Set Theory, Sequence and Series

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A set can be represented by various methods. 3 common methods used for representing set:

1. **Statement form:** In this representation, the well-defined description of the elements of the set is given. Below are some examples of the same.

1. The set of all even number less than 10.

2. Roster form: In this representation, elements are listed within the pair of brackets {} and are separated by commas. Below are two examples.

1. Let N is the set of natural numbers less than 5.

N = { 1 , 2 , 3, 4 }.

3. Set builder form: In Set-builder set is described by a property that its member must satisfy.

1. {x : x is even number divisible by 6 and less than 100}.

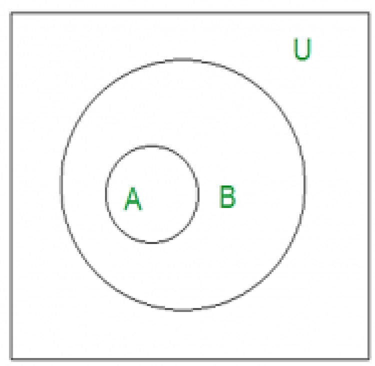
2. {x : x is natural number less than 10}.

An Equal sets

Two sets are said to be equal if both have same elements. For example A = {1, 3, 9, 7} and B = {3, 1, 7, 9} are equal sets. [Order of elements of a set doesn't matter].

Subset

A set A is said to be subset of another set B if and only if every element of set A is also a part of other set B. Denoted by '⊆'. 'A ⊆ B ' denotes A is a subset of B.



To prove A is the subset of B, we need to simply show that if x belongs to A then x also belongs to B. To prove A is not a subset of B, we need to find out one element which is part of set A but not belong to set B. 'U' denotes the universal set.

Size of a Set

Size of a set can be finite or infinite. Finite set: Set of natural numbers less than 100. Infinite set: Set of real numbers. Size of the set S is known as **Cardinality number**, denoted as |S|.

Let A be a set of odd positive integers less than 10. Solution : A = {1,3,5,7,9}, Cardinality of the set is 5, i.e.,|A| = 5. Note: Cardinality of a null set is 0.

Power Sets

The power set is the set all possible subset of the set S. Denoted by P(S).
What is the power set of {0,1,2}? Solution: All possible subsets
{∅}, {0}, {1}, {2}, {0,1}, {0,2}, {1,2}, {0,1,2}.
Note: Empty set and set itself is also the member of this set of subsets.

Union: Union of the sets A and B, denoted by A ∪ B, is the set of distinct element belongs to set A or set B, or both. Example : Find the union of A = {2, 3, 4} and B = {3, 4, 5};
Solution : A ∪ B = {2, 3, 4, 5}.

Intersection: The intersection of the sets A and B, denoted by A ∩ B, is the set of elements belongs to both A and B i.e. set of the common element in A and B.

Example: Consider the previous sets A and B. Find out A ∩ B. Solution : A ∩ B = {3, 4}.

Disjoint

Two sets are said to be disjoint if their intersection is the empty set .i.e sets have no common elements. Let A = {1, 3, 5, 7, 9} and B = { 2, 4 ,6 , 8} . A and B are disjoint set both of them have no common elements.

Set Difference

Difference between sets is denoted by 'A – B', is the set containing elements of set A but not in B. i.e all elements of A except the element of B.

Complement

Complement of a set A, denoted by A^c is the set of all the elements except A. Complement of the set A is U – A.

Addition & Subtraction

Addition of sets A and B, referred to as Minknowski Addition, is the set in whose elements are the sum of each possible pair of elements from the 2 sets (that is one element is from set A and other is from set B).

Set subtraction follows the same rule, but with the subtraction operation on the elements. It is to be observed that these operations are operable only on numeric data types

1. $A \cup B = n(A) + n(B) - n(A \cap B)$

2. $A - B = A \cap B$

Cartesian Products

Let A and B be two sets. Cartesian product of A and B is denoted by A × B, is the set of all ordered pairs (a,b), where a belong to A and b belong to B.

$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$.

Example 1. What is Cartesian product of A = {1,2} and B = {p, q, r}.
Solution : A × B ={(1, p), (1, q), (1, r), (2, p), (2, q), (2, r) };

The cardinality of A × B is N*M, where N is the Cardinality of A and M is the cardinality of B.

Note: A × B is not the same as B × A.

Properties of Union and Intersection of sets:

- **Associative Properties:** $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$
- **Commutative Properties:** $A \cup B = B \cup A$ and $A \cap B = B \cap A$
- **Identity Property for Union:** $A \cup \varphi = A$
- **Intersection Property of the Empty Set:** $A \cap \varphi = \varphi$
- **Distributive Properties:** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ similarly for intersection.

Types of Sets

The sets are further categorised into different types, based on elements or types of elements. These different types of sets in basic set theory are:

- Finite set: The number of elements is finite
- Infinite set: The number of elements are infinite
- Empty set: It has no elements
- Singleton set: It has one only element
- Equal set: Two sets are equal if they have same elements
- Equivalent set: Two sets are equivalent if they have same number of elements
- Power set: A set of every possible subset.
- Universal set: Any set that contains all the sets under consideration.
- Subset: When all the elements of set A belong to set B, then A is subset of B

Properties of Union of Sets

i) Commutative Law: The union of two or more sets follows the commutative law i.e., if we have two sets A and B then,

A∪B=B∪A

Example: A = {a, b} and B = {b, c, d}

So, A∪B = {a, b, c, d}

B∪A = {b, c, d, a}

Since, in both the union, the group of elements is same. Therefore, it satisfies commutative law.

A ∪ B = B ∪ A

ii) Associative Law: The union operation follows the associative law i.e., if we have three sets A, B and C then

(A ∪ B) ∪ C = A ∪ (B ∪ C)

Example: A = {a, b} and B = {b, c, d} and C = {a, c, e}

(A ∪ B) ∪ C = {a, b, c, d} ∪ {a, c, e} = {a, b, c, d, e}

A ∪ (B ∪ C) = {a, b} ∪ {b, c, d, e} = {a, b, c, d, e}

Hence, associative law proved.

iii) Identity Law: The union of an empty set with any set A gives the set itself i.e., A ∪ ∅ = A

Suppose, A = {a, b, c} and ∅ = {}

then, A ∪ ∅ = {a, b, c} ∪ {} = {a, b, c}

iv) Idempotent Law: The union of any set A with itself gives the set A i.e.,

A ∪ A = A

Suppose, A = {1, 2, 3, 4, 5}

then A ∪ A = {1, 2, 3, 4, 5} ∪ {1, 2, 3, 4, 5} = {1, 2, 3, 4, 5} = A

v) Domination Law: The union of a universal set U with its subset A gives the universal set itself.

A ∪ U = U

Suppose, A = {1, 2, 4, 7} and U = {1, 2, 3, 4, 5, 6, 7}

then A ∪ U = {1, 2, 4, 7} ∪ {1, 2, 3, 4, 5, 6, 7} = {1, 2, 3, 4, 5, 6, 7} = U

Hence, proved.

Example 1:

Let U be a universal set consisting of all the natural numbers until 20 and set A and B be a subset of U defined as A = {2, 5, 9, 15, 19} and B = {8, 9, 10, 13, 15, 17}. Find A ∪ B.

Solution:

Given,

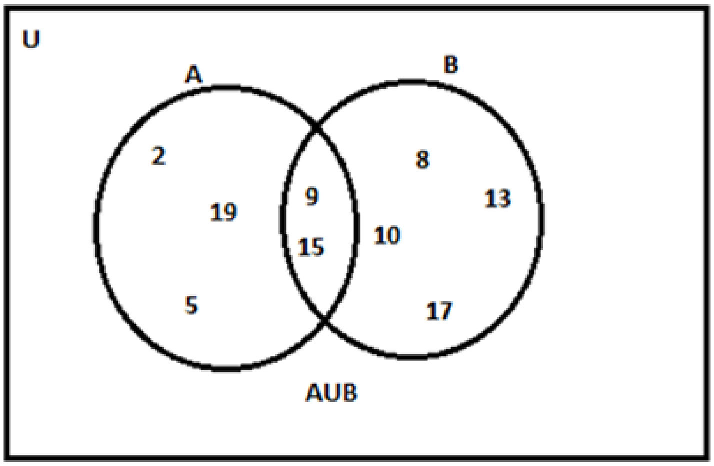
U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

A = {2, 5, 9, 15, 19}

B = {8, 9, 10, 13, 15, 17}

A ∪ B = {2, 5, 8, 9, 10, 13, 15, 17, 19}

This can be represented using the following Venn diagram:



Sets Operation (Set Theory)

- Commutative Laws:

$$(a)\ A \cap B = B \cap A$$
$$(b)\ A \cup B = B \cup A$$

- Associative Laws:

$$(a)\ (A \cap B) \cap C = A \cap (B \cap C)$$
$$(b)\ (A \cup B) \cup C = A \cup (B \cup C)$$

- Distributive Laws:

$$(a)\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$(b)\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- Double Complement Law:

$$(A^c)^c = A$$

- De Morgan's Laws:

$$(a)\ (A \cap B)^c = A^c \cup B^c$$
$$(b)\ (A \cup B)^c = A^c \cap B^c$$

- Absorption Laws:

$$(a)\ A \cup (A \cap B) = A$$
$$(b)\ A \cap (A \cup B) = A$$

- A and B are called **disjoint** iff $A \cap B = \varnothing$.
- Sets $A_1, A_2, \ldots, A_n$ are called **mutually disjoint**

  iff for all i,j = 1,2,…, n

  $$A_i \cap A_j = \varnothing \text{ whenever } i \neq j.$$
- *Examples:*

  1) A={1,2} and B={3,4} are disjoint.

  2) The sets of even and odd integers are disjoint.

  3) A={1,4}, B={2,5}, C={3} are mutually disjoint.

  4) A–B, B–A and $A \cap B$ are mutually disjoint.

Law	Description
Commutative law of Addition	a + b = b + a
Commutative law of Multiplication	a . b = b . a
Associative law of Addition	a + (b + c) = (a + b) + c a + (b + c) =(a + b) + c
Associative law of Multiplication	a .(b . c) = (a . b) . c . a . (b . c) =(a . b) . c
Distributive law	a . (b + c) = (a . b) + (a . c) a . (b + c) =(a . b) + (a . c)
Cancellation law of Addition	a + c = b + c ⇔ a = b
Cancellation law of Multiplication	a . c = b . c ⇔ a = b where c ! = 0

Name	Set Identity
Identity laws	$A \cap U = A$ $A \cup \emptyset = A$
Domination laws	$A \cup U = U$ $A \cap \emptyset = \emptyset$
Idempotent laws	$A \cup A = A$ $A \cap A = A$
Complementation Law	$\overline{(\overline{A})} = A$
Commutative laws	$A \cup B = B \cup A$ $A \cap B = B \cap A$
Associative laws	$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$
Distributive laws	$A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$
De Morgans laws	$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$ $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
Complement laws	$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$

Notation	Name	Meaning
A∪B	Union	Elements that belong to set A or set B or both A and B
A∩B	Intersection	Elements that belong to both set A and set B
A⊆B	Subset	Every element of set A is also in set B
A⊂B	Proper subset	Every element of A is also in B, but B contains more elements
A⊄B	Not a subset	Elements of set A are not elements of set B
A=B	Equal sets	Both set A and B have the same elements
A^c or A’	Complement	Elements not in set A but in the universal set
A-B or A\B	Set difference	Elements in set A but not in set B
P(A)	Power set	The set of all subsets of set A
A×B	Cartesian product	The set that contains all the ordered pairs from set A and B in that order
n(A) or |A|	Cardinality	The number of elements in set A
∅ or { }	Empty set	The set that has no elements
U	Universal set	The set that contains all the elements under consideration
N	The set of natural numbers	N={1,2,3,4,…}
Z	The set of integers	Z={…,-2,-1,0,1,2,…}
R	The set of real numbers	R={x|-∞<x<+∞}

Properties related to difference, union and intersection and the cardinal number of set

i) Union of Disjoint Sets:

If A and B are two finite sets and if A ∩ B = ∅, then n(A ∪ B) = n(A) + n(B)

In simple words if A and B are finite sets and these sets are disjoint then the cardinal number of Union of sets A and B is equal to the sum of the cardinal number of set A and set B.

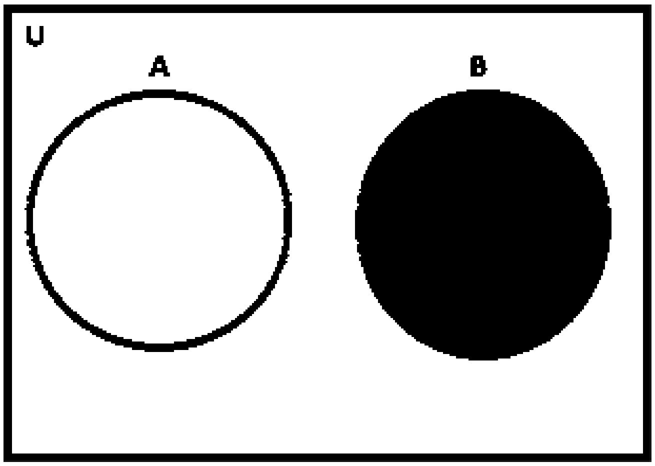


Figure 1- Disjoint sets

The union of the disjoint sets A and B represented by the Venn diagram is given by A ∪ B and it can be seen that A ∩ B = ∅ because no element is common to both the sets.

ii) Union of two sets:

If A and B are two finite sets, then

n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

Simply, the number of elements in the union of set A and B is equal to the sum of cardinal numbers of the sets A and B, minus that of their intersection.

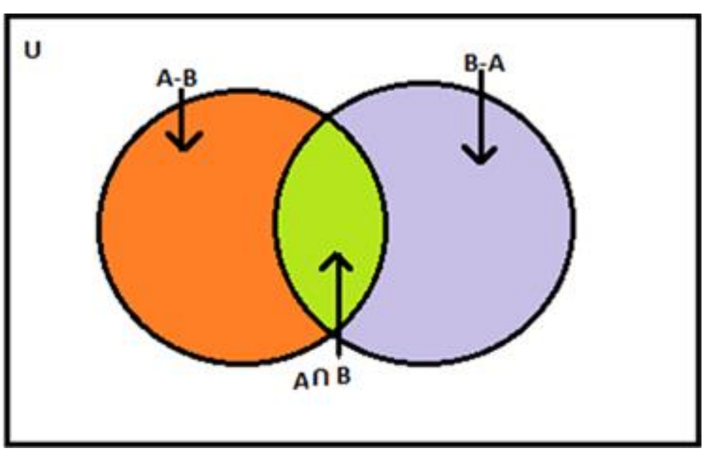


Figure 2- Union of two sets

In the figure given above the differently shaded regions depict the different disjoint sets i.e. A – B, B – A and A ∩ B are three disjoint sets as shown and the sum of these represents A ∪ B. Hence,

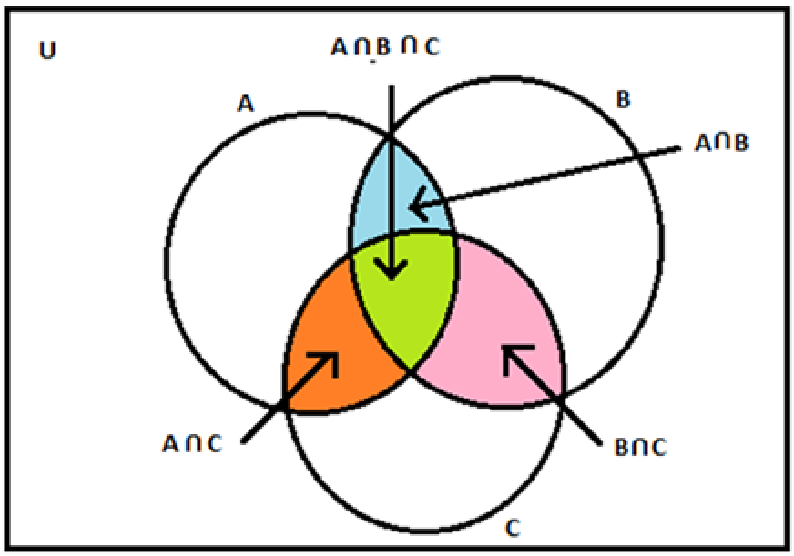
n (A ∪ B) = n (A – B) + n(B – A) + n(A ∩ B)

iii) Union of three sets

If A, B and C are three finite sets, then;

n(A ∪ B ∪ C) = n(A) + n(B) + n(C) – n(A ∩ B) – n(B ∩ C) – n(A ∩ C) + n(A ∩ B ∩ C)

This is clearly visible from the Venn diagram that the union of the three sets will be the sum of the cardinal number of set A, set B, set C and the common elements of the three sets excluding the common elements of sets taken in pairs of two.



Example: There is a total of 200 students in class XI. 120 of them study mathematics, 50 students study commerce and 30students study both mathematics and commerce. Find the number of students who

i) Study mathematics but not commerce

ii) Study commerce but not mathematics

iii) Study mathematics or commerce

Solution: The total number of students represents the cardinal number of the universal set. Let A denote the set of students studying mathematics and set B represent the students studying commerce.

Therefore,

n (U) = 200

n(A) = 120

n(B) = 50

$n(A \cap B) = 30$

The Venn diagram represents the number of students studying mathematics and commerce.
i) Here, we are required to find the difference of sets A and B.

$n(A) = n(A - B) + n(A \cap B)$

$n(A-B) = n(A) - n(A \cap B)$
$\Rightarrow n(A - B) = 120 - 30 = 90$

The number of students who study mathematics but not commerce is 90.

ii) Similarly here, we are required to find the difference of sets B and A

$n(B) = n(B - A) + n(A \cap B)$
$\Rightarrow n(B - A) = 50 - 30 = 20$

The number of students who study commerce but not mathematics is 20.

iii) The number of students who study mathematics or commerce

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(A \cup B) = 120 + 50 - 30 = 140$

Question ID:- 2495

Identify which one of the following statements is INCORRECT?

निम्नलिखित में से गलत कथन की पहचान कीजिए:

Options:-

- If $X \cap Y = \phi$, then $X - Y \neq X$/यदि $X \cap Y = \phi$, तो $X - Y \neq X$,

Option ID :- 9979,

- $X - Y = X \cap \bar{Y}$/$X - Y = X \cap \bar{Y}$,

Option ID :- 9980,

- **Complement of a Universal set is a null set/समष्टीय समुच्चय का पूरक रिक्त समुच्चय है, Option ID :- 9978,**

- **Intersection of two disjoint sets is a null set/दो असंयुक्त समुच्चयों का तर्कगुणनफल रिक्त समुच्चय है, Option ID :- 9977,**

Q-1) In an office, every employee likes at least one of tea, coffee and milk. The number of employees who like only tea, only coffee, only milk and all the three are all equal. The number of employees who like only tea and coffee, only coffee and milk and only tea and milk are equal and each is equal to the number of employees who like all three. Then, a possible value of the number of employees in the office is:

a) 65

b) 90

c) 77

d) 85

Q-2) In a survey of the market, it is noted that 300 families were randomly selected in a city. 142 buy Lipton tea and 139 buy Brookbond tea and 70 buy both types of tea. Find out the number of families who buy

(i) at least one brand of tea

(ii) exactly one of these brands

(iii) none of these brands

Q-3) In a class of 30 M.A. (Economics) students, 17 students have taken econometrics and 13 have taken econometrics but not mathematical economics. Find (a) the number of students who have taken econometrics and mathematical economics (b) how many of them have taken mathematical economics but also econometrics.

Refer to the live session for the solution of these questions

Sequence and Series

An itemized collection of elements in which repetitions of any sort are allowed is known as a sequence, whereas a series is the sum of all elements. An arithmetic progression is one of the common examples of sequence and series.

- In short, a sequence is a list of items/objects which have been arranged in a sequential way.
- A series can be highly generalized as the sum of all the terms in a sequence. However, there must be a definite relationship between all the terms of the sequence.

Sequence and Series Definition

A sequence is an arrangement of any objects or a set of numbers in a particular order followed by some rule. If a1, a2, a3, a4,……… etc. denote the terms of a sequence, then 1,2,3,4,…..denotes the position of the term.

A sequence can be defined based on the number of terms i.e. either finite sequence or infinite sequence.

If a1, a2, a3, a4, ……. is a sequence, then the corresponding series is given by

SN = a1+a2+a3 + .. + aN

Note: The series is finite or infinite depending if the sequence is finite or infinite.

Sequence: The sequence is defined as the list of numbers that are arranged in a specific pattern. Each number in the sequence is considered a term. For example, 5, 10, 15, 20, 25, … is a sequence. The three dots at the end of the sequence represents that the pattern will continue further. Here, 5 is the first term, 10 is the second term,15 is the third term and so on. Each term in the sequence can have a common difference, and the pattern will continue with the common difference. In the example given above, the common difference is 5. The sequence can be classified into different types, such as:

- Arithmetic Sequence
- Geometric Sequence
- harmonic Sequence
- Fibonacci Sequence

Series: The series is defined as the sum of the sequence where the order of elements does not matter. It means that the series is defined as the list of numbers with the addition symbol in between. The series can be classified as a finite series or infinite series which depends on the type of sequence whether it is finite or infinite. Note that, the finite series is a series where the list of numbers has an ending, whereas the infinite series is never-ending. For example, 1+3+5+7+.. is a series. The different types of series are:

- Geometric series
- Harmonic series
- Power series

Sequence	Series
Sequence relates to the organization of terms in a particular order (i.e. related terms follow each other) and series is the summation of the elements of a sequence.	Series can also be classified as finite and infinite series.
In sequence, the ordering of elements is the most important	In series, the order of elements does not matter
The elements in the sequence follow a specific pattern	The series is the sum of elements in the sequence
Example: 1, 2, 4, 6, 8, ... n are said to be in a Sequence and 1 + 2 + 4 + 6 + 8 . . . . n is said to be in a series.	A finite series can be represented as m1 + m2 + m3 + m4 + m5 + m6 + ... + mn
General Form: $[p_n]_{n=1}^{\infty}$	**General Form:** $S_n = \sum_{r=1}^{n} m_r$.
The order of a sequence matters. Hence, a sequence 5, 6, 7 is different from 7, 6, 5.	However, in case of series 5 + 6 + 7 is same as 7 + 6 + 5.

Types of Sequence and Series

- Arithmetic Sequences
- Geometric Sequences
- Harmonic Sequences
- Fibonacci Numbers

Arithmetic Sequences

A sequence in which every term is created by adding or subtracting a definite number to the preceding number is an arithmetic sequence.

- Arithmetic progression (A.P.) is a sequence in which each term except the first is obtained by adding a fixed number (positive or negative) to the preceding term.
- Thus any sequence a1 , a2 , a3 ... an , ... is called an arithmetic progression if $a_{n+1} = a_n + d$, n ∈ N, where d is called the **common difference** of the A.P., usually we denote the first term of an A.P by a and the last term by l
- The general term or the n th term of the A.P. is given by $a_n = a + (n – 1) d$ The n th term from the last is given by $a_n = l – (n – 1) d$

The sum Sn of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2}\left[2a + (n-1)\,d\right] = \frac{n}{2}(a + l)$$, where $l = a + (n-1)\,d$ is the last terms of the A.P.,

and the general term is given by $a_n = S_n – S_{n-1}$

The arithmetic mean for any n positive numbers a1 , a2 , a3 , ... an is given by

$$\text{A.M.} = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

If the terms of an A.P. are increased, decreased, multiplied or divided by the same constant, they still remain in A.P.

If *a*1, *a*2, *a*3 ... are in A.P. with common difference *d*, then

(i) *a*1 ± *k*, *a*2 ± *k*, *a*3 ± *k*, ... are also in A.P with common difference *d*.

(ii) *a*1 *k*, *a*2 *k*, *a*3 *k*, ... are also in A.P with common difference *dk* (*k* ¹ 0).

And $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}$ ... are also in A.P. with common difference *d*/*k* (*k* ¹ 0).

(iii) If *n*th term of any sequence is linear expression in *n*, then the sequence is A.P.

(iv) If sum of *n* terms of any sequence is a quadratic expression in *n*, then sequence is an A.P.

Geometric Sequences: A sequence in which every term is obtained by multiplying or dividing a definite number with the preceding number is known as a geometric sequence.

Geometric progression (G.P.) is a sequence in which each term except the first is obtained by multiplying the previous term by a non-zero constant called the **common ratio**. Let us consider a G.P. with first non-zero term *a* and common ratio *r*, i.e., *a*, *ar*, *ar*2, ... , *arn* – 1, ...

Here, common ratio $r = \frac{ar^{n-1}}{ar^{n-2}}$

The **general term** or ***n*th term** of G.P. is given by $an = ar^{n-1}$.

Last term *l* of a G.P. is same as the *n*th term and is given by $l = ar^{n-1}$.

and the *n*th term from the last is given by $a_n = \frac{l}{r^{n-1}}$

The sum S*n* of the first *n* terms is given by $S_n = \frac{a(r^n - 1)}{r - 1}$, or $S_n = na$

If *a*, G and *b* are in G.P., then G is called the **geometric mean** of the numbers *a* and *b* and is given by G = √*a b*

(i) Sum of the first n natural numbers:

$$\sum n = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

(ii) Sum of the squares of first n natural numbers.

$$\sum n^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(iii) Sum of cubes of first n natural numbers:

$$\sum n^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Harmonic Sequences

A series of numbers is said to be in harmonic sequence if the reciprocals of all the elements of the sequence form an arithmetic sequence.

Fibonacci Numbers

Fibonacci numbers form an interesting sequence of numbers in which each element is obtained by adding two preceding elements and the sequence starts with 0 and 1. Sequence is defined as, F0 = 0 and F1 = 1 and Fn = Fn-1 + Fn-2

A sequence of the numbers is called a Fibonacci sequence when the required term is obtained by adding the two terms before that.

The formula of the Fibonacci Sequence is

$a_n = a_{n-2} + a_{n-1}$, Where $n > 2$

Also known as the Recursive Formula.

	Arithmetic Series	Geometric Series
General Form	{a + (a + d) + (a + 2d) + (a + 3d) + ………}	$\sum_{n=1}^{\infty} ar^n = a + ar + ar^2 + \ldots . ar^n$
n^{th} term	$a_n = a + (n-1)d$	$a_n = a r^{n-1}$
Sum of n terms	$\frac{n[2a+(n-1)d]}{2}$	$S_n = \frac{a(1-r^n)}{1-r}$

Convergence of geometric series

Consider the geometric progression

$$1,\ \tfrac{1}{2},\ \tfrac{1}{4},\ \tfrac{1}{8},\ \tfrac{1}{16},\ \ldots .$$

We have $a = 1$ and $r = \frac{1}{2}$, and so we can calculate some sums. We get

$$\begin{aligned} S_1 &= & & 1 \\ S_2 &= 1 + \tfrac{1}{2} &&= \tfrac{3}{2} \\ S_3 &= 1 + \tfrac{1}{2} + \tfrac{1}{4} &&= \tfrac{7}{4} \\ S_4 &= 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} &&= \tfrac{15}{8} \\ &\vdots \end{aligned}$$

and there seems to be a pattern because

$$\begin{aligned} 1 &= 2 - 1 \\ \tfrac{3}{2} &= 2 - \tfrac{1}{2} \\ \tfrac{7}{4} &= 2 - \tfrac{1}{4} \\ \tfrac{15}{8} &= 2 - \tfrac{1}{8}. \end{aligned}$$

In each case, we subtract a small quantity from 2, and as we take successive sums the quantity gets smaller and smaller. If we were able to add 'infinitely many' terms, then the answer 'ought to be' 2 — or as near as we want to get to 2.

Let us see if we can explain this by using some algebra. We know that

$$S_n = \frac{a(1 - r^n)}{1 - r},$$

and we want to examine this formula in the case of our particular example where r = ½.

Now the formula contains the term r^n and, as $-1 < r < 1$, this term will get closer and closer to zero as n gets larger and larger. So, if $-1 < r < 1$, we can say that the 'sum to infinity' of a geometric series is:

$$S_\infty = \frac{a}{1 - r}$$

where we have omitted the term r^n. We say that this is the limit of the sums S_n as n ‘tends to infinity’.

Question: Find the sum to infinity of the geometric progression

$$1, \quad \frac{1}{3}, \quad \frac{1}{9}, \quad \frac{1}{27}, \quad \ldots .$$

For this geometric progression we have a = 1 and r = 1/3. As −1 < r < 1 we can use the formula, so that

$$S_\infty = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}.$$

Convergence or Divergence of geometric series

Method-1:

Calculate common ration ‘r’ [by the method mentioned in GP series].

- If If r < 1, then the series **converges**.
- If r > 1, then the series **diverges**.
- If r = 1, the root test is inconclusive, and the series may converge or diverge.

Method-2:

Ascertain Un i.e. the general form of the given series. And then apply lim n → ∞ in Un.

- If the value comes out to be finite or zero (anything other then ∞), then it is **convergent.**
- If the values come out to be infinity, then it is **divergent.**

Method-3:

Ascertain the Sum of the series using the formula of GP series or AP series depending upon the case.

- If the Sum is finite or zero (anything other then ∞), then it is **convergent.**
- If the Sum is infinite, then it is **divergent.**

Two series are given as under

(a) $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\ldots\ldots$ upto ∞

(b) $2+\sqrt{2}+1+\frac{1}{\sqrt{2}}+\frac{1}{2}+\frac{1}{2\sqrt{2}}+\ldots$ upto ∞

Choose the correct option:

A. (a) is convergent while (b) is divergent series
B. Both (a) and (b) are convergent series
C. (a) is divergent while (b) is convergent series
D. None of (a) and (b) are convergent

Answer ||| B

Solution |||

If $r < 1$, then the series converges. If $r > 1$, then the series diverges. If $r = 1$, the root test is inconclusive, and the series may converge or diverge.

Here in both the equations $r < 1$. In (a) part $r = ¼$ and in (b) part $r = 1/\sqrt{2}$. Thus both the series are convergent.

Thus, Option B is correct.