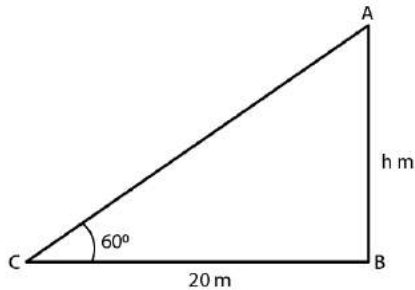


Exercise 12.1

Page No: 12.29

1. A tower stands vertically on the ground. From a point on the ground, 20 m away from the foot of the tower, the angle of elevation of the top the tower is 60° . What is the height of the tower?

Solution:



Given:

Distance between the foot of the tower and point of observation = 20 m = BC

Angle of elevation of the top of the tower = $60^\circ = \theta$

And, Height of tower (H) = AB

Now, from fig. ABC

ΔABC is a right angle triangle,

So,

$$\tan \theta = \frac{\text{Opposite side (AB)}}{\text{Adjacent side (BC)}}$$

$$\Rightarrow \tan C = \frac{AB}{BC}$$

$$AB = 20 \tan 60^\circ$$

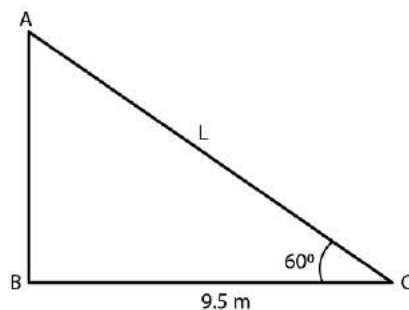
$$H = 20 \times \sqrt{3}$$

$$= 20\sqrt{3}$$

$$\therefore \text{Height of the tower} = 20\sqrt{3}\text{m}$$

2. The angle of elevation of a ladder against a wall is 60° and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.

Solution:



Given:

Distance between the wall and foot of the ladder = 9.5 m

Angle of elevation (θ) = 60°

Length of the ladder = $L = AC$

Now, from fig. ABC

ΔABC is a right angle triangle,

So,

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\cos 60^\circ = \frac{BC}{AC}$$

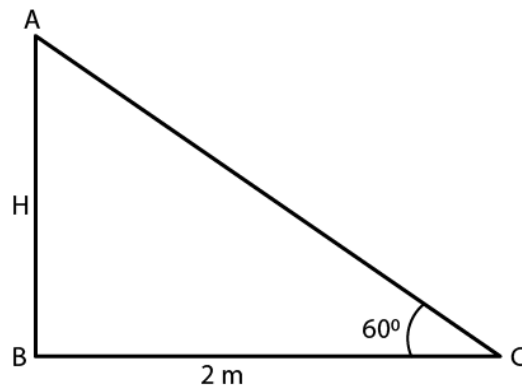
$$\frac{1}{2} = \frac{9.5}{AC}$$

$$AC = 2 \times 9.5 = 19\text{m}$$

Thus, length of the ladder (L) = 19 m

3. A ladder is placed along a wall of a house such that its upper end is touching the top of the wall. The foot of the ladder is 2 m away from the wall and the ladder is making an angle of 60° with the level of the ground. Determine the height of the wall.

Solution:



Given,

Distance between the wall and the foot of the ladder = 2m = BC

Angle made by ladder with ground (θ) = 60°

Height of the wall (H) = AB

Now, the fig. of ABC forms a right angle triangle.

So,

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\tan 60^\circ = AB/BC$$

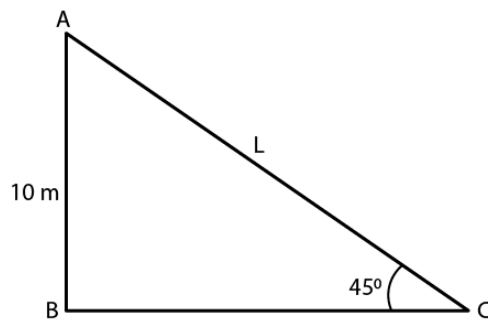
$$\sqrt{3} = \frac{AB}{2}$$

$$AB = 2\sqrt{3}$$

$$\therefore \text{Height of the wall} = 2\sqrt{3}\text{m}$$

4. An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.

Solution:



Given,

Height of the electric pole = 10 m = AB

The angle made by steel wire with ground (horizontal) $\theta = 45^\circ$

Let length of wire = L = AC

So, from the figure formed we have ABC as a right triangle.

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin 45^\circ = \frac{AB}{AC}$$

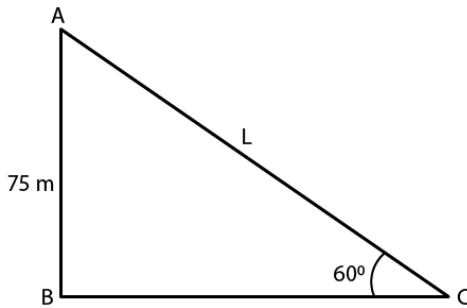
$$\frac{1}{\sqrt{2}} = \frac{10\text{m}}{L}$$

$$L = 10\sqrt{2}\text{m}$$

$$\therefore \text{Length of the wire (L)} = 10\sqrt{2}\text{m}$$

5. A kite is flying at a height of 75 meters from the ground level, attached to a string inclined at 60° to the horizontal. Find the length of the string to the nearest meter.

Solution:



Given,

Height of kite flying from the ground level = 75 m = AB

Angle of inclination of the string with the ground (θ) = 60°

Let the length of the string be $L = AC$

So, from the figure formed we have ABC as a right triangle.

Hence,

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin 60^\circ = \frac{AB}{AC}$$

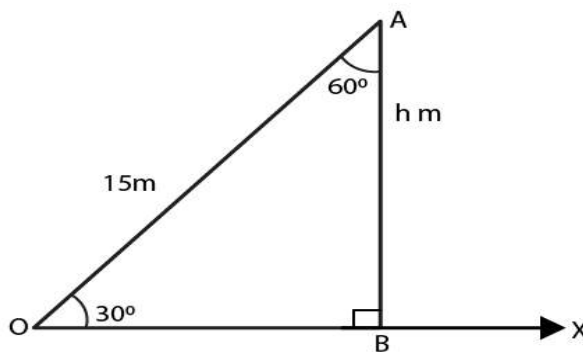
$$\frac{\sqrt{3}}{2} = \frac{75}{L}$$

$$L = 50\sqrt{3}\text{m}$$

$$\text{Length of string } L = 50\sqrt{3}\text{m}$$

6. A ladder 15 metres long reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall.

Solution:



Given,

The length of the ladder = 15m = AO
Angle made by the ladder with the wall = 60°

Let the height of the wall be h metres.
And the horizontal ground taken as OX.
Then from the fig. we have,

In right $\triangle ABO$, using trigonometric ratios
 $\cos (60^\circ) = AB/AO$

$$1/2 = h/15$$

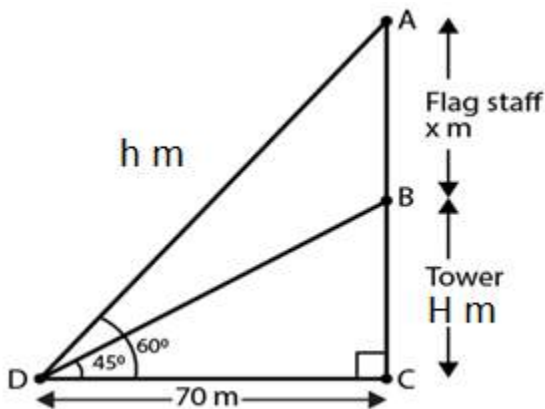
$$h = 15/2$$

$$h = 7.5\text{m}$$

Hence, the height of the wall is 7.5m

7. A vertical tower stands on a horizontal place and is surmounted by a vertical flag staff. At a point on the plane 70 meters away from the tower, an observer notices that the angles of elevation of the top and bottom of the flag-staff are respectively 60° and 45° . Find the height of the flag staff and that of the tower.

Solution:



Given,

A vertical tower is surmounted by flag staff.

Distance between observer and the tower = 70 m = DC

Angle of elevation of bottom of the flag staff = 45°

Angle of elevation of top of the flag staff = 60°

Let the height of the flag staff = h = AD

Height of tower = H = BC

If we represent the above data in the figure then it forms right angle triangles $\triangle ACD$ and $\triangle BCD$

When θ is angle in right angle triangle we know that

$\tan \theta = \text{opp. Side} / \text{Adj. side}$

Now,

$$\tan 45^\circ = BC / DC$$

$$1 = H / 70$$

$$\therefore H = 70 \text{ m}$$

Again,

$$\tan 60^\circ = \frac{AC}{DC}$$

$$\sqrt{3} = \frac{AB + BC}{70} = \frac{x + 70}{70}$$

$$x + 70 = 70\sqrt{3}$$

$$x = 70(\sqrt{3} - 1)$$

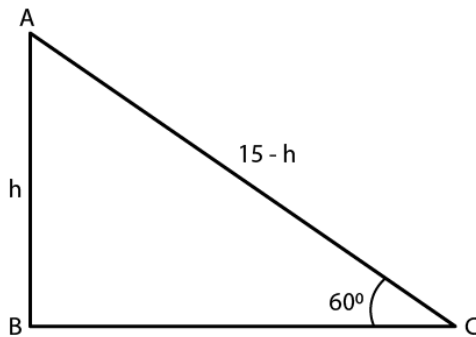
$$x = 70(1.732 - 1)$$

$$\therefore x = 51.24 \text{ m}$$

Therefore, the height of tower = 70 m and the height of flag staff = 51.24 m

8. A vertically straight tree, 15 m high, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break?

Solution:



Given,

The initial height of tree $H = 15 \text{ m} = AB + AC$

Let us assume that it is broken at point A.

And, the angle made by broken part with the ground (θ) = 60°

Height from ground to broken points = $h = AB$

So, we have

$$H = AC + h$$

$$\Rightarrow AC = (H - h) \text{ m}$$

We get a right triangle formed by the above given data,

So,

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin 60^\circ = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{H-h}$$

$$\sqrt{3}(15-h) = 2h$$

$$(2 + \sqrt{3})h = 15\sqrt{3}$$

$$h = \frac{15\sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

Rationalizing denominator, we have

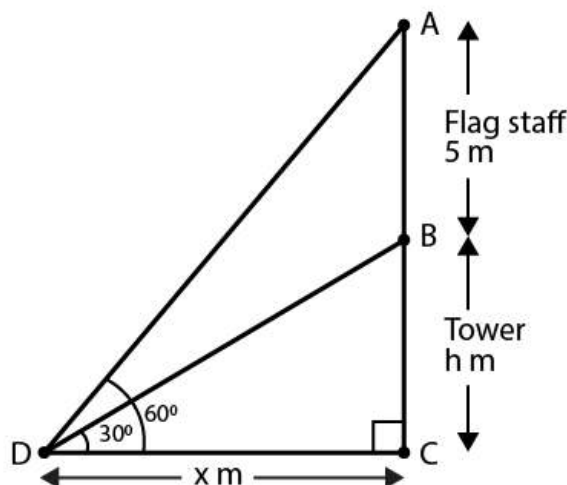
$$h = \frac{(15\sqrt{3})(2 - \sqrt{3})}{2^2 - (\sqrt{3})^2}$$

$$h = 15(2\sqrt{3} - 3)/1$$

Therefore, the height of broken point from the ground is $15(2\sqrt{3} - 3)$ m

9. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height 5 meters. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are respectively 30° and 60° . Find the height of the tower.

Solution:



Given,

Height of the flag staff = 5 m = AB

Angle of elevation of the top of flag staff = 60°

Angle of elevation of the bottom of the flagstaff = 30°

Let height of tower be 'h' m = BC

And, let the distance of the point from the base of the tower = x m

In right angle triangle BCD, we have

$$\tan 30^\circ = BC/DC$$

$$1/\sqrt{3} = h/x$$

$$x = h\sqrt{3} \dots\dots (i)$$

Now, in $\triangle ACD$,

$$\tan 60^\circ = AC/DC$$

$$\sqrt{3} = (5 + h)/x$$

$$\sqrt{3}x = 5 + h$$

$$\sqrt{3}(h\sqrt{3}) = 5 + h \quad [\text{using (i)}]$$

$$3h = 5 + h$$

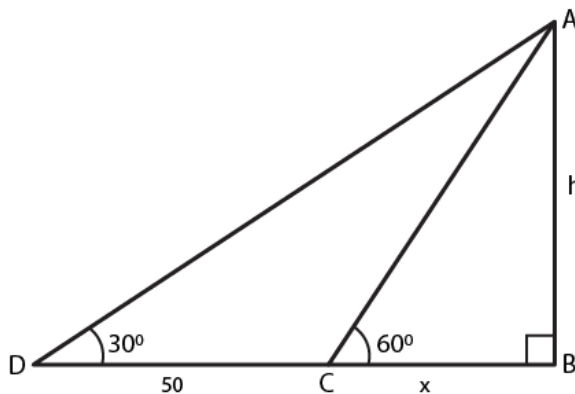
$$2h = 5$$

$$h = 5/2 = 2.5\text{m}$$

Therefore, the height of the tower = 2.5 m

10. A person observed the angle of elevation of a tower as 30° . He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60° . Find the height of the tower.

Solution:



Given,

The angle of elevation of the tower before he started walking = 30°

Distance walked by the person towards the tower = 50m

The angle of elevation of the tower after he walked = 60°

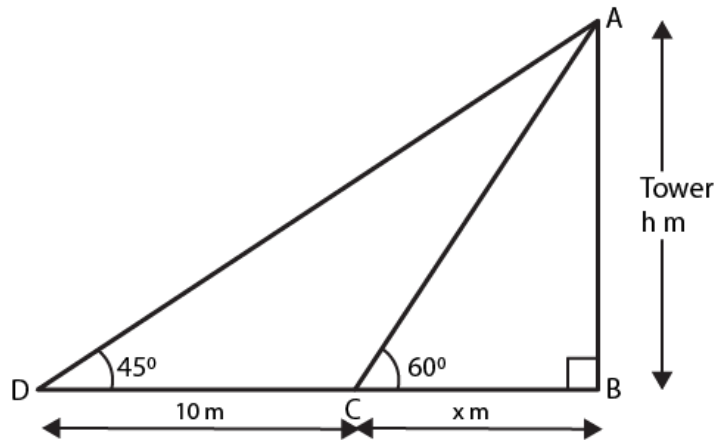
Let height of the tower (AB) = h m

Let the distance $BC = x$ m
 From the fig. in $\triangle ABC$,
 $\tan 60^\circ = AB/BC$
 $\sqrt{3} = h/x$
 $x = h/\sqrt{3} \dots(i)$

Now, in $\triangle ABD$
 $\tan 30^\circ = AB/BD$
 $1/\sqrt{3} = h/(50 + x)$
 $\sqrt{3}h = 50 + x$
 $\sqrt{3}h = 50 + (h/\sqrt{3})$ [using (i)]
 $3h = 50\sqrt{3} + h$
 $2h = 50\sqrt{3}$
 $h = 25\sqrt{3} = 25(1.73) = 43.25\text{m}$
 Therefore, the height of the tower = 43.25m

11. The shadow of a tower, when the angle of elevation of the sun is 45° , is found to be 10 m longer than when it was 60° . Find the height of the tower.

Solution:



Let the height of the tower(AB) = h m
 Let the length of the shorter shadow be x m
 Then, the longer shadow is $(10 + x)$ m
 So, from fig. In $\triangle ABC$
 $\tan 60^\circ = AB/BC$
 $\sqrt{3} = h/x$
 $x = h/\sqrt{3} \dots (i)$

Next, in $\triangle ABD$
 $\tan 45^\circ = AB/BD$
 $1 = h/(10 + x)$
 $10 + x = h$
 $10 + (h/\sqrt{3}) = h$ [using (i)]

$$10\sqrt{3} + h = \sqrt{3}h$$

$$h(\sqrt{3} - 1) = 10\sqrt{3}$$

$$h = 10\sqrt{3} / (\sqrt{3} - 1)$$

After rationalising the denominator, we have

$$h = [10\sqrt{3} \times (\sqrt{3} + 1)] / (3 - 1)$$

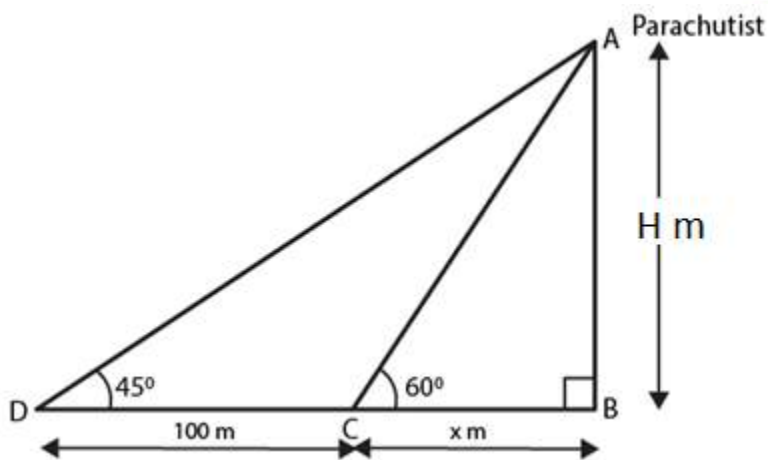
$$h = 5\sqrt{3}(\sqrt{3} + 1)$$

$$h = 5(3 + \sqrt{3}) = 23.66 \quad [\sqrt{3} = 1.732]$$

Therefore, the height of the tower is 23.66 m.

12. A parachute is descending vertically and makes angles of elevation of 45° and 60° at two observing points 100 m apart from each other on the left side of himself. Find the maximum height from which he falls and the distance of point where he falls on the ground from the just observation point.

Solution:



Let the parachute at highest point A and let C and D be points which are 100 m apart on ground where from then $CD = 100$ m

Angle of elevation from point D = $45^\circ = \alpha$

Angle of elevation from point C = $60^\circ = \beta$

Let B be the point just vertically down the parachute

Let us draw figure according to above data then it forms the figure as shown in which ABC and ABD are two triangles

Maximum height of the parachute from the ground

$AB = H$ m

Distance of point where parachute falls to just nearest observation point = x m

If in right angle triangle one of the included angles is θ then

$$\tan \alpha = \frac{AB}{BD}$$

$$\tan 45^\circ = \frac{H}{x+100}$$

$$100 + x = H \dots (a)$$

$$\tan \beta = \frac{AB}{BC}$$

$$\tan 60^\circ = \frac{H}{x}$$

$$H = \sqrt{3}x \dots (b)$$

From (a) and (b)

$$\sqrt{3}x = 100 + x$$

$$(\sqrt{3} - 1)x = 100$$

$$x = \frac{100}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$x = \frac{100(\sqrt{3} + 1)}{2}$$

$$x = 50(\sqrt{3} + 1)$$

$x = 136.6$ m in (b)

$$H = \sqrt{3} \times 136.6 = 236.6\text{m}$$

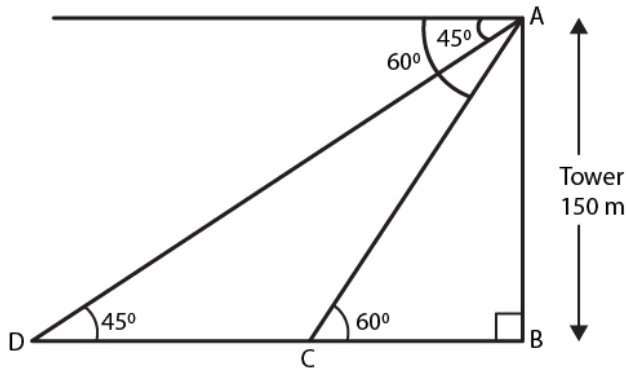
Therefore,

The maximum height of the parachute from the ground, $H = 236.6$ m

Distance between the two points where parachute falls on the ground and just the observation is $x = 136.6$ m

13. On the same side of a tower, two objects are located. When observed from the top of the tower, their angles of depression are 45° and 60° . If the height of the tower is 150 m, find the distance between the objects.

Solution:



Given,

The height of the tower (AB) = 150m

Angles of depressions of the two objects are 45° and 60° .

In $\triangle ABD$

$$\tan 45^\circ = AB / BD$$

$$1 = 150 / BD$$

$$BD = 150\text{m}$$

Next, in $\triangle ABC$

$$\tan 60^\circ = AB / BC$$

$$\sqrt{3} = 150 / BC$$

$$BC = 150 / \sqrt{3}$$

$$BC = 50\sqrt{3} = 50(1.732) = 86.6 \text{ m}$$

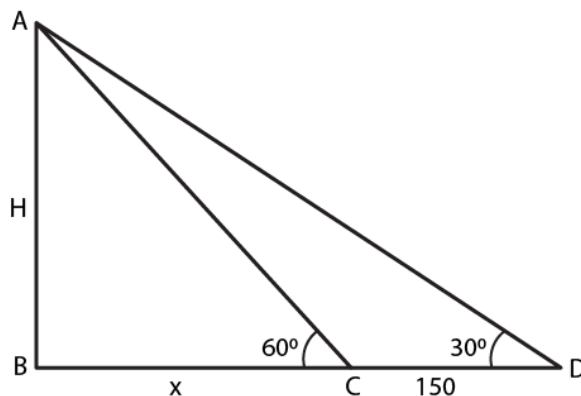
So,

$$DC = BD - BC = 150 - 86.6 = 63.4$$

Therefore, the distance between two objects = 63.4 m

14. The angle of elevation of a tower from a point on the same level as the foot of the tower is 30° . On advancing 150 meters towards the foot of the tower, the angle of elevation of the tower becomes 60° . Show that the height of the tower is 129.9 metres.

Solution:



Given,

The angle of elevation of top tower from first point D, $\alpha = 30^\circ$

On advancing through D to C by 150 m, then $CD = 150$ m

Angle of elevation of top of the tower from second point C, $\beta = 60^\circ$

Let height of tower $AB = H$ m

Representing the above data in form of figure then it form a figure as shown with $\angle B = 90^\circ$

$$\tan \alpha = \frac{AB}{BD}$$

$$\tan 30^\circ = \frac{H}{150 + x}$$

$$150 + x = H\sqrt{3} \dots (a)$$

$$\tan \beta = \frac{AB}{BC}$$

$$\tan 60^\circ = \frac{H}{x}$$

$$H = x\sqrt{3}$$

$$x = \frac{H}{\sqrt{3}} \dots (b)$$

Substituting (b) in (a), we have

$$150 + \frac{H}{\sqrt{3}} = H\sqrt{3}$$

$$H\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 150$$

$$H\left(\frac{3-1}{\sqrt{3}}\right) = 150$$

$$H = \frac{150 \times \sqrt{3}}{2}$$

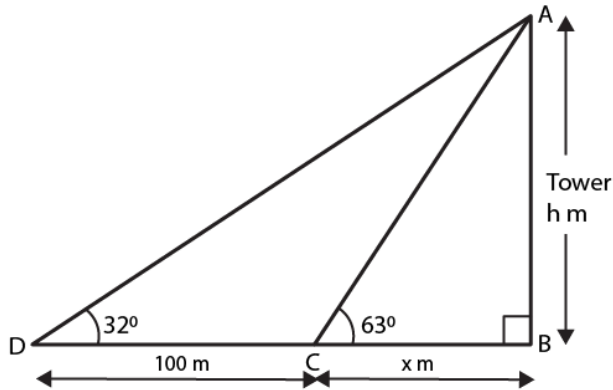
$$H = 75\sqrt{3}$$

$$H = 129.9$$

Therefore, the height of the tower = 129.9 m

15. The angle of elevation of the top of a tower as observed from a point in a horizontal plane through the foot of the tower is 32° . When the observer moves towards the tower a distance of 100 m, he finds the angle of elevation of the top to be 63° . Find the height of the tower and the distance of the first position from the tower. [Take $\tan 32^\circ = 0.6248$ and $\tan 63^\circ = 1.9626$]

Solution:



Let the height of the tower = h m

And the distance BC = x m

Then, from the fig.

In $\triangle ABC$

$$\tan 63^\circ = \frac{AB}{BC}$$

$$1.9626 = \frac{h}{x}$$

$$x = \frac{h}{1.9626}$$

$$x = 0.5095 h \dots (i)$$

Next, in $\triangle ABD$

$$\tan 32^\circ = \frac{AB}{BD}$$

$$0.6248 = \frac{h}{(100 + x)}$$

$$h = 0.6248(100 + x)$$

$$h = 62.48 + 0.6248x$$

$$h = 62.48 + 0.6248(0.5095 h) \dots [using (i)]$$

$$h = 62.48 + 0.3183h$$

$$0.6817h = 62.48$$

$$h = \frac{62.48}{0.6817} = 91.65$$

Using h in (i), we have

$$x = 0.5095(91.65) = 46.69$$

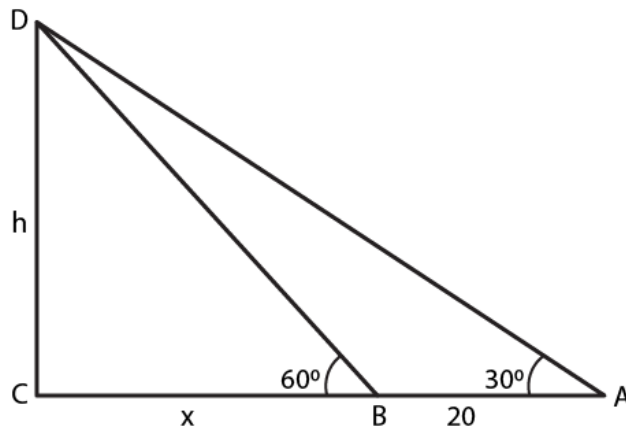
Therefore,

the height of the tower is 91.65 m

Distance of the first position from the tower = $100 + x = 146.69$ m

16. The angle of elevation of the top of a tower from a point A on the ground is 30° . On moving a distance of 20 meters towards the foot of the tower to a point B the angle of elevation increases to 60° . Find the height of the tower and the distance of the tower from the point A.

Solution:



Given,

Angle of elevation of top of the tower from point A, $\alpha = 30^\circ$.

Angle of elevation of top of tower from point B, $\beta = 60^\circ$.

And, the distance between A and B, $AB = 20$ m

Let height of tower $CD = 'h'$ m and distance between second point B from foot of the tower be ' x ' m

Representing the above data in form of figure with $\angle C = 90^\circ$,

$$\tan \alpha = \frac{CD}{AD}$$

$$\tan 30^\circ = \frac{h}{20 + x}$$

$$20 + x = h\sqrt{3} \dots (a)$$

$$\tan \beta = \frac{CD}{BD}$$

$$\tan 60^\circ = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \dots (b)$$

Substituting (b) in (a), we have

$$20 + \frac{h}{\sqrt{3}} = h\sqrt{3}$$

$$h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 20$$

$$h\left(\frac{3-1}{\sqrt{3}}\right) = 20$$

$$h = \frac{20 \times \sqrt{3}}{2}$$

$$h = 10\sqrt{3}$$

$$h = 17.32 \text{ m}$$

$$x = \frac{10\sqrt{3}}{\sqrt{3}}$$

$$x = 10 \text{ m}$$

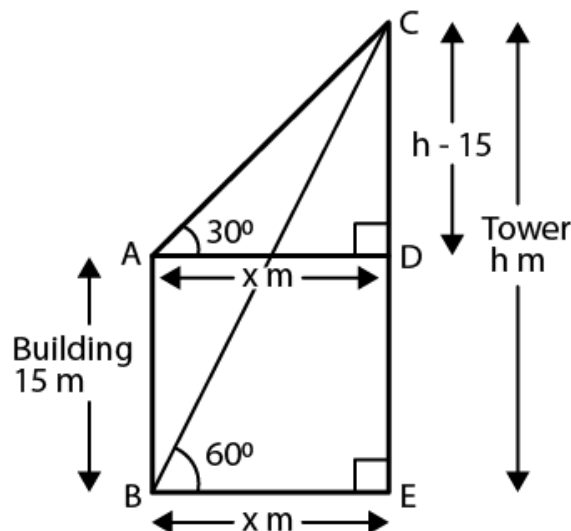
Therefore,

Height of the tower = 17.32 m

Distance of the tower from point A = (20 + 10) = 30 m

17. From the top of a building 15 m high the angle of elevation of the top of tower is found to be 30° . From the bottom of the same building, the angle of elevation of the top of the tower is found to be 60° . Find the height of the tower and the distance between the tower and the building.

Solution:



Given,

The height of the building = 15 m

The angle of elevation from the top of the building to top of the tower = 30°

The angle of elevation from the bottom of the building to top of the tower = 60°

Let the height of the tower = h m

And the distance between tower and building = x m

So, $AD = x$ [since, $AD \parallel BE$]

Similarly, $AB = DE$

Then, from fig.

In $\triangle BEC$,

$$\tan 60^\circ = CE/BE$$

$$\sqrt{3} = h/x$$

$$x = h/\sqrt{3} \dots (i)$$

Next, in $\triangle CDA$,

$$\tan 30^\circ = CD/DA$$

$$1/\sqrt{3} = (h - 15)/x$$

$$\sqrt{3}h - 15\sqrt{3} = x$$

$$\sqrt{3}h - 15\sqrt{3} = h/\sqrt{3} \quad [\text{using (i)}]$$

$$3h - 45 = h$$

$$2h = 45$$

$$h = 45/2 = 22.5$$

Putting h in (i), we get

$$x = 22.5/(\sqrt{3})$$

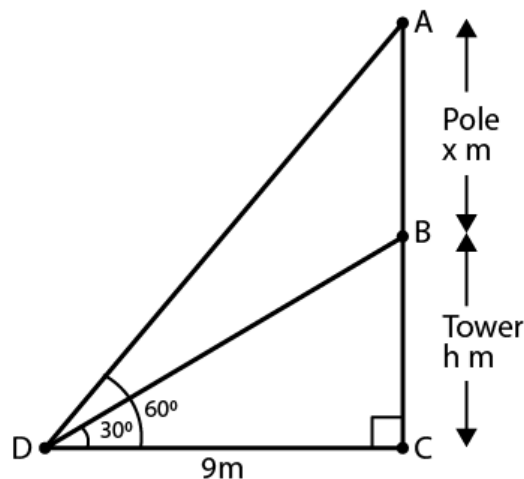
$$x = 22.5/(1.73) = 12.990$$

Therefore, the height of the building = 22.5 m

And, distance between tower and building = 12.990 m

18. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 m away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it.

Solution:



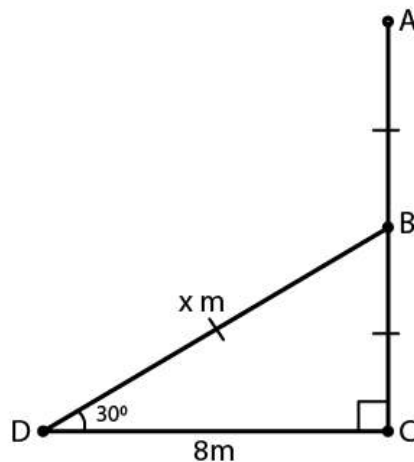
Let BC be the tower and AB be the flag pole on the tower
 Distance of the point of observation from foot of the tower DC = 9 m
 Angle of elevation of top of flag pole is 60°
 Angle of elevation of bottom of flag pole is 30°
 Let height of the tower = h m = BC
 Height of the pole = x m = AB
 From fig, we have
 In $\triangle BCD$,
 $\tan 30^\circ = BC/DC$
 $1/\sqrt{3} = h/9$
 $h = 9/\sqrt{3} = 3\sqrt{3}$

Next, in $\triangle ACD$
 $\tan 60^\circ = AC/DC$
 $\sqrt{3} = (x + h)/9$
 $x + h = 9\sqrt{3}$
 $x + 3\sqrt{3} = 9\sqrt{3}$
 $x = 6\sqrt{3}$ m

Therefore,
 Height of the tower = $3\sqrt{3}$ m
 And, height of the pole = $6\sqrt{3}$ m

19. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Solution:



Let the initial height of the tree be AC.
 And, due to storm the tree is broken at B.
 Let the bent portion of the tree be AB = x m and the remaining portion BC = h m
 So, the height of the tree AC = (x + h) m
 And, given DC = 8m

Now, in $\triangle BCD$
 $\tan 30^\circ = BC/DC$
 $1/\sqrt{3} = h/8$
 $h = 8/\sqrt{3}$

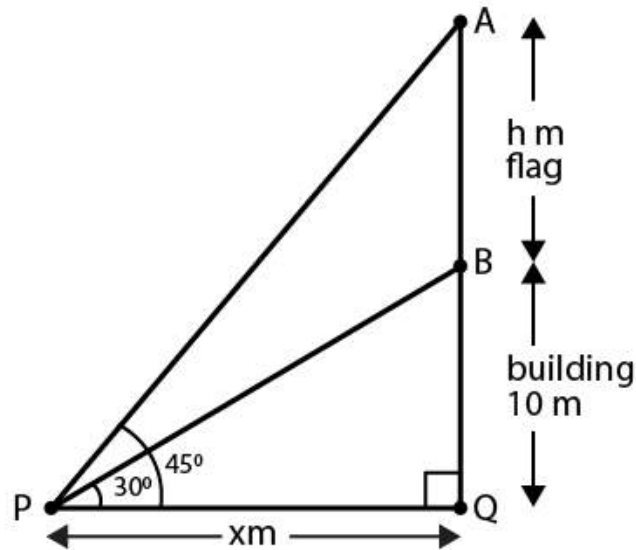
Next, in $\triangle BCD$
 $\cos 30^\circ = DC/BD$
 $\sqrt{3}/2 = 8/x$
 $x = 16/\sqrt{3} \text{ m}$

So, $x + h = 16/\sqrt{3} + 8/\sqrt{3}$
 $= 24/\sqrt{3} = 8\sqrt{3}$

Therefore, the height of the tree is $8\sqrt{3} \text{ m}$.

20. From a point P on the ground the angle of elevation of a 10 m tall building is 30° . A flag is hoisted at the top of the building and the angle of elevation of the top of the flag staff from P is 45° . Find the length of the flag staff and the distance of the building from the point P.

Solution:



Let the height of flag-staff(AB) = $h \text{ m}$

And, the distance $PQ = x \text{ m}$

Given,

Angle of elevation of top of the building = 30°

Angle of elevation of top of the flag staff = 45°

From the fig.

In $\triangle BQP$,

$\tan 30^\circ = BQ/PQ$

$1/\sqrt{3} = 10/x$

$x = 10\sqrt{3} \text{ m}$

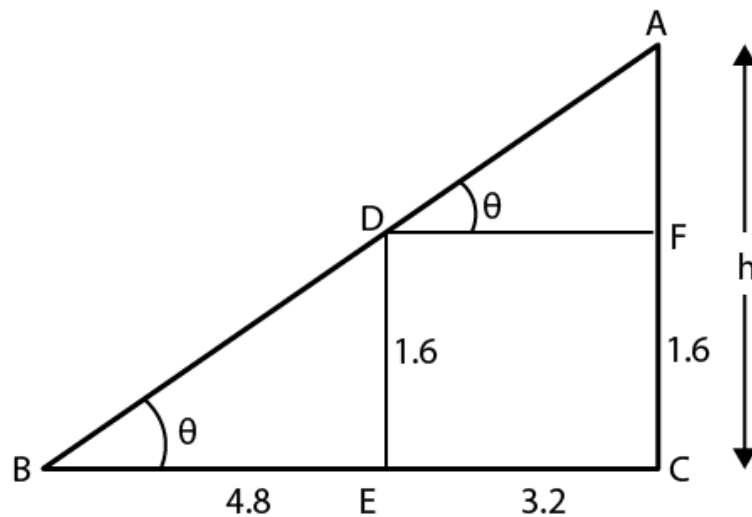
Next,

In $\triangle AQP$,
 $\tan 45^\circ = AQ/PQ$
 $1 = (h + 10)/x$
 $h + 10 = x = 10\sqrt{3}$
 $h = (10\sqrt{3} - 10) = 10(1.732) - 10 = 17.32 - 10$
 $= 7.32 \text{ m}$

Therefore, the distance of point P from building = $x = 10\sqrt{3} = 10(1.732) = 17.32\text{m}$

21. A 1.6 m tall girl stands at a distance of 3.2 m from a lamp post and casts a shadow of 4.8 m on the ground. Find the height of the lamp post by using (i) trigonometric ratio (ii) properties of similar triangles.

Solution:



Let AC be the lamp post of height 'h'
 DE is the tall girl and her shadow is BE.
 So, we have ED = 1.6 m, BE = 4.8 m and EC = 3.2

(i) By using trigonometric ratio

In $\triangle BDE$,
 $\tan \theta = 1.6/4.8$
 $\tan \theta = 1/3$
 Next, In $\triangle ABC$
 $\tan \theta = h/(4.8 + 3.2)$
 $1/3 = h/8$
 $h = 8/3 \text{ m}$

(ii) By using similar triangles

Since triangle BDE and triangle ABC are similar (by AA criteria), we have

$$\frac{AC}{BC} = \frac{ED}{BE}$$

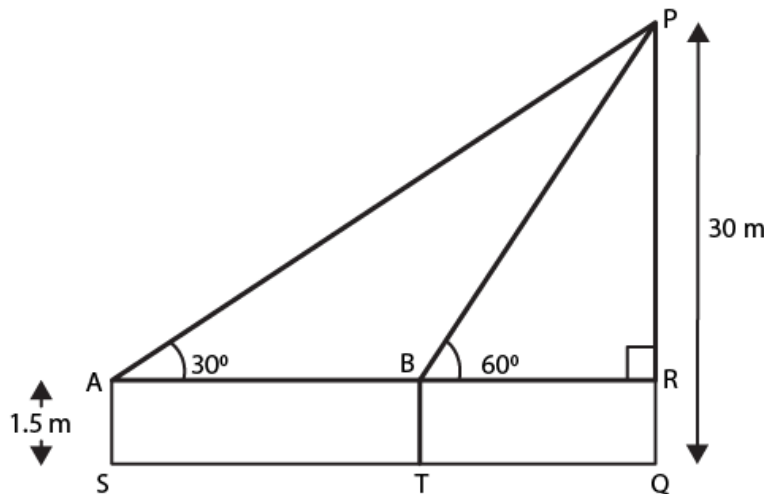
$$\frac{h}{4.8 + 3.2} = \frac{1.6}{4.8}$$

$$h = \frac{8}{3} \text{ m}$$

Therefore, the height of the lamp post is $h = 8/3$ m

22. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Solution:



Given,

The height of the tall boy (AS) = 1.5 m

The length of the building (PQ) = 30 m

Let the initial position of the boy be S. And, then he walks towards the building and reached at the point T.

From the fig. we have

$$AS = BT = RQ = 1.5 \text{ m}$$

$$PR = PQ - RQ = (30 - 1.5) \text{ m} = 28.5 \text{ m}$$

In $\triangle PAR$,

$$\tan 30^\circ = PR/AR$$

$$1/\sqrt{3} = 28.5/AR$$

$$AR = 28.5\sqrt{3}$$

In $\triangle PRB$,

$$\tan 60^\circ = PR/BR$$

$$\sqrt{3} = 28.5/BR$$

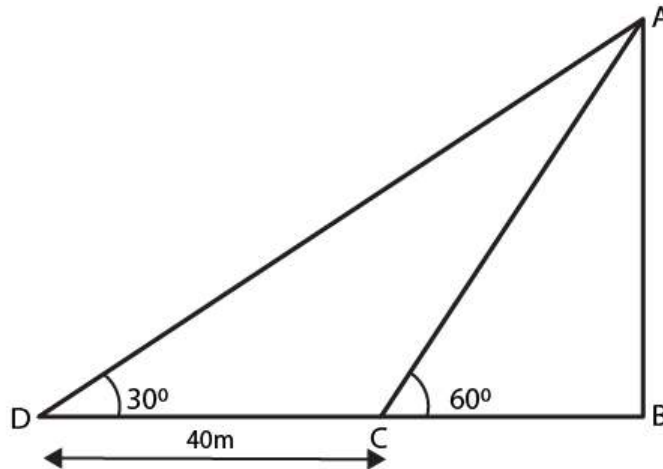
$$BR = 28.5/\sqrt{3} = 9.5\sqrt{3}$$

$$\text{So, } ST = AB = AR - BR = 28.5\sqrt{3} - 9.5\sqrt{3} = 19\sqrt{3}$$

Therefore, the distance which the boy walked towards the building is $19\sqrt{3}$ m.

23. The shadow of a tower standing on level ground is found to be 40 m longer when Sun's altitude is 30° than when it was 60° . Find the height of the tower.

Solution:



When the sun's altitude is the angle of elevation of the top of the tower from the tip of the shadow.

Let AB be h m and BC be x m. From the question, DC is 40 m longer than BC.

$$\text{So, } BD = (40 + x) \text{ m}$$

And two right triangles ABC and ABD are formed.

In $\triangle ABC$,

$$\tan 60^\circ = AB/BC$$

$$\sqrt{3} = h/x$$

$$x = h/\sqrt{3} \dots (i)$$

In $\triangle ABD$,

$$\tan 30^\circ = AB/BD$$

$$1/\sqrt{3} = h/(x + 40)$$

$$x + 40 = \sqrt{3}h$$

$$h/\sqrt{3} + 40 = \sqrt{3}h \quad [\text{using (i)}]$$

$$h + 40\sqrt{3} = 3h$$

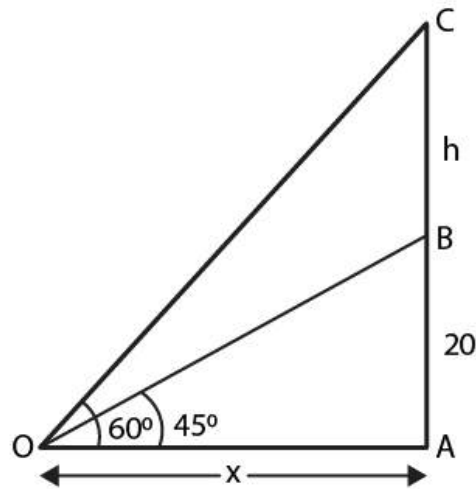
$$2h = 40\sqrt{3}$$

$$h = 20\sqrt{3}$$

Therefore, the height of the tower is $20\sqrt{3}$ m.

24. From a point on the ground the angle of elevation of the bottom and top of a transmission tower fixed at the top of 20 m high building are 45° and 60° respectively. Find the height of the transmission tower.

Solution:



Given,

Height of the building = 20 m = AB

Let height of tower above building = h = BC

Height of tower + building = (h + 20) m [from ground] = OA

Angle of elevation of bottom of tower, $\alpha = 45^\circ$

Angle of elevation of top of tower, $\beta = 60^\circ$

Let distance between tower and observation point = x m

Then, from the fig. we have

$$\tan \alpha = \frac{AB}{OA}$$

$$\tan 45^\circ = \frac{20}{x}$$

$$x = 20 \text{ m}$$

Next,

$$\tan \beta = \frac{CA}{OA}$$

$$\tan 60^\circ = \frac{h + 20}{x}$$

$$h + 20 = 20\sqrt{3} \text{ m}$$

$$h = 20(\sqrt{3} - 1) \text{ m}$$

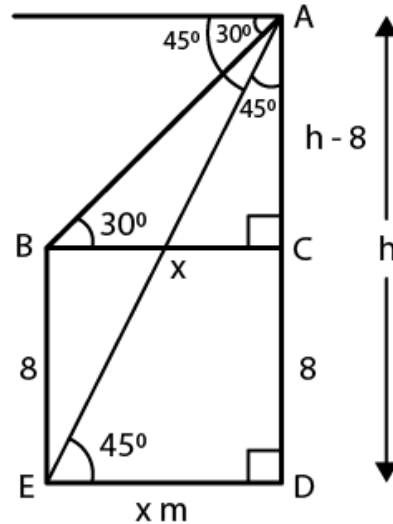
$$\text{Height of the tower } h = 20(\sqrt{3} - 1) \text{ m}$$

$$h = 20(1.73 - 1) = 20 \times .732 = 14.64$$

Therefore, the height of the tower is 14.64m

25. The angle of depression of the top and bottom of 8 m tall building from the top of a multistoried building are 30° and 45° . Find the height of the multistoried building and the distance between the two buildings.

Solution:



Let height of multistoried building 'h' m = AD

Height of the tall building = 8 m = BE

Angle of depression of top of the tall building from the multistoried building = 30°

Angle of depression of bottom of the tall building from the multistoried building = 30°

And, let the distance between the two buildings = 'x' m = ED

So, BC = x and CD = 8 m [As BCDE forms a rectangle]

AD = AC + CD

So, AC = (h - 8) m

From the fig. we have

In $\triangle BCA$

$$\tan 30^\circ = AC/BC$$

$$1/\sqrt{3} = (h - 8)/x$$

$$x = \sqrt{3}(h - 8) \dots \dots (i)$$

In $\triangle ADE$

$$\tan 45^\circ = AD/ED$$

$$1 = h/x$$

$$h = x$$

$$h = \sqrt{3}(h - 8) \text{ [using (i)]}$$

$$h = \sqrt{3}h - 8\sqrt{3}$$

$$(\sqrt{3} - 1)h = 8\sqrt{3}$$

$$h = 8\sqrt{3}/(\sqrt{3} - 1)$$

Rationalising the denominator by $(\sqrt{3} + 1)$, we have

$$h = 8\sqrt{3}(\sqrt{3} + 1)/(3 - 1)$$

$$h = 4\sqrt{3}(\sqrt{3} + 1)$$

$$h = 12 + 4\sqrt{3}$$

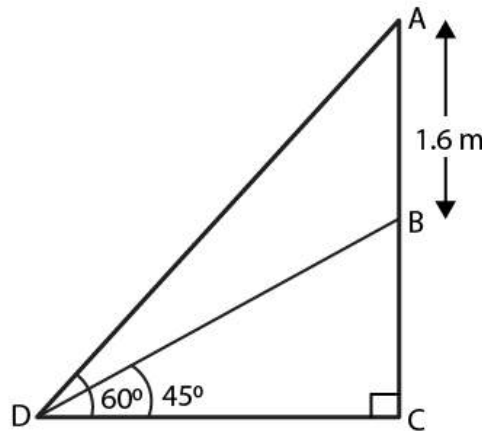
$$= 4(3 + \sqrt{3})$$

Thus, $x = h = 4(3 + \sqrt{3})$

Therefore, the height of the building is $4(3 + \sqrt{3})$ m and the distance between the building is also $4(3 + \sqrt{3})$ m

26. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.

Solution:



Let's assume AB as the statue, BC be the pedestal and D be the point on ground from where elevation angles are measured.

Given,

Angle of elevation of the top of statue is 60°

Angle of elevation of the top of the pedestal is 45°

So, from the fig. we have

In $\triangle BCD$,

$$\tan 45^\circ = BC / CD$$

$$1 = BC / CD$$

$$CD = BC$$

Next, In $\triangle ADC$

$$\tan 60^\circ = (AB + BC) / CD$$

$$\sqrt{3} = (AB + BC) / BC \quad [\text{As } CD = BC, \text{ found above}]$$

$$BC\sqrt{3} = AB + BC$$

$$AB = (\sqrt{3} - 1)BC$$

$$BC = AB / (\sqrt{3} - 1)$$

$$BC = 1.6 / (\sqrt{3} - 1)$$

Rationalising the denominator, we have

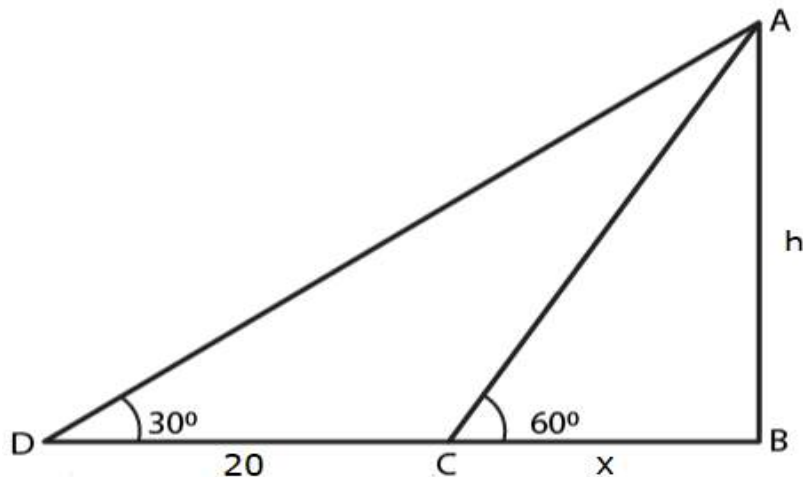
$$BC = 1.6(\sqrt{3} + 1) / 2$$

$$BC = 0.8(\sqrt{3} + 1) \text{ m}$$

Therefore, the height of pedestal is $0.8(\sqrt{3} + 1)$ m

27. A T.V. tower stands vertically on a bank of a river of a river. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° . From a point 20 m away this point on the same bank, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the river.

Solution:



Let AB be the T.V tower of height 'h' m on the bank of river and 'D' be the point on the opposite side of the river. An angle of elevation at the top of the tower is 30°

Let AB = h and BC = x

And, given CD = 20 m

From fig. we have

In $\triangle ACB$

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{x}$$

$$\sqrt{3}x = h$$

$$x = \frac{h}{\sqrt{3}}$$

Next, in $\triangle DBA$,

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{20 + x}$$

$$\sqrt{3}h = 20 + x$$

$$\sqrt{3}h = 20 + \frac{h}{\sqrt{3}}$$

$$\sqrt{3}h - \frac{h}{\sqrt{3}} = 20$$

$$\frac{2h}{\sqrt{3}} = 20$$

$$h = 10\sqrt{3} \text{ m}$$

And,

$$x = h/\sqrt{3}$$

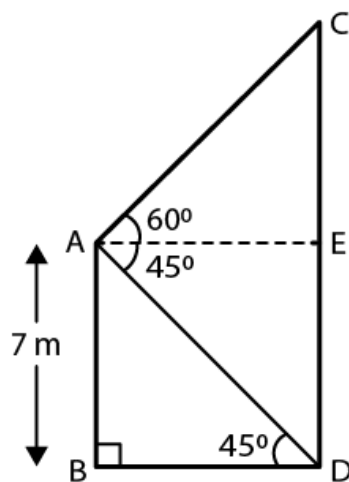
$$x = 10\sqrt{3}/\sqrt{3}$$

$$x = 10$$

Therefore, the height of the tower is $10\sqrt{3}$ m and width of the river is 10 m.

28. From the top of a 7 m high building, the angle of elevation of the top of a cable is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.

Solution:



Given

Height of the building = 7 m = AB

Height of the cable tower = CD

Angle of elevation of the top of the cable tower from the top of the building = 60°
 Angle of depression of the bottom of the building from the top of the building = 45°

Then, from the fig. we see that

$$ED = AB = 7 \text{ m}$$

And,

$$CD = CE + ED$$

So, In $\triangle ABD$, we have

$$AB/BD = \tan 45^\circ$$

$$AB = BD = 7$$

$$BD = 7$$

In $\triangle ACE$,

$$AE = BD = 7$$

$$\text{And, } \tan 60^\circ = CE/AE$$

$$\sqrt{3} = CE/7$$

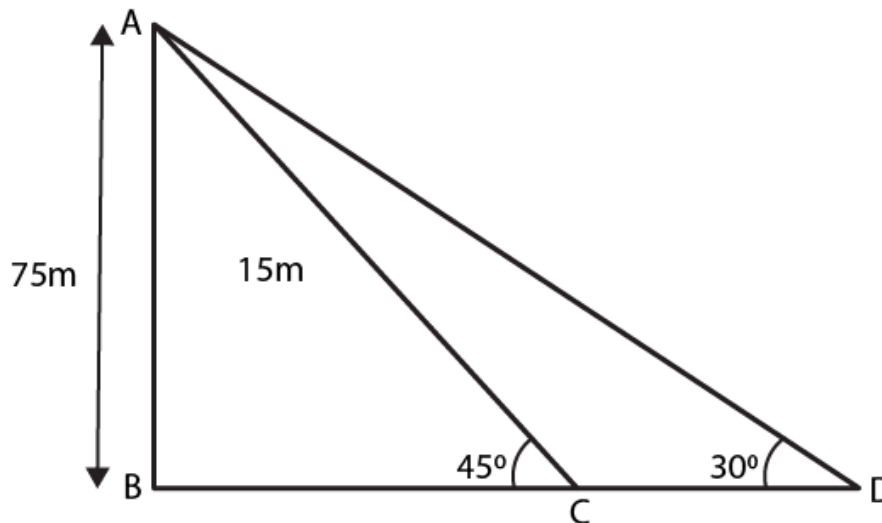
$$CE = 7\sqrt{3} \text{ m}$$

$$\text{So, } CD = CE + ED = (7\sqrt{3} + 7) = 7(\sqrt{3} + 1) \text{ m}$$

Therefore, the height of the cable tower is $7(\sqrt{3} + 1) \text{ m}$

29. As observed from the top of a 75 m tall lighthouse, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Solution:



Given;

Height of the lighthouse = 75m = 'h' m = AB

Angle of depression of ship 1, $\alpha = 30^\circ$

Angle of depression of bottom of the tall building, $\beta = 45^\circ$

The above data is represented in form of figure as shown

Let distance between ships be 'x' m = CD

If in right angle triangle one of the included angle is θ

$$\tan \alpha = \frac{AB}{DB}$$

$$\tan 30^\circ = \frac{75}{x + BC}$$

$$x + BC = 75\sqrt{3} \dots (1)$$

$$\tan \beta = \frac{AB}{BC}$$

$$\tan 45^\circ = \frac{75}{BC}$$

$$BC = 75 \dots (2)$$

Substituting (2) in (1)

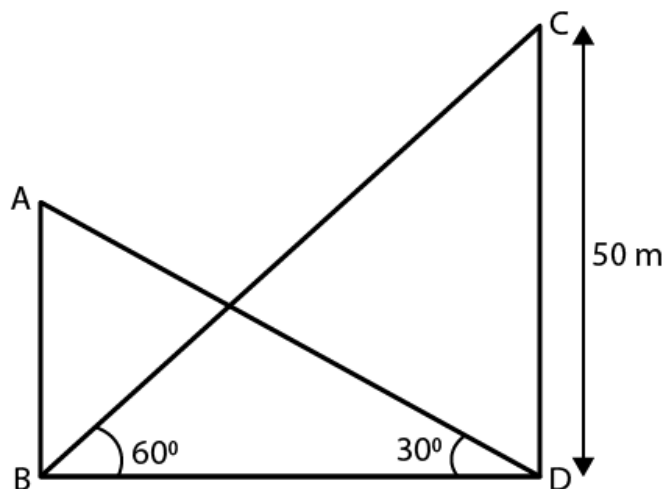
$$x + 75 = 75\sqrt{3}$$

$$x = 75(\sqrt{3} - 1)$$

Therefore, the distance between the two ships is $75(\sqrt{3} - 1)$ m

30. The angle of elevation of the top of the building from the foot of the tower is 30° and the angle of the top of the tower from the foot of the building is 60° . If the tower is 50 m high, find the height of the building.

Solution:



Let AB be the building and CD be the tower.

Given,

The angle of elevation of the top of the building from the foot of the tower is 30° .

And, the angle of elevation of the top of the tower from the foot of the building is 60° .

Height of the tower = $CD = 50$ m

From the fig. we have

In $\triangle CDB$,

$$CD/BD = \tan 60^\circ$$

$$50/BD = \sqrt{3}$$

$$BD = 50/\sqrt{3} \dots (i)$$

Next in $\triangle ABD$,

$$AB/BD = \tan 30^\circ$$

$$AB/BD = 1/\sqrt{3}$$

$$AB = BD/\sqrt{3}$$

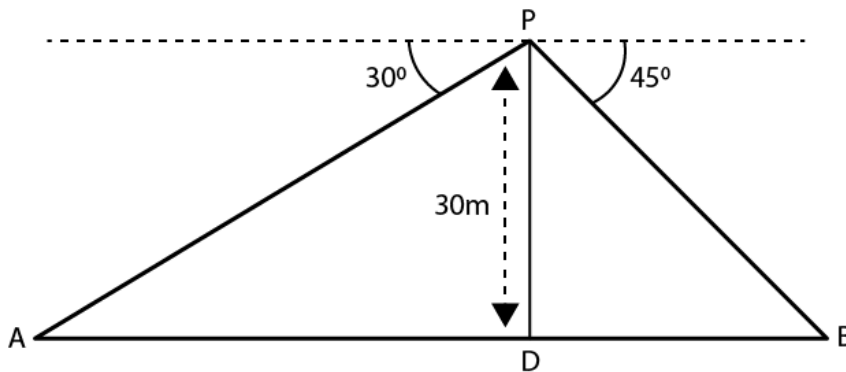
$$AB = 50/\sqrt{3}/(\sqrt{3}) \text{ [From (i)]}$$

$$AB = 50/3$$

Therefore, the height of the building is $50/3$ m.

31. From a point on a bridge across a river the angle of depression of the banks on opposite side of the river are 30° and 45° respectively. If the bridge is at the height of 30 m from the banks, find the width of the river.

Solution:



Given,

The bridge is at a height of 30 m from the banks.

Let, A and B represent the points on the bank on opposite sides of the river. And, AB is the width of the river. P is a point on the bridge which is at the height of 30 m from the banks.

Now, from the fig, we have

$$AB = AD + DB$$

In right $\triangle APD$,

$$\angle A = 30^\circ$$

$$\text{So, } \tan 30^\circ = PD/AD$$

$$1/\sqrt{3} = PD/AD$$

$$AD = \sqrt{3}(30)$$

$$AD = 30\sqrt{3} \text{ m}$$

Next, in right $\triangle PBD$

$$\angle B = 45^\circ$$

So, $\tan 45^\circ = PD/BD$

$$1 = PD/BD$$

$$BD = PD$$

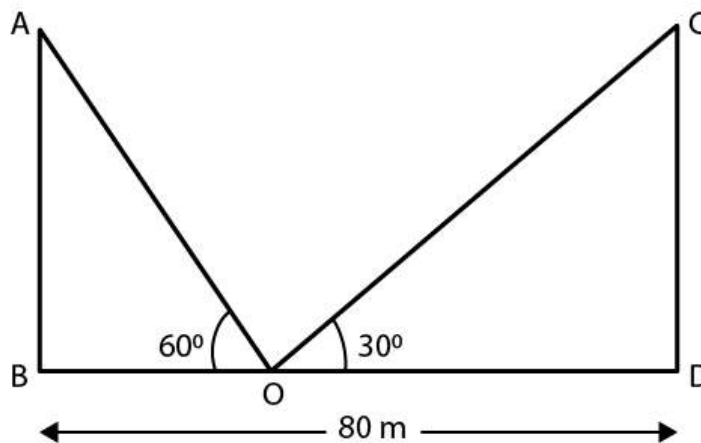
$$BD = 30 \text{ m}$$

We know that, $AB = AD + DB = 30\sqrt{3} + 30 = 30(\sqrt{3} + 1)$

Hence, the width of the river = $30(1 + \sqrt{3})\text{m}$

32. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point between them on the road the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Solution:



Given,

Distance between the poles = 80 m = BD

Let the point of observation of the angles be O.

The angles of elevation to the top of the points is 60° and 30°

Let AB and CD be the poles and O is the point on the road.

From the fig. we have

In $\triangle ABO$,

$$AB/BO = \tan 60^\circ$$

$$AB/BO = \sqrt{3}$$

$$BO = AB/\sqrt{3} \dots(i)$$

Next in $\triangle CDO$,

$$CD/DO = \tan 30^\circ$$

$$CD/(80 - BO) = 1/\sqrt{3}$$

$$\sqrt{3}CD = 80 - BO$$

$$\sqrt{3}AB = 80 - (AB/\sqrt{3}) \text{ [As } AB = CD \text{ and using (i)]}$$

$$3AB = 80\sqrt{3} - AB$$

$$4AB = 80\sqrt{3}$$

$$AB = 20\sqrt{3}$$

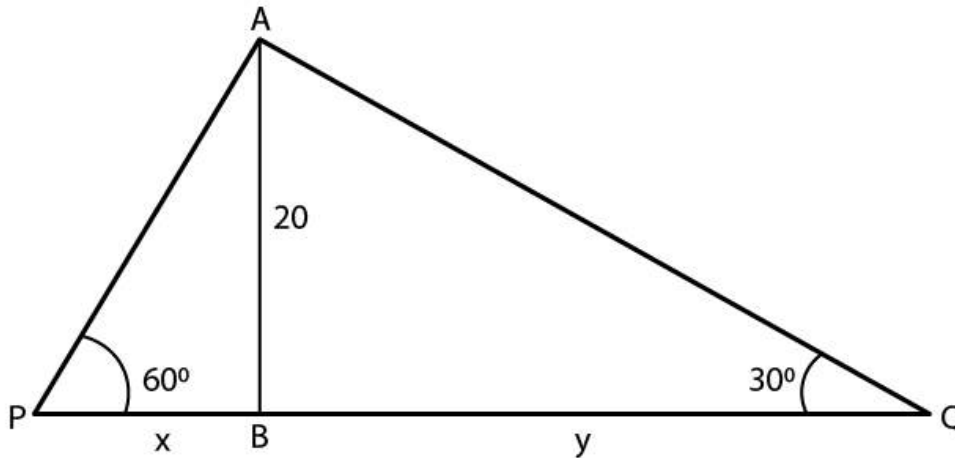
$$\text{So, } BO = 20\sqrt{3}/(\sqrt{3}) = 20 \text{ m}$$

$$\text{And, } DO = BD - BO = 80 - 20 = 60$$

Hence, the height of the poles is $20\sqrt{3}$ m and the distances of the points from the poles is 20m and 60 m.

33. A man sitting at a height of 20 m on a tall tree on a small island in the middle of a river observes two poles directly opposite to each other on the two banks of the river and in line with foot of tree. If the angles of depression of the feet of the poles from a point at which the man is sitting on the tree on either side of the river are 60° and 30° respectively. Find the width of the river.

Solution:



From the given data, the fig. is made

Let width of river = $PQ = (x + y)$ m

Height of tree (AB) = 20 m

So, in $\triangle ABP$

$$\tan 60^\circ = AB / BP$$

$$\sqrt{3} = 20 / x$$

$$x = 20 / \sqrt{3} \text{ m}$$

In $\triangle ABQ$,

$$\tan 30^\circ = AB / BQ$$

$$1 / \sqrt{3} = 20 / y$$

$$y = 20\sqrt{3}$$

$$\text{So, } (x + y) = 20 / \sqrt{3} + 20\sqrt{3}$$

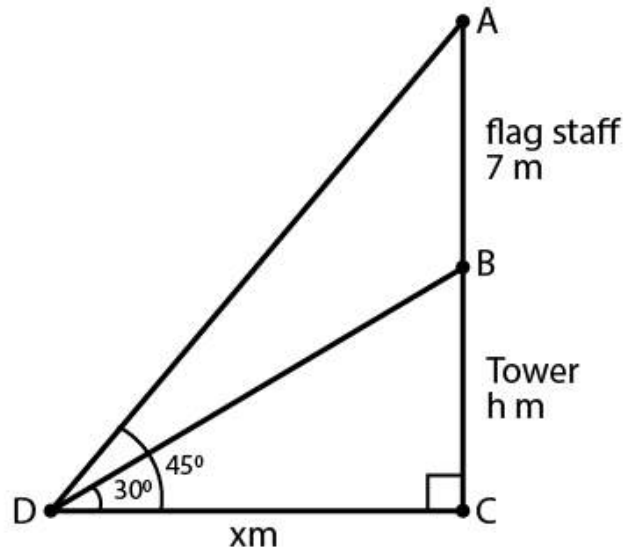
$$= [20 + 20(3)] / \sqrt{3}$$

$$= 80 / \sqrt{3}$$

Therefore, the width of the river is $80 / \sqrt{3}$ m.

34. A vertical tower stands on a horizontal plane and is surmounted by a flag staff of height 7m. From a point on the plane, the angle of elevation of the bottom of flag staff is 30° and that of the top of the flag staff is 45° . Find the height of the tower.

Solution:



Given,

The length of the flag staff = 7 m

Angles of elevation of the top and bottom of the flag staff from point D is 45° and 30° respectively.

Let the height of tower (BC) = h m

And, let DC = x m

So, in $\triangle BCD$

$$\tan 30^\circ = \frac{BC}{DC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = h\sqrt{3} \dots (i)$$

And, in $\triangle ACD$

$$\tan 45^\circ = \frac{AC}{DC}$$

$$1 = \frac{(7 + h)}{x}$$

$$x = 7 + h$$

$$h\sqrt{3} = 7 + h \quad [\text{from (i)}]$$

$$h(\sqrt{3} - 1) = 7$$

$$h = \frac{7}{(\sqrt{3} - 1)}$$

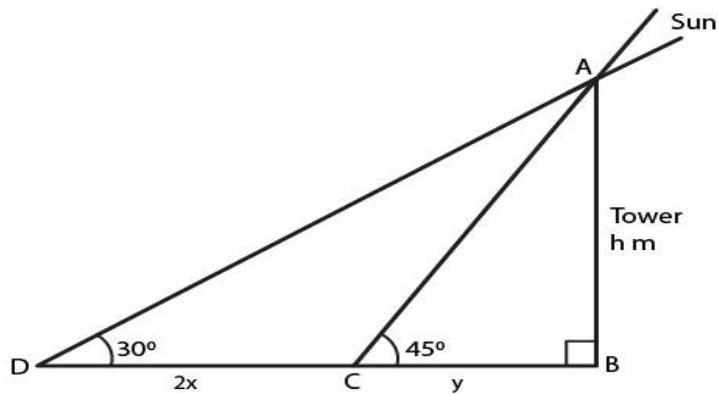
Now, rationalising the denominator we get

$$h = \frac{7(\sqrt{3} + 1)}{2} = \frac{7(1.732 + 1)}{2} = 9.562$$

Therefore, the height of the tower is 9.56 m

35. The length of the shadow of a tower standing on level plane is found to be $2x$ meters longer when the sun's attitude is 30° than when it was 60° . Prove that the height of tower is $x(\sqrt{3}+1)$ meters.

Solution:



From the question, the following fig. is made

Let the height of tower (AB) = h m

Let the distance BC = y m

Then, in $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{h}{y}$$

$$y = h$$

Next, in $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{2x + y}$$

$$2x + y = \sqrt{3}h$$

$$2x + h = \sqrt{3}h$$

$$2x = (\sqrt{3} - 1)h$$

$$h = \frac{2x}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$$

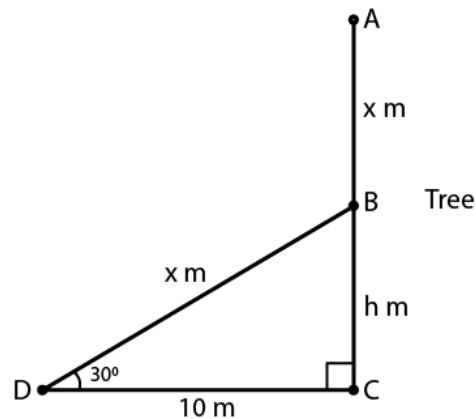
$$h = \frac{2x(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{2x(\sqrt{3} + 1)}{3 - 1} = x(\sqrt{3} + 1)$$

Therefore, the height of the tower is $x(\sqrt{3} + 1)$ m

Hence Proved

36. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 meters. Find the height of the tree.

Solution:



Let AC be the height of the tree which is $(x + h)$ m
 Given, the broken portion of the tree is making an angle of 30° with the ground.
 From the fig.

In $\triangle BCD$, we have
 $\tan 30^\circ = BC/DC$
 $1/\sqrt{3} = h/10$
 $h = 10/\sqrt{3}$

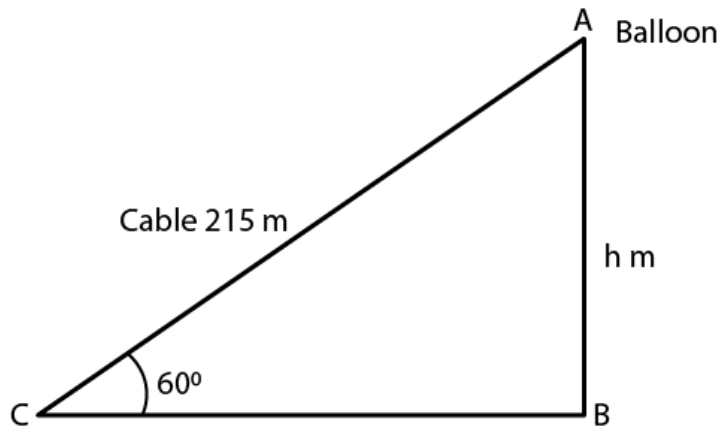
Next, in $\triangle BCD$
 $\cos 30^\circ = DC/BD$
 $\sqrt{3}/2 = 10/x$
 $x = 20/\sqrt{3}$ m

So,
 $x + h = 20/\sqrt{3} + 10/\sqrt{3}$
 $= 30/\sqrt{3}$
 $= 10\sqrt{3} = 10(1.732) = 17.32$

Therefore, the height of the tree is 17.32 m

37. A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at 60° to the horizontal. Determine the height of the balloon from the ground. Assume that there is no slack in the cable.

Solution:



Let the height of the balloon from the ground = h m

Given, the length of the cable = 215 m and the inclination of the cable is 60° .

In $\triangle ABC$

$$\sin 60^\circ = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{h}{215}$$

$$h = 215 \frac{\sqrt{3}}{2} = 185.9$$

Hence, the height of the balloon from the ground is 186m (approx).