

Euler's Equation of Motion

Euler's equation of motion for a steady flow of an ideal fluid along a streamline is a relationship between a flowing fluid's velocity, pressure, and density. It is based on Newton's Second Law of Motion, which asserts that linear momentum is preserved without an external force. The Bernoulli equation is obtained by integrating it and converting it to energy per unit weight of the following fluid.

Assumptions in Euler's Equation of Motion

Euler's Equation of Motion is based on the following assumptions:

- The substance isn't viscous (i.e., the frictional losses are zero).
- The substance is uniform and incompressible (i.e., the mass density of the fluid is constant).
- Continuous, steady, and streamlined flow is present.
- Over the section, the flow's velocity is constant.
- Except for gravity and pressure forces, there is no energy or force influencing the flow.
- Because no external force is applied (non-viscous flow), linear momentum is conserved.

What are Euler's Equations in Fluid Dynamics?

Euler's equation of motion is the equation of motion and continuity that deal with a purely theoretical fluid dynamics problem known as inviscid flow. Although some very low-viscosity incompressible fluids, like water or alcohol, can be addressed in specific flow regimes, Euler's equation of motion real fluids are nevertheless viscous. Due to this, Euler's equations describe an ideal situation where fluid drag and energy dissipation are low, which is nonphysical and cannot occur in the real world. To approach Euler's equation of motion, we can estimate the Navier-Stokes equations with extremely low viscosity.

2 - Dimensional, Steady Form:

Coordinates: (x, y)	Continuity:	$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$
Velocity Components: (u, v)	X - Momentum:	$\frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = -\frac{\partial p}{\partial x}$
Pressure: p	Y - Momentum:	$\frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} = -\frac{\partial p}{\partial y}$
Density: ρ		

Incompressible Form:

Continuity:	$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$
X - Momentum:	$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x}$
Y - Momentum:	$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y}$

Derive Euler's Equation of Motion

We shall simply consider gravity and pressure forces in Euler's equation of motion. Other factors will be ignored.

Assumptions:

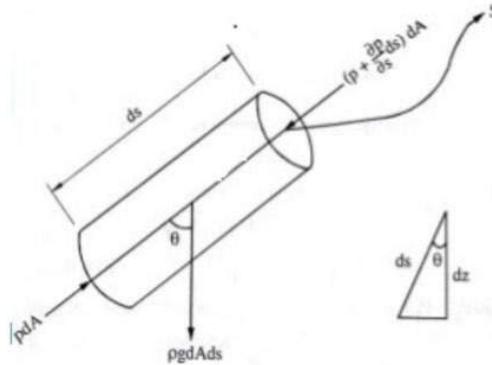
As stated here, the following presumptions are the foundation for Euler's equation of motion.

- The fluid has no viscosity. Frictional losses won't exist at all.
- The substance is uniform and incompressible.
- The fluid moves steadily, continuously, and in a streamline.
- The section's fluid flow velocity is consistent.
- Only the forces of gravity and pressure will be considered.

Imagine a fluid moving from point A to point B, and consider a very small cylindrical portion of this fluid flow with length dS and cross-sectional area dA , as shown in the figure below.

Consider the forces on the cylinder: Pressure force PdA , in the direction of fluid flow.

Pressure force $[P + (\partial P/\partial S) ds] dA$, in the opposite direction of fluid flow
 Weight of fluid element $(\rho g dA ds)$



Let's assume that the angle between the fluid flow direction and the fluid element's weight action line is θ .

As previously stated, Euler's equation of motion is founded on the fundamental premise of Newton's second law of motion. As a result, we can put the following equation here:

Net force in the direction of S over the fluid element = mass of the fluid element x acceleration in the direction of S.

$$PdA - \left(P + \frac{\partial P}{\partial S} ds \right) dA - \rho g dA ds \cos \theta = \rho dA ds a_s$$

$a_s = \text{Acceleration in direction of S.}$ $a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s} \frac{ds}{dt} + \frac{\partial V}{\partial t} = V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$ <p style="font-size: small;">(Where V is a function of S and t)</p> <p>If the flow is steady $\frac{\partial V}{\partial t} = 0$,</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; padding: 2px;">$a_s = V \frac{\partial V}{\partial s}$</div> <div style="font-size: 2em;">&</div> <div style="border: 1px solid black; padding: 2px;">$\cos \theta = \frac{dz}{ds}$</div> </div>

Simplify above equation by putting a_s & $\cos \theta$

$$\frac{\partial P}{\rho} + V dV + g dz = 0$$

Inviscid Approximation

Although Euler's equation of motion in fluid dynamics is unphysical, it can be used to describe an "almost inviscid" condition in which drag forces are significantly smaller than any externally applied forces. Euler's equations can be quickly derived from the Navier-Stokes equations in this flow regime. To begin, consider the viscous terms in the Navier-Stokes equation of motion:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot \left[\mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \right] + \nabla \cdot [\lambda (\nabla \cdot \mathbf{u}) \mathbf{I}] + \rho \mathbf{g}$$