common geological origin will have relationship between $\mathrm{I}_{\mathrm{p}}$ and $W_{L}$ which will be parallel to $A$-line.
- soil is divided into 3 broad divisions:
> coarse grain soil: when $50 \%$ or more of the total material by weight is retained on 75 microns IS sieve.
> Fine grain soil: open $50 \%$ or more of the total material by weightpasses the 75 MicronsIS sieve
> If the soil is highly organic and contains a large percentage of organic matter and particle decomposedvegetables,then it is kept in a separate category called peat.
- Organics soil is distinguished from inorganic one by color and odor, organic soil is dark in color, which have distinctive odor and odor is made more noticeable by heating the wet sample.
- If organic content is still doubtful, material can be oven dried and remixed with water and tested for liquid limit. If reduction in liquid limit is more than $25 \%$ of the liquid limit before oven drying, then it is an organic sample.


## COARSE GRAINED SOIL:

- It is done on the basic of (i) grain size, (ii) gradation characteristic, (iii) percentage of fines (<75-micron particles) present in the soil weight. when $50 \%$ or more of the total material by weight is retained on 75 microns IS sieve.
\%ofFines is less than $\mathbf{5 \%}$

| Gravel |  | Sand |  |
| :---: | :---: | :---: | :---: |
| Well graded <br> gravel (GW) | Poorly graded <br> gravel (GP) | Well graded <br> sand (SW) | Poorly graded <br> sand (SP) |


| More than half of the coarse fraction is retained on 4.75 mm sieve. (gravel>sand) |  | More than half of the coarse fraction passes over 4.75 mm sieve. (gravel<sand) |  |
| :---: | :---: | :---: | :---: |
| $C_{u}>4$ and $C_{c}$ is between $(1,3)$ | Otherwise poorly graded | $\mathrm{C}_{\mathrm{u}}>6$ and $\mathrm{C}_{\mathrm{c}}$ is between (1,3) | Otherwise poorly graded |

## \% Fines is greater than 12\%

- Gravel and sand are classified as specified above and type of fine is calculated based on the plasticity chart, name can be GC,GM, SC, SM.
- If fines get plotted in hazed zone it means fines $>12 \%$ and $I_{p}$ is between (4-7) then dual symbol is used, GM-GC, SM-SC.
\% Fines is between 5 to 12\%
- It is a border line case and dual symbol are used, first part indicate gradation and second part indicates nature of fines.
- Symbols can be: GW-GC or GW-GM, GP-GC or GP-GM,GW-GC or SW-GM,SW-GC or SW-SM
- if on plasticity chart soil lies near A line or hatched zone then we prefer non-plastic classification. Names will be like: GW-GM, not GM-GC.


## FINE GRAINED SOIL

- Fine grained soil is classified based on the plasticity chart.
- For soil less than 425 -micron size, $W_{L}$ and $W_{p}$ are determined and plotted on the plasticity chart which is used for naming the soil accordingly where it is plotted.
Note: if sand and gravels are present in equal amount, then dual symbols are used, for ex: GW-SW, GP-SP or GM-SM, GC-SC. If fines classification is also at border, then non-plastic classification is classified as already dual symbols are present.

CHAPTER-3 SOIL COMPACTION

## PROCTOR TEST



OMC = optimum moisture content

- The moisture content at maximum dry density is known as optimum moisture content.
- Maximum dry density is a function of type of soil, compaction effort and method of compaction.
- Zero air voids line:maximum dry density achieved at a given $\mathrm{m} / \mathrm{c}$ by removing all air voids is called zero air void dry density.

$$
\begin{gathered}
\gamma_{d}=\frac{G \gamma_{w}}{1+w G} \\
\text { here, } \\
\mathrm{w}=\text { water content of compacted soil } \\
\gamma_{d}=\text { dry unit weight } \\
\mathrm{G}=\text { specific gravity } \\
\gamma_{w}=\text { unit weight of water }
\end{gathered}
$$

- Note: line of optimum is almost parallel to zero air void dry density.
- Field control of compaction: In field we need to calculate $\gamma_{d}$ and water content. Proctor needle can be used to calculate the field density and then using water content we can calculate the dry density.

$$
\text { relative density }=\frac{\gamma_{\text {dry }}(\text { field })}{\gamma_{d_{\max }}(\text { lab })}
$$

Note: $\mathbf{1 0 \%}$ air content density and $\mathbf{9 0 \%}$ saturation density are same, and $0 \%$ air content and $100 \%$ saturation density are same. But, $\mathbf{1 0 \%}$ air void and $\mathbf{9 0 \%}$ saturation density are not same, although 0\% air void and 100\% saturation density are same.

## Comparison and data for proctor tests:

|  | Standard <br> proctor test | Modified <br> proctor test | IS light <br> compaction | Is heavy <br> compaction |
| :---: | :---: | :---: | :---: | :---: |
| Weight of hammer | 2.495 kg | 4.54 kg | 2.6 kg | 4.9 kg |
| Height of fall | 304.8 mm | 457.2 mm | 310 mm | 450 mm |
| Volume of mould | 944 cc | 944 cc | 1000 cc | 1000 cc |
| No of layers | 3 | 5 | 3 | 5 |
| No of blows in each <br> layer | 25 | 25 | 25 | 25 |
| Compaction energy <br> per unit volume. | $592.7 \mathrm{KJ} / \mathrm{m}^{3}$ | $2696.3 \mathrm{KJ} / \mathrm{m}^{3}$ | $593.01 \mathrm{KJ} / \mathrm{m}^{3}$ | $2703.88 \mathrm{KJ} / \mathrm{m}^{3}$ |

Note: energy in heavy is 4.54 times energy used in light compaction, the ratio is same for modified and standard proctor test as well.

## Effect of compactive effort:

- for a given type of soil higher is the compaction effort higher will be the dry density and lesser will be the OMC.
- On wet side of optimum increase in efforts will have little effect on dry density. If large compactive effort is applies on wet side of optimum, then density will not increase much instead pore pressure may built up which will create slope stability problem and induce time dependent settlement.
- Degree of saturation at OMC is almost same in all cases.


## Effect of type of soil:

- Coarse grained soil, well graded can be compacted to high dry density, especially when some fines are present, but as quantity of fines increases maximum dry density decreases.
- Poorly graded sand will have least maximum dry density.
- In case of clays, maximum dry density decreases, and OMC increases as plasticity of the soil increases.

- Increases in organic content will increase OMC but decreases the maximum dry density.


## Effect of compaction on property of soil:

| Property |  | Dry of optimum | Wet on optimum |
| :---: | :---: | :---: | :---: |
| Structure |  | Flocculated (random) | Dispersed (oriented) |
| Water deficiency |  | More | Less |
| Swell ability |  | More | Less |
| Construction pore water pressure |  | Less | More |
| Shrinkage |  | Less | More |
| Permeability <br> It is minimum at $\mathrm{w} / \mathrm{c}$ slightly more than OMC. |  | More (isotropic) | Less (anisotropic) $\left(K_{h}>K_{v}\right)$ |
| Sensitivity |  | More | Less |
| Shear strength |  | High | Low |
| Stress strain behaviour |  | Brittle <br> (High modulus of elasticity) high peak | Ductile <br> (Low modulus of elasticity) no peak |
| Compressibility | At low stress | Less | More |
|  | At higher stress | More | Less |

## Suitability of compaction equipment:

| Type of equipment | Suitability of soil type | Nature of projects |
| :---: | :---: | :---: |
| Rammers, tampers or frog hammer | All type of soil | In confined area such as fills behind the retaining wall, basement wall trench fill etc. |
| Smooth wheel roller | Crushed rock, gravel, sand, ballast | Road construction. <br> Not suitable for embankment Not suitable for soft subgrade and uniform sand |
| Pneumatic tyred roller | Sand, gravel, silts, clay of all compressibility | Not suitable for uniformly graded soil, base-subbase, embankments, earthen dam, air field. |
| Sheep foot roller | Clayey soil, silty and clayey sand, silty clay of all compressibility | Core of earthen dam |
| Vibratory roller | Sand, granular base coarse, asphalt | Embankment for oil storage tank and base course |
| Grid roller | Well graded coarse soil, weathered rock, crushed over size material. | Not suitable for clayey soil, silty clay or uniform subgrade |

## CHAPTER-4 EFFECTIVE STRESS \& CAPILLARITY

## STRESS CONDITIONS IN SOIL

Mathematically total stress in the soil mass can be expressed as follows
Total stress $(\sigma)=$ Effective stress $(\bar{\sigma})+$ Pore water pressure (u)

$$
\sigma=\bar{\sigma}+u
$$

- Total stress on a plane within a soil mass is the force per unit area transmitted in normal direction on a plane across the soil mass.
- Pore pressure is the pressure applied by the fluid in the pores of the soil mass. It is also called neutral stress because it acts on all sides of the particles and does not cause particles to stress against each other.
- Effective stress does not represent the contact stress because we are not dividing summation of normal force with the area of contact between grains rather, we divide it by total area. Hence it does not have any physical significance. It is just a term taken as ( $\sigma$-u).


## EFFECT OF EXTERNAL LOAD ON PORE PRESSURE:

- If soil is fully saturated and soil particles and water are incompressible. Also, soil is laterally restrained. At time $t=0$, all the external loads will be taken by water. Then slowly water leaks out and load will be transferred to soil particles. At time $t=0$, as soil particles and water both are incompressible so soil particles will try to occupy a position closer together, soil is laterally restrained hence it is not possible to rearrange without expulsion of pore water. Excess pore water pressure will be generated which is over and above the hydraulic water pressure.



## EFFECT OF CAPILLARY RISE IN THE SOIL

- Due to capillary effect, pore water pressure become negative and hence effective stress increases.
- Certain depth above the water table is completely saturated with capillary water and it is called capillary saturation zone and in this zone the pore water pressure in taken to vary linearly.
- The depth of capillary saturation zone in sand and silt in givenby,

$$
\frac{0.03}{0.2 \mathrm{D}_{10}}=\alpha \quad \bullet \quad \mathrm{D}_{10} \text { in } \mathrm{mm} \text { and } \alpha \rightarrow \text { meter. }
$$

$$
\begin{array}{ll}
\alpha=\frac{\mathrm{C}}{\mathrm{eD}_{10}} & \bullet \alpha \rightarrow \mathrm{~cm}, \mathrm{D} 10 \rightarrow \mathrm{~cm} \\
& \bullet \mathrm{C} \rightarrow \text { empirical constant (value } 0.1-0.5 \mathrm{~cm} 2 \text { ) }
\end{array}
$$

- Soil having same $D_{10}$ value can have different capillary rise depending on soil structure and zoological history.
- Due to increase in effective stress on account of capillary action shear strength of soil increase.
- Bulking of sand is due to capillary effect.


## EFFECT OF SEEPAGE ON EFFECTIVE STRESS

- When water seeping through the soil total head is dissipated as viscous friction producing a frictional drag in the direction of flows on the soil particles. This drag force results in seepage force on the soil mass which acts in the direction of flow.
- Component of seepage force acting vertically upward will reduce the effective vertical stress and component of this seepage force acting vertically downward will increase the effective vertical stress.
- Under no flow condition surface force acting is only equal to buoyant force acting in vertically upward direction and under flowing condition, apart from the buoyant force there will also be a seepage force acting in the flow direction and seepage force is equal to [Seepage pressure $\times$ Area]
- Seepage pressure $=h_{2} Y_{w}$
- Seepage force per unit volume $=\frac{h_{2} \gamma_{w} g \times 1}{b^{2} \times 1}=\frac{h_{2} \gamma_{w}}{b}=i \gamma_{w}$
- Resultant force on the soil mass in the vector summation of buoyant weight and seepage force.

- The above resultant force will be balanced by the contact forces and vertical component of the contact force will be equal to
[Buoyant weight $\pm$ vertical component of seepage force].


## No flow condition:



Datum

|  | Datum head | Pressure head | Total head |
| :---: | :---: | :---: | :---: |
| $A$ | H | $\mathrm{H}_{1}$ | $\mathrm{H}+\mathrm{H}_{1}$ |
| $B$ | 0 | $\mathrm{H}_{1}+\mathrm{H}$ | $\mathrm{H}+\mathrm{H}_{1}$ |
| C | $\mathrm{H}-\mathrm{Z}$ | $\mathrm{H}_{1}+\mathrm{Z}$ | $\mathrm{H}+\mathrm{H}_{1}$ |

Note: Pressure is not affected by datum assumed.

## Downward flow situation



|  | Datum head | Pressure head | Total head |
| :---: | :---: | :---: | :---: |
| $A$ | $H$ | $H_{1}$ | $H+H_{1}$ |
| $B$ | 0 | $H_{1}+\mathrm{H}-\mathrm{h}_{\mathrm{L}}$ | $\mathrm{H}+\mathrm{H}_{1}-\mathrm{H}_{L}$ |
| $C$ | $\mathrm{H}-\mathrm{Z}$ | $\mathrm{H}_{1}+\mathrm{H}-\frac{\mathrm{h}_{\mathrm{L}}}{\mathrm{H}} \cdot \mathrm{Z}$ | $\mathrm{H}+\mathrm{H}_{1}-\frac{\mathrm{h}_{\mathrm{L}}}{\mathrm{H}} \cdot \mathrm{Z}$ |

## Upward flow situation



|  | Datum head | Pressure head | Total head |
| :---: | :---: | :---: | :---: |
| $A$ | $H$ | $H_{1}$ | $H+H_{1}$ |
| $B$ | 0 | $H_{1}+H+h_{L}$ | $H+H_{1}+H_{L}$ |
| $C$ | $H-Z$ | $H_{1}+\frac{h_{L}}{H} \cdot Z+Z$ | $H+H_{1}+\frac{h_{L}}{H} \cdot Z$ |

If a force $F$ acting at top, then

$$
\begin{aligned}
& \bar{\sigma}=\frac{F}{A} \pm \frac{\text { Seepage force }}{A}+\frac{\text { Buoyant force }}{A} \\
& {\left[\begin{array}{l}
+\downarrow \text { Flow } \\
-\uparrow \text { Flow }
\end{array}\right] }
\end{aligned}
$$

$\bar{\sigma}=$ effective stress under no flow $\pm$ Seepage pressure (hL. $\gamma_{w}$ )
$h_{L} \rightarrow$ Loss already occurred up to the point under consideration in downward flow.
$h_{L} \rightarrow$ Loss yet to occur beyond the point under consideration in upward flow.
Effective vertical stress at any point for = [Total vertical stress - Pore water pressure] any direction of flow can be easily

## calculated as

Note: Buoyant force is equal to effective stress under no flow condition.
Variation of stressin case of water is above the soil mass


Variation of stress for Soil mass with surcharge


Variation of Stress with Soil mass with capillary fringes


Note: If the soil above H is saturated by ground water table instead of capillary water, effective stresses in the soil will be reduced and it will behave as submerged soil.

## QUICKSAND CONDITION

- In case of upward seepage flow if the upward seepage force becomes equal to buoyant weight of soil effective stress at every point in the soil will be equal to zero.
$(\bar{\sigma})_{c}=\gamma_{\text {sub }} z-\frac{h_{L}}{H} \cdot Z_{\gamma_{w}}=0 \Rightarrow i=\frac{\gamma_{\text {sub }}}{\gamma_{w}}$ and $\quad(\bar{\sigma})_{B}=\gamma_{\text {sub }} H-\frac{h_{L} H r_{w}}{H}=0 \Rightarrow i=\frac{\gamma_{\text {sub }}}{\gamma_{w}}$
- critical hydraulic gradient at which effective stress is zero at every point in the soil mass.

$$
\mathrm{i}_{\mathrm{c}}=\frac{\gamma_{\text {sub }}}{\gamma_{\mathrm{w}}}=\frac{\mathrm{G}-1}{1+\mathrm{e}}
$$

- In case of sand $[\mathrm{t}=\mathrm{C}+\bar{\sigma} \tan \phi], \mathrm{C}=0$ hence if $\bar{\sigma}=0$ then $\mathrm{T}=0$, sand will loose its shear strength completely. This condition is called quicksand condition.
- To safeguard against the quicksand condition during excavation the option are:
$\Rightarrow$ Lower the water table by pumping.
$\Rightarrow$ Increase the depth of embedment of sheet pile.
$\Rightarrow$ by helping certain depth of water in pit.
$\Rightarrow$ Sometimes are also add graded filter on down stress side of masonry dam because at that point there could be chance of quicksand condition taking place.
- Factor of safety against quicksandcondition is equal to,

$$
\begin{gathered}
\text { FoS }=\frac{\text { Buoyant force }}{\text { Seepage force }} \\
\text { FoS }=\frac{\text { Effective stress under no flow condition }}{\text { Seepage pressure }}
\end{gathered}
$$

Note: In case of clay if the pore water pressure exceeds the total stress, we will have a heaving problem and that may lead to cracking of clay. To safeguard against heave,wehave to ensure that stress is greater than pore water pressure and

$$
\text { FoS against heavying }=\frac{\text { Total stress }}{\text { Pore water pressure }}
$$

## CHAPTER-5 PERMEABILITY

## DARCY'S LAW

As per Darcy's law, for 1-D in saturated soils, velocity of flow is directly proportional to hydraulic gradient. Darcy law is valid when the flow through the soil is laminar, soil is saturated, flow is steady, and flow is irrotational.

Note: flow through the soil finer than coarse sand is generally laminar in the field, however flow through the gravel and rock fills generally becomes turbulent and Darcy's law is invalid. If the volume and shape of water passage varies with time due to impurity in flowing fluid, then also Darcy's law is invalid.

As per Darcy's law, $\mathrm{v} \propto \mathrm{i} \Rightarrow \mathrm{v}=\mathrm{ki}$
Here, $\mathbf{k}=$ permeability or coefficient of permeabilityand $v$ is discharge velocity.

## Discharge:

$$
\begin{array}{l|l}
\boldsymbol{Q}=\boldsymbol{k} \boldsymbol{i} \boldsymbol{A} & \text { Here, } \mathrm{k}=\text { coefficient of permeability }
\end{array}
$$

$$
A=\text { total area of flow }
$$

$\mathrm{i}=$ hydraulic gradient

## Seepage velocity:

$$
\frac{V_{\text {discharge }}}{n}=V_{\text {seepage }}
$$

Note: seepage velocity is generally small and hence in seepage condition, for calculation the total head taken is piezometric head only. And difference of piezometric head between
two points is the head loss between two points. i.e. difference in level of liquids in the two piezometers inserted at two points in the flow is the head loss between these two points.
Note: actual rise in piezometer is pressure head.

## Factors Affecting Permeability

$$
k=C_{s}\left(\frac{\gamma}{\mu}\right)\left(\frac{1}{S_{0}^{2}}\right)\left(\frac{e^{3}}{1+e}\right) S^{3} \left\lvert\, \begin{array}{cl}
\text { Here, } \mathrm{C}_{s}=\text { shape factor } & \mathrm{S}_{0}=\text { wetted surface area per } \\
\mu=\text { dynamic viscosity of fluid } & \text { unit volume of surface and it } \\
\mathrm{S}=\text { degree of saturation } & \text { is dependent on particle size } \\
\gamma=\text { unit weight of fluid } & \text { and soil fabric. It is assumed } \\
& \text { to be a measure of pore size. }
\end{array}\right.
$$

## (i) Grain size:

- Permeability varies approximately as the square of the grain size.
- According to Allen Hazen

$$
\mathrm{k}=\mathrm{CD}_{10}^{2}
$$

```
k = coefficient of permeability
(cm/sec)
c = constant (value = 100)
D}10= effective diameter (cm
```

(ii) Effect of void ratio:

- Void ratio and permeability can be related by the following expression.

$$
\mathrm{k}=\frac{\mathrm{e}^{3}}{1+\mathrm{e}}
$$

Where, e = void ratio of soil
(iii) Density and Viscosity of Fluid:

- This relation between the fluid property and permeability can be given as

$$
k \propto\left(\frac{\gamma}{\mu}\right)
$$

Note: intrinsic permeability $\left(k_{i}\right)$, unit is $\mathrm{M}^{2}$ or Darcy. It is independent of fluid property.

$$
k=k_{i} \frac{\gamma}{\mu}
$$

## DETERMINATION OF AVERAGE PERMEABILITY

- parallel to Bedding plane

Where, $\gamma=$ unit weight of fluid $\mu=$ dynamic viscosity of fluid.

- perpendicular to bedding plane
parallel to Bedding plane



$$
\mathrm{k}_{\mathrm{x}}=\frac{\Sigma \mathrm{k}_{\mathrm{i}} \mathrm{z}_{\mathrm{i}}}{\Sigma \mathrm{z}_{\mathrm{i}}}
$$

Here,

- $Z_{1}, Z_{2}$, and $Z_{3}=$ thickness of layer
- $v_{1}, v_{2}$ and $v_{3}=$ velocity through layers.
- $\mathrm{k}_{1}, \mathrm{k}_{2}$ and $\mathrm{k}_{3}=$ coefficient of permeability
- hydraulic gradient (i) will be same in all
- $\mathrm{q}_{1}, \mathrm{q}_{2}$ and $\mathrm{q}_{3}=$ discharge through the layer

- $Z_{1}, Z_{2}$, and $Z_{3}=$ thickness of layer
- $i_{1}, i_{2}$, and $i_{3}=$ hydraulic gradient for layers
- velocity(v) and discharge(q) will be same in all
- $k_{1}, k_{2}$ and $k_{3}=$ coefficient of permeability


## DETERMINATION OF PERMEABILITY

Permeability can be determined using the following methods:

## Laboratory Method

## (a) Constant Head Method:



$$
\mathrm{k}=\frac{\mathrm{VL}}{\mathrm{tHA}}
$$

Here,
$\mathrm{V}=$ velocity of fluid flowing through soil
(b) Falling Head Method:


$$
\mathrm{k}=2.303 \frac{\mathrm{aL}}{\mathrm{At}} \log _{10} \frac{\mathrm{~h}_{1}}{\mathrm{~h}_{2}}
$$

Here, $a=$ area of standpipe $h_{1}$ is height at time $t_{1}$

$$
\begin{array}{c|l}
L=\text { length of soil medium } & h_{2} \text { is height at time } t_{2}=t_{1}+t . \\
t=\text { time } & \text { Note: This test is used for relatively less } \\
H=\text { head } & \text { permeable soils where discharge is small. } \\
A=\text { Area of cross section } & \text { Basically, be suited for fine grained soil. }
\end{array}
$$

Note: If water travel from $h_{1}$ to $h_{2}$ in time $t$ and it travels from $h_{2}$ to $h_{3}$ in another time $t$, then

$$
h_{2}=\sqrt{h_{1} h_{3}}
$$

## Field Method

## Case I: Well in unconfined aquifer


here, $R=3000 s \sqrt{k}$
$S=$ drawdown at the well in (m).
$K=$ coefficient of permeability in 'm/sec'
$r=$ radius of well

(i) As per Dupit's Theory:

$$
\mathrm{k}=\frac{2.303 \mathrm{qlog}_{10}(\mathrm{R} / \mathrm{r})}{\pi\left(\mathrm{H}^{2}-\mathrm{h}^{2}\right)}
$$

(ii) As per Theim's theory:

$$
\mathrm{K}=\frac{2.3036 \mathrm{qlog}_{10}\left(\mathrm{r}_{1} / \mathrm{r}_{2}\right)}{\pi\left(\mathrm{h}_{1}^{2}-\mathrm{h}_{2}^{2}\right)}
$$

$\mathrm{q}=$ discharge pumping out
$\mathrm{h}=$ depth of water in the well measured above the impermeable layer. $\mathrm{H}=$ thickness of aquifer, measured from the impermeable layer to the initial level of water table.

## (i) As per Dupit's theory:

$$
\mathrm{K}=\frac{2.303 \mathrm{qlog}_{10}(\mathrm{R} / \mathrm{r})}{2 \pi \mathrm{D}(\mathrm{H}-\mathrm{h})}
$$

Here, $D=$ width of confined aquifer.
(ii) As per Theim's theory:

$$
\mathrm{K}=\frac{2.303 \mathrm{qlog}_{10}\left(\mathrm{r}_{1} / \mathrm{r}_{2}\right)}{2 \pi \mathrm{D}\left(\mathrm{~h}_{1}-\mathrm{h}_{2}\right)}
$$

$r_{1}$ and $r_{2}=$ radial distance of two wells
$h_{1}$ and $h_{2}=$ depth of water in both wells

## CHAPTER-6 SEEPAGE THROUGH SOIL

- Seepage in the process in which liquid leaks through a porous medium from high head towards low head. Seepage problems could be loss of water from reservoir, uplift force below the dam and piping problem etc.
- Hence the need to calculate amount of seepage, uplift pressure and exit gradient. All of there can be calculate using flow net.
- Flow net is graphical representation of path taken by water particles and the head variation along the path.
- The concept of flow net is based on Laplace equation.
- Laplace equation of 2-D flowAssumptions
- Soil is fullysaturated and Darcy's law is valid.
- Soil mass in homogeneous and isotropic ( $\mathrm{K}_{\mathrm{x}}=\mathrm{K}_{\mathrm{y}}$ )
- Both soil grains and pore fluid are incompressible.
- Steady state condition exists.


## Properties and use of flow net



- Flow lines are drawn in such a way that between two successive flows lines discharge in constant.
- Equipotential lines are locus of points having same total head i.e. piezometer inserted at any two point on the same equipotential line will rise to the same level.
- Equipotential lines are drawn in such a way that they have same potential drop between them.
- The loss between two equipotential lines $=\frac{\mathrm{H}}{\mathrm{N}_{\mathrm{d}}} . \mathrm{H} \rightarrow$ (different between upstream and downstream water level.)
- Actual rise in piezometer in the pressure head.
- Each flow field is and elementary square (average distance between two flow line and average distance between two equipotential line is same).
- Flow line and equipotential lines are orthogonal to each other for isotropic soil.
- Space between two adjacent flow line is called flow channel and area between two adjacent flow line and two equipotential line is called flow field.
- $N_{f}=$ No. of flow channel $=$ No. of flow lines - 1
- $N_{d}=$ No. of potential drop $=$ No. of equipotential lines -1

$$
\begin{gathered}
q=K \frac{h_{L}}{b_{1}} \times a_{1} \times 1=K \frac{h_{L}}{b_{2}} \times a_{2} \times 1=K \frac{h_{L}}{b_{3}} \times a_{3} \times 1 \\
q=h_{L} \times K \times 1 \Rightarrow Q=q N_{f} \\
Q=K \frac{N_{f}}{N_{d}} \cdot H \quad \text { Where } \frac{N_{f}}{N_{a}} \rightarrow \text { Shape factor }
\end{gathered}
$$

Note:The shape factor is a function of boundary conditions only. It will not change with permeability of soil, head causing flow, direction of flow or number of $N_{f}$ and $N_{d}$ changes. It will only change if boundary condition or extent of flow is changed.

- Exit gradient is maximum near the toe because flow field is smaller there
- $\frac{\mathrm{h}_{\mathrm{L}}}{\ell}=$ Exist gradient and $\mathrm{h}_{\mathrm{L}}=\frac{\mathrm{H}}{\mathrm{N}_{\mathrm{D}}}$

- If the exit gradient is more piping begins near the downstream toe and lengthens progressively towards the upstream side as the seeping water gradually washes away more and more of soil particle leaving a void or pipe in the soil.Piping can proceed backwards along the base of the dam or along the bedding plane in the soil strata where the resistance is minimum. Dam may collapse into the pipes created and it could be a catastrophic failure (sudden).
- FOS against piping $=\frac{i_{c}}{i_{e}}$
- $\mathrm{i}_{\mathrm{c}}$ is critical hydraulic gradient
- $i_{e}$ is the exit gradient.
- FOS is taken conservatively minimum as 6 .
- D/S vertical cut-off leads to larger flow field at exit hence ie falls.
- U/S cut-off piles helps in reducing the uplift pressure force below the dam as more no of equipotential drops could have already occurred by the time water reaches the base of dam thus uplifting pressure reduces.
- Provision of downstream cut-off will reduce exit gradient but will increase the uplift pressure.
- Provision of upstream pile will reduce the uplift pressure, but it will increase the exit gradient.


Note: Provision ofcut-off upstream or downstream will change the ( $n_{f} / n_{d}$ ) value and hence the discharge.

Flow net for confined flow: For solution of Laplace equation in 2-D we required four boundary conditions, if four boundary conditionone known then the flow net drawn is called flow net for confined flow. Example: flow under sheet pile, concrete or masonry dam.

Flow net for unconfined flow: In case of earthen dam top flow line is not known in advance thus we have only three boundary conditionsknown hence the flow net drawn for this is called flow net for unconfined flow.

## Locating the top flow line for seepage through earthen dam

According to casegrande the flow lines and
equipotential lines are 2 sets of parabola with
point ' $P^{\prime}$ as the common focus and top parabola
is called base parabola.
$\mathrm{DC}=0.3 \mathrm{BC}$
$\sqrt{\mathrm{x}^{2}+\mathrm{y}^{2}}=\mathrm{x}+\mathrm{S}$
$\Rightarrow \mathrm{y}^{2}=(\mathrm{x}+\mathrm{s})^{2}-\mathrm{x}^{2}$
$\mathrm{y}^{2}=\mathrm{s}^{2}+2 \mathrm{sx}$
also, $\sqrt{\mathrm{D}^{2}+\mathrm{H}^{2}}=\mathrm{D}+\mathrm{S}$
then find $(\mathrm{S})$

- Once the top flow line is located flow net cam be determined and seepage calculation line $\left(K \cdot H \frac{N_{t}}{N_{D}}=q\right)$ can be obtained. $Q=k i . A, q=k \times 1 \times S \times 1$

Andq = ks (per unit width)

- Note: $\frac{\mathrm{PA}}{\gamma}=\alpha$
because $A$ and $B$ both have equal potential
(applying Bernoulli) $\frac{\mathrm{PA}}{\gamma}+0=0+\alpha$


Flow through anisotropic soil

Anisotropic

Isotropic

- In case of anisotropic conditionwe transform the section into an equivalent isotropic system by reducing the horizontal dimension by a factor of $\left[\sqrt{\frac{k_{y}}{k_{x}}}\right.$ or $\left.\sqrt{\frac{k_{v}}{k_{H}}}\right]$. While keeping the vertical dimension intact. On the transformed section flow net can be drawn and flow lines and equipotential lines can be drawn.
- Transformed section can be used to determine the pore pressure head at any point but to determine the hydraulic gradient one should use the length of original section.

$$
\begin{gathered}
\mathrm{i}_{\text {Actual }}=\frac{\mathrm{h}_{\mathrm{L}}(\text { Between two points })}{\text { Original distance between those points }} \\
\mathrm{L}_{\mathrm{AB}}=\sqrt{\mathrm{y}^{2}+\left(\mathrm{x} \sqrt{\frac{\mathrm{k}_{\mathrm{x}}}{\mathrm{k}_{\mathrm{y}}}}\right)^{2}} \text { in actual case }
\end{gathered}
$$

## FLOW THROUGH NON-HOMOGENEOUS SECTION



- $\frac{\mathrm{k}_{1}}{\mathrm{k}_{2}}=\frac{\tan \alpha_{1}}{\tan \alpha_{2}}$
- To prevent the possibility of erosion and piping two approaches are used -
> Control seepage and seepage force.
> Used of protective filter.
- Seepage force and erosion can be prevented by provision of cut-off wall or increasing the length of flow path.
- Use of protective filter prevents erosion and helps in safeguarding against piping.
- Filter will prevent migration of particles without significant loss of head in the filter, if the void in the filter are large, particles will be washed out and if the void are small then seepage force may develop in the filter, to achieve this filter must have grain size that satisfies certain requirement.

| $\frac{D_{15} \text { filter }}{D_{85} \text { protective material }<5}$ | I.This will ensure no significant invasion of <br> particle from the protected material to the <br> filter occurs and thus governs the upper <br> limit of grain size of filter. |
| :--- | :--- |
| $4<\frac{D_{15} \text { (filter) }}{\mathrm{D}_{15} \text { (protected material) }<20}$II. This will ensure that seepage pressure does <br> not build up in the filter and governs the <br> lower limit of material size. <br> $\frac{\mathrm{D}_{50} \text { (filter) }}{\mathrm{D}_{50} \text { (protected layer) }<25}$ III.Additional criteria |  |

## CHAPTER-7 VERTICAL STRESS DISTRIBUTION IN SOIL

## BOUSSINESQ'S THEORY:

## Assumptions:

- Soil mass is elastic.
- Soil is homogeneous and isotropic.
- Soil is Semi Infinite.
- Soil is weightless and unstressed before the application ofload.
- Distribution of the stress is symmetrical about the vertical axis.

ISOBAR: It is curve or a contour connecting points of equal vertical pressure below the ground surface.

PRESSURE BULB: it is zone of the soil in which there is significant stress distribution and beyond that stress is less.


VERTICAL STRESS DUE TO POINT LOAD

$$
\sigma_{z}=\frac{3 Q}{2 \pi z^{2}}\left[\frac{1}{1+\left(\frac{r}{z}\right)^{2}}\right]^{\frac{5}{2}}
$$

- if the point is directly under the load, i.e. $r=0$, then

$$
\sigma_{z}=0.4775\left(\frac{Q}{z^{2}}\right)
$$

| if $r$ is varying and $z=$ constant | if $r=$ constant and $z$ varies |
| :---: | :---: |
|  |  |
| $\longleftarrow \mathrm{r}=0 \longrightarrow$ |  |
| maximum value will be at $r=0$, $\sigma_{z}=0.4775\left(\frac{Q}{z^{2}}\right)$ | $\begin{gathered} \text { Maximum value will be at angle } \\ 39^{\circ} 15^{\prime} . \\ \sigma_{z}=\mathbf{0 . 1 3 3 2}\left(\frac{\boldsymbol{Q}}{z^{2}}\right) \text { and } \sigma_{z}=\mathbf{0 . 0 8 8 8}\left(\frac{\boldsymbol{Q}}{r^{2}}\right) \end{gathered}$ |

## EQUIVALENT POINT LOAD METHOD

- In this method the pressure loading over an area is converted to a single point load and applied at centre of the area and then Boussinesue's equation for point load is used.


## VERTICAL PRESSURE DUE TO LINE LOAD

- Here type of loading can be boundary wall, railway line etc.

$$
\sigma_{z}=\frac{2 q^{\prime}}{\pi z}\left[\frac{1}{1+\left(\frac{r}{z}\right)^{2}}\right]^{2}
$$



## VERTICAL STRESS DUE TO STRIP LOAD

- The expression for vertical stress at any point $P$ under a strip load can be developed from the equation of line load.

Case I. Point P below the centre of the strip.


Case II. Point P not below the centre of strip


$$
\sigma_{z}=\frac{q}{\pi}(2 \theta+\sin 2 \theta)
$$

$$
\sigma_{z}=\frac{q}{\pi}\left[\left(\beta_{2}-\beta_{1}\right)+\sin \left(\beta_{2}-\beta_{1}\right) \cos \left(\beta_{2}+\beta_{1}\right)\right]
$$

## VERTICAL STRESS UNDER A CIRCULAR AREA

- Consider a uniform load of intensity q acting over a circular area of radius $R$ on the surface of a semi-infinite soil mass.
- Point $P$ is directly below the centre of the circle,

$$
\begin{aligned}
& \sigma_{z}=q\left(1-\cos ^{3} \theta\right) \\
& \cos \theta=\frac{Z}{\sqrt{z^{2}+R^{2}}}
\end{aligned}
$$



## NEWMARK'S INFLUENCE CHART METHOD

- A chart consisting of the number of circles and radiating lines, is so prepared that the influence of each area unit is the same at the centre of the circles i.e. each area unit causes the equal vertical stress at the centre of the diagram. Let it consist of 'm' number of concentric circles and ' $n$ ' number of radial lines.
- Therefore, the number of area units $=m \times n$
- Vertical stress is $\boldsymbol{\sigma}_{\mathbf{z}}=\boldsymbol{q n I}$ and Influence value, $\boldsymbol{I}=\frac{1}{m n}$.
- The plan of the loaded area is traced on a tracing paper and placed over the influence chart such that the point at which the stress is required should coincide with the centre of the chart
- Number of area units occupied by the plane of the loaded area on the influence chart including the fractional areas are counted.
- NOTE: we draw a line of 1 cm on the graph and scale of our loaded area will be $1 \mathrm{~cm}=\mathrm{Hm}$, here H is depth of the
 point below loading area.

EQUIVALENT STRESS AT POINT LOCATED BELOW THE CORNER OF RECTANGULAR AREA:

$$
\sigma_{z}=q I
$$

- here I is the influence factor based on the parameter m and $\mathrm{n}, m=\frac{L}{z}, n=\frac{B}{z}$
- Any point can be placed at corners of various rectangles and then net vertical stress is calculated based on superposition.


Vertical stress at a point within rectangular area

$$
=q\left[I_{2(1)}+I_{2(2)}+I_{2(3)}+I_{2(4)}\right]
$$

## LOAD DISPERSION THEORY (2V:1H)

- if $L>B$, rectangular loading,

$$
\sigma_{z}=\frac{\boldsymbol{Q}}{(\boldsymbol{B}+\boldsymbol{z})(\boldsymbol{L}+\boldsymbol{z})}
$$

- if $L \gg B$, strip loading then,

$$
\sigma_{z}=\frac{q B}{(B+z)}
$$



## WESTERGAARD'S THEORY

## Assumptions:

- soil as a thin sheet layered one over other.
- Poisson's effect is not considered, and lateral strain development is taken zero.
- Hisanalysis is best suited for stratified soil deposits.
- Vertical stress at any given point below the loading is given by

$$
\sigma_{z}=\frac{Q}{\pi z^{2}}\left[\frac{1}{1+\left(\frac{r}{z}\right)^{2}}\right]^{\frac{3}{2}}
$$

- if $r=0$, then

$$
\sigma_{z}=0.318 \frac{Q}{z^{2}}
$$

Note:Boussinesq's theory is preferred over westergaard's theory as the results obtained from it are more conservative i.e. safe for the design.

Note: the limiting point at which the stress due to bousnisque and wasterguard theory gives equal result is


$$
\begin{aligned}
& \text { if } \frac{r}{Z}<1.5 \Rightarrow k_{\mathrm{b}}>k_{w} \\
& \text { if } \frac{r}{Z}>1.5 \Rightarrow k_{w}>k_{\mathrm{B}} \\
& \text { if } \frac{r}{Z}=1.5 \Rightarrow k_{w}=k_{B}
\end{aligned}
$$

$$
\frac{r}{z}=1.5 .
$$

## CHAPTER-8 COMPRESSIBILITY \& CONSOLIDATION

## Compaction Vs Consolidations

| Compaction | consolidation |
| :--- | :--- |
| It is an instantaneous process. | It is time dependent. |
| Soil is always partially saturated. | Soil is completely saturated. |
| Densification is due to reduction in void at a <br> given moisture content. | Volume reduction is due to expulsion of <br> pore water from voids. |
| Roller, tempers are used. | Static load is applied. |

Types Of Settlement
Primary settlement: If the soil is initially partially saturated, expulsion of air as well as compression of pore air may take place with the application of external loads which is called Initial Compression. Itis animmediate phenomenon.After the initial compression, soil reaches into fully saturated state, further reduction in volume occurs due to expulsion of pore water i.e. water present in the soils. Immediate settlement can also occur if significant lateral strain takes place. This is due to deformation of soil under undrained condition. This immediate settlement can be calculated from elastic theory.
Primaryconsolidation: occurs due to expulsion of excess pore water pressure generated due to increase in total stress. It is a time dependent phenomenon.
For Example: When a structure is built over a layer of saturated clay.
When water table is lowered permanently in a structure overlaying a clay layer.
Magnitude of settlement due to $1^{\circ}$-consolidation depends on:

1. Compressibility of soil, 2. Magnitude of stress increase, 3. Thickness of soil layer, 4. Permeability of soil

## Secondary consolidation:

Experimentally, it has been shown that compression of soil layer does not cease when excess pore water pressure has been completely dissipated to zero. It continues at a gradually decreasing rate under constant effective stress, $2^{\circ}$-Consolidation is thought to be due to gradual readjustment of clay particles into a more stableconfiguration following the structural disturbance caused by the decrease in void ratio.
Rate of 2-consolidation is thought to be controlled by highly viscous film of absorbed water, surrounding the clay mineral particles in soil. A very slow viscous flow of adsorbed water takes place from the zones of film contact, allowingthe particles to move closer together. Viscosity of film increases as particles move closer, resulting in a decrease in the rate of compressionof the soil.

It is more important for peat, for organic soil. It is unimportant for pre-consolidated clay and stiff clay.

Note: In coarse grained soil, any volume change resulting from change in loading occurs immediately after the loading increases and water pressure dissipates rapidly due to high permeability. This is called drained loading. In fine soils with low permeability, the soil is undrained as load is applied, slow seepage occurs, and excess pore water dissipates leading to consolidation. Hence in coarse grained soil $1^{\circ}$ consolidation is $=0$.

## Normal And Over Consolidated Soil

Over consolidation ratio $(O C R)=\frac{\text { maximum applied effective stress in the past }}{\text { present applied effective stress }}$

- For over consolidated soil,OCR>1
- For normal consolidated soil, OCR=1
- For Under consolidated soil, OCR<1

Effective stress v/s void ratio (NC soil):


Note: coefficient of compressibility decreases with increase in effective stress, because with increase in loading soil become more densified hence resistance to further compression increases.

Effective stress v/s void ratio on logarithmic scale:


When soil is in normal consolidation region: $\bar{\sigma}_{0}$ and $\bar{\sigma}_{0}+\Delta \bar{\sigma}$ both lies in NCR.

$$
\mathrm{S}_{\mathrm{c}}=\Delta \mathrm{H}=\frac{\mathrm{C}_{\mathrm{C}} \mathrm{H}_{0}}{1+\mathrm{e}_{0}} \log _{10}\left(\frac{\bar{\sigma}_{0}+\Delta \bar{\sigma}}{\bar{\sigma}_{0}}\right)
$$

$$
\mathrm{c}_{\mathrm{c}}=\frac{-(\Delta \mathrm{e})}{\log _{10}\left(\frac{\bar{\sigma}_{1}}{\bar{\sigma}_{0}}\right)}
$$

## $\mathbf{C}_{\mathbf{c}}=$ compression index

$\Delta \mathrm{e}=$ change in void ratio
$\bar{\sigma}_{1}=\bar{\sigma}_{0}+\Delta \bar{\sigma}, \Delta \bar{\sigma}=$ change in effective stress
for undisturbed soil, $C_{c}=0.009\left(w_{L}-10\right)$
for remoulded soil, $C_{c}=0.007\left(w_{L}-10\right)$

When soil is in over consolidation region:
$\bar{\sigma}_{0}$ and $\bar{\sigma}_{0}+\Delta \bar{\sigma}$ both lies in OCR.
$\mathrm{S}_{\mathrm{c}}=\Delta \mathrm{H}=\frac{\mathrm{C}_{\mathrm{r}} \mathrm{H}_{0}}{1+\mathrm{e}_{0}} \log _{10}\left(\frac{\bar{\sigma}_{0}+\Delta \bar{\sigma}}{\bar{\sigma}_{0}}\right)$

When $\bar{\sigma}_{0}$ lies in NCR and $\bar{\sigma}_{0}+\Delta \bar{\sigma}$ lies in OCR,

$$
\Delta \mathrm{H}_{1}=\frac{\mathrm{C}_{\mathrm{r}} \mathrm{H}_{0}}{1+\mathrm{e}_{0}} \log _{10}\left(\frac{\bar{\sigma}_{\mathrm{p}}}{\bar{\sigma}_{0}}\right)
$$

then total settlement, $\Delta \mathrm{H}=\Delta \mathrm{H}_{1}+\Delta \mathrm{H}_{2}$ here, $\mathrm{e}^{\prime}$ and $\mathrm{H}^{\prime}$ are void ratio and height after entering the NC region.

$$
\Delta \mathrm{H}_{2}=\frac{\mathrm{C}_{\mathrm{C}} \mathrm{H}^{\prime}}{1+\mathrm{e}^{\prime}} \log _{10}\left(\frac{\bar{\sigma}_{0}+\Delta \bar{\sigma}}{\bar{\sigma}_{\mathrm{p}}}\right)
$$

$\bar{\sigma}_{\mathrm{P}}=$ pre-consolidation stress
Similarly swelling can be: $\Delta H=\frac{C_{5} H}{1+e_{1}} \log _{10}\left(\frac{\bar{\sigma}_{1}}{\bar{\sigma}_{2}}\right)$ here, $e_{1}$ is void ration corresponding to $\bar{\sigma}_{1}$.

Note: insitu e-Log $\bar{\sigma}$ curve is steeper than laboratory e-Log $\bar{\sigma}$ curve. This is because lab curve is obtained for sample collected from field in which unloading has taken place during sampling hence the laboratory curve will be recompression curve leading to flatter slope.

## Immediate Settlement

(a) Immediate settlement in sand:

| $\bar{\sigma}_{0}=$ Initial effective overburden pressure at centre of soil $\Delta \bar{\sigma}=$ decrease in effective stress at the centre of compressibility. | $\begin{aligned} & \mathrm{S}_{\mathrm{i}}=\frac{\mathrm{H}_{0}}{\mathrm{C}_{\mathrm{s}}} \log _{10}\left(\frac{\bar{\sigma}_{0}+\Delta \bar{\sigma}}{\bar{\sigma}_{0}}\right) \\ & \text { And } C_{s}=1.5 \frac{c_{r}}{\bar{\sigma}_{0}} \\ & \mathrm{C}_{\mathrm{r}}=\text { static cone resistance constant }\left(\mathrm{kN} / \mathrm{m}^{2}\right) \\ & \mathrm{H}_{0}=\text { initial thickness of soil } \end{aligned}$ |
| :---: | :---: |
| (b) Immediate settlement in clays: |  |
| $\begin{aligned} & \qquad S_{i}=\frac{q B\left(1-\mu^{2}\right) I_{t}}{E_{s}} \\ & \mathrm{E}_{\mathrm{s}}=\text { young's modulus of soil } \\ & \mathrm{I}_{\mathrm{t}}=\text { shape factor or influence factor } . \end{aligned}$ | $B=$ width of foundation <br> $\mathrm{q}=$ uniform pressure at the base of foundation $\mu=\text { Poisson's ratio of soil (0.3 to } 0.5 \text { ) }$ |

$\mathbf{2}^{\circ}$ Consolidation Settlement (Ss)
$\mathrm{S}_{\mathrm{S}}=\frac{\mathrm{C}_{\alpha} \mathrm{H}_{1}}{1+\mathrm{e}_{1}} \log _{10}\left(\frac{\mathrm{t}}{\mathrm{t}_{1}}\right)$
$\mathrm{C}_{\alpha}=\frac{\Delta \mathrm{e}}{\log _{10}\left(\frac{\mathrm{t}_{2}}{\mathrm{t}_{1}}\right)}$
$\mathrm{S}_{\mathrm{s}}=2^{\circ}$ settlement after time ' t ' from completion of $1^{\circ}$ consolidation.
$\mathrm{t}_{1}=$ time required for completion of $1^{\circ}$ consolidation.
$\mathrm{e}_{1}=$ void ratio after $1^{\circ}$ consolidation
$\mathrm{H}_{1}=$ thickness of soil after $1^{\circ}$ consolidation
$\mathrm{C} \alpha=2^{\circ}$ compression index. It is $4-6 \%$ of $\mathrm{C}_{\mathrm{c}}$.

One dimensional consolidation equation of Terzaghi: the rate of change of settlement is directly related to the rate of dissipation of pore water pressure. Terzaghi predicted the time rate of consolidation for 1-D consolidation using a mathematical theory, if stress is constant:

$$
\begin{array}{ll}
\frac{\partial u}{\partial t}=C_{v} \cdot \frac{\partial^{2} u}{\partial z^{2}} & \begin{array}{l}
C_{v}=\text { coefficient of consolidation } \\
u=\text { excess pore pressure. }
\end{array}
\end{array}
$$

- The coefficient of consolidation can be calculated as

$$
\begin{array}{ll}
C_{v}=\frac{K}{\gamma_{w} \cdot m_{v}} & \mathrm{~K}=\text { coefficient of permeability } \\
& \gamma_{w}=\text { unit weight of water } \\
& \mathrm{m}_{\mathrm{v}}=\text { coefficient of volume compressibility }
\end{array}
$$

- As per Terzaghi, $M_{v}$ and $C_{v}$ both are constant, but in reality $C_{v}$ decreases with stress increment.


## Degree Of Consolidation

The following methods are used in computation of degree of consolidation.
(i) If settlement is given:

$$
\% \mathrm{U}=\frac{\Delta \mathrm{h}}{\Delta \mathrm{H}} \times 100
$$

(ii) If void ratio is given:

$$
\% U=\frac{\mathrm{e}_{0}-\mathrm{e}}{\mathrm{e}_{0}-\mathrm{e}_{100}} \times 100
$$

Here, $\Delta \mathrm{h}$ is the settlement of soil after time ' t ' And $\Delta \mathrm{H}$ is the total settlement.
here, $\mathrm{e}_{0}=$ void ratio at the centre of soil at the beginning of consolidation.
$\mathrm{e}_{100}=$ void ratio at the centre of soil after the completion of consolidation.
$e=$ void ratio after time ' $t$ ' at the centre of the soil.

## (iii) If excess pore pressure is given:

$$
\% U=\frac{u_{i}-u}{u_{i}} \times 100
$$

Let ' $u$ ' be the excess pore pressure after the time ' $t$ ', Where, $u_{i}=$ initial excess pore pressure at the beginning of test.

## Time Factor

Mathematically, $\mathrm{T}_{\mathrm{v}}=\frac{\mathrm{C}_{\mathrm{v}} \mathrm{t}}{\mathrm{H}^{2}}$
Here, $\mathrm{T}_{\mathrm{v}}=$ time factor

$$
\begin{gathered}
T_{v}=\frac{\pi}{4} U^{2} \text {, when } U \leq 0.6 \\
T_{v}=0.9332(1-U)-0.0851 \text {, when } U>0.6
\end{gathered}
$$

$$
U=\text { degree of consolidation }
$$

$\mathrm{H}=$ length of drainage path, and $\mathrm{C}_{\mathrm{v}}=$ coefficient of consolidation

- for two-way drainage: $H=\frac{H}{2}$
- One way drainage: $H=H$

Note: for a particular degree of consolidation, in a soil $T_{v}$ and $C_{v}$ both are fixed. So time required will be directly proportional to $\mathrm{H}^{2}$. Time required for single drainage is 4 times the time required for double drainage.

## Determination of Coefficient of Consolidation

There are two methods to find $C_{v}$ which are based on time fitting of the curve approach.
(i) Taylor's square root of time fitting method: we will find tgo from the graph of dial gauge reading and square root t . as we know $\mathrm{T}_{(90)}$ is 0.848 , we find expression for $\mathrm{C}_{\mathrm{v}}$.

$$
\begin{array}{r}
\mathrm{C}_{\mathrm{v}}=\frac{\mathrm{T}_{90} \cdot \mathrm{H}^{2}}{\mathrm{t}_{90}}=\frac{0.848 \cdot \mathrm{H}^{2}}{\mathrm{t}_{90}} \\
\mathrm{C}_{\mathrm{v}}=\frac{0.848 \mathrm{H}^{2}}{\mathrm{t}_{90}}
\end{array}
$$

(ii) Casagrande logarithm of time fitting Method:
we will find $\mathrm{t}_{50}$ from the graph of dial gauge reading and Log $t$. as we know $\mathrm{T}_{50}$ is 0.197 , we find expression for Cv .

$$
c_{v}=\frac{0.197 \mathrm{H}^{2}}{\mathrm{t}_{50}}
$$

## Note:

- $\quad \mathrm{Cv}$ is inversely related to $\mathrm{W}_{\mathrm{L}}$ and I .
- Cv is inversely related to compressibility of soil.
- In over consolidated soil faster dissipation occurs and in NC soil slow dissipation will occur.
- Organic soils will dissipate slowly. As coefficient of permeability increase then faster will be rate of dissipation.


## CHAPTER-9 SHEAR STRENGTH OF SOIL

Shear strength is the capacity to resist shear stress. Generally, soil loaded in compression fails due to shear, not due to crushing of particles. Various strength property of soil like bearing capacity, earth pressure, slope stability depends on shear strength of soil.

## Mechanism of shear resistance:

frictional strength:it accounts forgrain to grain contact friction and interlocking between particles.

Cohesive strength:it account for true cohesion between particle and apparent cohesion between particles. True cohesion is due to cementation, electrostatics and electromagnetic attraction, primary valance bonding and adhesion. Apparent cohesion is generated due to capillary stresses.In saturated soil apparent cohesion will be zero.

Mohr Hypothesis: Shear strength on the failure plane at time of failure is a unique function of normal stress on that plane.

## COULOMB HYPOTHESIS

$$
\tau_{\mathrm{f}}=\mathrm{C}+\sigma_{f} \tan \varphi
$$

Where,
$\tau_{\mathrm{f}}=$ shear strength of soil
$\mathrm{C}=$ apparent cohesion of soil
$\sigma=$ Normal stress on the plane of
rupture


Graphical representation of Coulomb equation
$\varphi=$ Angle of internal friction

$$
\tau_{\mathrm{f}}=\mathrm{C}^{\prime}+\bar{\sigma}_{f} \tan \varphi^{\prime}
$$

Where, $\bar{\sigma}_{f}=(\sigma-\mu)=$ effective stress
$\sigma_{f}=$ total stress
$\mu=$ pore water pressure
$C^{\prime} \& \varphi^{\prime}$ are effective stress shear strength parameters.

- Hence,

$$
\tau_{\mathrm{f}}=\mathrm{C}+\sigma_{f} \tan \varphi
$$

C $\& \varphi$ are total stress parameter

$$
\tau_{\mathrm{f}}=\mathrm{C}^{\prime}+\bar{\sigma}_{f} \tan \varphi^{\prime}
$$

$C^{\prime} \& \varphi^{\prime}$ are effective stress parameter

Relation between angle of failure plane ( $\theta_{\boldsymbol{f}}$ ) and angle of shearing resistance ( $\varphi$ )
$\sigma_{1}=$ major principal stress
$\sigma_{2}=$ minor principal stress
From the diagram it is clear that, $2 \theta_{\mathrm{f}}=90$
$+\varphi$.
i.e. the failure plane makes an angle of $\left(45^{\circ}+\frac{\phi}{2}\right)$ degree with the major principal
plane.


RELATION BETWEEN MAJOR \& MINOR PRINCIPAL STRESS AT FAILURE IN A SOIL MASS ON THE BASIS OF MOHR COULOMB CRITERIA OF FAILURE

$$
\begin{gathered}
\sigma_{1 f}=\sigma_{3 f} \tan ^{2}\left(45+\frac{\phi}{2}\right)+2 C \tan \left(45+\frac{\phi}{2}\right) \\
\sigma_{1 f}=\sigma_{3 f} \frac{1-\sin \phi}{1+\sin \phi}+2 C \sqrt{\frac{1-\sin \phi}{1+\sin \phi}}
\end{gathered}
$$



Note: From above equation, a straight line can be plotted by taking $\left(\frac{\sigma_{1 f}-\sigma_{3 f}}{2}\right)$ on $y$-axis and $\left(\frac{\sigma_{1 f}+\sigma_{3 f}}{2}\right)$ on x-axis, Known as p-q plot.

## Maximum Obliquity Relationship:

angle of obliquity in the angle between the resultant stress on a plane and normal stress on the plane.

Failure Plane is the one on which angle of obiquity is maximum for cohesion less soil.

## DETERMINATION OF C AND $\boldsymbol{\oplus}$ :

- $C$ and $\phi$ are not the inherent properties of soil they are related to the type of test and the condition under which the test are performed. The test performed to determine cand $\phi$ simulates the field condition:
(i) rapid or slow construction.
(ii) drained or contained condition
(iii)Type of soil.
- The drain and undrained condition depend on the type of the loading and the soil type, and these are relative terms, if the rate of loading is greater the rate of dissipation of excess pore water pressure the condition is undrained, and excess pore pressure will exist. However, if the rate of loading if lessees then the maximum possible rate of excess pore pressure dissipation, we will have drained condition and excess pore pressure will not exist. Only hydrostatic pressure will exist.
- It we can measure the effective stress in the field then to calculate the shear strength parameter we use the effective stress approach and $c^{\prime}$ and $\phi^{\prime}$ are calculated in the laboratory and shear strength in the field is calculated as $\tau=c^{\prime}+\bar{\sigma} \tan \phi^{\prime}$. Here, $\bar{\sigma}$ is effective stress in the field.
- If we cannot measure the effective stress in the field then we used total stress approach and find C and $\phi$ in the laboratory and shear strength in the field is taken as $\tau=C+\sigma \tan \phi$ . Here, $\sigma$ is total vertical stress in the field.
- It the application of loading leads to positive pore pressure development then critical condition is the immediately after construction condition and undrained analysis is performed. However, if negative pore pressure develops due to load application, critical condition is the long term condition and drained analysis is performed corresponding to long term loading.
- NC clay and loose sand compresses on shearing and hence on shear load application under undrained condition +ve excess pore pressure develops. And in heavily over consolidate clay $(O C R>2)$ and in dense sand under undrained condition there is dilation tendency and hence, negative excess pore pressure develops.


## DIRECT SHEAR TEST:(shear box test)

dial gauge


- Normal load in the shear box test simulates the effective vertical stress in the field.
- Shear is normally applied at constant rate of strain. Strain controlled test is better because stress-Strain characteristic can be easily obtained in these tests as the shape of stress strain curve beyond peak point can be observed only in the strain-controlled test and the strain-controlled test is also easy to perform.
- Normal stress $\sigma=\frac{N}{A}$, shear stress $\tau=\frac{T}{A}$, where A is the Normal area of cross section of sample, $\mathrm{N} \rightarrow$ Normal load and T -shear load.
- As drainage cannot be controlled in this test hence rate of loading should be such that the pore water pressure does not develop i.e. we will be performing only drain controlled test. This test is thus good for free draining soil like sand and gravel.
- Direct shear test can be used for performing drain test for clay also. This is because sample is smaller in direct shear test hence drained testing will take lesser time as compared to triaxial test, further it is more convenient to subject a sample of clay to large strains in the direct shear test.
- As specimen fails along a predetermined plane, this soil is useful when soil has a predetermined fault joint.
Disadvantage:Drainage condition cannot be controlled, and pore water pressure cannot be measured.
- failure plain is always horizontal which may not be the weakest plane.
- Non-uniform stress distribution on shear plane.
- direction of principal planesare not known at every stage of test it is only after we have determined the $\phi$ value that the principal plane will be known.
- Area under normal load does not remain constant during test.


## Other Results that can be derived from direct shear test:




## TRIAXIAL TEST

- Triaxial test is most widely used shear strength test and is suitable for all type of soil.
- Drainage can be controlled whatever be the type of soil i.e. sand can be tested under undrained condition and clay can be test under drained condition.
- Pore water pressure can be measured and Volume changes
 can also be measured.
- In triaxial test at any time either one of these can be measured. Both of those are not done together.
- Failure plane is not predetermined.
- At every stage of testing principal plane and principal strains are known.
- Stress distribution on failure plane is fairly uniform.
- Triaxial cell in filled with water and specimen is sealed inside a rubber membrane and cell pressure is applied. It is called confining pressure $\left(\sigma_{c}\right)$. Confirming pressure is applied to simulate effective vertical stress in the plane.
- With cell pressure held constant additional axial stress is gradually applied. This additional axial stress ( $\sigma_{\mathrm{d}}$ ) is called deviator stress and it produces shear in the soil on all plane except the horizonal and vertical plane.
- At the time of failure major principle stress ( $\sigma_{1 f}=\sigma_{d}+\sigma_{c}$ ) and minor principle stress ( $\sigma_{3 f}$ $=\sigma_{c}$ ) are noted and by plotting deviator stress vs axial strain curve.
- During the application of loading cross sectional area of the specimen will change hence to calculate the deviator stress at the time of failure ( $\sigma_{a f}$ ) us need to know the crosssectional area of the specimen of the time of failure.

$$
\begin{array}{ll}
\sigma_{\mathrm{d} f}=\frac{P}{A} & \mathrm{P}=\text { deviator load at the time of failure. } \\
\mathrm{A}=\text { cross section area at the time of failure } .
\end{array}
$$

$$
A=\frac{A_{0}\left(1-\varepsilon_{V}\right)}{\left(1-\varepsilon_{a}\right)} \quad A=\frac{A_{0}}{1-\varepsilon_{a}}
$$

for vol. reduction and height reduction.
For undrained condition $\varepsilon_{V}=0$

Note:- C $\& \phi$ values obtained from drained test will be taken as effective stress shear strength parameter. Hence, In the field $\Rightarrow \tau=c+\bar{\sigma} \tan \phi$. and $\bar{\sigma}=\sigma-u_{h}$

Note: Calculation of shear strength by the formula " $c=c+\bar{\sigma} \tan \phi$ " is an approximation because shear strength of the soil should be calculated as shear stress on failure plane at failure, but we are calculating effective vertical stress.

## UNCONFINED COMPRESSION STRENGTH TEST

- No confining pressure is applied, it is used to test cohesive soil, load is rapidly applied hence it is undrained test and hence angle of internal friction will not get mobilised. As there is only one confining pressure $\sigma_{3}=0$. Only one mohr's circle is obtained.

$$
\left.\begin{array}{cc}
c=\frac{q_{u}}{2} \\
q_{u}=\frac{p}{A}=\frac{p}{\frac{A_{0}}{1-C_{d}}} & \varepsilon_{a}=\frac{\Delta h}{h} \\
\Rightarrow A_{0}=\frac{A_{0}}{1-\varepsilon_{a}}\left[\varepsilon_{v}=0, \quad\right. \text { since it is } \\
\text { undrained conditon }
\end{array}\right] \quad .
$$

- As no lateral pressure i.e. confining pressure exist, sand/cause grained soil cannot stand in the equipment and hence us test is not done for sand and cause grained soil.


## VANE SHEAR STRENGTH

- In a highly sensitive cohesive soil obtaining undisturbed specimen is difficult, because shear strength of such soils can be significantly effected during sampling and handling. For such soils vane shear test can be done in the field.


Top end not shearing the soil
$\tau_{f}=\frac{T}{\frac{\pi d^{2}}{2}\left[h+\frac{d}{6}\right]}$
Both top and bottom shearing the soil.
$\tau_{f}=\frac{T}{\frac{\pi d^{2}}{2}\left[h+\frac{d}{3}\right]}$

- The above test is also sometimes used to find out the liquid limit in the lab. The moisture at which shear strength is negligible it is called liquid limit moisture content.
- This test can also be used to find the sensitivity of soil, in which after initial failure if the vane is rotated several times the soil becomes simulated and the shear strength under remoulded conditions can be calculated.

$$
\text { Sensitivitg }=\frac{q_{u} \text { undisturbed }}{q_{u} \text { remoulded }}=\frac{2 c_{u} \text { undisturbed }}{2 \mathrm{c}_{u} \text { remoulded }}=\frac{\tau_{f} \text { undisturbed }}{\tau_{f} \text { remoulded }}
$$

## Types of Triaxial Test:

1. Consolidated drained test (CD Test) $\rightarrow$ Takes long time
2. Consolidated undrained test (Cu Test) $\rightarrow$ Takes 24 hr. in $1^{\text {st }}$ stage $\& 2 \mathrm{hr}$. in $2^{\text {nd }}$ stage.
3. Unconsolidated undrained (UU test) $\rightarrow$ Takes only 15 min .
4. Unconsolidated drained test $\rightarrow$ (Not a realistic one as it does not occur in field).

## CD test

- drainage is allowed in both stages. And loading is slow, to allow water to expel out and therefore pore pressure does not build up.
- Progressively bigger mohr's circles are formed because initial void ratio will progressively decrease as more and more higher confining pressure is applied and Hence greater confining Load has become effective progressively. Thus deviator stress required to cause failure will go on increasing as the confining pressure is being increased.
- for N-C clays: $\mathrm{C}=0$, failure envelope passing through origin.
- for $\mathrm{O}-\mathrm{C}$ clays: $\mathrm{C} \neq 0$
- we get effective stress parameter.


Use of CD test: for all cases involving coarse sand and gravel. Except when very rapid load like Bomb blast or earth quack vibration are applied.long term loading of any soil for
ex:earthen dam with steady seepage, foundation on clay long time after excavation, cut slopes serval years after excavation and embankment constructed very slowly.

## CU test

- In the first stage, drainage is permitted $\&$ in the second stage drainage is not permitted. I.e. volume change not allowed. We can measure total stress parameter and effective stress parameter. Because pore pressure can be measured at failure.
- Effective stress failure envelope will generally pass through the origin i.e. $c^{\prime}=0$. [not necessarily $\mathrm{c}=0$ ]

Use of CU: Most of the cases involving short term strength. For example:- Building, Embankment, Earth dam during rapid draw down, Sudden unloading condition.

## UU test:

- Drainage is not permitted in the $1^{\text {st }}$ and $2^{\text {nd }}$ stage. After application of confining pressure excess pore pressure is developed which is not allowed to dissipate and during deviator stress application also pore pressure develop which is not allowed to dissipate. Since load does not become effective in any stage, hence function is not mobilised, I.e. $\phi=0$

Circles of same dia.
$\Downarrow$
Failure occurat same deviator stress.


Uses of UU: Sudden loading condition such as rapid construction. rapid construction of earthen dam, rapid construction of building, Strength of soil in excavation, immediately after cut is made.

Note: Some time it is preformed UCS instead of UU test.

## STRESS STRAIN AND VOLUME CHANGE RELATIONSHIP FOR CLAYEY SOIL

Curve 1 [undisturbed sensitive clay]

- Sharp peak at low strain.
- Specimen shear along a well-defined plane
- Failure is called brittle failure.
- Failure is considered corresponding to peak condition.



## Curve 2 [remolded sensitive clay]

- Plastic failure
- Result in building of specimen
- Failure is decided on the basic of some pre determined strain.


## Result of C-D test



Post peak drop of in strength is more pronounced as degree of over consolidation increases.

## Curve 3 [Insensitive clay]

- Failure can be taken corresponding to horizontal portion of curve


## Result of C-U test




- N-C soil has volume decrease tendency on shearing but O-C soil has volume in case tendency on shearing. Hence, under undrained condition of volume changes are not allowed.
- +ve pore pressure develops in case of $\mathrm{N}-\mathrm{C}$ soil.
- -ve pore pressure develops in case of O-C soil.


## STRESS STAIN \& VOLUME CHANGE BEHAVIORS OF SAND

Result of C-D test.



Note: Peak value of deviator stress is generally called compressive strength of soil.


- It $\mathrm{e}=\mathrm{e}_{\text {cr }}$. There is no tendency of volume change and hence drained and undrained strength will be some.
- e >e cr, there is volume decrease tendency and hence +ve pore pressure develops under undrained condition and hence effective stress gets reduced hence undrained strength will be smaller than the drained strength.
- if e $>\mathrm{er}$, there is volume increase tendency on shearing and hence negative pore pressure develops under undrained condition hence effective stress increases and undrained strength will be greater than the drained strength.


## SOIL LIQUEFACTION

- Loose sand will have +ve pore pressure developed under undrained condition and hence $\bar{\sigma}$ decreases under undrained condition. If $\bar{\sigma}$ reduces to ' 0 ', sand will lose all of its shearing strength this phenomenon is called liquefaction.
- It occurs in case of saturated loose sand during pile driving, machine vibration, EQ shock \& explosive blasting.
- It occurs under high frequency of vibration (pore pressure increases progressively). There is a cumulative increase in pore water pressure under successive cycle of loading.

Note: Strength degradation of clay due to cyclic loading flows similar pattern to that of sand but degradation of strength is less than for cohesionless or slightly cohesive soil.

Note: strength decreases in clay with increase in plasticity index (IP). So under consolidated soil is prone to greater loss of strength as compared to $\mathrm{N}-\mathrm{C}$ or $\mathrm{O}-\mathrm{C}$ clay due to cyclic loading.[as under consolidated clay has pore pressure already present]

## PORE PRESSURE COEFFICIENT

$$
\begin{array}{c|l}
\Delta u=B\left[\Delta \sigma_{3}+A\left(p \sigma_{1}-\sigma r_{3}\right)\right] & \text { • } \begin{array}{c}
\text { \& } \mathrm{B} \text { and skemptan's pore pressure } \\
\text { coefficients } \\
B=\frac{\Delta u_{1}}{\Delta \sigma_{3}} \\
\hline A B=\frac{\Delta U_{2}}{\Delta \sigma_{L}-\Delta \sigma_{3}}
\end{array} \\
& \begin{array}{ll}
\text { - } \mathrm{B}=1 \text { for completely submerged soil } \\
& \text { - }=0 \text { for completely dry soil. } \\
& \text { - For pointily submerged soil, ( } \mathrm{B} \text { is very small) } \\
& \text { - The value of } \mathrm{B} \text { varies with the stress range. }
\end{array}
\end{array}
$$

value of $A$ \& $B$ can be calculated by UU test and if $B$ is known we can also use CU test to find out the value of $A$.


## CHAPTER-10 EARTH PRESSURE AND RETAINING WALLS

> In fluids the hydrostatic pressure acts equally in all direction at any point given. i.e. $\boldsymbol{\sigma}_{\mathrm{x}}$ $=\sigma_{y}=\sigma_{z}$. The horizontal and vertical pressure are same because water does not have any shear strength.
> In case of soil the vertical and the Horizontal stresses are not equal it is because the soil has shear strength due to either friction, cohesion, or both. Hence, $\boldsymbol{\sigma}_{x} \neq \boldsymbol{\sigma}_{y}$ (exceptions can be there based on different theory)
> Earth Pressure can be defined on the basic of three types. Terzaghi has performed various test based on which he defined these above earth pressure
(i) Rest Pressure: If the walls of a rigid structure are unyielding.i.e. there is no movement of the wall then the soil is said to be in state of rest.
(ii) Active condition: If the soil exerts the force on the wall (wall moves away from the soil) due to which it is called in the state of active pressure.
> Due to the wall movement the soil tries to expend and shear planes are developed with the mobilisation of shear away from the wall.

> The soil mass which is falling tries to move down wards and outward
(iii) Passive earth pressure: When the wall is pushed towards the back fill the soil is getting compacted and the shear resistance mobilised is towards the wall.
> The soil tries to move toward and inward and the pressure exceeds the rest pressure.


> The movement of top of retaining wall to generally active earth pressure condition, for dense soil is $0.2 \%$ and for loose condition is 0.5\%
> The yielding of the wall is given by $\frac{\Delta \mathrm{H}}{\mathrm{H}}$ ratio.
> To generate a passive condition for dense soil the yield shall be $2 \%$ and loose soil it should be by $5 \%$.

## Mohr circle:



Rest Pressure: $\epsilon_{\mathrm{x}}=\frac{\sigma_{\mathrm{x}}}{\mathrm{E}}-\mu\left(\frac{\sigma_{\mathrm{x}}}{\mathrm{E}}+\frac{\sigma_{\mathrm{y}}}{\mathrm{E}}\right)$ and $\epsilon_{\mathrm{x}}=0, \sigma_{\mathrm{x}}=\sigma_{\mathrm{y}}$

$$
\sigma_{H}=\sigma_{V}\left(\frac{\mu}{1-\mu}\right) \Rightarrow \mathrm{K}_{0}=\frac{\mu}{1-\mu}
$$

> It the soil is considered elastic, homogeneous, isotopic semi-infinite-then the value of earth pressure coefficient based on elastic theory is given by $\mathrm{K}_{0}=\frac{\mu}{1-\mu}$
> $\mu$ is considered as a constant but its actual value changes depending on drainage condition.
> Based on field measurement, earth pressure given by different scientist are as follows.
(i) $\mathrm{ALPAN} \Rightarrow \mathrm{NCC} \rightarrow \mathrm{K}_{0}=0.19+0.233 \log \left(\mathrm{I}_{\mathrm{p}}\right)$

$$
\mathrm{OCC} \rightarrow \mathrm{~K}_{0}=\mathrm{K}_{\mathrm{o}(\mathrm{NCC})} \times \sqrt{\mathrm{OCR}}
$$

(ii) Jaky $\Rightarrow \mathrm{K}_{0}=1-\sin \phi[$ for sandy type of soil.]

## Rankine theory



- It is considered that back of retaining wall is vertical and smooth and backfill is horizontal.
- It considers the stress in a state of plastic equilibrium.
- The soil is homogeneous, dry, cohesion (later this theory was modified to consider effect of water table and cohesion).
- Rupture surface is a plane surface.


## Active Earth Pressure

| $\sigma_{1}=\sigma_{3} \tan ^{2}\left(45+\frac{\phi}{2}\right)+2 \mathrm{c} \tan \left(45+\frac{\phi}{2}\right)$ |  |
| :---: | :---: | :---: |
| $\Rightarrow \mathrm{c}=0 \Rightarrow \sigma_{\mathrm{V}}=\sigma_{\mathrm{H}}\left(\frac{1+\sin \phi}{1-\sin \phi}\right)$ |  |
| $\frac{\sigma_{\mathrm{H}}}{\sigma_{\mathrm{V}}}=\mathrm{ka}=\frac{1-\sin \phi}{1+\sin \phi}$ | $\bullet 2 \theta=\phi+90$ <br> angle between normal to <br> failure plane and normal to <br> major principal plane. <br> $\bullet$ <br> $\theta=45+\frac{\phi}{2}$ |

## Analysis of various conditions:

## 1. single soil mass


2. stratified soil mass

3. external loading is acting

4. Water table rise upto ground


In short, add [water pressure $+\mathrm{K}_{\mathrm{a}}$ (effective vertical stress)]
5. C- $\phi$ soil(No water table)

Plastic equilibrium eq ${ }^{n} \Rightarrow \sigma_{1}=\sigma_{3} \tan ^{2}\left(45^{\circ}+\frac{\phi}{2}\right)+2 C \cdot \tan \left(45^{\circ}+\frac{\phi}{2}\right)$

$$
\begin{gathered}
\sigma_{V}=\sigma_{H}\left[\frac{1+\sin \phi}{1-\sin \phi}\right]+2 c \sqrt{\frac{1+\sin \phi}{1-\sin \phi}} \\
\sigma_{H}=\sigma_{V} \times k_{a}-2 c \sqrt{k_{a}}
\end{gathered}
$$

Note: Effect of cohesion is to reduces active earth pressure


Total active thrust $\Rightarrow\left(-2 c \sqrt{k_{a}} H\right)+\frac{K_{a} \gamma H^{2}}{2}=F_{R}$
For $\sigma_{H}=0 \Rightarrow K_{a}\left(\gamma z_{0}+q\right)-2 c \sqrt{k_{a}}=0$

$$
\Rightarrow \quad Z_{0}=\frac{2 c}{\gamma \sqrt{k_{a}}}
$$

It tension crack has not occurred, then force will decrease by
$2 \sqrt{k a}$


$$
F=\frac{1}{2} 2 c \sqrt{k a} \times \frac{2 c}{\gamma \sqrt{k a}}=\frac{2 c^{2}}{\gamma}
$$

## Box cutting:



- For design purpose cracks are assumed to be occurred.Because In practice tension cannot be taken on the wall, since tension cracks tends to develop within the soil and it may not remain adhered to the wall.
- So, in calculation of total thrust usually tension zone is ignored which results in a larges value of the active earth pressure thrust. As a theoretical concept no 'tension crack condition will result in a net active thrust value for a region upto a depth Ho equal to zero.
- $H_{0}=\frac{4 c}{\gamma \sqrt{R} a}$ Critical depth



## Passive Pressure:



$$
\begin{aligned}
& \text { If } c=0 \Rightarrow \sigma_{H}=\sigma_{V} \cdot \tan ^{2}\left(45^{\circ}+\frac{\phi}{2}\right) \\
& \Rightarrow K_{p}=\tan ^{2}\left(45^{\circ}+\frac{\phi}{2}\right)=\frac{1+\sin \phi}{1-\sin \phi} \\
& K p=\frac{1+\sin \phi}{1-\sin \phi} K P=\frac{1}{\mathrm{~K}_{\mathrm{a}}}
\end{aligned}
$$

## c- $\boldsymbol{\phi}$ soil:

$$
\sigma_{H}=\sigma_{\mathrm{V}} \cdot \mathrm{~K}_{\mathrm{p}}+2 \mathrm{c} \sqrt{k_{p}}
$$

Note: The effect of cohesion is to increase the passive earth pressure.


## Inclined Backfill

$$
\mathrm{P}_{\mathrm{a}}=\frac{1}{2} \mathrm{~K}_{\mathrm{a}} \gamma \mathrm{H}^{2} \cos \beta
$$

$$
k a=\frac{\cos \beta-\sqrt{\cos ^{2} \beta-\cos ^{2} \phi}}{\operatorname{con} \beta+\sqrt{\cos ^{2} \beta-\cos ^{2} \phi}} \cdot \cos \beta
$$



Resultant force will act at a distance $\frac{\mathrm{H}}{3}$
from bottom and will act parallel to the backfill inclination.

## Inclined Back of wall



## Columb's theory: assumptions: -

1. The back fill is assumed dry, cohesionless, homogenesis and isotropic.
2. The back of the wall and the backfill can be inclined.
3. The friction between the wall as the soil is considered.
4. Actual failure plane is curved in bottom region but assumed plane for analysis.
5. The failure plane is assumed to pass through heel
6. The sliding wedge is considered as a rigid body and $\lambda$ the angle of failure plane with the horizontal.
7. At the failure condition the trial wedge is in equilibrium under the action of three forces that are ( $\mathrm{W}, \mathrm{R}, \mathrm{Pa}$ ).

| Pa - path pressure forace | $w \rightarrow$ self-weight. | R- Reaction force at failure plane <br> which is inclined at an angle s <br> with the normal of the back of <br>  |
| :--- | :--- | :--- |

8. Value of $\delta$ is assumed as follows.

- For smooth wall, $\delta=\frac{\phi}{3}$
- For ordinary retaining wall made of concrete, $\delta=\frac{2 \phi}{3}$
- For retaining wall where provision of drainage of soil is provided through deep
 holes. $\delta=\frac{3}{4} \phi$
- If soil is subjected to vibration $\delta=0$.

9. The direction of all the forces is known and value of $w$ is also known hence using this data a force triangle can be drawn and the value of Pa can be found out.
10. For active earth pressure force the highest among all the trial planes is considered as the critical value and for the passive earth pressure force the minimum of all the trial planes is considered as the critical.

Note: Rankine earth Pressure theoryOverestimates the active earth pressure and underestimates the passive earth pressure. Retaining walls are designed for active earth pressure.

Note: Movement of water table upwards will increase the active earth pressure and will decrease the pressive earth pressure.

Note: On compacting the soil.Active earth pressure decreasesand Passive earth pressure increases.

## Braced System:

- In a Braced extraction the theory proposed by peck Hanson and Thorburn suggests that the earth pressure distribution on the backfill side of the sheet pile is represented by an apparent earth pressure diagram.
- An imaginary strut is assumed at the dredge line for the analysis
- Load carried by each is based on the tributary area which town up to half of the distance between the strut above and below the street level.



## Anchored Bulb head $\Rightarrow$


for granular and sandy type of soil


Note: In anchored bulb head the moment equilibrium is considered about the anchor level and resultant earth pressure diagram is as follows.

Sheet Piles: Sheet piles are also known as flexible retaining wall that are used to support the soil temporary after the excavation. They are driven into the soil and act as continuous wall which is retaining earth mass. These are generally made of steel which has a high
strength, it is having very light weight and is thin.


## Cantilever pile



- The resultant diagram is based on assuming that there will be some consolidation over a period of time which will take place, also pile is of flexible nature, Hence it will get deformed.
- Due to overturning there will be shrinkage, swelling loss of contact with the pimple or development of tuition cracks.


For pure cohesionless soil ( $\phi=0$ )

Approximate diagram for analysis:(most simplified diagram)


## CHAPTER-11 STABILITY OF SLOPES

## STABILITY OF INFINITE SLOPES

- Let's consider an infinite slope $A B$ having a slope angle of ' i ' with horizontal, failure of which takes place along the plane ' $C D$ ' that is at a depth of H below the surface.


| Vertical stress on plane | $\Rightarrow \sigma_{z}=\gamma \cdot H \cdot \operatorname{cosi}$ |
| :--- | :--- |
| $C D=\sigma_{z}=\frac{\gamma \cdot H \cdot b \cdot \cos i}{b \times 1}$ |  |
| Normal stress on plane $\mathrm{CD}, \sigma_{n}=\sigma_{z} \cdot \operatorname{cosi}$ | $\Rightarrow \sigma_{n}=\gamma \cdot H \cdot \cos ^{2} i$ |
| Tangential stress on plane $\mathrm{CD}, \tau=\sigma_{z} \cdot \operatorname{sini}$ | $\Rightarrow \tau=\gamma \cdot$ H.cosi. $\sin i$ |

F.O.S. $=\frac{s}{\tau}=\frac{c+\overline{\sigma_{n}} \tan \phi}{\tau}$

Case 1: Cohesionless soil $(c=0)$

$$
\begin{gathered}
\Rightarrow F O S=\frac{0+\sigma_{n} \tan \phi}{\tau}=\frac{\sigma_{n}=\gamma \cdot H \cdot \cos ^{2} i \cdot \tan \phi}{\gamma \cdot H \cdot \cos i \cdot \sin i} \\
\Rightarrow F O S=\frac{\tan \phi}{\tan i}
\end{gathered}
$$

Case 2: Cohesionless submerged soil mass

$$
\text { FOS }=\frac{\gamma^{\prime} \cdot H \cdot \cos ^{2} i \cdot \tan \phi}{\gamma^{\prime} \cdot H \cdot \cos i \cdot \sin i}=\frac{\tan \phi}{\tan i}
$$

Case 3: Slope subjected to steady seepage and water table is up to depth of ' $h$ ' above the failure plane.

$$
\begin{gathered}
\text { FOS }=\left(1-\frac{\gamma_{w} h}{\gamma_{e f f} H}\right) \frac{\tan \phi}{\operatorname{tani}}, \text { here } \gamma_{e f f}=\frac{\gamma \cdot(H-h)+\gamma_{\text {sub }} \cdot h}{H} \\
\text { If } h=H, F O S=\frac{\gamma^{\prime}}{\gamma_{e f f}} \times \frac{\tan \phi}{\operatorname{tani}}=\frac{1}{2} \frac{\tan \phi}{\tan i}
\end{gathered}
$$

Case 4: Cohesive soil having dry or moist slope

$$
F O S=\frac{C+\gamma \cdot H \cdot \operatorname{Cos}^{2} i \cdot \tan \phi}{\gamma \cdot H \cdot \operatorname{cosi} \cdot \operatorname{sini}}
$$

- If slope angle ' $i$ ' is less than frictional angle ' $\phi$ ' no failure takes place as for any value of normal stress shear stress is less than shear strength of the soil.

Here, $S_{n}=$ Taylor's stability number

## FOS with respect to height:

Here, $H_{c}=$ critical height

$$
\text { (F.O.S. })_{h}=\frac{H_{c}}{H}
$$

## FOS with respect to cohesion:

$$
\begin{aligned}
\mathrm{C} & =\text { effective cohesion } & \text { (F.O.S. })_{c}=\frac{c_{m}}{c} \\
\mathrm{C}_{\mathrm{m}} & =\text { mobilized cohesion } &
\end{aligned}
$$

## In terms of cohesion,

$$
S_{n}=\frac{c}{\gamma H_{c}}=\frac{c}{\gamma \times F O S \times H}
$$

Case 5: Cohesive soil in submerged condition

$$
F O S=\frac{C+\left(\gamma^{\prime} H\right) \cdot \operatorname{Cos}^{2} i \cdot \tan \phi}{\gamma^{\prime} \cdot H \cdot \operatorname{cosi} i \cdot \sin i}
$$

Case 6: Slope subjected to steady seepage and water table is up to depth of ' h ' above the failure plane.

$$
F O S=\frac{C+\left(\gamma_{e f f} H-\gamma_{w} h\right) \cdot \operatorname{Cos}^{2} i \cdot \tan \phi}{\gamma_{e f f} \cdot H \cdot \operatorname{cosi} \cdot \sin i}
$$

STABILITY OF FINITE SLOPES: Failure of finite slopes has following three modes
(i) Slope failure: In this case the failure surface passes through the sloping face. Observed in steep slopes when soil below the toe is hard and rigid comparison to soil above the toe.
(ii) Base failure: The failure surface passes below the toe. It is generally seen in case of flatter slopes where soil below the toe is comparatively softer then the soil above the toe.
(iii) Toe failure: In this case failure occurs along a surface that passes through the toe. In this failure soil above and below have same property.


## Swedish Slip Circle Method

This method is suitable for both cohesive and frictional soil. The soil is cut into a number of slices of equal with the forces between different slices is neglected and each slice acts as an independent column of unit thickness.


## Friction Circle Method

This method is based upon total stress analysis and uses the shearing angle of the soil in order toanalyse the stability of finite slopes.


## CHAPTER-12 SHALLOW FOUNDATION

## BASIC DEFINITION

## - Gross Pressure or Gross loading intensity $\left(\mathrm{q}_{\mathrm{g}}\right)$ :

It is the total pressure intensity at the base of footing.

$$
\boldsymbol{q}_{g}=\frac{\boldsymbol{P}}{\boldsymbol{B}^{2}}+\gamma * \boldsymbol{D}_{f}
$$

- Net pressure Intensity ( $\mathbf{q}_{\mathrm{n}}$ ):

It is generally the loading intensity at the base of footing more than the load intensity that the soil was originally subjected to that causes deformation in soil. Hence, net pressure intensity

$$
q_{n}=\frac{P}{B^{2}}=\underset{g}{q}-\gamma * D_{f}
$$

If after placing of load, foundation is not filled with soil again. Then net pressure intensity will be

$$
\boldsymbol{q}_{n}=\frac{\boldsymbol{P}}{\boldsymbol{B}^{2}}-\boldsymbol{\gamma} * \boldsymbol{D}_{f}
$$

## - Ultimate bearing capacity ( $\mathrm{qu}_{\mathrm{u}}$ :

The ultimate bearing capacity is the gross pressure at the base of the foundation at which the soil fails in shear.

## - Net Ultimate Bearing Capacity (qnu):

The net ultimate bearing capacity is the net pressure at the base of the foundation at which the soil fails in shear.

$$
\boldsymbol{q}_{\boldsymbol{n u}}=\boldsymbol{q}_{\boldsymbol{g}}-\gamma * \boldsymbol{D}_{\boldsymbol{f}}
$$

## - Net safe bearing capacity ( $\mathrm{qns}_{\mathrm{n}}$ ):

$$
q_{n s}=\frac{q_{n u}}{\text { F.O.S. }}
$$

Where $F=$ Factor of safety, which is usually taken as 3.0.

- Gross Safe Bearing Capacity ( $\mathrm{q}_{\mathrm{s}}$ ):

$$
\boldsymbol{q}_{s}=\boldsymbol{q}_{\boldsymbol{n s}}+\boldsymbol{\gamma} * \boldsymbol{D}_{\boldsymbol{f}} \text { or } \boldsymbol{q}_{\boldsymbol{n s}}=\frac{\boldsymbol{q}_{n u}}{\text { F.O.S. }}+\boldsymbol{\gamma} * \boldsymbol{D}_{\boldsymbol{f}}
$$

## - Safe bearing pressure( $\mathbf{q p s}^{\text {s }}$ :

Maximum net intensity of loading that can be allowed on soil without the settlement exceeding the permissible value. No factor of safety is used when dealing with settlement.

## - Allowable bearing pressure ( $\mathrm{q}_{\mathrm{a}, \mathrm{net}}$ )

Maximum net intensity of loading that can be imposed on the soil with no possibility of shear failure or the possibility of excessive settlement. It is the smaller of net safe bearing capacity ( $\mathbf{q n s}_{\text {s }}$ ) and safe bearing pressure ( $\mathbf{q p s}_{\text {) }}$.
Note: According to IS code Allowable bearing pressure ( $q_{a}$, net) is Allowable bearing capacity.
SHEAR FAILURE:The shear failure is in three modes.
(i) General shear failure
(ii) Local shear failure
(iii) Punching shear failure

General shear failure: It occurs in shallow footing on the soils having brittle type of shear stress curve.


- The soils that are of medium dense sand, silt and over consolidated clays represent general shear failure, these shown less compressibility.
- At the time of failure, the foundation gets tilted on one side
- Before the failure the settlement will be negligible, and the stress zone willbe extended upto the ground level. A state of plastic equilibrium is reached initially around the edges,
and it spread outward and downward. The portion of the soil below the foundation extents passive pressure on the surrounding soil.
- The shear planes developed are well defined.
- Bulging and heaving can be noticed in the surrounding of formation
- Density Index $>70 \%$; $\operatorname{SPT}$ value $(N)>30$
- The angle of frictional resistance $\phi>30^{\circ}$
- Void ratio < 0.55

Local Shear failure: The load settlement curve for a soil subjected to local shear failure represents the stress curve for a plastic material.

- Construable compression of the soil occurs the failure surface does not reach the ground surface and only a slight heaving observed due to excessive load.
- This failure is observed in loose sand and soft clay.
- No sudden failure is observed
- A well-defined slip surface is only observed below the foundation.

- The density index (30,70\%); $\phi \leq 28^{\circ}$;SPT value < 5
- void ratio $>0.75 ; \mathrm{UCS}<80 \mathrm{kN} / \mathrm{m}^{2}$
- for local shear failure, $c_{m}=\frac{2}{3} c, \quad \tan \phi_{m}=\frac{2}{3} \tan \phi$


## Punching shear failure

- The failure results in high compression of the soil accompanied by the shearing in vertical direction which is around the edge of footing.
- No heaving or budging is observed in the soil.
- The load settlement curve resembles the stress curve of a highly plastic material.
- This failure occurs in very loose sand and very soft clays.
- The zone \& beyond the vertical shear plane are not stressed.
- The density index < 30\%.



## TERZAGHI'S BEARING CAPACITY THEORY

## - The failure surface as given by Terzaghi is as follows:



## - Assumptions:

1. Footing is a strip footing ( $L \gg B$ )
2. Soil is Homogenous

3 2-D plane strain condition prevails.
4. Base of footing is rough.
5. The base of footing is laid down at shallow depth.
6. Loading is vertical and symmetric i.e. (moment $=0$ )
7.General shear failure occurs.
8. Ground is Horizontal.
9. Shearing resistance of soil between the ground surface and base of footing is neglected. Thus, footing considered as a surface footing with uniform surcharge $=\boldsymbol{\gamma} * \boldsymbol{D}_{\boldsymbol{f}}$ at the footing.
10. Shear strength of soil is goverend by Mohr's coulmb criteria.

- Failure zone in the soil mass is divided into three zones:

1. Zone (I) $\rightarrow$ zone of elastic equilibrium. Assumed to be a part of the footing.
2. Zone (II) $\rightarrow$ Radial shear zone. Its lower curved boundary has shape of logarithmic spiral.
3. Zone (III) $\rightarrow$ Rankine's passive zone. Boundary makes an angle of ( $45-\frac{\varphi}{2}$ ) with horizontal.

- The ultimate bearing capacity for strip footing:

$$
q_{u}=c \cdot N_{c}+q \cdot N_{q}+0.5 B_{f} \cdot \gamma \cdot N_{\gamma}
$$

$\mathrm{c}=$ Effective cohesion, $\mathrm{B}_{\mathrm{f}}=$ width of strip footing.
$\mathrm{q}=\gamma * D_{f}$ and $\mathrm{D}_{\mathrm{f}}=$ Depth of footing.
$\mathrm{N}_{\mathrm{c}}, \mathrm{N}_{\mathrm{q}}$ and $N_{\gamma}$ are the dimensionless numbers known as bearing capacity factors depending upon angle of friction of soil.

- Local shear failure: use $c^{\prime}$ and $\varphi^{\prime}$ in case of local shear failure.

Mobilised cohesion, $c_{m}=\frac{2}{3} c$ and Mobilised angle of shearing resistance, $\phi_{m}=\tan ^{-1}\left(\frac{2}{3} \tan \phi\right)$.

- Net ultimate bearing capacity for strip footing:

$$
q_{n u}=c \cdot N_{c}+q \cdot\left(N_{q}-1\right)+0.5 B_{f} \cdot \gamma \cdot N_{\gamma}
$$

- For clayey soil:

For clayey soil, $\varphi=0$ and $N_{c}=5.7, N_{q}=1, N_{\gamma}=0$.

$$
q_{u}=5.7 c+q
$$

## - Bearing Capacity of Square and Circular Footing

(a) Square footing:

$$
q_{u}=1.3 c \cdot N_{c}+q \cdot N_{q}+0.4 B_{f} \cdot \gamma \cdot N_{\gamma}
$$

Where, $B$ is the dimension of each side of the footing.

## (b) Circular footing

$$
q_{u}=1.3 c \cdot N_{c}+q \cdot N_{q}+0.3 D \cdot \gamma \cdot N_{\gamma}
$$

Where, D is the diameter of the footing.
(c) Rectangular footing:

$$
q_{u}=\left(1+0.3 \frac{B}{L}\right) c N_{c}+\gamma D_{f} N_{q}+0.5\left(1-0.2 \frac{B}{L}\right) \gamma B N_{\gamma}
$$

Where, $B$ and $L$ are the width and length of the footing respectively.

- Effect of Water Table on Bearing Capacity


$$
\begin{aligned}
& \text { Here, } \gamma_{1} \cdot D_{f}=y * \gamma_{t}+\left(D_{f}-y\right) * \gamma_{\text {sub }} \\
& \text { and } \gamma_{2} \cdot \boldsymbol{B}=\gamma_{\text {sub }}
\end{aligned}
$$

## MEYERHOFF'S BEARING CAPACITY THEORY

- It is most generalized theory in which the shape factor, depth factor and inclination factor are used to account for the shape of footing, variation in depth of footing, inclination of load and ground surface. the stress zone is considered to extend up tothe ground level hence this theory can be applied to deep foundation also.

$$
q_{u}=c \cdot N_{c} \cdot\left(s_{c} \cdot d_{c} \cdot i_{c}\right)+\gamma \cdot D_{f} \cdot N_{q}\left(s_{q} \cdot d_{q} \cdot i_{q}\right)+0.5 \gamma \cdot B_{f} \cdot\left(s_{\gamma} \cdot d_{\gamma} \cdot i_{\gamma}\right)
$$

- Meyerhoff has also given the concept of reduced area for the eccentric loading condition. It means on the footingthe line of action of the force is not passing through the centroid.Then the width and length are reduced such that loadactson the centerof the remaining area.


SKEMPTON'S ANALYSIS FOR COHESIVE SOILS

- Skempton analysis is suitable for soils having an angle of internal friction equal to zero. As per Skempton, $q_{n u}=c_{u} N_{c}$.
- Here $N_{c}$ is a function of shape of footing and $\frac{D_{f}}{B}$. This theory does not neglect the shear strength of soil above foundation of soil foundation level.
- The bearing capacity factors can be calculated using the following relations:
- For Strip Footing:

$$
N_{c}=5\left(1+0.2 \frac{D_{f}}{B}\right) \leq 7.5
$$

- For square and circular footings:

$$
N_{c}=6\left(1+0.2 \frac{D_{f}}{B}\right) \leq 9
$$

- For rectangular footing:

$$
\begin{gathered}
N_{c}=5\left(1+0.2 \frac{D_{f}}{B}\right)\left(1+0.2 \frac{B}{L}\right) ; \text { for } \frac{D_{f}}{B} \leq 2.5 \\
N_{c}=7.5\left(1+0.2 \frac{B}{L}\right) ; \text { for } \frac{D_{f}}{B}>2.5
\end{gathered}
$$

## RANKINE THEORY

- This Theory is applicable for cohesionless soil, it considers the equilibrium of the elements adjacent to each other at the corner of footing. Element 1 is assumed in active state and element 2 is in passive state.

$$
q=\gamma_{D f}\left[\frac{1+\sin \phi}{1-\sin \phi}\right]^{2}
$$



- If the footing is at zero depth, then bearing capacity from this formula is also zero which is not practical.
- This theory does not consider effective of shape, as well as the size of the Foundation and hence it is absolute nowadays. Also does not fill it for clay soil.


## S.P.T.

## Test Procedure:

- It is Suitable for Granular soil.
- Split spoon samples is used in the bore hole.
- Bore hole in advanced to a depth at which N -value is to be calculated.
- The split-spoon sampler is allowed to penetrate the soil by applying impact load of 65 Kg having a free fall of 75 cm .
- The sample ( is allowed to penetrate for 150 mm depth, but reading is not noted. (No. of blows required for 150 mm penetration is not Noted).
- Then the sampler is allowed to penetrate, further for 300 mm and No. of blows required to penetrate the sampler to this 300 mm is the SPT N -value.
- The next test is carried out at level 750 mm below the previous test reference level.
- If bore hole depth is large, then interval of next test is taken at a depth of (2.5 to 2 ) m or at the change of strata.
- The following values are refused (i) if for successive 10 number of blows there is no advancement in penetration of sampler. (ii) if for any 150 mm penetration the N value is more than 50, then refused. (iii) if for any 300 mm penetration N value is more than 100 it is refused.


## The following corrections are required for $\mathbf{N}$-values obtained above.

- Over burden correction
- It is necessary because the $N$-value will have effect on it due to confinement of soil at various depth.
- Two granular soils possessing the same relative density but having different confining pressures are tested, the one with a higher confining pressure will give higher N value.
- Since the confining pressure increases with depth, the $N$ values at shallow depths are underestimated and the N values at larger depths are overestimated
- Therefore, if no correction is applied to recorded $N$ values, the relative densities at shallow depths will be underestimated and at higher depths, it will be overestimated.
- If $\mathrm{N}_{0}=$ observed S.P.T. value Then,

$$
N_{1}=N_{0} \times \frac{350}{\bar{\sigma}+70}
$$

Where, $\bar{\sigma}=$ Effective stress at level of test $\left(\mathrm{KN} / \mathrm{m}^{2}\right)$,
$\mathrm{N}_{1}=$ Corrected N -value of overburden,
Overburden correction will not be applied if, $\bar{\sigma}>280 \mathrm{KN} / \mathrm{m}^{2}$.

## - Dilatancy Correction.

- It is applied to the already corrected N -values for overburden pressure, Dilatancy correction is required only if [N1>15] in saturated fine sand and silt (when water table is above test level).
- ( $\mathrm{N}>15$ ) basically represents the Dense sand which will have the tendency to dilate under rapid loading (undrained condition) and (-ve) pore water pressure will develop. Hence, observed ( N ) value will be more because shear resistance will increase.
- Corrected N value after Dilatancy Correction is $\mathrm{N}_{2}=15+0.5^{*}\left(\mathrm{~N}_{1}-15\right)$
- This correction becomes more significant for fine dense sand.


## Late load test:

- It is performed to calculate the ultimate bearing capacity of the soil, and pressure vs ultimate settlement graph is made.
- The size of the plate is kept between 30 cm and 75 cm , the smaller size plate is used for granular soil and for clayey soil large size are used. The width of the test pit should be greater than 5 times the plate width.
- Initially a pressure of $70 \mathrm{~kg} / \mathrm{cm}^{2}$ which is also known as the this is to stablish a perfect contact between plate and soil.

- The water table is kept below the test level so as to avoid any pore pressure development.
- The test is performed in the increment of of $1 / 5^{\text {th }}$ of assumed ultimate load upto the breaking point or upto a total settlement of 25 mm whichever occurs first.
- If a marked breakpoint is not visible then approximate value of ultimate bearing capacity is determined by double tangent method.


## Correction:

| Clay | Sand |
| :---: | :---: |
| $q_{u f} \equiv q_{u p}$ | $\frac{q_{u f}}{q_{a P}}=\frac{B_{f}}{B_{P}}$ |
| $\frac{S_{f}}{S_{p}}=\frac{B_{f}}{B_{p}}$. | $\frac{S_{f}}{S_{p}}=\left[\frac{B_{f}}{B_{P}} \times \frac{\left(B_{p}+03\right)}{\left(B_{f}+0.3\right)}\right]^{2}$ |

All values are in m .

- It the actual foundation level is below the plate load test level then the correction need to be performed for settlement.



## Settlement and pressure distribution:



Immediate settlement or elastic settlement:

| $S_{i}=\frac{q \cdot B I\left(1-\mu^{2}\right)}{E}$ | B - width <br>  |
| :--- | :--- |
| I - Influence factor <br>  <br>  | - modulus of elasticity of soil <br> q- not pressure ratio intensity |

- In flexible footing the contact pressure is uniform over thefoundation width whereas in rigid footing the settlement is uniform along the foundation width. To increases to settlement at any point the contact pressure at that particular point head $s$ to be increased.
- In case of granular soil E varies along the loaded area with lesser value at the corner compare to the centralpoint.


## CHAPTER-13 DEEP FOUNDATION

## CLASSIFICATION OF PILE:

- Based on material: steel, concrete, timber.
- Based on load transfer: end bearing, friction, combined.
- Based on installation method: bored, driven.
- Based on function and location: compression, tension, anchor, batten, compaction, fender/dolphin pile are provided in case of hydraulic structure.
- Based on soil displacement: displaced, non-displaced.


## PILE LOAD CAPACITY BY STATIC FORMULA:

$$
\begin{array}{cl}
Q_{U}=Q_{B}+Q_{S} & Q_{B}=\text { end bearing } \\
Q_{U}=\text { ultimate bearing capacity } & Q_{S}=\text { skin friction }
\end{array}
$$

| End bearing: |  |  |  |
| :---: | :---: | :---: | :---: |
| c- $\boldsymbol{\phi}$ soil | sand |  | clay |
| $\mathrm{q}_{\mathrm{u}}=\mathrm{cN}_{\mathrm{c}}+\overline{\mathrm{\sigma}} \mathrm{~N}_{\mathrm{q}}$ | $\mathrm{q}_{\mathrm{u}}=\bar{\sigma} \mathrm{N}_{\mathrm{q}}$ |  | $\mathrm{q}_{\mathrm{u}}=\mathrm{c}_{\mathrm{u}} \mathrm{N}_{\mathrm{c}}$ |
| $\begin{gathered} Q_{B}=q_{u} \times A_{b} \\ A_{b}=\text { area of the end } \end{gathered}$ | $\bar{\sigma}=$ Effective overburden pressure <br> at the tip of the pile <br> $\mathrm{L}=$ Length of embedment of the pile |  | $\mathrm{N}_{\mathrm{c}}=9$ for deep foundation. <br> $\mathrm{C}_{u}=$ undrained cohesion |
| Skin friction: |  |  |  |
| sand |  | clay |  |
| $\mathrm{Q}_{\mathrm{s}}=\mathrm{f}_{\mathrm{s}} * \mathrm{~A}_{\mathrm{s}}$ and $\mathrm{A}_{\mathrm{s}}=\pi \mathrm{DL}$ |  | $\mathrm{Q}_{\mathrm{s}}=\alpha * \mathrm{C}_{\mathrm{u}}$ and $\mathrm{A}_{\mathrm{s}}=\pi \mathrm{DL}$ |  |
| frictional reisistance $=\mathrm{f}_{\mathrm{s}}=\mu . \mathrm{N}$ |  | $\mathrm{C}_{\mathrm{u}}=$ undrained cohesion |  |
| $\begin{gathered} \mathrm{N}=\text { lateral earth pressure }=\mathrm{K} * \bar{\sigma}_{a v} \\ \mu=\text { coefficient of friction }=\tan \delta \end{gathered}$ |  | $\alpha=$ adhesion factor <br> $\alpha=1$ for loose soil and $\alpha=0.3$ for very stiff soil |  |

Table: Value of $K$ and $\delta$

| Pile material | $\boldsymbol{\Delta}$ | Values of K |  |
| :---: | :---: | :---: | :---: |
|  |  | Loose sand | Dense sand |
| Steel | $20^{\circ}$ | 0.5 | 1.0 |
| Concrete | $0.75 \phi$ | 1.0 | 2.0 |


| Timber | $0.67 \phi$ | 1.5 | 4.0 |
| :---: | :---: | :---: | :---: |

Arching effect in sand: it is observed that value of effective stress increases up to a certain depth of sand, after which it become constant, this is due to arching effect.

For loose sand, critical depth is 15 d .
For dense sand, critical depth is 20d.


Note: this critical depth concept is not applicable for clay strata, in clay arching effect is negligible.

Note: All these calculations are for driven piles, in case of bored cast in-situ pile: due to boring condition, the value of $K$ is very small. $K$ is usually taken as 0.5 and $\delta=\phi$. Rest of the calculations are same. Point bearing resistance of Board cast insitu piles $=0.5$ (point bearing resistance of driven pile).

## PILE CAPACITY USING DYNAMIC PILE FORMULA

## - Engineering News Formula

$\mathrm{Q}_{\mathrm{u}}=\frac{\mathrm{WH}}{\mathrm{F}(\mathrm{S}+\mathrm{C})}$

- W = load (in KN)
- Height of fall H (in cm)
$F=$ factor of safety is taken as 6 . last 5 blows of drop hammer).
- $\quad \mathrm{S}=$ set (penetration per blow of hammer) in cm (taken for
- C is taken as 2.5 cm for drop hammer and 0.25 cm for Single Acting Steam Hammer (SASH).
- Modified hilly formula
$\mathrm{Q}_{\mathrm{u}}=\frac{\mathrm{Wh} \eta}{\left(\mathrm{S}+\frac{\mathrm{C}}{2}\right)}$
$F=$ factor of safety is taken as 6 .
- $W=$ load (in KN)
- Height of fall H (in cm)
- $S=$ set (penetration per blow of hammer) in cm (last blows).
- C is total elastic compression per blow (elastic compression of pile + soil + dolly)
- $\eta$ is efficiency of blow, depends on coeff. Of restitution.


## PILE CAPACITY BASED ON SPT VALUES

$Q_{u}=40 N \cdot \frac{D_{f}}{B} \cdot A_{B}+2 \bar{N} A_{s}$

- $\mathrm{Q}_{\mathrm{u}}=$ ultimate load capacity for driven pile
and 40N. $\frac{D_{f}}{B}<400 N\left[\frac{K N}{m^{2}}\right]$
For H-pile
- Same formula is used for displacement piles also.

$$
Q_{u}=40 N \cdot \frac{D_{f}}{B} \cdot A_{B}+\bar{N} A_{s} \text { and } 40 N \cdot \frac{D_{f}}{B}<400 N\left[\frac{K N}{m^{2}}\right]
$$

- For bored piles value is $1 / 3^{\text {rd }}$ of driven pile.


## GROUP ACTION OF PILES

- When single pile is used as a driven pile, there is uncertainty regarding vertical installation of piles. Hence, in case of driven piles, minimum three piles used. Whereas in case of bored piles verticality can be ensured. Hence, single piles can also be used. However, we always provide group of piles.
- If pile cap is above the ground surface then it is called free standing pile group (used in case of expanding soil at top).
- Minimum spacing between pile:
(1) $2.5 \times$ Diameter $\rightarrow$ for point bearing piles [Centre to centre]
(2) $3 \times$ Diameter $\rightarrow$ for friction piles.
(3) $2 \times$ Diameter $\rightarrow$ for loose sand or fill deposits.

In case of non-circular piles, diameter of circumscribed circle will be taken as the diameter

## - Ultimate Bearing capacity of pile group for Clay:

- The group of pile may fail as: (a) Block failure (b) Individual pile failure
- In block failure, soil is bound by perimeter of pile group and embedded length of pile as one unit or block.
- Ultimate load capacity of the pile group for block failure is given by.
- $Q_{u g}=\left[C_{u b} * N_{C} *\right.$ area of base of block $+\alpha * C_{u} * L *$
perimeter of the block]
- $\mathrm{C}_{\mathrm{ub}}=$ Undrained strength of clay at the base of the pile group.
- $N_{c}=9$, for deep foundation.
- $C_{u}=$ average undrained strength of clay over the length of block.
- Value of $\alpha=1$, in case of pile group (soil-soil interaction)
- Whereas safe load capacity of pile group is given by $\boldsymbol{\operatorname { m i n }}\left(\boldsymbol{Q}_{u g}\right.$ and $\left.n * \boldsymbol{Q}_{u}\right)$.
- Ultimate Bearing capacity of pile group for in sands:
- It has been observed that group efficiency of driven piles in loose or medium dense sand is $>1$. This is because soil around and between the piles get compacted due to the vibration caused during the driving operations. Whereas in dense sand above phenomenon is not true.
- For design purpose we never take group efficiency greater than 1. Hence an efficiency factor of 1 is commonly assumed in design.


## - Group efficiency:

$$
\text { group efficiency }=\eta=\frac{Q_{u g}}{n * Q_{u}}
$$

- $\mathrm{n}=$ No. of piles
- $Q_{u g}=$ ultimate load capacity of pile group.
- $\mathrm{Q}_{\mathrm{u}}=$ ultimate load capacity of single pile.


## NEGATIVE SKIN FRICTION

- Negative skin friction or 'down drag' is a phenomenon, which occurs when a portion of soil layer surrounding a pile settles more than the pile. This condition can develop when a soft soil stratum located above the pile tip is subjected to a compressive loading, the soil may settle more than the pile, also by lowering of ground water table which includes consolidation of the soft soil.
- Normally friction between pile and soil helps in carrying the axial load. Where negative skin friction (or downward drag) developed, it increases the load acting on the pile because the weight of consolidating layer is transferred to pile by friction, thus imposing extra load on the pile.
- Negative skin friction can be reduced either by providing a casing around the pile or by providing a bitumen coating around the precast pile.
- Negative Skin Friction in Single Pile

| Cohesive soil: | $\mathrm{F}_{\mathrm{n}}=$ negative skin friction on single pile |
| :--- | :--- |
| $\boldsymbol{F}_{\boldsymbol{n}}=\boldsymbol{P} * \boldsymbol{L}_{\boldsymbol{s}} * \boldsymbol{\alpha} * \boldsymbol{C}$ | $\mathrm{P}=$ perimeter of pile |
| Cohesionless soil: | $\mathrm{L}_{\mathrm{s}}=$ length of pile in settling zone |
| $\boldsymbol{F}_{\boldsymbol{n}}=\mathbf{0 . 5} * \boldsymbol{P} * \boldsymbol{L}_{\boldsymbol{s}}^{\mathbf{2}} * \boldsymbol{\gamma} * \boldsymbol{K} * \boldsymbol{T a n \delta}$ | $\mathrm{C}_{\mathrm{u}}=$ undrained cohesion of compressible soil. |
|  | $\mathrm{K}=$ lateral earth pressure coefficient |
|  | $\delta=$ angleof friction between pile and soil $(0.5-0.66) \phi$ |
|  | $\alpha=$ adhesion factor |

- Negative skin friction in pile group
the magnitude of negative skin friction

| $\mathbf{F}_{\mathbf{n g}}=$ | (Frictional force on the block + <br> weight of soil enclosed in the block) <br> Or <br> (Negative skin friction due to single <br> pile $\times$ Number of files in the pile <br> group) | $\boldsymbol{C}_{\boldsymbol{u}} * \boldsymbol{L}_{\boldsymbol{c}} * \boldsymbol{P}_{\boldsymbol{g}}+\boldsymbol{\gamma r} * \boldsymbol{L}_{\boldsymbol{c}} * \boldsymbol{A}_{\boldsymbol{g}}$ |
| :--- | :---: | :---: |

## Effect of Negative skin friction on Factor of safety:

As it is necessary to subtract negative skin friction force from the total load that the pile can support. In such a case factor of safety will be as below:

$$
F O S=\frac{\text { ultimate load capacity of single pile or a group }}{\text { working load + negative skin friction }}
$$

## UNDER REAMED PILES

- These are provided on expansive soil which are bored into the soil and are cast insitu with a bubble. As per IS2911, maximum 2 bulbs can be provided.
- $\mathrm{Q}+\mathrm{W}=$ resistance $=\mathrm{Q}_{1}+\mathrm{Q}_{2}+\mathrm{Q}_{3}$
- $\quad Q_{1}=f_{B} * A_{B}$

$$
f_{B}=\overline{\sigma_{V}} * N_{q}(\text { for sand })
$$

$$
f_{B}=C * N_{c}(\text { for clay })
$$

- $\quad Q_{2}=f_{s} * A_{s}$

$$
\begin{aligned}
& A_{s}=\pi D L \\
& f_{s}= \bar{\sigma}_{V_{\text {av. }} .} * k * \tan \delta(\text { for sand }) \\
& f_{s}=C * \alpha(\text { for clay }) \\
& Q_{3}=\left(f_{B}\right)_{(\text {bulb level })} *\left[\frac{\pi}{4} D_{B}^{2}-\frac{\pi}{4} D^{2}\right]
\end{aligned}
$$



- Bulb helps in bearing and anchorage.
- The diameter of bulb should be between 2-3 times diameter of shaft.
- Vertical distance between bulbs shall be more than 1.5 times the bulb diameter.
- Horizontal pile spacing shall be more than 2 times the bulb diameter.


## SETTLEMENT OF PILE GROUP

- The settlement of the pile group is greater than that of individual pile, this is because a large volume of soil gets influenced by the group behaviour.
- In clay soil since the major part of settlement is due to primary consolidation hence the pile load test which is performed for short duration, do not give accurate results. Hence an equivalent raft theory is assumed for calculation of group settlement.
- 1. Friction pile (NCC): an equivalent raft a a depth of $2 / 3^{\text {rd }}$ of embedded length of the pile is covered to transform the load on the soil in case of friction pile and also displacement pile.


2. In case of end bearing pile the equivalent raft is considered at the tip of the pile.
3. For sandy soil the settlement is calculated empirically in reference to the settlement obtained from pile load test.

$$
\begin{array}{|l|l}
\hline \frac{S_{g}}{S_{i}}=\left[\frac{4 B+2.7}{B+3.6}\right]^{2} & \begin{array}{l}
\mathrm{B} \rightarrow \text { width of the bock }(\mathrm{m}) \text { (smaller) } \\
\mathrm{S}_{g} \rightarrow \text { group settlement } \\
\mathrm{S}_{\mathrm{i}} \rightarrow \text { individual file statement }
\end{array} \\
\hline
\end{array}
$$

- Note: Load carrying capacity of a pile group can be calculated either from shear Strength criteria and Settlement criteria and minimum value will be adopted.


## PILE LOAD TEST

- It is a direct method to evaluate the file capacity by doing institute test. IS911 states that if the number of files is more than 200 then at least two number of initial test needs to be performed. For routine test approximately $0.5-2 \%$ of total number of piles are considered for testing and initial test piles do not form the part of structural system and are abandoned after the test. Initial test is performed to obtain the allowable load and to check the settlement.
- Routine test is done on the piles which are part of the structure system, to assess the soil densification and the suitability against the working load.
- Safe load for the piles is taken as the minimum of the following three criteria:

1. $2 / 3^{\text {rd }}$ of ultimate load value corresponding to settlement of 12 mm in total.
2. $1 / 2^{\text {nd }}$ of the load corresponding to the settlement of $10 \%$ of pile diameter in total.
3. $2 / 3^{\text {rd }}$ of the load corresponding to net settlement of 6 mm .


CHAPTER-14 SOIL EXPLORATION

## Soil Sample

- Disturbed samples are those in which natural soil structure gets modified or destroyed during the sampling operation. But we can observe natural water content, minerals composition, consistency limits and specific gravity of solidsfrom those samples are called representative samples.
- Undisturbed samples are those in which original soil structure is preserved as well as mineral properties have not undergone any change. These are used for calculation of permeability, shear strength and consolidation characteristics.

Basic Terminology in a Sampler:

| (a) Inside clearance | $D_{1}=$ inner diameter of cutting edge |  |
| :---: | :---: | :---: |
| $\mathrm{C}_{i}=\frac{\mathrm{D}_{3}-\mathrm{D}_{1}}{\mathrm{D}_{1}} \times 100$ it should be $1-$ |  |  |
| 3\%. | $\mathrm{D}_{2}=$ outer diameter of cutting edge <br> $D_{3}=$ inner diameter of sampling tube | $\theta$ |
| (b) Outside Clearance |  | O |
| $C_{0}=\frac{D_{2}-D_{4}}{D_{4}} \times 100$ it should be $0-2 \%$. |  |  |
| (c) Area ratio $A_{r}=\frac{D_{2}^{2}-D_{1}^{2}}{D_{1}^{2}} \times 100$ <br> it should be < 20 For stiff clays and < <br> 10 For sensitive clays | $\mathrm{D}_{4}=$ outer diameter of sampling tube | 実 |

- In order to express the results, of the sampling operations and to have the satisfactory design requirements following design criteria shall be used for samplers.
- $\quad$ Recovery Ratio $=\frac{L}{H}$
- Where $L$ is length of the sample before withdrawal,
- $H$ is penetration of the sample in the soil mass
- If recovery Ratio is

$$
\begin{aligned}
& =1, \text { Good recovery } \\
& <1 \text {, compressed } \\
& >1 \text {, Soil is swelled }
\end{aligned}
$$

## GATE/ESE

Civil Engineering

Highway Engineering

Important Formula Notes


## IMPORTANT FORMULAS ON HIGHWAY ENGINEERING

## CHAPTER-1 GEOMETRIC DESIGN

## 1. HIGHWAY CROSS-SECTION ELEMENTS

### 1.1. Pavement surface characteristics

(i) Friction
> For the calculation of stopping distance, the longitudinal friction coefficient values of 0.35 to 0.40 have been recommended by IRC.
> IRC has recommended the lateral friction coefficient value of 0.15
(ii) Unevenness

| Value of Unevenness | Type of pavement |
| :---: | :---: |
| $<1500 \mathrm{~mm} / \mathrm{km}$ | Good |
| $1500-2500 \mathrm{~mm} / \mathrm{km}$ | Satisfactory |
| $2500-3500 \mathrm{~mm} / \mathrm{km}$ | Bad |
| $>3500 \mathrm{~mm} / \mathrm{km}$ | Unsatisfactory |

Bump integrater $(\mathrm{mm} / \mathrm{km})=630(\operatorname{IRI}\{\mathrm{~m} / \mathrm{km}\})^{1.12}$

### 1.2. Cross Slope or Camber

Cross slope or camber is the slope provided to the road in the transverse direction to drain off the rainwater from the road surface.


Recommended values of camber:

| Type of roads | Range of camber in areas <br> of |  |
| :---: | :---: | :---: |
|  | Heavy <br> rainfall | Low <br> rainfall |
| Cement concrete and high type bituminous <br> surface | $2 \%$ | $1.7 \%$ |
| Thin bituminous surface | $2.5 \%$ | $2 \%$ |
| Water bound macadam and gravel pavement | $3.0 \%$ | $2.5 \%$ |


| Earthen roads | $4.0 \%$ | $3.0 \%$ |
| :--- | :--- | :--- |


(a) PARABOLIC SPHERE $\left[\mathrm{Y}=2 \mathrm{X}^{3} / \mathrm{nW}\right]$

(b) STRAIGHT LINE CAMBER

(c) COMBINATION OF STRAIGHT AND PARABOLIC SHAPE

Note: VERTICAL SCALES IS ENLARGED IN THE ABOVE SKETCHES Shapes of cross slope

### 1.3. Width of Pavement or Carriageway

> The Portion of carriageway width intended for one line of traffic movement is called a traffic lane width of carriageway recommended by the IRC.

| Class of road | Width of the carriageway (m) |
| :---: | :---: |
| single lane road | 3.75 |
| Two lanes, without raised kerbs | 7.0 |
| Two lanes, with raised kerb | 7.5 |
| Intermediate carriageway (except on important roads) | 5.5 |
| Multilane pavements | 3.5 per lane |

### 1.4. Medians or Traffic separators


$>$ The IRC recommends a minimum desirable width of 5.0 m for medians of rural highways, which way be reduced to 3.0 m where land is restricted.
$>$ Width of median for bridge should be 1.2 to 1.5 metres.

### 1.5 Shoulder

$>$ Shoulder is provided to accommodate stopped vehicles and to provide lateral confinement to thr pavement layer.
$>$ Derirable width is 4.6 m with a minimum of 2.5 m for two lane rural road.
> Formation width of a single lane/ two lane NH section is 12 m as per IRC.
$>$ Slope of shoulder should be at least $0.5 \%$ steeper than slope of camber subjected to a minimum of $3 \%$.

### 1.6 Kerbs

> Kerbs indicate the boundary between the pavement and median or footpath or island or shoulder.

### 1.5. Right of way and land width

Right of way is the area of land acquired for the road, along its alignment. The width of the acquired land for the right of way is known as 'Land width'.


## 2. SIGHT DISTANCE

It is the length of the road visible ahead to the driver at any instance.

### 2.1. $\quad$ Stopping Sight Distance (SSD)

a. $\operatorname{SSD}=$ lag distance + breaking distance.

Case I:No gradient is present on the road

$$
S S D=v \times t_{r}+\frac{v^{2}}{2 g f b}
$$

> Lag distance $=v \times t_{r}$
> Braking distance $=\frac{v^{2}}{2 \times g \times f \times b}$
Here, $v=$ speed with which vehicle was moving initially
$\mathrm{t}_{\mathrm{r}}=$ reaction time (s) \{ as per IRC it should be 2.5 sec$\}$
$b=$ brake efficiency

Case II: Gradient is present on the road


$$
S S D=v \times t_{r}+\frac{v^{2}}{2 g(f b \pm \tan \theta)}
$$

$+\rightarrow$ up gradient

- $\rightarrow$ down gradient
$\tan \theta=n / 100$


## Speed (kmph) f (longitudinal friction as per IRC)

| $<30$ | 0.4 |
| :---: | :---: |
| 60 | 0.36 |
| $>80$ | 0.35 |

Note:
> Relation between types of roads and SSD.

| Road | SSD |
| :---: | :---: |
| One lane one way | SSD |
| Two-lane one way | SSD |
| Two-lane two way | SSD |
| One lane two way | 2SSD or $\left(\mathrm{SSD}_{1}+\mathrm{SSD}_{2}\right)$ |

### 2.2. OVERTAKING SIGHT DISTANCE

$>O S D=d_{1}+d_{2}+d_{3}$
Here, $\mathrm{d}_{1}=v_{\mathrm{b}} \mathrm{t}_{\mathrm{r}}{ }^{\prime} \quad\left\{v \& v_{\mathrm{b}}\right.$ is in $\left.\mathrm{m} / \mathrm{sec}\right) \mathrm{d}_{3}=v \mathrm{~T}\left\{\mathrm{t}_{\mathrm{r}}{ }^{\prime} \rightarrow 2 \mathrm{sec}\right.$ acc toIRC $\}$

$$
\mathrm{d}_{2}=2 \mathrm{~S}+\mathrm{v}_{\mathrm{b}} \mathrm{~T}\left\{\mathrm{~T}=\sqrt{\frac{4 \mathrm{~S}}{\mathrm{a}}}\right\}\left\{\mathrm{a} \rightarrow \mathrm{~m} / \mathrm{s}^{2}\right\}
$$


$>$ If $v_{b}$ is not given,

$$
\begin{gathered}
\mathrm{V}_{\mathrm{b}}=\mathrm{V}-16\left(\mathrm{~V}_{\mathrm{b}} \text { in } \mathrm{km} / \mathrm{hr}\right) \\
\mathrm{V}_{\mathrm{b}}=\mathrm{v}-4.5\left(\mathrm{v}_{\mathrm{b}} \text { in } \mathrm{m} / \mathrm{s}\right)
\end{gathered}
$$

$>\mathrm{S}=0.7 \mathrm{v}_{\mathrm{b}}+6 \quad\left(\mathrm{v}_{\mathrm{b}}\right.$ in $\left.\mathrm{m} / \mathrm{s}\right)$
$\mathrm{S}=0.2 \mathrm{~V}_{\mathrm{b}}+6\left(\mathrm{~V}_{\mathrm{b}}\right.$ in $\left.\mathrm{km} / \mathrm{hr}\right)$

### 2.2.1. IRC recommendation on OSD :

(i) on divided highways and roads with one-way traffic regulation $\rightarrow$ OSD $=d_{1}$ $+d_{2}$
(ii) on divided highways with four (or) more lanes, IRC suggests no need to provide OSD. However, SSD should always be provided
(iii) Effect of the gradient is not considered while calculating OSD
(iv) If OSD can not be provided throughout the length of the road, we provide overtaking zone at a certain interval

- The length of overtaking zone is five times OSD, Subjected to a minimum of three times OSD



### 2.3. Intermediate right distance (ISD)

> Mathematically, ISD = 2SSD
> In case OSD is not provided, we try to provide ISD.

## Note:

> Relation between sight distances is, OSD > ISD > SSD
> Height of driver and object considered during calculations are given in the table below:

| Sight distance | Height of driver | Height of object |
| :---: | :---: | :---: |
| SSD | 1.2 m | 0.15 m |
| ISD | 1.2 m | 1.2 m |
| OSD | 1.2 m | 1.2 m |

## 3. HORIZONTAL ALIGNMENT

Condition for No skidding and overturning:


P = centrifugal force
$\mathrm{w}=\mathrm{mg}=$ weight of vehicle
$R=$ radius of circular curve
$\mathrm{v}=$ Speed of vehicle
$\mathrm{g}=$ acceleration due to gravity.
$b=$ center to cernter distance between wheels of vehicle
$h=$ height of C.G of vehicle above the road level

$$
\Rightarrow \mathrm{f}=\frac{\mathrm{P}}{\mathrm{~W}}
$$

Here $\frac{P}{W}$ is the centrifugal ratio
i. When the road surface is flat (i.e., no cross slope)

> For no overturning:

$$
\frac{v^{2}}{g R} \leq \frac{b}{2 h}
$$

> For no skidding:

$$
\frac{v^{2}}{g R} \leq f
$$

> To avoid both:

$$
\frac{P}{m g} \leq \frac{b}{2 h} \leq f
$$

ii. When the vehicle is moving on a banked road

> For no overturning:

$$
\frac{v^{2}}{g R} \leq \frac{\tan \theta+\frac{b}{2 h}}{1-\frac{b}{2 h} \tan \theta}
$$

> For no skidding:

$$
\frac{\mathrm{v}^{2}}{\mathrm{gR}} \leq \frac{\mu+\tan \theta}{1-\mu \tan \theta}
$$

## 4. Super Elevation

> The rate of superelevation, ' e ', is expressed as the ratio of the height of the outer edge with respect to the horizontal width.


| Terrian | $\mathbf{e}_{\mathbf{m a x}}$ | $\mathbf{e}_{\text {min }}$ |
| :---: | :---: | :---: |
| Plain and Rolling | $7 \%$ | Camber |
| Hilly terrain (snowbound) | $7 \%$ | Camber |
| Hilly terrain (not bounded by snow) | $10 \%$ | Camber |
| Urban Area | $4 \%$ | Camber |

## (i) Equilibrium super elevation:

> If only super elevation counteracts, full centrifugal force is assumed to be zero. Therefore, pressure at the inner and outer tyres is the same.

$$
\mathrm{R}_{\mathrm{A}}=\mathrm{R}_{\mathrm{B}}
$$

## (ii) Design super elevation:

> Step 1. Calculate the equilibrium corresponding to $75 \%$ of the design speed (as per IRC for mixed traffic),
$e=\frac{(0.75 v)^{2}}{g R}$ (here $v$ is in $\left.m / s\right)$ or $e=\frac{V^{2}}{225 R}$ (here $V$ is in kmph)
If calculated $\mathrm{e}<0.07$ it is acceptable and if not then move to step 2.
> Step 2. Provide $e=0.07$, check for friction coefficient $\Rightarrow e+f=\frac{v^{2}}{g R}$
If $\mathrm{f}<0.15$ then accepted, otherewise move to step 3.
> Step 3. Calculate maximum permessible speed for maximum values of $f$ and $e$ by using $\Rightarrow 0.15+0.07=\frac{\mathrm{v}^{2}}{\mathrm{gR}}$ and restrict the speed for that particular section of the road.
> Note: minimum superelevation is provided for drainage of water hence camber I staken as the value of minimum superelevation. Minimum radius of curve beyond
which no superelevation is provide as per IRC is $\Rightarrow R_{\text {min }}=\frac{(0.75 \mathrm{v})^{2}}{\mathrm{gC}}$
$>$ Note: Rulling minimum radius $\Rightarrow R_{\text {rulling min }}=\frac{v_{\text {rulling }}{ }^{2}}{(e+f)_{\max } g}$
$\Rightarrow$ Note: absolute minimum radius $\Rightarrow R_{\text {absolute min }}=\frac{v_{\text {mindesign }}{ }^{2}}{(e+f)_{\max } g}$
(iii) Design Speed:

| Types of <br> road | Plain terrain |  | Rolling terrain |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Rolling design <br> speed | Minimum <br> speed | Rolling design <br> speed | Minimum <br> speed |
| NH and SH | 100 | 80 | 80 | 65 |
| MDR | 80 | 65 | 65 | 50 |
| ODR | 65 | 50 | 50 | 40 |
| VR | 50 | 40 | 40 | 35 |

## 5. Extra widening on the horizontal curve ( $\mathrm{W}_{\mathrm{e}}$ ):


> Extra widening is the sum of mechanical widening and psychological widening. Mathematically,

$$
\begin{gathered}
W_{e}=W_{\text {mech. }}+W_{\text {Psy }} \\
\Rightarrow W_{e}=\frac{\mathrm{nl}^{2}}{2 R}+\frac{V}{9.5 \sqrt{R}} \text { or } \frac{\mathrm{n}^{2}}{2 R}+\frac{v}{2.64 \sqrt{R}}
\end{gathered}
$$

here, $\mathrm{V}=$ speed of vehiclein kmph and $\mathrm{v}=$ speed of vehicle in $\mathrm{m} / \mathrm{s}$.
$I=$ length of wheel base, in $m$ (take $I=6 \mathrm{~m}$, if not given)
$\mathrm{n}=$ number of lanes.
Note: for single lane road only mechanical widening is provided.
Note: if radius of the curve is more than 300 m then extra widening is not provided as per IRC.

## 6. Transition curve

$>$ A transition curve has a radius that decreases from infinity at the tangent point to a designed radius of the circular curve.
$>$ IRC recommends a "spiral curve" as the shape of the transition curve.

- In the case of spiral curve, we have

> length of a curve $(I) \propto 1 / R$
> centrifugal force $(P) \propto 1 / R$
$>$ At the beginning of transition curve one leg of cambered section is made horizontal and by the ned of transition curve full superelevation is provided.
$>$ Extra widening is provided along the transition curve gradually $50 \%$ on each side of the road, on sharp curves, on hills extra widening is provided on inner side of the road.
> If transition curve is not provided then $2 / 3^{\text {rd }}$ of the superelevation and extra widening is provided on the straight portion itself and remaining $1 / 3^{\text {rd }}$ is provided in the circular portion.

### 6.1. Types of transition curves:

The types of transition curves commonly adopted in the horizontal alignment of highways are
(i) Spiral
(ii) Lemniscate
(iii) Cubic parabola


### 6.2. Length of transition curve:

(i) Rate of change of centrifugal acceleration:

$$
L_{s}=\frac{v^{3}}{C \times R}
$$

Where, $v=$ speed of vehicle in $\mathrm{m} / \mathrm{s}$
$R=$ radius of the curve
$\mathrm{L}_{\mathrm{s}}=$ length of transition curve, and

$$
\mathrm{C}=\frac{80}{75+\mathrm{V}}
$$

Where V is in $\mathrm{km} / \mathrm{hr}$
C is in $\mathrm{m} / \mathrm{s}^{3}$, Also, $0.5 \leq \mathrm{C} \leq 0.8$
ii. Rate of the introduction of superelevation:
> For plain terrain

$$
L_{s}=e \times N \times\left(w+w_{e}\right)
$$

> For hilly terrain,

$$
L_{s}=\frac{e \times N \times\left(w+w_{e}\right)}{2}
$$

Where $\mathrm{N}=150$ (minimum) for plain and rolling terrain
$\mathrm{N}=100$ (minimum) for built-up area
$N=60$ (minimum) for hilly area.
(iii) based onthe empirical formula:

$$
L_{S}=\frac{2.7 V^{2}}{R} o r L_{S}=\frac{35 v^{2}}{R} \text { (for plain and rolling terrain) }
$$

$$
L_{S}=\frac{V^{2}}{R} o r L_{S}=\frac{12.96 v^{2}}{R} \text { (for hilly terrain) }
$$

Here, V is in $\mathrm{km} / \mathrm{hr}$ and v is in $\mathrm{m} / \mathrm{s}$.
$R$ is in $m$.
$\mathrm{L}_{\mathrm{s}}$ is in m .

## Note: Insertion of Transition Curve

- When transition curves are introduced between the tangents and a circular curve of radius $R$, the circular curve is 'shifted' inwards from its original position by an amount $A B=S$ (the shift) as shown in the above figure such that the curve can meet tangentially.
- This is equivalent to have a circular curve of radius ( $R+S$ ) connecting the tangents replaced by two transition curves and a circular curve of radius $R$, although the tangent points are not the same, being $A$ and $B$.

The amount of shift $\mathrm{S}=\frac{L^{2}}{24 R}$ and $\mathrm{TC}=\mathrm{CD}=\frac{L}{2}$

## Setting out transition Curve

## To locate the tangent point $T$ :

1. Calculate the shift S from the expression below

$$
\mathrm{S}=\frac{L^{2}}{24 R}
$$

2. Calculate $V A=(R+S) \tan \frac{\Delta}{2}$
3. Since $T A=\frac{L}{2}$


Then $\mathrm{VT}=(\mathrm{R}+\mathrm{S}) \tan \frac{\Delta}{2}+\frac{L}{2}$

Measure this length back from V and mark/set the point T .
The next step depends on whether it is intended to set out the transition with tapes using the cubic spiral or cubic parabola, or by the theodolite using the cubic spiral.
4. Either calculate offsets from

$$
x=\frac{l^{3}}{6 L R} \quad \text { or } \quad x=\frac{y^{3}}{6 L R}
$$

Each peg is located by swinging a chord length from the preceding peg.

## 7. Setback distance (m)


> The clearance distance is required from the centre line of the horizontal curve to an obstruction on the inner side of the curve to provide adequate sight distance is called set back Distance (m).
> The set back ( m ) distance or clearance depends upon the following factors:
(i) Required sight distance, S
(ii) Radius of the horizontal curve, R
(iii) Length of the curve, $L_{c}$, may be greater or lesser than S (SSD).

Case 1: If, $L_{c}>S$
(a) For single-lane road

$\Rightarrow \frac{\alpha}{2}=\frac{\mathrm{S}}{2 \mathrm{R}} \times \frac{180}{\pi}$ degree $\left\{\because \frac{s}{2 R}\right.$ is in radian $\}$

$$
\Rightarrow \quad \mathrm{m}=\mathrm{R}(1-\cos \alpha / 2)
$$

(b) For two-lane road


$$
\begin{gathered}
\Rightarrow \frac{m=R-(R-d) \cos \alpha / 2}{4 d=w+w_{e}} \\
\Rightarrow \frac{\alpha}{2}=\frac{S}{2(R-d)} \times \frac{180}{\pi} \text { degree }
\end{gathered}
$$

Case 2: If, $L_{c}<S$
(a) For single-lane road

(b) For two-lane road


$$
m=R-(R-d) \cos \alpha / 2+\left[\frac{S-L_{c}}{2}\right] \sin \frac{\alpha}{2}
$$

$$
\Rightarrow \frac{\alpha}{2}=\frac{L_{c}}{2(R-d)} \times \frac{180}{\pi} \text { degree }
$$

## 8. GRADE COMPENSATION

> Grade compensation is not required for grades flatter than 4\%

$$
\left.\begin{array}{rl}
\text { Grade compensation } & =\mathrm{GC}=\frac{30+\mathrm{R}}{\mathrm{R}} \\
& =\frac{75}{\mathrm{R}}
\end{array}\right\}_{\text {minimum }}
$$

compensated grade $=$ Gradient -GC , should not be less than $4 \%$.
Note: Rulling gradient $=2 \times$ camber

## 9. Curve resistance:

$>$ loss of pulling power $=\mathrm{T}(1-\cos \theta)$
where $\mathrm{T}=$ tractive effort of vehicle
$\theta=$ angle of the turn
> IRC recommendation for vertical alignment gradients.

| Terrain | Rolling | Limiting | Exceptional |
| :---: | :---: | :---: | :---: |
| (i) Plain/Rolling | $3.3 \%$ | $5 \%$ | $6.7 \%$ |
| (ii) Mountainous | $5 \%$ | $6 \%$ | $7 \%$ |

## 10. VERTICAL ALIGNMENT

### 10.1. Summit curve


> Summit curves are convex upward or concave downwards.
> The ideal shape of the summit curve is circular because site distance is always constant for a circular curve. But generally, a square parabola is preferred due to best riding quality (rate of change of grade is constant) and simplicity of calculation.

## > Length of summit curve:

While designing the length of the parabolic summit curve, it is necessary to consider SSD and OSD separately.

Case 1: L > SSD or OSD
The general equation for length $L$ of the parabolic curve is given by:
$\mathrm{L}=\frac{\mathrm{NS}^{2}}{(\sqrt{2 \mathrm{H}}+\sqrt{2 \mathrm{~h}})^{2}}$
where $L=$ length of summit curve, $m$
$\mathrm{S}=$ sight distance
$\mathrm{N}=$ Deviation angle, equal to the algebraic difference in grades, radians or tangent of deviation angle.
$H=$ height of eye level of the driver above roadway surface, $m$
$h=$ height of the object above the road surface, $m$
For SSD ( $\mathrm{H}=1.2$ and $\mathrm{h}=0.15 \mathrm{~m}$ )

$$
\mathrm{L}=\frac{\mathrm{NS}^{2}}{(\sqrt{2 \times 1.2}+\sqrt{2 \times 0.15})^{2}} \Rightarrow \mathrm{~L}=\frac{\mathrm{NS}^{2}}{4.4}
$$

For OSD and ISD ( $\mathrm{H}=1.2 \mathrm{~m}$ and $\mathrm{h}=1.2 \mathrm{~m}$ )

$$
\mathrm{L}=\frac{\mathrm{NS}^{2}}{(\sqrt{2 \times 1.2}+\sqrt{2 \times 1.2})^{2}} \Rightarrow \mathrm{~L}=\frac{\mathrm{NS}^{2}}{9.6}
$$

Case 2: L < SSD or OSD
The general equation for the length of the parabolic summit curve, when it is less than the sight distance, is given by:

$$
L=2 S-\frac{(\sqrt{2 H}+\sqrt{2 h})^{2}}{N}
$$

For SSD ( $\mathrm{H}=1.2 \mathrm{~m}$ and $\mathrm{h}=0.15 \mathrm{~m}$ )

$$
L=2 S-\frac{4.4}{N}
$$

For OSD and ISD ( $\mathrm{H}=1.2 \mathrm{~m}$ and $\mathrm{h}=1.2 \mathrm{~m}$ )

$$
L=2 S-\frac{9.6}{N}
$$

### 10.2. Valley curve/sag curve

> Valley curves are concave upward convex downward.
> Cubic parabola is generally preferred for valley curves to introduce centrifugal force gradually. Howere IRC recommend square parabola. But for small deviation angles, a cubic parabola is similar toa square parabola. IRC has recommended providing square parabola due to its simplicity.
(a) Length of valley curve for comfort condition:

The length of transition curve, $L_{s}$ fulfilling allowable rate of change of centrifugal acceleration, C is given as

$$
L_{s}=\frac{v^{3}}{C R}
$$

Here,

$$
L_{s}=R \times N
$$

$\Rightarrow R=\frac{L_{s}}{N}$

$$
\Rightarrow L_{s}=\frac{v^{3}}{C \times \frac{L_{s}}{N}}
$$



$$
L_{s}=\sqrt{\frac{N v^{3}}{C}}
$$

if not given take $C=0.6 \mathrm{~m} / \mathrm{s}^{3}$
Length of valley curve from comfort condition criterion is given as

$$
\begin{gathered}
L=2 L_{s} \\
L=2 \sqrt{\frac{N v^{3}}{C}} \\
\hline
\end{gathered}
$$

## (b) Length of valley curve for headlight sight distance:

Case 1: When L > SSD


Here,

$$
\mathrm{h}_{1}+\mathrm{S} \tan \mathrm{a}=\mathrm{as}^{2}=\frac{\mathrm{NS}^{2}}{2 \mathrm{~h}}
$$

It the average height of the headlight is taken as $h_{1}=0.75 \mathrm{~m}$ and the beam angle $a=1^{\circ}$, by substituting these values in the above equation, we get

$$
L=\frac{N S^{2}}{(1.5+0.035 S)}
$$

Where $L=$ total length of valley curve, $m$
S = sight distance
$\mathrm{N}=$ deviation angle

Case 2: When L < SSD

$$
L=2 S-\frac{1.5+0.035 S}{N}
$$

Note: Length of valley curve is provided for the following factors.
(i) Comfort condition
(ii) Headlight sight distance
(iii) Cross drainage control.
(iv) Aesthetic appearance.

## CHAPTER-2 TRAFFIC ENGINEERING

## 1. Dimensions of vehicle (According to IRC)

| Dimension | Detail | Maximum dimension, (m) |
| :---: | :--- | :---: |
| Width | --- | 2.44 |
| Height | (a) Single decked vehicle | 3.81 |
|  | (b) Double decked vehicle | 4.72 |
| Length | (a) Single unit 2 axle | 10.67 |
|  | (b) Single unit more than two-axle | 12.19 |
|  | (c) Semi trailer-tractor | 15.24 |
|  | (d) Tractor and trailer | 18.29 |

## 2. TRAFFIC STUDIES

### 2.1. Traffic Volume Study

### 2.1.1. Counting of Traffic Volume

(i) Mechanical Counter
(ii) Manual Counting
(iii) Moving Car Method

$$
\text { Traffic volume, } \mathrm{q}=\frac{\mathrm{n}_{\mathrm{a}}+\mathrm{n}_{\mathrm{y}}}{\mathrm{t}_{\mathrm{a}}+\mathrm{t}_{\mathrm{y}}}
$$

And average journey time, $\overline{\mathrm{t}}=\mathrm{t}_{\mathrm{y}}-\frac{\mathrm{n}_{\mathrm{y}}}{\mathrm{q}}$
Here,
$\mathrm{q}=$ vehicle/minute in one direction
$\mathrm{n}_{\mathrm{a}}=$ number of vehicles met in travelling against the stream
$n_{y}=$ number of vehicles overtaking the test car minus number of vehicles overtaken by the test car.
$t_{a}=$ travel time of test vehicle travelling against the stream in minutes
$t_{y}=$ travel time of the test vehicle travelling with the stream in minutes

### 2.1.2. Presentation of traffic volume study data:

(i) AADT (Annual Average Daily Traffic): The average of 24 hr . volumes at a location is calculated over 365 days. It includes a seasonal variation of traffic.
(ii) ADT (Average Daily Traffic): includes a weekly variation of traffic
(iii) Trend chart: volume trend over a year is calculated
(iv) Traffic flow map along the route: It gives an idea of traffic at a glance
(v) $\mathbf{3 0}^{\text {th }}$ highest hourly volume: It is taken as design hourly volume or design capacity. It is exceeded only 29 times in a year.


### 2.2. Traffic speed study:

## (i) Spot speed:

(a) It is the instantaneous speed of a vehicle of a particular location.
(b) It is measured using the "Endoscope" pressure contact tube, doppler radar and loop deflector.
(ii) Average speed:
(a) Time mean speed: It is the arithmetic mean of spot speed.

$$
V_{t}=\frac{\sum_{1}^{n} V_{i}}{n}
$$

Here, $\mathrm{n} \rightarrow$ no. of the vehicle crossing the location in a given interval time, $V_{i} \rightarrow$ spot speed of an $i^{\text {th }}$ vehicle at that location
(b) Space mean speed: It is the harmonic mean of spot speed.

$$
V=\frac{1}{\frac{1}{n} \sum_{1}^{n} \frac{1}{V_{i}}}
$$

$\because \mathrm{AM} \geq \mathrm{HM} \Rightarrow \mathrm{V}_{\mathrm{t}} \geq \mathrm{V}$

### 2.2.1. Cumulative speed distribution curve


> For design purposes, $98^{\text {th }}$ percentile speed is taken.
> For maximum safe speed, $85^{\text {th }}$ percentile speed is taken.
> For minimum speed (to avoid congestion), the $15^{\text {th }}$ percentile speed is taken.

### 2.3. Speed and Delay Study

(a) Floating car or riding check method
(b) Interview technique
(c) Elevated observations and Photographic technique

### 2.4. Origin and Destination Studies

> It determines information like duration of travel, selection of route and length of the route.
> Various methods of collecting O and D data:
(i) Roadside interview method
(ii) License plate method
(iii) Return postcard method
(iv) Tag on car method
(v) Home interview method

### 2.5. TRAFFIC FLOW CHARACTERISTICS

(i)

(iii) Crossing

(iv) Weaving


Note:

| Number of lanes |  | Number of potential conflicts |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Road A | Road <br> B | Both roads <br> two way | A:1-way, <br> B:2-way | Both roads <br> two way |
| 2 | 2 | 24 | 11 | 6 |
| 2 | 3 | 24 | 11 | 8 |
| 2 | 4 | 32 | 17 | 10 |
| 3 | 3 | 24 | 13 | 11 |
| 4 | 4 | 44 | 25 | 18 |

### 2.6. Traffic capacity study:

> Traffic volume (q): no. of the vehicle crossing a given point in unit time (vehicle/hr).
> Traffic density (k); no. of vehicle occupying the unit length of road at a given time (vehicle/km)
> Time headway: The time interval between the passes of the rear bumper of the successive vehicle at a point is called "time headway".

$$
\text { traffic volume }(\mathrm{q})=\frac{1}{\text { average time headway }(\mathrm{hr} / \text { veh })}
$$

> Space headway:Distance between the rear bumper of the successive vehicle
traffic density $(k)=\frac{1}{\text { average space headway }(\mathrm{km} / \mathrm{veh})}$

### 2.6.1. Calculation of theoretical maximum capacity:

(i) from space headway:

$$
\mathrm{C}=\frac{1000 \mathrm{~V}}{\mathrm{~S}} \quad\{\mathrm{~S} \rightarrow \text { in } \mathrm{m}\}
$$

$\mathrm{C}=$ capacity of a single lane (veh/hr)
$V=$ speed (km/hr)
$S=$ space headway maximum of ( $\{v t+L\},\{0.7 v+L\}$ )

- In this case, the driver remains alert because the vehicle ahead can stop anytime. So, reaction time is considered as 0.7 sec instead of 2.5 sec .
- v is meter / sec
$L=$ length of vehicle (if not given, assume $L=6 \mathrm{~m}$ )
(ii) from time headway:

$$
\mathrm{C}=\frac{3600}{\mathrm{~h}_{\mathrm{t}}}\left[\mathrm{~h}_{\mathrm{t}} \rightarrow \text { in } \mathrm{sec}\right]
$$

## 3. TRAFFIC SIGNALS

(i) cycle: A signal cycle is one complete rotation to all indications provided

$$
\begin{array}{|l|l|l|}
\hline \mathrm{G} & \mathrm{~A} & \mathrm{R} \\
\hline
\end{array}
$$

## (ii) Intervals:

(a) change interval
> It warns traffic about the coming of the red signal.
> It gives time for traffic that has entered the intersection to clear it before green time starts.
(b) Clearance interval: It is used to clear the vehicle from the intersection.
(iii) Loss time:
(a) start-up loss:It is due to the reaction time of the driver.
(b) clearance loss: when the signal turns green to yellow, the latter Portion of amber is generally not utilised during this interval.

### 3.1. Detemination od cycles length:

Let the cycle length be c sec
No of cycle in $1 \mathrm{hr}=\frac{3600}{c}$
Total loss time in $\left.1 \mathrm{hr}=\frac{3600}{c} \sum_{\mathrm{i}=1}^{n} t_{l i}=\frac{3600}{c} n t_{l}\right\}$ (It losses in all phases are equal)
n is number of phase tei is loss in 1th Phase Per cycle.

Effective green time in $1 \mathrm{hr}=3600-\frac{3600}{c}\left(n t_{l}\right)$.

$$
\begin{gathered}
3600-\frac{3600\left(n t_{l}\right)}{c}=V_{c} \times h \\
1-\frac{n t e}{c}=\frac{V_{c}}{s} \\
\Rightarrow c=\frac{n t e}{1-\frac{v_{c}}{s}}
\end{gathered}
$$

$\mathrm{V}_{\mathrm{c}}=$ critical lane volume (sum. Of all phases) and $\mathrm{h}=$ saturation time head way.
> It losses and saturation time head away of different Phases and not same.

$$
c=\frac{\sum_{1}^{n} t e_{i}}{1-\sum_{1}^{n} \frac{v_{c i}}{s_{i}}}
$$

> It $\varepsilon \frac{v_{c i}}{s i} \geqslant 1$ Then number of lanes must be increased so that critical lane volume decrease. No. of lanes is so selected that the value of c is neither too small nor too large.
> Generallyue by to keep cycle time around 50-90 sec.

### 3.2. Webster Method

$$
\text { Optimum cycle length, } C_{o}=\frac{1.5 \times \mathrm{L}+5}{1-\mathrm{y}}
$$

Where,
Co=cycle time in sec
$\mathrm{L}=$ total time lost in a cycle length $=$ n.tsl + All red time
tsı $=$ start up loss per phase
Assume tsl $=2 \mathrm{sec}$, if not provided.
$\mathrm{L}=2 \mathrm{n}+$ All red time.
$\mathrm{n}=$ number of phasesi.e., 2 [we generally design for 2 phase signed]
$\mathrm{Y}=$ ratio of critical flow to saturation ratio

$$
\text { and, i.e., } Y=\sum_{1}^{n} y_{i}=\sum_{1}^{n} \frac{q_{c i}}{S_{i}}
$$

$\mathrm{q}_{\mathrm{ci}}=$ critical lane volume for $\mathrm{i}^{\text {th }}$ phase
$S_{i}=$ saturation flow per lane for $i^{\text {th }}$ phase

Effective green time for $\mathrm{i}^{\text {th }}$ phase is given as

$$
g_{i}=(C-L) \frac{y_{i}}{y}
$$

### 3.3. ROTARY

### 3.3.1. IRC guidelines for traffic rotary:

(i) According to IRC, the maximum traffic that a rotary can efficiently handle is 3000 veh/hr, and the minimum is 500 veh/hr.
(ii) Generally,the rotary is provided when 4 to 7 roads meet at an intersection.
(iii) Given mixed traffic conditions, it is suggested by the IRC that a rotary must be provided if intersecting traffic is about $50 \%$ of total traffic or fast turning traffic towards the right is at least $30 \%$ of total traffic.
(iv) Entry width is kept smaller than approach width to forcefully reduce the driver's speed while entering and increase it while exiting. The minimum entry width is 5 m .

### 3.2.2. Design parameters:

(i) design speed: for urban rotary: 30 kmph , for rural rotary: 40 kmph
(ii) Entry radius: for urban rotary: 20 m , for rural rotary: 25 m
(iii) Exit radius is generally taken as ( 1.5 to 2 ) times entry radius.
(iv) width at entry and exit: Acceleration to IRC, a two-lane road of 7 m width should be kept at 7 m for urban roads and 6.5 m for rural roads.

$\mathrm{L}=$ length of weaving section
$e_{1}=$ width of entry
$\mathrm{e}_{2}=$ width of exit
$\mathrm{w}=$ width of rotary
$\alpha=$ weaving angle
> The superelevation is not provided at a rotary, and thus stability is only provided by the Coefficient of friction. So,

$$
\mathrm{Rentry}=\frac{v^{2}}{127 f}
$$

- $\quad R_{\text {exit }}=(1.5$ to 2$) \times$ Rentry
- For central island,

$$
\mathrm{R}_{\mathrm{CI}}=1.33 \times \mathrm{R}
$$

- If not provided, take $\mathrm{e}_{1}=\mathrm{e}_{2}$
- $e=\frac{e_{1}+e_{2}}{2}$
- $w=e+3.5$
> The practical capacity of rotary is given by,

$$
P=\frac{280 w\left(1+\frac{e}{\omega}\right)\left(1-\frac{p}{3}\right)}{\left(1+\frac{\omega}{L}\right)}
$$

Here,
$\mathrm{w}=$ width of weaving section ( 6 to 18 m )
$\mathrm{p}=\frac{\text { weaving traffic }}{\text { total traffic }}=\frac{b+c}{b+c+a+d}$ In the range of 0.4 to 1 .
$a=$ left-turning traffic moving along left extreme lane
$b=$ right-turning traffic moving along right extreme lane
$\mathrm{c}=$ crossing/weaving traffic turning towards the right while entering the rotary
$\mathrm{d}=$ crossing/weaving traffic turning towards left while leaving the rotary

> The formula for practical capacity is applicable only when
(a) $6 \mathrm{~m} \leq \mathrm{w} \leq 18 \mathrm{~m}$
(b) $\frac{e}{\omega}=0.4$ to 1.0
(c) $\frac{\omega}{L}=0.12$ to 0.4
(d) $p=0.4$ to 1.0

## 4. PARKING

Two types:
(i) On-street or kerb parking
(ii) Off-street parking
> The number of parking spaces can be calculated as:

- For parallel parking: $N=\frac{L}{5.9}$
$L=$ length of kerb in $m$

- For $30^{\circ}$ parking: $N=\frac{L}{2.5}$

- For $60^{\circ}$ parking: $N=\frac{L-2.16}{2.89}$
- For $45^{\circ}$ parking: $N=\frac{L-1.77}{3.54}$
- For $9 \mathbf{0}^{\circ}$ parking: $N=\frac{L}{2.5}$


## 5. LIGHTING

Spacing $=\frac{\text { Lamp Lumen } \times \text { coefficient of utilization } \times \text { Ma int a inence factor }}{\text { Average Lux } \times \text { width of road }}$
Lamp $\rightarrow$ Lumen
Width $\rightarrow$ Meter
$>$ The Coefficient of utilization depends upon the ratio of the width of the road and mounting height.
6. TRAFFIC DISTRIBUTION

### 6.1.Greenshield's Distribution Model





$$
\begin{aligned}
& v=\frac{V_{f}}{K_{j}} K+V_{f} \\
& q=K V \\
& q_{\max }=\frac{V_{S F} \times K_{j}}{4}
\end{aligned}
$$

$\mathrm{V}_{\mathrm{SF}}=$ Free mean speed, $\mathrm{K}=$ traffic density, $\mathrm{V}=$ traffic velocity, $\mathrm{K}_{\mathrm{j}}=$ Traffic density at jam

### 6.2. Greenberg's Distribution Model

$$
\begin{gathered}
V=V_{S F} \ln \frac{k_{j}}{k} \\
q=k \times v=k \times V_{S F} \ln \frac{k_{j}}{k} \\
\text { at } k=\frac{k_{j}}{e}, v=v_{S F}
\end{gathered}
$$

### 6.3. Exponential Model

$$
\begin{aligned}
& V=V_{S F} e^{-k / k_{j}} \\
& \quad \therefore q_{\max }=\frac{V_{S F} \times k_{j}}{e}
\end{aligned}
$$

## 7. Vehicle Arrival Rate Distribution

### 7.1. Exponential Distribution


$>\mathrm{f}(\mathrm{t})=\lambda \mathrm{e}^{-\lambda \mathrm{t}}$
where, $\lambda=$ vehicle arrival rate vehicle/time
> Find the probability that $\mathrm{t} \geq \mathrm{h}$ sec.

$$
P[h \leq t \leq \infty]=\int_{h}^{\infty} \lambda e^{-\lambda t} d t=\lambda\left[\frac{e^{-\lambda t}}{-\lambda}\right]_{h}^{\infty}=e^{-\lambda h}
$$

$\Rightarrow \mathrm{P}[0 \leq \mathrm{t} \leq \mathrm{h}]=1-\mathrm{e}^{-\lambda \mathrm{h}}$

### 7.2. Poisson Distribution

> Probability of passing of ' $n$ ' vehicle in ' $t$ ' time is

$$
P(n, t)=\frac{(\lambda t)^{n} e^{-\lambda t}}{n!}
$$

> The probability of passing of zero vehicles in ${ }^{\prime} \mathrm{t}_{1}$ ' is

$$
P\left(0, t_{1}\right)=\frac{\left(\lambda t_{1}\right)^{0} e^{-\lambda t_{1}}}{1!}=e^{-\lambda t_{1}}
$$

i.e. probability that time headway is greater than $\mathrm{t}_{1}=e^{-\lambda t_{1}}$

## 8. DELAY ANALYSIS:

## assumptions:

1. Arrival Process is deterministic and vehicle arrive at a uniform rate.
2. System is unsaturated, i.e. total number of vehicle arriving in a period is less than total number of vechicles that can be served by the system, it implies that vechicles arriving in a cycle and cleared in the same cycle

$C=$ the cycle time,
$\mathrm{g}=$ effective green time.
$d_{i}=$ the delay for $\mathrm{i}^{\text {th }}$ vehicle.
$\mathrm{V}=$ slope of cumulative arrival line i.e. uniform rate of arrival $S=$ slope of cumulative divestiture line i.e. saturation from rate.

Assuming the number of vehicles to be large we have total delay $=\frac{1}{2} \times(c-g) \times l^{*}$ $\left(\simeq \sum d_{i}\right)$

No of vehicles arriving in time $t=n o$. of vehicles cleansed in time [ $\mathrm{t}-(\mathrm{c}-\mathrm{g})$ ]

$$
\begin{gathered}
\mathrm{Vt}=\mathrm{S}(\mathrm{t}-(\mathrm{c}-\mathrm{g})) \\
t=\frac{S(c-g)}{s-v}
\end{gathered}
$$

$$
\mathrm{i}^{*}=\mathrm{vt} \Rightarrow i^{*}=\frac{s v(c-g)}{s-v}
$$

$$
\text { Total delay }=\frac{1}{2} \times \frac{(c-g) \times s v(c-g)}{s-v}=\frac{1}{2} \frac{(c-g)^{2} s v}{(s-v)}
$$

av. Delay $\frac{1}{2} \frac{(c-g)^{2} s v}{(s-v) c v} \Rightarrow \mathrm{av} \cdot$ delay pc vehicle $=\frac{1}{2} \frac{(\mathrm{c}-\mathrm{g})^{2} \mathrm{~s}}{(\mathrm{~s}-\mathrm{v}) \mathrm{c}}$
total number of
vehicle in one cycle $=C V$
av. delay per vehicle $=\frac{c\left(1-\frac{g}{c}\right)^{2}}{2\left(1-\frac{v}{s}\right)}$
> Horizontal ordinate between cumulative arrival and cummulative departure line represents delay
> Vertical ordinate between cummulative arrival and cummulative departure line represents queue (cummulation).
> Area of triangle between cummulative arrival and cummulative departure line is equal to total delay of all vehicles.
> Saturation flow rate = arrival rate + rate of decrease of queue
9. PEAK HOURLY FACTOR
> It is used to represent the variation in hourly traffic, it is defined as the ratio of 60 min volume in Peak hour to 4 times peak 15 min volume.

$$
\begin{array}{|ll|l|l|l|}
\hline(P H F)_{15 \min }=\frac{V_{60}}{4 \times V_{15}} \\
\hline
\end{array} \quad \begin{array}{l|l|l|l}
1 & 1 & 1 & 1 \\
\hline 0 & 4 & 0 & 0
\end{array}
$$

(PHF) 15 min $=1 \quad$ (for uniform flow)
$=0.25$ ( for maximum variation)
> Similarly for 20 min

| 1 | 1 | 1 |
| :--- | :--- | :--- |
| 0 | 3 | 0 |

(PHF) ${ }_{20} \Rightarrow$ ( 0.33 to 1)

## 10. ACCIDENT STUDIES

It is of the following types:
(i) Moving vehicle collides with a parked vehicle
(ii) Two moving vehicles from different directions collide at an intersection.
(iii) Head-on collision.

### 10.1. When Moving Vehicle Collides with Parked Vehicle/Object


> Before collision: $\frac{1}{2} m_{A} v_{1}^{2}=\frac{1}{2} m_{A} v_{2}^{2}+f m_{A} g s_{1} \Rightarrow \quad V_{1}^{2}=2 f g s_{1}+V_{2}^{2}$

- At collision: (Assumption $\rightarrow$ Collision is purely inelastic)

Initial momentum $=$ final momentum
$M_{A} V_{2}+M_{B} \times 0=M_{A} V_{3}+M_{B} V_{3}$
$M_{A} V_{2}=\left(M_{A}+M_{B}\right) V_{3}$
$\Rightarrow V_{2}=\left(\frac{m_{A}+m_{B}}{m_{A}}\right) V_{3}=\left(1+\frac{M_{B}}{M_{A}}\right) V_{3}$
After collision: $\frac{1}{2}\left(M_{A}+M_{B}\right) V_{3}^{2}-0=f\left(M_{A}+M_{B}\right) g S_{2} \Rightarrow V_{3}=\sqrt{2 g f s_{2}}$

### 10.2. Two Vehicles from Different Directions Collide at An Intersection

> Before collision: In the E-W direction,

$$
\frac{1}{2} m_{A} v_{A_{1}}^{2}=\frac{1}{2} m_{A} v_{A_{2}}^{2}+f m_{A} g \cdot s_{A_{1}}
$$

$V_{A_{1}}^{2}=2 f g S_{A_{1}}+V_{A_{2}}^{2}$
in $\mathrm{N}-\mathrm{S}$ direction,
$\frac{1}{2} M_{B} V_{B_{1}}^{2}=\frac{1}{2} M_{B} V_{B_{2}}^{2}+f M_{B} g S_{B_{1}}$
$V_{B_{1}}^{2}=2 f g S_{B_{1}}+V_{B_{2}}^{2}$

> After collision: Momentum is conserved in E-W direction,
$M_{A} \times V_{A_{2}}+0=m_{A} V_{A_{3}} \cos \theta_{A}+m_{B} V_{B_{3}} \sin \theta_{B}$
$V_{A_{2}}=V_{A_{3}} \cos \theta_{A}+\left(\frac{m_{B}}{m_{A}}\right) V_{B_{3}} \sin \theta_{B}$
Momentum is conserved in N-S direction,
$0+M_{B} V_{B_{2}}=m_{A} V_{A_{3}} \sin \theta_{A}+m_{B} V_{B_{B}} \cos \theta_{B}$
$V_{B_{2}}=\left(\frac{m_{A}}{m_{B}}\right) V_{A_{3}} \sin \theta_{A}+v_{B_{3}} \cos \theta_{B}$
For vehicle $A: \frac{1}{2} m_{A} v_{A_{3}}^{2}-0=f m_{A} g S_{A_{2}} \Rightarrow \quad V_{A_{3}}=\sqrt{2 f g S_{A_{2}}}$
for vehicle $B: \frac{1}{2} m_{B} v_{B_{3}}^{2}-0=f m_{B} g S_{B_{2}} \Rightarrow \quad V_{B_{3}}=\sqrt{2 f g S_{B_{2}}}$

## CHAPTER-3 HIGHWAY MATERIALS

## 1. Group Index of Soil

Group index for soil can be defined as:

$$
G I=0.2 a+0.005 a c+0.01 b d
$$

Here,
$a=p-35$,
$b=p-15$, ( $a$ and $b$ areexpressed as a whole number from 0 to 40)
$\mathrm{c}=\mathrm{W}$ - 40 ,
$\mathrm{d}=\mathrm{I}_{\mathrm{p}}-10$ ( $\mathrm{c}, \mathrm{d}$ areexpressed as a whole number from 0 to 20)
$\mathrm{p}=\%$ passing 0.075 mm sieve,
$\mathrm{I}_{\mathrm{p}}=$ plastic index.
$W_{\mathrm{L}}=$ liquid limit.
Classification of soil based on group index:

| Group Index | Class |
| :---: | :---: |
| $0-1$ | Good |
| $2-4$ | Fair |
| $5-9$ | Poor |
| $10-20$ | very poor |

2. Strength Evaluation of Soil

## (i) Plate Bearing Test:

$$
\begin{aligned}
\mathrm{k}=\frac{\text { Pressure sustained }}{\text { Deformation }} \\
\Rightarrow \mathrm{k}=\frac{\mathrm{p}}{\Delta}, \text { Here, } \Delta=0.125 \mathrm{~cm}
\end{aligned}
$$

> The K value obtained from the plate bearing test have to be corrected under the following conditions:
(a) Correction for a soaked condition:

$$
\frac{\mathrm{P}_{\mathrm{s}}}{\mathrm{k}_{\mathrm{s}}}=\frac{\mathrm{P}}{\mathrm{k}}
$$

Here,
$\mathrm{K}_{\mathrm{s}}=$ modulus of subgrade reaction in soaked condition.
$\mathrm{P}_{\mathrm{s}}=$ pressure sustained in soaked condition.
(b) Correction for the size of plate:

$$
\mathrm{k}_{1} \mathrm{~d}_{1}=\mathrm{k}_{2} \mathrm{~d}_{2}
$$

## (ii) California Bearing Ratio Test:

$$
C B R(\%)=\frac{\text { Load(orpressure)sustainedbythespecimen }}{\text { at } 2.5 \text { or } 5 \text { mmpenetration }} \begin{gathered}
\text { Load(orpressure)sustainedbythestandardmaterialatthe } \\
\text { correspondingpenetrationlevel }
\end{gathered} \times 100
$$

$>$ If $\mathrm{CBR}_{2.5 \mathrm{~mm}}>\mathrm{CBR}_{5.0 \mathrm{~mm}}$ then ok and take $\mathrm{CBR}_{2.5 \mathrm{~mm}}$
But if $\mathrm{CBR}_{5.0 \mathrm{~mm}}>\mathrm{CBR}_{2.5 \mathrm{~mm}}$ then repeat the test and if the same result come again then report CBR 5.0 mm .
> Loading is applied through a 50 mm diameter plunger and at a rate of $1.25 \mathrm{~m} / \mathrm{min}$.
> The standard load values given below can be directly used to compute the CBR value of the material.

| Penetration <br> $\mathbf{( m m )}$ | Standard load (kg) | Unit standard load (kg/cm ${ }^{\mathbf{2}}$ ) |
| :---: | :---: | :---: |
| 2.5 | 1370 | 70 |
| 5 | 2055 | 105 |

## 3. Tests on Road Aggregates

## (i) Crushing test

$$
\text { Aggregate crushing value }=\frac{\mathrm{w}_{2}}{\mathrm{w}_{1}} \times 100
$$

Here,
$\mathrm{w}_{1}=$ weight of aggregates before the test (passing 12.5 mm and retaining on
10 mm size sieve).
$\mathrm{w}_{2}=$ weight of aggregate after testpassing 2.36 mm.
> Aggregate crushing value for a good quality aggregate:

- $\ngtr 30 \%$ for surface course
- $\ngtr 45 \%$ for base course
(ii) Los Angeles Abrasion Test:

$$
\binom{\text { Abrasion }}{\text { value }}=\frac{W_{2}}{W_{1}} \times 100
$$

$\mathrm{w}_{1}=$ initially weight of aggregate.
$\mathrm{w}_{2}=$ weight of abraded aggregate passing through 1.7 mm sieve.
> Abrasion value for a good quality aggregate

- $\ngtr 30 \%$ far surface course
- $\ngtr 50 \%$ for the base course
(iii) Impact Test

$$
\text { Aggregate impact value }=\frac{w_{2}}{w_{1}} \times 10
$$

$\mathrm{w}_{1}=$ weight of aggregates before the test
$=$ weight of aggregate passing 12.5 mm and retaining on 10 mm size sieve.
$\mathrm{w}_{2}=$ weight of aggregate after test
$=$ weight of aggregate passing 2.36 mm .
> Impact value for a good aggregate

- $\quad \ngtr 30 \%$ for the surface cause.
- $\ngtr 4 \%$ for bituminous macadam


## (iv) Shape Test

## a. Flakiness Index

$$
\text { Flakiness Index }=\frac{\mathrm{w}_{1}}{\mathrm{w}_{2}} \times 100
$$

$\mathrm{w}_{1}=$ Initial weight of aggregate.
$\mathrm{w}_{2}=$ weight of aggregate having least dimension less than 0.6 times the mean dimension.

## b. Elongation Index

Note: Elongation index test is performed only on a sample from which flaky particles have been removed.

$$
\text { Elongation Index }=\frac{\mathrm{w}_{2}}{\mathrm{w}_{1}} \times 100
$$

$\mathrm{w}_{1}=$ Initial weight of aggregate.
$w_{2}=$ weight of aggregate having greatest dimension more than 1.8 times the mean dimension.
> Elongated and flaky particles are less workable and are likely to break under small load, flakiness and elongation index value in excess of $15 \%$ is generally udesirable.
> MORTH has specified maximum value of combined index for coarse aggregate as 30\% for WMM, DBM, and BC coarse.
c. Angularity Number Test

$$
\text { Angularity number }=67-\frac{100 \mathrm{w}}{\mathrm{C} \times \mathrm{G}_{\mathrm{a}}}
$$

$\mathrm{w}=$ weight of aggregate.
C = weight of water filling the cylinder.
$\mathrm{G}_{\mathrm{a}}=$ specific gravity of aggregate.

## (v) Soundness test:

To determine resistance against weathering (durability) aggregate are subjected to alternate cycles of wetting and drying in saturated solution of sodium sulphate or magnesium sulphate for 16-18 hr and drying over an oven at 105-110 dergree till constant mass.
> After five cycles loss in weight should not be;
As per IRC recommandations: not more than $12 \%$ for sodium sulphate and not more than $18 \%$ for magnesium sulphate.
(vi) Water Absorption Test:
> The percentage of water absorbed with respect to the initial aggregate weight should not be more than $0.6 \%$ of the initial aggregate weight

## 4. Design of Bituminous mixes by Marshall Method

> The phase diagram for the bituminous mixture can be expressed as


So, total weight of mix $(\mathrm{w})=\mathrm{w}_{1}+\mathrm{w}_{2}+\mathrm{w}_{3}+\mathrm{w}_{\mathrm{b}}$
And total volume of mix $(\mathrm{V})=\mathrm{V}_{1}+\mathrm{V}_{2}+\mathrm{V}_{3}+\mathrm{V}_{\mathrm{b}}+\mathrm{V}_{\mathrm{V}}$
(i) Apparent specific gravity or theoretical specific gravity

It is the specific gravity calculated excluding air voids, it is denoted by 'Gt'.
Mathematically,

$$
\Rightarrow \mathrm{G}_{\mathrm{t}}=\frac{\mathrm{w}}{\frac{\mathrm{w}_{1}}{\mathrm{G}_{1}}+\frac{\mathrm{w}_{2}}{\mathrm{G}_{2}}+\frac{\mathrm{w}_{3}}{\mathrm{G}_{3}}+\frac{\mathrm{w}_{\mathrm{b}}}{\mathrm{G}_{\mathrm{b}}}}
$$

In terms of percentage, $\mathrm{G}_{\mathrm{t}}$ can be expressed as,

$$
\mathrm{G}_{\mathrm{t}}=\frac{100 \%}{\frac{\mathrm{w}_{1} \%}{\mathrm{G}_{1}}+\frac{\mathrm{w}_{2} \%}{\mathrm{G}_{2}}+\frac{\mathrm{W}_{3} \%}{\mathrm{G}_{3}}+\frac{\mathrm{w}_{\mathrm{b}} \%}{\mathrm{G}_{\mathrm{b}}}}
$$

## (ii) Mass Specific gravity:

The bulk specific gravity or the actual specific gravity of the mix is found out by:

$$
\mathrm{G}_{\mathrm{t}}=\frac{100 \%}{\frac{\mathrm{w}_{1} \%}{\mathrm{G}_{1}}+\frac{\mathrm{W}_{2} \%}{\mathrm{G}_{2}}+\frac{\mathrm{W}_{3} \%}{\mathrm{G}_{3}}+\frac{\mathrm{w}_{\mathrm{b}} \%}{\mathrm{G}_{\mathrm{b}}}+\mathrm{V}_{\text {air }}}
$$

It can also be calculated by following method, $G_{m}=\frac{W_{m}}{W_{m}-W_{w}}$ weight of sample in water $=$ weight of sample in air- total volume* unit weight of water here, $\mathrm{W}_{\mathrm{m}}=$ weight of mix in air andW $\mathrm{W}_{\mathrm{w}}=$ weight of mix in water
Note: $\mathbf{G}_{\mathrm{t}}>\mathbf{G}_{\mathrm{m}}$, always true.
(iii) Percentage air voids in the mix:

It is expressed as the ratio of the volume of the air void to the total volumes of the mix

$$
\Rightarrow \quad \% \mathrm{~V}_{\mathrm{v}}=\frac{\mathrm{V}_{\mathrm{v}}}{\mathrm{~V}} \times 100
$$

In terms of apparent and bulk specific gravity,

$$
\% V_{v}=\left(\frac{G_{t}-G_{m}}{G_{t}}\right) \times 100
$$

(iv) Percent voids in mineral aggregate (VMA):

It is expressed as

$$
\begin{aligned}
& \% V M A=\% V_{v}+\% V_{b} \\
\Rightarrow & \% V_{b}=\frac{w_{b}}{w} \times \frac{G_{m}}{G_{b}} \times 100
\end{aligned}
$$

(v) Percent voids filled with Bitumen (VFB)

$$
\mathrm{VFB}=\frac{\% \mathrm{~V}_{\mathrm{b}}}{\% \mathrm{~V}_{\mathrm{b}}+\% \mathrm{~V}_{\mathrm{v}}}
$$

### 4.1. Some Important Graphical Representations:

Optimum bitumen $\%=\frac{\alpha+\beta+\gamma}{3}$


## 5. PROPERTIES OF BITUMEN

### 5.1. Viscosity

$>$ It is the property of bitumen which resist flow due to internal friction.
$>$ It is estimated using plak viscometer or efflux viscometer.
$>\mathrm{VG} \rightarrow$ Viscosity Grade.
$>$ VG $30 \Rightarrow(100 \pm 20) 30=2400-3600\left\{\right.$ Viscosity in Poise at $\left.60^{\circ} \mathrm{C}\right\}$.

### 5.2. Ductility

$>$ Bitumen binder should be sufficiently ductile i.e. it shoued be capable of being stretched without breaking.
$>$ The distance in am the briquette can be stretched at $27^{\circ} \mathrm{C}$ under water at $5 \mathrm{~cm} / \mathrm{min}$ without breaking is called ductility
$>$ It very from 5-100 and minimum value of 50 is commonly specified.

### 5.3. Penetration

$>$ It is an indirect measure of hardness a penetration grade 80-100 means that the penetration of 100 gm needle when left for 5 sec at $25^{\circ} \mathrm{C}$ in bitumen was $8-10 \mathrm{~mm}$.

Note:Safe limit for heating bitumen is $50^{\circ}$ under flash point

### 5.4. Specific Gravity

It is estimated from Acnometer method at $27^{\circ} \mathrm{C}$. It varies from $0.97-1.02$

### 5.5. Loss on heating

It should not be more then $1 \%$ when 50 gm bitumen is heated at $163^{\circ} \mathrm{C}$ over an over for 5 hr .

### 5.6. Water content

To avoid foaming the maximum water content in bitumen should not exceed $0.2 \%$ by weight.

## Cut back bitumen:

> Bitumen in volatile dilution like, kerosene diesel Naphthaetc. used in cold weather.

## Emulsion

> Aqueous Bitumen, used in wet condition.
Grades of Tar $\rightarrow$ It varies from RT1 (Road Tar 1) (Lowest viscosity, used in surface paintings) to RT5 (highest viscosity, used in grouting)

| Bitumen | Tar |
| :---: | :---: |
| 1. obtained from fractional distillation of <br> crude oil | 1. obtained from destructive distillation of <br> coal or wood. |
| 2. Soluble in carbon di sulphide. | 2. Soluble in toluene (Ratny benzene) |
| 3. More rest, to temp \& water | 3. Insoluble in water. |
|  | 4. Contains more free carbon constant. |

## Asphalt Pavement dispesses

Bleeding:- Migration of excess bitumen from the mix do the road surface which is deposited as a thin shiny film making the road slippery.

It is caused by (a) excessive binder in mix design (b) low air void(c) use of low viscosity bitumen(d) too heavy tach coat

Note: apply hot sand and roll it during hot weather to bloat out extra asphalt binder at the surface.

Rabeling:- It is progressive disintegration of asphalt surface which is the result of dislodgment of aggregate particles in the mix at the surface, it occurs due to lack of sufficient cohesion within the asphalt mix due to inadequate binder content, low density, lack of fines and aging. HoweverRabling due to aging occurs after many years.

Stripping:- It is the braking of adhesion between aggregate and asphalt binder usually in the pressure of moisture.

Scaling:- Thin wearing course separates due to intrusion of moisture between binder and wearing course.

CHAPTER-4 PAVEMENT DESIGN

1. DIFFERENCES BETWEEN FLEXIBLE AND RIGID PAVEMENT

| Flexible Pavement | Rigid Pavement |
| :---: | :---: |
| 1. Low or negligible flexure strength. | 1. High flexural strength. |
| 2. Load is transferred from grain to grain <br> contact. | 2. Load is transferred from slab action. |
| 3. Joints are absent. | 3. Joints are present |
| 4. Low initial cost but high maintenance cost. | 4. High initial cost but low maintenance |
| cost. |  |

## 2. FLEXIBLE PAVEMENT:

### 2.1 Subgrade:

> it is prepared from natural soil by it to 95-98\% of proctor's density.
> Generally 500 mm is compacted, for rural low value road 300 mm .
> It is designed to receive stresses from the upper layer such that vertical compressive tress does not exceeds its permissible value.

### 2.2 Subbase:

> Its main function is to provide drainage and structural support ti the upper pavement layers.
> It also reduces the intrusion of fines into the pavement.
> High quality subgrade with steep slope may not require subbase.

### 2.3 Base course:

> If provides structural support by bearing high stresses coming from the top layer abd distributes them to the lower layers.
> It also contributes in sub surface drainage.


### 2.4 Surface coarse:

> It is also known as wearing coarse.
> It is of highest quality and generally bituminious mix is used.
> It provides an overall smooth surface, skid resistance for tyres and sustains environmental and weathering action. It also act as waterproof at surface.

### 2.5 Failue of flexible pavement:

Major mode of failure are:
> Fatigue cracking (alligator and crocodile cracking).
> Thermal cracking.
> Rutting (IRC consider rutting deformation due to subgrade deformation only).
> As per IRC37:2001 fatigue cracking should not occur on more than 20\% of the pavement area and rutting deformation depth should not be greater than 20 mm in design life of the pavement.
> As per IRC37:2012 fatigue cracking in 20\% area has been considered for traffic upto 30 msa and $10 \%$ for traffic is more than 30 msa . For rutting deformation should not be greater than 20 mm in $20 \%$ of length for traffic upto 30 msa and $10 \%$ of length for traffic beyond 30 msa .
Note: To control fatigue cracking we limit the tensile strain at the bottom of surface course and to control rutting we limit the axial compression strain at the subgrade layer.

## Note: different types of coats are-

> Seal coat: improve impermeability and skid resistance.
> Tack coat: provides bonding between two layers.
> Prime coat: pluges voids of base coarse and prepare the surface for applicationof tack coat.

## 3. RIGID PAVEMENT:

> In this case load is distributed due to flexure action of the slab, purpose of base coarse is to prevent mud pumping, provide drainage and reduce deflection
> Base/sub base course is optional in rigid pavement.

> Failure of rigid pavement: various modes are-

- Fatigue cracking
- Thermal cracking
- Mud pumping

Note: IRC gives design steps for fatigue cracking only, not for mud pumping.

## 4. LOAD AND TRAFFIC BASIC CONCEPT:

4.1 Stress at Any Depth Below the Wheel load
a = radius of the circular contact area

$$
\sigma_{z}=p\left[1-\frac{z^{3}}{\left(a^{2}+z^{2}\right)^{3 / 2}}\right]
$$



### 4.2 Equivalent Single Wheel Load (ESWL):

- It is the single wheel replacement of multiple wheels such that some vertical deflection or strain or stress is caused at any depth ' $z$ '.


Here,
$\mathrm{S}=$ centre to centre spacing
d = clear gap between wheels
Also, $S=d+2 a$.

## i. Relation between Depth and ESWL:

| Depth | ESWL |
| :---: | :---: |
| $1.0 \leq \mathrm{z} \leq \mathrm{d} / 2$ | P |
| $2 . \mathrm{d} / 2<\mathrm{z}<2 \mathrm{~S}$ | $\mathrm{P}<\mathrm{ESWL}<2 \mathrm{P}$ |
| $3 . \mathrm{z} \geq 2 \mathrm{~S}$ | 2 P |

## ii. Value of ESWL if Depth is $\mathbf{d} / \mathbf{2}<\mathbf{Z}<\mathbf{2 S}$

Graphically, it can be represented as

By similar triangle theorem in $\triangle \mathrm{OAB}$ and $\triangle \mathrm{OCD}$, we get, $\frac{O C}{O D}=\frac{A B}{O B}$
$\Rightarrow \frac{\log _{10} 2 P-\log _{10} P}{\log _{10} 2 S-\log _{10} d / 2}=\frac{\log _{10} P_{1}-\log _{10} P}{\log _{10} Z-\log _{10} d / 2}$
Where, $z=$ depth at which ESWL (i.e equal to $P_{1}$ ) is to be calculated.

### 4.3 Rigidity factor (RF)

> The factors show the degree of tension developed on the walls of the tyre.
> Mathematically, the rigidity factor is expressed as

$$
R F=\frac{\text { Contact pressure }}{\text { Tyre Pressure }}=\frac{C P}{T P}
$$

$>$ If $\mathrm{CP}>\mathrm{TP}$, it means tyres in compression.

> If CP $<\mathrm{TP}$, then tyres are in tension.

| Tyre Pressure | Contact Pressure | Rigidity Factor |
| :---: | :---: | :---: |
| $<7 \mathrm{~kg} / \mathrm{cm}^{2}$ | $>7 \mathrm{~kg} / \mathrm{cm}^{2}$ | $>1$ |
| $>7 \mathrm{~kg} / \mathrm{cm}^{2}$ | $<7 \mathrm{~kg} / \mathrm{cm}^{2}$ | $<1$ |
| $=7 \mathrm{~kg} / \mathrm{cm}^{2}$ | $=7 \mathrm{~kg} / \mathrm{cm}^{2}$ | $=1$ |

### 4.4 Equivalent axle load factor

> The damaging effect of wheel load $P_{1}$ with respect to standard axle load $P$ is given by 'Equivalent wheel load factor' (EWLF) or 'Equivalent axle load factor' (EALF) or 'vehicle damage factor' (VDF).

- If not given, take $P=80 \mathrm{KN}$ or 8000 kg
- Mathematically,

$$
E W L F=\left(\frac{P_{1}}{P}\right)^{4}
$$

Total number of standard axle $=\mathrm{N}_{1} . \mathrm{f}_{1}+\mathrm{N}_{2} . \mathrm{f}_{2}+\ldots \ldots$.
where $N_{i}$ is number of vehicle of $i^{\text {th }}$ class interval
and $\mathrm{f}_{\mathrm{i}}$ is EALF for $\mathrm{i}^{\text {th }}$ class interval.
Note: Total number of standard axles when divided by number of vehicles surveyed will give VDF.
Note: Minimum sample size to be surveyed for CVPD upto $\mathbf{3 0 0 0}$ is $\mathbf{2 0 \%}$, for CVPD 3000 to $\mathbf{6 0 0 0}$ it is $\mathbf{1 5 \%}$ and for CVPD more than 6000 it is $\mathbf{1 0 \%}$ of the vehicles.

### 4.5 Cummulative standard axles:

> In this, the number of repetitions of cumulative standard axle $\left(N_{s}\right)$ is calculated
as,
$N_{s}=\frac{365 A\left[\left(1+\frac{r}{100}\right)^{n}-1\right]}{(r / 100)} \times V D F \times L D F \times L S F$
Where,
A = Number of commercial vehicles per day when construction is complete.
$r=$ Rate of growth of traffic (5 as per IRC27:2012 and $7.5 \%$ as per IRC37:2001).
$\mathrm{n} \quad=$ Design life ( 15 years for NH and SH, 20 years for expressway for flexible and 30 years for rigid pavement).
VDF = Vehicle damage factor.
LDF = Lane distribution factor.
LSF = Load Safety factor.
Here, $A=P\left(1+\frac{r}{100}\right)^{x}$
Where, $x=$ construction period and, $P=$ Present day traffic count.

| Single Carriageway |  | Dual Carriageway |  |
| :---: | :---: | :---: | :---: |
| No. of Lane | LDF | No. of lane | LDF |


| 1 | 1 | 2 | 0.75 |
| :---: | :---: | :---: | :---: |
| 2 | 0.75 | 3 | 0.60 |
| 4 | 0.4 | 4 | 0.45 |

## 5. DESIGN OF FLEXIBLE PAVEMENTS

### 5.1.California bearing ratio (CBR) method

$$
t_{(c m)}=\sqrt{\frac{1.75 P}{C B R(\%)}-a^{2}}
$$

Where,
$P=$ Wheel load, kg. (If not given, take $P=4100 \mathrm{~kg}$ )
a = radius of contact area.
> IRC37:1970 has provided design charts to calculate the pavement thickness based on CBR value and anticipated traffic per day at the end of design life.
> IRC37:2012 has provide design chart for calculation of pavement thickness for various layers based on CBR value and cummulative number of standard axles.

### 5.2.California Resistance Value (CRV) Method.

The general formula that is used for this method is

$$
T_{(c m)}=\frac{0.166 \times T I \times(90-R)}{C^{1 / 5}}
$$

where,
TI = Traffic index
$\mathrm{R}=$ Stabliometer value
C = Cohesionmeter value
$\mathrm{TI}=1.35 \times(\text { Equivalent wheel load })^{0.11}$

EWL= summation of (AADT*EWLconstant)

| No. of axle | EWLconstant |
| :---: | :---: |
| 2 | 330 |
| 3 | 1070 |
| 4 | 2460 |
| 5 | 4620 |

$>$ In this method equivalency used is $\frac{T_{1}}{T_{2}}=\left[\frac{C_{2}}{C_{1}}\right]^{1 / 5}$, it means $\mathrm{T}_{1}$ thickness of material $\mathrm{C}_{1}$ is equivalent to $\mathrm{T}_{2}$ thicness of material $\mathrm{C}_{2}$.
> While designing forst we calculate the thickness of pavement corresponding to one material and then by using the equivalency we convert this thickness to thickness of different material to be used.

### 5.3 Triaxial Method

The general formula used in this method is given

$$
T_{1}(c m)=\sqrt{\left(\frac{3 P X Y}{2 \pi E_{s} \Delta}\right)^{2}-a^{2}}\left(\frac{E_{s}}{E_{p}}\right)^{3}
$$

Where,
$P=$ wheel load in kg (if not given, take $P=4100 \mathrm{~kg}$ )
X = Traffic Coefficient
$\mathrm{Y}=$ saturation Coefficient
$\mathrm{E}_{\mathrm{s}}=$ Modulus of Elasticity of soil subgrades in $\mathrm{kg} / \mathrm{cm}^{2}$
$E_{p}=$ modulus of elasticity of pavement in $\mathrm{kg} / \mathrm{cm}^{2}$
$\Delta=$ Design deflection in cm
$\mathrm{a}=$ Radius of the contact area of wheel load in cm .
> In this method equivalency used is $\frac{T_{1}}{T_{2}}=\left[\frac{E_{2}}{E_{1}}\right]^{1 / 3}$, it means $T_{1}$ thickness of material with modulus of elasticity $\mathrm{E}_{1}$ is equivalent to $\mathrm{T}_{2}$ thickness of material with modulus of elasticity $\mathrm{E}_{2}$.

### 5.4 Burmester Method

Assumptions involved in this method are
> It wheel load test/flexible plate is used, then deflection is given as

$$
\Delta=\frac{1.5 p a}{E s} \times F
$$

where, $\Delta=$ design deflection in cm , assumed to be 0.25 cm if not given.
$P=$ contact pressure in $\mathrm{kg} / \mathrm{cm}^{2}$
$\mathrm{a}=$ radius of contact area in cm .
Es = Modulus of Elasticity of soil sub grade
$F=$ deflection factor or displacement factor, it svalue depends on ratios $h / a$ and Es/Ep.
> For calculation of Es/Ep testing is done on subgrade and pavement layer of an existing pavement having same modulus of elasticity.
> In this plate load test/Rigid plate is used, then deflection is given as

$$
\Delta=\frac{1.18 p a}{E} \times F
$$

Here, $\Delta=$ deflection recorded in PBT in cm.
$\mathrm{P}=$ pressure used in PBT ( $\mathrm{kg} / \mathrm{cm}^{2}$ )
a= radius of plate used for test
$E=$ modulus of elasticity of subgrade
During testing $\mathrm{F}=1$.

## 6. DESIGN OF RIGID PAVEMENTS

### 6.1 Design Parameters

## i. Modulus of Subgrade Reaction, $k$

$>$ It is used to measure soil resistance in terms of deformation corresponding to the load applied.

$$
k=\frac{p}{\Delta}=\frac{p}{0.125 \mathrm{~cm}}
$$

Here k is in $\mathrm{kg} / \mathrm{cm}^{3}$
> The value of k depends on size of plate used in plate bearing test and for rigid pavement we generally use 75 cm diameter plate. Also k.a= constant.

## ii. Radius of Relative Stiffness, I

> The general formula for the radius of relative stiffness is given as

$$
l=\left[\frac{E h^{3}}{12 k\left(1-\mu^{2}\right)}\right]^{1 / 4}
$$

Where, $\mathrm{E}=$ Modulus of Elasticity of concrete $\mathrm{kg} / \mathrm{cm}^{2}$.
$\mathrm{h}=$ Thickness of concrete slab or pavement in cm .
$\mu=$ Poisson's ratio (0.15)
$\mathrm{k}=$ Modulus of sub grade reaction $\mathrm{kg} / \mathrm{cm}^{3}$.

## iii. Equivalent Radius of Resisting Section, b

> The general formula for the equivalent radius of the resisting section is given as.

$$
\begin{aligned}
& b=\sqrt{1.6 a^{2}+h^{2}}-0.675 h(\text { if } a<1.724 h) \\
& b=a(\text { if } a>1.724 h)
\end{aligned}
$$

Where $b=$ Equivalent radius of resisting section in cm .
$h=$ Thickness of pavement in cm
$\mathrm{a}=$ Radius of the contact area of wheel load in cm .

### 6.2 Wheel Load Stresses

### 6.2.1 Westergaard Analysis

$>$ Stress at the interior, $S_{\text {interior }}=\frac{0.316 P}{h^{2}}\left[4 \log _{10}\left(\frac{l}{b}\right)+1.069\right]$
$>$ Stress at the edge, $S_{\text {edge }}=\frac{0.572 P}{h^{2}}\left[4 \log _{10}\left(\frac{l}{b}\right)+0.359\right]$
> Stress at the corner, $S_{\text {corner }}=\frac{3 P}{h^{2}}\left[1-\left(\frac{a \sqrt{2}}{l}\right)^{0.6}\right]$

### 6.2.2 IRC Formula

- Sint is calculated using Westergaard formula.
- Sedge is calculated by the Teller formula. According to this,
$S_{\text {edge }}=\frac{0.529 P}{h^{2}}(1+0.54 \mu)\left[4 \log _{10}\left(\frac{l}{b}\right)+\log _{10} b-0.4048\right]$
- Scorner is calculated by Kelly's formula. According to which stress can be calculated as
$S_{\text {corner }}=\frac{3 P}{h^{2}}\left[1-\left(\frac{a \sqrt{2}}{l}\right)^{1.2}\right]$
where, $P=$ wheel load ( kg )
$\mu=$ Poisson's ratio of concrete
$a=$ radius of wheel load contact area (cm)
I = radius of relative stiffness (cm)
$\mathrm{b}=$ Equivalent radius of resisting section (cm)


### 6.2.3 Temperature Stresses

## Warping Stress

> Daily variation of temperature causes warping, stress variation due to temperature gradient is called warping stresses.

$$
\begin{array}{r}
S_{\text {interior }}=\frac{E \alpha t}{2}\left[\frac{C_{x}+\mu C_{y}}{1-\mu^{2}}\right] \\
S_{\text {edge }}=\max \left(\frac{C_{x} E \alpha t}{2}, \frac{C_{y} E \alpha t}{2}\right), \\
S_{\text {corner }}=\frac{E \alpha t}{3(1-\mu)} \sqrt{\frac{a}{l}}
\end{array}
$$

Where,
$\mathrm{E}=$ Modules of elasticity of concrete.
$\mathrm{a}=$ Coefficient of thermal expansion.
$\mu=$ Poisson's ratio.
$\mathrm{t}=$ temperature difference between the top and bottom of the slab.
a = radius of wheel load contact area.
I = radius of relative stiffness.
$C_{x}$ and $C_{y}$ are coefficients in $x$, and $y$-direction respectively depends on $\frac{L_{x}}{l}$ and $\frac{L_{y}}{l}$ Ratiorespectively.
$L_{x}$ and $L_{y}$ are the lengths of the slab in $x$ and $y$ directions, respectively which are almost equal to spacing longitudinal and transverse joints.

## Day-TimeNight-Time



## Frictional stress:

$>$ Seasonal variation caused frictional stress due to overall change in slab length.
$>$ In summer overall length of the slab increases and friction will try to compress the slab so compressive stresses are generated in summer, similarly tensile stresses are generated in winter.

## Frictional Stress

$$
\sigma_{c}-\frac{\gamma_{c} \mathbf{L} f_{a}}{2}
$$

$\sigma_{\mathrm{c}}$ Stress in Concrete $\gamma_{c}$ Unit Weight of Concrete

L=Slab Length
$f_{a}$ Average Coefficient of Friction

$>$ Average friction coefficient is taken as 1.5.

## NATURE OF STRESSES

Here, $\mathrm{C} \rightarrow$ compression and $\mathrm{T} \rightarrow$ tension

| Section |  | Wheel load <br> stresses | Warping stress |  | Frictional stress |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Night | Summers | Winters |  |
| Interior | Top |  | C | C | T | C | T |
|  | Bottom | T | T | C | C | T |
|  | Top | C | C | T | C | T |
|  | Bottom | T | T | C | C | T |


| Corner | Top | T | C | T | C | T |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Bottom | C | T | C | C | T |

Critical combination of stresses:
> Out of the various wheel load stresses, corner stress is maximum and interior stress is minimum and edge stress is intermediate.
> In case of temperature stresses interior stresses come out to be maximum and edge stresses come out to be minimum and edge stress is intermediate.

### 6.3 JOINTS

(i) Expansion joint:
> The prupose of expansion joint is to allow for expansion of pavement due to rise in pavement temperature with respect to construction tempearure.
> As per IRC maximum spacing between expansion joint is 140 m and maximum gap of joint is 2.5 cm .
> At expansion joint dowel bars are provided which develops bending, bearing and shearing stress and helps in load transfer. One end of dowel bar is bonded with concrete and the other end is free to move also filler is provided to seal the joint.

$$
\mathrm{L}_{\mathrm{e}} \alpha(\Delta \mathrm{~T})=\frac{\delta}{2} \sigma \gamma_{\mathrm{c}}
$$

$\mathrm{L}_{\mathrm{e}}=$ Spacing of expansion joint
$\delta=$ gap in expansion joint, filler material is assumed to be $50 \%$ compressible.
$\alpha=$ Coefficient of thermal expansion

## (ii) Contraction joint ( $L_{c}$ )

> It is provided to control cracks due to shrinkage, to regulate the crackes. i.e to ensure that crackesdevelopes at a predetermined location, the slab is weakened at certain location, these location are called contraction joint.
> Note: As per IRC maximum spacing between contraction joint can be 4.5 m .
a. When reinforcement is not provided

$$
\sigma B h=\frac{f \gamma_{c} B h L_{c}}{2} \Rightarrow L_{c}=\frac{2 \sigma}{f \gamma_{c}}
$$

$\mathrm{L}_{\mathrm{c}}=$ Spacing of contraction joint
$\sigma=$ Allowable stress in tension in cement concrete
$\gamma_{\mathrm{c}}=$ Unit weight of cement concrete
$\mathrm{f}=$ friction

## b. When reinforcement is provided

$$
L_{c}=\frac{2 \times \sigma_{s t} \times A_{s}}{B \times h \times \gamma_{c} \times f}
$$

$\sigma_{\text {st }}=$ Allowable tensile stress in steel
$A_{s}=$ Total Area of steel
(iii) Longitudinal Joints:
> Tie bars are provided at longitudinal joint to ensure that slab remains together fermally.
> Tie bars are not designed as load transfer device, load is transferred through aggregate interlocking to the adjacent slab.

$$
A_{s}=\frac{b \times h \times \gamma_{c} \times f}{\sigma_{s t}}
$$

$A_{s}=$ area of steel required per meter length of joint
$\mathrm{b}=$ Distance between the joint and nearest free edge
$\mathrm{h}=$ Thickness of pavement
$\mathrm{f}=$ friction coefficient
$\gamma_{\mathrm{c}}=$ unit wt of concrete
$\sigma_{\mathrm{st}}=$ Allowable working stress in tension for steel

## GATE/ESE

## Civil Engineering

## Irrigation

## Important Formula Notes

## IMPORTANT FORMULAS ON IRRIGATION

## CHAPTER-1 IRRIGATION TECHNIQUES \& QUALITY OF WATER

## I. TIME REQUIREMENT

$$
t=2.3 \times \frac{y}{f} \times \log _{10}\left(\frac{Q}{Q-f A}\right)
$$

Here,
Q = Discharge through supply ditch.
$y=$ depth of water flowing over the border strip.
$f=$ rate of infiltration of soil.
$A=$ Area of the land strip to be irrigated.
$t=$ time required to cover the given area A.

## II. QUALITY OF IRRIGATION WATER

$$
C_{s}=\frac{C . Q}{\left[Q-\left(C_{u}-P_{\text {eff }}\right)\right]}
$$

Where,
$\mathrm{Q}=$ the quantity of water applied
$C_{u}=$ consumptive use of water.
$P_{\text {eff }}=$ useful rainfall
$C_{u}-P_{\text {eff }}=$ Used up irrigation water
$C=$ Concentration of salt in irrigation water.
C. $\mathrm{Q}=$ Total salt applied to the soil with Q amount of irrigation water.
$C_{s}=$ The salinity concentration of the soil solution

## II. PROPORTION OF SODIUM IONS TO OTHER CATIONS (SAR)

$$
\mathrm{SAR}=\frac{\mathrm{Na}^{+}}{\sqrt{\frac{\mathrm{Ca}^{++}+\mathrm{Mg}^{++}}{2}}}
$$

classification of water based on its SAR value is as follows:

| SAR | Type of water |
| :---: | :---: |
| $0-10$ | Low sodium water $\left(\mathrm{S}_{1}\right)$ |
| $10-18$ | Medium sodium water $\left(\mathrm{S}_{2}\right)$ |


| $18-26$ | High sodium water $\left(\mathrm{S}_{3}\right)$ |
| :---: | :---: |
| $>26$ | Very high sodium water $\left(\mathrm{S}_{4}\right)$ |

## CHAPTER 2: SOIL MOISTURE AND PLANT RELATIONSHIP

## I. RELATION BETWEEN DUTY AND DELTA

$$
\begin{aligned}
\Delta & =\frac{8.64 B}{D} m \\
\Delta & =\frac{864 B}{D} \mathrm{~cm}
\end{aligned}
$$

$B$ is in days, and $D$ is in ha/m ${ }^{3}$

| Crop | Delta on the field (cm) |
| :---: | :---: |
| Sugarcane | 120 |
| Rice | 120 |
| Tobacco | 75 |
| Cotton | 50 |
| Wheat | 40 |
| Barley | 30 |
| Maize | 25 |
| Fodder | 22.5 |
| Peas | 15 |

## II. KHARIF-RABI RATIO OR CROP RATIO

It is the ratio (in terms of area) of Kharif season crop to rabi season crop

$$
\text { Crop ratio }=\frac{\text { Area irrigated in } \text { Kharif season }}{\text { Area irrigated in Rabi season }}
$$

## III. TIME FACTOR

The ratio of the actual operating period of a canal to the crop period is called the time factor of the canal.

$$
\text { Time factor }=\frac{\text { The time for which canal actually runs }}{\text { Crop period }}
$$

## IV. CAPACITY FACTOR

The ratio of mean supply discharge in a canal to its design full capacity is known as the capacity factor.

$$
\text { Capacity factor }=\frac{\text { Mean supply discharge }}{\text { Design discharge }}
$$

## V. IRRIGATION EFFICIENCIES

## a. Water Conveyance Efficiency ( $\eta_{c}$ )

This is the ratio of the water delivered into fields from the outlet point of the channel to the water entering into the channel at its starting point.


It also considers the conveyance or transit losses.

## b. Water Application Efficiency ( $\boldsymbol{\eta}_{\mathrm{a}}$ )

It is the ratio of the quantity of water stored in the root zone of the crops to the quantity of water delivered into the field. It may also be called on-farm efficiency, as it considers the water lost in the farm.


$$
\eta_{a}=\frac{\text { water stored in the root zone }}{\text { water delivered to the field }}=\frac{V_{R Z}}{V_{f}} \times 100
$$

Also,

$$
V_{R Z}=V_{f}-\text { Run off losses. }
$$

## c. Water Storage Efficiency ( $\boldsymbol{\eta}_{\mathbf{s}}$ )

It is the ratio of the water stored in the root zone during irrigation to the water needed in the root zone before irrigation.

$$
\eta_{s}=\frac{\text { Water actually stored in the root zone }}{\text { Water needed in the root zone for reaching field capacity }}=\frac{V_{R Z}}{V_{R_{R Z}}} \times 100
$$

$V_{R_{R Z}}=$ volume of water required in the root zone

## d. Water Use Efficiency $\left(\eta_{u}\right)$

It is the ratio of the water beneficially used, including leaching water, to the quantity of water delivered. Mathematically, it can be expressed as follow.

$$
\eta_{u}=\frac{\text { water beneficially used in irrigation }}{\text { Quantity of water delivered to field }}=\frac{V_{u}}{V_{f}} \times 100
$$

here,
$V_{u}=$ volume of water used by the plant
$V_{f}=$ volume of water supplied to the field.

## e. Water Distribution Efficiency ( $\boldsymbol{\eta}_{\mathrm{d}}$ )

The effectiveness of irrigation may also be measured by its water distribution efficiency, which is defined below.

$$
\eta_{d}=\left(1-\frac{d}{D}\right) \times 100
$$

$D=$ Mean depth of water stored during irrigation.
$\mathrm{d}=$ average of the absolute values of deviations from the mean.

## VI. CONSUMPTIVE USE OR EVAPOTRANSPIRATION ( $\mathbf{C u}$ )

Consumptive use for a particular crop can be defined as the total amount of water used in transpiration and evaporation from adjacent soils or plant leaves at any specified time.

$$
C_{u}=\frac{T+E}{B} \mathrm{~mm} / \mathrm{day}
$$

$\mathrm{T}=$ transpiration
E = Evaporation
$B=$ Base period
Values of monthly consumptive use are used to determine the irrigation requirement of the crop.
Frequency of irrigation $=\frac{\text { depth of root zone }}{\text { Consumptive use }}=\frac{d_{r z}}{C_{u}}\left(\frac{m m}{\frac{m m}{d a y}}\right)$

## VII. DETERMINATION OF CONSUMPTIVE USE

## a. Direct Method

1.Tank Lysimeter method
2.Field Experimental Plots
3.Inflow outflow studies: The consumptive use is obtained by studying inflow and outflow in a certain area.

$$
E=I+P-O+G_{s}-G_{e}
$$

Where,
$\mathrm{E}=$ Consumptive use

I = Total inflow
$\mathrm{P}=$ Precipitation in that area
$\mathrm{O}=$ Total outflow
$\mathrm{G}_{\mathrm{s}}=$ Groundwater storage in the starting
$\mathrm{G}_{\mathrm{e}}=$ Groundwater storage at the end of the year

## b. Indirect Method

1.Blaney - Criddle Formula: It states that the monthly consumptive use is given as

$$
C u=\frac{k \cdot p}{40}[1.8 t+32]
$$

$\mathrm{k}=$ crop factor, it is determined by experiment for each crop, under the environmental conditions of the particular area.
$\mathrm{t}=$ Mean monthly temperature is ${ }^{\circ} \mathrm{C}$
$p=$ monthly percent of annual daylight hours during the period.
2. Hargreaves Class Pan Evaporation Method: Evapotranspiration is related to pan evaporation by a constant $k$ called consumptive use coefficient. The formula can be written as

$$
\frac{\text { Evapotranspiration }(C u)}{\text { Pan Evaporation }\left(E_{p}\right)}=k
$$

Epcan be measured experimentally as well as empirically.
3.Penman Equation: This equation is derived by combining the energy balance and mass transfer approach of the computation of evaporation and transpiration, respectively. It is given as,

$$
E_{t}=\frac{A H_{n}+E_{a} \gamma}{A+\gamma}
$$

$\mathrm{E}_{\mathrm{t}}=$ Daily Potential evapotranspiration
A = Slope of the saturation vapour pressure vs temperature curve at the mean air temperature
$H_{n}=$ Net incoming solar radiation expressed in mm of evaporable water per day
$E_{a}=A$ parameter including wind velocity and saturation deficit
$Y=$ psychrometric constant

## VIII. IRRIGATION REQUIREMENTS

a. Effective rainfall ( $\mathbf{P e f f}$ ): It is that portion of natural rainfall that falls during the crop's growth period and is available for the evapotranspiration need of the crop.
b. Consumptive Irrigation Requirement (CIR): It is that part of consumptive use which has to be supplied by the provision of irrigation, and mathematically it is expressed as follows.

$$
\text { CIR }=C_{u}-P_{\text {eff }} .
$$

C. Net Irrigation Requirement (NIR): It takes into consideration the CIR as well as leaching requirement (i.e. to reduce the salinity of soil in the root zone). Mathematically, it is expressed as follow.

$$
\mathrm{NIR}=\mathrm{CIR}+\mathrm{LR}
$$

Where, LR = Leaching requirement of soil
d. Field Irrigation Requirement (FIR): It considers the surface runoff losses occurring over the field and is expressed as follows.

$$
F I R=\frac{N I R}{\eta_{a}}
$$

Here, $\eta_{a}=$ surface runoff losses.
e. Gross Irrigation Requirement (GIR): It considers the conveyance and transmission losses occurring in a canal and is expressed as follows.

$$
G I R=\frac{F I R}{\eta_{c}}
$$

## CHAPTER 3: WATER REQUIREMENTS OF CROP



## I. FIELD CAPACITY

The proportion of water apart from gravity water retains on the surfaces of soil grains by molecular attraction and by loose chemical bonds (i.e. adsorption). This water cannot be easily drained under the action of gravity and is known as field capacity.

$$
\begin{gathered}
F C=\frac{\text { Weight of } \text { water in certain volume of soil }}{\text { Weight of same volume of drysoil }} \times 100 \\
\qquad d_{w_{f}}=\frac{\gamma_{d}}{\gamma_{w}} \times d \times F C
\end{gathered}
$$

$\mathrm{d}_{\mathrm{wf}}=$ depth of water in the soil
$d=$ depth of root zone
$Y_{d}=$ Dry density of soil
$Y_{w}=$ Dry unit weight of soil

## II. PERMANENT WILTING POINT

Water content in the root zone below which plant can no longer extract sufficient water for its growth and wilts up.

$$
\text { Available moisture }=\text { Field Capacity }- \text { Permanent wilting point }
$$

## III. OPTIMUM MOISTURE CONTENT

The optimum level up to which the soil moisture can be allowed to be depleted in the root zone without fall in the crop yield represents the OMC.

## IV. ANALYSIS OF FREQUENCY OF IRRIGATION



Maximum storage capacity or available moisture

$$
=\frac{\gamma_{d}}{\gamma_{w}} \cdot d\left[\begin{array}{c}
\text { Field capacity } \\
\text { moisture content } \\
100
\end{array} \frac{\begin{array}{c}
\text { Wilting poin moisture } \\
\text { content }
\end{array}}{100}\right]
$$

## CHAPTER 4: DESIGN OF LINED \& UNLINED CANALS

## I. CALCULATION OF DESIGN CAPACITY OF A CANAL


( $\mathrm{Q}_{\mathrm{s}}$ )
$\mathrm{Q}_{\mathrm{k}}=$ Discharge required for Kharif season
$Q_{R}=$ Discharge required for Rabi season
Qz = Discharge required for Zaid season
Qs = Discharge required for sugarcane

$$
\text { Design discharge }=Q_{d}=\text { Maximum of }\left\{\begin{array}{l}
Q_{K}+Q_{S} \\
Q_{R}+Q_{S} \\
Q_{Z}+Q_{S}
\end{array}\right.
$$

## II. DESIGN OF ALLUVIAL CANAL

## a. Kennedy Theory

## Design Steps:

Step 1: For a given discharge, assume a trial depth and find the critical velocity as per the following expression.

$$
v_{0}=n C_{1} y^{C 2}
$$

The first depth can be assumed as per the given discharge value and, as suggested by Kennedy.

Step 2: for the given Discharge and an above-calculated $\mathrm{v}_{0}$, calculate the area required as following.

$$
A=\text { Area required }=\frac{\text { Disch arge }}{\text { Velocity }}=\frac{Q}{V_{0}}
$$

Step 3: Find the dimensions of the channel by assuming it to be a trapezoidal channel with the side slope of $\frac{1}{2} \mathrm{H}: 1 \mathrm{~V}$.

Step 4: Calculate the hydraulic radius as following

$$
\text { Hydraulic radius }=R=\frac{\text { Area }}{\text { Parameter }}=\frac{A}{P}
$$

Step 5: Using the above value of R, calculate the actual mean velocity of flow by either using Chezy's equation or manning's equation as following

$$
V=C \sqrt{R . S} \quad \text { (Chezy's formula) }
$$

Here, C = Chezy's constant
$\mathrm{R}=$ hydraulic radius
$S=$ slope of the canal.

$$
\begin{gathered}
V=\frac{1}{n} R^{2 / 3} S^{1 / 2} \text { (Manning's formula) } \\
C=\frac{\frac{1}{n}+23+\frac{0.00155}{S}}{1+\left(23+\frac{0.00155}{S}\right) \frac{n}{\sqrt{R}}} \quad \text { (Kutter's formula) }
\end{gathered}
$$

$\mathrm{n}=$ Kutter's rugosity coefficient.

If the actual mean velocity value calculated above equals the critical velocity of step 1 , the design is okay; else, repeat the above steps for the suitable trial depth.
Following is the table to assume first trial depth depending upon the given discharge value.

| $\mathrm{Q}\left(\mathrm{m}^{3} / \mathrm{sec}\right)$ | $\mathrm{Y}(\mathrm{m})$ |
| :---: | :---: |
| $0-20$ | 1.0 |
| $20-40$ | 2.0 |
| $40-80$ | 2.5 |
| $80-100$ | 3.0 |
| $>100$ | 3.5 |

> If the channel's bed slope value is not given, it can be taken as $s=1$ in 2500 to 1 in 5000 .
> If the value of Manning's coefficient is not given, it can be assumed as per the following range.

| $\mathrm{n}:$ manning's constant | Material |
| :---: | :---: |
| $0.022-0.025$ | Good Earthen channel |
| $0.025-0.030$ | Poor Earthen channel |
| $0.015-0.018$ | Concrete lined channel |

## b. Lacey's Theory

## Design procedure

Step 1:The velocity of flow is calculated as following.

$$
\mathrm{v}=\left(\frac{\mathrm{Qf}^{2}}{140}\right)^{1 / 6} \mathrm{~m} / \mathrm{s}
$$

Here, $f=$ silt factor

$$
f=1.76 \sqrt{d}
$$

$\mathrm{d}=$ average size (diameter) of silt particles in mm .
Step 2:Find the hydraulic radius using the following relation

$$
R=\frac{5}{2} \frac{v^{2}}{f}
$$

Step 3:For the given Discharge, calculate the area required as following

$$
A_{\text {req }}=\frac{Q}{v_{0}}
$$

Step 4:For an assumed cross-section of trapezium of side slope, $\frac{1}{2} \mathrm{H}: 1 \mathrm{~V}$, express the value of area required in forms of $B$ and $y$.


Step 5:Calculating the wetted perimeter for the known Discharge as following:

$$
\mathrm{P}=4.75 \sqrt{\mathrm{Q}}
$$

Now, express the known value of $P$ in terms of $B$ and $y$.
Step 6:Calculate the value of bed slope for known value if Discharge as per the following.
As per Lacey's theory, the scour depth can be calculated for the following cases.

## Case 1: For Wide channel (Regime channel, $\mathbf{P} \simeq B$ ).

Rr: Normal regime scour depth

$$
R_{r}=0.473\left(\frac{Q}{f}\right)^{1 / 3}
$$

Here, Q = Flood discharge.

## Case 2: For normal channel ( $R^{\prime}$ r: Normal scour depth)

$$
P_{r}^{\prime}=1.35\left(\frac{q^{2}}{f}\right)^{2 / 3}, m
$$

$\mathrm{q}=$ discharge/unit width.

## III. DESIGN OF LINED CANAL

## a. Triangular section with round bottom

This type of cross-section is to be used when Discharge through the lined canal is in the range of $\mathrm{Q} \leq 55 \mathrm{~m}^{3} / \mathrm{sec}$.


FB : Free board $\left\{\begin{array}{l}=0.75 \mathrm{~m} \text { for main canal } \\ =0.60 \mathrm{~m} \text { for branck canal }\end{array}\right.$
FSL: Full supply level
$\Rightarrow A_{f}=y^{2}(\theta+\cot \theta)$
$\Rightarrow \mathrm{P}=2 \mathrm{y}(\theta+\cot \theta)$
$\Rightarrow \frac{A}{P}=R \Rightarrow R=\frac{y}{2}$
b. Trapezoidal section with round corner

It is suitable for the canal having the Discharge, $\mathrm{Q}>55 \mathrm{~m}^{3} / \mathrm{sec}$


$$
\text { Area of flow }=A_{f}=B y+y^{2}(\theta+\cot \theta)
$$

| $P=B+2 y(\theta+\cot \theta)$ |  |
| :---: | :---: |
| Lining Material | Permissible $\mathrm{vf}^{\prime}$ |
| Cement concrete | $2.5 \mathrm{~m} / \mathrm{s}$ |
| Tile (Burn clay) | $1.8 \mathrm{~m} / \mathrm{s}$ |
| Stone (Boulder) | $1.5 \mathrm{~m} / \mathrm{s}$ |

## a. Self-weight

The self-weight of the dam is the major retarding Force that acts on the dam. It acts through the centre of gravity of the dam.


Where,
$Y_{c}=$ specific unit weight of concrete

## b. Force due to Water Pressure



Pressure at upstream: $P_{u}=\frac{\gamma_{w} H^{2}}{2}$
Which is acting at a distance of $\mathrm{H} / 3$ from the base of the dam.
Pressure at downstream: $P_{d}=\frac{\gamma_{w} h^{2}}{2}$
Acting at a distance of $\mathrm{h} / 3$ from the base of the dam.
c. Uplift pressure


Uplift Pressure

## d. Earthquake force

Earthquake wave may move in any direction, and for design purposes, it has to be released in vertical and horizontal components. Hence, two acceleration, i.e. one horizontal acceleration ( $\alpha_{h}$ ) and one vertical acceleration ( $\alpha_{v}$ ) are induced by an earthquake. The value of the basic seismic coefficient ( $\alpha_{0}$ ) according to the zone is given below.

| Seismic zones | Value of $\alpha_{0}$ |
| :---: | :---: |
| Zone I | 0.01 |
| Zone II | 0.02 |
| Zone III | 0.04 |
| Zone IV | 0.05 |
| Zone V | 0.08 |

The seismic force can be analysed as shown below:


C: Centre of Earth
F: Focus/Hypo centre of Earthquake
E: Epi-centre

Generally, the value of vertical acceleration is $75 \%$ of the horizontal acceleration.

$$
\alpha_{v}=75 \% \text { of } \alpha_{h}
$$

Where,

$$
\alpha_{h}=k \cdot g
$$

k : seismic co-efficient, which is equal to

$$
\mathrm{k}=\beta \mathrm{I} \alpha_{0}
$$

Where,
$\beta$ : soil foundation system factor (1 for gravity dams)
I: importance factor (3 for gravity dams)

## 1. Effect of $\alpha_{v}$ (Vertical acceleration)


$\alpha_{v}$ can act in an upward direction also, but we are taking for worst condition.

$$
\begin{gathered}
w^{\prime}=w-F_{I_{v}}(\uparrow) \\
\Rightarrow w^{\prime}=\mathrm{w}-\mathrm{M} \cdot \alpha_{v} \\
\Rightarrow w^{\prime}=w-\frac{w}{g} \alpha_{v} \\
\Rightarrow w^{\prime}=w\left(1-\frac{\alpha_{v}}{g}\right)
\end{gathered}
$$

## 2. Effect of $\boldsymbol{\alpha}_{\mathrm{h}}$ (horizontal acceleration)

Development of $F_{I_{H}}$ :


$$
F_{I_{H}}=\frac{w}{g} \times k . g
$$

$$
\Rightarrow \mathrm{F}=\mathrm{kw}(\rightarrow)
$$

Development of hydrodynamic Force:


$$
\mathrm{P}_{\mathrm{H}_{\mathrm{D}}}=0.555 \mathrm{k} \gamma_{\mathrm{w}} \mathrm{H}^{2} \text {, Acting at } \frac{4 \mathrm{H}}{3 \pi}=0.424 \mathrm{H} \text { from base. }
$$

## e. Wave pressure

Waves are generated on the reservoir's surface by the blowing winds, which causes pressure toward the downstream side.
Wave pressure depends upon the wave height. The equation may give wave height.
$h_{w}=0.032 \sqrt{V . F}+0.763-0.271(F)^{3 / 4}$ for $F<32 \mathrm{~km}$.
$h_{w}=0.032 \sqrt{\text { V.F }}$ for $F>32 \mathrm{~km}$
Where,
$h_{w}=$ height of the water from the top of the crest to the bottom of the trough in meters.
$\mathrm{V}=$ wind velocity in $\mathrm{km} / \mathrm{hr}$
$\mathrm{F}=$ fetch or straight length of a water expanse in km .


The maximum pressure intensity due to wave action may be given by $p_{w}=2.4 \gamma_{w} h_{w}$ and acts at $\frac{h_{w}}{2}$ meters above the still water surface.

SWL


SWL = Still water level.
$h_{w}=$ height of the wave.

$F w=2 \gamma_{w} h_{w}^{2} @ 3 h_{w} / s$ above SWL.

## f. Silt Pressure


$\mathrm{p}_{\mathrm{s}}=$ maximum active silt pressure
$\mathrm{k}_{\mathrm{a}}=$ active silt pressure coefficient $=\frac{1-\sin \theta}{1-\sin \theta}=\tan ^{2}(45-\phi / 2)$
$\phi=$ angle of internal friction of soil.
$\gamma_{s}=$ submerged unit weight of silt $=\gamma_{\text {sat }}-\gamma_{w}$
$h_{s}=$ height of silt load

$$
F_{s}=\frac{k_{a} \gamma_{s} h_{s}^{2}}{2} \text {, Acting at } \frac{h_{s}}{3} \text { from base. }
$$

In any absence of any data, silt pressure can be taken as
$F_{\mathrm{s}}=\frac{1}{2} 360 \mathrm{~h}_{\mathrm{s}}^{2}$ in kg.f

## g. Ice pressure

> The ice formed on the reservoir's water surface in cold countries may sometimes melt and expand.
> The dam face then resists the thrust exerted by the expanding ice.
> The magnitude of this Force varies from 250 to $1500 \mathrm{kN} / \mathrm{m}^{2}$ depending upon temperature variation. On average, a value of $500 \mathrm{kN} / \mathrm{m}^{2}$ may be allowed under ordinary conditions.

## h. Wind pressure

> Its value is taken as $1-1.5 \mathrm{kN} / \mathrm{m}^{2}$ of the exposed area


$$
F_{\text {wind }}=1.5 \times \mathrm{d}_{\mathrm{c}} \times 1 \mathrm{kN} @\left(\mathrm{H}+\frac{\mathrm{d}_{\mathrm{c}}}{2}\right) \text { from base. }
$$

## II. MODES OF FAILURE FOR A GRAVITY DAM

## a. Overturning:

> If the resultant of all the forces acting on a dam, any of its sections passes outside the toe, the dam shall rotate and overturn about the toe.
> The ratio of the righting moments about the toe to the overturning moments about the toe is called the factor of safety against overturning.


For no overturning about the toe, $\Sigma M_{R} \geq \Sigma M_{\text {o }}$

$$
\begin{gathered}
\Rightarrow \frac{\Sigma \mathrm{M}_{\mathrm{R}}}{\Sigma \mathrm{M}_{o}} \geq 1 \\
\text { FOS }=\frac{\Sigma M_{R}}{\Sigma M_{o}} \geq 1
\end{gathered}
$$

For design condition,

$$
F O S=\frac{\Sigma M_{R}}{\Sigma M_{o}} \simeq 1.5
$$

## b. Sliding:

> Sliding (or shear failure) will occur when the net horizontal Force above any plane in the dam or at the dam's base exceeds the frictional resistance developed at that level.


For no sliding failure, resisting Force $\geq$ sliding Force

$$
\begin{gathered}
\mu \Sigma \mathrm{V} \geq \Sigma \mathrm{H} \\
\text { FOS }_{\text {sliding }}=\frac{\mu \Sigma \mathrm{V}}{\Sigma H} \geq 1
\end{gathered}
$$

> The shear friction factor is used to check the stability of a dam against sliding when the bond strength of concrete is also considered. It is given by

$$
S F F=\frac{\text { Resisting Force }}{\text { Sliding Force }}=\frac{\mu \Sigma V+q \times B \times 1}{\Sigma H}
$$

$\mathrm{q}=$ bond strength of concrete $\left(\mathrm{kN} / \mathrm{m}^{3}\right)$
> For the same FOS against the sliding mode of failure, the weight of concrete used in the second case will be less thanthe first case, and therefore the second case gives us an economical design.

## c. Crushing/compression mode of failure

> A dam may fail by the failure of its materials, i.e. the compressive stresses produced may exceed the allowable stresses, and the dam material may get crushed.


Considering base area for 1 m length of dam ( $\mathrm{B} \times 1$ )


$$
\left(\sigma_{V_{\max }}\right)_{t o e}=\frac{\Sigma V}{B}\left(1+\frac{6 e}{B}\right)
$$

And, minimum compressive stress will be at the heel of the section, i.e. $x=-B / 2$

$$
\left(\sigma_{V_{\text {min }}}\right)_{\text {Heel }}=\frac{\Sigma V}{B}\left(1-\frac{6 e}{B}\right)
$$

Now, the resultant stress on base width will have the following distribution depending upon the value of $e$.

$\mathrm{e}=\mathrm{eccentricity}$ of the resultant force from the centre of the base.
$\Sigma \mathrm{V}=$ total vertical force
$B=$ base width.
Now, analysing a stress element @ Toe


Taking a small triangular section at the toe, the free body diagram of the stresses will be


For no crusting,
$\sigma_{1} \leq f\left(=\frac{\sigma_{y}}{\text { Fos }}\right)$
$f=$ failure strength of concrete.

$$
\Rightarrow \tau=\left(\sigma_{v}-\sigma_{2}\right) \tan \alpha
$$

## d. Tension/tensile mode of failure

$$
\left(\sigma_{V_{\text {min }}}\right)_{\text {Heel }}=\frac{\Sigma V}{B}\left(1-\frac{6 e}{B}\right)
$$

Case1: When the reservoir is full


Case 2: When the reservoir is empty


The above diagram shows that for no tension failure, resultant force must always pass through the middle third strip of the base width.


## III. ELEMENTARY PROFILE OF GRAVITY DAM

The elementary profile, subjected only to the external water pressure on the upstream side, will be a right-angled triangle with zero width at the water level and a base width $(B)$ at the bottom, i.e. the point where maximum hydrostatic water pressure acts.


Here,
$\mathrm{C}=\mathrm{U} / \mathrm{L}$ uplift pressure coefficient
$\mathrm{C}=1$, if uplift pressure is considered
$\mathrm{C}=0$, if uplift pressure is absent
Case 1: When the reservoir is empty
The eccentricity in this profile $=\frac{B}{2}-\frac{B}{3}=\frac{B}{6}$
In this case, there will be
a) No overturning failure
b) No sliding failure
c) No tension failure

The only mode of failure in an empty reservoir case is crushing.
For no crushing mode of failure,

$$
\begin{aligned}
& \left(\sigma_{V_{\max }}\right)_{t o e}=\frac{\Sigma V}{B}\left(1+\frac{6 e}{B}\right) \\
= & \frac{W}{B}(1+1)=\frac{2 W}{B} \leq f
\end{aligned}
$$



Case 2: When Reservoir is full

## (a) For no tension failure:

For no tension failure:

$$
\begin{gathered}
\mathrm{e} \leq \mathrm{B} / 6 \\
\Rightarrow \mathrm{~B} \geq \frac{\mathrm{H}}{\sqrt{\mathrm{~S}_{\mathrm{c}}-\mathrm{C}}} \\
\therefore \mathrm{~B}_{\min }=\frac{\mathrm{H}}{\sqrt{\mathrm{~S}_{\mathrm{c}}-\mathrm{C}}}
\end{gathered}
$$

The critical width will be corresponding to the case when uplift pressure intensity is zero.

$$
\mathrm{B}_{\text {Critical }}=\frac{\mathrm{H}}{\sqrt{\mathrm{~S}_{\mathrm{c}}}}
$$

Where,
$\mathrm{S}_{\mathrm{c}}=$ specific gravity of concrete
C = uplift pressure intensity factor
$\mathrm{H}=$ height of the dam.

## (b) For no overturning failure:

$$
\begin{gathered}
\because M_{r} \geq M_{o} \\
\Rightarrow B \geq \frac{H}{\sqrt{2\left(S_{c}-C\right)}}
\end{gathered}
$$

$$
\therefore \mathrm{B}_{\text {min }}=\frac{\mathrm{H}}{\sqrt{2\left(\mathrm{~S}_{\mathrm{c}}-\mathrm{C}\right)}}
$$

And,

$$
\mathrm{B}_{\text {Critical }}=\frac{\mathrm{H}}{\sqrt{2 \mathrm{~S}_{\mathrm{c}}}}
$$

## (c) For no sliding failure:

For no sliding failure,

$$
\begin{gathered}
\text { Resultant force } \geq \text { shear force } \\
\qquad B \geq \frac{H}{\mu\left(S_{c}-C\right)} \\
\therefore B_{\min }=\frac{H}{\mu\left(S_{c}-C\right)}
\end{gathered}
$$

When there is no upward force,

$$
\mathrm{B}_{\text {Critical }}=\frac{\mathrm{H}}{\mu \mathrm{~S}_{\mathrm{c}}}
$$

## (d) For no crushing failure:

For no crushing failure

$$
\begin{aligned}
& \Rightarrow H \leq \frac{f}{\gamma_{w}\left(S_{c}-C+1\right)} \\
& \Rightarrow H={\frac{f}{\gamma_{w}\left(S_{c}-C+1\right)_{\max }}}^{\Rightarrow}
\end{aligned}
$$

And, critical height for the dam is

$$
\Rightarrow H_{\text {Critical }}=\frac{f}{\gamma_{w}\left(S_{c}+1\right)}
$$

# CHAPTER 6: SEEPAGE THEORY 

## I. BLIGH'S CREEP THEORY FOR SEEPAGE FLOW


> According to Bligh's theory, the percolating water flows the outline of the base of the foundation of the hydraulic structure.
$>$ The length of the path thus traversed by water is called the length of the creep.
$>$ It is assumed that the loss of the head is proportional to the length of the creep.
$>$ If $\mathrm{H}_{2}$ is the total head loss between the upstream and downstream and L is the creep length, then the head per unit of creep length is called the hydraulic gradient.
> Now, for any point P , an impervious floor

$$
\mathrm{L}=\text { total length of creep. }
$$

$$
\mathrm{L}=2 \mathrm{~d}_{1}+\mathrm{b}_{1}+2 \mathrm{~d}_{2}+\mathrm{b}_{2}+2 \mathrm{~d}_{3}
$$

Let the length of creep $=L_{p}$
Head loss till $P=H_{2 p}=\frac{H}{L} \times L_{p}$.
Residual seepage head @ $P=H_{R}=H-\left(\frac{H}{L}\right) \times L_{p}=h$

## a. Safety against piping or undermining

According to Bligh's theory, safety against piping or undermining following conditions should be satisfied.

$$
\begin{gathered}
\frac{\mathrm{H}}{\mathrm{~L}} \leq \frac{1}{\mathrm{C}} \\
\Rightarrow \mathrm{~L} \geq \mathrm{CH}
\end{gathered}
$$

Here, C = Bligh's creep co-efficient.

| Type of soils | Value of C |
| :---: | :---: |
| Fine micaceous sand | 15 |
| Coarse-grained sand | 12 |
| Sand mixed with boulder and gravel | 5 to 9 |


| Light sound and mud | 8 |
| :---: | :---: |

## b. Safety against uplift pressure

## At any point $\mathbf{P} \rightarrow$



$$
\mathrm{t}_{\min }=\frac{\mathrm{h}}{\mathrm{G}-1} \text { and } \mathrm{t}_{\text {design }}=\frac{4}{3}\left(\frac{\mathrm{~h}}{\mathrm{G}-1}\right)
$$

Here, $\gamma_{\omega}=$ unit weight of water.
$\mathrm{G}=$ specific gravity of the floor material.
By the above expression, the thickness of the floor can be determined. This is generally increased by $33 \%$ to allow a suitable factor of safety.

## II. LANE'S WEIGHTED CREEP THEORY

> According to Lane's weighted creep theory, the weighted creep length is given as per the following expression.

$$
\mathrm{L}_{\omega}=\frac{\mathrm{N}}{3}+\mathrm{v}
$$

Where $\mathrm{N}=$ sum of horizontal creep length as per Bligh $\mathrm{V}=$ sum of vertical creep length as per Bligh.

## a. Safety against piping failure

The following condition is to be satisfied to avoid piping failure.

$$
\begin{gathered}
\quad \frac{\mathrm{L}}{\mathrm{~L}_{\omega}} \leq \frac{1}{\mathrm{C}_{1}} \\
\Rightarrow \quad \mathrm{~L}_{\omega} \geq \mathrm{C}_{1} H
\end{gathered}
$$

Here, $\mathrm{C}_{1}=$ weighted creep coefficient for any soil.

| Types of soil | Value of $\mathbf{G}$ |
| :---: | :---: |
| Very fine sand or silt | 8.5 |
| Fine sand | 7.5 |
| Coarse sand | 5.0 |


| Gravel and sand | 3.5 to 3.0 |
| :---: | :---: |
| Boulder, gravels and sand | 2.5 to 3.0 |
| Clayey soils | 3.0 to 1.6 |

## b. Safety against the uplift pressure

$\mathrm{t}_{\text {min }}=\frac{\mathrm{h}_{\omega}}{\mathrm{G}-1}$ and $\mathrm{t}_{\text {design }}=\frac{4}{3} \cdot \frac{\mathrm{~h}_{\omega}}{\mathrm{G}-1}$
Where $h_{\omega}=H-\frac{H}{L_{\omega}} \times L_{\omega_{p}}$

## III. KHOSLA'S THEORY

> Residual seepage head, as per Khosla's theory, is given by following potential functions

$$
\mathrm{P}=\frac{\mathrm{H}}{\pi} \cos ^{-1}\left(\frac{2 \mathrm{x}}{\mathrm{~b}}\right)
$$

Here,
$\phi=$ residual seepage head potential function, given as, $\phi=\frac{1}{\pi} \cos ^{-1}\left(\frac{2 \mathrm{x}}{\mathrm{b}}\right)$
$H$ = total seepage head,


## Critical Hydraulic Gradient

> As per the observations of Khosla's theory, the exit gradient at the downstream end of the floor is given by the following expression.

$$
\mathrm{G}_{\mathrm{E}}=\frac{\mathrm{H}}{\mathrm{~d}} \times \frac{1}{\pi \sqrt{\lambda}}
$$

There, $\mathrm{H}=$ total seepage head
$\mathrm{d}=$ depth of pile at the downstream end of the impervious floor.

$$
\begin{aligned}
& \lambda=\frac{1+\sqrt{1+\alpha^{2}}}{2} \\
& \text { Where, } \alpha=\frac{b}{d}
\end{aligned}
$$

$\mathrm{b}=$ total horizontal length of the floor.
Critical hydraulic gradient CHG $\left.\right|_{\text {For soil }}=(1-\eta)(G-1)$
Here $\eta=$ porosity
$\mathrm{G}=$ Specific gravity.
For no piping failure,

$$
\mathrm{G}_{\mathrm{E}} \leq \mathrm{CH} \cdot \mathrm{G}
$$

## CHAPTER 7: CROSS DRAINAGE

## I. RIVER TRAINING \& PROTECTION WORKS

> River training implies certain measures adopted on a river to stabilize the river channel along a certain alignment with a certain cross-section.

## II. MORPHOLOGY OF A RIVER

> River/Stream morphology describes the shape of river channels and how they change in shape with direction with respect to time.

## III. THALWEG OF A RIVER

> A thalweg or talweg is the line of lowest elevation within a Valley or watercourse.
IV. GROYNES/SPURS
> Groynes are constructed transverse to the river flow.
> It extends from the bank into the river up to a limit.
Types of Groynes: Groynesare classified based on function
a. Repelling/Reflecting Spur,
b. Deflecting Spur,
c. Attracting Spur
d. T-Shaped (Denehey), Hockey (Or Burma) Type, Kinked Type, Etc.

## V. MEANDERING OF RIVERS

> A meandering type of river flows in consecutive curves of reverse order connected with a short strait called a crossing.


$$
\begin{gathered}
M_{B}=153.42 \sqrt{ } \mathrm{Q} \\
M_{L}=53.61 \sqrt{ } \mathrm{Q} \\
M=8.84 \sqrt{ } \mathrm{Q}
\end{gathered}
$$

Where Q is in $\mathrm{m}^{3} / \mathrm{s}$;
$M_{B}, M_{L}, W$ in meter.

## a. Sinuosity or Tortuosity

> It is the ratio of the Actual length (along the curve) to the Meander Length (along a straight line) between the curve's endpoints of a meandering river.

## b. Effect of Meandering

> The meandering action increases the length of the stream or river and tends to reduce the slope.

## GATE/ESE

Civil Engineering

Open Channel Flow

Important Formula Notes

## IMPORTANT FORMULAS ON OPEN CHANNEL FLOW

## CHAPTER 1: UNIFORM FLOW

## 1. INTRODUCTION:

In case of open channel flow,
i. The pressure is atmospheric,
ii. The flow takes place under the force of gravity
iii. The flow takes place due to the slope of the bed of the channel only.
iv. Hydraulic Gradient Line coincides with the free surface of the water.
v. Froude's number is used for analysis.

## 2. TYPES OF CHANNELS

## (i) Artificial and Natural channel.

(ii) Prismatic and Non-Prismatic Channel: If the shape, cross-section and bed slope of the channel remain constant in the flow direction, then the channel is considered to be prismatic; otherwise channel is non-prismatic.
(iii)Rigid Boundary Channel
(a) Boundaries are non-deformable
(b) Sedimentation and deposition do not take place.
(c) Water has only one degree of freedom, i.e. depth of flow.
(iv)Mobile Boundary Channel
(a) Boundaries are deformable.
(b) Sedimentation and deposition take place.
(c) Water has four degrees of freedom: depth of flow, bed slope, width and layout of the channel.

## 3. CLASSIFICATION OF FLOW IN CHANNELS

The flow in an open channel is classified into the following types.
3.1. Steady and Unsteady Flow: If the flow characteristics such as depth of flow, the velocity of flow, rate of flow at any point in open channel flow do not change with respect to time, the flow is said to be a steady flow,
Mathematically: $\frac{\partial v}{\partial t}=0 ; \frac{\partial Q}{\partial t}=0 ; \frac{\partial y}{\partial t}=0$
Here, $v=$ velocity, $Q=$ rate of flow, $y=$ depth of flow.
If at any point in open channel flow, the velocity of flow, depth of flow or rate of flow changes with respect to time, the flow is said to be unsteady flow.
Mathematically: $\frac{\partial v}{\partial t} \neq 0 ; \frac{\partial Q}{\partial t} \neq 0 ; \frac{\partial y}{\partial t} \neq 0$
3.2. Uniform and Non-Uniform Flow: If for a given channel length, the velocity of flow, depth, the slope of channel and cross-section remain constant, the flow is known as uniform flow.

Otherwise, flow is non-uniform. Mathematically,
$\frac{\partial y}{\partial s}=0, \frac{\partial v}{\partial s}=0$ (Uniform flow)
$\frac{\partial \mathrm{y}}{\partial \mathrm{s}} \neq 0, \frac{\partial \mathrm{v}}{\partial \mathrm{s}} \neq 0 \quad$ (non-uniform flow)
Non-uniform flow in open channels is also called varied flow, which is classified as following.
(i)Rapidly Varied Flow (RVF): It is defined as that flow in which the depth of flow changes abruptly over a smaller length of the channel.
(ii)Gradually Varied Flow (GVF): If the depth of flow in a channel change gradually over a long length of the channel, the flow is said to be a gradually varied flow and denoted by GVF.

### 3.3. Laminar Flow and Turbulent Flow

(a) Laminar: If the Reynolds number ( $\mathrm{Re}_{\mathrm{e}}$ ) is less than 500.
(b) Turbulent: If $\mathrm{R}_{\mathrm{e}}$ is more than 2000.
(c) Transition: If Re lies between 500 to 2000.

### 3.4. Sub Critical, Critical and Super Critical Flow

The Froude number is defined as, $F_{r}=\frac{v}{\sqrt{g D}}$
Here,
$\mathrm{v}=$ mean velocity of flow.
$D=$ hydraulic depth of the channel and is equal to the ratio of wetted area to the top width of the channel $=\frac{A}{T}$, where $T=$ top width of the channel.

The flow is called critical flow if $\mathrm{Fr}_{\mathrm{r}}=1.0$
If $\mathrm{F}_{\mathrm{r}}>1.0$ the flow is called super critical or shooting or rapid flow.

| Type of Flow | Velocity | Depth | Froud No. (Fe) |
| :---: | :---: | :---: | :---: |
| Critical Flow | $\mathrm{v}_{\mathrm{c}}$ | $\mathrm{y}_{\mathrm{c}}$ | 1 |
| Super Critical Flow | $\mathrm{v}>\mathrm{v}_{\mathrm{c}}$ | $\mathrm{y}<\mathrm{y}_{\mathrm{c}}$ | $>1$ |
| Sub Critical Flow | $\mathrm{v}<\mathrm{v}_{\mathrm{c}}$ | $\mathrm{y}>\mathrm{y}_{\mathrm{c}}$ | $<1$ |

Here,
$\mathrm{v}_{\mathrm{c}}=$ critical velocity.
$\mathrm{y}_{\mathrm{c}}=$ critical depth.
3.5. Control Section: A section in which a fixed relationship exists between depth and discharge. For sub-critical flow, downstream section. (i.e. computation of water surface profile must start from downstream location and proceed upstream location). For super-critical flow control section is at the upstream section.

Note: In the case of critical conditions, disturbance wave will not travel at all.

## Pressure distribution.



Small slope
$\mathrm{P}=\gamma \mathrm{h}$

$\mathrm{P}=\gamma \mathrm{h} \operatorname{COS} \theta$

## Momentum equation in open channel flow:

$M_{2}-M_{1}=P_{1}-P_{2}+w \sin \theta-F_{t}$
$-\rho A_{1} V_{1}^{2}+\rho A_{2} V_{2}^{2}=\gamma A_{1} \bar{y}_{1}-\gamma A_{2} \bar{y}_{2}+w \sin \theta-\mathrm{F}_{\mathrm{f}}$

$M_{1}, M_{2}=$ Momentum per unit time
$P_{1}, P_{2}=$ Pressure force
$F_{f}=$ Friction force along the surface of contact between water and channel
$\mathrm{W}=$ Weight of water enclosed between sections.
$\theta=$ Bed slope.
Note: Momentum equation is generally used where the loss of internal energy is high because it deals with external forces only. For e.g. Hydraulic Jump.

Specific force: Sum of pressure force and momentum flux per unit "unit weight" of the fluid at a section.

Specific force $=\frac{P+M}{r}$ It is assumed to be constant if the channel is
horizontal and frictionless.


## 4. UNIFORM FLOW

### 4.1. Geometric Properties of Channel Section

(i) Depth of Flow: It is the vertical distance measured from the bed of the channel to the top surface of the water.
(ii)Depth of Flow Section: It is the normal distance measured from the bed of the channel to the top surface of the water.


Depth of flow = y
Depth of flow section= $D$
$D=y \cos \theta$
At very small $\Theta$ will be nearly equal to $y$, i.e., depth of flow and depth of flow section will be nearly equal
(iii)Wetted Area (A): Assume a rectangular channel of width ' $B$ ' having the depth of water as ' $y^{\prime}$.

Therefore, wetted Area (A) = By

(iv)Wetted Perimeter ( $\mathbf{P}$ ): Assume a rectangular channel of width ' $B$ ' and the depth of water as ' $y$ '.
Therefore, wetted perimeter $=B+2 y$
(v)Top Width (T): It is the width of the top surface of the channel. In the case of the rectangular channel,
$\mathrm{T}=\mathrm{B}$
Where $B$ is the width of the channel.
(vi)Hydraulic Radius (R): It is the ratio of wetted area and the wetted perimeter.
Mathematically, Hydraulic radius $(R)=\frac{A}{P}=\frac{B y}{B+2 y}$
For a very wide rectangular channel, $\mathrm{R}=\mathrm{y}$
(vii)Characteristics Length ( $\mathbf{L}_{\mathbf{c}}$ ): It is the ratio of wetted area and top width of the water surface.
Mathematically,
Characteristic length $\left(L_{c}\right)=\frac{A}{T}=\frac{B y}{B}$
$\Rightarrow \mathrm{L}_{\mathrm{c}}=\mathrm{y}$
(viii)Froude's Number: It is the number that governs the open channel flow.

Mathematically, Froude's number $F_{r}=\frac{v}{\sqrt{g_{\mathrm{c}}}}$

### 4.2. Discharge Through Open Channel by Chezy's Formula

According to Chezy's equation, $\mathrm{V}=\mathrm{C} \sqrt{\mathrm{RS}_{\mathrm{f}}}$
Here,
$\mathrm{V}=$ velocity of flow
C = Chezy's constant
$\mathrm{R}=$ hydraulic radius
$\mathrm{S}_{\mathrm{f}}=$ slope of total energy line (TEL)


Here,
$\mathrm{S}_{\mathrm{w}}=$ slope of water surface
$S_{0}=$ slope of the bed surface.
As, $\mathrm{S}_{\mathrm{o}}=\mathrm{S}_{\mathrm{f}}$
Therefore, $\mathrm{V}=\mathrm{C} \sqrt{R \mathrm{~S}_{\text {。 }}}$
Also, we know that $\mathrm{Q}=\mathrm{AV}$
Thus, $Q=A C \sqrt{R S_{0}}$

## Formulas for Chezy's Coefficient:

(i)Kutter's Formula

$$
C=\frac{23+\frac{0.00155}{S_{o}}+\frac{1}{n}}{1+\left(23+\frac{0.00155}{S_{o}}\right) \frac{n}{\sqrt{R}}}
$$

(ii)Bazin's Formula

$$
C=\frac{157.6}{1.81+\frac{\mathrm{m}}{\sqrt{\mathrm{R}}}}
$$

Here,
$\mathrm{m}=$ Bazin's constant
$\mathrm{R}=$ hydraulic radius.

## (iii)Manning's Formula

We know that, $\mathrm{V}=\mathrm{C} \sqrt{\mathrm{RS} \text { 。 }}$
Also, according to manning's formula, $V=\frac{1}{n} R^{2 / 3} \sqrt{S_{0}}$
So, from both the equations, we get
$C \sqrt{R S_{0}}=\frac{1}{n} R^{2 / 3} \sqrt{S_{0}}$
$\Rightarrow C=\frac{1}{n} R^{1 / 6}$
Here, $\mathrm{n}=$ Manning's coefficient.
4.3. Most Economical Sections of Channels: The wetted perimeter for a given discharge should be minimum.
The most economical section is also called the most efficient section as the discharge passing through a most economical section of channel for a given cross-sectional area, the slope of the bed and, the resistance coefficient is maximum.
(i) Most Economical Rectangular Channel: Assuming a rectangular section, having a width as ' B ' and the water depth is ' y '.


Geometric Parameter of Economical Rectangular Channel:
(i) $\mathrm{T}=\mathrm{B}=2 \mathrm{y}$
(ii) $P=B+2 y=4 y$
(iii) $\mathrm{A}=\mathrm{By}=2 \mathrm{y}^{2}$
(iv) Hydraulic radius $=R=\frac{A}{P}=\frac{y}{2}$
(v) Characteristic length $=L_{c}=\frac{A}{T}=y$
(vi) Froude's number $=\mathrm{F}_{\mathrm{r}}=\frac{\mathrm{v}}{\sqrt{\mathrm{gLc}}}=\frac{\mathrm{Q} / 2 \mathrm{y}^{2}}{\sqrt{\mathrm{gy}}}$

## (ii) Most Economical Trapezoidal Channel



For the most efficient section, various criteria are there,
First criteria: $\frac{B+2 m y}{2}=y \sqrt{1+m^{2}}$
$\frac{T}{2}=$ lenath of one side slone.
Here,
$\mathrm{m}=$ side slope of trapezoidal section
$B=$ bed width of the trapezoid
$y=$ depth of water
$\mathrm{T}=\mathrm{Top}$ width of the trapezoidal section.

Second Criteria: $R=\frac{y}{2}$
Here, $\mathrm{R}=$ hydraulic radius
Third Criteria: $\theta=60^{\circ}$
Fourth Criteria: $B=\frac{2 y}{\sqrt{3}}$

## Geometric Parameter

(i) Top width $=\mathrm{T}=\frac{4 \mathrm{y}}{\sqrt{3}}$
(ii) Area $=(B+m y) y=\sqrt{3} y^{2}$
(iii) Wetted perimeter $=P=2 \sqrt{3} y$
(iv) Hydraulic radius $=\mathrm{R}=\frac{\mathrm{y}}{2}$
(v) Characteristic length $=L_{c}=\frac{4}{T}=\frac{3 y}{4}$
(vi) Froude's number $=\frac{v}{\sqrt{g \mathrm{~L}_{\mathrm{c}}}}=\frac{\mathrm{Q} / \sqrt{3} \mathrm{y}^{2}}{\sqrt{\mathrm{~g} \cdot \frac{3 y}{4}}}$

## (iii) Most Economical Triangular Channel

Assuming the triangular section, with side slope as 'm'


## Geometric Parameters

(i) $\theta=45^{\circ}$
(ii) Top width $=\mathrm{T}=2 \mathrm{my}=2 \mathrm{y}$
(iii) Area $=A=y^{2}$
(iv) Perimeter $=P=2 \sqrt{2} y$
(v) Characteristic length $=L_{c}=\frac{y}{2}$
(vi) Hydraulic Radius $=\mathrm{R}=\frac{\mathrm{Y}}{2 \sqrt{2}}$
(vii) Froude's number $=F_{r}=\frac{Q / y^{2}}{\sqrt{g \times \frac{y}{2}}}$

Note: Out of all the channel sections, a semi-circular section has a minimum perimeter for a given area.

## CHAPTER-2:ENERGY DEPTH RELATIONSHIP

## 1. SPECIFIC ENERGY AND SPECIFIC ENERGY CURVE

(i) Specific energy is the total energy per unit weight above channel bed, i.e. it is the sum of potential energy head and kinetic energy head.


## Datum

(ii) Total energy always decreases in the direction of flow for the channel having some frictional losses, but specific energy can neither increase, decrease or remain constant along the direction of flow.
(iii) Specific Energy Curve is defined as the curve which shows the variation of specific energy with the depth of flow.

The specific energy of a flowing liquid is
$E=y+\frac{v^{2}}{2 g}=E_{p}+E_{k}$
Where,
$E_{p}=$ Potential energy of flow $=y$
$\mathrm{E}_{\mathrm{K}}=$ Kinetic energy of flow $=\frac{\mathrm{v}^{2}}{2 \mathrm{~g}}$
(iv) Considering a rectangular channel in which a steady but non-uniform flow is taking place.

Let,
$\mathrm{Q}=$ discharge through the channel.
$\mathrm{b}=$ width of the channel
$y=$ depth of flow
$\mathrm{q}=$ discharge per unit width.
Then, $\mathrm{q}=\frac{\mathrm{Q}}{\mathrm{B}}$ constant [ Q and b are constant]
Velocity of flow $=v=\frac{Q}{b \times h y}=\frac{q}{y}$

Substituting the value of v, we get,
$E=y+\frac{q^{2}}{2 g y^{2}}=E_{p}+E_{k}$
$\because E=E_{p}+E_{k}$
From the above equation, we get
$E_{p} \alpha y$
$E_{k} \alpha \frac{1}{y^{2}}$
Drawing the graph with the help of the above relation.

here, $\mathrm{y}_{\mathrm{c}}=$ critical depth
$y_{1}, y_{2}=$ Alternate depth at which specific energy is constant.
2. Criteria for Minimum Specific Energy (for a given discharge):

For minimum specific energy, $\frac{d E}{d y}=0$
Which gives, $\frac{Q^{2} T}{g^{3}}=1$
And, $F_{r}^{2}=1$ or $F_{r}=1$
Important: At critical depth, specific energy is minimum; it is to be noted that the condition $\frac{\mathrm{Q}^{2} \mathrm{~T}}{\mathrm{gA}^{3}}=1$ for minimum specific energy is applicable for all channel sections.

## 3. Criteria for Maximum Discharge (for given specific energy):

For discharge to be maximum, $\frac{d Q}{d y}=0$
Which gives, $\frac{Q^{2} T}{g^{3}}=1$

And, $\mathrm{F}_{\mathrm{r}}^{2}=1$ or $\mathrm{F}_{\mathrm{r}}=1$

## Features of Critical Flow:

(i) Specific energy and specific force is minimum for the given discharge
(ii) Discharge is maximum for the given specific energy and specific force.
(iii) Froude's number is equal to unity, Mathematically,
$\mathrm{F}_{\mathrm{r}}=1$ or $\frac{\mathrm{Q}^{2} \mathrm{~T}}{\mathrm{gA}^{3}}=1$

## 4. Critical Depth and Minimum Specific Energy for different sections

(i) Rectangular Section:

For rectangular section
$\Rightarrow y_{c}=\left(\frac{q^{2}}{g}\right)^{1 / 3}$
here, $\mathrm{q}=$ discharge per unit width.
As the discharge increases, $\mathrm{y}_{\mathrm{c}}$ also increases. This is a reason due to which specific energy curves shift upward as discharge increases.
Since, $\mathrm{F}_{\mathrm{r}}=1$
$\Rightarrow \frac{\mathrm{v}_{\mathrm{c}}{ }^{2}}{2 \mathrm{~g}}=\frac{\mathrm{y}_{\mathrm{c}}}{2}$
We know that for critical energy, $\mathrm{E}_{\mathrm{c}}$
$E_{c}=y_{c}+\frac{v_{c}^{2}}{2 g}$
from equations (1) and (2), we get
$E_{c}=y_{c}+\frac{y_{c}}{2} \quad \Rightarrow E_{c}=\frac{3}{2} y_{c}$

## (ii) Triangular Channel:

Assuming a triangular channel section having slope ' $m$ ' and critical depth ' $\mathrm{yc}_{\mathrm{c}}$ '


We know that for the critical section,
$\frac{\mathrm{Q}^{2} \mathrm{~T}}{\mathrm{gA}_{\mathrm{c}}^{3}}=1$

Top width of triangular section $=2 \mathrm{myc}=\mathrm{T}$
Critical area $=A_{c}=\mathrm{myc}^{2}$
putting values in equation (1), we get
We know that, $E_{c}=y_{c}+\frac{v_{c}{ }^{2}}{2 g}$
Putting the value of $\frac{v_{c}{ }^{2}}{2 g}$ in the above equation, we get
$E_{c}=y_{c}+\frac{y_{c}}{4}$
$\Rightarrow \mathrm{E}_{\mathrm{c}}=\frac{5}{4} \mathrm{y}_{\mathrm{c}}$

## 5. CHANNEL TRANSITION

There are two types of transitions possible in the channel section.
(i) Flow through hump (obstruction of small height)
(ii) Width contraction.

Assumptions are as follows.
(i) Channel is rectangular, horizontal and frictionless.
(ii) Flow can be subcritical or supercritical.

### 5.1. Flow Through Hump


a. Subcritical flow: Free surface drops down

b. Supercritical flow: Free surface rises at the hump


### 5.2. Width Contraction


a. Subcritical flow: Free surface falls down

b. Supercritical flow: Free surface rises up


## CHAPTER-3: GRADUALLY VARIED FLOW

1. GRADUALLY VARIED FLOW: A steady non-uniform flow in a prismatic channel with gradual changes in its water surface elevation is called gradually varied flow (GVF). The backwater produced by a dam or weir across a river and the drawdown produced at a sudden drop in a channel are a few typical examples of GVF.
The basic assumptions involved in the analysis of GVF are:
(i) The slope of the channel bottom is small.
(ii) The channel is prismatic, and there is no lateral inflow or outflow from the channel.
(iii) The pressure distribution at any section is assumed to be hydrostatic.
(iv) The head losses in gradually varied flow may be determined using the equations for head losses in uniform flows.
2. Differential Equation for GVF:


The total head at the channel section is given by the following equation:

$$
\begin{aligned}
& H=z+y+\frac{v^{2}}{2 g} \\
& \Rightarrow H=z+y+\frac{Q^{2}}{2 g A^{2}}
\end{aligned}
$$

Where,
$\mathrm{H}=$ elevation of the energy grade line above the datum
$z=$ elevation of the channel bottom
$y=$ flow depth
$v=$ mean flow velocity
Differentiating with respect to $x$, we have

$$
\frac{\mathrm{dH}}{\mathrm{dx}}=\frac{\mathrm{dz}}{\mathrm{dx}}+\frac{\mathrm{dy}}{\mathrm{dx}}+\frac{\mathrm{Q}^{2}}{2 \mathrm{~g}} \frac{\mathrm{~d}}{\mathrm{dx}}\left(\frac{1}{\mathrm{~A}^{2}}\right)
$$

$\Rightarrow \frac{d y}{d x}=\frac{S_{0}-S_{f}}{1-F_{r}^{2}}$
Where, $\frac{d y}{d x}$ is the slope of the water surface relative to the bottom of the channel.

## 3. Classification of Slopes in GVF:

The classification of slopes is made based on two parameters, namely normal depth ( $\mathrm{y}_{\mathrm{n}}$ ) and critical depth ( $\mathrm{y}_{\mathrm{c}}$ ). They are classified into the following five categories:
(i) Mild slope (M): When $y_{n}>y_{c}$. Thus, the flow will be subcritical at normal depth.
(ii) Steep slope (S): When $\mathrm{y}_{\mathrm{n}}<\mathrm{y}_{\mathrm{c}}$. Thus, the flow will be supercritical at normal depth.
(iii)Critical slope (C): When $\mathrm{y}_{\mathrm{n}}=\mathrm{y}_{\mathrm{c}}$.
(iv) Horizontal slope (H): This profile exists for the channels having a horizontal bed slope. When the bottom slope is horizontal, normal depth is infinite.
(v) Adverse slope (A): In this type of profile, the bottom slope is negative. Thus, normal depth for such flow does not exist.
4. Water Surface Profiles in GVF:

The channel is divided into three zones by normal depth and critical depth. For each zone, a different water profile exists. Thus, there will be 12 different types of surface profiles:
(i) three for the mild slope,
(ii) three for the steep slope,
(iii) two for the critical slope (zone 2 does not exist since $y_{n}=y_{c}$ ),
(iv) two for the horizontal slope (zone 1 does not exist since $y_{n}=\infty$ ), and
(v) two for the adverse slope (there is no zone 1 since $y_{n}$ does not exist).

The characteristics of the surface water profile can be summarised in the following table.

| Slope | Profile | Condition | Slope of <br> water <br> surface <br> dy | Type of <br> flow | Important <br> features |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Mild <br> $\left(y_{n}>y_{c}\right)$ | $M_{1}$ | $y>y_{n}>y_{c}$ | Positive | Subcritical | Produced when <br> water is obstructed <br> by weirs, dams etc. |
|  | $M_{2}$ | $y_{n}>y>y_{c}$ | Negative | Subcritical | Produced when a <br> sudden drop in <br> channel bed is <br> observed |


|  | $M_{3}$ | $y_{n}>y_{c}>y$ | Positive | Supercritical | Produced when a <br> supercritical stream <br> enters a mild slope. <br> Generally observed <br> near sluice gates, <br> spillways etc. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $S_{1}$ | $y^{\prime}>y_{c}>y_{n}$ | Positive | Subcritical | Produced when <br> supercritical flow <br> from a steep <br> channel is |
| terminated by deep |  |  |  |  |  |
| pool created by |  |  |  |  |  |
| weirs, dams etc. |  |  |  |  |  |$|$

The above profiles can be diagrammatically represented as:


Horizontal bed $S_{0}=0$


## CHAPTER-4: RAPIDLY VARIED FLOW

## 1. HYDRAULIC JUMP OR STANDING WAVE

In rapidly varied flow, a sudden change in depth occurs at a particular stretch of a channel, and this change takes place over a short reach.
When flow condition changes from supercritical to subcritical, a sudden increase in depth of flow takes place, known as hydraulic jump or a standing wave.
Applications of the hydraulic jump are as an energy dissipator, desalination of seawater, to aerate the water bodies, to reduce uplift pressure.

## 2. Sequent Depth or Conjugate Depth


3. Hydraulic Jump Equation for Horizontal, Frictionless and Rectangular Channel Assumptions
(i) Depth of flow before and after the jump is uniform and pressure distribution is hydrostatic.
(ii) Length of the jump is so small that the effect of frictional force is negligible and hence neglected.
(iii) Channel bed is horizontal, or slope is so gentle that weight component of water mass comprising the jump is very less and hence neglected.

i. $\frac{y_{1}{ }^{2}}{2}+\frac{q^{2}}{g y_{1}}=\frac{y_{2}{ }^{2}}{2}+\frac{q^{2}}{g y_{2}}$
ii. $\frac{2 q^{2}}{g}=y_{1} y_{2}\left(y_{1}+y_{2}\right)$
iii. $\frac{\mathrm{y}_{2}}{\mathrm{y}_{1}}=\frac{1}{2}\left[-1+\sqrt{1+8 \mathrm{~F}_{1}^{2}}\right]$
or
$\frac{\mathrm{y}_{1}}{\mathrm{y}_{2}}=\frac{1}{2}\left[-1+\sqrt{1+8 \mathrm{~F}_{2}^{2}}\right]$
iv. Energy loss $=E_{L}=\frac{\left(y_{2}-y_{1}\right)^{3}}{4 y_{1} \cdot y_{2}}$
v. $\mathrm{F}_{2}^{2}=\frac{8 \mathrm{~F}_{1}^{2}}{\left(\sqrt{1+8 \mathrm{~F}_{1}^{2}}-1\right)^{3}}$
here, $y_{1}$ and $y_{2}$ are sequent depths, and $F_{1}$ and $F_{2}$ are the Froude no before and after the jump, respectively
$F_{1}^{2}=\frac{q^{2}}{\mathrm{gy}_{1}^{3}}$ and $F_{2}^{2}=\frac{q^{2}}{\mathrm{gy}_{2}^{3}}$
$q$ is the discharge per unit width

## 4. Classifications of Hydraulic Jump

Type of Jump
(i) Undular jump
(ii) Weak jump
(iii) Oscillating jump
(iv) Steady jump
(v) Strong jump

## 5. Length of Hydraulic Jump

For the rectangular channel from experiments, it has been found equal to 5 to 7 times the height of the hydraulic jump.
6. Location of Hydraulic Jump

Tailwater depth plays an important role in the formation of a hydraulic jump.
Depth downstream of a controlled structure such as sluice gate etc., control by a downstream channel is known as tailwater depth.
Consider a sluice gate having gate opening 'a', depth at vena contracta as ' $\mathrm{ya}^{\prime}$. Sequent depth corresponding to $y_{a}$ is $y_{2}$ and $y_{t}$ as tailwater depth.


Based on the value of $y_{t}$ and $y_{2}$ there are three possible types of jumps.
(i) When $y_{t}=y_{2} \quad$ free hydraulic jump
(ii) When $\mathrm{y}_{\mathrm{t}}<\mathrm{y}_{2} \quad$ free repelled jump
(iii) When $\mathrm{y}_{\mathrm{t}}>\mathrm{y}_{2}$ submerged/drowned jump

## GATE/ESE

Civil Engineering

Railway \& Airport

Important Formula Notes

## IMPORTANT FORMULAS ON RAILWAY AND AIRPORT



1. DEPTH OF BALLAST
$D_{B}=\frac{S S-W S}{2}$
Here, SS =center to center spacing between sleepers
WS = Width of sleeper
$\mathrm{D}_{\mathrm{B}}=$ Depth of ballast
2. COMPOSITE SLEEPER INDEX

$$
\mathrm{CSI}=\frac{\mathrm{S}+10 \mathrm{H}}{20}
$$

S : strength of timber at $12 \%$ moisture constant.
H : hardness of timber at $12 \%$ moisture content.
3. CONING OF WHEEL:

- The wheels of the train are coned at a slope of 1 in 20.


## - purpose ofconing:

to keep the train in central position, if the train moves outside in any direction the diameter of the wheel on that direction increases and on other rail it decreases so distance travelled by two will become unequal, it directs back the rail in central position so coning of wheels act as automatic handle for the train.

- when train moves on curved tracks the distance of the outer rail is more than inner rail, on curves due to centrifugal force the train is forced to move in outward direction so the diameter of wheel of the outer rail become more than the diameter
of wheel on the inner rail so distance travelled by outer wheel is more than the inner wheel.
- advantage of coning the wheel are:It reduce the wear and tear of the wheel flange and rails which is due to rubbing action of flanges with inside faces of railhead.



## 4. ADZING OF SLIPPERS:

For the effective use of coning of wheels the rails are not laid horizontally, these are laid at a slope of 1 in 20 on the slippers, this is known as adzing of slippers or tilting of rails.
5. SLEEPER SPACING AND DENSITY

- Number of sleepers per rail length is termed as slipper density.
- In India, sleeper density is represented by $(M+x)$ where $M$ is theof rail length in meters and $x$ is a number which varies from 3 to 7 . for example $B G$ track sleeper density is represented by $(M+5)$, length of the rail is 13 meter and so $M+5$ is 18 therefore 18 sleepers will be used per rail length.
- slippers spacing on a particular rail line depends upon, the strength of rail, the type and design of sleeper, depth of the ballast cushion and its type, bearing capacity of formation, and the axle load, volume and speed of traffic.


## 6. GAUGE DISTANCE

i. Broad Gauge(B.G): 1.676 m
ii. Standard Gauge(S.G): 1.435 m (used in USA and UK)
iii. Meter Gauge(M.G.): 1.00 m
iv. Narrow Gauge(N.G.): 0.762 m

## 7. MAXIMUM SPEED ON A RAILWAY TRACK BY MARTIN FORMULA:

- Martin's formula :
a) On a Transition Curve
> For BG/MG
$V_{\text {max }}=4.35 \sqrt{(\mathrm{R}-67)}$
> For NG
$\mathrm{V}_{\max }=3.6 \sqrt{(\mathrm{R}-61)}$, [subjected to a maximum of 50 kmph .]
b) On a Non- Transition Curve
> $\mathrm{V}_{\text {max }}=0.8$ ( $\mathrm{V}_{\text {max }}$ of transition curve)
c) For High-Speed Trains (>100kmph)
> $\mathrm{V}_{\text {max }}=4.58 \sqrt{ }(\mathrm{R})$


## 8. THE RADIUS/DEGREE OF THE CURVE,

Angle subtended at the centre by an arc of 30.5 m .

$$
\begin{aligned}
& D=\frac{360^{\circ}}{2 \pi \times R} \times 30.5=\frac{1750}{R} \\
& \mathrm{R}=\text { Radius of Curve in meter. } \\
& \mathrm{D}=\text { degree of curve. }
\end{aligned}
$$

## 9. SUPER ELEVATION:


> $\mathrm{G}=$ gauge in 'milimeter'. G is the dynamic gauge, which is equal to the sum of the gauge and the width of the railhead in millimetres. It is equal to 1750 mm for BG tracks and 1058 mm for MG tracks.
$>\mathrm{V}=$ velocity in $\mathrm{km} / \mathrm{h}$
> $\mathrm{R}=$ radius in 'meter'
> $\mathrm{e}=$ is in 'milimeter'

- Different trains have different speeds on the railway track. The actual cant is provided for average speed (weighted mean), which is also called equilibrium speed.

$$
\mathrm{e}_{\text {act/eqn } / \mathrm{avg}}=\frac{\mathrm{GV}_{\text {avg }}^{2}}{127 \mathrm{R}}
$$

> Permissible value of actual cant (in mm)

| Gauge | Speed $\mathbf{\leq 1 2 0} \mathbf{~ k m} / \mathbf{h}$ | Speed > 120 km/h |
| :---: | :---: | :---: |
| BG | 165 | 185 |
| MG | 100 | - |
| NG | 76 | - |

## 10. CANT DEFICIENCY

- For a high-speed train, the cant requirement will be more than the actual cant provided, so the train will be forced to move on a lower cant value.
- This deficiency of cant for high-speed train movement is called the cant deficiency. $e_{\text {theoretical }}=e_{\text {actual }}+$ cant deficiency


## > Limits of cant deficiency

| $\mathbf{C m}$ | Speed $\leq \mathbf{1 0 0} \mathbf{~ k m} / \mathbf{h r}$ | Speed $\mathbf{> 1 0 0} \mathbf{~ k m} / \mathbf{h r}$ |
| :---: | :---: | :---: |
| BG | 76 | 100 |
| MG | 51 | - |
| NG | 38 | - |

## 11. CANT EXCESS

- Cant excess on the other hand occurs when train travels on the curve at a speed less than the equilibrium speed.
- It is the difference between the actual cant and the theoretical cant required for such lower speed.
- Limits of cant excess are: broad gauge 75 mm and meter gauge 65 mm
- Note: booked speed of goods trains should be taken into account for working out cantexcess for a particular section.


## 12. NEGATIVE SUPERELEVATION

- If the level of outer rail is below the level of inner rail it is known as negative superelevation, when a main line on a curve has a turn out of opposite curvature leading to a branch line then the level of outer rail in branch line is always kept lower than the level of inner rail this leads to negative superelevation for branchline.
- following steps are to be followed while calculating the speed on mainline, branchline and negative superelevation on branch line:

1. The equilibrium superelevation or Canton branchline is calculated by formula, $e=\frac{G V^{2}}{127 R^{\prime}}$ after assuming a speed on branchline.
2. The permissible can't deficiency is deduced from the equilibrium cant as obtained in step 1.
3. The difference obtained (equilibrium cant- permissible cant deficiency) will give the negative superelevation to be used on the branch line.This negative superelevation is also equal to the maximum superelevation permitted on the main curved track.
4. The restricted speed on curved track is obtained by adding permissible deficiency in maximum cant on the main track.

## 13. GRADE COMPENSATION

> If the gradient is provided on a curved location, the gradient value is reduced to compensate for curve resistance.
> The grade compensation are:

- For BG: 0.04\% per degree of curve.
- For MG: 0.03\% per degree of curve.
- For NG: 0.02\% per degree of curve.


## 14. TRANSITION CURVE

> The ideal shape of the transition curve is one which changes its radius from infinity to that of circular curve at a constant rate of change with distance travelled.

$$
\left\lvert\, \propto \frac{1}{R}\right.
$$

$I=$ length of the curve
$R=$ radius of the curve

> Cubic parabola is used for railways because deflection allowed in tracks are below 9 degree and within this limit cubic parabola traces spiral curve and sighting out of parabola is also easy.
> Cubical parabola equation for transition curve:

$$
y=\frac{x^{3}}{6 R_{c} L}
$$

## > Length of transition curve:

Length of transition curve is maximum of these three:
(i) Based on rate of change of cant deficiency:- $\mathrm{L}=0.0073 \mathrm{C}_{\mathrm{d}} \mathrm{V}_{\mathrm{m}}$.
(ii) Based on rate of actual cant:- $\mathrm{L}=0.0073 \mathrm{C}_{\mathrm{a}} \mathrm{V}_{\mathrm{m}}$.
(iii) Desirable length of cant gradient:- $\mathrm{L}=0.72 \mathrm{Ca}$.
> Here $\mathrm{V}_{\mathrm{m}}$ is in kmph.
> Spiral and Deflection Angles:


$$
\begin{aligned}
\tan \phi_{\mathrm{S}} & =\frac{\mathrm{x}^{2}}{2 R \mathrm{~L}} \text { and } \\
\tan \alpha & =\frac{1}{3} \tan \phi_{\mathrm{S}}=\left(\frac{\mathrm{x}^{2}}{6 R \mathrm{~L}}\right) \\
\phi & =\text { spiral angle }
\end{aligned}
$$

$$
\alpha=\text { deflection angle }
$$

> Shift $\rightarrow$ because of the transition curve. The path shifted w.r.t. circular curve

$$
S=\frac{L^{2}}{24 R}
$$

$\mathrm{L}=$ length of transition curve

## 15. POINT AND CROSSING

Point \& crossing are special arrangement which are used to move trains from one track to another track.
> They are weakest link in the track
> High manganese steel is used to meet these.
> Tounge rail is kept 6 mm higher than the stock rail. Hence vertical wear of tounge rail should not be $>6 \mathrm{~mm}$.
> Crossing is represented in number rather than in angle,



## > Design of turnout

> Curve Lead (CL) :- It is the distance measured along stock rail between toe of switch to TNC (Theoretical Nose of crossing).
> Switch Lead (SL) :- Distance between heel of switch and toe of switch.
> Lead (L):- Distance between heal of switch and TNC, measured along stock rail.
Case I: when curve starts from toe of switch and end at TNC (Full portion is curved).


Case II: Curve is between TNC and Heel of switch $\Rightarrow$


$$
S L=d \cot \beta
$$

$$
\begin{gathered}
\text { In } \Delta A E F \Rightarrow \frac{A E}{E F}=\cot \left(\frac{\alpha+\beta}{2}\right) \\
A E=(G-d) \cot \left(\frac{\alpha+\beta}{2}\right) \\
\text { In } \Delta A E F \\
A F=(G-d) \operatorname{cosec}\left(\frac{\alpha+\beta}{2}\right) \\
\text { Also, } \frac{A F}{2}=R \sin \left(\frac{\alpha-\beta}{2}\right) \\
\Rightarrow R=\frac{(G-d) \operatorname{cosec}\left(\frac{\alpha+\beta}{2}\right)}{2 \sin \left(\frac{\alpha-\beta}{2}\right)} \Rightarrow R=\frac{(G-d)}{(\cos \beta-\cos \alpha)} .
\end{gathered}
$$

Case III: IRS turnout with straight switches and apportion of straight leg crossing

$$
\begin{gathered}
\text { Lead }=(G-d-K \sin \alpha) \cot \left(\frac{\alpha+\beta}{2}\right)+K \cos \alpha \\
R_{0}=\frac{G-d-K \cot \beta}{\cos \beta-\cos \alpha}
\end{gathered}
$$

> Crossing:- device at junction of two rail allowing the movement of wheel flange from one track to other track.

- There is a gap over which wheel trade jump.


Diamond crossing:- when track crosses each other a diamond shape in formed. Called diamond crossing. It consists of 2 acute angle, 2 obtuse angle. Crossing, 4 check rail, 6 point rail, two splice rail.

- Normally diamond crossing is not provided on curves, if provided then there will be no cant in 50 m of crossing and speed will be restricted accordingly.
- No speed restriction is imposed on the straight track on which diamond crossing exist.
- Design of diamond crossing means calculation of diagonal length.


$$
\mathrm{AC}=\mathrm{G} \cot \frac{\alpha}{2} \operatorname{In} \Delta \mathrm{ACE}
$$

$$
\text { In } \triangle A F D, F D=A F \tan \frac{\alpha}{2}
$$

$$
\mathrm{BD}=2 \mathrm{FD}=2 \frac{\mathrm{G}}{2} \operatorname{cosec} \frac{\alpha}{2} \cdot \tan \frac{\alpha}{2}
$$

$$
\mathrm{BD}=\mathrm{G} \sec \frac{\alpha}{2}
$$

Cross over:- when two parallel tracks are connected by 2 sets of turnout, the connecting portion is known as cross over.

- It consist of two pair of switch, two acute angle crossing and 4 check rail.
- Design
(i) when intermediate portion is straight.

$B D=D-G, \quad B D-C B=C D$.
$D-G-\sec a=C D$.
Also, $\mathrm{DE}=\mathrm{CD} \cot \mathrm{a}$,

$$
\begin{gathered}
D E=[D-G-G \sec a] \cot a \\
D E=(D-G) N-G \sqrt{1+N^{2}}
\end{gathered}
$$

Total length of cross over $=C L+S+C L$.

$$
\mathrm{L}=4 \mathrm{GN}+\mathrm{DN}-\mathrm{GN}-\mathrm{G} \sqrt{1+\mathrm{N}^{2}}
$$

(ii) When intermediate portion of track is curved

$$
\begin{aligned}
& \text { Also, } \mathrm{O}_{2} \mathrm{~A}=\left(\mathrm{R}_{1}+\mathrm{R}_{2}-\mathrm{D}\right) \text { and } \mathrm{R}_{1}=\mathrm{R}_{0_{1}}-\frac{\mathrm{G}}{2} \text { and } \mathrm{R}_{2}=\mathrm{R}_{0_{2}}-\frac{\mathrm{G}}{2} \\
& \mathrm{O}, \mathrm{~A}=\sqrt{\left(\mathrm{O}, \mathrm{O}_{2}\right)^{2}-\left(\mathrm{AO}_{2}\right)^{2}}=>\mathrm{O}, \mathrm{~A}=\sqrt{\left(\mathrm{R}_{1}+R_{2}\right)^{2}-\left(R_{1}+R_{2}-\mathrm{D}\right)^{2}} \\
& \mathrm{Also}, \mathrm{R}_{1}=\mathrm{G}+2 \mathrm{C}_{1} \mathrm{~N}_{1}^{2} \text { and } \mathrm{R}_{2}=\mathrm{G}+2 \mathrm{GN}_{2}^{2}
\end{aligned}
$$

## 16. TRACTION AND TRACTIVE EFFORTS

- Tractive effort: This is the pull applied by engine on the driving wheelsfor the movement of train, it is equal to or little in excess ofhauling capacity. If engine apply more power than hauling capacity than driving wheel will slip.
- Hauling capacity: This is the maximum value of friction force that can be obtained between rail and driving wheel.
- $F=\mu * n * W_{d}$, here $\mathrm{n}=$ pair of driving wheel. $\mathrm{W}_{\mathrm{d}}=$ axle load. $\mu=$ coeddicient of friction.
- 4-6-2 Means 3driving axle, 2-6-2 means3 driving axle,2-8-2 means 4driving axle.


## TRACTIVE RESISTANCE

## 1. train resistance.

- Resistance independent of speed (Rolling resistance).

$$
\mathrm{R}_{\mathrm{T} 1}=0.0016 \mathrm{~W}
$$

- Resistance dependent on speed.

$$
\mathrm{R}_{\mathrm{T} 2}=0.00008 \mathrm{WV}
$$

- Atmospheric resistance.

$$
\mathrm{R}_{\mathrm{T} 3}=0.0000006 \mathrm{WV}^{2}
$$

2. resistance to track profile.

- Due to gradient.

$$
\mathrm{R}_{\mathrm{g}}=W \tan \theta=\frac{x}{100}
$$

- Due to curvature

$$
\mathrm{R}_{\mathrm{c}}=0.0004 \mathrm{DW} \text { or } 0.0003 \mathrm{DW} \text { or } 0.0002 \mathrm{DW}
$$

## 3. Resistance due to starting and acceleration.

4. wind Resistance.

Note: for numerical purpose

$$
H C=0.0016 W+0.00008 W V+0.0000006 W V^{2}+W \tan \theta+0.0004 W D
$$

## AIRPORT PLANNING AND DESIGN

## 1. RUNWAY ORIENTATION

> Runway is usually oriented in the direction of prevailing wind, i.e. direction of wind opposite to the direction of landing/takeoff.
> The length of runway requirements will be greater if landing and takeoff operations are performed along the wind direction.
> Wind parameters should be collected for at least 5 years, if available 10 years data is used.
> The normal wind component is called a cross-wind component. The permissible limit is $15 \mathrm{~km} / \mathrm{hr}$ for the smaller plane and for the bigger plane is $25 \mathrm{~km} / \mathrm{hr}$.
> The percentage of time during which the cross-wind component remains within the permissible limit in a year is called wind coverage. It should be $95 \%$ for normal airport and 98-100\% for busy airports.
> Percentage of period for which wind intensity is less than 6.4 kmph is called calm period.

## Note: Important Points related to Wind- Rose Diagrams:

> Wind parameters (direction, duration and intensity) are graphically represented by the wind rose diagrams.
> Type 1: showing direction and duration od wind, circle represents duration and radial line represents direction. The best direction of runway is along the direction of largest line on the wind rose diagram.
> Type 2: showing direction, duration and intensity. Here circle represent wind intensity to some scale, data written in box represent the \% of time wind in that direction and intensity. We take a strip of width equal to crosswind component allowed and place the strip on the graph and runway is oriented along center line of strip. Runway is oriented in a direction where sum of all thr percentage inside the strip is maximum.

## 2. BASIC RUNWAY LENGTH

## Condition under which basic runway length is assumed:

> Airport altitude is at MSL.
$>$ The temperature at the airport is standard $15^{\circ} \mathrm{C}$.
> The runway is levelled in the longitudinal direction.
> The aircraft is loaded through its full loading capacity.
> The speed of wind should be zero on the runway.
> There is no wind blowing enroute to destination
> Enroute temperature is standard.

## Calculation ofbasic runway length:

- Normal landing case : aircraft should come to stop within $60 \%$ of landing distance 1 runway length.



## - Normal take off :



- Engine failure case: If the engine has failed at speed less than designated engine failure speed, the aircraft deaccelerate with use of the runway.
If engine fails after the designated speed, the aircraft has no option but to take off and turn in turning zone and land again for normal take off.


## Corrections:

## I. Correction for elevation

> ICAO recommends that the basic runway length be increased by $7 \%$ per 300 m rise in elevation above MSL.

## II. Correction for temperature

> The standard atmospheric temperature at rest altitude is calculated as

$$
=15^{\circ} \mathrm{C}-[0.0065 \times \text { change in Elevation above MSL }]
$$

$>$ The local body temperature of a particular area is called airport reference temperature $\&$ which is defined for the hottest month of the year.

$$
A R T=T_{a}+\left(\frac{T_{m}-T_{a}}{3}\right)
$$

$\mathrm{T}_{\mathrm{a}} \rightarrow$ monthly mean of avg. Daily temperature.
$\mathrm{T}_{\mathrm{m}} \rightarrow$ monthly mean of max. Daily temperature.
> ICAO recommends that the basic runway length after corrected for elevation should be increased by $1 \%$ for every $1^{\circ} \mathrm{C}$ rise of art above the std. The atmospheric temperature at that elevation (air density).
Note: If the total correction for elevation \& temperature should be less than or equal to $35 \%$ of basic runway length.

## iii. Correction for gradient

> After corrected runway length for elevation \& temperature, runway length should be increased by $20 \%$ for every $1 \%$ of the effective gradient.

## Note:

> In the case of takeoff elevation, temperature \& gradient corrections are necessary.
> In the case of landing, only elevation correction is necessary.

## 3. Turning radius at taxiway

1) $R=\frac{v^{2}}{125 f}$
2) $R=\frac{0.388 w^{2}}{\left(\frac{T}{2}-s\right)}$
3) $R \geq 180 \mathrm{~m}$ for supersonic

$$
\begin{aligned}
\mathrm{R} & \rightarrow \text { in "m" } \\
\mathrm{V} & \rightarrow \text { in } \mathrm{km} / \mathrm{hr} . \\
\Rightarrow \text { maximum } \mathrm{f} & \rightarrow \text { friction weight }=0.13 \\
\mathrm{w} & \rightarrow \text { wheel base in " } \mathrm{m} " \\
\mathrm{~T} & \rightarrow \text { width of Taxiway in " } \mathrm{m} \text { " (22.5) }
\end{aligned}
$$

4) $R>120 \mathrm{~m}$ for Subsonic jets.

Note: strut of main gear must be 6 m from the edge of pavement.

## 4. GEOMETRIC DESIGN OF RAILWAY.

(I) Runway length: It is calculated after knowing basic runway length and applying the elevation, temperature and gradient corrections.
(II) Runway width:It depends on the size of aircraft, it can be 18 to 45 meters. Also. Outside engine should not be damaged by dust of shoulder of runways.
(III) Longitude and effective gradient: This affects the fuel uses and runway length. For A,B,C type runway longitudinal gradient should be less than 1.5\% and effectively gradient should be less than $1 \%$. For D,E type runways longitudinal gradient should be less than $2 \%$ and effective gradient should be less than $2 \%$.
(IV) Rate of change of longitudinal gradient: For smooth movement,0.1\% per 30 meter length for $A, B$ type.
(V) Safety area: It is extended at least 60 m beyond runway end on each side. It's minimum width for $A, B, C$ type is 150 meter and for $D, E$ it is 78 meters.
Note: for instrumental runway, width should not be less than 300 meters. It gives feel of openness to Pilot for safe lending.
(VI) Sight distance: If two runway or runway and taxiway intersect, then sufficient sight distance must be present. Ant two points 3 meters above should be visible from half the runway length.
(VII) Taxiway: Main function of taxiway is to provide access to the aircraft from the runway to the loading apron or Service hanger and back. Both operations should not interfere with each other, i.e. coming and going aircraft should not interfere. Length of taxiway should be least to save fuel.width of taxiway is 7.5 meter to 22.5 meters. It is much less than the runway width,because speed of aircraft is lower. Longitudinal gradients should also be not more than 1.5\%. Otherwise, fuel consumption will be high. Speed of aircraft is less so will be Sight distance,two objects Of3m height should be visible from 300m Apart.
(VIII) Apron: It is the field area for parking of aircraft, for Loading and unloading of passengers and cargo. This area is nearby terminal building or hangars. It should have adequate gradients for drainage.
(IX) Hanger: Hanger is the covered area for repair and servicing the aircraft.size of hangar depends on size of the aircraft and turning radius. Number of hangers depends on the peak hour volume.
(X) Airport parking system:(i) frontal Parking (ii) Open area parking (iii) finger system (iv) Satellite system.
(XI) Gate: it is defined as an aircraft parking space adjacent to the terminal building and used by a single aircraft for loading, unloading of passengers baggage and mails.
(XII) Gate capacity. It is defined as the ability of a specified number of gates to accommodate aircraft loading and unloading operations under condition of continuous demand. It is inverse of the weighted average gate occupancy time for all the aircraft served.
The Gate capacity for a single gate is:

$$
\mathrm{G}_{\mathrm{c}}=\frac{1}{\text { weighted service time }}
$$



## GATE/ESE

Civil Engineering

## Strength of Materials

Important Formula Notes



## IMPORTANT FORMULAS TO REMEMBER

## CHAPTER 1: STRESS AND STRAIN

1. STRESS: When a material is subjected to an external force, a resisting force is set up in the component. The internal resistance force per unit area acting on a material is called the stress at a point. It is a tensor quantity having unit of $\mathrm{N} / \mathrm{m}^{2}$ or Pascal.

Stress $=\frac{\text { Force }}{\text { Area }}$

Units of stress: $\mathrm{N} / \mathrm{m}^{2}$ or Pa.

- $\quad 1 \mathrm{~Pa}=1 \mathrm{~N} / \mathrm{m}^{2}$
- $1 \mathrm{MPa}=10^{6} \mathrm{~Pa}$ and $1 \mathrm{GPa}=10^{9} \mathrm{~Pa}$


$$
\sigma_{0}=\frac{\mathbf{P}}{\mathbf{A}_{0}}
$$

$A_{0}$ is area of section before loading.

$$
\sigma_{\mathrm{A}}=\frac{\mathbf{P}}{\mathbf{A}}
$$

$A$ is area of section after application of load

Normal Stress: Either tensile or compressive in nature.


Its magnitude is given by:
Normal stress $=\frac{\text { Normal force }}{\text { area }} \sigma_{n}=\frac{F_{n}}{A}$


The tensile forces are termed as (+ve) while the compressive forces are termed as negative (-ve).

Shear Stress: Stress produced due toforces which are tangential to the surface and parallel to the area concerned, is called shear stress.


Tangential
Force

$$
\text { Shear stress }=\frac{\tan \text { gential force }}{\text { area }} \sigma_{\mathrm{t}}=\frac{\mathrm{F}_{\mathrm{t}}}{\mathrm{~A}}
$$



$$
\tau=\frac{\text { Shear resistance }}{\text { Shear area }}=\frac{P_{s}}{A}
$$

## - Bulk Stress



$$
\text { Bulk Stress }=\frac{\text { Normal Inward Force }}{\text { Area }} ; \sigma_{B}=\frac{P}{A}
$$

If the normal inward forces from the 3 directions are $P_{1}, P_{2}$ and $P_{3}$ respectively, then Bulk stress will be,

$$
\sigma_{B}=\frac{\frac{\left(P_{1}+P_{2}+P_{3}\right)}{3}}{A}
$$

True stress: It is the applied load divided by the instantaneous cross-sectional area (the changing area with respect to time) of the specimen at that load.

$$
\text { True stress }\left(\sigma_{T}\right)=\frac{\text { Load }(P)}{\text { Instantenous } \operatorname{area}(\mathrm{A})}
$$

## Cartesian co-ordinate system:



## Stress tensor is given by:

$$
\sigma=\left[\begin{array}{lll}
\sigma_{x x} & \tau_{x y} & \tau_{x z} \\
\tau_{y x} & \sigma_{y y} & \tau_{y z} \\
\tau_{z x} & \tau_{z y} & \sigma_{z z}
\end{array}\right]
$$

$\sigma_{x x}, \sigma_{y y}$ and $\sigma_{z z}$ are the tensile stresses on the $x, y$ and $z$ faces while various shear stress component in a 3-dimensional body is as follows $\tau_{x y}, \tau_{y x}, \tau_{y z}, \tau_{z y}, \tau_{z x}, \tau_{x z}$. Here, first subscript indicates the direction of the normal to the surface while second subscript indicates the direction of the stress.
For conditions of equilibrium:

$$
\tau_{x y}=\tau_{y x}, \tau_{y z}=\tau_{z y}, \tau_{z x}=\tau_{x y}
$$

Strain:The ratio of change in dimension of the body to the original dimension of body is known as strain. It is a dimensionless entity.

Strain may be of following types:


Normal strain: strain $(\mathrm{e})=\frac{\text { change in length }}{\text { Original lenth }}=\frac{\Delta \mathrm{L}}{\mathrm{L}}$
True strain: It equals to the natural log of the quotient of instantaneous length over the original length.

$$
\varepsilon_{\mathrm{T}}=\ln \left(\frac{\mathrm{L}}{\mathrm{~L}_{\mathrm{i}}}\right)=\ln (1+\mathrm{e})
$$

## - Shear strain:



$$
\text { Shear Strain }=\frac{\text { tangential dislacement }}{\text { original normallength }}
$$

$$
\text { Shear strain: } \gamma=\frac{\tau}{\mathrm{G}}
$$

Where $\gamma$ is the shear strain and G is the modulus of Rigidity.

## Lateral strain:



- Bulk Strain or Volumetric Strain


Normal inward forces compress the solid
Bulk Strain $=\frac{-(\text { change in volume })}{\text { originalvolume }}$

$$
\varepsilon_{B}=\frac{-\delta V}{V}
$$

## True Stress and True Strain:

- The true stress is defined as the ratio of the load to the cross-section area at the instant of loading.

$$
\left(\sigma_{T}\right)=\frac{\text { load }}{\text { Instantaneous area }}=\sigma(1+\varepsilon)
$$

Where $\sigma$ and $\varepsilon$ is the engineering stress and engineering strain respectively.

- The true strain is defined as

$$
\left(\varepsilon_{T}\right)=\int_{L_{0}}^{L} \frac{d l}{l}=\ln \left(\frac{L}{L_{0}}\right)=\ln (1+\varepsilon)=\ln \left(\frac{A_{0}}{A}\right)=2 \ln \left(\frac{d_{0}}{d}\right)
$$

Lo- original length, L-successive values of the length as it changes

- The volume of the specimen is assumed to be constant during plastic deformation.
2.Poisson's ratio: Within Elastic limit, Poisson's Ratio is defined as,

$$
\mu=-\frac{\text { Lateral strain }}{\text { Longitudinal strain }}
$$

| Material | $\boldsymbol{\mu}$ |
| :--- | :--- |
| Cork | Zero |
| Concrete | 0.1 to 0.2 |
| Metals | $\frac{1}{4}$ to $\frac{1}{3}$ |
| Rubber, Clay, Paraffin | $0.5 \rightarrow$ Behaves like perfect plastic material |

## 3.Three-Dimensional Stress System:



Total strain in the direction of $x$ due to stresses $\sigma_{1}, \sigma_{2}$ and $\sigma_{3}$ is:

$$
e_{1}=\frac{\sigma_{1}}{E}-\mu\left(\frac{\sigma_{2}}{E}+\frac{\sigma_{3}}{E}\right)
$$

Similarly, total strain in the direction of $y$ due to stresses $\sigma_{1}, \sigma_{2}$ and $\sigma_{3}$ is:

$$
e_{2}=\frac{\sigma_{2}}{E}-\mu\left(\frac{\sigma_{1}}{E}+\frac{\sigma_{3}}{E}\right)
$$

and total strain in the direction of $z$ due to stress $\sigma_{1}, \sigma_{2}$ and $\sigma_{3}$ is:

$$
\mathrm{e}_{3}=\frac{\sigma_{3}}{\mathrm{E}}-\mu\left(\frac{\sigma_{1}}{\mathrm{E}}+\frac{\sigma_{2}}{\mathrm{E}}\right)
$$

## Relationship between stress and strain:

Consider a two-dimensional figure ABCD, subjected to two mutually perpendicular stresses $\sigma_{1}$ and $\sigma_{2}$.


Let, $\mathrm{e}_{1}=$ Total strain in x -direction and $\mathrm{e}_{2}=$ Total strain in y -direction.

$$
\begin{aligned}
& \text { Thus: } \mathrm{e}_{1}=\frac{\sigma_{1}}{\mathrm{E}}-\mu \frac{\sigma_{2}}{\mathrm{E}} \\
& \text { and } \mathrm{e}_{2}=\frac{\sigma_{2}}{\mathrm{E}}-\mu \frac{\sigma_{1}}{\mathrm{E}}
\end{aligned}
$$

4.Stress-strain curve: The stress strain curve for mild steel is shown in figure.


A $=$ Proportional Limit
Oa $=$ Linear Deformation
B = Elastic Limit
Ob = Elastic Deformation
C = Yield Point
bd = Perfect Plastic Yielding
$\mathrm{C}^{\prime}=$ Lower Yield Point
de $=$ Strain hardening
E = Ultimate Strength
ef $=$ Necking
F = Rupture Strength/ Fracture strength
The slope of stress-strain curve is called the young's modulus of elasticity (E).

Slope of stress-strain curve: $E=\frac{\sigma}{\varepsilon}$
i.e., $\sigma=\varepsilon$.E

This equation is known as Hooke's law. Thus, the modulus of elasticity (E) is the constant of proportionality which is defined as the intensity of stress that causes unit strain.
Stress strain curve for different materials:


Approximate stress-strain curves for certain materials:


Fig. Linear-elastic curve


Fig. Elasto-plastic with strain hardening


Fig. Perfectly-plastic


Fig. non-linear curve


Fig. Elasto-plastic or visco-plastic

Fig. Perfectly Rigid (Ideal rigid))

## 5.Properties of Metals:

Ductility: It is that properly of material due to which a metal piece can be drawn into wires of thin section under tensioning effect.

Brittleness: It is the lack of the ductility. Such metal doesn't show necking before fracture.
Malleability: It is that property of metal due to which a metal can be drawn into a thin sheet of negligible section by pressing/forging through the compression process.
Proof stress: A proof stress is a level of stress at which a material undergoes plastic deformation. It is often defined as the point when the material undergoes an amount of plastic deformation equal to 0.2 percent.


Resilience: It is defined as the maximum energy that can be absorbed up to the elastic limit, without creating a permanent distortion.

Modulus of Resilience: The modulus of resilience is defined as the maximum energy that can be absorbed per unit volume without creating a permanent distortion.


Toughness:It is defined as the amount of energy per unit volume that a material can absorb before rupturing.

Modulus of toughness: The modulus of toughness is the amount of strain energy per unit volume (i.e., strain energy density) that a material can absorb just before it fractures.


Hardness: Hardness is defined as the resistance of a material to local plastic deformation achieved from indentation of a predetermined geometry indenter onto a flat surface of metal under a predetermined load.
Creep: Creep (Cold Flow) is the tendency of a solid material to move slowly or deform permanently under the influence of persistent mechanical stresses.
Fatigue: Fatigue strength is the highest stress that a material can withstand for a given number of cycles without breaking.
Endurance limit: An endurance or fatigue limit which is defined as the maximum stress below which the steel could presumably endure an infinite number of cycles without failure.


Elasticity: It is that property of metals due to which original dimensions will be recovered offer loading. Within elastic limits the stress-strain curve may be linear or non-linear.


Bulk Modulus: When a body is subjected to three mutually perpendicular like and equal direct stresses, then the ratio of direct stress to the volumetric strain is termed as bulk modulus.

$$
\mathrm{K}=\frac{\text { Bulk stress }}{\text { Bulk strain }}=-\frac{\Delta \mathrm{P}}{\Delta \mathrm{~V} / \mathrm{V}}
$$

## 6.Inter relationship of Elastic Constants:

$$
\begin{gathered}
E=3 \mathrm{~K}(1-2 \mu) \\
E=2 G(1+\mu) \\
E=\frac{9 K G}{3 K+G} \\
\mu=\frac{3 K-2 G}{6 K+2 G}=\frac{1}{m}
\end{gathered}
$$

Here, $\mu=$ Poisson's ratio and $m=1 / \mu$.

|  | $\mu$ | $G$ | $K$ |
| :--- | :---: | :---: | :---: |
| Min limit | 0 | $\frac{\mathrm{E}}{2}$ | $\frac{\mathrm{E}}{3}$ |
| Max limit | $\frac{1}{2}$ | $\frac{\mathrm{E}}{3}$ | $\infty$ |

For metals:

| $\mu$ | G | K |
| :---: | :---: | :---: |
| $\frac{1}{4}$ | 0.4 E | 0.67 E |
| $\frac{1}{3}$ | 0.375 E | E |

## 7.Materials based on elastic properties:

Homogeneous Material: When a material exhibits same elastic properties at any point in a given directions than the material is known as homogenous material i.e., elastic properties are independent of location.


Isotropic Material: When a material exhibits same elastic properties at any direction at a given point than the material is known as Isotropic Material i.e., elastic properties of material are independent of direction.


Homogenous \& isotropic material: When a material exhibits Same elastic properties at any direction at a every point than the material is known as homogeneous Isotropic Material.


Anisotropic Material: When a material exhibits different elastic properties at every direction at a every point than the material is known as Isotropic Material i.e., they exhibit direction dependent elastic property.


Orthotropic Material: When a material exhibits Same elastic properties at only orthogonal direction at a given point than the material is known as OrthotropicMaterial.

| Material | No. of independent elastic constants |
| :--- | :--- |
| Isotropic | 2 |
| Orthotropic | 9 |
| Anisotropic | 21 |

## 8.Analysis of stresses in different bars

(a). Elongation in Bars of Varying Sections:


Total change in the length of the bar:

$$
d \mathrm{~L}=\mathrm{dL}_{1}+\mathrm{dL}_{2}+\mathrm{dL}_{3}=\frac{\mathrm{PL}_{1}}{\mathrm{~A}_{1} \mathrm{E}}+\frac{\mathrm{PL}_{2}}{\mathrm{~A}_{2} \mathrm{E}}+\frac{\mathrm{PL}_{3}}{\mathrm{~A}_{3} \mathrm{E}}=\frac{\mathrm{P}}{\mathrm{E}}\left[\frac{\mathrm{~L}_{1}}{\mathrm{~A}_{1}}+\frac{\mathrm{L}_{2}}{\mathrm{~A}_{2}}+\frac{\mathrm{L}_{3}}{\mathrm{~A}_{3}}\right]
$$

(b). Elongation of Tapering Circular Rod: A bar uniformly tapering from a diameter $\mathrm{D}_{1}$ at one end to a diameter $D_{2}$ at the other and $\mathrm{L}=$ Total length of the barand $\mathrm{E}=$ Young's modulus.


$$
\Delta \mathrm{L}=\frac{4 \mathrm{PL}}{\pi \mathrm{ED}_{1} \mathrm{D}_{2}}
$$

(c). Elongation of Tapering Rectangular Bar

$\therefore$ Total extension: $\Delta=\frac{\mathrm{PL}}{\left(\mathrm{b}_{2}-\mathrm{b}_{1}\right) \mathrm{tE}} \log _{\mathrm{e}} \frac{\mathrm{b}_{2}}{\mathrm{~b}_{1}}$
(d). Elongation of uniform section bar due to self-weight:


## 9.Analysis of Compound bars:



For the composite bar the following two points are important:

- The extension or compression in each bar is equal. Hence deformation per unit length i.e., strain in each bar is equal.
- The total external load on the composite bar is equal to the sum of the loads carried by each different material

$$
P_{1}=\frac{P}{1+\frac{A_{2} E_{2}}{A_{1} E_{1}}}
$$

and

> Rod)

$$
\mathrm{P}_{2}=\frac{\mathrm{P}}{1+\frac{\mathrm{A}_{1} \mathrm{E}_{1}}{\mathrm{~A}_{2} \mathrm{E}_{2}}}
$$

( $\mathrm{P}_{2}$ - Load resisted by

Tube)

## Equivalent modulus of a compound bar:



Equivalent modulus: $E=\frac{A_{1} E_{1}+A_{2} E_{2}}{A_{1}+A_{2}}$
10.Thermal Stresses: Thermal stresses are due to the change in temperature of the component and the component is mechanically restrained to expand or contract.

Free expansion of uniform bars:


Coefficient of Thermal Expansion (or contraction): The coefficient ( $\alpha$ ) is a property of the material and has a unit reciprocal of temperature change.

Common values of $\alpha$ are:

| Material | Coefficient of Thermal Expansion |
| :--- | :--- |
| Steel | $10 \times 10^{-6}$ to $18 \times 10^{-6} /{ }^{\circ} \mathrm{C}$ |
| Copper | $17 \times 10^{-6} /{ }^{\circ} \mathrm{C}$ |
| Aluminium and Aluminium Alloys | $23 \times 10^{-6} /{ }^{\circ} \mathrm{C}$ |

## Thermal stresses:

(a). When bar is constrained (supports unyielding):

$$
\sigma_{\mathrm{Th}}=\frac{\Delta_{\mathrm{Th}} \mathrm{E}}{\mathrm{~L}}=\mathrm{L} \alpha \mathrm{~T} \cdot \frac{\mathrm{E}}{\mathrm{~L}}=\mathrm{E} \alpha \mathrm{~T} \text { and }(\Delta \mathrm{L})_{\text {Total }}=0
$$

(b). When Supports yield: If the support yields by an amount ' $a$ '. In this case, the total amount of expansion checked will be $\left(\Delta_{\mathrm{t}}-\mathrm{a}\right)$. Hence the resulting temperature stress in:

$$
\sigma_{\mathrm{Th}}=\left(\Delta_{\mathrm{Th}}-\mathrm{a}\right) \frac{\mathrm{E}}{\mathrm{~L}}=(\mathrm{L} \alpha \mathrm{~T}-\mathrm{a}) \frac{\mathrm{E}}{\mathrm{~L}}
$$

## Prismatic bar with spring:



Spring force: $\mathrm{R}_{\mathrm{s}}=\frac{\alpha \text { TEA }}{1+\frac{\mathrm{K}_{\text {bar }}}{\mathrm{K}_{\text {spring }}}}$

Temperature stresses in bars of tapering section:


Thermal stresses in compounds and composite bars:
(a). Thermal stresses in compound bars (Bars in Series):


Sum of thermal deformations $=$ Sum of axial deformations

$$
\left(\alpha_{1} L_{1} T+T \alpha_{2} L_{2}\right)=\frac{\sigma_{1} L_{1}}{E_{1}}+\frac{\sigma_{2} L_{2}}{E}
$$

(b). Temperature stresses in composite bar (parallel bars):


Sum of axial deformations $=$ difference in thermal deformations.

$$
\left(\Delta_{\mathrm{s}}\right)_{\text {axial }}+\left(\Delta_{\mathrm{c}}\right)_{\text {axial }}=\operatorname{LT}\left(\alpha_{c}-\alpha_{s}\right)
$$

St. Venant Principle: As per this principle, stress in the immediate vicinity of load application is not uniform over the cross section. It is the only after a length greater than the width of the section that the stress will be uniform.


## CHAPTER 2: SHEAR FORCE AND BENDING MOMENT

1.Introduction: The algebraic sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force. The algebraic sum of the moments of all the forces acting to the right of left of the section is known as bending moment.

A shear force diagram is one which shows the variation of the shear force along the length of the beam. A bending moment diagram is one which shows the variation of the bending moment along the length of the beam.

## 2.Types of supports:

## (1). Simple Supports

## (a). Roller Support:

Number of restricted motions by support = Number of reactions at any support Hence, number of rection in any roller support is 1 .


## (b). Hinge Support (or) Pin Support:

Number of restricted motions support = Number of reactions at any support Hence, number of rection is any hinged support is 2.


$$
\begin{gathered}
\mathrm{R}=\sqrt{\left(\mathrm{R}_{\mathrm{AH}}\right)^{2}+\left(\mathrm{R}_{\mathrm{AV}}\right)^{2}} \\
\tan \theta=\frac{\mathrm{R}_{\mathrm{AV}}}{\mathrm{R}_{\mathrm{AH}}}
\end{gathered}
$$

## Fixed Supports:

Clamped Supports (or) Built-in Supports:


Axial Load at any support $= \pm R_{H}$ at that point
Shear force at any support $= \pm R_{V}$ at that point
Moment reaction at any support $= \pm$ Moment reaction at that support

## 3.Sign Convention for Shear Force and Bending Moment:

Shear force: If moving from left to right, then take all upward forces as positive and downward as negative.

| Left to right | Right to left |
| :---: | :---: |
| $\uparrow(+\mathrm{ve})$ | $\downarrow(+\mathrm{ve})$ |
| $\downarrow(-\mathrm{ve})$ | $\uparrow(-\mathrm{ve})$ |

Bending moment: If moving from left to right, take clockwise moment as positive and anticlockwise as negative.

| Left to right | Right to left |
| :--- | :--- |
| $\Sigma_{(+\mathrm{ve})}$ | $\mathbf{\Sigma}_{(+\mathrm{ve})}$ |
| $(-\mathrm{ve})$ | $)_{(-\mathrm{ve})}$ |

## Points of Concern for SFD and BMD (Critical Points):

(i). Starting and end points of beam.
(ii). Point where concentrated point load or concentrated moment is acting.
(iii). Starting and end point of distributed load (UDL or UVL).
(iv). Point where SFD changes sign.

## Relationship between SF, BM and load:

(i). Slope of SFD=intensity of distributed load.

$$
\frac{\mathrm{dV}}{\mathrm{dx}}=\mathrm{W}_{\mathrm{x}}
$$

(ii). Slope of BMD $=$ Shear force at that section.

$$
\frac{d M}{d x}=V_{X}
$$

| Loading | Shape of SFD | Shape of BMD |
| :---: | :---: | :---: |
| No load | Straight line | Inclined straight line |
| UDL | Inclined straight line | $2^{\circ}$ Curve |
| UVL | $2^{\circ}$ curve | $3^{\circ}$ curve |


| Loading | Shear Force diagram $\frac{d V}{d x}=W$ | Bending Moment diagram, $\frac{d M}{d x}=V$ |
| :---: | :---: | :---: |
|  |  |  |
| - Loading is negative and constant $\Rightarrow \mathrm{SF}$ slope is negative and constant <br> - SF is positive decreasing $\Rightarrow$ Bending moment slope is decreasing <br> - If load intensity is UDL $\Rightarrow$ SFD is learner $\rightarrow$ BMD is parabolic <br> - Slope of BMD at any section is equal to SFD ordinate at that section <br> - Slope of SFD at any section is equal to load intensity at that section <br> - If load intensity is $n$-degree curve, SFD will be $(n+1)$ degree curve and BMD will be $(n+2)$ degree curve. |  |  |



- Loading is negative and increasing $\Rightarrow$ SFD slope is negative and increasing.
- SF is positive and decreasing $\Rightarrow$ BMD slope is positive and decreasing.
- If load intensity is UVL (uniformly varied load) SFD is parabolic and BMD is cubic.

- $\quad$ SF is positive and decreasing $\Rightarrow$ BMD slope is positive and decreasing
- Loading is negative and decreasing $\Rightarrow$ SFD slope is positive and decreasing


## CHAPTER 3: BENDING STRESS

1.Bending Theory: The following are the important assumptions in the theory of bending:

1. The material of the beam is homogeneous and isotropic.
2. The value of Young's modulus of elasticity is the same in tension and compression.
3. The transverse sections which were plane before bending, remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bending into circular arcs with a common centre of curvature.
5. The radius of curvature is large compared with the dimensions of the cross-section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

$$
\frac{\mathrm{M}}{\mathrm{I}}=\frac{\sigma}{\mathrm{R}}=\frac{\mathrm{E}}{\mathrm{R}}
$$

This equation is known asBending Equation.

## 2. Section Modulus:

Section modulus is defined as the ratio of moment inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It is denoted by the symbol $Z$.

$$
\mathrm{Z}=\frac{\mathrm{I}}{\mathrm{Y}_{\max }}
$$

I = M.O.I. about neutral axis
$y_{\max }=$ Distance of the outermost layer from the neutral axis.

$$
\mathrm{M}=\sigma_{\max } \cdot \mathrm{Z}
$$

In the above equation, $M$ is the maximum bending moment (or moment of resistance offered by the section). Hence moment of resistance offered the section is maximum when section modulus $Z$ is maximum. Hence section modulus represents the strength of the section.

## Section Modulus for Various Shapes or Beam Sections:

## Rectangular Section:

Moment of inertia of a rectangular section about an axis passing through its C.G. (or through N.A. is given by).

$$
\mathrm{I}=\frac{\mathrm{bd}^{3}}{12}
$$

Distance of outermost layer from N.A. is given by,

$$
y_{\max }=\frac{\mathrm{d}}{2}
$$


$\therefore$ Section modulus is given by:

$$
Z=\frac{I}{Y_{\max }}=\frac{b d^{3}}{12 \times(d / 2)}=\frac{b d^{2}}{6}
$$

Hollow Rectangular Section: $\quad Z=\frac{1}{6 D}\left[B D^{3}-\mathrm{bd}^{3}\right]$

## Circular Section:

$$
Z=\frac{I}{Y_{\max }}=\frac{\frac{\pi}{64} d^{4}}{\left(\frac{d}{2}\right)}=\frac{\pi}{32} d^{3}
$$

## Hollow Circular Section:

$$
Z=\frac{\pi}{32 D}\left[D^{4}-d^{4}\right]
$$

Strength of a section: The strength of a section means the moment of resistance offered by the section and moment of resistance is given by:

Maximum stress induced $\leq$ permissible Stress.

$$
\begin{gathered}
\frac{M}{Z_{\text {N.A }}} \leq \sigma_{\text {per }} \\
M \leq\left\{\left(\sigma_{\text {per }} \cdot Z_{\text {N.A }}\right)=M_{R}\right\} \\
M_{R}=\sigma_{\text {per }} \cdot Z_{\text {N.A }}
\end{gathered}
$$

For unit radius of bend:

$$
\mathrm{EI}_{\mathrm{N.A}} \uparrow=\mathrm{M}_{\mathrm{R}}(\uparrow)=(\theta \& \delta) \downarrow=\text { chances of beams failures }(\downarrow)
$$

Section Modulus should be considered in design of beams based on strength criterion whereas flexural rigidity is considered in design of beam based on rigidity criterion.

For a given cross-section area:

$$
{ }^{(\mathrm{z})}{ }^{<(\mathrm{z})}{ }^{<(\mathrm{z})}{ }^{<(\mathrm{z})}
$$

For a given cross-section area and given $\mathrm{BM}:\left(\sigma_{\mathrm{b}}\right)_{\max } \propto \frac{1}{\mathrm{Z}_{\text {N.A }}}$


For a given cross-section area of given material: $M_{R} \propto Z_{\text {N.A. }}$


Greater the value of section modulus, stronger will be the section.

## Ratio of depth of width of the strongest beam that can be cut from a circular log:


3. Beam of uniform strength: It is the beam ofuniform moment of resistance.

| Constant width |  | Constant depth |  |
| :---: | :---: | :---: | :---: |
| Simply supported $d_{x}=\sqrt{\frac{3 W}{f b}} \sqrt{x}=c \sqrt{x}$ | (a) LONGITUDINAL SECTION $\square$ <br> $\overline{4}$ $b$ $\downarrow$ $\downarrow$ <br> (B) PLAN | Simple Supported $\mathrm{b}_{\mathrm{x}}=\frac{3 \mathrm{~W}}{\sigma \mathrm{~d}^{2}} \cdot \mathrm{x}=\mathrm{c}^{\prime} \mathrm{x}$ |  |
| Cantilever beam $M=$ constant $\left[\left(\sigma_{b}\right)_{\max }\right]_{x-x}= \pm \frac{6 M}{b d^{2}}$ <br> M=Varying $d_{x}=d \sqrt{\frac{x}{L}}$ |  | Cantilever beam $b_{x}=b \frac{x}{L}$ |  |

4.Flitched beams (Beams of composite sections): Beams that are made of more than one material are called composite beams. The common examples are:
(a). Bimetallic beams
(b). Sandwich beams
(c). Fletched beams
(d). Reinforced concrete beams

5.Symmetrical Section: Let us take the examples of two timber pieces strengthened by a steep strip sandwiched between them. Such a beam is commonly known as a fletched beam.


From Statics:

$$
M=M_{1}+M_{2}
$$

At any section distance $y$ from the N.A., the strain in both the materials will remain the same (Strain Compatibility condition) since they are in contact. Hence,

$$
\begin{gathered}
\mathrm{e}_{1}=\mathrm{e}_{2} \\
\frac{\sigma_{1}}{\mathrm{E}_{1}}=\frac{\sigma_{2}}{\mathrm{E}_{2}} \\
\therefore \sigma_{2}=\sigma_{1} \frac{\mathrm{E}_{2}}{\mathrm{E}_{1}}=\mathrm{m} \mathrm{\sigma}_{1}
\end{gathered}
$$

Where $m=\frac{E_{2}}{E_{1}}$ is known as the modular ratio.

## 6.Unsymmetrical section:


(a) Original section

(b) Equivalent section
$\mathrm{Z}_{1}{ }^{\prime}=\frac{\sigma_{2}}{\sigma_{1}} \mathrm{Z}_{2}=\mathrm{mZ}_{2}$
Since $Z_{1}{ }^{\prime}$ and $Z_{2}$ are proportional to $b$. Thus:
Hence $b^{\prime}=m b=\frac{E_{2}}{E_{1}} b$

## CHAPTER4: SHEAR STRESS IN BEAMS

## 1.Shear stress at a section:

The shear stress is given by:

$$
\tau=\frac{F}{I b} . A \bar{y}
$$

Assumptions: The above analysis is based on the following assumptions:

1. For all values of $\mathrm{y}, \tau$ is uniform across the width of the cross-section, irrespective of its shape.
2. $F=\frac{d m}{d x}$ is derived from the assumption that bending stress varies linearly across the section and is zero at the centroid.
3. The material is homogeneous and isotropic, and the value of $E$ is the same for tension as well as compression.

Assumption No. 1 is not strictly correct because the tangential value must be zero at the boundaries of the section. Hence it is understood that $\tau$ is the average value across the section. Hence it is understood that $q$ is the average value across the action. Regarding assumption No 2, the stress curve is not a straight line passing through the centroid of the section.
2.Shear stress distribution and relations for different sections:

| (a). Rectangular section: $\begin{aligned} & \tau=\frac{\mathrm{F}}{2 \mathrm{I}}\left(\frac{\mathrm{~d}^{2}}{2}-\mathrm{y}^{2}\right) \\ & \frac{\tau_{\text {max }}}{\tau_{\text {mean }}}=\frac{3}{2}=1.5 \end{aligned}$ |  |
| :---: | :---: |
| (b). Circular section: $\tau=\frac{\mathrm{F}}{12 \mathrm{I}}\left\{4\left(\mathrm{r}^{2}-\mathrm{y}^{2}\right)\right\}=\frac{\mathrm{F}}{31}\left(\mathrm{r}^{2}-\mathrm{y}^{2}\right)$ $\tau_{\max }=\frac{4}{3} \tau_{\mathrm{avg}}$ |  |



## 4.Shear stress distribution over other sections:



(d) I-Section

(e) T-Section

(f) L-Section

(g) Cross

(h) Built-up Section

(i) Square

(j) Triangle

(k) Rolled Section

## 5.Difference between the Bending and Shearing Stresses:

| Ben | Sh |
| :---: | :---: |
| 1. Bending stress acts perpendicular to the cross-section of beam. <br> 2. Bending stress varies linearly over the depth of beam. <br> 3. At extreme fibers bending stress is maximum. <br> 4. At neutral axis $\sigma_{\mathrm{b}}$ is zero. <br> 5. $\quad \sigma_{\mathrm{b}}=\frac{\mathrm{MY}}{\mathrm{I}_{\text {N.A. }}} \operatorname{or} \frac{\mathrm{EY}}{\mathrm{R}} \operatorname{or}\left(\sigma_{\mathrm{b}}\right)_{\max }\left[\frac{\mathrm{y}}{\mathrm{y}_{\max }}\right]$ | 1. Shear stress acts parallel to crosssection of beam. <br> 2. Shear varies parabolically over the depth of beam. <br> 3. At extreme fiber shear stress in zero. <br> 4. At neutral axis shear stress is not zero but at neutral axis it become maximum in case of rectangular, circular, Square, T cross-section \& I crosssection beams. |


| 6. $\left(\sigma_{b}\right)_{\max }=\frac{M}{Z_{\text {N.A. }}}$ or $\frac{E y_{\max }}{R}$ <br> 7. Bending stress distribution consists of two similar triangles for any crosssection of beam. | 5. $\tau_{\mathrm{s}}=\frac{\mathrm{F}}{\mathrm{I}_{\text {N.A. }}}\left[\frac{\mathrm{A} \overline{\mathrm{y}}}{\mathrm{b}}\right]$ <br> 6. $\tau_{\max }=k \tau_{\text {avg. }}$. Where $\tau_{\text {avg }}=\frac{\mathrm{p}}{\mathrm{A}}$ and $\begin{aligned} & \mathrm{K}=\frac{3}{2} \Rightarrow \\ & \mathrm{k}=\frac{4}{3} \Rightarrow \\ & \mathrm{k}=\frac{9}{8} \Rightarrow \end{aligned}$ <br> 7. Shape of shear stress variation varies from cross-section to cross-section. |
| :---: | :---: |

## 6.Shear Center:

- A lateral load acting on a beam will produce bending without twisting only if it acts through the Shear center.

For a doubly symmetric section Shear centerand Centroid coincide.


## Locating Shear Centre



- Moment due to applied force $=F_{y} \cdot e_{z}$
- Force on shaded element = t.t.ds = Q.ds
- Moment due to that force $=\mathrm{Q} . \mathrm{ds} . \mathrm{r}_{\mathrm{t}}$
- Total resisting moment $=\int_{0}^{s_{1}} Q r_{t} \cdot d s$.
- Equating applied moment and resisting moment $\therefore \int_{0}^{s_{1}} Q r_{t} d s=F_{y} e_{z}$

$$
Q=-\frac{F_{y}}{I_{u}} \cdot{ }_{0}^{s} y \cdot t \cdot d s \quad \therefore-\frac{F_{y}}{I_{z z}} \cdot s_{0}^{s_{1}}\left(r_{t} \cdot \int_{0}^{s} y \cdot t \cdot d s\right) d s=F_{y} \cdot e_{z}
$$

$$
\therefore e_{z}=-\frac{1}{I_{z z}} \int_{0}^{s_{1}}\left(r_{i} \cdot \int_{0}^{s} y \cdot t \cdot d s\right) d s
$$

## Unsymmetric Loading of Thin-Walled Members:



Beam loaded in a vertical plane of symmetry deforms in the symmetry plane without twisting.

$$
\sigma_{x}=-\frac{M y}{I} \quad \tau_{\text {ave }}=\frac{V Q_{z}}{I t}
$$



Beam without a vertical plane of symmetry bends and twists under loading.


If the shear load is applied such that the beam does not twist, then the shear stress distribution satisfies

$$
\tau_{\text {ave }}=\frac{V Q_{z}}{I t} \quad V=\int_{B}^{D} Q d s \quad F=\int_{A}^{B} Q d s=-\int_{D}^{E} Q d s=-F^{\prime}
$$


$F$ and $F^{\prime}$ indicate a couple $F h$ and the need for the application of a torque as well as the shear load.

$$
F h=V e
$$



When the force P is applied at a distance ' $e$ ' to the left of the web center-line, the member bends in a vertical plane without twisting.
For symmetric sections subject to bending about one axis.
Elements parallel to bending axis-Linear distribution.
Elements normal to bending axis-Parabolic distribution.

For unsymmetric sections shear flow in all elements is parabolic.
When moving from one element to another the end value of shear in one element equals the initial value for the subsequent element (from equilibrium).

## Shear Centres for Some Other Sections



Location of shear centre for important sections:

| Cross section | Shear centre location |
| :--- | :--- |
| a. Channel section | $e=\frac{b^{2} h^{2} t}{4 I}$ |
|  |  |


| C. Open Circular Slit | $\mathrm{e}=2 \mathrm{R}(>\mathrm{R})$ |
| :---: | :--- |

## CHAPTER 5: TRANSFORMATION OF STRESSES

1.Stress tensor:A tensor is a multi-dimensional array of numerical values that can be used to describe the physical state or properties of a material.

(a)

(b)

$$
[\sigma]=\left[\begin{array}{ccc}
\sigma_{x x} & \tau_{x y} & \tau_{\mathrm{xz}} \\
\tau_{y \mathrm{x}} & \sigma_{y y} & \tau_{\mathrm{yz}} \\
\tau_{\mathrm{zx}} & \tau_{\mathrm{zy}} & \sigma_{\mathrm{zz}}
\end{array}\right]
$$

## Sign conventions:

A pair of shear stresses on parallel planes forming a clockwise couple is positive and a pair with counter clockwise couple, negative. Clockwise angle is taken as positive and counter clockwise as negative.

## 2.Direct Stress Condition:


(a)

(b)

(c)

Normal stress on a plane at any angle $\theta$ :

$$
\sigma_{\theta}=\sigma_{\mathrm{x}} \cos ^{2} \theta
$$

Shear stress:

$$
\tau_{\theta}=-\frac{1}{2} \sigma_{x} \sin 2 \theta
$$

Resultant stress:

$$
\sigma_{r}=\sigma_{x} \cos \theta
$$

Inclination with the normal stress:

$$
\begin{gathered}
\tan \phi=\frac{\sigma_{x} \sin \theta \cos \theta}{\sigma_{x} \cos ^{2} \theta}=\tan \theta \\
\phi=\theta
\end{gathered}
$$

i.e., it is always in the direction of the applied stress.

## 3.Bi-axial Stress Condition:



Normal stress on a plane at any angle $\theta$ :

$$
\sigma_{\theta}=\frac{1}{2}\left(\sigma_{x}+\sigma_{y}\right)+\frac{1}{2}\left(\sigma_{x}-\sigma_{y}\right) \cos 2 \theta
$$

Shear stress on a plane at any angle $\theta$ :

$$
\tau_{\theta}=-\left(\sigma_{x}-\sigma_{y}\right) \sin \theta \cos \theta=-\frac{1}{2}\left(\sigma_{x}-\sigma_{y}\right) \sin 2 \theta
$$

Resultant stress:

$$
\sigma_{r}=\sqrt{\sigma_{x}^{2} \cdot \cos ^{2} \theta+\sigma_{y}^{2} \sin ^{2} \theta}
$$

For greatest obliquity or inclination of the resultant with the normal stress:

$$
\tan \theta=\sqrt{\frac{\sigma_{x}}{\sigma_{y}}} \text { and } \tan \phi_{\max }=\frac{\sigma_{y}-\sigma_{x}}{2 \sqrt{\sigma_{x} \sigma_{y}}}
$$

The angle of inclination of the resultant with $\sigma_{\mathrm{x}}$ :

$$
\tan \alpha=\frac{\sigma_{y}}{\sigma_{x}} \tan \theta
$$

Maximum shear stress is given by:

$$
\tau_{\max }= \pm \frac{1}{2}\left(\sigma_{x}-\sigma_{y}\right)
$$

## 4.Pure Shear Stress Condition:

Let an element of a body be acted upon by shear stresses on its two perpendicular faces as shown in Fig. Let $d x, d y$ and ds be the lengths of the sides $A B, B C$ and $A C$ respectively.


Considering unit thickness of the body and resolving:
Normal stress:

$$
\sigma_{\theta}=\tau \sin \theta \cos \theta+\tau \sin \theta \cos \theta=\tau \cdot \sin 2 \theta
$$

Shear stress:

$$
\tau_{\theta}=\tau \cos 2 \theta
$$

Resultant stress:

$$
\sigma_{r}=\sqrt{\sigma_{\theta}^{2}+\tau_{\theta}^{2}}=\tau \sqrt{(\sin 2 \theta)^{2}+(\cos 2 \theta)^{2}}=\tau
$$

Inclination with the direction of shear stress planes:

$$
\begin{gathered}
\tan \phi=\frac{\sin 2 \theta}{\cos 2 \theta}=\tan 2 \theta \\
\phi=2 \theta
\end{gathered}
$$

## 5.Biaxial and Shear Stresses Condition:



Normal stress on a plane angle $\theta$ :

$$
\sigma_{\theta}=\frac{1}{2}\left(\sigma_{x}+\sigma_{y}\right)+\frac{1}{2}\left(\sigma_{x}-\sigma_{y}\right) \operatorname{Cos} 2 \theta+\tau \sin 2 \theta
$$

Shear stress on a plane at angle $\theta$ :

$$
\tau_{\theta}=-\frac{1}{2}\left(\sigma_{x}-\sigma_{y}\right) \sin 2 \theta+\tau \cos 2 \theta
$$

Angle of planes having maximum and maximum values of direct stress:

$$
\tan 2 \theta=\frac{2 \tau}{\sigma_{x}-\sigma_{y}}
$$

## Sum of direct stresses on two mutually perpendicular planes:

$$
\sigma_{\theta}+\sigma_{\left(\theta+90^{\circ}\right)}=\sigma_{x}+\sigma_{y}
$$

The sum of direct stresses $\sigma_{x}$ and $\sigma_{y}$ is constant, the sum of direct stresses on two mutually perpendicular planes at a point at angle $\theta$ and $\left(\theta+90^{\circ}\right)$ remains constant and equal to $\sigma_{x}+$ $\sigma_{y}$.

## 6.Principal stresses:

Three mutually perpendicular planes, on each of which the resultant stress is wholly normal. These are known as principal planes and the normal stress across these planes, as principal stresses.

$$
\tan 2 \theta=\frac{2 \tau}{\sigma_{x}-\sigma_{y}}
$$

Which provides two values of $2 \theta$ differing by $180^{\circ}$ or two values of $\theta$ differing by $90^{\circ}$. Thus, the two principal planes are perpendicular to each other.


Principal stresses are given by:

$$
\sigma_{1,2}=\frac{1}{2}\left(\sigma_{x}+\sigma_{y}\right) \pm \sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau^{2}}
$$

## 7.Maximum (principal) shear stresses:

For maximum value of $\tau_{\theta}: \quad \tan 2 \theta=-\frac{\sigma_{x}-\sigma_{y}}{2 \tau}$
Maximum shear stress: $\tau_{\max }=\frac{1}{2}\left(\sigma_{1}-\sigma_{2}\right)=\frac{1}{2} \sqrt{\left(\sigma_{x}-\sigma_{y}\right)^{2}+4 \tau^{2}}$
Principal planes are given by:

$$
\tan 2 \theta_{\mathrm{p}}=\frac{2 \tau}{\sigma_{\mathrm{x}}-\sigma_{\mathrm{y}}}
$$

and planes of maximum shear stress:

$$
\tan 2 \theta_{s}=-\frac{\sigma_{x}-\sigma_{y}}{2 \tau}
$$

Multiplying the two:

$$
\begin{gathered}
\tan 2 \theta_{\mathrm{p}} \cdot \tan 2 \theta_{\mathrm{s}}=-1 \\
\text { which means } 2 \theta_{\mathrm{s}}=2 \theta_{\mathrm{p}}+90^{\circ} \\
\text { i.e., } \theta_{\mathrm{s}}=\theta_{\mathrm{p}}+45^{\circ} .
\end{gathered}
$$

This indicates that the planes of maximum shear stress lie at $45^{\circ}$ to the planes of principal axes.

The maximum value of shear stress lies in the planes at $45^{\circ}$ to the principal planes:

$$
\tau_{\max }=\frac{1}{2}\left(\sigma_{1}-\sigma_{2}\right)=\frac{1}{2} \sqrt{\left(\sigma_{1}-\sigma_{2}\right)^{2}+4 \tau^{2}}
$$

## 8.Normal stress on the planes of maximum shear stress:

$$
\sigma_{\theta}=\frac{1}{2}\left(\sigma_{x}+\sigma_{y}\right)
$$

9.Plane Stress Condition: It is used by the designer for designing thin sheets. In this condition, stress in one of the directions is neglected but strain is assumed to exist in that direction.
10.Plane strain condition: In plane strain condition designer assumes strain in one of the directions is zero but for preventing this deformation stress is required in that direction. Designing of thick plate, thick pressure vessel designers use this method.
11.Strain Rosette: A strain gauge rosette is a term for an arrangement of two or more strain gauge that are positioned closely to measure strains along different directions of the component under evaluation.

## 12.Mohr's circle:

Let $C R$ and $C S$ represent two perpendicular planes $B D$ and $A B$ respectively so that $O L=\sigma_{x}$, $\mathrm{OM}=\sigma_{y}$ and $L R$ and MS each equal to T in the clockwise and counter-clockwise directions respectively (Fig.9). Now if it is desired to find stresses on an inclined plane at angle $\theta$ clockwise with plane BD, a radial line CP may be drawn at angle $2 \theta$ in the clockwise direction with CR. Then ON and NP will represent the direct and shear components respectively on the plane AD and the resultant is given by OP.

Thus, the procedure may be summarised as follows:
$\square$ Take OL and OM as the direct components of the two perpendicular stresses $\sigma_{x}$ and $\sigma_{y}$.At $L$ and $M$ draw $\perp$ LR and MS on the x-axis each equal to $T$ using the same scale as for the direct stresses. For the stress system shown in Fig.8, LR is taken upwards as the direction on plane $B D$ is clockwise and MS downwards as the direction on plane $A B$ is counter clockwise.Bisect LM at C and draw a circle with C as centre and radius equal to CR (= CS).Rotate the radial line $C R$ through angle $2 \theta$ in the clockwise direction if $\theta$ is taken clockwise and let it take the position CP.Draw NP $\perp$ on x-axis. Join OP.
It can be proved that ON and NP represent the normal and the shear stress components on the inclined plane AD.


Major principal stress $=\frac{1}{2}\left(\sigma_{x}+\sigma_{y}\right)+\frac{1}{2} \sqrt{\left(\sigma_{x}-\sigma_{y}\right)^{2}+4 \tau^{2}}$
Minor principal stress $=\frac{1}{2}\left(\sigma_{x}+\sigma_{y}\right)-\frac{1}{2} \sqrt{\left(\sigma_{x}-\sigma_{y}\right)^{2}+4 \tau^{2}}$

Radius of the circle:

$$
\mathrm{R}=\sqrt{\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right)^{2}+\tau^{2}}
$$

Centre of Mohr's circle:

$$
C=\left(\frac{\sigma_{x}+\sigma_{y}}{2}, 0\right)
$$

## Mohr's circle different cases:

(a). When both the stresses are tensile in nature and no shear stress:



Example- Thin cylinder stress case.

$$
\text { Hoop stress: } \sigma_{h}=\frac{P D}{2 t} \quad \text { (tensile) }
$$

$$
\text { Longitudinal stress : } \sigma_{1}=\frac{\mathrm{PD}}{4 \mathrm{t}} \quad \text { (tensile) }
$$

(b). When one is tensile and other is compressive in nature and no shear stress:

(c). When both the stresses are tensile in nature and are equal:

Example- Spherical pressure vessel case. $\sigma_{h}=\sigma_{I}=\frac{P D}{4 t} \quad$ (tensile)



## 14.Strain analysis:




$$
\varepsilon_{\theta}=\frac{1}{2}\left(\varepsilon_{x}+\varepsilon_{y}\right)+\frac{1}{2}\left(\varepsilon_{y}-\varepsilon_{y}\right) \cos 2 \theta+\frac{1}{2} \varphi \sin 2 \theta
$$

$$
\gamma=-\frac{1}{2}\left(\varepsilon_{x}-\varepsilon_{y}\right) \sin 2 \theta+\varphi \sin ^{2} \theta
$$

Compare the results with bi-axial and shear stresses conditions:In a linear system: $\varepsilon_{\theta}=\varepsilon_{x} \cdot \cos ^{2} \theta$ or $\varepsilon_{x}\left(\frac{1+\cos 2 \theta}{2}\right)$.In a pure shear system and for $\theta=45^{\circ}, \varepsilon 45^{\circ}=\phi / 2$

## 15.Principal strains:

The maximum and the minimum values of strains on any plane at a point are known as the principal strains and the corresponding planes as the principal planes for strains.

$$
\tan 2 \theta=\frac{\varphi}{\sigma_{x}-\sigma_{y}}
$$

For shear strain to be maximum or minimum:

$$
\tan 2 \theta=-\frac{\varepsilon_{\mathrm{x}}-\varepsilon_{\mathrm{y}}}{\varphi}
$$

The planes of maximum shear strain are inclined at $45^{\circ}$ to the planes of maximum shear strain as in case of maximum shear stress.

Sum of direct strains on two mutually perpendicular planes:

$$
\varepsilon_{\theta}+\varepsilon_{\theta+90^{\circ}}=\varepsilon_{x}+\varepsilon_{y}
$$

## Relation between principal stress and strain:

$$
\sigma_{2}=\frac{\mathrm{E}\left(\mu \varepsilon_{1}+\varepsilon_{2}\right)}{1-\mu^{2}} \text { and } \sigma_{1}=\frac{\mathrm{E}\left(\mu \varepsilon_{2}+\varepsilon_{1}\right)}{1-\mu^{2}}
$$

## Types of strain rosette:

Let $\epsilon_{x}$ and $\epsilon_{y}$ be the linear strains in $x$ and $y$ directions and $\phi$ be the shear strain at the point under consideration. Then linear strains in any three arbitrary chosen directions at angles $\theta_{1}$, $\theta_{2}$ and $\theta_{3}$ made with the $x$-axis will be:

$$
\begin{aligned}
& \varepsilon_{\theta_{1}}=\varepsilon_{x} \cdot \cos ^{2} \theta_{1}+\varepsilon_{y} \cdot \sin ^{2} \theta_{1}+\varphi \cdot \sin \theta_{1} \cdot \cos \theta_{1} \\
& \varepsilon_{\theta_{2}}=\varepsilon_{x} \cdot \cos ^{2} \theta_{2}+\varepsilon_{y} \cdot \sin ^{2} \theta_{2}+\varphi \cdot \sin \theta_{2} \cdot \cos \theta_{2} \\
& \varepsilon_{\theta_{3}}=\varepsilon_{x} \cdot \cos ^{2} \theta_{3}+\varepsilon_{y} \cdot \sin ^{2} \theta_{3}+\varphi \cdot \sin \theta_{3} \cdot \cos \theta_{3}
\end{aligned}
$$

## Rectangular strain Rosette:



$$
\theta_{1}=\theta^{\circ}, \theta_{2}=45^{\circ} \text { and } \theta_{2}=90^{\circ}
$$

The above equations can be written:

$$
\begin{aligned}
& \epsilon_{0^{\circ}}=\epsilon_{x} \text { and } \epsilon_{90^{\circ}}=\epsilon_{y} \\
& \varepsilon_{45^{\circ}}=\frac{1}{2}\left(\varepsilon_{x}+\varepsilon_{y}+\varphi\right)
\end{aligned}
$$

From which:
$\epsilon_{\mathrm{x}}=\epsilon 0^{\circ}, \epsilon_{\mathrm{y}}=\epsilon 90^{\circ}$ and $\Phi=2 \epsilon 45^{\circ}-\left(\epsilon_{\mathrm{x}}+\epsilon_{\mathrm{y}}\right)$

## Delta strain rosette:



$$
\begin{gathered}
\epsilon_{x}=\epsilon_{0^{\circ}} \text { and } \varepsilon_{y}=\frac{1}{3}\left(2 \varepsilon_{60^{\circ}}+2 \varepsilon_{120^{\circ}}-\varepsilon_{0^{\circ}}\right) \\
\\
\text { and } \varphi=\frac{2}{\sqrt{3}}\left(\varepsilon_{60^{\circ}}-\varepsilon_{120^{\circ}}\right)
\end{gathered}
$$

16.Strain tensor: Strain tensor is used to define the state of strain at a point (i.e., different strains developed on three mutual perpendicular planes, passing through a point.
$\epsilon \rightarrow$ [Normal Strain]
$\mathrm{T} \rightarrow$ [Shear strain]

$$
\underset{\substack{\text { Strain } \\
\text { Trensor } \\
\text { at anoint } \\
\text { in } \\
\text { 3D. }}}{[\varepsilon]_{3 D}}=\left[\begin{array}{ccc}
\varepsilon_{x x} & \frac{\phi_{x y}}{2} & \frac{\phi_{x z}}{2} \\
\frac{\phi_{x y}}{2} & \varepsilon_{y y} & \frac{\phi_{y z}}{2} \\
\frac{\phi_{x z}}{2} & \frac{\phi_{y z}}{2} & \varepsilon_{z z}
\end{array}\right]_{3 \times 3}
$$

## For the 2-D:

$$
[\varepsilon]_{2 D}=\left[\begin{array}{ll}
\varepsilon_{x x} & \frac{\phi_{x y}}{2} \\
\frac{\phi_{x y}}{2} & \varepsilon_{y y}
\end{array}\right]
$$

## CHAPTER 6:Torsion Of Circular Shaft

1.Torsion: A shaft is said to be under pure torsion when it is subjected to two equal \& opposite couples in a plane perpendicular to the longitudinal axis of the shaft.


## Assumptions:

The following assumptions have been made in developing the equations for stresses and deformations in a bar subjected to pure torsion:

1. Shaft is loaded with twisting couples in planes that are perpendicular to the axis of the shaft.
2. Torsion is uniform along the length i.e., all normal cross-section which are the same axial distance suffer equal relative rotations.
3. Circular sections remain circular. Thus, radii remain straight after torsion.
4. Plane normal sections of shaft remain plane after twisting, i.e., no warping or distortion of parallel planes normal to the axis of the shaft takes place.
5. Stress is proportional to strain, i.e., stresses do not exceed proportional limit.
6. Material is homogenous and isotropic.

## 2.Torsion formula:



$$
\frac{\tau_{r}}{r}=\frac{\tau}{R}=\frac{T_{R}}{J}=\frac{G \theta}{L}
$$

Where:
$\theta=$ twist angle in radians
$\tau=$ shear stress developed in the material
$\mathrm{T}=$ torque applied
$\mathrm{G}=$ modulus of rigidity
$I_{p}=$ Polar moment of inertia

(a)

(b)

## Sign convention:

Right hand thumb rule $\rightarrow$ If right hand fingers represent direction of torque applied and thumb is pointing towards the section, the torque is taken positive.

## 3.Polar moment of inertia:

| Cross section | Polar moment of Inertia and maximum torque | Shear stress variation |
| :---: | :---: | :---: |
| Solid circular shaft | $\begin{aligned} & I_{z z}=\frac{\pi}{32} D^{4} \\ & Z_{P}=\frac{J}{R}=\frac{J}{D / 2}=\frac{\pi}{16} D^{3} \\ & T=\tau \times \frac{\pi}{2} \times \frac{D^{3}}{8}=\frac{\pi}{16} \tau D^{3} \end{aligned}$ |  |
| Hollow circular <br> shaft | $\begin{aligned} & \mathrm{I}_{z 2}=\frac{\pi}{32}\left(\mathrm{D}^{4}-\mathrm{d}^{4}\right) \\ & \mathrm{Z}_{\mathrm{p}}=\frac{\pi}{16}\left(\frac{D^{4}-d^{4}}{D}\right) \\ & T=\tau \frac{\pi}{16}\left[\frac{D_{0}^{4}-D_{i}^{4}}{D_{0}}\right] \end{aligned}$ |  |

## 4.Connection of shafts:

## (i). Series connection:



Here,

$$
\begin{aligned}
& \mathrm{T}_{1}=\mathrm{T}_{2}=\mathrm{T} \\
& \theta_{\mathrm{AC}}=\theta_{\mathrm{AB}}+\theta_{\mathrm{BC}}
\end{aligned}
$$

## (ii). Parallel connection:



Composite Shaft (Shaft made of two materials):


Torques: $\mathrm{T}_{1}=\mathrm{T}\left(\frac{\mathrm{G}_{1} \mathrm{~J}_{1}}{\mathrm{G}_{1} \mathrm{~J}_{1}+\mathrm{G}_{2} \mathrm{~J}_{2}}\right)$ and $\mathrm{T}_{2}=\mathrm{T}\left(\frac{\mathrm{G}_{2} \mathrm{~J}_{2}}{\mathrm{G}_{1} \mathrm{~J}_{1}+\mathrm{G}_{2} \mathrm{~J}_{2}}\right)$

Twist angle: $\theta=\frac{\mathrm{TL}}{\mathrm{G}_{1} \mathrm{~J}_{1}+\mathrm{G}_{2} \mathrm{~J}_{2}}$
Ratio of shear stresses: $\frac{\tau_{1}}{\tau_{2}}=\frac{T_{1} D_{1}}{T_{2} D_{2}} \cdot \frac{J_{2}}{J_{1}}=\frac{G_{1}}{G_{2}} \cdot \frac{D_{1}}{D_{2}}$

## 5.Strain energy due to torsion:

Total strain energy:

$$
\begin{gathered}
\mathrm{U}=\frac{1}{2} \mathrm{~T} \cdot \theta=\text { Areaunder } \mathrm{T}-\theta \text { diagram } \\
\mathrm{U}=\frac{\tau^{2}}{4 \mathrm{G}} \times \text { Volume of the shaft }
\end{gathered}
$$

## For hollow shaft:

$$
\mathrm{U}=\frac{\tau^{2}}{4 \mathrm{G}}\left(\frac{\mathrm{D}^{2}+\mathrm{d}^{2}}{\mathrm{D}^{2}}\right) \times \text { Volume of the shaft }
$$

For the thin tube: $D \approx d$

$$
\mathrm{U} \approx \frac{\tau^{2}}{2 \mathrm{G}} \times \text { Volume of the shaft }
$$

6.Torsional stiffness $\left(K_{t}\right)$ : Torsional stiffness is defined as the amount of torque or twisting couple required to produce a twist of unit radian. And it represented by ' $K$ '.

$$
\mathrm{K}_{\mathrm{t}}=\frac{\mathrm{T}}{\theta}
$$

For a given twisting couple ' $T$ '

$$
\mathrm{K}_{\mathrm{t}}=\frac{\mathrm{GJ}}{\mathrm{~L}} \uparrow \Rightarrow \theta \downarrow \Rightarrow \varphi \downarrow \Rightarrow \gamma \downarrow \tau_{\text {induced }} \downarrow \Rightarrow \text { Chances of torsional failure } \downarrow
$$

Torsional Stiffness is used to compare angular twist of two different shafts which are subjected to same twisting moment whereas Torsional Rigidity is used to compare angular twist of two different shafts which are having same length and subjected to same torque.
7.Combined bending and torsion: Consider any point on the cross-section of as shaft.

Let, $\mathrm{T}=$ Torque at the section
D = Diameter of the shaft
$M=B . M$. at the section
Major principal stress: $\sigma_{1}=\frac{16 \mathrm{~T}}{\pi \mathrm{D}^{3}}\left(\mathrm{M}+\sqrt{\mathrm{M}^{2}+\mathrm{T}^{2}}\right)$
Minor principal stress: $\sigma_{2}=\frac{16}{\pi D^{3}}\left(M-\sqrt{M^{2}+\mathrm{T}^{2}}\right)$

Maximum shear stress

$$
\begin{gathered}
\tau_{\max }=\frac{\text { Major principal stress }- \text { Minor principal stress }}{2} \\
\tau_{\max }=\frac{16}{\pi \mathrm{D}^{3}}\left(\sqrt{\mathrm{M}^{2}+\mathrm{T}^{2}}\right)
\end{gathered}
$$

## For a hollow shaft:

Major principal stress:

$$
\sigma_{1}=\frac{16 D_{0}}{\pi\left[D_{0}^{4}-D_{i}^{4}\right]}\left(M+\sqrt{M^{2}+T^{2}}\right)
$$

Minor principal stress:

$$
\sigma_{2}=\frac{16 D_{0}}{\pi\left[D_{0}^{4}-D_{i}^{4}\right]}\left(M-\sqrt{M^{2}+T^{2}}\right)
$$

Maximum shear stress:

$$
\tau_{\max }=\frac{16 \mathrm{D}_{0}}{\pi\left[\mathrm{D}_{0}^{4}-\mathrm{D}_{\mathrm{i}}^{4}\right]}\left(\sqrt{\mathrm{M}^{2}+\mathrm{T}^{2}}\right)
$$

## CHAPTER 7: COLUMNS

## 1.Introduction:

## Column:

$\square$ These are the vertical slender members subjected to an axial compressive load is called a column and used to carry loads of beams, slabs etc. Stanchions are steel columns made of rolled steel sections, commonly used in buildings. The failure of columns is due to the buckling at loads considerably less than those required to cause failure by crushing.

Struts: The term strut is commonly used for compression member in a roof truss, it may either be in vertical position or in inclined position.
Note-1: A compression member is generally considered to be a column when its unsupported length is more than 10 times its least lateral dimension.

## 2.Types of Columns

i. Short Columns: Short columns fail under direct compression, also called as crushing (at ultimate strength).
ii. Long Columns: Long columns fail at loads considerably lower than those required to cause crushing due to elastic instability, also called as buckling. The buckling behaviour is explained using Euler's theory.
iii. Intermediate Columns: Intermediate columns fail by a combination of crushing and buckling. This behaviour is complex, and several semi-empirical formulations are used for analysissuch as Rankine's formula.
3.Slenderness ratio: The buckling tendency of a column varies with the ratio of the length to least lateral dimension. The ratio is known is known slenderness ratio. It is given by the following relation:

Slenderness ratio ( S ) $=\frac{\mathrm{L}_{\mathrm{e}}}{\mathrm{K}}=\frac{\text { Effective length of member }}{\text { Least radius of gyration }}$
Its numerical value indicates whether the member falls into the class of columns or struts.

| S.No. | Type of column | Slenderness ratio |
| :---: | :---: | :---: |
| 1 | Short Columns | $0-40$ |
| 2 | Intermediate columns | $40-125$ |
| 3 | Long Columns | $>125$ |

4.Radius of gyration: It is defined as the distance from the axis of rotation to a point where the total mass of the body is supposed to be concentrated so that the moment of inertia about the axis may remain the same. The radius of gyration of a section is given by:

$$
\mathrm{K}=\sqrt{\frac{\mathrm{I}}{\mathrm{~A}}}
$$

5.Euler's theory: The struts which fails by buckling can be analysed by Euler's theory.

Assumptions:The Euler's theory is based on the following assumptions:
(i). Axis of the column is perfectly straight when unloaded.
(ii). The line of thrust coincides exactly with the unstrained axis of the column.
(iii). Flexural rigidity El is uniform.
(iv) Material is isotropic and homogeneous.
(v). The buckling value of $\mathrm{P}=\mathrm{P}_{\mathrm{E}}$ is assumed to obtain for all degrees of flexure.

Usually, the two assumptions are not really realised in practice. The column may have initial curvature, or crookedness. The theory, therefore, refers to an ideal column and not to a real one.
6.Equivalent length (Le): The Effective length Le for any column is the length of the equivalent pinned end column, i.e., it is the pinned end column having a deflection curve that exactly matches all part of deflection curve of an original column.

| Condition and effective length | Diagram |
| :---: | :---: |
| a. Strut with pinned ends: <br> $P_{e}=\frac{\pi^{2} E I}{L^{2}}$ and $L_{e}=L$ |  |
| b. One end fixed and the other free: $\mathrm{P}_{\mathrm{e}}=\frac{\pi^{2} \mathrm{EI}}{4 \mathrm{~L}^{2}}$ and $\mathrm{L}_{\mathrm{e}}=2 \mathrm{~L}$ |  |
| c.Strut with fixed ends: $\mathrm{P}_{\mathrm{e}}=\frac{4 \pi^{2} \mathrm{EI}}{\mathrm{~L}^{2}} \text { and } \mathrm{L}_{\mathrm{e}}=\mathrm{L} / 2$ |  |
| d. One and fixed and other hinged: $\mathrm{P}_{\mathrm{e}}=\frac{2 \pi^{2} \mathrm{EI}}{\mathrm{~L}^{2}} \text { and } \mathrm{L}_{\mathrm{e}}=\frac{\mathrm{L}}{\sqrt{2}}$ |  |

7.Rankine's Formula: Rankine proposed an empirical formula for columns which coven all Lasts ranging from very short to very long struts.

$$
\frac{1}{\mathrm{P}_{\mathrm{r}}}=\frac{1}{\mathrm{P}_{\mathrm{c}}}+\frac{1}{\mathrm{P}_{\mathrm{E}}}
$$

Where
$P_{c}=\sigma_{c} \times A=$ Ultimate load for a strut.
$P_{E}=\frac{\pi^{2} E I}{L^{2}}=$ Eulerian crippling load for the standard case.
In the above relation, $\frac{1}{\mathrm{P}_{\mathrm{c}}}$ is constant for a material.

$$
P_{r}=\frac{\sigma_{c} \cdot A}{1+\left(\frac{\sigma_{c}}{\pi^{2} E}\right)\left(\frac{L}{k}\right)^{2}}=\frac{\sigma_{\mathrm{c}} \cdot \mathrm{~A}}{1+\mathrm{a}\left(\frac{\mathrm{~L}}{\mathrm{k}}\right)^{2}}
$$

Above equation is the Rankine's formula for the standard case of column hinged at ends.
Where Rankine's constant: $a=\frac{\sigma_{c}}{\pi^{2} \mathrm{E}}$
Values of $\sigma_{c}$, and a for the materials commonly used for columns and struts:

| Material | $\sigma_{\mathbf{c}}$ |  | $\mathbf{a}$ |
| :--- | :---: | :---: | :---: |
|  | Kg/cm | N/mm | (For hinged ends) |
| 1. Wrought Iron. | 2550 | 255 | $1 / 9000$ |
| 2. Cast Iron | 5670 | 567 | $1 / 1400$ |
| 3.Mild steel | 3300 | 330 | $1 / 7500$ |
| 4.Strong timber | 500 | 50 | $1 / 750$ |

## CHAPTER 8:Combined Stresses \& Springs

## 1.Combined bending and direct stresses (Eccentric Loading):

A column subjected by a compressive load $P$ whose line of action is at a distance of 'e' from the axis of the column. Here 'e' is known as eccentricity of the load. The eccentric load will cause direct stress and bending stress.

| Colum cross section |  |
| :---: | :---: |
| (a). Rectangular cross section: $\sigma_{\max }=\frac{\mathrm{P}}{\mathrm{~A}}+\frac{6 \mathrm{P} \cdot \mathrm{e}}{\mathrm{~A} \cdot \mathrm{~b}}$ <br> and $\sigma_{\min }=\frac{\mathrm{P}}{\mathrm{~A}}\left(1-\frac{6 \times \mathrm{e}}{\mathrm{~b}}\right)$ | Evaluation <br> Plan <br> (a) <br> (b) <br> (c) |
| (b). Rectangular cross section subjected to load eccentric to both axes: <br> Resultant stress $=\frac{P}{A} \pm \frac{M_{x} \times x}{I_{y y}} \pm \frac{M_{x-y}}{I_{x x}}$ |  |

2.Conditions of eccentricity for which there is no tensile stresses in columns:

| Shape of column | Condition |
| :--- | :---: |
| a. Middle Third rule | Rectangular cross section: |
|  | $e_{x} \geq \frac{b}{6}$ |
|  | and |
|  | $e_{y} \geq \frac{d}{6}$ |



## 3.SPRINGS:

A spring is defined as an elastic machine element, which deflects under the action of the load and returns to its original shape when the load is removed.

## Helical spring:



$$
\text { Solid length }=\mathrm{N}_{\mathrm{t}} \mathrm{~d}
$$

Where
$\mathrm{N}_{\mathrm{t}}=$ total number of coils

$$
\text { Totalgap }=\left(N_{t}-1\right) \times \text { Gap between adjacent coils }
$$

## Spring deflection:

## Assumptions:

(1) The Bending \& shear effects may be neglected.
(2) For the purpose of derivation of formula, the helix angle is considered to be so small that it may be neglected.

$$
\text { Spring deflection: } \delta=\frac{8 \mathrm{~W} \cdot \mathrm{D}^{3} \mathrm{n}}{\mathrm{G} \cdot \mathrm{~d}^{4}}
$$

Where, W = axial load
$\mathrm{D}=$ mean coil diameter
d = diameter of spring wire
n = number of active coils

$$
\text { spring index: } C=\frac{D}{d} \text { (For circular wires) }
$$

I = length of spring wire
$\mathrm{G}=$ modulus of rigidity

$$
\text { Spring stiffness: } \mathrm{k}=\frac{\mathrm{G} \cdot \mathrm{~d}^{4}}{8 . D^{3} \mathrm{n}}=\frac{\mathrm{G} \cdot \mathrm{~d}}{8 . \mathrm{C}^{3} \mathrm{n}}
$$

$$
\tau=\frac{8 W D}{\pi d^{3}}\left(1+\frac{d}{2 D}\right)
$$

$$
\mathrm{k}_{\mathrm{sh}}=\left(1+\frac{\mathrm{d}}{2 \mathrm{D}}\right)=\text { shear stress correction factor }
$$


(a) Torsional shear stress diagram.

(b) Direct shear stress diagram.

(c) Resultant torsional shear and direct shear stress diagram.

## Wahl's correction factor:

$$
\mathrm{K}_{\mathrm{w}}=\frac{4 \mathrm{c}-1}{4 \mathrm{c}-4}+\frac{0.615}{\mathrm{c}}
$$

If we take into account, the Wahl's factor than the formula for the shear stress becomes

$$
\tau_{\max }=\frac{8 \mathrm{WD}}{\pi \mathrm{~d}^{3}} \mathrm{~K}_{\mathrm{w}}
$$

The Wahl's stress factor (K) may be considered as composed of two sub-factors, $\mathrm{K}_{\text {sh }}$ and $\mathrm{K}_{\mathrm{c}}$, such that

$$
\mathrm{K}_{\mathrm{w}}=\mathrm{K}_{\mathrm{sh}} \times \mathrm{K}_{\mathrm{c}}
$$

where $K_{\text {sh }}=$ Stress factor due to shear, and
$K_{c}=$ Stress concentration factor due to curvature.
4.Strain Energy: The strain energy is defined as the energy which is stored within a material when the work has been done on the material.

$$
\mathrm{U}=\frac{\mathrm{T}^{2} \mathrm{~L}}{2 \mathrm{GJ}} \text { and } \mathrm{L}=\pi \mathrm{Dn}
$$

So, after substitution:

$$
\mathrm{U}=\frac{16 \mathrm{~T}^{2} \mathrm{Dn}}{\mathrm{G} \cdot \mathrm{~d}^{4}}
$$

## 5.Connection of springs:



## Series connection:

$$
\frac{1}{\mathrm{k}_{\mathrm{eq}}}=\frac{1}{\mathrm{k}_{1}}+\frac{1}{\mathrm{k}_{2}}
$$

For n number of springs:

$$
\frac{1}{\mathrm{~K}_{\mathrm{eq}}}=\frac{1}{\mathrm{~K}_{1}}+\frac{1}{\mathrm{~K}_{2}}+\frac{1}{\mathrm{~K}_{3}}+\ldots \frac{1}{\mathrm{~K}_{\mathrm{n}}}
$$

## Spring in parallel:


$\therefore$ For n number of springs

$$
\mathrm{K}_{\mathrm{eq}}=\mathrm{K}_{1}+\mathrm{K}_{2}+\mathrm{K}_{3}+\ldots+\mathrm{K}_{\mathrm{n}}
$$

Equivalent stiffness of the spring in parallel connection is the sum of the stiffness of the individual spring.

## CHAPTER 9: THIN AND THICK SHELLS

1.Introduction: Thin pressure vessel is defined as a closed cylindrical or spherical container designed to hold or store fluids at a pressure substantially different from ambient pressure.


Common examples of pressure vessels are steam boilers, reservoirs, tanks, working chambers of engines, gas cylinders etc.

## 2.Thin cylindrical shell subject to internal pressure:

## Assumptions:

(i).Stresses are assumed to be distributed uniformly.
(ii). Area is calculated considering the pressure vessel as thin.
(iii). Radial stresses are neglected.
(iv). Biaxial state of stress is assumed to be applicable.

## 3.Circumferential stress (or Hoop stress) $\sigma_{H}: \sigma_{H}=\frac{P d}{2 t}$

4.Longitudinal stress (or axial stress) $\sigma_{\mathrm{L}} \sigma_{\mathrm{L}}=\frac{\mathrm{Pd}}{4 \mathrm{t}}$

From here, we can say that

$$
\sigma_{\mathrm{L}}=\frac{\sigma_{\mathrm{H}}}{2}
$$

Thus, the magnitude of the longitudinal stress is one half of the circumferential stress, both the stresses being of tensile nature.
Note:The radial stress is negligible as compare to the axial stress and hoop stress. Hence the third stress is neglected.
5. Maximum shear stress in the plane of $\sigma_{H}$ and $\sigma_{L}$ :

$$
\tau_{\max }=\frac{\sigma_{H}-\sigma_{\mathrm{L}}}{2}=\frac{1}{2}\left(\frac{\mathrm{Pd}}{2 \mathrm{t}}-\frac{\mathrm{Pd}}{4 \mathrm{t}}\right)=\frac{\mathrm{Pd}}{8 \mathrm{t}}
$$

$$
\text { Absolute } \tau_{\max }=\frac{\mathrm{Pd}}{4 \mathrm{t}}
$$

## 6.Strain:

Hoop strain or Circumferential strain: $\varepsilon_{\mathrm{c}}=\frac{\Delta \mathrm{d}}{\mathrm{d}}=\frac{\mathrm{Pd}}{4 \mathrm{tE}}[2-\mu]$
Longitudinal Strain or axial strain: $\varepsilon_{\mathrm{L}}=\frac{\Delta \mathrm{L}}{\mathrm{L}}=\frac{\mathrm{Pd}}{4 \mathrm{tE}}[1-2 \mu]$
Note:Theratio of circumferential strain and longitudinal strain under given loading condition is

$$
\frac{\text { circumeferential strain }\left(\varepsilon_{\mathrm{c}}\right)}{\text { Iongitudinalstrain }\left(\varepsilon_{\mathrm{L}}\right)}=\frac{\frac{\mathrm{Pd}}{4 \mathrm{tE}}(2-\mu)}{\frac{\mathrm{Pd}}{4 \mathrm{tE}}(1-2 \mu)}=\frac{(2-\mu)}{(1-2 \mu)}
$$

## Volumetric Strain or Change in the Internal Volume:

volumetric strain $=$ longitudinal strain $+2 \times$ circumferential strain

Volumetric Strain:

$$
\varepsilon_{\mathrm{v}}=\frac{\Delta \mathrm{V}}{\mathrm{~V}}=\frac{\mathrm{Pd}}{4 \mathrm{tE}}[5-4 \mu]
$$

## 6.Thin spherical shells under internal pressure:

$$
\text { Hoop stress: } \sigma_{\mathrm{h}}=\frac{\mathrm{Pd}}{4 \mathrm{t}^{\prime}}
$$

$$
\text { Longitudinal stress: } \sigma_{\mathrm{L}}=\frac{\mathrm{Pd}}{4 \mathrm{t}^{\prime}}
$$

In plane shear stress in a spherical pressure vessel:

$$
\text { Maximum shear stress }=\frac{\sigma_{1}-\sigma_{2}}{2}=\frac{P \times d}{4 t}-\frac{P \times d}{4 t}=0
$$

$$
\text { Absolute } \tau_{\max }=\frac{\mathrm{Pd}}{8 \mathrm{t}}
$$

$$
\varepsilon_{\mathrm{c}}=\varepsilon_{\mathrm{L}}=\frac{\mathrm{Pd}}{4 \mathrm{tE}}(1-\mu)
$$

Volumetric strain:

$$
\varepsilon_{\mathrm{v}}=\frac{\mathrm{dV}}{\mathrm{~V}}=\frac{3 \mathrm{Pd}}{4 \mathrm{tE}}(1-\mu)
$$

Volumetric strain in spherical shell is thrice of the longitudinal strain or hoop strain.
7.Thin spherical shells under external pressure:

$$
\varepsilon_{\mathrm{V}}=\frac{\mathrm{dV}}{\mathrm{~V}}=-\frac{3 \mathrm{Pd}}{4 \mathrm{tE}}(1-\mu) .
$$

## 8.Thin cylinder with hemispherical ends:

If there is no distortion of the junction of hemispherical ends with cylindrical ends under pressure, then:

$$
\frac{\mathrm{t}^{\prime}}{\mathrm{t}}=\frac{1-\mu}{2-\mu}
$$

Thus, the thickness of cylindrical wall must be greater than thickness of hemispherical ends.

## 9.Strengthening of cylindrical pressure vessel:

Wire Winding: A tube can be strengthened against the internal pressure by winding it with wire under tension and putting the tube wall in compression. Since the vessel is in compression, as the pressure is applied, the resultant hoop stress produced is much less as it would have been in the absence of wire.
10.Thick pressure vessels: If the ratio of the of diameter to its thickness is less than 20 then it is said to be thick pressure vessel. In thick pressure vessels stresses vary from maximum at inner surface to minimum at outer surface.
(i). In thick cylinders hoop stress due to inside pressure is:
(a) Maximum inside
(b) Minimum outside
(c) And tensile throughout.

(ii). In thick cylinders longitudinal stress due to inside pressure is constant and tensile throughout the thickness.
(iii) Radial pressure is maximum inside, zero outside and compressive throughout.
11.Analysis of thick shells using Lame's theorem: In this theorem the material is assumed to be homogeneous and isotropic and the longitudinal stress are assumed to be constant throughout.
(a). Longitudinal stress:

(b). Hoop stress: Hoop stress in thick cylinder pressure varies maximum at inside to minimum outside. Hoop stress at a radial distance of $R$ is

$$
\sigma_{H}=\frac{B}{R^{2}}+A
$$

Where A \& B are Lame's constant and they and they are always positive.

(c). Radial Pressure: Radial stress at a radial distance of $R$ is

$$
P_{R}=\frac{B}{R_{2}}-A \text { (Compression) }
$$

Boundary condition are:
at $R=R_{o} \Rightarrow P_{R}=0$

$$
\therefore \frac{\mathrm{B}}{\mathrm{R}_{0}^{2}}=\mathrm{A}
$$

At $R=R_{i} ; P_{R}=P$

$$
P=\frac{B}{R_{i}^{2}}-A
$$

By solving these two conditions we can find out value of A \& B.
12.Thick sphere: Hoopand longitudinal stress are equal and varies from maximum at inner face to minimum at outer face.

$$
\begin{gathered}
\sigma_{H}=\frac{B}{R^{3}}+A \text { (Tensile) } \\
P_{R}=\frac{2 B}{R^{3}}-A \text { (compressive) }
\end{gathered}
$$

at $R=R_{i} \quad P_{R}=P$
$R=R_{0} \quad P_{R}=0 \quad$ get A \& B
Lame's constants are both positive for inside pressure and both negative for outside pressure.

## CHAPTER10: DEFLECTION OF BEAMS

1.Introduction: Theeffect of bending results in the deflection of the beam. This is the stiffness aspect of the beam. For design purpose, a beam should be so designed that it has adequate stiffness so that the deflections are within the permissible limits.

## 2.Differential equation of the deflection curve of beam:

$$
\frac{d^{2} y}{d x^{2}}=\frac{d}{d x}\left(\frac{d y}{d x}\right)=\frac{d \theta}{d x}=-\frac{M}{E I}
$$

This equation can be integrated in each particular case to find the angle of rotation $\theta$ (usually called the slope) or the deflection $y$ provided the bending moment $M$ is known.

Sign Conventions: The following sign conventions are adopted:
(i) $x$ is positive when measured towards the right.
(ii) y is positive when measured downwards.
(iii) $\theta$ is positive when the rotation is clockwise from the $x$-axis.
(iv) $M$ is positive when sagging, i.e., when it produces compression in the upper portion of the beam.

## Methods of determining deflection of beams:

(a). Double integration method: $\mathrm{EI} \frac{\mathrm{dy}}{\mathrm{dx}}=\mathrm{EI} \cdot \theta=-\int \mathrm{M}$
(b). Area moment method (Mohr's method): This method utilizes the properties of the area of the bending moment diagram and the moment of that area.

- The moment-area method is a semi graphical procedure that utilizes the properties of the area under the bending moment diagram. It is the quickest way to compute the deflection at a specific location if the bending moment diagram has a simple shape.


Moment Diagram

## Theorems of Area-Moment Method

## - Theorem 1

$>$ The change in slope between the tangents drawn to the elastic curve at any two points $A$ and $B$ is equal to the product of $1 / E I$ multiplied by the area of the moment diagram between these two points

$$
\theta_{A B}=(1 / E I) *(\text { Area between } A \text { and } B)
$$

## - Theorem 2

> The deviation of any point $B$ relative to the tangent drawn to the elastic curve at any other point $A$, in a direction perpendicular to the original position of the beam, is equal to the product of $1 / E I$ multiplied by the moment of an area about $B$ of that part of the moment diagram between points $A$ and $B$.

$$
t_{B / A}=(1 / E I) *(\text { Area between } A \text { and } B) * X_{B}^{-}
$$

and

## $t_{A / B}=(1 / E I)($ Area between $A$ and $B) * X^{-} A$

Method of Superposition: The method of superposition, in which the applied loading is represented as a series of simple loads for which deflection formulas are available. Then the desired deflection is computed by adding the contributions of the component loads(principle of superposition).
Note:
(i). Non- zero slope cross-section should be a cross-section where slope \& deflection are to be determined.
(ii). $\bar{X}$-should be measured from origin (i.e., non-zero slope cross-section).
(c). Conjugate beam method (method of elastic weights): A conjugate beam is an imaginary secondary beam, which when loaded with the M/EI diagram of the real beam, yield
directly the slope and deflection of the real beam in the form of shear force and bending moment of the conjugate beam.
Preposition 1: Similarity between S.F. of beam and slope of real beam.

$$
\mathrm{F}_{\mathrm{x}}=\int \frac{\mathrm{d}^{2} \mathrm{y}}{\mathrm{dx}^{2}} d x=\frac{\mathrm{dy}}{\mathrm{dx}}=\text { slope }
$$

Preposition 2: Similarity between B.M. of conjugate beam and deflection of real Shear force.

$$
M_{x}^{\prime}=\int_{0}^{x} \frac{d y}{d x} d x=y=\text { deflection }
$$

| Rules | Existing support condition <br> of actual beam | Corresponding support condition <br> for the conjugate beam |
| :---: | :---: | :---: |
| Rule -1 | Fixed end | Free end |
| Rule -2 | Free end | Fixed end |
| Rule -3 | Simple support at the end | Simple support at the end |
| Rule -4 | Simple support not at the end | Unsupported hinge |
| Rule -5 | Unsupported hinge | Simple support |

## (d). Castigliano's first theorem (deflection from strain energy):

It may be observed that though the differentiation with respect to load may be carried out before or after the integration, the calculations are simplified if the differentiation is carried out before the integration. In case of a beam,

$$
\mathrm{U}=\int_{0}^{1} \frac{\mathrm{M}^{2} \cdot \mathrm{dx}}{2 \mathrm{EI}}
$$

Deflection: $\delta_{i}=\frac{\partial \mathrm{U}}{\partial \mathrm{W}_{\mathrm{i}}}=\frac{1}{\mathrm{EI}} \int_{0}^{1} \mathrm{M} \cdot \frac{\partial \mathrm{M}}{\partial \mathrm{W}_{\mathrm{i}}} \mathrm{dx}$
The slope $\theta_{i}$ of a beam at a point can also be obtained by applying a virtual couple $M_{i}$ at the point and putting the same to zero before integration.

$$
\theta_{\mathrm{i}}=\frac{\partial \mathrm{U}}{\partial \mathbf{M}_{\mathrm{i}}}=\int_{0}^{1} \frac{\mathrm{M}}{\mathrm{EI}} \frac{\mathrm{M}}{\partial \mathrm{M}_{\mathrm{i}}} \mathrm{dx}=\frac{1}{\mathrm{EI}} \int_{0}^{1} \mathrm{M} \cdot \frac{\partial \mathrm{M}_{\partial \mathrm{W}}}{\partial \mathrm{~W}_{\mathrm{i}}} \mathrm{dx}
$$

(e). Maxwell's reciprocal deflection theorem: "The deflection of any point P resulting from application of a load at any other point $Q$ is the same as the deflection of $Q$ resulting from the application of the same load at $\mathrm{P}^{\prime \prime}$.

## Deflections and slopes of different beams under different loadings:

## Case (1): Deflection and slope under constant bending moment M:

$$
\theta_{\max }=\left[\frac{1}{\mathrm{~K}_{1}}\right]\left[\frac{\mathrm{ML}}{\mathrm{EI}_{\mathrm{N} . \mathrm{A} .}}\right]
$$

And

$$
\mathrm{y}_{\max }=\left[\frac{1}{\mathrm{~K}_{2}}\right]\left[\frac{\mathrm{ML}^{2}}{\mathrm{EI}_{\text {N.A. }}}\right]
$$

Where $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ are constants.

## Case (2): In presence of concentrated point loads:

$$
\begin{gathered}
\theta_{\max }=\left[\frac{1}{\mathrm{~K}_{1}}\right]\left[\frac{\mathrm{WL}^{2}}{\mathrm{EI}_{\text {N.A. }}}\right] \\
\mathrm{Y}_{\max }=\left[\frac{1}{\mathrm{~K}_{2}}\right]\left[\frac{\mathrm{WL}^{3}}{\mathrm{EI}_{\text {N.A. }}}\right]
\end{gathered}
$$

## Case (3): In presence of distributed loads:

$$
\mathrm{Q}_{\max }=\left[\frac{1}{\mathrm{~K}_{1}}\right]\left[\frac{\mathrm{wL}^{3}}{\mathrm{EI}_{\text {N.A. }}}\right]
$$

$$
\mathrm{Y}_{\max }=\left[\frac{1}{\mathrm{~K}_{2}}\right]\left[\frac{\mathrm{wL}^{4}}{\mathrm{EI}_{\text {N.A. }}}\right]
$$

$$
\mathrm{y}_{\max }=\theta_{\max }\left[\frac{\mathrm{k}_{1}}{\mathrm{k}_{2}}\right] \mathrm{L}
$$

| S. No. | Type of beam | K1 | K2 |
| :---: | :---: | :---: | :---: |
| 1 |  | 1 | 2 |
| 2 | ( ¢ $_{\text {M }}$ | 2 | 8 |
| 3 |  | 2 | 3 |
| 4 |  | 16 | 48 |


| 5 |  | - | 192 |
| :---: | :---: | :---: | :---: |
| 6 | 丰 | 6 | 8 |
| 7 |  | 24 | $\frac{384}{5}$ |
| 8 |  | - | 384 |
| 9 |  | 24 | 30 |
| 10 |  | $\theta_{\mathrm{B}}=\frac{\mathrm{Wb}}{3 \mathrm{EI}}\left[\mathrm{I}_{\mathrm{NA} .}-\mathrm{ab}\right]$ <br> (Not maximum) | $Y_{B}=\frac{w a^{2} b^{2}}{3 E I_{\text {N.A. }}{ }^{L}}$ |

## GATE/ESE

Civil Engineering

## Structural Analysis

## Important Formula Notes

## IMPORTANT FORMULAS ON STRUCTURAL ANALYSIS

## CHAPTER 1: INDETERMINACY \& STABILITY OF A STRUCTURE

## 1. External Indeterminacy

Mathematically, external indeterminacy can be expressed as follows.

$$
\mathrm{D}_{\mathrm{sc}}=\mathrm{r}-\mathrm{s}
$$

Where,
$r=$ total number of unknown support reactions.
$S=$ total number of equilibrium equations available.
$\mathrm{S}=3$ (For 2D structure) and 6 (For 3D structure)
2. Internal Indeterminacy

## Case 1: Beam

There is no internal indeterminacy for beams because if we know the support reactions, we can find the axial force, shear force and bending moment at any section in the beam.

## Case 2: Trusses

The internal indeterminacy for the trusses can be determined by following expression.

$$
\begin{aligned}
& D_{s i}=m-(2 j-3) ; \text { for plane truss } \\
& D_{s i}=m-(3 j-6) ; \text { for space truss }
\end{aligned}
$$

Where,
$\mathrm{m}=$ number of members
$\mathrm{j}=$ number of joints

## 3. KINEMATIC INDETERMINACY ( $\mathrm{D}_{\mathrm{K}}$ ):

It is defined as the number of independent displacements at all joints in a structure. Displacements are counted always only at the joints. Displacement includes slopes and deflection. Wherever the cross-section area, changes or material changes then it is treated as a joint in any structure.
The kinematic indeterminacy can be determined for various cases as follows.

## Case 1: Beams

## Example:


$\rightarrow$ Displacement at $A$ and $B$ in x-direction is zero
$\rightarrow$ Displacement at $A$ and $B y$-direction is zero
$\rightarrow$ Rotation at $A$ and $B$ is zero
$\therefore$ Degree of freedom $=D_{k}=0$
$D_{k}($ inextensible $)=D_{k}$ (extensible) - Number of independent displacements prevented.
Note: It not given in the question, then assume that members are extensible.
Example:


Sol.
Degree of freedom $D_{k}=2 \times 3-5+4$ (Due to internal Hinge) $=5$
Ignoring axial deformation, $D_{K}=5-2=3$
Case 2: Truss
At each joint in a truss number of independent displacements are only two (horizontal and vertical displacement). Rotation of a member in a truss is not considered because it implies that the member buckled. Rigid body rotation is not counted because it is not unknown.

$D_{k}$ at $A=0$
$D_{k}$ at $B=1$
$D_{k}$ at $C=1$
$D_{k}$ at $D=2$
$D_{k}$ at $E=2$
$D_{k}$ at $F=2$
So, degree of freedom $=0+1+1+2+2+2=8$

## Case 3: Frames

(i) Count only one rotation for all members meeting at a rigid joint.
(ii) Count rotation of all members meeting at a pin joint.

$D_{k}$ at $A=0$
$D_{k}$ at $B=2$
$D_{k}$ at $C=1$
$D_{k}$ at $D=1$
$D_{k}$ at $E=5$
$D_{k}$ at $F=6$
$\mathrm{D}_{\mathrm{k}}$ at $\mathrm{G}=3$
$\mathrm{D}_{\mathrm{k}}$ at $\mathrm{H}=3$
$D_{k}$ at $I=3$
$D_{k}$ at $\mathrm{J}=3$
$D_{k}$ at $K=3$
$D_{k}$ at $L=3$
$\therefore \mathrm{D}_{\mathrm{k}}$ when extensible $=0+2+1+1+5+6+3+3+3+3+3+3=33$ degree
$D_{k}$ when inextensible
$=D_{k}$ (extensible) - Number of independent displacements prevented.
= $33-14=29$ degrees.

## Example:



Sol.
Total degree of freedom $D_{k}=3 \times 5-3+4$ (Due to internal Hinge) $=16$
If members are considered inextensible then, $D_{k}=16-5=8$

## 4. STABILITY OF STRUCTURE

The stability of structure includes external stability and internal stability. The external stability deals with support reaction and internal stability deals within the structure.

### 4.1. External Stability

Minimum number of reactions required for a structure to be stable externally is 3 .
These three reactions must be non-concurrent and non-parallel.
If three reactions are parallel then rigid body translation take place. If they are concurrent, then rigid body rotation takes place.

If the structure becomes unstable due to the improper arrangement of three reactions, then it is known as geometrically unstable structure.

If structure becomes unstable due to less than 3 support reactions, then it is called statically unstable structure.

### 4.2. Internal Stability

Internal stability of various cases is explained through the following examples:
Case 1: Beams
$\rightarrow$ Internal floating hinge


The above structure is internally unstable.

## Case 2: Trusses

In case of trusses if following condition exist then it is classified as unstable truss.

$$
\mathrm{m}<(2 \mathrm{j}-3)
$$

Where,
$\mathrm{m}=$ number of members in truss structure.
$j$ = number of joints in truss structure.
Case 3: Frames
If reactions are parallel to each other, then the frame structure will be termed as unstable.


The above shown structure is unstable due to presence of reactions which are parallel.

## CHAPTER 2: ANALYSIS OF A TRUSS

Definition: A truss is an assembly of beams or other elements that creates a rigid structure. In engineering, a truss is a structure that consists of two force members only, where the members are organized so that the whole assembly should behave as a single object.

## 1. ASSUMPTIONS USED IN TRUSS ANALYSIS

(i) Members of the truss will be subjected to axial force only.
(ii) Members are initially straight and load is acting only on joints
(iii) All joints can be assumed as frictionless hinges.
(iv) All the members of truss are assumed in the same plane called the middle plane of truss.

## 2. ANALYSIS OF STATICALLY DETERMINATE AND STABLE TRUSSES

There are two methods of analysis for statically determinate and stable trusses.
(i) Method of Section
(ii) Method of Joints

## Methods of section

The advantage of this method is that, force in an intermediate member can be found directly without finding force in any other members. Equilibrium of sections of truss is considered in method of section. The procedure of this method comprises of following steps.
(i) Determine the value of support reaction.
(ii) Cut the member under consideration by a section (1)-(1) and consider equilibrium of either left hand side of (1)-(1) or R.H.S. of (1)-(1) and use $\Sigma F_{x}=0, \Sigma F_{y}=0$ and $\Sigma M$ $=0$ to find unknown forces in members.
(iii) Cut the member such that entire truss is divided into two separate parts.

(iv) Preferably, don't cut more than 3 members (because, in method of section, we have 3 equilibrium equation which are $\Sigma F_{x}=0, \Sigma F_{y}=0 \Sigma M=0$ with 3 equilibrium equation we can easily find 3 unknown values.
(v) Cut the member such that all the cut members do not meet at one joint. If they meet at one joint, $\Sigma \mathrm{M}=0$ becomes useless equation and it becomes method of joints problem.

## Methods of Joints

The free body diagram of any joint is a concurrent force system in which the summation of moment will be no help. Recall that only two equilibrium equations can be written as $\Sigma F_{x}=0$ and $\Sigma F_{y}=0$. This means that to solve completely for the forces acting on a joint, we must select a joint with not more than two unknown forces involved. This can be started by selecting a joint acted on by only two members. We can assume any unknown member to be either tension or compression. If negative value is obtained, this means that the force is opposite in action to that of assumed direction. One the forces in on joint are determined their effect on adjacent joints are known. We then continue solving on successive joints until all members have been found.

## 3. ZERO FORCE MEMBERS

Zero force members in a truss are members which do not have any force in them. There are two rules that may be used to find zero force members in a truss. They are as follows.

Case 1: At a two-member joint which are not parallel and there are no other external loads or reaction at the joint then both members are zero force members.
Case 2: At a three-member joint, if two of those members are parallel and there are no other external loads (or reaction) at the joint then the member that is not parallel is a zero-force member.

## CHAPTER 3: METHODS OF STRUCTURAL ANALYSIS

- Force Method: The force method of analysis, also known as the method of consistent deformation, uses equilibrium equations and compatibility conditions to determine the unknowns in statically indeterminate structures. This means that there is one reaction force that can be removed without jeopardizing the stability of the structure.
- Types of Force Method:


## Castigliano's Method or Strain Energy Method

Virtual work method

## Column Analogy Method

Flexibility Matrix Method

- Displacement Method: In the displacement method of analysis, the primary unknowns are the displacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure.
- Types of Displacement Method:

Slope Deflection Method.
Moment Distribution Method.
Direct Stiffness Method.

- Approximate Methods:

Portal Method.
Cantilever Method.
Points of Inflection Method.
Kani's Method.

## CHAPTER 4: FORCE METHOD OF ANALYSIS

## 1. Energy methods:

Energy methods are based on linear elastic behaviour of material and conservation of energy i.e. work done by external forces is equal to the energy stored in the structure under the load.
Strain Energy in various cases is given by following expressions.
In Axial tension or compression, $U=\frac{P^{2} L}{2 A E}$
In Bending, $U=\frac{M^{2} L}{2 E I}$
In Torsion, $U=\frac{T^{2} L}{2 G J}$

## 2. Castigliano's Method

As per Castigliano's theory

$$
\Delta=\frac{\partial U}{\partial P}
$$

And,

$$
\theta=\frac{\partial U}{\partial M}
$$

This relation can also be used in finding deflection in the beams as explained in the following example.

Example: Find rotation and deflection at free end in the beam shown in the figure below:

(a) Rotation at the free end:

Bending Moment at a distance x from the free end $M_{X}=-M$
So, the strain energy stored in the beam

$$
U=\int_{0}^{L / 2} \frac{M^{2} d x}{2 E I}+\int_{L / 2}^{L} \frac{M^{2} d x}{4 E I}
$$

So, rotation at the free end,

$$
\theta_{B}=\frac{\partial U}{\partial M}=\int_{0}^{L / 2} \frac{M d x}{E I}+\int_{L / 2}^{L} \frac{M d x}{2 E I}=\frac{3 M L}{4 E I}
$$

(b) Deflection at free end:

Applying vertical load $P$ at the free end


Bending Moment at a distance x from free end $M_{X}=-M-P x$
So, Strain energy stored in the beam

$$
U=\int_{0}^{L / 2} \frac{(-M-P x)^{2} d x}{2 E I}+\int_{L / 2}^{L} \frac{(-M-P x)^{2} d x}{4 E I}
$$

So, Deflection at free end,

$$
\begin{gathered}
\Delta_{B}=\left.\frac{\partial U}{\partial P}\right|_{P=0}=\int_{0}^{L / 2} \frac{(M+P x) x d x}{E I}+\left.\int_{L / 2}^{L} \frac{(M+P x) x d x}{2 E I}\right|_{P=0} \\
\\
\Rightarrow \Delta_{B}=\int_{0}^{L / 2} \frac{M x d x}{E I}+\int_{L / 2}^{L} \frac{M x d x}{2 E I}=\frac{5 M L^{2}}{16 E I}
\end{gathered}
$$

3. Unit Load Method

Deflection at a point as per unit load method is given by

$$
\Delta=\int \frac{M_{x} m_{x} d x}{E I}
$$

Where,
$M_{x}$ is the bending moment due to external loading.
$m_{x}$ is the bending moment due to virtual unit load.
$E I$ is the flexural rigidity of the beam.
The application of unit load method is explained using the example given below.

## 4. Maxwell Law of Reciprocal Theorem

This law states that in a linearly elastic structure, the deflection at any point A due to loading at some point $B$ will be equal to deflection at $B$ due to loading at $A$.
Betti's Theorem: This is a generalised case of Maxwell reciprocal theorem. As per this theorem the virtual work done by P system of forces in going through the deformation of Q system of forces is equal to virtual work done by Q system of forces in going through the deformation of $P$ systems of forces.


Virtual work done by $P$ system of forces due to the displacements caused by Q system of forces $=P_{1} \delta_{1 Q}+P_{2} \delta_{2 Q}$

Similarly,
Virtual work done by Q system of forces due to the displacements caused by P system of forces $=Q_{1} \Delta_{1 P}+Q_{2} \Delta_{2 P}$

As per Maxwell-Betti's Theorem

$$
P_{1} \delta_{1 Q}+P_{2} \delta_{2 Q}=Q_{1} \Delta_{1 P}+Q_{2} \Delta_{2 P}
$$

## 5. THEOREM OF LEAST WORK

This is a special case of Castigliano's theorem. This theorem states that for any statically indeterminant structure, the redundant should be such that strain energy of the system is minimum.

Thus,

$$
\frac{\partial U}{\partial R}=0
$$

Where,
$U=$ Strain energy stored in the system
$R=$ Redundant force

## 6. DEFLECTION OF STATICALLY DETERMINATE TRUSSES

Two methods mainly used to calculate deflection in trusses are
(i) Castigliano's Method
(ii) Unit load method

### 6.1. Castigliano's Method

For getting the deflection in case of truss, there are two theorems. According to these theorem deflection and slope can be determined as follows.

## (i) Castigliano's $I^{\text {st }}$ theorem:

$$
W=\frac{\partial U}{\partial \delta}
$$

Here,
$\mathrm{w}=$ load
$\partial u=$ change in strain energy
$\partial \delta=$ variation in deflection.

## (ii) Castigliano's II ${ }^{\text {nd }}$ theorem

It states, that the first partial derivative of total strain energy with respect to a load at any point in the structure gives deflection at that point in the direction of load.

$$
\delta=\frac{\partial U}{\partial P} \quad \theta=\frac{\partial U}{\partial M}
$$

## Application of Castigliano's theorem:

(i) To find absolute deflection of a joint in a truss.

$\mathrm{U}=$ Strain energy in all members

$$
\mathrm{U}=\frac{\Sigma \mathrm{P}^{2} \mathrm{I}}{2 \mathrm{AE}}
$$

Where, $\mathrm{P}_{1}, \mathrm{P}_{2} \ldots \mathrm{P}_{\mathrm{n}}=$ force in members due to applied load w . and $I_{1}, I_{2} \ldots . L_{n}=$ length of each member.
From Castigliano's II theorem

$$
\Rightarrow \quad \delta_{E}=\frac{\partial U}{\partial W}=\Sigma \frac{2 P \frac{\partial P}{\partial W} l}{2 A E}=\Sigma \frac{P k l}{A E}
$$

Where, $k=k_{1}, k_{2}=\frac{\partial P_{1}}{\partial w}, \frac{\partial P_{2}}{\partial w}=$ force in all members due to unit load applied at a point where we have to find deflection ( $\delta$ ).


If we want to find relative displacement of any two joints $B$ and $E$, apply unit loads at $B$ and $E$ in the direction $B E$. Find forces in all members due to this load then relative displacement of two joints $B$ and $E$ is

$$
\delta_{\mathrm{BE}}=\Sigma \frac{\mathrm{PkI}}{\mathrm{AE}}
$$

Where,
$P=P_{1}, P_{2}$ etc forces in all member due to applied loads unit loads
$K=$ forces in all members due to unit loads applied at $B$ and $E$.
If we want to find rotation of any member FG, apply unit couple at $G$ and $F$ (these two forces form unit couple i.e., $1 / a \times a=1$ ). Find forces in all members due to these two loads, then rotation of member is given as

$$
\theta_{\mathrm{GF}}=\Sigma \frac{\mathrm{PkI}}{\mathrm{AE}}
$$

Where, $P_{1}, P_{2} \ldots P_{n}=$ force in all members.
$k=$ forces in all member due to unit couple applied at $G$ and $F$.

### 6.2. Unit Load Method

This method is based on method of virtual work. From virtual work principle external work done on a body is equal to internal work done by the body.
If a unit virtual load produces internal stresses $u_{i}$ in the member and the real displacement of the $\mathrm{i}^{\text {th }}$ member is $\mathrm{dli}_{\mathrm{i}}$ then the internal virtual work done is equal to $\Sigma u_{i} d l_{i}$. If the virtual load at any point is 1 and the displacement at that point due to external forces is $\Delta$ then,

$$
1 \times \Delta=\Sigma u_{i} d l_{i}
$$

## (i) Due to external loading:

Deflection of truss due to external loading is given by

$$
\Delta=\sum \frac{P u L}{A E}
$$

Where,
$P=$ Force in the member due to external loading
$u=$ Force in the member due to unit load applied in the direction at the point where deflection is to be calculated after removal of external loading
$L=$ Length of the member
$\mathrm{AE}=$ Axial rigidity of the member

## (ii) Temperature Change Case:

Due to temperature change,

$$
d l_{i}=l_{i} \alpha_{i} \Delta T_{i}
$$

So,

$$
\Delta=\Sigma u_{i} l_{i} \alpha_{i} \Delta T_{i}
$$

Where,
$\Delta=$ Deflection of truss due to temperature change
$u_{i}=$ Forces in the member due to unit load at the point where deflection is to be computed.
$l_{i}=$ length of the member
$a_{i}=$ Coefficient of linear expansion
$\mathrm{T}_{\mathrm{i}}=$ Temperature increment
(iii) Fabrication Error (Lack of Fit Case): If the member is shorter or longer in length, it will induce stresses in truss. In this case, the joint deflection is calculated as

$$
\Delta=\Sigma u_{i} d L_{i}
$$

Where,
$\Delta=$ Deflection of truss due to fabrication error
$u_{i}=$ Forces in the member due to unit load at the point where deflection is to be computed.
$\mathrm{dl}_{\mathrm{i}}=$ Fabrication error

Definition: Slope deflection method is a classical method which is very useful in analysis of indeterminate structure like continuous beams and plane frames. In this method the unknown loads are written in terms of the displacement by using the load-displacement relations, then these equations are solved for the displacements using the joint equilibrium conditions. After computation of displacements, loads are computed using load displacement relations.

## Important Results:

(i) Angular Displacement at A:


The load displacement relations are

$$
\begin{aligned}
& M_{A B}=\frac{4 E I}{L} \theta_{A} \\
& M_{B A}=\frac{2 E I}{L} \theta_{A}
\end{aligned}
$$

## (ii) Angular Displacement at B:


(iii) Relative linear displacement, $\delta$ :


## Sign Convention:

(i) Clockwise moment is taken as positive.
(ii) If $\delta$ gives clockwise rotation to member, it is considered as positive.

FIXED END MOMENTS FOR SOME STANDARD CASES

| Beam | Fixed End Moments |
| :---: | :---: |
|  | $M_{A}=M_{B}=\frac{P I}{8}$ |
|  | $M_{A}=\frac{P b^{2} a}{L} \quad M_{B}=\frac{P a^{2} b}{L}$ |
|  | $M_{A}=\frac{w L^{2}}{12} \quad M_{B}=\frac{w L^{2}}{12}$ |
|  | $M_{A}=\frac{w L^{2}}{20} \quad M_{B}=\frac{w L^{2}}{30}$ |
|  | $M_{A}=\frac{6 E I \Delta}{L^{2}} \quad M_{B}=\frac{6 E I \Delta}{L^{2}}$ |
|  | $M_{A}=\frac{5 w L^{2}}{96} \quad M_{B}=\frac{5 w L^{2}}{96}$ |
|  | $M_{A}=\frac{3 E I \Delta}{L^{2}}, M_{\mathrm{B}}=0$ |

## Slope Deflection Equations:

$$
M_{A B}=M_{F A B}+\frac{4 E I}{L} \theta_{A}+\frac{2 E I}{L} \theta_{B}-\frac{6 E I \delta}{L^{2}}
$$

$$
\Rightarrow M_{A B}=M_{F A B}+\frac{2 E I}{L}\left(2 \theta_{A}+\theta_{B}-\frac{3 \delta}{L}\right)
$$

And,

$$
\begin{aligned}
& M_{B A}=M_{F B A}+\frac{2 E I}{L} \theta_{A}+\frac{4 E I}{L} \theta_{B}-\frac{6 E I \delta}{L^{2}} \\
& \Rightarrow M_{B A}=M_{F B A}+\frac{2 E I}{L}\left(\theta_{A}+2 \theta_{B}-\frac{3 \delta}{L}\right)
\end{aligned}
$$

## CHAPTER 6: DISPLACEMENT METHOD OF ANALYSIS (MOMENT DISTRIBUTION

## METHOD)

## 1. INTRODUCTION

It is a displacement method for analysis of statically indeterminate beams and frames developed by Hardy Cross. The method only accounts for flexural effects and ignores axial and shear effects. In this method it is assumed in the beginning that all joints of the structure are fixed. Then by locking and unlocking each joint in succession, the internal moments are distributed such that each joint attains its final position.

## 2. IMPORTANT DEFINITIONS

### 2.1. Stiffness Factor

Stiffness factor can be defined as the moment required to produce unit rotation in the beam. Stiffness factor for various cases is defined as follows.
Case 1: Far end is fixed


Stiffness factor $=s=\frac{4 E I}{l}$
Case 2: Far end is hinged


Stiffness factor $=s=\frac{3 E I}{l}$

### 2.2. Relative stiffness (k)

Relative stiffness is the relative value of the stiffness factor. It is value for various cases can be expressed as follows.

Case 1: Far end is fixed

$$
\mathrm{k}=\frac{\mathrm{I}}{\mathrm{~L}}
$$

Case 2: Far end is hinged

$$
\mathrm{k}=\frac{3}{4} \frac{\mathrm{I}}{\mathrm{~L}}
$$

Case 3: Far end is free

$$
K=0
$$

### 2.3. Distribution Factor

It is the ratio in which the applied moment is distributed to various members meeting at a rigid point. Sum of distribution factor of all members meeting at a rigid joint is one. If far end is free, its $D, k$ and distribution factor is zero.

$$
D F=\frac{K}{\Sigma K}
$$

Where,
$K=$ Relative stiffness of the member
$\Sigma K=$ Summation of relative stiffness of all members meeting at a joint

### 2.4. Carry over moment

It is the moment developed at one end due to applied moment at the other end.
It is developed to make the slope zero. It is exerted by the fixed support on the beam. It is developed to make slope zero not to keep the structure in equilibrium. Various case for carry over moment are as follows.
Case 1: Far end is fixed

$$
\mathrm{COM}=\frac{\mathrm{M}}{2}
$$

Case 2: Far end is hinged

$$
C O M=0
$$

### 2.5. Carry Over Factor

Carry over factor can be defined as the ratio of carry over moment and applied moment.
Carry over factor for various cases can be given as follows.
Case 1: Far end is fixed

$$
\mathrm{COF}=\frac{\mathrm{M}}{\frac{2}{M}}=\frac{1}{2}
$$

Case 2: Far end is hinged

$$
C O F=\frac{0}{M}=0
$$

## 3. ANALYSIS OF BEAMS USING MOMENT DISTRIBUTION METHOD

### 3.1. Sign convention

(i) Clockwise end moments and clockwise rotations are taken as positive. Anticlockwise end moments and anti-clockwise rotations are taken as negative.
(ii) Bending moment which is sagging in nature are taken as positive and hogging bending moment is taken as negative.

### 3.2. Procedure

Step 1: Find out Distribution factors and fixed end moments.
Step 2: Assume all joints to be initially locked. Then Determine the moment needed to bring each joint in equilibrium. Release the joints and distribute the counterbalancing moment into the connecting span at each joint. Carry these moments in each span over to its other end.
Repeat the same cycle until the moment equilibrium at the joint achieved.

## CHAPTER 7: ARCHES \& CABLES

## 1. TYPES OF ARCHES

There are three types of arches depending upon the number of hinges provided.
(i) Three hinged arch (Determinate)
(ii) Two hinged arch (Indeterminate to 1 degree)
(iii) Fixed arch (Indeterminate to 3 degree)

### 1.1. Three hinged Arch

The three hinged arches are statically determinate structure as equations of equilibrium alone are sufficient to find all the unknown quantities.


## Circular Arch:

From the properly of a circle the radius $r$ of the circular arch of span $L$ and rise $h$ may be found as

$$
\begin{aligned}
& \frac{L}{2} \times \frac{L}{2}=h(2 R-h) \\
& \Rightarrow R=\frac{L^{2}}{8 h}+\frac{h}{2}
\end{aligned}
$$

Taking origin at A, the coordinates of any pont $d$ on the arch may be defined as

$$
\begin{aligned}
x & =\left[\frac{L}{2}-R \sin \theta\right] \\
y & =R \cos \theta-(R-h) \\
\Rightarrow y & =h-R(1-\cos \theta)
\end{aligned}
$$

## Parabolic Arch:

Taking spring point as the origin, its equation is given by

$$
y=\frac{4 h_{x}}{L^{2}}(L-x)
$$

Bending moment at the section $X-X$

$$
\begin{gathered}
\mathrm{B} \mathrm{M}_{\mathrm{x}}-\mathrm{x}=+\mathrm{V}_{\mathrm{A}} \times \mathrm{x}-\mathrm{H}_{\mathrm{A}} \times \mathrm{y} \\
\Rightarrow \mathrm{BM} \mathrm{x}-\mathrm{x}=\text { Beam moment }-\mathrm{H} \text {-moment }
\end{gathered}
$$

When compared with a beam of similar span, bending moment at any section in a three hinged arch is less by an amount of ' $\mathrm{H} \times \mathrm{y}^{\prime}$ ' or moment dur to horizontal force.

## 2. Two hinged arches

A two hinged arch is an indeterminate arch. The horizontal thrust is determined using Castigliano's theorem of least energy.


Assuming the redundant to be H , As per Castigliano's theorem

$$
\frac{\partial U}{\partial H}=0
$$

Which gives the following condition

$$
H=\frac{\int_{0}^{1} \frac{M_{x} y d x}{E I_{C}}}{\int_{0}^{1} \frac{y^{2} d x}{E I_{C}}}
$$

Where,
$M_{x}=$ beam moment at any section $x-x$
$I_{c}=$ Moment of inertia of the cross section of the arch at the crown.
(i) Horizontal Thrust in case of circular arch subjected to point load

$$
H=\frac{W}{\pi} \sin ^{2} \alpha
$$

(ii) Horizontal Thrust in case of circular arch subjected to UDL

$$
H=\frac{4}{3} \frac{w R}{\pi}
$$

(iii) Horizontal Thrust in case of parabolic arch subjected to a point load at centre

$$
H=\frac{25}{128} \frac{w L}{H}
$$

(iv) Horizontal Thrust in case of parabolic arch subjected to a UDL

$$
H=\frac{w l^{2}}{8 h}
$$

If there is rib shortening, temperature rise by $t^{\circ} \mathrm{C}$ and yielding of supports then horizontal thrust is given by

$$
H=\frac{\int \frac{M_{x} y d x}{E I_{C}}+\alpha t l}{\int \frac{y^{2} d x}{E I_{C}}+\frac{l}{A E}+k}
$$

Where,
$\alpha \mathrm{tl}=$ due to increase in temperature
I/AE= due to rib shortening
$\mathrm{K}=$ yielding of support/unit horizontal thrust.
In a two hinged parabolic arch as the temperature increase, horizontal thrust increases. If the effect of rib shortening and yielding of support are considered, then horizontal thrust decreases.

## 3. CABLES:

Suspension Cables are generally used to support suspension bridges, roofs and cable car system. They are used to transmit load from one structure to another. Cables are deformable structure so they can undergo change in shape according to externally applied load. Thus, bending moment and shear force at every point in the cable is zero.

### 3.1. CABLES SUBJECTED TO CONCENTRATED LOADS:

When cables are subjected to concentrated loads, it may take shape of several straight-line members subjected to tension. The shape cable takes is known as funicular polygon. The cable will always be subjected to pure tensile forces having the funicular shape of load.
Consider the cable subjected to concentrated loads $P_{1}$ and $P_{2}$ as shown in the figure. Due to the loading, cable assumes the shape ACDB.


The tension at each member can be easily determined using the equation of equilibrium at each joint and the geometry of the structure.

### 3.2 CABLES SUBJECTED TO UNIFORMLY DISTRIBUTED LOAD:

The tension in the cable at any distance x will be

$$
\begin{aligned}
T & =\sqrt{\left(F_{H}\right)^{2}+\left(w_{0} x\right)^{2}} \\
\Rightarrow T & =\sqrt{\left(\frac{w_{0} L^{2}}{2 h}\right)^{2}+\left(w_{0} x\right)^{2}}
\end{aligned}
$$

Tension will be maximum when x is maximum i.e., $\mathrm{x}=\mathrm{L}$.

$$
T_{\max }=w_{0} L \sqrt{1+\left(\frac{L}{2 h}\right)^{2}}
$$

## CHAPTER 8: INFLUENCE LINE DIAGRAM

## 1. MULLER BRESLAU PRINCIPLE:

As per this principle, "If an internal stress component or reaction component is considered to act through and tends to deflect a structure than the deflected shape of
the structure will be the influence line for the stress or reaction component to some scale."

Note: This Principle gives for quantitative and qualitative deflected shape for determinate structure and qualitative deflected shape for indeterminate structures.
2. MAXIMUM SHEAR FORCE AND BENDING MOMENT FOR A BEAM SUBJECTED TO MOVING LOADS

### 2.1. Due to Single Point Load

The influence line diagram for a single point load for Shear force and Bending moment is


So, bending moment will be maximum if the load is at the section. For maximum negative shear force the load should be just to the left of section and for maximum positive shear force the load should be just right to the section.

## Absolute Maximum shear force and Bending Moment:

## Absolute maximum shear force:

For absolute maximum negative shear force, $x / L$ should be maximum. Thus, for absolute maximum negative shear force the value of $x$ would be $L$ i.e. at support B. For absolute maximum positive shear force, $\left(1-\frac{x}{L}\right)$ should be maximum. Thus, for absolute maximum positive shear force the value of $x$ would be zero i.e. at support A.

## For absolute maximum bending moment:

$$
\begin{gathered}
M_{x}=\frac{W x(L-x)}{L} \\
\frac{d M_{X}}{d x}=\frac{W(L-2 x)}{L}=0
\end{gathered}
$$

Thus, absolute maximum bending moment will occur at mid span in case of point load.

### 2.2. Due to Uniformly Distributed Load Longer than span

Maximum negative shear force occurs when the load covers portion AC only and maximum positive shear force occurs when the load covers the portion CB only. Maximum bending moment at any section will be due to UDL covering the entire span.

## Absolute Maximum Value of SF and BM anywhere in the span:

For Absolute maximum Shear Force:
On observing the influence line diagram for Shear force, it is clear that maximum negative shear force will occurs at support $B$, when the UDL covers the entire span and maximum positive shear force will occurs at support A, when the UDL covers the entire span.

For Absolute maximum bending moment:
Maximum bending moment at any section,

$$
\begin{gathered}
M_{x}=\frac{1}{2} \times w \times \frac{x(L-x)}{L} \times L \\
\frac{d M_{X}}{d x}=\frac{w(L-2 x)}{2}=0 \\
\Rightarrow x=\frac{L}{2}
\end{gathered}
$$

Thus, absolute maximum bending moment will occur at mid span in case of UDL larger than the span.

### 2.3. UDL shorter than the span

For UDL shorter than the span, maximum negative shear force will take place when entire UDL is just left of the section and maximum positive shear force will take place when entire UDL is just right to the section.

For maximum bending moment at C


Load should be placed such that

$$
\frac{a}{b}=\frac{x}{L-x}
$$

### 2.4. Due to Train of Concentrated Loads

Maximum Bending Moment at a section: Due to train of concentrated loads maximum bending moment will occur at the section if the loads are placed such that the average loading to the left of the section is equal to average loading to the right of the section.
Maximum Bending Moment under a wheel load: Maximum bending moment under a wheel load occurs if the loads are placed such that the load and the resultant of the loading is equidistant from the centre of span.
Maximum bending moment will occur under $W_{3}$ if the loads are placed as shown below.


## CHAPTER 9: MATRIX METHOD OF ANALYSIS

## 1. DISPLACEMENT METHOD/STIFFNESS METHOD:

In this method displacements at the joints are taken as unknowns and equation are expressed in terms of these unknown displacement. Additional joint equilibrium equations are developed to find the unknown displacement. This method is suitable when the Kinematic indeterminacy is less than the static indeterminacy.

### 1.1. Stiffness (k)

It is the load required to produce unit displacement. Stiffness for various cases are as follows.

(1) Axial stiffness $\left(k_{11}\right)=\frac{A E}{l}$
(2) Transverse stiffness $\left(k_{22}\right)=\frac{12 E I}{\beta^{3}}$
(3) Flexural stiffness $\left(\mathrm{k}_{33}\right)=\frac{4 E I}{\mathrm{l}}$
(4) Torsional stiffness $\left(\mathrm{k}_{44}\right)=\frac{\mathrm{GJ}}{\mathrm{l}}$

### 1.2. Procedure to Construct Stiffness Matrix

To get first column of stiffness matrix, fix all the coordinates and give unit displacement at the $1^{\text {st }}$ coordinate and find forces developed at all other coordinates similarly to get the second column of stiffness matric apply unit displacement at coordinate 2 and find forces at all coordinates.


The cantilever beam shown in the figure above will be subjected to three displacements (1), (2) and (3).
When the unit displacement is given in direction of (1) i.e., horizontal deflection only,
$\mathrm{K}_{11}=$ Force at (1) due to unit displacement at (1) $=\frac{A E}{L}$
$\mathrm{K}_{21}=$ Force at (2) due to unit displacement at (1) $=0$
$\mathrm{K}_{31}=$ Force at (3) due to unit displacement at (1) $=0$
When the unit displacement is given in direction of (2) i.e., vertical deflection only,

$\mathrm{K}_{12}=$ Force at (1) due to unit displacement at (2) $=0$
$\mathrm{K}_{22}=$ Force at (2) due to unit displacement at (2) $=\frac{2 E I}{l^{3}}$
$\mathrm{K}_{32}=$ Force at (3) due to unit displacement at $(2)=-\frac{6 E I}{l^{2}}$
When the unit displacement is given in direction of (3) i.e., rotation only,

$\mathrm{K}_{13}=$ Force at (1) due to unit displacement at (3) $=0$
$\mathrm{K}_{23}=$ Force at (2) due to unit displacement at (3) $=-\frac{6 E I}{l^{2}}$
$\mathrm{K}_{33}=$ Force at (3) due to unit displacement at (3) $=\frac{4 E I}{l}$
So, the stiffness matrix is

$$
K=\left[\begin{array}{ccc}
\frac{A E}{L} & 0 & 0 \\
0 & \frac{2 E I}{l^{3}} & -\frac{6 E I}{l^{2}} \\
0 & -\frac{6 E I}{l^{2}} & \frac{4 E I}{l}
\end{array}\right]
$$

## 2. FLEXIBILITY MATRIX METHOD:

In this method, forces are taken as unknown and equations are expressed in terms of these forces. Additional equation called compatibility condition are developed to find all
the unknown forces. This method is suitable when the static indeterminacy is less than kinematic indeterminacy.

### 2.1. Flexibility ( $\delta$ )

Flexibility is defined as the displacement produced due to unit force. It is the inverse of stiffness. Flexibility for various cases are as follows
(a) Axial flexibility $=\frac{1}{\frac{A E}{l}}=\frac{l}{A E}$
(b) Transverse flexibility $=\frac{1}{\frac{A E}{l^{3}}}=\frac{l^{3}}{12 A E}$
(c) Flexural flexibility $=\frac{1}{\frac{4 A E}{l}}=\frac{l}{4 E I}$
(d) Torsional flexibility $=\frac{1}{\frac{G J}{l}}=\frac{l}{G J}$

### 2.2. Procedure to construct Flexibility Matrix

To get the first column of flexibility matrix, apply unit force at coordinate (1) and find displacement at all coordinates in the released structure. Similarly, to get II column of the flexibility matrix apply unit force at coordinate (2) and find displacement at all coordinates in the released structure.


The cantilever beam shown in the figure above is subjected unit forces in three directions.
When the unit force is applied in direction of (1)
$\delta_{11}=$ displacement at coordinate (1) due to unit load at coordinate (1) $=\frac{l}{A E}$
$\delta_{21}=$ displacement at coordinate (2) due to unit load at coordinate (1) $=0$
$\delta_{31}=$ displacement at coordinate (3) due to unit load at coordinate $(1)=0$
When the unit load is applied in the direction of (2)

$\delta_{12}=$ displacement at coordinate (1) due to unit load at coordinate $(2)=0$
$\delta_{22}=$ displacement at coordinate (2) due to unit load at coordinate $(2)=\frac{b^{3}}{3 E I}$
$\delta_{32}=$ displacement at coordinate (3) due to unit load at coordinate (2) $=-\frac{t^{2}}{2 E I}$ When the unit load is applied in the direction of (3)

$\delta_{13}=$ displacement at coordinate (1) due to unit load at coordinate (3) $=0$
$\delta_{23}=$ displacement at coordinate (2) due to unit load at coordinate (3) $=-\frac{l^{2}}{2 E I}$
$\delta_{33}=$ displacement at coordinate (3) due to unit load at coordinate (3) $=\frac{l}{E I}$
So, the flexibility matrix is

$$
[\delta]=\left[\begin{array}{ccc}
\frac{l}{A E} & 0 & 0 \\
0 & \frac{l^{3}}{3 E I} & -\frac{l^{2}}{2 E I} \\
0 & -\frac{l^{2}}{2 E I} & \frac{l}{E I}
\end{array}\right]
$$

## GATE/ESE

Civil Engineering

Surveying

## Important Formula Notes

## IMPORTANT FORMULAS ON SURVEYING

## CHAPTER 1: FUNDAMENTALS OF SURVEYING

## 1. INTRODUCTION

- Surveying: Surveying is the art of determining the relative positions of points on above or beneath the surface of the earth by means of direct or indirect measurements of distance, direction and elevation.
- All measurements of lengths in surveying are either horizontal or are reduced to horizontal distances, i.e., the plotted measurements are projections on horizontal plane.
- The primary objective of a survey is creating a map or a plan to represent an area on the horizontal plane, to layout or mark out the proposed structure.


## 2. CLASSIFICATIONS OF SURVEYS

Primarily, Surveying can be divided into two classes:

- Plane Surveying: It is that type of surveying where the surface of the earth is considered plane and spheroidal shape is neglected.
- Geodetic Surveying: It is that type of survey where the shape of the earth (oblate spheroid) is considered. When, for a triangle having an area of 195.5 sq. km., the spherical excess become greater than one second.


### 2.1. Classification based on the object of survey

- Topographical surveying: Details on man-made and natural features on earth surface including their elevation details, used to obtain map of the area.
- Engineering survey: Done for engineering works to obtain data for design of projects such as building, roads, railways, reservoirs etc.
- Cadastral survey: Used to establish property boundaries.
- Military survey: Used for determining points of strategic importance.
- Mine survey: Used for exploring areas containing mineral wealth.
- Geological survey: Used for determining different strata in the earth's crust.
- Marine or Hydrographic survey: It deals with bodies of water for the purpose of the navigation, harbour works or for the determination of the mean sea level.
- Astronomical survey: These surveys are done in order to determine position of celestial bodies like stars and planets.


### 2.2. Classification based on instruments used

(i) Chain survey, (ii) Theodolite survey, (iii) Traverse survey, (iv) Triangulation survey, (v) Tacheometric survey, (vi) Plane table survey, (vii) Photogrammetric survey, (viii) Aerial survey

## 3. PRINCIPLES OF SURVEYING

- Working from Whole to Part: main objective of working from whole to part is to localise the error and prevent error accumulation. If we work from part to whole, errors will get accumulated, and it get maximised to greater extent.
- Location of a point with reference to two reference points: At least two points of reference are used to locate the relative positions of the points to be surveyed.

4. SCALE: Scale can be represented by (i) Numerical scale (ii) Graphical scale.

- Numerical scale is of two type (a) Engineering scale: $1 \mathrm{~cm}=10 \mathrm{~km}$ (b) Representative scale (R.F): 1:100000
- Graphical scale is a line drawn on map and marking ground distance directly on it. Graphical scale has advantage over numerical scale because distance on the map can be calculated even though map has shrunk because scale also shrunk in the same ratio with the shrinkage of the map.
- Larger is the denominator smaller is the scale. Normally map has smaller scale and plan has larger scale. In simple words zoomed out picture is of larger scale.

5. VERNIER: Vernier is a device used for measuring the reading which are fractional part of smallest division on main scale.

- Exact measurement $=$ reading before index mark on main scale reading $+(\mathrm{N} \times$ least count)

Here, ' N ' numbering of division which is matching with main scale division at the time of measurement.

- In direct vernier $n$ division of vernier equal to ( $n-1$ ) division of main scale and least count for direct vernier is $\mathrm{S}-\mathrm{V}$, where S is size of one division on main scale and V is size of one division on vernier scale. It comes out to be L.C. $=\mathrm{S} / \mathrm{n}$.
- In retrograde vernier $n$ division on vernier scale are equal to ( $n+1$ ) division on main scale. Here least count will be V-S, which also comes out to be $\mathrm{S} / \mathrm{n}$.
- Least Count: It is the smallest measurement that can be made using any measuring device.

6. SHRUNK SCALE: original scale is always larger than shrunk scale, S.F. is always less than 1.

Shrinkage factor or shrinkage ratio $=$ SF $=\frac{\text { Shrunk Length }}{\text { OriginalLength }}=\frac{\text { ShrunkScale }}{\text { OriginalScale }}=\frac{\text { ShurnkR.F. }}{\text { OriginaIR.F. }}$
7. ERRORS DUE TO USE OF WRONG SCALES

Correct length $=\frac{\text { R.F of wrong scale }}{\text { R.F of correct scale }} \times$ measured length
Correct area $=\left(\frac{\text { R.F of wrong scale }}{\text { R.F of correct scale }}\right)^{2} \times$ calculated area.

## CHAPTER-2-LINEAR MEASUREMENT

1. METHOD OF LINEAR MEASURMENT:

- Direct Measurement: Chain or tape.
- Measurements by Optical Means: Tacheometry.
- Electronic distance measuring instrument (EDMI): Total station.

2. INSTRUMENTS FOR CHAINING
2.1. Chain: There are following types of chains.

- Metric chain: 20 m [100 links]/30m [150 links].
- Gunter's chain: 66 ft [100 links].
- Engineers chain: 100 ft [100 links].
- Revenue chain: 33 ft [16 links].
2.2. Tape: Tapes are classified depending on material of the tape.
- Cloth Tape: It is rarely used for accurate measurements because it is affected by moisture, likely to twist and has the problem of stretching.
- Metallic Tape: It is made of brass and copper. It is superior to cloth tape.
- Steel Tape: It is made of steel and superior to metallic tape.
- Invar Tape: It is made from an alloy of nickel (36\%) and steel (64\%). Its coefficient of thermal expansion is very low.


### 2.3. Location Devices

(i) Arrows: These are used by leader to mark end points of the chain length which is later collected by the follower.
(ii) Pegs: Wooden pegs are used to mark definite points on the ground semi permanently or temporarily.

### 2.4. Ranging devices

(i) Ranging Rod: Used to locate intermediate points along a straight line. It has a length of either 2 m or 3 m ; the 2 m length being more common.
(ii) Offset Rod: It is ranging rod with slot made at right angle, it can be used to establish perpendicular offset with respect to survey line.

### 2.5 Instrument used for setting perpendicular lines

- cross staff/open cross staff
- French cross staff: can be used to establish angles at $45 / 135$ degree as well.
- Optical square: index mirror->silvered fully, horizon mirror->half silvered.
- prism square: similar arrangement as that of optical square only difference is mirror are replaced with prism.
- Note: Simple clinometer is used to measure slope of ground or vertical angle.


## 3. DIFFERENT STATIONS AND LINES IN CHAIN SURVEYING

- Main station: Main station is the point where two sides of a triangle meet these lines decides boundary of survey area, ex: -A, B, C, D.
- Main Line: Line joining main stations.
- Base Line: Biggest central line which will divide the whole area into two parts.
- Check Line: Any line used to check accuracy of survey is known as check line.
- Tie station/subsidiary station/auxiliary station: These are stations on main line.
- Tie Line: Line joining tie station, used for detailing of features in an area.



## 4. ERROR AND DIFFERENT CORRECTIONS:

- Error = Measured value - True value
- Correction = True value - measured value
- Correction = -Error

NOTE: Remember SS, if tape is actually SHORT then correction will be SUBTRACTIVE.

### 4.1. Correction Due to Incorrect Length of Chain/Tape

$\therefore$ correct length $=\left(\frac{\text { actual length }}{\text { nominal length }}\right) \times$ measured length

### 4.2. Correction Due to Slope:

$\theta=$ Slope of the ground,
$L_{o}=$ Measured length,
$\mathrm{C}_{\text {slope }}=\mathrm{L}_{\mathrm{o}}(1-\cos \theta)$,
correction due to slope is always negative.
$C_{\text {slope }}=-\frac{h^{2}}{2 \mathrm{~L}}-\frac{h^{4}}{8 L^{3}}$, if higher terms are ignored then $C_{\text {slope }}=-\frac{h^{2}}{2 \mathrm{~L}}$

## Note: Hypotenuse's allowance $=\mathbf{L}(\mathbf{s e c} \boldsymbol{\theta}-1)$

### 4.3. Correction Due to Temperature

$\mathrm{T}_{\mathrm{m}}=$ Temperature at the time of measurement
$\mathrm{T}_{0}=$ Temperature at the time of standardization
$\alpha=$ Coefficient of Thermal Expansion
$\mathrm{C}_{\mathrm{Temp}}=\mathrm{L} \alpha\left(\mathrm{T}_{\mathrm{m}}-\mathrm{T}_{\mathrm{o}}\right)$

### 4.4. Correction Due to Pull

$P_{0}=$ Pull at time of standardization
$\mathrm{P}_{\mathrm{m}}=$ pull at time of measurement
$A_{x}=$ cross section area of tape.
$E_{x}=$ modulus of elasticity of tape.
$\mathrm{L}_{0}=$ measured length
$\mathrm{C}_{\text {pull }}=\frac{\left(P_{m}-P_{0}\right) L_{0}}{A_{x} E_{x}}$

### 4.5. Correction Due to Sag

W = total weight of tape
$\mathrm{w}=$ weight per m length
$\mathrm{P}_{\mathrm{m}}=$ pull at time of measurement
$\mathrm{n}=$ number of bays.

$$
\mathrm{C}_{\text {sag }}=\frac{\left(W^{2} L_{0}\right)}{\left(24 n^{2} P_{m}^{2}\right)}=\frac{\left(w^{2} L_{0}^{3}\right)}{\left(24 n^{2} P_{m}^{2}\right)}
$$

This correction is always negative.
Normal Tension: It is the value of pull such that Positive Pull Correction= Negative Sag Correction

$$
\frac{\left(P_{m}-P_{0}\right) L_{0}}{A_{x} E_{x}}=\frac{W^{2} L_{0}}{24 P_{m}^{2}}
$$

### 4.6. Correction due to Misalignment

$$
\begin{aligned}
& \mathrm{h}=\text { perpendicular deviation } \\
& \mathrm{C}_{\mathrm{h}}=\left(\sqrt{L_{1}^{2}-h^{2}}+\sqrt{L_{2}^{2}-h^{2}}\right)-\left(L_{1}-L_{2}\right)
\end{aligned}
$$

This correction is always negative.

### 4.7. Correction due to Mean Sea Level

D = Equivalent length at MSL
$h=$ Mean equivalent of the base line above MSL
R = Radius of earth
$C_{m s l}=-\frac{L h}{R}$
5. LIMITING LENGTH OF OFFSET: The maximum length of offset allowed in chain survey due to which no error is reflected on the map. Maximum length of error allowed on the map is 0.025 cm .

### 5.1. Error in Laying Direction



Here,
$P=$ actual point on ground
$P_{1}=$ Point located on paper
$\theta=$ error in laying direction
Scale of map, $1 \mathrm{~cm}=\mathrm{S} \mathrm{m}$
Limiting length of offset, $\mathrm{I}=\frac{0.025 S}{\sin \theta}$

### 5.2. Error in Laying Direction as Well as in Linear Measurement



Here,
$\mathrm{P}=$ Actual point on ground
$P_{1}=$ point located on paper
$\theta=$ error in laying direction
$x=$ error in linear measurement
Scale of map, $1 \mathrm{~cm}=\mathrm{S} \mathrm{m}$

$$
\text { Limiting length of offset, } I=\frac{\sqrt{(0.025 S)^{2}-x^{2}}}{\sin \theta}
$$

## CHAPTER-3-COMPASS SURVEYING

1. INTRODUCTION: Objective of the compass survey is to find bearing for any line or to relate different features in terms of angular measurement on horizontal plans.
2. BEARING AND ANGLES: The horizontal angle measured for a survey line with respect to fixed direction (meridian) is called bearing.


There are two methods for designation of bearing:
2.1. Whole Circle Bearing (WCB): In this system, the bearing of a line is measured with magnetic north in clockwise direction. The value of the bearing thus varies from $0^{\circ}$ to $360^{\circ}$. Prismatic compass is graduated on this system.

2.2. Quadrantal Bearing System (QBS) or Reduced Bearing System: In this system, the bearing of a line is measured eastward or westward from north or south, whichever is nearer. The angle varies between $0^{\circ}$ to $90^{\circ}$. This system is used in surveyor's compass.


## 3. MERIDIAN

3.1. True Meridian: Line joining the true north and true south pole of earth along the earth's curvature is known as true meridian. Bearing measured for any line with respect to true meridian is known as true bearing. It is always fixed.
3.2. Magnetic Meridian: Line joining the magnetic north and magnetic south pole of earth along the magnetic flux line is called magnetic meridian. The bearing measured for any line with respect to magnetic meridian is known as magnetic bearing.
3.3. Grid Meridian: For survey of a country the true meridian passing through central area is taken as reference meridian for the whole country and such a reference meridian is called grid meridian. Ex: 82.5-degree East, Allahabad. The bearing measured for any line with respect to Grid meridian is known as grid bearing.
3.4. Arbitrary meridian: it is the meridian taken in arbitrary direction generally it is taken in the direction of a well-defined point at the end of the day. The bearing measured for any line with respect to arbitrary meridian is known as arbitrary bearing.
4. MAGNETIC DECLINATION: Magnetic declination at a place is the horizontal angle between true meridian and magnetic meridian. Its value keeps on changing. Eastward declination is taken as positive, while westward declination is taken as negative.
Note: Declination at a place changes from time to time and declination at same time changes from place to place.

Isogonic Lines: Lines connecting the points having same magnetic declination.
Agonic Lines: Lines connecting the points having zero magnetic declination.
Variations in magnetic declinations are categorized as following:

## (i) Diurnal Variation:

- Daily variation in magnetic declination
- More at magnetic poles and less at equator.
- More in summer, less in winters.
- More in day, less at night.
- It also changes year to year.
(ii) Annual Variation:
- Yearly variation in magnetic declination. It is not same as annular rate of change of secular variation.
- caused due to revolution of earth around sun.


## (iii) Secular Variation:

- Magnitude of change is very high and this variation follows sine curve
- Time period is approximately 250-300 Years.


## (iv) Irregular Variation:

- Random variation in magnetic declination.
- These occur due to "magnetic storms" like earthquakes and other solar influences.


## Conversion of true bearing to magnetic bearing and reverse:

True bearing $=$ Magnetic bearing $+\delta_{E}$
Note: Care should be taken such that the bearing mentioned are in whole circle system and eastward declination is positive and westward is negative.

5. ANGLE OF DIP: Vertical angle of magnetic flux lines measured with respect to earth's surface is known as angle of dip.
Isoclinic Lines: Lines connecting the points having same angle of dip. Ex: Iatitude.
Aclinic Lines: Lines connecting the points having zero angle of dip. Ex: equator.
Angle of dips has following value:
$\theta=0^{\circ}$ at equator (as flux lines are parallel to earth's surface at equator)
$\theta=90^{\circ}$ at pole (as flux lines are perpendicular to earth's surface at poles)
6. MEASUREMENTS IN COMPASS SURVEY:
(i) Fore bearing: measured in the direction of traverse.
(ii) Back bearing: measured from in the direction opposite to traverse.


Line $A B \rightarrow$ Fore Bearing $=\theta_{A}$ \& Back Bearing $=\theta_{B}$
Line $B A \rightarrow$ Fore Bearing $=\theta_{B} \&$ Back Bearing $=\theta_{A}$
Note: The difference of fore bearing and back bearing for a line is always equal to $180^{\circ}$, provided both the stations are free from local attraction.
7. INCLUDED ANGLE: Included angle is the angle measured in clockwise direction from previous line to the next line. Mathematically, included angle $=$ F.B. of next line - B.B. of previous line $\{+360\}$ if a negative value is obtained, simply add 360 to get required value of included angle.


## 8. CALCULATION OF BEARING FROM INCLUDED ANGLE:

Mathematically, F.B. of next line = B.B. of previous line + included angle \{-360\}
If value comes out to be more than 360, simply subtract 360 to get WCB of F.B. of next line.
9. LOCAL ATTRACTION: Local attraction is a term used to denote any influence which prevents the needle from pointing to magnetic north in a given locality. Influence on the actual reading can be due to heavy steel, nickel objects, electric poles, transmission lines, steel pens, steel buttons, etc.

- Local attraction affects all bearings taken at a point equally. So, the included angle between two survey lines is unaffected.
- For a particular line, if the difference between fore bearing and back bearing is found to be $180^{\circ}$, the lines/measurement associated with this station are said to be free from the local attraction.
- Correction for local attraction is done using concept of included angle, starting from the station where local attraction is zero. For a closed polygon, firstly we need to calculate and correct the error found while closing the polygon. For interior angles, Theoretical Sum $=(2 n-40) 90^{\circ}$ and for exterior angles, Theoretical Sum $=(2 n+$ $40) 90^{\circ}$. Error is distributed equally to all the included angles and then correct bearing is found from station where local attraction is zero.

10. ERRORS IN COMPASS SURVEY: The errors in compass survey may be classified as:
A. Instrumental Error: These are those which arise due to faulty adjustments of the instruments. For example:

- The needle not being perfectly straight.
- Pivot being bent.
- Improper balancing weight.
- Plane of sight not being vertical.
B. Personal Error: These may be due to following reasons:
- Inaccurate levelling of the compass.
- Inaccurate centring.
- Inaccurate bisection of the signals.
- Carelessness in readings and recordings.
C. Error due to Natural Causes: These may be due to the following reasons:
- Variations in declinations.
- Local attractions due to proximity of magnetic material.
- Magnetic change in the atmosphere due to clouds and storms.


## 11. DIFFERENT TYPE OF COMPASS USED:

| Item | Prismatic compass | Surveyor's Compass |
| :---: | :--- | :--- |
| (1) Magnetic <br> Needle | The needle is of 'broad needle' <br> type. Therefore, the needle <br> does not act as an index. | The needle is of 'edge bar' type. <br> Thus, the needle acts as the index <br> also. |
| (2) <br> Graduated <br> ring | (i) The graduated ring is <br> attached to the needle. The ring <br> does not rotate along with the <br> line of sight. | (i) The graduated ring is attached <br> to the box and not to the needle. <br> The graduated ring rotates along <br> with the line of sight. |


|  | (ii) The graduations are in the WCB system, $0^{\circ}$ is at the South end and readings are marked in clockwise direction. <br> (iii)The graduations are engraved inverted. | (ii) The graduations are in the QB system, having $0^{\circ}$ at N and S and $90^{\circ}$ at East and West but East and West are interchanged. <br> (iii)The graduations are engraved erect. |
| :---: | :---: | :---: |
| (3) Sighting <br> Vanes | (i) The object vane consists of a metal vane with vertical hair. <br> (ii) The eye vane consists of a small metal vane with silt. | (i) The object vane consists of a metal vane with vertical hair. <br> (ii) The eye vane consists of a metal vane with a fine slit. |
| (4) Reading | (i) The reading is taken with the help of a prism provided at the eye slit. <br> (ii) Sighting and reading taking can be done simultaneously from one position of the observer. | (i) The reading is taken by directly seeing through the top of the glass. <br> (ii) Sighting and reading taking cannot be done simultaneously from one position of the observer. |
| (5) Tripod | Tripod may or may not be provided. The instrument can be used even by holding suitably in hand. | The instrument cannot be used without a tripod. |

## 12. THEODOLITE:

A theodolite is a versatile instrument basically designed to measure horizontal and vertical angles. It is also used to give horizontal and vertical distances using stadia hairs. Magnetic bearing of lines can be measured by attaching a trough compass to the theodolite.

### 12.1. Definitions and Terms

(1) The vertical axis: The vertical axis is the axis about which the instrument can be rotated in a horizontal plane. This is the axis about which the lower and upper plates rotate.
(2) The horizontal axis: The horizontal or trunnion as is the is about which the telescope and the vertical circle rotate in vertical plane.
(3) The line of sight or line of collimation: It is the line passing through the intersection of the horizontal and vertical crosshairs and the optical centre of the object glass its continuation.
(4) The axis of level tube: The axis of the level tube or the bubble line is a straight line tangential to the longitudinal curve of the level tube at its centre. The axis of the level tube is horizontal when the bubble is central.
(5) Centring: The process of setting the theodolite exactly over the station mark is known as centring.
(6) Transiting: It is the process of turning the telescope in vertical plane through $180^{\circ}$ about the trunnion axis. Since the line of sight is reversed in this operation, it is also known as plunging or reversing
(7) Swinging the telescope: It is the process of turning the telescope in horizontal plane. If the telescope is rotated in clock-wise direction, is known as right swing. If telescope is rotated in the anti-clockwise direction, it is known as the left swing.
(8) Face left observation: If the face of the vertical circle is to the left of the observer, the observation of the angle (horizontal or vertical) is known as face left observation
(9) Face Right observation: If the face of the vertical circle is to the right of the observer, be observation is known as face right observation.
(10) Telescope normal: A telescope is said to be normal or direct when the face of the vertical circle is to the left.
(11) Telescope inverted: A telescope is said to be inverted or reversed if the vertical circle is to the right.
(12) Changing face: It is an operation of bringing the face of the telescope from left to right and vice versa.

### 12.2. THE DESIRED RELATIONSHIP BETWEEN THE FUNDAMENTAL LINES OF THE THEODOLITE

The line of sight, axis, and circles of the theodolite is as follows:


The desired relationship between the fundamental lines of the theodolite is as follows:

1) The axis of the plate level must lie in a plane perpendicular to the vertical axis. If this condition exists, the vertical axis will be truly vertical when the bubble is in the center of its run.
2) The line of collimation must be perpendicular to the horizontal axis at its intersection with the vertical axis. Also, if the telescope is an external focusing type, the optical axis, the axis of the objective slide, and the line of collimation must coincide. If this condition exists, the line of sight will generate a vertical plane when the telescope is rotated about the horizontal axis.
3) The horizontal axis must be perpendicular to the vertical axis. If this condition exists, the line of sight will generate a vertical plane when the telescope is plunged.
4) The altitude level (or telescope level) axis must be parallel to the line of collimation. If this condition exists, the vertical angles will be free from index error due to a lack of parallelism.
5) The vertical circle vernier must read zero when the line of collimation is horizontal. If this condition exists, the vertical angles will be free from index error due to Displacement of the vernier.
6) The axis of the striding level (if provided) must be parallel to the horizontal axis. If this condition exists, the line of sight (if in adjustment) will generate a vertical plane when the telescope is plunged, the bubble of striding level being in the centre of its run.

### 12.3 Permanent Adjustment of Theodolite:

The permanent adjustments of a theodolite are done in a prescribed order one adjustment does not affect any other adjustment. The order in which adjustments are to be done is the following.

1. Plate level test to make the plate level at the centre when the vertical axis is truly vertical.
2. Cross hair ring test to make the line of collimation coincide with the optical axis and also to ensure that the line of collimation generates a vertical plane when the telescope is transited.
3. Spire test to make the horizontal axis perpendicular to the vertical axis.
4. Collimation test to make the line of collimation perpendicular to the horizontal axis.
5. Telescope bubble test to centre the telescope bubble when the line of sight is horizontal.
6. Vertical Vernier test to ensure that the vertical circle reads zero when the line of sight is horizontal.

## CHAPTER-4-TRAVERSING

1. TRAVERSING: Traversing is the type of survey in which several connected survey lines form the framework and the direction and lengths of survey lines are measured with the help of an angle measuring instrument and a tape respectively.
It can be of two types: open loop or closed loop
For closed loop starting and end point are of known location, end point can be same as starting point or any other point of known coordinates. For open loop finishing point is of unknown location.

## 2. MESURMENT OF TRAVERSING

(i) Linear measurements: Linear measurements are taken by chain or tape, tacheometry, EDMI.
(ii) Angular measurements: The following methods are Generally used for the measurement of angles in a theodolite traverse.

- Loose needle Method: In loose needle method, the direction of the magnetic meridian is established at each traverse station and the direction of the line is measured with reference to the magnetic meridian. In other words, the magnetic
bearing of each line is measured at each station. The loose needle method is also known as the free needle method.
- Fast Needle Method: In fast needle method, the magnetic meridian is established only at the starting station and the magnetic bearing of the first line is measured. The magnetic bearings of all other lines are determined indirectly from the magnetic bearing of the first line and the included angles. However, the magnetic bearing of the first line has the accuracy of the compass, the difference of bearings of the two adjacent lines has the accuracy of the theodolite. The method is more accurate than the loose needle method and is generally preferred in the field.
- Method of included angle: Traversing by the method of included angles is the generally used method. In this method, magnetic bearing of any of the one line (generally, the initial line) is measured in the field. All the included angles are also measured. Bearing of all the other lines are determined from the bearing of the initial line and the included angles. This method is more accurate than the fast needle method. An included angle is one of the two angles formed at a station by the two traverse lines meeting there.
Note: All angles are measured clockwise. This is done because in theodolite the graduations increase in clockwise direction.
- Method of direct angles: This method is similar to the method of included angle. In this method, directly angles or the angles to the right are measured. The method is generally used in an open traverse.
- Method of deflection angles: The method of deflection angles is mainly used for the open traverse conducted for the survey of roads, railways, canals, pipelines, sewers, etc. where the traverse lines make small deflection angles. Deflection angles are the angles which a line makes with the prolongation of the preceding line.


## 3. PLOTTING A TRAVERSE SURVEY

(i) Angle and Distance Method: The distances between the stations are plotted to a scale and the angles between the lines are plotted by some angle plotting or method such as a protractor or the chord or the tangent method. This method is suitable for small surveys and its accuracy for plotting is not as good as the co-ordinate method.
(ii) Co-ordinate Method: This is the most accurate and the most practical method of plotting traverses. The survey stations are plotted by calculating their co-ordinates. Its most important advantage being that that the closing error can be eliminated by balancing before plotting.

## 4. LATITUDE AND DEPARTURE:

(i) Latitude (L): Projection of a line on North-South (N-S) axis is called latitude. It is considered positive on north axis (northing) and considered negative on south axis (southing).
(ii) Departure (D): Projection of a line on East-West (E-W) axis is called departure. It is considered positive on East axis (easting) and negative on west axis (westing).


Here,

$$
\begin{gathered}
\text { Latitude }=I \cos \theta \\
\text { Departure }=I \sin \theta
\end{gathered}
$$

## 5. INDEPENDENT COORDINATE:

Coordinate of different point with respect to single origin is called independent coordinate.

$$
\begin{aligned}
& \mathrm{A} \equiv\left(\mathrm{~L}_{1}, \mathrm{D}_{1}\right) \\
& B \equiv\left[\left(L_{1}+L_{2}\right),\left(\mathrm{D}_{1}+\mathrm{D}_{2}\right)\right] \\
& \mathrm{C} \equiv\left[\left(\mathrm{~L}_{1}+\mathrm{L}_{2}+\mathrm{L}_{3}\right),\left(\mathrm{D}_{1}+\mathrm{D}_{2}-\mathrm{D}_{3}\right)\right] \\
& D \equiv\left[\left(L_{1}+L_{2}+L_{3}-L_{4}\right),\left(D_{1}+D_{2}-D_{3}-D_{4}\right)\right] \sim(0,0) .
\end{aligned}
$$

6. CLOSING ERROR: After completion of the survey of a closed traverse at time of plotting, if the end point doesn't coincide exactly with the starting point, the closing error is introduced.
Hence, $\Sigma L \neq 0$ and $\Sigma D \neq 0$


Here,

$$
\begin{aligned}
& \text { error= } \mathrm{A}^{\prime} \mathrm{A}, \text { correction= } \mathrm{AA}^{\prime} \\
& \mathrm{e}_{\llcorner }=\text {total error in latitude } \\
& \mathrm{e}_{\mathrm{D}}=\text { total error in departure }
\end{aligned}
$$

The magnitude of closing error, $\mathrm{e}=\sqrt{e_{L}^{2}+e_{D}^{2}}$

$$
\text { Direction }=\theta=\tan ^{-1}\left(\frac{e_{D}}{e_{L}}\right)
$$

## For a closed Traverse:

$$
\Sigma L=0 \text { and } \Sigma \Sigma=0
$$

$$
\text { Where } \Sigma L=\text { Sum of all latitude }
$$

$$
\Sigma \mathrm{D}=\text { Sum of all departure }
$$

## 7. METHODS TO CORRECT CLOSING ERROR

(i) Bowditch method, (ii) Transit method, (iii) Graphical method

### 7.1. Bowditch Method

This method is suitable when linear and angular measurement are both taken with an equal degree of precision.
The basis of this method is on the assumptions that
a) the errors in linear measurements are proportional to $\sqrt{l}$ and
b) the errors in angular measurements are inversely proportional to $\sqrt{l}$, where $I$ is the length of a line.
Correction to lat. (or dep.) of any side $=$ Total error in lat. (or dep.)
$\times \frac{\text { Length of that side }}{\text { Perimeter of the traverse }}$
Let,
$C_{L}=$ correction to latitude or any side
$C_{D}=$ correction to departure on any side
$\Sigma \mathrm{L}=$ total error in latitude
$\Sigma \mathrm{D}=$ total error in departure
$\Sigma I=$ length of the perimeter, and $I=$ length of any side

$$
\text { We have, } C_{L}=\Sigma L \frac{l}{\Sigma l} \text { and } C_{D}=\sum D \frac{l}{\Sigma l}
$$

### 7.2. Transit Method

This method may by employed where angular measurements are more precise than the linear measurements.
According to this rule, the total error in latitude and in departures is distributed in proportion to the latitudes and departures of the sides. It is claimed that the angles are less affected by corrections applied by transit method than by those by Bowditch's method.
Correction to lat.(or dep.)=Total Error in lat.(or dep.) $\times$ $\frac{\text { Latitude (or departure) of that line }}{\text { Arithmetic sum of latitudes (or departures) }}$
Let,
L = latitude of any line
D = departure of any line
$\mathrm{L}_{\mathrm{T}}=$ arithmetic sum of latitudes
$\mathrm{D}_{\mathrm{T}}=$ arithmetic sum of departure

$$
\text { We have, } C_{L}=\Sigma L \cdot \frac{L}{L_{T}} \text { and } C_{D}=\Sigma D \cdot \frac{D}{D_{T}}
$$

### 7.3. Graphical Method

It is a simple method for the application of Bowditch method graphically without calculations. The closing error $A A^{\prime}$ is distributed linearly to all the sides in proportion to their length by the graphical construction. The ordinate $\mathrm{aA}^{\prime}$ is considered equal to the closing error and the corresponding errors $\mathrm{bB}^{\prime}, \mathrm{cC}^{\prime}$ and $\mathrm{dD}^{\prime}$ are found by constructing similar triangles. The lines $\mathrm{D}^{\prime} \mathrm{D}, \mathrm{C}^{\prime} \mathrm{C}, \mathrm{B}^{\prime} \mathrm{B}$ are drawn parallel to the closing error $\mathrm{A}^{\prime} \mathrm{A}$ and are made equal to $\mathrm{dD}^{\prime}, \mathrm{cC}^{\prime}$ and $\mathrm{bB}^{\prime}$ respectively. The polygon $A B C D$ thus obtained represents the adjusted traverse.


CHAPTER-5-LEVELLING

1. INTRODUCTION: Levelling is that branch of surveying the objective of which is

- to find the elevations of given points with respect to a given or assumed datum.
- to establish points at a given elevation or at different elevations with respect to a given or assumed datum.
- Levelling deals with measurements in a vertical plane.

Level Surface: A curved surface in which each point is perpendicular to the direction of gravity at any point. The surface of still water is a truly level surface.

Datum: A datum is a reference level surface for measuring the elevations of the points.
Mean Sea Level is considered a standard datum. It is the mean of 19 years tidal level data considered at Mumbai high.
Horizontal Plane: A plane tangential to the level surface. It is perpendicular to the plumb line.
Elevation: The vertical distance of a point above or below a datum. Vertical distance is always measured along the direction of plumb line. Altitude is vertical distance above the datum, above datum elevation and elevation both are same.

Line of collimation: Line of sight has to be horizontal while taking reading, when LOS become horizontal i.e., perpendicular to plumb line it is called as line of collimation.

## 2. TERMS AND ABBREVIATIONS IN LEVELLING WORK

(i) Reduced level (RL): Reduced level of a point is its height or depth above or below the assumed datum. It is the elevation of the point.
(ii) Benchmark (BM): It is any station or a point which has known reduced level. Reduced levels of other points are found with respect to reduced level of a benchmark.

- GTS Benchmark: Great Triangular Survey benchmark, they are established by survey of India throughout the country with highest precision.
- Permanent Benchmark: These are established at closer intervals between widely spaced GTS BM by SPWD or Survey of India.
- Temporary Benchmark: All the benchmarks which are used for temporary purposes are called temporary benchmarks.
(iii) Station: In levelling, a station is that point where the level rod is held. It is not where the level is set up. It is the point whose elevation is to be determined or the point that is to be established at a given elevation.
(iv) Height of Instrument (HI): The elevation of the plane of sight (Line of Sight) with respect to the assumed datum. It does not mean the height of the telescope above the ground where level stands.
(v) Back Sight (BS): The first reading taken after instrument is set up, with staff held at a point of known elevation. The objective of back sighting is to ascertain the height of the plane of sight.
(vi) Fore Sight (FS): The last reading taken from instrument position. After fore sight, either the instrument is shifted, or the work is closed.
(vii) Intermediate Sight (IS): Every reading except back sight and fore sight is called intermediate sight. Both FS and IS are taken at point of unknown elevation.
(viii) Turning Point (TP): A change point where the instrument is shifted and both FS and BS are taken. The RL will be calculated based on the old HI while the new HI will be calculated from the RL and BS data. This new HI will be used for further stations until another TP is encountered.
(ix) Rocking: to ensure verticality of staff, staff is waved slightly towards instrument and then away from the instrument this process is called rocking of staff, during the process smallest reading should be recorded.
(x) Shimmering: during very intense sunshine, air near the earth surface shimmers hence avoid taking reading for staff height up to 0.5 m .
(xi) Parallax: it is apparent movement of image relative to cross hair.
(xii) Use of Inverted Staff: when the point whose elevation is to be determined whose elevation is much above the line of sight, ex:- soffit level of beam or slab. Then the determination of level of that point is done by taking inverted staff reading. Staff is kept inverted with 0 m at top and hanging downwards. Readings are recorded with a negative sign in level book field book.
R.L. $=$ H.I.- (-staff reading) $=$ H.I. + staff reading.


## 3. OPTICAL DEFECTS IN LENS OF TELESCOPE:

(i) chromatic aberration: It occurs in telescope due to dispersion of white light. White light splits into components of colours, which results in formation of rainbow near the image and focusing become difficult. If chromatic aberration is absent, it is called Achromatic.
(ii) Spherical aberration: This occurs due to the defect in spherical surface of lens. If spherical aberration is absent, it is called Applanation.

## Note: These defects are removed by using combination of (concave lens + convex lens), and liquid used between them is Canada Balsam.

4. THE LEVEL TUBE: The level tube or bubble tube is most important part of the surveying instrument as it is used to make LOS horizontal and plumb line intersect coincide with vertical axis.

Let ' $\theta$ ' be the rotation given to the level tube, and the staff intercept caused by this movement be ' S ', when the staff is at a distance of ' $D$ ' from the instrument. So,

$$
\tan \theta=\theta=\frac{s}{D^{\prime}}
$$

because rotation will be very small.


Within the level tube, the bubble moves by ' $n$ ' divisions. If ' $I$ ' is the length of one of the divisions of the bubble tube and ' $R$ ' is the radius of internal curve of tube,

$$
\theta=\frac{n l}{R}
$$

Combining the two expressions of $\theta$, we get, $\theta=\frac{n l}{R}=\frac{S}{D}$
Sensitiveness: The angular value of one division of the bubble tube.

$$
\text { Sensitiveness }(\alpha)=\frac{\theta}{n}
$$

Substituting the expression of $\theta$, we get, $\alpha=\frac{l}{R}=\frac{s}{n D}$, lesser is value of alpha better is the sensitiveness of the instrument.

Note: Sensitivity is expressed as seconds/division, if length of one division is not given then it is not a definite quantity. If not specified, then length of one division is taken as 2 mm .

Note: Length of air bubble changes with temperature and also under action of gravity.
Note: Liquid used in tube should be stable and non-freezing at room temperature, its viscosity should be low. Generally, used liquids are Chloroform, Spirit and Synthetic Alcohol.

## The Sensitiveness of a Bubble Tube Can Be Increased By

(i) Increasing the radius of curvature of the tube.
(ii) Increasing the diameter of the tube.
(iii) Increasing the length of the air bubble.
(iv) Decreasing the roughness of the walls.
(v) Decreasing the viscosity of the liquid.
(vi) by decreasing the temperature.

## 5. METHODS OF RECORDING READING AND REDUCED LEVELS CALCULATION:

5.1. Height of Instrument Method: In this method, the height of the instrument (HI) is calculated for each setting of the instrument by adding back sight to the elevation of the benchmark.

The elevation of reduced level of the turning point is then calculated by subtracting the FS from HI. For the next setting of the instrument the HI is obtained by adding the back sight taken on TP1 to its RL. The process continues till the R.L of the last point is obtained by subtracting the staff reading from height of the last setting of the instrument.

If there are some intermediate points, the R.L of these points is calculated by subtracting the intermediate sight from the height of the instrument for that setting.

Arithmetic Check: The difference between the sum of back sights and the sum of fore sights should be equal to the difference between the last and the first RL thus

$$
\Sigma B S-\Sigma F S=\text { Last RL }- \text { First RL }
$$

Note: If both BS and FS exist at a station, it is a TP and the height of the instrument will change. The RL will be calculated based on the old HI while the new HI will be calculated from the RL and BS data. This new HI will be used for further stations until another TP is encountered.
5.2. Rise and Fall Method: In this method, the height of instrument is not calculated. The difference between the levels of two consecutive stations is found by comparing the staff readings on the two points for the same setting of the instrument. The difference between their staff reading indicates a rise or fall as the staff reading at the point is smaller or greater than that at the preceding point. The figures for 'rise' and 'fall' worked out thus for all the points given the vertical distance of each point above or below the preceding one and the level of the next will be obtained by adding its rise or subtracting its fall.

Arithmetic Check: The difference between the sum of back sights and sum of fore sights should be equal to the difference between the sum of rise and the sum of fall and should also be equal to the difference between the R.L of last and first point.

$$
\Sigma \text { BS }-\Sigma \text { FS }=\Sigma \text { Rise }-\Sigma \text { Fall }=\text { Last RL }- \text { First RL }
$$

Rise and fall method is better than HI method because,

- It gives better visualisation of terrain.
- R.L. of intermediate sight are also checked.
- Note: staff reading in the methods cannot be checked.


## 7. CORRECTIONS

### 7.1. Correction Due to Earth's Curvature:

Radius of earth, $\mathrm{R}=\mathrm{OA}$
Distance, $d=A C$
Curvature correction, $\mathrm{C}_{\mathrm{c}}=\mathrm{BC}$
$\therefore \mathrm{Cc}=$ curvature correction $=0.07849 \mathrm{~d}^{2}$

## This correction is always negative.

$\therefore C_{c}=-0.0785 \mathrm{~d}^{2}$

Where, $\mathrm{C}_{\mathrm{c}}$ is in ' m ' and d is in ' km '


### 7.2. Correction Due to Refraction:

Correction due to refraction $=C_{R}=\frac{1}{7} C_{C}$
$\Rightarrow C_{R}=\frac{1}{7} \times 0.0785 d^{2}$
$\Rightarrow C_{R}=0.0112 \mathrm{~d}^{2}$
This correction is always positive.
Here, $C_{R}$ is in ' $m$ ' and $d$ is in ' $k m$ '

### 7.3. Combined Correction (C):

$C=C_{c}+C_{R}$
$C=-0.0785 d^{2}+0.0112 d^{2}=>C=-0.0673 d^{2}$
Here, C is in ' m ' and d is in ' km '

## 8. DISTANCE OF VISIBLE HORIZON

$$
d=\sqrt{\frac{C_{C}}{0.06728}} k m, d=3.8553 \sqrt{C_{C}} k m
$$



Here, $\mathrm{C}_{\mathrm{c}}=$ dip of horizon
9. RECIPROCAL LEVELLING: It eliminates the errors due to the curvature of the earth, atmospheric refraction and collimation.

Case 1: When instrument is near $A$,


Staff reading at $A=h_{a}$
Staff reading at $B=h_{b}$
Corrected staff reading at $B=h_{b}-e$
Exact height difference $=H=\left(h_{b}-e\right)-h_{a}$
Case 2: When instrument is near station $B$,


Staff reading at $A=h^{\prime}$

Staff reading at $B=h^{\prime}$
Corrected staff reading at $\mathrm{A}=\mathrm{ha}^{\prime}-\mathrm{e}$
Exact height difference $=H=h_{b^{\prime}}-\left(h_{b}{ }^{\prime}-e\right)$
From equation 1 and 2, we get
Exact height difference ( H ), $H=\frac{\left(h_{b}-h_{a}\right)+\left(h_{b}^{\prime}-h_{a}^{\prime}\right)}{2}$
Here e is total error due to collimation, refraction and curvature.
$e=e_{L}+e_{c}+e_{r}$
$e_{L}=$ collimation error (assumed upwards positive)
$\mathrm{e}_{\mathrm{c}}=$ curvature error (+ve)
$e_{r}=$ refraction error (-ve)
Firstly, calculate correct height difference and then put in any equation to find e and the put values of curvature error and refraction error to calculate error due to collimation.

Collimation Error: This is a type of instrumental error due to which line of collimation (line of sight of the equipment) may not be horizontal even if the levelling bubble is at centre.
11. Contour: Contour is the line joining points of equal or same elevation (RL).

### 11.1.USES OF CONTOURS:

- Proper and precise location of engineering works such as roads, canals. etc.
- In location of water supply, water distribution and to solve the problems of steam pollution.
- In planning and designing of dams, reservoirs, aqueducts, transmission lines, etc.
- In selection of sites for new industrial plants.
- Determining the intervisibility of stations.
- Determining the profile of the country along any direction.
- To estimate the quantity of cutting filling, and the capacity of reservoirs.
11.2.CONTOUR INTERVAL: Elevation difference between two consecutive contours known as contour interval. It is always kept same for a map, so that difference features of area can be easily identified.
11.3.HORIZONTAL EQUIVALENT: Horizontal distance between two consecutive contours at any point called horizontal equivalent. Generally, for natural feature horizontal equivalent are irregular.


### 11.4.SOME IMPORTANT POINTS:

- The horizontal distance between any two contour lines indicates the amount of slope and varies inversely on the amount of slope. Thus, contours are spaced equally for uniform slope, closely for steep slope contours, and widely for moderate slope.
- The variation of the vertical distance between any two contour lines is assumed to be uniform.
- Terrain's steepest slope on a contour is represented along the normal of the contour at that point. Thus, they are perpendicular to ridge and valley lines where they cross such lines.
- Contours do not passthrough permanent structures such as buildings.
- Contours of different elevations cannot cross each other (caves and overhanging cliffs are the exceptions).
- Contours of different elevations cannot unite to form one contour (vertical cliff is an exception).
- The contour line must close itself but need not be necessarily within the limits of the map.
- A closed contour lines on a map represent either depression or hill. For example, a set of ring contours with higher values inside depicts a hill, whereas the lower value inside depicts a depression (without an outlet).
- Contours deflect uphill at valley lines and downhill at ridgelines. Contour lines in the U-shape cross a ridge, and in V-shape cross a valley at right angles. The concavity in contour lines is towards the higher ground in the case of the ridge and towards the lower ground in the case of the valley


## CHAPTER-6-TACHEOMETRY

1. INTRODUCTION: Both horizontal and vertical distances are measured without the use of a chain or tape. It is mostly used for contouring. It is extremely useful for rough terrain that is when chaining is difficult. Its accuracy is less than chaining for flat terrain and more than chaining for rough terrain.
2. BASIC PRINCIPLE OF TACHEOMETRY: The reading against the middle hair is used for finding differences in elevation, the reading against the top and bottom hairs are used to find horizontal distances.


Mathematical equation of tacheometry is as follows.


Note: Telescope for which multiplying constant, $\mathrm{k}=100$ and additive constant, $\mathrm{c}=0$, if the telescope is fixed with Analytic lens.
3. DISTANCE AND ELEVATION FORMULA FOR HORIZONTAL LINE OF SIGHT


Here,
i = stadia internal (size of image)
$\mathrm{s}=$ staff intercept (size of object)
$\mathrm{f}=$ focal length
$d=$ distance of vertical axis of instrument from optical centre of objective lens.
$u=$ distance of object from objective lens
$v=$ distance of image from objective lens
$\mathrm{D}=$ distance between instrument to staff along line of sight.

$$
\begin{gathered}
\Rightarrow D=d+f+\left(\frac{f}{i}\right) s \\
\Rightarrow D=\left(\frac{f}{i}\right) s+(f+d) \\
\Rightarrow D=K s+C
\end{gathered}
$$

Here, $K=$ multiplying constant $=\left(\frac{f}{i}\right)$ and $C=$ additive constant $=(f+d)$

## 4. DISTANCE \& ELEVATION FORMULA FOR INCLINED LINE OF SIGHT

There can be two cases for inclined line of sight

## Case 1: When staff kept vertical (along gravity)

## (a) Angle of elevation:



BM
Staff intercept S = $\left.\mathrm{S}_{3}-\mathrm{S}_{1}\right)$
Staff intercept perpendicular to Line of Sight $=\operatorname{Sos} \theta$
Tacheometry equation

$$
\mathrm{L}=\mathrm{KS} \cos \theta+\mathrm{C}
$$

Horizontal distance (D)

$$
\begin{aligned}
& D=L \cos \theta \\
& D=K S \cos ^{2} \theta+C \cos \theta
\end{aligned}
$$

Elevation (V)

$$
V=L \sin \theta \quad \rightarrow \quad V=\frac{1}{2} K S \sin 2 \theta+C \sin \theta
$$

And,

$$
R L \text { of } B=R L \text { of } B M+B S+V-S_{2}
$$

## (b) Angle of Depression:



Staff intercept $\mathrm{S}=\left(\mathrm{S}_{3}-\mathrm{S}_{1}\right)$
Staff intercept perpendicular to Line of Sight $=S \cos \theta$
Tacheometry equation

$$
\mathrm{L}=\mathrm{kS} \cos \theta+\mathrm{C}
$$

Horizontal distance (D)

$$
\begin{gathered}
D=L \cos \theta \\
D=K S \cos ^{2} \theta+C \cos \theta
\end{gathered}
$$

Depression (V)

$$
\begin{gathered}
V=L \sin \theta \\
V=\frac{1}{2} K S \sin 2 \theta+C \sin \theta
\end{gathered}
$$

And,

$$
R L \text { of } B=R L \text { of } B M+B S-V-S_{2}
$$

## Case 2: When staff kept normal to Line of Sight

## (a) Angle of elevation:



Staff intercept perpendicular to Line of Sight (S) = ( $\left.\mathrm{S}_{3}-\mathrm{S}_{1}\right)$
Tacheometry equation

$$
L=K S+C
$$

Horizontal distance (H)

$$
\begin{gathered}
D=L \cos \theta+S_{2} \sin \theta \\
D=(K S+C) \cos \theta+S_{2} \sin \theta
\end{gathered}
$$

Elevation (V)

$$
\begin{gathered}
V=L \sin \theta \\
V=(K S+C) \sin \theta
\end{gathered}
$$

And,

$$
R L \text { of } B=R L \text { of } B M+B S+V-S_{2} \cos \theta
$$

(b) Angle of Depression:


Staff intercept perpendicular to Line of Sight (S) = $\left(\mathrm{S}_{3}-\mathrm{S}_{1}\right)$
Tacheometry equation

$$
L=K S+C
$$

Horizontal distance (D)

$$
\begin{gathered}
D=L \cos \theta+S_{2} \sin \theta \\
D=(K S+C) \cos \theta+S_{2} \sin \theta
\end{gathered}
$$

Depression (V)

$$
\begin{gathered}
V=L \sin \theta \\
V=(K S+C) \sin \theta
\end{gathered}
$$

And,
$R L$ of $B=R L$ of $B M+B S-V-S_{2} \cos \theta$

## CHAPTER-7-PHOTOGRAMMETRY

1. INTRODUCTION: Photogrammetry is the practice of determining the geometric properties of objects from photographic images. Photogrammetry is called horizontal when camera axis is horizontal and if the camera axis is vertical then it is called vertical photogrammetry.
2. HORIZONTAL PHOTOGRAMMETRY


Let an object AB be photographed by a camera of focal length $f$ at a distance of $D$ from the camera.

$$
\begin{gather*}
\frac{k b}{K B}=\frac{O k}{O K}=\frac{O b}{O B} \\
\frac{x}{X}=\frac{f}{D}=\frac{o b}{O B} \\
\text { from } \Delta \text { Oab and } \triangle \mathrm{OAB} \\
\frac{a b}{A B}=\frac{o b}{O B} \\
\frac{y}{Y}=\frac{o b}{O B} \tag{2}
\end{gather*}
$$

From equation (1) and (2), we get

$$
\frac{x}{X}=\frac{y}{Y}=\frac{f}{D}
$$

As we know, Scale of photograph $=\frac{\text { Photo distane }}{\text { Ground distance }}$ $\Rightarrow$ Scale of horizontal photograph, $\mathrm{S}=\frac{x}{X}=\frac{y}{Y}=\frac{f}{D}$

## 3. HORIZONTAL DISTANCE BETWEEN TWO POINTS


(i) For point A

$$
\begin{gathered}
\text { Scale }=\frac{x_{A}}{X_{A}}=\frac{y_{A}}{Y_{A}}=\frac{f}{\left(H-h_{A}\right)} \\
\Rightarrow X_{A}=\left[\frac{\left(H-h_{A}\right)}{f} \times x_{A}\right] \quad \text { and } Y_{A}=\left[\frac{\left(H-h_{A}\right)}{f} \times y_{A}\right]
\end{gathered}
$$

Here,
$f$ is the focal length, distance between $O$ and picture plate.
$X_{A}$ and $Y_{A}$ are the horizontal distance at ground in a and $y$ direction for point $A$. $x_{A}$ and $y_{A}$ are the horizontal distance at photo in $x$ and $y$ direction for point $A$.
(ii) Similarly, for point $B$

$$
\begin{gathered}
\text { Scale }=\frac{x_{B}}{X_{B}}=\frac{y_{B}}{Y_{B}}=\frac{f}{H-h_{B}} \\
\Rightarrow X_{B}=\left[\left(\frac{H-h_{B}}{F}\right) \times x_{B}\right] \text { and } Y_{B}=\left[\left(\frac{H-h_{B}}{F}\right) \times y_{B}\right]
\end{gathered}
$$

$\therefore$ Horizontal distance between A and $\mathrm{B}=\sqrt{\left(X_{A}-X_{B}\right)^{2}+\left(Y_{A}-Y_{B}\right)^{2}}$

## 4. TYPES OF SCALE

(i) Average Scale: All the points of photograph are assumed to be having average elevation above mean sea level.

$$
S_{a v g}=\frac{x}{X}=\frac{y}{Y}=\frac{f}{H-h_{a v g}}
$$

(ii) Datum Scale: All the points of photograph are assumed to be projected on MSL ( $\mathrm{RL}=0$ )

$$
S_{D}=\frac{x}{X}=\frac{y}{Y}=\frac{f}{H}
$$

## 5. IMPORTANT DEFINITIONS

(i) Exposure Station: Point in the atmosphere occupied by centre of camera lenses at the instant of photography.
(ii) Flying Height: Vertical distance between exposure station and mean sea level.
(iii) Flight Line: Line traced by exposure station in atmosphere (track of aircraft).
(iv) Photo Principal Plane: Point on photograph obtained by projecting camera axis to intersect at a point on photograph.
(v) Photo Nadir Point: Point on photograph obtained by dropping vertical line from camera centre.
(vi) Horizontal Point: Point of intersection of horizontal line through centre of lenses and principal line on photograph.
(viii) Azimuth: Clockwise horizontal angle measured about ground nadir point from true north to the principal plane of photograph.
(viii) Swing: Angle measured in plane of photograph from $+Y$ axis clockwise to photo nadir point.
(ix) Iso Centre: Point on photo where bisection of tilt falls on photo.
6. RELIEF DISPLACEMENT: Any displacement on photo plate which represents height of object on ground is known as relief displacement. It exists because photos are a perspective projection.


Here,
h = height of object
d = radial distance to top of object - radial distance to bottom of object
$r=$ radial distance to top of object
$H=$ height of flight as measured from the bottom of the object
If the average elevation of ground level is havg then relief displacement would be,

$$
d=\frac{h r}{H-h_{a v g}}
$$

Here,
h = height of object
$\mathrm{H}=$ flying height
havg = elevation of ground level
$r=$ distance of image of top from principle point of positive plate.
7. PARALLAX: Parallax is the displacement of two images in successive photographs.

The same tower $A B$ is photographed from two positions, $\mathrm{O}_{1}$ and $\mathrm{O}_{2}$. The camera positions are at a distance $B$ apart from each other.


( $\mathrm{p}_{\text {top }}$ )

From $\Delta o a_{1} a_{2}$ and $\triangle \mathrm{AO}_{1} \mathrm{O}_{2}$,
$\frac{p_{\text {top }}}{F}=\frac{B}{H-h_{\text {top }}}$
Or parallax for top, $p_{\text {top }}=\frac{B F}{H-h_{\text {top }}}$
Similarly, parallax for bottom, $p_{\text {Bottom }}=\frac{B F}{H-h_{\text {Bottom }}}$
Here, $B=$ length of air base, $F=$ focal length of camera
$h_{\text {top }}=$ elevation of tower top, $h_{\text {Bottom }}=$ elevation of tower bottom
$\mathrm{H}=$ flying height

## 8. OVERLAP IN THE PHOTOGRAPHS

Longitudinal overlap $=55$ to $65 \%$
Lateral Overlap $=15$ to $35 \%$
for maximum rectangular area, to be covered by one photograph, the rectangle should have the dimension in the flight to be one-half the dimension normal to the direction of flight. $W=2 B$

## 9. INTERVAL BETWEEN EXPOSURES

$\mathrm{T}=(3.6 \mathrm{~L}) / \mathrm{V}$
T= time interval between exposures in sec.
$\mathrm{V}=$ ground speed of airplane KMPH.
$L=$ ground distance covered by each photograph in the direction of flight in meters.
10. NUMBER OF PHOTOGRAPH TO COVER A GIVEN AREA

$$
\mathrm{N}=\frac{\mathrm{A}}{\mathrm{a}}
$$

$A=$ Total area to be photographed
a = net ground area covered by each photography
$\mathrm{N}=$ number of photographs required.
$a=L \times W$
$\mathrm{L}=\left(1-\mathrm{P}_{\mathrm{L}}\right) . \mathrm{S} . \mathrm{I}$
$\mathrm{W}=\left(1-\mathrm{P}_{\mathrm{w}}\right) . \mathrm{s} \cdot \mathrm{w}$
$a=I . W \cdot S^{2}(1-P L) *(1-P W)$
Where, $\mathrm{I}=$ length of photograph in direction of flight
$\mathrm{w}=$ width of photograph
$\mathrm{P}_{\mathrm{L}}=$ \% overlap in longitudinal direction
$\mathrm{P}_{\mathrm{w}}=\%$ overlap in transverse direction
$S=$ Scale of Photograph $=\frac{H}{f}$

- If instead of total area A, the rectangular dimensions L1 $\times \mathrm{L} 2$ (Parallel and Transverse to flight) are given then, the number of photographs required are given as follows.
$\mathrm{N}=\mathrm{N} 1 \times \mathrm{N} 2$

$$
\mathrm{N}_{1}=\frac{\mathrm{L}_{1}}{\left(1-\mathrm{P}_{1}\right) \mathrm{s} \times 1}+1, \mathrm{~N}_{2}=\frac{\mathrm{L}_{2}}{\left(1-\mathrm{P}_{\mathrm{w}}\right) \mathrm{s} \cdot \mathrm{w}}+1
$$

Let $\quad \mathrm{L} 1=$ Dimension of area parallel to the direction flight
L2 = Dimension of area Transverse the direction of flight
N1 = Number of Photographs in each strip
N2 = Number of strips required.
$\mathrm{N}=$ Total number of photographs to cover the whole area.

## 11. CRAB AND DRIFT

Crab: If the line of photographs is not parallel to fight line, the phenomenon is known as crab. At the instant of exposure, the focal plane of the camera is not square with the line of flight. Crabbing should be eliminated by rotating the camera axis since it reduces the effective coverage of photographs.
Drift: When aircraft is swayed away from its pre-planned flight line then it is known as drift. In case of excessive drifting, refights may need to be made due to gaps in camera coverage.

## CHAPTER-8-CURVES

1. INTRODUCTION: The purpose of the curve is to gradually negotiate the change in direction of the two intersecting straight lines.


## 2. TERMINOLOGY USED IN SIMPLE CIRCULAR CURVES



Point of curve (PC): It is the beginning point where alignment changes from straight line to a curve.

Back tangent: It is the straight line at the beginning of curve tangent to the point of curve.

Point of Tangency (PT): It is the end point where alignment changes from a curve to tangent.
Forward Tangent: It is the straight line at the end of curve tangent to the point of tangency.
Point of Intersection (PI): The point where back and forward tangent intersects when produced is called Point of intersection.
Tangent Length (T): It is the distance from PC to PI or PI to PT.
Deflection Angle ( $\boldsymbol{\Delta}$ ): It is the angle between the back tangent when produced and forward tangent.

Radius of Curvature ( $\mathbf{R}$ ): It is the radius of curve.
Long Chord (LC): It is the straight-line joining point of curve to point of tangency.
Length of curve (L): It is the total length of curve from PC to PT.
Mid Ordinate (M): It is the ordinate from mid-point of long chord to mid-point of curve.

Apex/ External Distance (E): The distance between POI and apex of curve is known as external distance.
3. DESIGNATION OF A CURVE: A curve is designated by either radius of curve or degree of curve. The degree of a curve is the angle subtended at the centre by a chord or arc of specified length.


For 20 m arc length

$$
R=20 \times \frac{360}{2 \pi D}=\frac{1146}{D}
$$

For 30 m arc length

$$
R=30 \times \frac{360}{2 \pi D}=\frac{1719}{D}
$$

4. ELEMENTS OF SIMPLE CIRCULAR CURVE

Formulas to calculate various elements of circular curve are as under:
(i) Tangent Length: $T=R \tan \left(\frac{\Delta}{2}\right)$
(ii) Length of curve: $L=R \times \Delta \times \frac{\pi}{180}$
(iii) Length of Long Chord: $L C=2 R \sin \left(\frac{\Delta}{2}\right)$
(iv) Mid ordinate: $M=R\left(1-\cos \frac{\Delta}{2}\right)$
(v) Apex/External distance: $E=R\left(\sec \frac{\Delta}{2}-1\right)$
(vi) Chainage:
(a) Chainage at PC $=$ Chainage at PI - length of tangent
(b) Chainage at PT $=$ Chainage at PC + length of curve
(c) Chainage at apex point = Chainage of PC + Half the curve length
(vii) Intermediate chord length: All intermediate chords are normal chord except the first and last chord. The tangent points (point of curve and point of tangency) will not be full station i.e. the chainage will not be multiple of full chains. The distance between PC and first peg will be less than normal chord known as sub chord. Same case will be for the distance between last peg and PT.
(a) Length of First chord:

$$
\mathrm{C}_{1}=\left[\left(\begin{array}{l}
\text { Multiple of chainlength } \\
\text { just greater then } \\
\text { chainge at } \mathrm{T}_{1}
\end{array}\right)-\text { Chainage at } \mathrm{T}_{1}\right]
$$

(b) Length of Last chord:

$$
C_{n}=\left[\text { Chainage at } T_{2}-\left(\begin{array}{l}
\text { Multiple of chain length } \\
\text { just less then } \\
\text { chainge at } T_{2}
\end{array}\right)\right]
$$

(c) Number of intermediate (normal) chord:

$$
\mathrm{n}=\left[\left(\frac{L-C_{1}-C_{n}}{\text { Chain length }}\right)\right]
$$

(d) All other chords:

$$
C_{2}=C_{3}=------=C_{n-1}=1 \text { chain length. }
$$

## 5. SETTING OUT OF SIMPLE CIRCULAR CURVE

6.1 Linear Method

Linear method makes use of chain or tape only for setting of curves. Following are the methods available:
(i) Offset from the long chord

$$
\begin{gathered}
O_{o}=R-\sqrt{R^{2}-\left(\frac{L}{2}\right)^{2}} \\
O_{x}=\sqrt{\left(R^{2}-x^{2}\right)}-\left(R-O_{0}\right)
\end{gathered}
$$


(ii) Radial offset method

$$
\left.\Rightarrow O_{x}=\sqrt{\left(R^{2}+x^{2}\right)}-R \ldots . . \text { exact }\right)
$$

Approximately,

$$
o_{x}=\frac{x^{2}}{2 R}
$$



## (iii) Perpendicular offsets method

$$
\Rightarrow O_{x}=R-\sqrt{\left(R^{2}-x^{2}\right)} \ldots . . \text { (exact) }
$$

Approximately,

$$
O_{x}=\frac{x^{2}}{2 R}
$$


(iv) Successive Bisection of Arcs: Join the tangent point $T_{1}$ and $T_{2}$ and bisect the long chord at $D$. Erect the perpendicular DC. Join $T_{1} C$ and $T_{2} C$ and bisect them at $D_{1}$ and $D_{2}$ respectively. At $D_{1}$ and $D_{2}$, set out perpendicular offsets $C_{1} D_{1}=C_{2} D_{2}=$ $R\left(1-\cos \frac{\phi}{4}\right)$ to get points $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ on the curve. More points can be obtained by successful bisection of chords.


## (v) Offsets from the chord produced

This method is very useful for long curves generally used on highway curves when theodolite is not available.


Since $T_{1} V$ is the tangent to the circle at $T_{1}$

$$
\begin{gathered}
\angle T_{1} O A=2 \angle A_{1} T_{1} A=2 \delta \\
T_{1} A=R \times 2 \delta \\
\Rightarrow \delta=\frac{T_{1} A}{2 R}
\end{gathered}
$$

Now,

$$
\operatorname{arc} A_{1} A=O_{1}=T_{1} A \times \delta
$$

Substituting the value of $\delta$, we get

$$
A_{1} A=\frac{T_{1} A^{2}}{2 R}
$$

Taking Arc $T_{1} A=$ chord $T_{1} A$, we get

$$
O_{1}=\frac{C_{1}^{2}}{2 R}
$$

In the similar manner, all other offsets can be obtained. They will be given as

$$
\begin{gathered}
O_{2}=\frac{C_{2}}{2 R}\left(C_{1}+C_{2}\right) \\
O_{n}=\frac{C_{n}}{2 R}\left(C_{n-1}+C_{n}\right)
\end{gathered}
$$

### 6.2 Angular Methods

These methods make use of angle measuring instruments such as theodolite with or without use of distance measuring instruments. Following are the angular methods.
(i) Rankine Method of deflection Angles


A deflection angle at any point is the angle at P.C. between the back tangent and the chord from P.C. to that point. Rankine method is based on the principle that the deflection angle to any point on a circular curve is equal to one half the angle subtended by the arc from P.C. to that point.
From the property of circle,

$$
\begin{gathered}
\angle V T_{1} A=\frac{1}{2} \angle T_{1} O A \\
\Rightarrow \angle T_{1} O A=2 \angle V T_{1} A=2 \delta_{1}
\end{gathered}
$$

Now,

$$
\begin{aligned}
& \frac{\angle T_{1} O A}{C_{1}}=\frac{180^{\circ}}{\pi R} \\
& \angle T_{1} O A=2 \delta_{1}=\frac{180^{\circ} C_{1}}{\pi R} \\
& \Rightarrow \delta_{1}=\frac{90^{\circ} C_{1}}{\pi R}
\end{aligned}
$$

For the first chord $\mathrm{T}_{1} \mathrm{~A}$, the deflection angle $=$ its tangential angle

$$
\Delta_{1}=\delta_{1}
$$

For the second point B,

$$
\begin{aligned}
& \Delta_{2}=\angle V T_{1} B=\angle A_{1} T_{1} A+\angle A T_{1} B \\
\Rightarrow & \Delta_{2}=\delta_{1}+\delta_{2}=\Delta_{1}+\delta_{2}
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
& \Delta_{3}=\Delta_{2}+\delta_{3} \\
& \Delta_{n}=\Delta_{n-1}+\delta_{n}
\end{aligned}
$$

## (ii) Two Theodolite Method

In this method two theodolite are used one at P.C. and other at P.T. This method is based on the principle that the angle between the tangent and the chord is equal to the angle which the chord subtends in the opposite segment.


But $\angle A T_{2} T_{1}$ is the angle subtended by the chord $\mathrm{T}_{1} \mathrm{~A}$ in the opposite segment.

$$
\angle A T_{2} T_{1}=\angle V T_{1} A=\Delta_{1}
$$

Similarly, $\angle V T_{1} B=\angle T_{1} T_{2} B=\Delta_{2}$

## (iii) Tacheometric Method



In this method, a point on the curve is fixed by the deflection angle from the rear tangent and measuring tacheometrically, the distance of that point from $T_{1}$.

$$
\begin{gathered}
T_{1} A=L_{1}=2 R \sin \sin \Delta_{1} \\
T_{1} B=L_{2}=2 R \sin \sin \Delta_{2} \\
T_{1} T_{2}=L_{n}=2 R \sin \sin \Delta_{n}=2 R \sin \sin \frac{\Delta}{2}=L
\end{gathered}
$$

Knowing the lengths, staff intercepts can be calculated.

## 5. TRANSITION CURVE:

- When a vehicle moves on a curve, a centrifugal force acts on it. Thus, sudden transition from a straight path to a circular curve of radius R will introduce the centrifugal force suddenly. Hence a sudden lateral shock will be felt by the passengers. To avoid this, we introduce a curve of varying radius between straight path circular curve such that the radius changes from infinity (i.e. straight line) to a radius R of circular curve. Thus, curve of varying radius is called transitions curve.
- For a transition curve L.R=constant, ideal transition curve are clothoid, the glover spiral or Euler spiral. For ease of setting out we use cubic spiral, however nowadays with electronic instruments even cubic spiral or clothoid can also be set up easily.


## Insertion of Transition Curve

- When transition curves are introduced between the tangents and a circular curve of radius $R$, the circular curve is 'shifted' inwards from its original position by an amount $A B=S$ (the shift) as shown in the above figure such that the curve can meet tangentially.
- This is equivalent to have a circular curve of radius ( $\mathrm{R}+\mathrm{S}$ ) connecting the tangents replaced by two transition curves and a circular curve of radius $R$, although the tangent points are not the same, being A and B .

The amount of shift $\mathrm{S}=\frac{L^{2}}{24 R}$ and $\mathrm{TC}=\mathrm{CD}=\frac{L}{2}$

## Setting out transition Curve

## To locate the tangent point $T$ :

5. Calculate the shift S from the expression below

$$
S=\frac{L^{2}}{24 R}
$$

6. Calculate $\mathrm{VA}=(\mathrm{R}+\mathrm{S}) \tan \frac{\Delta}{2}$
7. Since $\mathrm{TA}=\frac{L}{2}$

$$
\text { Then } \mathrm{VT}=(\mathrm{R}+\mathrm{S}) \tan \frac{\Delta}{2}+\frac{L}{2}
$$

Measure this length back from V and mark/set the point T .
The next step depends on whether it is intended to set out the transition with tapes using the cubic spiral or cubic parabola, or by the theodolite using the cubic spiral.
8. Either calculate offsets from

$$
x=\frac{l^{3}}{6 L R} \quad \text { or } \quad x=\frac{y^{3}}{6 L R}
$$

Each peg is located by swinging a chord length from the preceding peg.


## CHAPTER-9-THEORY OF ERRORS

## 1. INTRODUCTION

Errors in the surveying may be of three types:
(i) Gross Error/ Mistakes
(ii) Systematic Error/ Cumulative Error
(iii) Random Error/ Compensating Error

Gross errors are not the errors but results of mistakes that are due to the carelessness of observer.

Systematic errors follow some pattern and can be expressed by functional relationship based on some deterministic system. Like the gross errors, the systematic errors must also be removed from the measurement by applying necessary corrections.
After all mistake and systematic errors have been detected and removed from the measurements, there will still be some errors in the measurements called the random errors or accidental errors. The random errors are treated using probability models.

## 2. PRECISION AND ACCURACY:

Precision: degree of fineness of care with which any physical measurement is done. Précised value represents set of observation that are closely grouped and have small deviation from the true value.

Accuracy: degree of perfection. A value is said to be accurate if it is close to the true


## 3. BASIC DEFINITIONS

(i) Observed Value: Observed value is the value derived from an observation after correcting from all the errors.
(ii) True Value: The true value of a quantity is the value of quantity which is free from all errors.
(iii) Most probable value: The most probable value of a quantity is the value which has maximum chances of being true value.
(iv) Standard Deviation: Standard deviation also called the root-means square (R.M.S) error, is a measure of spread of a distribution and for the population, assuming the observations are of equal reliability it is expressed as

$$
\sigma_{n-1}=\sqrt{\left[\frac{\sum(\bar{x}-x)^{2}}{(n-1)}\right]}
$$

(v) Variance: It is used as a measure of dispersion or spread of a distribution. It is equal to square of standard deviation. Mathematically,

$$
V=\left[\frac{\sum(\bar{x}-x)^{2}}{(n-1)}\right]=\sigma^{2}
$$

(vi) True Error: It is the difference between true value and observed value of a quantity.
(vii) Residual Error: It is the difference between the most probable value and observed value of a quantity.
v = Most Probable Value - Observed Value
(viii)Standard Error of mean: The standard error of mean $\sigma_{m}$ is given by

$$
\begin{aligned}
\sigma_{m} & = \pm \sqrt{\left[\frac{\sum v^{2}}{n(n-1)}\right]} \\
& = \pm \frac{\sigma}{\sqrt{n}}
\end{aligned}
$$

(ix) Most Probable Error: The most probable error is defined as the error for which there are $50 \%$ chances of the true error will be less than the probable error and $50 \%$ chances that true error will be more than the probable error. The most probable error is given by

$$
\mathrm{e}= \pm 0.6745 \sqrt{\frac{\sum v^{2}}{(n-1)}} \quad \Rightarrow e= \pm 0.6745 \sigma
$$

(x) Most Probable Error of Mean: It is given by, $e_{m}= \pm \frac{e}{\sqrt{n}}$
(xi) Confidence Limits: The range of values within which true value should lie is called confidence interval and its bounds are called the confidence limits.
(xii) Maximum error: it is not possible to exactly estimate the maximum error, as the probability distribution extends to infinity. But the value corresponding to
99.9\% error is assumed to be maximum error in surveying. It corresponds to $\pm 3.29 \sigma$.

## (xiii) Different percentage error:

- $90 \%$ error ( $\mathrm{E}_{90}$ ) corresponds to $\pm 1.645 \sigma$
- $95 \%$ error ( $E_{95}$ ) corresponds to $\pm 1.96 \sigma$
- $68.3 \%$ error ( $\mathrm{E}_{68.3}$ ) corresponds to $\pm 1.0 \sigma$
- $95.5 \%$ error (E95.5) corresponds to $\pm 2.0 \sigma$
- $99.7 \%$ error (E99.7) corresponds to $\pm 3.0 \sigma$
(xiv) Weights: The weight of a quantity is measure of its relative trust worthiness of the set of observations. In other words, weight indicates the relative precision of a quantity within a set of observations. The greater the precision of an observation, the greater will be its weight. The weights are always expressed in numbers. The greater number indicates higher precision and trust in comparison to the lower number.


## Allocation of weights:

- The weights are assigned depending upon the degree of precision. The weights are takes inversely proportional to the variance or square of standard errors (or probable errors). In other words, "For an observation repeated a great number of times, the weight is inversely proportional to the square of the probable error".
- The weights of the quantities measured in similar conditions are assigned in direct proportion to the number of times ( $n$ ) the quantity measured. For example, if an angle $A$ is measured four times, it will have a weight of 4 in comparison to another angle $B$ which is measured only once.
- The weights are sometimes allocated by personal judgment depending on the field prevailing and environmental conditions. The lower weight is allocated to the observations made in difficult terrain under varying environmental conditions of temperature, winds, humidity, etc. Whereas the observation made on a level terrain under stable conditions is given greater weight.
- The weight of a level line is taken as inversely proportional to the length (L) of the route.


## For observations having unequal weight:

- Standard deviation of weighted observation $\sigma_{n-1}= \pm \sqrt{\frac{\sum w_{i} v_{i}^{2}}{(n-1)}}$
- Most Probable error of single observation of weight $\mathrm{w}_{\mathrm{i}}= \pm 0.6745 \sqrt{\frac{\sum w v^{2}}{w_{i}(n-1)}}$
- Most Probable error of weighted arithmetic mean $= \pm 0.6745 \sqrt{\frac{\sum w v^{2}}{\sum w \times(n-1)}}$


## 3. LAWS OF WEIGHTS

(i) The weight of arithmetic mean of a number of observations of equal weight is equal to the number of observations.
(ii) The weight of weighted arithmetic mean of observation is equal to sum of weight of the observations.
(iii) The weight of algebraic sum of two or more quantities is equal to the reciprocal of the sum of the reciprocal of individual weights.
If measurement ' $\mathrm{x}_{1}$ ' taken with weight ' $\mathrm{w}_{1}$ ' and ' $\mathrm{x}_{2}$ ' taken with ' $\mathrm{w}_{2}$ ' are added or subtracted then

| Function | Result [s] | Weight of Result [w] |
| :---: | :---: | :---: |
| Addition | $\left[x_{1}+x_{2}\right]$ | $\frac{1}{\frac{1}{w_{1}}+\frac{1}{w_{2}}}$ |
| Subtraction | $\left[x_{1}-x_{2}\right]$ Or $\left[x_{2}-x_{1}\right]$ | $\frac{1}{\frac{1}{w_{1}}+\frac{1}{w_{2}}}$ |

(iv) If measurement ' $\mathrm{x}_{1}$ ' taken with weight ' $\mathrm{w}_{1}$ ' is multiplied /divided by a constant ' $\mathrm{k}^{\prime}$, then

| Function | Result [s] | Weight of Result [w] |
| :---: | :---: | :---: |
| Multiplication | $\mathrm{k} \cdot \mathrm{x}_{1}$ | $\frac{\mathrm{w}_{1}}{\mathrm{k}^{2}}$ |
| Division | $\frac{\mathrm{x}_{1}}{\mathrm{k}}$ | $\mathrm{w}_{1} \cdot \mathrm{k}^{2}$ |

(v) If an equation is multiplied by its own weight, the weight of the resulting quantity is the reciprocal of the weight of the equation.
(vi) The weight of an equation remains the same if the signs of all the terms of the equation are changed or the equation is added to or subtracted from a constant.

## 4. PROBABLE ERROR IN COMPUTED QUANTITY

The probable error in computed quantities follows the following laws depending upon the relation between the computed quantity and observed quantity.
(i) If the computed quantity is equal to sum or difference of the observed quantity plus/minus a constant, the probable error of the computed quantity is the same as of the observed quantity.
Let, $\mathrm{x}=$ observed quantity, $\mathrm{y}=$ computed quantity, $\mathrm{c}=$ constant
such that $y= \pm x \pm c$
Then, $\quad e_{y}=e_{x}$
(ii) If a computed quantity is equal to an observed quantity multiplied by a constant, then the probable error of computed quantity is equal to the probable error of observed quantity multiplied by the constant.
$x=$ observed quantity, $y=$ computed quantity, $k=$ constant
such that, $y=k x$
Then, $\mathrm{e}_{\mathrm{y}}=\mathrm{ke} \mathrm{e}_{\mathrm{x}}$
(iii) If the computed quantity is equal to algebraic sum of two or more quantities, then the probable error of computed quantity is equal to the square root of sum of square of the probable error of observed quantities.
Let, $x_{1}, x_{2}, x_{3} \ldots$. are observed quantities and $y=x_{1} \pm x_{2} \pm x_{3} \ldots$.
Then, $e_{y}=\sqrt{e_{x 1}^{2}+e_{x 2}^{2}+e_{x 3}^{2}+\cdots}$
(iv) If the computed quantity is a function of observed quantity, its probable error is obtained by multiplying the probable error of the observed quantity with its differentiation with respect to that quantity.
Let, $x=$ observed quantity, $y=$ computed quantity
Such that $\quad y=f(x)$
Then, $\quad e_{y}=\frac{d y}{d x} e_{x}$
(v) If the computed quantity is a function of two or more observed quantities, its probable error is equal to the square root of summation of the squares of the probable error of the observed quantities multiplied by its differentiation with respect to that quantity.
Let, $x_{1}, x_{2}, x_{3} \ldots$. are observed quantities and $y=f\left(x_{1}, x_{2}, x_{3} \ldots\right)$
Then, $\quad e_{y}=\sqrt{\left(e_{x 1} \frac{d y}{d x_{1}}\right)^{2}+\left(e_{x 2} \frac{d y}{d x_{2}}\right)^{2}+\left(e_{x 3} \frac{d y}{d x_{3}}\right)^{2}}$

## 5. DISTRIBUTION OF ERROR OF THE FIELD MEASUREMENTS

Whenever observations are made in field, a check for closing error is necessary. The closing error is distributed to the observed angles as per following rules:
(i) The correction to be applied is inversely proportional to the weight of the observation.
(ii) The correction to be applied to an observation is directly proportional to the square of probable error.
(iii) The correction to be applied is proportional to length of line.

## 6. PRINCIPLE OF LEAST SQUARES

This principle states that the most probable value of a quantity evaluated from a number of observations of equal weight is the one for which the sum of square of residual error $\left(\Sigma v^{2}\right)$ is minimum. And if the observations are of unequal weight, then most probable value is the one for which the sum of product of weight and square of residual error i.e., $\Sigma\left(w v^{2}\right)$ is a minimum value.

Normal Equation Method: This method is used to find out most probable value of an indirectly observed quantity. A normal equation is a conditional equation from which MPV of any one quantity can be determined by assigning a particular set of values to the remaining quantities. The rules for formation of normal equation are as follows:
Rule 1: If observations are of equal weight, then to form a normal equation for each of the unknown quantities, multiply each observation equation by the algebraic coefficient of that unknown quantity in that equation, and add the results.

Rule 2: If observations are of unequal weight, then to form a normal equation for each of the unknown quantities, multiply each observation equation by the product of the algebraic coefficient of that unknown quantity in that equation and the weight of that observation and add the result.

## CHAPTER-10-PLANE TABLE

1. PLANE TABLE SURVEYING: Plane table surveying is a graphical method of surveying in which the field observations and plotting are done simultaneously. It is simpler and cheaper than theodolite survey. It is suitable for small scale maps.

## 2. Advantages

- There is no possibility of omitting the necessary measurements.
- Surveyor can compare the plotted work with actual features of the area.
- Simpler and cheaper than theodolite survey.
- It is most suitable for large scale maps.
- No great skill is required.
- It is useful in magnetic areas where compass may not be used.


## 3 Disadvantages

- It is not very accurate.
- It is not suitable in monsoon.
- Equipment is inconvenient to transport.


## 4. Instruments Required

- Alidade: Alidade is useful for establishing a line of sight, nowadays telescopic alidade is also used, when points too high or low are to be sighted, the range and accuracy are considerably increased by providing a telescope.
- Drawing board: It is made from well-seasoned wood. Normally, rectangular in shape with size $75 \mathrm{~cm} \times 60 \mathrm{~cm}$
- Plumbing fork: It is used for centring.
- Spirit level: It is used for ascertaining if the table is properly level.
- Trough compass: it is required for drawing line showing magnetic meridian on the paper.


## 5. Principle of Plane Table Surveying

All the rays drawn through various details should pass through the survey station. The direction of the plane table at each station must be identical i.e., at each survey station the table must be oriented in the direction of magnetic north.
6. Temporary Adjustment of Plane Table

Following three distinct operations at each survey station are carried out for the temporary adjustments of a plane table.

- Centring: This process ascertains the fact that the point on paper represents the station point on ground. Exact centring is required for large scale map only.
- Levelling: For levelling the table, ordinary spirit level may be used.
- Orientation: The process by which the position occupied by the board at various survey stations are kept parallel is known as the orientation.


## 7. Methods of Plane Tabling

There are four distinct method of plane tabling.

- Method of Radiation
- Method of Intersection
- Method of traversing
- Method of resection


## 8. Method of plane table orientation

- By trough compass: It cannot be used when local attraction is suspected.
- By back sighting: Most accurate method of orientation.
- Resection.

9. Resection: This method of orientation is employed when the plane table occupies a position not yet plotted on the drawing sheet.
Resection can be defined as the process of locating the instrument station occupied by the plane table by drawing rays from the stations whose positions are already plotted on the drawing sheet. The point representing the resection of two rays will be the station to be located, provided the orientation at the station to be plotted is correct, which is seldom achieved. This problem can be solved by any of the methods such as resection after orientation by back ray, by two points, or by three points. This method is employed when surveyor feels that some important details can be plotted easily by choosing any station other than the triangulation stations. The position of such a station is fixed on the drawing sheet by resection.

## 10. Errors in Plane Tabling

The various sources of error may be classified as:

- Instrumental errors
- Errors in manipulation and sighting
- Errors in plotting


## CHAPTER-11- GPS, GIS \& REMOTE SENSING

## 1. GLOBAL POSITIONING SYSTEM (GPS):

- GPS is a worldwide radio navigation system consisting of satellite, computers and receivers, operated by the US department of defence. A radio navigation system allows a user to determine her position in three dimensions as well as time. Such a system is run with the help of satellites launched for this purpose.
- The GPS is formed from 24 satellites, the satellites are positioned in six earth centred orbital plane with 4 satellite in each plane. Each satellite takes 12 hr to complete one full orbit. GPS uses these satellites as a reference point to calculate positions accurately.
- The distance between a user and the satellites can be computed using the satellite signals. As there are four unknowns i.e., three unknowns of position in threedimensional space and one unknown of time, for the accurate position of a point, we need a minimum of four satellites. The satellites have atomic clocks onboard to provide accurate times.
- GPS can be used for determining position, navigation, tracking, mapping and precise time determination.
- With handheld instruments and precise positioning available with the technology, GPS can be used to undertake survey work accurately for many purposes like establishing control points for geodetic work.


## 2. GEOGRAPHIC INFORMATION SYSTEM (GIS):

- GIS is a computer-based information system that enables, captures, manipulates, analyses and presents the geographical reference data.
- GIS is used for analysing and manipulating spatial data which can be used to help produce maps and other products in standardized formats. Such data can also be used for research activities.


## 3. REMOTE SENSING:

It is broadly defined as the science and art of collecting information about objects, area without being in physical contact with them. There are two types of remote sensing.
i) Passive Remote Sensing: In this type of remote sensing, the instrument, such as a camera, doesn't generate or emit radiation. The source of the radiation is generally the sun and the reflected radiation from the object is used to determine its properties. A photographic camera, or a remote sensing satellite are examples of such systems.
ii) Active Remote Sensing: The sensing equipment emits radiation and the reflection coming back from the object is used to determine the properties of the object. Radar and sonar are types of active remote sensing systems.

### 3.1. Interaction of EM Radiation with Earth's Surface

Electromagnetic energy that strikes or encounters matter is called as incident radiation. The Electromagnetic radiation striking may get :
(a) Reflected (scattered): The unpredictable diffusion of radiation by atmospheric particles.
(b) Absorbed: Atmospheric gases absorb incident radiation as per their characteristic absorption spectra. For example, oxygen absorbs in the ultraviolet spectrum.
(c) Transmitted: The electromagnetic waves that pass through the atmosphere undisturbed, without being affected.
It depends on various factors such as:
(a) Wavelength of radiation, (b) Angle of incidence, (c) Surface roughness, (d) Condition and composition of surface material

Incident energy $=$ Transmitted + Absorbed + Reflected energy
$E_{I \lambda}=E_{T \lambda}+E_{A \lambda}+E_{R \lambda}$
$E_{\mathrm{R} \lambda}=\mathrm{E}_{\mathrm{I} \lambda}-\left(\mathrm{E}_{\mathrm{A} \lambda}+\mathrm{E}_{\mathrm{T} \lambda}\right)$
$\frac{E_{R \lambda}}{E_{I \lambda}}=1-\left(\frac{E_{A \lambda}}{E_{I \lambda}}+\frac{E_{I \lambda}}{E_{I \lambda}}\right)$
Reflectance $=1$ - (transmittance + absorbance $)$
$\zeta=1-(\alpha-r)$

### 3.2. Remote Sensing Platforms

Two types of platforms have been in use in remote sensing.
(i) Air Borne Platforms: Aircrafts have been used as a remote sensing platform for obtaining photographic images. Multi spectral scanners, ocean colour
radiometer, photography in various spectra can all be done with aircraft mounted sensors. While expensive, these systems provide flexibility in deployment and higher fidelity as compared to space-based platforms.
(ii) Space Based Platforms: Satellites offer a larger field of view, a systematic and repetitive coverage of an area. These attributes make space-based platforms immensely useful for monitoring natural resources.

## 4. ADVANTAGES OF REMOTE SENSING:

- Provides data of large areas
- Provides data of very remote and inaccessible regions
- Able to obtain imagery of any area over a continuous period of time through which the any anthropogenic or natural changes in the landscape can be analysed
- Relatively inexpensive when compared to employing a team of surveyors
- Easy and rapid collection of data
- Rapid production of maps for interpretation


## 5. DISADVANTAGES OF REMOTE SENSING:

- The interpretation of imagery requires a certain skill level.
- Needs cross verification with ground (field) survey data.
- Data from multiple sources may create confusion.
- Objects can be misclassified or confused.
- Distortions may occur in an image due to the relative motion of sensor and source.


## 6. APPLICATIONS OF REMOTES SENSING:

- Land use and land cover mapping
- Crop identification
- Flood plain mapping
- District level mapping
- Urban growth studies
- Ground water mapping
- Waste land mapping
- Disaster management: effect of earthquake, Tsunami etc.

