BYJU'S EXAM PREP

GATE 2022

Mechanical Engineering

Forenoon Shift

Questions with Detailed Solution

1. After playing __________ hours of tennis, I am feeling __________ tired to walk back.

[MCQ: 1 Mark]

A. Too/too B. Too/two

C. Two/two D. Two/too

Ans. D

Sol. After playing **two** hours of tennis, I am feeling **too** tired to walk back.

2. The average of the monthly salaries of M, N and S is ₹ 4000. The average of the monthly salaries of N, S and P is ₹ 5000. The monthly salary of P is ₹ 6000.

What is the monthly salary of M as a percentage of the monthly salary of P?

[MCQ: 1 Mark]

A. 50% B. 75%

C. 100% D. 125%

Ans. A

Sol. Given,

Average of the monthly salaries of M, N, S = 4000 Rs

Average of monthly salaries of N, S, P = 5000 Rs

Monthly salary of P = 6000 Rs

$$\frac{M+N+S}{3} = 4000 \quad ....(i)$$

$$\frac{M+S+P}{3} = 5000 \quad ....(ii)$$

From equation (i) and (ii)

N + S = 12000 - M

N + S = 15000 - P

12000 – M = 15000 – P

12000 – M = 15000 – 6000

M = 3000 Rs

Salary of M as a percentage of the monthly salary of $P = \frac{M}{P}\%$

$$= \frac{3000}{6000}\% = 50\%$$

3. A person travelled 80 km in 6 hours. If the person travelled the first part with a uniform speed of 10 kmph and the remaining part with a uniform speed of 18 kmph.

What percentage of the total distance is travelled at a uniform speed of 10 kmph?

[MCQ: 1 Mark]

A. 28.25 B. 37.25

C. 43.75 D. 50.00

Ans. C

Sol. Given,

Total distance $D_1 + D_2 = 80$ km,

Total time taken $t_1 + t_2 = 6$ hours

First part with a uniform speed = 10 kmph

So, $D_1 = 10 \times t_1$

Remaining part with a uniform speed of 18 kmph

So, $D_2 = 18 \times t_2$

as $D_1 + D_2 = 80$

$10\,t_1 + 18\,t_2 = 80$

$10\,t_1 + 18\,(6 - t_1) = 80$

$t_1 = \frac{7}{2}$ hours

First part distance $= 10 \times t_1 = 10 \times \frac{7}{2} = 35\text{km}$

So percentage of the total distance is travelled at uniform speed of so kmph

$$= \frac{35}{80} \times 100 = 43.75\%$$

4. Four girls P, Q, R and S are studying languages in a university. P is learning French and Dutch. Q is learning Chinese and Japanese. R is learning Spanish and French. S is learning Dutch and Japanese.

Given that: French is easier than Dutch; Chinese is harder than Japanese; Dutch is easier than Japanese, and Spanish is easier than French.

Based on the above information, which girl is learning the most difficult pair of languages?

[MCQ: 1 Mark]

A. P B. Q

C. R D. S

Ans. B

Sol. Languages as per difficulty are arranged as:

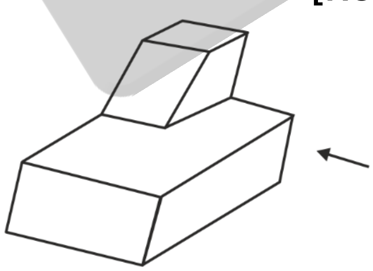
Chines > Japanese > Dutch > French > Spanish

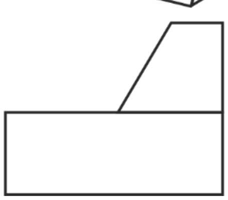
Since Q is learning Chinese and Japanese, Q is learning most difficult pair of language.

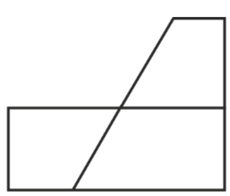
5. A block with a trapezoidal cross-section is placed over a block with rectangular cross section as shown above.

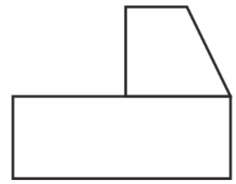
Which one of the following is the correct drawing of the view of the 3D object as viewed in the direction indicated by an arrow in the above figure?

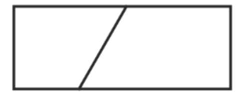
[MCQ: 1 Mark]



A. 

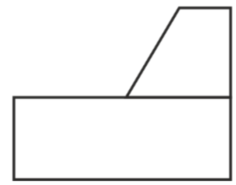
B. 

C. 

D. 

Ans. A

Sol.



This is the correct drawing of the view of the 3D object as viewed in the direction indicated by an arrow in the above figure.

6. Humans are naturally compassionate and honest. In a study using strategically placed wallets that appear "lost", it was found that wallets with money are more likely to be returned than wallets without money. Similarly, wallets that had a key and money are more likely to be returned than wallets with the same amount of money alone. This suggests that the primary reason for this behavior is compassion and empathy. Which one of the following is the CORRECT logical inference based on the information in the above passage?

[MCQ: 2 Marks]

A. Wallets with a key are more likely to be returned because people do not care about money

B. Wallets with a key are more likely to be returned because people relate to suffering of others

C. Wallets used in experiments are more likely to be returned than wallets that are really lost

D. Money is always more important than keys

Ans. B

Sol. Empathy and compassion in humans made these relate to suffering of others and here wallets with a key are more likely to be returned.

7. A rhombus is formed by joining the midpoints of the sides of a unit square.
What is the diameter of the largest circle that can be inscribed within the rhombus?

[MCQ: 2 Marks]

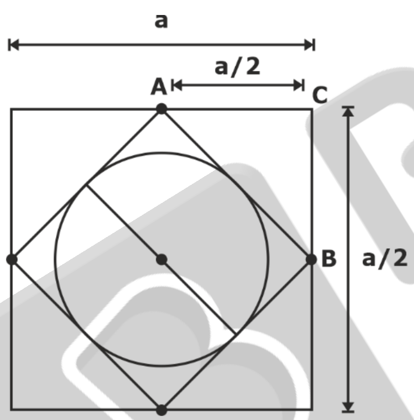
A. $\frac{1}{\sqrt{2}}$ B. $\frac{1}{2\sqrt{2}}$

C. $\sqrt{2}$ D. $2\sqrt{2}$

Ans. A

Sol. Given,

Unit square a = 1 unit



In Δ ABC,

$AC^2 + BC^2 = AB^2$

$$AB = D = \sqrt{AC^2 + BC^2}$$

$$= \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$$

Diameter of largest circle $= \frac{a}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ unit

8. An equilateral triangle, a square and a circle have equal areas.
What is the ratio of the perimeters of the equilateral triangle to square to circle?

[MCQ: 2 Marks]

A. $3\sqrt{3} : 2 : \sqrt{\pi}$

B. $\sqrt{(3\sqrt{3})} : 2 : \sqrt{\pi}$

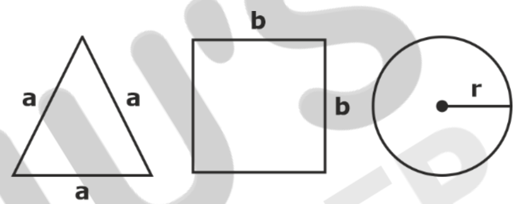
C. $\sqrt{(3\sqrt{3})} : 4 : 2\sqrt{\pi}$

D. $\sqrt{(3\sqrt{3})} : 2 : 2\sqrt{\pi}$

Ans. B

Sol. Given,

Area of equilateral triangle = Area of square = Area of circle



$$\text{Area} = \frac{\sqrt{3}\,a^2}{4} = b^2 = \pi r^2$$

$$\frac{(3)^{1/4}\,a}{2} = b = \sqrt{\pi}\,r = k \text{ (let)} \qquad \text{...(i)}$$

Perimeter = 3a = 4b = 2πr ...(ii)

From (i)

$$a = \frac{2k}{3^{1/4}},\ \ b = k,\ r = \frac{k}{\sqrt{\pi}}$$

Putting in equation (ii)

$$\frac{3 \times 2k}{3^{1/4}} = 4k = \frac{2\pi k}{\sqrt{\pi}}$$

$$\frac{3k}{3^{1/4}} = 2k = \sqrt{\pi}\,k$$

$$\frac{3}{3^{1/4}} : 2 : \sqrt{\pi}$$

$$\sqrt{(3\sqrt{3})} : 2 : \sqrt{\pi}$$

9. Given below are three conclusions drawn based on the following three statements.
Statement 1: All teachers are professors.
Statement 2: No professor is a male.
Statement 3: Some males are engineers.

Conclusion I: No engineer is a professor.

Conclusion II: Some engineers are professors.

Conclusion III: No male is a teacher.

Which one of the following options can be logically inferred?

[MCQ: 2 Marks]

A. Only conclusion III is correct

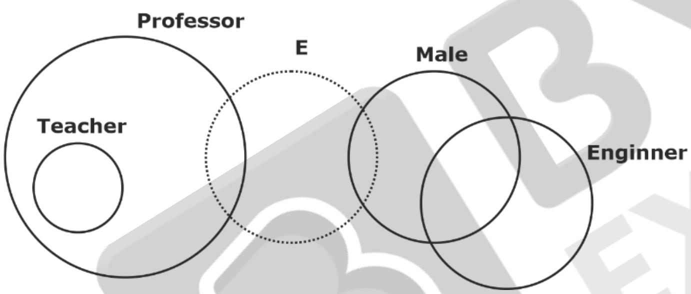
B. Only conclusion I and conclusion II are correct

C. Only conclusion II and conclusion III are correct

D. Only conclusion I and conclusion III are correct

Ans. A

Sol. Given,



So only conclusion III is correct.

Conclusion I & conclusion II may or may not be correct.

10. In a 12-hour clock that runs correctly, how many times do the second, minute, and hour hands of the clock coincide, in a 12-hour duration from 3 PM in a day to 3 AM the next day?

[MCQ: 2 Marks]

A. 11 B. 12

C. 144 D. 2

Ans. A

Sol. In 1 hour, the hour and minute hand coincide 1 time. So in 12 hours they should coincide with 12 times. But before 11 AM to 1 AM they coincide 1 times at 12'o-clock so, they coincide 11 times between 3 PM and 3 AM.

11. The limit $p = \lim_{x\to\pi}\left(\frac{x^2+\alpha x+2\pi^2}{x-\pi+2\sin x}\right)$ has a finite value for a real α. The value of α and the corresponding limit p are

[MCQ: 1 Mark]

A. $\alpha = -3\pi$, and $p = \pi$

B. $\alpha = -2\pi$, and $p = 2\pi$

C. $\alpha = \pi$, and $p = \pi$

D. $\alpha = 2\pi$, and $p = 3\pi$

Ans. A

Sol. $p = \lim_{x\to\pi}\left(\frac{x^2+\alpha x+2\pi^2}{x-\pi+2\sin x}\right)$

To solve limit numerator should be zero

$x^2 + \alpha x + 2\pi^2 = 0$

Put $x = \pi$

$\pi^2 + \alpha\pi + 2\pi^2 = 0$

$\alpha\pi + 3\pi^2 = 0$

$\alpha = -3\pi$

Put in equation (i)

$$p = \lim_{x\to\pi}\left(\frac{x^2-3\pi x+2\pi^2}{x-\pi+2\sin x}\right)$$

Apply L – Hospital rule,

$$p = \lim_{x\to\pi}\left(\frac{2x-3\pi}{1+2\cos x}\right)$$

$$p = \left(\frac{2\pi-3\pi}{1+2\cos x}\right)_{x=\pi} = \pi$$

12. Solution of $\nabla^2 T = 0$ in a square domain ($0 < x < 1$ and $0 < y < 1$) with boundary conditions: $T(x, 0) = x$; $T(0, y) = y$; $T(x, 1) = 1 + x$; $T(1, y) = 1 + y$ is

[MCQ: 1 Mark]

A. T(x, y) = x − xy + y

B. T(x, y) = x + y

C. T(x, y) = −x + y

D. T(x, y) = x + xy + y

Ans. B

Sol. T(x, 0) = x ⇒ option (c) is not correct

T(0, y) = y ⇒ all options satisfied.

T(x, 1) = 1 + x ⇒ only option (b) is satisfied.

T(1, y) = 1 + y ⇒ only option (b) is satisfied.

13. Given a function φ = 1/2 (x^2 + y^2 + z^2) in three-dimensional Cartesian space, the value of the surface integral $\oint\!\!\oint_s \hat{n}.\nabla\varphi dS$, where S is the surface of a sphere of unit radius and $\hat{n}$ is the outward unit normal vector on S, is

[MCQ: 1 Mark]

A. 4π

B. 3π

C. 4π/3

D. 0

Ans. A

Sol. Given,

$$\phi = \frac{x^2 + y^2 + z^2}{2}$$

By Gauss divergence theorem,

$$\int\int_s \hat{n}.\nabla\phi ds = \int\int\int_v Div\left(\vec{\nabla}\phi\right) dV$$

$$Div\left(\vec{\nabla}\phi\right) = \vec{\nabla}.\left(\vec{\nabla}\phi\right)$$

$$= \nabla^2.\phi$$

$$\text{So, } \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$= \frac{1}{2}\left[2 + 2 + 2\right] = 3$$

Put value in above equation

$$I = \int\int\int_v 3.dv = 3 \times \int\int\int_v dv$$

$$= 3V = 3 \times \frac{4}{3}\pi r^3$$

Given, sphere of unit radius , r =1

I = 4π

14. The Fourier series expansion of x^3 in the interval −1 ≤ x < 1 with periodic continuation has

[MCQ: 1 Mark]

A. only sine terms

B. only cosine terms

C. both sine and cosine terms

D. only sine terms and a non-zero constant

Ans. A

Sol. Given,

Fourier series of x^3 in the interval −1 ≤ x ≤ 1

As we know,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \frac{\cos \pi 2x}{l} + \sum_{n=1}^{\infty} b_n \frac{\sin n\pi x}{l}$$

If f(x) is an odd function in (−l, l) then,

$\boxed{a_o = 0}$, $\boxed{a_n = 0}$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

So f(x) contains only sine term.

15. If $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-2 & k+5 \end{bmatrix}$ is a symmetric matrix, the value of k is ____________.

[MCQ: 1 Mark]

A. 8

B. 5

C. −0.4

D. $\frac{1+\sqrt{1561}}{12}$

Ans. A

Sol. $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix}$

Given, A is a symmetric matrix,

So, A = A^T

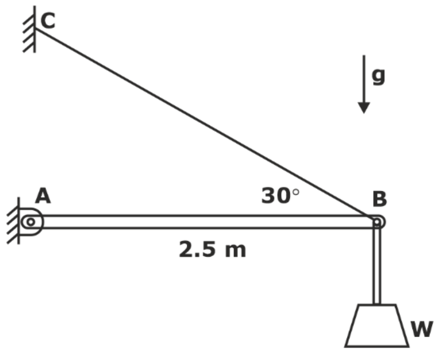
$$\begin{bmatrix} 10 & 2k+5 \\ 3k-3 & k+5 \end{bmatrix} = \begin{bmatrix} 10 & 3k-3 \\ 2k+5 & k+3 \end{bmatrix}$$

Equate both matrix

2k + 5 = 3k − 3

k = 8

16. A uniform light slender beam AB of section modulus EI is pinned by a frictionless joint A to the ground and supported by a light inextensible cable CB to hang a weight W as shown. If the maximum value of W to avoid buckling of the beam AB is obtained as $\beta\pi^2EI$, where π is the ratio of circumference to diameter of a circle, then the value of β is



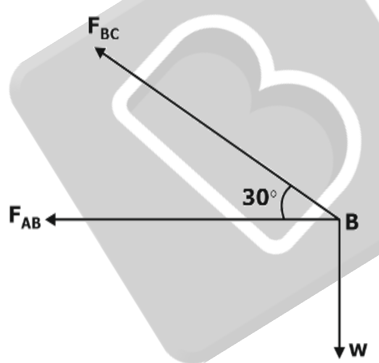
[MCQ: 1 Mark]

A. 0.0924 m^{-2} B. 0.0713 m^{-2}

C. 0.1261 m^{-2} D. 0.1417 m^{-2}

Ans. A

Sol.



$F_{BC}\sin 30° = w$

$F_{BC} = 2w$

$$F_{AB} = F_{BC}\cos 30° = 2w \times \frac{\sqrt{3}}{2}$$

$$F_{AB} = w\sqrt{3} \qquad \text{....(i)}$$

$$P_{or} = F_{AB} = \frac{\pi^2 EI}{(2.5)^2}$$

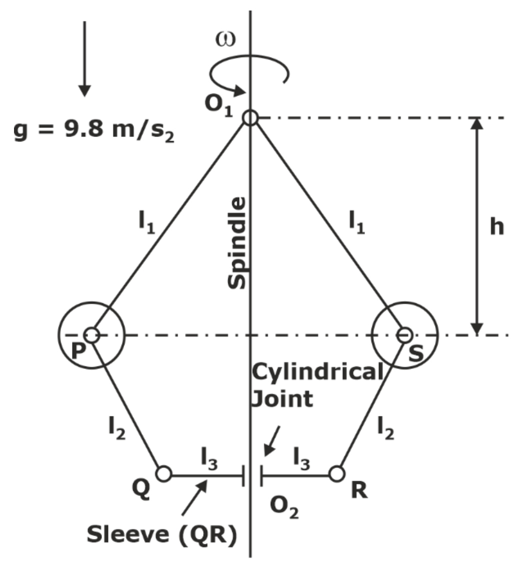
$$P_{or} = F_{AB} = \frac{\pi^2 EI}{(2.5)^2}$$

$$w\sqrt{3} = \frac{\pi^2 \times EI}{(2.5)^2}$$

$$w = \frac{\pi^2 EI}{\sqrt{3}\times(2.5)^2} = \beta\pi^2 EI$$

$$\beta = \frac{1}{\sqrt{3}\times 6.25} = 0.0924\ m^{-2}$$

17. The figure shows a schematic of a simple Watt governor mechanism with the spindle O_1O_2 rotating at an angular velocity ω about a vertical axis. The balls at P and S have equal mass. Assume that there is no friction anywhere and all other components are massless and rigid. The vertical distance between the horizontal plane of rotation of the balls and the pivot O_1 is denoted by h. The value of h = 400 mm at a certain ω. If ω is doubled, the value of h will be __________ mm.



[MCQ: 1 Mark]

A. 50 B. 100

C. 150 D. 200

Ans. B

Sol. Given, watt governor,

$h = 400$ mm

$$\omega^2 = \frac{g}{h}$$

$$h \propto \frac{1}{\omega^2}$$

$$h_1\omega_1^2 = h_2\omega_2^2$$

$$400 \times (\omega)^2 = h_2 \times (2\omega)^2$$

$h_2 = 100$ mm

18. A square threaded screw is used to lift a load W by applying a force F. Efficiency of square threaded screw is expressed as

[MCQ: 1 Mark]

A. The ratio of work done by W per revolution to work done by F per revolution

B. W/F

C. F/W

D. The ratio of work done by F per revolution to work done by W per revolution

Ans. A

Sol. Given,

Square threaded screw, efficiency

$$\eta = \left[\frac{\text{work done by w per revolution}}{\text{work done by F per revolution}}\right]$$

19. A CNC worktable is driven in a linear direction by a lead screw connected directly to a stepper motor. The pitch of the lead screw is 5 mm. The stepper motor completes one full revolution upon receiving 600 pulses. If the worktable speed is 5 m/minute and there is no missed pulse, then the pulse rate being received by the stepper motor is

[MCQ: 1 Mark]

A. 20 kHz B. 10 kHz

C. 3 kHz D. 15 kHz

Ans. B

Sol. Given,

Pitch of lead screw = 5 mm

$$\text{Pulses per revolution} = \frac{600\,\text{pulses}}{\text{revolution}}$$

$$\text{Work table speed} = \frac{5\,\text{m}}{\text{minute}}$$

$$\text{Table speed} = \text{BLU} \times \frac{\text{Pulse}}{\text{sec}}$$

$$\text{BLU} = \frac{\text{no. of start} \times \text{pitch} \times \text{Gear Ratio}}{\text{no. of puses per revolution}}$$

$$= \frac{1 \times 5}{600}$$

$$\text{BLV} = \frac{5}{600}\frac{\text{mm}}{\text{pulse}}$$

$$10^3 \times \frac{5}{60} = \frac{5}{600} \times \frac{\text{pulse}}{\text{sec}}$$

f = pulse per sec = 10^4 Hz

= 10 kHz

20. The type of fit between a mating shaft of diameter $25.0^{+0.010}_{-0.010}$ mm and a hole of diameter $25.015^{+0.015}_{-0.015}$ mm is ______

[MCQ: 1 Mark]

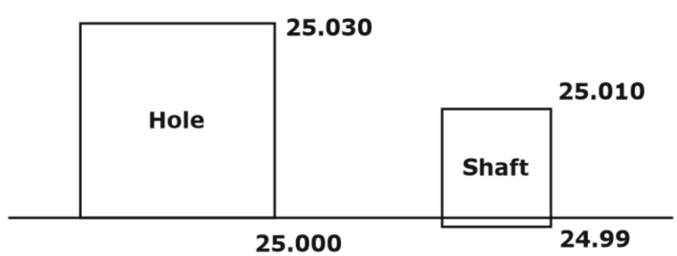
A. Clearance B. Transition

C. Interference D. Linear

Ans. B

Sol. Shaft of diameter = $25.0^{+0.010}_{-0.010}$

Hole of diameter = $25.015^{+0.015}_{-0.015}$



So transition fit.

21. In a linear programming problem, if a resource is not fully utilized, the shadow price of that resource is

[MCQ: 1 Mark]

A. positive B. negative

C. zero D. infinity

Ans. C

Sol. Shadow price indicates additional unit of resources in order to maximize profit under the resource constraint. If a resource is not completely used i.e., there is slack then its marginal return is zero.

22. Which one of the following is NOT a form of inventory?

[MCQ: 1 Mark]

A. Raw materials

B. Work-in-process materials

C. Finished goods

D. CNC Milling Machines

Ans. D

Sol. Form of inventory:

(i) work-in-process inventory

(ii) Spare part inventory

(iii) Waste inventory

(iv) Raw material inventory

(v) Finished good inventory

So, CNC milling machines is not form of inventory.

23. The Clausius inequality holds good for

[MCQ: 1 Mark]

A. any process

B. any cycle

C. only reversible process

D. only reversible cycle

Ans. B

Sol. By Clausius inequality,

$$\oint \frac{\delta Q}{T} \le 0 \text{ (for any cycle)}$$

$$\oint \frac{\delta Q}{T} = 0 \text{ (for reversible cycle)}$$

24. A tiny temperature probe is fully immersed in a flowing fluid and is moving with zero relative velocity with respect to the fluid. The velocity field in the fluid is $\vec{V} = (2x)\hat{i} + (y + 3t)\hat{j}$, and the temperature field in the fluid is $T = 2x^2 + xy + 4t$, where x and y are the spatial coordinates, and t is the time. The time rate of change of temperature recorded by the probe at

(x = 1, y = 1, t = 1) is ________.

[MCQ: 1 Mark]

A. 4 B. 0

C. 18 D. 14

Ans. C

Sol. Given,

Velocity field $\vec{V} = 2x\hat{i} + (y + 3t)\hat{j}$

Temperature field $T = 2x^2 + xy + 4t$

Where, x, y are the spatial coordinates, t is time

(x = 1, y = 1, t = 1)

Time rate of change of temperature

$$\frac{DT}{Dt} = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} + \frac{\partial T}{\partial t}$$

u = 2x, v = y + 3t , from velocity filed.

$$\frac{DT}{Dt} = (2x)[4x + y] + (y + 3t)[x] + 0 + 4$$

Put values of x, y, t

$= 2(4 + 1) + (1 + 3)(1) + 4$

$= 2 \times 5 + 4 + 4 = 18$

25. In the following two-dimensional momentum equation for natural convection over a surface immersed in a quiescent fluid at temperature T_∞ (g is the gravitational acceleration, β is the volumetric thermal expansion coefficient, ν is the kinematic viscosity, u and v are the velocities in x and y directions, respectively, and T is the temperature $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2}$, then term $g\beta(T - T_\infty)$ represents

[MCQ: 1 Mark]

A. Ratio of inertial force to viscous force.

B. Ratio of buoyancy force to viscous force.

C. Viscous force per unit mass.

D. Buoyancy force per unit mass.

Ans. D

Sol. $\beta = \frac{1}{V}\frac{\partial V}{\partial T}$

β is the volumetric thermal expansion which, signify the buoyancy forces. The term $g\beta(T - T_\infty)$ represents buoyant force per unit mass.

26. Assuming the material considered in each statement is homogeneous, isotropic, linear elastic, and the deformations are in the elastic range, which one or more of the following statement(s) is/are TRUE?

[MSQ: 1 Mark]

A. A body subjected to hydrostatic pressure has no shear stress.

B. If a long solid steel rod is subjected to tensile load, then its volume increases

C. Maximum shear stress theory is suitable for failure analysis of brittle materials

D. If a portion of a beam has zero shear force, then the corresponding portion of the elastic curve of the beam is always straight.

Ans. A & B

Sol. In hydrostatic condition shear stress on all planes are equal to zero. All planes are principal planes. Under hydrostatic loading condition ductile material may behave as brittle.

Under uniaxial loading application of tensile stress will lead to volume increase and application of compressive stress will lead to volume decrease.

For biaxial or triaxial loading volume will increase if $\sigma_x + \sigma_y + \sigma_z > 0$ most suitable theory of failure for brittle material = Maximum principal stress theory , Most suitable theory of failure for ductile material = Maximum distortion energy theory.

If shear force is zero in certain portion of beam, bending moment will be constant in that portion i.e., beam will be circular or straight in that portion.

27. Which of the following heat treatment processes is/are used for surface hardening of steels?

[MSQ: 1 Mark]

A. Carburizing B. Cyaniding

C. Annealing D. Carbonitriding

Ans. A, B & D

Sol. Heat treatment process used for surface hardening of steels

(i) Carburizing : Carburizing is a heat treatment process in which iron or steel absorbs carbon while the metal is heated in the presence of a carbon-bearing material, such as charcoal or carbon monoxide. The intent is to make the metal harder.

(ii) Cyaniding : Cyaniding is a case-hardening process that is fast and efficient; it is mainly used on low-carbon steels. The part is heated to 871–954 °C (1600–1750 °F) in a bath of sodium cyanide and then is quenched and rinsed, in water or oil, to remove any residual cyanide.

(iii) Carbonitriding : Carbonitriding is a metallurgical surface modification technique that is used to increase the surface hardness of a metal, thereby reducing wear. During the process, atoms of carbon and nitrogen diffuse interstitially into the metal, creating barriers to slip, increasing the hardness and modulus near the surface.

28. Which of the following additive manufacturing technique(s) can use a wire as a feedstock material?

[MSQ: 1 Mark]

A. Stereolithography

B. Fused deposition modelling

C. Selective laser sintering

D. Directed energy deposition processes

Ans. B & D

Sol. Fused deposition modelling is a 3D printing process that uses continuous filament of thermo plastic material filament in fed is the form of wire form a large spool.

In stereolithography, liquid form of polymer material is present and UV falls upon the material. Due to UV, material hardened and forms a layer. Here, wire feed cannot be used.

Also is selection laser sintering resin powder of polymer is hardened with a the help of laser.

In directed energy deposition process, feed stock material is in the form of wire or powder form is used.

29. Which of the following methods can improve the fatigue strength of a circular mild steel (MS) shaft?

[MSQ: 1 Mark]

A. Enhancing surface finish

B. Shot peening of the shaft

C. Increasing relative humidity

D. Reducing relative humidity

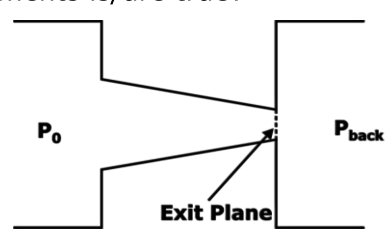
Ans. A, B & D

Sol. Surface roughness increases surface matches and hence these is more concentration of stresses during fatigue loading and hence improving surface finish will result in less cracks and enhanced surface finish.

Shot peening is a method of improving fatigue stress by inducing compressive stresses which decreases chances of crack and this cold working technique increases the fatigue strength of material.

Decreasing relative humidity will decrease the moisture content and hence chances of corrosion in less. Corrosion decreases fatigue strength by degrading the material.

30. The figure shows a purely convergent nozzle with a steady, inviscid compressible flow of an ideal gas with constant thermophysical properties operating under choked condition. The exit plane shown in the figure is located within the nozzle. If the inlet pressure (P_0) is increased while keeping the back pressure (P_{back}) unchanged, which of the following statements is/are true?



[MSQ: 1 Mark]

A. Mass flow rate through the nozzle will remain unchanged.
B. Mach number at the exit plane of the nozzle will remain unchanged at unity.
C. Mass flow rate through the nozzle will increase.
D. Mach number at the exit plane of the nozzle will become more than unity.

Ans. B & C

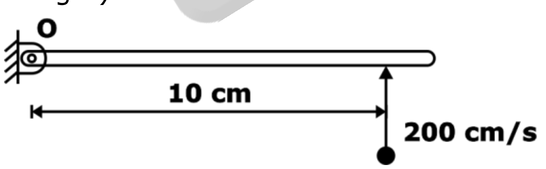
Sol. $$\frac{\dot{m}}{A} = \sqrt{\frac{\gamma}{R}}\frac{P_0}{\sqrt{T_0}}\frac{1}{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\dot{m} \propto P_0$$

(Hence on increasing P_0, mass flow rate will increase)

Mach number at exit place of nozzle will remain unchanged at unity because of choked condition.

31. The plane of the figure represents a horizontal plane. A thin rigid rod at rest is pivoted without friction about a fixed vertical axis passing through O. Its mass moment of inertia is equal to 0.1 kg·cm^2 about O. A point mass of 0.001 kg hits it normally at 200 cm/s at the location shown, and sticks to it. Immediately after the impact, the angular velocity of the rod is ___________ rad/s (in integer).



[NAT: 1 Mark]

Ans. 10

Sol. Range (10 to 10)

Given,

Rigid rod mass moment of inertia = 0.1 kg/cm^2

According to law of conservation of angular momentum about the point O.

Before impact angular momentum = After impact angular momentum

$$mvr = (I_{net})\,\omega$$

$$mvr = (I_{rod} + I_{mass})\,\omega$$

Moment of inertia of point mass about point O

$$= mr^2 = 0.001 \times (10)^2$$

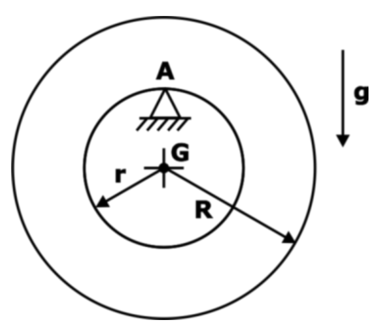
$$= 0.1 \text{ kg cm}^2$$

$$0.001 \times 200 \times 10 = (0.1 + 0.1) \times \omega$$

$$T = \beta\pi\sqrt{\frac{R}{g}}$$

32. A rigid uniform annular disc is pivoted on a knife edge A in a uniform gravitational field as shown, such that it can execute small amplitude simple harmonic motion in the plane of the figure without slip at the pivot point. The inner radius r and outer radius R are such that $r^2 = R^2/2$, and the acceleration due to gravity is g. If the time period of small amplitude simple harmonic motion is given by

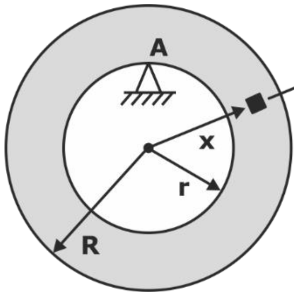
$T = \beta\pi\sqrt{\frac{R}{g}}$, where π is the ratio of circumference to diameter of a circle, then β = _________ (round off to 2 decimal places).



[NAT: 1 Mark]

Ans. 2.66

Sol. Range: (2.62 to 2.70)



Consider a ring of thickness 'dx' at a radius of x.

Area of the disk will be $\pi(R^2 - r^2)$

Mass per unit area will be $\dfrac{m}{\pi(R^2 - r^2)}$

Mass of the differential element will be

$$d\mu = \frac{m}{\pi(R^2 - r^2)} \times 2\pi x\, dx = \frac{2m}{(R^2 - r^2)} x \cdot dx$$

Moment of inertia of this differential disk will be

$$dI_y = (d_m)x^2 = \frac{2m}{(R^2 - r^2)} x\, dx \times x^2$$

$$I_y = \int_r^R \frac{2m}{(R^2 - r^2)} x^3\, dx$$

$$I_y = \frac{2m}{(R^2 - r^2)} \int_r^R x^3\, dx = \frac{2m}{(R^2 - r^2)} \left(\frac{1}{4}\right)(R^4 - r^4)$$

$$I_y = \frac{2m}{(R^2 - r^2)} \times \left(\frac{1}{4}\right)(R^2 + r^2)\,(R^2 - r^2)$$

$$\boxed{I_y = \frac{m}{2}(R^2 + r^2)}$$

Putting the relation given

$$r = \frac{R}{\sqrt{2}}$$

$$I_y = \frac{3}{4} mR^2$$

Using parallel axis theorem to get mass moment of inertia about the hinge points A, $\perp^r$ to the plane.

$$I_A = I_y + mr^2 = \frac{3}{4} mR^2 + mr^2$$

$$I_A = \frac{3}{4} mR^2 + \frac{mR^2}{2} = \frac{5}{4} mR^2$$

Applying D'Alembert's principle

θ $I\ddot{\theta}$ mg rθ

$$I\ddot{\theta} + mgr\sin\theta = 0$$

$$[\because \sin\theta \simeq \theta]$$

$$\frac{5}{4} mR^2 \ddot{\theta} + mg\frac{R}{\sqrt{2}}\theta = 0$$

$$\ddot{\theta} + \frac{4g}{5\sqrt{2}R}\theta = 0$$

$$\ddot{\theta} + \omega_n^2 \theta = 0$$

$$\omega_n = \frac{2}{\sqrt{5\sqrt{2}}}\sqrt{\frac{g}{R}}$$

$$T = \frac{2\pi}{\omega_n} = \frac{2\pi}{\frac{2}{\sqrt{5\sqrt{2}}}\sqrt{\frac{g}{R}}} = \sqrt{5\sqrt{2}}\,\pi\sqrt{\frac{R}{g}}$$

Comparing the equation with time period given in question

$$\beta = \sqrt{5\sqrt{2}}$$

$$= 2.659 \simeq 2.66$$

33. Electrochemical machining operations are performed with tungsten as the tool, and copper and aluminum as two different workpiece materials. Properties of copper and aluminum are given in the table below.

Ignore overpotentials, and assume that current efficiency is 100% for both the workpiece materials. Under identical conditions, if the material removal rate (MRR) of copper is 100 mg/s, the MRR of aluminum will be ________ mg/s(round-off to two decimal places).

[NAT: 1 Mark]

Ans. 28.54

Sol. Range (27 to 30)

Given,

Material Removal Rate (MRR) of copper = 100 mg/sec

Current efficiency = 100%

Material	Atomic mass (amu)	Valency	Density (g/cm^3)
Copper	63	2	9
Aluminum	27	3	2.7

$$MRR = \frac{I}{\rho ZF}$$

$$\frac{(MRR)}{(MRR)_2} = \left(\frac{A_1}{\rho_1 z_1}\right)\left(\frac{\rho_2 z_2}{A_2}\right) = \left(\frac{A_1}{A_2}\right)\left(\frac{\rho_2}{\rho_2} \times \frac{z_2}{z_1}\right)$$

$$\frac{(MRR)_{Cu}}{(MRR)_{Alu.}} = \left(\frac{A_{Cu}}{A_{Al}}\right)\left(\frac{\rho_{Al}}{\rho_{Cu}} \times \frac{Z_{Al}}{Z_{Cu}}\right)$$

$$\frac{(MRR \times \rho)_{Cu}}{(MRR \times \rho)_{Alumin.}} = \left(\frac{A_{Cu}}{A_{Al}}\right) \times \left(\frac{Z_{Al}}{Z_{Cu}}\right)$$

$$\frac{100}{(MRR \times \rho)_{Aluminium}} = \frac{63}{27} \times \frac{3}{2}$$

$$\boxed{(MRR)_{Aluminium} = 28.57\ \frac{mg}{sec}}$$

34. A polytropic process is carried out from an initial pressure of 110 kPa and volume of 5 m3 to a final volume of 2.5 m^3. The polytropic index is given by n = 1.2. The absolute value of the work done during the process is ________ kJ (round off to 2 decimal places).

[NAT: 1 Mark]

Ans. 408.875

Sol. Range (404 to 414)

Given,

Polytropic process,

Initial pressure = 110 kPa

Initial volume = 5 m^3

Final volume = 2.5 m^3

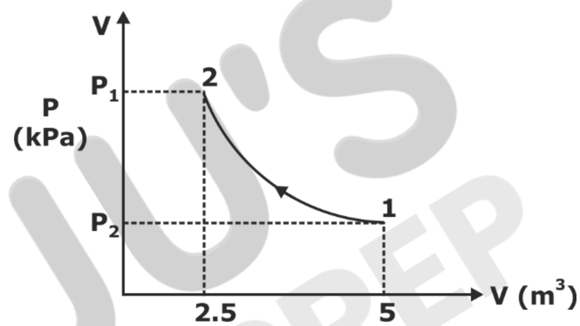
Poly tropic index = n = 1.2

$$P_1V_1^n = P_2V_2^n$$

$$P_2 = \left(\frac{V_1}{V_2}\right)^n \times P_1$$

$$= \left(\frac{5}{2.5}\right)^{1.2} \times 110$$

$$\boxed{P_2 = 252.71\ kPa}$$



$$\text{Poly tropic work done} = \frac{(P_2V_2 - P_1V_1)}{n-1}$$

$$W_{1-2} = \left(\frac{252.71 \times 2.5 - 110 \times 5}{0.2}\right)$$

$$= -408.875$$

35. A flat plate made of cast iron is exposed to a solar flux of 600 W/m^2 at an ambient temperature of 25°C. Assume that the entire solar flux is absorbed by the plate. Cast iron has a low temperature absorptivity of 0.21. Use Stefan-Boltzmann constant = 5.669 × 10^{-8} $W/m^2–K^4$. Neglect all other modes of heat transfer except radiation.

Under the aforementioned conditions, the radiation equilibrium temperature of the plate is __________ °C (round off to the nearest integer).

[NAT: 1 Mark]

Ans. 218

Sol. Range (210 to 225)

Given,

Solar flux $= 600 \frac{W}{m^2}$ (incident)

Ambient temperature = 25°C

Cast iron absorptivity = 0.21

Stefan Boltzmann constant

$= 5.67 \times 10^{-8} \frac{W}{m^2 k^4}$

At a given instant of time, from energy balance,

$$G_s = \varepsilon\sigma(T_5^4 - T_\infty^4)$$

$$\alpha = \varepsilon$$

$$T_s = 491.33 \text{ K}$$

$$T_s = 218^\circ\text{C}$$

Eq. temperature of plate = 218°C.

36. The value of the integral

$$\oint\left(\frac{6z}{2z^4 - 3z^3 + 7z^2 - 3z + 5}\right)dz$$

evaluated over a counter-clockwise circular contour in the complex plane enclosing only the pole $z = i$, where i is the imaginary unit, is

[MCQ: 2 Marks]

A. $(-1 + i)\pi$ B. $(1 + i)\pi$

C. $2(1 - i)\pi$ D. $(2 + i)\pi$

Ans. A

Sol. Given,

$$\oint\left(\frac{6z}{2z^4 - 3z^3 + 7z^2 - 3z + 5}\right)$$

Pole z = i,

Factorization of denominator

$= 2z^4 - 3z^3 + 7z^2 - 3z + 5$

$= 2z^4 - 7z^2 + 3z(z^2 + 1) + 5$

$= (2z^4 + 7z^2 + 5) - 3z(z^2 + 1)$

$= (2z^4 + 2z^2 + 5z^2 + 5) - 3z(z^2 + 1)$

$= (2z^2 + 5 - 3z)(z^2 + 1)$

$$\text{Res}(z = i) = \lim_{z \to i} \frac{6z}{(z+i)(z-i)(2z^2 - 3z + 5)} \text{ b}$$

$$= \lim_{z \to i} \frac{6z}{(z+i)(2z^2 - 3z + 5)}$$

$$= \frac{6i}{2i(2i^2 - 3i + 5)}$$

$$\text{Res}(z = i) = \frac{3}{3 - 3i} = \frac{1}{1 - i} = \left(\frac{1+i}{2}\right)$$

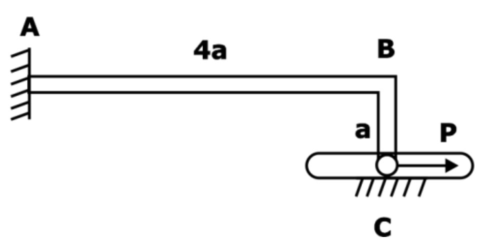
Cauchy residue theorem,

$= 2\pi i$ (sum of residue)

$$= 2\pi i\left(\frac{1+i}{2}\right) = \pi i(1 + i)$$

$= \pi(-1 + i)$

37. An L-shaped elastic member ABC with slender arms AB and BC of uniform cross-section is clamped at end A and connected to a pin at end C. The pin remains in continuous contact with and is constrained to move in a smooth horizontal slot. The section modulus of the member is same in both the arms. The end C is subjected to a horizontal force P and all the deflections are in the plane of the figure. Given the length AB is $4a$ and length BC is a, the magnitude and direction of the normal force on the pin from the slot, respectively, are



[MCQ: 2 Marks]

A. 3P/8, and downwards

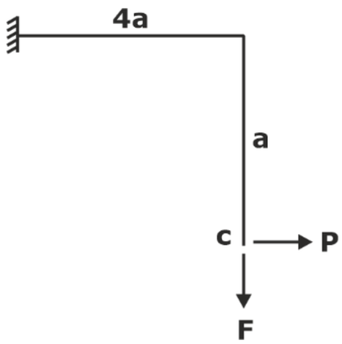
B. 5P/8, and upwards

C. P/4, and downwards

D. 3P/4, and upwards

Ans. A

Sol. FBD of member



Where F is the force at point C which will move member downwards

Δ_1 = Deflection upwards by force P

Δ_2 = Deflection downwards by force F

$$\Delta_1 = \frac{(Pa)\,(4a)^2}{2EI} = \frac{8Pa^3}{EI}$$

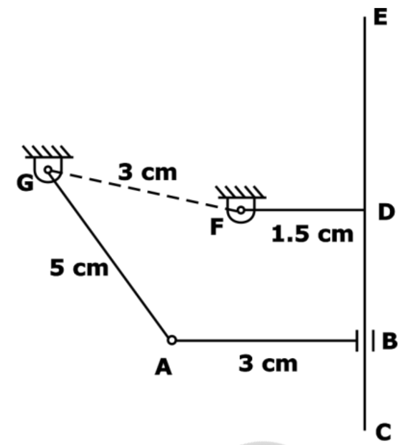
$$\Delta_2 = \frac{F \times (4a)^3}{3EI} = \frac{64Fa^3}{3EI}$$

As, $\Delta_1 = \Delta_2$

$$\frac{8Pa^3}{EI} = \frac{64Fa^3}{3EI}$$

$$F = \frac{3P}{8}\text{(downwards)}$$

38. A planar four-bar linkage mechanism with 3 revolute kinematic pairs and 1 prismatic kinematic pair is shown in the figure, where AB $\perp$ CE and FD $\perp$ CE. The T-shaped link CDEF is constructed such that the slider B can cross the point D, and CE is sufficiently long. For the given lengths as shown, the mechanism is



[MCQ: 2 Marks]

A. a Grashof chain with links AG, AB, and CDEF completely rotatable about the ground link FG

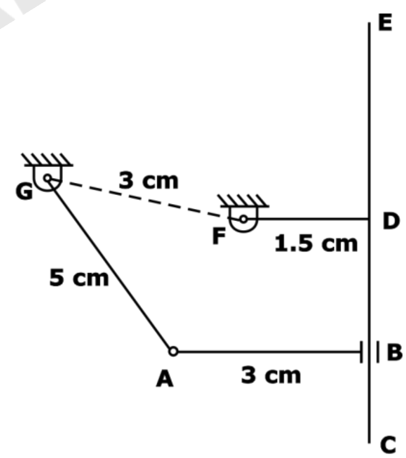
B. a non-Grashof chain with all oscillating links

C. a Grashof chain with AB completely rotatable about the ground link FG, and oscillatory links AG and CDEF

D. on the border of Grashof and non-Grashof chains with uncertain configuration(s)

Ans. B

Sol.



Here all lengths of links are shown

s = 1.5 cm

l = 5 cm

s + l = 1.5 + 5 = 6.5 cm

Sum of other two links:

p + q = 3 cm + 3 cm = 6 cm

s + l > (p + q)

⇒ Grashoff's law is not satisfied, therefore non Grashoff's chain.

⇒ All are oscillating links or triple rocker mechanism.

39. Consider a forced single degree-of-freedom system governed by $\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = \omega_n^2 \cos(\omega t)$, where ζ and ω_n are the damping ratio and undamped natural frequency of the system, respectively, while ω is the forcing frequency. The amplitude of the forced steady state response of this system is given by $[(1 - r^2)^2 + (2\zeta r)^2]^{-1/2}$, where $r = \omega/\omega_n$. The peak amplitude of this response occurs at a frequency $\omega = \omega_p$. If ω_d denotes the damped natural frequency of this system, which one of the following options is true?

[MCQ: 2 Marks]

A. $\omega_p < \omega_d < \omega_n$ B. $\omega_p = \omega_d < \omega_n$
C. $\omega_d < \omega_n = \omega_p$ D. $\omega_d < \omega_n < \omega_p$

Ans. A

Sol. A force single degree of freedom system is given as

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = \omega_n^2 \cos\omega t$$

Amplitude is given as, $A = \dfrac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

Where, $r = \dfrac{\omega}{\omega_n}$

The peak amplitude of this response occurs at

$\dfrac{\omega}{\omega_n} = 1$

$\omega = \omega_n$ ...(i)

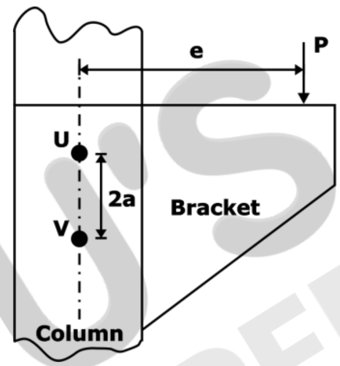
Damped frequency, is $\omega_d = \sqrt{(1-\zeta^2)}\omega_n$

hence, $\omega_d < \omega_n$ ...(ii)

$\omega_p < \omega_n$

Therefore, $\omega_p < \omega_d < \omega_n$

40. A bracket is attached to a vertical column by means of two identical rivets U and V separated by a distance of 2a = 100 mm, as shown in the figure. The permissible shear stress of the rivet material is 50 MPa. If a load P = 10 kN is applied at an eccentricity $e = 3\sqrt{7}\,a$, the minimum cross-sectional area of each of the rivets to avoid failure is ____________ mm^2.



[MCQ: 2 Marks]

A. 800 B. 25
C. $100\sqrt{7}$ D. 200

Ans. A

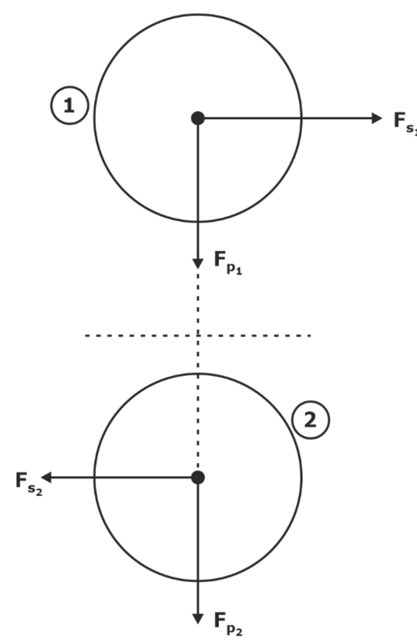
Sol. Given,

U and V distance 2a = 100 mm

Permissible sheal stress = 50 MPa

Load P = 10 kN

Eccentricity $e = 3\sqrt{7}a$



Primary force on each rivets

$$F_{P_1} = F_{P_2} = \frac{10}{2} = 5\text{ kN}$$

Secondary force $= F_s = \frac{Pe\,r_1}{(r_1^2 + r_2^2)}$

$$= \frac{10 \times 10^3 \times 3\sqrt{7}a \times a}{(a^2 + a^2)}$$

$$= \frac{10 \times 10^3 \times 3\sqrt{7} \times a^2}{2a^2} = 39.68\text{ kN}$$

Resultant force $(R) = \sqrt{F_P^2 + F_s^2}$

$$= \sqrt{(5)^2 + (39.686)^2}$$

R = 40 kN

$$(\tau_{ind})_{max} = \frac{R}{A} \le \tau_{per}$$

$$A \ge \frac{R}{\tau_{per}}$$

Minimum cross-sectional area $\ge \frac{40 \times 10^3}{50}$

$$\ge 800\text{ mm}^2$$

41. In Fe-Fe$_3$C phase diagram, the eutectoid composition is 0.8 weight % of carbon at 725 °C. The maximum solubility of carbon in α-ferrite phase is 0.025 weight % of carbon. A steel sample, having no other alloying element except 0.5 weight % of carbon, is slowly cooled from 1000 °C to room temperature. The fraction of pro-eutectoid α-ferrite in the above steel sample at room temperature is

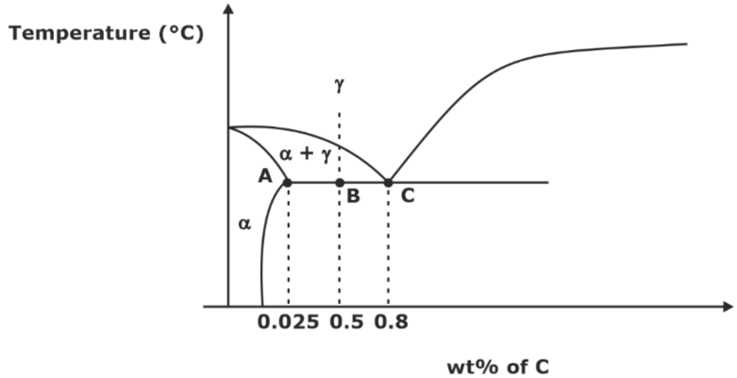
[MCQ: 2 Marks]

A. 0.387 B. 0.864

C. 0.475 D. 0.775

Ans. A

Sol. Given,



Mass Factor of pro eutectoid $\alpha = \frac{BC}{CA}$

$$= \frac{(0.8 - 0.5)}{(0.8 - 0.025)}$$

= 0.378

42. Activities A to K are required to complete a project. The time estimates and the immediate predecessors of these activities are given in the table. If the project is to be completed in the minimum possible time, the latest finish time for the activity G is ____________ hours.

Activity	Time(hours)	Immediate predecessors
A	2	–
B	3	–
C	2	–
D	4	A
E	5	B
F	4	B
G	3	C
H	10	D, E
I	5	F
J	8	G
K	3	H, I, J

[MCQ: 2 Marks]

A. 5 B. 10

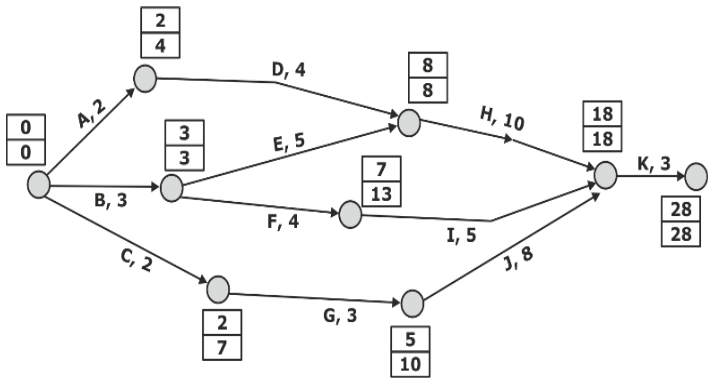
C. 8 D. 9

Ans. B

Sol. Given,

Activity A to K are required to complete a project.

Latest finish time for the activity G = 10



43. A solid spherical bead of lead (uniform density = 11000 kg/m^3) of diameter d = 0.1 mm sinks with a constant velocity V in a large stagnant pool of a liquid (dynamic viscosity = 1.1 × 10^{-3} kg-m^{-1}·s^{-1}). The coefficient of drag is given by c_D = 24/Re, where the Reynolds number (Re) is defined on the basis of the diameter of the bead. The drag force acting on the bead is expressed as $D = (c_D)(0.5\rho V^2)$ ($\pi d^2/4$), where ρ is the density of the liquid. Neglect the buoyancy force. Using g = 10 m/s^2, the velocity V is ___________ m/s.

[MCQ: 2 Marks]

A. 1/24 B. 1/6

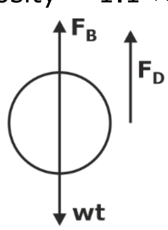
C. 1/18 D. 1/12

Ans. C

Sol. Given,

Diameter = d = 0.1 mm

Dynamic viscosity = 1.1 × 10^{-3}



$F_B + F_D$ = weight of bead

As neglected the buoyancy force

$$C_D \times \frac{1}{2}\rho A_1 V^2 = \rho_b g V_{body}$$

$$\frac{24}{\frac{\rho V d}{\mu}} \times \frac{1}{2}\rho A_p V^2 = \rho_b g \times \left(\frac{\pi}{6}d^3\right)$$

$$\frac{12\mu A_p V}{d} = \rho_b g \times \frac{\pi}{6}d^3$$

$$\frac{12\mu}{d} \times \frac{\pi}{4}d^2 \times V = \rho_b g \frac{\pi}{6}d^3$$

$$V = \frac{\rho_b g \times \pi d^3 \times 4 \times d}{6 \times 12\mu \times \pi d^2}$$

$$V = \frac{\rho_b g \times d^2 \times 4}{6 \times 12 \times \mu}$$

$$= \frac{11000 \times 10 \times (0.1 \times 10^{-3})^2 \times 4}{6 \times 12 \times 1.1 \times 10^{-3}}$$

$$V = \frac{1}{18}\ \text{m/sec}$$

44. Consider steady, one-dimensional compressible flow of a gas in a pipe of diameter 1 m. At one location in the pipe, the density and velocity are 1 kg/m^3 and 100 m/s, respectively. At a downstream location in the pipe, the velocity is 170 m/s. If the pressure drop between these two locations is 10 kPa, the force exerted by the gas on the pipe between these two locations is ____________ N.

[MCQ: 2 Marks]

A. 350π^2 B. 750π

C. 1000π D. 3000

Ans. B

Sol. Given,

By momentum equation in flow direction

$$(P_1A_1 - P_2A_2) - F = m(V_2 - V_1)$$

$$(P_1 - P_2)A - F = \rho_L A_1 V_1(V_2 - V_1)$$

$$(P_1 - P_2) - \frac{F}{A} = \rho_L V_1(V_2 - V_1)$$

$$10\times10^3 - \frac{F}{A} = 1\times100(170-100)$$

$$10\times10^3 - 100\times70 = \frac{F}{A}$$

$$F = \frac{\pi}{4}D^2\times3000$$

$$F = \frac{\pi}{4}\times(1)^2\times3000$$

Force exerted = 750π

45. Consider a rod of uniform thermal conductivity whose one end (x = 0) is insulated and the other end ($x = L$) is exposed to flow of air at temperature T_∞ with convective heat transfer coefficient h. The cylindrical surface of the rod is insulated so that the heat transfer is strictly along the axis of the rod. The rate of internal heat generation per unit volume inside the rod is given as $\dot{q} = \cos\frac{2\pi x}{L}$. The steady state temperature at the mid-location of the rod is given as T_A. What will be the temperature at the same location, if the convective heat transfer coefficient increases to 2h?

[MCQ: 2 Marks]

A. $T_A + \frac{\dot{q}L}{2h}$

B. $2T_A$

C. T_A

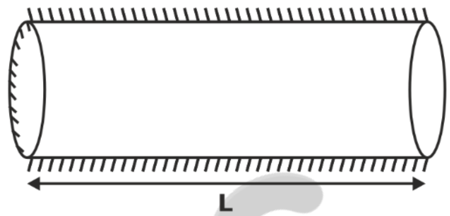
D. $T_A(1 - \frac{\dot{q}L}{4\pi h}) + \frac{\dot{q}L}{4\pi h}T_\infty$

Ans. C

Sol. Given,

Rate of internal heat generation per unit volume,

$$\dot{q} = \cos\frac{2\pi x}{L}$$



$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}_g}{k} = 0$$

$$\frac{\partial T}{\partial x} = -\frac{1}{k}\int\cos\frac{2\pi x}{L}dx$$

$$\frac{\partial T}{\partial x} = \frac{-\sin\frac{2\pi x}{L}}{\frac{2\pi k}{L}} + C_1$$

Boundary condition

1. at x = 0, $\frac{\partial T}{\partial x} = 0$

2. at x = L, $-kA\frac{\partial T}{\partial x}\Big|_{x=L} = hA\Delta T$

Using boundary condition (1)

$C_1 = 0$

Using Boundary condition (2)

$$-kA\frac{dT}{dx}\Big|_{x=L} = hA\Delta T$$

$$-kA\left[-\sin\frac{2\pi\times L}{L}\right] = h\Delta T$$

$$-kA\left[\frac{-\sin\frac{2\pi\times L}{L}}{\frac{2\times k}{L}}\right] = h\Delta T$$

$\Delta T = 0$

∴ since ΔT = 0 at the open surface so no convection is taking place.

So, h has no effect on calculating temperature of any point in the body.

46. The system of linear equations in real (x, y) given by

$$(x \;\; y)\begin{bmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{bmatrix} = (0 \;\; 0)$$

involves a real parameter α and has infinitely many non-trivial solutions for special value(s) of α. Which one or more among the following options is/are non-trivial solution(s) of (x, y) for such special value(s) of α?

[MSQ: 2 Marks]

A. x = 2, y = – 2

B. x = −1, y = 4

C. x = 1, y = 1

D. x = 4, y = – 2

Ans. A & B

Sol. For non-trivial solution

$|A| = 0$

$$\begin{vmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{vmatrix} = 0$$

$2 - (5 - 2\alpha)\,\alpha = 0$

$2 - 5\alpha + 2\alpha^2 = 0$

$2\alpha^2 - 5\alpha + 2 = 0$

$$\alpha = \frac{5 \pm \sqrt{25-16}}{2\times 2} = 2, \frac{1}{2}$$

Case-1:

when $\alpha = 2$

$$\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{array}{l} 2x + 2y = 0 \\ x + y = 0 \end{array}\right\} x + y = 0$$

Case-2:

When $\alpha = \frac{1}{2}$

$$\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 2 & 4 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x + \frac{1}{2}y = 0$$

$4x + y = 0$

47. Let a random variable X follow Poisson distribution such that

Prob (X = 1) = Prob (X= 2).

The value of Prob (X = 3) is ___________ (round off to 2 decimal places).

[NAT: 2 Marks]

Ans. 0.18

Sol. Range (0.17 to 0.19)

Given,

$$\text{Probability of poison distribution} P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

Prob (x = 1) = prob (x = 2)

$$\frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$2\lambda = \lambda^2$

$\lambda^2 - 2\lambda = 0$

$\lambda(\lambda - 2) = 0$

$\lambda = 0, \lambda = 2$

$$\text{Prob}(x=3) = \frac{e^{-\lambda}\,\lambda^x}{x!} = \frac{e^{-2}(2)^3}{3!} = 0.18$$

48. Consider two vectors:

$$\vec{a} = 5\hat{i} + 7\hat{j} + 2k$$

$$\vec{b} = 3\hat{i} - \hat{j} + 6k$$

Magnitude of the component of $\bar{a}$ orthogonal to $\bar{b}$ in the plane containing the vectors $\bar{a}$ and $\bar{b}$ is ___________ (round off to 2 decimal places).

[NAT: 2 Marks]

Ans. 8.32

Sol. Range (7.90 to 8.70)

Given,

$$\vec{a} = 5\hat{i} + 7\hat{j} + 2k$$

$$\vec{b} = 3\hat{i} - \hat{j} + 6k$$

the angle between $\vec{a}$ and $\vec{b}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{(15 - 7 + 12)}{\sqrt{5^2 + 7^2 + 2^2}\cdot\sqrt{3^2 + 1^2 + 6^2}}$$

$\cos\theta = 0.333$

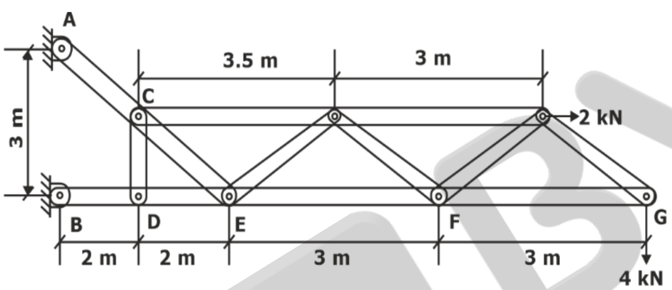
$\theta = 70.5°$

Magnitude of $\vec{a}$ orthogonal to $\vec{b}$ in plane of $\vec{a}$ and $\vec{b}$

= |a| sinθ

$= \sqrt{5^2 + 7^2 + 2^2} \times \sin 70.5^\circ$ = 8.32

49. A structure, along with the loads applied on it, is shown in the figure. Self-weight of all the members is negligible and all the pin joints are friction-less. AE is a single member that contains pin C. Likewise, BE is a single member that contains pin D. Members GI and FH are overlapping rigid members. The magnitude of the force carried by member CI is ________ kN (in integer).



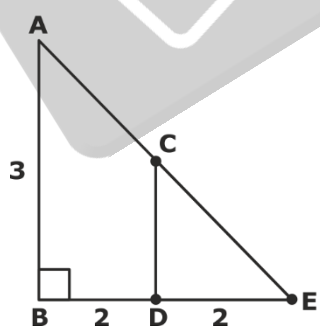
[NAT: 2 Marks]

Ans. 18

Sol. Range (18 to 18)

Apply method section,

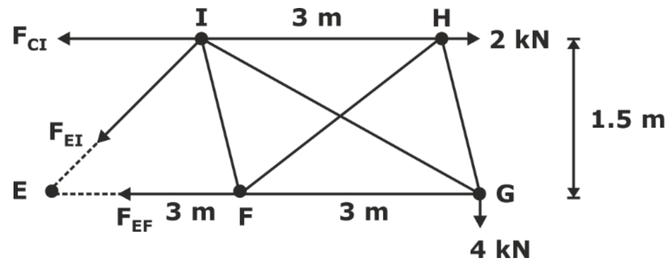
First find the length of member CD:



$$\frac{AB}{BE} = \frac{CD}{DE}$$

$$\frac{3}{4} = \frac{CD}{2}$$

CD = 1.5 m

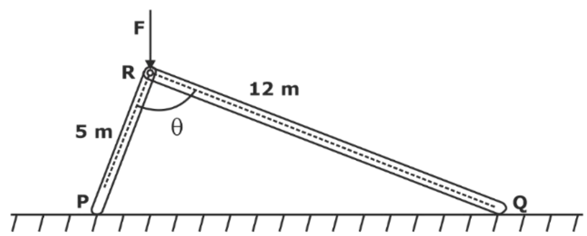


$M_E = 0$ (for equilibrium truss)

$F_{CI} \times 1.5 = 2 \times 1.5 + 4 \times 6$

$F_{CI} = 18$ kN

50. Two rigid massless rods PR and RQ are joined at frictionless pin-joint R and are resting on ground at P and Q, respectively, as shown in the figure. A vertical force F acts on the pin R as shown. When the included angle $\theta < 90°$, the rods remain in static equilibrium due to Coulomb friction between the rods and ground at locations P and Q. At $\theta = 90°$, impending slip occurs simultaneously at points P and Q. Then the ratio of the coefficient of friction at Q to that at P (μ_Q/μ_P) is __________ (round off to two decimal places).

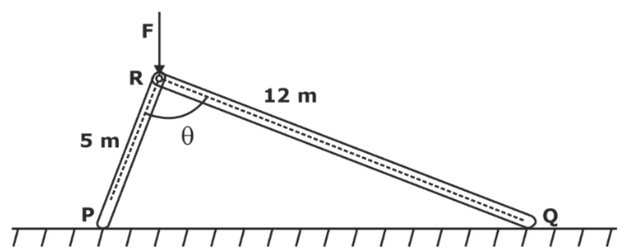


[NAT: 2 Marks]

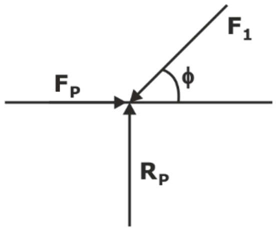
Ans. 5.76

Sol. Range (5.70 to 5.80)

At $\theta = 90°$,



At point P



$R_P = F_1 \sin\phi$

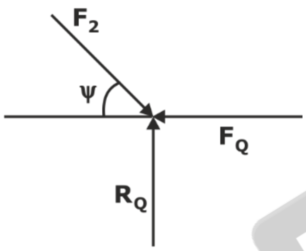
$F_P = F_1 \cos\phi$

$\mu_p R_P = F_1 \cos\phi$

$\mu_p F_1 \sin\phi = F_1 \cos\phi$

$\mu_p = \dfrac{1}{\tan\phi}$

At point Q



$R_Q = F_2 \sin\psi$

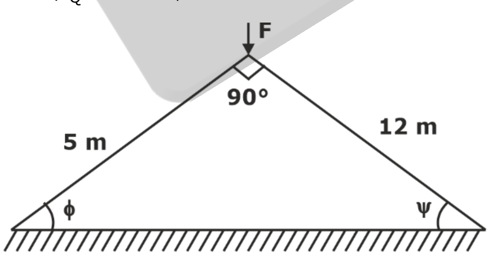
$F_Q = F_2 \cos\psi$

$\mu_Q R_Q = F_2 \cos\psi$

$\mu_Q F_2 \sin\psi = F_2 \cos\psi$

$\mu_Q = \dfrac{1}{\tan\psi}$

$\dfrac{\mu_P}{\mu_Q} = \dfrac{\tan\psi}{\tan\phi}$

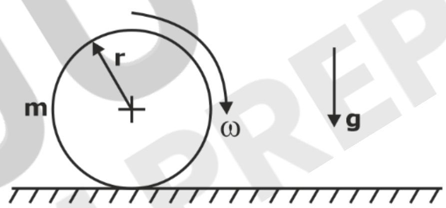


$\tan\phi = \dfrac{12}{5}$

$\tan\psi = \dfrac{5}{12}$

$\dfrac{\mu_Q}{\mu_P} = \dfrac{\tan\phi}{\tan\psi} = 5.76$

51. A cylindrical disc of mass m = 1 kg and radius r = 0.15 m was spinning at ω = 5 rad/s when it was placed on a flat horizontal surface and released (refer to the figure). Gravity g acts vertically downwards as shown in the figure. The coefficient of friction between the disc and the surface is finite and positive. Disregarding any other dissipation except that due to friction between the disc and the surface, the horizontal velocity of the center of the disc, when it starts rolling without slipping, will be __________ m/s (round off to 2 decimal places).

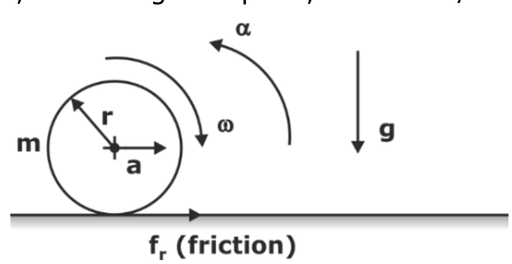


[NAT: 2 Marks]

Ans. 0.25

Sol. Range: (0.24 to 0.26)

Given : Cylindrical disc, m = 1 kg, r = 0.15 m, initial angular speed, ω = 5 rad/s



$\Sigma F_x = ma$

$\Rightarrow \quad f_r = ma \qquad \text{...(i)}$

$\Sigma\, \text{Torque} = I\alpha$

$f_r \times r = I\alpha \qquad \left(\because I = \dfrac{mr^2}{2}\right)$

$\Rightarrow \quad f_r = \dfrac{mr\alpha}{2} \qquad \text{...(ii)}$

From equation (i) and (ii)

$$ma = \frac{mr\alpha}{2}$$

$$a = \frac{r\alpha}{2}$$

When rolling starts

$$v = \omega' r$$

$$v = u + at \text{ and } \omega' = \omega - \alpha t$$

$$\Rightarrow \quad v = 0 + at = r(\omega - \alpha t) \quad \text{...(iii)}$$

$$at = r\omega - r\alpha t$$

$$\frac{r\alpha t}{2} = r\omega - r\alpha t$$

$$\Rightarrow \quad \alpha t = \frac{2}{3}\omega$$

From equation (iii),

$$v = r(\omega - \alpha t) = r\left(\omega - \frac{2}{3}\omega\right)$$

$$= \frac{r\omega}{3} = \frac{0.15 \times 5}{3} = 0.25 \text{ m/s}$$

52. A thin-walled cylindrical pressure vessel has mean wall thickness of t and nominal radius of r. The Poisson's ratio of the wall material is 1/3. When it was subjected to some internal pressure, its nominal perimeter in the cylindrical portion increased by 0.1% and the corresponding wall thickness became $\bar{t}$. The corresponding change in the wall thickness of the cylindrical portion, i.e. 100 × $(\bar{t} - t)/t$, is ______%. (round off to 3 decimal places).

[NAT: 2 Marks]

Ans. –0.06

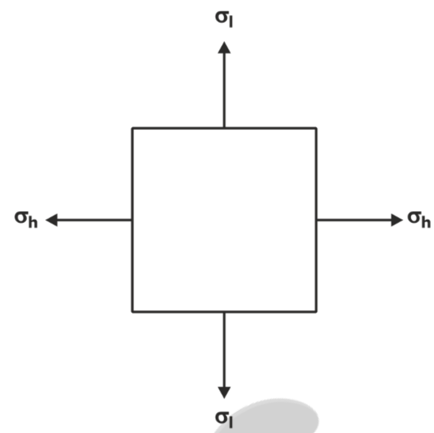
Sol. Range (–0.07 to –0.05)

Given,

Poisson's ratio of the wall material $\mu = \frac{1}{2}$

Circumferential strain = 0.1%,

Thin walled cylindrical pressure vessel,



Circumferential strain $\varepsilon_c = \frac{\sigma_h}{E} - \frac{\mu\sigma_L}{E} = \frac{0.1}{100}$

$$\sigma_h = \frac{Pr}{2t}, \quad \sigma_l = \frac{\sigma_h}{z}$$

$$\varepsilon_c = \frac{Pr}{2tE}(2 - \mu) = \frac{0.1}{100} \quad \text{...(i)}$$

Radial strain $= \varepsilon_r = \frac{\delta t}{t} = \frac{\sigma_r}{E} - \frac{\mu\sigma_h}{E} - \frac{\mu\sigma_L}{E}$

$$\frac{\delta t}{t} = \frac{1}{E}[0 - \mu(\sigma_h + \sigma_L)]$$

$$\frac{\delta t}{t} = \frac{1}{E}\left[-\mu \times \frac{3\sigma_h}{2}\right] == \frac{-3\mu Pr}{2tE}$$

$$\frac{\delta t}{t} = \frac{-3\mu Pr}{2tE} \quad \text{...(ii)}$$

From equation (i)

$$\frac{Pr}{2tE}(2 - \mu) = 0.001$$

$$\frac{Pr}{2tE} = \frac{0.001}{2 - \mu} = \frac{0.001}{2 - \frac{1}{3}} = 6 \times 10^{-4}$$

Put value in equation (ii)

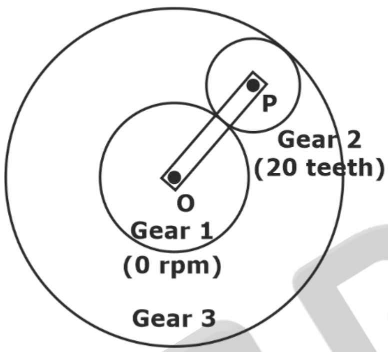
$$\frac{\delta t}{t} = -3\mu\left[\frac{Pr}{2tE}\right]$$

$$\frac{\delta t}{t} = -3 \times \frac{1}{3} \times 6 \times 10^{-4}$$

$$\frac{\delta t}{t} = -0.06\%$$

$$\left(\frac{\bar{t} - t}{t}\right) \times 100 = -0.06\%$$

53. A schematic of an epicyclic gear train is shown in the figure. The sun (gear 1) and planet (gear 2) are external, and the ring gear (gear 3) is internal. Gear 1, gear 3 and arm OP are pivoted to the ground at O. Gear 2 is carried on the arm OP via the pivot joint at P, and is in mesh with the other two gears. Gear 2 has 20 teeth and gear 3 has 80 teeth. If gear 1 is kept fixed at 0 rpm and gear 3 rotates at 900 rpm counter clockwise (ccw), the magnitude of angular velocity of arm OP is ________ rpm (in integer).



[NAT: 2 Marks]

Ans. 600

Sol. Range (600 to 600)

Given,

Module of all Gear same

$T_1 + 2T_2 = T_3$

$T_1 + 2 \times 20 = 80$

$T_1 = 40$

Arm (OP)	Gear 1 (40)	Gear 2 (20)	Gear 3 (80)
O	$+x$	$-x\dfrac{40}{20}$	$-x\dfrac{40}{20}\times\dfrac{20}{80}$
y	$y + x$	$y - 2x$	$y-\dfrac{x}{2}$

If gear 1 is kept fixed = 0 rpm

$y + x = 0$

$y = -x$

Given N_3 = 900 (ccw)

$y-\dfrac{x}{2} = -900$ rpm taking clockwise positive.

$y+\dfrac{y}{2} = -900$

$\dfrac{3y}{2} = -900$

$y = -600$ (ccw)

Here magnitude of angular velocity of arm OP = 600 rpm.

54. Under orthogonal cutting condition, a turning operation is carried out on a metallic workpiece at a cutting speed of 4 m/s. The orthogonal rake angle of the cutting tool is 5°. The uncut chip thickness and width of cut are 0.2 mm and 3 mm, respectively. In this turning operation, the resulting friction angle and shear angle are 45º and 25°, respectively. If the dynamic yield shear strength of the workpiece material under this cutting condition is 1000 MPa, then the cutting force is ______________ N (round off to one decimal place).

[NAT: 2 Marks]

Ans. 2573.40

Sol. Range (2570 to 2576)

Given,

Orthogonal cutting condition,

Cutting speed = 4 m/sec

Rake angle = α = 5°

Uncut chip thickness = 0.2 mm

Width of cut = 3 mm

Friction angle = β = 45°

Sheal angle = α = 25°

Yield shear strength = 1000 MPa,

From merchant diagram,

Cutting force F_C = R cos (β – α)

Shear force F_s = R cos (φ + β – α)

$$\text{Shear force } F_s = \frac{\tau \times bt}{\sin\phi}$$

$$= \frac{1000 \times 0.2 \times 3}{\sin 25^\circ} = 1419.72 \text{ N}$$

$$\frac{F_C}{F_s} = \frac{\cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)}$$

$$\frac{F_C}{1419.72} = \frac{\cos(45-5)}{\cos(25+45-5)}$$

Cutting force F_C = 2573.40 N

55. A 1 mm thick cylindrical tube, 100 mm in diameter, is orthogonally turned such that the entire wall thickness of the tube is cut in a single pass. The axial feed of the tool is 1 m/minute and the specific cutting energy (u) of the tube material is 6 J/mm^3. Neglect contribution of feed force towards power. The power required to carry out this operation is __________ kW (round off to one decimal place).

[NAT: 2 Marks]

Ans. 31.416

Sol. Range (31.20 to 31.60)

Given,

Thickness of cylindrical tube = 1mm,

Mean diameter of tube = 100mm,

axial feed of the tool = 1 m/minute,

specific cutting energy (u) = 6 J/mm^3,

Power required = specific cutting energy × material removal rate

$$= 4 \times (2\pi r) t \times \text{axial feed}$$

$$= 4 \times \pi D t \times f$$

$$= 6 \times \pi \times 100 \times 1 \times \frac{1000}{60}$$

= 31.416 kW

56. A 4 mm thick aluminum sheet of width w = 100 mm is rolled in a two-roll mill of roll diameter 200 mm each. The workpiece is lubricated with a mineral oil, which gives a coefficient of friction, μ = 0.1. The flow stress (σ) of the material in MPa is $\sigma = 207 + 414\,\varepsilon$, where ε is the true strain. Assuming rolling to be a plane strain deformation process, the roll separation force (F) for maximum permissible draft (thickness reduction) is __________ kN (round off to the nearest integer).

Use: $F = 1.15\bar{\sigma}\left(1 + \frac{\mu L}{2\bar{h}}\right)wL$, where $\bar{\sigma}$ is average flow stress, L is roll-workpiece contact length, and $\bar{h}$ is the average sheet thickness.

[NAT: 2 Marks]

Ans. 351

Sol. Range (340 to 360)

Given,

Width W = 100 mm

Roll diameter D = 200 mm

Coefficient of friction μ = 0.1

Flow stress σ = 207 + 414 ε

h_o = 4 mm

$$\Delta h_{max} = \mu^2 R$$

$$h_o - h_f = 0.1^2 \times 100$$

$4 - h_f = 1$

h_f = 3 mm

Contact length, $L = \sqrt{R\Delta h}$

$$L = \sqrt{100 \times 1} = 10 \text{ mm}$$

$\sigma_f = 207 + 414\,\varepsilon$

$$\sigma_{avg.} = \frac{1}{\varepsilon}\int_0^{\varepsilon} \sigma_f \, d\varepsilon$$

$$= \frac{1}{\varepsilon}\int_0^{2} (207 + 414\varepsilon)\, d\varepsilon$$

$$= \frac{1}{\varepsilon}\left(207\varepsilon + \frac{414\varepsilon^2}{2}\right)_0^{\varepsilon}$$

$$= \frac{1}{\varepsilon}\left(207\varepsilon + \frac{414}{2}\varepsilon^2\right)$$

$$= 207 + 207\varepsilon$$

$$\sigma_{avg} = 207(1+\varepsilon)$$

true strain, $\varepsilon = \ln\left(\frac{h_0}{h_f}\right) = \ln\left(\frac{4}{3}\right) = 0.2876$

$$\sigma_{avg} = 207(1+0.2876) = 266.53 \text{ MPa}$$

$$F = 1.15\bar{\sigma}\left(1 + \frac{\mu L}{2\bar{h}}\right)wL$$

$$F = 1.15\bar{\sigma}\left(1 + \frac{0.1\times 10}{2\times\left(\frac{4+3}{2}\right)}\right)\times 100\times 10$$

$$= 1.15\times 266.53\left(1 + \frac{0.1\times 10}{2\times\left(\frac{4+3}{2}\right)}\right)\times 100\times 10$$

F = 350.3 kN

57. Two mild steel plates of similar thickness, in butt-joint configuration, are welded by gas tungsten arc welding process using the following welding parameters.

Welding voltage	20V
Welding current	150 A
Welding speed	5 mm/s

A filler wire of the same mild steel material having 3 mm diameter is used in this welding process. The filler wire feed rate is selected such that the final weld bead is composed of 60% volume of filler and 40% volume of plate material. The heat required to melt the mild steel material is 10 J/mm^3. The heat transfer factor is 0.7 and melting factor is 0.6. The feed rate of the filler wire is ___________ mm/s (round off to one decimal place).

[NAT: 2 Marks]

Ans. 10.695

Sol. Range (10.50 to 10.90)

Given,

Welding voltage = 20 V,

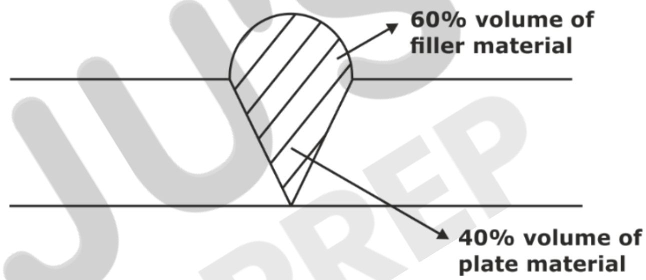
Welding current = 150 A,

Welding speed = 5 mm/s,

heat required to melt the mild steel material =10 J/mm^3,

heat transfer factor = 0.7,

melting factor =0.6,



Average power input $= \eta_{H.T.} \times V \times I$

= 0.7 × 20 × 150 = 2100 J

Melting efficiency $= \frac{H_m}{\frac{\eta_H VI}{A_b V_b}} = \frac{10}{\frac{2100}{A_b V_b}}$

Where, V_b = welding speed

$$0.6 = \frac{10}{2100}\times A_b V_b$$

$$A_b V_b = \frac{0.6\times 2100}{10} = 126$$

In question given final weld bead is composed of 60% of volume of filles so

$A_b V_b$ × 0.6 = deposition of filler

$$126\times 0.6 = \frac{\pi}{4}d^2 \times f$$

$$126\times 0.6 = \frac{\pi}{4}\times (3)^2 \times f$$

$$f = 10.695 \text{ mm/sec}$$

58. An assignment problem is solved to minimize the total processing time of four jobs (1, 2, 3 and 4) on four different machines such that each job is processed exactly by one machine and each machine processes exactly one job. The minimum total processing time is found to be 500 minutes. Due to a change in design, the processing time of Job 4 on each machine has increased by 20 minutes. The revised minimum total processing time will be _____________ minutes (in integer).

[NAT: 2 Marks]

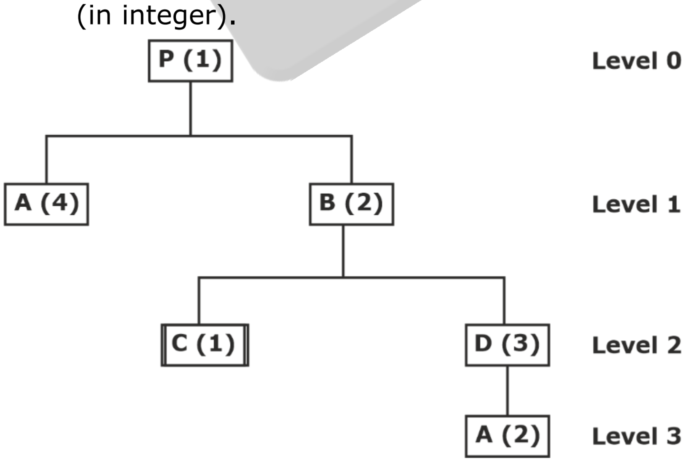
Ans. 520

Sol. Range (520 to 520)

The minimum processing time for an assignment problem is given 500 minutes. Due to change in design, the time of one job increases by 20 minutes, then the total minimum time would also increased by 20 minutes.

Hence, new processing time for assignment problem is 500 + 20 = 520 minutes

59. The product structure diagram shows the number of different components required at each level to produce one unit of the final product P. If there are 50 units of on hand inventory of component A, the number of additional units of component A needed to produce 10 units of product P is __________ (in integer).



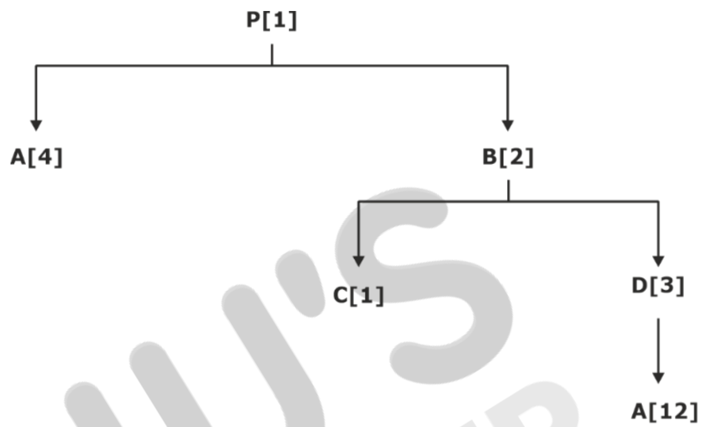
[NAT: 2 Marks]

Ans. 110

Sol. Range (110 to 110)

Given,

On hand inventory of component A = 50 units,



Number of unit required for A

$= 1 \times 4 \times 10 + (2 \times 3 \times 2 \times 10) - 50$

$= 110$

60. Consider a one-dimensional steady heat conduction process through a solid slab of thickness 0.1 m. The higher temperature side A has a surface temperature of 80 °C, and the heat transfer rate per unit area to low temperature side B is 4.5 kW/m2. The thermal conductivity of the slab is 15 W/m.K. The rate of entropy generation per unit area during the heat transfer process is _________ W/m^2.K (round off to 2 decimal places).

[NAT: 2 Marks]

Ans. 1.184

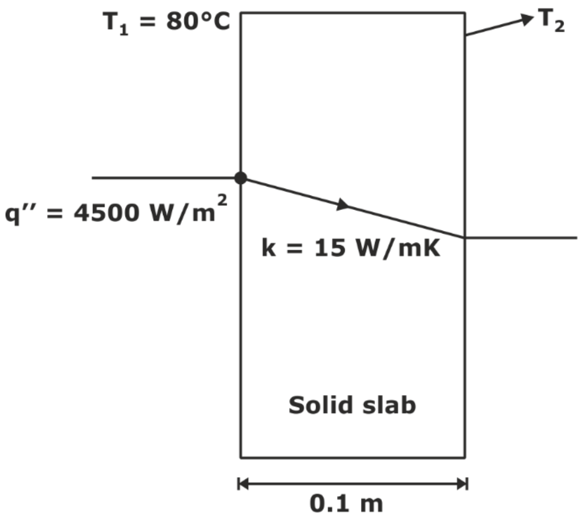
Sol. Range (1.12 to 1.24)

Given,

solid slab of thickness = 0.1 m,

higher side temperature T_1=80°C,

heat transfer rate per unit area =45 kW/m^2



Fourier's law of heat conduction

$$q'' = \frac{k(T_1 - T_2)}{x}$$

$$4500 = \frac{15 \times (80 - T_2)}{0.1}$$

$$T_2 = 50°C = 323\text{ K}$$

Steady state entropy change

$$\left(\frac{ds}{dt}\right)_{C.V.} = \dot{m}_i s_i - \dot{m}_e s_e + \frac{\delta \dot{Q}}{T} + (\delta \dot{s})_{gen.}$$

$$0 = 0 + 0 + \frac{\dot{Q}}{T_h} - \frac{\dot{Q}}{T_C} + (\delta \dot{s})_{gen}$$

$$0 = \frac{4500}{353} - \frac{4500}{323} + (\delta \dot{s})_{gen}$$

$$(\delta \dot{s})_{gen} = \left(\frac{4500}{323}\right) - \left(\frac{4500}{353}\right) = 1.184\text{ W/m}^2\text{K}$$

61. In a steam power plant based on Rankine cycle, steam is initially expanded in a high-pressure turbine. The steam is then reheated in a reheater and finally expanded in a low-pressure turbine. The expansion work in the high-pressure turbine is 400 kJ/kg and in the low-pressure turbine is 850 kJ/kg, whereas the pump work is 15 kJ/kg. If the cycle efficiency is 32%, the heat rejected in the condenser is _________ kJ/kg (round off to 2 decimal places).

[NAT: 2 Marks]

Ans. 2624.37

Sol. Range (2620 to 2630)

Given,

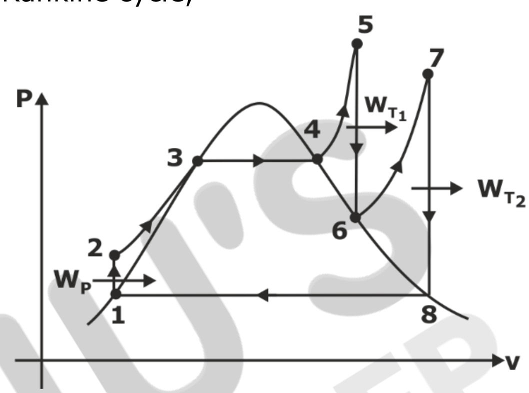
High pressure turbine work = 400 kJ/kg

low pressure turbine work = 850 kJ/kg

Pump work = 15 kJ/kg

Cycle efficiency = 32%

Rankine cycle,



$$\eta = \frac{\text{Net Work done}}{\text{Heat supplied}} = \frac{W_{net}}{Q_s} = \frac{W_{T_1} + W_{T_2} - W_P}{Q_s}$$

$$0.32 = \frac{400 + 850 - 15}{Q_s}$$

$$Q_s = 3859.37\text{ kJ/kg}$$

$$Q_s = W_{net} + Q_R$$

$$3859.37 = 1235 + Q_R$$

$$Q_R = 2624.37\text{ kJ/kg}$$

62. An engine running on an air standard Otto cycle has a displacement volume 250 cm^3 and a clearance volume 35.7 cm^3. The pressure and temperature at the beginning of the compression process are 100 kPa and 300 K, respectively. Heat transfer during constant-volume heat addition process is 800 kJ/kg. The specific heat at constant volume is 0.718 kJ/kg.K and the ratio of specific heats at constant pressure and constant volume is 1.4. Assume the specific heats to remain constant during the cycle. The maximum pressure in the cycle is _______ kPa (round off to the nearest integer).

[NAT: 2 Marks]

Ans. 4811

Sol. Range (4780 to 4825)

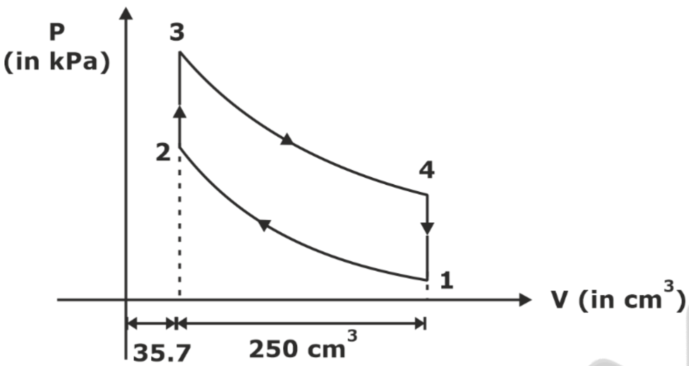
Given,

Air standard Otto cycle,

displacement volume = 250 cm^3

clearance volume =35.7 cm^3

Constant-volume heat addition process = 800 kJ/kg,



Compression ratio is defined as the ratio of volume before compression to that of volume after compression.

$$CR = r = \frac{V_d + V_c}{V_c} = \frac{250 + 35.7}{35.7} = 8$$

Heat supplied = $C_V (T_3 - T_2)$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 (r)^{\gamma-1}$$

$$= 300(8)^{0.4} = 689.31\ K$$

T_2 = 689.31 K

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_2 = P_2 \left(\frac{V_1}{V_2}\right)^{\gamma} = 100 \times (r)^{1.4} = 100 \times (8)^{1.4}$$

$$P_2 = 1838.81\ kPa$$

800 = 0.718 × (T_3 – 689.31)

T_3 = 1803.51 K

At 2 – 3 constant volume process

$P \propto T$

$$\frac{P_2}{T_2} = \frac{P_3}{T_3}$$

$$\frac{1838.81}{689.31} = \frac{P_3}{1803.51}$$

P_3 = 4811.060 kPa

63. A steady two-dimensional flow field is specified by the stream function

$$\Psi = kx^3y$$

where x and y are in meter and the constant k = 1 $m^{-2}s^{-1}$. The magnitude of acceleration at a point (x, y) = (1 m, 1 m) is ________ m/s^2 (round off to 2 decimal places).

[NAT: 2 Marks]

Ans. 4.242

Sol. Range (4.20 to 4.28)

Given,

Stream function y = kx^3y

k = 1 $m^{-2}s^{-1}$,

$$u = -\frac{\partial\psi}{\partial y} = -x^3,\ v = \frac{\partial\psi}{\partial x} = 3x^2y$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$= -x^3(3x^2) + 3x^2y(0)$

$$a_x = 3x^5 = 3\ m/sec$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$= -x^3(6xy) + 3x^2y(3x^2)$

$a_y = -6x^4y + 3x^4y = -6 + 3 = -3$

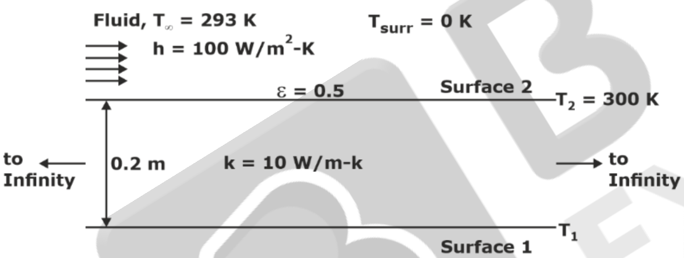
$$a_{net} = \sqrt{a_x^2 + a_y^2} = \sqrt{(3)^2 + (3)^2} = 4.242$$

magnitude of acceleration at a point (x, y)

$$\vec{a} = 4.242\ m/s^2$$

64. Consider a solid slab (thermal conductivity, k = 10 W·m^{-1}·K^{-1}) with thickness 0.2 m and of infinite extent in the other two directions as shown in the figure. Surface 2, at 300 K, is exposed to a fluid flow at a free stream temperature (T_∞) of 293 K, with a convective heat transfer coefficient (h) of 100 W·m^{-2}·K^{-1}. Surface 2 is opaque, diffuse and gray with an emissivity (ε) of 0.5 and exchanges heat by radiation with very large surroundings at 0 K. Radiative heat transfer inside the solid slab is neglected. The Stefan-Boltzmann constant is 5.67 × 10^{-8} W·m^{-2}·K^{-4}. The temperature T_1 of Surface 1 of the slab, under steady-state conditions, is __________ K (round off to the nearest integer).

[NAT: 2 Marks]



Ans. 318.6

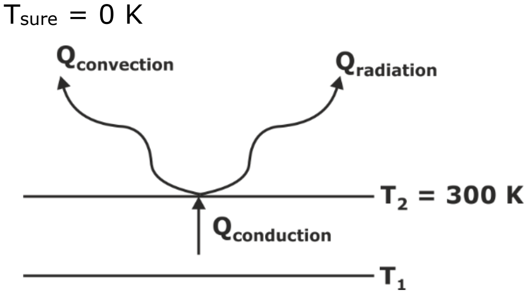
Sol. Range (315 to 320)

Given,

Free stream temperature (T_∞) = 293 K,

Very large surrounding temperature

T_{sure} = 0 K



Under steady state,

$Q_{conduction} = Q_{convection} + Q_{radiation}$

$$\frac{kA(T_1 - T_2)}{x} = hA(T_2 - T_\infty) + \sigma\varepsilon A(T_2^4 - T_{surr}^4)$$

$$\frac{10 \times (T_1 - 300)}{0.2}$$

$$= 100(300 - 293) + 0.5 \times 5.67 \times 10^{-8}$$

$$\times (300^4 - 0^4)$$

$$T_1 = 318.592 \text{ K}$$

65. During open-heart surgery, a patient's blood is cooled down to 25°C from 37°C using a concentric tube counter-flow heat exchanger. Water enters the heat exchanger at 4°C and leaves at 18 °C. Blood flow rate during the surgery is 5 L/minute.

Use the following fluid properties:

Fluid	Density (kg/m^3)	Specific heat (J/kg-K)
Blood	1050	3740
Water	1000	4200

Effectiveness of the heat exchanger is __________ (round off to 2 decimal places).

[NAT: 2 Marks]

Ans. 0.424

Sol. Range (0.40 to 0.44)

Given,

Concentric tube counter-flow heat exchanger,

Hot fluid (blood),

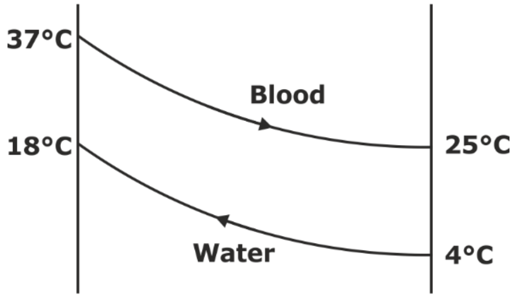
patient's blood is cooled down from 37°C to 25°C,

Blood flow rate during the surgery

= 5 L/minute,

Cold fluid (water),

Water enters the heat exchanger at 4°C and leaves at 18°C,



By applying energy balance:

$C_h\,(37 - 25) = C_C\,(18 - 4)$

$C_h \times 12 = C_C \times 14$

$$\frac{C_C}{C_n} = \frac{4}{12} = \frac{1}{3}$$

Cold fluid having minimum heat capacity.

$$\text{Effectiveness} = \frac{Q_{actual}}{Q_{max}} = \frac{C_C(T_{C_o} - T_{C_i})}{C_{min}(T_{h_i} - T_{C_i})}$$

$$= \frac{(18-4)}{(37-4)} = \frac{14}{33} = 0.4242$$

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