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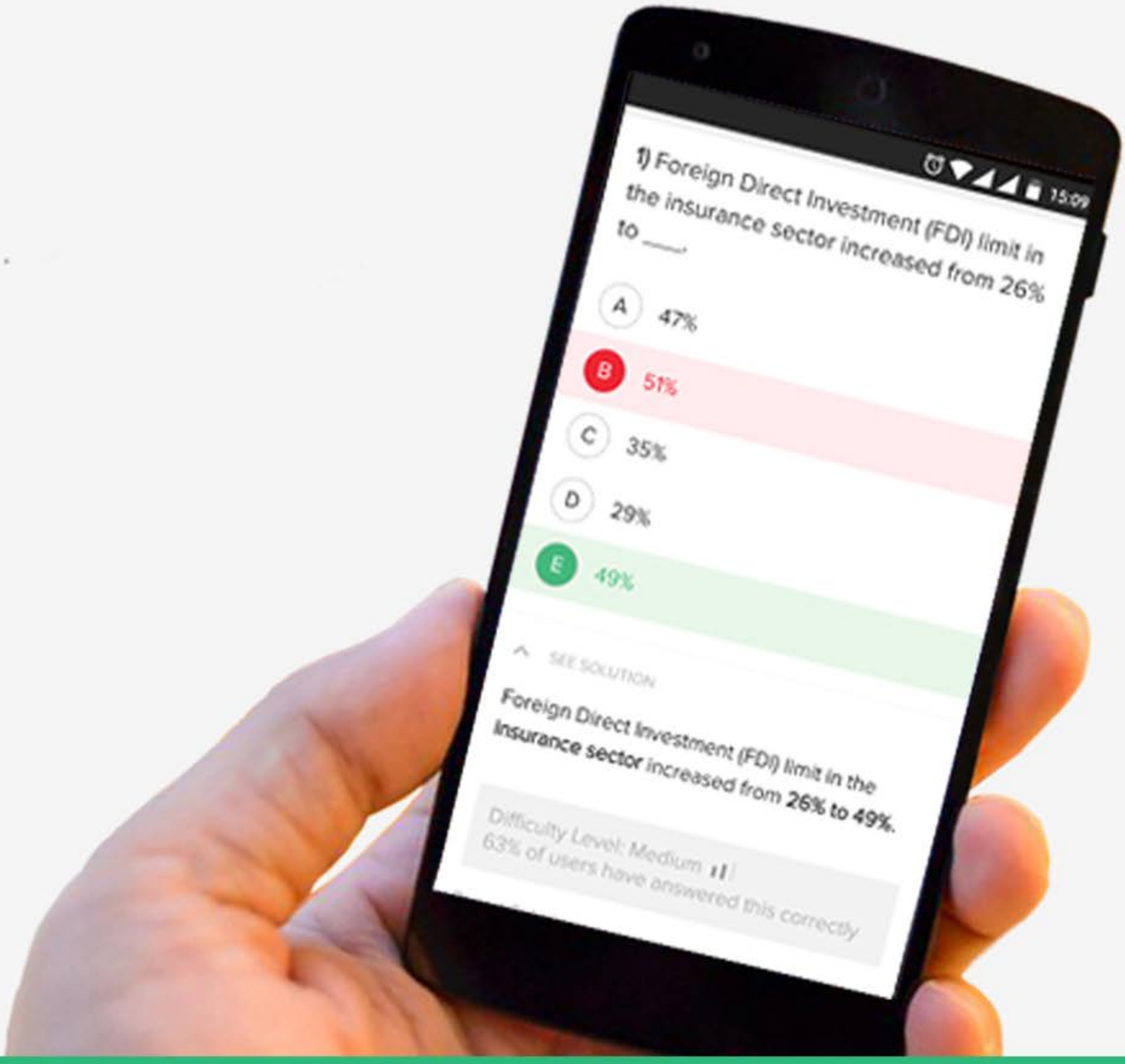
Formulas on
CONTROL SYSTEMS
for GATE EE Exam



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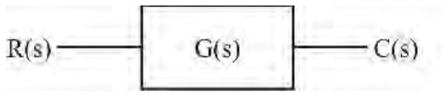
Control Systems

Open Loop Control System:

- In this system the output is not feedback for comparison with the input.
- Open loop system faithfulness depends upon the accuracy of input calibration.

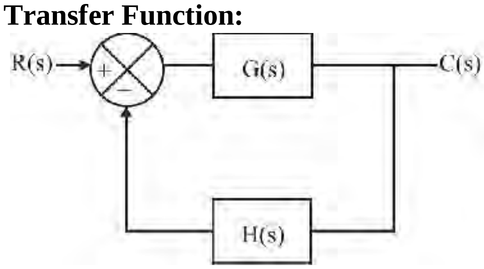


$$\text{Transfer function } G(s) = \frac{C(s)}{R(s)}$$

When a designer designs, he simply design open loop system.

Closed Loop Control System: It is also termed as feedback control system. Here the output has an effect on control action through a feedback. Ex. Human being

Transfer Function:



$$\text{Transfer function} = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Comparison of Open Loop and Closed Loop control systems:

Open Loop:

1. Accuracy of an open loop system is defined by the calibration of input.
2. Open loop system is simple to construct and cheap.
3. Open loop systems are generally stable.
4. Operation of this system is affected due to presence of non-linearity in its elements.

Closed Loop:

1. As the error between the reference input and the output is continuously measured through feedback. The closed system works more accurately.
2. Closed loop systems is complicated to construct and it is costly.
3. It becomes unstable under certain conditions.
4. In terms of performance the closed loop system adjusts to the effects of non-linearity present.

Transfer Function: The transfer function of an LTI system may be defined as the ratio of Laplace transform of output to Laplace transform of input under the assumption

$$G(s) = \frac{Y(s)}{X(s)}$$

- The transfer function is completely specified in terms of its poles and zeros and the gain factor.

- The T.F. function of a system depends on its elements, assuming initial conditions as zero and is independent of the input function.
- To find a gain of system through transfer function put $s = 0$

Example: $G(s) = \dfrac{s+4}{s^2+6s+9}$ Gain $= \dfrac{4}{9}$

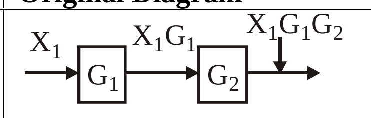
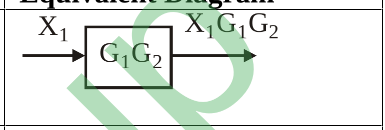
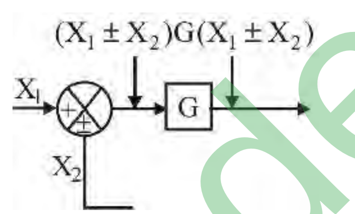
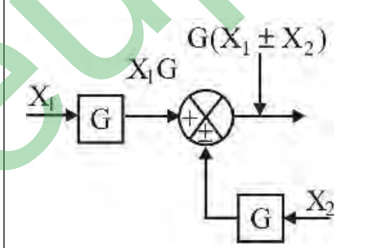
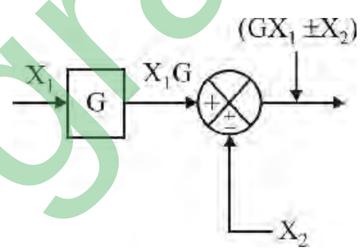
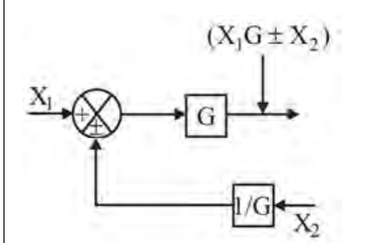
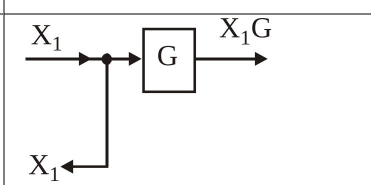
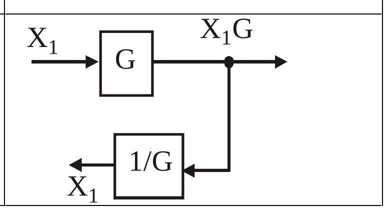
If a step, ramp or parabolic response of T.F. is given, then we can find Impulse Response directly through differentiation of that T.F.

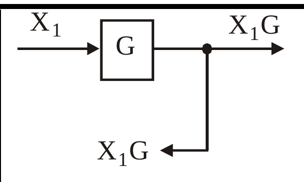
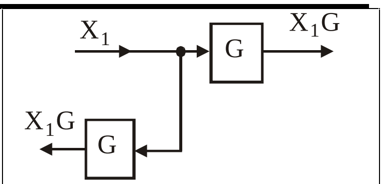
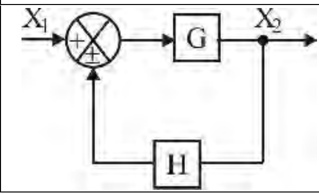
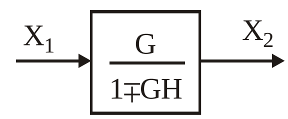
$$\frac{d}{dt}\text{(Parabolic Response)} = \text{Ramp Response}$$

$$\frac{d}{dt}\text{(Ramp Response)} = \text{Step Response}$$

$$\frac{d}{dt}\text{(Step Response)} = \text{Impulse Response}$$

Block Diagram Reduction:

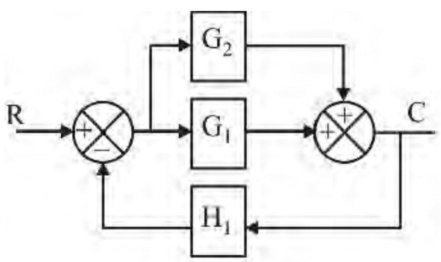
Rule	**Original Diagram**	**Equivalent Diagram**
1. Combining blocks in cascade		
2. oving a summing point after a block		
3. Moving a summing point ahead of block		
4. Moving a take off point after a block		

5. Moving a take off point ahead of a block		
6. Eliminating a feedback loop		

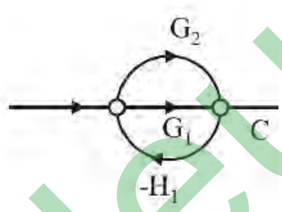
$(GX_1 \pm X_2)$

Signal Flow Graphs:

- It is a graphical representation of control system.
- Signal Flow Graph of Block Diagram:



Block diagram



Signal Flow Graph

Mason's Gain Formula: Transfer function = $\frac{\Sigma\, p_k\, \Delta_k}{\Delta}$

$p_k \rightarrow$ Path gain of k^{th} forward path

Δ = 1 – [Sum of all individual loops] + [Sum of gain products of two non-touching loops] – [Sum of gain products of 3 non-touching loops] + ……….

$\Delta_k \rightarrow$ Value of Δ obtained by removing all the loops touching k^{th} forward path as well as non-touching to each other

Some Laplace and Z Transforms

$F(s)$	$f(nT)$	$F(z)$
$\frac{1}{s}$	$1(nT)$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	nT	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s+a}$	e^{-anT}	$\frac{z}{z-e^{-aT}}$
$\frac{a}{s(s+a)}$	$1-e^{-anT}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$\frac{a}{s^2(s+a)}$	$(anT-1+e^{-anT})/a$	$\frac{z[z(aT-1+e^{-aT})+(1-(1+aT)e^{-aT})]}{a(z-1)^2(z-e^{-aT})}$

Laplace Transform:

$$F(s) = \mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt$$

Inverse Laplace Transform:

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi}\int_{c-i\infty}^{c+i\infty} e^{st}F(s)\,ds$$

Fourier Transform:

$$F(j\omega) = \mathcal{F}[f(t)] = \int_0^\infty f(t)e^{-j\omega t}dt$$

Inverse Fourier Transform:

$$f(t) = \mathcal{F}^{-1}\{F(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{-j\omega t}d\omega$$

Star Transform:

$$F^*(s) = \mathcal{L}^*[f(t)] = \sum_{i=0}^{\infty} f(iT)e^{-siT}$$

Z Transform:

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{i=-\infty}^{\infty} x[n]z^{-n}$$

Inverse Z Transform:

$$x[n] = Z^{-1}\{X(z)\} = \frac{1}{2\pi j}\oint_C X(z)z^{n-1}dz$$

Modified Z Transform:

$$X(z,m) = \mathcal{Z}(x[n],m) = \sum^{\infty} x[n+m-1]z^{-n}$$

Final Value Theorem:

$$x(\infty) = \lim_{s\to 0} sX(s)$$
$$x[\infty] = \lim_{z\to 1}(z-1)X(z)$$

Initial Value Theorem:

$$x(0) = \lim_{s\to\infty} sX(s)$$

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$$e^{j\omega} = \cos(\omega) + j\sin(\omega)$$

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$$(a * b)(t) = \int_{-\infty}^{\infty} a(\tau)b(t-\tau)d\tau$$

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$$\mathcal{L}[f(t) * g(t)] = F(s)G(s)$$
$$\mathcal{L}[f(t)g(t)] = F(s) * G(s)$$

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$$|A - \lambda I| = 0$$
$$Av = \lambda v$$
$$wA = \lambda w$$

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$$dB = 20\log(C)$$

Unit Step Function:

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

Unit Ramp Function: $r(t) = tu(t)$

Unit Parabolic Function: $p(t) = \frac{1}{2}t^2u(t)$

Closed-Loop Transfer Function:

$$H_{cl}(s) = \frac{KGp(s)}{1 + KGp(s)Gb(s)}$$

Open-Loop Transfer Function:

$$H_{ol}(s) = KGp(s)Gb(s)$$

Characteristic Equation:

$$F(s) = 1 + H_{ol}$$

Time Response of 2nd order system :

Step i/P :

- C(t) = $1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sin \omega_n \sqrt{1-\zeta^2}\, t \pm \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$
- e(t) = $\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sin \omega_d t \pm \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$
- $e_{ss} = \lim_{t\to\infty} \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sin \omega_d t \pm \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$

→ ζ → Damping ratio ; $\zeta\omega_n$ → Damping factor

$\zeta < 1$ (Under damped) :-

$$C(t) = 1- = \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \text{ Sin}\left(\omega_d t \pm \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)$$

$\zeta = 0$ (un damped) :-

$c(t) = 1 - \cos \omega_n t$

$\zeta = 1$ (Critically damped) :-

$C(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$

$\zeta > 1$ (over damped) :-

$$C(t) = 1 - \frac{e^{-\left(\zeta - \sqrt{\zeta^2-1}\right)\omega_n t}}{2\sqrt{\zeta^2-1}\left(\zeta - \sqrt{\zeta^2-1}\right)}$$

$$T = \frac{1}{\left(\zeta - \sqrt{\zeta^2-1}\right)\omega_n}$$

$T_{undamped} > T_{overdamped} > T_{underdamped} > T_{criticaldamp}$

Time Domain Specifications :

- Rise time $t_r = \frac{\pi - \phi}{\omega_n\sqrt{1-\zeta^2}}$ $\quad \phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$
- Peak time $t_p = \frac{n\pi}{\omega_d}$
- Max over shoot % $M_p = e^{-\zeta\omega_n/\sqrt{1-\zeta^2}} \times 100$
- Settling time $t_s = 3T$ 5% tolerance

  $= 4T$ 2% tolerance

- Delay time $t_d = \frac{1+0.7\zeta}{\omega_n}$
- Damping factor[2] $\zeta^2 = \frac{(\ln M_p)^2}{\pi^2 + (\ln M_p)^2}$
- Time period of oscillations $T = \frac{2\pi}{\omega_d}$
- No of oscillations $= \frac{t_s}{2\pi/\omega_d} = \frac{t_s \times \omega_d}{2\pi}$
- $t_r \approx 1.5\, t_d \quad t_r = 2.2\, T$
- Resonant peak $M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$; $\omega_r = \omega_n \sqrt{1-2\zeta^2}$ $\left.\begin{matrix}\omega_n > \omega_r \\ \omega_b > \omega_n\end{matrix}\right] \omega_r < \omega_n < \omega_b$
- Bandwidth $\omega_b = \omega_n \left(1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}\right)^{1/2}$

Static error coefficients :

- Step i/p : $e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \frac{SR(s)}{1+GH}$

  $e_{ss} = \frac{1}{1+K_p}$ (positional error) $\quad K_p = \lim_{s\to 0} G(s)\,H(s)$

- Ramp i/p (t) : $e_{ss} = \frac{1}{K_v}$ $\quad K_v = \lim_{s\to 0} S\,G(s)H(s)$

- Parabolic i/p ($t^2/2$) : $e_{ss} = 1/K_a$ $\quad K_a = \lim_{s\to 0} s^2\, G(s)H(s)$

Type < i/p → $e_{ss} = \infty$
Type = i/p → e_{ss} finite
Type > i/p → $e_{ss} = 0$

- Sensitivity $S = \frac{\partial A/A}{\partial K/K}$ sensitivity of A w.r.to K.
- Sensitivity of over all T/F w.r.t forward path T/F G(s) :
  Open loop: $S = 1$

  Closed loop : $S = \frac{1}{1+G(s)H(s)}$

- Minimum 'S' value preferable
- Sensitivity of over all T/F w.r.t feedback T/F H(s) : $S = \frac{G(s)H(s)}{1+G(s)H(s)}$

Stability RH Criterion :

- Take characteristic equation 1+ G(s) H(s) = 0
- All coefficients should have same sign
- There should not be missing 's' term . Term missed means presence of at least one +ve real part root

- If char. Equation contains either only odd/even terms indicates roots have no real part & posses only imag parts there fore sustained oscillations in response.
- Row of all zeroes occur if
  (a) Equation has at least one pair of real roots with equal image but opposite sign
  (b) has one or more pair of imaginary roots
  (c) has pair of complex conjugate roots forming symmetry about origin.

Position Error Constant:

$$K_p = \lim_{s \to 0} G(s)$$
$$K_p = \lim_{z \to 1} G(z)$$

Velocity Error Constant:

$$K_v = \lim_{s \to 0} sG(s)$$
$$K_v = \lim_{z \to 1} (z-1)G(z)$$

Acceleration Error Constant:

$$K_a = \lim_{s \to 0} s^2 G(s)$$
$$K_a = \lim_{z \to 1} (z-1)^2 G(z)$$

General System Description:

$$y(t) = \int_{-\infty}^{\infty} g(t,r)x(r)dr$$

Convolution Description:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Transfer Function Description:

$$Y(s) = H(s)X(s)$$
$$Y(z) = H(z)X(z)$$

State-Space Equations:

$$x'(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Transfer Matrix:

$$C[sI - A]^{-1}B + D = \mathbf{H}(s)$$
$$C[zI - A]^{-1}B + D = \mathbf{H}(z)$$

Transfer Matrix Description:

$$\mathbf{Y}(s) = \mathbf{H}(s)\mathbf{U}(s)$$
$$\mathbf{Y}(z) = \mathbf{H}(z)\mathbf{U}(z)$$

Mason's Rule:

$$M = \frac{y_{out}}{y_{in}} = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

State-Space Methods:

1.General State Equation Solution:

$$x(t) = e^{At-t_0}x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau$$

$$x[n] = A^n x[0] + \sum_{m=0}^{n-1} A^{n-1-m}Bu[n]$$

2. General Output Equation Solution:

$$y(t) = Ce^{At-t_0}x(t_0) + C\int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$$

$$y[n] = CA^n x[0] + \sum_{m=0}^{n-1} CA^{n-1-m}Bu[n] + Du[n]$$

3. Time-Variant General Solution:

$$x(t) = \phi(t, t_0)x(t_0) + \int_{t_0}^{t} \phi(\tau, t_0)B(\tau)u(\tau)d\tau$$

$$x[n] = \phi[n, n_0]x[t_0] + \sum_{m=n_0} \phi[n, m+1]B[m]u[m]$$

4. Impulse Response Matrix:

$$G(t,\tau) = \begin{cases} C(\tau)\phi(t,\tau)B(\tau) & \text{if } t \geq \tau \\ 0 & \text{if } t < \tau \end{cases}$$

$$G[n] = \begin{cases} CA^{k-1}N & \text{if } k > 0 \\ 0 & \text{if } k \leq 0 \end{cases}$$

Root Locus:

The Magnitude Equation:

$$1 + KG(s)H(s) = 0$$
$$1 + K\overline{GH}(z) = 0$$

The Angle Equation:

$$\angle KG(s)H(s) = 180^\circ$$
$$\angle K\overline{GH}(z) = 180^\circ$$

Number of Asymptotes: $N_a = P - Z$

Angle of Asymptotes:

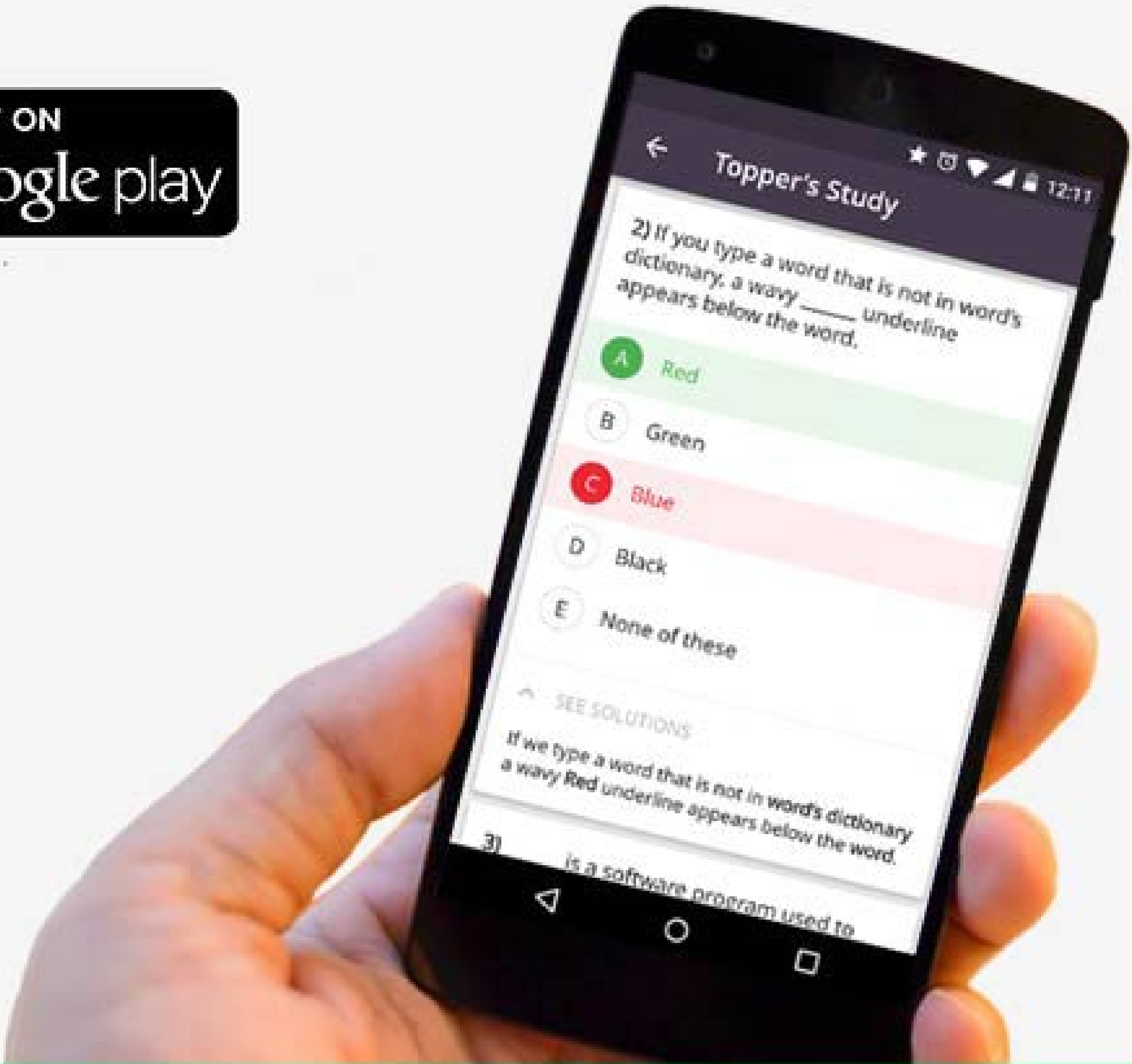
$$\phi_k = (2k+1)\frac{\pi}{P-Z}$$

Breakaway Point Locations:

$$\frac{G(s)H(s)}{ds} = 0 \text{ or } \frac{\overline{GH}(z)}{dz} = 0$$

Controllers and Compensators:

PID: $D(s) = K_p + \frac{K_i}{s} + K_d s$ $\qquad D(z) = K_p + K_i \frac{T}{2}\left[\frac{z+1}{z-1}\right] + K_d\left[\frac{z-1}{Tz}\right]$


Topper's Study
12:11
2) If you type a word that is not in word's dictionary, a wavy ______ underline appears below the word.
A Red
B Green
C Blue
D Black
E None of these
SEE SOLUTIONS
If we type a word that is not in word's dictionary a wavy Red underline appears below the word.
3) ______ is a software program used to