# Force Method of Analysis of Indeterminate Structure

In the structural analysis, there are two methods available one is known as the Force method, and the other is the Displacement method. In the force method of analysis of indeterminate structure, forces and moments are unknowns that we need to find. And in the displacement method of structure analysis, the effect of these forces and moments like rotations and deformations are unknown, and we need to find them.

The force method of analysis of indeterminate structure can be further classified into some other methods like the Unit load method, Castigliano's method, strain energy method, Flexibility matrix method, etc. With the help of these methods, unknown forces and moments are found in the structure. In the case of determinate structures, the force method allows us to find forces and moments by using only equilibrium equations, and it doesn't require material information. Material information like stress-strain relationships is needed only to calculate deflections in the structure. However, equilibrium equations, compatibility conditions, and material information are required to analyze the structure for indeterminate structures.

## Analysis of Statically Indeterminate Structures by the Forced Method

The force analysis of indeterminate structures is based on finding the structures' unknown forces and moments. In the force method, an indeterminate structure is first converted into a determinate structure by replacing some redundant forces with unit forces causing the same effect on the structure. After that, with the superposition theorem's help, the value of the replaced force will be calculated by achieving the compatibility of the considered structure member. In force method of analysis of indeterminate structure, these redundant forces are solved mathematically by representing them in a matrix representation, and this matrix is known as the flexibility matrix.

The force method of analysis is also known as the method of consistent deformation. This method uses equilibrium and compatibility equations to determine the unknowns in an indeterminate structure. In the force method, unknowns are the redundant forces and moments. A redundant force in the

structure can be a support reaction force, or it may be an internal member force or moments. If anyone or all the redundant forces are removed from the structure, it will never cause any type of instability to the structure. The force method establishes a set of compatibility equations, which depend upon the total number of redundant forces or moments in the structure. By solving these compatibility equations simultaneously, redundant forces can be calculated. And now redundant forces are not unknown forces. Hence, the structure becomes determinate, which can be analyzed with the help of equilibrium equations alone.

## Steps in Force Method of Analysis of Indeterminate Structure

The following steps are used in the force method to analyze the indeterminate structures:

- Step 1: Determine the degree of static indeterminacy of the structure.

- Step 2: Transform the structure into a statically determinate system by releasing some static constraints equal to the degree of static indeterminacy, n. This is accomplished by releasing external support conditions or by creating internal hinges. The system thus formed is called the basic determinate structure.

- Step 3: For a given released constraint j, introduce an unknown redundant force R corresponding to the type and direction of the released constraint.

- Step 4: Apply the given loading or imposed deformation to the basic determinate structure. Use a suitable method to calculate displacements at each released constraint in the basic determinate structure.

- Step 5: Solve all the redundant forces by imposing the compatibility conditions of the original structure. These conditions transform the basic determinate structure back to the original structure by finding the combination of redundant forces that make displacement at each released constraint equal to zero.

## Type of Force Method of Analysis of Indeterminate Structure

As we know, the force method of analysis of indeterminate structure is a method of finding the unknown parameter of the structure. And these

unknowns will basically be in terms of force and moments when it is analyzed by the force method. And there are many methods of analysis in which unknowns are forces and moments. So, these methods can be classified as the types of force methods listed below.

1. Virtual work/Unit load Method

2. Method of consistent deformation

3. Three-moment theorem

4. Castigliano's theorem of minimum strain energy

5. Maxwell-Mohr's equation

6. Column Analogy Method

## Force Method of Analysis of Frames

Frames are structures that can take horizontal and vertical loads over them. It can be analyzed with the help of force, displacement, or any approximate structure analysis method. In the case of the frame, the suitability of the method used for analysis depends on its type, configuration (either it is a portal bay or multi-bay frame), and degree of indeterminacy.
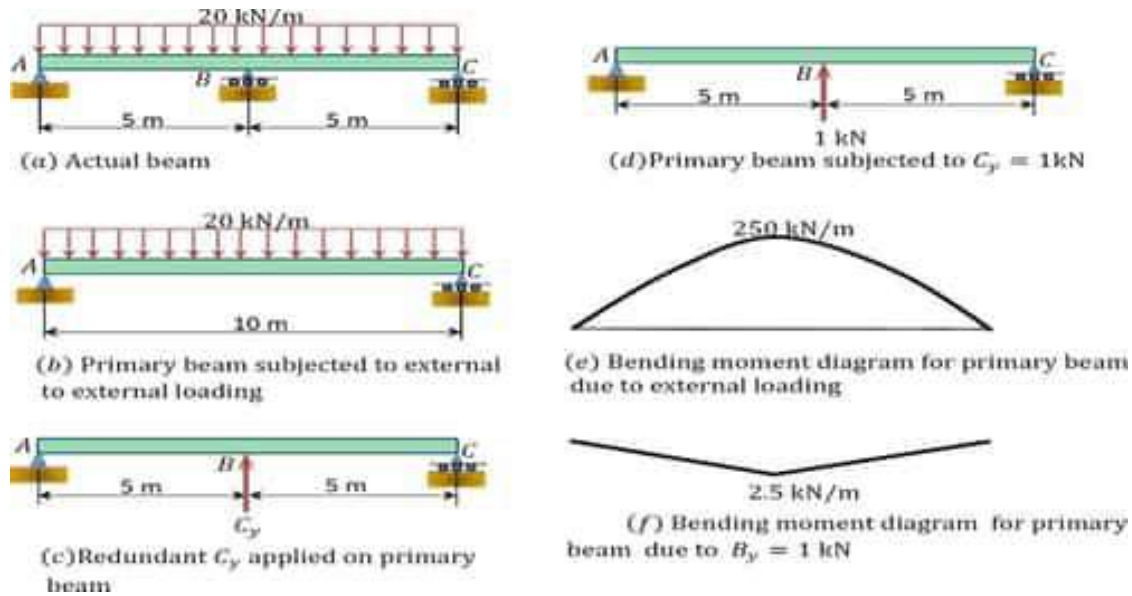
The method of analysis to adopt depends upon the type of frame, its configuration (portal bay or multi-bay) in the multi-storied frame, and the degree of indeterminacy.

The force method of analysis of indeterminate structure can also be used for finding the unknown parameters of a frame. Analysis of a frame is similar to analyzing the beam with the force method. In analyzing frames by this method, first, we need to calculate the determinacy of the frame and replace the redundant force with the unifying force. After that, with the help of compatibility conditions of the frame, unknown redundant forces can be found. And when redundant forces are known, frames become a determinate structure that equilibrium equations can analyze.

## Force Method of Analysis Example

As we know, structure analysis can be done with the help of many available methods, and among those methods force method is also a method to analyze the structure like beams, trusses, and frames. The suitability of any method will depend upon many factors, such as indeterminacy of the structure, type of structure, etc. Here is an example of the force method of

analysis of indeterminate structure, which helps to understand all the concepts related to this method.



(a) Actual beam

(d) Primary beam subjected to $C_y = 1kN$

(b) Primary beam subjected to external to external loading

(e) Bending moment diagram for primary beam due to external loading

(c) Redundant $C_y$ applied on primary beam

(f) Bending moment diagram for primary beam due to $B_y = 1$ kN

## Comparison between Displacement and Force Methods of Analysis

As we already discussed, Force and Displacement methods both are methods of analysis of an indeterminate structure. And in the force method of analysis of indeterminate structure, forces and moments are the unknown parameters of the structure. And in the displacement analysis method of indeterminate structure, rotations and deformations are desired parameters. So based on that, we can say that we need to find the degree of static determinacy, and in the case of the displacement method, we need to find out the degree of freedom associated with the structure. So in both methods of analysis of indeterminate structure, we need to establish the Force-Displacement equation, based on which unknown structure parameters are found.

The force method of analysis of indeterminate structure is used when the degree of static indeterminacy is less than the degree of freedom of structure, and the displacement method of analysis of indeterminate structure is used when the degree of freedom is less than the degree of static indeterminacy of the structure. And some other comparisons of both methods are listed below.

- In the case of the force method, the main governing equations are the compatibility equations, while in the displacement method, the main governing equation is the equilibrium equation.

- In the case of the force method, force-displacement relations are shown in the flexibility matrix, and in the case of the displacement method, force-displacement relations are shown in the stiffness matrix.