

Power system formula notes

Work done = $F \cdot d \cos \alpha$

Where F = force applied, d = displacement,

α = angle between F & d

Energy: It is capacity to do the work.

Unit: watt second $1 \text{ w} - \text{s} = 1 \text{ Joule} = 1\text{N} - \text{m}$ (Newton - meters)

Electrical energy generally expressed in kilo watt hours (kwh)

$1 \text{ kwh} = 3.6 \times 10^6 \text{ J}$

Kinetic energy (KE): $\frac{1}{2}mv^2$ (Jules)

Potential Energy (PE): Mgh (Jules)

Thermal Energy: Internal energy present in system by virtue of its temperature.

Unit: Calories $1 \text{ Cal} = 4.186 \text{ J}$

Power: it is time rate of change of energy

$$P = \frac{dw}{dt} = \frac{du}{dt} \quad u = \text{work}, w = \text{energy}$$

Unit: Watt $1 \text{ Watt} = 1 \text{ J/s}$

Note: Electric motor ratings are expressed in horse power (hp)

$1 \text{ hp} = 745.7 \text{ W}$ and also $1 \text{ metric horse power} = 735 \text{ Watt}$.

Electric parameter:

$$\text{Let } v = \sqrt{2V} \sin \omega t$$

$$i = \sqrt{2} I \sin(\omega t - \phi)$$

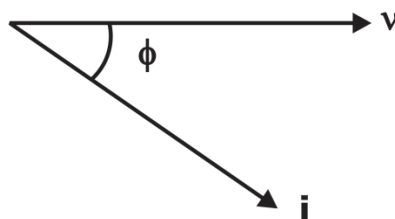
where v = instantaneous voltage

i = instantaneous value current

V = rms value of voltage

In Phasor representation

$$V = V \angle 0, i = I \angle -\phi$$

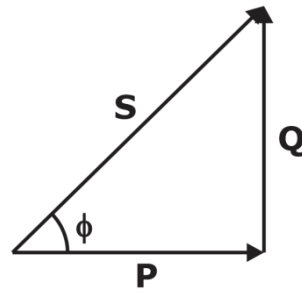


$$S = P + jQ = VI \cos \phi + jVI \sin \phi \quad VI^* \text{ (for this relation } Q \text{ will be positive for lagging VAR)}$$

Where S = complex power of apparent power

P = Active power

Q = Reactive power



For balanced 3 phase system

$$P = 3 | V_P | | I_P | \cos \phi_P = \sqrt{3} | V_L | | I_L | \cos \phi_P$$

$$Q = 3 | V_P | | I_P | \sin \phi_P = \sqrt{3} | V_L | | I_L | \sin \phi_P$$

Where V_L = line voltage

V_P = phase voltage

Note: in γ connection $V_P = \frac{V_L}{\sqrt{3}}$ & $I_P = I_L$

Δ connection $V_P = V_L$ & $I_P = \frac{I_L}{\sqrt{3}}$

Hydro power:

$$P = \rho g W h \text{ (watt)}$$

Where ρ = water density (1000 kg/m³)

$$g = 9.81 \text{ m/s}^2$$

W = discharge rate (m³/sec)

h = head of water

Tidal power

$$P = \rho g h^2 A/T \text{ (watt)}$$

Where h = tidal head

A = area of basin

T = period of tidal cycle

Wind power

$$P = 0.5 \rho A V^3 \text{ (watt)}$$

ρ = air density (1201 g/m³ at NTP)

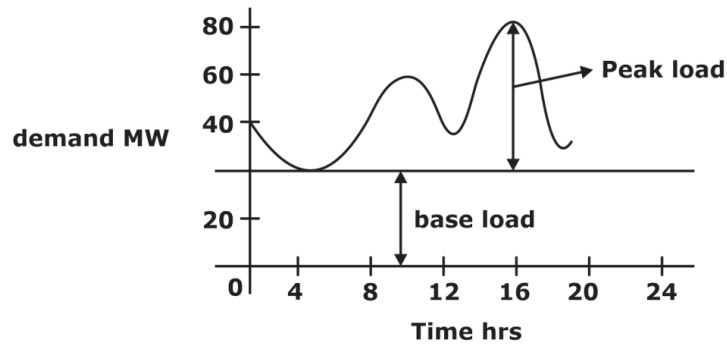
V = Wind speed in (m/s)

A = Swept area by blade (m²)

Load Curve: It is graph between the power demands of the system w.r.t. to time.

(i) Base Load: The unvarying load which occur almost the whole day.

(ii) Peak load: The various peak demands of load over and above the base load.



Designation Capacity	Capital cost	Fuel cost	Typical annual load factor	Type of plant
Base load	High	Low	65-75	Nuclear, thermal
Peak load	Low	High	5-10	Gas based, small hydro, pump storage

Operational factors:

1. Demand Factor = $\frac{\text{Maximum demand}}{\text{Connected load}}$
2. Average load = $\frac{\text{energy consumed in a given period}}{\text{Hours in that time period}}$
3. Load factor = $\frac{\text{Average demand}}{\text{Maximum load}}$
4. Diversity factor = $\frac{\text{sum of individual max demands}}{\text{Maximum demand on power station}}$
5. Plant Capacity factor = $\frac{\text{Average demand}}{\text{Installed capacity}}$
6. Reserve Capacity = Plant capacity - max. demand
7. Plant use factor = $\frac{\text{Actual energy produced}}{\text{Plant capacity} \times \text{hours (the plant has been in operation)}}$

Thermal Power Station:-

→ Thermal efficiency, $\eta_{\text{Thermal}} = \frac{\text{Heat equivalent of mech - energy Transmitted to Turbine shaft}}{\text{Heat of coal combustion}}$

→ Thermal efficiency = $\eta_{\text{boiler}} \times \eta_{\text{turbine}}$

→ Overall efficiency, $\eta_{\text{overall}} = \frac{\text{Heat equivalent of electrical o / p}}{\text{Heat of combustion of coal}}$

→ Overall efficiency, Thermal efficiency × Electrical efficiency.

→ Energy output = coal consumption × calorific value = coal consumption × 6500 k. cal

$$\eta = \frac{\text{Output in k.cal}}{\text{Input in k.cal}}$$

Water Power equation:-

Water Head: The difference of water level is called the water head.

Gross Head : The total head between the water level at inlet and tail race is called as gross head

Rated Head: Head utilized in doing work on the turbine

Net Head: It is the sum of the Rated Head and the loss of head in guide passage and entrance

H = Head of water in meter

Q = Quantity of water in m³/sec or lit/sec.

W = specific gravity of water

= 1 kg/lit when 'Q' represented in lit/sec.

= 100 kg/m³ when 'Q' represented in m³/sec.

η = efficiency of the system

Effective work done = WQH × η kg - m/sec.

→ Metric output = $\frac{WQH \times \eta}{75}$ (H.P)

1 H.P = 75 kg-m/sec

→ Metric output in watt = $\frac{WQH \times \eta}{75} \times 735.5$

→ Output = $\frac{WQH}{102} \times \eta$ kw

→ Volume of water available per annum = catchment area × Annual Rainfall

→ Electric energy generated = weight × head × overall η.

GAS TURBINE POWER PLANT:

→ The thermal efficiency of gas turbine plant is about 22% to 25%

→ The air fuel ratio may be of the order of 60: 1 in this case.

→ Engine efficiency $\eta_{engines} = \frac{\eta_{overall}}{\eta_{alt}}$

→ Thermal efficiency $\eta_{the} = \frac{\eta_{engine}}{mech.\eta \text{ of engIn d}}$

→ Heat produced by fuel per day = coal consumption/day × calorific value

Terms and Definitions :-

1. Connected load :-

It is the sum of ratings in kilo watts of equipment installed in the consumer's premises

2. Demand :-

It is the load or power drawn from the source of supply at the receiving end averaged over a specified period.

3. Maximum Demand :-

Maximum demand (M.D) of a power station is the maximum load on the power station in a given period.

4. Average load :-

If the number of KWH supplied by a station in one daily average load.

$$\text{Daily average load} = \frac{\text{KWH delivered in one day}}{24}$$

$$\text{Monthly average load} = \frac{\text{KWH delivered in one month}}{30 \times 24}$$

$$\text{Yearly average load} = \frac{\text{KWH delivered in one year}}{365 \times 24}$$

5. Plant capacity :-

It is the capacity or power for which a plant or station is designed. It should be slightly more than M.D. it is equal to sum of the ratings of all the generators in a power station

6. Firm Power :-

It is the power which should be always be available even under emergency

7. Prime Power :-

It is the maximum power (may be thermal or hydraulic or mechanical) continuously available for conversion into electrical power.

8. Dump power:-

This is the term usually used in hydro electric plant and it represents the power in excess of the load requirements. It is made available by surplus water.

9. Spill Power:-

Is that power which is produced during floods in a hydro power station.

10. Cold reserve:-

Is that reserve generating capacity which is not in operation but can be made available for service.

11. Hot reserve:-

It that reserve generating capacity which is in operation but not in service

12. Spinning reserve:-

Is that reserve generating capacity which is connected to bus-bars and is ready to take the load.

Load factor:-

It is defined as the ratio of number of units actually generated in a given period to the number of units that could have been generated with maximum demand.

$$\rightarrow \text{Load factor} = \frac{\text{Average load or Average Demand}}{\text{Maximum Demand}}$$

$$= \frac{\text{Energy generated in a given period}}{(\text{Maximum Demand}) \times (\text{Hours of operation in the given period})}$$

→ The load factor will be always less than one (<1)

Demand factor:-

It is defined as the ratio of maximum demand on the station to the total connected load to the station.

$$\rightarrow \therefore \text{Demand factor} = \frac{\text{Maximum Demand on the station}}{\text{Total connected load to the station}}$$

→ Its value also will be always less than one (<1)

Diversity Factor:–

Diversity factor may be defined as “the sum of individual maximum demand to the station to the maximum demand on the power station”.

$$\rightarrow \text{Diversity factor} = \frac{\text{sum of individual consumers maximum demand}}{\text{Maximum demand on the station.}}$$

→ Its value will be always greater than one (>1)

Plant Factor or Plant Use Factor:–

$$\text{Plant factor} = \frac{\text{station output in kwh}}{\sum (KW_1)H_1 + (KW_2)H_2 + (KW_3)H_3 + \dots}$$

Where KW_1, KW_2, KW_3 etc. are the kilowatt ratings of each generator and H_1, H_2, H_3 etc. are the number of hours for which they have been worked.

Capacity Factor or plant capacity factor or capability factor:–

→ It is defined as the ratio of average demand on the station to the maximum installed capacity.

$$\text{i.e. capacity factor} = \frac{\text{Average demand on the station}}{\text{Max. installed capacity of the station}}$$

→ **Coincidence factor:–**

It is the reciprocal of diversity factor and is always less than 1

$$\rightarrow \text{Utilization factor} = \frac{\text{Maximum demand}}{\text{Plant capacity}}$$

$$\rightarrow \text{Operation factor} = \frac{\text{Service hours}}{\text{Total duration}}$$

$$\rightarrow \text{Use factor} = \frac{\text{Actual energy produced}}{\text{Plant capacity} \times \text{Time (hrs) the plant has been in operation}}$$

D.C. Distribution calculations

Uniformly loaded Distributor fed at one end.

→ Fig (a) shows the single lien diagram of a 2 – wire d. c. distributor AB fed at one end A and loaded uniformly with i amperes per metre length.

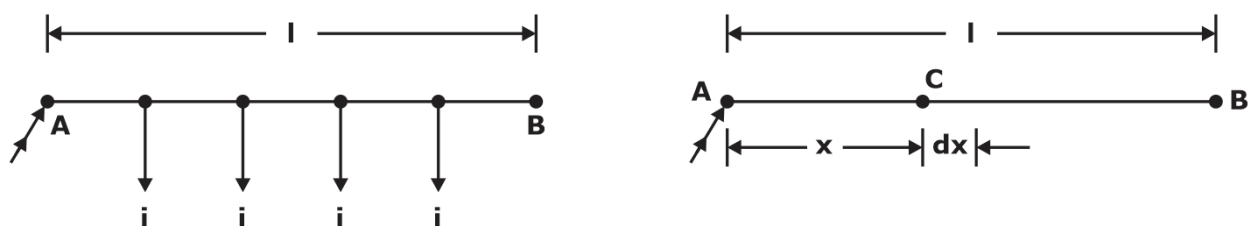


Fig. (a)

→ Then the current at point c is.

$$= \delta l - ix \text{ amperes}$$

$$= i(l - x) \text{ amperes.}$$

→ Total voltage drop is the distributor up to point C is

$$V = \int_0^x ir(l-x) dx = ir(lx - \frac{x^2}{2})$$

→ Voltage drop over the distributor AB

$$= \frac{1}{2} irl^2 = \frac{1}{2} IR$$

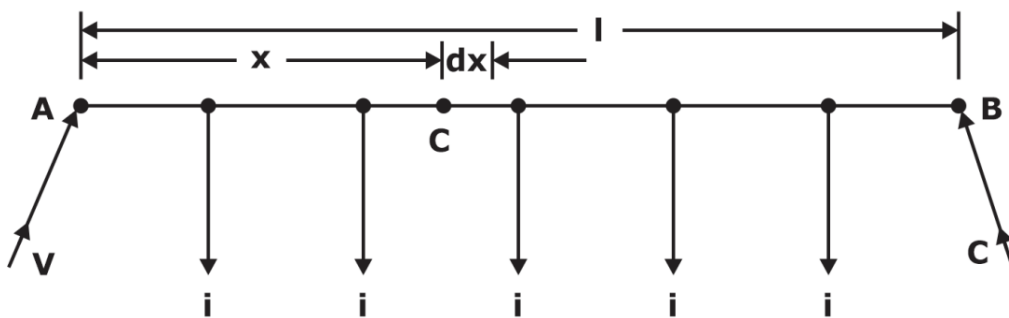
Where $il = I$, the total current entering at point A

$rl = R$, the total resistance of the distributor.

Uniformly loaded distributor fed at both ends.

(i) Distributor fed at both ends with equal voltages

Current supplied from each feeding point = $\frac{9l}{2}$



→ Voltage drop up to point C = $\frac{ir}{2}(lx - x^2)$

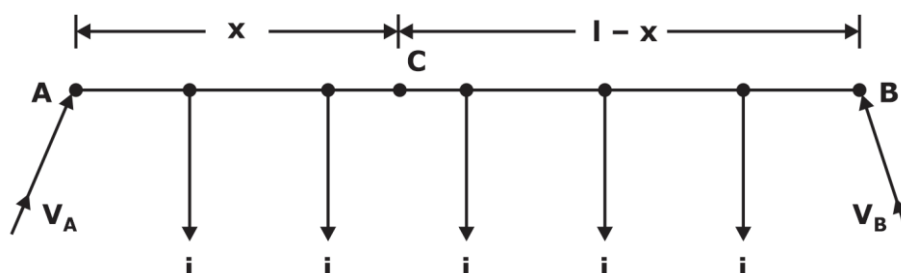
→ Max. voltage drop = $\frac{1}{8} IR$

→ Min. voltage = $V - \frac{IR}{8}$ volts

(ii) Distributor fed at both ends with unequal voltages:-

The point of minimum potential C is situated at a distance x meters from the feeding point A.

Voltage drop in section AC = $\frac{irx^2}{2}$ volts.



$$\rightarrow X = \frac{V_A - V_B}{i r l} + \frac{l}{2}$$

Performance of Lines

→ By performance of lines is meant the determination of efficiency and regulation of lines.

The efficiency of lines is defined as

$$\rightarrow \% \text{ efficiency} = \frac{\text{Power delivered at the receiving end}}{\text{Power sent from sending end}} \times 100$$

$$\rightarrow \% \text{ efficiency} = \frac{\text{Power delivered at the receiving end}}{\text{Power delivered at the receiving end} + \text{losses}} \times 100$$

Where V_r' is the receiving end voltage under no load condition and V_r the Receiving end voltage under full load condition.

Effect of Earth on a 3 – φ lines :-

S. No.	Line Description	R	L	X_L	C	X_C
1.	Length Increases	Increases	Increases	Increases	Increases	Decreases
2.	Distance of separation	No change	Increases	Increases	Decreases	Increases
3.	Radius of conductor	Decreases	Decreases	Decreases	Increases	Decreases
4.	Symmetrical spacing	Does not depend	Decreases	Decreases	Increases	Decreases
5.	Unsymmetrical spacing	Does not depend	Increases	Increases	Decreases	Increases
6.	Effect of earth is taken into account	No change	No change	No change	Increases	Decreases
7.	Height of the conductor increases	No change	No change	No change	Decreases	Increases

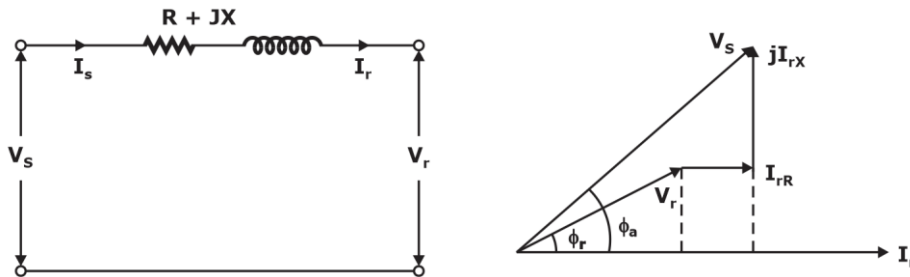
Short Transmission Line

→ The equivalent circuit and vector diagram for a short transmission line are shown in fig.

$$V_s = \sqrt{V_r^2 + \frac{2I_r R \cos \phi_r}{V_r} + \frac{2I_r X \sin \phi_r}{V_r} + \frac{I_r^2}{V_r^2} (R^2 + X^2)}$$

→ In practice the last term under the square root sign is generally negligible; therefore.

$$V_s = V_r \left\{ 1 + \left(\frac{2I_r R}{V_r} \cos \phi_r + \frac{2I_r X}{V_r} \sin \phi_r \right) \right\}^{1/2}$$



The terms within the simple brackets is small as compared to unity. Using binomial expansion and limiting only to second term,

$$V_s \approx V_r + I_r R \cos \phi_r + I_r X \sin \phi_r$$

→ The receiving end voltage under no load V_r' is the same as the sending end voltage under full load condition.

$$\% \text{ regulation} = \frac{V_s - V_r}{V_r} \times 100 = \left(\frac{I_r R}{V_r} \cos \phi_r + \frac{I_r X}{V_r} \sin \phi_r \right) \times 100$$

$$\text{Regulation per unit} = \frac{I_r R}{V_r} \cos \phi_r + \frac{I_r X}{V_r} \sin \phi_r = V_r \cos \phi_r + V_x \sin \phi_r$$

→ Where V_r and V_x are the per unit values of resistance and reactance of the line.

$$V_s = AV_r + BI_r$$

$$I_s = CV_r + DI_r$$

$$A = \frac{V_s}{V_r} \mid I_r = 0$$

This means A is the voltage impressed at the sending end per volt at the receiving end when receiving end is open. It is dimensionless.

$$B = \frac{V_s}{V_r} \mid V_r = 0$$

B is the voltage impressed at the sending end to have one ampere at the short circuited receiving end. This is known as transfer impedance in network theory.

$$C = \frac{V_s}{V_r} \mid I_r = 0$$

C is the current in amperes into the sending end per volt on the open – circuited receiving end. It has the dimension of admittance.

$$D = \frac{I_s}{I_r} | V_r = 0$$

D is the current at the sending end for one ampere of current at the short circuited receiving end

→ The constants A, B, C, and D are related for a passive network as follow

$$AD - BC = 1$$

→ The sending end voltage and current can be written from the equivalent network as,

$$V_s = V_r + I_r Z$$

$$I_s = I_r$$

→ The constants for short transmission lines are,

$$A = 1$$

$$B = Z$$

$$C = 0$$

$$D = 1$$

$$\rightarrow \% \text{ regulation} = \frac{V_{S/A} - V_r}{V_r} \times 100$$

$$\rightarrow \% \eta = \frac{\text{Power received at the receiving end}}{\text{Power received per at the receiving end} + \text{losses}} \times 100$$

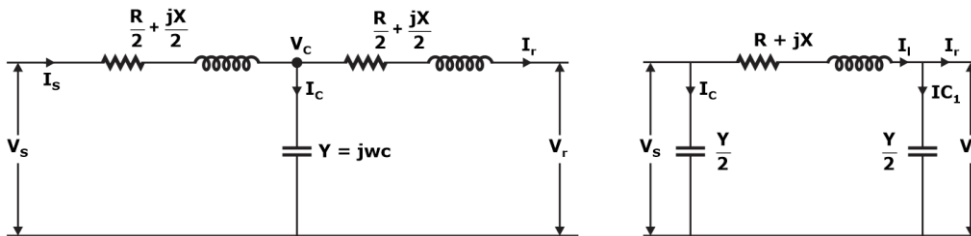
Where R is the resistance per phase of the line.

Medium Length Lines:-

→ Transmission lines with length between 80 km and 160 km are categorized as medium lines

Where the parameters are assumed to be lumped.

→ The two configurations are known as nominal -T and nominal - π respectively.



A, B, C, D constant for nominal - T

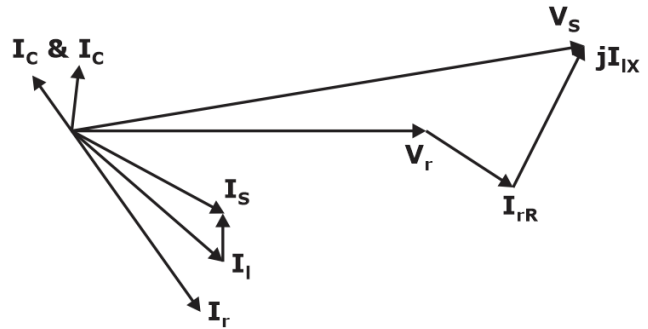
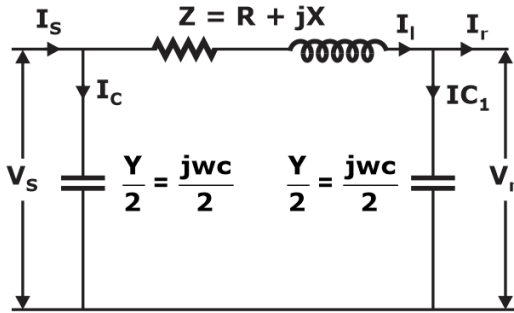
$$A = 1 + \frac{YZ}{2}$$

$$B = Z \left(1 + \frac{YZ}{2} \right)$$

$$C = Y$$

$$D = \left(1 + \frac{YZ}{2} \right)$$

Nominal - π



$$V_r' = \frac{|V_s| \left(\frac{-2j}{\omega C} \right)}{R + jX - \frac{j}{\omega C/2}}$$

$$\% \text{ regulation} = \frac{V_r L_{Vr}}{V_r} \times 100$$

$$\% \eta = \frac{P}{P + 3I_r^2 R} \times 100$$

A, B, C, D constants for nominal - π

$$A = 1 + \frac{YZ}{2}$$

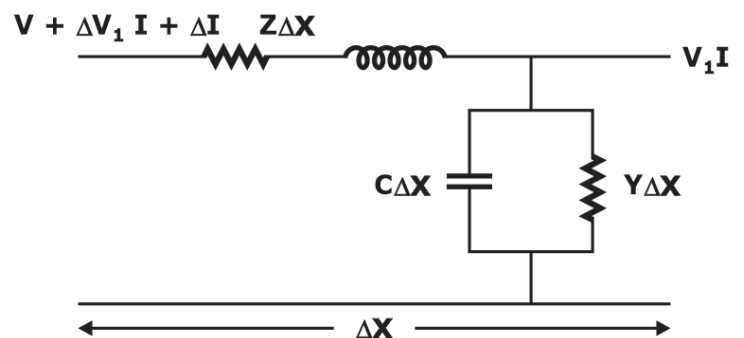
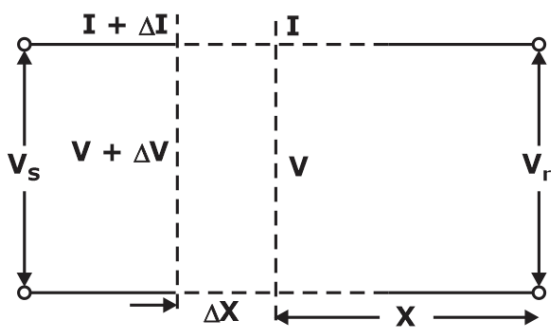
$$B = Z$$

$$C = Y \left(1 + \frac{YZ}{4} \right)$$

$$D = \left(1 + \frac{YZ}{2} \right)$$

Long Transmission Lines :-

→ In case the lines are more than 160 km long



→ Let Z = series impedance per unit length

Y = shunt admittance per unit length

l = length of line

Z = zl = total series impedance

Y = yl = total shunt admittance.

V = Ae^{rx} + Be^{-rx}

$$I = \frac{I}{Z_c} (Ae^{rx} - Be^{-rx})$$

$$V = \frac{V_r + I_r Z_c}{2} e^{rx} + \frac{V_r - I_r Z_c}{2} e^{-rx}$$

$$I = \frac{1}{Z_c} \left[\frac{V_r + I_r Z_c}{2} e^{rx} - \frac{V_r - I_r Z_c}{2} e^{-rx} \right]$$

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + j\omega L}{g + j\omega C}}$$

→ The propagation constant $r = \alpha + j\beta$; the real part is known as attenuation constant and the quadrature component β the phase constant and is measured in radians per unit length.

$$V = \frac{V_r + I_r Z_c}{2} e^{\alpha x} \cdot e^{j\beta x} + \frac{V_r - I_r Z_c}{2} e^{-\alpha x} \cdot e^{-j\beta x}$$

$$V_s = V_r \cosh rl + I_r Z_c \sinh rl$$

$$I_s = V_r \frac{\sinh rl}{Z_c} + I_r \cosh rl$$

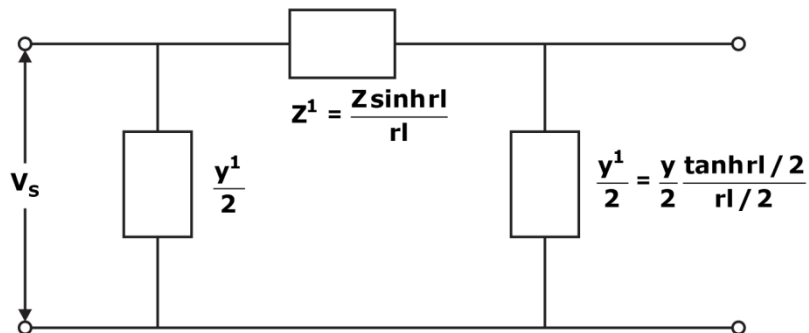
$$A = \cosh rl$$

$$B = Z_c \sinh rl$$

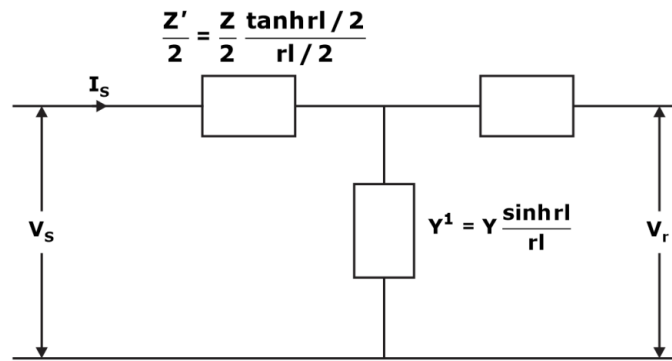
$$C = \frac{\sinh rl}{Z_c}$$

$$D = \cosh rl$$

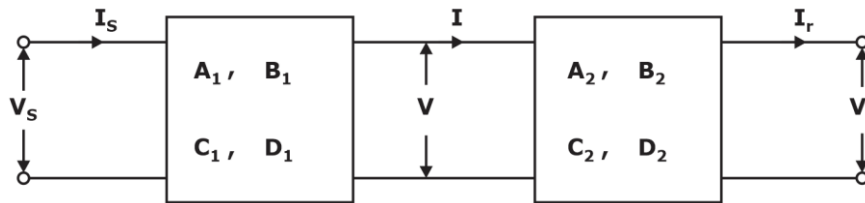
The equivalent Circuit Representation of a Long Line equivalent – π Representation.



Equivalent – T Representation of Long Line.

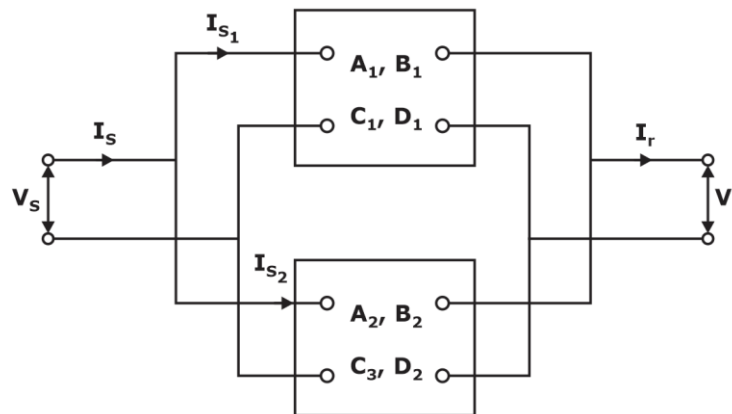


Constants for Two networks in Tandem



equivalent $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$

Constants for networks in parallel



Equivalent Single Network Parameters

$$\begin{cases} A = \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} \\ B = \frac{B_1 \cdot B_2}{B_1 + B_2} \\ A = D = \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} = \frac{D_1 B_2 + D_2 B_1}{B_1 + B_2} \\ C = C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} \end{cases}$$

FAULTS:

→ Percentage reactance $\%X = \frac{IX}{V} \times 100$ | = full load current

V = phase voltage

X = reactance in ohms per phase

→ Alternatively percentage reactance (%X) (can also be expressed in terms of KVA and KV under

$$\% = \frac{(KVA)}{10(KV)^2}$$

Where X is the reactance in ohms.

→ If X is the only reactance element in the circuit then short circuit current is given by

$$I_{sc} = \frac{V}{X} = I \times \left(\frac{100}{\%X} \right)$$

i.e short circuit current is obtained by multiplying the full load current by 100/%X

$$\text{Short - circuit KVA} = \text{Base KVA} \times \frac{100}{\%X}$$

Symmetrical components in terms of phase currents:-

→ The unbalanced phase current in a 3-phase system can be expressed in terms of symmetrical components as under.

$$\vec{I}_R = \vec{I}_{R1} + \vec{I}_{R2} + \vec{I}_{R0}$$

$$\vec{I}_Y = \vec{I}_{Y1} + \vec{I}_{Y2} + \vec{I}_{Y0}$$

$$\vec{I}_B = \vec{I}_{B1} + \vec{I}_{B2} + \vec{I}_{B0}$$

Where the positive phase current ($\vec{I}_{R1}, \vec{I}_{Y1}, \& \vec{I}_{B1}$)

Negative phase sequence currents ($\vec{I}_{R2}, \vec{I}_{Y2}, \& \vec{I}_{B2}$) and

Zero phase sequence currents ($\vec{I}_{R0}, \vec{I}_{Y0}, \& \vec{I}_{B0}$)

→ The operator 'a' is one, which when multiplied to a vector rotates the vector through 120° in the anticlockwise direction.

$$\rightarrow A = -0.5 + j 0.866 \quad ; \quad a^2 = -0.5 - j 0.866$$

$$a^3 = 1$$

→ Properties of operator 'a' :

$$1 + a + a^2 = 0$$

$$a - a^2 = j\sqrt{3}$$

→ Positive sequence current \vec{I}_{B1} in phase B leads \vec{I}_{R1} by 120° and therefore $\vec{I}_{B1} = a \vec{I}_{R1}$

similarly, positive sequence current in phase Y is 240° ahead of $\vec{I}_{Y1} = a^2 \vec{I}_{R1}$

$$\vec{I}_R = \vec{I}_{R1} + \vec{I}_{R2} + \vec{I}_{R0}$$

$$\vec{I}_Y = \vec{I}_{Y1} + \vec{I}_{Y2} + \vec{I}_{Y0} = a^2 \vec{I}_{R2} + \vec{I}_{R0}$$

$$\vec{I}_B = a \vec{I}_{R1} + a^2 \vec{I}_{R2} + \vec{I}_{R0} = \vec{I}_{B0} + \vec{I}_{B1} + \vec{I}_{B2}$$

→ Zero sequence current:

$$\vec{I}_R + \vec{I}_Y + \vec{I}_B = \vec{I}_{R1}(1 + a + a^2) + \vec{I}_{R2}(1 + a + a^2) + 3\vec{I}_{R0} = 3\vec{I}_{R0}$$

$$\therefore \vec{I}_{R0} = \frac{1}{3} [\vec{I}_R + \vec{I}_R + \vec{I}_R]$$

→ Positive sequence current:

$$\vec{I}_R + a\vec{I}_Y + a^2\vec{I}_B = \vec{I}_{R1}(1 + a^3 + a^3) + \vec{I}_{R2}(1 + a^2 + a^4) + \vec{I}_{R0}(1 + a + a^2) = 3\vec{I}_{R1}$$

$$\therefore \vec{I}_{R1} = \frac{1}{3} [\vec{I}_R + a\vec{I}_Y + a^2\vec{I}_B]$$

→ Negative sequence current:-

$$\vec{I}_R + a^2\vec{I}_Y + a\vec{I}_B(1 + a^4 + a^2) + \vec{I}_{R2}(1 + a^3 + a^3) + \vec{I}_{R0}(1 + a^2 + a) = 3\vec{I}_{R2}$$

$$\therefore \vec{I}_{R2} = \frac{1}{3} [\vec{I}_R + a^2\vec{I}_Y + a\vec{I}_B]$$

Single Line to – Ground Fault:

$$\rightarrow \vec{V}_R = 0 \text{ and } \vec{I}_B = \vec{I}_Y = 0$$

The sequence currents in the red phase in terms of line currents shall be:-

$$\vec{I}_1 = \frac{1}{3} [\vec{I}_R + a\vec{I}_Y + a^2\vec{I}_B] = \frac{1}{3} \vec{I}_R$$

$$\vec{I}_0 = \frac{1}{3} [\vec{I}_R + \vec{I}_Y + \vec{I}_B] = \frac{1}{3} \vec{I}_R$$

$$\vec{I}_2 = \frac{1}{3} [\vec{I}_R + a^2\vec{I}_Y + a\vec{I}_B] = \frac{1}{3} \vec{I}_R$$

$$\rightarrow \text{Fault current:- Fault current, } \vec{I}_R = 3\vec{I}_0 = \frac{3\vec{E}_R}{Z_0 + Z_1 + Z_2}$$

Phase voltage at fault

Since the generated emf system is of positive sequence only, the sequence components of emf in R-phase are:

$$\rightarrow \vec{E}_0 = 0; \vec{E}_2 = 0 \text{ and } \vec{E}_1 = \vec{E}_R$$

This is expected because R-phase is shorted

$$\Rightarrow \vec{V}_1 + \vec{V}_2 + \vec{V}_0 = 0$$

The sequence voltage at the fault for R-phase are: to ground.

$$\vec{V}_1 = \frac{\vec{Z}_2 + \vec{Z}_0}{\vec{Z}_1 + \vec{Z}_2 + \vec{Z}_0} \vec{E}_R$$

$$\vec{V}_2 = \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2 + \vec{Z}_0} \vec{E}_R$$

$$\vec{V}_0 = \frac{\vec{Z}_0}{\vec{Z}_1 + \vec{Z}_2 + \vec{Z}_0} \vec{E}_R$$

∴ The phase voltages at fault are :

$$\vec{V}_R = \vec{V}_0 + \vec{V}_1 + \vec{V}_2 = 0$$

$$\vec{V}_Y = \vec{V}_0 + a^2\vec{V}_1 + a\vec{V}_2$$

$$\vec{V}_B = \vec{V}_0 + a\vec{V}_1 + a^2\vec{V}_2$$

Line-To-Line fault:-

The condition created by this fault lead to:

$$\rightarrow \vec{V}_Y = \vec{V}_B := 0 \text{ and } \vec{I}_Y + \vec{I}_B = 0$$

Again taking R-phase as the reference, we have

$$\rightarrow \vec{I}_0 = \frac{1}{3}(\vec{I}_R + \vec{I}_Y + \vec{I}_B) = 0$$

$$\vec{I}_Y = \vec{I}_B$$

Expressing in terms of sequence components of red line, we have

$$\vec{V}_0 + a^2\vec{V}_1 + a\vec{V}_2 = \vec{V}_0 + a\vec{V}_1 + a^2\vec{V}_2$$

$$\Rightarrow \vec{V}_1 = \vec{V}_2$$

$$\text{Also, } \Rightarrow \vec{I}_Y + \vec{I}_B = 0 \Rightarrow \vec{I}_1 + \vec{I}_2 = 0 [\because I_0 = 0]$$

Fault current:

$$I_1 = -I_2 = \frac{\vec{E}_R}{\vec{Z}_1 + \vec{Z}_2}$$

$$I_Y = \frac{-j\sqrt{3}\vec{E}_R}{\vec{Z}_1 + \vec{Z}_2}$$

→ Phase voltages:- since the generated emf system is of positive phase sequence only, the sequence components of emf in R-phase are:

$$\vec{E}_0 = 0 : \vec{E}_2 = 0 \text{ and } \vec{E}_1 = \vec{E}_R$$

→ The sequence voltages at the fault for R-phase are :

$$\vec{V}_1 = \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R$$

$$\vec{V}_2 = \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R$$

$$\vec{V} = 0$$

→ The phase voltages at the fault are :

$$\vec{V}_R = \frac{2\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \cdot \vec{E}_R$$

$$\vec{V}_Y = \frac{-\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \cdot \vec{E}_R$$

$$\vec{V}_B = \frac{-\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \cdot \vec{E}_R$$

→ Double Line- To - Ground Fault:-

The conditions created by this fault lead to:

$$\vec{I}_R = 0; \vec{V} = \vec{V}_B = 0$$

$$\vec{V}_1 = \vec{V}_2 = \vec{V}_0 = \frac{1}{3} \vec{V}_R$$

Also, $\vec{I}_R = \vec{I}_1 + \vec{I}_2 + \vec{I}_0 = 0$

→ Fault current:

$$\rightarrow \vec{I}_F = \vec{I}_Y + \vec{I}_B = 3\vec{I}_0 = \frac{-3\vec{Z}_2\vec{E}_R}{\vec{Z}_0\vec{Z}_1 + \vec{Z}_0\vec{Z}_2 + \vec{Z}_1\vec{Z}_2}$$

Phase voltages:- the sequence voltages for phase R are:

$$\rightarrow \vec{V}_1 = \vec{E}_R - \vec{I}_1\vec{Z}_1 : \vec{V}_2 = 0 - \vec{I}_2\vec{Z}_2 : \vec{V}_0 = 0 - \vec{I}_0\vec{Z}_0$$

Now $\vec{V}_1 = \vec{V}_2 = \vec{V}_0 = \frac{1}{3} \vec{I}_R$

$$\rightarrow \therefore \vec{V}_R = 3\vec{V}_2 : \vec{V}_Y = 0 \text{ and } \vec{V}_B = 0$$

TRANSIENTS IN SIMPLE CIRCUITS:

1. D.C sources

(a) Resistance only:- As soon as switch is closed, the current in the circuit will be determined according to ohms law.

$$I = \frac{V}{R}$$

Now transients will be there in the circuit.

(b) Inductance only :- when switch s is closed the current in the circuit will be given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{sL} = \frac{V}{L} \cdot \frac{1}{s^2}$$

$$i(t) = \frac{V}{L} t$$

(c) Capacitance only:- when switch s is closed, the current in the circuit is given

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot Cs = VC$$

Which is an impulse of strength (magnitude) VC

(d) R-L circuit: when switch s is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \frac{1}{R + sL} = \frac{V}{s} \cdot \frac{1/L}{s + R/L}$$

$$= \frac{V}{L} \left[\frac{1}{s} - \frac{1}{s + R/L} \right] \frac{L}{R}$$

$$= \frac{V}{R} \left[\frac{1}{s} - \frac{1}{s + R/L} \right]$$

$$i(t) = \frac{V}{R} \left[1 - \exp\left(\frac{-R}{L} t\right) \right]$$

(e) R-L circuit: After the switch s is closed, current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \frac{1}{R + 1/CS}$$

$$= \frac{V \left(\frac{1}{RC}\right) CS}{S S + 1/RC} = \frac{V}{R} \cdot \frac{1}{S + 1/RC}$$

$$i(t) = \frac{V}{R} \cdot e^{-t/CR}$$

→ R-L-C circuit: – After the switch S is closed, the current in the circuit is given by

$$I(s) = \frac{V}{S} \frac{1}{R + LS + 1CS}$$

$$I(s) = \frac{V}{L} \frac{1}{(s + a - b)(s + a + b)}$$

$$i(t) = \frac{V}{2bL} \{e^{-(a-b)t} - e^{-(a+b)t}\}$$

where $\frac{R}{2L} = a$ and $\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = b$; then

→ There are three conditions based on the value of b

- * If $\frac{R^2}{4L^2} > \frac{1}{LC}$, b is real
- * If $\frac{R^2}{4L^2} = \frac{1}{LC}$, b is zero
- * If $\frac{R^2}{4L^2} < \frac{1}{LC}$, b is imaginary

Case I: when b is real

$$\rightarrow i(t) = \frac{V}{2\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \cdot L} \left\{ \exp\left\{-\left[\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right]t\right\} - \exp\left\{-\left[\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right]t\right\} \right\}$$

Case II: when $b = 0$

The expression for current becomes

$$\rightarrow i(t) = \frac{V}{2bL} \{e^{-at} - e^{-at}\} \text{ which is indeterminate.}$$

→ Now at $b = 0$

$$i(t) = \frac{V}{L} t e^{-at} = \frac{Vt}{L} - (R/2L)t$$

Case III. When b is imaginary

$$\rightarrow i(t) = \frac{V}{2bL} \{e^{-at} \cdot e^{jkt} - e^{-at} \cdot e^{-jkt}\} = \frac{V}{2bL} e^{-at} \cdot 2 \sin kt$$

$$= \frac{V}{2L\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}} e^{-at} \cdot 2 \sin\left(\sqrt{\frac{-R^2}{4L^2} + \frac{1}{LC}} t\right)$$

A.C source:

→ R-L circuit: when switch is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = V_m \left\{ \frac{\omega \cos \phi}{s^2 + \omega^2} \right\} \frac{1}{R + sL}$$

$$= \frac{V_m}{L} \left\{ \frac{\omega \cos \phi}{s^2 + \omega^2} + \frac{s \sin \phi}{s^2 + \omega^2} \right\} \frac{1}{s + R/L}$$

→ R-L circuit connected to an ac source

Let $\frac{R}{L} = a$; then

$$I(s) = \frac{V_m}{L} \left\{ \frac{\omega \cos \phi}{(s+a)(s^2 + \omega^2)} + \frac{s \sin \phi}{(s+a)(s^2 + \omega^2)} \right\}$$

$$i(t) = \frac{V_m}{\sqrt{(\sqrt{R^2 + \omega^2 L^2})^2}} \left\{ \sin(\omega t + \phi - \theta) - \sin(\phi - \theta)e^{-at} \right\}$$

Where $\theta = \tan^{-1} \frac{\omega L}{R}$

Circuit Breaker ratings:

→ The value of resistor required to be connected across the breaker contacts which will

given no transient oscillation, is $R = 0.5 \sqrt{\frac{L}{C}}$

Where L,C are the inductance and capacitance up to the circuit breaker

→ The average RRRV = $\frac{2V_r}{\pi \sqrt{LC}}$

→ Maximum value of RRRV = $\omega_n E_{peak}$

→ Where $\omega_n = 2 \pi f_n$,

→ Natural frequency of oscillations, $f_n = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$

Where L, C are the reactance and capacitance up to the location of circuit breaker

→ Frequency of demand oscillation, $f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$

Breaking capacity:

→ Symmetrical breaking current = r.m.s value of a.c component

$$= \frac{X}{\sqrt{2}}$$

→ Asymmetrical breaking current = r.m.s value of total current.

$$= \sqrt{\left(\frac{X}{\sqrt{2}}\right)^2 + Y^2}$$

Where X = maximum value of a.c component

Y = d.c component

→ Is the rated service line voltage in volts, then for 3-phase circuit? Breaking capacity

$$= \sqrt{3} \times V \times I \times 10^{-6} \text{ MVA}$$

$$\rightarrow \text{String efficiency} = \frac{\text{Voltage across the string}}{n \times \text{voltage across the unit near power conductor}}$$

Where, n = no of insulators

Making capacity:-

→ Making capacity = 2.55 × symmetrical breaking capacity.

The Universal Relay Torque Equation:-

→ The universal relay torque equation is given as follows

$$T = K_1 I^2 + K_2 V^2 + K_3 VI (\theta - \tau) + K$$

Distance Relays:

Impedance relays:

From the universal torque equation putting $K_3 = 0$ and giving negative sign to voltage term, it becomes

$$\rightarrow T = K_1 I^2 - K_2 V^2 \text{ (Neglecting spring torque)}$$

For the operation of the relay the operating torque should be greater than the restraining torque i.e

$$K_1 I^2 > K_2 V^2$$

→ Here V and I are the voltage and current quantities fed to the relay.

$$\rightarrow \frac{V^2}{I^2} < \frac{K_1}{K_2}$$

$$\rightarrow Z < \sqrt{\frac{K_1}{K_2}}$$

→ Z < constant (design impedance)

This means that the impedance relay will operate only if the impedance seen by the relay is less than a pre-specified value (design impedance). At threshold condition,

$$Z = \sqrt{\frac{K_1}{K_2}}$$

Reactance Relay:

The directional element is so designed that its maximum torque angle is 90° i.e. in the universal torque equation.

$$T = K_1 I^2 - K_3 VI \cos (\theta - \tau) = K_1 I^2 - K_3 VI \cos (\theta - 90) = K_1 I^2 - K_3 VI \sin \theta$$

For the operation of the relay

$$KI^2 > K_3 VI \sin \theta$$

$$\frac{VI}{I^2} \sin \theta < K_1 / K_3$$

$$Z \sin \theta < \frac{K_1}{K_3}$$

$$X < \frac{K_1}{K_3}$$

The mho relay:-

→ In the relay the operating torque is obtained by the V - I element and restraining torque due to the voltage element

$$T = K_3 VI \cos(\theta - \tau) K_2 V^2$$

→ For relay to operate

$$K_3 VI \cos(\theta - \tau) K_2 V^2$$

$$\frac{V^2}{VI} < \frac{K_3}{K_2} \cos(\theta - \tau)$$

$$Z < \frac{K_3}{K_2} \cos(\theta - \tau)$$
