

GATE 2022

CS & IT Engineering

► Questions with
Detailed Solutions

1. The _____ is too high for it to be considered _____.
- A. fair / fare B. faer / fair
C. fare / fare D. fare / fair

[MCQ: 1 Mark]

Ans. D

Sol. • Fare means the money paid for a journey on public transport.

- Fair means treating people equally.

2. A function $y(x)$ is defined in the interval $[0, 1]$ on the x -axis as

$$y(x) = \begin{cases} 2 & \text{if } 0 \leq x < \frac{1}{3} \\ 3 & \text{if } \frac{1}{3} \leq x < \frac{3}{4} \\ 1 & \text{if } \frac{3}{4} \leq x \leq 1 \end{cases}$$

Which one of the following is the area under the curve for the interval $[0, 1]$ on the x -axis?

- A. $\frac{5}{6}$ B. $\frac{6}{5}$
C. $\frac{13}{6}$ D. $\frac{6}{13}$

[MCQ: 1 Mark]

Ans. C

Sol. Total Area \Rightarrow

$$\begin{aligned} &= \left\{ \frac{1}{3} * 2 \right\} + \left\{ \left(\frac{3}{4} - \frac{1}{3} \right) * 3 \right\} + \left\{ \left(1 - \frac{3}{4} \right) * 1 \right\} \\ &= \frac{2}{3} + \left\{ \frac{5}{12} * 3 \right\} + \frac{1}{4} \\ &= \frac{2}{3} + \frac{5}{4} + \frac{1}{4} \\ &= \frac{2}{3} + \frac{6}{4} \\ &= \frac{13}{6} \end{aligned}$$

3. Let r be a root of the equation $x^2 + 2x + 6 = 0$. Then the value of the expression $(r + 2)(r + 3)(r + 4)(r + 5)$ is

- A. 51 B. -51
C. 126 D. -126

[MCQ: 1 Mark]

Ans. D

Sol. $x^2 + 2x + 6 = 0$

$$x^2 + 2x = -6$$

Now when we solve $(r + 2)(r + 3)(r + 4)(r + 5)$ we get.

$$\begin{aligned} &\Rightarrow (r^2 + 2r + 4r + 8)(r^2 + 3r + 5r + 15) \\ &\Rightarrow (4r + 2)(6r + 9) \\ &\Rightarrow 6\{4(r^2 + 2r) + 3\} \\ &\Rightarrow 6(-24 + 3) \\ &\Rightarrow 6(-21) \\ &\Rightarrow -126. \end{aligned}$$

4. Given below are four statements.

Statement 1: All students are inquisitive.

Statement 2: Some students are inquisitive.

Statement 3: No student is inquisitive.

Statement 4: Some students are not inquisitive.

From the given four statements, find the two statements that **CANNOT BE TRUE** simultaneously, assuming that there is at least one student in the class.

- A. Statement 1 and Statement 3
B. Statement 1 and Statement 2
C. Statement 2 and Statement 4
D. Statement 3 and Statement 4

[MCQ: 1 Mark]

Ans. A

Sol. Statement 1, says that all students are inquisitive while statement B says that no student is inquisitive, these two statements cannot be true simultaneously.

5. A palindrome is a word that reads the same forwards and backwards. In a game of words, a player has the following two plates painted with letters.



From the additional plates given in the options, which one of the combinations of additional plates would allow the player to construct a five-letter palindrome.

The player should use all the five plates exactly once. The plates can be rotated in their plane.

- A.
- B.
- C.
- D.

[MCQ: 1 Mark]

Ans. B

Sol. As per the question we have to make a five letter palindrome so the correct answer will be.



After rotating the plates we get:



6. Some people believe that "what gets measured, improves". Some others believe that "what gets measured, gets gamed". One possible reason for the difference in the beliefs is the work culture in organizations. In organizations with good work culture, metrics help improve outcomes. However, the same

metrics are counterproductive in organizations with poor work culture.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

- A. Metrics are useful in organizations with poor work culture
- B. Metrics are useful in organizations with good work culture
- C. Metrics are always counterproductive in organizations with good work culture
- D. Metrics are never useful in organizations with good work culture

[MCQ: 2 Marks]

Ans. B

Sol. In the passage it is stated that in organizations with good work culture, metrics help improve outcomes' so we can clearly say that option B supports the passage.

7. In a recently conducted national entrance test, boys constituted 65% of those who appeared for the test. Girls constituted the remaining candidates and they accounted for 60% of the qualified candidates.

Which one of the following is the correct logical inference based on the information provided in the above passage?

- A. Equal number of boys and girls qualified
- B. Equal number of boys and girls appeared for the test
- C. The number of boys who appeared for the test is less than the number of girls who appeared
- D. The number of boys who qualified the test is less than the number of girls who qualified

[MCQ: 2 Marks]

Ans. D

Sol. Suppose the number of students be 100 Now as per the given Condition:

Number of boys who appeared for test =65

Number of girls who appeared for test
=100 – 65 = 35.

Now if 60% of the qualified students are girls then rest 40% are boys.

- 8.** A box contains five balls of same size and shape. Three of them are green coloured balls and two of them are orange coloured balls. Balls are drawn from the box one at a time. If a green ball is drawn, it is not replaced. If an orange ball is drawn, it is replaced with another orange ball.

First ball is drawn. What is the probability of getting an orange ball in the next draw?

- A. $\frac{1}{2}$ B. $\frac{8}{25}$
C. $\frac{19}{50}$ D. $\frac{23}{50}$

[MCQ: 2 Marks]

Ans. D

Sol. Case 1: If first ball drawn is green then, while drawing second ball (orange), we will be left with 4 balls

$$\frac{3}{5} + \frac{2}{4} = \frac{3}{10}$$

Case 2: If the first ball drawn is orange, then, in second draw we still have 5 balls as ball can be replaced.

$$\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

so total probability will be:

$$\frac{3}{10} + \frac{4}{25} = \frac{23}{50}$$

- 9.** The corners and mid-points of the sides of a triangle are named using the distinct letters

P, Q, R, S, T and U, but not necessarily in the same order. Consider the following statements:

- The line joining P and R is parallel to the line joining Q and S.
- P is placed on the side opposite to the corner T.
- S and U cannot be placed on the same side.

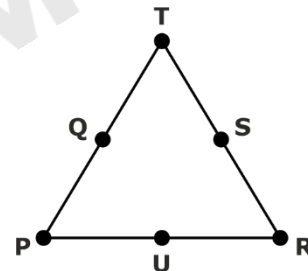
Which one of the following statements is correct based on the above information?

- A. P cannot be placed at a corner
B. S cannot be placed at a corner
C. U cannot be placed at a mid-point
D. R cannot be placed at a corner

[MCQ: 2 Marks]

Ans. B

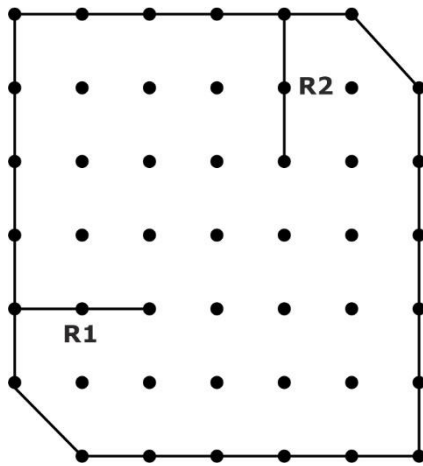
Sol. When we draw the diagram based on the given condition, we get



This clearly depicts that S cannot be on any of the corners. However, there are more possible arrangements but in all those arrangements S cannot be placed on the corners according to the criteria provided in the question.

- 10.** A plot of land must be divided between four families. They want their individual plots to be similar in shape, not necessarily equal in area. The land has equally spaced poles, marked as dots in the below figure. Two ropes, R1 and R2, are already present and cannot be moved.

What is the least number of **additional** straight ropes needed to create the desired plots? A single rope can pass through three poles that are aligned in a straight line.

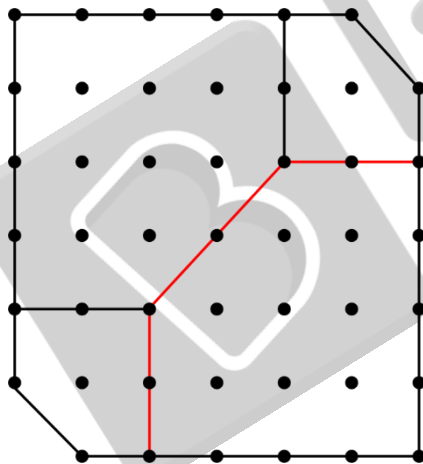


- A. 2
B. 4
C. 5
D. 3

[MCQ: 2 Marks]

Ans. D

Sol.



We need only 3 lines, as it is clearly mentioned in the question that shape should be similar and area should not be equal.

11. Which one of the following statements is TRUE for all positive functions $f(n)$?
- A. $f(n^2) = \theta(f(n)^2)$, when $f(n)$ is a polynomial
- B. $f(n^2) = o(f(n)^2)$
- C. $f(n^2) = O(f(n)^2)$, when $f(n)$ is an exponential function

D. $f(n^2) = \Omega(f(n)^2)$

[MCQ: 1 Mark]

Ans. A

Sol. Option A always holds good because if we square the input variable, then the highest order in the polynomial will also get squared.

Ex: $f(n) = n^{\text{constant}}$.

Option B is false as it is not true when $f(n)$ is a polynomial function

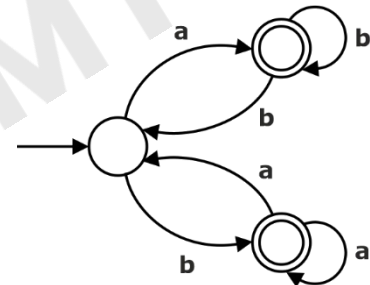
Ex: $n^2 = o(n^2)$ is false.

Option C is false as an exponential function may be increasing or decreasing, so this condition may not always be true.

Option D is false as it is not true for a decreasing function.

Thus, A is correct answer.

12. Which one of the following regular expressions correctly represents the language of the finite automaton given below?

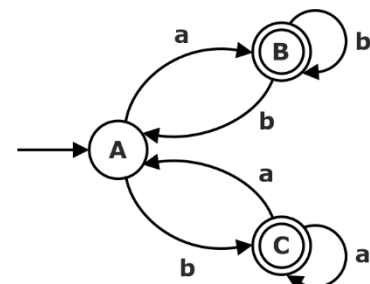


- A. $ab^*bab^* + ba^*aba^*$
- B. $(ab^*b)^*ab^* + (ba^*a)^*ba^*$
- C. $(ab^*b + ba^*a)^*(a^* + b^*)$
- D. $(ba^*a + ab^*b)^*(ab^* + ba^*)$

[MCQ: 1 Mark]

Ans. D

Sol.



- The regular expression for above finite state automaton will be:

$$R.E = R.E \text{ of state 'B'} + R.E \text{ of state 'C'}$$

- Resolving the loops on state A. So, Regular expression with respect to state 'A' after resolving loop:

$$R.E \text{ of state 'A'} = (ab^*b + ba^*a)^*$$

$$\therefore \text{Regular expression} = R.E = R.E \text{ of state 'B'} + R.E \text{ of state 'C'} = R.E \text{ of state 'A'} (ab^* + ba^*)$$

$$= (ba^*a + ab^*b)^* (ab^* + ba^*)$$

Hence option D is the correct regular expression for above NFA.

- 13.** Which one of the following statements is TRUE?

- A. The LALR(1) parser for a grammar G cannot have reduce-reduce conflict if the LR(1) parser for G does not have reduce-reduce conflict.
- B. Symbol table is accessed only during the lexical analysis phase.
- C. Data flow analysis is necessary for run-time memory management.
- D. LR(1) parsing is sufficient for deterministic context-free languages.

[MCQ: 1 Mark]

Ans. D

Sol. Option A is false as merging the states or combining the states with the same LR(0) items and different lookaheads in LR(1) may also lead to reduce-reduce conflict in the LALR(1) parser.

Option B is false as symbol table is associated with all phases of compilation and will be used at different stages.

Option C is false as Data flow analysis is applied for optimizations, analysing the code and finding minimum number of registers etc.

It is not necessary for runtime memory management.

Option D is correct as the LR parser can recognize any deterministic context-free language in linear-bounded time. So, LR(1) parsing is sufficient for DCFL.

- 14.** In a relational data model, which one of the following statements is TRUE?

- A. A relation with only two attributes is always in BCNF.
- B. If all attributes of a relation are prime attributes, then the relation is in BCNF.
- C. Every relation has at least one non-prime attribute.
- D. BCNF decompositions preserve functional dependencies.

[MCQ: 1 Mark]

Ans. A

Sol. A. A relation with only two attributes is always in BCNF. It is true.

B. This statement is also false.

We take an example as

$R(A,B,C)$ with FD's

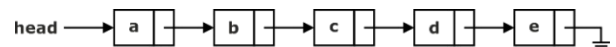
$A \rightarrow C; B \rightarrow C; C \rightarrow A$

In this, all are prime attributes but not in BCNF.

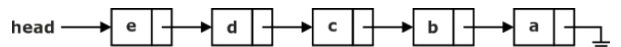
C. This statement is also false.

D. No, BCNF decomposition does not preserve function dependency as every lossless decomposition is not dependency preserving.

- 15.** Consider the problem of reversing a singly linked list. To take an example, given the linked list below,



the reversed linked list should look like



Which one of the following statements is TRUE about the time complexity of algorithms that solve the above problem in $O(1)$ space?

- A. The best algorithm for the problem takes $\theta(n)$ time in the worst case.
- B. The best algorithm for the problem takes $\theta(n \log n)$ time in the worst case.
- C. The best algorithm for the problem takes $\theta(n^2)$ time in the worst case.
- D. It is not possible to reverse a singly linked list in $O(1)$ space.

[MCQ: 1 Mark]

Ans. A

Sol. The given linked list is as below:

head \rightarrow

a

 \rightarrow

b

 \rightarrow

c

 \rightarrow

d

 \rightarrow

e

 \rightarrow /

Now, we need to find the time complexity of reversing the linked list using $O(1)$ space complexity.

We can use three pointers and one scan of linked list would provide us the reversed linked list.

Code:

```
Struct node *p, *q, *r;  
    p = head;  
    q = NULL;  
    r = NULL;
```

```
while (p != Null)
```

```
{
```

```
    r = q;
```

```
    q = p;
```

```
    p = p  $\rightarrow$  next;
```

```
    q  $\rightarrow$  next = r;
```

```
}
```

```
head = q;
```

```
return head;
```

The time complexity of above code is $\theta(n)$ and space complexity is $O(1)$.

Hence, A is the correct answer.

- 16.** Suppose we are given n keys, m hash table slots, and two simple uniform hash functions h_1 and h_2 . Further suppose our hashing scheme uses h_1 for the odd keys and h_2 for the even keys. What is the expected number of keys in a slot?

- A. $\frac{m}{n}$
- B. $\frac{n}{m}$
- C. $\frac{2n}{m}$
- D. $\frac{n}{2m}$

[MCQ: 1 Mark]

Ans. B

Sol. Here, number of keys = n

And number of hash table slots = m

Also, we have been provided with 2 simple uniform hash functions.

Now, for uniform hash function, irrespective of the number of hash function, the expected number of keys in slot = n/m

So, B is the correct answer.

- 17.** Which one of the following facilitates transfer of bulk data from hard disk to main memory with the highest throughput?
- A. DMA based I/O transfer
 - B. Interrupt driven I/O transfer
 - C. Polling based I/O transfer
 - D. Programmed I/O transfer

[MCQ: 1 Mark]

Ans. A

Sol. • There are three modes in which data get transferred to/from I/O devices/main memory:

1. Programmed I/O
2. Interrupt initiated or interrupt driven I/O
3. DMA (Direct memory access).

- In the first 2 modes data get transferred in the CPU intervention, which waste lots of clock cycles of the CPU.
- In DMA there will be direct channel gets established between peripheral devices or I/O devices and main memory since CPU intervention is avoided.
Hence DMA transfer gives maximum throughput specially, when the data gets transferred between fast I/O devices such as disk to main memory.
Hence, DMA based I/O transfer will be preferred.
- Direct memory Access (DMA) transfer the block of data between the memory and peripheral devices (I/O device) of the system, without the participation of the CPU.
- So, to transfer bulk data from hard disk to main memory with highest throughput then we have to transfer data with the help of DMA without any intervention of CPU.

18. Let R1 and R2 be two 4-bit registers that store numbers in 2's complement form. For the operation $R1+R2$, which one of the following values of R1 and R2 gives an arithmetic overflow?

- A. R1 = 1011 and R2 = 1110
- B. R1 = 1100 and R2 = 1010
- C. R1 = 0011 and R2 = 0100
- D. R1 = 1001 and R2 = 1111

[MCQ: 1 Mark]

Ans. B

Sol. 4 bit has the range of 2's complement as
 $= -2^{n-1}$ to $2^{n-1} - 1$
 $= -2^{4-1}$ to $2^{4-1} - 1$
 $= -8$ to 7

B R1=1100 ; R2=1010

In 2's Complement: R1 = 0100; R2=0110

In decimal R1 = -5; R2 = -6

Overflow = $R1+R2 = -5 - 6 = -11$

As -11 is not represented in 4 bits, it is not in the range.

Hence option B causes overflow.

19. Consider the following threads, T_1 , T_2 , and T_3 executing on a single processor, synchronized using three binary semaphore variables, S_1 , S_2 , and S_3 , operated upon using standard `wait()` and `signal()`. The threads can be context switched in any order and at any time.

T_1	T_2	T_3
<pre>while (true) { wait(S₃); print("C"); signal(S₂); }</pre>	<pre>while(true) { wait(S₁); print("B"); signal(S₃); }</pre>	<pre>while(true) { wait(S₃); print("A"); signal(S₁); }</pre>

Which initialization of the semaphores would print the sequence BCABCABCA...?

- A. $S_1 = 1$; $S_2 = 1$; $S_3 = 1$
- B. $S_1 = 1$; $S_2 = 1$; $S_3 = 0$
- C. $S_1 = 1$; $S_2 = 0$; $S_3 = 0$
- D. $S_1 = 0$; $S_2 = 1$; $S_3 = 1$

[MCQ: 1 Mark]

Ans. C

Sol. Initially if $S_1 = 1$, $S_2 = 0$, $S_3 = 0$ Process T_2 will execute successfully and perform wait. (S_1); while T_1 and T_3 remain stuck at wait (S_3) & wait (S_2) respectively.

T_2 prints B, then process T_1 gets executed as T_2 performs signal (S_3), T_1 performs wait (S_3) and prints C. After T_1 performs signal (S_2), process T_3 gets executed performs wait (S_3) and prints A.

This pattern keeps on repeating and prints the sequence BCABCA....

- 20.** Consider the following two statements with respect to the matrices $A_{m \times n}$, $B_{n \times m}$, $C_{n \times n}$ and $D_{n \times n}$.

Statement 1: $\text{tr}(AB) = \text{tr}(BA)$

Statement 2: $\text{tr}(CD) = \text{tr}(DC)$

where $\text{tr}()$ represents the trace of a matrix.

Which one of the following holds?

- A. Statement 1 is correct and Statement 2 is wrong.
- B. Statement 1 is wrong and Statement 2 is correct.
- C. Both Statement 1 and Statement 2 are correct.
- D. Both Statement 1 and Statement 2 are wrong.

[MCQ: 1 Mark]

Ans. C

Sol. Suppose take an example as

$$A = \begin{bmatrix} 1 & 2 \end{bmatrix}_{1 \times 2} \quad B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{2 \times 1} \quad C = \begin{bmatrix} 1 & 5 \\ 2 & 6 \end{bmatrix}_{2 \times 2}$$

$$D = \begin{bmatrix} 7 & 8 \\ 1 & 3 \end{bmatrix}_{2 \times 2}$$

$$S1: \text{tr}(AB) = \text{tr}(BA)$$

$$AB = [11]_{1 \times 1}$$

$$BA = \begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}_{2 \times 2}$$

$$\text{tr}(AB) = 11$$

$$\text{tr}(BA) = 8 + 3 = 11$$

$$S2: \text{tr}(CD) = \text{tr}(DC)$$

$$DC = \begin{bmatrix} 23 & 83 \\ 7 & 23 \end{bmatrix}$$

$$CD = \begin{bmatrix} 12 & 23 \\ 20 & 34 \end{bmatrix}$$

$$\text{tr}(CD) = 12 + 34 = 46$$

$$\text{tr}(DC) = 23 + 23 = 46$$

Hence both statements are true.

Therefore, option C is correct.

- 21.** What is printed by the following ANSI C program?

```
#include <stdio.h>
int main(int argc, char *argv[])
{
    int x = 1, z[2] = {10, 11};
    int *p = NULL;
    p = &x;
    *p = 10;
    p = &z[1];
    *(&z[0] + 1) += 3;
    printf("%d, %d, %d\n", x, z[0], z[1]);
    return 0;
}
```

A. 1, 10, 11

B. 1, 10, 14

C. 10, 14, 11

D. 10, 10, 14

[MCQ: 1 Mark]

Ans. D

Sol. Here: $\overset{x}{\boxed{1}}_{1000}$

$$\text{And: } z[2] = \begin{bmatrix} 10 & 11 & 14 \end{bmatrix}_{\begin{smallmatrix} 2000 & 2002 \end{smallmatrix}}$$

Now, $p = \&x$

$$\text{So: } \overset{p}{\boxed{1000}}$$

Now, $*p = 10$

$$\Rightarrow *1000 = 10$$

Now, $p = \&z[1]$

$$= 2002$$

Now, $*(&z[0] + 1) += 3$

$$\Rightarrow *(2000 + 1) += 3$$

$$\Rightarrow *(2002) += 3$$

$$\Rightarrow *(2002) = *(2002) + 3$$

$$= 11 + 3$$

$$= 14$$

So, $\text{printf}("\%d, \%d, \%d\n", x, z[0], z[1])$

gives output: 10, 10, 14

- Recursive languages are closed under complementary operations.
- Every CFL is also a Recursive Language. Hence Complement of CFL must be recursive language.

Option D: True

- Regular Language closed under intersection operation (\cap)
so, $L_1 \cap L_2 = \text{regular}$
- Every regular is also DCFL.

24. Let WB and WT be two set associative cache organizations that use LRU algorithm for cache block replacement. WB is a write back cache and WT is a write through cache. Which of the following statements is/are FALSE?

- Each cache block in WB and WT has a dirty bit.
- Every write hit in WB leads to a data transfer from cache to main memory.
- Eviction of a block from WT will not lead to data transfer from cache to main memory.
- A read miss in WB will never lead to eviction of a dirty block from WB.

[MSQ: 1 Mark]

Ans. A, B, D

Sol. Option A: False

- In write-through cache, when data is updated, it is written to both the cache and main memory simultaneously.
- So, WT cache does not require dirty bit to indicate modification of data.
- A WB must necessarily have a dirty bit to avoid redundant writes to Main Memory.

Option B: False

- In write-back cache, when data is updated, it is written in only to the cache

memory. So, while write hit in WB, it writes the data in cache only.

- A WB cache's primary use is to increase throughput or useful work. Multiple writes to the same cache block will not be reflected immediately.
- After the final write operation to that block then before block get evicted the block data first need to transfer from cache to main memory in order to update the main memory. Hence Every write hit in WB leads to a data transfer from cache to main memory is incorrect statement.

Option C: True

In write-through a cache line can always be invalidated without writing back since main memory already has an up-to-data copy of the line.

Option D: False

A read miss in WB can lead to eviction of a dirty block from WB, it depends on page replacement algo. It does not depends on read or write miss operation.

25. Consider the following three relations in a relational database.

Employee (eId, Name), Brand (bId, bName), Own (eId, bId)

Which of the following relational algebra expressions return the set of eIds who own all the brands?

- $\pi_{eId}(\pi_{eId, bId}(\text{Own})/\pi_{bId}(\text{Brand}))$
- $\pi_{eId}(\text{Own}) - \pi_{eId}((\pi_{eId}(\text{Own}) \times \pi_{bId}(\text{Brand})) - \pi_{eId, bId}(\text{Own}))$
- $\pi_{eId}(\pi_{eId, bId}(\text{Own})/\pi_{bId}(\text{Own}))$
- $\pi_{eId}((\pi_{eId}(\text{Own}) \times \pi_{bId}(\text{Own}))/\pi_{bId}(\text{Brand}))$

[MSQ: 1 Mark]

Ans. A, B

Sol. Consider an example as:

Employee

eId	Name
1	x
2	y
3	z
4	x

Brand

bId	bName
11	truke
12	boat
13	MI

Own

eId	bId
1	11
1	12
3	13
3	12
2	12

A. $\Pi_{eId}(\Pi_{eId, bId}(\text{Own}) / \Pi_{bId}(\text{Brand}))$.

This will search for eId in relation to that is associated with all the three values {11,12,13} since none of the employees are associated with all the three brands. Hence it is correct.

$$\begin{aligned}
 & B. \Pi_{eId}(\text{Own}) - \Pi_{eId}((\Pi_{eId}(\text{Own}) \times \\
 & \quad \Pi_{bId}(\text{Brand})) - \Pi_{eId, bId}(\text{Own})) \\
 & \quad \Pi_{eId, bId}(\text{Own}) \\
 & = \{(1,11), (1,12), (3,13), (3,12), (2,12)\}, \\
 & \quad \Pi_{eId}(\text{Own}) \times \Pi_{bId}(\text{Brand}) \\
 & = \{(1,11), (1,12), (1,13), (3,11), (3,12), \\
 & \quad (3,13), (2,11), 2(2,12), (2,13)\}. \\
 & \quad (\Pi_{eId}(\text{Own}) \times \Pi_{bId}(\text{Brand}) - \Pi_{eId, bId}(\text{Own})) \\
 & = \{(1,13), (3,11), (2,11), (2,13)\}. \\
 & = \Pi_{eId}(\text{Own}) - \Pi_{eId}\{1,2,3\} \\
 & = \{1,2,3\} - \{1,2,3\} = \text{empty set} \\
 & \text{Hence A, B are correct.} \\
 & \text{The rest option are incorrect.}
 \end{aligned}$$

26. Which of the following statements is/are TRUE with respect to deadlocks?

- A. Circular wait is a necessary condition for the formation of deadlock.
- B. In a system where each resource has more than one instance, a cycle in its wait-for graph indicates the presence of a deadlock.
- C. If the current allocation of resources to processes leads the system to unsafe state, then deadlock will necessarily occur.
- D. In the resource-allocation graph of a system, if every edge is an assignment edge, then the system is not in deadlock state.

[MSQ: 1 Mark]

Ans. A, D

Sol. When every edge is the assignment edge, it implies no process needs any more resources thus deadlock doesn't exist in the system. Circular wait is a necessary condition for deadlock but not sufficient.

27. Which of the following statements is/are TRUE for a group G ?

- A. If for all $x, y \in G$, $(xy)^2 = x^2 y^2$, then G is commutative.
- B. If for all $x \in G$, $x^2 = 1$, then G is commutative. Here, 1 is the identity element of G .
- C. If the order of G is 2, then G is commutative.
- D. If G is commutative, then a subgroup of G need not be commutative.

[MSQ: 1 Mark]

Ans. A, B, C

Sol. if for all x, y belongs to G , $(xy)^2 = x^2 y^2$ then G is commutative.

Whenever we have a condition $g^2 = e$ in a group, its equivalent to $g = g^{-1}$.

The proof is simply:

$$ab = (ab)^{-1} = b^{-1} a^{-1} = ba.$$

All groups with less than 6 elements are abelian.

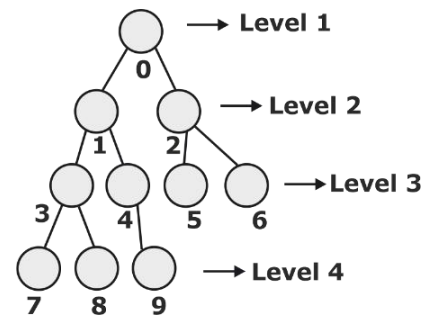
28. Suppose a binary search tree with 1000 distinct elements is also a complete binary tree. The tree is stored using the array representation of binary heap trees. Assuming that the array indices start with 0, the 3rd largest element of the tree is stored at index_____.

[NAT: 1 Mark]

Ans. 509

Sol. We have been provided with a Binary Search tree with 1000 distinct elements and the tree is represented using array representation of binary heap tree.

Let us consider 10 elements initially,
So, its structure would be:



So, we have 4 levels here and the largest element in BST is the rightmost element.

So, maximum element would be index 6, the second maximum would be index 2 and the third maximum would be at index 5.

Now, with 1000 elements; we would have 10 levels as $2^{10} = 1024$ elements.

With the same approach as above, the largest element in BST is the right most element with index $2^9 - 2$ (as the indexing of array starts with 0).

So, the largest element is 510 index element. Its, parent is the second largest and its left child is the third largest which is of index $510 - 1 = 509$.

29. Consider the augmented grammar with $\{+, *, (,), id\}$ as the set of terminals.

$S' \rightarrow S$

$S \rightarrow S + R \mid R$

$R \rightarrow R * P \mid P$

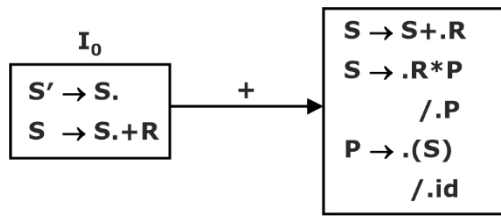
$P \rightarrow (S) \mid id$

If I_0 is the set of two LR(0) items $\{[S' \rightarrow S.], [S \rightarrow S. + R]\}$, then $\text{goto}(\text{closure}(I_0), +)$ contains exactly _____ items.

[NAT: 1 Mark]

Ans. 5

Sol. Here, the LR (0) items in I_0 set is as below.
We need to find $\text{goto}(\text{closure}(I_0), +)$



Hence, $\text{goto}(\text{closure}(I_0), +)$ contains 5 items.

- 30.** Consider a simple undirected graph of 10 vertices. If the graph is disconnected, then the maximum number of edges it can have is _____.

[NAT: 1 Mark]

Ans. 36

Sol. To make a graph disconnected, we require $n-1C_2$ edges.

Therefore $\frac{(n-1)(n-2)}{2}$

Here $n = 10$, putting in formula:

$$\frac{(10-1)(10-2)}{2} \Rightarrow \frac{(9)(8)}{2} = 36.$$

- 31.** Consider a relation $R(A, B, C, D, E)$ with the following three functional dependencies.
 $AB \rightarrow C$; $BC \rightarrow D$; $C \rightarrow E$;
 The number of superkeys in the relation R is _____.

[NAT: 1 Mark]

Ans. 8

Sol. $(AB)^+ = \{A, B, C, D, E\}$

Candidate key = AB

The number of super keys = $2^{5-2} = 2^3 = 8$.

- 32.** The number of arrangements of six identical balls in three identical bins is _____.

[NAT: 1 Mark]

Ans. 7

Sol. Here, we need to find the arrangements of six identical balls in three identical bins.
 So, this problem belong to the category of indistinguishable objects into indistinguishable boxes.

Given that, we have 3 bins and 6 balls. We would solve the problem case by case.

The following cases are possible for this problem :

Case I : When all of the three bins are used.

Now, in this case the distribution of balls in the bins is as follows:

Bin 1	Bin 2	Bin 3
4	1	1
3	2	1
2	2	2

So, in case I we have 3 ways.

Case II : When 2 bins are used.

Now, in this case distribution of balls in the bins is as follows :

Bin 1	Bin 2	Bin 3
5	1	empty
4	2	empty
3	3	empty

So, in case II we have 3 ways.

Case III : When only 1 bin is used

Now, in this case, the distribution of balls in the bins is as follows :

Bin 1	Bin 2	Bin 3
6	empty	empty

So, in case III we have 1 way.

Thus, total number of arrangement = $3 + 3 + 1 = 7$.

- 33.** A cache memory that has a hit rate of 0.8 has an access latency 10 ns and miss penalty 100 ns. An optimization is done on the cache to reduce the miss rate. However, the optimization results in an increase of cache access latency to 15 ns, whereas the miss penalty is not affected. The minimum hit rate (rounded off to two decimal places) needed after the optimization such that it should not increase the average memory access time is _____.

[NAT: 1 Mark]

Ans. 0.85 to 0.85

- Sol.** • Given information of the system without optimization:

Hit rate of cache memory (H_C) = 0.8Cache memory access time (T_C) = 10 nsMain memory (miss penalty) access time (T_M) = 100 ns

$$\begin{aligned}\therefore \text{EMAT} &= H_C \times T_C + (1 - H_C) \times T_M \\ &= 0.8 \times 10 + (1 - 0.8) \times 100 \\ &= 8 + 0.2 \times 100 \\ &= 28 \text{ ns}\end{aligned}$$

- Now after system optimization

Cache memory access time (T_C) = 15 nsLet ' H_x ' be the cache hit rate after optimization.

$$\begin{aligned}\therefore \text{EMAT} &= H_x \times T_C + (1 - H_x) \times T_M \\ 28 &\geq H_x \times 15 + (1 - H_x) \times 100 \\ 28 &\geq 15 \times H_x + 100 - 100 \times H_x \\ 28 &\geq 100 - 85 H_x \\ 85 \times H_x &\geq 100 - 28 \\ H_x &= 72/85 = 0.847\end{aligned}$$

Hence the minimum hit rate is 0.847 after system optimization.

- 34.** The value of the following limit is _____.

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{1 - e^{2\sqrt{x}}}$$

[NAT: 1 Mark]

Ans. -0.5

Sol. $\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{1 - e^{2\sqrt{x}}}$

Apply L'Hospital's Rule.

$$\begin{aligned}&= \frac{\frac{1}{2\sqrt{x}}}{-e^{2\sqrt{x}} \cdot 2 \cdot \frac{1}{2\sqrt{x}}} \\ &= -\frac{1}{2e^{2\sqrt{x}}} = -\frac{1}{2} = -0.5\end{aligned}$$

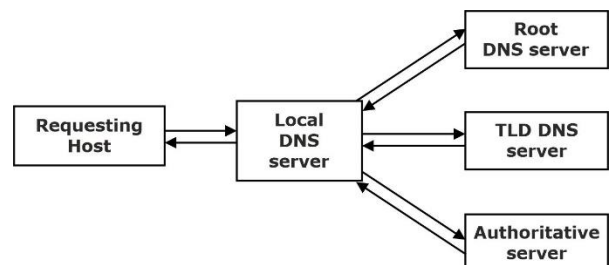
- 35.** Consider the resolution of the domain name www.gate.org.in by a DNS resolver. Assume that no resource records are cached anywhere across the DNS servers and that iterative query mechanism is used in the resolution. The number of DNS query-response pairs involved in completely resolving the domain name is _____.

[NAT: 1 Mark]

Ans. 4

- Sol.** (I) The root, TLD, and authoritative DNS servers all belong to the hierarchy of DNS servers. There is another important type of DNS server called the local DNS server. A local DNS server does not strictly belong to the hierarchy of servers but is nevertheless central to DNS architecture.

- (II) For given URL "www.gate.org.in", there will be "4" pair's of queries.
"3" iterative queries + 1 recursive query.



36. Which one of the following is the closed form for the generating function of the sequence $\{a_n\}_{n \geq 0}$ defined below ?

$$a_n = \begin{cases} n+1, & n \text{ is odd} \\ 1, & \text{otherwise} \end{cases}$$

- A. $\frac{x(1+x^2)}{(1-x^2)^2} + \frac{1}{1-x}$ B. $\frac{x(3-x^2)}{(1-x^2)^2} + \frac{1}{1-x}$
 C. $\frac{2x}{(1-x^2)^2} + \frac{1}{1-x}$ D. $\frac{x}{(1-x^2)^2} + \frac{1}{1-x}$

[MCQ: 2 Marks]

Ans. A

Sol. Generating function $G(x)$ of a_n is

$$\begin{aligned} G(x) &= \sum_{n=0}^{\infty} a_n x^n \\ &= \sum_{n \text{ is odd}} (n+1)x^n + \sum_{n \text{ is even}} (1)x^n \\ &= \sum_{n \text{ is odd}} (n)x^n + \sum_{n \text{ is odd}} (1)x^n + \sum_{n \text{ is even}} (1)x^n \\ &= \text{solving the above equation, we get.} \end{aligned}$$

$$\frac{x(1+x^2)}{(1-x^2)^2} + \frac{1}{1-x}$$

37. Consider a simple undirected unweighted graph with at least three vertices. If A is the adjacency matrix of the graph, then the number of 3-cycles in the graph is given by the trace of

- A. A^3 B. A^3 divided by 2
 C. A^3 divided by 3 D. A^3 divided by 6

[MCQ: 2 Marks]

Ans. D

Sol. Suppose $A[i][j]$ be the adjacency matrix of the graph. If we calculate A^3 , then the number of triangles in the undirected graph is equal to $\text{trace } A^3 / 6$ where $\text{trace } (A)$ is the sum of the diagonal elements of matrix A .

Trace of a graph represented as adjacency matrix $A[x][x]$ is:

$$\text{Trace } A[x][x] = A[0][0] + A[1][1] + \dots + A[v-1][v-1]$$

$$\text{Count of triangles} = \text{trace}(A^3) / 6.$$

A triangle is a cyclic path of length three, so $A^3[i][i]$ represents a triangle beginning and ending with vertex i . Since a triangle has 3 vertexes and it is Counted for every vertex, we need to divide the result by 3. Now since the graph is undirected, so we divide by 2 also, Therefore, the number of triangles is $\text{trace}(A^3) / 6$.

38. Which one of the following statements is FALSE?

- A. The TLB performs an associative search in parallel on all its valid entries using page number of incoming virtual address.
 B. If the virtual address of a word given by CPU has a TLB hit, but the subsequent search for the word results in a cache miss, then the word will always be present in the main memory.
 C. The memory access time using a given inverted page table is always same for all incoming virtual addresses.
 D. In a system that uses hashed page tables, if two distinct virtual addresses V_1 and V_2 map to the same value while hashing, then the memory access time of these addresses will not be the same.

[MCQ: 2 Marks]

Ans. C

Sol. When a memory reference takes place, the virtual address is matched by the memory mapping unit and then the inverted Page table is searched for a match and then finally the corresponding frame number is obtained. If the match is found in the k^{th} entry then

physical address of the process is sent as real address otherwise segmentation fault is generated.

So finding a match, memory access time varies. Therefore, the statement is false.

39. Let $R_i(z)$ and $W_i(z)$ denote read and write operations on a data element z by a transaction T_i , respectively. Consider the schedule S with four transactions.

S:

$R_4(x)R_2(x)R_3(x)R_1(y)W_1(y)W_2(x)W_3(y)R_4(y)$

Which one of the following serial schedules is conflict equivalent to S ?

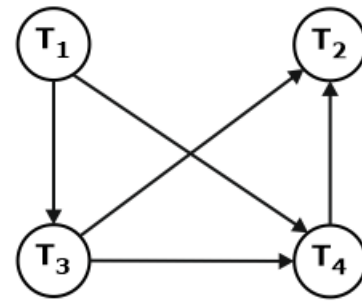
- A. $T_1 \rightarrow T_3 \rightarrow T_4 \rightarrow T_2$
- B. $T_1 \rightarrow T_4 \rightarrow T_3 \rightarrow T_2$
- C. $T_4 \rightarrow T_1 \rightarrow T_3 \rightarrow T_2$
- D. $T_3 \rightarrow T_1 \rightarrow T_4 \rightarrow T_2$

[MCQ: 2 Marks]

Ans. A

Sol.

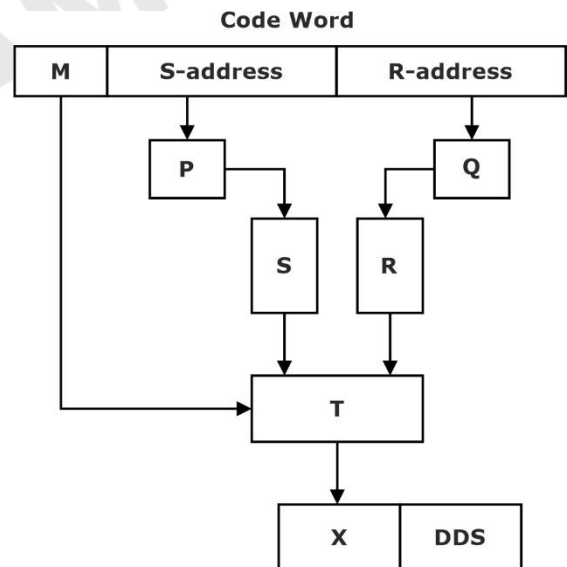
T_1	T_2	T_3	T_4
			$R(x)$
	$R(x)$		
		$R(x)$	
$R(y)$			
$W(y)$			
	$W(x)$		
		$W(y)$	
			$R(y)$



$T_1 - T_3 - T_4 - T_2$.

Therefore, option A is correct

40. Consider a digital display system (DDS) shown in the figure that displays the contents of register X. A 16-bit code word is used to load a word in X, either from S or from R. S is a 1024-word memory segment and R is a 32-word register file. Based on the value of mode bit M, T selects an input word to load in X. P and Q interface with the corresponding bits in the code word to choose the addressed word. Which one of the following represents the functionality of P, Q, and T?



- A. P is 10:1 multiplexer;
Q is 5:1 multiplexer;
T is 2:1 multiplexer
- B. P is 10:2¹⁰ decoder;
Q is 5:2⁵ decoder;
T is 2:1 encoder

- C. P is $10:2^{10}$ decoder;
 Q is $5:2^5$ decoder;
 T is 2:1 multiplexer
 D. P is 1:10 de-multiplexer;
 Q is 1:5 de-multiplexer;
 T is 2:1 multiplexer

[MCQ: 2 Marks]

Ans. C

Sol. S is 1024-word memory. It means there is 1024 words segment to represent it. We need 10 bits.

So, at P, we require a decoder of 10×2^{10} . R is a 32-word register. It means we have 32 registers to represent it. We require 5 bits.

So, at Q, we require a 5×2^5 decoder.

Now at T, we have to select one from S and R for this we need. 2×1 MUX.

Hence option C is correct.

- 41.** Consider three floating point numbers A, B and C stored in registers R_A , R_B and R_C , respectively as per IEEE-754 single precision floating point format. The 32-bit content stored in these registers (in hexadecimal form) are as follows.

$R_A =$	$R_B =$	$R_C =$
0xC1400000	0x42100000	0x41400000

Which one of the following is FALSE?

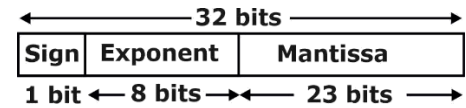
- A. $A + C = 0$ B. $C = A + B$
 C. $B = 3C$ D. $(B - C) > 0$

[MCQ: 2 Marks]

Ans. B

Sol. (i) First find the equivalent decimal value for each floating-point number stored in R_A , R_B and R_C according to 32-bit single precision floating point format:

- (ii) 32-bit single precision floating point format:



- (iii) $R_A: 0 \times C1400000$

1	10000010	100000.....00
---	----------	---------------

\therefore sign bit = 1

Bias Exponent = 1

Mantissa = 1.10000.....

Note: Bias value in IEEE single precision format is 127.

\therefore Equivalent Decimal value = $(-1)^S (1.M) \times 2^{BE - \text{Bias}}$

$$= (-1)^1 (1.1000...) \times 2^{130-127}$$

$$= 1.1 \times 2^3 = -1100$$

$$R_A = -12$$

- (ii) $R_B: 0 \times 42100000$

0	10000100	001000.....00
---	----------	---------------

$S = 0$

$BE = 132$

Mantissa = 1.00100...

\therefore Equivalent Decimal value = $(-1)^0 (1.00100...) \times 2^{132-127}$

$$= + (1.001...) \times 2^{+5}$$

$$= 100100 = 36$$

- (iii) $R_C: 0 \times 41400000$

0	10000010	10000.....00
---	----------	--------------

$S = 0$

$BE = 130$

Mantissa = 1.10000.....

Equivalent Decimal value = $(-1)^0 \times (1.1000...) \times 2^{130+127}$

$$= + (1.1000...) \times 2^{+3}$$

$$= + (1100) = 12$$

Option A: True

$$A + C = 0$$

$$(-12) + (+12) = 0$$

Option B: False

$$C = A + B$$

$$(+12) = (-12) + (+36)$$

$$(+12) \neq (+24)$$

Option C: True

$$B = 3C$$

$$(+36) = 3 \times (+12)$$

$$(+36) = (+36)$$

Option D: True

$$(B-C) > 0$$

$$(+36 - 12) > 0 = (+24) > 0$$

Hence, option B is the correct answer.

- 42.** Consider four processes P, Q, R, and S scheduled on a CPU as per round robin algorithm with a time quantum of 4 units. The processes arrive in the order P, Q, R, S, all at time $t = 0$. There is exactly one context switch from S to Q, exactly one context switch from R to Q, and exactly two context switches from Q to R. There is no context switch from S to P. Switching to a ready process after the termination of another process is also considered a context switch. Which one of the following is NOT possible as CPU burst time (in time units) of these processes?

- A. $P = 4, Q = 10, R = 6, S = 2$
- B. $P = 2, Q = 9, R = 5, S = 1$
- C. $P = 4, Q = 12, R = 5, S = 4$
- D. $P = 3, Q = 7, R = 7, S = 3$

[MCQ: 2 Marks]

Ans. D

Sol. Given processes arrived in the order P,Q,R,S,P,Q,R,S....

Now analyzing the data provided in the question we can conclude that:

- If there is no context switch from S to P then P should be completed in 1 time quantum.
- There are exactly 2 context switches from Q to R, therefore both should require more than 1-time quantum
- Q requires more time than R because there is exactly 1 context switch from R to Q.
- S should be completed in 1-time quantum because 1 context switch from R to S.
- Exactly 1 context switch from S to Q.

By analyzing all the mentioned points we can conclude that:

- $0 < P \leq 1$ Time Quantum
- $0 < S \leq 1$ Time Quantum
- $1 \text{ time Quantum} < R \leq 2 \text{ time Quantum}$
- $2 \text{ time Quantum} < Q$.

Therefore by eliminating options based on above conditions we get:

$P=3, Q=7, R=7, S=3$. As the false. option, rest all the three options follow the conditions concluded.

- 43.** What is printed by the following ANSI C program?

```
#include<stdio.h>

int main(int argc, char *argv[])
{
    int a[3][3][3] =
        {{1, 2, 3, 4, 5, 6, 7, 8, 9},
         {10, 11, 12, 13, 14, 15, 16, 17, 18},
         {19, 20, 21, 22, 23, 24, 25, 26, 27}};
    int i = 0, j = 0, k = 0;
    for( i = 0; i < 3; i++ ){
        for(k = 0; k < 3; k++ )
            printf("%d ", a[i][j][k]);
        printf("\n");
    }
    return 0;
}
```

- A. 1 2 3
10 11 12
19 20 21
- B. 1 4 7
10 13 16
19 22 25
- C. 1 2 3
4 5 6
7 8 9
- D. 1 2 3
13 14 15
25 26 27

[MCQ: 2 Marks]

Ans. A**Sol.** The array $a[3][3][3]$ is:

$\{ \{1,2,3,4,5,6,7,8,9\},$
 $\{10,11,12,13,14,15,16,17,18\},$
 $\{19,20,21,22,23,24,25,26,27\} \}$

So, we have three 2-D array, each of which contains three 1-D array which in turn contains 3 elements.

The representation of three 2-D arrays are as follows:

The first 2-D array,

1	2	3
4	5	6
7	8	9

The second 2-D array,

10	11	12
13	14	15
16	17	18

The third 2-D array,

19	20	21
22	23	24
25	26	27

Now, so in $a[3][3][3]$; the first index represent 2D array, the second index represents 1D array and the third index represents elements in the 1D array.

Now, the loop goes like as:

$i = 0, j = 0, k = 0$ so $a[0][0][0] = 1$

$i = 0, j = 0, k = 1$ so $a[0][0][1] = 2$

$i = 0, j = 0, k = 2$ so $a[0][0][2] = 3$

Then:

$i = 1, j = 0, k = 0$ so $a[1][0][0] = 10$

$i = 1, j = 0, k = 1$ so $a[1][0][1] = 11$

$i = 1, j = 0, k = 2$ so $a[1][0][2] = 12$

Finally:

$i = 2, j = 0, k = 0$ so $a[2][0][0] = 19$

$i = 2, j = 0, k = 1$ so $a[2][0][1] = 20$

$i = 2, j = 0, k = 2$ so $a[2][0][2] = 21$

Hence, A is the correct answer.

44. What is printed by the following ANSI C program?

```
#include<stdio.h>

int main(int argc, char *argv[]){
    char a = 'P';
    char b = 'x';
    char c = (a & b) + '*';
    char d = (a | b) - '-';
    char e = (a ^ b) + '+';
    printf("%c %c %c\n", c, d, e);
    return 0;
}
```

ASCII encoding for relevant characters is given below

A	B	C	...	Z
65	66	67	...	90

a	b	c	...	z
97	98	99	...	122

*	+	-
42	43	45

- A. z K S B. 122 75 83
C. * - + D. P x +

[MCQ: 2 Marks]

Ans. A**Sol.** Here:

char a = 'P' and char b = 'x';

Now, char c = (a & b) + '*';

= ('P' & 'x') + 42

P = 80 = 01010000

X = 120 = 01111000

01010000 = 80

So, c = 80 + 42 = 122

= z

Then char d = (a | b) - '-';

= ('P' | 'x') - 45;

P = 80 = 01010000

X = 120 = 01111000

01111000 = 120

So; d = 120 - 45 = 75

= k

Finally, char e = (a ^ b) + '+'

= ('P' ^ 'x') + 43;

P = 80 = 01010000

X = 120 = 01111000

00101000 = 40

So; e = 40 + 43 = 80

= S

Here, A is answer.

- 45.** Consider solving the following system of simultaneous equations using LU decomposition.

$$x_1 + x_2 - 2x_3 = 4$$

$$x_1 + 3x_2 - x_3 = 7$$

$$2x_1 + x_2 - 5x_3 = 7$$

where L and U are denoted as

$$L = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$

Which one of the following is the correct combination of values for L_{32} , U_{33} , and x_1 ?

- A. $L_{32} = 2$, $U_{33} = -\frac{1}{2}$, $x_1 = -1$
B. $L_{32} = 2$, $U_{33} = 2$, $x_1 = -1$
C. $L_{32} = -\frac{1}{2}$, $U_{33} = 2$, $x_1 = 0$
D. $L_{32} = -\frac{1}{2}$, $U_{33} = -\frac{1}{2}$, $x_1 = 0$

[MCQ: 2 Marks]

Ans. D**Sol.** As we know, $L*U = M$

$$M = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 1 & -5 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ L_{11} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \quad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{33} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} * \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{22} & L_{21}U_{13} + U_{23} \\ U_{11}L_{31} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + U_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 1 & -5 \end{bmatrix}$$

$$U_{11} = 1; U_{12} = 1 \quad U_{13} = -2$$

$$L_{21} = 1; U_{22} = 3 - 1 = 2; U_{23} = -1 + 2 = 1$$

$$L_{31} = 2; L_{31}U_{12} + L_{32}U_{22} = 1$$

$$2 * 1 + L_{32} * 2 = 1$$

$$L_{32} = -\frac{1}{2}$$

$$U_{33} = -5 + 4 + \frac{1}{2} = -1 + \frac{1}{2} = -\frac{1}{2}$$

Therefore, option D is correct.

46. Which of the following is/are undecidable?

- A. Given two Turing machines M_1 and M_2 , decide if $L(M_1) = L(M_2)$.
- B. Given a Turing machine M , decide if $L(M)$ is regular.
- C. Given a Turing machine M , decide if M accepts all strings.
- D. Given a Turing machine M , decide if M takes more than 1073 steps on every string.

[MSQ: 2 Marks]

Ans. A, B, C

Sol. Option A: Undecidable

Equivalence of Language of two Turing machine that is

$L(M_1) = L(M_2)$ is undecidable problem.

Option B: Undecidable

- Regularity problem for R.E Language is undecidable.
- So, whether Language of Turing machine is regular or not is undecidable problem.

Option C: Undecidable

- Completeness problem ($L = E^*$) is undecidable for RE language.
- By applying Rice's Theorem also, we can say that problem is undecidable due non-trivial property.
- Non-trivial property means some Turing machine will accept all string, but other

Turing machine will not accept all string. So, we cannot decide always yes or no. Hence Undecidable problem.

Option D: Decidable

- Given problem is decidable because we can always decide whether Turing machine takes 1073 steps on all input strings or not.
- If Turing machine runs 1073 steps on all the inputs strings, then yes accept the strings as a member of the given problem. After 1073 steps whether it halts or not does not matter, we have to find that whether Turing machine taking more than 1073 steps or not.
- If Turing machine halts within 1073 steps then reject the strings and it will not be the member of given problem.

47. Consider the following languages:

$$L_1 = \{a^n w a^n \mid w \in \{a, b\}^*\}$$

$$L_2 = \{w x w^R \mid w, x \in \{a, b\}^*, |w|, |x| > 0\}$$

Note that w^R is the reversal of the string w .

Which of the following is/are TRUE?

- A. L_1 and L_2 are regular.
- B. L_1 and L_2 are context-free.
- C. L_1 is regular and L_2 is context-free.
- D. L_1 and L_2 are context-free but not regular.

[MSQ: 2 Marks]

Ans. A, B, C

Sol. • $L_1 = \{a^n w a^n \mid w \in (a, b)^*\}$

(i) Condition on 'n' not given so, 'n' can be either $n > 0$ or $n = 0$

(a) Consider $n = 0$

By putting 'n' as epsilon, L_1 can generate all the strings over alphabet $(a, b)^*$

\therefore Regular expression = $(a + b)^*$

(b) Consider $n > 0$

We can't put 'n' as epsilon.

Try to generate all the string by putting "n = 1"

∴ Regular expression

$= a(a + b)^*a$

(ii) Hence given language L_1 is regular language.

- $L_2 = \{w \times w^R / w, x \in \{a, b\}^*, |w|, |x| > 0\}$

(i) Here $|w|$ and $|x| > 0$ so, we cannot put w as epsilon.

(ii) Try to generate all the strings without violating the condition.

∴ Regular expression: $a(a + b)^*a + b(a + b)^*b$

Hence, L_2 is also regular language.

Note: Every Regular language is also context – Free Language.

Hence, option A, B and C are correct.

48. Consider the following languages:

$L_1 = \{ww | w \in \{a, b\}^*\}$

$L_2 = \{a^n b^n c^m | m, n \geq 0\}$

$L_3 = \{a^m b^n c^n | m, n \geq 0\}$

Which of the following statements is/are FALSE?

- A. L_1 is not context-free but L_2 and L_3 are deterministic context-free.
- B. Neither L_1 nor L_2 is context-free.
- C. L_2 , L_3 and $L_2 \cap L_3$ all are context-free.
- D. Neither L_1 nor its complement is context-free.

[MSQ: 2 Marks]

Ans. B, C, D

Sol. • $L_1 = \{ww / w \in (a,b)^*\}$ is CSL as we have to perform comparisons of two strings in forward manner and match string 'w' with

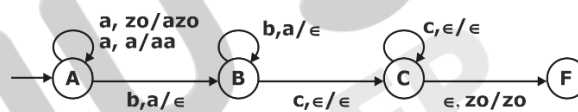
another string 'w' in forward manner so, it is not possible using PDA.

- $L_2 = \{a^n b^n c^m / m, n \geq 0\}$ is DCFL .

(I) We have to check the equality between the number of a's and number of b's.

(II) Push and Pop operation clear in the given language. Start pushing alphabet a's into the stack and start popping as soon as b's appears in input. Reach to final state by skipping any number of c's.

DPDA:



- $L_3 = \{a^m b^n c^n / m, n \geq 0\}$ is DCFL.

(i) We have to check the equality between the number of b's and c's.

(ii) Push and Pop operation clear in the given language. Start skipping any number of a's, after that start pushing alphabet b's into the stack and start popping as soon as c's appears in input.

Option A: True

Option B: False

L_1 is context sensitive language

L_2 is context free language

Option C: False

L_2 is context – free Language

L_3 is context – free Language

but

$L_2 \cap L_3 = \{a^n b^n c^m / m, n \geq 0\} \cap \{a^m b^n c^n / m, n \geq 0\}$

$= \{a^n b^n c^n / n \geq 0\}$

∴ $L_2 \cap L_3$ is context – sensitive language.

Option D: False

L_1 is CSL but L_1 complement can be CFL

$$\therefore L_1 = \{ww, w \in (a,b)^*\}$$

$$\therefore \bar{L}_1 = \Sigma^* - L = \Sigma^* - \{ww/w \in (a,b)^*\}$$

$$\bar{L}_1 = \{ww^R/w \in (a,b)^*\} \cup \{\text{some regular}\} = \text{CFL} \cup \text{regular} = \text{CFL}$$

49. Consider a simple undirected weighted graph G , all of whose edge weights are distinct. Which of the following statements about the minimum spanning trees of G is/are TRUE?

- A. The edge with the second smallest weight is always part of any minimum spanning tree of G .
- B. One or both of the edges with the third smallest and the fourth smallest weights are part of any minimum spanning tree of G .
- C. Suppose $S \subseteq V$ be such that $S \neq \phi$ and $S \neq V$. Consider the edge with the minimum weight such that one of its vertices is in S and the other in $V \setminus S$. Such an edge will always be part of any minimum spanning tree of G .
- D. G can have multiple minimum spanning trees.

[MSQ: 2 Marks]

Ans. A, B, C

Sol. Here, we have been provided with a simple undirected weighted graph G all of whose edge weights are distinct.

Now, as all of the edge weights are distinct, the graph cannot have multiple minimum spanning trees.

The smallest edge is always a part of the MST.

The second and third smallest edge will be part of the MST if the number of vertices are greater than 3 and 4 respectively.

Thus, with the above discussion, we can conclude that option A, B, C are correct.

50. The following simple undirected graph is referred to as the Peterson graph.



Which of the following statements is/are TRUE?

- A. The chromatic number of the graph is 3.
- B. The graph has a Hamiltonian path.
- C. The following graph is isomorphic to the Peterson graph.



- D. The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)

[MSQ: 2 Marks]

Ans. A, B, C

Sol. The size of the largest independent set of the given graph is 4

\therefore only option D is false.

The Peterson graph has a Hamiltonian path but no Hamiltonian cycle.

The chromatic number of the Peterson graph is 3.

51. Consider the following recurrence:

$$f(1) = 1;$$

$$f(2n) = 2f(n) - 1, \text{ for } n \geq 1;$$

$$f(2n+1) = 2f(n) + 1, \text{ for } n \geq 1.$$

Then, which of the following statements is/are TRUE?

A. $f(2^n - 1) = 2^n - 1$

B. $f(2^n) = 1$

C. $f(5 \cdot 2^n) = 2^{n+1} + 1$

D. $f(2^n + 1) = 2^n + 1$

[MSQ: 2 Marks]

Ans. A, B, C

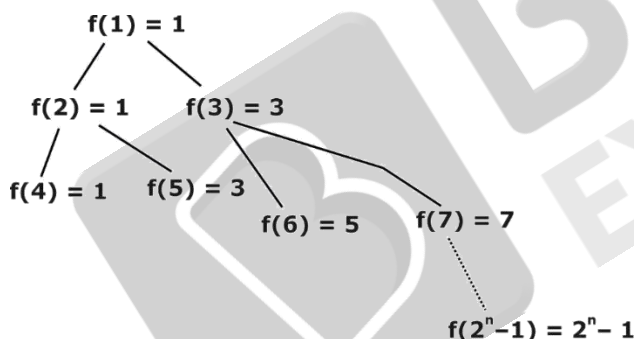
Sol. We have the following recurrence:

$$f(1) = 1$$

$$f(2n) = 2 \times f(n) - 1; n \geq 1$$

$$f(2n+1) = 2 \times f(n) + 1, n \geq 1$$

We can prove option A by using mathematical induction and by drawing a tree as follows:



Now, option B follows from the given base case $f(1) = 1$ and the recurrence.

$$f(2n) = 2 \times f(n) - 1$$

$$f(2) = 2 \times f(1) - 1 = 1.$$

$$f(4) = 1 \text{ and so on.}$$

Hence, $f(2^n) = 1$ is true.

For option D:

We can prove it wrong by calculating the value of $f(5)$.

$$f(2) = 1, f(5) = 2 \times f(2) + 1 = 3, \text{ which is not equal to } 2^2 + 1 = 5.$$

Option C

$$f(5) = 5$$

$$f(10) = 2 \times f(5) - 1 = 9$$

$$f(20) = 2 \times f(10) - 1$$

$$= 17$$

.

$$\begin{aligned} f(5 \times 2^n) &= 2^{n-1} (5) - 2^{n-2} - 2^{n-3} \dots - 2 - 1 \\ &= 2^{n-1} (6) + (1 + 2 + \dots + 2^{n-1}) \\ &= 6 \times 2^{n-1} - 2^n + 1 \\ &= 4 \times 2^{n-1} + 1 \\ &= 2^{n+1} + 1. \end{aligned}$$

So C is correct.

52. Which of the properties hold for the adjacency matrix A of a simple undirected unweighted graph having n vertices?

A. The diagonal entries of A^2 are the degrees of the vertices of the graph.

B. If the graph is connected, then none of the entries of $A^{n-1} + I_n$ can be zero.

C. If the sum of all the elements of A is at most $2(n-1)$, then the graph must be acyclic.

D. If there is at least a 1 in each of A 's rows and columns, then the graph must be connected.

[MSQ: 2 Marks]

Ans. A

Sol. In simple undirected adjacency matrix representation A should have diagonal entries as zero (0) because no self loops allowed in simple graphs. In option "A" let's consider some random node 'X', if X to A edge is present, then entry is 1, so for each edge between X and any vertex V_i there should be one (1) in the matrix entry. Therefore A^2

diagonal elements = degree of the vertices of the graph so only option A is correct.

Option B is not correct in all the cases.

Option C not correct, if there is an isolated vertex present and even though cycle present, in this scenario the sum of all the elements of A is at most $2(n - 1)$

Option D is incorrect, if there are 2 components with each having more than one vertex, then atleast one entry of each row and column is non zero.

53. Which of the following is/are the eigenvector(s) for the matrix given below?

$$\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$$

A. $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

B. $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

C. $\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$

D. $\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

[MSQ: 2 Marks]

Ans. A, C, D

Sol. $\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$

As we know, $A\lambda = \lambda X$

Check options.

A. $\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} 9 - 6 - 4 \\ 8 - 6 - 1 \\ -20 + 15 + 5 \\ -32 + 21 + 12 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

It is true.

C. $\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$

$$= \begin{pmatrix} 9 - 4 - 8 \\ 8 - 6 - 2 \\ -20 + 16 + 10 \\ -32 + 14 + 24 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 6 \\ 6 \end{pmatrix} = 3 \begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$$

It is true.

D. $\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

$$= \begin{pmatrix} -6 + 6 \\ -6 + 9 \\ 15 - 24 \\ 21 - 21 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -9 \\ 0 \end{pmatrix} = 3 \begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$$

Hence options A, C, D are correct.

54. Consider a system with 2 KB direct mapped data cache with a block size of 64 bytes. The system has a physical address space of 64 KB and a word length of 16 bits. During the execution of a program, four data words P, Q, R, and S are accessed in that order 10 times

(i.e., PQRSPQRS...). Hence, there are 40 accesses to data cache altogether. Assume that the data cache is initially empty and no other data words are accessed by the program. The addresses of the first bytes of P, Q, R, and S are 0xA248, 0xC28A, 0xCA8A, and 0xA262, respectively. For the execution of the above program, which of the following statements is/are TRUE with respect to the data cache?

- A. Every access to S is a hit.
- B. Once P is brought to the cache it is never evicted.
- C. At the end of the execution only R and S reside in the cache.
- D. Every access to R evicts Q from the cache.

[MSQ: 2 Marks]

Ans. A, B, D

Sol. Given that

- Direct mapped cache size = 2 KB
- Block size is given as 64 bytes
- Physical memory space size is 64 KB
- Word length of CPU is given as 16 bits.

Now,

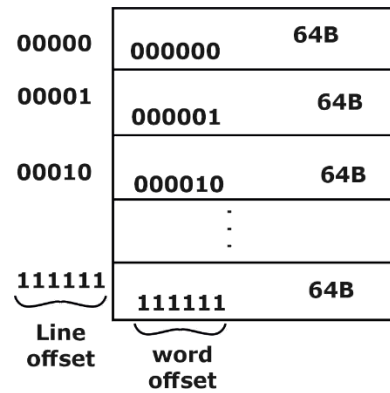
(I) Number of address bits required to access main memory:

$$= \log_2 64k = \log_2 2^{16} = 16 \text{ bits.}$$

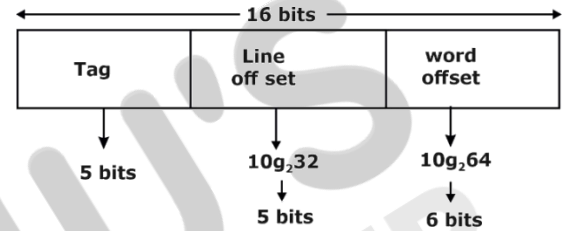
(II) Number of lines in direct mapped

$$\text{Cache} = \frac{\text{Cache Size}}{\text{Block Size}} = \frac{2KB}{64B}$$

$$= \frac{2^{11}B}{2^6B} = 2^5 = 32 \text{ lines}$$

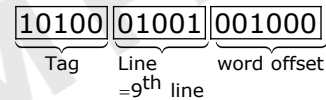


(iii) Address Format:

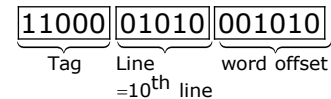


(iv) Convert each data words according to address format:

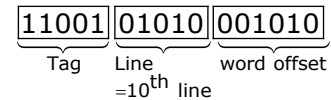
(a) P (A248):



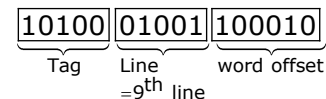
(b) Q (C28A):



(c) R (CA8A):



(d) S (A262):



- Tag bits of data word P and S are matched so, both P and S will be hit in cache line 9. Hence P and S will not be replaced by each other because both are resided in same block.

- Tag bits of Q and R data word is not matching so, for every miss of Q and R they replace each other.

Line 0	
	:
Line 9	P & S
Line 10	Q R Q R
Line 31	

- According to above direct mapped cache data word P and S are mapped to same cache line '9' and data word Q and R mapped to same cache line '10'.
- One P brought to cache, it will be hit for remaining access requires so, it is never evicted.
- At the end of execution P, S and R data words reside in the cache. Thus, option C is incorrect.
- Every access to S will be hit because we are accessing data words in particular order "PQRS PQRS..." so, first time when 'P' made request for data then entire will be transferred from Main memory to cache memory.
- P and S are in same blocks so, every access to S will be a hit as 'S' is already present in cache line 9.

Hence, option A, B and D are correct.

55. Consider routing table of an organization's router shown below:

Subnet Number	Subnet Mask	Next Hop
12.20.164.0	255.255.252.0	R1
12.20.170.0	255.255.254.0	R2

12.20.168.0	255.255.254.0	Interface 0
12.20.166.0	255.255.254.0	Interface 1
default		R3

Which of the following prefixes in CIDR notation can be collectively used to correctly aggregate all of the subnets in the routing table?

- A. 12.20.164.0/20 B. 12.20.164.0/22
C. 12.20.164.0/21 D. 12.20.168.0/22

[MSQ: 2 Marks]

Ans. A

Sol. Option A: If you use network ID as 12.20.164.0/20, it will cover all the subnets given in the table.

CIDR:

12.20.164.0/20

$$= \underbrace{12.20.1010}_{\text{Fixed Network bits}} \underbrace{0100.0000 \ 0000}_{\text{host bits}}$$

Now,

$$\text{Subnet } 12.20.164.0 = 12.20.1010 \ 0100.0000 \ 0000$$

Subnet 12.20.170.0

$$= \underbrace{12.20.1010}_{\text{Fixed}} \underbrace{1000.0000 \ 0000}_{\text{host}}$$

Subnet 12.20.160.0

$$= \underbrace{12.20.1010}_{\text{Fixed}} \underbrace{0110.0000 \ 0000}_{\text{host}}$$

Hence option A is correct answer.

Option B: if you use network ID as 12.20.164.0/22, then it will not cover subnet IP address 12.20.170.0

Hence, this is incorrect answer.

Option C: if you use network ID as 12.20.164.0/21, then it will not cover subnet IP address 12.20.170.0

Hence, this is incorrect answer.

Option D: If you use network ID as 12.20.168.0/22, it will not cover subnet IP address 12.20.164.0.

Hence, this is incorrect answer.

- 56.** Consider the relational database with the following four schemas and their respective instances.

Student(sNo, sName, dNo) Dept(dNo, dName)
Course(cNo, cName, dNo) Register(sNo, cNo)

Student		
<u>sNo</u>	sName	dNo
S01	James	D01
S02	Rocky	D01
S03	Jackson	D02
S04	Jane	D01
S05	Milli	D02

Dept	
<u>dNo</u>	dName
D01	CSE
D02	EEE

Course		
<u>cNo</u>	sName	dNo
C11	DS	D01
C12	OS	D01
C21	DE	D02
C22	PT	D02
C23	CV	D03

Register	
<u>cNo</u>	<u>cNo</u>
S01	C11
S01	C12
S02	C11
S03	C21
S03	C22
S03	C23
S04	C11
S04	C12
S05	C11
S05	C21

SQL Query:

SELECT * FROM Student AS S WHERE NOT
EXIST

(SELECT cNo FROM Course WHERE dNo =
"D01"

EXCEPT

SELECT cNo FROM Register WHERE sNo =
S.sNo)

The number of rows returned by the above
SQL query is _____.

[NAT: 2 Marks]

Ans. 2

Sol. For sNo=S01

The inner query will have {C11, C12} Except
{C11, C12}

Hence not non-exist Clause will return true
and the tuple sNo=S01 will be added.

Similarly, for sNo=S02.

The inner query will have {C11, C12} Except
{C11}

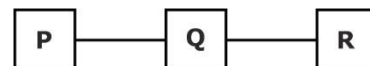
The non-exist Clause will return false thus it
will not add.

Similarly, for S03, S04, and S05.

The resultant has 2 tuples. that is

sNo	sName	dNo
S01	James	D01
S04	Jane	D01

- 57.** Consider a network with three routers P, Q, R shown in the figure below. All the links have cost of unity.



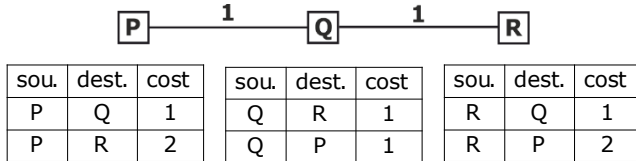
The routers exchange distance vector routing information and have converged on the routing tables, after which the link Q–R fails.

Assume that P and Q send out routing updates at random times, each at the same average rate. The probability of a routing loop formation (rounded off to one decimal place) between P and Q, leading to count-to-infinity problem, is_____.

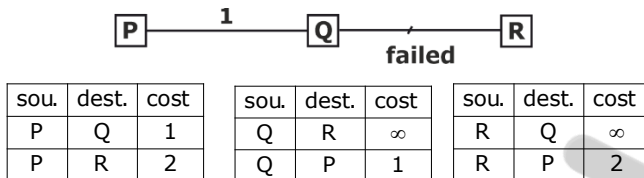
[NAT: 2 Marks]

Ans. 0.5

Sol.



Assume link Q – R failed



Point 1: To make routing table correct, if node Q share its routing table to node 'p' then node 'p' will also know that link Q-R failed hence there will be no looping problem.

Point 2: If node 'P' shares its routing table to node Q before Q sharing then Q will think that node P have some path to reach node 'R' and it update its table according. Hence, there will be a count to infinity problem.

$$\therefore \text{Probability} = \frac{1}{2} = 0.5$$

58. Let $G(V, E)$ be a directed graph, where $V = \{1, 2, 3, 4, 5\}$ is the set of vertices and E is the set of directed edges, as defined by the following adjacency matrix A.

$$A[i][j] = \begin{cases} 1, & 1 \leq j \leq i \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

$A[i][j] = 1$ indicates a directed edge from node i to node j . A directed spanning tree of G , rooted at $r \in V$, is defined as a subgraph T of G such that the undirected version of T is a

tree, and T contains a directed path from r to every other vertex in V . The number of such directed spanning trees rooted at vertex 5 is_____.

[NAT: 2 Marks]

Ans. 24

Sol. Here, we have been provided with a directed graph with 5 vertices.

The edges are directed and defined by the adjacency matrix A.

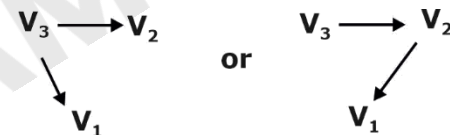
$$A[i][j] = \begin{cases} 1, & 1 \leq j \leq i \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

Now, we need to find the number of directed spanning tree rooted at vertex 5.

When we consider a graph of two elements, that is V_1 and V_2 , we get 1 possible spanning tree.

$$V_2 \rightarrow V_1$$

Now, when we consider a graph of three elements, that is V_1, V_2 and V_3 , we get 2 possible spanning trees.



Similarly, when we consider a graph of four elements, we get only $3 \times 2 \times 1 = 6$ possible spanning trees.

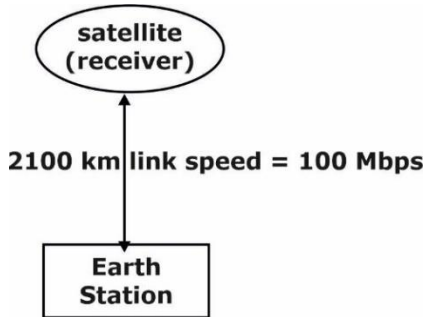
Finally, when we consider a graph of five elements, we get: $4 \times 3 \times 2 \times 1 = 24$ possible directed spanning trees.

59. Consider a 100 Mbps link between an earth station (sender) and a satellite (receiver) at an altitude of 2100 km. The signal propagates at a speed of 3×10^8 m/s. The time taken (in milliseconds, rounded off to two decimal places) for the receiver to completely receive a packet of 1000 bytes transmitted by the sender is_____.

[NAT: 2 Marks]

Ans. 7.07 to 7.09

Sol.



∴ Distance between Earth station and Satellite = 2100 km

Bandwidth = 100 Mbps

Pocket size = 1000 bytes

Now,

Total time = Transmission Time + Propagation Time

$$(i) \text{ Transmission time} = \frac{\text{Pocket Size}}{\text{Bandwidth}}$$

$$= \frac{1000 \times 8 \text{ bits}}{100 \times 10^6 \text{ bits / sec}}$$

$$= 80 \times 10^{-6} \text{ sec}$$

$$= 0.08 \text{ m sec.}$$

$$(ii) \text{ Propagation time} = \frac{\text{Length}}{\text{speed}}$$

$$= \frac{2100 \times 1000 \text{ m}}{3 \times 10^8 \text{ m / sec}}$$

$$= 7000 \times 10^{-6} \text{ sec}$$

$$= 7 \text{ m sec}$$

∴ Total time = 0.08 + 7 = 7.08 m sec.

- 60.** Consider the data transfer using TCP over a 1 Gbps link. Assuming that the maximum segment lifetime (MSL) is set to 60 seconds, the minimum number of bits required for the sequence number field of the TCP header, to prevent the sequence number space from wrapping around during the MSL is_____.

[NAT: 2 Marks]

Ans. 33 to 33

Sol. Given that

Bandwidth = 1 Gbps = 10^9 bps

MSL (maximum segment lifetime) = 60 sec

Now,

In 1 second 10^9 bits are transmitted

∴ 60 second 60×10^9 bits

$$= \frac{60 \times 10^9}{8} \text{ bytes}$$

$$= 7.5 \times 10^9 \text{ bytes.}$$

- TCP generate sequence number for each byte so, it will generate 7.5×10^9 Sequence number in every 60 seconds.

∴ Minimum sequence bits in TCP header will be = $\log_2 [\text{Ceil}(7.5 \times 10^9)]$
 $= \log_2 [\text{ceil}(2^{32.8})] = 33 \text{ bits.}$

- 61.** A processor X_1 operating at 2 GHz has a standard 5-stage RISC instruction pipeline having a base CPI (cycles per instruction) of one without any pipeline hazards. For a given program P that has 30% branch instructions, control hazards incur 2 cycles stall for every branch. A new version of the processor X_2 operating at same clock frequency has an additional branch predictor unit (BPU) that completely eliminates stalls for correctly predicted branches. There is neither any savings nor any additional stalls for wrong predictions. There are no structural hazards and data hazards for X_1 and X_2 . If the BPU has a prediction accuracy of 80%, the speed up (rounded off to two decimal places) obtained by X_2 over X_1 in executing P is_____.

[NAT: 2 Marks]

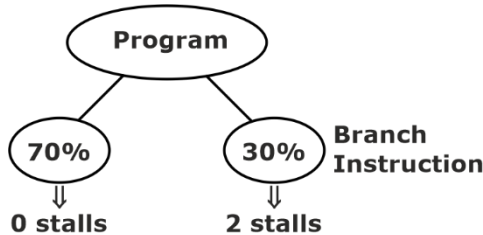
Ans. 1.42 to 1.45

Sol. Given that

5-stage RISC instruction pipeline cycle time

$$= \frac{1}{2 \text{ GHz}} = 0.5 \text{ ns}$$

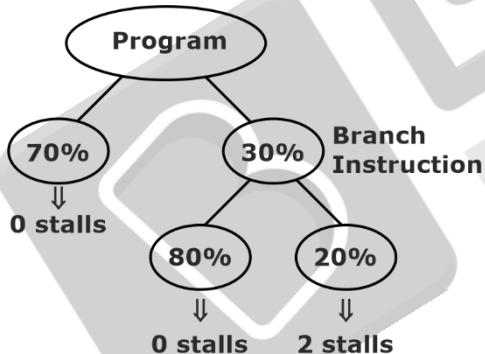
(I) X_1 : without branch prediction



Average CPI = (1 + number of stalls per Instruction)

$$\begin{aligned} \therefore \text{Average instruction execution time } (X_1) &= (1 + \# \text{ stalls/instruction}) * \text{cycle time} \\ &= (1 + 0.6) * 0.5 \text{ n sec} \\ &= 0.8 \text{ n sec} \end{aligned}$$

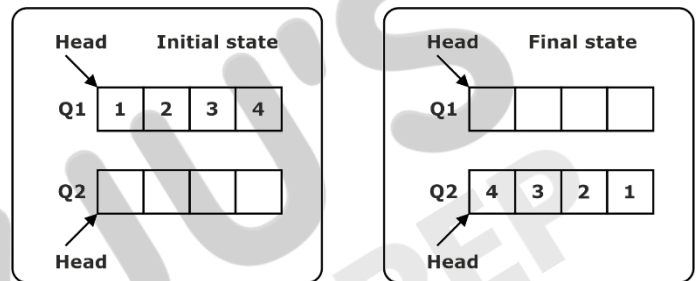
(II) X_2 : with branch prediction



$$\begin{aligned} \therefore \text{Average instruction execution time } (X_2) &= (1 + \# \text{ stalls/instruction}) * \text{cycle time} \\ &= (1 + (0.2 * 0.3 * 2)) * 0.5 \text{ n sec} \\ &= 0.56 \text{ n sec} \end{aligned}$$

$$\begin{aligned} \therefore \text{speed up} &= \frac{\text{ET without prediction}}{\text{ET with prediction}} \\ &= \frac{0.8}{0.56} \\ &= 1.428 \end{aligned}$$

62. Consider the queues Q_1 containing four elements and Q_2 containing none (shown as the Initial State in the figure). The only operations allowed on these two queues are Enqueue(Q , element) and Dequeue(Q). The minimum number of Enqueue operations on Q_1 required to place the elements of Q_1 in Q_2 in reverse order (shown as the Final State in the figure) without using any additional storage is _____.



[NAT: 2 Marks]

Ans. 0

Sol. Here, we need to find the minimum number of Enqueue operations on Q_1 required to place the elements of Q_1 in Q_2 in reverse order without using any additional storage.

So, we have initially



Now, we do the following operation :-

ENQ (Q_2 , DEQ(Q_1))



ENQ (Q_2 , DEQ (Q_1))



Now, we do ENQ (Q_2 , DEQ(Q_2))



Now, ENQ(Q_2 , DEQ(Q_1))



Then, $ENQ(Q_2, DEQ(Q_2)) \Rightarrow Q_2 : \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$

Then, $ENQ(Q_2, DEQ(Q_2)) \Rightarrow Q_2 : \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$

and, $ENQ(Q_2, DEQ(Q_1)) \Rightarrow Q_2 : \begin{bmatrix} 3 & 2 & 1 & 4 \end{bmatrix}$

Again, $ENQ(Q_2, DEQ(Q_2))$

$\Rightarrow Q_2 : \begin{bmatrix} 2 & 1 & 4 & 3 \end{bmatrix}$

Similarly, $ENQ(Q_2, DEQ(Q_2))$

$\Rightarrow Q_2 : \begin{bmatrix} 1 & 4 & 3 & 2 \end{bmatrix}$

Finally, $ENQ(Q_2, DEQ(Q_2))$

$\Rightarrow Q_2 : \begin{bmatrix} 4 & 3 & 2 & 1 \end{bmatrix}$

Thus, 0 enqueue in Q_1 .

- 63.** Consider two files systems A and B, that use contiguous allocation and linked allocation, respectively. A file of size 100 blocks is already stored in A and also in B. Now, consider inserting a new block in the middle of the file (between 50th and 51st block), whose data is already available in the memory. Assume that there are enough free blocks at the end of the file and that the file control blocks are already in memory. Let the number of disk accesses required to insert a block in the middle of the file in A and B are n_A and n_B , respectively, then the value of $n_A + n_B$ is_____.

[NAT: 2 Marks]

Ans. 153

- Sol.** In contiguous allocation we know that random access is possible so we will directly jump to block 51st and from 51st to 100th block we will perform right shift to make space for new block, this process will require the read and write of 50 blocks (51st to 100th) therefore

$2 \times 50 = 100$ block accesses required here. Now to insert new block we require 1 write operation, hence in contiguous we require a total of $2 \times 50 + 1 = 101 = n_A$ block accesses.

For Linked allocation, we have to read 50 blocks to find the middle element as linked allocation supports sequential access only. Then first we will make the pointer of newly added block to point to 51st block and then pointer of 50th block to point to newly added block $\therefore n_B = 50 + 1 + 1 = 52$

$n_A + n_B = 101 + 52 = 153$.

- 64.** Consider a demand paging system with four page frames (initially empty) and LRU page replacement policy. For the following page reference string

7, 2, 7, 3, 2, 5, 3, 4, 6, 7, 7, 1, 5, 6, 1

the page fault rate, defined as the ratio of number of page faults to the number of memory accesses (rounded off to one decimal place) is_____.

[NAT: 2 Marks]

Ans. 0.6

Sol. String provided:

7, 2, 7, 3, 2, 5, 3, 4, 6, 7, 7, 1, 5, 6, 1.

Frames provided: 4.

$\begin{matrix} \times & \times & \checkmark & \times & \checkmark & \times & \checkmark & \times & \times & \times & \checkmark & \times & \times & \times & \times \\ 7, & 2, & 7, & 3, & 2, & 5, & 3, & 4, & 6, & 7, & 7, & 1, & 5, & 6, & 1 \end{matrix}$

7	4 5
2	6
3	1
8	7

Total page faults = 9.

Total excess = 15.

Therefore, page fault rate = $9/15 = 0.6$.

65. Consider the following grammar along with translation rules.

$$S \rightarrow S_1 \# T \quad \{S_{\text{val}} = S_{1.\text{val}} * T_{\text{val}}\}$$

$$S \rightarrow T \quad \{S_{\text{val}} = T_{\text{val}}\}$$

$$T \rightarrow T_1 \% R \quad \{T_{\text{val}} = T_{1.\text{val}} \div R_{\text{val}}\}$$

$$T \rightarrow R \quad \{T_{\text{val}} = R_{\text{val}}\}$$

$$R \rightarrow \text{id} \quad \{R_{\text{val}} = \text{id}_{\text{val}}\}$$

Here # and % are operators and id is a token that represents an integer and id_{val} represents the corresponding integer value. The set of non-terminals is $\{S, T, R, P\}$ and a subscripted non-terminal indicates an instance of the non-terminal.

Using this translation scheme, the computed value of S_{val} for root of the parse tree for the expression $20\#10\%5\#8\%2\%2$ is

_____.

[NAT: 2 Marks]

Ans. 80

Sol. The expression provided is :

$$20 \# 10 \% 5 \# 8 \% 2 \% 2$$

We can solve this expression using rule of associativity and precedence.

Note: # is equivalent to *,

% is equivalent to \div .

Now, % is having higher precedence then # as % is farther from the start symbol. Also, both # and % are left associative.

So,

$$\begin{aligned} &20 \# (10 \% 5) \# ((8 \% 2) \% 2) \\ &= 20 * (10 \div 5) * ((8 \div 2) \div 2) \\ &= 20 * 2 * (4 \div 2) \\ &= 20 * 2 * 2 \\ &= 80 \end{aligned}$$
