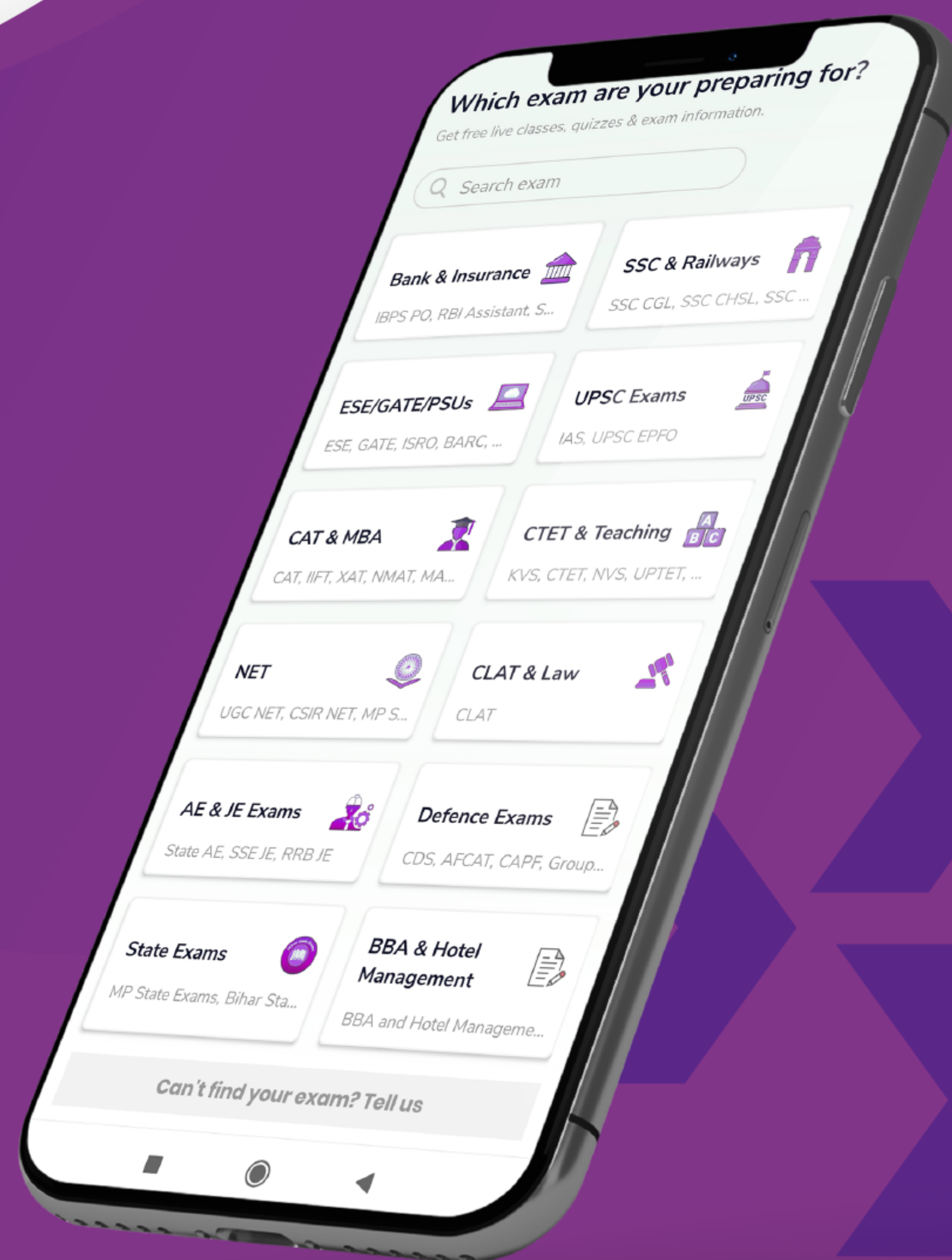


Formula Sheet On Number Factors



NUMBER OF FACTORS OF A NUMBER

If N is a composite number such that $N = a^p b^q c^r \dots$ where a, b, c are prime factors of N and $p, q, r \dots$ are positive integers, then the number of factors of N is given by the expression.

$$= (p + 1) (q + 1) (r + 1) \dots$$

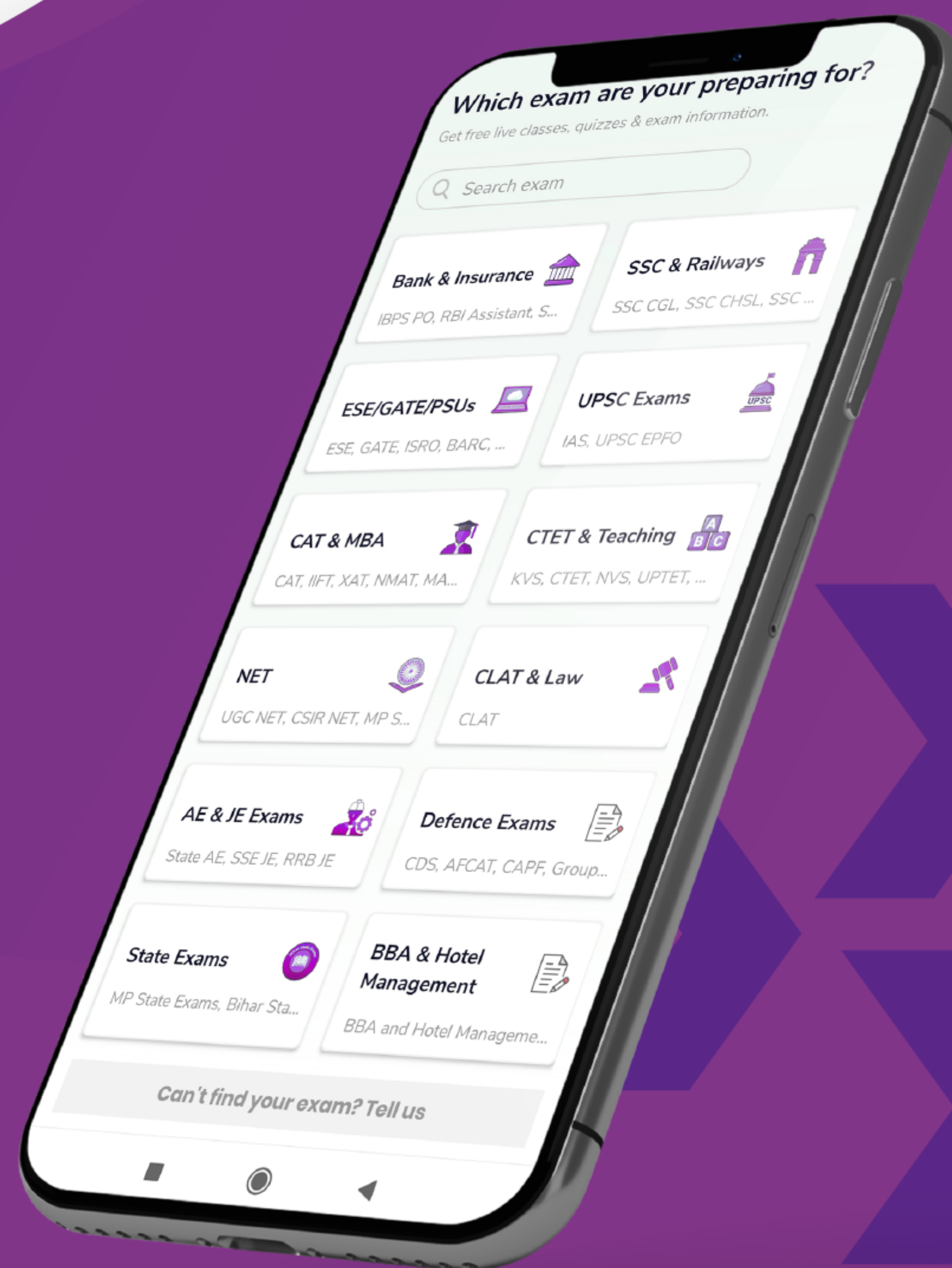
Example 1: Find the number of factors of 140

Solution: $140 = 2^2 \times 5^1 \times 7^1$

Hence 140 has $(2 + 1) (1 + 1) (1 + 1)$, i.e., 12 factors.

Please note that the figure arrived at by using the above formula includes 1 and the given number N also as factors. So, if you want to find the number of factors the given number has excluding 1 and the number itself, we find out $(p + 1) (q + 1) (r + 1)$ and then subtract 2 from that figure.

In the above example, the number 140 has 10 factors excluding 1 and 140 itself.



Product of all the factors of a number:

From the above given example of number 48. The total number of factors will be 10 i.e. 1, 2, 3, 4, 6, 8, 12, 16, 24, 48. If we try to find the product of all the factors of 48 then we have to multiply all the factors. But we can see that

Multiplication of 1 and 48 = 48;

Multiplication of 2 and 24 = 48;

Multiplication of 3 and 16 = 48;

Multiplication of 4 and 12 = 48;

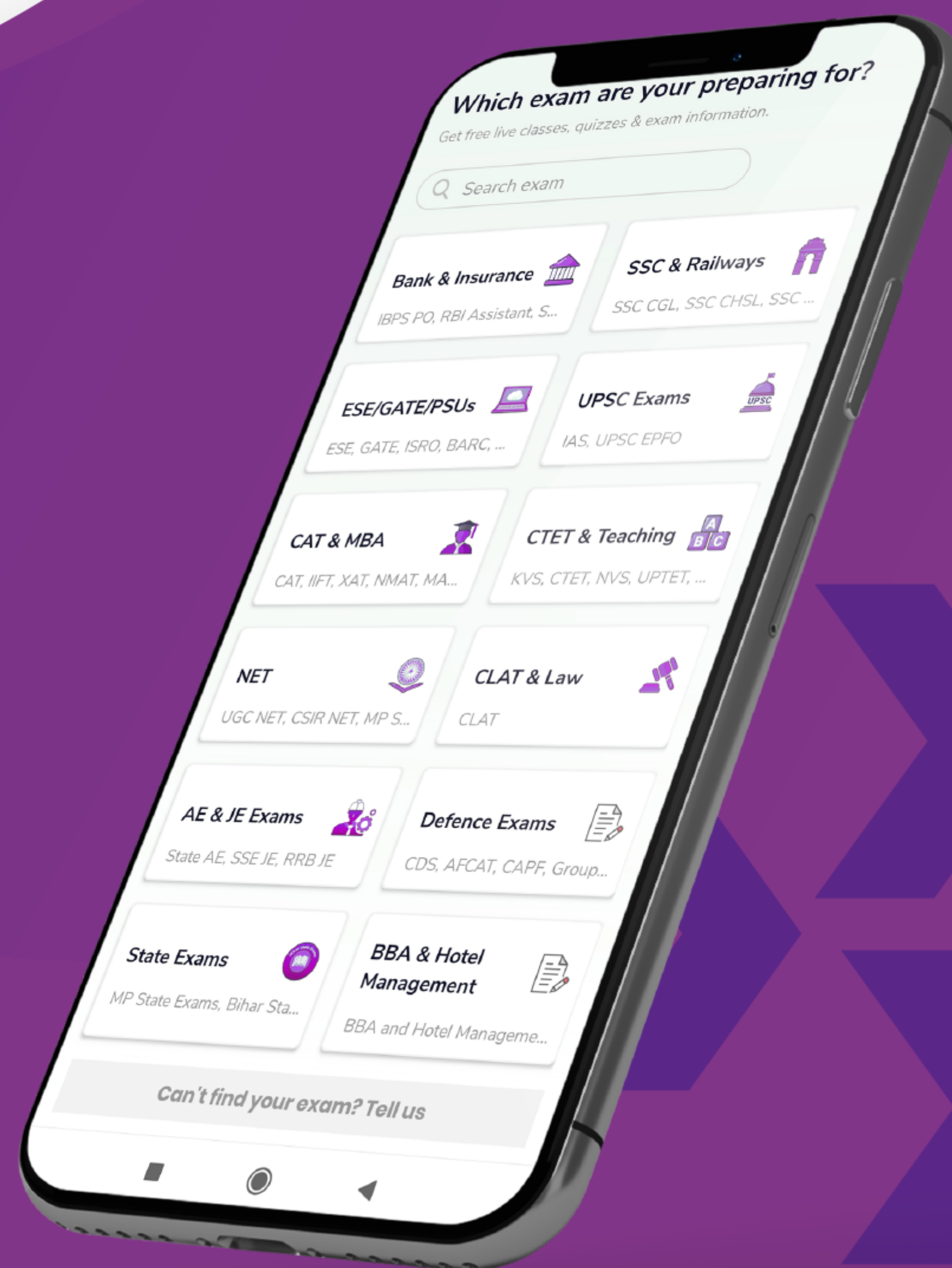
Multiplication of 6 and 8 = 48

Thus, Multiplication of a factor from the beginning and a corresponding factor from the end, results into the number itself. Here, in the example 48 has total 10 factors and in the product, we have to multiply 48 to itself total 5 times ($10 \div 2 = 5$ times).

So, in general case

Product of all the factors of a number = Number $\left(\frac{\text{No. of total factors}}{2}\right)$

So, product of all the factors of 48 = $48^{\frac{(10)}{2}} = 48^5$



Number of odd factors of a number:

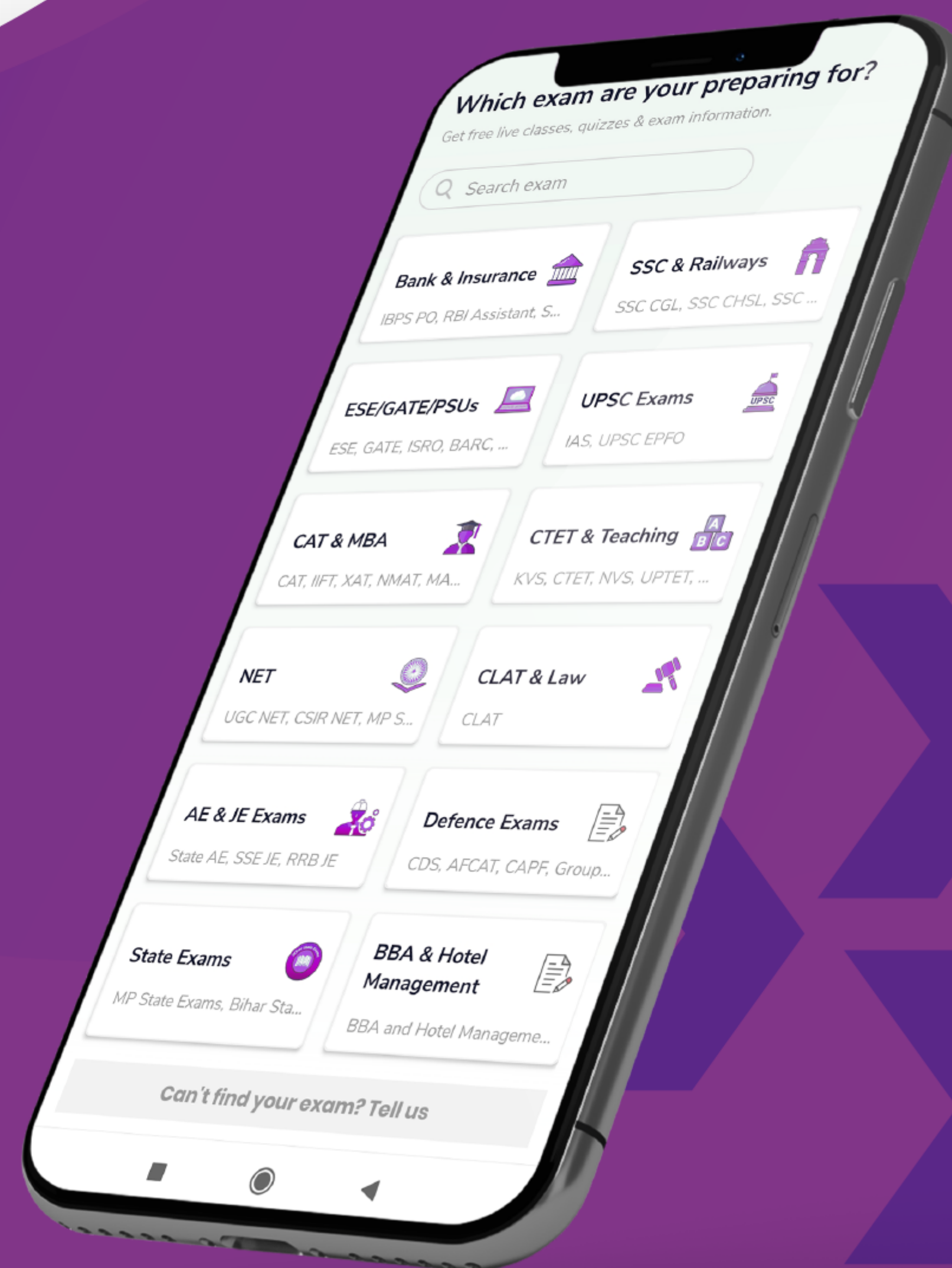
Number of odd factors of a number: The total number of odd factors of any given number can be found by calculating all the possible combinations of powers of odd primenumbers as base such as 3, 5, 7..... etc.

Number of even factors of a number: The total number of even factors of any given number can be found by calculating all the factors and subtracting the number of odd factors from it.

By another method, Number of even factors can be found by keeping the power of 2 and ignoring 2^0 for our calculations and finding the total number of factors as usual.

Number of factors of a number that are perfect squares:

A perfect square or simply square is obtained when a number is multiplied to itself. Thus, to find the total number of factors that are perfect square, we have to look for the power 0, 2, 4, 6,..... occurring in the factorization.



If a number $N = a^p b^q c^r \dots$ where a, b, c, \dots are prime numbers and p, q, r are positive integers, then,

The sum of all the factors of N (including 1 and the number itself) is:

$$\left(\frac{a^{p+1} - 1}{a - 1}\right) \left(\frac{b^{q+1} - 1}{b - 1}\right) \left(\frac{c^{r+1} - 1}{c - 1}\right) \dots$$

The above can be verified by an example.

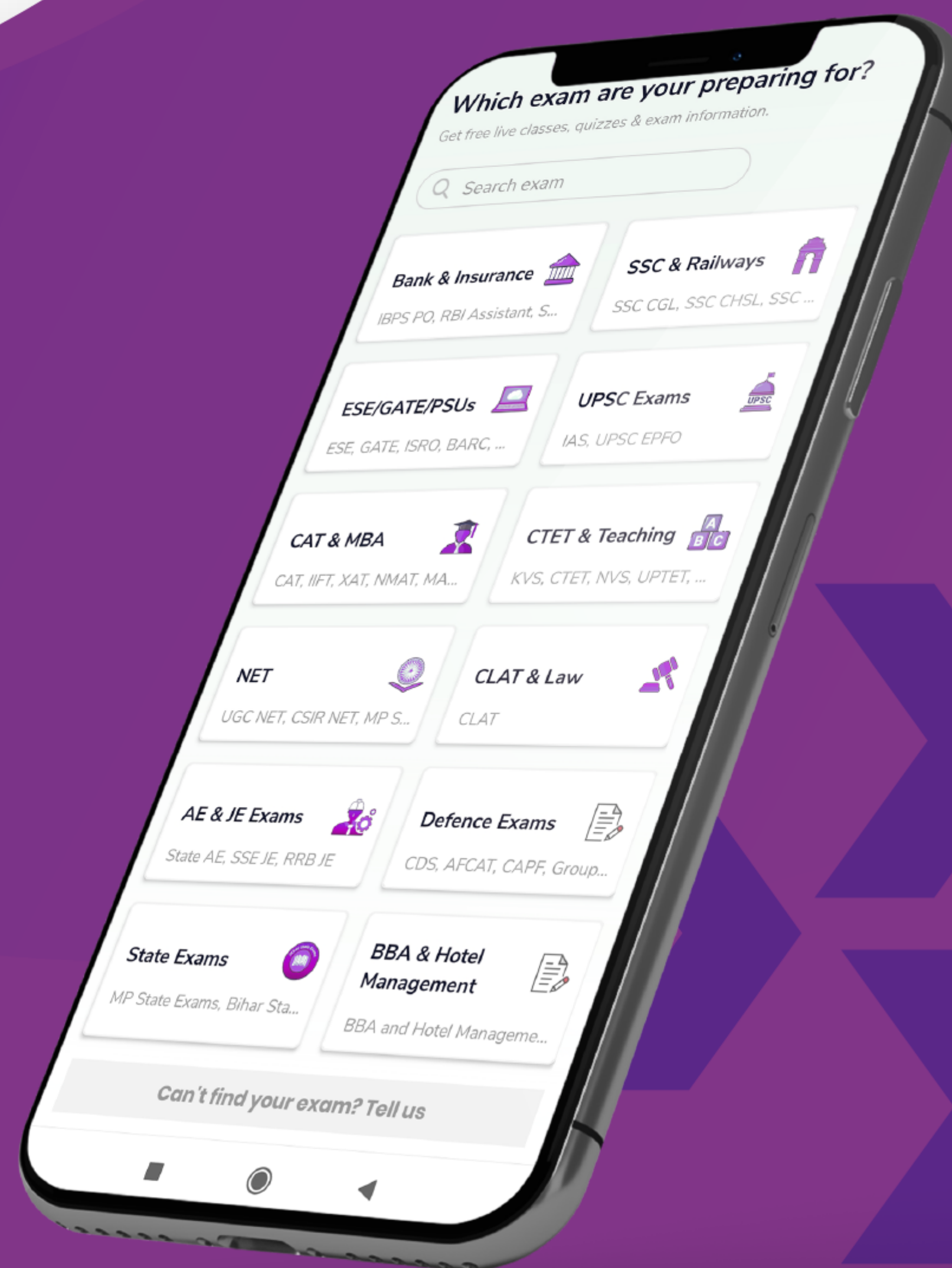
Consider the number 48, when resolved into prime factors, $48 = 2^4 \times 3^1$. Here $a = 2, b = 3, p = 4, q = 1$.

Hence, sum of all the factors: $= \left(\frac{2^{4+1}-1}{2-1}\right) \left(\frac{3^{1+1}-1}{3-1}\right) = \frac{31}{1} \times \frac{8}{2} = 124$

The list of factors of 48 are:

1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

If these factors are added, the sum is 124 and tallies with the above result.



SUM OF ALL THE FACTORS OF NUMBER:

If a number $N = a^p b^q c^r \dots$ where a, b, c, \dots are prime numbers and p, q, r are positive integers, then,

The sum of all the factors of N (including 1 and the number itself) is:

$$\left(\frac{a^{p+1} - 1}{a - 1}\right) \left(\frac{b^{q+1} - 1}{b - 1}\right) \left(\frac{c^{r+1} - 1}{c - 1}\right) \dots$$

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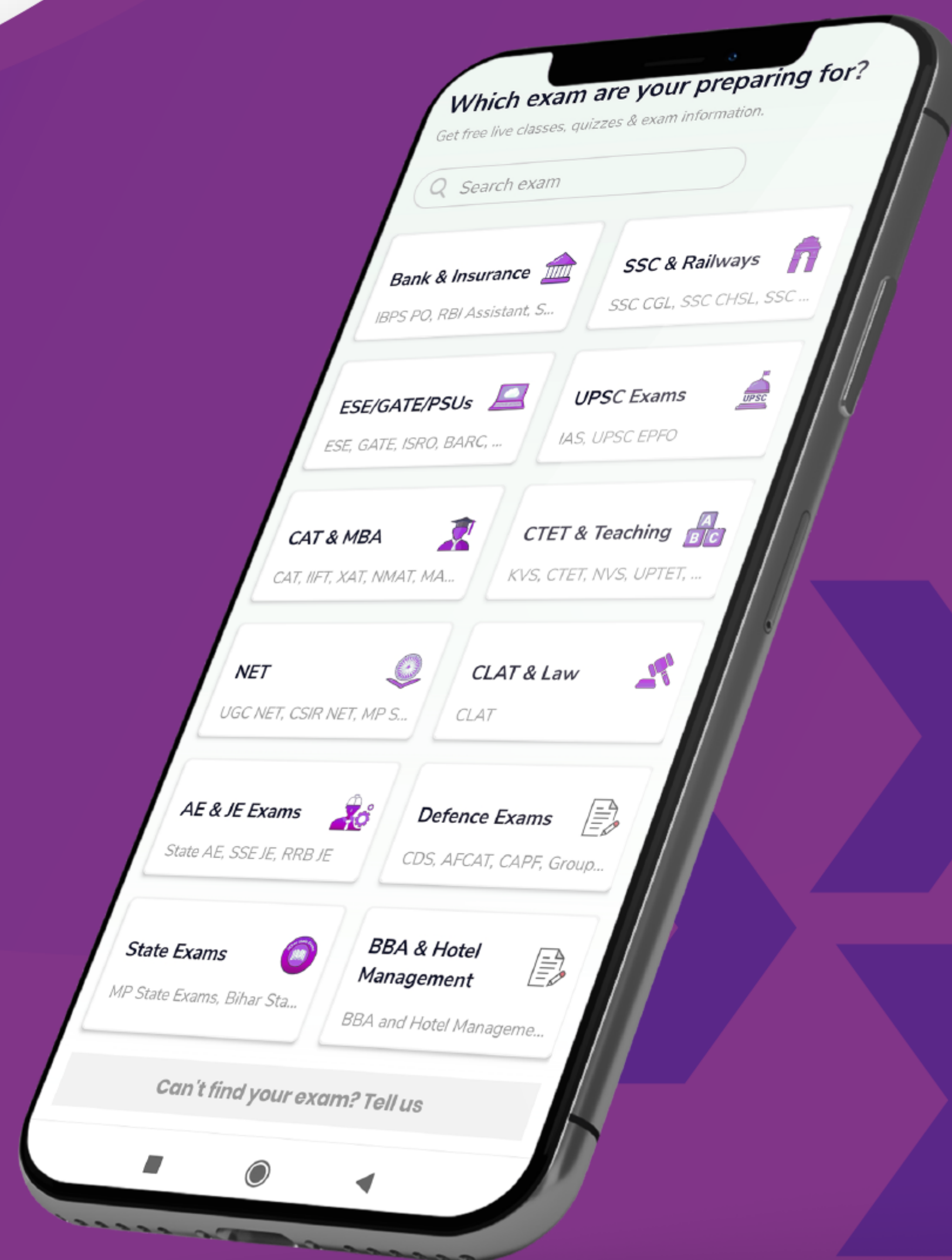
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The list of factors of 48 are:

1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

If these factors are added, the sum is 124 and tallies with the above result.



SUM OF EVEN FACTORS OF A NUMBER:

To find the sum of even factors of a number, we will omit the factor 2^0 .

SUM OF ODD FACTORS OF A NUMBER:

To find the sum of odd factors of a number, we will omit all the powers of 2 altogether. Taking into consideration all other factors with their powers.

SUM OF PERFECT SQUARE FACTORS OF NUMBER:

SUM OF PERFECT SQUARE FACTORS OF A NUMBER: When we find the sum of factors that are perfect squares, we only consider the powers that make perfect squares i.e. $0, 2, 4, \dots$ and follow the usual method.

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