Exercise 15.1 Page No: 15.11

1. Find the circumference and area of a circle of radius of 4.2 cm.
Solution:

Given,
Radius (r) = 4.2 cm
We know that,
Circumference of a circle = $2\pi r$
$= 2 \times (22/7) \times 4.2 = 26.4 \text{ cm}^2$
Area of a circle = πr^2
$= (22/7) \text{ x } 4.2^2$
$= 22 \text{ x } 0.6 \text{ x } 4.2 = 55.44 \text{ cm}^2$
Hence, the circumference and area of the circle is 26.4 cm^2 and 55.44 cm^2.

2. Find the circumference of a circle whose area is 301.84 cm^2.
Solution:

Given,
Area of the circle = 301.84 cm^2
We know that,
Area of a Circle = $\pi r^2 = 301.84 \text{ cm}^2$
$(22/7) \times r^2 = 301.84$
$r^2 = 13.72 \text{ x } 7 = 96.04$
$r = \sqrt{96.04} = 9.8$
So, the radius is = 9.8 cm.
Now, Circumference of a circle = $2\pi r$
$= 2 \times (22/7) \times 9.8 = 61.6$ cm
Hence, the circumference of the circle is 61.6 cm.

3. Find the area of a circle whose circumference is 44 cm.
Solution:

Given,
Circumference = 44 cm
We know that,
Circumference of a circle = $2\pi r$ = 44 cm
$2 \times (22/7) \times r = 44$
r = 7 cm
Now, Area of a Circle = πr^2
$= (22/7) \times 7 \times 7$
$= 154 \text{ cm}^2$
Hence, area of the Circle = 154 cm^2

4. The circumference of a circle exceeds the diameter by 16.8 cm. Find the circumference of the circle.

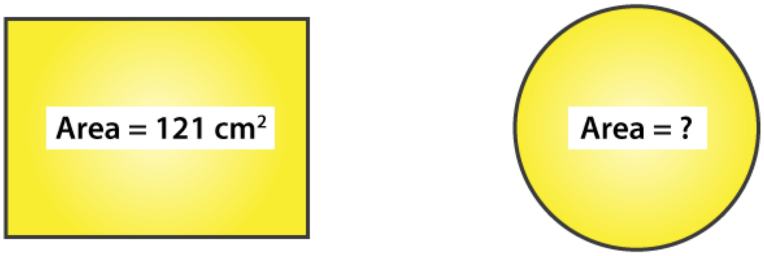
Solution:

Let the radius of the circle be r cm
So, the diameter (d) = 2r [As radius is half the diameter]
We know that,
Circumference of a circle (C) = $2\pi r$
From the question,
Circumference of the circle exceeds its diameter by 16.8 cm
C = d + 16.8
$2\pi r = 2r + 16.8$ [d = 2r]
$2\pi r - 2r = 16.8$
$2r(\pi - 1) = 16.8$
$2r(3.14 - 1) = 16.8$
r = 3.92 cm
Thus, radius = 3.92 cm
Now, the circumference of the circle (C) = $2\pi r$
$C = 2 \times 3.14 \times 3.92$
= 24.64 cm
Hence, the circumference of the circle is 24.64 cm.

5. A horse is tied to a pole with 28 m long string. Find the area where the horse can graze.
Solution:

Given,
Length of the string (l) = 28 m
Area the horse can graze is the area of the circle with a radius equal to the length of the string.
We know that,
Area of a Circle = πr^2
$= (22/7) \times 28 \times 28 = 2464\ m^2$
Hence, the area of the circle which is same as the area the horse can graze is 2464 m^2

6. A steel wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form of a circle, find the area of the circle.
Solution:



Given,
Area of the square = $a^2 = 121\ cm^2$

We know that,
Area of the circle = πr^2
Area of a square = a^2
121 cm^2 = a^2
So, a = 11 cm
Thus, each side of the square = 11 cm
Now, the perimeter of the square = 4a
= 4 × 11 = 44 cm
From the question, its understood that
Perimeter of the square = Circumference of the circle
We know that, circumference of a circle (C) = $2\pi r$
$4a = 2\pi r$
$44 = 2(22/7)r$
r = 7 cm
Now, area of the Circle = πr^2
= (22/7) × 7 × 7 = 154 cm^2
Hence, the area of the circle is 154 cm^2.

7. The circumference of two circles are in the ratio of 2:3. Find the ratio of their areas.
Solution:

Let's consider the radius of two circles C_1 and C_2 be r_1 and r_2.
We know that, Circumference of a circle (C) = $2\pi r$
And their circumference will be $2\pi r_1$ and $2\pi r_2$.
So, their ratio is = r_1: r_2
Given, circumference of two circles is in a ratio of 2: 3
r_1: r_2 = 2: 3
Then, the ratios of their areas is given as
$= \pi r_1^2 : \pi r_2^2$

$= \left(\frac{r1}{r2}\right)^2$

$= \left(\frac{2}{3}\right)^2$

= 4/9
Hence, ratio of their areas = 4: 9.

8. The sum of the radii of two circles is 140 cm and the difference of their circumference is 88 cm. Find the diameters of the circles.
Solution:

Let the radii of the two circles be r_1 and r_2.
And, the circumferences of the two circles be C_1 and C_2.

We know that, circumference of circle (C) = $2\pi r$
Given,
Sum of radii of two circle i.e., $r_1 + r_2 = 140$ cm ... (i)
Difference of their circumference,
$C_1 - C_2 = 88$ cm
$2\pi r_1 - 2\pi r_2 = 88$ cm
$2(22/7)(r_1 - r_2) = 88$ cm
$(r_1 - r_2) = 14$ cm
$r_1 = r_2 + 14$..... (ii)
Substituting the value of r_1 in equation (i), we have,
$r_2 + r_2 + 14 = 140$
$2r_2 = 140 - 14$
$2r_2 = 126$
$r_2 = 63$ cm
Substituting the value of r_2 in equation (ii), we have,
$r_1 = 63 + 14 = 77$ cm
Therefore,
Diameter of circle 1 = $2r_1 = 2 \times 77 = 154$ cm
Diameter of circle 2 = $2r_2 = 2 \times 63 = 126$ cm

9. Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of radii 15cm and 18cm.
Solution:

Given,
Radius of circle 1 = $r_1 = 15$ cm
Radius of circle 2 = $r_2 = 18$ cm
We know that, the circumference of a circle (C) = $2\pi r$
So, $C_1 = 2\pi r_1$ and $C_2 = 2\pi r_2$
Let the radius be r of the circle which is to be found and its circumference (C)
Now, from the question
$C = C_1 + C_2$
$2\pi r = 2\pi r_1 + 2\pi r_2$
$r = r_1 + r_2$ [After dividing by 2π both sides]
$r = 15 + 18$
$r = 33$ cm
Thus, the radius of the circle = 33 cm

10. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having its area equal to the sum of the areas of two circles.
Solution:

Given,
Radii of the two circles are 6 cm and 8 cm
Area of circle with radius 8 cm = $\pi (8)^2 = 64\pi$ cm^2
Area of circle with radius 6cm = $\pi (6)^2 = 36\pi$ cm^2

Sum of areas = $64\pi + 36\pi = 100\pi$ cm^2
Let the radius of the circle be x cm
Area of the circle = 100π cm^2 (from above)
$\pi x^2 = 100\pi$
$x = \sqrt{100} = 10$ cm
Therefore, the radius of the circle is 10 cm.

11. The radii of two circles are 19 cm and 9 cm respectively. Find the radius and area of the circle which has circumferences is equal to sum of the circumference of two circles.
Solution:
Given,
Radius of circle 1 = r_1 = 19 cm
Radius of circle 2 = r_2 = 9 cm
We know that, the circumference of a circle (C) = $2\pi r$
So, $C_1 = 2\pi r_1$ and $C_2 = 2\pi r_2$
Let the radius be r of the circle which is to be found and its circumference (C)
Now, from the question
$C = C_1 + C_2$
$2\pi r = 2\pi r_1 + 2\pi r_2$
$r = r_1 + r_2$ [After dividing by 2π both sides]
$r = 19 + 9$
$r = 28$ cm
Thus, the radius of the circle = 28 cm
So, the area of required circle = $\pi r^2 = (22/7) \times 28 \times 28 = 2464$ cm^2

12. The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of ₹50 per metre.
Solution:

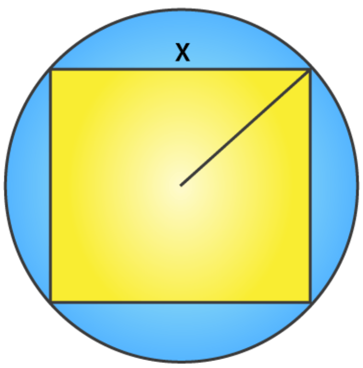
Given,
Area of the circular playground = 22176 m^2
And the cost of fencing per metre = ₹50
If the radius of the ground is taken as r.
Then, its area = πr^2
$\pi r^2 = 22176$
$r^2 = 22176(7/22) = 7056$
Taking square root on both sides, we have
r = 84 m
We know that, fencing is done only on the circumference of the ground
Circumference of the ground = $2\pi r = 2(22/7)84 = 528$ m
So, the cost of fencing 528 m = ₹50 x 528 = ₹26400
Therefore, the cost of fencing the ground is ₹26400.

13. The side of a square is 10 cm. Find the area of the circumscribed and inscribed circles.
Solution:

For circumscribed circle:
Radius = diagonal of square/ 2
Diagonal of the square = side x $\sqrt{2}$
= $10\sqrt{2}$ cm
Radius = $(10 \times 1.414)/ 2 = 7.07$ cm
Thus, the radius of the circumcircle = 7.07 cm
Then, its area is = $\pi r^2 = (22/7) \times 7.07 \times 7.07 = 157.41$ cm^2

Therefore, the Area of the Circumscribed circle is 157.41 cm^2
For inscribed circle:
Radius = side of square/ 2
= 10/ 2 = 5 m
Then, its area is = $\pi r^2 = 3.14 \times 5 \times 5 = 78.5$ cm^2
Thus, the area of the circumscribed circle is 157.41 cm^2 and the area of the inscribed circle is 78.5 cm^2.

14. If a square is inscribed in a circle, find the ratio of areas of the circle and the square.
Solution:



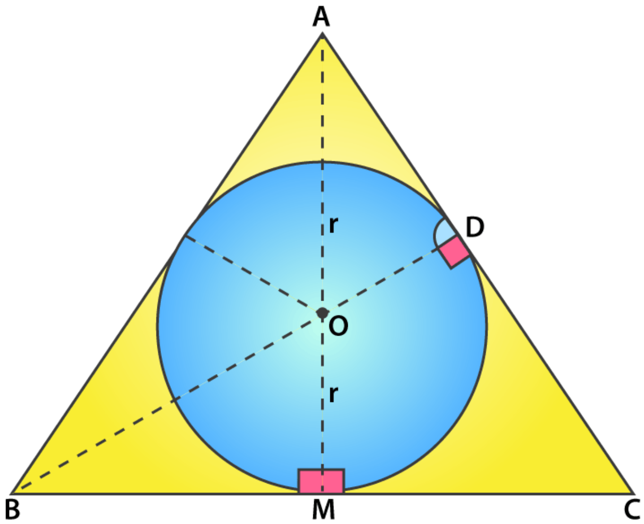
Let side of square be x cm which is inscribed in a circle.
Given,
Radius of circle (r) = 1/2 (diagonal of square)
= $1/2(x\sqrt{2})$
r = $x/\sqrt{2}$
We know that, area of the square = x^2
And, the area of the circle = πr^2

$$= \pi\left(\frac{x}{\sqrt{2}}\right)^2 = \frac{\pi * x^2}{2}$$

$$\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi}{2} = \pi : 2$$

Therefore, the ratio of areas of the circle and the square = $\pi : 2$

15. The area of circle inscribed in an equilateral triangle is 154 cm^2. Find the perimeter of the triangle.
Solution:



Let the circle inscribed in the equilateral triangle be with a centre O and radius r.
We know that, Area of a Circle = πr^2
But, the given that area is 154 cm^2.
$(22/7) \times r^2 = 154$
$r^2 = (154 \times 7)/22 = 7 \times 7 = 49$
r = 7 cm
From the figure seen above, we infer that
At point M, BC side is tangent and also at point M, BM is perpendicular to OM.
We know that,
In an equilateral triangle, the perpendicular from vertex divides the side into two halves.
BM = ½ x BC
Consider the side of the equilateral triangle be x cm.

$$BM = \frac{1}{2}x = \frac{x}{2}$$

$$OB^2 = BM^2 + MO^2$$

$$OB = \sqrt{r^2 + \frac{x^2}{4}} = \sqrt{49 + \frac{x^2}{4}}$$

$$BD = \frac{\sqrt{3}}{2}(\text{side}) = \frac{\sqrt{3}}{2}x = OB + OD$$

$$\frac{\sqrt{3}}{2}x - r = \sqrt{49 + \frac{x^2}{4}}, r = 7$$

After solving the above equation we get,

$x = 14\sqrt{3}$ cm

Perimeter $= 3x = 3 \times 14\sqrt{3} = 42\sqrt{3}$ cm

Therefore, the perimeter of the triangle is found to be $42\sqrt{3}$ cm = 42(1.73) = 72.7 cm

Exercise 15.2 Page No: 15.24

1. Find, in terms of π, the length of the arc that subtends an angle of 30° at the centre of a circle of radius of 4 cm.
Solution:

Given,
Radius = 4 cm
Angle subtended at the centre 'O' = 30°
We know that,
Length of arc = θ/360 × 2πr cm
Length of arc = 30/360 × 2π∗4 cm = 2π/3 cm
Thus, the length of arc that subtends an angle of 30° degrees is 2π/3 cm

2. Find the angle subtended at the centre of a circle of radius 5 cm by an arc of length 5π/3 cm.
Solution:

Given,
Radius = 5 cm
Length of arc = 5π/3 cm
We know that,
Length of arc = θ/360 ∗ 2πr cm
5π/3 cm = θ/360 ∗ 2πr cm
Solving the above, we get
θ = 60°
Thus, the angle subtended at the centre of circle is 60°

3. An arc of length 20π cm subtends an angle of 144° at the centre of a circle. Find the radius of the circle.
Solution:

Given,
Length of arc = 20π cm
And. θ = Angle subtended at the centre of circle = 144°
We know that,
Length of arc = θ/360 ∗ 2πr cm
θ/360 ∗ 2πr cm = 144/360 ∗ 2πr cm = 4π/5 ∗ r cm
From the question, we can equate
20π cm = 4π/5 ∗ r cm
r = 25 cm.
Thus, the radius of the circle is 25 cm.

4. An arc of length 15 cm subtends an angle of 45° at the centre of a circle. Find in terms of π, the radius of the circle.
Solution:

Given,
Length of arc = 15 cm
θ = Angle subtended at the centre of circle = 45°
We know that,
Length of arc = $\theta/360 * 2\pi r$ cm
= $45/360 * 2\pi r$ cm
From the question, we can equate
15 cm = $45/360 * 2\pi * r$ cm
$15 = \pi r/4$
Radius = $15*4/\pi = 60/\pi$
Therefore, the radius of the circle is $60/\pi$ cm.

5. Find the angle subtended at the centre of a circle of radius 'a' cm by an arc of length (aπ/4) cm.
Solution:

Given,
Radius = a cm
Length of arc = $a\pi/4$ cm
θ = angle subtended at the centre of circle
We know that,
Length of arc = $\theta/360 * 2\pi r$ cm
From the question, we can equate
$\theta/360 * 2\pi a$ cm = $a\pi/4$ cm
$\theta = 360/(2 \times 4)$
$\theta = 45°$
Hence, the angle subtended at the centre of circle is 45°

6. A sector of a circle of radius 4 cm subtends an angle of 30°. Find the area of the sector.
Solution:

Given,
Radius = 4 cm
Angle subtended at the centre O = 30°
We know that,
Area of the sector = $\theta/360 * \pi r^2$
= $30/360 * \pi 4^2 = 1/12 * \pi 16 = 4\pi/3$
Therefore, the area of the sector of the circle = $4\pi/3$ cm^2

7. A sector of a circle of radius 8 cm contains an angle of 135º. Find the area of sector.
Solution:

Given,
Radius = 8 cm
Angle subtended at the centre O = 135°
We know that,
Area of the sector = $\theta/360 * \pi r^2$

Area of the sector = $135/360 * \pi 8^2$
= 24π cm^2
Therefore, Area of the sector calculated is 24π cm^2

8. The area of a sector of a circle of radius 2 cm is π cm^2. Find the angle contained by the sector.
Solution:

Given,
Radius = 2 cm
Angle subtended at the centre 'O'
Area of sector of circle = π cm^2
We know that,
Area of the sector = $\theta/360 * \pi r^2$
= $\theta/360 * \pi 3^2$
= $\pi\theta/90$
From the question, we can equate
$\pi = \pi\,\theta/90$
On solving, we have
$\theta = 90°$
Hence, the angle subtended at the centre of circle is 90°

9. The area of a sector of a circle of radius 5 cm is 5π cm^2. Find the angle contained by the sector.
Solution:

Given,
Radius = 5 cm
Angle subtended at the centre 'O'
Area of sector of circle = 5π cm^2
We know that,
Area of the sector = $\theta/360 * \pi r^2$
= $\theta/360 * \pi 5^2$
= $5\pi\theta/72$
From the question, we can equate
$5\pi = 5\pi\theta/72$
On solving, we have
$\theta = 72°$
Hence, the angle subtended at the centre of circle is 72°

10. Find the area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.
Solution:

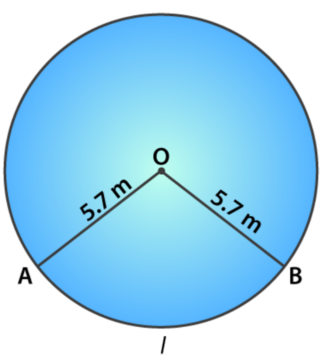
Given,
Radius = 5 cm
Length of arc = 3.5 cm
Let θ = angle subtended at the centre of circle
We know that,

Length of arc $= \theta/360 * 2\pi r$ cm
$= \theta/360 * 2\pi(5)$
From the question, we can equate
$3.5 = \theta/360 * 2\pi(5)$
$3.5 = 10\pi * \theta/360$
$\theta = 360 \times 3.5/ (10\pi)$
$\theta = 126/ \pi$
Now, the area of the sector $= \theta/360 * \pi r^2$
$= (126/ \pi)/ 360 * \pi(5)^2$
$= 126 \times 25 / 360 = 8.75$
Hence, the area of the sector = 8.75 cm^2

11. In a circle of radius 35 cm, an arc subtends an angle of 72° at the centre. Find the length of the arc and area of the sector.
Solution:

Given,
Radius = 35 cm
Angle subtended at the centre = 72°
We know that,
Length of arc $= \theta/360 * 2\pi r$ cm
$= 72/360 * 2\pi(35) = 14\pi = 14(22/7) = 44$ cm
Next,
Area of the sector $= \theta/360 * \pi r^2$
$= 72/360 * \pi\ 35^2$
$= (0.2) \times (22/7) \times 35 \times 35$
$= 0.2 \times 22 \times 5 \times 35$
Area of the sector $= (35 \times 22) = 770$ cm^2
Hence, the length of arc = 44cm and the area of the sector is 770 cm^2.

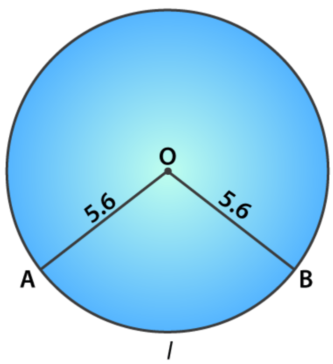
12. The perimeter of a sector of a circle of radius 5.7 m is 27.2 m. Find the area of the sector.
Solution:



Given,
Radius = 5.7 cm = OA = OB [from the figure shown above]
Perimeter of the sector = 27.2 m
Let the angle subtended at the centre be θ

We know that,
Length of arc = θ/360 ∗ 2πr m
Now, Perimeter of the sector = θ/360 ∗ 2πr + OA + OB
27.2 = θ/360 ∗ 2π x 5.7 cm + 5.7 + 5.7
27.2 – 11.4 = θ/360 ∗ 2π x 5.7
15.8 = θ/360 ∗ 2π x 5.7
θ = 158.8°
So, the area of the sector = θ/360 ∗ πr^2
Area of the sector = 158.8/360 ∗ π 5.7^2
Solving the above, we get
Area of the sector = 45.03 m^2

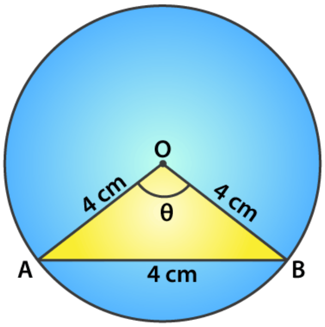
13. The perimeter of a certain sector of a circle of radius is 5.6 m and 27.2 m. Find the area of the sector.
Solution:



Given,
Radius of the circle = 5.6 m = OA = OB
Perimeter of the sector = (AB arc length) + OA + OB = 27.2
Let the angle subtended at the centre be θ
We know that,
Length of arc = θ/360 ∗ 2πr cm
θ/360 ∗ 2πr cm + OA + OB = 27.2 m
θ/360 ∗ 2πr cm + 5.6 + 5.6 = 27.2 m
Solving the above, we get
θ = 163.64°
Now, the Area of the sector = θ/360 ∗ πr^2
Area of the sector = 163.64/360 ∗ π 5.6^2 = 44.8
Therefore, the area of the sector = 44.8 m^2

Exercise 15.3 Page No: 15.32

1. AB is a chord of a circle with centre O and radius 4 cm. AB is of length 4 cm and divides the circle into two segments. Find the area of the minor segment.
Solution:



Given,
Radius of the circle with centre 'O', r = 4 cm = OA = OB
Length of the chord AB = 4 cm
So, OAB is an equilateral triangle and angle AOB = 60°
Thus, the angle subtended at centre θ = 60°
Area of the minor segment = (Area of sector) - (Area of triangle AOB)

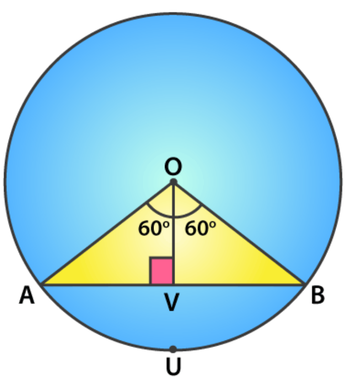
$$= \frac{\theta}{360} \times \pi r^2 - \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{60}{360} \times \pi 4^2 - \frac{\sqrt{3}}{4}(4)^2$$

$= (8\pi/3 - 4\sqrt{3}) = 8.37 - 6.92 = 1.45\ cm^2$

Therefore, the required area of the segment is $(8\pi/3 - 4\sqrt{3})\ cm^2$

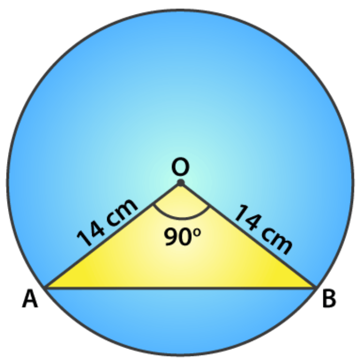
2. A chord PQ of length 12 cm subtends an angle 120° at the centre of a circle. Find the area of the minor segment cut off by the chord PQ.
Solution:



Given, ∠POQ = 120° and PQ = 12 cm
Draw OV ⊥ PQ,

PV = PQ × (0.5) = 12 × 0.5 = 6 cm
Since, ∠POV = 120°
∠POV = ∠QOV = 60°
In triangle OPQ, we have
sin θ = PV/ OA
sin 60° = 6/ OA
√3/2 = 6/ OA
OA = 12/ √3 = 4√3 = r
Now, using the above we shall find the area of the minor segment
We know that,
Area of the segment = area of sector OPUQO – area of △OPQ
= $\theta/360 \times \pi r^2 - ½ \times PQ \times OV$
= $120/360 \times \pi(4\sqrt{3})^2 - ½ \times 12 \times 2\sqrt{3}$
= $16\pi - 12\sqrt{3} = 4(4\pi - 3\sqrt{3})$
Therefore, the area of the minor segment = $4(4\pi - 3\sqrt{3})$ cm^2

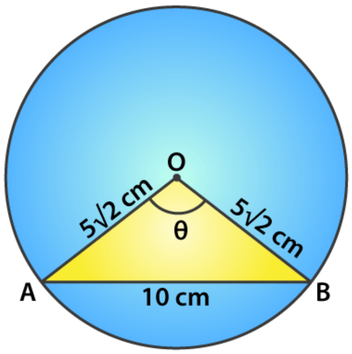
3. A chord of a circle of radius 14 cm makes a right angle at the centre. Find the areas of the minor and major segments of the circle.
Solution:



Given,
Radius (r) = 14 cm
Angle subtended by the chord with the centre of the circle, θ = 90°
Area of minor segment = $\theta/360 \times \pi r^2 - ½ \times r^2 \sin\theta$
= $90/360 \times \pi(14)^2 - ½ \times 14^2 \sin(90)$
= $¼ \times (22/7)(14)^2 – 7 \times 14$
= 56 cm^2
Area of circle = πr^2
= $22/7 \times (14)^2 = 616$ cm^2
Thus,
The area of the major segment = Area of circle – Area of the minor segment
= 616 – 56
= 560 cm^2

4. A chord 10 cm long is drawn in a circle whose radius is 5√2 cm. Find the area of both the segments.

Solution:



Given,
Radius of the circle, r = 5√2 cm = OA = OB
Length of the chord AB = 10 cm
In triangle OAB,

$$= OA^2 + OB^2$$

$$=(5\sqrt{2})^2 + (5\sqrt{2})^2$$

$$= 50 + 50$$

$$= 100$$

$$AB^2 = 100$$

We see that the Pythagoras theorem is satisfied.
So, OAB is a right angle triangle.
Angle subtended by the chord with the centre of the circle, θ = 90°
Area of minor segment = area of sector - area of triangle
= $\theta/360 \times \pi r^2$ - ½ x $r^2 \sin\theta$
= 90/360 × (3.14) $5\sqrt{2}^2$ - ½ x $(5\sqrt{2})^2$ sin 90
= [¼ x 3.14 x 25 x 2] – [½ x 25 x 2 x 1]
= 25(1.57 - 1)
= 14.25 cm^2

Area of circle = πr^2 = 3.14 x $(5\sqrt{2})^2$ = 3.14 x 50 = 14.25 cm^2

Thus, Area of major segment = Area of circle – Area of minor segment
= 157 – 14.25
= 142.75 cm^2

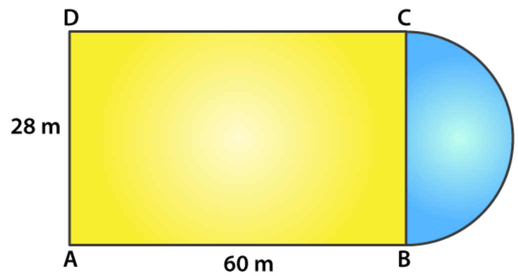
5. A chord AB of circle of radius 14 cm makes an angle of 60° at the centre of a circle. Find the area of the minor segment of the circle.
Solution:

Given,
Radius of the circle (r) = 14 cm = OA = OB
Angle subtended by the chord with the centre of the circle, $\theta = 60°$
In triangle AOB, angle A = angle B [angle opposite to equal sides OA and OB = x]
By angle sum property,
$\angle A + \angle B + \angle O = 180$
$x + x + 60° = 180°$
$2x = 120°, x = 60°$
All angles are 60° so the triangle OAB is an equilateral with OA = OB = AB
Area of the minor segment = area of sector - area of triangle OAB
$= \theta/360 \times \pi r^2 - \sqrt{3}/4\ (\text{side})^2$
$= 60/360 \times (22/7)\ 14^2 - \sqrt{3}/4\ (14)^2$
$= [1/6 \times 22 \times 2 \times 14] - \sqrt{3} \times (7)^2$
$= 102.7 – 84.9 = 17.8$
Therefore, the area of the minor segment = 17.80 cm^2

Exercise 15.4

1. A plot is in the form of a rectangle ABCD having semi-circle on BC as shown in Fig.15.64. If AB = 60m and BC = 28m, find the area of the plot.

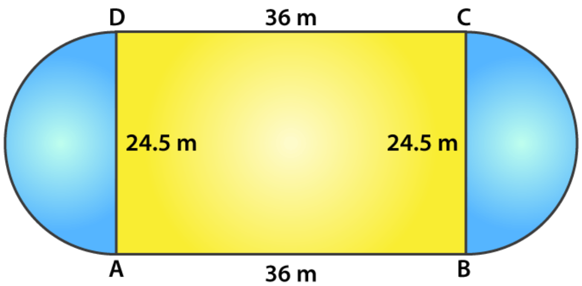
Solution:



Given, ABCD is a rectangle
So, AB = CD = 60 m
And, BC = AD = 28 m
For the radius of the semi-circle = BC/2 = 28/2 = 14 m
Now,
Area of the plot = Area of rectangle ABCD + Area of semi – circle
$= (l \times b) + \frac{1}{2}\pi r^2$
$= (60 \times 28) + \frac{1}{2}(22/7)(14)^2$
$= 1680 + 308$
$= 1988\ cm^2$

2. A play ground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of the rectangle are 36m and 24.5m, find the area of the playground.

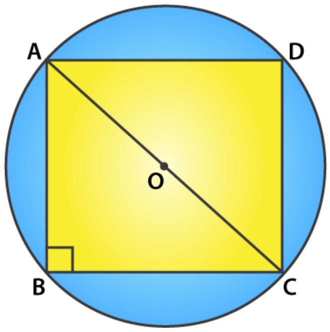
Solution:



Given,
Length of rectangle = 36 m
Breadth of rectangle = 24.5 m
Radius of the semi-circle = breath/2 = 24.5/2 = 12.25 m
Now,
Area of the playground = Area of the rectangle + 2 x area of semi-circles
$= l \times b + 2 \times \frac{1}{2}(\pi r^2)$
$= (36 \times 24.5) + (22/7) \times 12.25^2$
$= 882 + 471.625 = 1353.625$

Thus, the area of the playground is 1353.625 m^2

3. Find the area of the circle in which a square of area 64 cm^2 is inscribed.
Solution:

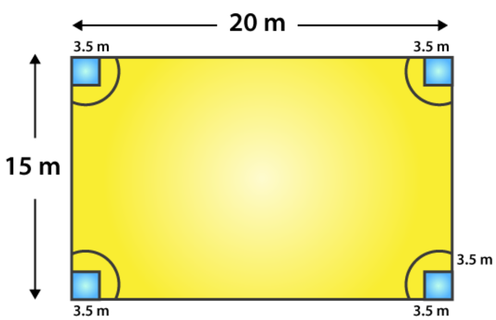


Given,
Area of square inscribed the circle = 64 cm^2
$Side^2 = 64$
Side = 8 cm
So, AB = BC = CD = DA = 8 cm

In triangle ABC, by Pythagoras theorem we have
$AC^2 = AB^2 + BC^2$
$AC^2 = 8^2 + 8^2$
$AC^2 = 64 + 64 = 128$
$AC = \sqrt{128} = 8\sqrt{2}$ cm

Now, as angle B = 90^o and AC being the diameter of the circle
The radius is $AC/2 = 8\sqrt{2}/2 = 4\sqrt{2}$ cm
Thus, the area of the circle = $\pi r^2 = 3.14(4\sqrt{2})^2$
= 100.48 cm^2

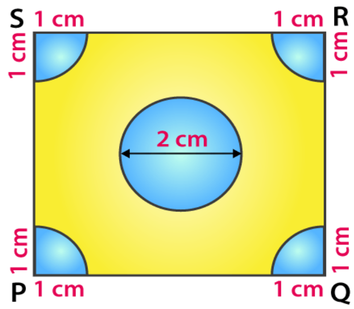
4. A rectangular piece is 20m long and 15m wide. From its four corners, quadrants of radii 3.5m have been cut. Find the area of the remaining part.
Solution:



Given,
Length of the rectangle = 20 m

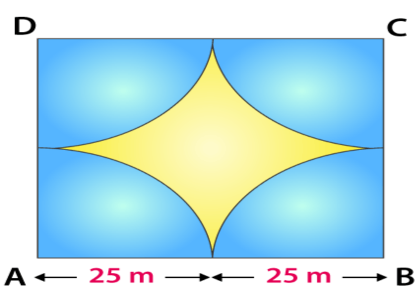
Breadth of the rectangle = 15 m
Radius of the quadrant = 3.5 m
So,
Area of the remaining part = Area of the rectangle – 4 x Area of one quadrant
$= (l \times b) - 4 \times (\frac{1}{4} \times \pi r^2)$
$= (l \times b) - \pi r^2$
$= (20 \times 15) - (22/7)(3.5)^2$
$= 300 - 38.5$
$= 261.5 \text{ m}^2$

5. In fig. 15.73, PQRS is a square of side 4 cm. Find the area of the shaded square.
Solution:



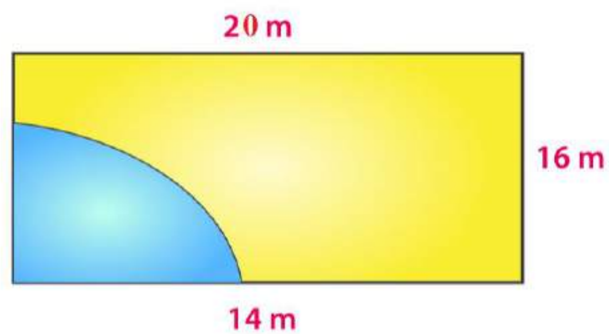
We know that, each quadrant is a sector of 90° in a circle of 1 cm radius. In other words its 1/4th of a circle.
So, its area = $\frac{1}{4} \times \pi r^2$
$= \frac{1}{4} \times (22/7)(1)^2 = 22/28 \text{ cm}^2$
And, the area of the square = side^2 [Given, side = 4 cm]
$= 4^2 = 16 \text{ cm}^2$
Area of the circle = $\pi r^2 = \pi(1)^2 = 22/7 \text{ cm}^2$ [Given, diameter = 2 cm, so radius = 1cm]
Thus,
The area of the shaded region = area of the square – area of the circle – 4 x area of a quadrant
$= 16 - 22/7 - (4 \times 22/28)$
$= 16 - 22/7 - 22/7 = 16 - 44/7$
$= 68/7 \text{ cm}^2$

6. Four cows are tethered at four corners of a square plot of side 50m, so that they just cannot reach one another. What area will be left un-grazed?
Solution:



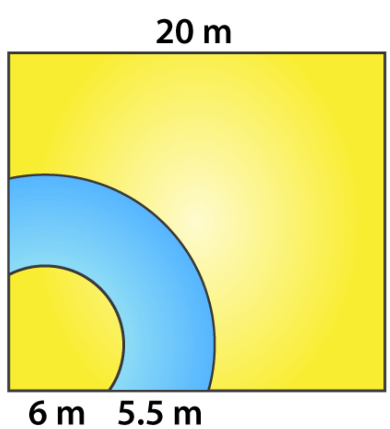
Given,
Side of square plot = 50 m
Radius of a quadrant = 25 m
So, we can tell
Area of plot left un-grazed = Area of the plot – 4 x (area of a quadrant)
= Side^2 – 4 x (¼ x πr^2)
= 50^2 - 22/7 x $(25)^2$
= 2500 – 1964.28
= 535.72 m^2

7. A cow is tied with a rope of length 14 m at the corner of a rectangle field of dimensions 20 m x 16 m, find the area of the field in which the cow can graze.
Solution:



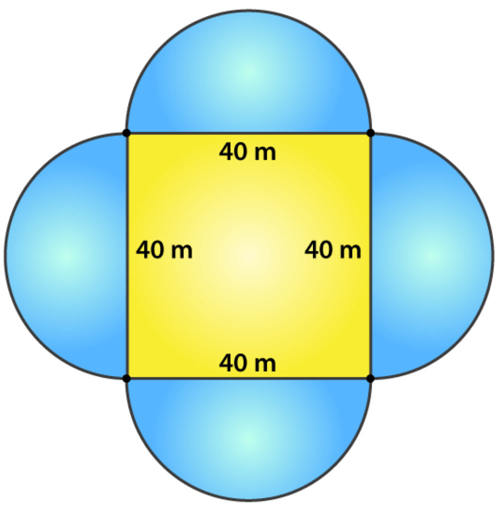
The dotted portion indicated the area over which the cow can graze.
It's clearly seen that, the shaded area is the area of a quadrant of a circle of radius equal to the length of the rope.
Thus, the required area = ¼ x πr^2
= ¼ x 22/7 x 14 x 14
= 154
Hence, the area of the field in which the cow can graze is 154 cm^2

8. A calf is tied with a rope of length 6 m at the corner of a square grassy lawn of side 20 m. If the length of the rope is increased by 5.5 m, find the increase in area of the grassy lawn in which the calf can graze.
Solution:



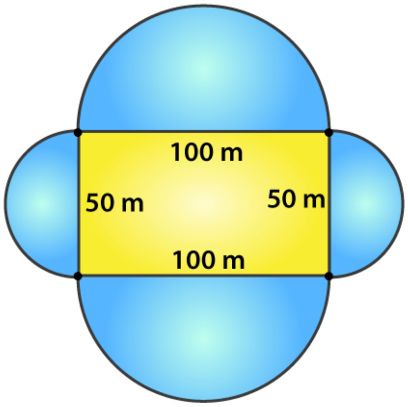
Given,
The initial length of the rope = 6 m
Then the rope is said to be increased by 5.5m
So, the increased length of the rope = (6 + 5.5) = 11.5 m
We know that, the corner of the lawn is a quadrant of a circle.
Thus,
The required area = ¼ x $\pi(11.5)^2$ - ¼ x $\pi(5.5)^2$
= ¼ x 22/7 (11.5^2 - 6^2)
= ¼ x 22/7 (132.25 - 36)
= ¼ x 22/7 x 96.25
= 75.625 cm^2

9. A square tank has its side equal to 40 m. There are four semi-circular grassy plots all around it. Find the cost of turfing the plot at Rs 1.25 per square meter.
Solution:



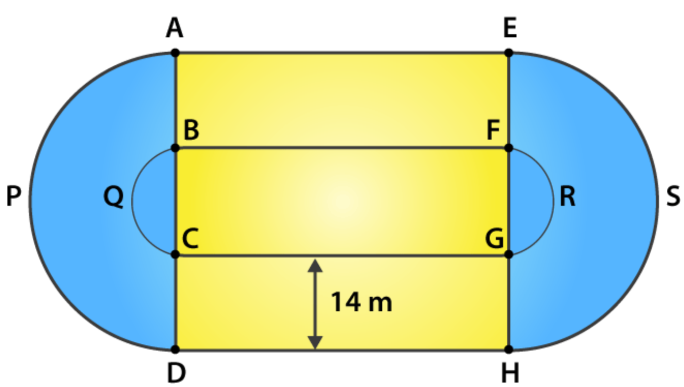
Given,
Side of the square tank = 40 m
And, the diameter of the semi-circular grassy plot = side of the square tank = 40 m
Radius of the grassy plot = 40/2 = 20 m
Then,
The area of the four semi-circular grassy plots = 4 x ½ πr^2
= 4 x ½ $(3.14)(20)^2$
= 2512 m^2
Rate of turfing the plot = Rs 1.25 per m^2
So, the cost for 2512 m^2 = (1.25 x 2512) = Rs 3140

10. A rectangular park is 100 m by 50 m. It is surrounded by semi-circular flower beds all round. Find the cost of levelling the semi-circular flower beds at 60 paise per square meter.
Solution:



Given,
Length of the park = 100 m and the breadth of the park = 50 m
The radius of the semi-circular flower beds = half of the corresponding side of the rectangular park
Radius of the bigger flower bed = 100/2 = 50 m
And the radius of the smaller flower bed = 50/2 = 25 m
Total area of the flower beds = 2[Area of bigger flower bed + Area of smaller flower bed]
$= 2[\frac{1}{2}\pi(50)^2 + \frac{1}{2}\pi(25)^2]$
$= \pi[(50)^2 + (25)^2]$
$= 3.14 \times [2500 + 625]$
$= 9812.5\ m^2$
Now, rate of levelling flower bed = 60 paise per m^2
Therefore,
The total cost of levelling = 9812.5 x 60 = 588750 paise
= Rs 5887.50

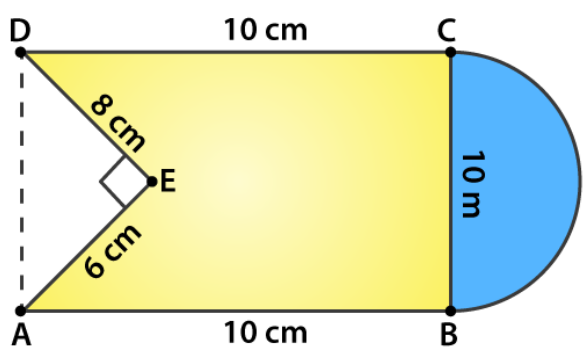
11. The inner perimeter of a running track (show in Fig.) is 400 m. The length of each of the straight portion is 90 m and the ends are semi-circles. If the track is everywhere 14 m wide, find the area of the track. Also, find the length of the outer running track.
Solution:



Let the radius of the inner semi-circle = r
And that of the outer semi-circle = R

Given,
Length of the straight portion = 90 m
Width of the track = 14 m
The inner perimeter of the track = 400 m
But,
Inner perimeter of the track = BF + FRG + GC + CQB = 400
$90 + \pi r + 90 + \pi r = 400$
$2\pi r + 180 = 400$
2 x 22/7 x r = 220
r = 35 m
So, the radius of the outer semi-circle = 35 + 14 = 49 m
Now,
Area of the track = 2[Area of the rectangle AEFB + Area of semi-circle APD – Area of semi-circle BQC]
= 2[90 x 14 + ½ x 22/7 x 49^2 – ½ x 22/7 x 35^2]
= 2[1260 + 11 x 7 x 49 – 11 x 5 x 35]
= 2 [1260 + 3773 - 1925]
= 3 x 3108
= 6216 m^2
Thus,
The length of outer running track = AE + APD + DH + HSE
= $90 + \pi R + 90 + \pi R$
= $180 + 2\pi R$
= 180 + 2 x 22/7 x 49
= 180 + 308
= 488 m

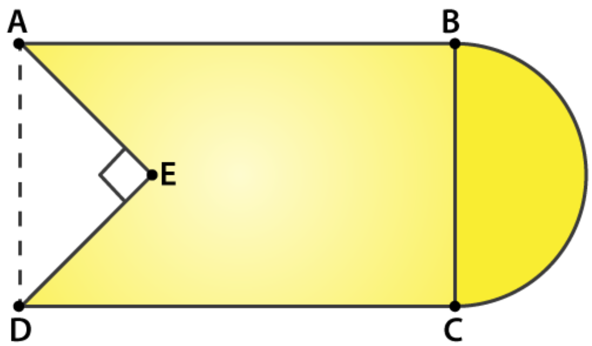
12. Find the area of Fig. 15.76 in square cm, correct to one place of decimal.
Solution:



The radius of the semi-circle = 10/2 = 5 cm
It's seen that,
The area of figure = Area of square + Area of semi-circle – Area of triangle AED
= 10 x 10 + ½ πr^2 – ½ x 6 x 8
= 100 + ½ $(22/7)(5)^2$ - 24

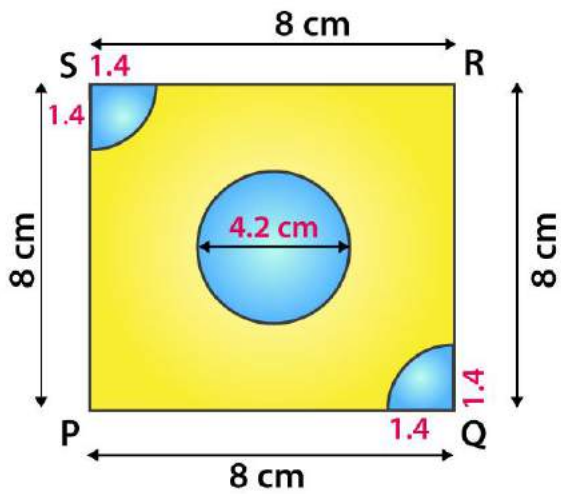
$= (700 + 275 – 168)/7$
$= (807)/7$
$= 115.3\ cm^2$

13. From a rectangular region ABCD with AB = 20 cm, a right angle AED with AE = 9 cm and DE = 12 cm, is cut off. On the other end, taking BC as diameter, a semicircle is added on outside the region. Find the area of the shaded region. (π = 22/7)
Solution:



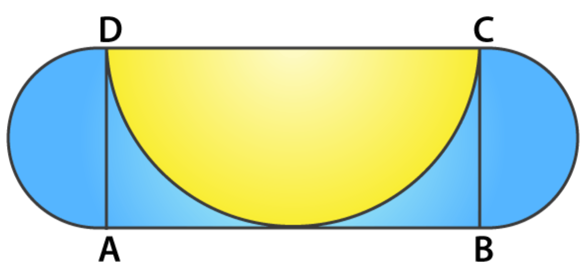
Given,
Length of the rectangle ABCD = 20 cm
AE = 9 cm and DE = 12 cm
The radius of the semi-circle = BC/ 2 or AD/2
Now, using Pythagoras theorem in triangle AED
$AD = \sqrt{(AE^2 + ED^2)} = \sqrt{(9^2 + 12^2)}$
$= \sqrt{(81 + 144)}$
$= \sqrt{(225)} = 15$ cm
So, the area of the rectangle = 20 x 15 = 300 cm^2
And, the area of the triangle AED = ½ x 12 x 9 = 54 cm^2
The radius of the semi-circle = 15/2 = 7.5 cm
Area of semi-circle = $\frac{1}{2}\pi(15/2)^2 = \frac{1}{2} \times 3.14 \times 7.5^2 = 88.31\ cm^2$
Thus,
The area of the shaded region = Area of the rectangle ABCD + Area of semi-circle – Area of triangle AED
$= 300 + 88.31 – 54$
$= 334.31\ cm^2$

14. From each of the two opposite corners of a square of side 8 cm, a quadrant of a circle of radius 1.4 cm is cut. Another circle of radius 4.2 cm is also cut from the center as shown in Fig. Find the area of the remaining (shaded) portion of the square. (Use π = 22/7)
Solution:



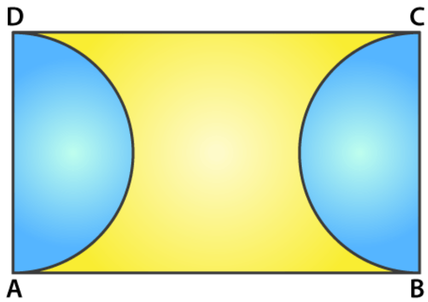
Given,
Side of the square = 8 cm
Radius of circle = 4.2 cm
Radius of the quadrant = 1.4 cm
Thus,
Area of the shaded potion = Area of square – Area of circle – 2 x Area of the quadrant
$= \text{side}^2 – \pi r^2 – 2 \times ½\, \pi r^2$
$= 8^2 – \pi(4.2)^2 – 2 \times ½\, \pi(1.4)^2$
$= 64 – 22/7(4.2 \times 4.2) – 22/7(1.4 \times 1.4)$
$= 64 – 388.08/7 – 21.56/7$
$= 5.48 \text{ cm}^2$

15. ABCD is a rectangle with AB = 14 cm and BC = 7 cm. Taking DC, BC and AD as diameters, three semi-circles are drawn as shown in the figure. Find the area of the shaded region.
Solution:



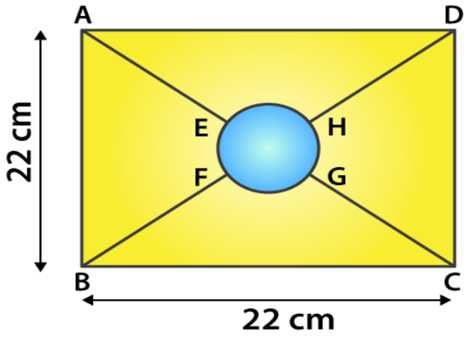
Given,
ABCD is a rectangle with AB = 14 cm and BC = 7 cm
It's seen that,
The area of shaded region = Area of rectangle ABCD + 2 x area of semi-circle with AD and BC as diameters – area of the semi-circle with DC as diameter
$= 14 \times 7 + 2 \times ½\, \pi(7/2)^2 – ½\, \pi(7)^2$
$= 98 + 22/7 \times (7/2)^2 – ½\, (22/7)(7)^2$
$= 98 + 77/2 – 77$
$= 59.5 \text{ cm}^2$

16. ABCD is rectangle, having AB = 20 cm and BC = 14 cm. Two sectors of 180° have been cut off. Calculate:
(i) the area of the shaded region.
(ii) the length of the boundary of the shaded region.
Solution:



Given,
Length of the rectangle = AB = 20 cm
Breadth of the rectangle = BC = 14 cm
(i) Area of the shaded region = Area of rectangle – 2 x Area of the semi-circle
= l x b – 2 x ½ πr^2
= 20 x 14 – (22/7) x 7^2
= 280 – 154
= 126 cm^2
(ii) Length of the boundary of the shaded region = 2 x AB + 2 x circumference of a semi-circle
= 2 x 20 + 2 x πr
= 40 + 2 x (22/7) x 7
= 40 + 44
= 84 cm

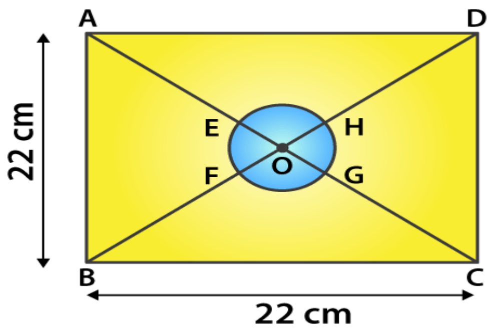
17. The square ABCD is divided into five equal parts, all having same area. The central part is circular and the lines AE, GC, BF and HD lie along the diagonals AC and BD of the square. If AB = 22 cm, find:
(i) the circumference of the central part. (ii) the perimeter of the part ABEF.
Solution:



Given,
Side of the square = 22 cm = AB
Let the radius of the centre part be r cm.

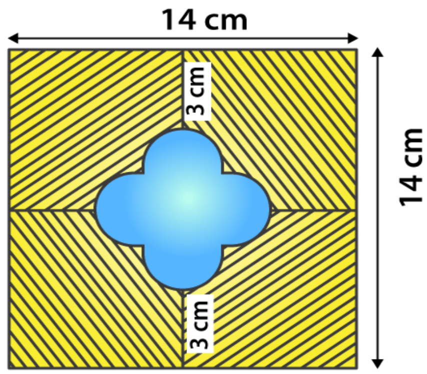
Then, area of the circle = 1/5 x area of the square
$\pi r^2 = 1/5 \times 22^2$
$22/7 \times r^2 = (22 \times 22)/ 5$
r = 154/5 = 5.55 cm
(i) Circumference of central part = $2\pi r$ = 2(22/7)(5.55) = 34.88 cm

(ii) Let O be the center of the central part. Then, its clear that O is also the center of the square as well.
Now, in triangle ABC
By Pythagoras theorem
$AC^2 = AB^2 + BC^2 = 22^2 + 22^2 = 2 \times 22^2$
$AC = 22\sqrt{2}$
Since diagonals of a square bisect each other
$AO = ½ AC = ½ (22\sqrt{2}) = 11\sqrt{2}$ cm



And,
$AE = BF = OA – OE = 11\sqrt{2} – 5.55 = 15.51 – 5.55 = 9.96$ cm
EF = ¼(Circumference of the circle) = $2\pi r/4$
= ½ x 22/7 x 5.55 = 8.72 cm
Thus, the perimeter of the part ABEF = AB + AE + EF + BF
= 22 + 2 x 9.96 + 8.72
= 50.64 cm

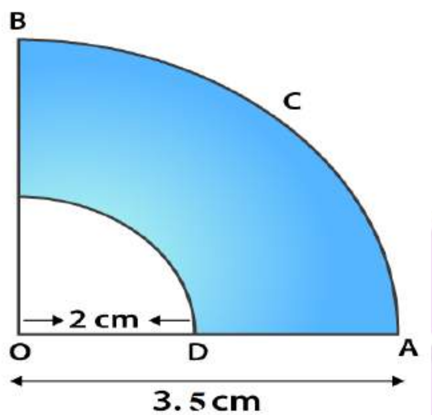
18. In figure, find the area of the shaded region. (Use π = 3.14)
Solution:



The side of the square = 14 cm
So, it's area = $14^2 = 196$ cm^2

Let's assume the radius of each semi-circle be r cm.
Then,
r + 2r + r = 14 – 3 – 3
4r = 8
r = 2
The radius of each semi-circle is 2 cm.
Area of 4 semi-circles = (4 x ½ x 3.14 x 2 x 2) = 25.12 cm^2
Now, length of side of the smaller square = 2r = 2 x 2 = 4 cm
Thus, the area of smaller square = 4x4 = 16 cm^2
Area of unshaded region = Area of 4 semi-circles + Area of smaller square
= (25.12 + 16) = 41.12 cm^2
Therefore,
The area of shaded region = Area of square ABCD – Area of unshaded region
= (196 – 41.12) = 154.88 cm^2

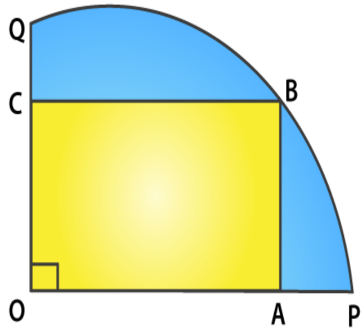
19. OACB is a quadrant of a circle with center O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB (ii) shaded region.
Solution:



Given,
Radius of small quadrant, r = 2 cm
Radius of big quadrant, R = 3.5 cm
(i) Area of quadrant OACB = ¼ πR^2
= ¼ $(22/7)(3.5)^2$
= 269.5/28 = 9.625 cm^2

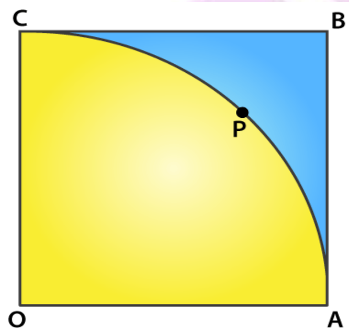
(ii) Area of shaded region = Area of big quadrant – Area of small quadrant
= ¼ $\pi(R^2 - r^2)$
= ¼ $(22/7)(3.5^2 - 2^2)$
= ¼ (22/7)(12.25 - 4)
= ¼ (22/7)(8.25)
= 6.482 cm^2

20. A square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 21 cm, find the area of the shaded region.
Solution:



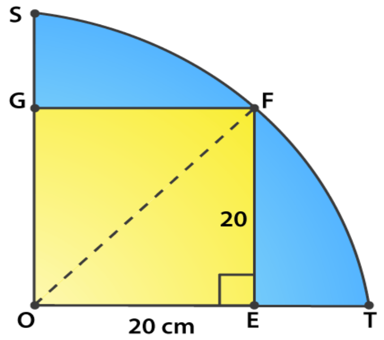
Given,
Side of the square = 21 cm = OA
Area of the square = $OA^2 = 21^2 = 441\ cm^2$
Diagonal of the square OB = $\sqrt{2}$ OA = $21\sqrt{2}$ cm
And, from the fig. its seen that
The diagonal of the square is equal to the radius of the circle, r = $21\sqrt{2}$ cm
So, the area of the quadrant = $\frac{1}{4}\pi r^2 = \frac{1}{4}(22/7)(21\sqrt{2})^2 = 693\ cm^2$
Thus,
The area of the shaded region = Area of the quadrant – Area of the square
= 693 – 441
= $252\ cm^2$

21. OABC is a square of side 7 cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region.
Solution:



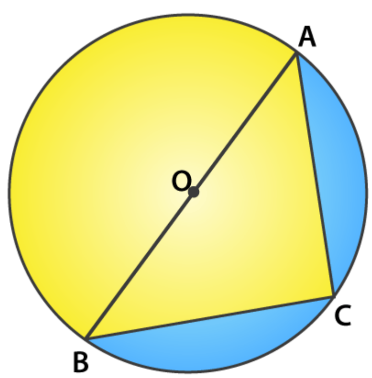
Given,
OABC is a square of side 7 cm
So, OA = AB = BC = OC = 7 cm
Area of square OABC = $side^2 = 7^2 = 49\ cm^2$
And given, OAPC is a quadrant of a circle with centre O.
So, the radius of the quadrant = OA = OC = 7 cm
Area of the quadrant OAPC = $90/360 \times \pi r^2$
= $\frac{1}{4} \times (22/7) \times 7^2$
= $77/2 = 38.5\ cm^2$
Thus,
Area of shaded portion = Area of square OABC – Area of quadrant OAPC
= $(49 – 38.5) = 10.5\ cm^2$

22. OE = 20 cm. In sector OSFT, square OEFG is inscribed. Find the area of the shaded region.
Solution:



It's seen that, OEFG is a square of side 20 cm.
So its diagonal = $\sqrt{2}$ side = $20\sqrt{2}$ cm
And, the radius of the quadrant = diagonal of the square
Radius of the quadrant = $20\sqrt{2}$ cm
So,
Area of the shaded portion = Area of quadrant – Area of square
$= \frac{1}{4} \pi r^2 - \text{side}^2$
$= \frac{1}{4} (22/7)(20\sqrt{2})^2 - (20)^2$
$= \frac{1}{4} (22/7)(800) - 400$
$= 400 \times 4/7 = 1600/7 = 228.5 \text{ cm}^2$

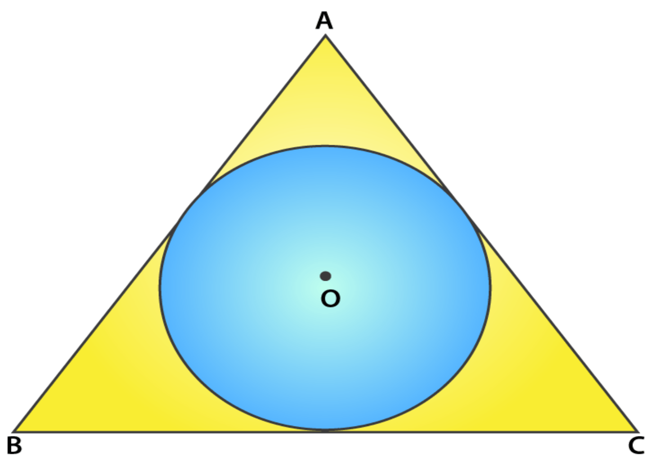
23. Find the area of the shaded region in Fig., if AC = 24 cm, BC = 10 cm and O is the center of the circle
Solution:



Given,
AC = 24 cm and BC = 10 cm
Since, AB is the diameter of the circle
Angle ACB = 90^o
So, using Pythagoras theorem
$AB^2 = AC^2 + BC^2 = 24^2 + 10^2 = 576 + 100 = 676$
AB = $\sqrt{676}$ = 26 cm
Thus, the radius of the circle = 26/2 = 13 cm

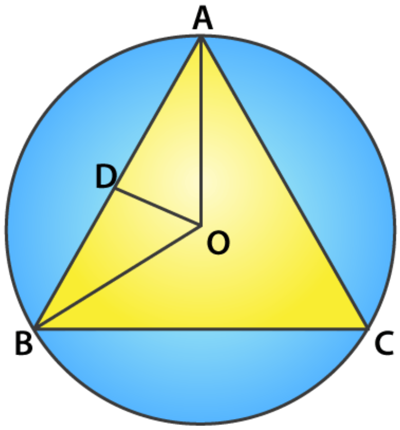
The area of shaded region = Area of semi-circle – Area of triangle ACB
$= ½ \pi r^2 – ½ \times b \times h$
$= ½ (22/7)13^2 – ½ \times 10 \times 24$
$= 265.33 – 120$
$= 145.33 \text{ cm}^2$

24. A circle is inscribed in an equilateral triangle ABC of side 12 cm, touching its sides (fig.,). Find the radius of the inscribed circle and the area of the shaded part.
Solution:



Given,
An equilateral triangle of side = 12 cm
Area of the equilateral triangle = $\sqrt{3}/4(\text{side})^2$
$= \sqrt{3}/4(12)^2 = 36\sqrt{3} \text{ cm}^2$
Perimeter of triangle ABC = 3 x 12 = 36 cm
So,
The radius of incircle = Area of triangle/ ½ (perimeter of triangle)
$= 36\sqrt{3}/ ½ \times 36$
$= 2\sqrt{3}$ cm
Therefore,
Area of the shaded part = Area of equilateral triangle – Area of circle
$= 36\sqrt{3} - \pi r^2$
$= 36(1.732) – (3.14)(2\sqrt{3})^2$
$= 62.352 – 37.68$
$= 24.672 \text{ cm}^2$

25. In fig., an equilateral triangle ABC of side 6 cm has been inscribed in a circle. Find the area of the shaded region. (Take π = 3.14).
Solution:



Given,
Side of the equilateral triangle = 6 cm
And,
The area of the equilateral triangle = $\sqrt{3}/4(\text{side})^2$
$= \sqrt{3}/4(6)^2$
$= \sqrt{3}/4(36)$
$= 9\sqrt{3}\ cm^2$
Let us mark the center of the circle as O, OA and OB are the radii of the circle.
In triangle BOD,
$\sin 60^o = BD/ OB$
$\sqrt{3}/2 = 3/ OB$
$OB = 2\sqrt{3}\ cm = r$

Therefore,
The area of shaded region = Area of the circle – area of the equilateral triangle
$= \pi r^2 - 9\sqrt{3}$
$= 3.14 \times (2\sqrt{3})^2 - 9\sqrt{3}$
$= 3.14 \times 12 – 9 \times 1.732$
$= 37.68 – 15.588$
$= 22.092\ cm^2$