CAT Geometry Questions

Q- In a triangle ABC, the lengths of the sides AB and AC equal 17.5 cm and 9 cm respectively. Let D be a point on the line segment BC such that AD is perpendicular to BC. If AD = 3 cm, then what is the radius (in cm) of the circle circumscribing the triangle ABC?

(A) 17.05
(B) 27.85
(C) 22.45
(D) 32.25
(E) 26.25

Solution: The correct option is E.

The formula for the circumradius of a triangle

$$R = \frac{a \times b \times c}{4 \times (\text{Area of the triangle})}$$

$$\text{or } R = \frac{a \times b \times c}{4 \times \left(\frac{1}{2} \times b \times AD\right)} = \frac{a \times c}{2 \times AD}$$

$$= \frac{17.5 \times 9}{2 \times 3} = 26.25 \text{ cm}$$

Q-Consider a square ABCD with midpoints E, F, G, H of AB, BC, CD and DA respectively. Let L denote the line passing through F and H. Consider points P and Q, on L and inside ABCD, such that the angles APD and BQC both equal 120°. What is the ratio of the area of ABQCDP to the remaining area inside ABCD?

(A) $\frac{4\sqrt{2}}{3}$

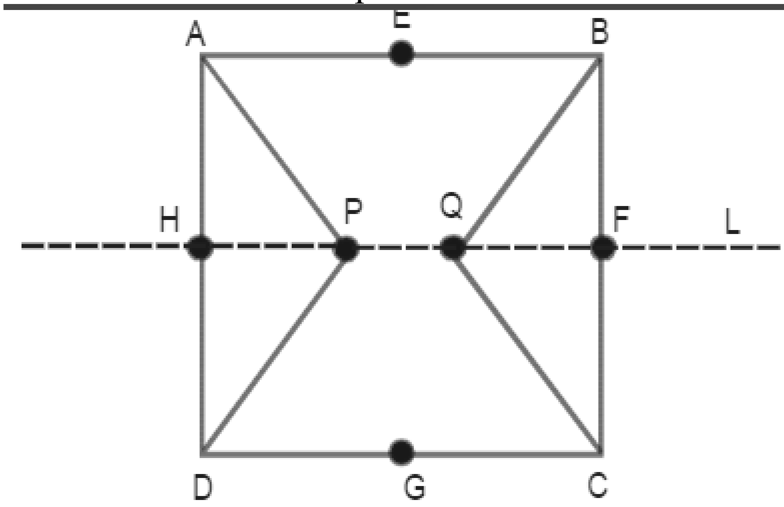
(B) $2+\sqrt{3}$

(C) $\frac{10-3\sqrt{3}}{9}$

(D) $1+\frac{1}{\sqrt{3}}$

(E) $2\sqrt{3}-1$

Solution: The correct option is E.



Let , the length of AH = ‘x’ cm

By symmetry of the figure given above, we can conclude that ΔAPD and ΔBQC will have the same area.

$\because \angle APD$ is 120° and line ‘L’ divides the square ABCD in 2 equal halves, therefore

$\angle APH = \angle HPD = 60°$

In $\Delta AHP : \frac{AH}{HP} = \tan 60° = \sqrt{3} \Rightarrow HP = \frac{x}{\sqrt{3}}$ cm

Area of $\Delta APD = 2 \times \text{area}(\Delta AHP)$

$= 2 \times \frac{1}{2} \times x \times \frac{x}{\sqrt{3}} = \frac{x^2}{\sqrt{3}}$ cm

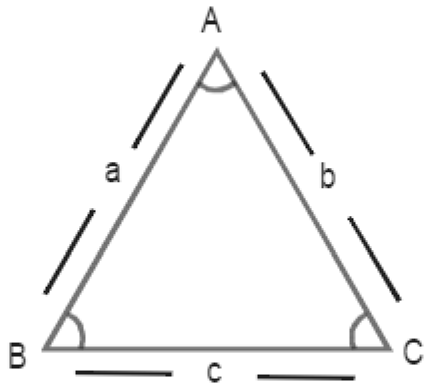
Area of ABQCDP = area (ABCD) – 2 area (ΔAPD)

$$= 4x^2 - \frac{2x^2}{\sqrt{3}} = \frac{2x^2\left(2\sqrt{3}-1\right)}{\sqrt{3}}$$

Required Ratio $= \dfrac{\dfrac{2x^2\left(2\sqrt{3}-1\right)}{\sqrt{3}}}{\dfrac{2x^2}{\sqrt{3}}} = 2\sqrt{3}-1$

Alternate method:

Concepts used:



$$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Q-Two circles, both of radii 1 cm, intersect such that the circumference of each one passes through the centre of the other. What is the area (in sq. cm.) of the intersecting region?

(A) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$

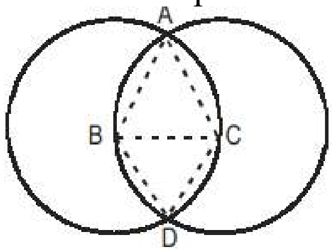
(B) $\frac{2\pi}{3}+\frac{\sqrt{3}}{2}$

(C) $\frac{4\pi}{3}-\frac{\sqrt{3}}{2}$

(D) $\frac{4\pi}{3}+\frac{\sqrt{3}}{2}$

(E) $\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$

Solution: The correct option is E.



It is given that AB = BC = AC = BD = DC = 1 cm.

Therefore, ΔABC is an equilateral triangle.

Hence, $\angle ACB = 60^\circ$

Now area of sector $\widehat{AB} = \frac{60}{360}\times\pi(1)^2 = \frac{\pi}{6}$

Area of equilateral triangle $\Delta ABC = \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}$

Area of remaining portion in the common region $\widehat{ABC}$ excluding ABC

$= 2\times\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)$

Hence, the total area of the intersecting region =

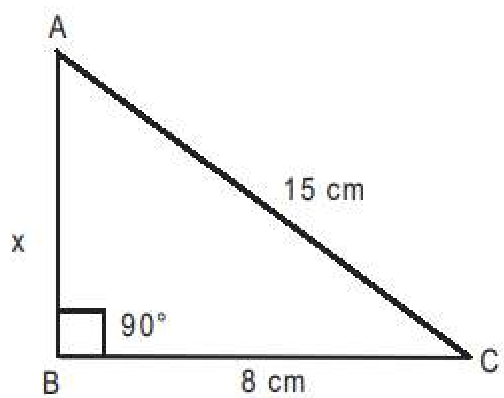
$2\times\frac{\sqrt{3}}{4}\times(1)^2+4\times\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)$

$=\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$ sq. cm.

Q-Consider obtuse-angled triangles with sides 8 cm, 15 cm and x cm. If x is an integer, then how many such triangles exist?
(A)5
(B)21
(C)10
(D)15
(E)14

Solution: The correct option is C.
Let the three sides of the triangle be 8 cm, 15 cm, and x cm. As 15 is greater than 8, hence either x or 15 will be the largest side. Therefore, there will be the following 2 cases:

Case I:

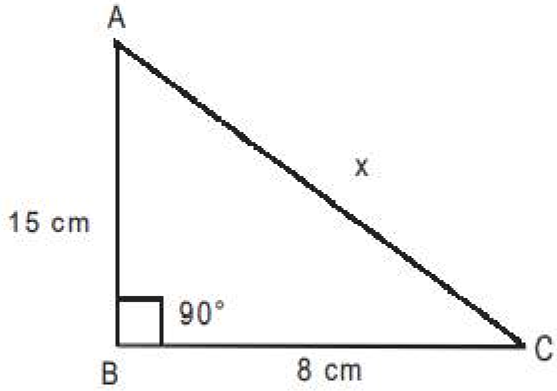


Consider the right ΔABC above,

$$x = \sqrt{15^2 - 8^2} = 12.68 \text{ cm}$$

The triangle will be obtuse for all values of x<12.68. But as the sum of two sides of triangle must be greater than the third side hence (x+8)>15 or x>7. Thus the permissible values of x are 8,9,10,11 and 12.

Case II:



In the right ΔABC above, $x = \sqrt{15^2 + 8^2} = 17$.

For all values of x > 17, ΔABC will be obtuse. But, as the length of third side should be less than the sum of other two sides, hence x < (15 + 8) or x < 23. The permissible values of x are: 18, 19, 20, 21 and 22. From Case I and II, x can take 10 values.