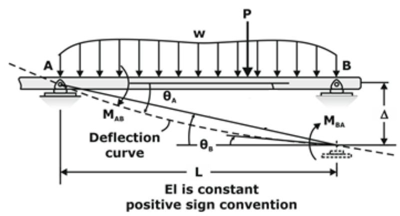
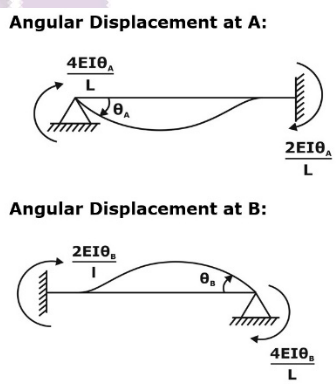
Slope Deflection Equation

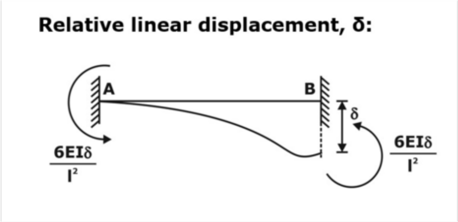
The slope deflection equation gives the relationship between end moments and the rotation of the member. A member's rotation, deflection, and end moments are the design's main unknown parameters in the structural analysis. The slope-deflection equations can be obtained using the superposition principle by considering separately the moments developed at each support due to each displacement θ_A, θ_B, and Δ and the loading. These equations, later modified based on the requirement of analysis, are known as the modified slope deflection equations.



Slope Deflection Equation Derivation

Slope deflection equations are very useful for the view of structural analysis. These equations are used in the slope deflection method of analysis of indeterminate structures. The derivation of slope deflection equations is subjected to the actual problem statement, and that can be derived by considering the fixed end moments and rotation of different support conditions, which are given below:





Slope Deflection Formula

Slope deflection formulas are based on slope deflection equations, which are used to analyze indeterminate structures. These equations are based on some special equations and equilibrium conditions. And moments and rotations can find out by solving these equations simultaneously. These formulas are derived based on considering the following sign conventions:

(i) Clockwise moment is taken as positive.

(ii) If δ gives clockwise rotation to a member, it is considered positive.

With the help of the superposition principle, the slope deflection equation can be written as

$$M_{AB} = M_{FAB} + \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B - \frac{6EI\delta}{L^2}$$

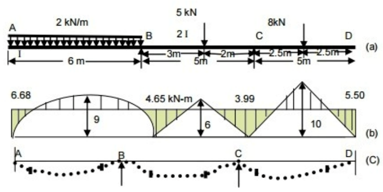
$$\Rightarrow M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\theta_A + \frac{4EI}{L}\theta_B - \frac{6EI\delta}{L^2}$$

$$\Rightarrow M_{BA} = M_{FBA} + \frac{2EI}{L}\left(\theta_A + 2\theta_B - \frac{3\delta}{L}\right)$$

Slope Deflection Equation for Continuous Beams

Continuous beams have more than two support; hence, the number of support reactions will be more than the number of available equilibrium equations. So such beams are indeterminate and can be analysed with the help of slope deflection equations. This can be understood with the help of the following example:



$$M_{AB} = \frac{2E(2I)}{6}[\theta_B] - 6 = \frac{EI}{3}\theta_B - 6 \quad \ldots(1)$$

$$M_{BA} = 2\frac{E(I)}{6}[2\theta_B] + 6 = 2\frac{EI}{3}\theta_B + 6 \quad \ldots(2)$$

$$M_{BC} = \frac{2E(2I)}{5}[2\theta_B + \theta_C] - 2.4 = \frac{4EI}{5}(2\theta_B + \theta_C) - 2.4 \quad \ldots(3)$$

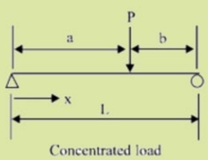
$$M_{CB} = \frac{2E(2I)}{5}[2\theta_C + 2_B] + 3.6 = \frac{4EI}{5}(\theta_B + 2\theta_C) + 3.6 \quad \ldots(4)$$

$$M_{CD} = \frac{2E(2I)}{5}[2\theta_C] - 5 = \frac{4EI}{5}\theta_C - 5 \quad \ldots(5)$$

$$M_{DC} = \frac{2E(I)}{5}[\theta_C] + 5 = \frac{2EI}{5}\theta_C + 5 \quad \ldots(6)$$

Slope Deflection Equation for Simply Supported Beam

A simply supported beam is a beam supported by hinge support at one end, and roller support supports another. Simply supported beams can be analyzed with the help of equilibrium equations alone; hence, such a beam is determinate. Slope deflection equations can be used to determine the slope and deflection of beams under the action of external loading. This can be understood with the help of the following example:



Concentrated load

$0 \le x \le a$:

$$\text{Slope} = \frac{Pb}{6EIL}(3x^2 + b^2 - L^2)$$

$$\text{Deflection} = \frac{Pb}{6EIL}(x^3 + b^2x - L^2x)$$

$a \le x \le L$:

$$\text{Slope} = \frac{Pa}{6EIL}[L^2 - a^2 - 3(L - x)^2]$$

$$\text{Deflection} = \frac{Pa(L - x)}{6EIL}(x^2 + a^2 - 2Lx)$$