IMPORTANT FORMULAS TO REMEMBER

CHAPTER 1: FLUID PROPERTIES

Introduction: Fluid is a substance that deforms continuously under the application of shear (tangential) stress no matter how small the stress may be.

Fluid properties:

Mass density (ρ): It is defined as mass per unit volume.

$$\boxed{\rho = \frac{M}{V}}\ kg/m^3$$

Weight density or specific weight (w): It is defined as weight per unit volume.

$$\boxed{w = \frac{weight}{volume} = \frac{m \times g}{V} = (\rho g)}$$

Specific gravity or (relative density S): It is defined as ratio between density of fluid and density of standard fluid i.e., water

$$\boxed{S = \frac{\rho}{\rho_w}}$$

Specific gravity of Hg is 13.6.

Specific volume (v): It is defined as volume per unit mass.

$$\boxed{v = \frac{V}{m} = \frac{1}{\rho}}\ \left(m^3/kg\right)$$

Viscosity (μ): The viscosity of a fluid is a measure of its resistance to deformation at a given rate. Viscosity can be conceptualized as quantifying the frictional force that arises between adjacent layers of fluid that are in relative motion.

Causes of viscosity

- intermolecular force of cohesion.
- molecular momentum exchange.

Kinematic viscosity (ν):

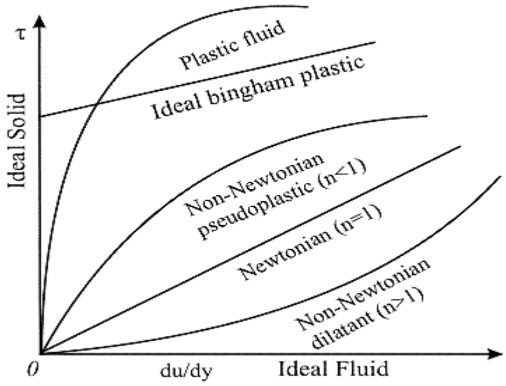
$$\boxed{\nu = \frac{\mu}{\rho}}$$

Newton's Law of Viscosity:

$$\boxed{\tau = \mu \frac{du}{dy}}$$

μ = dynamic viscosity of fluid

Fluid Flow Behaviour:



- General Relationship between shear stress and velocity gradient is given by:

$$\boxed{\tau = \mu \left(\frac{du}{dy}\right)^n + B}$$

- The fluids in which the apparent viscosity increases with increases with velocity gradient(n>1) the fluid is termed as dilatant (shear thickening) fluid.
- The fluids in which the apparent viscosity decreases with increases with velocity gradient(n<1) the fluid is termed as pseudoplastic (shear thinning) fluid.

Surface Tension (σ): Surface tension is the apparent interfacial tensile stress (force per unit length of interface) that acts whenever a liquid has a density interface, such as when the liquid contacts a gas, vapour, second liquid, or a solid. Surface tension is due to intermolecular force.

Gauge pressure inside a water droplet:

$$\boxed{P_{gauge} = \frac{2\sigma}{R}}\ N/m^2$$

Gauge pressure for soap bubble:

$$\boxed{P_{gauge} = \frac{4\sigma}{R}}\ N/m^2$$

Gauge pressure inside a jet of water:

$$\boxed{P_{gauge} = \frac{\sigma}{R}}\ N/m^2$$

Capillarity: This phenomenon is due to interplay of terms of cohesion and adhesion.

Capillarity rise $\boxed{h = \dfrac{4\sigma\cos\theta}{\rho g d}}$

For water: $\theta = 0°$, capillary rise: $h=+ve$.

For Hg: $\theta = 130°$, capillary fall: $h= -ve$.

Vapour pressure and Cavitation: The vapour molecules execute a partial pressure in the space above liquid, known as vapour pressure. Cavitation is formation of vapor bubbles in the liquid flowing through any Hydraulic Turbine.

Compressibility and Bulk Modulus:

- Bulk modulus of elasticity:

$$\boxed{(k) = -\frac{\Delta P}{(\Delta V / V)} = \rho\left(\frac{dp}{d\rho}\right)}$$

- Compressibility(C) of fluid $= \dfrac{1}{\text{Bulk modulus of elasticity}}$

$$\boxed{\begin{array}{l} C = \dfrac{1}{K} = \dfrac{1}{\rho}\left(\dfrac{d\rho}{dp}\right) \\ C = -\dfrac{1}{V}\left(\dfrac{\Delta V}{\Delta P}\right) \end{array}}$$

Isothermal Bulk Modulus: $\boxed{K_T = \rho RT}$

Adiabatic Bulk Modulus: $\boxed{K_A = \gamma(\rho RT)}$

CHAPTER 2: FLUID STATICS

Introduction: Fluid statics deals with fluids at rest. The fluid can be either gaseous or liquid.

Pressure: Pressure is defined as normal force per unit area and its SI unit is N/m^2 or Pascal (Pa).

$$\boxed{\text{Pressure} = \frac{\text{Normal force}}{\text{Area}}}$$

Hydrostatic law: The variation of pressure in a fluid in vertical direction is directly proportional to specific weight.

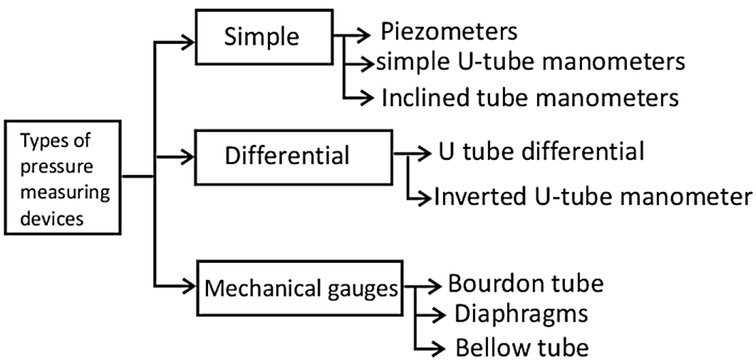
$$\boxed{\frac{dP}{dh} = \rho g}$$

$$\boxed{P_{Gauge} = \rho g h \ N/m^2 \text{or Pascal}}$$

- As we move vertically down is a fluid the pressure increase as +ρgh. As we move vertically up in a fluid the pressure decreases as −ρgh.
- There is no charge g pressure in horizontally same level.

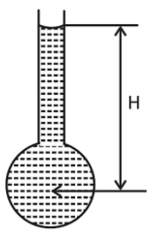
Pascal's Law: According to Pascal's Law in a static fluid, the pressures at a point is equally distributed in all the direction. Ex: hydraulic lift, hydraulic break etc**.**

Pressure measurement:

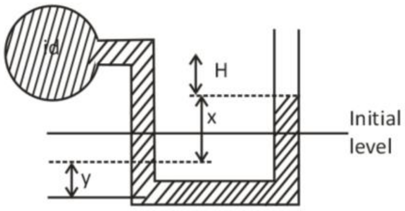


Piezometers: A Piezometer is a simple glass tube that is open at both the ends. Piezometers can't be used to measured very high pressure and gas pressures.

$$\boxed{P = \rho g H}$$



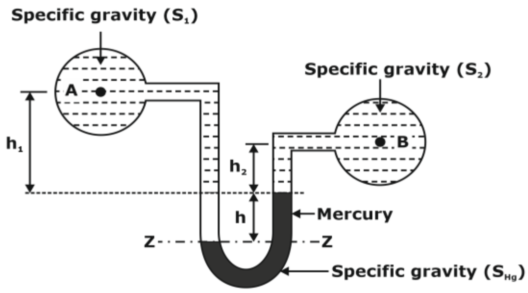
Simple U-tube manometer: It consists of a glass tube with one end open to the atmosphere and other end connected to a point at which pressure is to be measured.



$P_A + \rho_o g (H + x) + \rho_m gy — \rho_m gy — \rho_m gx = 0$

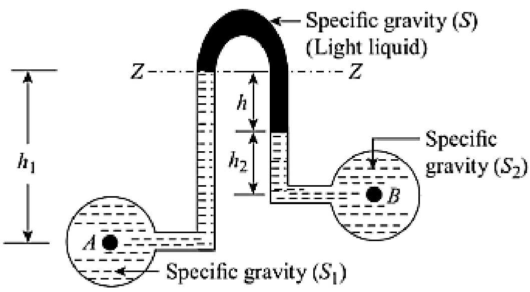
Differential Manometer: A differential manometer is used to measure the difference in pressures in two pipes or two points in the pipeline.

Upright U-tube differential manometer:



$$\boxed{P_A - P_B = \rho_2 gh_2 + (\rho_{Hg} - \rho_1)gh - \rho_1 gh_1}$$

Inverted U-tube differential manometer:



$$\boxed{P_A - P_B = \rho_1 gh_1 - \rho_2 gh_2 - \rho gh}$$

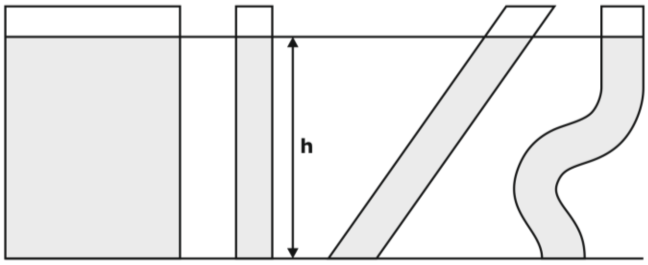
When both the pipes are at the same level i.e. $h_1 = h_2$:

$$\boxed{P_A - P_B = (\rho_1 - \rho_2)gh_1 - \rho gh}$$

Inverted U-tube manometers are used when the pipelines are underground and in these manometers the density of manometric fluid is less than the density of flowing fluid ($\rho_m < \rho$).

The Hydrostatic Paradox:

The pressure at any point depends only upon the depth below the free surface and unit weight of the liquid. It is independent form the size and shape of the container.



Hydrostatic forces: When a fluid is in contact with a surface is exerts a normal force on the surface which is termed as the hydrostatic force.

Hydrostatic forces on submerged plane surfaces:

Plane inclined surface at angle θ:

Force on the surface is given by: $\boxed{F = \rho g A\bar{h}}$

Centre of pressure for the vertical submerged surface is given by:

$$\boxed{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}}$$

h = vertical distance of centre of Pressure from free surface

I_G = moment of inertia at centroidal axis.

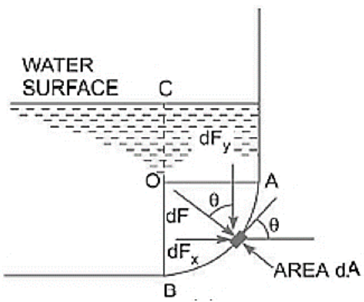
Plane vertical surface (θ = 90°):

$$\boxed{\begin{array}{l} F = \rho g A\bar{h} \\ h^* = \dfrac{I_G \sin^2\theta}{A\bar{h}} + \bar{h} \end{array}}$$

Plane horizontal surface (θ = 0°):

$$\boxed{\begin{array}{l} F = \rho g A\bar{h} \\ h^* = \bar{h} \end{array}}$$

Hydrostatic forces on curved surfaces:



Horizontal component of force on curved surface: Hydrostatic force on the vertical projection area and this force will act at center of pressure of the corresponding area.

Horizontal force: $\boxed{F_X = \gamma A\bar{h}}$

Where,

A = Projected Area

$\bar{h}$ = depth of centroid of an area.

Resultant Force: $\boxed{F_R = \sqrt{F_X^{\ 2} + F_Y^{\ 2}}}$

The angle from the horizontal at which this force will act:

$$\boxed{\tan\alpha = \frac{F_Y}{F_H}}$$

Vertical component of force on curved surface: The vertical component of force on a curved surface is equal to weight of the fluid contained by the curved surface till the free surface and this force will act at the center of gravity of the corresponding weight.

Buoyancy & floatation:

Center of buoyancy (B): Center of buoyancy is defined as the point of application of buoyancy force and this force will act at the centroid of volume of fluid displaced.

Principal of floatation:

Weight of Body = Buoyant Force (F_B)

$\rho_{body}.g.A.h = \rho_f.g.A.x$

(a). If $\rho_{body} > \rho_f$, then body will be submerged totally in the fluid and will rest at the body of the container.

(b). If $\rho_{body} = \rho_f$, then body will be submerged totally in the fluid. In this body will float in the liquid and remains at rest at any point in the fluid if given slight displacement from original position.

(c). If $\rho_{body} < \rho_f$, then body will be partially submerged in the fluid and will float.

For floating bodies:

$$F_B = W$$

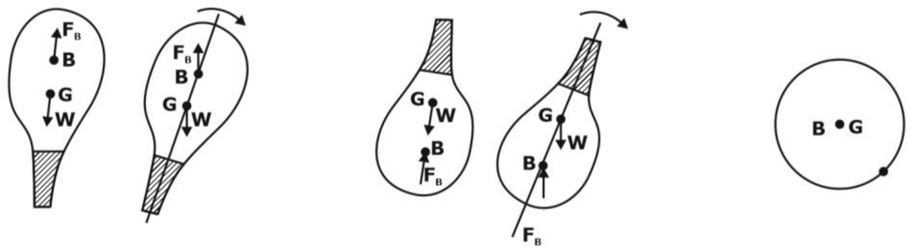
$$\frac{V_{sub}}{V_{total}} = \frac{\rho_{ave,body}}{\rho_f}$$

Stability conditions of a completely submerged bodies under angular deflection:

(i). Stable Equilibrium-B above G.

(ii). Unstable Equilibrium-B below G.

(iii). Neutral Equilibrium-B and G at same point.



(a) STABLE EQUILIBRIUM

(b) UNSTABLE EQUILIBRIUM

(c) NEUTRAL EQUILIBRIUM

Stability conditions of partially submerged bodies under angular deflection:

Metacenter (M) it is defined as the point of intersection of normal Axis and new line of Action of buoyancy force.

(i). Stable Equilibrium-M above G.

(ii). Unstable Equilibrium-M below G**.**

(iii). Neutral Equilibrium-Mand G at same point.

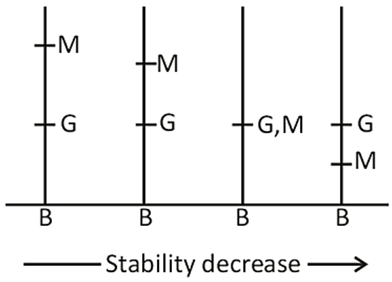
Metacentric height (GM):

Metacentric Height (GM): it is defined as the vertical distance between center of gravity "G" and metacenter "M" which is the intersection point of the lines of action of the buoyant force through the body before and after rotation.

$$(BM)_{xx} = \frac{I_{xx}}{V_{disp}} \; and \; (BM)_{y-y} = \frac{I_{yy}}{V_{disp}}$$

$$(BM)_{xx} > (BM)_{yy}$$

$$(BM)_{pitching} > (BM)_{rolling}$$



Determination of the metacentric height:

Experimental Method:

Consider a ship is floating in water, let w be a movable weight placed centrally on the deck of the ship and W be the total weight of the ship including w.

$$\boxed{\overline{GM} = \frac{wx}{W} \times \frac{l}{d}}$$

If l is the length of the pendulum and d is the distance moved by it on the horizontal scale.

Theoretical Method:

$$\boxed{\overline{GM} = \pm(\overline{BM} - \overline{BG})}$$

In the above expression:

(i). Positive (+Ve) sign is used when the metacentre M lies above the G.

(ii). Negative (- Ve) sign is used when the metacentre M lies below the G.

Where: $\overline{BM} = \frac{I_{body,freesurafce}}{V_{disp}}$

Time period of Oscillation of the floating body:

The time period of the oscillation is given by the following expression:

$$\boxed{T = 2\pi\sqrt{\frac{K_G^{\,2}}{g\overline{GM}}}}$$

Since $\left(\overline{GM}\right)_{pitching} > \left(\overline{GM}\right)_{rolling}$

Increasing the metacentric height gives the greater stability but reduces the time period of the roll so the ship will be less comfortable for the passengers.

CHAPTER 3: FLUID KINEMATICS

Fluid kinematics: It describes the motion of fluid and its consequences without any consideration of the forces invaded.

Flow field: It is the region in which the flow parameters i.e. pressure, velocity, etc. are defined at each and every point at any instant of time.

Approaches to study fluid motion:

Lagrangian approach: In the Lagrangian description of fluid flow, individual fluid particles are "marked," and their positions, velocities, etc. are described as a function of time.

Eulerian approach: In the Eulerian description of fluid flow, individual fluid particles are not identified. Instead, a control volume is defined.

Types of fluid flow:

Steady & Unsteady flow:

For steady flow: $\boxed{\left(\frac{\partial V}{\partial t}\right)_{x_0,y_0,z_0} = 0, \left(\frac{\partial P}{\partial t}\right)_{x_0,y_0,z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} = 0}$

For unsteady flow: $\boxed{\left(\frac{\partial V}{\partial t}\right)_{x_0,y_0,z_0} \neq 0, \left(\frac{\partial P}{\partial t}\right)_{x_0,y_0,z_0} \neq 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0,y_0,z_0} \neq 0}$

Uniform & Non-uniform flow:

For uniform flow: $\boxed{\vec{V} = V(t)}$ and $\boxed{\left(\frac{dV}{ds}\right)_{t=constant} = 0}$

For non-uniform flow: $\boxed{\left(\frac{dV}{ds}\right)_{t=constant} \neq 0}$

Laminar & Turbulent flows:

Laminar flow is the flow in which the fluid particles move along well-defined paths or steam line & all the streamlines are straight & parallel.

Turbulent floe is the flow in which the fluid particles move in a zig zag which results in eddies formation which results in energy loss.

Compressible & incompressible flow:

Compressible flow: $\boxed{\rho \neq constant}$

Incompressible flow: $\boxed{\rho = constant}$

Rotational & Irrotational flows: Rotational flow is the flow in which the fluid particles while flowing along streamlines also rotate about their own axis.

Irrotational flow is the flow in which the fluid particles while flowing along the streamlines do not rotate about their own axis.

Flow pattern description:

Streamlines: It is defined as an imaginary curve or line in the flow field so that the tangent to the curve at any point represents the direction of the instantaneous velocity at that point.

$$\boxed{\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}}$$ equation of streamline in 3D

$$\begin{vmatrix} i & j & k \\ dx & dy & dz \\ u & v & w \end{vmatrix}=0$$

Streak line: It is the focus of all fluid particles which at same instant of time all of which had passed through a common point

Path line: It represents the path traced by an inert tracer fluid particle over a period of time.

Note:

For steady flow, Streamlines, Streak lines & path lines are identical.

Timeline: A timeline is a set of adjacent fluid particles that were marked at the same (earlier) instant in time

Continuity equation:

By continuity equation:

Mass flow rate at inlet = mass flow rate at outlet

Fluid in compressible: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

If fluid is Incompressible i.e. ρ = constant

$A_1V_1 = A_2V_2$

Continuity Equation for Steady and 3 – D Incompressible Flow:

$$\boxed{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0}$$

Continuity Equation for Steady and 2 – D Incompressible Flow:

$$\boxed{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0}$$

Acceleration of fluid flow:

$$a_x=\frac{Du}{Dt}=u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\frac{\partial u}{\partial t}$$

Similarly, acceleration in y and z directions is given by:

$$a_y=\frac{Dv}{Dt}=u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+\frac{\partial v}{\partial t}$$

$$a_z=\frac{Dw}{Dt}=u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}+\frac{\partial w}{\partial t}$$

$$\underset{\text{Total acceleration}}{\frac{D(\ )}{Dt}} = \underset{\substack{\text{because of unsteadiness}\\ \text{(local / temporal acceleration)}}}{\frac{\partial(\ )}{\partial t}} + \underbrace{u\frac{\partial(\ )}{\partial x}+v\frac{\partial(\ )}{\partial y}+w\frac{\partial(\ )}{\partial z}}_{\text{because of non – uniformity}} = \frac{\partial(\ )}{\partial t} + (\vec{V}.\nabla)$$

(convective acceleration)

(i). The total differential D/Dt is known as the **material or substantial derivative** with respect to time.

(ii). The first term $\frac{\partial}{\partial t}$ in the right-hand side of is known as **temporal or local derivative** which expresses the rate of change with time, at a fixed position.

(iii). The last three terms in the right-hand side of the equation, are together known as **convective derivative** which represents the time rate of change due to change in position in the field.

Total acceleration: The total acceleration is given by the following vector:

$$\boxed{\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}}$$

Thus, magnitude of the total acceleration is given by:

$$A = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$$

$$\boxed{\vec{A} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \vec{V}.\nabla\vec{V}}$$

Angular velocity: Angular velocity is given by:

$$\boxed{\vec{\omega} = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}}$$

$$\boxed{\vec{\omega} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix}}$$

Where $\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$, $\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$ and $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$.

Vorticity (Ω): Vorticity (ξ) in the simplest form is defined as a vector which is equal to two times the rotation vector. It is given by:

$$\boxed{\vec{\Omega} = 2\vec{\omega}}$$

Circulation (Γ): Circulation is defined as the line integral of tangential component of velocity vector along a closed curve.

$$\boxed{\text{Circulation}(\Gamma) = \text{Vorticity} \times \text{Area}}$$

Velocity potential(φ): It is a scalar function of space & time such that its negative derivate with respect to any direction gives the fluid velocity in that direction.

$$\boxed{u = -\frac{\partial \phi}{\partial x} \text{ and } v = -\frac{\partial \phi}{\partial y}}$$

The continuity equation is given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi=0$$

$$\nabla^2\phi=0$$

- For flow to be possible velocity potential function must satisfy Laplace equation and velocity potential function only exists for irrotational flow.
- Slope for equipotential line: $\boxed{\left(\frac{dy}{dx}\right)_{\varphi=c}=-\frac{u}{v}}$.

Stream function(Ψ): It is defined as the scalar function of space and time, such that its patrial derivative with respect to any direction gives the velocity component at right angles to that direction.

$$\left.u=\frac{\partial\Psi}{\partial y}\ \text{ and } v=-\frac{\partial\psi}{\partial x}\right\}\text{where, } \psi\rightarrow \text{stream function}.$$

In cylindrical polar co-ordinates: $\boxed{u_r=\frac{1}{r}\frac{\partial\psi}{\partial\theta}\ \text{and}\ u_\theta=-\frac{\partial\psi}{\partial r}}$

- For flow to be possible stream function must exist.
- For irrotational flow stream function must satisfy Laplace equation.
- Slope of equistream line: $\boxed{\left(\frac{dy}{dx}\right)_{\psi=c}=\frac{v}{u}}$.

For 2 – D compressible steady flow: Discharge per unit width between two points in a flow = absolute difference between values of stream function through those two points, i.e.

$$\boxed{\frac{Q_2-Q_1}{b}=\left|\Psi_2-\Psi_1\right|}$$

Relation between equipotential line and streamline:

$$\boxed{\left(\frac{dy}{dx}\right)_{\varphi=c}\times\left(\frac{dy}{dx}\right)_{\psi=c}=\frac{v}{u}\times\left(-\frac{u}{v}\right)=-1}$$

Hence, equipotential lines and streamlines are orthogonal to each other everywhere in flow field except at stagnation point where $V=0$.

CHAPTER 4: FLUID DYNAMICS

Introduction: The dynamic behaviour of fluid is analysed by Newton's Second law of motion, which relates acceleration with the forces.

Equation of motion for fluid flow:

According to newton's second law:

$$\boxed{\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}}$$

For uniform flow: $\frac{d\vec{v}}{dt}$

$$\boxed{\vec{F} = \vec{v}\frac{dm}{dt}}$$

- Net force $F_x = F_g + F_p + F_v + F_T + F_C$
- Reynold's equation of motion: $F_x = F_g + F_p + F_v + F_T$
- Navier-Stokes equation of motion: $F_x = F_g + F_p + F_v$,
- Euler equation of motion: $F_x = F_g + F_p$,

Euler's Equation: This is the equation of motion in which the forces due to gravity and pressure are taken into consideration.

Assumptions:

(a). Flow is non-viscous, $F_v=0$ i.e., viscous forces are zero.

(b). Flow is ideal.

$$\boxed{\frac{dp}{\rho} + vdv + gdz = 0}$$

This equation is known as Euler's equation of motion.

Bernoulli's Theorem:

Assumptions:

(i). Fluid is Non-viscous.

(ii). Steady flow.

(iii). incompressible fluid.

(iv). ideal fluid.

(v). irrotational flow.

$$\boxed{\frac{P}{\rho} + \frac{v^2}{2} + gz = const.}$$

Where,

$\frac{P}{\rho g}$ = Pressure energy of fluid per unit weight of fluid.

$\frac{v^2}{2g}$ = kinetic energy per unit weight or kinetic head.

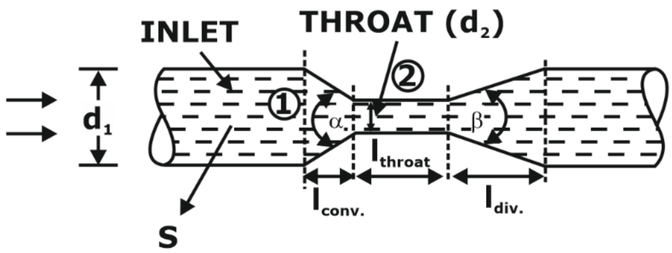
Z = potential energy per unit weight or potential head

Classical Bernoulli's equation is also known as 'mechanical energy conservation Bernoulli's theorem state that for a non-viscous, steady and incompressible flow along a streamline the summation of kinetic energy, potential energy and Pressure energy is a constant.

$$\boxed{\frac{P}{\rho g}+z=Piezometric\,head}$$

Applications of Bernoulli's equation:

Venturi meter: It is a converging-diverging device that is used to measure discharge.



Geometric details: $d_2=\left(\frac{1}{3}to\frac{1}{2}\right)d_1$ [To avoid cavitation]

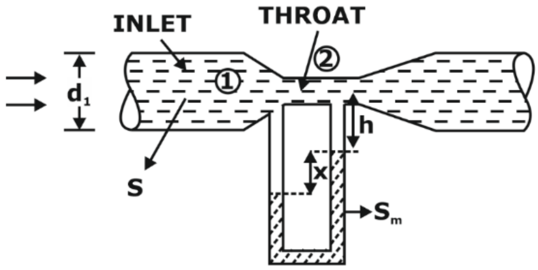
Angle of convergence (α) = 20 — 22°

Angle of divergence (β) = must be below 7° (To avoid Boundary layer separation).

$$Q_{actual}=C_d\times\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh}$$

$$C_d=\frac{Q_{actual}}{Q_{theoretical}}$$

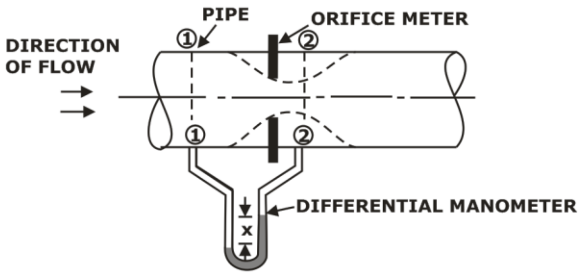
Calculation of h: The value of h is calculated with the help of the deflection of the manometric column.



Case 1: S_m > S (flowing fluid), then: $\boxed{h=x\left(\frac{S_m}{S}-1\right)}$

Case 2: S_m < S (flowing fluid), then: $\boxed{h=x\left(1-\frac{S_m}{S}\right)}$

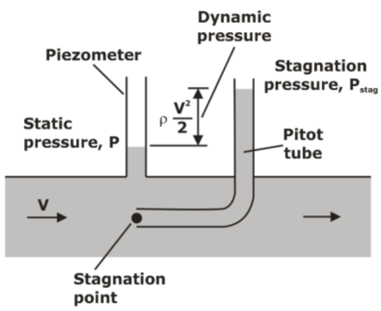
Orifice-meter: An orifice meter is essentially a thin circular plate with a sharp edged concentric circular hole in it.



Coefficient of contraction (C_c): $\boxed{C_c = \frac{\text{Actual Area at vena-contracta}}{\text{Theoretical Area at vena-contracta}} = \frac{A_2}{A_o}}$

Discharge: $\boxed{Q = C_d \cdot \frac{A_1 A_o}{\sqrt{A_1^2 - A_o^2}} \times \sqrt{2gh}}$

Pitot tube: A right-angled glass tube, large enough for capillary effects to be negligible, is used for the purpose. One end of the tube faces the flow while the other end is open to the atmosphere



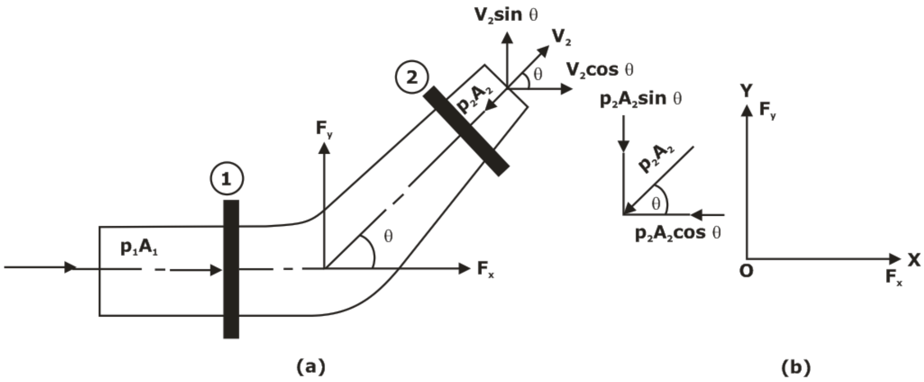
$$\boxed{V_1 = \sqrt{2gh} = \sqrt{2g\left(\text{stagnation head} - \text{static head}\right)}}$$

$$h = x\left[\frac{S_m}{S} - 1\right]$$

Momentum equation:

F.dt = d(mv)

which is known as the impulse-momentum equation and states that impulse of a force F acting on a fluid of mass m in a short interval of time dt is equal to the change of momentum d(mv) in the direction of force.



$\therefore$ $F_x = \rho Q (V_1 - V_2 \cos\theta) + p_1A_1 - p_2A_2 \cos\theta$

$\therefore$ $F_y = \rho Q (- V_2 \sin\theta) - p_2A_2 \sin\theta$

Resultant force: $\boxed{F_R = \sqrt{F_x^2 + F_y^2}}$

The angle made by the resultant force with horizontal direction is given by:

$$\boxed{\tan\theta = \frac{F_y}{F_x}}$$

Moment of momentum equation: It states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

According to moment of momentum principle:

Resultant torque = rate of change of moment of momentum

$T = \rho Q [V_2r_2 - V_1r_1]$

Applications:

1. For analysis of flow problems in turbines and centrifugal pumps.
2. For finding torque exerted by water on sprinkler

Vortex flow: Vortex flow is defined as the flow of fluid along a curved path or the flow of a rotating mass of fluid is known as vortex flow.

Equation of Motion for Vortex Flow:

$$\boxed{dp = \rho \frac{v^2}{r} dr - \rho g dz}$$

Plane vortex flows: It is the flow where streamlines are concentric circles.

$V_r = 0$ and $V_\theta \neq 0$ and $\frac{\partial V_\theta}{\partial \theta} = 0$

Forced Vortex Flow: In this type of flow, external torque is required to rotate the fluid mass.

$$\boxed{v = w \times r \Rightarrow \omega = \frac{v}{r} = \text{constant}}$$

$$P_2 - P_1 = \frac{\rho}{2}\left[v_2^2 - v_1^2\right] - \rho g\left[z_2 - z_1\right] \quad \left\{\begin{matrix} \because & v_2 = \omega r_2 \\ & v_1 = \omega r_1 \end{matrix}\right\}$$

where r = Radius of fluid particle from the axis of rotation.

For free surface of the liquid, then $P_1 = P_2$:

$$\boxed{\left[z_2 - z_1\right] = \frac{1}{2g}\left[v_2^2 - v_1^2\right]}$$

Note: In case of forced vortex, the rise of liquid level at the ends is equal to the fall of liquid level at the axis of rotation.

$$\boxed{\text{volume of paraboloid} = \frac{1}{2}\left[\text{Area of cross} - \text{section} \times \text{Height of parabola}\right]}$$

Examples: Flow of liquid inside the impeller of a centrifugal pump, Flow of water through the runner of a turbine

Free Vortex Flow: Total mechanical energy remains constant in the entire flow field i.e. does not vary from streamline to streamline. There is neither any addition nor any destruction of energy in the flow field.

$$vr = \frac{\text{Constant}}{m} = \text{Constant}$$

Examples: Flow of liquid through a hole provided at the bottom of a container, Flow of liquid around a circular bend in a pipe, A whirlpool in a river, Flow of fluid in a centrifugal pump casing.

CHAPTER 5: LAMINAR FLOW THROUGH PIPES & PLATES

Introduction: Laminar flow (streamline flow) occurs when a fluid flows in parallel layers, with no disruption between the layers.

Reynold's number: $\boxed{R_e = \frac{\rho VD}{\mu} = \frac{VD}{\upsilon}}$

Where, V=mean velocity of flow in pipe.

μ = dynamic viscosity of the liquid ($N - s/m^2$)

v = Kinematic viscosity of the liquid (m^2/s)

D= diameter → for pipe, D = length → for plate

Hydraulic Diameter: For the flow through non – circular pipes characteristic dimension is given by:

$$\boxed{D_h = \frac{4A_c}{P}}$$

Where,

A_c = Cross – section area of the pipe

P = Perimeter of the pipe

Pipe	Plate
$R_e < 2000$ laminar $2000 < R_e < 4000$ Transient $R_e > 4000$ turbulent	$R_e < 5 \times 10^5$ Laminar $R_e > 5 \times 10^5$ turbulent [transient is small so neglected]

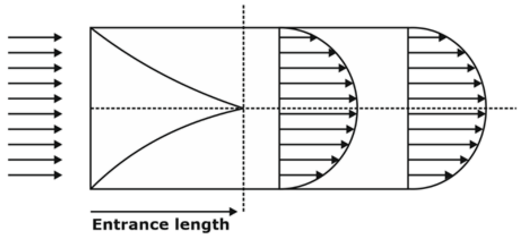
Hagen-Poiseuille flow [Laminar flow in a pipe]:

Assumptions:

(1). steady flow.

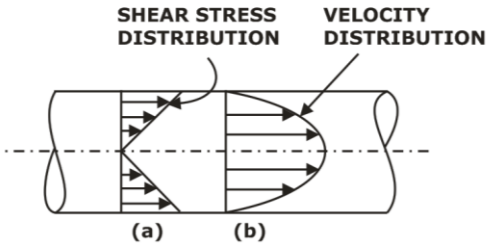
(2). flow is fully developed.

Velocity profile is constant w.r.t length in the fully developed flow.



$$\boxed{\frac{\partial P}{\partial x} = \text{constant}}$$

Laminar flow through circular pipe:



Shear stress: $\boxed{\tau = \frac{-\partial P}{\partial x} \times \frac{r}{2}}$

It varies linearly along the pipe and it becomes maximum at r=R.

Velocity variation: $\boxed{u = \frac{-R^2}{4\mu}\left(\frac{\partial P}{\partial x}\right)\left(1 - \frac{r^2}{R^2}\right)}$

Velocity varies parabolically along the pipe and it becomes maximum at r = 0.

Maximum velocity: $\boxed{U_{max} = \frac{-R^2}{4\mu}\left(\frac{\partial P}{\partial x}\right)}$

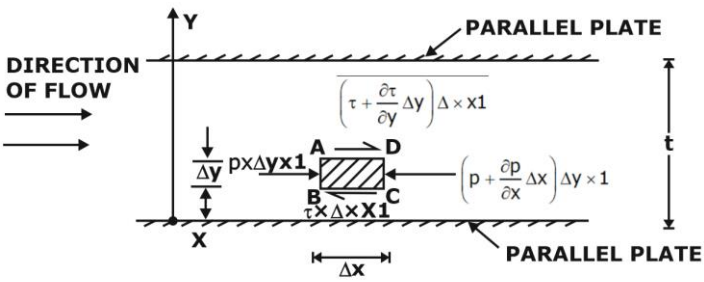
$\boxed{u_{avg.} = \frac{u_{\max}}{2}}$

Pressure drop: $\boxed{P_1 - P_2 = \frac{32\mu vL}{D^2} = \frac{8\mu vL}{R^2}}$

Friction factor: $\boxed{f = \frac{64}{R_e}}$

coefficient of friction: $\boxed{f' = \frac{f}{4} = \frac{16}{\text{Re}}}$

Laminar flow between two fixed parallel plates [with unit width]:



Laminar flow between parallel plates when one of the plates is moving and other one is stationary is known as **"Couette"** flow.

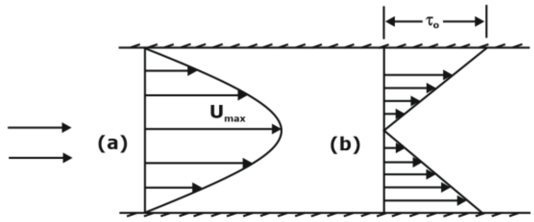
$\boxed{\frac{\partial P}{\partial x} = \frac{\partial \tau}{\partial y}}$

Velocity Distribution: $u = \frac{1}{2\mu}\left(\frac{-\partial P}{\partial x}\right)\left(By - y^2\right)$

Maximum velocity: $u_{max} = -\frac{t^2}{8\mu}\left(\frac{\partial P}{\partial x}\right)$

$V_{avg.} = \frac{2}{3}U_{max} = Average\ velocity$

Shear Stress distribution: $\tau = \left(\frac{-\partial P}{\partial x}\right)\left(\frac{t}{2} - y\right)$



Discharge per unit width: $Q = \frac{-1}{12\mu}\left(\frac{\partial P}{\partial x}\right)t^3$

Pressure difference: $P_1 - P_2 = \frac{12\mu V_{avg.}L}{t^2}$

Correction factors:

Momentum correction factor (β): It is defined as the ratio of momentum per second based on actual velocity to the momentum per second based on average velocity across a section. It is denoted by β.

$$\beta = \frac{1}{AV^2}\int_0^R u^2 dA$$

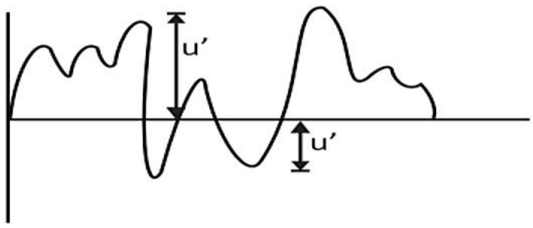
Kinetic energy correction factor (α): It is defined as the ratio of kinetic energy of flow per second based on actual velocity to the kinetic energy of the flow per second based on average velocity across a same section.

$$\alpha = \frac{1}{AV^3}\int_0^R u^3 dA$$

Correction-factor	Laminar	Turbulent
α	2	1.33
β	1.33	1.2

CHAPTER 6: TURBULENT FLOW

Introduction: In case of turbulent flow there is huge order intermission fluid particles and due to this, various properties of the fluid are going to change with space and time.



$\bar{u}$ = mean or average component of velocity

Any parameter = Average component + fluctuating component

$$\boxed{u = u' + \bar{u}}$$

Shear stress in turbulent flow:

Shear stress due to turbulence: $\tau_t = \eta \dfrac{d\bar{u}}{dy}$ (Boussinesq Hypothesis).

Kinematic eddy viscosity: $\varepsilon = \dfrac{\eta}{\rho}$

Total shear stress: $\boxed{\tau = \tau_v + \tau_t = \mu \dfrac{du}{dy} + \eta \dfrac{d\bar{u}}{dy}}$

Reynold's hypothesis: $T_{turbulent.} = \rho\, u'\, v'$

According to Prandtl Mixing length hypothesis: $u' = v' = l \dfrac{du}{dy}$

l = Prandtl's mixing length

$$\tau_{turb.} = \rho l^2 \left(\frac{du}{dy}\right)^2$$

$$\tau_{total} = \underbrace{\tau_{laminar}}_{\text{Negligible}} + \tau_{turb.}$$

Relation of Shear Stress with Coefficient of friction: $\boxed{f = \dfrac{2\tau_0}{\rho V^2}}$

Velocity distribution in turbulent flow:

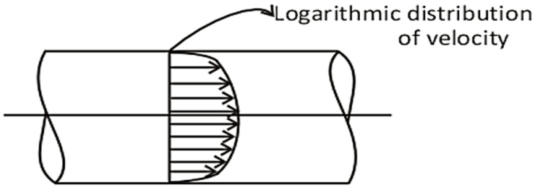
Shear or Fictitious velocity: $\boxed{u^* = \sqrt{\dfrac{\tau_0}{\rho}}}$

$$\boxed{u = u_{max} + 2.5\, u^* \log_e \left(\frac{y}{R}\right)}$$

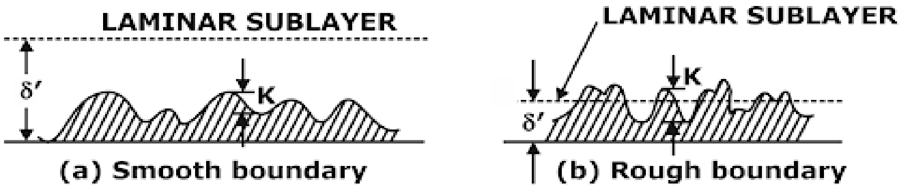
The above equation is known as 'Prandtl's universal velocity distribution equation for turbulent flow in pipes.

$$\frac{u_{max} - u}{u^*} = 5.75 \log_e(R / y)$$

The difference between the maximum velocity u_{max}, and local velocity u at any point i.e. (u_{max} - u) is known as 'velocity defect'.



Hydrodynamically smooth & rough pipes:



K = average height of roughness

δ' = height of Laminar sublayer

Nikuradse's conditions for smooth & rough boundary.

Nikuradse's conditions	In terms of roughness Reynolds number $\left(\frac{u^* k}{\nu}\right)$
$\frac{K}{\delta'} < 0.25 \rightarrow smooth$	If $\frac{u^* k}{\nu} < 4$, boundary is considered smooth.
$0.25 < \frac{K}{\delta'} < 6 \rightarrow Transition$	If $4 < \frac{u^* k}{\nu} < 100$, boundary is in transition stage
$\frac{K}{\delta'} > 6 \rightarrow Rough$	If $\frac{u^* k}{\nu} > 100$, the boundary is rough

Laminar sublayer thickness: $\delta' = \frac{11.6\upsilon}{V_s}$

shear friction velocity or hypothetical velocity: $V_s = \sqrt{\frac{\tau}{\rho}} = V.\sqrt{\frac{f}{8}}$ =

and V is the average velocity.

Velocity distribution for turbulent flow:

Velocity distribution in rough pipes: $\frac{u}{u^*} = 5.75\log_{10}\left(\frac{y}{k}\right) + 8.5$

Average velocity in Rough pipes: $\boxed{\dfrac{\bar{U}}{u^*} = 5.75\log_{10}\left(\dfrac{R}{K}\right) + 4.75}$

Velocity distribution in rough pipes: $\boxed{\dfrac{u}{u^*} = 5.75\ \log_{10}\left(\dfrac{u^* y}{v}\right) + 5.5}$

Average velocity in smooth pipes: $\boxed{\dfrac{\bar{U}}{u^*} = 5.75\ \log_{10}\left(\dfrac{u^* R}{v}\right) + 1.75}$

Now: $\boxed{\dfrac{u - \bar{U}}{u^*} = 5.75\log_{10}(y / R) + 3.75}$

Velocity distribution for turbulent flow with average velocity in smooth pipes for power law:

$\boxed{\dfrac{u}{u_{max}} = \left(\dfrac{y}{R}\right)^{1/n}}$ and $\boxed{n = \dfrac{1}{7} \Rightarrow \dfrac{u}{u_{max}} = \left(\dfrac{y}{R}\right)^{1/7}}$

Resistance of smooth and rough pipes:

The term of the right-hand side is called co-efficient of friction f. Thus:

$\boxed{f = \phi\left[R_e, \dfrac{k}{D}\right]}$

This equation shows that friction co-efficient is a function of Reynolds number and k/D ratio, where k is the average height of pipe wall roughness protrusions.

Friction factor in turbulent flow:

(a). For Smooth pipes:

The value of 'f' for smooth pipe for Reynolds number varying from 4000 to 100000 is given by the relation:

$\boxed{f = \dfrac{.0791}{(R_e)^{1/4}}}$

When, Re = 10^5 to 4×10^7 $\Rightarrow$ $\boxed{f = 0.0032 + \dfrac{0.221}{Re^{0.232}}}$

(b). For rough pipes:

$\boxed{\dfrac{1}{\sqrt{4f}} = 2\log_{10}\left(\dfrac{R}{K}\right) + 1.74}$ When, Re > 6×10^8

CHAPTER 7: LOSSES THROUGH PIPES

Introduction: Major loss is due to friction and calculated by Darcy Weisbach equation and Chezy's formula**.**

Minor loss into five types of losses i.e., sudden expansion loss, sudden contraction loss, exit loss, entry loss and bend losses.

Major losses:

Darcy Weisbach equation: $\boxed{h_f = \frac{fLV^2}{2gD} = \frac{4f'LV^2}{2gD}}$

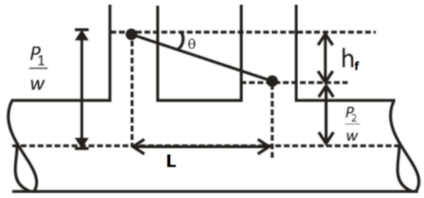
Where, f is friction factor

f = 4f',

where f' is friction coefficient

V is mean velocity of flow in the pipe, D is diameter of the pipe, L is length of the pipe

Chezy's formula:



$\boxed{V = C\sqrt{mi}}$

Where, C = Chezy's constant

V = Average velocity

i = hydraulic slope

Hydraulic mean depth: $\boxed{m = \frac{Area\ of\ cross-section}{wetted\ perimeter} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4}}$

Minor losses:

Sudden expansion loss (h_l)$_{se}$: $\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$

Sudden contraction loss: $\boxed{(h_l)_{S.C.} = \left(\frac{1}{C_C} - 1\right)^2 \frac{V_2^2}{2g} = k\frac{V_2^2}{2g}}$

Entry losses: $\boxed{(h_l)_{entry} = 0.5\frac{V^2}{2g}}$

Exit losses: $\boxed{(h_l)_{exit} = \frac{V^2}{2g}}$

Bend losses: $\boxed{(h_l)_{Bend} = K\frac{V^2}{2g}}$

Hydraulic gradient line (HGL) & total energy line (TEL):

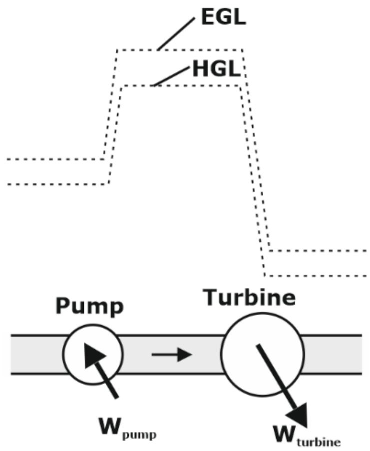
(i). The line that represents the sum of the static pressure and the elevation heads $\frac{p}{\rho g}+z$ is called the hydraulic grade line.

(ii). The line that represents the total head of the fluid $\frac{p}{\rho g}+\frac{V^2}{2g}+z$ is called the energy grade line.

(iii). The difference between the heights of EGL and HGL is equal to the dynamic head $\frac{V^2}{2g}$

.

(iv). For *stationary bodies* such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid. The elevation of the free surface *z* in such cases represents both the EGL and the HGL since the velocity is zero and the static pressure (gage) is zero.
(v). *A steep jump or droop* occurs in EGL and HGL whenever mechanical energy is added to the fluid (by a pump or mechanical energy is removed from the fluid (by a turbine) respectively.



Pipes in series & parallel

(a). Pipes in series:

$$\boxed{Q_1 = Q_2 + Q_3 + Q_4 \ldots\ldots\ldots Q_n = Q = const.}$$

$$\boxed{h_l = h_{l_1} + h_{l_2} + h_{l_3} \ldots\ldots h_{l_n}}$$

Equivalent length is given by (Dupit's formula):

$$\boxed{\frac{L_e}{D_e^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots\ldots\ldots}$$

(b). Pipes in Parallel:

$Q = Q_1 + Q_2 + Q_3 \ldots\ldots\ldots\; Q_n$

$h_{l_1} = h_{l_2} = h_{l_3} = \ldots\ldots\ldots h_{l_n}$

For "n" pipes → *Discharge through each pipe* $= \frac{Q}{n}$

$h_{l_e} = (h_l)$

$$\boxed{\frac{fL_eQ^2}{12D_e^5} = \frac{fLQ^2}{n^2 12D^5} \Rightarrow \frac{L_e}{D_e^5} = \frac{L}{n^2D^5}}$$

(When all pipes are similar).

$$\boxed{\sqrt{\frac{D_e^5}{L_e}} = \sqrt{\frac{D_1^5}{L_1}} + \sqrt{\frac{D_2^5}{L_2}} + \sqrt{\frac{D_3^5}{L_3}} + \ldots\ldots}$$

(When pipes are different & head loss is same).

Power transmission through pipes:

$$P_{th.} = \rho gQH$$
$$P_{act} = \rho gQ\left(H - h_f\right)$$

Efficiency is given by:

$$\boxed{\eta = \frac{P_{act}}{P_{th.}} = \frac{H-h_f}{H}}$$

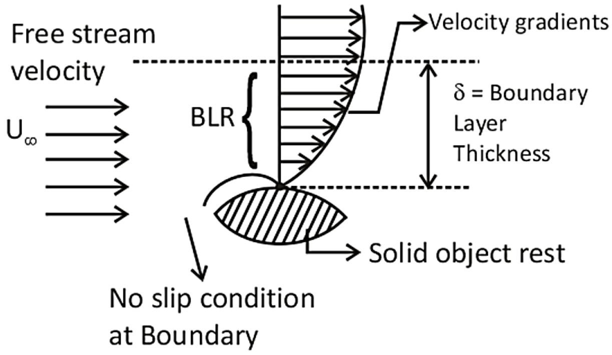
Condition for maximum power transmission:

$\boxed{h_f = \frac{H}{3}}$ and $\boxed{\eta_{\max} = \frac{H-h_f}{H} = 66.67\%}$

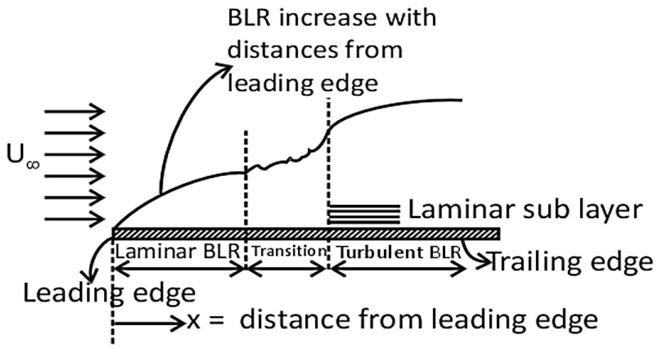
CHAPTER 8: BOUNDARY LAYER THEORY

Introduction: It is a narrow thin region near the solid boundary where velocity gradient exists.

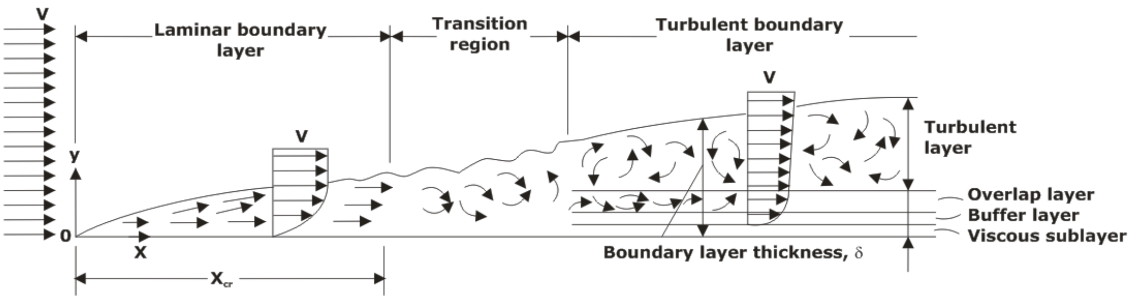
Growth of boundary layer over a flat plate:



The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the **boundary layer.**



Growth of boundary layer over a flat plate:



Boundary conditions:

(i). at $y = 0$, $u = 0$

(ii). $y = \delta$, $u = u_\infty$

(iii). $y = \delta$, $\frac{du}{dy} = 0$

(iv). $x = 0$, $\delta = 0$

Boundary layer thickness (δ): It is defined as the vertical distance from boundary up to the point where velocity becomes 99% of free stream velocity.

For numerical, at y = δ; u = u_∞

Displacement thickness (δ*): It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation. It is denoted by δ*.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$$

Momentum thickness (θ): It is defined as "the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation".

$$\theta = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$$

Energy thickness (δ_e): It is defined as "the distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation".

$$\delta_e = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u^2}{u_\infty^{\ 2}}\right) dy$$

Shape Factor(H): It is defined as the ratio of displacement thickness to momentum thickness.

$$H = \frac{\delta^*}{\theta}$$

Flow over a flat plate: Von-Karman momentum integral equation:

$$\frac{\tau_0}{\rho v_\infty^2} = \frac{d\theta}{dx}$$

Reynolds number is given by: $\mathrm{Re} = \frac{\rho u_\infty L}{\mu}$

If, Re < 5×10^5 → Laminar flow.

Re > 5×10^5 → turbulent flow.

Coefficient of Drag (C_D)

$$F_D = \frac{C_D^{\ *}}{2} \rho A U^2$$

Where, F_D is the drag force experienced by the motion of fluid over flat plate C_D^* = local skin friction coefficient.

Analysis of turbulent boundary layer:

(a). If Reynold number is more than 5×10^5 and less than 10^7 the thickness of boundary layer and drag co-efficient are given as:

$$\delta = \frac{0.37x}{\left(R_{e_x}\right)^{1/5}} \quad \text{and} \quad C_D = \frac{0.072}{\left(R_{e_L}\right)^{1/5}}$$

where x = Distance from the leading edge

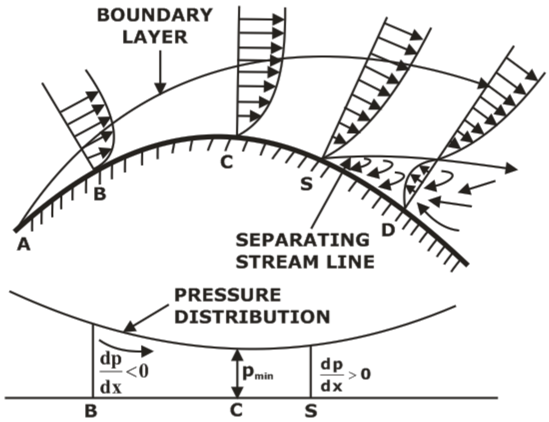
R_{e_x} = Reynold number for length x

R_{e_L} = Reynold number at the end of the plate.

(b). If Reynold number is more than 10^7 but less than 10^9, Schlichting gave the empirical equation as:

$$C_D = \frac{0.455}{\left(\log_{10} R_{e_L}\right)^{2.58}}$$

Boundary layer separation:



The separation point S is determined from the condition: $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$

1. If $\left(\frac{\partial u}{\partial y}\right)_{y=0} > 0$, the flow has separated, and it is the necessary condition for flow separation.

2. If $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$, the flow is on the verge of separation. This is the sufficient condition for the flow separation to occur.

3. If $\left(\frac{\partial u}{\partial y}\right)_{y=0} < 0$, the flow will not separate, or flow will remain attached with the surface.

CHAPTER 9: DIMENSIONLESS ANALYSIS

Introduction: length(L), Mass(M), Time(T) dimensions are called fundamental dimension.

Some important dimensions:

Area	A	L^2
Volume	V	L^3
Velocity	v	LT^{-1}
Angular velocity	ω	T^{-1}
Acceleration	a	LT^{-2}
Angular Acceleration	α	T^{-2}
Discharge	Q	L^3T^{-1}
Acceleration due to Gravity	g	LT^{-2}
Force	F	MLT^{-2}
Weight	W	MLT^{-2}
Dynamic viscosity	μ	$ML^{-1}T^{-1}$
Kinematic viscosity	ϑ	L^2T^{-1}
Pressure	p	$ML^{-1}T^{-2}$
Surface Tension	σ	MT^{-2}
Work/Energy	E	ML^2T^{-2}
Power	P	ML^2T^{-3}

Methods of dimensional analysis:

(i). Rayleigh method:

It is used for determining the expression for a variable which depends maximum on three or four variables.

Let X be a variable which depends on X_1, X_2, X_3 then, according to Rayleigh method

$X=f(X_1, X_2, X_3)$

$$X = KX_1^{a}X_2^{b}X_3^{c}$$

(ii). Buckingham π-theorem:

Buckingham's π theorem states that "If there are n variables in a problem and these variables contain m primary dimensions (for example M, L, T) the equation relating all the variables will have (n-m) dimensionless groups".

Buckingham referred these groups as π groups. The final equation obtained is in the form of $\pi_l = f(\pi_2, \pi_3 \ldots\ldots \pi_{n-m})$.

Method of selecting repeating variables: The number of repeating variables is equal to the number of fundamental dimensions of the problem. The choice of repeating variables is governed by following considerations:

(a). Dependent variable should not be repeating variable

(b). Repeating variable should be selected in such a way that one of the variables should be geometric property, other should be flow property and third variable should be fluid property.

Various forces in fluid mechanics:

Inertia force (F_i): $\boxed{F_i = \rho l^2 v^2}$

Surface tension force (F_s): It is equal to the product of surface tension and length of surface of the flowing fluid.

$F_s = \sigma \times l$

Gravity force (F_g): It is equal to the product of mass and acceleration due to gravity of the flowing fluid.

$F_g = \rho l^3 g$

Pressure force (F_P): It is equal to the product of pressure intensity and cross-sectional area of the flowing fluid

$F_P = Pl^2$

Viscous force (F_v): It is equal to the product of shear stress (τ) due to viscosity and surface area of the flow.

$F_v = \mu l v$

Elastic force (F_e): It is equal to the product of elastic stress and area of the flowing fluid.

$F_e = Kl^2$

Dimensionless numbers:

Dimensionless Number with Symbol	Group Variables	Significance	Field of Use
1. Reynolds number (R_e)	$\frac{\rho VL}{\mu}$	$\frac{\text{inertia force}}{\text{viscous force}}$	Laminar viscous flow in confined passages where viscous effects are predominant.
2. Froude number (F_r)	$\frac{V}{\sqrt{lg}}$	$\frac{\text{inertia force}}{\text{gravitational force}}$	Free surface flows where effects of gravity are important.
3. Mach number (M)	$\frac{V}{\sqrt{K/\rho}}$	$\frac{\text{inertia force}}{\text{elastic force}}$	High speed flow where compressible effects are important.
4. Weber number (W)	$\frac{\rho l V^2}{\sigma}$	$\frac{\text{inertia force}}{\text{surface tension force}}$	Small surface waves, capillary and sheet flow where surface tension is important.
5. Euler number (E)	$\frac{p}{\rho V^2}$	$\frac{\text{inertia force}}{\text{pressure force}}$	Conduit flow where pressure variations are significant.

Similitude & modeling:

Geometric similarity:

Length ratio: $\frac{l_m}{l_p} = \frac{b_m}{b_p} = \frac{t_m}{t_p} = L_r$

Area ratio: $\frac{A_m}{A_P} = \frac{l_m b_m}{l_p b_p} = L_r^{\ 2}$

Volume ratio: $\frac{V_m}{V_p} = L_r^{\ 3}$

Kinematic similarity: Kinematic similarity means the similarity of motion between the model and prototype.

$$\boxed{\frac{V_{1m}}{V_{1P}} = \frac{V_{2m}}{V_{2p}} = \frac{V_{3m}}{V_{3p}}}$$

Dynamic Similarity: A model and prototype are said to be in dynamic similarity, if the ratio of forces at corresponding points in model and prototype are same.

$$\frac{(F_v)_{1m}}{(F_V)_{1p}} = \frac{(F_i)_{1m}}{(F_i)_{1p}} = \frac{(F_g)_{1m}}{(F_g)_{1p}} = \frac{(F_p)_{1m}}{(F_P)_{1p}}$$

$$\frac{(F_i)_{1m}}{(F_v)_{1m}} = \frac{(F_i)_{1p}}{(F_v)_{1p}} \Rightarrow (\mathrm{R_e})_{1m} = (\mathrm{R_e})_{1p} \rightarrow \text{Reynolds model law}$$

Model laws: The laws on which the models are designed for dynamic similarity are called model laws or laws of similarity. The followings are the model laws:

Reynold's Model Law: Reynold's model law is the law in which models are based on Reynold's number.

$$[\mathrm{R_e}]_m = [\mathrm{R_e}]_P \quad \text{or} \quad \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_P V_P L_P}{\mu_P}$$

Froude's Model Law:

$$(\mathrm{F_e})_{\text{model}} = (\mathrm{F_e})_{\text{prototype}} \Rightarrow \frac{V_m}{\sqrt{g_m L_m}} = \frac{V_P}{\sqrt{g_P L_P}}$$

$$\boxed{\frac{V_P}{V_m} = \sqrt{\frac{L_P}{L_m}} = \sqrt{L_r}}$$

$$\boxed{a_r = 1}$$

$$\boxed{Q_r = L_r^{2.5}}$$

Distorted Models: A model is said to be distorted if it is not geometrically similar to its prototype. For a distorted model different scale ratio for the linear dimensions are adopted. For example, in case of rivers, harbours, reservoirs etc., two different scale ratios, one for horizontal dimensions and other for vertical dimensions are taken.

CHAPTER 10: FLOW THROUGH JET

Introduction: A jet is a stream of fluid that is projected into a surrounding medium, usually from some kind of a nozzle, aperture or orifice.

Force exerted by fluid jet:

(a). Force exerted on a stationary flat plate held normal to jet:

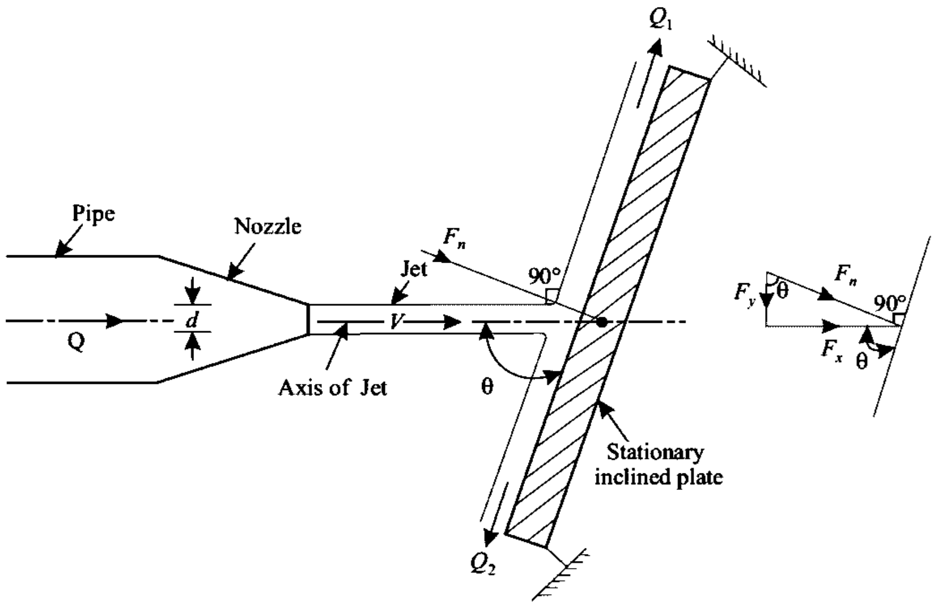
$F_x = \rho a V^2$

V = velocity of jet,

d = diameter of the jet

a = area of cross section of the jet

(b). Force of Jet Impinging on An Inclined Fixed Plate:



Force normal to plate: $F_n = \rho aV(V\sin\theta - 0) = \rho aV^2 \sin\theta$

Horizontal component in x direction:

$$\boxed{F_x = F_n \sin\theta = \rho aV^2 \sin^2\theta}$$

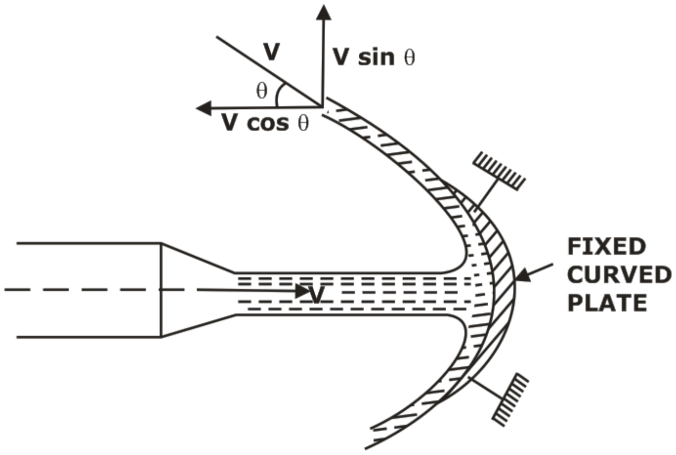
Horizontal component in x direction:

$$\boxed{F_x = F_n \cos\theta = \rho aV^2 \sin\theta\cos\theta}$$

$\boxed{Q_1 = \frac{Q}{2}(1+\cos\theta)}$ and $\boxed{Q_2 = \frac{Q}{2}(1-\cos\theta)}$

Force exerted on stationary curved plate:

(a). Jet strikes the curved plate at the centre:



Force in x-direction is given by:

$$\boxed{F_x = \rho AV(V_{1x} - V_{2x})}$$

$$\boxed{F_x = \rho AV(V - (-V\cos\theta)) = \rho aV^2(1 + \cos\theta)}$$

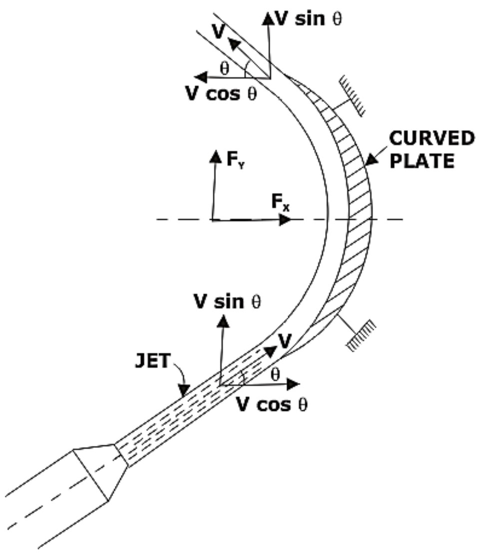
Force in y-direction is given by:

$$\boxed{F_y = \rho AV(V_{1y} - V_{2y})}$$

$$\boxed{F_y = \rho AV[0 - V\sin\theta] = -\rho AV^2 \sin\theta}$$

Negative sign indicates that force is in downward direction.

(b). Jet strikes the stationary curved plate at one end tangentially when the plate is symmetrical:



Force in horizontal direction is given by:

$$\boxed{F_x = \rho AV(V_{1x} - V_{2x})}$$

$$\boxed{F_x = \rho AV(V\cos\theta - (-V\cos\theta)) = 2\rho aV^2 \cos\theta}$$

Force in y – direction is given by:

$$\boxed{F_y = \rho AV(V_{1y} - V_{2y})}$$

$$\boxed{F_y = \rho AV(V\sin\theta - V\sin\theta) = 0}$$

(c). Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical:

$$F_x = \rho AV(V_{1x} - V_{2x})$$

$$\boxed{F_x = \rho AV(V\cos\theta - (-V\cos\phi)) = \rho aV^2(\cos\theta + \cos\phi)}$$

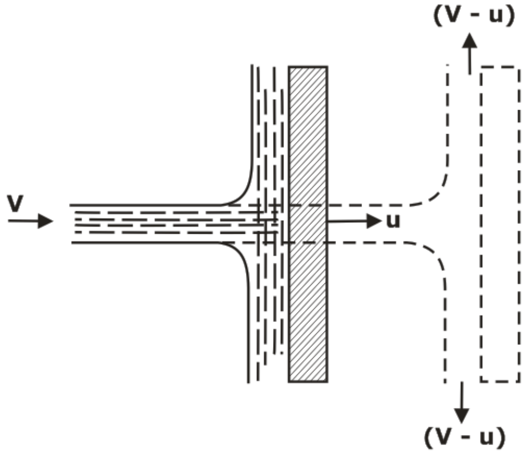
Force in y direction is given by:

$$F_y = \rho AV(V_{1y} - V_{2y})$$

$$\boxed{F_y = \rho AV(V\sin\theta - (-V\sin\phi)) = \rho aV^2(\sin\theta + \sin\phi)}$$

Force exerted on moving plate:

(a). Force exerted on a moving flat plate held normal to jet:

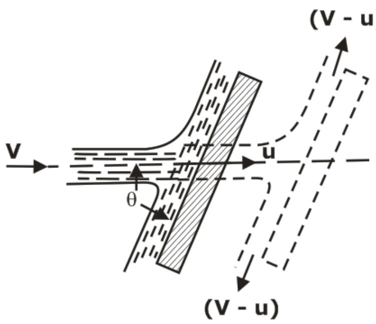


Horizontal component of force in x – direction:

$$\boxed{F_x = \rho a(V - u)[(v - u) - 0] = \rho a(V - u)^2}$$

$$\boxed{\text{Work done} = F_x.u = \rho a(V - u)^2 u}$$

(b). Force of Jet Impinging on An Inclined moving Plate:



$$\boxed{F_n = \rho a(V - u)[(v - u)\sin\theta - 0] = \rho a(V - u)^2 \sin\theta}$$

$F_x = F_n\sin\theta$ and $F_y = F_n\cos\theta$

$$\boxed{F_x = \rho a(V - u)\sin\theta \times \sin\theta = \rho a(V - u)^2 \sin^2\theta}$$

$$\boxed{F_y = \rho a(V-u)\sin\theta \times \cos\theta = \rho a(V-u)^2 \sin\theta\cos\theta}$$

$$\boxed{\text{Work done} = F_x \times u = \rho a(V-u)^2 u \sin^2\theta}$$

Force exerted by jet on curved plate(vane) when the plate/vane is moving in the direction of jet

(a). When single curved vane is used:

Horizontal component of force:

$$\boxed{F_x = \rho a(V-u)^2(1+\cos\theta)}$$

Work done one the vane per sec:

Work done/sec = $\rho a(V-u)^2(1+\cos\theta)$ u

Kinetic energy of issuing jet:

$$\boxed{KE = \frac{1}{2}mV^2 = \frac{1}{2}\rho AV \times V^2 = \frac{1}{2}\rho AV^3}$$

Efficiency is given as:

$$\boxed{\eta = \frac{\text{Work done / sec}}{\text{Kinetic energy}} = \frac{\rho a(v-u)^2(1+\cos\theta)u}{\frac{1}{2}\rho AV^3}}$$

At u = V/3 maximum efficiency will be obtained.

(b). When series of vane is used:

Horizontal component is given by:

$F_x = \rho aV(V-u)\ (1+\cos\theta)$

Work done one the vane per sec = $\rho aV(V-u)\times(1+\cos\theta)u$

Kinetic energy of issuing jet:

$$KE = \frac{1}{2}mV^2 = \frac{1}{2}\rho AV \times V^2 = \frac{1}{2}\rho AV^3$$

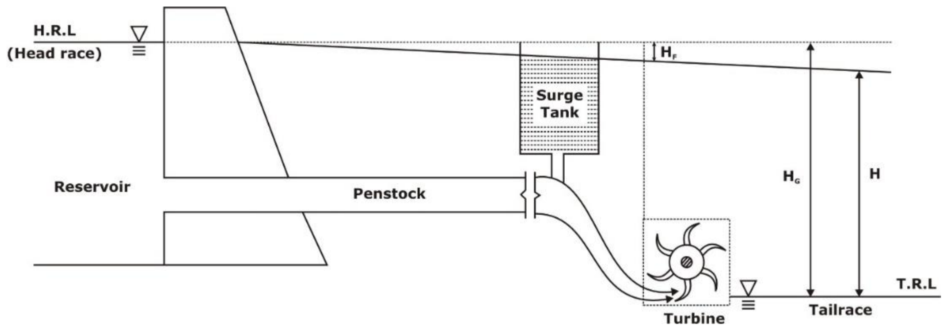
$$\boxed{\eta = \frac{\text{Work done / sec}}{\text{Kinetic energy}} = \frac{\rho aV(V-u)(1+\cos\theta)u}{\frac{1}{2}\rho AV^3}}$$

At u=V/2, maximum efficiency will be obtained

Maximum efficiency is given by: $\boxed{\eta = \frac{1+\cos\theta}{2}}$

CHAPTER 11: HYDRAULIC MACHINERY

Hydraulic turbine:



Classification of turbines:

(a). On the basis of energy available at inlet:

(i). Kinetic energy only: Impulse turbine

Ex: Pelton Wheel (tangential flow)

(ii). Kinetic energy + Pressure energy = Impulse Reaction Turbine.

Ex: Francis (mixed flow) and Kaplan (Axial flow)

(b). On the basis of head, discharge and specific speed:

Turbine	Head	Discharge	Specific speed
Pelton	High (H > 250 m)	Low	Low
Francis	Medium (50 < H < 250)	Medium	Medium
Kaplan	Low (H < 50 m)	High	High

Type of Heads:

Gross Head (H_G): It is elevation difference between tail race level (TRL) and head race level (HRL). It is the total head under which power plant is working.

Net head (H): It is calculated by subtracting the heat loss in penstock from gross head. It is the net head under which turbine is working.

$H_{net} = H_G - h_f$

$$\boxed{H_{net} = H_G - \left(\frac{fLV^2}{2gD}\right)}$$

Types of Power:

Waterpower/Hydraulic (power power at inlet to turbine):

$$\boxed{\text{Water power} = \dot{m}gh = \rho QgH}$$

Power available at exit of nozzle (KE/sec):

Power = $\frac{1}{2}\dot{m}v_1^2$, (v₁= velocity of jet at the exit of nozzle)

Thus: $\rho QgH = \frac{1}{2}\dot{m}v_1^2$ (if there is no head loss in nozzle)

$$\boxed{gH = \frac{V_1^2}{2} \Rightarrow V_1 = \sqrt{2gH}}$$

Practically : $\boxed{V_1 = C_v\sqrt{2gH}}$

$$\boxed{\text{Runner power} = \rho Q\left(V_{w1}u_1 \pm V_{w2}u_2\right)}$$

Final power available on the shaft:

Shaft power (SP) = (RP) – (Mechanical frictional losses) = Final power output

Efficiencies

(a). Volumetric efficiency $\left[\eta_{vol}\right]$

$$\eta_{vol} = \frac{\text{Discharge striking the runner}}{\text{Discharge supplied by penstock}} = \frac{Q - \Delta Q}{Q}$$

$\Delta Q \Rightarrow$ leakage loss (some amount of water directly goes to tailrace without striking the runner).

(b). Hydraulic efficiency $\left(\eta_n\right)$:

$$\boxed{\eta_n = \frac{\text{Runner Power}}{\text{Water Power}} = \frac{\text{R.P.}}{\text{W.P.}}}$$

(c). Mechanical efficiency $\left(\eta_m\right)$:

$$\boxed{\eta_m = \frac{\text{S.P.}}{\text{R.P.}}}$$

(d). Overall efficiency $\left(\eta_o\right)$:

$$\boxed{\eta_o = \frac{\text{S.P.}}{\text{R.P.}} \times \frac{\text{R.P.}}{\text{W.P.}} = \eta_m \times \eta_H} \qquad \left[\text{when } \eta_{vol} \text{ is not given}\right]$$

$$\boxed{\eta_o = \eta_{vol} \cdot \eta_m \cdot \eta_h} \qquad \left[\text{when } \eta_{vol} \text{ is given}\right]$$

Pelton wheel (Tangential flow runner):

Bucket velocity is given by: $\boxed{u = \frac{\pi DN}{60}}$

and $u = u_1 = u_2$

Waterpower is given by: $\boxed{\text{W.P.} = \rho QgH}$

Runner Power: $\boxed{P = \text{Work Done / sec} = \text{F.u.} = \rho{\cdot}aV_1Vr_1\left(1 + k\cos\phi\right)\cdot u}$

Hydraulic efficiency is given by:

$$\boxed{\eta_h = \frac{\rho\cdot a\cdot V_1Vr_1\left(1 + k\cos\phi\right)\cdot u}{\frac{1}{2}\dot{m}V_1^2}}$$

Condition for maximum hydraulic efficiency: $\boxed{u = \frac{V_1}{2}}$

$$\boxed{(\eta_h)_{max} = \frac{1 + k\cos\phi}{2}}$$

Design Data for Pelton turbine

(a). $\phi = 10 - 20^\circ$ = exit blade angle

δ = angle of deflection = 180- ϕ (Generally 160 – 170°).

(b). Jet ratio (m) = $\frac{D}{d} = \frac{\text{Runner dia}}{\text{Jet dia}}$

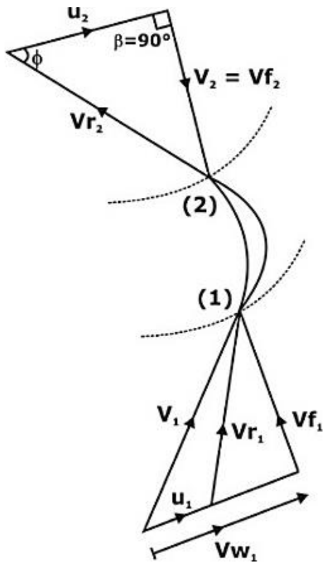
(c). No. of vanes (Bucket) $= \left[\frac{m}{2} + 15\right] = \left[\frac{D}{2d} + 15\right]$

Francis turbine: Francis turbine is an inward flow reaction turbine. To maximise runner power, V_2 should be minimum. It can be done by converting absolute velocity direction into radial direction to the runner at exit.

Runner Power per unit weight: $\boxed{\frac{RP}{\dot{m}g} = \frac{\rho Q V_{w_1} \cdot u_1}{\rho Q g} = \frac{V_{w_1} \cdot u_1}{g}}$

$$\boxed{\text{Runner head} = \frac{RP}{\dot{m}g} = \frac{V_{w1} \cdot u_1}{g}}$$

Hydraulic efficiency: $\boxed{\eta_h = \frac{RP}{HP} = \frac{\rho Q \cdot V_{w_1} u_1}{\rho Q g H} = \frac{V_{w_1} \cdot u_1}{gH}}$



Axial flow reaction turbine: Kaplan and propeller turbines are the example of axial flow reaction turbine.

Performance of turbine:

Unit speed (Nu): $\boxed{Nu = \frac{N}{\sqrt{H}}}$

Unit discharge (Q_u): $\boxed{Q_u = \frac{Q}{\sqrt{H}}}$

Unit power (P_u): $\boxed{Pu = \frac{P}{H^{3/2}}}$

Specific speed of Turbine (N_s):

It is defined as the speed of a turbine which is identical in shape, geometrical dimensions, blade angles, gate opening etc., with the actual turbine but of such a size that it will develop unit power when working under unit head.

$$\boxed{N_S = \frac{N\sqrt{P}}{H^{5/4}}}$$

Degree of Reaction (R): Degree of reaction is defined as the ratio of pressure energy change inside a runner to the total energy change inside the runner. It is represented by 'R'.

$$\boxed{R = \frac{\text{Change of pressure energy inside the runner}}{\text{Change of total energy inside the runner}}}$$

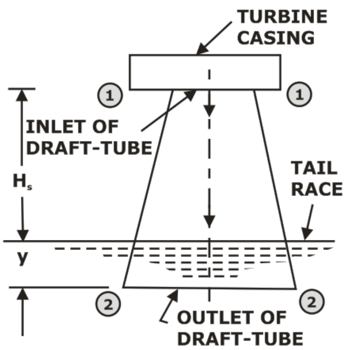
$$\boxed{R = 1 - \frac{\left(V_1^2 - V_2^2\right)}{2g\,H_e}}$$

(i) For a Pelton turbine: $u_1 = u_2$ and $V_{r2} = V_{r1}$

Degree of reaction: $\boxed{R = 0}$

(ii). For a Francis turbine: $\boxed{R = 1 - \frac{\cot\alpha}{2\left(\cot\alpha - \cot\theta\right)}}$

Draft tube: It is a diverging tube fitted at the exit of runner which connect exit of the runner to the tailrace. It's diverging section helps in converging of velocity head into pressure head, but the angle of divergence should not be greater than 6-8°.



Efficiency of Draft tube: $\boxed{\eta_d = \dfrac{\left(\dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g}\right) - h_f}{\left(\dfrac{V_1^2}{2g}\right)}}$

Hydraulic pumps: The device which converts Mechanical energy into hydraulic energy or pressure energy is called hydraulic pumps.

Types of pumps: The pumps are classified as follows:

(1). Positive displacement Pumps: Positive displacement pumps are further classified into two categories:

(a). Rotary Pumps: Rotary pumps include Gear Pump, Lobe pump, Vane Pump, Screw Pump.

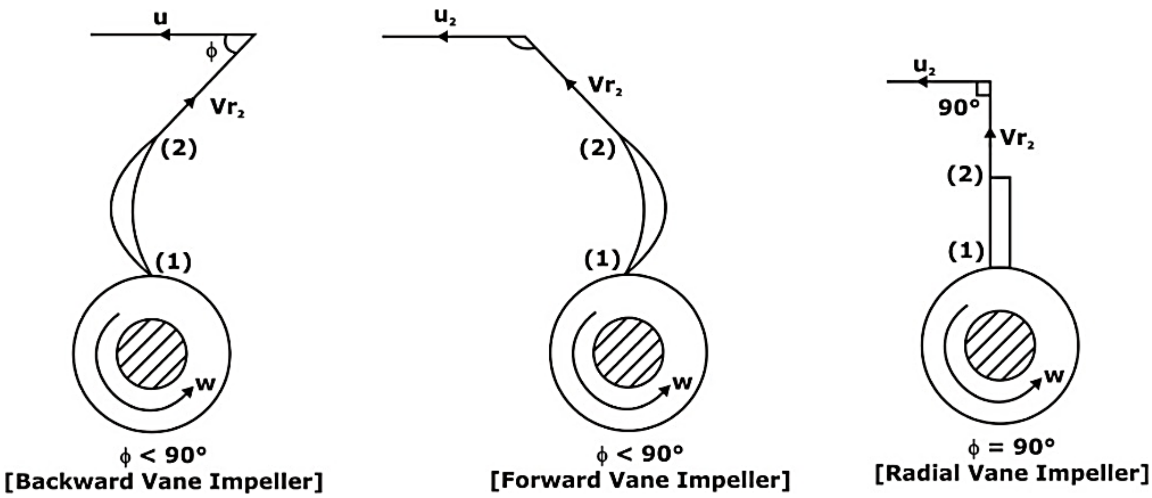
(b). Reciprocating pumps: Reciprocating pumps include piston and diaphragm pumps.

(2). Dynamic Pumps: Dynamic pumps include centrifugal and axial pumps.

Centrifugal pump: When certain mass of fluid is rotated about centre of rotation, increase in head takes place.

Component of Centrifugal Pump:

(i). Casing **(ii).** Impeller



Types of Head:

Static Head (Hs): $\boxed{H_S = h_s + h_d}$

Friction head loss (HF): $\boxed{H_F = h_{fs} + h_{fd}}$

Manometric head (Hm): Actual head produced by the pump. It is given by:

For ideal pump: $\boxed{H_m = \dfrac{V_{w2}u_2}{g}}$

Power: Power requirement of pump = shaft power (SP)

Impeller power (IP) = SP – (mechanical frictional losses)

$$\boxed{IP = \rho Q V_{w2} \cdot u_2}$$

Output power:

Waterpower (Manometric power) = $\boxed{\rho Q g H_m}$

Note:

$$\boxed{SP > IP > MP}$$

Efficiencies:

(a). Volumetric Efficiency (η_v): $\boxed{\eta_v = \dfrac{Q}{Q + \Delta Q}}$

ΔQ = amount of discharge not delivered

(b). Manometric efficiency (η_{mano}):

$$\boxed{\eta_{man} = \frac{\text{Mechanical.Power.}}{\text{I.P.}} = \frac{\rho Q \cdot g H_m}{\rho Q V_{w2} u_2} = \frac{g \cdot H_m}{V_{w2} . u_2}}$$

(c). Mechanical Efficiency (η_{mech}): $\boxed{\eta_{mech} = \dfrac{\text{I.P.}}{\text{S.P.}}}$

(d). Overall Efficiency (η_o): $\boxed{\eta_o = \eta_{mano} \cdot \eta_{mech} = \dfrac{\rho Q g H_m}{\text{S.P.}}}$

Minimum starting speed of pump: $\boxed{N \geq \dfrac{60}{2\pi}\sqrt{\dfrac{2gH_m}{r_2^2 - r_1^2}}}$

Specific speed of pump (N_s): The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver one cubic metre of liquid per second against a head of one metre. It is denoted by 'N_s'.

$$\boxed{N_S = \frac{N\sqrt{Q}}{H_m^{3/4}}}$$

Non-dimensional form of specific speed ⇒ It is called the Shape number of the pump.

$$\boxed{\text{Shape no} = \frac{N\sqrt{Q}}{(gH_m)^{3/4}}}$$

Model-Prototype relationship

1. $\boxed{\left.\dfrac{H_m}{D^2N^2}\right|_m = \left.\dfrac{H_m}{D^2N^2}\right|_p}$

2. $\boxed{\left.\dfrac{Q}{D^3N}\right|_m = \left.\dfrac{Q}{D^3N}\right|_p}$

3. $\boxed{\left.\frac{P}{D^5N^3}\right|_m = \left.\frac{Q}{D^5N^3}\right|_p}$

Priming of pump: Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe up to the delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump.

Cavitation: Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure and causing damage to the components.

Thomas cavitation parameter:

$$\sigma = \frac{H_b - H_s}{H} = \frac{(H_{atm} - H_v) - H_s}{H}$$

where H_b = Barometric pressure head in m of water,

H_{atm} = Atmospheric pressure head in m of water,

H_v = Vapour pressure head in in of water.

H_s = Suction pressure at the outlet of reaction turbine in m of water or height of turbine runner above the tail water surface,

H = Net head on the turbine in m

If the value of σ is greater than σ_c the cavitation will not occur in that turbine or pump.

Net positive suction head (NPSH): Net positive suction head is defined as the sum of absolute pressure head and kinetic head at the inlet to the pump minus absolute vapour pressure head.

$$NPSH = \frac{P_1}{\rho g} - \frac{P_v}{\rho g} + \frac{V_s^2}{2g}$$

$$\boxed{NPSH = \frac{P_{atm}}{\rho g} - h_s - h_{fs} - \frac{P_{vap}}{\rho g}}$$

Minimum NPSH is given by: $\boxed{\sigma = \frac{NPSH}{H_m}}$

Where σ is the Thomas cavitation parameter.

To avoid cavitation ⇒ $\boxed{\sigma > \sigma_c}$

σ_c = critical cavitation factor

Multi-stage Pumps

(a). Pumps in series:

Q = constant

Total head = $n \cdot H_m$

Where n = no. of stages/no. of impellers/no. of pumps in series

(b). Pumps in parallel:

H_m = constant

Total discharge = n · Q

Where n = no. of pumps in parallel

Reciprocating Pump: In the reciprocating pumps the mechanical energy is converted to hydraulic energy (or pressure energy) by sucking the liquid into a cylinder in which a piston is reciprocating (moving backwards and forward) which exerts the thrust on the fluid and increases the pressure energy.

Single acting Reciprocating Pump:

Theoretical discharge: Q_{th} = Volume/sec

Volume per cycle = A · L

Cycle per second $= \frac{N}{60}$

Volume per second $= AL \cdot \left(\frac{N}{60}\right)$

Discharge from a pump:

For single acting pump: $\boxed{Q_{th} = \frac{ALN}{60}}$

For double acting pump: $\boxed{Q_{th} = 2\left(\frac{ALN}{60}\right)}$

Work done/sec= $\rho g Q(h_s+h_d)$

Slip of reciprocating pump: Slip of a pump is defined as the difference between the theoretical discharge and actual discharge of the pump.

$\boxed{Slip = Q_{th} - Q_{act}}$

Percentage slip is given by:

$$\boxed{\text{Percentage slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100}$$

****