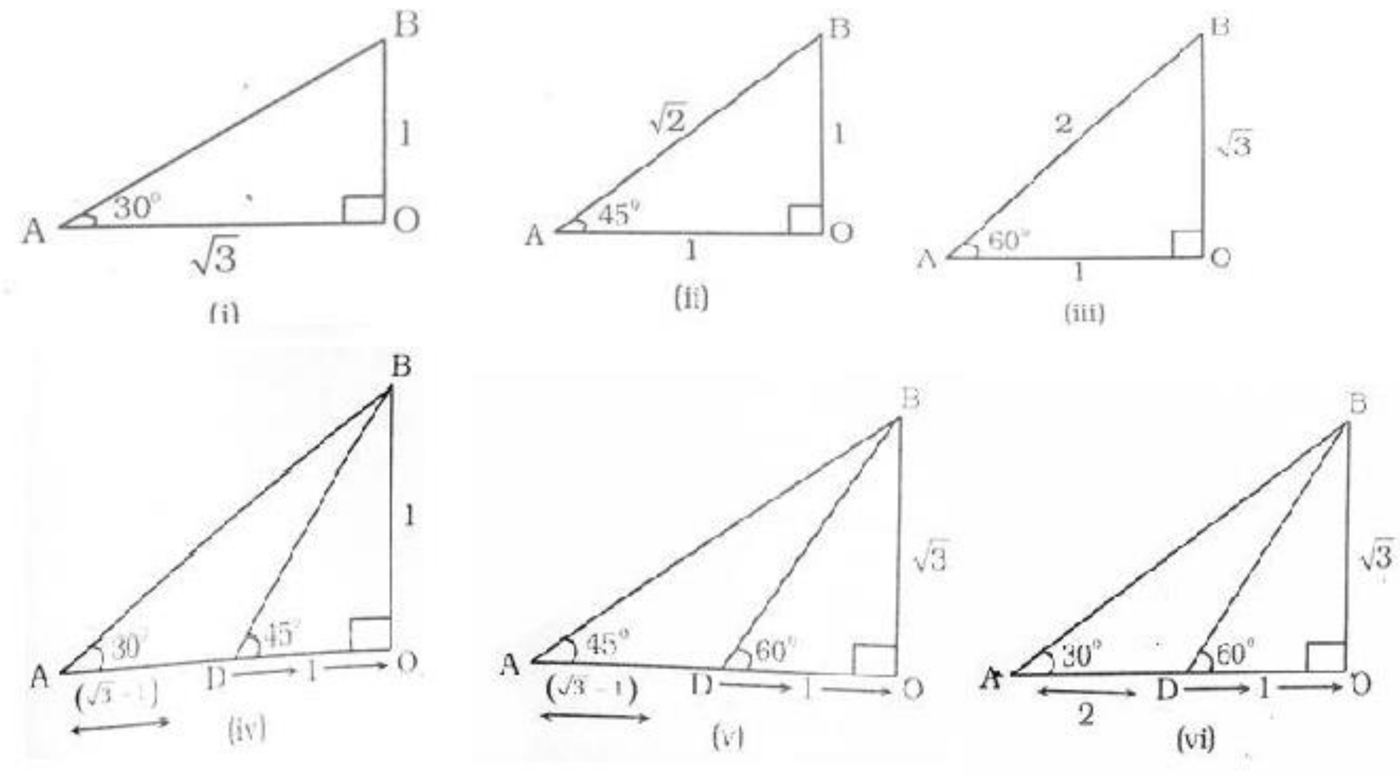
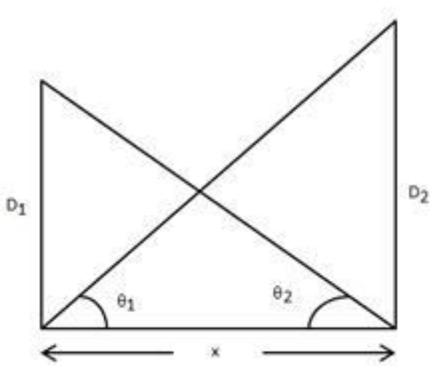
Defence Exam Notes: Heights & Distance

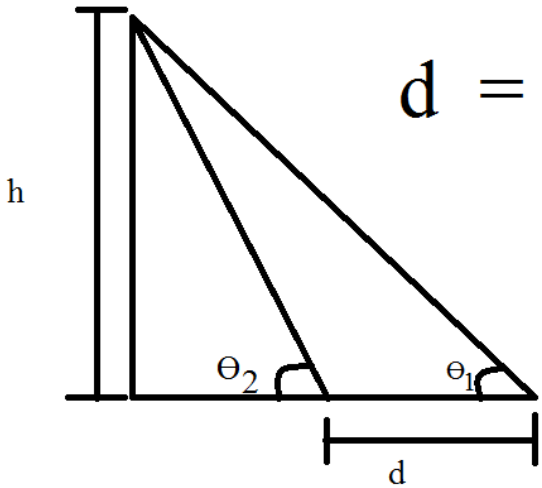
Here are some ratio figure which you have to remember



Important short tricks are :



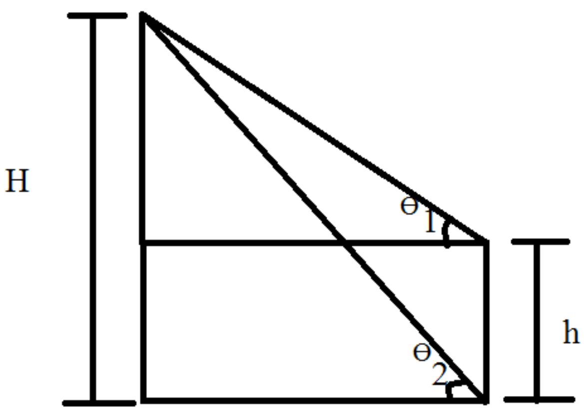
Note: $x = \sqrt{D_1 \times D_2}$ only when the sum of angle i.e $\theta_1 + \theta_2 = 90^0$



$$d = h(\cot \Theta_1 - \cot \Theta_2)$$

d = distance between two point

h = height of the tower or any object



$$H = \frac{h\cot\Theta 1}{\cot\Theta 1 - \cot\Theta 2}$$

H = height of tall tower or object

h = height of small tower or object

Some Important questions are as follows:

Example 1: The angle of elevation of the top of a tower at a distance of 500 m from its foot is 30°. The height of the tower is :

(a) $\frac{500(\sqrt{3}-1)}{3}m$

(b) $\frac{500(\sqrt{3}+1)}{3}$

(c)500

(d) $\frac{500\sqrt{3}}{3}m$

Ans. (d)

$$\tan 30^\circ = \frac{BC}{AB} = \frac{h}{500}$$

$$\Rightarrow h = 500 \times \frac{1}{\sqrt{3}} = \frac{500\sqrt{3}}{3}$$

Short trick:

Solve it with ratio, as the angle of elevation is 30° then ratio between P: B: H is 1:√3:2 so √3= 500 then 1= 500/√3 and height is equal to $\frac{500\sqrt{3}}{3}m$

Example 2: The banks of a river are parallel. A swimmer starts from a point on one of the banks and swims in a straight line inclined to the bank at 45^0 and reaches the opposite bank at a point 20 m from the point opposite to the starting point. The breadth of the river is :

(a) 20 m

(b) 28.28 m

(c) 14.14 m

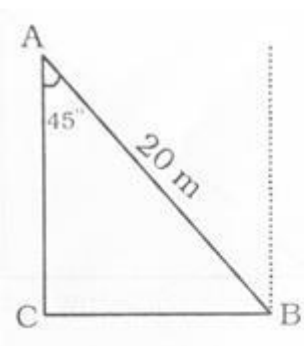
(d) 40 m

Ans. (c) 14.14 m

Solution:

Let A be the starting point and B, the endpoint of the swimmer. Then AB = 20m & $\angle BAC = 45^0$

$$\sin 45^\circ = \frac{BC}{AB}$$



$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{20} \Rightarrow BC = 10\sqrt{2} = 14.14m$$

Short Method;

AS the angle of elevation is 45° then the ratio of P: B: H i.e. 1:1:√2

here √2 =20 then 1 =20/√2

Question 3: A man from the top a 50m high tower, sees a car moving towards the tower at an angle of depression of 30^0. After some time, the angle of depression becomes 60^0. The distance (in m) travelled by car during this time is –

(a) $50\sqrt{3}$

(b) $\frac{50\sqrt{3}}{3}$

(c) $\frac{100\sqrt{3}}{3}$

(d) $100\sqrt{3}$

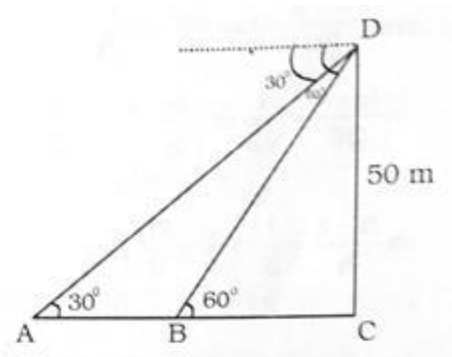
Ans. (c)

Solution:

$BC = \frac{50}{\sqrt{3}}, AC = 50\sqrt{3}$

AB = AC – BC

$= \frac{100\sqrt{3}}{3}$



Example 4:A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite side of the bank is 60^0. When he moves 50m away from the bank, the angle of elevation becomes 30^0. The height of the tree and width of river respectively are :

(a) $25, 25\sqrt{3}\,m$

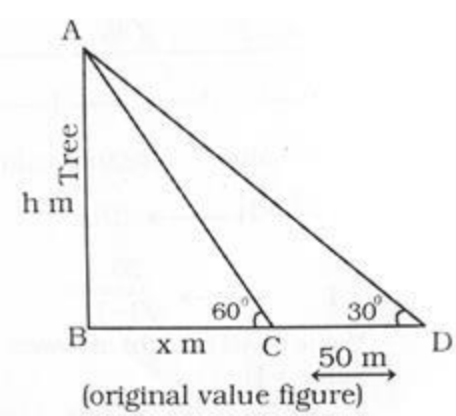
(b) $25\sqrt{3}, 25\sqrt{3}\,m$

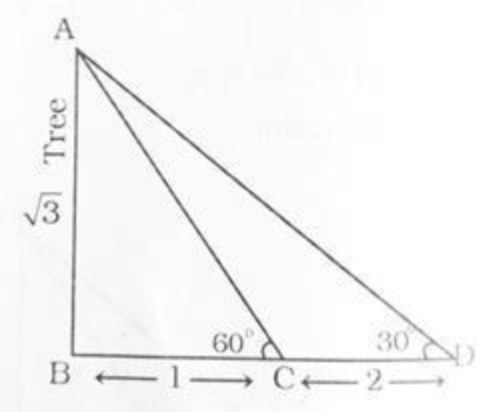
(c) $25\sqrt{3}\,m, 25\,m$

(d) None of these

Answer: c)

Solution:





(Ratio value figure)

Ratio value original value

$$\begin{aligned} 2 &\rightarrow 50 \\ \therefore \quad 1 &\rightarrow 25 \\ \text{and } \sqrt{3} &\rightarrow 25\sqrt{3} \end{aligned}$$

height of the tree= h (ratio value = $\sqrt{3}$)= $25\sqrt{3}\ m$

and width of the river = x (ratio value = 1) = 25 m

Example 5: From the top of a pillar of height 80 m the angle of elevation and depression of the top and bottom of another pillar are 30^0 and 45^0 respectively. The height of second pillar (in metre) is:

(a) $80\sqrt{3}$ m

(b) $\frac{80}{\sqrt{3}}(\sqrt{3}-1)m$

(c) $\frac{80}{\sqrt{3}}(\sqrt{3}+1)m$

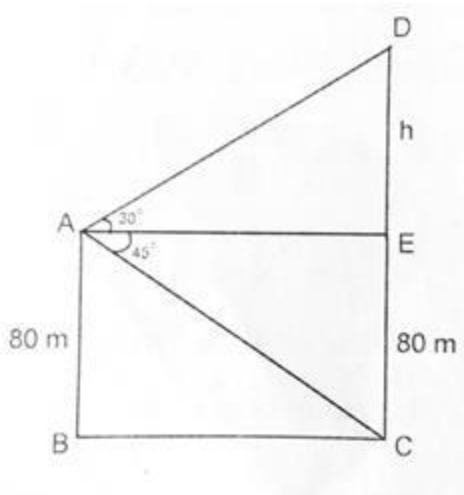
(d) $\frac{80}{\sqrt{3}}m$

Answer: (c)

Solution:

Let AB and CD are pillars.

Let DE = h



In $\Delta ADE, \tan 30^0 = \frac{h}{AE}$

$\Rightarrow AE = h\sqrt{3}$ ....(i)

$In\ \Delta ACE, \tan 45^0 = \frac{80}{AE}$

$\Rightarrow AE = 80 \Rightarrow h\sqrt{3} = 80\ [From(i)]$

$\Rightarrow h = \frac{80}{\sqrt{3}}$

Required height

$= 80 + \frac{80}{\sqrt{3}} = \frac{80}{\sqrt{3}}\left(\sqrt{3}+1\right)m$

Example 6: Two poles of equal height are standing opposite to each other on either side of a road, which is 28m wide. From a point between them on the road, the angles of elevation of the tops are 30^0 and 60^0. The height of each pole is:

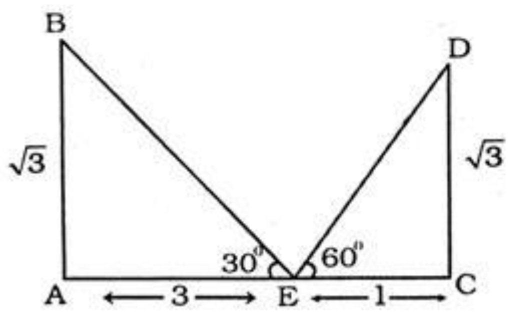
(a) $6\sqrt{3}\ m$

(b) $5\sqrt{3}\ m$

(c) $4\sqrt{3}\ m$

(d) $7\sqrt{3}$

Ans. (d)



Let AB and CD be the pole and AC be the road.

Let AE = x, then EC = 28-x and AB = CD = h. Then let AB = CD=√3

then, EC =1 and AE = 3

AC (ratio value) = 3 + 1 = 4

4 = 28 then 1 =7

and √3=7√3 so height of tower is 7√3.

Example 7: There are two vertical posts, one on each side of a road, just opposite to each other. One post is 108 metre high. From the top of this post, the angles of depression of the top and foot of the other post are 30^0 and 60^0 respectively. The height of the other post is :

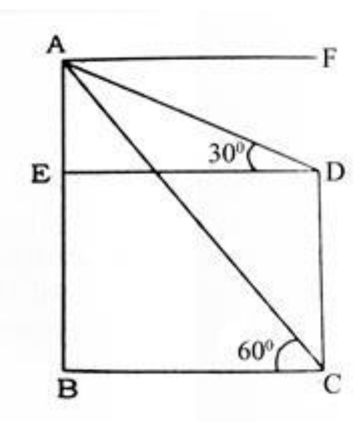
(a)36

(b)72

(c)76

(d)80

Ans (b)



The height of greater Lower i.e. AB = 108 = H

$$H = \frac{h\cot\theta_1}{\cot\theta_1 - \cot\theta_2}$$

$$108 = \frac{h\,\text{co+}\,30^\circ}{\cot 30^\circ - \cot 60^\circ}$$

$$108 = \frac{h\sqrt{3}}{\sqrt{3} - \frac{1}{\sqrt{3}}} \Rightarrow 108 = \frac{h \times \sqrt{3} \times \sqrt{3}}{2}$$

$$h \Rightarrow 72m$$

so height of tower is 72

Example 8: An aeroplane when flying at height of 5000 m from the ground passes vertically above another aeroplane at an instant, when the angles of elevation of the two aeroplanes from the same point on the ground are 60^0 and 45^0 respectively. The vertical distance between the aeroplanes at that instant is:

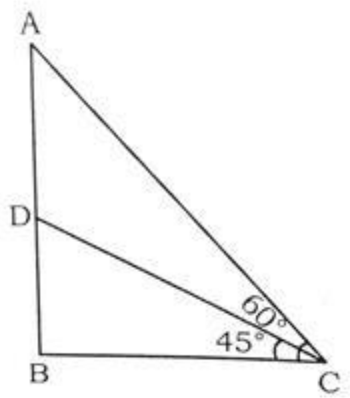
(a) $5000\left(\sqrt{3}-1\right)$

(b) $5000\left(3-\sqrt{3}\right)m$

(c) $5000\left(1-\frac{1}{\sqrt{3}}\right)m$

(d)4500 m

Ans (c)



$\angle ACB = 60^0$
$\angle DCB = 450^0$
$AB = 5000m$
$From \triangle ABC \text{ the angle is } 60^0$
$So, the AB = \sqrt{3}$
$\sqrt{3} = 5000$
$1 = \frac{5000}{\sqrt{3}}$
$AD = \sqrt{3} - 1$
$\Rightarrow 5000 - \frac{5000}{\sqrt{3}} = 5000\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$

In this question we have two triangle ABC and triangle DBC. In triangle ABC we apply the ratio according to 60° and in triangle DBC we apply ratio according to the 45°. That why we take AB=√3 and DB =1.

Example 9: A boy standing in the middle of a square field which is of length 50√3 m, observes a flying bird in the north at an angle of elevation of 30^0 and after 2 minutes, he observes the same bird in the south at an angle of elevation of 60^0. If the bird flies all along in a straight line at a height of then its speed in km/h is:

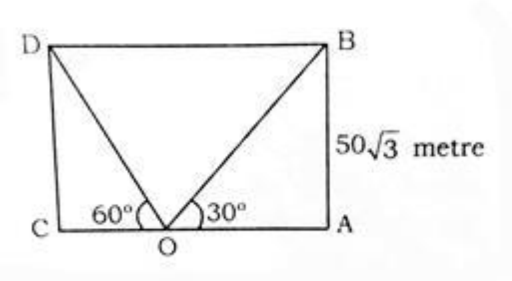
(a) 4.5

(b) 3

(c) 9

(d) 6

Ans.(d)



In ABO

According to the ratio method

AB	:	AO	:	BO
1		$\sqrt{3}$		2
$50\sqrt{3}$		150		$100\sqrt{3}$

From triangle DCO

DC	:	CO	:	DO
$\sqrt{3}$	:	1	:	2
$50\sqrt{3}$		50		100

DO cot AO = 150 + 50 = 200 m

Speed = $\frac{D}{t} = \frac{200}{2} = 100\,\text{m/minute}$

$= \frac{100}{1000} \times 60\,kmph = 6\,km/h$

Example 10: A tree is broken by the wind. If the top of the tree struck the round at an angle of 30° and at a distance of 30 m from the root, then the height of the tree is :

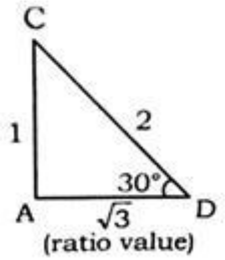
(a) $25\sqrt{3}\,m$

(b) $30\sqrt{3}\,m$

(c) $15\sqrt{3}\,m$

(d) $20\sqrt{3}\,m$

Ans. (b)



√3=30

1= 10√3 & 2 =20√3

so total height is 1+2 =10√3+20√3= 30√3

Example 11: The angle of elevation of a cloud from height h above the level of water in a lake is a and the angle of the depression of its image in the lake is b. Then, the height of the cloud above the surface of the lake is :

(a) $h \cot \beta$

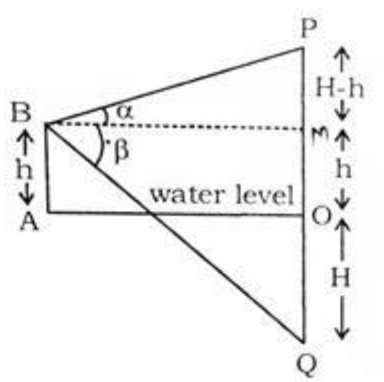
(b) $h(\cot \alpha + \cot \beta)$

(c) $h \cot \alpha$

(d) $h \frac{(\cot \alpha + \cot \beta)}{(\cot \alpha - \cot \beta)}$

Ans. (d)

Let P be the cloud at height H above the level of the water in the lake Q its image in the water



$\therefore OQ = OP = H$,

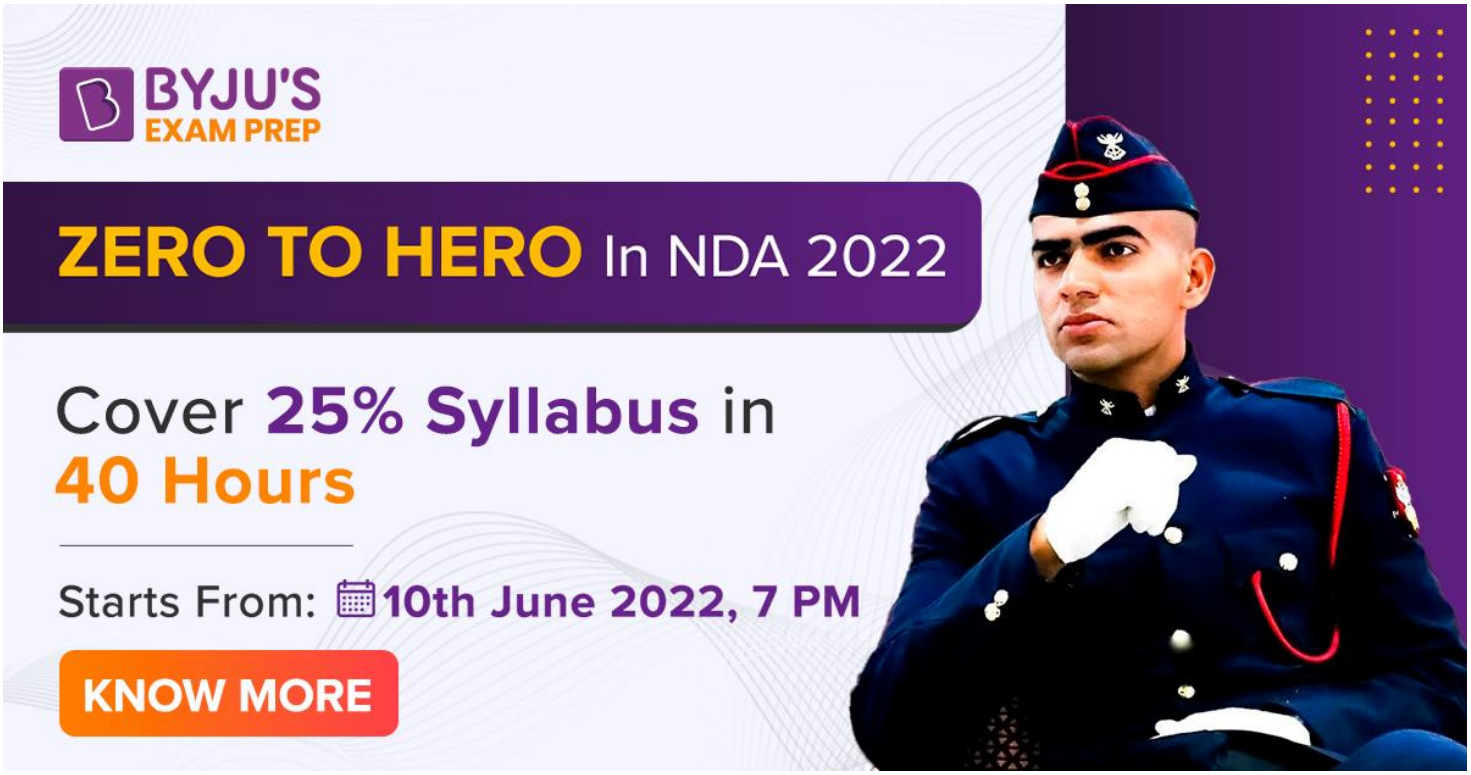
B is at a point at a height AB = h, above the water, Angle of elevation of P and depression of Q from B are respectively

In triangle PBM

$$\tan \alpha = \frac{H-h}{BM}$$
$$\therefore BM = (H-h)\cot \alpha \quad \ldots\ldots(i)$$
$$In \triangle QMB,$$
$$\tan \beta = \frac{QM}{BM}$$
$$\therefore BM = (H+h)\cot \beta \quad \ldots\ldots.(ii)$$

From equations (i) and (ii),

$$(H-h)\cot \alpha = (H+h)\cot \beta$$
$$\Rightarrow H(\cot \alpha - \cot \beta) = h(\cot \alpha + \cot \beta)$$
$$\therefore H = \frac{h(\cot \alpha + \cot \beta)}{\cot \alpha - \cot \beta}$$



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