Exercise 2.1 Page No: 2.33

1. Find the zeros of each of the following quadratic polynomials and verify the relationship between the zeros and their coefficients:

(i) $f(x) = x^2 – 2x – 8$

Solution:

Given,

$f(x) = x^2 – 2x – 8$

To find the zeros, we put f(x) = 0

⇒ $x^2 – 2x – 8 = 0$

⇒ $x^2 - 4x + 2x - 8 = 0$

⇒ $x(x - 4) + 2(x - 4) = 0$

⇒ $(x - 4)(x + 2) = 0$

This gives us 2 zeros, for

x = 4 and x = -2

Hence, the zeros of the quadratic equation are 4 and -2.

Now, for verification

Sum of zeros = - coefficient of x / coefficient of x^2

4 + (-2)= - (-2) / 1

2 = 2

Product of roots = constant / coefficient of x^2

4 x (-2) = (-8) / 1

-8 = -8

Therefore, the relationship between zeros and their coefficients is verified.

(ii) $g(s) = 4s^2 – 4s + 1$

Solution:

Given,

$g(s) = 4s^2 – 4s + 1$

To find the zeros, we put g(s) = 0

⇒ $4s^2 – 4s + 1 = 0$

⇒ $4s^2 - 2s - 2s + 1= 0$

⇒ $2s(2s - 1) - (2s - 1) = 0$

⇒ $(2s - 1)(2s – 1) = 0$

This gives us 2 zeros, for

s = 1/2 and s = 1/2

Hence, the zeros of the quadratic equation are 1/2 and 1/2.

Now, for verification

Sum of zeros = - coefficient of s / coefficient of s^2

1/2 + 1/2 = - (-4) / 4

1 = 1

Product of roots = constant / coefficient of s^2

1/2 x 1/2 = 1/4

1/4 = 1/4

Therefore, the relationship between zeros and their coefficients is verified.

(iii) $h(t)=t^2 - 15$

Solution:

Given,
$h(t) = t^2 - 15 = t^2 + (0)t - 15$
To find the zeros, we put $h(t) = 0$
$\Rightarrow t^2 - 15 = 0$
$\Rightarrow (t + \sqrt{15})(t - \sqrt{15}) = 0$
This gives us 2 zeros, for
$t = \sqrt{15}$ and $t = -\sqrt{15}$
Hence, the zeros of the quadratic equation are $\sqrt{15}$ and $-\sqrt{15}$.
Now, for verification
Sum of zeros = - coefficient of t / coefficient of t^2
$\sqrt{15} + (-\sqrt{15}) = -(0) / 1$
$0 = 0$
Product of roots = constant / coefficient of t^2
$\sqrt{15} \times (-\sqrt{15}) = -15/1$
$-15 = -15$
Therefore, the relationship between zeros and their coefficients is verified.

(iv) $f(x) = 6x^2 - 3 - 7x$

Solution:

Given,
$f(x) = 6x^2 - 3 - 7x$
To find the zeros, we put $f(x) = 0$
$\Rightarrow 6x^2 - 3 - 7x = 0$
$\Rightarrow 6x^2 - 9x + 2x - 3 = 0$
$\Rightarrow 3x(2x - 3) + 1(2x - 3) = 0$
$\Rightarrow (2x - 3)(3x + 1) = 0$
This gives us 2 zeros, for
$x = 3/2$ and $x = -1/3$
Hence, the zeros of the quadratic equation are 3/2 and -1/3.
Now, for verification
Sum of zeros = - coefficient of x / coefficient of x^2
$3/2 + (-1/3) = -(-7) / 6$
$7/6 = 7/6$
Product of roots = constant / coefficient of x^2
$3/2 \times (-1/3) = (-3) / 6$
$-1/2 = -1/2$
Therefore, the relationship between zeros and their coefficients is verified.

(v) $p(x) = x^2 + 2\sqrt{2}x - 6$

Solution:

Given,

$p(x) = x^2 + 2\sqrt{2}x - 6$

To find the zeros, we put p(x) = 0

$\Rightarrow \quad x^2 + 2\sqrt{2}x - 6 = 0$

$\Rightarrow \quad x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0$

$\Rightarrow \quad x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2}) = 0$

$\Rightarrow \quad (x - \sqrt{2})(x + 3\sqrt{2}) = 0$

This gives us 2 zeros, for

$x = \sqrt{2}$ and $x = -3\sqrt{2}$

Hence, the zeros of the quadratic equation are $\sqrt{2}$ and $-3\sqrt{2}$.

Now, for verification

Sum of zeros = - coefficient of x / coefficient of x^2

$\sqrt{2} + (-3\sqrt{2}) = -(2\sqrt{2}) / 1$

$-2\sqrt{2} = -2\sqrt{2}$

Product of roots = constant / coefficient of x^2

$\sqrt{2} \times (-3\sqrt{2}) = (-6) / 2\sqrt{2}$

$-3 \times 2 = -6/1$

$-6 = -6$

Therefore, the relationship between zeros and their coefficients is verified.

(vi) $q(x)=\sqrt{3}x^2 + 10x + 7\sqrt{3}$

Solution:

Given,

$q(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$

To find the zeros, we put q(x) = 0

$\Rightarrow \quad \sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}x^2 + 3x + 7x + 7\sqrt{3}x = 0$

$\Rightarrow \quad \sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$

$\Rightarrow \quad (x + \sqrt{3})(\sqrt{3}x + 7) = 0$

This gives us 2 zeros, for

$x = -\sqrt{3}$ and $x = -7/\sqrt{3}$

Hence, the zeros of the quadratic equation are $-\sqrt{3}$ and $-7/\sqrt{3}$.

Now, for verification

Sum of zeros = - coefficient of x / coefficient of x^2

$-\sqrt{3} + (-7/\sqrt{3}) = -(10)/\sqrt{3}$

$(-3-7)/\sqrt{3} = -10/\sqrt{3}$

$-10/\sqrt{3} = -10/\sqrt{3}$

Product of roots = constant / coefficient of x^2

$(-\sqrt{3}) \times (-7/\sqrt{3}) = (7\sqrt{3}) / \sqrt{3}$

$7 = 7$

Therefore, the relationship between zeros and their coefficients is verified.

(vii) $f(x) = x^2 – (\sqrt{3} + 1)x + \sqrt{3}$

Solution:

Given,

$f(x) = x^2 – (\sqrt{3} + 1)x + \sqrt{3}$

To find the zeros, we put f(x) = 0

$\Rightarrow \quad x^2 – (\sqrt{3} + 1)x + \sqrt{3} = 0$

$\Rightarrow \quad x^2 - \sqrt{3}x - x + \sqrt{3} = 0$

$\Rightarrow \quad x(x - \sqrt{3}) - 1 (x - \sqrt{3}) = 0$

$\Rightarrow \quad (x - \sqrt{3})(x - 1) = 0$

This gives us 2 zeros, for

$x = \sqrt{3}$ and $x = 1$

Hence, the zeros of the quadratic equation are $\sqrt{3}$ and 1.

Now, for verification

Sum of zeros = - coefficient of x / coefficient of x^2

$\sqrt{3} + 1 = - (-(\sqrt{3} +1)) / 1$

$\sqrt{3} + 1 = \sqrt{3} +1$

Product of roots = constant / coefficient of x^2

$1 \times \sqrt{3} = \sqrt{3} / 1$

$\sqrt{3} = \sqrt{3}$

Therefore, the relationship between zeros and their coefficients is verified.

(viii) $g(x)=a(x^2+1)–x(a^2+1)$

Solution:

Given,

$g(x) = a(x^2+1)–x(a^2+1)$

To find the zeros, we put g(x) = 0

$\Rightarrow \quad a(x^2+1)–x(a^2+1) = 0$

$\Rightarrow \quad ax^2 + a − a^2x – x = 0$

$\Rightarrow \quad ax^2 − a^2x – x + a = 0$

$\Rightarrow \quad ax(x − a) − 1(x – a) = 0$

$\Rightarrow \quad (x – a)(ax – 1) = 0$

This gives us 2 zeros, for

$x = a$ and $x = 1/a$

Hence, the zeros of the quadratic equation are a and 1/a.

Now, for verification

Sum of zeros = - coefficient of x / coefficient of x^2

$a + 1/a = - (-(a^2 + 1)) / a$

$(a^2 + 1)/a = (a^2 + 1)/a$

Product of roots = constant / coefficient of x^2

$a \times 1/a = a / a$

$1 = 1$

Therefore, the relationship between zeros and their coefficients is verified.

(ix) $h(s) = 2s^2 – (1 + 2\sqrt{2})s + \sqrt{2}$

Solution:

Given,

$h(s) = 2s^2 – (1 + 2\sqrt{2})s + \sqrt{2}$

To find the zeros, we put $h(s) = 0$

$\Rightarrow \quad 2s^2 – (1 + 2\sqrt{2})s + \sqrt{2} = 0$

$\Rightarrow \quad 2s^2 – 2\sqrt{2}s – s + \sqrt{2} = 0$

$\Rightarrow \quad 2s(s – \sqrt{2}) -1(s - \sqrt{2}) = 0$

$\Rightarrow \quad (2s - 1)(s - \sqrt{2}) = 0$

This gives us 2 zeros, for

$x = \sqrt{2}$ and $x = 1/2$

Hence, the zeros of the quadratic equation are $\sqrt{3}$ and 1.

Now, for verification

Sum of zeros = - coefficient of s / coefficient of s^2

$\sqrt{2} + 1/2 = - (-(1 + 2\sqrt{2})) / 2$

$(2\sqrt{2} + 1)/2 = (2\sqrt{2} +1)/2$

Product of roots = constant / coefficient of s^2

$1/2 \text{ x } \sqrt{2} = \sqrt{2} / 2$

$\sqrt{2} / 2 = \sqrt{2} / 2$

Therefore, the relationship between zeros and their coefficients is verified.

(x) $f(v) = v^2 + 4\sqrt{3}v - 15$

Solution:

Given,

$f(v) = v^2 + 4\sqrt{3}v – 15$

To find the zeros, we put $f(v) = 0$

$\Rightarrow \quad v^2 + 4\sqrt{3}v – 15 = 0$

$\Rightarrow \quad v^2 + 5\sqrt{3}v - \sqrt{3}v - 15 = 0$

$\Rightarrow \quad v(v + 5\sqrt{3}) - \sqrt{3} (v + 5\sqrt{3}) = 0$

$\Rightarrow \quad (v - \sqrt{3})(v + 5\sqrt{3}) = 0$

This gives us 2 zeros, for

$v = \sqrt{3}$ and $v = -5\sqrt{3}$

Hence, the zeros of the quadratic equation are $\sqrt{3}$ and $-5\sqrt{3}$.

Now, for verification

Sum of zeros = - coefficient of v / coefficient of v^2

$\sqrt{3} + (-5\sqrt{3}) = - (4\sqrt{3}) / 1$

$-4\sqrt{3} = -4\sqrt{3}$

Product of roots = constant / coefficient of v^2

$\sqrt{3} \text{ x } (-5\sqrt{3}) = (-15) / 1$

$-5 \text{ x } 3 = -15$

$-15 = -15$

Therefore, the relationship between zeros and their coefficients is verified.

(xi) $p(y) = y^2 + (3\sqrt{5}/2)y – 5$

Solution:

Given,
$p(y) = y^2 + (3\sqrt{5}/2)y – 5$
To find the zeros, we put $f(v) = 0$
$\Rightarrow \quad y^2 + (3\sqrt{5}/2)y – 5 = 0$
$\Rightarrow \quad y^2 - \sqrt{5}/2\ y + 2\sqrt{5}y - 5 = 0$
$\Rightarrow \quad y(y - \sqrt{5}/2) + 2\sqrt{5}\ (y - \sqrt{5}/2) = 0$
$\Rightarrow \quad (y + 2\sqrt{5})(y - \sqrt{5}/2) = 0$
This gives us 2 zeros, for
$y = \sqrt{5}/2$ and $y = -2\sqrt{5}$
Hence, the zeros of the quadratic equation are $\sqrt{5}/2$ and $-2\sqrt{5}$.
Now, for verification
Sum of zeros = - coefficient of y / coefficient of y^2
$\sqrt{5}/2 + (-2\sqrt{5}) = - (3\sqrt{5}/2) / 1$
$-3\sqrt{5}/2 = -3\sqrt{5}/2$
Product of roots = constant / coefficient of y^2
$\sqrt{5}/2 \times (-2\sqrt{5}) = (-5) / 1$
$- (\sqrt{5})^2 = -5$
$-5 = -5$
Therefore, the relationship between zeros and their coefficients is verified.

(xii) $q(y) = 7y^2 - (11/3)y – 2/3$

Solution:

Given,
$q(y) = 7y^2 - (11/3)y – 2/3$
To find the zeros, we put $q(y) = 0$
$\Rightarrow \quad 7y^2 - (11/3)y – 2/3 = 0$
$\Rightarrow \quad (21y^2 - 11y -2)/3 = 0$
$\Rightarrow \quad 21y^2 - 11y - 2 = 0$
$\Rightarrow \quad 21y^2 - 14y + 3y - 2 = 0$
$\Rightarrow \quad 7y(3y - 2) – 1(3y + 2) = 0$
$\Rightarrow \quad (3y - 2)(7y + 1) = 0$
This gives us 2 zeros, for
$y = 2/3$ and $y = -1/7$
Hence, the zeros of the quadratic equation are 2/3 and -1/7.
Now, for verification
Sum of zeros = - coefficient of y / coefficient of y^2
$2/3 + (-1/7) = - (-11/3) / 7$
$-11/21 = -11/21$
Product of roots = constant / coefficient of y^2
$2/3 \times (-1/7) = (-2/3) / 7$
$- 2/21 = -2/21$
Therefore, the relationship between zeros and their coefficients is verified.

2. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeros are as given. Also, find the zeros of these polynomials by factorization.

(i) -8/3 , 4/3

Solution:

A quadratic polynomial formed for the given sum and product of zeros is given by:

$f(x) = x^2 + -(\text{sum of zeros})\ x + (\text{product of roots})$

Here, the sum of zeros is = -8/3 and product of zero= 4/3

Thus,

The required polynomial f(x) is,

⇒ $x^2 - (-8/3)x + (4/3)$

⇒ $x^2 + 8/3x + (4/3)$

So, to find the zeros we put f(x) = 0

⇒ $x^2 + 8/3x + (4/3) = 0$

⇒ $3x^2 + 8x + 4 = 0$

⇒ $3x^2 + 6x + 2x + 4 = 0$

⇒ $3x(x + 2) + 2(x + 2) = 0$

⇒ $(x + 2)(3x + 2) = 0$

⇒ $(x + 2) = 0$ and, or $(3x + 2) = 0$

Therefore, the two zeros are -2 and -2/3.

(ii) 21/8 , 5/16

Solution:

A quadratic polynomial formed for the given sum and product of zeros is given by:

$f(x) = x^2 + -(\text{sum of zeros})\ x + (\text{product of roots})$

Here, the sum of zeros is = 21/8 and product of zero = 5/16

Thus,

The required polynomial f(x) is,

⇒ $x^2 - (21/8)x + (5/16)$

⇒ $x^2 - 21/8x + 5/16$

So, to find the zeros we put f(x) = 0

⇒ $x^2 - 21/8x + 5/16 = 0$

⇒ $16x^2 - 42x + 5 = 0$

⇒ $16x^2 - 40x - 2x + 5 = 0$

⇒ $8x(2x - 5) - 1(2x - 5) = 0$

⇒ $(2x - 5)(8x - 1) = 0$

⇒ $(2x - 5) = 0$ and, or $(8x - 1) = 0$

Therefore, the two zeros are 5/2 and 1/8.

(iii) -2√3, -9

Solution:

A quadratic polynomial formed for the given sum and product of zeros is given by:

$f(x) = x^2 + -(\text{sum of zeros})\ x + (\text{product of roots})$

Here, the sum of zeros is = $-2\sqrt{3}$ and product of zero = -9

Thus,
The required polynomial f(x) is,
⇒ $x^2 - (-2\sqrt{3})x + (-9)$
⇒ $x^2 + 2\sqrt{3}x – 9$
So, to find the zeros we put f(x) = 0
⇒ $x^2 + 2\sqrt{3}x – 9 = 0$
⇒ $x^2 + 3\sqrt{3}x - \sqrt{3}x – 9 = 0$
⇒ $x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$
⇒ $(x + 3\sqrt{3}) (x - \sqrt{3}) = 0$
⇒ $(x + 3\sqrt{3}) = 0$ and, or $(x - \sqrt{3}) = 0$
Therefore, the two zeros are $-3\sqrt{3}$and $\sqrt{3}$.

(iv) -3/2√5, -1/2

Solution:

A quadratic polynomial formed for the given sum and product of zeros is given by:
f(x) = x^2 + -(sum of zeros) x + (product of roots)
Here, the sum of zeros is = -3/2√5 and product of zero = -1/2
Thus,
The required polynomial f(x) is,
⇒ $x^2 - (-3/2\sqrt{5})x + (-1/2)$
⇒ $x^2 + 3/2\sqrt{5}x – 1/2$
So, to find the zeros we put f(x) = 0
⇒ $x^2 + 3/2\sqrt{5}x – 1/2 = 0$
⇒ $2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$
⇒ $2\sqrt{5}x^2 + 5x – 2x - \sqrt{5} = 0$
⇒ $\sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = 0$
⇒ $(2x + \sqrt{5}) (\sqrt{5}x - 1) = 0$
⇒ $(2x + \sqrt{5}) = 0$ and, or $(\sqrt{5}x - 1) = 0$
Therefore, the two zeros are $-\sqrt{5}/2$ and $1/\sqrt{5}$.

3. If α and β are the zeros of the quadratic polynomial f(x) = $x^2 – 5x + 4$, find the value of 1/α + 1/β – 2αβ.

Solution:

From the question, it’s given that:
α and β are the roots of the quadratic polynomial f(x) where a = 1, b = -5 and c = 4
So, we can find
Sum of the roots = α+β = -b/a = - (-5)/1 = -5
Product of the roots = αβ = c/a = 4/1 = 4
To find, 1/α +1/β – 2αβ
⇒ [(α +β)/ αβ] – 2αβ
⇒ (-5)/ 4 – 2(4) = -5/4 – 8 = -27/ 4

4. If α and β are the zeros of the quadratic polynomial p(y) = $5y^2 – 7y + 1$, find the value of 1/α+1/β.

Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial f(x) where a =5, b = -7 and c = 1
So, we can find
Sum of the roots = α+β = -b/a = - (-7)/5 = 7/5
Product of the roots = αβ = c/a = 1/5
To find, 1/α +1/β

⇒ (α +β)/ αβ

⇒ (7/5)/ (1/5) = 7

5. If α and β are the zeros of the quadratic polynomial f(x)=x^2 – x – 4, find the value of 1/α+1/β–αβ.
Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial f(x) where a = 1, b = -1 and c = - 4
So, we can find
Sum of the roots = α+β = -b/a = - (-1)/1 = 1
Product of the roots = αβ = c/a = -4 /1 = - 4
To find, 1/α +1/β – αβ

⇒ [(α +β)/ αβ] – αβ

⇒ [(1)/ (-4)] – (-4) = -1/4 + 4 = 15/ 4

6. If α and β are the zeroes of the quadratic polynomial f(x) = x^2 + x – 2, find the value of 1/α – 1/β.
Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial f(x) where a = 1, b = 1 and c = - 2
So, we can find
Sum of the roots = α+β = -b/a = - (1)/1 = -1
Product of the roots = αβ = c/a = -2 /1 = - 2
To find, 1/α - 1/β

⇒ [(β - α)/ αβ]

⇒ $\frac{\beta-\alpha}{\alpha\beta} = \frac{\beta-\alpha}{\alpha\beta} \times \frac{(\alpha-\beta)}{\alpha\beta} = \frac{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}{\alpha\beta} = \frac{\sqrt{1+8}}{2} = \frac{\sqrt{9}}{2} = \frac{3}{2}$

7. If one of the zero of the quadratic polynomial f(x) = $4x^2$ – 8kx – 9 is negative of the other, then find the value of k.
Solution:

From the question, it's given that:
The quadratic polynomial f(x) where a = 4, b = -8k and c = - 9
And, for roots to be negative of each other, let the roots be α and – α.
So, we can find
Sum of the roots = α - α = -b/a = - (-8k)/1 = 8k = 0 [∵ α - α = 0]

⇒ k = 0

8. If the sum of the zeroes of the quadratic polynomial $f(t)=kt^2 + 2t + 3k$ is equal to their product, then find the value of k.
Solution:

Given,
The quadratic polynomial $f(t)=kt^2 + 2t + 3k$, where a = k, b = 2 and c = 3k.
And,
Sum of the roots = Product of the roots
$\Rightarrow$ (-b/a) = (c/a)
$\Rightarrow$ (-2/k) = (3k/k)
$\Rightarrow$ (-2/k) = 3
$\therefore$ k = -2/3

9. If α and β are the zeros of the quadratic polynomial $p(x) = 4x^2 – 5x – 1$, find the value of $\alpha^2\beta+\alpha\beta^2$.
Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial p(x) where a = 4, b = -5 and c = -1
So, we can find
Sum of the roots = α+β = -b/a = - (-5)/4 = 5/4
Product of the roots = αβ = c/a = -1/4
To find, $\alpha^2\beta+\alpha\beta^2$
$\Rightarrow$ αβ(α +β)
$\Rightarrow$ (-1/4)(5/4) = -5/16

10. If α and β are the zeros of the quadratic polynomial $f(t)=t^2– 4t + 3$, find the value of $\alpha^4\beta^3+\alpha^3\beta^4$.
Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial f(t) where a = 1, b = -4 and c = 3
So, we can find
Sum of the roots = α+β = -b/a = - (-4)/1 = 4
Product of the roots = αβ = c/a = 3/1 = 3
To find, $\alpha^4\beta^3+\alpha^3\beta^4$
$\Rightarrow$ $\alpha^3\beta^3$ (α +β)
$\Rightarrow$ $(\alpha\beta)^3$ (α +β)
$\Rightarrow$ $(3)^3$ (4) = 27 x 4 = 108

Exercise 2.2 Page No: 2.43

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also, verify the relationship between the zeros and coefficients in each of the following cases:

(i) f(x) = $2x^3 + x^2 - 5x + 2$; 1/2, 1, -2

Solution:

Given, f(x) = $2x^3 + x^2 - 5x + 2$, where a= 2, b= 1, c= -5 and d= 2

For x = 1/2

f(1/2) = $2(1/2)^3 + (1/2)^2 - 5(1/2) + 2$

= 1/4 + 1/4 - 5/2 + 2 = 0

⇒ f(1/2) = 0, hence x = 1/2 is a root of the given polynomial.

For x = 1

f(1) = $2(1)^3 + (1)^2 - 5(1) + 2$

= 2 + 1 - 5 + 2 = 0

⇒ f(1) = 0, hence x = 1 is also a root of the given polynomial.

For x = -2

f(-2) = $2(-2)^3 + (-2)^2 - 5(-2) + 2$

= -16 + 4 + 10 + 2 = 0

⇒ f(-2) = 0, hence x = -2 is also a root of the given polynomial.

Now,

Sum of zeros = -b/a

1/2 + 1 – 2 = - (1)/2

-1/2 = -1/2

Sum of the products of the zeros taken two at a time = c/a

(1/2 x 1) + (1 x -2) + (1/2 x -2) = -5/ 2

1/2 - 2 + (-1) = -5/2

-5/2 = -5/2

Product of zeros = - d/a

1/2 x 1 x (– 2) = -(2)/2

-1 = -1

Hence, the relationship between the zeros and coefficients is verified.

(ii) g(x) = $x^3 - 4x^2 + 5x - 2$; 2, 1, 1

Solution:

Given, g(x) = $x^3 - 4x^2 + 5x - 2$, where a= 1, b= -4, c= 5 and d= -2

For x = 2

g(2) = $(2)^3 - 4(2)^2 + 5(2) - 2$

= 8 - 16 + 10 - 2 = 0

⇒ f(2) = 0, hence x = 2 is a root of the given polynomial.

For x = 1

g(1) = $(1)^3 - 4(1)^2 + 5(1) - 2$

= 1 - 4 + 5 - 2 = 0

⇒ g(1) = 0, hence x = 1 is also a root of the given polynomial.

Now,

Sum of zeros = -b/a

$1 + 1 + 2 = - (-4)/1$

$4 = 4$

Sum of the products of the zeros taken two at a time = c/a

$(1 \times 1) + (1 \times 2) + (2 \times 1) = 5/ 1$

$1 + 2 + 2 = 5$

$5 = 5$

Product of zeros = - d/a

$1 \times 1 \times 2 = -(-2)/1$

$2 = 2$

Hence, the relationship between the zeros and coefficients is verified.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeros as 3, -1 and -3 respectively.

Solution:

Generally,

A cubic polynomial say, f(x) is of the form $ax^3 + bx^2 + cx + d$.

And, can be shown w.r.t its relationship between roots as.

$\Rightarrow$ f(x) = k [x^3 – (sum of roots)x^2 + (sum of products of roots taken two at a time)x – (product of roots)]

Where, k is any non-zero real number.

Here,

$f(x) = k [x^3 – (3)x^2 + (-1)x – (-3)]$

$\therefore f(x) = k [x^3 – 3x^2 - x + 3)]$

where, k is any non-zero real number.

3. If the zeros of the polynomial $f(x) = 2x^3 – 15x^2 + 37x – 30$ are in A.P., find them.

Solution:

Let the zeros of the given polynomial be α, β and γ. (3 zeros as it's a cubic polynomial)

And given, the zeros are in A.P.

So, let's consider the roots as

$\alpha = a – d$, $\beta = a$ and $\gamma = a + d$

Where, a is the first term and d is the common difference.

From given f(x), a= 2, b= -15, c= 37 and d= 30

$\Rightarrow$ Sum of roots = $\alpha + \beta + \gamma = (a - d) + a + (a + d) = 3a = (-b/a) = -(-15/2) = 15/2$

So, calculating for a, we get $3a = 15/2$ $\Rightarrow a = 5/2$

$\Rightarrow$ Product of roots = $(a - d) \times (a) \times (a + d) = a(a^2 – d^2) = -d/a = -(30)/2 = 15$

$\Rightarrow a(a^2 – d^2) = 15$

Substituting the value of a, we get

$\Rightarrow (5/2)[(5/2)^2 – d^2] = 15$

⇒ $5[(25/4) - d^2] = 30$
⇒ $(25/4) - d^2 = 6$
⇒ $25 - 4d^2 = 24$
⇒ $1 = 4d^2$
∴ d = 1/2 or -1/2

Taking d = 1/2 and a = 5/2
We get,
the zeros as 2, 5/2 and 3

Taking d = -1/2 and a = 5/2
We get,
the zeros as 3, 5/2 and 2

Exercise 2.3 Page No: 2.57

1. Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing f(x) by g(x) in each of the following:

(i) $f(x) = x^3 - 6x^2 + 11x - 6$, $g(x) = x^2 + x + 1$

Solution:

Given,

$f(x) = x^3 - 6x^2 + 11x - 6$, $g(x) = x^2 + x + 1$

$$
\begin{array}{r|rrrr}
 & x & -7 & & \\
\hline
x^2+x+1 & x^3 & -6x^2 & +11x & -6 \\
- & x^3 & +x^2 & +x & \\
\hline
 & & -7x^2 & +10x & -6 \\
- & & -7x^2 & -7x & -7 \\
\hline
 & & & 17x & +1
\end{array}
$$

Thus,

$q(x) = x - 7$ and $r(x) = 17x + 1$

(ii) $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$, $g(x) = 2x^2 + 7x + 1$

Solution:

Given,

$f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$ and $g(x) = 2x^2 + 7x + 1$

$$
\begin{array}{r|rrrrr}
 & 5x^2 & -9x & -2 & & \\
\hline
2x^2+7x+1 & 10x^4 & +17x^3 & -62x^2 & +30x & -3 \\
- & 10x^4 & +35x^3 & +5x^2 & & \\
\hline
 & & -18x^3 & -67x^2 & +30x & -3 \\
- & & -18x^3 & -63x^2 & -9x & \\
\hline
 & & & -4x^2 & +39x & -3 \\
- & & & -4x^2 & -14x & -2 \\
\hline
 & & & & 53x & -1
\end{array}
$$

Thus,

$q(x) = 5x^2 – 9x - 2$ and $r(x) = 53x - 1$

(iii) $f(x) = 4x^3 + 8x^2 + 8x + 7$, $g(x)= 2x^2 – x + 1$

Solution:

Given,

$f(x) = 4x^3 + 8x^2 + 8x + 7$ and $g(x)= 2x^2 – x + 1$

$$\begin{array}{r|rrrr}
 & 2x & +5 & & \\
\hline
2x^2 - x + 1 & 4x^3 & +8x^2 & +8x & +7 \\
- & & & & \\
 & 4x^3 & -2x^2 & +2x & \\
\hline
 & & 10x^2 & +6x & +7 \\
 & & - & & \\
 & & 10x^2 & -5x & +5 \\
\hline
 & & & 11x & +2
\end{array}$$

Thus,

$q(x) = 2x + 5$ and $r(x) = 11x + 2$

(iv) $f(x) = 15x^3 – 20x^2 + 13x – 12$, $g(x) = x^2 – 2x + 2$

Solution:

Given,
$f(x) = 15x^3 – 20x^2 + 13x – 12$ and $g(x) = x^2 – 2x + 2$

$$\begin{array}{r|rrrr}
 & 15x & +10 & & \\
\hline
x^2 - 2x + 2 & 15x^3 & -20x^2 & +13x & -12 \\
- & & & & \\
 & 15x^3 & -30x^2 & +30x & \\
\hline
 & & 10x^2 & -17x & -12 \\
 & & - & & \\
 & & 10x^2 & -20x & +20 \\
\hline
 & & & 3x & -32
\end{array}$$

Thus,
$q(x) = 15x + 10$ and $r(x) = 3x - 32$

2. Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm:

(i) $g(t) = t^2 – 3$; $f(t) = 2t^4 + 3t^3 – 2t^2 – 9t – 12$
Solution:

Given,
$g(t) = t^2 – 3$; $f(t) = 2t^4 + 3t^3 – 2t^2 – 9t – 12$

$$
\begin{array}{rlllll}
 & 2t^2 & +3t & +4 & & \\
\hline
t^2-3 \;\big) & 2t^4 & +3t^3 & -2t^2 & -9t & -12 \\
- & & & & & \\
 & 2t^4 & +0t^3 & -6t^2 & & \\
\hline
 & & 3t^3 & +4t^2 & -9t & -12 \\
 & - & & & & \\
 & & 3t^3 & +0t^2 & -9t & \\
\hline
 & & & 4t^2 & +0t & -12 \\
 & & - & & & \\
 & & & 4t^2 & +0t & -12 \\
\hline
 & & & & & 0
\end{array}
$$

Since, the remainder r(t) = 0 we can say that the first polynomial is a factor of the second polynomial.

(ii) $g(x) = x^3 – 3x + 1$; $f(x) = x^5 – 4x^3 + x^2 + 3x + 1$
Solution:

Given,
$g(x) = x^3 – 3x + 1$; $f(x) = x^5 – 4x^3 + x^2 + 3x + 1$

$$
\begin{array}{rllllll}
 & x^2 & & & & & \\
\hline
x^3-3x+1 \;\big) & x^5 & +0x^4 & -4x^3 & +x^2 & +3x & +1 \\
- & & & & & & \\
 & x^5 & +0x^4 & -3x^3 & +x^2 & & \\
\hline
 & & & & +0x^2 & +3x & +1 \\
 & & - & & & & \\
 & & & -x^3 & +0x^2 & +3x & -1 \\
\hline
 & & & & & & 2
\end{array}
$$

Since, the remainder r(x) = 2 and not equal to zero we can say that the first polynomial is not a factor of the second polynomial.

(iii) $g(x) = 2x^2 – x + 3$; $f(x) = 6x^5 − x^4 + 4x^3 – 5x^2 – x – 15$
Solution:

Given,

$g(x) = 2x^2 - x + 3$; $f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

$$
\begin{array}{r|rrrrrr}
 & 3x^3 & +x^2 & -2x & -5 & & \\
2x^2 - x + 3 & 6x^5 & -x^4 & +4x^3 & -5x^2 & -x & -15 \\
 & - & & & & & \\
 & 6x^5 & -3x^4 & +9x^3 & & & \\
\hline
 & & 2x^4 & -5x^3 & -5x^2 & -x & -15 \\
 & & - & & & & \\
 & & 2x^4 & -x^3 & +3x^2 & & \\
\hline
 & & & -4x^3 & -8x^2 & -x & -15 \\
 & & & - & & & \\
 & & & -4x^3 & +2x^2 & -6x & \\
\hline
 & & & & -10x^2 & +5x & -15 \\
 & & & & - & & \\
 & & & & -10x^2 & +5x & -15 \\
\hline
 & & & & & & 0
\end{array}
$$

Since, the remainder $r(x) = 0$ we can say that the first polynomial is not a factor of the second polynomial.

3. Obtain all zeroes of the polynomial $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$, if two of its zeroes are -2 and -1.
Solution:

Given,

$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$

If the two zeros of the polynomial are -2 and -1, then its factors are (x + 2) and (x + 1)

$\Rightarrow (x+2)(x+1) = x^2 + x + 2x + 2 = x^2 + 3x + 2$ …… (i)

This means that (i) is a factor of f(x). So, performing division algorithm we get,

$$
\begin{array}{r|llllll}
 & 2x^2 & -5x & -3 & & \\
\hline
x^2+3x+2 & 2x^4 & +x^3 & -14x^2 & -19x & -6 \\
 & - & & & & \\
 & 2x^4 & +6x^3 & +4x^2 & & \\
\hline
 & & -5x^3 & -18x^2 & -19x & -6 \\
 & & - & & & \\
 & & -5x^3 & -15x^2 & -10x & \\
\hline
 & & & -3x^2 & -9x & -6 \\
 & & & - & & \\
 & & & -3x^2 & -9x & -6 \\
\hline
 & & & & & 0
\end{array}
$$

The quotient is $2x^2 - 5x - 3$.

$\Rightarrow f(x) = (2x^2 - 5x - 3)(x^2 + 3x + 2)$

For obtaining the other 2 zeros of the polynomial

We put,

$2x^2 - 5x - 3 = 0$

$\Rightarrow (2x + 1)(x - 3) = 0$

$\therefore x = -1/2$ or 3

Hence, all the zeros of the polynomial are -2, -1, -1/2 and 3.

4. Obtain all zeroes of $f(x) = x^3 + 13x^2 + 32x + 20$, if one of its zeros is -2.
Solution:

Given,
$f(x) = x^3 + 13x^2 + 32x + 20$
And, -2 is one of the zeros. So, $(x + 2)$ is a factor of $f(x)$,
Performing division algorithm, we get

$$
\begin{array}{r|llll}
 & x^2 & +11x & +10 & \\
\hline
x+2 & x^3 & +13x^2 & +32x & +20 \\
 & - & & & \\
 & x^3 & +2x^2 & & \\
\hline
 & & 11x^2 & +32x & +20 \\
 & & - & & \\
 & & 11x^2 & +22x & \\
\hline
 & & & 10x & +20 \\
 & & & - & \\
 & & & 10x & +20 \\
\hline
 & & & & 0
\end{array}
$$

⇒ $f(x) = (x^2 + 11x + 10)(x + 2)$

So, putting $x^2 + 11x + 10 = 0$ we can get the other 2 zeros.

⇒ $(x + 10)(x + 1) = 0$

∴ $x = -10$ or -1

Hence, all the zeros of the polynomial are -10, -2 and -1.

5. Obtain all zeroes of the polynomial $f(x) = x^4 - 3x^3 - x^2 + 9x - 6$, if the two of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$.

Solution:

Given,

$f(x) = x^4 - 3x^3 - x^2 + 9x - 6$

Since, two of the zeroes of polynomial are $-\sqrt{3}$ and $\sqrt{3}$ so, $(x + \sqrt{3})$ and $(x - \sqrt{3})$ are factors of f(x).

⇒ $x^2 - 3$ is a factor of f(x). Hence, performing division algorithm, we get

$$
\begin{array}{r|l}
 & x^2 \quad -3x \quad +2 \\
\hline
x^2 - 3 & x^4 \quad -3x^3 \quad -x^2 \quad +9x \quad -6 \\
 & - \\
 & x^4 \quad +0x^3 \quad -3x^2 \\
\hline
 & \quad -3x^3 \quad +2x^2 \quad +9x \quad -6 \\
 & - \\
 & \quad -3x^3 \quad +0x^2 \quad +9x \\
\hline
 & \quad 2x^2 \quad +0x \quad -6 \\
 & - \\
 & \quad 2x^2 \quad +0x \quad -6 \\
\hline
 & \quad 0
\end{array}
$$

⇒ f(x)= $(x^2 - 3x + 2)(x^2 - 3)$

So, putting $x^2 - 3x + 2 = 0$ we can get the other 2 zeros.

⇒ $(x - 2)(x - 1) = 0$

∴ x = 2 or 1

Hence, all the zeros of the polynomial are $-\sqrt{3}$, 1, $\sqrt{3}$ and 2.

6. Obtain all zeroes of the polynomial f(x)= $2x^4 - 2x^3 - 7x^2 + 3x + 6$, if the two of its zeroes are $-\sqrt{(3/2)}$ and $\sqrt{(3/2)}$.

Solution:

Given,

f(x)= $2x^4 - 2x^3 - 7x^2 + 3x + 6$

Since, two of the zeroes of polynomial are $-\sqrt{(3/2)}$ and $\sqrt{(3/2)}$ so, $(x + \sqrt{(3/2)})$ and $(x - \sqrt{(3/2)})$ are factors of f(x).

⇒ $x^2 - (3/2)$ is a factor of f(x). Hence, performing division algorithm, we get

$$\begin{array}{r|lllll} & 2x^2 & -2x & -4 & & \\ x^2-\frac{3}{2} & 2x^4 & -2x^3 & -7x^2 & +3x & +6 \\ & - & & & & \\ & 2x^4 & +0x^3 & -3x^2 & & \\ \hline & & -2x^3 & -4x^2 & +3x & +6 \\ & & - & & & \\ & & -2x^3 & +0x^2 & +3x & \\ \hline & & & -4x^2 & +0x & +6 \\ & & & - & & \\ & & & -4x^2 & +0x & +6 \\ \hline & & & & & 0 \end{array}$$

⇒ $f(x)= (2x^2 - 2x - 4)(x^2 – 3/2)= 2(x^2 - x - 2)(x^2 – 3/2)$

So, putting $x^2 - x - 2 = 0$ we can get the other 2 zeros.

⇒ $(x - 2)(x + 1) = 0$

∴ $x = 2$ or -1

Hence, all the zeros of the polynomial are $-\sqrt{(3/2)}$, -1, $\sqrt{(3/2)}$ and 2.

7. Find all the zeroes of the polynomial $x^4 + x^3 – 34x^2 – 4x + 120$, if the two of its zeros are 2 and -2.

Solution:

Let,

$f(x) = x^4 + x^3 – 34x^2 - 4x + 120$

Since, two of the zeroes of polynomial are −2 and 2 so, $(x + 2)$ and $(x – 2)$ are factors of f(x).

⇒ $x^2 – 4$ is a factor of f(x). Hence, performing division algorithm, we get

$$\begin{array}{r|lllll} & x^2 & +x & -30 & & \\ x^2-4 & x^4 & +x^3 & -34x^2 & -4x & +120 \\ & - & & & & \\ & x^4 & +0x^3 & -4x^2 & & \\ \hline & & x^3 & -30x^2 & -4x & +120 \\ & & - & & & \\ & & x^3 & +0x^2 & -4x & \\ \hline & & & -30x^2 & +0x & +120 \\ & & & - & & \\ & & & -30x^2 & +0x & +120 \\ \hline & & & & & 0 \end{array}$$

$\Rightarrow f(x) = (x^2 + x - 30)(x^2 - 4)$

So, putting $x^2 + x - 30 = 0$ we can get the other 2 zeros.

$\Rightarrow (x + 6)(x - 5) = 0$

$\therefore x = -6$ or 5

Hence, all the zeros of the polynomial are 5, -2, 2 and -6.