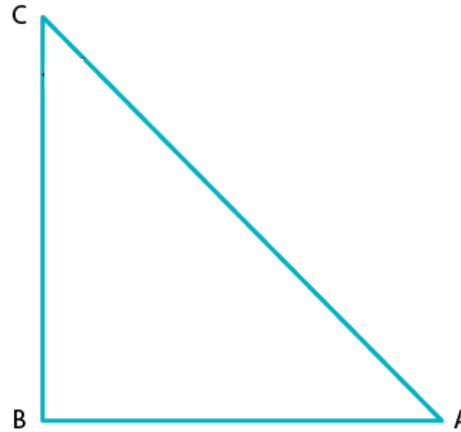


Exercise 5.1

1. In each of the following, one of the six trigonometric ratios is given. Find the values of the other trigonometric ratios.



(i) $\sin A = 2/3$

Solution:

We have,

$$\sin A = 2/3 \dots\dots\dots (1)$$

As we know, by sin definition;

$$\sin A = \text{Perpendicular/ Hypotenuse} = 2/3 \dots\dots(2)$$

By comparing eq. (1) and (2), we have

Opposite side = 2 and Hypotenuse = 3

Now, on using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the values of perpendicular side (BC) and hypotenuse (AC) and for the base side as (AB), we get

$$\Rightarrow 3^2 = AB^2 + 2^2$$

$$AB^2 = 3^2 - 2^2$$

$$AB^2 = 9 - 4$$

$$AB^2 = 5$$

$$AB = \sqrt{5}$$

Hence, Base = $\sqrt{5}$

By definition,

$$\cos A = \text{Base/Hypotenuse}$$

$$\Rightarrow \cos A = \sqrt{5}/3$$

Since, cosec A = 1/sin A = Hypotenuse/Perpendicular

$$\Rightarrow \text{cosec A} = 3/2$$

And, sec A = Hypotenuse/Base

$$\Rightarrow \sec A = 3/\sqrt{5}$$

And, tan A = Perpendicular/Base

$$\Rightarrow \tan A = 2/\sqrt{5}$$

And, $\cot A = 1/\tan A = \text{Base/Perpendicular}$
 $\Rightarrow \cot A = \sqrt{5}/2$

(ii) $\cos A = 4/5$

Solution:

We have,
 $\cos A = 4/5$ (1)
 As we know, by cos definition
 $\cos A = \text{Base/Hypotenuse}$ (2)
 By comparing eq. (1) and (2), we get
 Base = 4 and Hypotenuse = 5

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and hypotenuse (AC) and for the perpendicular (BC), we get

$$5^2 = 4^2 + BC^2$$

$$BC^2 = 5^2 - 4^2$$

$$BC^2 = 25 - 16$$

$$BC^2 = 9$$

$$BC = 3$$

Hence, Perpendicular = 3

By definition,

$$\sin A = \text{Perpendicular/Hypotenuse}$$

$$\Rightarrow \sin A = 3/5$$

Then, $\operatorname{cosec} A = 1/\sin A$

$$\Rightarrow \operatorname{cosec} A = 1/(3/5) = 5/3 = \text{Hypotenuse/Perpendicular}$$

And, $\sec A = 1/\cos A$

$$\Rightarrow \sec A = \text{Hypotenuse/Base}$$

$$\sec A = 5/4$$

And, $\tan A = \text{Perpendicular/Base}$

$$\Rightarrow \tan A = 3/4$$

Next, $\cot A = 1/\tan A = \text{Base/Perpendicular}$

$$\therefore \cot A = 4/3$$

(iii) $\tan \theta = 11/1$

Solution:

We have, $\tan \theta = 11$ (1)

By definition,

$$\tan \theta = \text{Perpendicular/ Base} \dots (2)$$

On Comparing eq. (1) and (2), we get;

Base = 1 and Perpendicular = 11

Now, using Pythagoras theorem in ΔABC .

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and perpendicular (BC) to get hypotenuse(AC), we get;

$$AC^2 = 1^2 + 11^2$$

$$AC^2 = 1 + 121$$

$$AC^2 = 122$$

$$AC = \sqrt{122}$$

Hence, hypotenuse = $\sqrt{122}$

By definition,

$$\sin \theta = \text{Perpendicular/Hypotenuse}$$

$$\Rightarrow \sin \theta = 11/\sqrt{122}$$

And, $\operatorname{cosec} \theta = 1/\sin \theta$

$$\Rightarrow \operatorname{cosec} \theta = \sqrt{122}/11$$

Next, $\cos \theta = \text{Base/ Hypotenuse}$

$$\Rightarrow \cos \theta = 1/\sqrt{122}$$

And, $\sec \theta = 1/\cos \theta$

$$\Rightarrow \sec \theta = \sqrt{122}/1 = \sqrt{122}$$

And, $\cot \theta = 1/\tan \theta$

$$\therefore \cot \theta = 1/11$$

(iv) $\sin \theta = 11/15$

Solution:

We have, $\sin \theta = 11/15$ (1)

By definition,

$$\sin \theta = \text{Perpendicular/ Hypotenuse} \dots (2)$$

On Comparing eq. (1) and (2), we get;

Perpendicular = 11 and Hypotenuse= 15

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base (AB), we have

$$15^2 = AB^2 + 11^2$$

$$AB^2 = 15^2 - 11^2$$

$$AB^2 = 225 - 121$$

$$AB^2 = 104$$

$$AB = \sqrt{104}$$

$$AB = \sqrt{(2 \times 2 \times 2 \times 13)}$$

$$AB = 2\sqrt{(2 \times 13)}$$

$$AB = 2\sqrt{26}$$

Hence, Base = $2\sqrt{26}$

By definition,

$$\cos \theta = \text{Base/Hypotenuse}$$

$$\therefore \cos \theta = 2\sqrt{26}/ 15$$

- And, $\operatorname{cosec} \theta = 1/\sin \theta$
 $\therefore \operatorname{cosec} \theta = 15/11$
- And, $\sec \theta = \text{Hypotenuse}/\text{Base}$
 $\therefore \sec \theta = 15/2\sqrt{26}$
- And, $\tan \theta = \text{Perpendicular}/\text{Base}$
 $\therefore \tan \theta = 11/2\sqrt{26}$
- And, $\cot \theta = \text{Base}/\text{Perpendicular}$
 $\therefore \cot \theta = 2\sqrt{26}/11$

(v) $\tan \alpha = 5/12$

Solution:

- We have, $\tan \alpha = 5/12 \dots (1)$
 By definition,
 $\tan \alpha = \text{Perpendicular}/\text{Base} \dots (2)$
 On Comparing eq. (1) and (2), we get
 Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in ΔABC
 $AC^2 = AB^2 + BC^2$

Putting the value of base (AB) and the perpendicular (BC) to get hypotenuse (AC), we have

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

$$AC = 13 \quad [\text{After taking sq root on both sides}]$$

Hence, Hypotenuse = 13

- By definition,
 $\sin \alpha = \text{Perpendicular}/\text{Hypotenuse}$
 $\therefore \sin \alpha = 5/13$
- And, $\operatorname{cosec} \alpha = \text{Hypotenuse}/\text{Perpendicular}$
 $\therefore \operatorname{cosec} \alpha = 13/5$
- And, $\cos \alpha = \text{Base}/\text{Hypotenuse}$
 $\therefore \cos \alpha = 12/13$
- And, $\sec \alpha = 1/\cos \alpha$
 $\therefore \sec \alpha = 13/12$
- And, $\tan \alpha = \sin \alpha / \cos \alpha$
 $\therefore \tan \alpha = 5/12$
- Since, $\cot \alpha = 1/\tan \alpha$
 $\therefore \cot \alpha = 12/5$

(vi) $\sin \theta = \sqrt{3}/2$

Solution:

We have, $\sin \theta = \sqrt{3}/2 \dots \dots \dots (1)$

By definition,

$$\sin \theta = \text{Perpendicular} / \text{Hypotenuse} \dots (2)$$

On Comparing eq. (1) and (2), we get;

$$\text{Perpendicular} = \sqrt{3} \text{ and Hypotenuse} = 2$$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of perpendicular (BC) and hypotenuse (AC) and get the base (AB), we get;

$$2^2 = AB^2 + (\sqrt{3})^2$$

$$AB^2 = 2^2 - (\sqrt{3})^2$$

$$AB^2 = 4 - 3$$

$$AB^2 = 1$$

$$AB = 1$$

Thus, Base = 1

By definition,

$$\cos \theta = \text{Base} / \text{Hypotenuse}$$

$$\therefore \cos \theta = 1/2$$

And, $\operatorname{cosec} \theta = 1/\sin \theta$

Or $\operatorname{cosec} \theta = \text{Hypotenuse} / \text{Perpendicular}$

$$\therefore \operatorname{cosec} \theta = 2/\sqrt{3}$$

And, $\sec \theta = \text{Hypotenuse} / \text{Base}$

$$\therefore \sec \theta = 2/1$$

And, $\tan \theta = \text{Perpendicular} / \text{Base}$

$$\therefore \tan \theta = \sqrt{3}/1$$

And, $\cot \theta = \text{Base} / \text{Perpendicular}$

$$\therefore \cot \theta = 1/\sqrt{3}$$

(vii) $\cos \theta = 7/25$

Solution:

We have, $\cos \theta = 7/25 \dots \dots \dots (1)$

By definition,

$$\cos \theta = \text{Base} / \text{Hypotenuse}$$

On Comparing eq. (1) and (2), we get;

$$\text{Base} = 7 \text{ and Hypotenuse} = 25$$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC)

$$25^2 = 7^2 + BC^2$$

$$BC^2 = 25^2 - 7^2$$

$$BC^2 = 625 - 49$$

$$BC^2 = 576$$

$$BC = \sqrt{576}$$

$$BC = 24$$

Hence, Perpendicular side = 24

By definition,

$$\sin \theta = \text{perpendicular/Hypotenuse}$$

$$\therefore \sin \theta = 24/25$$

Since, $\operatorname{cosec} \theta = 1/\sin \theta$

Also, $\operatorname{cosec} \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \operatorname{cosec} \theta = 25/24$$

Since, $\sec \theta = 1/\operatorname{cosec} \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = 25/7$$

Since, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 24/7$$

Now, $\cot = 1/\tan \theta$

So, $\cot \theta = \text{Base/Perpendicular}$

$$\therefore \cot \theta = 7/24$$

(viii) $\tan \theta = 8/15$

Solution:

We have, $\tan \theta = 8/15$ (1)

By definition,

$\tan \theta = \text{Perpendicular/Base}$ (2)

On Comparing eq. (1) and (2), we get;

Base = 15 and Perpendicular = 8

Now, using Pythagoras theorem in ΔABC

$$AC^2 = 15^2 + 8^2$$

$$AC^2 = 225 + 64$$

$$AC^2 = 289$$

$$AC = \sqrt{289}$$

$$AC = 17$$

Hence, Hypotenuse = 17

By definition,

Since, $\sin \theta = \text{perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 8/17$$

Since, $\operatorname{cosec} \theta = 1/\sin \theta$

Also, $\operatorname{cosec} \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \operatorname{cosec} \theta = 17/8$$

Since, $\cos \theta = \text{Base/Hypotenuse}$

$$\therefore \cos \theta = 15/17$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = 17/15$$

Since, $\cot \theta = 1/\tan \theta$

Also, $\cot \theta = \text{Base/Perpendicular}$
 $\therefore \cot \theta = 15/8$

(ix) $\cot \theta = 12/5$

Solution:

We have, $\cot \theta = 12/5$ (1)

By definition,

$$\cot \theta = 1/\tan \theta$$

$$\cot \theta = \text{Base/Perpendicular} \dots\dots (2)$$

On Comparing eq. (1) and (2), we have

Base = 12 and Perpendicular side = 5

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and perpendicular (BC) to get the hypotenuse (AC);

$$AC^2 = 12^2 + 5^2$$

$$AC^2 = 144 + 25$$

$$AC^2 = 169$$

$$AC = \sqrt{169}$$

$$AC = 13$$

Hence, Hypotenuse = 13

By definition,

Since, $\sin \theta = \text{perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 5/13$$

Since, $\text{cosec } \theta = 1/\sin \theta$

Also, $\text{cosec } \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \text{cosec } \theta = 13/5$$

Since, $\cos \theta = \text{Base/Hypotenuse}$

$$\therefore \cos \theta = 12/13$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = 13/12$$

Since, $\tan \theta = 1/\cot \theta$

Also, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 5/12$$

(x) $\sec \theta = 13/5$

Solution:

We have, $\sec \theta = 13/5$ (1)

By definition,

$$\sec \theta = \text{Hypotenuse/Base} \dots\dots\dots (2)$$

On Comparing eq. (1) and (2), we get

Base = 5 and Hypotenuse = 13

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

And, putting the value of base side (AB) and hypotenuse (AC) to get the perpendicular side (BC)

$$13^2 = 5^2 + BC^2$$

$$BC^2 = 13^2 - 5^2$$

$$BC^2 = 169 - 25$$

$$BC^2 = 144$$

$$BC = \sqrt{144}$$

$$BC = 12$$

Hence, Perpendicular = 12

By definition,

Since, $\sin \theta = \text{perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 12/13$$

Since, $\text{cosec } \theta = 1/\sin \theta$

Also, $\text{cosec } \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \text{cosec } \theta = 13/12$$

Since, $\cos \theta = 1/\sec \theta$

Also, $\cos \theta = \text{Base/Hypotenuse}$

$$\therefore \cos \theta = 5/13$$

Since, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 12/5$$

Since, $\cot \theta = 1/\tan \theta$

Also, $\cot \theta = \text{Base/Perpendicular}$

$$\therefore \cot \theta = 5/12$$

(xi) $\text{cosec } \theta = \sqrt{10}$

Solution:

We have, $\text{cosec } \theta = \sqrt{10}/1 \dots\dots\dots (1)$

By definition,

$\text{cosec } \theta = \text{Hypotenuse/ Perpendicular} \dots\dots\dots(2)$

And, $\text{cosec } \theta = 1/\sin \theta$

On comparing eq.(1) and(2), we get

Perpendicular side = 1 and Hypotenuse = $\sqrt{10}$

Now, using Pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

Putting the value of perpendicular (BC) and hypotenuse (AC) to get the base side (AB)

$$(\sqrt{10})^2 = AB^2 + 1^2$$

$$AB^2 = (\sqrt{10})^2 - 1^2$$

$$AB^2 = 10 - 1$$

$$AB = \sqrt{9}$$

$$AB = 3$$

So, Base side = 3

By definition,

Since, $\sin \theta = \text{Perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 1/\sqrt{10}$$

Since, $\cos \theta = \text{Base/Hypotenuse}$

$$\therefore \cos \theta = 3/\sqrt{10}$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = \sqrt{10}/3$$

Since, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 1/3$$

Since, $\cot \theta = 1/\tan \theta$

$$\therefore \cot \theta = 3/1$$

(xii) $\cos \theta = 12/15$

Solution:

We have; $\cos \theta = 12/15$ (1)

By definition,

$\cos \theta = \text{Base/Hypotenuse}$ (2)

By comparing eq. (1) and (2), we get;

Base = 12 and Hypotenuse = 15

Now, using Pythagoras theorem in ΔABC , we get

$$AC^2 = AB^2 + BC^2$$

Putting the value of base (AB) and hypotenuse (AC) to get the perpendicular (BC);

$$15^2 = 12^2 + BC^2$$

$$BC^2 = 15^2 - 12^2$$

$$BC^2 = 225 - 144$$

$$BC^2 = 81$$

$$BC = \sqrt{81}$$

$$BC = 9$$

So, Perpendicular = 9

By definition,

Since, $\sin \theta = \text{perpendicular/Hypotenuse}$

$$\therefore \sin \theta = 9/15 = 3/5$$

Since, $\text{cosec } \theta = 1/\sin \theta$

Also, $\text{cosec } \theta = \text{Hypotenuse/Perpendicular}$

$$\therefore \text{cosec } \theta = 15/9 = 5/3$$

Since, $\sec \theta = 1/\cos \theta$

Also, $\sec \theta = \text{Hypotenuse/Base}$

$$\therefore \sec \theta = 15/12 = 5/4$$

Since, $\tan \theta = \text{Perpendicular/Base}$

$$\therefore \tan \theta = 9/12 = 3/4$$

Since, $\cot \theta = 1/\tan \theta$

Also, $\cot \theta = \text{Base/Perpendicular}$

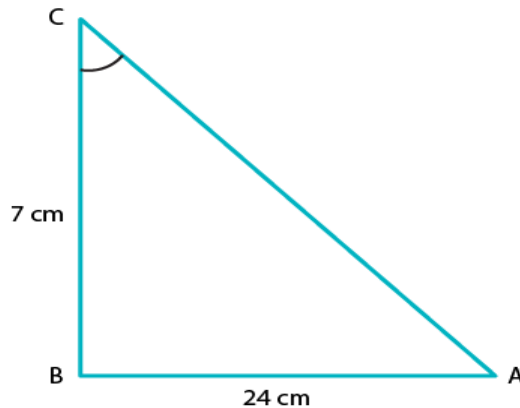
$$\therefore \cot \theta = 12/9 = 4/3$$

2. In a $\triangle ABC$, right angled at B, $AB = 24$ cm , $BC = 7$ cm. Determine

(i) $\sin A$, $\cos A$

(ii) $\sin C$, $\cos C$

Solution:



- (i) Given: In $\triangle ABC$, $AB = 24$ cm, $BC = 7$ cm and $\angle ABC = 90^\circ$
To find: $\sin A$, $\cos A$

By using Pythagoras theorem in $\triangle ABC$ we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 24^2 + 7^2$$

$$AC^2 = 576 + 49$$

$$AC^2 = 625$$

$$AC = \sqrt{625}$$

$$AC = 25$$

Hence, Hypotenuse = 25

By definition,

$\sin A =$ Perpendicular side opposite to angle A/ Hypotenuse

$$\sin A = BC/ AC$$

$$\sin A = 7/ 25$$

And,

$\cos A =$ Base side adjacent to angle A/Hypotenuse

$$\cos A = AB/ AC$$

$$\cos A = 24/ 25$$

- (ii) Given: In $\triangle ABC$, $AB = 24$ cm and $BC = 7$ cm and $\angle ABC = 90^\circ$
To find: $\sin C$, $\cos C$

By using Pythagoras theorem in $\triangle ABC$ we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 24^2 + 7^2$$

$$AC^2 = 576 + 49$$

$$AC^2 = 625$$

$$AC = \sqrt{625}$$

$$AC = 25$$

Hence, Hypotenuse = 25

By definition,

$\sin C = \text{Perpendicular side opposite to angle } C / \text{Hypotenuse}$

$$\sin C = AB / AC$$

$$\sin C = 24 / 25$$

And,

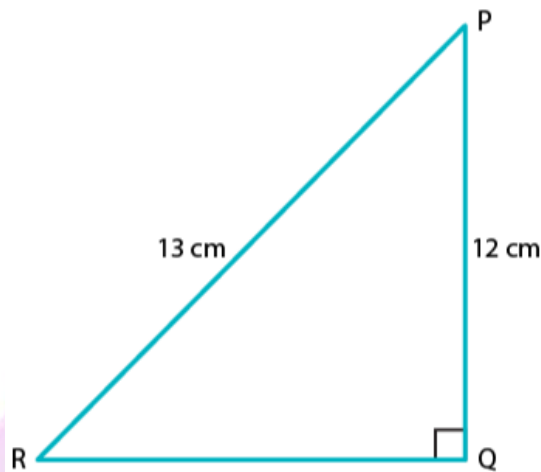
$\cos C = \text{Base side adjacent to angle } C / \text{Hypotenuse}$

$$\cos A = BC / AC$$

$$\cos A = 7 / 25$$

3. In fig. 5.37, find $\tan P$ and $\cot R$. Is $\tan P = \cot R$?

Solution:



By using Pythagoras theorem in ΔPQR , we have

$$PR^2 = PQ^2 + QR^2$$

Putting the length of given side PR and PQ in the above equation

$$13^2 = 12^2 + QR^2$$

$$QR^2 = 13^2 - 12^2$$

$$QR^2 = 169 - 144$$

$$QR^2 = 25$$

$$QR = \sqrt{25} = 5$$

By definition,

$\tan P = \text{Perpendicular side opposite to } P / \text{Base side adjacent to angle } P$

$$\tan P = QR / PQ$$

$$\tan P = 5 / 12 \dots\dots\dots (1)$$

And,

$\cot R = \text{Base} / \text{Perpendicular}$

$$\cot R = QR / PQ$$

$$\cot R = 5 / 12 \dots\dots (2)$$

When comparing equation (1) and (2), we can see that R.H.S of both the equation is equal.

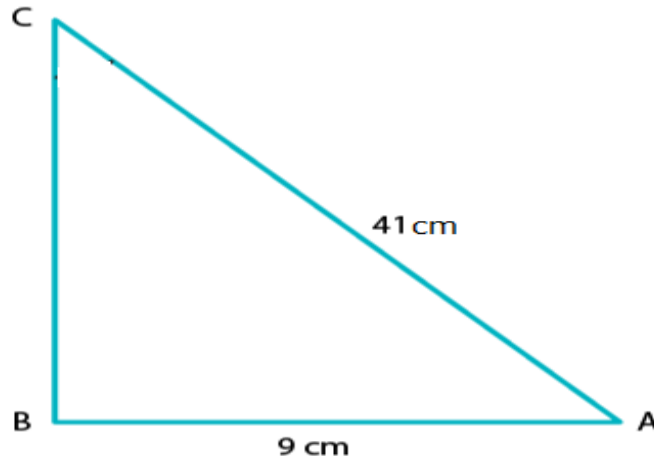
Therefore, L.H.S of both equations should also be equal.

$$\therefore \tan P = \cot R$$

Yes, $\tan P = \cot R = 5/12$

4. If $\sin A = 9/41$, compute $\cos A$ and $\tan A$.

Solution:



Given that, $\sin A = 9/41$ (1)

Required to find: $\cos A$, $\tan A$

By definition, we know that

$$\sin A = \text{Perpendicular/ Hypotenuse} \dots \dots \dots (2)$$

On Comparing eq. (1) and (2), we get;

Perpendicular side = 9 and Hypotenuse = 41

Let's construct ΔABC as shown below,
And, here the length of base AB is unknown.

Thus, by using Pythagoras theorem in ΔABC , we get;

$$AC^2 = AB^2 + BC^2$$

$$41^2 = AB^2 + 9^2$$

$$AB^2 = 41^2 - 9^2$$

$$AB^2 = 168 - 81$$

$$AB = 1600$$

$$AB = \sqrt{1600}$$

$$AB = 40$$

\Rightarrow Base of triangle ABC , $AB = 40$

We know that,

$$\cos A = \text{Base/ Hypotenuse}$$

$$\cos A = AB/AC$$

$$\cos A = 40/41$$

And,

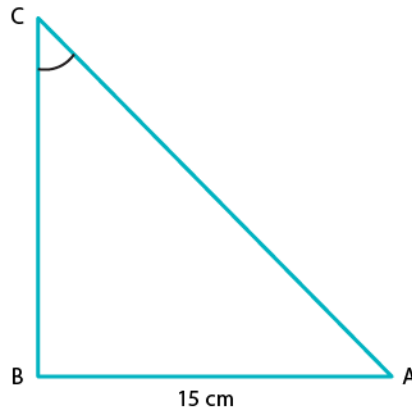
$$\tan A = \text{Perpendicular/ Base}$$

$$\tan A = BC/AB$$

$$\tan A = 9/40$$

5. Given $15\cot A = 8$, find $\sin A$ and $\sec A$.

Solution



We have, $15\cot A = 8$

Required to find: $\sin A$ and $\sec A$

As, $15 \cot A = 8$

$$\Rightarrow \cot A = 8/15 \dots\dots(1)$$

And we know,

$$\cot A = 1/\tan A$$

Also by definition,

$$\cot A = \text{Base side adjacent to } \angle A / \text{Perpendicular side opposite to } \angle A \dots (2)$$

On comparing equation (1) and (2), we get;

$$\text{Base side adjacent to } \angle A = 8$$

$$\text{Perpendicular side opposite to } \angle A = 15$$

So, by using Pythagoras theorem to $\triangle ABC$, we have

$$AC^2 = AB^2 + BC^2$$

Substituting values for sides from the figure

$$AC^2 = 8^2 + 15^2$$

$$AC^2 = 64 + 225$$

$$AC^2 = 289$$

$$AC = \sqrt{289}$$

$$AC = 17$$

Therefore, hypotenuse = 17

By definition,

$$\sin A = \text{Perpendicular/Hypotenuse}$$

$$\Rightarrow \sin A = BC/AC$$

$$\sin A = 15/17 \text{ (using values from the above)}$$

Also,

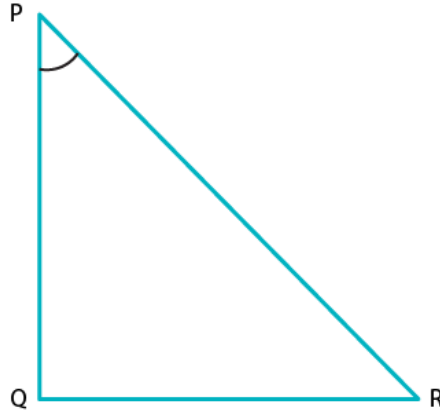
$$\sec A = 1/\cos A$$

$$\Rightarrow \sec A = \text{Hypotenuse/ Base side adjacent to } \angle A$$

$$\therefore \sec A = 17/8$$

6. In ΔPQR , right-angled at Q, $PQ = 4\text{cm}$ and $RQ = 3\text{ cm}$. Find the value of $\sin P$, $\sin R$, $\sec P$ and $\sec R$.

Solution:



Given:

ΔPQR is right-angled at Q.

$PQ = 4\text{cm}$

$RQ = 3\text{cm}$

Required to find: $\sin P$, $\sin R$, $\sec P$, $\sec R$

Given ΔPQR ,

By using Pythagoras theorem to ΔPQR , we get

$$PR^2 = PQ^2 + RQ^2$$

Substituting the respective values,

$$PR^2 = 4^2 + 3^2$$

$$PR^2 = 16 + 9$$

$$PR^2 = 25$$

$$PR = \sqrt{25}$$

$$PR = 5$$

\Rightarrow Hypotenuse = 5

By definition,

$\sin P = \frac{\text{Perpendicular side opposite to angle P}}{\text{Hypotenuse}}$

$$\sin P = \frac{RQ}{PR}$$

$\Rightarrow \sin P = \frac{3}{5}$

And,

$\sin R = \frac{\text{Perpendicular side opposite to angle R}}{\text{Hypotenuse}}$

$$\sin R = \frac{PQ}{PR}$$

$\Rightarrow \sin R = \frac{4}{5}$

And,

$\sec P = \frac{1}{\cos P}$

$\sec P = \frac{\text{Hypotenuse}}{\text{Base side adjacent to } \angle P}$

$$\sec P = \frac{PR}{PQ}$$

$\Rightarrow \sec P = \frac{5}{4}$

Now,

$$\begin{aligned} \sec R &= 1/\cos R \\ \sec R &= \text{Hypotenuse/ Base side adjacent to } \angle R \\ \sec R &= PR/ RQ \\ \Rightarrow \sec R &= 5/3 \end{aligned}$$

7. If $\cot \theta = 7/8$, evaluate

(i) $(1+\sin \theta)(1-\sin \theta) / (1+\cos \theta)(1-\cos \theta)$

(ii) $\cot^2 \theta$

Solution:

(i) Required to evaluate: $\frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$, given $\cot \theta = 7/8$

Taking the numerator, we have

$$(1+\sin \theta)(1-\sin \theta) = 1 - \sin^2 \theta \quad [\text{Since, } (a+b)(a-b) = a^2 - b^2]$$

Similarly,

$$(1+\cos \theta)(1-\cos \theta) = 1 - \cos^2 \theta$$

We know that,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow 1 - \cos^2 \theta = \sin^2 \theta$$

And,

$$1 - \sin^2 \theta = \cos^2 \theta$$

Thus,

$$(1+\sin \theta)(1-\sin \theta) = 1 - \sin^2 \theta = \cos^2 \theta$$

$$(1+\cos \theta)(1-\cos \theta) = 1 - \cos^2 \theta = \sin^2 \theta$$

$$\begin{aligned} \Rightarrow \frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)} &= \frac{\cos^2 \theta}{\sin^2 \theta} \\ &= (\cos \theta / \sin \theta)^2 \end{aligned}$$

And, we know that $(\cos \theta / \sin \theta) = \cot \theta$

$$\begin{aligned} \Rightarrow \frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)} &= (\cot \theta)^2 \\ &= (7/8)^2 \\ &= 49/ 64 \end{aligned}$$

(ii) Given,

$$\cot \theta = 7/8$$

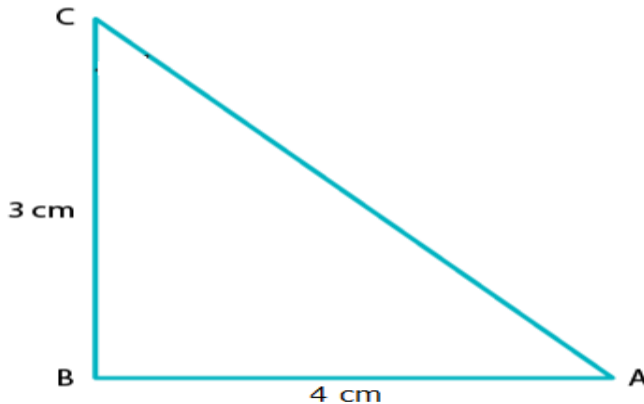
So, by squaring on both sides we get

$$(\cot \theta)^2 = (7/8)^2$$

$$\therefore \cot \theta^2 = 49/64$$

8. If $3\cot A = 4$, check whether $(1-\tan^2 A)/(1+\tan^2 A) = (\cos^2 A - \sin^2 A)$ or not.

Solution:



Given,

$$3\cot A = 4$$

$$\Rightarrow \cot A = 4/3$$

By definition,

$$\tan A = 1/\cot A = 1/(4/3)$$

$$\Rightarrow \tan A = 3/4$$

Thus,

Base side adjacent to $\angle A = 4$

Perpendicular side opposite to $\angle A = 3$

In $\triangle ABC$, Hypotenuse is unknown

Thus, by applying Pythagoras theorem in $\triangle ABC$

We get

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 4^2 + 3^2$$

$$AC^2 = 16 + 9$$

$$AC^2 = 25$$

$$AC = \sqrt{25}$$

$$AC = 5$$

Hence, hypotenuse = 5

Now, we can find that

$$\sin A = \text{opposite side to } \angle A / \text{Hypotenuse} = 3/5$$

And,

$$\cos A = \text{adjacent side to } \angle A / \text{Hypotenuse} = 4/5$$

Taking the LHS,

$$\text{L.H.S} = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

Putting value of $\tan A$

We get,

$$\text{L.H.S} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

Take L.C.M of both numerator and denominator;

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{\frac{16 - 9}{16}}{\frac{16 + 9}{16}}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{7}{25}$$

Thus, LHS = 7/25

Now, taking RHS

$$\text{R.H.S} = \cos^2 A - \sin^2 A$$

Putting value of sin A and cos A

$$\text{R.H.S} = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$$

$$\cos^2 A - \sin^2 A = \frac{16-9}{25}$$

$$\cos^2 A - \sin^2 A = \frac{7}{25}$$

Therefore,

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

Hence Proved

9. If $\tan \theta = a/b$, find the value of $(\cos \theta + \sin \theta) / (\cos \theta - \sin \theta)$

Solution:

Given,

$$\tan \theta = a/b$$

And, we know by definition that

$$\tan \theta = \text{opposite side} / \text{adjacent side}$$

Thus, by comparison

Opposite side = a and adjacent side = b

To find the hypotenuse, we know that by Pythagoras theorem that

$$\text{Hypotenuse}^2 = \text{opposite side}^2 + \text{adjacent side}^2$$

$$\Rightarrow \text{Hypotenuse} = \sqrt{a^2 + b^2}$$

So, by definition

$$\sin \theta = \text{opposite side} / \text{Hypotenuse}$$

$$\sin \theta = a / \sqrt{a^2 + b^2}$$

And,

$$\cos \theta = \text{adjacent side} / \text{Hypotenuse}$$

$$\cos \theta = b / \sqrt{a^2 + b^2}$$

Now,

After substituting for cos θ and sin θ , we have

$$\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{(a + b) / \sqrt{(a^2 + b^2)}}{(a - b) / \sqrt{(a^2 + b^2)}}$$

$$\therefore \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} = \frac{(a + b)}{(a - b)}$$

Hence Proved.

10. If $3 \tan \theta = 4$, find the value of
Solution:

$$\frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$$

Given, $3 \tan \theta = 4$
 $\Rightarrow \tan \theta = 4/3$

From, $\frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$ let's divide the numerator and denominator by $\cos \theta$.

We get,

$$\begin{aligned} & (4 - \tan \theta) / (2 + \tan \theta) \\ \Rightarrow & (4 - (4/3)) / (2 + (4/3)) && \text{[using the value of } \tan \theta \text{]} \\ \Rightarrow & (12 - 4) / (6 + 4) && \text{[After taking LCM and cancelling it]} \\ \Rightarrow & 8/10 = 4/5 \end{aligned}$$

$$\therefore \frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta} = 4/5$$

11. If $3 \cot \theta = 2$, find the value of

$$\frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta}$$

Solution:

Given, $3 \cot \theta = 2$
 $\Rightarrow \cot \theta = 2/3$

From, $\frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta}$ let's divide the numerator and denominator by $\sin \theta$.

We get,

$$\begin{aligned} & (4 - 3 \cot \theta) / (2 + 6 \cot \theta) \\ \Rightarrow & (4 - 3(2/3)) / (2 + 6(2/3)) && \text{[using the value of } \tan \theta \text{]} \\ \Rightarrow & (4 - 2) / (2 + 4) && \text{[After taking LCM and simplifying it]} \\ \Rightarrow & 2/6 = 1/3 \end{aligned}$$

$$\therefore \frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta} = 1/3$$

12. If $\tan \theta = a/b$, prove that $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{a^2 - b^2}{a^2 + b^2}$

Solution:

Given, $\tan \theta = a/b$

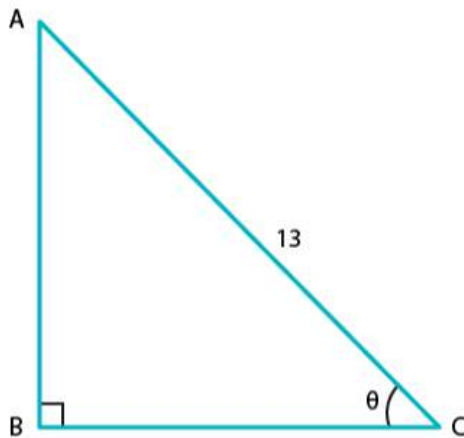
From LHS, $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta}$ let's divide the numerator and denominator by $\cos \theta$.

And we get,

$$\begin{aligned} & (a \tan \theta - b) / (a \tan \theta + b) \\ \Rightarrow & (a(a/b) - b) / (a(a/b) + b) && \text{[using the value of } \tan \theta \text{]} \\ \Rightarrow & (a^2 - b^2)/b^2 / (a^2 + b^2)/b^2 && \text{[After taking LCM and simplifying it]} \\ \Rightarrow & (a^2 - b^2) / (a^2 + b^2) \\ & = \text{RHS} \\ & \text{- Hence Proved} \end{aligned}$$

13. If $\sec \theta = 13/5$, show that $\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} = 3$

Solution:



Given,

$$\sec \theta = 13/5$$

We know that,

$$\sec \theta = 1 / \cos \theta$$

$$\Rightarrow \cos \theta = 1 / \sec \theta = 1 / (13/5)$$

$$\therefore \cos \theta = 5/13 \dots\dots (1)$$

By definition,

$$\cos \theta = \text{adjacent side} / \text{hypotenuse} \dots\dots (2)$$

Comparing (1) and (2), we have

$$\text{Adjacent side} = 5 \text{ and hypotenuse} = 13$$

By Pythagoras theorem,

$$\begin{aligned} \text{Opposite side} &= \sqrt{(\text{hypotenuse})^2 - (\text{adjacent side})^2} \\ &= \sqrt{13^2 - 5^2} \\ &= \sqrt{169 - 25} \\ &= \sqrt{144} \\ &= 12 \end{aligned}$$

Thus, opposite side = 12

By definition,

$$\tan \theta = \text{opposite side} / \text{adjacent side}$$

$$\therefore \tan \theta = 12/5$$

From, $\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta}$ let's divide the numerator and denominator by $\cos \theta$.

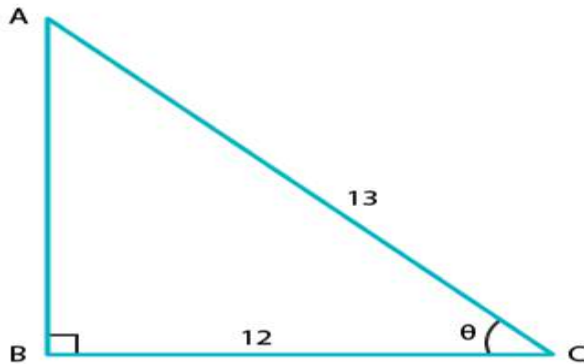
We get,

$$\begin{aligned} &(2 \tan \theta - 3) / (4 \tan \theta - 9) \\ \Rightarrow &(2(12/5) - 3) / (4(12/5) - 9) && \text{[using the value of } \tan \theta \text{]} \\ \Rightarrow &(24 - 15) / (48 - 45) && \text{[After taking LCM and cancelling it]} \\ \Rightarrow &9/3 = 3 \end{aligned}$$

$$\therefore \frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} = 3$$

14. If $\cos \theta = 12/13$, show that $\sin \theta(1 - \tan \theta) = 35/156$

Solution:



Given, $\cos \theta = 12/13$ (1)

By definition we know that,

$\cos \theta = \text{Base side adjacent to } \angle \theta / \text{Hypotenuse}$ (2)

When comparing equation (1) and (2), we get

Base side adjacent to $\angle \theta = 12$ and Hypotenuse = 13

From the figure,

Base side BC = 12

Hypotenuse AC = 13

Side AB is unknown here and it can be found by using Pythagoras theorem

Thus by applying Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$13^2 = AB^2 + 12^2$$

Therefore,

$$AB^2 = 13^2 - 12^2$$

$$AB^2 = 169 - 144$$

$$AB^2 = 25$$

$$AB = \sqrt{25}$$

$$AB = 5 \dots (3)$$

Now, we know that

$\sin \theta = \text{Perpendicular side opposite to } \angle \theta / \text{Hypotenuse}$

Thus, $\sin \theta = AB/AC$ [from figure]

$$\Rightarrow \sin \theta = 5/13 \dots (4)$$

And, $\tan \theta = \sin \theta / \cos \theta = (5/13) / (12/13)$

$$\Rightarrow \tan \theta = 5/12 \dots (5)$$

Taking L.H.S we have

$$\text{L.H.S} = \sin \theta (1 - \tan \theta)$$

Substituting the value of $\sin \theta$ and $\tan \theta$ from equation (4) and (5)

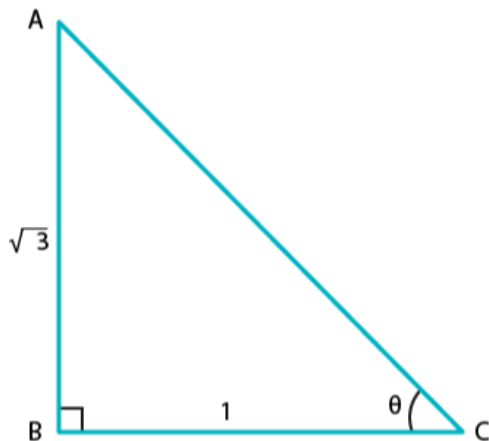
We get,

$$\begin{aligned} \Rightarrow \text{L.H.S} &= \frac{5}{13} \left(1 - \frac{5}{12} \right) \\ \text{L.H.S} &= \frac{5}{13} \left(\frac{1 \times 12}{1 \times 12} - \frac{5}{12} \right) \quad [\text{Taking LCM}] \\ \text{L.H.S} &= \frac{5}{13} \left(\frac{12-5}{12} \right) \\ \text{L.H.S} &= \frac{5}{13} \left(\frac{7}{12} \right) \\ \text{L.H.S} &= \frac{5 \times 7}{13 \times 12} \\ \text{L.H.S} &= 35/156 \end{aligned}$$

Therefore it's shown that $\sin \theta(1 - \tan \theta) = 35/156$

15. If $\cot \theta = \frac{1}{\sqrt{3}}$, show that $\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{3}{5}$

Solution:



Given, $\cot \theta = 1/\sqrt{3}$ (1)

By definition we know that,

$$\cot \theta = 1/\tan \theta$$

And, since $\tan \theta = \text{perpendicular side opposite to } \angle \theta / \text{Base side adjacent to } \angle \theta$

$$\Rightarrow \cot \theta = \text{Base side adjacent to } \angle \theta / \text{perpendicular side opposite to } \angle \theta \dots\dots (2)$$

[Since they are reciprocal to each other]

On comparing equation (1) and (2), we get

Base side adjacent to $\angle \theta = 1$ and Perpendicular side opposite to $\angle \theta = \sqrt{3}$

Therefore, the triangle formed is,

On substituting the values of known sides as $AB = \sqrt{3}$ and $BC = 1$

$$AC^2 = (\sqrt{3})^2 + 1$$

$$AC^2 = 3 + 1$$

$$AC^2 = 4$$

$$AC = \sqrt{4}$$

Therefore, $AC = 2 \dots (3)$

Now, by definition

$$\sin \theta = \frac{\text{Perpendicular side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2} \dots\dots(4)$$

And, $\cos \theta = \frac{\text{Base side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC}$

$$\Rightarrow \cos \theta = \frac{1}{2} \dots\dots (5)$$

Now, taking L.H.S we have

$$\text{L. H. S} = \frac{1 - \cos^2 \theta}{2 - \sin^2 \theta}$$

Substituting the values from equation (4) and (5), we have

$$\text{L. H. S} = \frac{1 - \left(\frac{1}{2}\right)^2}{2 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\text{L. H. S} = \frac{1 - \frac{1}{4}}{2 - \frac{3}{4}}$$

Now by taking L.C.M in numerator and denominator, we get

$$\text{L. H. S} = \frac{\frac{(4 \times 1) - 1}{4}}{\frac{(4 \times 2) - 3}{4}}$$

$$\text{L. H. S} = \frac{\frac{4 - 1}{4}}{\frac{8 - 3}{4}}$$

$$\text{L. H. S} = \frac{3}{4} \times \frac{4}{5}$$

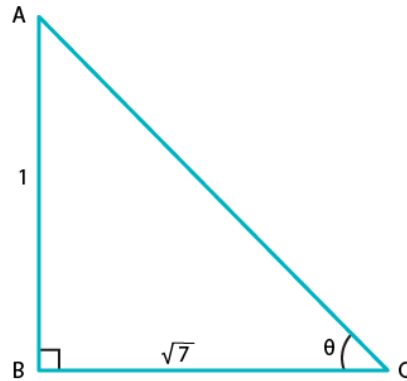
$$\text{L. H. S} = \frac{3}{5} = \text{R. H. S}$$

Therefore,
$$\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{3}{5}$$

16. If $\tan \theta = \frac{1}{\sqrt{7}}$, then show that $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = \frac{3}{4}$

Solution:

Given, $\tan \theta = 1/\sqrt{7}$ (1)



By definition, we know that

$\tan \theta = \text{Perpendicular side opposite to } \angle \theta / \text{Base side adjacent to } \angle \theta$ (2)

On comparing equation (1) and (2), we have

Perpendicular side opposite to $\angle \theta = 1$

Base side adjacent to $\angle \theta = \sqrt{7}$

Thus, the triangle representing $\angle \theta$ is,

Hypotenuse AC is unknown and it can be found by using Pythagoras theorem

By applying Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 1^2 + (\sqrt{7})^2$$

$$AC^2 = 1 + 7$$

$$AC^2 = 8$$

$$AC = \sqrt{8}$$

$$\Rightarrow AC = 2\sqrt{2}$$

By definition,

$\sin \theta = \text{Perpendicular side opposite to } \angle \theta / \text{Hypotenuse} = AB / AC$

$$\Rightarrow \sin \theta = 1 / 2\sqrt{2}$$

And, since $\operatorname{cosec} \theta = 1/\sin \theta$

$$\Rightarrow \operatorname{cosec} \theta = 2\sqrt{2} \text{ (3)}$$

Now,

$\cos \theta = \text{Base side adjacent to } \angle \theta / \text{Hypotenuse} = BC / AC$

$$\Rightarrow \cos \theta = \sqrt{7} / 2\sqrt{2}$$

And, since $\sec \theta = 1/\cos \theta$

$$\Rightarrow \sec \theta = 2\sqrt{2} / \sqrt{7} \text{ (4)}$$

Taking the L.H.S of the equation,

$$\text{L. H. S} = \frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta}$$

Substituting the value of cosec θ and sec θ from equation (3) and (4), we get

$$\text{L. H. S} = \frac{[(2\sqrt{2})]^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{[(2\sqrt{2})]^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$\text{L. H. S} = \frac{(8) - \left(\frac{8}{7}\right)}{(8) + \left(\frac{8}{7}\right)} = \frac{56-8}{56+8}$$

[Taking L.C.M and simplifying]

$$\text{L. H. S} = \frac{48}{64}$$

Therefore,

$$\text{L.H.S} = 48/64 = 3/4 = \text{R.H.S}$$

Hence proved that $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = \frac{3}{4}$

17. If sec $\theta = 5/4$, find the value of
Solution:

$$\frac{\sin \theta - 2\cos \theta}{\tan \theta - \cot \theta}$$

Given,

$$\sec \theta = 5/4$$

We know that,

$$\sec \theta = 1/\cos \theta$$

$$\Rightarrow \cos \theta = 1/(5/4) = 4/5 \dots\dots (1)$$

By definition,

$$\cos \theta = \text{Base side adjacent to } \angle \theta / \text{Hypotenuse} \dots (2)$$

On comparing equation (1) and (2), we have

$$\text{Hypotenuse} = 5$$

$$\text{Base side adjacent to } \angle \theta = 4$$

Thus, the triangle representing $\angle \theta$ is ABC.

Perpendicular side opposite to $\angle\theta$, AB is unknown and it can be found by using Pythagoras theorem

By applying Pythagoras theorem, we have

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ AB^2 &= AC^2 - BC^2 \\ AB^2 &= 5^2 - 4^2 \\ AB^2 &= 25 - 16 \\ AB &= \sqrt{9} \\ \Rightarrow AB &= 3 \end{aligned}$$

By definition,

$$\begin{aligned} \sin \theta &= \text{Perpendicular side opposite to } \angle\theta / \text{Hypotenuse} = AB / AC \\ \Rightarrow \sin \theta &= 3 / 5 \dots\dots(3) \end{aligned}$$

Now, $\tan \theta = \text{Perpendicular side opposite to } \angle\theta / \text{Base side adjacent to } \angle\theta$

$$\Rightarrow \tan \theta = 3 / 4 \dots\dots(4)$$

And, since $\cot \theta = 1 / \tan \theta$

$$\Rightarrow \cot \theta = 4 / 3 \dots\dots(5)$$

Now,

Substituting the value of $\sin \theta$, $\cos \theta$, $\cot \theta$ and $\tan \theta$ from the equations (1), (3), (4) and (5) we have,

$$\begin{aligned} \frac{\sin \theta - 2 \cos \theta}{\tan \theta - \cot \theta} &= \frac{\frac{3}{5} - 2\left(\frac{4}{5}\right)}{\frac{3}{4} - \frac{4}{3}} \\ &= \frac{12}{7} \end{aligned}$$

Therefore,

$$\frac{\sin \theta - 2 \cos \theta}{\tan \theta - \cot \theta} = \frac{12}{7}$$

18. If $\tan \theta = 12/13$, find the value of
Solution:

$$\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$$

Given,

$$\tan \theta = 12/13 \dots\dots\dots (1)$$

We know that by definition,

$$\tan \theta = \text{Perpendicular side opposite to } \angle\theta / \text{Base side adjacent to } \angle\theta \dots\dots\dots (2)$$

On comparing equation (1) and (2), we have

$$\text{Perpendicular side opposite to } \angle\theta = 12$$

$$\text{Base side adjacent to } \angle\theta = 13$$

Thus, in the triangle representing $\angle \theta$ we have,

Hypotenuse AC is the unknown and it can be found by using Pythagoras theorem

So by applying Pythagoras theorem, we have

$$AC^2 = 12^2 + 13^2$$

$$AC^2 = 144 + 169$$

$$AC^2 = 313$$

$$\Rightarrow AC = \sqrt{313}$$

By definition,

$$\sin \theta = \text{Perpendicular side opposite to } \angle \theta / \text{Hypotenuse} = AB / AC$$

$$\Rightarrow \sin \theta = 12 / \sqrt{313} \dots (3)$$

And, $\cos \theta = \text{Base side adjacent to } \angle \theta / \text{Hypotenuse} = BC / AC$

$$\Rightarrow \cos \theta = 13 / \sqrt{313} \dots (4)$$

Now, substituting the value of $\sin \theta$ and $\cos \theta$ from equation (3) and (4) respectively in the equation below

$$\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{2 \times \frac{13}{\sqrt{313}} \times \frac{12}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2 - \left(\frac{12}{\sqrt{313}}\right)^2}$$

$$= \frac{312}{\frac{313}{25}}$$

$$= \frac{312}{25}$$

Therefore,

$$\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{312}{25}$$

Exercise 5.2

Evaluate each of the following:

1. $\sin 45^\circ \sin 30^\circ + \cos 45^\circ \cos 30^\circ$

Solution:

$$\sin 45^\circ \sin 30^\circ + \cos 45^\circ \cos 30^\circ$$

Value of trigonometric ratios are:

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \quad \sin 30^\circ = \frac{1}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Substituting in the given equation, we get

$$\begin{aligned} & \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}+1}{2} \\ &= \frac{\sqrt{3}+1}{2\sqrt{2}} \end{aligned}$$

2. $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

Solution:

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

By trigonometric ratios we have ,

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \cos 60^\circ = \frac{1}{2}$$

Substituting the values in given equation

$$\begin{aligned} &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} \\ &= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \end{aligned}$$

3. $\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

Solution:

$$\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

We know that by trigonometric ratios

$$\cos 60^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 45^\circ = \frac{1}{\sqrt{2}}$$

Substituting the values in given equation

$$\begin{aligned} & \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}} \end{aligned}$$

4. $\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$

Solution:

$$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$$

We know that by trigonometric ratios

$$\sin 30^\circ = \frac{1}{2} \quad \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 90^\circ = 1$$

Substituting the values in given equation, we get

$$\begin{aligned} &= \left[\frac{1}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{\sqrt{3}}{2}\right]^2 + 1 \\ &= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 \\ &= \frac{5}{2} \end{aligned}$$

5. $\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$

Solution:

$$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$$

We know that by trigonometric ratios

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 60^\circ = \frac{1}{2} \quad \cos 90^\circ = 0$$

Substituting the values in given equation

$$\begin{aligned} & \left[\frac{\sqrt{3}}{2} \right]^2 + \left[\frac{1}{\sqrt{2}} \right]^2 + \left[\frac{1}{2} \right]^2 + 0 \\ &= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} \\ &= \frac{3}{2} \end{aligned}$$

6. $\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ$

Solution:

$$\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ$$

We know that by trigonometric ratios

$$\tan 30^\circ = \frac{1}{\sqrt{3}} \quad \tan 60^\circ = \sqrt{3}$$

$$\tan 45^\circ = 1$$

Substituting the values in given equation

$$\begin{aligned} & \left[\frac{1}{\sqrt{3}} \right]^2 + [\sqrt{3}]^2 + 1 \\ &= \frac{1}{3} + 3 + 1 \\ &= \frac{13}{3} \end{aligned}$$

7. $2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$

Solution:

$$2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$$

We know that by trigonometric ratios

$$\sin 30^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan 60^\circ = \sqrt{3}$$

Substituting the values in given equation

$$= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2$$

$$= 2\left(\frac{1}{4}\right) - 3\left(\frac{1}{2}\right) + 3$$

$$= \frac{1-3+6}{2}$$

$$= 2$$

8. $\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + (1/2) \sin^2 90^\circ - 2\cos^2 90^\circ + (1/24) \cos 20^\circ$

Solution:

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2} \sin^2 90^\circ - 2\cos^2 90^\circ + \frac{1}{24} \cos 20^\circ$$

We know that by trigonometric ratios

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 90^\circ = 1$$

$$\cos 90^\circ = 0$$

$$\cos 0^\circ = 1$$

Substituting the values in given equation

$$\begin{aligned} & \left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4\left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2}[1]^2 - 2[0]^2 + \frac{1}{24}[1]^2 \\ &= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} \\ &= \frac{48}{24} \\ &= 2 \end{aligned}$$

9. $4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - \tan^2 45^\circ) + 5\cos^2 45^\circ$

Solution:

$$4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - \tan^2 45^\circ) + 5\cos^2 45^\circ$$

We know that by trigonometric ratios we have ,

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\tan 60^\circ = \sqrt{3} \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Substituting the values in given equation

$$\begin{aligned} &= 4 \cdot \frac{18}{16} - 6 + \frac{5}{2} \\ &= \frac{1}{4} - 6 + \frac{5}{2} \\ &= \frac{14}{2} - 6 = 7 - 6 = 1 \end{aligned}$$

10. $(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ)$

Solution:

$$(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ) (\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ)$$

We know that by trigonometric ratios,

$$\operatorname{cosec} 45^\circ = \sqrt{2} \sec 30^\circ = \frac{2}{\sqrt{3}}$$

$$\sin 30^\circ = \frac{1}{2} \cot 45^\circ = 1$$

$$\sec 60^\circ = 2$$

Substituting the values in given equation

$$\begin{aligned} & \left([\sqrt{2}]^2 \cdot \left[\frac{2}{\sqrt{3}} \right]^2 \right) \left(\left[\frac{1}{2} \right]^2 + 4(1) - (2)^2 \right) \\ &= (2 \cdot (4/3))[(1/4) + 4 - 4] = (8/3) \cdot (1/4) \\ &= 2/3 \end{aligned}$$

11. $\operatorname{cosec}^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$

Solution:

$$\operatorname{cosec}^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$$

Using trigonometric values, we have

$$= (2)^3 \times \left(\frac{1}{2}\right) \times (1^3) \times (1^2) \times (\sqrt{2}^2) \times (\sqrt{3})$$

$$= 8 \times \left(\frac{1}{2}\right) \times (1) \times (1) \times (2) \times (\sqrt{3})$$

$$= 8\sqrt{3}$$

12. $\cot^2 30^\circ - 2\cos^2 60^\circ - (3/4)\sec^2 45^\circ - 4\sec^2 30^\circ$

Solution:

Using trigonometric values, we have

$$\begin{aligned} & \cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ \\ &= (\sqrt{3}^2) - 2\left(\frac{1}{2}\right)^2 - \left(\frac{3}{4} \times \sqrt{2}^2\right) - \left(4 \times \left(\frac{2}{\sqrt{3}}\right)^2\right) \\ &= 3 - \frac{1}{2} - \frac{3}{2} - \frac{16}{3} \\ &= \frac{-13}{3} \end{aligned}$$

13. $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

Solution:

$$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$$

Using trigonometric values, we have

$$\begin{aligned} &= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right) \left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right) \\ &= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right) \left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right) \\ &= \left(\left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right) \\ &= \frac{9}{4} - \frac{1}{2} \\ &= \frac{7}{4} \end{aligned}$$

$$\frac{\sin 30^\circ - \sin 90^\circ + 2 \cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$$

14.

Solution:

Given,

$$\frac{\sin 30^\circ - \sin 90^\circ + 2 \cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$$

Using trigonometric values, we have

$$\begin{aligned} &= \frac{\frac{1}{2} - 1 + 2}{\frac{1}{\sqrt{3}} \times \sqrt{3}} \\ &= \frac{3}{2} \end{aligned}$$

15. $4/\cot^2 30^\circ + 1/\sin^2 60^\circ - \cos^2 45^\circ$

Solution:

$$\begin{aligned} &\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cos^2 45^\circ \\ &= \frac{4}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= \frac{4}{3} + \frac{4}{3} - \frac{1}{2} \\ &= \frac{16-3}{6} \\ &= \frac{13}{6} \end{aligned}$$

16. $4(\sin^4 30^\circ + \cos^2 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - \sin^2 60^\circ$

Solution:

Using trigonometric values, we have

$$\begin{aligned}
 & 4(\sin^4 30^\circ + \cos^2 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - \sin^2 60^\circ \\
 &= 4\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^2\right) - 3\left(\left(\frac{1}{\sqrt{2}}\right)^2 - 1\right) - \left(\frac{\sqrt{3}}{2}\right)^2 \\
 &= 4\left(\frac{1}{16} + \frac{1}{4}\right) + \frac{3}{2} - \frac{3}{4} \\
 &= \frac{8}{4} = 2
 \end{aligned}$$

17.
$$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

Solution:

Using trigonometric values, we have

$$\begin{aligned}
 & \frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ} \\
 &= \frac{(\sqrt{3})^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 + 5(0)}{2 + 2 - (\sqrt{3})^2} \\
 &= 3 + 2 + 4 \\
 &= 9
 \end{aligned}$$

18.
$$\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

Solution:

Using trigonometric values, we have

$$\begin{aligned}
 & \frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ} \\
 &= \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} + \frac{1}{2} - \frac{\frac{\sqrt{3}}{2}}{1} - \frac{\frac{\sqrt{3}}{2}}{1} \\
 &= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \\
 &= \frac{\sqrt{2} + 1 - 2\sqrt{3}}{2}
 \end{aligned}$$

$$19. \frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5\sin 90^\circ}{2\cos 0^\circ}$$

Solution:

Using trigonometric values, we have

$$\begin{aligned} & \frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5\sin 90^\circ}{2\cos 0^\circ} \\ &= \frac{1}{2} + \frac{2}{1} - \frac{5(1)}{2(1)} \\ &= \frac{5}{2} - \frac{5}{2} \\ &= 0 \end{aligned}$$

Find the value of x in each of the following: (20-25)

20. $2\sin 3x = \sqrt{3}$

Solution:

Given,

$$\begin{aligned} 2 \sin 3x &= \sqrt{3} \\ \sin 3x &= \sqrt{3}/2 \\ \sin 3x &= \sin 60^\circ \\ 3x &= 60^\circ \\ x &= 20^\circ \end{aligned}$$

Exercise 5.3

1. Evaluate the following:

(i) $\sin 20^\circ / \cos 70^\circ$

(ii) $\cos 19^\circ / \sin 71^\circ$

(iii) $\sin 21^\circ / \cos 69^\circ$

(iv) $\tan 10^\circ / \cot 80^\circ$

(v) $\sec 11^\circ / \operatorname{cosec} 79^\circ$

Solution:

(i) We have,
 $\sin 20^\circ / \cos 70^\circ = \sin (90^\circ - 70^\circ) / \cos 70^\circ = \cos 70^\circ / \cos 70^\circ = 1$ [$\because \sin (90 - \theta) = \cos \theta$]

(ii) We have,
 $\cos 19^\circ / \sin 71^\circ = \cos (90^\circ - 71^\circ) / \sin 71^\circ = \sin 71^\circ / \sin 71^\circ = 1$ [$\because \cos (90 - \theta) = \sin \theta$]

(iii) We have,
 $\sin 21^\circ / \cos 69^\circ = \sin (90^\circ - 69^\circ) / \cos 69^\circ = \cos 69^\circ / \cos 69^\circ = 1$ [$\because \sin (90 - \theta) = \cos \theta$]

(iv) We have,
 $\tan 10^\circ / \cot 80^\circ = \tan (90^\circ - 10^\circ) / \cot 80^\circ = \cot 80^\circ / \cot 80^\circ = 1$ [$\because \tan (90 - \theta) = \cot \theta$]

(v) We have,
 $\sec 11^\circ / \operatorname{cosec} 79^\circ = \sec (90^\circ - 79^\circ) / \operatorname{cosec} 79^\circ = \operatorname{cosec} 79^\circ / \operatorname{cosec} 79^\circ = 1$ [$\because \sec (90 - \theta) = \operatorname{cosec} \theta$]

2. Evaluate the following:

(i) $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

Solution:

We have, [$\because \sin (90 - \theta) = \cos \theta$ and $\cos (90 - \theta) = \sin \theta$]

$$\begin{aligned} & \left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2 \\ &= \left(\frac{\sin(90^\circ - 41^\circ)}{\cos 41^\circ}\right)^2 + \left(\frac{\cos(90^\circ - 49^\circ)}{\sin 49^\circ}\right)^2 \\ &= \left(\frac{\cos 41^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\sin 49^\circ}{\sin 49^\circ}\right)^2 \end{aligned}$$

$$= 1^2 + 1^2 = 1 + 1$$

$$= 2$$

(ii) $\cos 48^\circ - \sin 42^\circ$

Solution:

We know that, $\cos(90^\circ - \theta) = \sin \theta$.

So,

$$\cos 48^\circ - \sin 42^\circ = \cos(90^\circ - 42^\circ) - \sin 42^\circ = \sin 42^\circ - \sin 42^\circ = 0$$

Thus the value of $\cos 48^\circ - \sin 42^\circ$ is 0.

(iii) $\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\cos 35^\circ}{\sin 55^\circ} \right)$

Solution:

We have,

[$\because \cot(90 - \theta) = \tan \theta$ and $\cos(90 - \theta) = \sin \theta$]

$$\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\cos 35^\circ}{\sin 55^\circ} \right)$$

$$= \frac{\cot(90^\circ - 50^\circ)}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\cos(90^\circ - 55^\circ)}{\sin 55^\circ} \right)$$

$$= \frac{\tan 50^\circ}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\sin 55^\circ}{\sin 55^\circ} \right)$$

$$= 1 - 1/2(1)$$

$$= 1/2$$

(iv) $\left(\frac{\sin 27^\circ}{\cos 63^\circ} \right)^2 - \left(\frac{\cos 63^\circ}{\sin 27^\circ} \right)^2$

Solution:

We have,

[$\because \sin(90 - \theta) = \cos \theta$ and $\cos(90 - \theta) = \sin \theta$]

$$\left(\frac{\sin 27^\circ}{\cos 63^\circ} \right)^2 - \left(\frac{\cos 63^\circ}{\sin 27^\circ} \right)^2$$

$$= \left(\frac{\sin(90^\circ - 63^\circ)}{\cos 63^\circ} \right)^2 - \left(\frac{\cos(90^\circ - 27^\circ)}{\sin 27^\circ} \right)^2$$

$$= \left(\frac{\cos 63^\circ}{\cos 63^\circ} \right)^2 - \left(\frac{\sin 27^\circ}{\sin 27^\circ} \right)^2$$

$$= 1 - 1$$

$$= 0$$

$$(v) \frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$$

Solution:

We have,

$$[\because \cot (90 - \theta) = \tan \theta \text{ and } \tan (90 - \theta) = \cot \theta]$$

$$\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$$

$$\begin{aligned} &= \tan (90^\circ - 55^\circ) / \cot 55^\circ + \cot (90^\circ - 12^\circ) / \tan 12^\circ - 1 \\ &= \cot 55^\circ / \cot 55^\circ + \tan 12^\circ / \tan 12^\circ - 1 \\ &= 1 + 1 - 1 \\ &= 1 \end{aligned}$$

$$(vi) \frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$$

Solution:

We have ,

$$[\because \sin (90 - \theta) = \cos \theta \text{ and } \sec (90 - \theta) = \operatorname{cosec} \theta]$$

$$\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$$

$$\begin{aligned} &= \sec (90^\circ - 20^\circ) / \operatorname{cosec} 20^\circ + \sin (90^\circ - 31^\circ) / \cos 31^\circ \\ &= \operatorname{cosec} 20^\circ / \operatorname{cosec} 20^\circ + \cos 12^\circ / \cos 12^\circ \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

$$(vii) \operatorname{cosec} 31^\circ - \sec 59^\circ$$

Solution:

We have,

$$\operatorname{cosec} 31^\circ - \sec 59^\circ$$

Since, $\operatorname{cosec} (90 - \theta) = \cos \theta$

So,

$$\operatorname{cosec} 31^\circ - \sec 59^\circ = \operatorname{cosec} (90^\circ - 59^\circ) - \sec 59^\circ = \sec 59^\circ - \sec 59^\circ = 0$$

Thus,

$$\operatorname{cosec} 31^\circ - \sec 59^\circ = 0$$

$$(viii) (\sin 72^\circ + \cos 18^\circ) (\sin 72^\circ - \cos 18^\circ)$$

Solution:

We know that,

$$\sin (90 - \theta) = \cos \theta$$

So, the given can be expressed as

$$\begin{aligned}
 & (\sin 72^\circ + \cos 18^\circ) (\sin (90 - 18)^\circ - \cos 18^\circ) \\
 &= (\sin 72^\circ + \cos 18^\circ) (\cos 18^\circ - \cos 18^\circ) \\
 &= (\sin 72^\circ + \cos 18^\circ) \times 0 \\
 &= 0
 \end{aligned}$$

(ix) $\sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ$

Solution:

We know that,
 $\sin (90 - \theta) = \cos \theta$

So, the given can be expressed as
 $\sin (90 - 55)^\circ \sin (90 - 35)^\circ - \cos 35^\circ \cos 55^\circ$
 $= \cos 55^\circ \cos 35^\circ - \cos 35^\circ \cos 55^\circ$
 $= 0$

(x) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

Solution:

We know that,
 $\tan (90 - \theta) = \cot \theta$

So, the given can be expressed as
 $\tan (90 - 42)^\circ \tan (90 - 67)^\circ \tan 42^\circ \tan 67^\circ$
 $= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$
 $= (\cot 42^\circ \tan 42^\circ)(\cot 67^\circ \tan 67^\circ)$
 $= 1 \times 1$ [$\because \tan \theta \times \cot \theta = 1$]
 $= 1$

(xi) $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$

Solution:

We know that,
 $\sin (90 - \theta) = \cos \theta$ and $\cos (90 - \theta) = \sin \theta$

So, the given can be expressed as
 $\sec 50^\circ \sin (90 - 50)^\circ + \cos (90 - 50)^\circ \operatorname{cosec} 50^\circ$
 $= \sec 50^\circ \cos 50^\circ + \sin 50^\circ \operatorname{cosec} 50^\circ$
 $= 1 + 1$ [$\because \sin \theta \times \operatorname{cosec} \theta = 1$ and $\cos \theta \times \sec \theta = 1$]
 $= 2$

3. Express each one of the following in terms of trigonometric ratios of angles lying between 0° and 45°

(i) $\sin 59^\circ + \cos 56^\circ$

(ii) $\tan 65^\circ + \cot 49^\circ$

(iii) $\sec 76^\circ + \operatorname{cosec} 52^\circ$

(iv) $\cos 78^\circ + \sec 78^\circ$

(v) $\operatorname{cosec} 54^\circ + \sin 72^\circ$

(vi) $\cot 85^\circ + \cos 75^\circ$

(vii) $\sin 67^\circ + \cos 75^\circ$

Solution:

$$= \text{R.H.S}$$

- Hence Proved

LL L.H.S = $\sin \left(\frac{B + C}{2} \right)$

$$\Rightarrow \left(\frac{B + C}{2} \right) = \text{MQ} (90^\circ - A/2) \quad [\text{From (ii)}]$$

$$= \cos (A/2)$$

$$= \text{R.H.S}$$

- Hence Proved

7. Prove that:

(i) $\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = 1$

(ii) $\sin 48^\circ \sec 48^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ = 2$

(iii) $\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec} 42^\circ = 0$

(iv) $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2$

Solution:

(i) Taking L.H.S = $\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ$
 $= \tan (90^\circ - 70^\circ) \tan (90^\circ - 55^\circ) \tan 45^\circ \tan 55^\circ \tan 70^\circ$
 $= \cot 70^\circ \cot 55^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ \quad [\because \tan (90 - \theta) = \cot \theta]$
 $= (\tan 70^\circ \cot 70^\circ)(\tan 55^\circ \cot 55^\circ) \tan 45^\circ \quad [\because \tan \theta \times \cot \theta = 1]$
 $= 1 \times 1 \times 1 = 1$

- Hence proved

(ii) Taking L.H.S = $\sin 48^\circ \sec 48^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ$
 $= \sin 48^\circ \sec (90^\circ - 48^\circ) + \cos 48^\circ \operatorname{cosec} (90^\circ - 48^\circ)$
 $\quad \quad \quad [\because \sec (90 - \theta) = \operatorname{cosec} \theta \text{ and } \operatorname{cosec} (90 - \theta) = \sec \theta]$
 $= \sin 48^\circ \operatorname{cosec} 48^\circ + \cos 48^\circ \sec 48^\circ \quad [\because \operatorname{cosec} \theta \times \sin \theta = 1 \text{ and } \cos \theta \times \sec \theta = 1]$
 $= 1 + 1 = 2$

- Hence proved

(iii) Taking the L.H.S,

$$\begin{aligned} & \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec} 20^\circ \\ &= \frac{\sin(90^\circ - 20^\circ)}{\cos 20^\circ} + \frac{\operatorname{cosec}(90^\circ - 70^\circ)}{\sec 70^\circ} - 2 \cos(90^\circ - 20^\circ) \operatorname{cosec} 20^\circ \\ &= \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sec 70^\circ}{\sec 70^\circ} - 2 \sin 20^\circ \times \frac{1}{\sin 20^\circ} \quad \left[\begin{array}{l} \sin(90^\circ - \theta) = \cos \theta \\ \operatorname{cosec}(90^\circ - \theta) = \sec \theta \\ \cos(90^\circ - \theta) = \sin \theta \end{array} \right] \\ &= 1 + 1 - 2 \\ &= 2 - 2 \\ &= 0 \end{aligned}$$

- Hence proved

(iv) Taking L.H.S,

$$\begin{aligned} & \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ \\ &= \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec}(90^\circ - 59^\circ) \\ &= \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ \sec 59^\circ \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

- Hence proved

8. Prove the following:

(i) $\sin \theta \sin(90^\circ - \theta) - \cos \theta \cos(90^\circ - \theta) = 0$

Solution:

Taking the L.H.S,

$$\begin{aligned} & \sin \theta \sin(90^\circ - \theta) - \cos \theta \cos(90^\circ - \theta) \\ &= \sin \theta \cos \theta - \cos \theta \sin \theta \quad [\because \sin(90^\circ - \theta) = \cos \theta \text{ and } \cos(90^\circ - \theta) = \sin \theta] \\ &= 0 \end{aligned}$$

(ii)

$$\frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta} = 2$$

Solution:

Taking the L.H.S,

$$\begin{aligned} & \frac{\cos(90^\circ-\theta) \sec(90^\circ-\theta) \tan \theta}{\operatorname{cosec}(90^\circ-\theta) \sin(90^\circ-\theta) \cot(90^\circ-\theta)} + \frac{\tan(90^\circ-\theta)}{\cot \theta} \\ &= \frac{\sin \theta \operatorname{cosec} \theta \tan \theta}{\sec \theta \cos \theta \tan \theta} + \frac{\cot \theta}{\cot \theta} \\ &= \frac{1 \times \tan \theta}{1 \times \tan \theta} + 1 = \frac{\tan \theta}{\tan \theta} + 1 \quad [\because \operatorname{cosec} \theta \times \sin \theta = 1 \text{ and } \cos \theta \times \sec \theta = 1] \\ &= 1 + 1 \\ &= 2 = \text{R.H.S} \end{aligned}$$

- Hence Proved

(iii) $\frac{\tan(90^\circ-A) \cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$

Solution:

Taking the L.H.S,

[$\because \tan(90^\circ - \theta) = \cot \theta$]

$$\begin{aligned} \text{L.H.S.} &= \frac{\tan(90^\circ-A) \cot A}{\operatorname{cosec}^2 A} - \cos^2 A \\ &= \frac{\cot A \cot A}{\operatorname{cosec}^2 A} - \cos^2 A \\ &= \frac{\cot^2 A}{\operatorname{cosec}^2 A} - \cos^2 A = \frac{\frac{\cos^2 A}{\sin^2 \theta}}{\frac{1}{\sin^2 A}} - \cos^2 A \end{aligned}$$

$$\begin{aligned} &= \frac{\cos^2 A \times \sin^2 A}{\sin^2 A \times 1} - \cos^2 A = \cos^2 A - \cos^2 A \\ &= 0 = \text{R.H.S} \end{aligned}$$

- Hence Proved

