CSIR-NET 2022

Mathematical Science

Part A

1. Find the missing number.

3		9
7	2	2
4		1

1		6
5	7	3
4		8

9		8
2	1	7
6		3

4		5
8	?	1
2		3

A. 8
B. 5
C. 3
D. 1

2. If P = 7326515 × 7326525, Q = 7326514 × 7326526 and R = 7326513 × 7326527, then which one is largest?
A. P
B. Q
C. R
D. All are same

3. A force of 10 N acts on a block of mass 10 kg kept on a flat surface. If the coefficient of friction is 0.2, what is the frictional force acting on the body? Take g= 10 m/s^2.
A. 20 N
B. 2 N
C. 10 N
D. None of the above

4. The table given below shows the number of various animals in four different national parks in a country.

National Parks	Tigers	Elephants	Other animals
Jim Corbett national park	270	1125	3105
Kaziranga national park	180	820	2200
Gir national park	210	1240	2150
Pench national park	175	925	1900

Total number of animals in Kaziranga national park is what percent more or less than total number of animals in Gir national parks?
A. $11\frac{1}{9}$ %
B. $17\frac{1}{2}$ %
C. $15\frac{1}{7}$ %
D. $16\frac{2}{3}$ %

5. If in a concave mirror an object is placed between Radius of curvature and focus the image will be formed at?
A. Between radius of curvature and focus
B. Beyond radius of curvature
C. At focus
D. Between pole and focus

6. Suppose there are X gloves of different colours in a box. If you take out one glove at a time, what is the maximum number of gloves that you have to take out before a matching pair is found? Assume X is an even number.
A. X/2
B. X - 1
C. X + 1
D. X

7. How many different colour of shirt can be made from orange, blue, green, white and red?
A. 31
B. 32
C. 28
D. 20

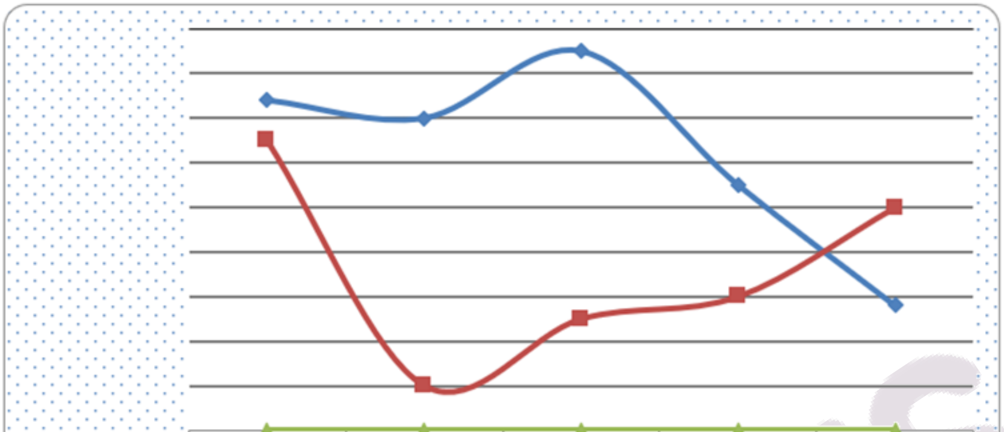
8. How many triangles are there in the following figures?

A. 4
B. 8
C. 6
D. 10

9. If June 1, 2021 was Tuesday, which day was it in June 1, 2012?
A. Monday
B. Sunday
C. Thursday
D. Friday

10. In a certain code language 'TCS' is coded as 'VAU' and 'GOOGLE' is coded as 'IMQENC' ; then 'GRADEUP' can be coded as
A. ITCBGWR
B. EPYBCSN
C. IPCBGSR
D. ITCFGWR

11. Distance travelled by two vehicles (in km) in five days is given below:



	Monday	Tuesday	Wednesday	Thursday	Friday
A	740	700	850	550	280
B	650	100	250	300	500
Column1					

The average distance travel (in km) by:

A. A is greater than that of B

B. A is less than that of B

C. A is equal to or less than that of B

D. B is equal to or less than that of A

12. Divide Rs.21236/- between P and Q, so that P's share at the end of 5 years may equal to Q's share at the end of 7 years, compound interest being 6% p.a.

A. Rs.11200, Rs. 15000

B. Rs.11000, Rs. 12000

C. Rs.11236, Rs. 10000

D. Rs.10236, Rs. 9000

13 .Find the smallest number which on adding 31 is exactly divisible by 34, 48 and 58.

A. 23655

B. 23636

C. 23633

D. 23542

14. A motorist, after driving a distance of 90 km on the 2nd day, finds that the ratio of the distance travelled by him on the 1st two days is 3 : 5. If he travels a distance of 46 km on the 3rd day, then the ratio of distance travelled on the 3rd day and the 1st day?

A. 52 : 63

B. 44 : 48

C. 46 : 54

D. 46 : 52

15. Pooja's only brother Anil is the husband of Vijay's mother Sushila. How is Pooja's mother Neha related to Vijay?

A. Maternal grandmother

B. Mother-in-law

C. Paternal Aunt

D. Paternal grandmother

16. Five chair number 1 to 5 are placed around a round table. Starting from chair number 3, a person keeps going around the table anticlockwise. After crossing 19 chairs, the person will reach the chair number?

A. 2 B. 3
C. 5 D. 1

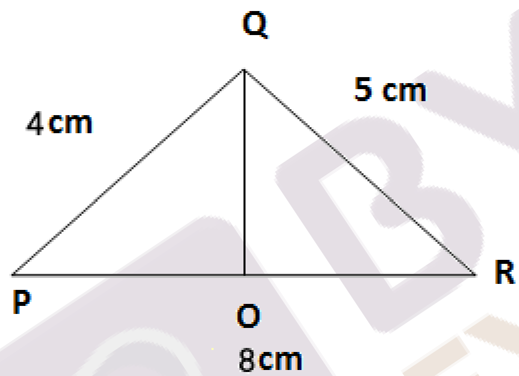
17. A Car X takes 2hrs more than car Y to travel a distance of 600km. Due to a failure in car Y the average speed of car Y becomes $3/4^{th}$ of the original speed and it takes 1hr more than car X to cover the same distance. Calculate the original speed of car Y?

A. 70 kmph B. 88 kmph
C. 66 kmph D. 56 kmph

18. How many digits are there in 7^{12} when it is expressed in the decimal form?

A. 11 B. 10
C. 8 D. 9

19. O is the point on PR in the following triangle ∠ POQ = ∠ROQ



The value of QO (in cm) is

A. 2 B. 5
C. 3 D. 4

20. Find the probability of choosing a team of 11 students from 8 boys and 8 girls if it is given that the number of boys should always be greater the number of girls.

A. 1/3 B. 2/3
C. 4/9 D. 1/2

Part B

21. The singular solution of the differential equation $x^6\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - 4y = 0$ is given by

A. $2x^2y^2 = -1$
B. $4x^4y = -1$
C. $2xy^4 = -1$
D. $4x^4y^4 = -1$

22. The solution of the integral equation $g(\xi) = \xi + \int_0^1 \xi u^2 \cdot g(u) \cdot du$ is given by

A. $9(x) = \frac{3x}{4}$
B. $9(x) = \frac{4x}{3}$
C. $9(x) = \frac{2x}{3}$
D. $9(x) = \frac{3r}{2}$

23. The external of the function $J(y) = \int_0^1 [3y'^2 + 4xy]dx : y(0) = 0, y(1) = 1, y \in C^2[0,1]$ is

A. $\frac{1}{9}(x^3 + 8x)$
B. $\frac{1}{9}x^2 + \frac{17}{18}x$
C. $\frac{1}{18}x^3 + \frac{1}{18}x$
D. $\frac{17}{18}x^3 + \frac{16}{18}x$

24. Let f(x) be a continuous function such that f(a).f(b) > 0. For two real numbers a and b then
A. at least one root of f(x) = 0 lies in (a, b)
B. no root lies in (a, b)
C. either no root or an even number of roots lies
D. None of these.

25. Let A be a 4 × 4 matrix. Such that both A & Adj (A) are non-null matrix. It det A = 0 then the rank (A) is
A. 1
B. 3
C. 4
D. 2

26. If f(z) is analytic on Δ, open unit disc such that f(0) = 0, |f(z)| < 1 for all z ∈ Δ, f(z) is analytic at z = 1 and f(z) = 1 . Then
A. $|f'(1)| \le 1$
B. $|f'(1)| \ge 1$
C. $|f'(1)| = 1$
D. $|f'(x)| < 1$

27. What is the remainder when 8^{103} divided by 103?
A. 8
B. 7
C. 6
D. 10

28. For the differential equation

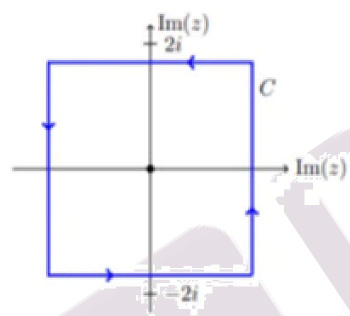
$$x^2(1-x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$$

A. x = 1 is an ordinary point
B. x = 1 is a regular singular point
C. x = 0 is an irregular singular point
D. x = 0 is an ordinary point

29. Extremals y = y(x) for the variational problem $v[y(x)]=\int_0^1 (y+y')^2\,dx$ satisfy the differential equation

A. y'' + y = 0
B. y'' – y= 0
C. y'' + y' = 0
D. y' + y = 0

30. $\int_C \frac{\cos z}{z(z^2+8)}dz$ over the contour shown



A. $\frac{\pi i}{8}$
B. $\frac{\pi i}{4}$
C. 0
D. $2\pi i$

31. Which of the following is/are correct?
A. Every cyclic group is isomorphic to $\mathbb{Z}$
B. $\exists$ an infinite group G such that each element of group has a finite order.
C. $\nexists$ an infinite group G such that each element of group has a finite order.
D. Both A & B are true

32. What would be the degree of the differential equation $(y'+7y)^2=4x(x+7y)$; when $(y'-x)\neq 0$?
A. 0
B. 1
C. 2
D. 4

33. The extremal of the variational problem $\int_0^5 (9y^2 + y'^2 - 3x)\,dx$ with the boundary condition $P(0,0)\,\&\,Q(5,0)$

A. $y(x) = e^x - 1$
B. $y(x) = 0$
C. $y(x) = x$
D. $y(x) = e^{-x}$

34. The number of divisor of zero in $\mathbb{Z}_{15}$ is
A. 2
B. 3
C. 7
D. 8

35. The number of subring of $\mathbb{Z}_{40}$ is
A. 4
B. 6
C. 8
D. 10

36. The number maximal ideal of $\frac{\mathbb{Z}[i]}{<2+3i>}$ is
A. 2
B. 3
C. 4
D. 0

37. Let x_i's be independent random variable such that x_i's are symmetric about 0 and variance of x_i's 2i – 1 for $i \geq 1$, then
$\lim_{n\to\infty} P(x_1 + x_2 + ... + x_n > n\log n) = ?$
i.e. $\lim_{n\to\infty} P(s_n > n\log n) =$?
A. 0
B. 1
C. n
D. $\frac{1}{2}$

38. Consider the polynomial $p(x) = x^4 + 4$ in the ring $\mathbb{Q}[x]$ of polynomial in the variable x with the coefficient in the field $\mathbb{Q}$ of rational numbers. Then
A. The set of zeros of $p(x)$ in $\mathbb{C}$ forms a group under multiplication.
B. $p(x)$ is reducible in the ring $\mathbb{Q}[x]$
C. The splitting field of $p(x)$ has degree 3 over $\mathbb{Q}$
D. The splitting field of $p(x)$ has degree 4 over $\mathbb{Q}$

Part C

39. Which of the conditions below imply that a function f: $[0,1] \longrightarrow R$ is necessarily of bounded variation?
A. f is a monotone function on [0,1]
B. f is a continuous and monotone function on [0,1]
C. f has a derivative at each $x \in (0,1)$
D. f has bounded derivative on the interval (0,1)

40. Let y be a non – zero vector in an inner product space v. Then which of the following are subspaces of v?
A. $\{x \in v: <x, y> = 0\}$
B. $\{x \in v: <x, y> = 1\}$
C. $\{x \in v: <x, z> = 0\}$ for all z such that $<z, y> = 0\}$
D. $\{x \in v: <x, y> = 1\}$ for all z such that $<z,y> = 1\}$

41. Let, $f(z) = \frac{1-e^z}{1+e^z}$ then
A. No essential singularity in $\mathbb{C}$
B. $z = \infty$ is a non-isolated singularity
C. z = $(2n + 1)\pi i$ are simple poles
D. $z = \infty$ is isolated essential singularity

42. Pick out the true statements:
A. Let R be commutative ring with unit. Let M be an ideal such that every element of R not in M is a unit. The R/M is a field.
B. Let R be as above and let M be an ideal such that R/M is an integral domain. Then M is a prime ideal.
C. Let R = C [0, 1] be the ring of real-valued continuous function on [0, 1] with respect to pointwise addition and pointwise multiplication. Let $M\{f \in R \mid f(0) = f(1) = 0\}$, then M is a maximal ideal.
D. None of these

43. Let $y_1(x)$ be any non-trivial real valued solution $y''(x) + xy(x) = 0, \ 0 < x < \infty$. Let $y_2(x)$ be the solution of $y''(x) + y(x) = x^2 + 2, y(0) = y'(0) = 0.$ Then
A. y(x) has infinitely many zeros.
B. $y_2(x)$ has infinitely many zeros
C. $y_1(x)$ has finitely many zeros
D. $uy_2(x)$ has finitely many zeros.

44. If the value of $x = 2.71828$ is replaced by 2.71937, then which of the following(s) is/are true?
A. The absolute error is 2.71937
B. The percentage error is 0.02%
C. The percentage error is 0.04%
D. The relative error is 0.00129

45. Consider the Iteration scheme
$$x_{n+1} = \frac{1}{2}\frac{x_n^2 + b}{x_n}$$
The which of the following(s) is/are correct with this scheme?
A. The iteration scheme can be used to compute the root of $\sqrt{b}$
B. For b = 3 and taking initial approximation $x_0 = 2$, the first iteration is 1.45
C. For b = 3 and taking initial approximation $x_0 = 2$, the second iteration is 1.7559
D. For b = 3 and taking initial approximation $x_0 = 2$, the second iteration is 1.7595

46. If y(x) be the solution of the differential equation $y'' - y = 0$, then which of the following(s) is/are correct?
A. $|y(x)| \to \infty$ as $x \to \infty$
B. $|y(x)| \to \infty$ as $x \to -\infty$
C. $|y(x)| \to$ finite as $x \to 0$
D. All of the above

47. Let G be a group of order 65 then which of the following(s) is/are true?
A. There exists one 13-sylow subgroup of G.
B. There exists are 5-sylow subgroup of G.
C. G is not a simple group
D. Groups give in A and B are cyclic.

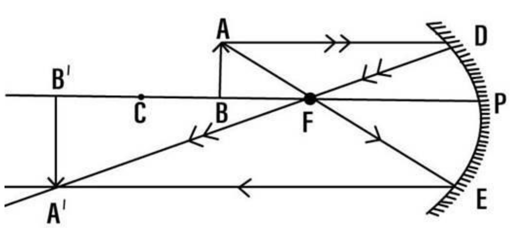
48. The value of the integral $\int_C \frac{|z|}{z} dz$, where C is a circle $|z| = 1$ is not
A. 0
B. $2\pi i$
C. $-2\pi i$
D. 2π

49. If $A \in M_2(\mathbb{R})$ be a matrix which is not diagonal matrix which of the following statements are true?
A. If tr A = –1 and det A = 1 then $A^3 = I$
B. If $A^3 = I$ then true A = –1 and det A = 1
C. If $A^3 = I$ then A is diagonalizable over $\mathbb{R}$
D. All of the above

50. Consider the series
$$\frac{1}{\sqrt{1-x^2}} = 1 + \frac{1}{2}x^2 + \frac{1.3}{2.4}x^4 + \frac{1.3.5}{2.6.8}x^6 + \ldots..$$
Which of the following(s) is/are true?
A. The radius of convergence of the series is 1
B. The interval of convergence of the series is $\frac{1}{2}$
C. The interval of convergence of the series is $-1 \le x \le 1$
D. The interval of convergence of the series is $-1 < x < 1$

ANSWERS

1. Ans. B.
Central Element = Sum of the left column element – Sum of the right column element
Thus,
x = 4 + 8 + 2 – (5 + 1 + 3)
x = 5
Missing number = 5
Hence, option B is the correct answer.
2. Ans. A.
We know,
$(a^2 – b^2) = (a - B) \times (a + B)$
P = 7326515 × 7326525 = (7326520- 5) × (7326520+ 5)
Q = 7326514 × 7326526 = (7326520 - 6) × (7326520 + 6)
R = 7326513 × 7326527 = (7326520 - 7) × (7326520 + 7)
Hence, P = (7326520 - 5) × (7326520 + 5) is the largest among P, Q and R.
So, option A is correct.
3. Ans. C.
We know that the maximum value of static friction on a body= μ*N
Here, μ is the coefficient of friction and N is the normal reaction.
Normal reaction on a body kept on flat surface= mg= 10*10= 100 N
Maximum value of static friction= 0.2*100= 20 N
Since the object is in rest.
Static frictional force is always equal to or less than the applied force. Therefore, friction force acting on the body= Force applied on the body= 10 N.
Hence the correct option is (C)
4. Ans. A.
Total number of animals in Kaziranga national park
= 180 + 820 + 2200
= 3200
Total number of animals in Gir national park
= 210 + 1240 + 2150
= 3600
Required Percentage = $\frac{3600-3200}{3600} \times 100\% = \frac{100}{9}\% = 11\frac{1}{9}\%$
Hence option A is correct.
5. Ans. B.



R = radius of curvature
F = focus
P = pole
AB = object
A'B' = Image

6. Ans. D.

Suppose we take four different colours of gloves i.e. Green, Blue, Red and Yellow
First we take out green color gloves
Then Blue color gloves
Then Red color gloves
At last yellow color gloves
If all four colors of gloves are taken out then fifth one will be again any of the given four color gloves, and then so on…
It means if we have four colors of different gloves then in fifth time the color of gloves started matching.
So, here we can conclude that the maximum number of gloves that you have to take out before a matching pair is found is X number of gloves.
Hence option D is correct.

7. Ans. A.

Combination formula

$^nC_r = \frac{n!}{r!(n-r)!}$

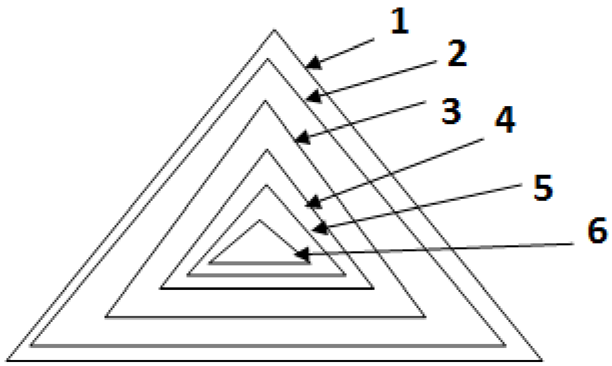
^nC_r = number of combination
n = total number of objects in the set
r = number of choosing objects from the set
Orange, blue, green, white and red
Total five different colour
We can use 1, 2, 3, 4 and 5 colour
It does not matter which colour is used
So, total combinations possible

$= {}^5C_5 + {}^5C_4 + {}^5C_3 + {}^5C_2 + {}^5C_1$

$= \frac{5!}{5!} + \frac{5!}{4!\times 1!} + \frac{5!}{3!\times 2!} + \frac{5!}{2!\times 3!} + \frac{5!}{1!\times 4!}$

= 1 + 5 + 10 + 10 + 5
= 31
Answer
There are 31 different colours of shirts can be made.

8. Ans. C.



Answer
When we count it is clear that
There are 6 triangles in the given figure.
Hence, option C is the correct answer.

9. Ans. D.

In a year, number of weeks = 52 extra day = 1
From 2012 to 2021, there are 10 years.
So number of extra days = 10(1) = 10
While 2012 is a leap year,
Having one more extra day apart from the normal extra day.
Thus, number of extra days = 10 + 1 = 11
Out of these 11 extra days, 7 days form a week and so 4 day remains.
Hence, June 1, 2012 is 4 day less then June 1, 2021 i.e., it is Friday.
Answer
June 1, 2012 is Friday
Hence, option D is the correct answer.

10. Ans. C.

Given, TCS is coded as VAU. The coding pattern is as follows

T $+2 \rightarrow$ V

C $-2 \rightarrow$ A

S $+2 \rightarrow$ U

Given, GOOGLE is coded as IMQENC. The coding pattern is as follows

G $+\mathbf{2} \rightarrow$ I

O $-\mathbf{2} \rightarrow$ M

O $+\mathbf{2} \rightarrow$ Q

G $-2 \rightarrow$ E

L $+2 \rightarrow$ N

E $-2 \rightarrow$ C

Similarly, GRADEUP is coded as follows:

G $+2 \rightarrow$ I

R $-2 \rightarrow$ P

A $+2 \rightarrow$ C

D $-2 \rightarrow$ B

E $+2 \rightarrow$ G

U $-2 \rightarrow$ S

P $+2 \rightarrow$ R

Therefore, GRADEUP is coded as IPCBGSR
GRADEUP is coded as IPCBGSR
Hence, option C is the correct answer.

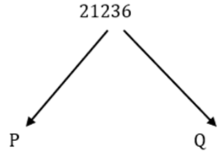
11. Ans. A.

Average distance travelled by A = $\frac{740+700+850+550+280}{5} = \frac{3120}{5} = 624$

Average distance travelled by B = $\frac{650+100+250+300+500}{5} = \frac{1800}{5} = 360$

The average distance travelled by A is greater than that of B.
Hence option A is correct.

12. Ans. C.



$P \times \left[1 + \frac{6}{100}\right]^5 = Q \times \left[1 + \frac{6}{100}\right]^7$

$P \times \left[\frac{53}{50}\right]^5 = Q \times \left[\frac{53}{50}\right]^7$

$\frac{P}{Q} = \frac{2809}{2500}$

5309 units = 21236

1 unit = 4

Therefore,

P : Q

2809 × 4 : 2500 × 4

11236 : 10000

Hence option C is correct.

13. Ans. C.

First we find the least common multiple (L.C.M) of 34, 48 and 58

Therefore, LCM of 34, 48 and 58 = 2 × 2 × 2 × 2 × 3 × 17 × 29 = 23664

Therefore, the required number = 23664 – 31 = 23633

Hence option C is correct.

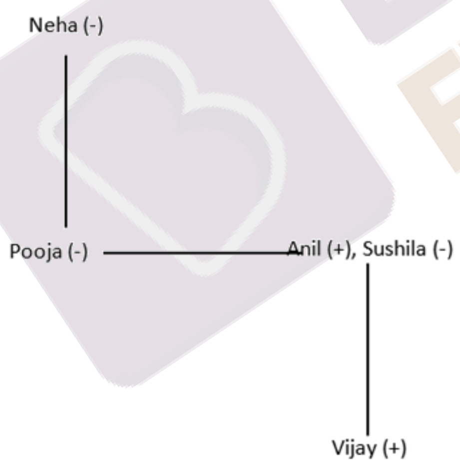
14. Ans. C.

Distance covered on the first day = $\frac{3}{5} \times 90 = 54$ km

Required Ratio = 46 : 54

Hence option C is correct.

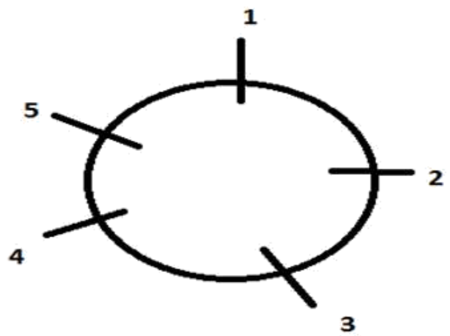
15. Ans. D.



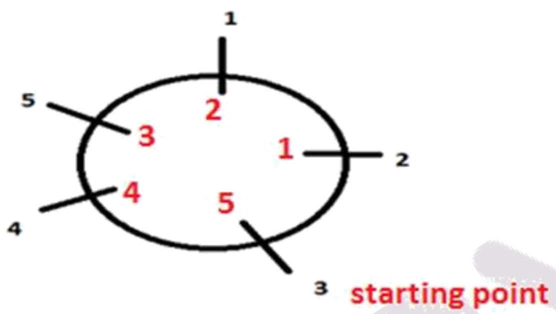
From the diagram, we can see that Neha is the paternal grandmother of Vijay.

Hence option D is correct.

16. Ans. B.



Given
Starting from chair number 3 moving in anti-clockwise



After crossing 19 chairs which means it goes 20 tables
$\frac{20}{5} = 4$ Rounds [The black digit is the chair no.]
After moving 4 rounds it will goes in chair number 3.
And for 20th position, it doesn't matter for the chair position whether it is clock or anti clockwise.
Therefore, after crossing 19 tables, the person will reach the chair number 3.
Hence, option B is the correct answer.

17. Ans. C.

Time taken by car X = Time taken by car Y + 2hr (given)
$T_X = T_Y + 2hr$ ……………………….(1)

$\frac{600}{S_X} = \frac{600}{S_Y} + 2$

Now due to failure,
$T_X = T_Y' - 1hr$ (T_Y' is the new time taken by car Y)

$\frac{600}{S_X} = \frac{600}{\frac{3}{4}S_Y} - 1$ ……………………….(2)

Equating eq (1) and (2)

$\frac{600}{S_Y} + 2 = \frac{600}{\frac{3}{4}S_Y} - 1$

$\frac{600}{S_Y} + 3 = \frac{600}{S_Y} * \frac{4}{3}$

$\frac{600}{S_Y}\left(\frac{4}{3}-1\right)=3$

$S_y = 66.67$ kmph

18. Ans. A.

Approximation method could be used

$7^{12} = (7^2)^6 = (49)^6$

We take simplest value

$50^6 = 5^6 \times 1000000 = 15625000000$, which is 11 digits.

Answer

11 digits are there in 7^{12}

19. Ans. B.

In ΔPOQ and ΔROQ

∠POQ = ∠ROQ (Given)

∠ Q = ∠ Q (common)

By Angle – Angle Concept

Δ POQ is similar to ΔROQ

QO/QR=PQ/PO

QO/5=4/4

QO = 5 cm

Answer

The value of QO is 5cm

20. Ans. D.

Required Probability = $\frac{{}^8C_6\,{}^8C_5 + {}^8C_7\,{}^8C_4 + {}^8C_8\,{}^8C_3}{{}^{16}C_{11}} = \frac{28\times 56 + 8\times 70 + 1\times 56}{4368}$

$= \frac{1568+560+56}{4368} = \frac{2184}{4368}$

$= \frac{1}{2}$

Hence, Option D is correct.

21. Ans. B.

$x^6p^2 - 2xp - 4p = 0$

$\therefore B^2 - 4AC = 4x^2 - (x^6)(-4y)$

$= 4x^2(1 + 4x^4y)$

$\therefore B^2 - 4AC = 0 \Rightarrow 1 + 4x^4y = 0$

$\Rightarrow 4x^4y = -1$

$\therefore$ singular solution of given DE is $4x^{4y} = -1$

22. Ans. B.

Let solution of given IF is g(x)= kx.

$$\therefore g(\xi) = \xi + \int_0^1 \xi u^2 (ku)\cdot du$$

$$= \xi + \xi k \cdot \int_0^1 u^3 \cdot du$$

$$= \xi + k\xi\left[\frac{u^4}{4}\right]_0^1$$

$$= \xi + \frac{k}{4}\xi(1-0)$$

$$= \xi\left(1+\frac{k}{4}\right)$$

$$\therefore k = 1+\frac{k}{4} \Rightarrow k-\frac{k}{4} = 1 \Rightarrow \frac{3k}{4} = 1 \Rightarrow k = \frac{4}{3}$$

$$\therefore g(x) = \frac{4}{3}x$$

Answer is option B

23. Ans. A.

f(x, y, y') = $3y'^2 + 4xy$

Euler's equation

$$\frac{\partial f}{\partial y} - \frac{d}{\partial x}\left(\frac{\partial f}{\partial y'}\right) = 0$$

(4x) – 6y" = 0

∴ 6y" = 4x

$$y'' = \frac{4}{6}x$$

$$y' = \frac{2}{3}\cdot\frac{x^2}{2} + A$$

$$y = \frac{1}{3}\cdot\frac{x^3}{3} + Ax + B$$

$$y(0) = 0 \Rightarrow B = 0$$

$$y(1) = 1 \Rightarrow 1 = \frac{1}{9}(1) + A \Rightarrow A = \frac{8}{9}$$

$$\therefore y(x) = \frac{1}{9}x^3 + \frac{8}{9}x$$

24. Ans. C.

Option A.

f(x) = x & x∈(2, 3)

f(a). f(b) > 0

f(x) has no roots in (2, 3)

∴ option A is false

Option. B

f(x) = x^2 x∈(–1, 1)

f (–1). f (1) > 0

and f(x) has roots in (–1, 1)

∴ option B is false

Option. C

f(x) is continuous and f(a). f(b) > 0
⇒ either both are negative or both are positive.
⇒ So, graph at f(x) either not intersect x-axis or cut even number of times
∴ option C is true.

25. Ans. B.

Det (A) = 0 $\Rightarrow$ rank (A) $\neq 4$
$\therefore \text{rank}(A) < 4$

Result: rank(A) $\leq n-2 \Rightarrow$ rank (adj A) = 0
If rank (adj A) = 0 $\Rightarrow$ adj A is null matrix
but given that adjA is non-null matrix.
∴ rank(A) > n-2 = 2
∴ 2 < rank (A) < 4
∴ rank (A) = 3
∴ Answer is option (B)

26. Ans. B.

Let f(x) = z^2
Option A, C, D are false
Answer is option B

27. Ans. A.

gcd (8, 103) = 1
By Euler's theorem,
If gcd (a, n) = 1 then $a^{\phi(n)} \equiv 1$ (mod n)
$\therefore 8^{\phi(103)} \equiv 1 \pmod{103}$
$\therefore 8^{102} \equiv 1 \pmod{103}$
$\therefore 8^{103} \equiv 8 \pmod{103}$
Answer is option A

28. Ans. B.

P(x)y" + q(x)y' + r(x)y = O
∴ P(x) = 0 are singular point.
Here P(x) = $x^2(1 - x)$
∴ x = 0, 1 are singular point
∴ option (a) & (d) are false.

$A(x) = \frac{q(x)}{P(x)} = \frac{1}{x(1-x)}$ & $B(x) = \frac{r(x)}{p(x)} = \frac{1}{x^2(1-x)}$

For x = 0,

$$\lim_{x\to 0} x.A(x) = \lim_{x\to 0}\frac{1}{1-x} = 1$$

$$\lim_{x\to 0} x^2.B(x) = \lim_{x\to 0}\frac{1}{1-x} = 1$$

∴ x = 0 is regular singular point

∴ option (c) is false.
Far x = 1

$$\lim_{x\to 1}(x-1)A(x) = \lim_{x\to 1}\frac{1}{(-x)} = -1$$

$$\lim_{x\to 1}(x-1)^2.B(x) = \lim_{x\to 1}\frac{(x-1)}{(-x^2)} = 0$$

∴ x = 1 is regular singular point

∴ Answer is option (B)

29. Ans. B.

F (x, y, y') = $(y + y')^2$

By Euler's Equation

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

2(y + y') - 2(y' + y'') = 0

2y - 2y'' = 0

Therefore, y'' – y = 0

Therefore, Answer b option is (B)

30. Ans. B.

Z=0 is lies inside C.

$$\int_C \frac{\cos z}{z(z^2+8)}dz = 2\pi i f(0) where f(z) = \frac{\cos z}{(z^2+8)}$$

$$= 2\pi i\left(\frac{1}{8}\right)$$

$$= \frac{\pi i}{4}$$

Answer is option B.

31. Ans. B.

Option A,

Let $\mathbb{Z}_6$ is cyclic group but $\mathbb{Z}_6 \not\approx \mathbb{Z}$

Option A is false.

Option B & C,

Let $\mathbb{Q}/\mathbb{Z}$ it is infinite group in which every element has order finite.

Option B is true & C is false.

Answer is option B.

32. Ans. C.

The degree of the DE is the highest power of the highest derivative provided all the derivatives are in natural powers.

33. Ans. B.

From the point $P(0,0)\ \&\ Q(5,0)$ satisfies in only option B

Answer is option B

$$f(x,y,y')=\int_0^5 (9y^2+y'^2-3x)\,dx$$

By Euler's equation

$$\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$$

$$18y-2y''=0$$

$$y(x)=C_1e^{3x}+C_2e^{-3x}$$

$$y(0)=0\Rightarrow C_1+C_2=0\Rightarrow C_2=-C_1$$

$$y(5)=0\Rightarrow C_1e^{3x}-C_1e^{-3x}=0\Rightarrow C_1=0$$

$$y(x)=0$$

Answer is option B

34. Ans. C.

Now the number of units in $\mathbb{Z}_{15}=\Phi(15)=8.$

We know that every non zero element in a finite ring having no divisor of zero is a unit. Therefore the number of divisor of zero is=15-8=7.

35. Ans. C.

The number of subring of $\mathbb{Z}_n$ is τ(n).

Hence the number of subring of $\mathbb{Z}_{40}$ is τ(40)=τ(8)×τ(5)=8.

36. Ans. A.

$\frac{\mathbb{Z}[i]}{<3+i>}\cong\mathbb{Z}_{10}$. The number of maximal ideal of $\frac{\mathbb{Z}[i]}{<2+3i>}$ is ω(10), where ω is the number of distinct prime divisor of 10.

ω(10)=2. So the number of $\frac{\mathbb{Z}[i]}{<2+3i>}$ maximal ideal is 2.

37. Ans. A.

Given x_i's are independent random variables such that

X_i's are symmetric at about zero,

i.e. $E(x_i) = 0$

Since, $s_n=\sum_{i=1}^{n}x_i$

$E(s_n) = 0$

and $v(s_n) = v(x_1) + v(x_2) + \ldots\ldots + v(x_n)$

$= 1 + 3 + 5 + \ldots\ldots + 2n - 1$

$$=\frac{n}{2}[2\times1+(n-1)2]$$

$$=\frac{n}{2}[2+2n-2]=n^2$$

Hence, the required probability of using CLT

$$= \lim_{n\to\infty} P\left[\frac{s_n - E(s_n)}{\sqrt{v(s_n)}} \geq \frac{n\log n - E(s_n)}{\sqrt{v(s_n)}}\right] \sim N(0,1)$$

$$= \lim_{n\to\infty} P\left(z > \frac{n\log n}{\sqrt{n^2}}\right)$$

$$= \lim_{n\to\infty} P\left(z > \frac{n\log n}{n}\right)$$

$$= \lim_{n\to\infty} P(z > \log n)$$

$$= P(z > \infty)$$

= 0

38. Ans. B.

$p(x) = x^4 + 4 = x^4 + 4x^2 + 4 - 4x^2 = (x+2)^2 - (2x)^2 = (x+2+2x)(x+2-2x)$

So, it is reducible over $\mathbb{Q}$.

Option B is true.

Root of $p(x)$ are $1 \pm i, -1 \pm i$

Splitting field $\mathbb{Q}(i)$

$\dim[\mathbb{Q}(i):\mathbb{Q}] = 2$

option C, D are false.

$(1+i)(1-i) = 1 - i^2 = 2 \notin \text{group}$

Option A is false.

39. Ans. A. B. D.

We know that if f is a monotone function on [a,b] then f is of bounded variation on [a,b]. Option (A) and (B) are correct.

Also, if derivative of f(x) is bounded on (a,b) then f is of bounded variation

Option (D) is also correct

40. Ans. A. C.

$\{x \in v: \langle x, y\rangle = 0\}$

$y\ (\neq 0) \in v$ is subspace

$\therefore \{x \in v: \langle x, y\rangle = 0\} = S^{\perp}$

$\therefore S^{\perp}$ is subspace of v.

Option (B),

$A = \{x \in v: \langle x, y\rangle = 1\}$ is not subspace since $0 \notin A$.

Option (C)

$\{x \in v: \langle x, z\rangle = 0 \propto z$ such that: $\langle z, y\rangle = 0\}$

$\langle z, y\rangle = 0 \Rightarrow x \in S^{\perp}$

$x \perp z \Rightarrow x \in s^{\perp\perp}$

$s^{\perp\perp}$ is a subspace.

Option (D)
Not subspace.

Since 0 $\notin$ given set in option (D)

41. Ans. B. C.

$$f(z)=\frac{1-e^z}{1+e^z}$$

Poles of f(z) are obtained by equating to denominator of f(z)

$1 + e^z = 0 \Rightarrow e^z = -1$

$\Rightarrow z=(2n+1)\pi i, n\in\mathbb{Z}$

are the simple poles.

and $z = \infty$ is limit point of $(2n+1)\pi i$ (poles).

$z = \infty$ is non isolated essential singularity.

42. Ans. A. B.

Let R be a commutative ring with unity

R – M = set of all units in R $\Rightarrow$ M contain all non-units

Now, ideal in a ring is generated by non-unit on contain only non-unit.

Now as all the non-unit belongs to M, i.e., no one unit left in R so that we can not make more ideal

$\Rightarrow$ M is maximal ideal.

Mathematically,

Let $x\in R/M$

Assume M is not maximal. For an ideal K in R satisfying

$M\subset K\subset R$. K = M + $\langle x\rangle$

Two cases arose

Case-I $\Rightarrow$ x is unit, then

$\langle x\rangle = R$

Case-II $\Rightarrow$ x is non-unit $\langle x\rangle = 1$

As M contain all non-units, it is clear that M + $\langle x\rangle = M$

$\Rightarrow$ either K = R or K = M

Hence M is maximal and therefore R/M is a field.

So, option A is true.

(Result: In a commutative ring with unity if M is maximal then R/M is a field)

R is commutative ring with unity.

Given R/M is an integral domain = M is a prime ideal.

So option B is true.

R = C [0, 1] is ring of real valued continuous functions on [0, 1] w.r.t_0. pointwise addition and print wise multiplication.

Let $M=\{f\in R: f(0)\,f(1)=0\}$. Consider $M'=\{f\in R: f(0)=0\}$

Now, $M\subseteq M'$ and we know m' is maximal ideal and hence M is not a maximal ideal.

So option C is not true.

43. Ans. A. D.

$y_1(x)$ is any non-trivial solution of $y'' + xy = 0$ where $0 < x < \infty$
Since, x can take any positive value. So let us observe the nature of the solution of (1) by finding out the solution of $y'' + y = 0$ is $y(x) = c_1 \cos x + c_2 \sin x$
Clearly, y(x) has infinitely many zeros in R.
So, we can say that $y_1(x)$ has infinitely many zeros.
Hence option A is correct.
The solution of $y'' + y = x^2 + 2, y(0) = y'(0) = 0$ is $y_2(x)$, which is given $y_2(x)$ = C.F + P.I
C.F is the solution of $y'' + y = 0$
i.e. C.F = $C_1' \cos x + C_2' \sin x$

$$\text{P.I.} = \frac{1}{D^2+1}(x^2+2)$$

$= (1 + D^2)^{-1} (x^2 + 2)$

$$= \left(1 - D^2 + \frac{D^4}{4} \ldots\ldots\right)(x^2+2)$$

$= x^2 + 2 - 2$
$= x^2$

Hence, $y_2(x) = C_1' \cos x + C_2' \sin x + x^2$
$\Rightarrow y_2'(x) = -C_1' \sin x + C_2' \cos x + 2x$
Since, $y_2(0) = y_2'(0) = 0$
$\Rightarrow C_1' = 0$ and $C_2' = 0$
$\Rightarrow y_2(x) = x^2$ and it has only one zeros in R
Hence, $y_2(x)$ has finitely many zeros.
Therefore, option A and D are correct

44. Ans. A. C.

Let $x_T = 2.71828$ and $x_A = 2.71937$
Therefore, the percentage error is

$$E_P = \left|\frac{x_T - x_A}{x_T}\right| \times 100$$

i.e., $E_P = \left|\frac{2.71828 - 2.71937}{2.71828}\right| \times 100 = 0.04\%$

Hence option A and C are correct.

45. Ans. A. B. D.

Obviously the given iteration scheme is for finding the root of $x^2 = b$
For $x_0 = 2$, b = 3, the first iteration is

$$x_1 = \frac{2^2+3}{2\times 2} = 1.45$$

For $x_0 = 2$, b = 3, the second iteration is

$$x_2 = \frac{(1.45)^2 + 3}{2 \times 1.45} = 1.7595$$

Hence option A, B and D are correct.

46. Ans. A. B. C. D.

The general solution of the differential equation y” – y = 0 is $y(x) = a\,e^{x} + be^{-x}$ where a and b are arbitrary constant.

Now, $\lim_{x\to\infty} |y(x)| = \infty$, since $\lim_{x\to\infty} e^{x} = \infty$

Also $\lim_{x\to-\infty} |y(x)| = \infty$ since $\lim_{x\to-\infty} e^{x} = \infty$

Again, $\lim_{x\to 0} |y(x)| = a + b$, finite.

Hence, A, B, C, D are correct.

47. Ans. A. B. C. D.

We have 0(G) = 65 = 13.5

The number of 13-sylow subgroup of G is given by $n_{13} = 1 + 13k$, k = 0,1,2,…. Where n_{13}/0(G).

For k = 0, $n_{13} = 1$ and n_{13}/0(G) = 65

Hence there exists exactly one 13-sylow subgroup then it is normal in G. Note that G is not simple, Since G contains only are normal 13-sylow subgroup.

As 13-sylow subgroup of G of order 13(prime), so it is cyclic.

Now, the number of 5-sylow subgroup of G is given by $n_5 = 1 + 5k$, k = 0,1,2,……, where n_5/0(G).

For k = 0, $n_5 = 1$ and $n_5|65$

So, there exists only are 5-sylow subgroup of G of order 5. As 5-sylow subgroup of G of order 5 (prime)

So it is cyclic.

48. Ans. A. C. D.

$$\int_C \frac{|z|}{z} dz. \qquad |z| = 1$$

$$= \int_0^{2\pi} \frac{1}{e^{i\theta}} ie^{i\theta} d\theta \qquad \Rightarrow z = e^{i\theta}, 0 < \theta < 2\pi$$

$$\Rightarrow dz = ie^{i\theta} d\theta$$

$$= i\int_0^{2\pi} d\theta$$

$$= 2\pi i$$

49. Ans. A. B.

Given, tr(A) = –1 and det A = 1

$C_A(x) = x^2 + x + 1$

By Cayley Hamilton theorem

$A^2 + A + I = 0$

$\therefore A^2 = -A - I$

$\therefore A^3 = A^2A = (-A - I)A$

$= -A^2 - A$

$A^3 = I$

$\therefore$ option (A) is correct

for option B,
Given $A^3 = I$
So, $P(x) = x^3 – 1$ is an annihilating polynomial for A.
$\therefore M_A(x) / (x^3 – 1)$
$M_A(x) = (x – 1)$ or $M_A(x) = x^2 + x + 1$
Since A is not diagonal matrix
$\therefore M_A(x) = x^2 + x + 1$
Moreover $C_A(x) = x^2 + x + 1$
clearly now, tr A = –1 and det A = 1
option (B) is correct
Option (C),
Take A = $\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$
clearly $A^3 = I$
but A is not diagonalizable over $\mathbb{R}$ as
Eigen value of A are not in $\mathbb{R}$.
$\therefore$ option (C) is false

50. Ans. A. D.

Let, $x^2 = y$ then the given series becomes $1 + \frac{1}{2}y + \frac{1.3}{2.4}y^2 + \frac{1.3.5}{2.6.8}y^3 + \ldots\ldots = \sum a_n y^n (say)$

Where $a_0 = 1$ $a_n = \frac{1.3.5\ldots\ldots(2n-1)}{2.4.6\ldots\ldots 2n} for n \geq 1$

$$\frac{1}{R} = \lim_{n\to\infty} \frac{1.3.5\ldots\ldots(2n+1)2.4.6\ldots.(2n)}{2.4.6\ldots\ldots(2n+2)1.3.5\ldots\ldots(2n-1)}$$

$$= \lim_{n\to\infty} \frac{2n(2n+1)}{(2n+1)(2n+2)} = 1 \quad \therefore R = 1$$

Therefore, the radius of convergence is 1.
Hence the interval of convergence is
{y ∈ $\mathbb{R}$ - 1< y <1} i.e. {x ∈R: -1 < x< 1}
At x = ± 1, the series is divergent.