

1. Find the convolution

$$y(t) = e^{-t} u(t-3) * e^{-t} u(t+5)$$

- A. $e^3 e^{-(t+3)} u(t+3)$
- B. $e^{-3} e^{-(t+3)} u(t+3)$
- C. $e^2 (t+2) e^{-(t+2)} u(t+2)$
- D. $e^{-2} (t+2) e^{-(t+2)} u(t+2)$

Answer ||| C

$$\text{Solution } ||| f(t) * h(t) = y(t)$$

$$\underbrace{e^{-t} u(t)}_{f(t)} * \underbrace{e^{-t} u(t)}_{h(t)} = t e^{-t} u(t)$$

$$y(t) = e^{-(t-3+3)} u(t-3) * e^{-(t+5-5)} u(t+5)$$

$$y(t) = e^{-3} \underbrace{e^{-(t-3)} u(t-3)}_{f(t-3)} * e^5 \underbrace{e^{-(t+5)} u(t+5)}_{h(t+5)}$$

$$y(t) = e^2 [f(t-3) * h(t+5)]$$

$$y(t) = e^2 y(t-3+5)$$

$$y(t) = e^2 y(t+2)$$

$$y(t) = e^2 (t+2) e^{-(t+2)} u(t+2)$$

2. find the Fourier transform of its integral $g(x)$

$$= \int_{-\infty}^x f(y) dy \text{ is } \hat{g}(k) = -\frac{i}{k} \hat{f}(k) + \pi \hat{f}(0) \delta(k) \quad \text{if } f(x) \text{ has Fourier transform } \hat{f}(k) \text{ find the } h'(x)?$$

- A. $f(x) - \hat{f}(0) \delta(x)$
- B. $f(x) - \sqrt{\pi} \hat{f}(0) \delta(x)$
- C. $f(x) - \sqrt{2} \hat{f}(0) \delta(x)$
- D. $f(x) - \sqrt{2\pi} \hat{f}(0) \delta(x)$

Answer ||| D

Solution ||| Given integral $g(x) = \int_{-\infty}^x f(y) dy$ is $\hat{g}(k) = -\frac{i}{k} \hat{f}(k) + \pi \hat{f}(0) \delta(k)$

First notice that

$$\lim_{x \rightarrow -\infty} g(x) = 0$$

$$\lim_{x \rightarrow +\infty} g(x) = \int_{-\infty}^{\infty} f(x) dx = \sqrt{2\pi} \hat{f}(0)$$

Therefore, subtracting a suitable multiple of the step function from the integral the resulting function

$$H(x) = g(x) - \sqrt{2\pi} \hat{f}(0) \sigma(x) \text{ delays to 0 at both } \pm\infty.$$

We know that,

$$\hat{h}(k) = \hat{g}(k) - \pi \hat{f}(0) \delta(k) + \frac{i}{k} \hat{f}(0)$$

On the other hand,

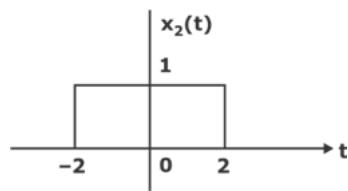
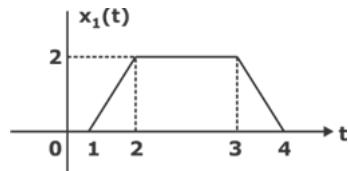
$$H'(x) = f(x) - \sqrt{2\pi} \hat{f}(0) \delta(x)$$

Since $h(x) \rightarrow 0$ as $|x| \rightarrow \infty$, we can apply differentiation rule and we finally conclude that
 $Ik \hat{h}(k) = \hat{f}(k) - \hat{f}(0)$

By combining $h'(x)$ and $ik \hat{h}(k)$ we get,

$$\hat{g}(k) = -\frac{i}{k} \hat{f}(k) + \pi \hat{f}(0) \delta(k)$$

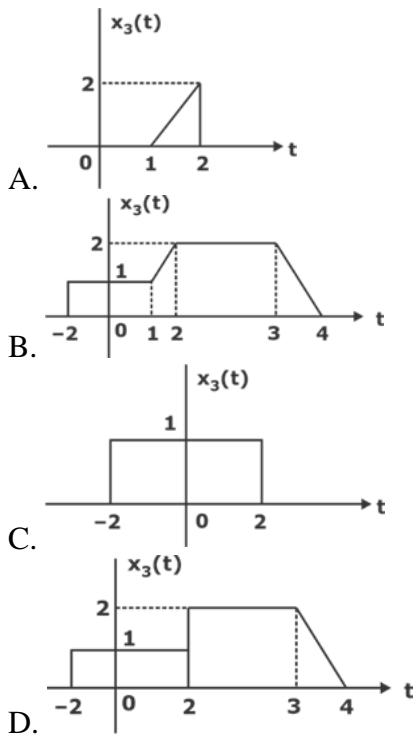
3. If $x_1(t)$ and $x_2(t)$ are two continuous time signals represented as



and $x_3(t)$ is defined as

$$x_3(t) = x_1(t) \cdot x_2(t)$$

representation of $x_3(t)$ is-



Answer ||| A

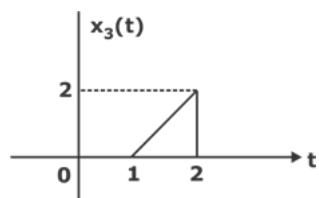
Solution |||

$x_2(t)$ is defined for $t = -2$ to 2.

$x_1(t)$ is defined for $t = 1$ to 4.

so common interval is $t = 1$ to 2.

so $x_3(t)$ will be present for $t = 1$ to 2.



4. Laplace transform of $x(t) = u(t(t^2 - 4))$

A. $x(s) = \frac{1}{s} (e^{2s} + e^{-s} - 1)$

B. $x(s) = \frac{1}{s} (e^s + e^{-2s} - 1)$

C. $x(s) = \frac{1}{s} (e^{2s} + e^{-2s} - 1)$

D. $x(s) = \frac{1}{2s} (e^{2s} + e^{-2s} - 1)$

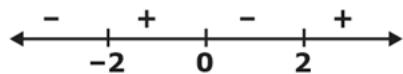
Answer ||| C

Solution |||

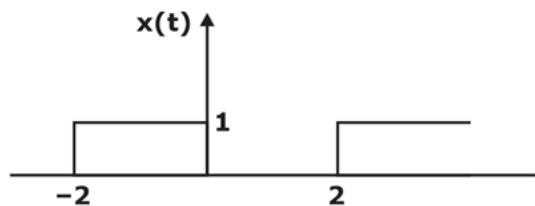
$$x(t) = u(t(t^2 - 4))$$

$$x(t) = 1 \text{ if } t(t^2 - 4) > 0$$

$$t(t+2)(t-2) > 0$$



$$x(t) = 1 \text{ if } \begin{cases} t > 2 \\ -2 < t < 0 \end{cases}$$



$$x(t) = u(t+2) - u(t) + u(t-2)$$

Apply Laplace transform

$$X(s) = \frac{e^{2s}}{s} - \frac{1}{s} + \frac{e^{-2s}}{s}$$

$$x(s) = \frac{1}{s} (e^{2s} + e^{-2s} - 1)$$

5. Consider the z-transform $X(z) = \frac{z(2z-5)}{(z-2)^2}$ with ROC given as $|z| < 2$. The corresponding inverse z-transform is:

- A. $-2^n u(-n - 1) + 3n 2^{n-1} u(-n - 1)$
 B. $-2^{n+1} u(-n - 1) + n 2^{n-1} u(-n - 1)$
 C. $-3.2^{n-1} u(-n - 1) - n 2^{n+1} u(-n - 1)$
 D. $2^n u(-n - 1) - n 2^{n+1} u(-n - 1)$

Answer ||| B

Solution |||

$$\frac{X(z)}{z} = \frac{2z - 5}{(z - 2)^2} = \frac{A}{z - 2} + \frac{B}{(z - 2)^2}$$

$$\frac{X(z)}{z} = \frac{2}{z - 2} - \frac{1}{(z - 2)^2}$$

$$X(z) = \frac{2z}{z - 2} - \frac{z}{(z - 2)^2}$$

$$-(2)^n u(-n - 1) \leftrightarrow \frac{z}{z - 2}, |z| < 2$$

$$\frac{n}{2}(2)^n u(-n - 1) \leftrightarrow \frac{-z}{(z - 2)^2}, |z| < 2$$

$$x(n) = 2[-(2)^n u(-n - 1)] + \frac{n}{2}(2)^n u(-n - 1)$$

$$= -(2)^{n+1} u(-n - 1) + n.(2)^{n-1} u(-n - 1)$$

6. The z transform of $x[n] = \{2, 4, 5, 7, 0, 1\}$

- A. $2z^2 + 4z + 5 + 7z + z^3, z \neq \infty$
 B. $2z^{-2} + 4z^{-1} + 5 + 7z + z^3, z \neq \infty$
 C. $2z^{-2} + 4z^{-1} + 5 + 7z + z^3, 0 < |z| < \infty$
 D. $2z^2 + 4z + 5 + 7z^{-1} + z^{-3}, 0 < |z| < \infty$

Answer ||| D

Solution |||

$$x[n] = 2\delta[n + 2] + 4\delta[n + 1] + 5\delta[n] + 7\delta[n - 1] + \delta[n - 3]$$

$$X[z] = 2z^2 + 4z + 5 + 7z^{-1} + z^{-3}, \\ 0 < |z| < \infty$$

7. The Fourier transform of $x(t) = \frac{2}{jt}$ is

A. $X(\omega) = \begin{cases} 2\pi \omega & \geq 0 \\ -2\pi \omega & < 0 \end{cases}$

B. $X(\omega) = \begin{cases} -2\pi \omega & \geq 0 \\ 2\pi \omega & < 0 \end{cases}$

C. $2\pi(u(\omega) - \frac{1}{2})$

D. $2\pi(u(\omega) - u(-\omega))$

Answer ||| B

Solution ||| $\text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$

Using Duality

$$\frac{2}{jt} \leftrightarrow \text{sgn}(-\omega) 2\pi$$

Or, $X(\omega) = \begin{cases} -2\pi \omega & \geq 0 \\ 2\pi \omega & < 0 \end{cases}$

8. Consider the following Laplace transforms of certain signals. For which of the following, final value theorem is not applicable?

A. $\frac{s-1}{s+2}$

B. $\frac{s+1}{(s+2)(s+3)}$

C. $\frac{s+1}{s-2}$

D. $\frac{s+1}{s+2}$

Answer ||| C

Solution |||

For the final value theorem of the Laplace transform to be holds good, the system must be stable.

$$\frac{s+1}{s-2}$$

Therefore, from the options $\frac{s+1}{s-2}$ is unstable because the pole lie on the right hand side of the s-plane and therefore the final value theorem cannot be applied.

9.What is the one-sided z transform of $x(n) = n^2$

- A. $(z+1) / (z-1)^3$
- B. $z(z-1) / (z-1)^3$
- C. $z(z+1) / (z-1)^3$
- D. $-z(z+1) / (z-1)^3$

Answer ||| C

Solution ||| Multiplying the discrete time signal by a discrete unit step signal gives, $x(n) = n^2 u(n)$

Using z transform, $Z\{n^m u(n)\} = (-z d/dz)^m U(z)$

$$(-z d/dz) U(z) = -z [d(z/z-1)/dz] = -z [z-1 - z/(z-1)^2] = z/(z-1)^2$$
$$(-z d/dz)^2 U(z) = -z d/dz [(-z d/dz) U(z)] = -z d/dz [z/(z-1)^2] = -z [(z-1)(z-1-2z)/(z-1)^4]$$
$$= z(z+1)/(z-1)^3$$

10.Mathe the following List I and List II

List I - List II

(input-o/p relation) - (property of system)

A) $y(n) = x(n) - 1$ Non-linear, Non-causal

B) $y(n) = x(n^2) - 2$ Linear, Non-causal

C) $y(n) = x^2(-n) - 3$ Linear, Causal

D) $y(n) = x^2(n) - 4$ Non-linear, Causal

A. 1 4 3 2

B. 3 2 1 4

C. 1 2 3 4

D. 3 4 1 2

Answer ||| B

Solution |||

$y(n) = x(n) \rightarrow$ Linear & Causal

$y(n) = x(n^2) \rightarrow$ Linear & Non-causal

$y(n) = x^2(-n) \rightarrow$ Non-linear & Non-causal

$y(n) = x^2(n) \rightarrow$ Non-linear & Causal

11. The Fourier transform of signal $\sum_{m=0}^{\infty} a^m \delta(t - m), |a| < 1$, is

A. $\frac{a}{1 - ae^{-j\omega}}$

B. $\frac{a}{1 + ae^{-j\omega}}$

C. $\frac{1}{1 + ae^{-j\omega}}$

D. $\frac{1}{1 - ae^{-j\omega}}$

Answer ||| D

Solution |||

$$X(j\omega) = \int_0^{\infty} \sum_{m=0}^{\infty} a^m \delta(t - m) e^{-j\omega t} dt$$

$$= \left[\sum_{m=0}^{\infty} (ae^{-j\omega})^m \right]_{t=m} \int_{-\infty}^{\infty} \delta(t - t_0) f(t) dt = f(t_0)$$

$$= \sum_{m=0}^{\infty} (ae^{-j\omega})^m = \frac{1}{1 - ae^{-j\omega}}$$

12. Fourier transform of Gaussian surface

A. $X(\omega) = e^{\omega^2/4\pi}$

B. $X(\omega) = e^{-\omega^2/2\pi}$

C. $X(\omega) = e^{-\omega^2/4\pi}$

D. $X(\omega) = e^{-\omega^2/\pi}$

Answer ||| C

Solution |||

$$x(t) = e^{-\pi t^2} \quad \dots \dots \dots (1)$$

$$\frac{dx(t)}{dt} \xleftarrow{FT} j\omega X(s)$$

$$\frac{d}{dt} \left(e^{-\pi t^2} \right) \xleftarrow{FT} j\omega X(\omega)$$

$$-2\pi t e^{-\pi t^2} \xleftarrow[-2\pi]{FT} j\omega X(\omega) \quad \dots \dots \dots (2)$$

$$t.e^{-\pi t^2} \xlongequal{} j \frac{dX(\omega)}{d\omega} \quad \dots \dots \dots (3)$$

$$t.x(t) \xlongequal{} j \frac{dX(\omega)}{d\omega} \quad \{$$

{Using frequency differentiation property}

Comparing equations (2) & (3)

$$j \frac{dx(\omega)}{d\omega} = \frac{j\omega X(\omega)}{-2\pi}$$

$$\frac{dX(\omega)}{X(\omega)} = -\frac{1}{2\pi} \omega d\omega$$

[Integrating both sides]

$$\ln X(\omega) = -\frac{1}{2\pi} \frac{\omega^2}{2}$$

[Taking Antilog]

$$X(\omega) = e^{-\omega^2/4\pi}$$

$$\int_0^t f(t) dt$$

13. If $F(s)$ is the Laplace transform of function $f(t)$, then Laplace transform of

- A. $\frac{1}{s} F(s)$
- B. $\frac{1}{s} F(s) - f(0)$
- C. $sF(s) - f(0)$
- D. $\int F(s) ds$

Answer ||| A

$$L \left[\int_0^t \int_0^t \dots \int_0^t f(t) dt^n \right] = \frac{1}{s^n} F(s)$$

Solution ||| From definition, we know

14. Any arbitrary signal multiplied with another arbitrary signal is always a –

- A. Non- Linear System
- B. Linear System
- C. Can be linear or, non-linear
- D. None of the above

Answer ||| A

Solution ||| A linear system should always have constant magnitude. But in the question it is talking about arbitrary signals so, the resultant system should be Non-Linear as the magnitude is not constant.

15.Z – transform of $(n \cdot 2^n) =$

- A. $2z / (z - 1)^2$
- B. $2z / (z - 2)^2$
- C. $z / (2z - 1)^2$
- D. $2z / (z + 2)^2$

Answer ||| B

$$\text{Solution } ||| Z\{n\} = \frac{z}{(z-1)^2}$$

$$Z\{n \cdot a^n\} = \frac{\frac{z}{a}}{\left(\frac{z}{a}-1\right)^2} = \frac{\frac{z}{a}}{\frac{(z-a)^2}{a^2}} = \frac{az}{(z-a)^2}$$

hence $Z\{n \cdot 2^n\} = \frac{2z}{(z-2)^2}$

16.A discrete-time signal

$$x[n] = \delta[n-3] + 2\delta(n-5)$$

has z -transform $X(z)$. If $Y(z) = X(-z)$ is the z -transform of another signal $y[n]$, then

- A. $y[n] = x[n]$
- B. $y[n] = x[-n]$
- C. $y[n] = -x[n]$
- D. $y[n] = -x[-n]$

Answer ||| C

$$x[n] = \delta[n-3] + 2\delta(n-5)$$

$$\therefore x(z) = z^{-3} + 2z^{-5}$$

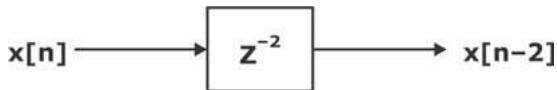
$$x(-z) = (-z)^{-3} + 2(-2)^{-5}$$

$$y(z) = -z^{-3} - 2z^{-5}$$

$$\therefore y[n] = -\delta[n-3] - 2\delta[n-5]$$

Solution ||| $\therefore y[n] = -x[n]$

17. Consider the system given below



Determine whether the signal is causal linear and memoryless.

- A. Memoryless, causal, and linear
- B. Not memoryless, non-causal and nonlinear
- C. Not memoryless, causal, and linear
- D. Not memoryless, causal, and nonlinear

Answer ||| C

Solution |||

$$y[n] = x[n-2]$$

At $n = 0$

$$y[0] = x[-2]$$

So, it has memory

Also, present output depends on past inputs so it's a causal system.

Also, it's a linear system

So, option C is correct

18. The output of a linear system for a step input is te^{-t} . Then the transfer function is.

- A. $\frac{1}{(s+1)^2}$
- B. $\frac{s}{s+1}$
- C. $\frac{s^2}{s+1}$
- D. $\frac{s}{(s+1)^2}$

Answer ||| D

Solution |||

$$y(t) = te^{-t}$$

$$y(s) = \frac{-d}{ds} \left[\frac{1}{s+1} \right]$$

$$= \frac{1}{(s+1)^2}$$

$$x(s) = \frac{1}{s}$$

$$TF = \frac{y(s)}{x(s)} = \frac{\frac{1}{(s+1)^2}}{\frac{1}{s}} = \frac{s}{(s+1)^2}$$

19. Let $x(t) \longleftrightarrow x(\omega) = \begin{cases} 1 & |\omega| \leq 1 \\ 0 & |\omega| > 1 \end{cases}$

consider $y(t) = \frac{d^2x(t)}{dt^2}$. Then value of $\int_{-\infty}^{\infty} y(t)^2 dt$ is _____

A. $\frac{3}{\pi}$

B. $\frac{2}{3}$

C. $\frac{1}{5\pi}$

D. $\frac{1}{6\pi^2}$

Answer ||| C

Solution |||

$$y(t) = \frac{d^2x}{dt^2}$$

$$y(\omega) = (j\omega)^2 x(\omega) \quad |y(\omega)| = \omega^2 |x(\omega)|$$

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |Y(\omega)|^2 d\omega$$

$$= \frac{1}{2\pi} \int_{-1}^1 \omega^4 d\omega$$

$$= \frac{1}{2\pi} \left. \frac{\omega^5}{5} \right|_{-1}^1 = \frac{1}{5\pi}$$

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{5\pi}$$

Now,

20. If the Fourier transform of $x(t)$ is $\frac{2}{\omega} \sin(\pi\omega)$ then what is the Fourier transform of $e^{j5t}x(t)$?

A. $\frac{2}{\omega-5} \sin(\pi\omega)$

B. $\frac{2}{\omega} \sin\{\pi(\omega-5)\}$

C. $\frac{2}{\omega+5} \sin\{\pi(\omega+5)\}$

D. $\frac{2}{\omega-5} \sin\{\pi(\omega-5)\}$

Answer ||| D

Solution |||

If $x(t) \xrightarrow{\text{FT}} X(j\omega)$

$e^{j\omega_0 t}x(t) \xrightarrow{\text{FT}} X(j(\omega - \omega_0))$

Therefore, if $x(t) \xrightarrow{\text{FT}} \frac{2}{\omega} \sin(\pi\omega)$

Then $e^{j5t}x(t) \xrightarrow{\text{FT}} \frac{2}{\omega-5} \sin\{\pi(\omega-5)\}$

21. Match List-I with List-II and select the correct answer using the code given below the Lists:

List-I

- A) Even signal
- B) Causal signal
- C) Periodic signal
- D) Energy signal

List-II
$$x(n) = \left(\frac{1}{4}\right)^n u(n)$$

1) $x(-n) = x(n)$

2) $x(t) = u(t)$

3) $x(n) = x(n+N)$

4)

A. A-2; B-3; C-4; D-1

B. A-1; B-3; C-4; D-2

C. A-2; B-4; C-3; D-1

D. A-1; B-4; C-3; D-2

Answer ||| A

Solution ||| 1) Even signal $x(n) = x(-n)$

2) Causal system is one in which output at any time depends only on present and/or past values of input.

3) Periodic signal is one which satisfies $x(n) = x(n+N)$
 $N \rightarrow$ Fundamental period.

$$x(n) = \left(\frac{1}{4}\right)^n u(n) \Big|_{n < \infty}$$

4) Energy signal is absolutely summable i.e.

22. Inverse Laplace transform of $\frac{s+2}{s^2+4}$ is

- A. $\cos 2t + \sin 2t$
- B. $\cos 2t - \sin 2t$
- C. $\cos t + \sin t$
- D. $\cos t - \sin t$

Answer ||| A

Solution |||

$$\frac{s+2}{s^2+4} = \frac{s}{s^2+4} + \frac{2}{s^2+4}$$

So,

$$L^{-1}\left(\frac{s}{s^2+4} + \frac{2}{s^2+4}\right) = \cos 2t + \sin 2t$$

23.Z-transform of $x[n] = n\left(-\frac{1}{2}\right)^n u[n]$ is:

- A. $\frac{z}{2(z+0.5)^2}; |z| > \frac{1}{2}$
- B. $-\frac{z}{2(z+0.5)^2}; |z| > \frac{1}{2}$
- C. $\frac{z}{2(z+0.5)}; |z| > \frac{1}{2}$
- D. $-\frac{z}{2(z+0.5)}; |z| > \frac{1}{2}$

Answer ||| B

Solution |||

Given:

$$\begin{aligned} x[n] &= n\left(-\frac{1}{2}\right)^n u[n] \\ \left(-\frac{1}{2}\right)^n u[n] &\xrightarrow{\text{z-transform}} \frac{z}{z+0.5}; \quad |z| > \frac{1}{2} \\ n\left(-\frac{1}{2}\right)^n u[n] &\xrightarrow{\text{z-transform}} -z \frac{d}{dz}\left(\frac{z}{z+0.5}\right) \\ &= -\frac{0.5z}{(z+0.5)^2}; \quad |z| > \frac{1}{2} \\ &= -\frac{z}{2(z+0.5)^2}; \quad |z| > \frac{1}{2} \end{aligned}$$

24.Express the z-transform of $y(n) = \sum_{k=-\infty}^n X(k)$ in terms of $X(z)$.

- A. $X(z)$
- B. $(1 + z^{-1}) X(z)$

- C. $\frac{X(z)}{1 - z^{-1}}$
 D. $\frac{X(z)}{1 + z^{-1}}$

Answer ||| C

Solution |||

$$y(n) = \sum_{k=-\infty}^n X(k)$$

$$y(n-1) = \sum_{k=-\infty}^{n-1} X(k)$$

$$y(n) - y(n-1) = x(n)$$

Apply Z transform on both sides

$$Y(Z) - Z^{-1} Y(Z) = X(Z)$$

$$Y(Z) = \frac{1}{1 - Z^{-1}} X(Z)$$

25. Given that

$$\mathcal{L}[F(t)] = \frac{s+2}{s^2+1}, \quad \mathcal{L}[g(t)] = \frac{s^2+1}{(s+3)(s+2)}$$

$$h(t) = \int_0^t F(\tau) g(t-\tau) d\tau$$

Area of $h(t)$ is

- A. 2
 B. 1
 C. 1/3
 D. 1/5

Answer ||| C

Solution |||

$$h(t) = \int_0^t F(\tau) g(t - \tau) d\tau$$

$$H(s) = F(s) \cdot g(s)$$

$$H(s) = \frac{s+2}{s^2+1} \times \frac{s^2+1}{(s+3)(s+2)}$$

$$H(s) = \frac{1}{s+3}$$

$$\text{Area of } h(t) = H(s)|_{s=0}$$

$$\text{Area of } h(t) = \frac{1}{0+3} = \frac{1}{3}$$

26. Laplace transform of $e^{-at} f(t)$ is

- A. $F(s)e^{at}$
- B. $F(s-a)$
- C. $F(s+a)$
- D. $\frac{F(s)}{s} + a$

Answer ||| C

Solution ||| We have to find: $LT[e^{-at}f(t)]$

We know that, $LT[e^{-at}f(t)] \Rightarrow F(s+a)$ [Complex shift property of fourier transform]

27. The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of $t f(t)$ is

- A. $-\frac{s}{(s^2+s+1)^2}$
- B. $-\frac{2s+1}{(s^2+s+1)^2}$

C. $\frac{s}{(s^2 + s + 1)^2}$

D. $\frac{2s + 1}{(s^2 + s + 1)^2}$

Answer ||| D

Solution ||| If $L[f(t)] = \bar{f}(s)$ then

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \bar{f}(s)$$

In this problem,
Given,

$$L[f(t)] = \frac{1}{s^2 + s + 1} = \bar{f}(s)$$

$$L[t f(t)] = (-1)^1 = \frac{d^1}{ds^1} \bar{f}(s)$$

We need

$$\begin{aligned} &= -\frac{d}{ds} \bar{f}(s) = -\frac{d}{ds} \left[\frac{1}{s^2 + s + 1} \right] \\ &= -\left[\frac{-1}{(s^2 + s + 1)^2} \right] \times (2s + 1) = \left[\frac{2s + 1}{(s^2 + s + 1)^2} \right] \end{aligned}$$

28. The region of convergence of the z-transform of $2^n u(n) - 3^n u(-n - 1)$ is

- A. $|z|>1$
- B. $|z|<1$
- C. $2<|z|<3$
- D. doesn't exist

Answer ||| C

Solution ||| $2^n u(n) \leftrightarrow \frac{1}{1-2/z}, |z|>2$

$$3^n u(-n - 1) \leftrightarrow \frac{1}{1-3/z}, |z|<3$$

So ROC is $2<|z|<3$

29. Match list I (Discrete time signal) with List II (energy of the signal) and select the correct answer using the codes given below.

List-I (Discrete time signal)

P. $2^n u[-n]$

Q. $2^n u[-n-1]$

R. $\left(\frac{1}{3}\right)^n u[n]$

S. $u[n] - u[n-4]$

List II (Energy of the signal)

1) $9/8$

2) $4/3$

3) 4

4) $1/3$

A. P-4, Q-3, R-2, S-1

B. P-2, Q-4, R-1, S-3

C. P-4, Q-1, R-3, S-2

D. P-1, Q-4, R-2, S-3

Answer ||| B

Solution |||

$$(P) \quad x^2[n] = 2^{2n} u[-n] = \left(\frac{1}{4}\right)^{-n} u[-n]$$

$$\text{Energy} \Rightarrow E_1 = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^{-n} = \frac{1}{1 - 1/4} = 4/3$$

$$(Q) \quad x^2[n] = 2^{2n} u[-n-1] = \left(\frac{1}{4}\right)^{-n} u[-n-1]$$

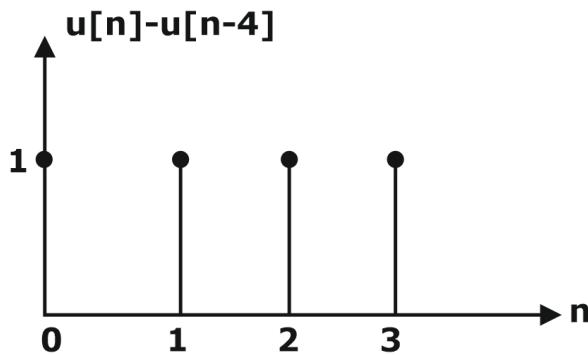
$$\text{Energy} \Rightarrow E_2 = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^{-n} = \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= \frac{1/4}{1 - 1/4} = 1/3$$

$$(R) E_3 = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{2n} = \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$= \frac{1}{1 - 1/9} = 9/8$$

(S)



$$E_4 = (1)^2 + (1)^2 + (1)^2 + (1)^2$$

$$= 4$$

30. Fourier transform of $(1/t)$ is:

- A. $j\pi \operatorname{sgn}(\omega)$
- B. $-j\pi \operatorname{sgn}(\omega)$
- C. $\pi \operatorname{sgn}(\omega)$
- D. $-\pi \operatorname{sgn}(\omega)$

Answer ||| B

Solution |||

$$\operatorname{sgn}(t) = \begin{cases} 1; & t \geq 0 \\ -1; & t < 0 \end{cases}$$

$$\begin{array}{ccc} \text{sgn}(t) & \longleftrightarrow & \frac{2}{j\omega} \\ \frac{2}{jt} & & 2\pi \text{sgn}(-\omega) \end{array}$$

or $\frac{1}{t} \longleftrightarrow -j\pi \text{sgn}(\omega)$

31.if $f(t) = \frac{2}{1} \sqrt{\frac{t}{\pi}}$

And Laplace transform of $f(t) = F(s) = s^{-3/2}$

$g(t) \text{ if } g(t) = \sqrt{\frac{1}{\pi t}}$ is
Then Laplace transform of

- A. $\frac{2}{\sqrt{s}}$
- B. $\frac{1}{\sqrt{s}}$
- C. \sqrt{s}
- D. $2\sqrt{s}$

Answer ||| B

Solution |||

$$g(t) = f \frac{(t)}{2t}$$

$$\mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s) ds = \int_s^{\infty} s^{3/2} ds$$

$$= \left[\frac{s^{-3/2} + 1}{-3/2 + 1} \right]_s^{\infty}$$

$$= -2 \left[0 - s^{-1/2} \right]$$

$$= 2s^{-1/2}$$

$$\mathcal{L}\{g(t)\} = \mathcal{L}\left\{\frac{f(t)}{2t}\right\} = \frac{1}{\sqrt{s}}$$

32. The impulse response of a linear time invariant system is $H(n) = (1, \underline{2}, 1, -1)$. The responses for the input signal $x(n) = (\underline{1}, 2, 3, 1)$ is

- A. {1, 8, 4, 8, 3, -1, 2}
- B. {1, 4, 8, 3, 8, -1, 2}
- C. {1, 4, 8, 8, 3, -2, -1}
- D. {1, 8, 3, 8, 8, 4, -1}

Answer ||| C

Solution ||| $x(n) = \{1, 2, 3, 1\}$

$$h(n) = \{1, \underline{2}, 1, -1\}$$

\backslash	$x[n]$	1	2	3	1
$h[n]$	1	1	2	3	1
1	1	2	3	1	
2	2	4	6	2	
1	1	2	3	1	
-1	-1	-2	-3	-1	

The sum of each diagonal element will give the response of the input.

$$x(n) * h(n) = (1, 4, 8, 8, 3, -2, -1)$$

33. The signal $x(n) = (j)^n + (j)^{-n}$ is:

- A. Energy signal
- B. Power signal
- C. Both 'a' and 'b'
- D. Neither energy nor power signal

Answer ||| B

Solution |||

$$x(n) = (j)^n + (j)^{-n} \quad [e^{j\theta} = \cos\theta + j\sin\theta]$$

$$= e^{jn\pi/2} + e^{-jn\pi/2} \quad \text{i.e. } j = e^{j\pi/2}$$

$$= 2\cos\frac{n\pi}{2} \rightarrow \text{It is a periodic signal}$$

∴ All periodic signals are power signals.

34. The even part of the discrete time sequence

$$x[n] = \left\{ 4, 3, \underset{\uparrow}{2}, -3, -2 \right\} \text{ is } \underline{\quad}$$

- A. $\left\{ 2, 1.5, \underset{\uparrow}{1}, -1.5, -1 \right\}$
- B. $\left\{ 1, 0, \underset{\uparrow}{2}, 0, 1 \right\}$
- C. $\left\{ 0, 1, \underset{\uparrow}{2}, 0, 1 \right\}$
- D. $\left\{ -1, -1.5, \underset{\uparrow}{1}, 1.5, 2 \right\}$

Answer ||| B

Solution |||

Even part of $x[n]$ is-

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x[n] = \left\{ 4, 3, 2, \underset{\uparrow}{-3}, -2 \right\}$$

$$x[-n] = \left\{ -2, -3, 2, \underset{\uparrow}{3}, 4 \right\}$$

$$x_e[n] = \frac{1}{2} \left\{ 2, 0, \underset{\uparrow}{4}, 0, 2 \right\}$$

$$x_e[n] = \left\{ 1, 0, \underset{\uparrow}{2}, 0, 1 \right\}$$

35. Z – transform of $(-4)^n$ is

- A. $z/(z-4)$
- B. $z/(4-z)$
- C. $z/(z+4)$
- D. $(z-4)/z$

Answer ||| C

Solution ||| $Z[(-4)^n] = \frac{z}{z+4}$

36. The signal $x(t) = e^{-at}|t| \cdot \cos 2t$ is _____

- A. even signal
- B. odd signal
- C. neither even nor odd signal
- D. odd for some values of t

Answer ||| A

Solution |||

$$x(t) = e^{-at}|t| \cdot \cos 2t$$

$$\text{Let } x_1(t) = e^{-at}|t|$$

$$x_2(t) = \cos 2t$$

$$x(t) = x_1(t) \cdot x_2(t)$$

$$x_1(t) = x_1(-t)$$

So, $x_1(t)$ is even signal

$$x_2(t) = x_2(-t)$$

So, $x_2(t)$ is even signal

since, $x(t)$ is multiplication of two even signal,

Hence, $x(t)$ will also be even signal.

37. Laplace transform of $\cos(\omega t)$ is $\frac{s}{s^2 + \omega^2}$. The Laplace transform of $e^{-2t} \cos(4t)$ is

A. $\frac{s-2}{(s-2)^2 + 16}$

B. $\frac{s+2}{(s-2)^2 + 16}$

C. $\frac{s-2}{(s+2)^2 + 16}$

D. $\frac{s+2}{(s+2)^2 + 16}$

Answer ||| D

Solution ||| Laplace transformation of $e^{-2t} \cos(4t)$ is,

$$L(e^{at} \cos bt) = \frac{s-a}{(s-a)^2 + b^2}$$

We know that,

Putting, a = -2 and b = 4

$$L(e^{-2t} \cos 4t) = \frac{s+2}{(s+2)^2 + 4^2}$$

$$L(e^{-2t} \cos 4t) = \frac{s+2}{(s+2)^2 + 16}$$

38. Fourier transform and Laplace transform are related through: -

- A. Time domain
- B. Frequency domain
- C. Both time and frequency domains
- D. None of these

Answer ||| B

Solution |||

Fourier transformer and Laplace Transforms are frequency domain analysis.

39. Consider the following statements.

1) Initial value Theorem of a z-transform is $x(0) = \lim_{z \rightarrow \infty} z x(z)$

2) Final value Theorem of a z-transform is $x(\infty) = \lim_{z \rightarrow 1} (z-1) x(z)$.

Which of the above statements are correct?

- A. only 1
- B. only 2
- C. Both 1 and 2
- D. None

Answer ||| C

Solution |||

Both 1 and 2 are correct as per the definitions of IVT and FVT.

40.The Laplace transform of $t^5 e^{2t}$ is :

- A. $\frac{120}{(s-2)^6}$
- B. $\frac{120}{(s-2)^5}$
- C. $\frac{720}{(s-2)^6}$
- D. $\frac{120}{(s+2)^6}$

Answer ||| A

Solution |||

Given:

$$x(t) = t^5 e^{2t}$$

we know that

$$L(t^n) = \frac{n!}{s^n + 1}$$

$$\text{So, } L(t^5) = \frac{5!}{s^6}$$

using frequency shifting property

$$L(t^5 e^{2t}) = \frac{5!}{(s-2)^6}$$

$$L(t^5 e^{2t}) = \frac{120}{(s-2)^6}$$

41.The power content in the signal

$$s(t) = 8 \sin\left(20\pi t - \frac{\pi}{4}\right) + 5 \cos(15\pi t)$$

- A. 89
- B. 22.25

C. 44.5

D. 76.5

Answer ||| C

Solution |||

The power is

$$P = \frac{8^2}{2} + \frac{5^2}{2}$$

$$= \frac{64 + 25}{2}$$

= 44.5 watt

42. The Laplace transform of $x(t) = \cos^3(3t)u(t)$

A. $\frac{s(s^2 + 1)}{(s^2 + 9)(s^2 + 81)}$

B. $\frac{s(s^3 + 63)}{(s^2 + 9)(s^2 + 81)}$

C. $\frac{s(s^2 + 16)}{(s^2 + 9)(s^2 + 81)}$

D. $\frac{s^2 + 16}{(s^2 + 9)(s^2 + 81)}$

Answer ||| B

Solution |||

$$x(t) = \cos^3(3t) u(t)$$

$$\therefore \cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\therefore \cos^3\theta = \frac{\cos 3\theta + 3\cos\theta}{4}$$

$$\therefore x(t) = \frac{1}{4} [\cos 9t + 3\cos 3t] u(t)$$

$$\cos(bt) \leftrightarrow \frac{s}{s^2 + b^2}$$

$$\therefore X(s) = \frac{1}{4} \left[\frac{s}{s^2 + 81} + \frac{3s}{s^2 + 9} \right]$$

$$= \frac{s}{4} \left[\frac{s^2 + 9 + 3s^2 + 243}{(s^2 + 81)(s^2 + 9)} \right]$$

$$= \frac{s}{4} \left[\frac{4s^2 + 252}{(s^2 + 81)(s^2 + 9)} \right]$$

$$= \frac{s(s^2 + 63)}{(s^2 + 81)(s^2 + 9)}$$

43. Determine the time period of the signal $x(t) = 15 \sin\left(4t + \frac{\pi}{6}\right) + 23 \cos\left(12t + \frac{\pi}{4}\right)$

- A. $\frac{\pi}{4}$
- B. $\frac{\pi}{2}$
- C. 2π
- D. Non-periodic

Answer ||| B

Solution |||

$$\omega_1 = 4 \text{ rad/s}, \omega_2 = 12 \text{ rad/s}$$

$$\omega_0 = \text{HCF}(\omega_1, \omega_2)$$

$$\Rightarrow \omega_0 = \text{HCF}(4, 12) = 4 \text{ rad/s}$$

$$\Rightarrow \text{Time period, } T = \frac{2\pi}{\omega_0} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ s}$$

44. The inverse Fourier transform of

$$\frac{2(j\omega)^2 + 12j\omega + 14}{(j\omega)^2 + 6j\omega + 5} \text{ is}$$

- A. $2\delta(t) + (e^{-t} - 2e^{-5t}) u(t)$
- B. $2t + (e^{-t} - 2e^{-5t}) u(t)$
- C. $2\delta(t) + (e^{-t} - e^{-5t}) u(t)$
- D. $2t + (e^{-t} - e^{-5t}) u(t)$

Answer ||| C

Solution |||

$$X(j\omega) = \frac{2(j\omega)^2 + 12j\omega + 14}{(j\omega)^2 + 6j\omega + 5}$$

$$= \frac{2}{5 + j\omega} + \frac{1}{1 + j\omega}$$

(Using partial fraction method)

Taking inverse transform, we get

$$x(t) = 2\delta(t) + (e^{-t} - e^{-5t}) u(t)$$

45. The signal

$$x[n] = 2e^{j3\pi n}$$

- A. an energy signal
- B. a power signal
- C. neither an energy nor a power signal
- D. an even signal

Answer ||| B

Solution ||| This signal is periodic and periodic signals are always power signals.

46. Given the Laplace transform of $f(t) = F(s)$, the Laplace transform of $[f(t)e^{-at}]$ is equal to

A. $F(s+a)$

B. $\frac{F(s)}{(s+a)}$

C. $e^{as}F(s)$

D. $e^{-as}F(s)$

Answer ||| A

Solution ||| If $Lf(t) = F(s)$

then according to the Laplace Shifting Property

$$L[e^{at} f(t)] = F(s-a)$$

So, according to the above property

$$[f(t)e^{-at}] = F(s+a)$$

47. A discrete time signal is given as $x[n] = \cos \frac{\pi n}{9} + \sin \left[\frac{\pi n}{7} + \frac{1}{2} \right]$. The period N for the periodic signal is

A. 126

B. 32

C. 252

D. 64

Answer ||| A

Solution |||

A discrete time signal is

$$x[n] = \cos \frac{\pi n}{9} + \sin \left[\frac{\pi n}{7} + \frac{1}{2} \right]$$

$$x[n] = x_1[n] + x_2[n]$$

$$x_1[n] = \cos \frac{\pi n}{9}$$

$$N_1 = \frac{2\pi}{\omega_0} \times K$$

$$N_1 = \frac{2\pi}{\pi/9} \times K = 18K$$

$$\boxed{N_1 = 18} \text{ for } K = 1$$

$$x_2[n] = \sin \left[\frac{\pi n}{7} + \frac{1}{2} \right]$$

$$N_2 = \frac{2\pi}{\omega_0} \times K$$

$$N_2 = \frac{2\pi}{\pi/7} \times K$$

$$N_2 = 14K$$

$$\boxed{N_2 = 14} \text{ for } K = 1$$

$$N = \text{LCM}(N_1 N_2) = 126$$

48.Z-Transform of the function

$\sum_{k=-\infty}^0 \delta(n-k)$ has the following region of convergence

- A. $|z| > 1$
- B. $|z|=1$
- C. $|z| < 1$
- D. $0 < |Z| < 1$

Answer || C

Solution || $x(n) = \sum_{k=-\infty}^0 \delta(n-k)$

$$X(Z) = \sum_{k=-\infty}^0 z^{-k} = \dots + z^3 + z^2 + z + 1$$

$$\frac{1}{1-z}, |z| < 1$$

49. Consider the signal

$$y(t) = \int_{-\infty}^t x(t) dt$$

This signal is

- A. Linear and time invariant
- B. Linear and time variant
- C. Non-linear and time invariant
- D. Non-linear and time variant

Answer ||| B

Solution ||| For linearity;

Whenever there is an introduction of integration in a signal then that signal is always be linear.

For time variance;

Since this signal is time scaled, so it is time variant.

$$y(t) = \frac{4t}{(1+t^2)^2}$$

50. Find the Fourier Transform of

- A. $-j\pi \omega e^{j\omega}$
- B. $2j\pi \omega e^{j\omega}$
- C. $j2\pi \omega e^{-j\omega}$
- D. $-2j\pi \omega e^{-j\omega}$

Answer ||| D

Solution |||

$$e^{-a|t|} \xrightarrow{\text{F.T.}} \frac{2a}{a^2 + \omega^2}$$

By duality,

$$\frac{2a}{a^2 + t^2} \xrightarrow{\text{F.T.}} 2\pi e^{-a|-j\omega|}$$

Put $a = 1$

$$\therefore \frac{2}{1+t^2} \xrightarrow{\text{F.T.}} 2\pi e^{-|j\omega|}$$

$$\frac{d}{dt} \left(\frac{2}{1+t^2} \right) \xrightarrow{\text{F.T.}} (j\omega)(2\pi e^{-|\omega|})$$

$$\frac{-4t}{(1+t^2)^2} \xrightarrow{\text{F.T.}} j2\pi\omega e^{-|\omega|}$$

$$\therefore \frac{4t}{(1+t^2)^2} \xrightarrow{\text{F.T.}} -j2\pi\omega e^{-|\omega|}$$

51.The Laplace Transform

- A. Is a real valued function of a complex argument
- B. Is a complex valued function of a complex argument
- C. Is a real valued function of a real argument
- D. Is a complex valued function of a real argument

Answer ||| B

Solution ||| The Laplace Transform is a complex valued function of a complex argument as
 $s = \sigma + j\omega$

$$52.\text{Inverse Laplace transform is } \frac{s^2 - 3s + 4}{s^3}$$

- A. $1 - 2t + 4t^2$
- B. $1 + 3t + 2t^2$
- C. $1 - 6t - 3t^2$
- D. $1 - 3t + 2t^2$

Answer ||| D

Solution |||

$$\Rightarrow L^{-1} \left(\frac{s^2 - 3s + 4}{s^3} \right)$$

$$\Rightarrow L^{-1} \left(\frac{1}{s} \right) - 3L^{-1} \left(\frac{1}{s^2} \right) + 4L^{-1} \left(\frac{1}{s^3} \right)$$

$$\Rightarrow 1 - 3t + \frac{4t^2}{2}$$

$$\Rightarrow 1 - 3t + 2t^2$$

53. A system defined by linear constant coefficient differential equation is linear if

- A. A system is at initial rest
- B. system has a linear impulse response
- C. $y(-\infty) = y(\infty) = 0$
- D. Auxiliary condition are zero

Answer ||| D

Solution |||

A system described by a linear constant coefficient differential equation is linear if auxiliary conditions are zero.

54. Consider the signal

$$x(n) = \begin{cases} b^n; & 0 \leq n \leq N-1 \\ 0; & \text{elsewhere} \end{cases}$$

The ROC of Z.T of $x(n)$ is

- A. $|z| > |b|$
- B. $|z| < |b|$
- C. $|z| > 0$
- D. $0 < |z| < \infty$

Answer ||| C

Solution |||

$x(n)$ is finite length sequence.

$$\begin{aligned} x(n) &\xrightarrow{\text{ZT}} X(z) \\ &= 1 + bz^{-1} + b^2z^{-2} + \dots + b^{N-1}z^{-(N-1)} \end{aligned}$$

$X(z)$ is finite for all z , except $z = 0$

$$\therefore |z| > 0$$

55. The Fourier transform of $e^{-\pi t^2}$ is $e^{-\pi f^2}$ then Fourier transform of $e^{-\alpha t^2}$ is

A. $\sqrt{\frac{\pi}{\alpha}} e^{-\frac{\pi^2 f^2}{\alpha}}$

B. $\frac{1}{\alpha} e^{-\alpha f^2}$

C. $\frac{1}{\sqrt{\pi \alpha}} e^{\alpha \pi^2 f^2}$

D. $\sqrt{\alpha \pi} e^{-\frac{f^2}{\pi^2 \alpha}}$

Answer ||| A

Solution |||

$$X(\omega) = \int_{-\infty}^{\infty} e^{-\alpha t^2} e^{-j\omega t} dt$$

$$\int_{-\infty}^{\infty} e^{-(\alpha t^2 + j\omega t)} dt$$

$$= e^{-\omega^2/4\alpha} \int_{-\infty}^{\infty} e^{-\left(t\sqrt{\alpha} + \frac{j\omega}{2\sqrt{\alpha}}\right)^2} dt$$

$$p = t\sqrt{\alpha} + \frac{j\omega}{2\sqrt{\alpha}}, dp = \sqrt{\alpha}dt$$

$$X(\omega) = \frac{e^{-\omega^2/4\alpha}}{\sqrt{\alpha}} \int_{-\infty}^{\infty} e^{-p^2} dp \quad \left[\int_{-\infty}^{\infty} e^{-p^2} dp = \sqrt{\pi} \right]$$

$$= \frac{\sqrt{\pi}}{\sqrt{\alpha}} e^{-\omega^2/4\alpha}$$

$$F[e^{-\alpha t^2}] = \frac{\sqrt{\pi}}{\sqrt{\alpha}} e^{-\omega^2/4\alpha}$$

$$= \frac{\sqrt{\pi}}{\sqrt{\alpha}} e^{-\frac{4\pi^2 f^2}{4\alpha}}$$

$$= \frac{\sqrt{\pi}}{\sqrt{\alpha}} e^{-\frac{\pi^2 f^2}{\alpha}}$$

56. Find the Inverse Laplace Transform of

$$F(s) = \frac{2s^2 + 6s + 5}{s^3 - 6s^2 + 11s - 6}$$

- A. $\frac{1}{2}e^t - e^{2t} + \frac{5}{2}e^{3t}$
- B. $te^t + 3e^{2t} + \frac{2}{5}e^{3t}$
- C. $-\frac{1}{2}e^t + e^{2t} - \frac{3}{2}e^{3t}$
- D. $\frac{1}{2}e^t + e^{2t} + \frac{2}{3}e^{3t}$.

Answer ||| A

Solution |||

$$\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6} = \frac{A}{(s-1)} + \frac{B}{(s-2)} + \frac{C}{(s-3)}$$

$$2s^2 - 6s + 5 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$$

Putting

$$s=1, A = \frac{1}{2}$$

$$s=2, B = -1$$

$$s=3, C = \frac{5}{2}.$$

$$\begin{aligned} L^{-1}\left(\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}\right) &= \frac{1}{2}L^{-1}\left(\frac{1}{s-1}\right) - L^{-1}\left(\frac{1}{s-2}\right) + \frac{5}{2}L^{-1}\left(\frac{1}{s-3}\right) \\ &= \frac{1}{2}e^t - e^{2t} + \frac{5}{2}e^{3t} \end{aligned}$$

57. Find the time period of the discrete time signal

$$x(n) = e^{j7\pi n} + -3e^{j3\pi \left(n+\frac{1}{2}\right)/5}$$

- A. $N = 10$
- B. $N = 2$
- C. $N = 5$
- D. Not periodic signal.

Answer || A

Solution ||

$$x(n) = e^{j7\pi n} + -3e^{j3\pi \left(n+\frac{1}{2}\right)/5}$$

$\downarrow \quad \quad \quad \downarrow$

$$x_1(n) \quad \quad \quad x_2(n)$$

$$x(n) = e^{j7\pi n} \quad W_o = 7\pi$$

$$N_1 = \frac{2\pi}{W_o} \times m$$

$$N_1 = \frac{2\pi}{2\pi} \times m$$

$$N_1 = \frac{2}{7} \times m$$

Where $m = \text{minimum integer value to make 'N' integer.}$

For $m = 7$

$$N_1 = 2$$

$$x_2(n) = 3e^{j\pi \left(n+\frac{1}{2}\right)/5}$$

We know that due to shifting time period is not changes.

$$W_o = \frac{3\pi}{5}$$

$$N_2 = \frac{2\pi}{W_o} \times m$$

$$N_2 = \frac{2\pi}{2\pi} \times 5 \times m$$

$$N_2 = \frac{10}{3} \times m$$

$$N_2 = 10$$

So the time period of signal $x(n)$.

$$N = \text{LCM of } (N_1 N_2)$$

$$N = \text{LCM of } (2, 10) = 10$$

$$N = 10$$

58. The impulse response for the discrete time system $y[n] = 0.5(x(n) + x(n-2)) + x(n-1)$ is given by

A. $0.5\delta(n) + 0.5\delta(n-1) + 0.5\delta(n-2)$

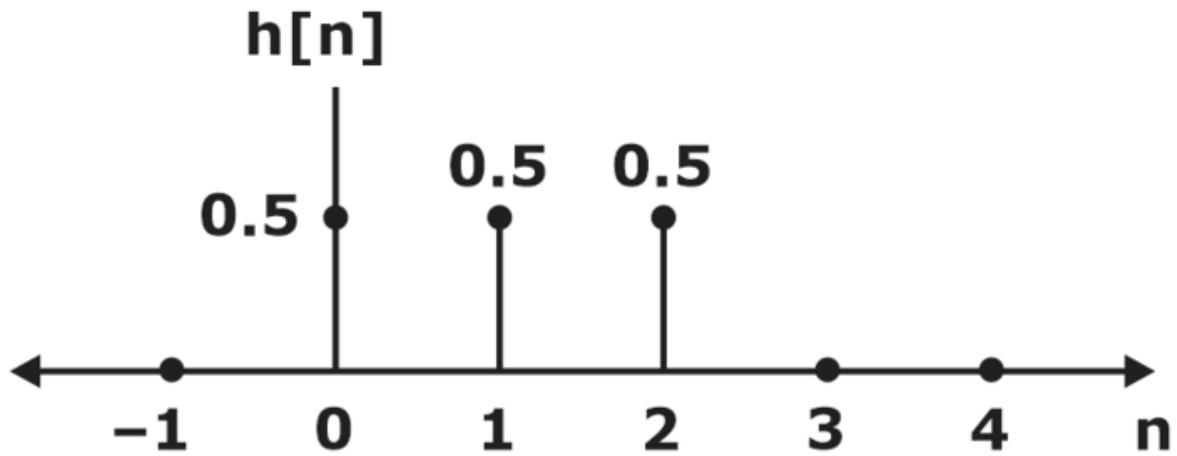
B. $0.5\delta(n+1) + 0.5\delta(n) + 0.5\delta(n-1)$

C. $\delta(n) + \delta(n-1) + \delta(n+2)$

D. none of these

Answer ||| A

Solution |||



$$\text{Given } y[n] = 0.5[x(n) + x(n-1) + x(n-2)]$$

To find $h[n]$, we have to take

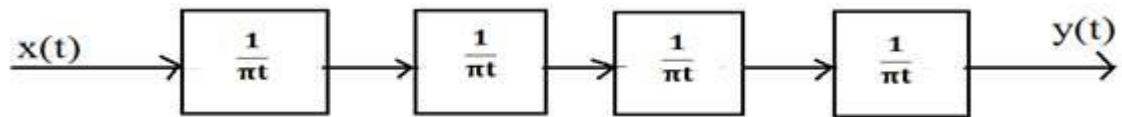
$$X[n] = \delta[n]$$

$$Y[n] = h[n] = 0.5[\delta(n) + \delta(n-1) + \delta(n-2)]$$

$$h[n] = 0.5\delta(n) + 0.5\delta(n-1) + 0.5\delta(n-2)$$

$$\therefore h[n] = \begin{cases} 0.5 & \text{for } n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

59. Consider the cascaded structure shown below and find the function $y(t)$ in terms of $x(t)$



- A. $x(t)$
- B. $x^2(t)$
- C. $-x(t)$
- D. $x^3(t)$

Answer ||| A

Solution ||| The above block is Hilbert Transform whose response is given by $\frac{1}{\pi t}$
 If $h(t) = 1/\pi t$, then we get output which is phase shifting by 90°
 Four time phase shifting by 90° means shift by 360° , hence sign will not be changes here

$$F(s) = \frac{s+1}{(s+2)(s+3)}$$

60.The initial value of $F(t)$, with transform

- A. 0
- B. 1
- C. ∞
- D. $\frac{1}{6}$

Answer ||| B

Solution |||

$$F(s) = \frac{s+1}{(s+2)(s+3)}$$

$$F(0) = \lim_{s \rightarrow \infty} sF(s) = 1$$

61.z-transform is particularly used in:

- A. Finding the frequency response of discrete-time LTI signal
- B. Evaluating the stability of discrete-time LTI systems
- C. Analysisg discrete-time LTI systems
- D. All of these

Answer ||| D

Solution ||| z-transform for finding the frequency response and evaluating the stability of discrete-time LTI systems

62.Show that the z- transform of w (z) if the given $w_n = x[n] = n + 1$. use convolution to show z- transform?

- A. 1
- B. $n+1$
- C. n
- D. 0

Answer ||| B

Solution ||| Given $w_n = x[n] = n + 1$

Let both $x_n = 1$ and $y_n = 1$ be the unit step sequence and both

$$X(z) = \frac{z}{z-1} \text{ and } y(z) = \frac{z}{z-1}$$

$$W(z) = x(z).y(z) = \frac{z}{z-1} \cdot \frac{z}{z-1} = \frac{z^2}{(z-1)^2}$$

$$\begin{aligned} \text{So that } w_n &= x_n * y_n = \sum_{i=0}^n x_i y_{n-i} \\ &= \sum_{i=0}^n 1 \end{aligned}$$

$$w_n = n+1$$

63. Find the inverse Laplace transform of $\frac{s+3}{s^2+2s+2}$.

- A. $e^{-2t}(\cos t + 2 \sin t)$
- B. $e^{-t}(\cos t + 2 \sin t)$
- C.
- D. $e^{-2t}(2\cos t + \sin t)$

Answer ||| B

Solution |||

$$\frac{s+3}{s^2+2s+2} = \frac{s+1+2}{(s+1)^2+1} = \frac{s+1}{(s+1)^2+1} + \frac{2}{(s+1)^2+1}$$

So,

$$L^{-1}\left(\frac{s+3}{s^2+2s+2}\right) = e^{-t} \cos t + 2e^{-t} \sin t$$

64. The convolution of $x_1(t) = \sin(4t)u(t)$ and $x_2(t) = \cos(3t)u(t)$ is:

A. $\frac{3}{7} \cos 3t u(t) - \frac{3}{7} \sin 4t u(t)$

B. $\frac{3}{7} \cos 3t u(t) - \frac{3}{7} \cos 4t u(t)$

C. $\frac{4}{7} \sin 3t u(t) - \frac{4}{7} \cos 4t u(t)$

D. $\frac{4}{7} \cos 3t u(t) - \frac{4}{7} \cos 4t u(t)$

Answer ||| D

Solution |||

$$x_1(t) = \sin 4t u(t) \xrightarrow{\text{LT}} X_1(s) = \frac{4}{s^2 + 16}$$

$$x_2(t) = \cos 4t u(t) \xrightarrow{\text{LT}} X_2(s) = \frac{s}{s^2 + 9}$$

$$\text{LT}[x_1(t) * x_2(t)] = X_1(s) \cdot X_2(s) = \frac{4}{s^2 + 16} \cdot \frac{s}{s^2 + 9}$$

$$X_1(s)X_2(s) = \frac{4}{7} \left[\frac{s}{s^2 + 9} - \frac{s}{s^2 + 16} \right]$$

Applying Inverse Laplace transform,

$$x_1(t) * x_2(t) = \frac{4}{7} \cos 3t u(t) - \frac{4}{7} \cos 4t u(t)$$

65. Given that $F(s)$ is the one sided Laplace transform of $f(t)$, the Laplace transform

of $\int\limits_0^t f(\tau) d\tau$
is

A. $sF(s) - f(0)$

B. $\frac{1}{2} F(s)$

C. $\int\limits_0^s F(\tau) d\tau$

D. $\frac{1}{s} [F(s) - f(0)]$

Answer ||| D

Solution ||| $F(s) = L \{ f(t) \}$

$$\text{so } F(s) = \frac{1}{s} [F(s) - f(0)]$$

Option (D) is the correct choice.

66. In Context with Laplace transform and Fourier transform. The correct statement is:

- A. Laplace transform is a special case of Fourier transform
- B. Laplace transform exists for both stable and unstable systems while Fourier transform exists only for Stable systems
- C. Laplace transform and Fourier transform are zero for unstable signal
- D. Laplace transform and Fourier transform are not related to each other.

Answer ||| B

Solution |||

Fourier transform = Laplace transform|Real part=0

i.e. Fourier transform is special case of Laplace transform.

Laplace transform exist for Stable as well as unstable signal while Fourier transform exist only for stable signals.

67. For the discrete time sequence $x[n] = 2(n-1) u[n-1] - 2(n-2) u[n-2]$.

The value of $x[3]$ is_____

- A. 1
- B. 0
- C. 2
- D. 3

Answer ||| C

Solution |||

$$x[n] = 2(n-1) u[n-1] - 2(n-2) u[n-2]$$

at $n = 3$

$$x[3] = 2(3-1) u[3-1] - 2(3-2) u[3-2]$$

$$= 2(2) - 2(1)$$

$$= 4 - 2 = 2$$

68.What is the nature of a Ramp Signal?

- A. Energy Signal
- B. Power signal
- C. neither energy signal nor power signal
- D. Both 'a' and 'b'

Answer ||| C

Solution |||

Ramp signal $R(t) = t \cdot u(t)$

$$\text{Energy, } E = \int_0^{\infty} t^2 dt \rightarrow ' \infty '$$

$$\text{Power, } P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{\infty} t^2 dt \rightarrow ' \infty '$$

\therefore Energy, Power both values are infinite.

So, it is a neither energy signal nor power signal.

69.Find the inverse Z- transform of $x(z) = \log(1+z)$, $|z|<1$.

- A. $\frac{(-1)^n}{n} u[-n+1]$
- B. $\frac{(-1)^n}{n} u[-n-1]$
- C. $\frac{n}{(-1)^n} u[n-1]$
- D. None of these

Answer ||| B

Solution || Given

$$X(z) = \log(1+z)$$

$$x(n) \Leftarrow X(z), \text{ ROC } |Z| < 1$$

$$n x(n) \Leftarrow -z \frac{d}{dz} X(z), \text{ ROC } |Z| < 1$$

$$n x(n) \Leftarrow -z \frac{d}{dz} [\log(1+z)], \text{ ROC } |Z| < 1$$

$$n x(n) \Leftarrow -z \cdot \frac{1}{1+z}, \text{ ROC } |Z| < 1$$

$$n x(n) \Leftarrow -\frac{z}{1+z}, \text{ ROC } |Z| < 1$$

$$n x(n) \Leftarrow -\frac{1}{1+z^{-1}}, \text{ ROC } |Z| < 1$$

$$n x(n) = -\left\langle -(-1)^n [u(-n-1)] \right\rangle$$

$$n x(n) = (-1)^n [u(-n-1)]$$

$$x(n) = \frac{(-1)^n}{n} [u(-n-1)]$$

70. The inverse Laplace transform:

$$L^{-1} \left[\frac{s+3}{(s^2 + 6s + 13)^2} \right] \text{ equals :}$$

A. $\frac{1}{4} te^{-3t} \sin 2t$

B. $\frac{1}{4} te^{3t} \sin 2t$

C. $\frac{1}{2} te^{-3t} \sin t$

D. $\frac{1}{2} te^{3t} \sin 2t$

Answer || A

Solution ||

$$L^{-1} \left[\frac{s+3}{(s^2 + 6s + 13)^2} \right]$$

$$= L^{-1} \left[\frac{s+3}{[(s+3)^2 + 4]^2} \right]$$

$$= e^{-3t} L^{-1} \left[\frac{s}{(s^2 + 4)^2} \right] \{ \text{Shifting property} \}$$

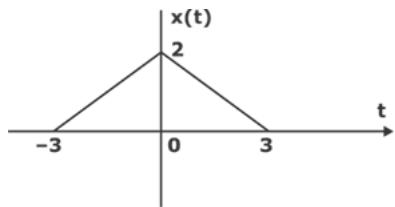
By a standard result: $L^{-1} \left[\frac{s}{(s^2 + a^2)^2} \right] = \frac{ts \sin at}{2a}$ we have:

$$L^{-1} \left[\frac{s}{(s^2 + 4)^2} \right] = \frac{ts \sin 2t}{4}$$

Thus :

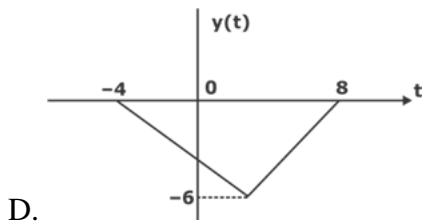
$$L^{-1} \left[\frac{s+3}{(s^2 + 6s + 13)^2} \right] = \frac{1}{4} t e^{-3t} \sin 2t$$

71. A signal $x(t)$ is shown in the following figure.



The plot for a transformed signal $y(t) = -3x\left(\frac{t-2}{2}\right)$ will be –

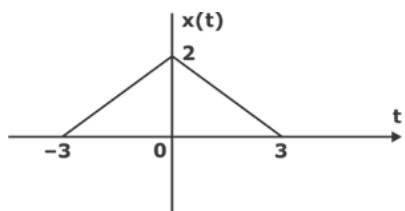
- A.
-
- | t | y(t) |
|----|------|
| -8 | -8 |
| -6 | -6 |
| -4 | -4 |
| 0 | -4 |
| 4 | -4 |
- B.
-
- | t | y(t) |
|----|------|
| -8 | -8 |
| -4 | -4 |
| 0 | -3 |
| 4 | -4 |
- C.
-
- | t | y(t) |
|----|------|
| -8 | -8 |
| -4 | -6 |
| 0 | -6 |
| 4 | -6 |



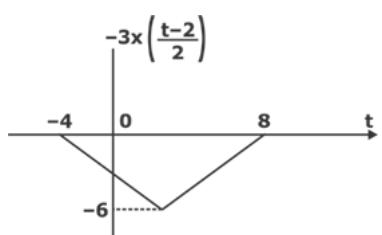
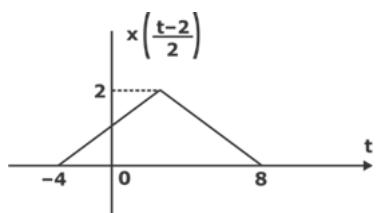
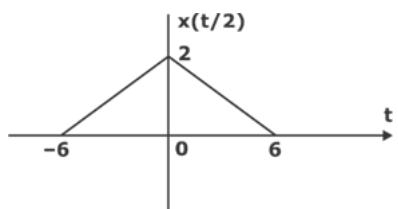
Answer ||| D

Solution |||

$$y(t) = -3x\left(\frac{t-2}{2}\right)$$



$$\begin{aligned} x(t) &\xrightarrow[\text{time scaling}]{t \rightarrow t/2} x\left(\frac{t}{2}\right) \xrightarrow[\text{time shifting}]{t \rightarrow (t-2)} x\left(\frac{t-2}{2}\right) \\ &\xrightarrow[\text{amplitude scaling}]{-3} -3x\left(\frac{t-2}{2}\right) \end{aligned}$$



72. Consider the signal $x(t) = \frac{d}{dt} \{\cos(t) \operatorname{sgn}(t)\}$ $x(t)$ is _____

- A. Even signal
- B. odd signal
- C. Neither even nor odd signal
- D. odd with some conditions

Answer ||| A

Solution |||

$$x(t) = \frac{d}{dt} \{\cos(t) \operatorname{sgn}(t)\}$$

$$\text{Let } x_1(t) = \cos(t)$$

$$x_1(-t) = \cos(-t) = \cos t$$

$$x_1(t) = x_1(-t)$$

Hence, $x_1(t)$ is even signal.

$$x_2(t) = -\operatorname{sgn}(t)$$

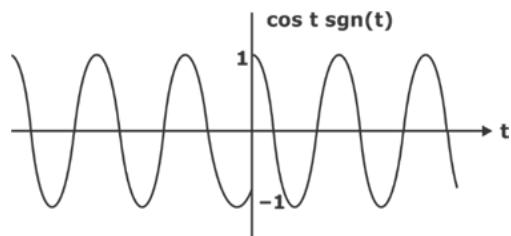
$$x_2(t) = -x_2(-t)$$

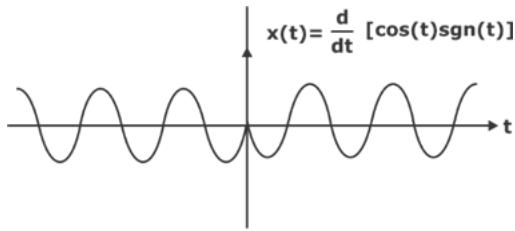
so, $x_2(t)$ is odd signal.

Since, multiplication of even and odd signal results as an odd signal.

$$\text{So, } x(t) = \frac{d}{dt}(\text{odd}) = \text{even signal}$$

Alternate method:





73. The unilateral Z Transform of $x(n) = 3^n u(-n)$ is

- A. 1
- B. $\frac{1}{1-3Z^{-1}}$, $|z|<3$
- C. $\frac{1}{1-3Z^{-1}}$, $|z|>3$
- D. 0

Answer ||| A

Solution |||

Unilateral Z transform is given by $X(Z) = \sum_{n=0}^{+\infty} x(n)Z^{-n}$

$x(n)$ is defined from $-\infty$ to 0 but unilateral Z Transform is defined from 0 to ∞

therefore, $x(n) = \delta(n)$

$$\delta(n) \xleftrightarrow{Z.T} 1$$

74. A continuous time signal is defined as,

$$x(t) = 4 \cos\left(\frac{2\pi}{3}t + 40^\circ\right) + 3 \sin\left(\frac{4\pi}{5}t + 20^\circ\right).$$

The fundamental time period of $x(t)$ is

- A. 30π sec
- B. 15π sec
- C. 15 sec
- D. 30 sec

Answer ||| C

Solution |||

$$x(t) = \underbrace{4 \cos\left(\frac{2\pi}{3}t + 40^\circ\right)}_{x_1(t)} + \underbrace{3 \sin\left(\frac{4\pi}{5}t + 20^\circ\right)}_{x_2(t)}$$

$$\omega_1 = \frac{2\pi}{3} \Rightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2\pi/3} = 3$$

$$\omega_2 = \frac{4\pi}{5} \Rightarrow T_2 = \frac{2\pi}{4\pi/5} = \frac{5}{2}$$

$$T = \text{LCM}(T_1, T_2)$$

$$\frac{T_1}{T_2} = \frac{3}{5/2} = 6/5$$

$$T = 3 \times 5 \text{ or } 6 \times 5/2$$

= 15 sec.

75. If $\delta(t)$ has Fourier transform 1, then 1 has Fourier transform

- A. $\delta(-f)$
- B. $\delta(f)$
- C. Both A and B are true
- D. $-\delta(f)$

Answer ||| C

Solution |||

By duality property

$$\text{If } \delta(t) \xrightarrow{\text{FT}} 1$$

$$\text{then } 1 \xrightarrow{\text{FT}} \delta(-f)$$

But $\delta(-f) = \delta(f)$ as $\delta(f)$ is an even function.

76. Find the given convolution

$$3e^{-2t} * \delta(t - 3)$$

- A. $3e^{-6} \delta(t - 3)$
- B. $e^{-2t} \delta(t - 3)$
- C. $3e^{-2t+3}$
- D. $3e^{-2(t-3)}$

Answer ||| D

Solution ||| $\because x(t) * \delta(t - t_0) = x(t - t_0)$

Let $x(t) = 3e^{-2t}$

$$x(t) * \delta(t - 3) = x(t - 3)$$

$$\text{Hence, } 3e^{-2t} * \delta(t - 3) = 3e^{-2(t-3)}$$

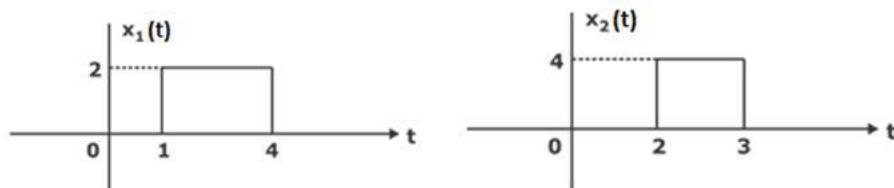
77. Discrete-time signal is obtained by sampling from:

- A. Discontinuous-time signal
- B. Continuous-time signal
- C. Continuous-displacement of signal
- D. Discontinuous-displacement of signal

Answer ||| B

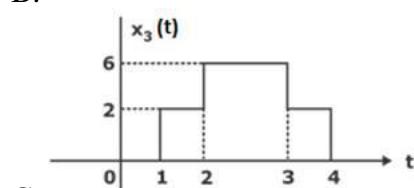
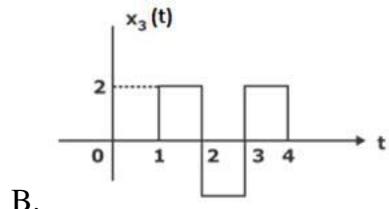
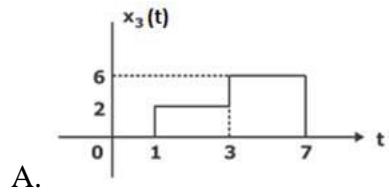
Solution ||| A discrete-time signal is a function of a continuous argument that can be obtained by sampling from a continuous-time signal.

78. If two signals $x_1(t)$ and $x_2(t)$ are defined as



$$x_3(t) = x_1(t) + x_2(t)$$

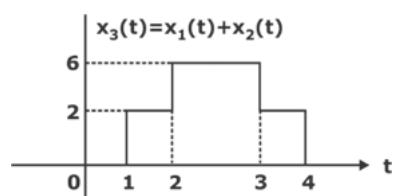
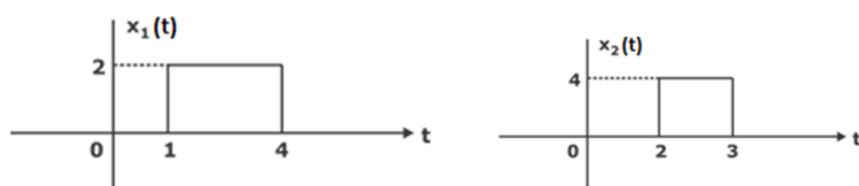
representation of $x_3(t)$ is-



D. None of these

Answer ||| C

Solution |||



79. Consider an energy signal $x(t)$, over the range $-3 \leq t \leq 3$ with energy $E = 12J$. Then the signal energy is $2x(t)$ is _____

- A. 24J
- B. 12J
- C. 48J
- D. 6J

Answer ||| C

Solution |||

$$E = \int_{-3}^3 |x(t)|^2 dt$$

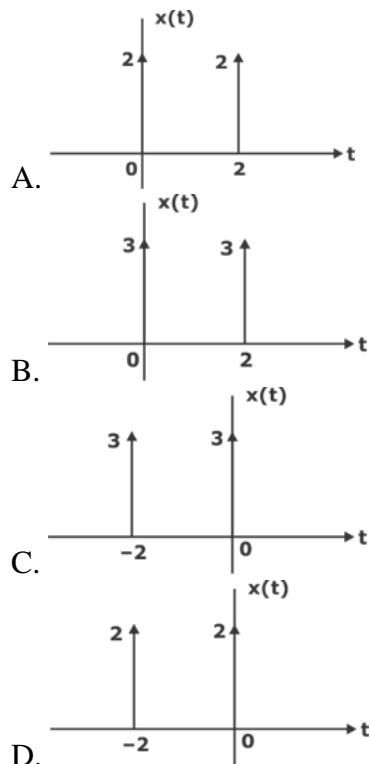
$$E_1 = \int_{-3}^3 |2x(t)|^2 dt = 4 \int_{-3}^3 |x(t)|^2 dt$$

$$E_1 = 4E = 4 \times 12J$$

$$E_1 = 48J$$

80. The graph of $x(t)$ is _____

If $x(t) = 6\delta(2t) + 12\delta(4t - 8)$



Answer ||| B

Solution |||

$$x(t) = 6\delta(2t) + 12\delta(4t - 8)$$

$$= \frac{6}{2} \delta(t) + 12 \delta(4(t-2))$$

$$= \frac{6}{2} \delta(t) + \frac{12}{4} \delta(t-2)$$

$$= 3 \delta(t) + 3 \delta(t-2)$$

81. Calculate the phase delay and group delay of an LTI system with phase response $\theta(\omega) = -\omega T/2$.

- A. $P(\omega) = -T/2, D(\omega) = T$
- B. $P(\omega) = T/2, D(\omega) = T/2$
- C. $P(\omega) = T/2, D(\omega) = -T/2$
- D. $P(\omega) = -T, D(\omega) = T/2$

Answer ||| B

Solution ||| $\theta(\omega) = -\omega T/2$ (given)

Phase Delay:-

$$P(\omega) = -\frac{\theta(\omega)}{\omega} = \frac{\omega T / 2}{\omega}$$

P(ω) = T/2

Group Delay:-

$$D(\omega) = -\frac{d\theta(\omega)}{d\omega} = \frac{d\left(\frac{\omega T}{2}\right)}{d\omega}$$

D(ω) = T/2

82. If $x(t)$ is causal and $y(t) = x(t/3)$ then

- A. $y(t)$ is causal
- B. $y(t)$ is non-causal
- C. If $y(t)$ is linear then it is causal
- D. If $x(t)$ is linear then $y(t)$ is causal

Answer ||| B

Solution |||

$$y(t - t_0) = x\left(\frac{t - t_0}{3}\right)$$

o/p of delayed i/p is $x\left(\frac{t}{3} - t_0\right)$

Delayed o/p is $x\left(\frac{t - t_0}{3}\right)$

Both are not equal so, $y(t)$ is non-causal.

\therefore Ans is option (B)

83. Modified z-transform is used

- A. for systems having zero dead-time
- B. for systems having dead-time which is an integer multiple of sampling time
- C. for systems having dead-time which is not an integer multiple of sampling time
- D. for systems having large time constant

Answer ||| C

Solution |||

Modified z-transform is used for the system having dead time which is not an integer multiple of sampling time.

84. The Fourier transform of $x(t) = e^{-3|t|} \sin 2t$ is

A. $\frac{\frac{1}{2}j}{3-j+j\omega} - \frac{\frac{1}{2}j}{3-j+3j\omega}$

B. $\frac{2}{(3+j\omega)^2 + 4} - \frac{2}{(3-j\omega)^2 + 4}$

C. $\frac{3-j+j\omega}{(3-2j+j\omega)(3-j\omega+j)}$

D. $\frac{1}{(j\omega + 3 - 2)} + \frac{1}{-j + 3 + 3j\omega}$

Answer ||| B

Solution ||| $x(t) = e^{-3t} \sin(2t)u(t) + e^{3t} \sin(2t)u(-t)$

$$x_1(t) = e^{-3t} \sin(2t)u(t) \xleftrightarrow{F} \frac{2}{(3+j\omega)^2 + 4}$$

Let

$$\begin{aligned} x_2(t) &= e^{-3t} \sin(2t)u(-t) = -x_1(-t) = -X(-j\omega) \\ &= \frac{2}{(3-j\omega)^2 + 4} \end{aligned}$$

$$X(j\omega) = X_1(j\omega) + X_2(j\omega)$$

$$= \frac{2}{(3+j\omega)^2 + 4} - \frac{2}{(3-j\omega)^2 + 4}$$

85. For the signal $g(t) = e^{(3+2j)t}$ the real conjugate symmetric part is

- A. $\sin 2t \cos h(3t)$
- B. $\sin 3t \cos h(2t)$
- C. $\sin 3t \sin h(3t)$
- D. $\cos 2t \cos h(3t)$

Answer ||| D

Solution |||

Conjugates symmetric part of $g(t)$ is

$$g_{CS.}(t) = \frac{g(t) + g^*(-t)}{2}$$

$$g_{CS.}(t) = \frac{e^{(3+2j)t} + e^{-(3-2j)t}}{2}$$

$$g_{CS.}(t) = \frac{e^{3t}(\cos 2t + j \sin 2t) + e^{-3t}(\cos 2t - j \sin 2t)}{2}$$

$$g_{CS.}(t) = [\cos 2t + j \sin 2t] \left(\frac{e^{3t} + e^{-3t}}{2} \right)$$

$$g_{C.S.}(t) = \cos 2t \left(\frac{e^{3t} + e^{-3t}}{2} \right)$$

real part of

$$= \cos 2t \cos h(3t)$$

86. An LTI system with frequency response $H(\omega) = \frac{1}{4 + j\omega}$, and output $y(t) = e^{-2t}u(t) - e^{-4t}u(t)$. Find the value of input signal at $t = 2$

A. 2

B. $\frac{2}{e^2}$

C. $\frac{2}{e^4}$

D. $\frac{2}{e^6}$

Answer ||| C

Solution |||

Given output $y(t) = e^{-2t}u(t) - e^{-4t}u(t)$

Apply Fourier transform to $y(t)$

$$Y(\omega) = \frac{1}{2 + j\omega} - \frac{1}{4 + j\omega} = \frac{2}{(2 + j\omega)(4 + j\omega)}$$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{2}{j\omega + 2}$$

Apply inverse fourier transform

Then, $x(t) = 2e^{-2t}u(t)$

$$x(2) = 2e^{-4}u(2) = \frac{2}{e^4}$$

87. Consider the z-transform $X(z) = 5z^2 + 4z^{-1} + 3$; $0 < |z| < \infty$. The inverse z-transform $x[n]$ is

A. $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$

B. $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$

C. $5u[n+2] + 3u[n] + 4u[n-1]$

D. $5u[n - 2] + 3u[n] + 4u[n + 1]$

Answer ||| A

$$\delta[n - n_0] \xleftarrow{z} z^{-n_0}$$

$$X(z) = 5z^2 + 4z^{-1} + 3; 0 < |z| < \infty$$

Solution |||
 $x(n) = 5\delta(n+2) + 4\delta(n-1) + 3\delta(n)$

88. Find the z-Transform of unit step sequence $u(n)$?

- A. $1 / 1 - z^{-1}$
- B. $1 / 1 - z^{-2}$
- C. $1 / 1 + z^{-1}$
- D. $1 + z^{-1}$

Answer ||| A

Solution ||| To find the z-Transform of unit step sequence $u(n)$

$$\begin{aligned} &= 1 + z^{-1} + z^{-2} + z^{-3} + \dots \\ &= 1 / 1 - z^{-1} \end{aligned}$$

89. Determine whether the given signal is energy/power signal $x(t) = e^{-3|t|}$

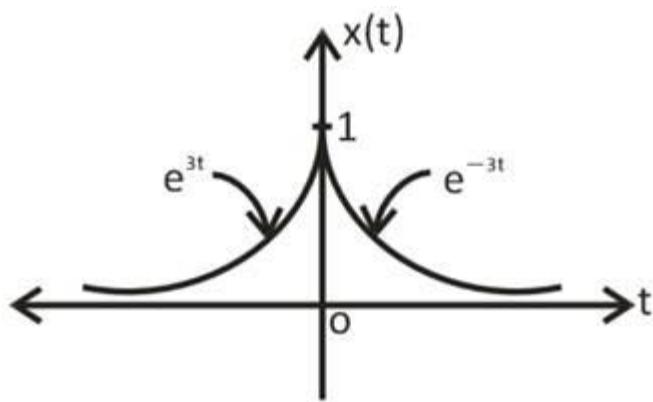
- A. Energy signal and power signal
- B. Power signal and not energy signal
- C. Energy signal but not power signal
- D. Neither energy nor power signal

Answer ||| C

Solution |||

$$x(t) = e^{3t}, t < 0$$

$$e^{-3t}, t \geq 0$$



$$\text{Energy } E = \int_{-\infty}^{\infty} \{x(t)\}^2 dt$$

Since from the signal waveform, signal is even.

$$E = \alpha \int_0^{\infty} \{x(t)\}^2 dt$$

$$= 2 \int_0^{\infty} (e^{-3t})^2 dt = 2 \int_0^{\infty} e^{-6t} dt$$

$$E = 2 \left[\frac{e^{-6t}}{-6} \right]_0^{\infty}$$

$$E = \frac{-3}{2} \{ e^{-\infty} - e^0 \}$$

$$E = \frac{1}{3}$$

è Finite energy

$$P = \frac{\text{Energy}}{\text{time}} = \frac{1/3}{t \rightarrow \infty} = 0$$

Power = 0

$\therefore x(t)$ is energy signal (as $E = \text{finite}$) but not power signal (as $P = 0$)

90. Consider the signal, $y(n) = \text{odd part of } x(n + 1)$, the signal is

- A. linear, Non-causal, stable, time variant
- B. linear, causal, stable, time invariant
- C. Non-linear, Non causal, unstable, time variant
- D. Non-linear, non-causal, stable, time invariant

Answer || A

Solution ||

$$y(x) = \frac{x(n+1) - x(-n+1)}{2}$$

$$y(x) = \frac{1}{2} \begin{bmatrix} x(n+1) - x(-n+1) \\ \downarrow \quad \downarrow \\ x_1(n) \quad x_2(n) \end{bmatrix}$$

$y(n)$ is the linear combination of $x_1(n)$ & $x_2(n)$.

so $y(n)$ is a linear system.

$x_1(n) = x(n+1)$ à causal system

$x_2(n) = x(-n+1)$ à non-causal system.

So the $y(n)$ is a non-causal system.

Let $x(n)$ is a stable i.e.

$$\sum_{n=-\infty}^{\infty} |x(n)| \leq \infty$$

$$B_x \leq \infty$$

Than also

$$\sum_{n=-\infty}^{\infty} |x(n+1)| \leq \infty$$

$$B_1 \leq \infty$$

Also

$$\sum_{n=-\infty}^{\infty} x(-n+1) \leq \infty$$

So

$$\sum_{n=-\infty}^{\infty} \frac{x(n+1) - x(-n+1)}{2} \leq \infty$$

$$B_1 - B_2 \leq \infty$$

So the system is stable.

$$x_1(n) = x(n+1)$$

Shift the input by ' n_o ' unit.

$$= x(n - n_o + 1) \dots \dots \dots (1)$$

Shift the output by ' n_o ' unit.

$$x(n - n_o) = x(n - n_o + 1) \dots \dots \dots (2)$$

$$\text{eq. (1)} = \text{eq. (2)}$$

so the $x_1(n)$ is time -Invariant.

$$x_2(n) = x_1(-n+1)$$

shift the input by ' n_o ' unit.

$$= x(-n - n_o + 1) \text{ LHS}$$

Shift the output by ' n_o ' unit.

$$x_2(n - n_o) = (-n - n_o + 1)$$

$$x_2(n - n_o) = (-n - n_o + 1) \text{ RHS.}$$

$$\text{LHS} \neq \text{RHS}$$

So the $x_2(n)$ is time variant.

$$y_1(n) = \frac{1}{2} [x_1(n) - x_2(n)]$$

So the system is time variant

91.The z - transform of the signal

$$x(n) = \begin{cases} 1, & n = -1 \\ 2, & n = 0 \\ -1, & n = 1 \\ 1, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

- A. $z+2 - z^{-1} + z^{-2}$
- B. $z^{-1}+2 - z + z^2$
- C. $z+2z^2 - z^{-1} + z^{-2}$
- D. $z+2 - z^{+1} + z^{-2}$

Answer ||| A

Solution ||| Bi lateral Z – transform is defined as

$$Z[x(n)] = \sum_{n=-\infty}^{+\infty} x(n) Z^{-n}$$

Input $x(n)$ for $n = -1$ to 2 ;

$$Z[x(n)] = \sum_{n=-1}^2 x(n) Z^{-n}$$

$$X(z) = x(-1)z^1 + x(0)z^0 + x(1)z^{-1} + x(2)z^{-2};$$

$$\text{Ans. } X(z) = Z + 2Z^{-1} + Z^{-2}$$

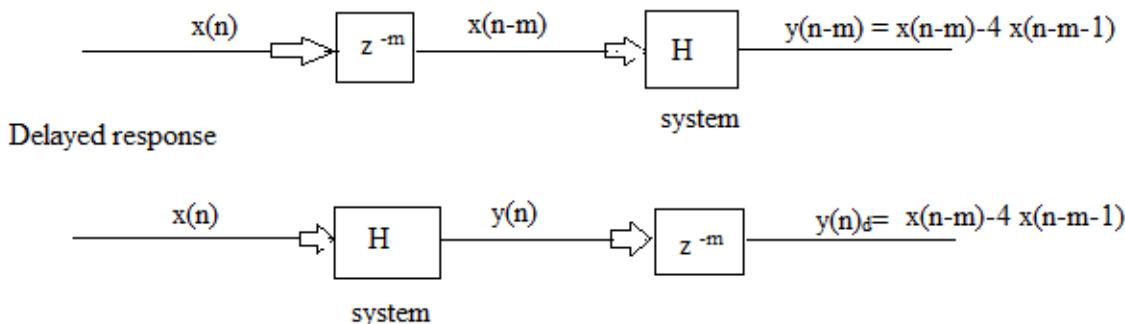
92.Which of the following options best describes the system $y(n) = x(n) - 4x(n-1)$ based on memory and time variance or invariance?

- A. Time variant, static system
- B. Time invariant, static system
- C. Time variant, dynamic system
- D. Time invariant, dynamic system

Answer ||| D

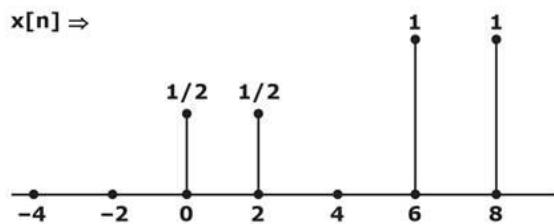
Solution ||| Since, the system has the term $x(n-1)$, it requires a finite memory. Hence, it is a dynamic system.

Response for delayed input



Since, $y(n-m) = y(n)_d$, the system is time invariant. And as output depends upon present as well as previous state so it is dynamic system.

93. Consider a discrete time signal as shown in the figure below



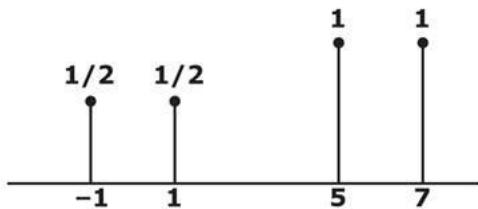
If $y[n] = n x[2n + 1]$, then the value of $\sum_{n=-\infty}^{\infty} y[n]$ will be _____

- A. 0
- B. 1
- C. 2
- D. 3

Answer ||| A

Solution |||

$$x[n+1] \Rightarrow$$



$$\Rightarrow x[2n + 1] = 0$$

$$\Rightarrow n x[2n + 1] = y[n] = 0$$

$$\sum_{n=-\infty}^{\infty} y(n) = 0$$

94. The Laplace transform of $(t+1)^2$ is

A. $\frac{1}{s^3} + \frac{2}{s^2} + \frac{1}{s}$

B. $\frac{2}{s^3} + \frac{1}{s^2} + \frac{1}{s}$

C. $\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}$

D. $2 \left[\frac{1}{s^3} + \frac{1}{s^2} + \frac{1}{s} \right]$

Answer ||| C

Solution |||

$$f(t) = (t+1)^2$$

$$\Rightarrow \text{Laplace } \{f(t)\} = L(t^2) + L(2t) + L(1) = 2/s^3 + 2/s^2 + 1/s$$

95. The function which has its Fourier transform, Laplace transform and z-transform unity is

- A. Gaussian
- B. impulse
- C. sinc
- D. pulse

Answer ||| B

$$\begin{aligned}\delta(t) &\xleftarrow{\text{F.T.}} 1 \\ \delta(t) &\xleftarrow{\text{L.T.}} 1 \\ \text{Solution } ||| \quad \delta(t) &\xleftarrow{\text{Z.T.}} 1\end{aligned}$$

96. Consider each of the signal respectively

$$x_1(t) = \int_{-\infty}^{\frac{6t}{7}} x(\tau) d\tau$$

$$x_2(t) = \int_{-\infty}^{\frac{3t}{7}} x(\tau) d\tau$$

Which of the following signal is/are causal?

- A. Both are causal
- B. Both are non-causal
- C. 1 is causal and 2 is non-causal
- D. 1 is non-causal and 2 is causal

Answer ||| C

Solution ||| For causality

The present output must be depends upon present/past input, otherwise it is non – causal.
So for A

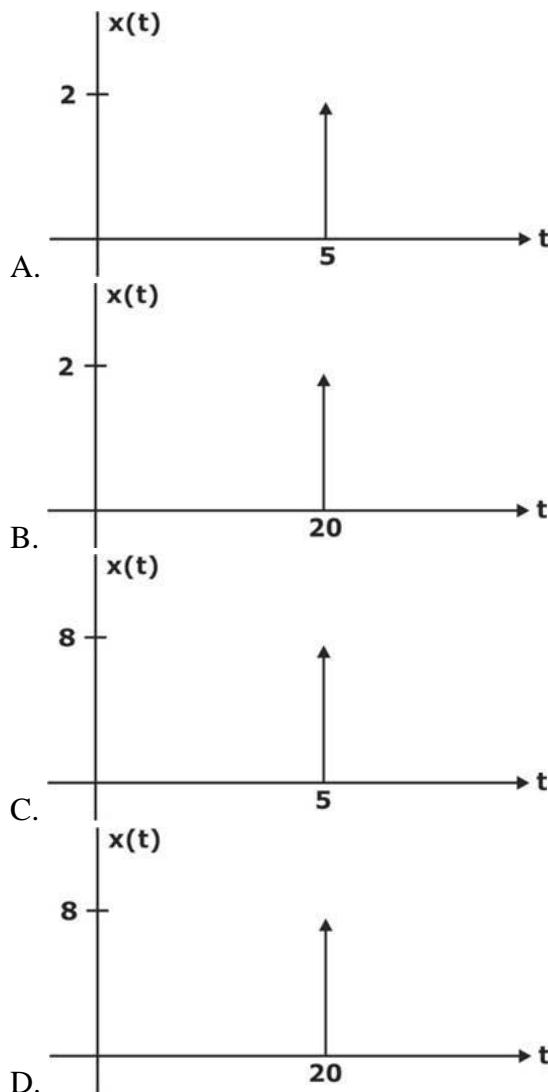
If we put $t = \dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots$ then $x(\tau) d\tau$ will always present inside those limit

But for B

If we put $t = 2$ then $x(\tau) d\tau$ will cross the limits of 2 and it will present in 6 which is not property of causality.

97. If $\delta(t)$ is an unit impulse function then which of the following is correct representation of $x(t)$.

$$x(t) = 8\delta(4t - 20)$$



Answer ||| A

Solution |||

$$x(t) = 8\delta(4t - 20)$$

$$= 8\delta[4(t - 5)]$$

$$= \frac{8}{4} \delta(t - 5)$$

$$= 2\delta(t - 5)$$

98. Consider a continuous time system A, modeled by the equation $y(t) = tx(t) + 4$ and a discrete time system B modeled by the equation $y[n] = x^2[n]$. These systems are

- A. A-time invariant and B-time invariant
- B. A-time varying and B-time invariant
- C. A-time invariant and B-time varying
- D. A-time varying and B-time varying

Answer ||| B

Solution ||| $y(t) = tx(t) + 4$

for $x(t) = x(t - t_0)$

$$y'(t) = tx(t - t_0) + 4 \quad \dots(i)$$

for $y(t) = y(t - t_0)$

$$y(t - t_0) = (t - t_0)x(t - t_0) + 4 \quad \dots(ii)$$

equation (i) \neq equation (ii)

Hence A is time varying

$$y(n) = x^2[n]$$

$$y(n) \rightarrow x(n - n_0)$$

$$y'(n) = x^2(n - n_0) \quad \dots(iii)$$

For $y(n) = (y(n - n_0))$

$$y(n - n_0) = x^2(n - n_0) \quad \dots(iv)$$

equation (iii) = equation (iv)

Hence B is time invariant

99. A discrete signal $x(n)$ has Z transform $X(z)$ which is given by

$$X(z) = \frac{3z^2 + \frac{3}{4}z}{z^2 + \frac{1}{4}z - \frac{1}{8}}$$

$x(n)$ is a causal signal, then the value of $|x(2) + x(3)|^{-1}$

- A. 3.55
- B. 0.28
- C. 4.89
- D. 6.32

Answer ||| A

Solution |||

$$\begin{aligned}
 & z^2 + \frac{1}{4}z - \frac{1}{8} \\
 & \left(\begin{array}{c} 3z^2 + \frac{3}{4}z \\ 3z^2 + \frac{3}{4}z - \frac{3}{8} \end{array} \right) \quad \left(3 + \frac{3}{8}z^{-2} - \frac{3}{32}z^{-3} \right) \\
 & \underline{- \qquad (+)} \\
 & \qquad \qquad + \frac{3}{8} \\
 & + \frac{3}{8} + \frac{3}{32}I^{-1} - \frac{3}{64}Z^{-2} \\
 & \underline{\qquad (-) \qquad (-) \qquad (+)} \\
 & - \frac{3}{32}z^{-1} + \frac{3}{64}z^{-2} \\
 & - \frac{3}{32}z^{-1} + \frac{3}{128}z^{-2} + \frac{3}{8 \times 32}z^{-3} \\
 & \underline{\qquad (+) \qquad (-) \qquad (-)} \\
 & \qquad \qquad \frac{3}{128}z^{-2} - \frac{3}{5 \times 32}z^{-3}
 \end{aligned}$$

So,

$$x(2) = \frac{3}{8}$$

$$x(3) = \frac{-3}{12}$$

$$x(2) + x(3) = \frac{3}{8} - \frac{3}{32}$$

$$= \frac{12 - 3}{32} = \frac{9}{32}$$

$$|x(2) + x(3)|^{-1} = \frac{32}{9}$$

100. Fundamental time period of signal, $x(t) = 2\sqrt{2}\pi t + \cos 2\pi t$ is

- A. 2 sec
 - B. 4 sec
 - C. $\sqrt{2}$ sec
 - D. None of the above

Answer ||| D

Solution |||

$$x(t) = 2\sqrt{2}\pi t + \cos 2\pi t$$

i.e., $\omega_1 = 2\sqrt{2}\pi$

$$\omega_1 = 2\pi$$

$$\frac{\omega_1}{\omega_2} = \frac{2\sqrt{2}\pi}{2\pi} = \sqrt{2} = \text{irrational}$$

So, $x(t)$ is non-periodic.