# CDS I 2021 PYSP Mathematics: Solution

1. Ans. B.

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{n(n+1)}=\frac{99}{100}$$
$$\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{n}-\frac{1}{n+1}\right)=\frac{99}{100}$$
$$1-\frac{1}{n+1}=\frac{99}{100}$$
$$\frac{n}{n+1}=\frac{99}{100}$$
$$n=99$$

2. Ans. B.

speed of the train $=\frac{200+100}{10}=30\,m/s$

3. Ans. A.

let their incomes are 7m, 9m and 10m and their expenditures are 8n, 9n and 15n. then their savings are $(7m-8n),(9m-9n)$ and $(10m-15n)$. Now, given that

$$7m-8n=\frac{1}{4}(7m)\Rightarrow 21m=32n \qquad \text{...(i)}$$

Now ratio their savings are $\Rightarrow(7m-8n):(9m-9n):(10m-15n)$

$$\Rightarrow 7\left(\frac{32}{21}\right)n-8n:9\left(\frac{32}{21}\right)n-9n::12\left(\frac{32}{21}\right)n-15n$$
$$\Rightarrow\frac{56}{21}:\frac{99}{21}:\frac{69}{21}$$
$$\Rightarrow 56:99:69$$

4. Ans. C.

the ratio of boys and girls in the class is $3:1$, let the number of boys and girls in the class are 3k and k. Given that the average score of the class is p and average score of the boys is $(p+1)$, let the average score of the girls is , then total score of the class

$\Rightarrow (4k)\times p = 3k(p+1)+k(x)$

$\Rightarrow x = p-3$

5. Ans. D.

change after adding 3 to both the numerator and denominator of all the fractions

$\frac{5}{6}-\frac{2}{3}=\frac{3}{6}$

$\frac{6}{7}-\frac{3}{4}=\frac{3}{28}$

$\frac{7}{8}-\frac{4}{5}=\frac{3}{40}$

$\frac{8}{9}-\frac{5}{6}=\frac{3}{54}$

We can see the minimum change is with fraction $\frac{5}{6}$.

6. Ans. C.

let $f(x)=4x^3+12x^2-x-3$

Now $f\left(-\frac{1}{2}\right)=4\times\frac{(-1)}{8}+12\times\frac{1}{4}-\frac{(-1)}{2}-3=0$

Hence $x=-\frac{1}{2}$ or $2x+1=0$ is a factor of $f(x)$

And $f\left(\frac{1}{2}\right)=4\times\frac{1}{8}+12\times\frac{1}{4}-\frac{1}{2}-3=0$

Hence $x=\frac{1}{2}$ or $2x-1=0$ is a factor of $f(x)$

7. Ans. B.

given that the sum of the roots and the product of roots are 6.

$-\frac{(-6)}{p}=\frac{q}{p}=6\Rightarrow q=6 \text{ and } p=1$

So, $(p+q)=6+1=7$

8. Ans. C.

if a quadratic equation $ax^2+bx+c=0$ has equal roots,

the discriminant, $D=0 \Rightarrow b^2=4ac$

$$(2k)^2=4\times4\times3k$$
$$k(k-12)=0$$
$$k=0,12$$

9. Ans. B.

given that $x+\frac{1}{x}=\frac{5}{2}$

$$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=\frac{25}{4}-2=\frac{17}{4}$$

$$x-\frac{1}{x}=\sqrt{\left(x+\frac{1}{x}\right)^2-4}=\sqrt{\left(\frac{5}{2}\right)^2-4}=\frac{3}{2}$$

$$x^4-\frac{1}{x^4}=\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)$$
$$=\frac{5}{2}\times\frac{3}{2}\times\frac{17}{4}=\frac{255}{16}$$

10. Ans. C.

for a quadratic equation $ax^2+bx+c=0$ to be perfect square,

the discriminant, $D=0 \Rightarrow b^2=4ac$

$$(12k-24)^2 = 4(6k)(16)$$
$$12\times12\left(k^2-4k+4\right) = 4\times6\times16k$$
$$3k^2-12k+12 = 8k$$
$$3k^2-20k+12 = 0$$
$$(k-6)(3k-2) = 0$$
$$k = 6, \frac{2}{3}$$

11. Ans. C.

$$a = \frac{b+\sqrt{b^2-2bx}}{b-\sqrt{b^2-2bx}}$$
$$\frac{a+1}{a-1} = \frac{b}{\sqrt{b^2-2bx}}$$
$$\frac{a^2+2a+1}{a^2-2a+1} = \frac{b^2}{b^2-2bx}$$
$$\frac{a^2-2a+1}{a^2+2a+1} = \frac{b-2x}{b} = 1-\frac{2x}{b}$$
$$x = \frac{b}{2}\left(1-\frac{a^2-2a+1}{a^2+2a+1}\right)$$
$$= \frac{b}{2}\left(\frac{4a}{a^2+2a+1}\right)$$
$$= \frac{2ab}{(a+1)^2}$$

12. Ans. A.

we know that the unit digits of $7^{4k+1} \equiv 7, 7^{4k+2} \equiv 9, 7^{4k+3} \equiv 3, 7^{4k+4} \equiv 1$

$$67^{32} \equiv 7^{32} \equiv 7^{4\times7+4} \equiv 1$$

13. Ans. B.

$$p = \frac{\sqrt{3q+2}+\sqrt{3q-2}}{\sqrt{3q+2}-\sqrt{3q-2}}$$

$$\frac{p+1}{p-1} = \frac{\sqrt{3q+2}}{\sqrt{3q-2}}$$ [by componendo-divindo]

$$\frac{p^2+2p+1}{p^2-2p+1}=\frac{3q+2}{3q-2}$$ [by squaring both sides]

$$\frac{2\left(p^2+1\right)}{4p}=\frac{6q}{4}$$ [by componendo-divindo]

$$p^2+1=3pq$$
$$p^2-3pq+1=0$$
$$p^2-3pq+2=1$$

14. Ans. D.

we have $a+b+c=0$

Squaring both sides

$$a^2+b^2+c^2+2(ab+bc+ca)=0$$
$$a^2+b^2+c^2=-2(ab+bc+ca) \qquad ...(ii)$$

$$a+b+c=0 \Rightarrow a+b=-c$$

Cubing both sides

$$(a+b)^3=(-c)^3$$
$$a^3+b^3+3ab(a+b)=-c^3$$
$$a^3+b^3+c^3=-3ab(a+b) \qquad ...(ii)$$
$$a^3+b^3+c=-3ab(-c)$$
$$a^3+b^3+c=3abc \qquad ...(iii)$$

15. Ans. A.

$$\begin{aligned}27^{27}-15^{27}&=(24+3)^{27}-(12+3)^{27}\\&=\left(24k_1+3^{27}\right)-\left(12k_2+3^{27}\right)\\&=12\left(2k_1-k_2\right)\end{aligned}$$

Here, we can see that it is always divisible by 6.

16. Ans. D.

since all the numbers are different then total number of terms = $= 3 \times 4 \times 5 = 60$

17. Ans. B.

prime numbers between 50 to 100 are 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

$59 - 53 = 67 - 61 = 79 - 73 = 6$

Such number of pairs are 3.

18. Ans. A.

here

$$\frac{x}{y} + \frac{y}{x} = 2$$
$$x^2 + y^2 - 2xy = 0$$
$$(x - y)^2 = 0$$
$$x = y$$

Since $x \neq y$ then, there are no such pairs.

19. Ans. B.

we have to find remainder of $\frac{2^{1000000}}{7}$

We know $R\left[\frac{2^3}{7}\right] = R\left[\frac{8}{7}\right] = 1$

$$R\left[\frac{2^{1000000}}{7}\right] = R\left[\frac{(2^3)^{333333} \times 2}{7}\right] = R\left[\frac{(2^3)^{333333}}{7}\right] \times R\left[\frac{2}{7}\right] = 1 \times 2 = 2$$

20. Ans. D.

we know that the rule of divisibility by 13 is "if $n_1 n_2 \cdots\cdots$ is a number N, then if the number formed by the alternative sum and difference of blocks of 3-3 digits from right to left is divisible by 13 then N is divisible by 13".

So, for $413283P759387$ is divisible by 13

$387-759+83P-132+4=83P-500=330+P$ must be divisible by 13.

$330+P=13(26)-8+P$ must be divisible by 13. So, P must be 8.

21. Ans. D.

$$\frac{1}{bc(a-b)(a-c)}+\frac{1}{ca(b-c)(b-a)}+\frac{1}{ab(c-a)(c-b)}$$
$$\Rightarrow \frac{a(c-b)+b(a-c)+c(b-a)}{abc(a-b)(b-c)(c-a)}$$
$$\Rightarrow \frac{ac-ab+ba-bc+cb-ca}{abc(a-b)(b-c)(c-a)}$$
$$\Rightarrow 0$$

22. Ans. A.

given that $x(x-1)(x-2)(x-3)+1=k^2$

$$\begin{aligned} k^2 &= x(x-1)(x-2)(x-3)+1 \\ &= (x^2-3x)(x^2-3x+2)+1 \\ &= y(y+2)+1 \qquad [t=x^2-3x] \\ &= y^2+2y+1 \\ &= (y+1)^2 \end{aligned}$$
$$k=y+1$$
$$k=x^2-3x+1$$

23. Ans. C.

$$x=\frac{12}{7-\frac{6}{7-\frac{3}{5-x}}}$$

$$x=\frac{12}{7-\frac{6(5-x)}{35-7x-3}}$$

$$x=\frac{12(32-7x)}{224-49x-30+6x}$$

$$x=\frac{384-84x}{194-43x}$$

$$194x-43x^2=384-84x$$

$$43x^2-278x+384=0$$

$$(x-2)(43x-192)=0$$

Integer value of $x=2$.

24. Ans. A.

$$\Rightarrow \frac{8x}{1-x^4}-\frac{4x}{x^2+1}+\frac{x+1}{x-1}-\frac{x-1}{x+1}$$

$$\Rightarrow \frac{8x}{(1-x)(1+x)(1+x^2)}-\frac{4x}{x^2+1}+\frac{(x+1)^2-(x-1)^2}{(x-1)(x+1)}$$

$$\Rightarrow \frac{8x-4x(x^2+1)}{(1-x)(1+x)(1+x^2)}+\frac{4x}{(1-x)(1+x)}$$

$$\Rightarrow \frac{8x-4x(x^2+1)+4x(x^2-1)}{(1-x)(1+x)(1+x^2)}$$

$$\Rightarrow \frac{8x-4x^3-4x+4x^3-4x}{(1-x)(1+x)(1+x^2)}$$

$$\Rightarrow 0$$

25. Ans. D.

factorizing both polynomials

$$x^3-19x+30=(x-2)(x-3)(x+5)$$

$$x^2-5x+6=(x-2)(x-3)$$

HCF $=(x-2)(x-3)$

26. Ans. C.

Statement-I: $x \propto z \Rightarrow x = k_1 z$ and $y \propto z \Rightarrow y = k_2 z$

$$x^2 - y^2 = k_1^2 z^2 - k_2^2 z^2$$
$$= \left(k_1^2 - k_2^2\right) z^2$$
$$\left(x^2 - y^2\right) \propto z^2$$

Statement-I: $x \propto \frac{1}{z} \Rightarrow xz = k_1$ and $y \propto \frac{1}{z} \Rightarrow yz = k_2$

$$(xz)(yz) = k_1 k_2 = k$$
$$xyz^2 = k$$
$$xy = \frac{k}{z^2}$$
$$xy \propto \frac{1}{z^2}$$

27. Ans. B.

If $(x-k)$ is the HCF of $x^2 + ax + b$ and $x^2 + cx + d$, then $x = k$ will satisfy both $x^2 + ax + b = 0$ and $x^2 + cx + d = 0$ equations.

Then, $k^2 + ak + b = 0$ ...(i)

And $k^2 + ck + d = 0$ ...(ii)

By equation (i) and (ii)

$$\left(k^2 + ak + b\right) - \left(k^2 + ck + d\right) = 0$$
$$k(a-c) + (b-d) = 0$$
$$k = \frac{d-b}{a-c}$$

28. Ans. C.

let $x = a^2 \text{ and } y = b^2$ then it satisfy both the equations, now

$$\frac{x}{a^2} - \frac{y}{b^2} = \frac{a^2}{a^2} - \frac{b^2}{b^2} = 0$$

Or

You can also put $x=y=a=b=1$.

29. Ans. A.

we should be add

$$=\frac{2x-5}{(x^2-5x+6)(x-4)}-\frac{1}{(x-2)(x-4)}$$
$$=\frac{2x-5}{(x-2)(x-3)(x-4)}-\frac{1}{(x-2)(x-4)}$$
$$=\frac{1}{(x-2)(x-4)}\left[\frac{2x-5}{x-3}-1\right]$$
$$=\frac{1}{(x-2)(x-4)}\left(\frac{x-2}{x-3}\right)$$
$$=\frac{1}{(x-3)(x-4)}$$
$$=\frac{1}{x^2-7x+12}$$

30. Ans. B.

$$\frac{(x^3-1)(x^2-9x+14)}{(x^2+x+1)(x^2-8x+7)}=\frac{(x-1)(x^2+x+1)(x-7)(x-2)}{(x^2+x+1)(x-7)(x-1)}=(x-2)$$

31. Ans. A.

let a man works a unit of work in days then,

$y=(x-1)(x+1)$ and $z=(x+2)(x-1)$

And $y:z=9:10$

$$\frac{(x-1)(x+1)}{(x+2)(x-1)}=\frac{9}{10}$$
$$\frac{x+1}{x+2}=\frac{9}{10}$$
$$10x+10=9x+18$$
$$x=8$$

32. Ans. C.

let a person can clean 1 unit of floor in one day. then,

20 persons clean 20 floors ≡ 20 days

20 persons clean 1 floor ≡ 1 day

1 person cleans 1 floor ≡ 20 days

1 person cleans 16 floors ≡ $20 \times 16$ days = 320 days

16 persons clean 16 floors ≡ $\frac{320}{16}$ days = 20 days

Or

$$\frac{M_1 \times D_1}{W_1} = \frac{M_2 \times D_2}{W_2}$$
$$\frac{20 \times 20}{20} = \frac{16 \times D_2}{16}$$
$$D_2 = 20$$

33. Ans. B.

liquid in 80 litres of mixture = $25\% \; of \; 80 = 20$

And water in the mixtue = 80 – 20 = 60

Let $x$ litres of water is added in the mixture, so

$$\frac{20}{80+x} = 20\%$$
$$\frac{20}{80+x} = \frac{1}{5}$$
$$80 + x = 100$$
$$x = 20 \, litres$$

34. Ans. B.

let the price of X's goods is $x$ then,

Y's goods price $= \frac{x}{1-0.25} = \frac{4x}{3}$

Z's goods price $= \frac{x}{1+0.25} = \frac{4}{5}x$

Percentage cheaper of Z's goods than Y's good

$= \dfrac{(4x/3)-(4x/5)}{4x/3} \times 100\% = 40\%$

35. Ans. B.

let the cost price is CP and selling price is SP.

$$100 \times CP = 80 \times SP$$
$$\frac{SP}{CP} = \frac{80}{100} = \frac{4}{5}$$
$$\frac{SP-CP}{SP} = \frac{1}{5}$$

%age profit = $\dfrac{SP-CP}{SP} \times 100\% = \dfrac{1}{5} \times 100\% = 20\%$

36. Ans. A.

let the speed of the man is v and time take by him is t.

Then , distance

$$\Rightarrow vt = \frac{4v}{5}(t+12)$$
$$\Rightarrow 5t = 4t + 48$$
$$\Rightarrow t = 48 \text{ minutes}$$

37. Ans. C.

Average speed, $v_{av} = \dfrac{\text{total distance}}{\text{total time}} = \dfrac{600+900}{5+10} = \dfrac{1500}{15} = 100\,kmph$

38. Ans. D.

let the principal amount is Rs. P and the rate of interest is 12% for 2 years. Then,

$CI - SI = 72$

$P\left[\left(1+\frac{12}{100}\right)^2 - 1\right] - \frac{P\times 12\times 2}{100} = 72$

$P\left(\frac{212}{100}\right)\left(\frac{12}{100}\right) - \frac{24P}{100} = 72$

$P\left(\frac{144}{10000}\right) = 72$

$P = 5,000$

39. Ans. A.

let the sum of amount is Rs. P and rate of simple interest is r% and invested it for 5 years, then

$SI = \frac{P\times r\times 5}{100} = \frac{P\times r}{20}$

According to the question if the rate of interest was 5% more then, new S.I.

$\frac{P\times(r+5)\times 5}{100} = \frac{P\times r}{20} + 500$

$\frac{P}{4} = 500$

$P = 2000$

40. Ans. B.

if successive discounts are 20%, 10% and 5% then,

overall discounts $= [1-(1-0.2)(1-0.1)(1-0.05)]\times 100\% = 31.6\%$

41. Ans. A.

we know that for real solution $b^2 \ge 4ac$

$4y^2 \sin^4\theta \ge 4y^2$

$\sin^4\theta \ge 1$

$\sin\theta \ge 1$

But, $\sin\theta = 1$

So, $x^2 + y^2 - 2xy\sin^2\theta = 0$

$$x^2 + y^2 - 2xy = 0$$
$$(x-y)^2 = 0$$
$$x = y$$

42. Ans. B.

let $p = 2 - 2\sin x - \sin^2 x$ , $0 \le \theta \le \frac{\pi}{2}$

$$p = 2 - \left(1 + 2\sin x + \sin^2 x\right) + 1$$
$$p = 3 - (1 + \sin x)^2$$
$$p|_{max} = 3 - (1+0)^2 = 2 \qquad [\sin x|_{min} = 0]$$
$$p|_{min} = 3 - (1+1)^2 = -1 \qquad [\sin x|_{max} = 1]$$

So, the required ratio will be -2.

43. Ans. A.

we have $p = \sin^2\theta + \cos^4\theta$ for $0 \le \theta \le \frac{\pi}{2}$

$$\begin{aligned} p &= \sin^2\theta + \cos^4\theta \\ &= \sin^2\theta + \left(1 - \sin^2\theta\right)^2 \\ &= 1 + \sin^2\theta - 2\sin^2\theta + \sin^4\theta \\ &= 1 - \sin^2\theta + \sin^4\theta \\ &= \cos^2\theta + \sin^4\theta \end{aligned}$$
$$\begin{aligned} 2p &= 1 + \sin^4\theta + \cos^4\theta \\ &= 1 + \left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin\theta\cos\theta \\ &= 1 + 1 - \sin 2\theta \end{aligned}$$
$$p = 1 - \frac{1}{2}\sin 2\theta$$

As we know that in $0 \le \theta \le \frac{\pi}{2}$

$$0 \le \sin 2\theta \le 1$$

$p|_{max} = 1 - \frac{1}{2}(0) = 1$ and $p|_{min} = 1 - \frac{1}{2}(1) = \frac{1}{2}$

We can see that p can never be more that 1 and less than $\frac{1}{2}$.

44. Ans. B.

given that $0 \le \theta \le \frac{\pi}{2} \Rightarrow 0 \le 2\theta \le \pi$

We can write it as $0 \le 2\theta \le \frac{\pi}{2}$ $or$ $\frac{\pi}{2} \le 2\theta \le \pi$

Taking sine

$$\sin 0 \le \sin 2\theta \le \sin\frac{\pi}{2}$$
$$0 \le 2\sin\theta\cos\theta \le 1$$
$$0 \le \sin\theta\cos\theta \le \frac{1}{2}$$

45. Ans. C.

we have $3\sin^2\theta + 4\cos^2\theta = 3\left(\sin^2\theta + \cos^2\theta\right) + \cos^2\theta = 3 + \cos^2\theta$

$$\left(3\sin^2\theta + 4\cos^2\theta\right)_{\min} = 3 + \cos^2\theta\,|_{\min} = 3$$

46. Ans. A.

we have $5^{x-3} = 8 = 2^3$

Taking log both side at base 10

$$\log_{10} 5^{x-3} = \log_{10} 2^3$$
$$(x-3)\log_{10} 5 = 3\log_{10} 2$$
$$(x-3)\log_{10}(10/2) = 3\log_{10} 2$$
$$(x-3)(1-\log_{10} 2) = 3\log_{10} 2$$
$$x-3 = \frac{3\log_{10} 2}{1-\log_{10} 2}$$
$$x = \frac{3\log_{10} 2}{1-\log_{10} 2} + 3$$
$$x = \frac{3}{1-\log_{10} 2}$$

47. Ans. D.

let n = 1 then $5^{2n}-1=5^2-1=24$

n = 2 then $5^4-1=\left(5^2-1\right)\left(5^2+1\right)=24\left(5^2-1\right)$

n = 3 then $5^6-1=\left(5^2-1\right)\left(5^4+5^2+1\right)=24\left(5^4+5^2+1\right)$

we can observe that we get always factor of 24 and factors of 24 are 1,2,3,4,6,8,12,24.

So, number of natural numbers that divides $5^{2n}-1$ are 8.

48. Ans. C.

let the let two alternate natural numbers are a and a+2, then according to the question,

$$\frac{1}{a}+\frac{1}{a+2}=\frac{7}{24}$$
$$\frac{2a+2}{a(a+2)}=\frac{7}{24}$$
$$7a^2-34a-48=0$$
$$(a-6)(7a+8)=0$$
$$a=6,\ a\neq\frac{-8}{7}$$

So, the numbers are 6 and 8. And their sum is 14.

49. Ans. A.

$$\begin{aligned}15-4\sqrt{14}&=15-2\times2\times\sqrt{7}\times\sqrt{2}\\&=8-2\times2\sqrt{2}\times\sqrt{7}+7\\&=\left(2\sqrt{2}\right)^2-2\left(2\sqrt{2}\right)\left(\sqrt{7}\right)+\left(\sqrt{7}\right)^2\\&=\left(2\sqrt{2}+\sqrt{7}\right)^2\ or\ \left(\sqrt{7}+2\sqrt{2}\right)^2\end{aligned}$$

Or let $\sqrt{a}-\sqrt{b}$ is the square root of $15-4\sqrt{14}$

$$15-4\sqrt{14}=\left(\sqrt{a}-\sqrt{b}\right)^2$$

$$15-2\sqrt{56}=a+b-2\sqrt{ab}$$

By comparing we get $a+b=15 \quad and \quad ab=56$

By observation we get $(a,b)=8,7$ or $(a,b)=7,8$

So, square root of $15-4\sqrt{14}$ is $2\sqrt{2}-\sqrt{7}$ or $\sqrt{7}-2\sqrt{2}$ .

50. Ans. A.

we can write $31.25=\frac{3125}{100}=\frac{125}{4}=\frac{1000}{32}$

$$\log_{10}31.25=\log_{10}\left(\frac{1000}{32}\right)=\log_{10}10^3-\log_{10}2^5=3-5\log_{10}2$$

51. Ans. A.

let the side of the cube is $s$ and radius of the cube is $r$. Then

$$6s^2=4\pi r^2$$
$$\frac{s^2}{r^2}=\frac{2\pi}{3}$$



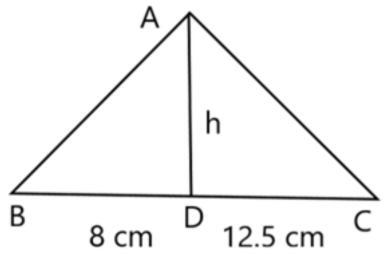
If  is the volume of the cube and $y$ is the volume of the sphere.

$$\frac{x}{y}=\frac{s^3}{4\pi r^3/3}=\frac{3s^3}{4\pi r^3}$$
$$\frac{x^2}{y^2}=\frac{9}{16\pi^2}\times\left(\frac{s^2}{r^2}\right)^3$$
$$=\frac{9}{16\pi^2}\times\left(\frac{2\pi}{3}\right)^3$$
$$=\frac{\pi}{6}$$

52. Ans. D.

ABC is a triangle right angled at A, then $BC=\sqrt{AB^2+AC^2}$

area of the triangle ABC,




$$\frac{1}{2}AD \times BC = \frac{1}{2}AB \times AC$$
$$h(20.5) = \left(\sqrt{h^2 + 64}\right)\left(\sqrt{156.25 + h^2}\right)$$
$$420.25h^2 = \left(h^2 + 64\right)\left(156.25 + h^2\right)$$
$$420.25h^2 = h^4 + 220.25h^2 + 10000$$
$$h^4 - 200h^2 + 10000 = 0$$
$$\left(h^2 - 100\right)^2 = 0$$
$$h^2 = 100$$
$$h = 10\,cm$$

53. Ans. C.

Two isosceles triangles have equal vertical angles and let their heights and bases are $h_1, h_2$ and $b_1, b_2$ , then $\frac{h_1}{h_2} = \frac{l_1}{l_2}$. Now,

$$\frac{A_1}{A_2} = \frac{\frac{1}{2}l_1 h_1}{\frac{1}{2}l_2 h_2} = \frac{4.84}{5.29}$$
$$\frac{h_1^2}{h_2^2} = \frac{2.2^2}{2.3^2}$$
$$\frac{h_1}{h_2} = \frac{22}{23}$$

54. Ans. B.

$\Delta ABC$ is similar to $\Delta DEF$. Then

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ then

$$\frac{BC + CA}{EF + FD} = \frac{AB + BC + CA}{DE + EF + FD} = \frac{40}{30} = \frac{4}{3}$$

55. Ans. B.

If The diagonal AC and BD intersect at P. then

$$\frac{AP}{PC}=\frac{BP}{PD}$$
$$\frac{4}{4(x-1)}=\frac{2x-1}{2x+4}$$
$$2x+4=2x^2-3x+1$$
$$2x^2-5x-3=0$$
$$(2x+1)(x-3)=0$$
$$x=3, x\neq\frac{-1}{2}$$

56. Ans. B.

perimeter of a semicircular park with radius r,

$$\pi r+2r=360$$
$$r=70\ m$$

Area of semicircle $A=\frac{1}{2}\pi r^2=7700\ m^2$

57. Ans. D.

perimeter of the rhombus = circumference of circle

$4a=2\pi(70)$ [ ¨ = side of rhombus]

$$a=110cm$$

58. Ans. A.

angle obtained by sector on the centre of the circle

$$\theta=\frac{\text{length of the arc}}{\text{radius}}rad=\frac{55}{21}rad$$

Area of the sector, $A=\pi r^2\times\frac{\theta}{2\pi}=(21)^2\times\frac{1}{2}\times\frac{55}{21}=577.5cm^2$

59. Ans. B.

the height of the water in the vessel raised due the volume of the sphere.

$$\frac{4}{3}\pi(3)^3 = \pi(6)^2 h$$
$$h = 1cm$$

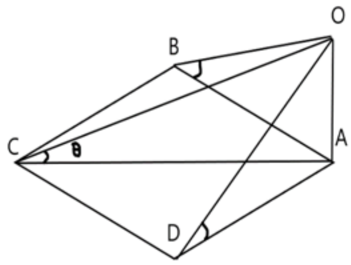
60. Ans. C.

surface area of the cloth and the tent remains same/

$$bh = \pi rl$$
$$3h = \frac{22}{7}\times 6\times 7$$
$$h = 44\,m$$

61. Ans. C.

let the angle B and angle D are $60^o$ and $45^o$ as shown in the figure. Then,




$$\tan 45^o = \frac{OA}{OD} \Rightarrow OA = OD$$
$$\tan 60^o = \frac{OA}{OB} \Rightarrow OA = \sqrt{3}OB$$

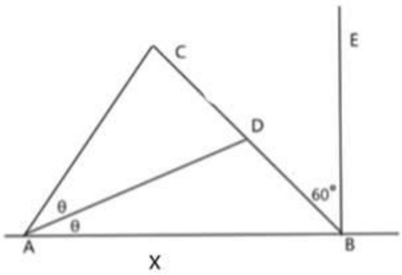
And $OC = \sqrt{OB^2 + OD^2} = \sqrt{\frac{OA^2}{3} + OA^2} = \frac{2}{\sqrt{3}}OA$

Now, $\cot\theta = \frac{OC}{OA} = \frac{2}{\sqrt{3}}$

62. Ans. B.

let the pole is BC which makes an angle of 60° with the vertical, then $\angle CBA = 30^o$

And CD = BD




AB = x meter and AD is common in both triangles

Then both the triangles are congruence then

It means, $\angle ADB = \angle ADC$

Now as we now AD cut the CB then

$$\angle ADB + \angle ADC = 180°$$
$$2\angle ADB = 180°$$
$$\angle ADB = 90° = \angle ADC$$

Now Triangle ADB is right angle triangle

Then

$$AB\cos 30° = BD$$
$$\frac{\sqrt{3}}{2}x = BD = CD$$

Now length of pole = $BD + DC = \sqrt{3}x$

63. Ans. D.

given that $6 + 8\tan\theta = \sec\theta$ and $8 - 6\tan\theta = k\sec\theta$

Add both after squaring,

$$\left(36 + 96\tan\theta + 64\tan^2\theta\right) + \left(64 - 96\tan\theta + 36\tan^2\theta\right) = \left(1 + k^2\right)\sec^2\theta$$
$$100\left(1 + \tan^2\theta\right) = \left(1 + k^2\right)\sec^2\theta$$
$$100 = 1 + k^2$$
$$k^2 = 99$$

64. Ans. C.

$$(1+\cot\theta-\csc\theta)(1+\tan\theta+\sec\theta)$$
$$=\frac{\sin\theta+\cos\theta-1}{\sin\theta}\times\frac{\cos\theta+\sin\theta+1}{\cos\theta}$$
$$=\frac{(\sin\theta+\cos\theta)^2-1}{\sin\theta\cos\theta}$$
$$=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$
$$=2$$

65. Ans. C.

given $\sec\theta+\cos\theta=\frac{5}{2}$, where $0\le\theta\le 90^o$

$$\frac{1+\cos^2\theta}{\cos\theta}=\frac{5}{2}$$
$$2\cos^2\theta-5\cos\theta+2=0$$
$$(2\cos\theta-1)(\cos\theta-2)=0$$
$$\cos\theta=\frac{1}{2},\cos\theta\ne 2$$

$$\sin^2\theta=1-\cos^2\theta=1-\frac{1}{4}=\frac{3}{4}$$

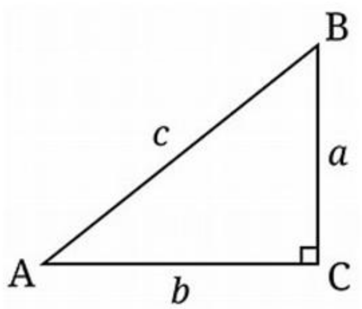
66. Ans. A.

let $\alpha=\beta=0^o$ that satisfy both the equation $\cos\theta+\cos\beta=2$ and $\sin\theta+\sin\beta=0$.

Then $\cos 2\alpha-\cos 2\beta=1-1=0$

67. Ans. D.

given ABC be a triangle right angled at C, then $\angle C=90^o$



$$\tan A+\tan B$$
$$=\tan A+\tan\left(90^o-A\right)$$
$$=\tan A+\cot A$$
$$=\frac{a}{b}+\frac{b}{a}$$
$$=\frac{a^2+b^2}{ab}$$
$$=\frac{c^2}{ab}$$

68. Ans. D.

given that, $\csc\theta-\cot\theta=m$ ...(i)

$$\csc^2\theta-\cot^2\theta=1$$
$$(\csc\theta+\cot\theta)(\csc\theta-\cot\theta)=1$$
$$\csc\theta+\cot\theta=\frac{1}{m} \qquad ...(ii)$$

From equation (i) and (ii)

$$\csc\theta=\frac{1}{2}\left(m+\frac{1}{m}\right)$$

69. Ans. A.

$p=\sec\theta-\tan\theta=\frac{1-\sin\theta}{\cos\theta}$ and $q=\csc\theta+\cot\theta=\frac{1+\cos\theta}{\sin\theta}$

$$p+q(p-1)$$
$$=(p-q)+pq$$
$$=\left(\frac{1-\sin\theta}{\cos\theta}-\frac{1+\cos\theta}{\sin\theta}\right)+\left(\frac{1-\sin\theta}{\cos\theta}\times\frac{1+\cos\theta}{\sin\theta}\right)$$
$$=\frac{\sin\theta-\sin^2\theta-\cos\theta-\cos^2\theta}{\sin\theta\cos\theta}+\frac{1-\sin\theta+\cos\theta-\sin\theta\cos\theta}{\sin\theta\cos\theta}$$
$$=\frac{\sin\theta-\cos\theta-1}{\sin\theta\cos\theta}+\frac{1-\sin\theta+\cos\theta-\sin\theta\cos\theta}{\sin\theta\cos\theta}$$
$$=\frac{-\sin\theta\cos\theta}{\sin\theta\cos\theta}$$
$$=-1$$

70. Ans. A.

$$\cos 57^{o} = \cos(90-33)^{o} = \sin 33^{o} > \sin 1^{o}$$

So, statement I is true.

$$\cos 60^{o} = \cos\left(90^{o} - 30^{o}\right) = \sin 30^{o} < \sin 57^{o}$$

So, statement is not correct.

71. Ans. A.

If the internal and external radii are 5 cm and 6 cm respectively, then volume of the sphere

$$V = \frac{4}{3}\pi\left(6^3 - 5^3\right) = \frac{1144}{3} cm^3$$

Mass of the sphere, $M = V \times d = \frac{1144}{3} \times 3 = 1144\, cm^3$

72. Ans. C.

the volume of the largest cylinder when it is rolled along its width, so

$$2\pi r = 22 \Rightarrow r = 3.5$$

$$x = \pi(3.5)^2(44)$$

And the volume of the largest cylinder when it is rolled along its height, so

$$2\pi r = 44 \Rightarrow r = 7$$

$$y = \pi(7)^2(22)$$

Required ratio, $\frac{x}{y} = \frac{\pi(3.5)^2(44)}{\pi(7)^2(22)} = \frac{1}{2}$

73. Ans. C.

curved surface area = 550 $cm^2$

$$\pi rl = 550$$
$$\frac{22}{7} \times rl = 550$$
$$rl = 175 \qquad ...(i)$$

height of cone = 24 cm

$$\sqrt{l^2 - r^2} = 24$$
$$l^2 - r^2 = 576 \qquad ...(ii)$$

Divide equation (ii) by equation (i)

$$\frac{l^2 - r^2}{lr} = \frac{576}{175}$$
$$\frac{l}{r} - \frac{r}{l} = \frac{576}{175}$$
$$\frac{1}{x} - x = \frac{567}{175} \qquad \left[\frac{r}{l} = x\right]$$
$$175x^2 + 576x - 175x = 0$$
$$(25x - 7)(7x + 25) = 0$$

$$\frac{r}{l} = x = \frac{7}{25}$$

74. Ans. C.

let the radius of the cone is r. since the volume remains equal.

$$\frac{1}{3}\pi \times r^2 \times 21 = 22^3$$
$$r = 22\,cm$$

75. Ans. B.

volume of both the vessel should be equal

$$\frac{1}{3}\pi(5)^2(24) = \pi(10)^2 h$$
$$h = 2\,cm$$

76. Ans. C.

let the edge of third cube is h, and the volume of the cube remains constant

$$24^3 = 12^3 + 16^3 + h^3$$
$$h^3 = 24^3 - 12^3 - 16^3$$
$$= 4^3\left(6^3 - 3^3 - 4^3\right)$$
$$= 4^3(125)$$
$$h = 4 \times 5 = 20\,cm$$

Now, the surface area of the third cube $= 6h^2 = 6 \times 400 = 2400\,cm^2$

77. Ans. D.

. If R is the outer radius and r is the inner radius of the pipe and length is 14 cm. then

Difference between outside and inside surface area = $= 2\pi(R - r)h$

$$44 = 2 \times \frac{22}{7} \times (R - r) \times 14$$
$$R - r = 0.5$$

Volume of the pipe $= \pi\left(R^2 - r^2\right)h$

$$99 = \frac{22}{7} \times (R + r) \times (R - r) \times 14$$
$$9 = 4 \times (R + r) \times 0.5$$
$$R + r = \frac{9}{2} = 4.5\,cm$$

78. Ans. C.

let the radius of base and height of the cylinder is $2x$ and $3x$ respectively.

And the volume of the cylinder

$$V = \pi r^2 h$$
$$1617 = \frac{22}{7} \times (4x^2) \times (3x)$$
$$x^3 = \frac{49 \times 7}{8} = \frac{7^3}{2^3}$$
$$x = \frac{7}{2}$$

Curved surface area of the con $= 2\pi rh = 2 \times \frac{22}{7} \times 2x \times 3x = \frac{3 \times 8 \times 11}{7} \times \frac{49}{4} = 462\, cm^2$

79. Ans. A.

let the radius of the wire is r. since volume remains equal.

sphere of radius = 30 mm = 3 cm

$$\frac{4}{3}\pi (3)^3 = \pi r^2 (144)$$
$$r^2 = \frac{36}{144} = \frac{1}{4}$$
$$r = \frac{1}{2} = 0.5\, cm$$

80. Ans. B.

let the height and radius of cone are h and r. Since bases of cone and hemisphere are equal then radius of hemisphere is r.

And given that the volumes are equal, then

$$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$
$$h = 2r$$
$$\frac{h}{r} = 2$$

81. Ans. B.

let the circle touches the side of quadrilateral AB, BC, CD and DA at P, Q, R and S.

$$AB = AP + PB = 9$$
$$BC = BQ + QC = 8$$
$$CD = CR + RD = 12$$
$$DA = DS + SA = ?$$

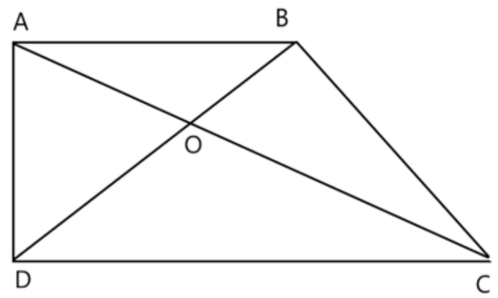
we know that the tangents from a point to the circle are equal. So,

$$AP = AS$$
$$BP = BQ$$
$$CQ = CR$$
$$DR = DS$$

Now,

$$\begin{aligned} DA &= DS + SA \\ &= DR + AP \\ &= (12 - CR) + (9 - PB) \\ &= 21 - (CQ + BQ) \\ &= 21 - 8 \\ &= 13 \end{aligned}$$

82. Ans. D.




given that ABCD is a trapezium in which AB is parallel to DC.

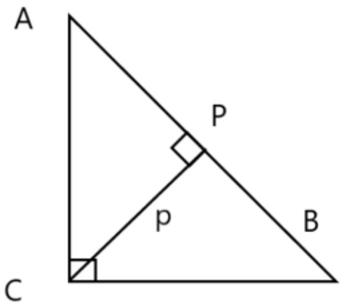
$\angle AOB = \angle DOC$ (vert. opposite angles)

$\angle ABO = \angle BDC$ (alternate angles)

So, $\Delta AOB \cong \Delta COD$ (AAA)

Now, $\dfrac{\Delta AOB}{\Delta COD} = \dfrac{AB^2}{CD^2} = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

83. Ans. C.




If BC = 6 cm and CA = 8 cm, then

$AB = \sqrt{6^2 + 8^2} = 10\,cm$

Area of the triangle ABC

$$\frac{1}{2} CA \times CB = \frac{1}{2} AB \times PC$$
$$8 \times 6 = 10 \times p$$
$$p = 4.8\,cm$$

84. Ans. C.

let the base and height of the right-angled triangle is b and h.

$b^2 + h^2 = 13^2$ ...(i)

Perimeter $\Rightarrow b + h + 13 = 30 \Rightarrow b + h = 17$ ...(ii)

$$(b+h)^2 = 17^2$$
$$b^2 + h^2 + 2bh = 289$$
$$2bh = 120$$
$$bh = 60$$

Or $b = 5, h = 12$

Now, the area of the triangle $A = \frac{1}{2} bh = 30\,cm^2$

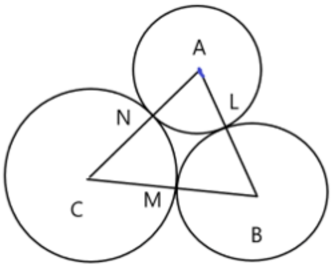
85. Ans. C.

we know that $PT^2 = PA \times PB$

$12^2 = 9 \times PB$
$PB = 16\,cm$

$AB = PB - PA = 16 - 9 = 7\,cm$

86. Ans. C.




let the sides AB, AC and BC of a triangle ABC are 4 cm, 6 cm and 8 cm. let the touching pointing of the circles are L, M and N such that AN = AL, BL = BM and CN = CM.

Let AN = a cm then CN = CM = 6 – a cm

Then BM = BL = 8 – (6 – a) = 2 + a

And AL = AN = 4 – ( 2 + a) = 2 - a

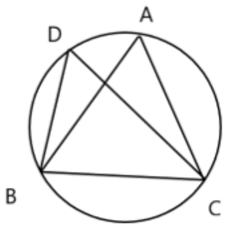
But AN = a then

a = 2 – a

a = 1 cm

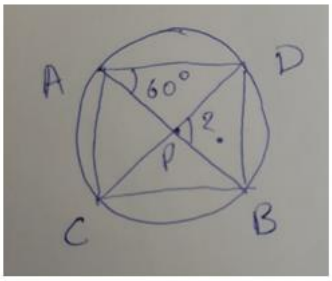
sum of the radii of the circle = AN + BM + CL = 1 + 5 + 3 = 9 cm

87. Ans. C.




ABC is a equilateral triangle then each angle will be $60^\circ$. We know that angle made by same chord on the circumference of circle is equal.
So $\angle BDC = 60^\circ$.

88. Ans. D.



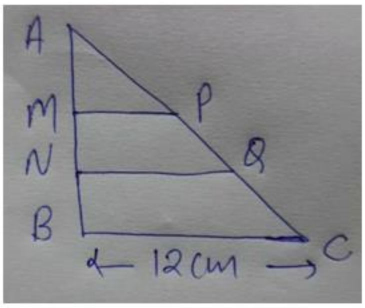

we know that the intersecting points of diameter of circle intersects at center of circle.

We know that angle made by a chord on centre is two times to the angle made on it circumference.

So, $\angle BPD = 120^{o}$

89. Ans. C.

given that $AM = MN = NB$ and $MP \| NQ \| BC$




$$\frac{AB}{BC} = \frac{AN}{NQ} = \frac{AM}{MP}$$
$$\frac{3AM}{12} = \frac{2AM}{NQ} = \frac{AM}{MP}$$
$$\frac{1}{4} = \frac{2}{NQ} = \frac{1}{MP}$$
$$MP = 4, NQ = 8$$
$$(MP + NQ) = 4 + 8 = 12cm$$

90. Ans. C.

The sides of a right-angled triangle are in the ratio $x:(x-1):(x-18)$

$$x^2 = (x-1)^2 + (x-18)^2$$
$$x^2 = 2x^2 - 38x + 325$$
$$x^2 - 38x + 325 = 0$$
$$(x-13)(x-25) = 0$$
$$x = 13, 25$$

$$x \neq 13 \;\; as \;\; x - 13 < 0$$

So, the ratio of sides $25:24:7$

So, perimeter $25+24+7=56$ units.

91. Ans. A.

| Year | I | III | IV | III+IV |
|---|---|---|---|---|
| 2000 | 160 | 70 | 90 | 160 |
| **2001** | **200** | **85** | **160** | **245** |
| 2002 | 135 | 44 | 95 | 139 |
| 2003 | 240 | 120 | 80 | 200 |
| 2004 | 180 | 85 | 95 | 180 |
| 2005 | 210 | 100 | 92 | 192 |

In year 2001.

92. Ans. C.

percentage drop in total production in 2004 compared to 2001
$$= \frac{695-585}{595} \times 100\% = \frac{110}{595} \times 100\%$$

percentage drop in total production in 2000 compared to 2001
$$= \frac{695-475}{595} \times 100\% = \frac{220}{595} \times 100\%$$

required ratio $= \frac{110}{220} = \frac{1}{2}$

93. Ans. A.

total number of tablets produced each year

2000: 160+80+70+90+75 = 475

2001: 200+150+85+160+100 = 695

2002: 135+35+44+95+85 = 394

2003: 240+95+120+80+120 = 655

2004: 180+110+85+95+115 = 585

2005: 210+150+100+92+110 = 662

(2003, 2005): |655 – 662| = 7

(2001, 2005): |695 – 662| = 33

(2003, 2004): |655 – 585| = 70

(2000, 2002): |475 – 394| = 81

(2003, 2005) is minimum

94. Ans. B.

total production of each tablet over years 2000-2005.

Type I: 160 + 200 + 135+ 240 + 180 + 210 = 1125

Type II: 80+150+35+95+110+150 = 620

Type III: 70+85+44+120+85+100 = 504

Type IV: 90+160+95+80+95+92 = 612

Type V: 75+100+85+120+115+110 = 605

So the least is type III

95. Ans. C.

given that the average $\bar{a}_m = p$, $\bar{a}_n = q$, where $p \le q$ and $\overline{a_{m+n}} = c$

Sum of m observation $= mp$ and sum of the n observation $= nq$ and

So, sum of $(m+n)$ observation $= \frac{mp+nq}{m+n} = c$

$$\begin{aligned} p + \frac{n(q-p)}{m+n} &= c \quad or \quad q - \frac{m(q-p)}{m+n} = c \\ p + k_1 &= c \qquad\qquad q - k_2 = c \\ p &\le c \qquad\qquad\quad q \ge c \\ p \le c &\le q \end{aligned}$$

Where $q \geq p \Rightarrow q - p \geq 0 \Rightarrow k_1, k_2 \in I$

96. Ans. B.

average marks of the students $= \frac{21+27+19+26+32}{5} = \frac{125}{5} = 25$

After adding 5 grace marks to each student.

The revised average marks = 25 +5 = 30

97. Ans. D.

first ten composite numbers are $4,6,8,9,10,12,14,15,16,18$

Mean $= \frac{4+6+8+9+10+12+14+15+16+18}{10} = \frac{112}{10} = 11.2$

98. Ans. B.

arranging the number in the ascending order

-1, 2,2,3,6, 8, 9

There are 7 terms, so the median $= \left(\frac{n+1}{2}\right) th\, term = \frac{7+1}{2} th\, term = 4th\, term = 3$

99. Ans. D.

total number of students are 90.

Number of students who scored less than or equals to 50% marks = 5+8+10+13+18 = 54

%age of such students $= \frac{54}{90} \times 100\% = 60\%$

100. Ans. C.

| | Year 2015 | Year 2016 | Change in 2016 over 2015 | %age change |
|---|---|---|---|---|
| Country A | 35 | 38 | 3 | 8.57% |
| Country B | 45 | 47 | 2 | 4.44% |
| Country C | 88 | 93 | 5 | 5.68% |
| Country D | 75 | 79 | 2 | 5.33 % |
| Country E | 58 | 60.9 | 2.9 | 5% |

Only A, C, D and E