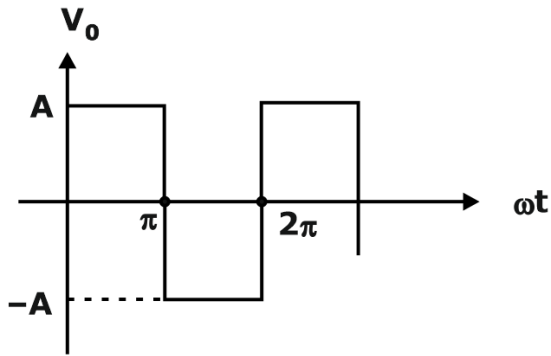


## POWER ELECTRONICS FORMULA NOTES

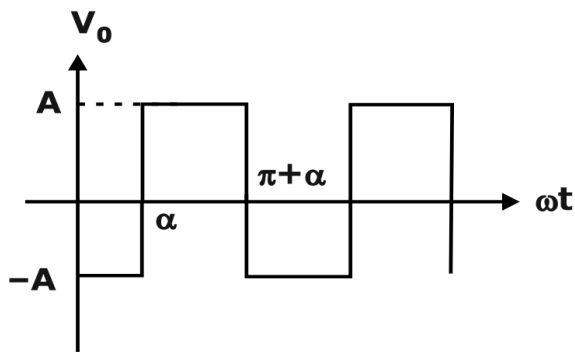
Important Fourier series representations :

1)



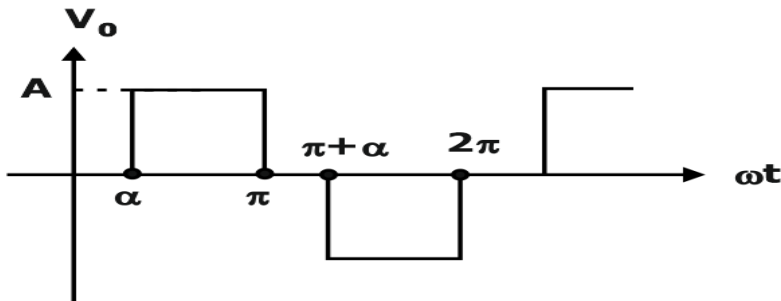
$$V_0 = \sum_{n=1,3,5} \frac{4A}{n\pi} \sin n\omega t$$

2)



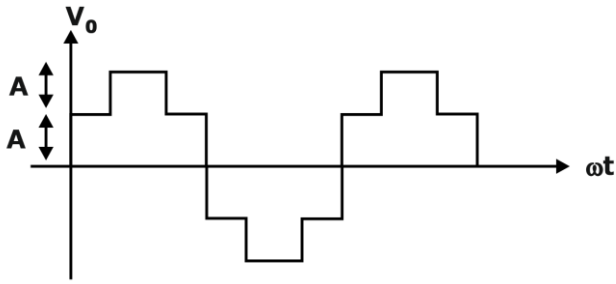
$$V_0 = \sum_{n=1,3,5} \frac{4A}{n\pi} \sin(n\omega t - n\alpha)$$

3)



$$V_0 = \sum_{n=1,3,5} \frac{4A}{n\pi} \cos\left(\frac{n\alpha}{2}\right) \sin\left(n\omega t - \frac{n\alpha}{2}\right)$$

4)



$$V_o = \sum_{n=1,5,7} \frac{6A}{n\pi} \sin n\omega t$$

Each pulse width of 60° Duration

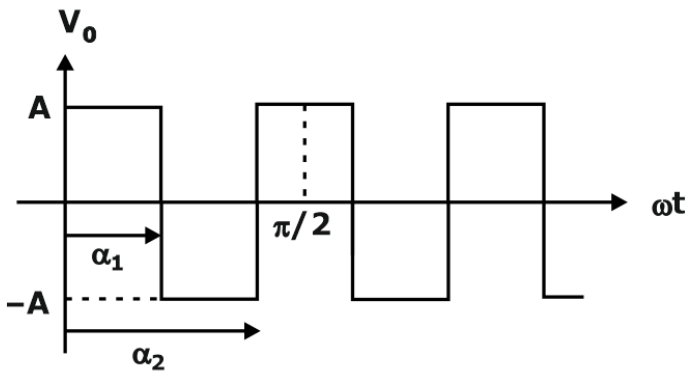
5) For output of half wave uncontrolled rectifier Fourier series expression is

$$V_o = \frac{A}{\pi} + \frac{A}{2} \sin \omega t + \sum_{n=2,4,6} \frac{2A}{\pi(1-n^2)} \cos n\omega t \quad \text{Where } A = \text{Amplitude of signal}$$

6) For output of Full wave uncontrolled rectifier Fourier series expression is

$$V_o = \frac{2A}{\pi} + (\text{fundamental} = 0) + 2 \sum_{n=2,4,6} \frac{2A}{\pi(1-n^2)} \cos n\omega t$$

7)



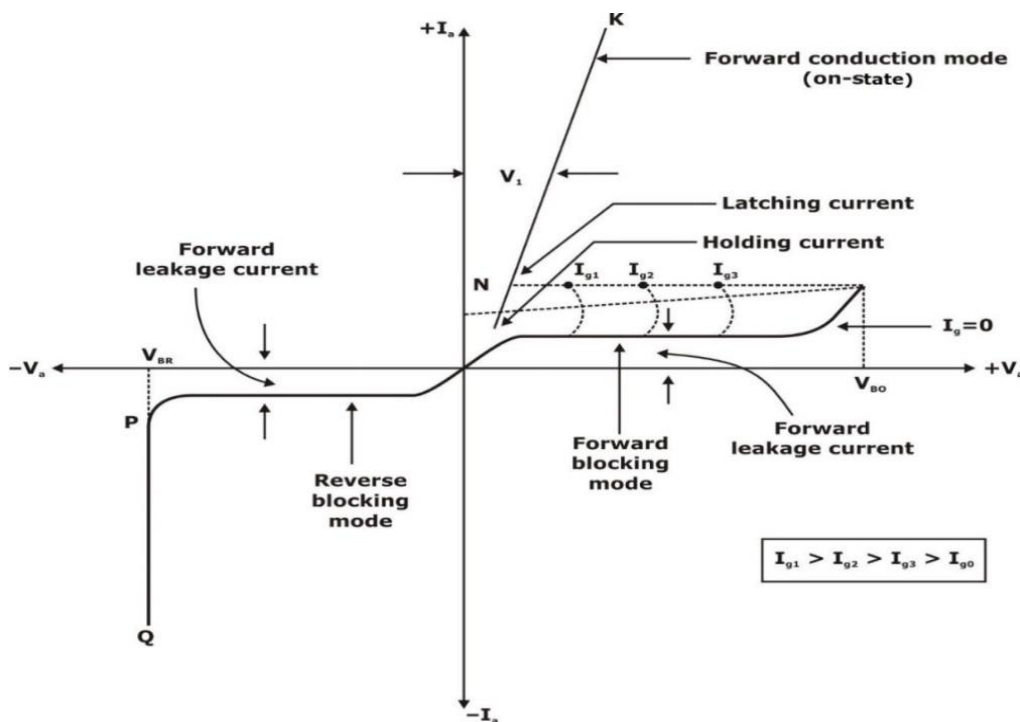
$$V_o = \frac{4A}{n\pi} (1 - 2 \cos n\alpha_1 + 2 \cos n\alpha_2)$$

Diode	BJT/MOSFET/IGBT	SCR and GTO	Triac
-------	-----------------	-------------	-------

<ul style="list-style-type: none"> <li>• Uncontrolled device</li> <li>• Unipolar and unidirectional</li> </ul>	<ul style="list-style-type: none"> <li>• Fully controlled</li> <li>• Unipolar and unidirectional (Without body diodes)</li> </ul>	<ul style="list-style-type: none"> <li>• SCR-Semi controlled</li> <li>• GTO- Fully controlled</li> <li>• Both are Bipolar and unidirectional</li> </ul>	<ul style="list-style-type: none"> <li>• Semi controlled device</li> <li>• Bipolar and Bidirectional</li> </ul>
--	---	---	---

- Conduction losses in BJT is less than MOSFET.
- Switching time in MOSFET is less than BJT.
- Majority carrier devices: MOSFET and Schottky diode are having Positive Temperature Coefficient property.
- Minority carrier devices: SCR, BJT, GTO, IGBT, Power diode and having Negative Temperature Coefficient property.

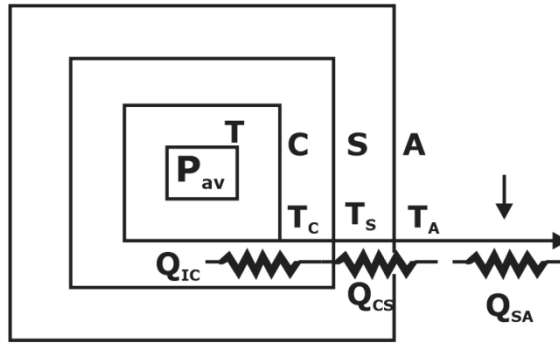
**Static V-I characteristics of SCR**



- Latching Current: Minimum current required for conduction even after the gate pulse is removed.
- Holding Current: Minimum Current below which SCR is turned off

- Usually Latching current is 1.5 to 3 times of Holding current

**Heat Sink Model:**



$$P_{avg} = \frac{T_J - T_C}{Q_{JC}} = \frac{T_C - T_S}{Q_{CS}} = \frac{T_S - T_A}{Q_{SA}}$$

Rating of thyristor  $\propto \sqrt{P_{avg}}$

**Charge stored in depletion region:**

Let  $Q_R$  be the charge stored in depletion region of power diode.

$$Q_R = \frac{1}{2} \cdot I_{RM} \cdot t_{rr}$$

$$I_{RM} = \frac{2Q_R}{t_{rr}} = t_a \cdot \frac{di}{dt}$$

If  $t_a \approx t_{rr}$ ,  $t_{rr} = \sqrt{\frac{2Q_R}{di/dt}}$

$$I_{RM} = t_{rr} \cdot \frac{di}{dt} = \sqrt{2Q_R} \left( \frac{di}{dt} \right)$$

$$t_{rr} \propto \sqrt{Q_R}$$

$$Q_R \propto I_f$$

$$t_{rr} \propto \sqrt{I_f}$$

$$I_{RM} \propto \sqrt{I_f}$$

**Relation Between  $\alpha$  and  $\beta$ :**

$$\alpha = \frac{I_C}{I_E} \quad \beta = \frac{I_C}{I_B}$$

$$\frac{I_E}{I_C} = 1 + \frac{I_B}{I_C}$$

$$\frac{1}{\alpha} = 1 + \frac{1}{\beta}$$

$$\alpha = \frac{\beta}{\beta + 1} \quad \beta = \frac{\alpha}{1 + \alpha}$$

**Design of Snubber circuit:**

For Inductor (L):

$$\left(\frac{di}{dt}\right)_{\max} = \frac{V_s}{L}$$

$$L = \frac{V_s}{\left(\frac{di}{dt}\right)_{\max}}$$

For resistor ( $R_s$ ):

$$\left(\frac{dv_a}{dt}\right)_{\max} = R_s \left(\frac{di}{dt}\right)_{\max}$$

$$\text{or } R_s = \frac{L}{V_s} \left(\frac{dv_a}{dt}\right)_{\max}$$

For Capacitor ( $C_s$ ):

$$C_s = \left(\frac{2\xi}{R_s}\right)^2 L \quad \text{where } 0.5 < \xi < 1$$

### Design of Snubber circuit:

$$\text{String efficiency} = \frac{\text{Actual voltage/current rating of string.}}{n \times \text{individual voltage/current rating of SCR}}$$

Where n is the number of SCR in string.

Derating factor, DRF = 1 - string efficiency.

### Parallel Operation of Thyristors:

When current required by the load is more than the rated current of a single thyristor, SCR's are connected in parallel in a string

### Series Operation of Thyristors:

Consider n thyristor connected in series, Let SCR<sub>1</sub> has minimum leakage current  $I_{bmn}$ . SCR with lower leakage current blocks more voltage.

Remaining (n-1) SCRs have the same leakage current  $I_{bmx}$

$$I_{bmx} > I_{bmn}$$

$$R_s = \frac{nV_{bm} - V_s}{(n-1)\Delta I_b}$$

Here  $V_{bm}$  is the maximum permissible blocking voltage as SCR1.

$R_s$  is the static equalizing resistance

$$\text{Similarly Static equalizing capacitance } C = \frac{(n-1)\Delta Q_T}{nV_{bm} - V_s}$$

$\Delta Q_T$  = difference in recovery charge

### Ratings of Thyristors:

**1)  $I_{Trms}$  Rating:** The actual Thyristor rms in a converter must always be less than thyristor RMS ratings.

$$[(I_T)_{rms} \text{ value in a converter}] < (I_T)_{rms} \text{ rating.}$$

**2) ( $I_{Tavg}$ ) Rating:** (average on-state current ratings)

$$(I_{Tavg}) \text{ rating} = \frac{(I_T)_{rms} \text{ Rating}}{\text{Form Factor of thyristor current waveform}}$$

**Average rating of a thyristor depends on:**

- Conduction angle of thyristor increases which decrease the form factor and then increase the average thyristor rating.
- Type of load: Smoothness of thyristor current waveform increase the FF decreases and therefore  $(I_{Tavg})_{Rating}$  increases.
- Type of converter: because FF of thyristor waveform depends on average value of converter.

**11.3. I<sup>2</sup>t Rating of thyristor:** specified to select a proper fuse for overcurrent protection.

$I^2t$  current Rating of thyristor >  $I^2t$  current Rating of Fuse.

**11.4. Surge current rating of thyristor:**

General values

$$(I_T)_{rms} = 35A$$

$$(I_s)_{rms} = 2000A \text{ for one cycle and } 3000A \text{ for } 2MW$$

N-cycle surge current rating:  $(I_m)$ : It is the surge current that the SCR can withstand for n-cycles.

$$\boxed{(I_{sn})^2 \left(\frac{nT}{2}\right) = I^2t \text{ rating of thy}}$$
 from the equation, we can find the value of ' $I_{sn}$ '

One-cycle surge current rating ( $I_s$ ): It is the surge current that the SCR can withstand

for a cycle.  $I_{s1}^2 = (I_{sn})^2$   $\boxed{I_{s1} = \sqrt{n} I_{sn}}$

Sub-cycle surge current rating: It is the surge current that the SCR can withstand for

1/nth period of a cycle.  $(I_{s/n})^2 = (I_{sn})^2$   $\boxed{\frac{I_s}{n} = \sqrt{n} I_{s1}}$

**RECTIFIERS**

- For n-pulse converter:
  - Source current has  $nk \pm 1$  Harmonics  $k=1,2,3,\dots$
  - Output voltage has  $nk$  Harmonics.

**Single Phase Half Wave controlled rectifier:**

R-load:

$$\text{Average output voltage } V_{o,avg} = \frac{V_m}{2\pi} (1 + \cos \alpha)$$

$$\text{RMS output voltage } V_{o,rms} = \sqrt{\frac{V_m^2}{4\pi} \left( (\pi - \alpha) + \frac{\sin 2\alpha}{2} \right)}$$

RL-Load:

$$\text{Average output voltage } V_{o,avg} = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta)$$

$$\text{RMS output voltage } V_{o,avg} = \frac{V_m}{2\sqrt{\pi}} \left( \sqrt{(\beta - \alpha) + \frac{1}{2}(\sin 2\alpha - \sin 2\beta)} \right)$$

$$\text{Circuit Turnoff time: } t_c = \frac{2\pi - \beta}{\omega}$$

**For a constant output current refer below table:**

	1- $\phi$ Full conv.	3 $\phi$ Full conv.	1 $\phi$ Semi conv.	3 $\phi$ semi conv
Output voltage	$\frac{2V_m}{\pi} \cos \alpha$	$\frac{3V_{m,line}}{\pi} \cos \alpha$	$\frac{V_m}{\pi} (1 + \cos \alpha)$	$\frac{3V_{m,line}}{2\pi} (1 + \cos \alpha)$
Fundamental source current RMS ( $I_{s1}$ )	$\frac{2\sqrt{2}}{\pi} I_0$	$\frac{\sqrt{6}}{\pi} I_0$	$\frac{2\sqrt{2}}{\pi} I_0 \cos\left(\frac{\alpha}{2}\right)$	$\frac{\sqrt{6}}{\pi} I_0 \cos\left(\frac{\alpha}{2}\right)$
Source current RMS ( $I_s$ )	$I_0$	$\sqrt{\frac{2}{3}} I_0$	$I_0 \sqrt{\frac{\pi - \alpha}{\pi}}$	$\alpha \leq 60^\circ \rightarrow I_0 \sqrt{\frac{2}{3}}$ $\alpha > 60^\circ \rightarrow I_0 \sqrt{\frac{\pi - \alpha}{\pi}}$
Displacement power factor(DPF)	$\cos \alpha$	$\cos \alpha$	$\cos\left(\frac{\alpha}{2}\right)$	$\cos\left(\frac{\alpha}{2}\right)$

- Distortion factor(DF) =  $\frac{I_{s1}}{I_s}$
- Input power Factor= DF\*DPF
- Total Harmonic distortion =  $\sqrt{\left(\frac{1}{DF}\right)^2 - 1}$

**For R-Load refer below table:**

	3 $\phi$ Half wave rectifier	3 $\phi$ full wave rectifier
--	------------------------------	------------------------------

<p>Continuous</p>	<p><math>\alpha &lt; 30^0</math></p> $\frac{3V_{m,line}}{2\pi} \cos \alpha$ <p><b>Hint:</b> Integrate from <math>30 + \alpha</math> to <math>150 + \alpha</math> and Time period <math>T=120^0</math> and function take in phase, You will get above formula Like this</p> $\frac{1}{\left(\frac{2\pi}{3}\right)} \int_{30+\alpha}^{150+\alpha} V_{m,phase} \sin \omega t \, d\omega t$	<p><math>\alpha &lt; 60^0</math></p> $\frac{3V_{m,line}}{\pi} \cos \alpha$ <p><b>Hint:</b> Integrate from <math>60 + \alpha</math> to <math>120 + \alpha</math> and Time period <math>T=60^0</math> and function take in line, You will get above formula</p>
<p>Discontinuous</p>	<p><math>\alpha \geq 30^0</math></p> $V_0 = \frac{3V_{m,phase}}{2\pi} \left(1 + \cos\left(\alpha + \frac{\pi}{6}\right)\right)$ <p><b>Hint:</b> Integrate from <math>30 + \alpha</math> to <math>180^0</math> and Time period <math>T=120^0</math> and function take in phase, You will get above formula</p>	<p><math>\alpha \geq 60^0</math></p> $V_0 = \frac{3V_{m,line}}{\pi} \left(1 + \cos\left(\alpha + \frac{\pi}{3}\right)\right)$ <p><b>Hint:</b> Integrate from <math>60 + \alpha</math> to <math>180^0</math> and Time period <math>T=60^0</math> and function take in line, You will get above formula</p>

**Effect of Source Inductance:**

- **1 φ** Half wave:

$$V_0 = \frac{V_m}{2\pi} (1 + \cos \alpha) - fL_s I_0$$

$$I_0 = \frac{V_m}{\omega L_s} (\cos \alpha - \cos(\alpha + \mu))$$

- **1 φ** Full wave:

$$V_0 = \frac{2V_m}{\pi} \cos \alpha - 4fL_s I_0$$

$$I_0 = \frac{V_m}{2\omega L_s} (\cos \alpha - \cos(\alpha + \mu))$$

$$\text{Regulation} = \frac{\cos \alpha - \cos(\alpha + \mu)}{2 \cos \alpha}$$



Displacement power factor:  $\cos\left(\alpha + \frac{\mu}{2}\right)$

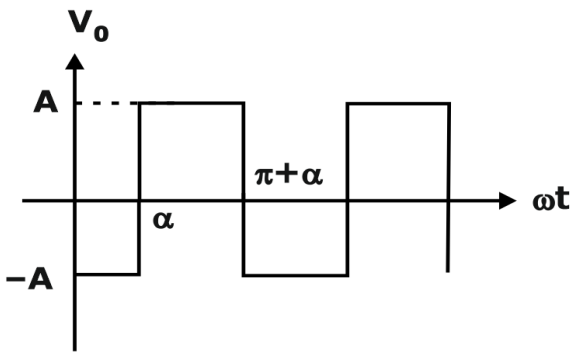
➤ 3ϕ Full Wave:

$$V_0 = \frac{3V_{m,line}}{\pi} \cos \alpha - 6fL_s I_0$$

$$I_0 = \frac{V_{m,line}}{2\omega L_s} (\cos \alpha - \cos(\alpha + \mu))$$

**Single Phase Full converter:**

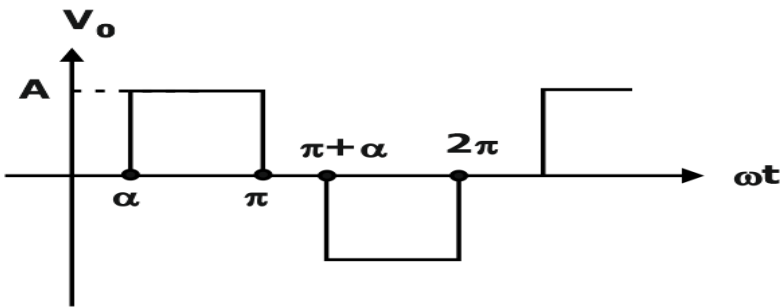
Source current waveform:



Fundamental source current is  $i_{s1} = \frac{2\sqrt{2}}{\pi} I_0$

**Single phase half controlled or Semi converter:**

Source current waveform:



➤ In this there are two configurations:

Symmetrical configuration: On one leg one thyristor and one diode

Unsymmetrical configuration: on one leg two thyristors or two diodes

$\gamma$  = represents conduction in below table

D- Diode, T-Thyristor,  $\alpha$  -Firing angle

Symmetrical configuration	Unsymmetrical configuration	Full converter with Freewheeling diode
---------------------------	-----------------------------	--

$\gamma_T = \pi$	$\gamma_T = \pi - \alpha$	$\gamma_T = \pi - \alpha$
$\gamma_D = \pi$	$\gamma_D = \pi + \alpha$	$\gamma_D = 2\alpha$

**3-Phase:**

Phase Voltage reference:

$$V_{an} = V \angle 0$$

$$V_{bn} = V \angle -120$$

$$V_{cn} = V \angle +120$$

Line Voltage reference

$$V_{AB} = V \angle 0$$

$$V_{BC} = V \angle -120$$

$$V_{CA} = V \angle +120$$

**3Phase half wave controlled Rectifiers:**

- Take phase voltage reference in the integration function for the below mentioned limits for calculations
- For R-Load  $\alpha < 30^\circ$ 
  - $\alpha < 30^\circ$  : Continuous conduction:  $30 + \alpha$  to  $150 + \alpha$
  - $\alpha \geq 30^\circ$  : Discontinuous conduction:  $30 + \alpha$  to  $180^\circ$
- For current stiff load:
  - Without Freewheeling diode:  $30 + \alpha$  to  $150 + \alpha$
  - With Freewheeling diode:  $\alpha < 30^\circ$  :  $30 + \alpha$  to  $150 + \alpha$
  - $\alpha \geq 30^\circ$  :  $30 + \alpha$  to  $180^\circ$
- ✓ Mentioned limits are useful while calculating output voltage average or RMS values for those particular conditions. Use phase as reference while doing calculations of average and RMS

**3Phase Full wave controlled Rectifiers:**

- Take Line voltage reference in the integration function for the below mentioned limits for calculations
- Limits are  $60 + \alpha$  to  $120 + \alpha$  for calculating output voltage average or RMS values
- Circuit Turnoff time:

$$\alpha \leq 60^\circ, t_c = \frac{240^\circ - \alpha}{\omega}$$

$$\alpha > 60^\circ, t_c = \frac{180^\circ - \alpha}{\omega}$$

**3Phase Semi converter:**

- $\alpha < 60^\circ$  it is 6 Pulse converter
- $\alpha \geq 60^\circ$  it is 3 Pulse converter
- Freewheeling Action Duration:

$\alpha < 60^\circ$ , Duration = zero (No freewheeling action)

$$\alpha \geq 60^\circ, \text{Duration} = 3 \left( \alpha - \frac{\pi}{3} \right)$$

➤ Limits for calculating output voltage average or RMS values (Line voltages are reference)

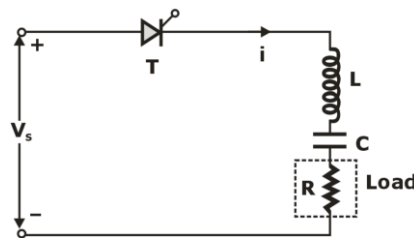
$\alpha < 60^\circ$ :  $60^\circ + \alpha$  to  $120^\circ \rightarrow V_{AB}$  reference

:  $120^\circ$  to  $180^\circ + \alpha \rightarrow V_{AC}$  reference

$\alpha \geq 60^\circ$ :  $60^\circ + \alpha$  to  $240^\circ \rightarrow V_{AC}$  reference

### Commutation Techniques

#### 1) Class A Commutation (Load Commutation/self-commutation)



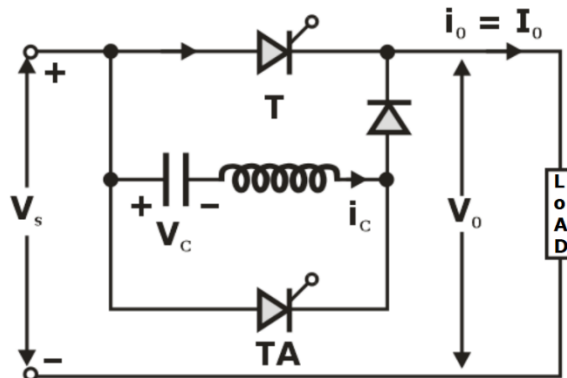
- For successful Load commutation, circuit must be under damped
- For under damped  $I = I_p e^{-\alpha t} \sin \omega_r t$

$$I_p = \frac{V_s}{\omega_r L}, \alpha = \text{Damping factor} = \frac{R}{2L}, \omega_r^2 = \omega_0^2 - \alpha^2$$

$$\omega_r \text{ is ringing frequency, } \omega_0 = \text{Natural frequency} = \frac{1}{\sqrt{LC}}$$

- Conduction time of thyristor,  $t_c = \frac{\pi}{\omega_r}$

#### 2) Class B Commutation/Current Commutation/Resonant pulse commutation:



- Voltage across capacitor  $V_c = V_s \cos \omega t$
- Circuit turn-off time for the main thyristor ( $T_1$ );  $t_c = C \frac{V_{ab}}{I_0}$

$$V_{ab} = V_s \cos \omega_0(t_3 - t_2)$$

Where  $t_3$  = time when the main thyristor is turned off

$t_2$  = time when auxiliary thyristor is turned off

$$\omega_0(t_3 - t_2) = \sin^{-1} \left( \frac{I_0}{I_p} \right)$$

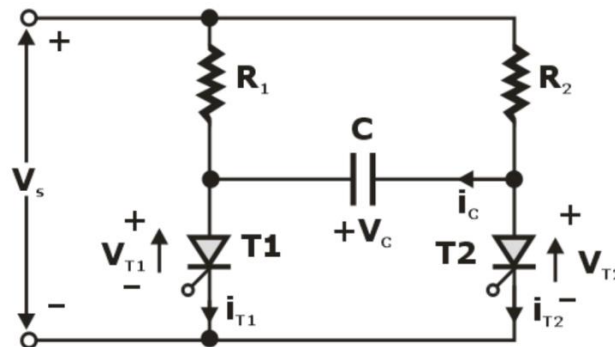
- Main thyristor peak current =  $I_0$
- Auxiliary Thyristor peak current =  $V_s \sqrt{\frac{C}{L}}$
- Conduction time of auxiliary thyristor =  $\pi \sqrt{LC}$
- Conduction time of main thyristor =  $\pi \sqrt{LC} + \sqrt{LC} \sin^{-1} \left( \frac{I_0}{I_p} \right)$

Minimum Conduction time of main thyristor =  $\pi \sqrt{LC}$

Maximum Conduction time of main thyristor =  $\frac{3}{2} \pi \sqrt{LC}$

- Time for which capacitor current exists =  $\frac{C}{I_0} (V_{ab} + V_s)$

**Class C Commutation (Impulse/Complementary commutation):**



When  $T_1$  is turned on at  $t=0$

- The charging current  $I_s = \frac{V_s}{R_2} \cdot e^{-t/R_2 C}$

• Voltage across capacitor

$$V_c(t) = V_s(1 - e^{-t/R_2 C})$$

When  $T_1$  is to be turned-off,  $T_2$  is turned-on at  $T_1$

- The charging current  $I_c(t) = -\frac{2V_s}{R_1} \cdot e^{-t/R_1 C}$

• The Voltage across capacitor

$$V_c(t) = V_s[2e^{-t/R_1C} - 1]$$

- Maximum current through thyristor T<sub>1</sub>

$$I_{T_1(max)} = V_s \left[ \frac{1}{R_1} + \frac{2}{R_2} \right]$$

- Maximum current through thyristor T<sub>2</sub>,

$$I_{T_2(max)} = V_s \left[ \frac{2}{R_1} + \frac{1}{R_2} \right]$$

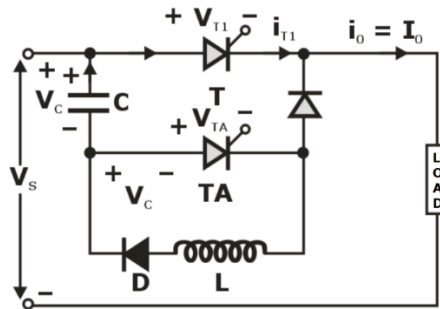
Circuit turn-off time t<sub>c1</sub> for thyristor T<sub>1</sub>

$$t_{c1} = R_1 C \ln(2)$$

Circuit turn-off time t<sub>c2</sub> for thyristor T<sub>2</sub>

$$t_{c2} = R_2 C \ln(2)$$

**Class D Commutation (Voltage commutation):**



- Maximum thyristor current Peak =  $I_0 + V_s \sqrt{\frac{C}{L}}$

- Auxiliary Thyristor peak current =  $I_0$

- Capacitor peak current =  $V_s \sqrt{\frac{C}{L}}$

- Circuit turn-off time for main thyristor T<sub>1</sub> is  $t_c = C \frac{V_s}{I_0}$

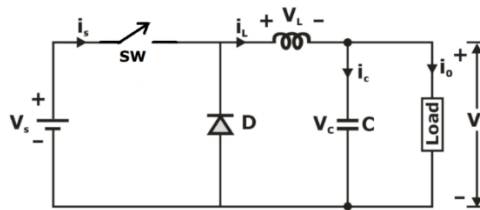
For R-load it  $t_c = RC \ln 2$

- Circuit turn-off time for main thyristor (TA)

$$t_{c1} = \frac{\pi}{2\omega_0}$$

### DC-DC Converters

#### Buck Converter:



In Buck regulator, the average output voltage  $V_0$  is less than the input voltage  $V_S$ .

$$\Delta I = \frac{(V_S - V_0) T_{ON}}{L}$$

$$\Delta I = \frac{V_0 T_{OFF}}{L}$$

$$V_0 = V_S \frac{T_{ON}}{T} = V_S \alpha$$

Where  $\Delta I = I_2 - I_1$  is the peak to peak current ripple of the inductor L.

The peak to peak ripple current is 
$$\Delta I = \frac{V_S \alpha (1 - \alpha)}{fL}$$

The peak to ripple voltage of the capacitor is 
$$\Delta V_C = \frac{V_S \alpha (1 - \alpha)}{8LCf^2}$$

#### Condition for continuous inductor current and capacitor voltage:

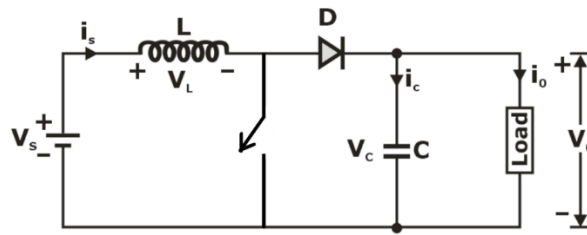
If  $I_L$  is average inductor current, the inductor ripple current  $\Delta I = 2I_L$ , which gives the critical

value of the inductor  $L_c$  as 
$$L_c = L = \frac{(1 - \alpha)R}{2f}$$

If  $V_C$  is the average capacitor voltage, the capacitor ripple voltage  $\Delta V_C = 2V_0$ , which gives

the critical value of capacitor  $C_c$  as 
$$C_c = C = \frac{1 - \alpha}{16Lf^2}$$

#### Boost Converter:



$$\Delta I = \frac{V_s T_{ON}}{L} = \frac{(V_o - V_s) T_{OFF}}{L}$$

where  $\Delta I = I_2 - I_1$  is peak to peak ripple current of the inductor L.

The average output voltage,

$$V_o = V_s \frac{T}{T_{OFF}} = \left( \frac{1}{1 - \alpha} \right) V_s$$

The peak to peak current ripple is,  $\Delta I = \frac{V_s \alpha}{fL}$

The peak to peak ripple voltage of capacitor,  $\Delta V_C = \frac{I_o \alpha}{fC}$

**Condition of continuous inductor current and capacitor voltage:**

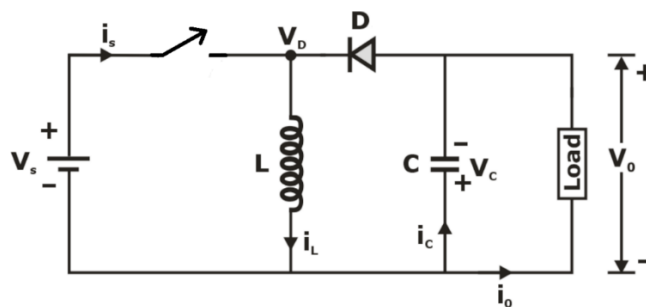
If  $V_C$  is the average capacitor voltage, the capacitor ripple voltage  $\Delta V_C = 2V_o$ , which gives

the critical value of the capacitor  $C_c$  as  $C_c = \frac{\alpha}{2fR}$

If  $I_L$  is average inductor current, the inductor ripple current  $\Delta I = 2I_L$ , which gives the critical

value of the inductor  $L_c$  as  $L_c = L = \frac{\alpha(1 - \alpha)^2 R}{2f}$

**Buck Boost Converter:**



$$\Delta I = \frac{V_s T_{ON}}{L} = \frac{-V_o T_{OFF}}{L}$$

where  $\Delta I = I_2 - I_1$  is the peak to peak ripple current of inductor L.

The average output voltage is,  $V_o = -\frac{V_s \alpha}{1 - \alpha}$

The peak to peak current ripple is,  $\Delta I = \frac{V_s \alpha}{fL}$

peak to peak ripple voltage of the capacitor is,  $\Delta V_C = \frac{I_o \alpha}{fC}$

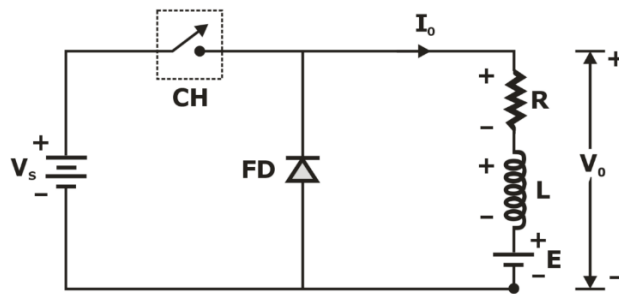
**Condition of continuous inductor current and capacitor voltage:**

If  $V_c$  is the average capacitor voltage, the capacitor ripple voltage,  $\Delta V_c = 2V_0$ , which gives the critical value of the capacitor  $C_c$  as  $C_c = \frac{\alpha}{2fR}$ .

If  $I_L$  is average inductor current, the inductor ripple current  $\Delta I = 2I_L$ , which gives the critical value of the inductor  $L_c$  as  $L_c = L = \frac{(1-\alpha)^2 R}{2f}$

Expression for $V_0$	BUCK	BOOST	BUCK BOOST
In CCM	$V_0 = \alpha V_s$	$V_0 = \frac{V_s}{1-\alpha}$	$V_0 = -\frac{\alpha V_s}{1-\alpha}$
In DCM	$V_0 = \frac{\alpha}{\beta} V_s$	$V_0 = \frac{\beta V_s}{\beta-\alpha}$	$V_0 = -\frac{\alpha V_s}{\beta-\alpha}$

**Steady State analysis of Type A Chopper:**



Average output voltage

$$V_0 = \alpha V_s$$

$$V_{or} = \sqrt{\alpha} V_s \text{ (Rms value of output voltage)}$$

$$I_{max} = \frac{V_s}{R} \left[ \frac{1 - e^{-T_{on}/T_a}}{1 - e^{-T/T_a}} \right] - \frac{E}{R}$$

$$I_{min} = \frac{V_s}{R} \left[ \frac{e^{T_{on}/T_a} - 1}{e^{T/T_a} - 1} \right] - \frac{E}{R}$$

Where,  $T_a$  = load time constant

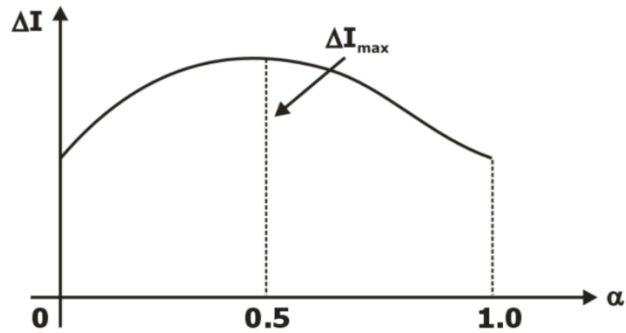
$$T_a = \frac{L}{R}$$

Current ripple,

$$(\Delta I) = I_{max} - I_{min}$$



$$\Delta I = \frac{V_S}{R} \left[ \frac{(1 - e^{-T_{on}/T_a})(1 - e^{-T_{off}/T_a})}{(1 - e^{-T/T_a})} \right]$$



$$T_{on} = \alpha T$$

$$T_{off} = (1 - \alpha) T$$

Per unit ripple (or) Ripple is a function of duty cycle 'α'. Ripple is minimum at α = 0, increases maximum at α = 0.5 and decrease at α = 1.0. For α = 0.5, ripple would be maximum.

$$(\Delta I)_{max} = \frac{V_S}{R} \left( \frac{(1 - e^{-0.5x})(1 - e^{-0.5x})}{1 - e^{-x}} \right) \quad \left( \text{Let, } \frac{T}{T_a} = x \right)$$

$$(\Delta I)_{max} = \frac{V_S}{R} \tanh\left(\frac{R}{4fL}\right)$$

### Inverters

**Series Inverters:** In a series inverter, the commutating elements L and C are connected in series with the load resistance R. The load resistance R can also be in parallel with C. The value of L and C are such that those form an underdamped circuit i.e.

$$R^2 < \frac{4L}{C}$$

$$f = \left[ \frac{1}{2\left(\frac{T}{2} + T_{off}\right)} \right] \text{ is the frequency of output voltage.}$$

Where,  $\frac{T}{2}$  is the time period of oscillations.

$T_{off}$  is the time gap between turn-off one thyristor and turn-on of the second thyristor.

$$\frac{T}{2} = \frac{\pi}{\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}}$$

The period of oscillation

**Bridge Inverter:** Bridge circuits are commonly used in DC-AC conversion. Moreover, an output transformer is not essential in a bridge circuit.

**1φ Half Bridge Inverter** - The output voltage volt  $V_0 = \sum_{n=1,3,5,\dots}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t$

**1φ Full Bridge Inverter**- The output voltage

$$V_0 = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t$$

Where, n = order of harmonic

$\omega=2\pi f$ , is frequency of the output voltage in red/sec

**Key points:**

- The load impedance ( $Z_n$ ) is

$$Z_n = \left[ R^2 + \left( n\omega L - \frac{1}{n\omega C} \right)^2 \right]^{1/2}$$

- Phase angle,  $\phi_n = \tan^{-1} \left[ \frac{n\omega L - \frac{1}{n\omega C}}{R} \right]$

**3phase Full Bridge VSI:**

	180° Conduction	120° Conduction
Line Voltage RMS	$V_s \sqrt{\frac{2}{3}}$	$V_s \frac{1}{\sqrt{2}}$
Phase voltage RMS	$V_s \frac{\sqrt{2}}{3}$	$V_s \frac{1}{\sqrt{6}}$
Fundamental line voltage RMS	$V_s \frac{\sqrt{6}}{\pi}$	$V_s \frac{3}{\pi\sqrt{2}}$
Fundamental phase voltage RMS	$V_s \frac{\sqrt{2}}{\pi}$	$V_s \frac{1}{\pi} \sqrt{\frac{3}{2}}$

**180° Conduction:**

1) Pole Voltages =  $V_{A0} = \sum_{n=1,3,5,\dots}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t$

$$2) \text{ Line Voltages} = \sum_{n=1,3,5} \left( \frac{4V_s}{n\pi} \cos\left(n \frac{\pi}{6}\right) \right) \sin(n(\omega t + 30^\circ))$$

When  $n=3, 9, 15$  Line voltage = 0, So Line voltages are free from Triplet harmonics

$$3) \text{ Phase Voltage} = \sum_{n=6k \pm 1} \frac{2V_{dc}}{n\pi} \sin n\omega t$$

$n = 6k \pm 1$  is due to stepped waveform

**120° Conduction:**

- 1) Pole and Phase Voltage are of same waveform
- 2) Triplet harmonics are absent in Phase and pole voltages
- 3) Line voltage contains  $n = 6k \pm 1$  Harmonics

**Pulse Width Modulation:**

- Let  $N$  = number of pulses per half cycle

$$\text{Each pulse width} = \frac{2d}{N}$$

Then Output voltage Expression is

$$V_0 = \sum_{n=1,3,5} \left( N \frac{4V_s}{n\pi} \sin n\gamma \sin \frac{nd}{N} \right) \sin n\omega t$$

$$\text{Where } \gamma = \frac{\pi - 2d}{N + 1} + \frac{d}{N}$$

- Number of pulses per half cycle  $N = \frac{f_c}{2f}$

$f$  = reference input frequency

$f_c$  = Carrier input frequency

- Modulation Index  $m_a = \frac{V_{Ref}}{V_{carrier}}$

- Relation between Pulse width and modulation index

$$\frac{2d}{N} = \frac{\pi}{N} (1 - m_a)$$

**Amplitude Modulation Depth:**

$$m_0 = \frac{\hat{V}_m}{\hat{V}_c}$$

Where  $V_m, V_c$  are the modulating and carrier signal voltage, respectively.

For sinusoidal PWM, the amplitude modulation depth must be less than 1.0

**Output Voltages by Sinusoidal PWM:**

- In single phase half bridge VSI

$$\text{Fundamental peak pole voltage} = \hat{V}_{Ao1} = m_a \frac{V_s}{2}$$

- In single phase Full bridge VSI

$$\text{Fundamental peak pole voltage} = \hat{V}_{Ao1} = m_a V_s$$

- In Three phase Full bridge VSI

$$\text{Peak Fundamental Phase voltage } \hat{V}_{an1} = m_a \frac{V_{DC}}{2}$$

The fundamental line-line rms voltage is given by

$$V_{LL01} = \frac{\sqrt{3}}{2\sqrt{2}} m_a V_{DC}$$

- If peak value of carrier input and zero crossing of reference sinusoidal coincidence then,

$$\text{Number of Pulses per half cycle will be } N = \frac{f_c}{2f}$$

- If Zero Crossing of carrier input and reference sinusoidal coincidence then, Number of Pulses

$$\text{per half cycle will be } N = \frac{f_c}{2f} - 1$$

- If N is the number of pulses per half cycle then the predominant harmonics in the output is  $2N \pm 1$

\*\*\*

