## GATE 2022

## Civil Engineering

## Shift-2

## Questions with Detailed Solution

1. The movie was funny and I.......
A. could help laughing
B. couldn't help laughed
C. couldn't help laughing
D. could helped laughed
[MCQ: 1 Mark]
Ans. (C)
Sol. The movie was funny and I couldn't help laughing.
2. $x: y: z=\frac{1}{2}: \frac{1}{3}: \frac{1}{4}$

What is the value of $\frac{x+z-y}{y}$ ?
A. 0.75
B. 1.25
C. 2.25
D. 3.25
[MCQ: 1 Mark]
Ans. (B)
Sol.

$$
\begin{aligned}
& x: y: z=\frac{1}{2}: \frac{1}{3}: \frac{1}{4} \\
& \Rightarrow \frac{x}{y}=\frac{1 / 2}{1 / 3}=\frac{3}{2} \\
& \Rightarrow \frac{y}{z}=\frac{1 / 3}{1 / 4}=\frac{4}{3} \\
& \frac{x+z-y}{y}=\frac{x}{y}+\frac{z}{y}-1 \\
&= \frac{3}{2}+\frac{3}{4}-1 \\
&= \frac{5}{4}=1.25
\end{aligned}
$$

3. Both the numerator and the denominator of $\frac{3}{4}$ are increased by a positive integer, $x$, and those of $\frac{15}{17}$ are decreased by the same integer. This operation results in the same
value for both the fractions. What is the value of $x$ ?
A. 1
B. 2
C. 3
D. 4
[MCQ: 1 Mark]
Ans. (C)
Sol.
$\frac{3+x}{4+x}=\frac{15-x}{17-x}$
$\Rightarrow(x+3)(x-17)=(x-15)(x+4)$
$\Rightarrow x^{2}-14 x-51=x^{2}-11 x-60$
$\Rightarrow 3 x-9=0$
$x=3$
4. A survey of 450 students about their subjects of interest resulted in the following outcome.

- 150 students are interested in Mathematics.
- 200 students are interested in Physics.
- 175 students are interested in Chemistry.
- 50 students are interested in Mathematics and Physics.
- 60 students are interested in Physics and Chemistry.
- 40 students are interested in Mathematics and Chemistry.
- 30 students are interested in Mathematics, Physics and Chemistry.
- Remaining students are interested in Humanities.
Based on the above information, the number of students interested in
Humanities is
A. 10
B. 30
C. 40
D. 45
[MCQ: 1 Mark]

Ans. (D)
Sol.
The given data can be shown in the venn diagram as below:-


Total no. of students interested in physics, chemistry and maths all together
$=120+20+30+30+105+10+90=$ 405.

Total no. of students $=450$
$\therefore$ No. of students interested in humanities $=$ $450-405=45$
5. For the picture shown above, which one of the following is the correct picture representing reflection with respect to the mirror shown as the dotted line?

A.

B.

C.

[MCQ: 1 Mark]
Ans. (A)
Sol.
The image will be symmetrical about the dotted line so that on folding about the dotted line, both pictures overlap each other.

6. In the last few years, several new shopping malls were opened in the city. The total number of visitors in the malls is impressive. However, the total revenue generated through sales in the shops in these malls is generally low.
Which one of the following is the CORRECT logical inference based on the information in the above passage?
A. Fewer people are visiting the malls but spending more
B. More people are visiting the malls but not spending enough
C. More people are visiting the malls and spending more
D. Fewer people are visiting the malls and not spending enough
Ans. (B)
[MCQ: 2 Mark]

## Sol.

The inference drawn from the passage is that more people are visiting the malls but not spending enough.
7. In a partnership business the monthly investment by three friends for the first six months is in the ratio 3: 4: 5. After six months, they had to increase their monthly investments by $10 \%, 15 \%$ and $20 \%$, respectively, of their initial monthly investment. The new investment ratio was kept constant for the next six months.
What is the ratio of their shares in the total profit (in the same order) at the end of the year such that the share is proportional to
their individual total investment over the year?
A. $22: 23: 24$
B. $22: 33: 50$
C. $33: 46: 60$
D. $63: 86: 110$
[MCQ: 2 Mark]
Ans. (D)
Sol.
Let the initial investments be $3 x, 4 x$ and $5 x$, respectively.
Now, after increasing their monthly investment by $10 \%, 15 \%$ and $20 \%$, respectively, the monthly instalment will be:
$3.3 x, 4.6 x$ and $6 x$, respectively
Ratio of their profits at the end of year
$=$ Ratio of their investments
$=(3 x+3.3 x):(4 x+4.6 x):(5 x+6 x)$
$=6.3 \mathrm{x}: 8.6 \mathrm{x}: 11 \mathrm{x}$
$=63: 86: 110$
8. Consider the following equations of straight lines:
Line L1: $2 x-3 y=5$
Line L2: $3 x+2 y=8$
Line L3: $4 x-6 y=5$
Line L4: $6 x-9 y=6$
Which one among the following is the correct statement?
A. L1 is parallel to L 2 and L 1 is perpendicular to L3
B. L 2 is parallel to L 4 and L 2 is perpendicular to L1
C. L3 is perpendicular to L4 and L3 is parallel to L2
D. L 4 is perpendicular to L 2 and L 4 is parallel to L3
[MCQ: 2 Mark]

Ans. (D)

## Sol.

$L_{1}: 2 x-3 y=5 \Rightarrow$ slope $=2 / 3$
$L_{2}: 3 x+2 y=8 \Rightarrow$ slope $=-3 / 2$
$L_{3}: 4 x-6 y=5 \Rightarrow$ slope $=4 / 6=2 / 3$
$L_{4}: 6 x-9 y=6 \Rightarrow$ slope $=6 / 9=2 / 3$
Slope of $L_{1}=$ slope of $L_{3}=$ slope of $L_{4}$
$\Rightarrow L_{1}, L_{3}$ and $L_{4}$ are parallel to each other
slope of $L_{4} \times$ slope of $L_{2}=\frac{2}{3} \times\left(\frac{-3}{2}\right)=-1$.
$\Rightarrow L_{2}$ and $L_{4}$ are perpendicular to each other
9. Given below are two statements and four conclusions drawn based on the statements.

Statement 1: Some soaps are clean.
Statement 2: All clean objects are wet.
Conclusion I: Some clean objects are soaps.
Conclusion II: No clean object is a soap.
Conclusion III: Some wet objects are soaps. Conclusion IV: All wet objects are soaps.

Which one of the following options can be logically inferred?
A. Only conclusion I is correct
B. Either conclusion I or conclusion II is correct
C. Either conclusion III or conclusion IV is correct
D. Only conclusion I and conclusion III are correct
[MCQ: 2 Mark]
Ans. (D)
Sol. The possible representations are:-


We can observe that:-

- Some clean objects are soaps
- Some wet objects are soaps

The above two criteria are fulfilled in both the diagrams, i.e. only conclusions I and III are correct.
10. An ant walks in a straight line on a plane leaving behind a trace of its movement. The initial position of the ant is at point $P$ facing east.

The ant first turns $72^{\circ}$ anticlockwise at $P$, and then does the following two steps in sequence exactly FIVE times before halting.

1. moves forward for 10 cm .
2. turns 1440 clockwise.


The pattern made by the trace left behind by the ant is
A.

B.

C.

D.


Ans. (C)
Sol. The path is shown below:

11. The function $f(x, y)$ satisfies the Laplace equation
$\nabla^{2} f(x, y)=0$
on a circular domain of radius $r=1$ with its center at point P with coordinates
$x=0, y=0$. The value of this function on the circular boundary of this domain is equal to 3.

The numerical value of $f(0,0)$ is:
A. 0
B. 2
C. 3
D. 1
[MCQ: 1 Mark]
Ans. (C)
Sol.

$f(x, y)=3$ within the circular region.
Given point $P$ also lies in the circular domain Hence, $f(0,0)=3$
12. $\int\left(x-\frac{x^{2}}{2}+\frac{x^{3}}{3}-\frac{x^{4}}{4}+\ldots\right) d x$ is equal to
A. $\frac{1}{1+\mathrm{x}}+$ Constant
B. $\frac{1}{1+\mathrm{x}^{2}}+$ Constant
C. $-\frac{1}{1+x}+$ Constant
D. $-\frac{1}{1+\mathrm{x}^{2}}+$ Constant
[MCQ: 1 Mark]
Ans. (*)

## Sol.

$\int\left(x-\frac{x^{2}}{2}+\frac{x^{3}}{3}-\frac{x^{4}}{4}+\ldots\right) d x$
$=\frac{x^{2}}{2}-\frac{x^{3}}{6}+\frac{x^{4}}{12}-\frac{x^{5}}{20}+\ldots$
$\frac{1}{1+x}=1-x+x^{2}-x^{3}+\ldots$
$\frac{1}{1+x^{2}}=1-x^{2}+x^{4}-x^{6}+\ldots$
$-\frac{1}{1-x}=-\left(1+x+x^{2}+\ldots\right)$
$-\frac{1}{1-x^{2}}=-\left(1+x^{2}+x^{4}+\ldots\right)$
None of the options are matching
Marks to all
13. For a linear elastic and isotropic material, the correct relationship among Young's modulus of elasticity (E), Poisson's ratio (v), and shear modulus ( G ) is
A. $G=\frac{E}{2(1+V)}$
B. $G=\frac{E}{(1+2 v)}$
C. $E=\frac{E}{2(1+V)}$
D. $E=\frac{G}{(1+2 v)}$
[MCQ: 1 Mark]
Ans. A
Sol. $E=2 G(1+\mu)=3 K(1-2 \mu)$
$\Rightarrow G=\frac{E}{2(1+\mu)}$
14. Read the following statements relating to flexure of reinforced concrete beams:
I. In over-reinforced sections, the failure strain in concrete reaches earlier than the yield strain in steel.
II. In under-reinforced sections, steel reaches yielding at a load lower than the load at which the concrete reaches failure strain.
III. Over-reinforced beams are recommended in practice as compared to the under-reinforced beams.
IV. In balanced sections, the concrete reaches failure strain earlier than the yield strain in tensile steel.

Each of the above statements is either True or False.

Which one of the following combinations is correct?
A. I (True), II (True), III (False), IV (False)
B. I (True), II (True), III (False), IV (True)
C. I (False), II (False), III (True), IV (False)
D. I (False), II (True), III (True), IV (False)
[MCQ: 1 Mark]

## Ans. A

## Sol. Under reinforced section ( $X_{u}<X_{u}$, max $)$

- In this case, steel yields before the crushing of concrete and the failure is ductile.
- In construction, under-reinforced sections are preferred as it gives warning before the collapse.

Balanced section ( $X_{u}=X_{u, \max }$ )

- In this case, the yielding of steel and crushing of concrete take place simultaneously.


## Over-reinforced section( $X_{u}>X_{u, m a x}$ )

- In this case, crushing of concrete occurs before yielding steel, and sudden failure occurs.

It means statement I, II is true, and Statement III, IV is false.
15. Match all the possible combinations between Column $X$ (Cement compounds) and Column Y (Cement properties):

## Column X

Column Y
(i) C3S
(P) Early age strength
(ii) C 2 S
(Q) Later age strength
(iii) C3A
(R) Flash setting
(S) Highest heat of hydration
(T) Lowest heat of hydration
A. (i) - (P), (ii) - (Q) and (T), (iii) - (R) and (S)
B. (i) - (Q) and (T), (ii) - (P) and (S), (iii) - (R)
C. (i) - (P), (ii) - (Q) and (R), (iii) - (T)
D. (i) - (T), (ii) - (S), (iii) - (P) and (Q)
[MCQ: 1 Mark]
Ans. (A)
Sol.
$\mathrm{C}_{3} \mathrm{~S} \rightarrow$ responsible for early age strength
$\mathrm{C}_{2} \mathrm{~S} \rightarrow$ give lowest heat of hydration and provide later age strength
$\mathrm{C}_{3} \mathrm{~A} \rightarrow$ Flash setting and highest heat of hydration
16. Consider a beam $P Q$ fixed at $P$, hinged at $Q$, and subjected to a load $F$ as shown in figure (not drawn to scale). The static and kinematic degrees of indeterminacy, respectively, are

A. 2 and 1
B. 2 and 0
C. 1 and 2
D. 2 and 2
[MCQ: 1 Mark]
Ans. (A)
Sol.


This is the case of general loading
No. of support reactions $=3+2=5$
No. of equilibrium equation available $=3$.
Static indeterminacy $=5-3=2$
Degrees of freedom at $\mathrm{P}=0$
Degrees of freedom at $\mathrm{Q}=3-2$
Kinematic indeterminacy $=0+1=1$
17. Read the following statements:
( P ) While designing a shallow footing in sandy soil, monsoon season is considered for critical design in terms of bearing capacity.
(Q) For slope stability of an earthen dam, sudden drawdown is never a critical condition.
(R) In a sandy sea beach, quicksand condition can arise only if the critical hydraulic gradient exceeds the existing hydraulic gradient.
(S) The active earth thrust on a rigid retaining wall supporting homogeneous cohesionless backfill will reduce with the lowering of water table in the backfill.

Which one of the following combinations is correct?
A. (P)-True, (Q)-False, (R)-False, (S)-False
B. (P)-False, (Q)-True, (R)-True, (S)-True
C. (P)-True, (Q)-False, (R)-True, (S)-True
D. (P)-False, (Q)-True, (R)-False, (S)-False
[MCQ: 1 Mark]
Ans. A
18. Stresses acting on an infinitesimal soil element are shown in the figure (with $\sigma_{z}>\sigma_{x}$ ). The major and minor principal stresses are $\sigma_{1}$ and $\sigma_{3}$, respectively. Considering the compressive stresses as positive, which one of the following expressions correctly represents the angle between the major principal stress plane and the horizontal plane?

A. $\tan ^{-1}\left(\frac{\tau_{\mathrm{zx}}}{\sigma_{1}-\sigma_{\mathrm{x}}}\right)$
B. $\tan ^{-1}\left(\frac{\tau_{\mathrm{zx}}}{\sigma_{3}-\sigma_{x}}\right)$
C. $\tan ^{-1}\left(\frac{\tau_{z x}}{\sigma_{1}-\sigma_{x}}\right)$
D. $\tan ^{-1}\left(\frac{\tau_{2 x}}{\sigma_{1}-\sigma_{3}}\right)$
[MCQ: 1 Mark]
Ans. *

Sol.


As given in the question, compressive stress is considered positive, the Mohr circle will be as shown below


$$
\sigma-\text { axis }
$$



$$
\begin{aligned}
& \tan \theta=\frac{\left(\sigma_{1}-\sigma_{2}\right)}{\tau_{\mathrm{zx}}} \\
& \Rightarrow \theta=\tan ^{-1}=\left(\frac{\sigma_{1}-\sigma_{2}}{\tau_{\mathrm{zx}}}\right)
\end{aligned}
$$

19. Match Column $X$ with Column $Y$ :

## Column X

(P) Horton
(Q) Penman
method
(R) Chezy's
(III) Evapotranspiration formula
(S) Lacey's
(IV) Infiltration theory
(T) Dicken's (V) Flow velocity formula

Which one of the following combinations is correct?
A. (P)-(IV),
(Q)-(III),
(R)-(V),
(S)-(I),
(T)-(II)
B. (P)-(III),
(Q)-(IV),
(R)-(V),
(S)-(I), (T)-(II)
C. (P)-(IV),
(Q)-(III),
(R)-(II),
(S)-(I), (T)-(V)
D. (P)-(III),
(Q)-(IV), (T)-(II)
(R)-(I), (S)-(V),
[MCQ: 1 Mark]
Ans. (A)
Sol. Horton equation- Infiltration
Penman method- Evapotranspiration
Chezy's formula- Flow velocity
Lacey's theory- Design of alluvial channel
Dicken's formula- Maximum flood discharge
20. In a certain month, the reference crop evapotranspiration at a location is 6 $\mathrm{mm} /$ day. If the crop coefficient and soil coefficient are 1.2 and 0.8 , respectively, the actual evapotranspiration in mm/day is
A. 5.76
B. 7.20
C. 6.80
D. 8.00
[MCQ: 1 Mark]

Ans. (A)
Sol. Actual evapotranspiration
$=\mathrm{k}_{\mathrm{c}} \times \mathrm{k}_{\mathrm{s}} \times$ reference evapotranspiration
$\Rightarrow \mathrm{k}_{\mathrm{s}}=0.8, \mathrm{k}_{\mathrm{c}}=1.2$
$\Rightarrow$ Actual evapotranspiration
$=0.8 \times 1.2 \times 6$
$=5.76 \mathrm{~mm}$
21. The dimension of dynamic viscosity is:
A. $\mathrm{ML}^{-1} \mathrm{~T}^{-1}$
B. $M L^{-1} \mathrm{~T}^{-2}$
C. $\mathrm{ML}^{-2} \mathrm{~T}^{-2}$
D. $M L^{0} \mathrm{~T}^{-1}$
[MCQ: 1 Mark]

## Ans. A

Sol. $\quad \tau=\mu=\frac{d v}{d y}$

$$
\begin{aligned}
& \mathrm{F} / \mathrm{A}=\mu \frac{\mathrm{V}}{\mathrm{Y}} \\
& \frac{\left[\mathrm{MLT}^{-2}\right]}{\left[\mathrm{L}^{2}\right]}=\mu \frac{\left[\mathrm{LT}^{-1}\right]}{[\mathrm{L}]} \\
& \mu=\mathrm{ML}^{-1} \mathrm{~T}^{-1}
\end{aligned}
$$

22. A process equipment emits $5 \mathrm{~kg} / \mathrm{h}$ of volatile organic compounds (VOCs). If a hood placed over the process equipment captures $95 \%$ of the VOCs, then the fugitive emission in $\mathrm{kg} / \mathrm{h}$ is
A. 0.25
B. 4.75
C. 2.50
D. 0.48
[MCQ: 1 Mark]
Ans. (A)
Sol. Total VOCs $=5 \mathrm{~kg} / \mathrm{h}$
VOCs captured $=95 \%$ of total VOCs
$=\frac{95}{100} \times 5=4.75 \mathrm{~kg} / \mathrm{h}$
Fugitive emission = VOCs escaped
= Total VOCs - VOCs captured
= 5-4.75
$=0.25 \mathrm{~kg} / \mathrm{h}$
23. Match the following attributes of a city with the appropriate scale of measurements.

## Attribute

(P) Average temperature $\left({ }^{0} \mathrm{C}\right)$ of a city
(Q) Name of a city
(R) Population density of a city
(S) Ranking of a city based on ease of

Business
Which one of the following combinations is correct?
A. (P)-(I), (Q)-(III), (R)-(IV), (S)-(II)
B. (P)-(II), (Q)-(I), (R)-(IV), (S)-(III)
C. (P)-(II), (Q)-(III), (R)-(IV), (S)-(I)
D. (P)-(I), (Q)-(II), (R)-(III), (S)-(IV)
[MCQ: 1 Mark]
Ans. (A)

## Sol. Attribute Scale of measurement

Average temperature $\left({ }^{\circ} \mathrm{C}\right)$ of a city Interval

Name of a cityNominal
Population density of a city Ratio
Ranking of a city based on ease of business Ordinal
24. If the magnetic bearing of the Sun at a place at noon is $S 2^{\circ} \mathrm{E}$, the magnetic declination (in degrees) at that place is
A. $2^{\circ} \mathrm{E}$
B. $2^{\circ} \mathrm{W}$
C. $4^{\circ} \mathrm{E}$
D. $4^{\circ} \mathrm{W}$
[MCQ: 1 Mark]

Ans. (A)
Sol.



TB at Noon $=180^{\circ}$
MB at noon $=180^{\circ}-2^{\circ}=178^{\circ}$
Magnetic decimation
$=\mathrm{TB}-\mathrm{MB}=180^{\circ}-178^{\circ}=+2^{\circ}$
$+2^{\circ}$ means $2^{\circ} \mathrm{E}$
25. $P$ and $Q$ are two square matrices of the same order. Which of the following statement(s) is/are correct?
A. If $P$ and $Q$ are invertible, then $[P Q]^{-1}=Q^{-}$ ${ }^{1} \mathrm{P}^{-1}$
B. If $P$ and $Q$ are invertible, then $[Q P]^{-1}$ $=\mathrm{P}^{-1} \mathrm{Q}^{-1}$
C. If $P$ and $Q$ are invertible, then $[P Q]^{-1}$ $=P^{-1} Q^{-1}$
D. If $P$ and $Q$ are not invertible, then $[P Q]^{-1}$ $=Q^{-1} P^{-1}$
[MCQ: 1 Mark]
Ans. (A, B)
Sol. $[P Q]^{-1}=Q^{-1} P^{-1}$ when both $P$ and $Q$ are invertible
$[\mathrm{QP}]^{-1}=\mathrm{P}^{-1} \mathrm{Q}^{-1}$ when both P and Q are invertible
26. In a solid waste handling facility, the moisture contents (MC) of food waste, paper waste, and glass waste were found to be MCf, MCp, and MCg, respectively. Similarly, the energy contents (EC) of plastic waste, food waste, and glass waste were found to be ECpp, ECf, and ECg, respectively. Which of the following statement(s) is/are correct?
A. MCf $>\mathrm{MCp}>\mathrm{MCg}$
B. ECpp > ECf $>\mathrm{ECg}$
C. MCf < MCp < MCg
D. ECpp < ECf < ECg
[MCQ: 1 Mark]
Ans. (A, B)
Sol. Typical specific weight and moisture content data for residential, commercial, industrial, and agricultural wastes.

|  | Specific weight, lb/yd ${ }^{\mathbf{3}}$ |  | Moisture content, \% by weight |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Range | Typical | Range | Typical |
| Residential (uncompacted) |  |  |  |  |
| *Food wastes (mixed) | 220-810 | 490 | 50-80 | 70 |
| *Paper | 70-220 | 150 | 4-10 | 6 |
| Cardboard | 70-135 | 85 | 4-8 | 5 |
| Plastics | 70-220 | 110 | 1-4 | 2 |
| Textiles | 70-170 | 110 | 6-15 | 10 |
| Rubber | 170-340 | 220 | 1-4 | 2 |
| Leather | 170-440 | 270 | 8-12 | 10 |
| Yard wastes | 100-380 | 170 | 30-80 | 60 |
| Wood | 220-540 | 400 | 15-40 | 20 |
| *Glass | 270-810 | 330 | 1-4 | 2 |
| Tin cans | 85-270 | 150 | 2-4 | 3 |
| Aluminium | 110-405 | 270 | 2-4 | 2 |
| Other metals | 220-1940 | 540 | 2-4 | 3 |
| Dirt, ashes, etc. | 540-1685 | 810 | 6-12 | 6 |
| Ashes | 1095-1400 | 1255 | 6-12 | 6 |
| Rubbish | 150-305 | 220 | 5-20 | 15 |
| Residential yard wastes |  |  |  |  |
| Leaves (loose and dry) | 50-250 | 100 | 20-40 | 30 |
| Green grass (loose and moist) | 350-500 | 400 | 40-80 | 60 |
| Green grass (wet and compacted) | 1000-1400 | 1000 | 50-90 | 80 |
| Yard waste (shredded) | 450-600 | 500 | 20-70 | 50 |
| Yard waste (composted) | 450-650 | 550 | 40-60 | 50 |


| Municipal |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| In compactor truck | $300-760$ | 500 | $15-40$ | 20 |
| In Landfill Normally compacted | $610-840$ | 760 | $15-40$ | 25 |
| In landfill Well compacted | $995-1250$ | 1010 | $15-40$ | 25 |
| Commercial |  |  |  |  |
| Food wastes (wet) | $800-1600$ | 910 | $50-80$ | 70 |
| Appliances | $250-340$ | 305 | $0-2$ | 1 |

Moisture content:
Mcf > Mcp > Mcg
Typical values for short residue and energy content of residential of residential MSW.

|  | Inert residue ${ }^{\text {a }}$ percent |  | Energy ${ }^{\text {b }}$ (Btu/lb) |  |
| :---: | :---: | :---: | :---: | :---: |
| Component | Range | Typical | Range | Typical |
| Organic |  |  |  |  |
| *Food wastes | 2.8 | 5.0 | 1,500-3,000 | 2,000 |
| *Paper | 4-8 | 6.0 | 5,000-8,000 | 7,200 |
| Cardboard | 3.6 | 5.0 | 6,000-7,500 | 7,000 |
| Plastics | 6-20 | 10.0 | 12,000-10,000 | 14,000 |
| Textiles | 2-4 | 2.5 | 6,500-8,000 | 7,500 |
| Rubber | 8-20 | 10.0 | 9,000-12,000 | 10,000 |
| Leather | 8-20 | 10.0 | 6,500-8,500 | 7,500 |
| Yard wastes | 2-6 | 4.5 | 1,000-8,000 | 2.000 |
| Wood | 0.6.2 | 1.5 | 7,500-8.500 | 8.000 |
| Misc. organics | - | - | - | - |
| Inorganic |  |  |  |  |
| *Class | 96.99 | 980 | 50-100 ${ }^{\text {c }}$ | 60 |
| Tin cans | 96.99 | 98.0 | 100-500 ${ }^{\text {c }}$ | 300 |
| Aluminium | 90.99 | 96.0 | - | - |
| Other metal | 94.99 | 98.0 | 100-500 ${ }^{\text {c }}$ | 300 |
| Dirt, ashes | 60.80 | 70.0 | 1,000-5,000 | 3.000 |
| Municipal solid wastes etc. |  |  | 4,000-6,000 | 5,000 |
| a After complete combustion <br> ${ }^{\mathrm{b}}$ As discarded basis <br> ${ }^{\text {c Energy content is from coatings, labels and attached minerals }}$ |  |  |  |  |

## Energy content:

ECpp > ECf > ECg
27. To design an optimum municipal solid waste collection route, which of the following is/are NOT desired:
A. Collection vehicle should not travel twice down the same street in a day.
B. Waste collection on congested roads should not occur during rush hours in morning or evening.
C. Collection should occur in the uphill direction.
D. The last collection point on a route should be as close as possible to the waste disposal facility.
[MCQ: 1 Mark]
Ans. (C)
Sol. Some guidelines that should be taken into consideration when laying out routes are as follows:
(i) Existing policies and regulations related to such items as the point of collection and frequency of collection must be identified.
(ii) Existing system characteristics such as crew size and vehicle types must be coordinated.
(iii) Wherever possible, routes should be laid out so that they begin and end near arterial streets, using topographical and physical barriers as route boundaries.
(iv) In hilly areas, routes should start at the top of the grade and proceed downhill as the vehicle becomes loaded. Proceeding uphill will require large tractive force (vehicles gets loaded) and more fuel.
(v) Routes should be laid out so that the last container to be collected on the route is located nearest to the disposal site.
(vi) Waste generated at traffic congested locations should be collected as early in the day as possible as it will cause the generation of foul smell and the breeding of insects for a long time during congestion.
(vii) Sources at which extremely large quantities of waste are generated should be serviced during the start of the day as the residual of large wastes cause the generation of foul smell and the breeding of insects.
(viii) Scattered pickup points (where small quantities of solid wastes are generated) that receive the same collection frequency should, if possible, be serviced during one trip or on the same day.
28. For a traffic stream, $v$ is the space mean speed, $k$ is the density, $q$ is the flow, $\mathrm{V}_{\mathrm{f}}$ is the free-flow speed, and $k_{j}$ is the jam density. Assume that the speed decreases linearly with density.
Which of the following relation(s) is/are correct?
A. $q=k_{j} k-\left(\frac{k_{j}}{v_{f}}\right) k^{2}$
B. $q=v_{f} k-\left(\frac{v_{f}}{k_{j}}\right) k^{2}$
C. $q=v_{f} v-\left(\frac{v_{f}}{k j}\right) v^{2}$
D. $q=k_{j} v-\left(\frac{k_{j}}{v_{f}}\right) v^{2}$
[MCQ: 1 Mark]
Ans. (B,D)

Sol. v = Space mean speed
$\mathrm{k}=$ density
q =flow
$\mathrm{V}_{\mathrm{f}}=$ free flow speed
$\mathrm{k}_{\mathrm{j}}=$ jam density
Since it is given speed decreasing linearly with density


Let $\mathrm{v}=\mathrm{ak}+\mathrm{b}$
At $\mathrm{k}=0$
$\mathrm{v}=\mathrm{v}_{\mathrm{f}}$
$\mathrm{V}_{\mathrm{f}}=\mathrm{a}$.
at $\mathrm{k}=\mathrm{kj}$,
$\mathrm{v}=0$
$b=-\frac{v_{f}}{k_{j}}$
by (1), (2), (3)
$v=v_{f}-\frac{v_{f}}{k_{j}} \cdot k$
$\therefore$ know q $=\mathrm{kv}$
By (4) and (5)
$q=v_{f} \cdot k-\frac{v_{f}}{k_{j}} k^{2}$
$q=v_{f} \cdot\left(\frac{q}{v}\right)-\frac{v_{f}}{k_{j}}\left(\frac{q}{v}\right)^{2}$
$q=q\left(\frac{v_{f}}{v}\right)-\frac{v_{f}}{k_{j}}\left(\frac{q}{v}\right)^{2}$
$1=\left(\frac{v_{f}}{v}\right)-\frac{q}{v^{2}}\left(\frac{v_{f}}{k_{j}}\right)$

$$
\begin{aligned}
& v^{2}=v_{f} v-q\left(\frac{v_{f}}{k_{j}}\right) \\
& q=k_{j} v-\left(\frac{k_{j}}{v_{f}}\right) v^{2}
\end{aligned}
$$

Hence B and D are correct
29. The error in measuring the radius of a 5 cm circular rod was $0.2 \%$. If the cross-sectional area of the rod was calculated using this measurement, then the resulting absolute percentage error in the computed area is $\qquad$ .
(round off to two decimal places)
[NAT: 1 Mark]
Ans. (0.39 to 0.41)
Sol. R = radius,
$A=$ area
$\mathrm{R}=5 \mathrm{~cm}$
$e_{r}=0.2 \%$ of $r$
$e_{r}=\frac{0.2}{100} \times 5=0.01 \mathrm{~cm}$
$\mathrm{A}=\pi \mathrm{r}^{2}$
$\mathrm{e}_{\mathrm{A}}=$ error in area
$e_{A}=2 \pi r\left(e_{r}\right)$
$\%$ Error in area $=\frac{\mathrm{e}_{\mathrm{A}}}{\mathrm{A}} \times 100=\frac{2 \pi r\left(\mathrm{e}_{\mathrm{r}}\right)}{\pi \mathrm{r}^{2}} \times 100$
\% Error in area $=\frac{2 e_{r}}{r} \times 100$
$\%$ Error in area $=\frac{2 \times 0.01}{5} \times 100=0.40$
30. The components of pure shear strain in a sheared material are given in the matrix form:
$\varepsilon=\left[\begin{array}{cc}1 & 1 \\ 1 & -1\end{array}\right]$
Here, $\operatorname{Trace}(\varepsilon)=0$. Given, $\mathrm{P}=$ Trace $\left(\varepsilon^{8}\right)$ and $\mathrm{Q}=\operatorname{Trace}\left(\varepsilon^{11}\right)$.

The numerical value of $(P+Q)$ is $\qquad$ . (in integer)
[NAT: 1 Mark]
Ans. (32-32)
Sol. $\varepsilon=\left[\begin{array}{cc}1 & 1 \\ 1 & -1\end{array}\right]$
$\varepsilon^{2}=\left[\begin{array}{cc}1 & 1 \\ 1 & -1\end{array}\right]\left[\begin{array}{cc}1 & 1 \\ 1 & -1\end{array}\right]$
$=\left[\begin{array}{ll}2 & 0 \\ 0 & 2\end{array}\right]=2 I$
$\varepsilon^{8}=\left(\varepsilon^{2}\right)^{4}=(2 \mathrm{I})^{4}=16 \mathrm{I}=\left[\begin{array}{cc}16 & 0 \\ 0 & 16\end{array}\right]$
$\mathrm{P}=$ Trace $\left(\varepsilon^{8}\right)=32$

$$
\begin{aligned}
& \varepsilon^{11}=\varepsilon^{10} \varepsilon \\
& =\left(\varepsilon^{2}\right)^{5} \varepsilon \\
& =(2 I)^{5} \varepsilon \\
& =32 \varepsilon
\end{aligned}
$$

$=\left[\begin{array}{cc}32 & 32 \\ 32 & -32\end{array}\right]$
$\mathrm{Q}=\operatorname{Trace}\left(\varepsilon^{11}\right)=0$
$\mathrm{P}+\mathrm{Q}=32$
31. The inside diameter of a sampler tube is 50 mm . The inside diameter of the cutting edge is kept such that the Inside Clearance Ratio (ICR) is $1.0 \%$ to minimize the friction on the
sample as the sampler tube enters into the soil.

The inside diameter (in mm) of the cutting edge is $\qquad$ .
(round off to two decimal places)
[NAT: 1 Mark]
Ans. (49.49-49.52)
Sol. $\mathrm{D}_{3}=50 \mathrm{~mm}$, inside clearance ratio $=1 \%$


Inside clearances ratio $=\frac{D_{3}-D_{1}}{D_{1}}$

$$
\Rightarrow 0.01=\frac{50-D_{1}}{D_{1}}
$$

$$
\mathrm{D}_{1}=49.50 \mathrm{~mm}
$$

32. A concentrically loaded isolated square footing of size $2 \mathrm{~m} \times 2 \mathrm{~m}$ carries a concentrated vertical load of 1000 kN . Considering Boussinesq's theory of stress distribution, the maximum depth (in m ) of the pressure bulb corresponding to $10 \%$ of the vertical load intensity will be
$\qquad$ . (round off to two decimal places)
[NAT: 1 Mark]
Ans. (4.35-4.39)
Sol. vertically load intensity (Q) at top
$\mathrm{Q}=\frac{\text { Load }}{\text { area }}=\frac{1000}{2^{2}}$

$$
\mathrm{Q}=250 \mathrm{kN} / \mathrm{m}^{2}
$$

At depth (Z), $\sigma_{z}=10 \%$ (250)

$$
\sigma_{2}=25 \mathrm{kN} / \mathrm{m}^{2}
$$

By Boussinesq's theory

$$
\begin{aligned}
& \sigma_{z}=\frac{3}{2 \pi} \frac{\mathrm{Q}}{\mathrm{z}^{2}}\left[\frac{1}{1+\frac{r^{2}}{z^{2}}}\right]^{5 / 2} \\
& \text { For, } r=0, \Rightarrow \sigma_{z}=\frac{3}{2 \pi} \frac{Q}{z^{2}} \\
& \Rightarrow 25=\frac{3}{2 \pi} \times \frac{1000}{z^{2}} \\
& z=4.37 \mathrm{~m}
\end{aligned}
$$

33. In a triaxial unconsolidated undrained (UU) test on a saturated clay sample, the cell pressure was 100 kPa . If the deviatoric stress at failure was 150 kPa , then the undrained shear strength of the soil is
$\qquad$ kPa . (in integer)
[NAT: 1 Mark]
Ans. (75-75)
Sol. Cell pressure $=\sigma_{c}=100 \mathrm{kPa}$
Deviatoric stress at Failure $\sigma_{d}=\Delta \sigma=150$
kPa
$\sigma_{1}=150+100=250 \mathrm{kPa}$
undrained shear strength
$=c_{u u}=\tau_{f}=\frac{\sigma_{1}-\sigma_{2}}{2}$
$=\frac{(100+150)-100}{2}$
$\mathrm{C}_{\mathrm{uu}}=75 \mathrm{kPa}$
34. A flood control structure having an expected life of $n$ years is designed by considering a flood of return period $T$ years. When $T=n$, and $n \rightarrow \infty$, the structure's hydrologic risk of failure in percentage is. (round off to one decimal place)
[NAT: 1 Mark]

Ans. (63.0 to 63.5)
Sol. Risk $=1-q^{n}$

$$
\begin{aligned}
& q=1-p \\
& P=\frac{1}{T} \\
& \Rightarrow q=1-\frac{1}{T}
\end{aligned}
$$

For $\mathrm{T}=\mathrm{n} \rightarrow \infty$
$\Rightarrow \operatorname{Risk}(R)=\left[1-\left(1-\frac{1}{n}\right)^{n}\right]$

$$
\lim _{x \rightarrow \infty}\left(1+\frac{1}{x}\right)^{x}=e
$$

$$
\begin{aligned}
& \Rightarrow \lim _{n \rightarrow \infty}\left(1+\left(-\frac{1}{n}\right)^{-n}\right)^{-1}=e^{-1} \\
& \Rightarrow R=1-e^{-1} \\
& R=1-\frac{1}{e}=0.6321 \\
& \Rightarrow R=63.2 \%
\end{aligned}
$$

35. The base length of the runway at the mean sea level (MSL) is 1500 m . If the runway is located at an altitude of 300 m above the MSL, the actual length (in m ) of the runway to be provided is $\qquad$ . (round off to the nearest integer)
[NAT: 1 Mark]
Ans. (1602-1606)
Sol. As per the provision, elevation must be increased by $7 \%$ per 300-meter rise in MSL.
$\mathrm{L}=$ Basic runway length $=1500 \mathrm{~m}$
Altitude $=300 \mathrm{~m}$
L' = actual length
$L^{\prime}=L+\frac{7}{100} \times \frac{300}{300} \times L$
$L^{\prime}=1500+\frac{7}{100} \times \frac{300}{300} \times 1500$
$\mathrm{L}^{\prime}=1605 \mathrm{~m}$
36. Consider the polynomial $f(x)=x^{3}-6 x^{2}+$ $11 x-6$ on the domain $S$ given by $1 \leq x \leq$
37. The first and second derivatives are $f^{\prime}(x)$ and $f^{\prime \prime}(x)$.
Consider the following statements:
I. The given polynomial is zero at the boundary points $x=1$ and $x=3$.
II. There exists one local maxima of $f(x)$ within the domain S .
III. The second derivative $f^{\prime \prime}(x)>0$ throughout the domain S .
IV. There exists one local minima of $f(x)$ within the domain S .
The correct option is:
A. Only statements I, II and III are correct.
B. Only statements I, II and IV are correct.
C. Only statements I and IV are correct.
D. Only statements II and IV are correct.
[MCQ: 2 Marks]
Ans. (B)
Sol. $f(x)=x^{3}-6 x^{2}+11 x-6, \quad S: 1 \leq x \leq 3$
$f^{\prime}(x)=3 x^{2}-12 x+11$
$f^{\prime \prime}(x)=6 x-12$
$f(x)=0$
$\Rightarrow x^{3}-6 x^{2}+11 x-6=0$
$\Rightarrow x^{3}-x^{2}-5 x^{2}+5 x+6 x-6=0$
$\Rightarrow x^{2}(x-1)-5 x(x-1)+6(x-1)=0$
$\Rightarrow(x-1)\left(x^{2}-5 x+6\right)=0$
$\Rightarrow(x-1)(x-2)(x-3)=0$
$x=1,2,3$
Hence, Statement 1 is correct
$f^{\prime}(x)=0$
$\Rightarrow 3 x^{2}-12 x+11=0$
$x=\frac{12 \pm \sqrt{12^{2}-4 \times 3 \times 11}}{6}=\frac{12 \pm 2 \sqrt{3}}{6}=2 \pm \frac{1}{\sqrt{3}}$
$=2+\frac{1}{\sqrt{3}}, 2-\frac{1}{\sqrt{3}}$
$=2.577,1.423$
$f^{\prime \prime}(x)=6 x-12$
$f^{\prime}(2.577)=+v e \Rightarrow$ point of local minima
$\mathrm{f}^{\prime}(1.423)=-\mathrm{ve} \Rightarrow$ point of local maxima
There exists one local minima and one local maxima within the domain S .
Hence statements II and IV are correct.
$\mathrm{f}^{\prime \prime}(\mathrm{x})>0$
$\Rightarrow 6 x-12>0$
$x>2$
$\mathrm{f}^{\prime \prime}(\mathrm{x})>0$ for $2<\mathrm{x} \leq 3$. Hence, Statement III is incorrect.
38. An undamped spring-mass system with mass $m$ and spring stiffness $k$ is shown in the figure. The natural frequency and natural period of this system are $\omega \mathrm{rad} / \mathrm{s}$ and T s , respectively. If the stiffness of the spring is doubled and the mass is halved, then the natural frequency and the natural period of the modified system, respectively, are

A. $2 \omega \mathrm{rad} / \mathrm{s}$ and $\mathrm{T} / 2 \mathrm{~s}$
B. $\omega / 2 \mathrm{rad} / \mathrm{s}$ and 2 T s
C. $4 \omega \mathrm{rad} / \mathrm{s}$ and $\mathrm{T} / 4 \mathrm{~s}$
D. $\omega \mathrm{rad} / \mathrm{s}$ and T s
[MCQ: 2 Marks]
Ans. (A)
Sol. $\omega=\sqrt{\frac{k}{m}}$
$T=\frac{2 \pi}{\omega}$

When stiffness is doubled, and mass is halved
$\omega^{\prime}=\sqrt{\frac{2 k}{m / 2}}=2 \sqrt{\frac{k}{m}}=2 \omega$
$\mathrm{T}^{\prime}=\frac{2 \pi}{\omega^{\prime}}=\frac{2 \pi}{2 \omega}=\frac{\mathrm{T}}{2}$
38. For the square steel beam cross-section shown in the figure, the shape factor about $z$ - $z$ axis is $S$ and the plastic moment capacity is $M_{p}$. Consider yield stress

$\mathrm{fy}=250 \mathrm{MPa}$ and $\mathrm{a}=100 \mathrm{~mm}$.
A. $S=2.0, M_{p}=58.9 \mathrm{kN}-\mathrm{m}$
B. $S=2.0, M_{p}=100.2 \mathrm{kN}-\mathrm{m}$
C. $S=1.5, M_{p}=58.9 \mathrm{kN}-\mathrm{m}$
D. $S=1.5, M_{p}=100.2 \mathrm{kN}-\mathrm{m}$
[MCQ: 2 Marks]
Ans. A
Sol. For diamond section, shape factor (S) is 2 .


Plastic moment capacity ( $\mathrm{M}_{\mathrm{P}}$ )
$M_{P}=($ shape factor $) \times M_{g}$
$M_{p}=S f_{y} Z_{e}$
$Z_{e}=$ elastic section modulus

$$
\begin{aligned}
& z_{e}=\frac{I_{z z}}{y_{\max }} \\
& z_{e}=\frac{\frac{a^{4}}{12}}{a / \sqrt{2}}=\frac{\sqrt{2} \mathrm{a}^{3}}{12} \mathrm{~mm}^{3} \\
& \text { Now, } M_{P}=S f_{y} Z_{e} \\
& =2 \times 250 \times \frac{\sqrt{2} \times(100)^{3}}{3} \\
& M_{\mathrm{P}}=58.93 \mathrm{kNm}
\end{aligned}
$$

39. A post-tensioned concrete member of span 15 m and cross-section of 450 mm 450 mm is prestressed with three steel tendons, each of cross-sectional area 200 mm 2 . The tendons are tensioned one after another to a stress of 1500 MPa . All the tendons are straight and located at 125 mm from the bottom of the member. Assume the prestress to be the same in all tendons and the modular ratio to be 6 . The average loss of prestress, due to elastic deformation of concrete, considering all three tendons is
A. 14.16 MPa
B. 7.08 MPa
C. 28.32 MPa
D. 42.48 MPa
[MCQ: 2 Marks]
Ans. (A)
Sol. $\mathrm{L}=15 \mathrm{~m}$
$B=450 \mathrm{~mm}$
$D=450 \mathrm{~mm}$
$N=3$
As $=200 \mathrm{~mm}^{2}$
$\mathrm{e}=1500 \mathrm{MPa}$
$\mathrm{m}=6$

e = eccentricity
$e=\frac{450}{2}-125=100 \mathrm{~mm}$
$A_{s}=$ Area of steel of each bar $=200 \mathrm{~mm}^{2}$
$\mathrm{p}_{0}=1500 \mathrm{MPa}$
$P$ - force in each tandon
$\mathrm{P}=\mathrm{p}_{0} \times \mathrm{As}$
$p=1500 \times \frac{200}{100}=300 \mathrm{kN}$
This force will be in each wire

$f_{c 1}=f_{c 2}=\frac{P_{3}}{A}+\frac{P_{3} e^{2}}{I}$
$f_{c 1}=f_{c 2}=\frac{300 \times 1000}{450 \times 450}+\frac{300 \times 1000 \times 100^{2}}{450 \times \frac{450^{3}}{12}}$
$=2.36 \mathrm{MPa}$
Loss due to elastic shortening $=m \times f_{c, a v g}$
Loss due to elastic shortening
$=6 \times\left(\frac{2.36+2.36}{2}\right)$
Loss due to elastic shortening
$=14.16 \mathrm{~N} / \mathrm{mm}^{2}$
Loss of stress due to elastic shortening in wire number 1 (due to $\mathrm{P}_{2}$ and $\mathrm{P}_{3}$ )
$P_{2}+P_{3}=2 P=600 \mathrm{kN}$
In wire - 1
$f_{c 1}=f_{c 2}=\frac{P_{2}+P_{3}}{A}+\frac{P_{2}+P_{3}}{I} e^{2}=4.72 \mathrm{~N} / \mathrm{mm}^{2}$
$\Delta=$ fixing wire at different location
We are neglecting self-weight because selfweight will create gain of stress

$f_{c 1}=f_{c 2}=\frac{P}{A}+\frac{P e}{I} \times y$
$=\frac{P}{A}+\frac{P^{2}}{I}$
Loss of stress due to elastic shorting in wire number $3=0$

Loss of stress due to elastic sharing in wire number 2 (due to $P_{3}$ force)

Loss of stress $=m \times f_{c}$, avg
Loss of stress $=6 \times 4.72=28.32 \mathrm{~N} / \mathrm{mm}^{2}$
Average loss in 3 wires $=\frac{0+14.16+28.32}{3}$
Average loss in 3 wires $=14.16 \mathrm{~N} / \mathrm{mn}^{2}$
40. Match the following in Column $\times$ with Column Y:

| Column X |  | Column Y |  |
| :--- | :--- | :--- | :--- |
| $(P)$ | In a triaxial <br> lompression <br> test, with <br> increase of <br> axial strain in <br> loose sand <br> under drained <br> shear <br> condition, the <br> volumetric <br> strain | (I) | decreases. |


| (Q) | In a triaxial <br> compression <br> test, with <br> increase of <br> axial strain in <br> loose sand <br> under <br> undrained <br> shear <br> condition, the <br> excess pore <br> water pressure | increases. |  |
| :--- | :--- | :--- | :--- |
| (R) | In a triaxial <br> compression <br> test, the pore <br> pressure <br> parameter 'B' <br> for a saturated <br> soil | (III) | remains |
| same. |  |  |  |
| (S) | For shallow <br> strip footing in <br> pure saturated <br> clay, Terzaghi's <br> bearing <br> capacity factor <br> (Na) due to <br> surcharge | (IV) | is always 0.0. |
|  |  | (V) |  |
|  | is always 1.0. |  |  |

## Sol.

- In case pf loose sand as axial strain is increased volume of sand decreases for drained shear condition.
- As volume of voids decrease in loose sand on the axial strain, the excess pore
water pressure increases for undrained condition
- Terzaghi's bearing capacity factor ( $\mathrm{N} \gamma$ ) is one for pure clay.
- The pore pressure parameter $B$ is one for saturate

41. A soil sample is underlying a water column of height h1, as shown in the figure. The vertical effective stresses at points $A, B$, and $C$ are $\sigma_{A}^{\prime}, \sigma_{B}^{\prime}, \sigma_{C}^{\prime}$ respectively. Let $y_{\text {sat }}$ and $y^{\prime}$ be the saturated and submerged unit weights of the soil sample, respectively, and $y_{w}$ be the unit weight of water. Which one of the following expressions correctly represents the sum $\left(\sigma_{A}^{\prime}+\sigma_{B}^{\prime}+\sigma_{C}^{\prime}\right)$ ?

A. $\left(2 h_{2}+h_{3}\right) y^{\prime}$
B. $\left(2 h_{1}+h_{2}+h_{3}\right) y^{\prime}$
C. $\left(h_{2}+h_{3}\right)\left(y_{\text {sat }}-y_{w}\right)$
D. $\left(h_{1}+h_{2}+h_{3}\right) y_{\text {sat }}$
[MCQ: 2 Marks]

## Ans. A

Sol. effective stress at $A, \sigma_{A}^{\prime}=0$
Effective stress at $B, \sigma_{B}^{\prime}=h_{2} \times y^{\prime}$

Effective stress at $C, \sigma_{C}^{\prime}=\left(h_{2}+h_{3}\right) y^{\prime}$
$\Rightarrow \sigma_{\mathrm{A}}^{\prime}+\sigma_{\mathrm{B}}^{\prime}+\sigma_{\mathrm{C}}^{\prime}=0+\mathrm{h}_{2} \gamma^{\prime}+\left(\mathrm{h}_{2}+\mathrm{h}_{3}\right) \gamma^{\prime}$
$=\left(2 h_{2}+h_{2}\right) \gamma^{\prime}$
42. A 100 mg of HNO 3 (strong acid) is added to water, bringing the final volume to 1.0 litre. Consider the atomic weights of $\mathrm{H}, \mathrm{N}$, and O , as $1 \mathrm{~g} / \mathrm{mol}, 14 \mathrm{~g} / \mathrm{mol}$, and $16 \mathrm{~g} / \mathrm{mol}$, respectively. The final pH of this water is (Ignore the dissociation of water.)
A. 2.8
B. 6.5
C. 3.8
D. 8.5
[MCQ: 2 Marks]
Ans. (A)
Sol. $\mathrm{HNO}_{3} \rightarrow \mathrm{H}^{+}+\mathrm{NO}_{3}^{-}$
1 mol of $\mathrm{HNO}_{3}$ gives 1 mol of $\mathrm{H}^{+}$
No. of moles of $\mathrm{HNO}_{3}$ in 100 mg of $\mathrm{HNO}_{3}=\frac{100}{1+14+3 \times 16} \mathrm{~m} \mathrm{~mol}$
$=1.587 \times 10^{-3} \mathrm{~mol}$
Concentration of $\mathrm{HNO}_{3}$ in $\mathrm{mol} / \mathrm{lit}=1.587 \times$ $10^{-3} \mathrm{ml} /$ lit
$\left[\mathrm{H}^{+}\right]=\left[\mathrm{HNO}_{3}\right]$
$=1.587 \times 10^{-3} \mathrm{~mol} / \mathrm{lit}$
$\mathrm{pH}=-\log _{10}\left[\mathrm{H}^{+}\right]$
$=3-\log _{10} 1.587$
$=2.8$
43. In a city, the chemical formula of biodegradable fraction of municipal solid waste (MSW) is $\mathrm{C}_{100} \mathrm{H}_{250} \mathrm{O}_{80} \mathrm{~N}$. The waste has to be treated by forced-aeration composting process for which air requirement has to be estimated.
Assume oxygen in air (by weight)
$=23 \%$, and density of air
$=1.3 \mathrm{~kg} / \mathrm{m}^{3}$. Atomic mass:
$\mathrm{C}=12$
$\mathrm{H}=1, \mathrm{O}=16, \mathrm{~N}=14$.
C and H are oxidized completely whereas N is converted only into $\mathrm{NH}_{3}$ during oxidation.

For oxidative degradation of 1 tonne of the waste, the required theoretical volume of air (in $\mathrm{m}^{3} /$ tonne) will be (round off to the nearest integer)
A. 4749
B. 8025
C. 1418
D. 1092
[MCQ: 2 Marks]
Ans. (A)
Sol. Balanced chemical reaction:
$4 \mathrm{C}_{100} \mathrm{H}_{250} \mathrm{O}_{80} \mathrm{~N}+487 \mathrm{O}_{2} \rightarrow 400 \mathrm{CO}_{2}+$ $494 \mathrm{H}_{2}+4 \mathrm{NH}_{3}$
$\Rightarrow 4$ moles of MSW requires 487 moles of oxygen
$\Rightarrow 1$ mole of MSW requires 121.75 moles of oxygen

No. of moles of MSW in 1 tonne
$=\frac{1000 \times 100}{100 \times 12+250+80 \times 16+14}$
$=\frac{1000 \times 100}{2744}$ moles
Moles of $\mathrm{O}_{2}$ required
$=\frac{1000 \times 1000}{2744} \times 121.75$
Wt. of $\mathrm{O}_{2}$ required
$=\frac{1000 \times 1000}{2744} \times 121.75 \times 32 \mathrm{gm}$
$=\frac{121.75 \times 32 \times 1000}{2744} \mathrm{~kg}$
$=1420 \mathrm{~kg}$
Mass of air $=\frac{1420}{0.23} \approx 6174 \mathrm{~kg}$
Density of air $=1.3 \mathrm{~kg} / \mathrm{m}^{3}$

Volume of air $=\frac{6174}{1.3}$

$$
\approx 4749 \mathrm{~m}^{3}
$$

44. A single-lane highway has a traffic density of 40 vehicles $/ \mathrm{km}$. If the time-mean speed and space-mean speed are 40 kmph and 30 kmph, respectively, the average headway (in seconds) between the vehicles is
A. 3.00
B. 2.25
C. $8.33 \times 10^{-4}$
D. $6.25 \times 10^{-4}$
[MCQ: 2 Marks]
Ans. (A)
Sol. $\mathrm{k}=40 \mathrm{veh} / \mathrm{km}$
$\mathrm{v}_{\mathrm{t}}=40 \mathrm{kmph}$
$\mathrm{v}_{\mathrm{s}}=30 \mathrm{kmph}$
$\mathrm{q}=\mathrm{k} \times \mathrm{v}_{\mathrm{s}}$
$=40 \times 30$
$=1200 \mathrm{veh} / \mathrm{hr}$
$\mathrm{q}=\frac{3600}{\mathrm{H}_{\mathrm{t}}}$
$1200=\frac{3600}{H_{t}}$
$H_{t}=3 \mathrm{sec}$
45. Let y be a non-zero vector of size $2022 \times 1$. Which of the following statement(s) is/are TRUE?
A. $\mathrm{yy}^{\top}$ is a symmetric matrix.
B. $y^{\top} y$ is an eigenvalue of $y y^{\top}$.
C. $\mathrm{yy}^{\top}$ has a rank of 2022.
D. $\mathrm{yy}{ }^{\top}$ is invertible.
[MSQ: 2 Marks]
Ans. (A, B)
Sol.

$$
y=\left[\begin{array}{c}
a_{1} \\
a_{2} \\
\vdots \\
\vdots \\
a_{2022}
\end{array}\right]
$$

$$
y^{\top}=\left[\begin{array}{lllll}
a_{1} & a_{2} & a_{3} & \ldots \ldots & a_{2022}
\end{array}\right]
$$

$$
y y^{\top}=\left[\begin{array}{c}
a_{1} \\
a_{2} \\
\vdots \\
: \\
\vdots \\
a_{2022}
\end{array}\right]\left[\begin{array}{lllll}
a_{1} & a_{2} & a_{3} & \ldots \ldots . & a_{2022}
\end{array}\right]
$$

$$
=\left[\begin{array}{ccccc}
a_{1}^{2} & a_{1} a_{2} & a_{1} a_{3} & \ldots \ldots . & a_{1} \\
a_{2022} \\
a_{2} a_{1} & a_{2}^{2} & a_{2} a_{3} & \ldots \ldots . & a_{2}
\end{array} a_{2022} .\left[\begin{array}{cccc}
a_{3} a_{1} & a_{3} a_{2} & a_{3}^{2} & \ldots \ldots . \\
\vdots & \vdots & \vdots & a_{3} \\
a_{2022} \\
\vdots & \vdots & \vdots & \vdots \\
a_{2022} a_{1} & a_{2022} a_{2} & \ldots & \ldots \\
\vdots \\
a_{2022}^{2}
\end{array}\right]\right.
$$

We can see that $\mathrm{yy}^{\top}$ is a symmetric matrix
$\operatorname{det}\left(y y^{\top}\right)=\left|\begin{array}{ccccc}a_{1}^{2} & a_{1} a_{2} & a_{1} a_{3} & \ldots \ldots . & a_{1} a_{2022} \\ a_{2} a_{1} & a_{2}^{2} & a_{2} a_{3} & \ldots \ldots . & a_{2} \\ a_{2022} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{2022} a_{1} & \ldots . . & \ldots . . & \ldots . . & a_{2022}^{2}\end{array}\right|$
$=\left(a_{1} a_{2} \ldots . a_{2022}\right)\left|\begin{array}{ccccc}a_{1} & a_{2} & a_{3} & \ldots . . & a_{2022} \\ a_{1} & a_{2} & a_{3} & \ldots . . & a_{2022} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{1} & a_{2} & a_{3} & \ldots . & a_{2022}\end{array}\right|$
Hence, $\operatorname{det}\left(y y^{\top}\right)=0$
i.e., rank of $\left(\mathrm{yy}^{\top}\right)<2022$
$\mathrm{yy}^{\top}$ is not invertible
2021 rows are identical after elementary transformation hence out of 2022 eigen values 0 eigen value will occur 2021 times.

Remaining eigen value $=$ Trace $\left(\mathrm{yy}^{\top}\right)$
$=a_{1}^{2} a_{2}^{2}+\ldots a_{2022}^{2}$
$=y^{\top} y$
46. Which of the following statement(s) is/are correct?
A. If a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the deflection at any point is equal to the load applied at that point.
B. If a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the load at any point is equal to the deflection at that point.
C. If a structure is acted upon by two force system $\mathrm{Pa}_{\mathrm{a}}$ and $\mathrm{Pb}_{\mathrm{b}}$, in equilibrium separately, the external virtual work done by a system of forces Pb during the deformations caused by another system of forces $\mathrm{Pa}_{\mathrm{a}}$ is equal to the external virtual work done by the Pa system during the deformation caused by the $\mathrm{Pb}_{\mathrm{b}}$ system.
D. The shear force in a conjugate beam loaded by the M/EI diagram of the real beam is equal to the corresponding deflection of the real beam.
[MSQ: 2 Marks]
Ans. (A, B, C)

## Sol.

Castigliano's first theorem states that the first partial derivative of the total internal energy (strain energy) in a structure with respect to any particular deflection component at a point is equal to the force applied at that point and in the direction corresponding to that deflection component.
$\frac{\partial \mathrm{U}}{\partial \delta}=\mathrm{P}$
Castigliano's second theorem states that the first partial derivative of the total internal energy in a structure with respect to the force applied at any point is equal to the deflection at the point of application of that force in the direction of its line of action.

$$
\frac{\partial \mathrm{u}}{\partial \mathrm{P}}=\delta
$$



From Betti's theorem, $\mathrm{Pa} \Delta_{\mathrm{b}}=\mathrm{Pb} \Delta_{\mathrm{a}}$
i.e. virtual work done by a system of forces $\mathrm{Pb}_{\mathrm{b}}$ during the deformations caused by another system of forces Pa is equal to the virtual work done by a system of forces $\mathrm{Pa}_{\mathrm{a}}$ during the deformations caused by $\mathrm{P}_{\mathrm{b}}$ system.
Hence A, B and C are correct.
47. Water is flowing in a horizontal, frictionless, rectangular channel. A smooth hump is built on the channel floor at a section and its height is gradually increased to reach choked condition in the channel. The depth of water at this section is $y_{2}$ and that at its upstream section is $y_{1}$. The correct statement(s) for the choked and unchoked conditions in the channel is/are
A. In choked condition, $y_{1}$ decreases if the flow is supercritical and increases if the flow is subcritical. B. In choked condition, $\mathrm{y}_{2}$ is equal to the critical depth if the flow is supercritical or subcritical.
C. In unchoked condition, $y_{1}$ remains unaffected when the flow is supercritical or subcritical.
D. In choked condition, $\mathrm{y}_{1}$ increases if the flow is supercritical and decreases if the flow is subcritical.
[MSQ: 2 Marks]
Ans. A, B, C
48. The concentration ( $x, t$ ) of pollutants in a one-dimensional reservoir at position $x$ and time $t$ satisfies the diffusion equation
$\frac{\partial s(x, t)}{\partial t}=D \frac{\partial^{2} s(x, t)}{\partial x^{2}}$
on the domain $0 \leq x \leq L$, where $D$ is the diffusion coefficient of the pollutants. The initial condition $s(\mathrm{x}, 0)$ is defined by the step-function shown in the figure.


The boundary conditions of the problem are given by $\frac{\partial s(x, t)}{\partial t}=0$ at the boundary point $x$ $=0$ and $\mathrm{x}=\mathrm{L}$ at all times. Consider $\mathrm{D}=0.1$ $\mathrm{m}^{2} / \mathrm{s}$. $\mathrm{S}_{0}=5 \mu \mathrm{~mol} / \mathrm{m}$, and $\mathrm{L}=10$,
The steady concentration $\tilde{S}\left(\frac{L}{2}\right)=S\left(\frac{L}{2}, \infty\right)$ at the center $x=\frac{L}{2}$ of the reservoir (in $\mu / \mathrm{mol} / \mathrm{m}$ ) $\qquad$ . (in integer)
[NAT: 2 Marks]
Ans. (2-2)

Sol. $\frac{\partial^{2} S}{\partial x^{2}}(x, t)=\frac{1}{D} \frac{\partial S}{\partial t}(x, t)$
Under steady state condition
$\Rightarrow \frac{\partial \mathrm{S}}{\partial \mathrm{t}}=0$
$\Rightarrow \frac{\partial^{2} \mathrm{~S}}{\partial \mathrm{x}^{2}}=0$
$\Rightarrow \frac{d^{2} s}{d x^{2}}=0$
$\Rightarrow \frac{\mathrm{ds}}{\mathrm{dx}}=\mathrm{A}$
$\Rightarrow S=A_{x}+B$
At $x=0, \quad S=S_{0}$
i) $\Rightarrow S_{0}=O+B$
$B=S_{0}$
And at $x=L \quad S=0$
i) $\Rightarrow \mathrm{O}=\mathrm{AL}+\mathrm{S}_{0}$

$$
\begin{equation*}
A=-\frac{S_{0}}{L} \tag{2}
\end{equation*}
$$

Put $A$ and $B$ in (1)
i)
$S=-\frac{S_{0}}{L} \cdot x+S_{0}$
$S=(x, t)=-\frac{S_{0}}{L} \cdot x+S_{0}$
At $x=\frac{L}{2}$
$S\left(\frac{L}{2}, \infty\right)=\frac{-S_{0}}{L} \cdot \frac{L}{2}+S_{0}=\frac{-S_{0}}{L}+S_{0}$
$S\left(\frac{L}{2}, \infty\right)=\frac{S_{0}}{2}$
Given
$S\left(\frac{L}{2}, \infty\right)=\frac{5}{2}=2.5$
$S\left(\frac{L}{2}, \infty\right)=2$ (only integer part)
49. A pair of six-faced dice is rolled thrice. The probability that the sum of the outcomes in each roll equals 4 in exactly two of the three attempts is $\qquad$ . (round off to three decimal places)
[NAT: 2 Marks]
Ans. (0.018-0.020)
Sol. Required outcomes
$\equiv(1,3)(2,2)$ and $(3,1)$
Probability of success, $\mathrm{p}=\frac{3}{36}=\frac{1}{12}$
Probability of failure, $\mathrm{q}=\frac{11}{12}$
Probability of 2 success in 3 attempts
$=3 c_{2} p^{2} q^{1}$
$=3 c_{2} \times\left(\frac{1}{12}\right)^{2}\left(\frac{11}{12}\right)$
$=0.01909$
$\approx 0.019$
50. Consider two linearly elastic rods HI and IJ, each of length $b$, as shown in the figure. The rods are co-linear, and confined between two fixed supports at H and J. Both the rods are initially stress free. The coefficient of linear thermal expansion is a for both the rods. The temperature of the rod IJ is raised by $\Delta T$, whereas the temperature of rod HI remains unchanged. An external horizontal force $P$ is now applied at node $I$. It is given that $\mathrm{a}=10^{-60} \mathrm{C}^{-1}, \Delta \mathrm{~T}$
$=50^{\circ} \mathrm{C}, \mathrm{b}=2 \mathrm{~m} \quad \mathrm{AE}=10^{6} \mathrm{~N}$. The axial rigidities of the rods HI and IJ are 2 AE and $A E$, respectively.


To make the axial force in rod HI equal to zero, the value of the external force P (in N) is $\qquad$ (round off to the nearest integer)
[NAT: $\mathbf{2}$ Marks]
Ans. (47-53)
Sol. To have zero axial force in rod HI, axial deformation of HI rod shall be zero.
$\Rightarrow \Delta_{\text {axial load }}=\Delta_{\text {temp }}$


By temperature raise of rod IJ
$\Delta_{\text {temp }}=l a \Delta t$
$\Delta_{\text {temp }}=\mathrm{ba} \Delta \mathrm{t}$
By force ' $P$ ', deflection is
$\Delta_{\text {axial load }}=\frac{\mathrm{Pb}}{\mathrm{AE}}$
$\Rightarrow \frac{\mathrm{Pb}}{\mathrm{AE}}=\mathrm{ba} \Delta \mathrm{t}$
$P=A E \times \quad a \Delta t$
$=10^{6} \times\left(10^{-6} \times 50\right)=50 \mathrm{~N}$
51. The linearly elastic planar structure shown in the figure is acted upon by two vertical concentrated forces. The horizontal beams UV and WX are connected with the help of the vertical linear spring with spring constant $\mathrm{k}=20 \mathrm{kN} / \mathrm{m}$. The fixed supports are provided at $U$ and $X$. It is given that flexural rigidity
$\mathrm{EI}=10^{5} \mathrm{kN}-\mathrm{m}^{2}, \mathrm{P}=100 \mathrm{kN}$, and $\mathrm{a}=5 \mathrm{~m}$. Force Q is applied at the center of beam WX such that the force in the spring VW becomes zero.


The magnitude of force Q (in kN ) is
$\qquad$ . (round off to the nearest integer)
[NAT: 2 Marks]
Ans. (620-660)
Sol. For force in spring to be zero, deflection of point $V$ and deflection of point $w$ shall be equal

$\Rightarrow \delta_{v}=\delta_{w}$
$\frac{\mathrm{P}(2 \mathrm{a})^{3}}{3 \mathrm{EI}}=\frac{\mathrm{Qa}^{3}}{3(2 \mathrm{EI})}+\frac{\mathrm{Qa}^{2} \times \mathrm{a}}{2(2 \mathrm{EI})}$
$\frac{8}{3} P=\frac{25}{6} Q$
$\Rightarrow Q=\frac{32}{5} P$
$\mathrm{Q}=\frac{32 \times 100}{5} \mathrm{P}$
Q $=640 \mathrm{kN}$
52. A uniform rod KJ of weight w shown in the figure rests against a frictionless vertical wall at the point K and a rough horizontal surface at point J. It is given that $\mathrm{w}=10$ $\mathrm{kN}, \mathrm{a}=4 \mathrm{~m}$ and $\mathrm{b}=3 \mathrm{~m}$.


The minimum coefficient of static friction that is required at the point $J$ to hold the rod in equilibrium is $\qquad$ . (round off to three decimal places)
[NAT: 2 Marks]
Ans. (0.350-0.40)
Sol.


Free body diagram

$$
\begin{aligned}
& \Sigma v=0 \Rightarrow V_{j}=10 k N \\
& \Sigma M_{k}=0 \\
& \Rightarrow V_{j} \times 3-10 \times 1.5-\mu_{\mathrm{s}} V_{j} \times 4=0 \\
& \Rightarrow 10 \times 3-15-\mu_{\mathrm{s}} \times 10 \times 4=0
\end{aligned}
$$

$$
\mu_{\mathrm{s}}=0.375
$$

53. The activities of a project are given in the following table along with their durations and dependency.

| Activities | Duration <br> (days) | Depends <br> on |
| :---: | :---: | :---: |
| A | 10 | - |
| B | 12 | - |
| C | 5 | A |
| D | 14 | A |
| E | 10 | B, C |

The total float of the activity $E$ (in days) is
$\qquad$ . (in integer)
[NAT: 2 Marks]
Ans. (1 to 1)
Sol.

$$
(10,11)
$$

$(15,16)$
$(0,0)$


Total float of the activity E

$$
\begin{aligned}
& \mathrm{F}_{\mathrm{T}}=\mathrm{T}_{\mathrm{L}}^{5}-\mathrm{T}_{E}^{4}-\mathrm{t}_{\mathrm{E}} \\
& =26-15-10=1 \text { day } \\
\left\{\mathrm{F}_{\mathrm{T}}^{\mathrm{ij}}=\right. & \left.\mathrm{T}_{\mathrm{T}}^{\mathrm{j}}-\mathrm{T}_{E}^{\mathrm{i}}-\mathrm{t}_{\mathrm{ij}}\right\}
\end{aligned}
$$

54. A group of total 16 piles are arranged in a square grid format. The center-to center spacing (s) between adjacent piles is 3 m . The diameter (d) and length of embedment of each pile are 1 m and 20 m , respectively.
55. A saturated compressible clay layer of thickness $h$ is sandwiched between two sand layers, as shown in the figure. Initially, the total vertical stress and pore water pressure at point $P$, which is located at the mid-depth of the clay layer, were 150 kPa and 25 kPa , respectively. Construction of a building caused an additional total vertical stress of 100 kPa at P . When the vertical effective stress at $P$ is 175 kPa , the percentage of consolidation in the clay layer at $P$ is $\qquad$ . (in integer)
[NAT: 2 Marks]

[NAT: 2 Marks]
Ans. (50-50)
Sol. At point ' $p$ ' initial effective stress
$\sigma_{\mathrm{P}}^{\prime}=150-25=125 \mathrm{kPa}$
After construction of building, effective stress at P .
$\sigma_{\mathrm{P} 2}^{\prime}=125+100=225 \mathrm{kPa}$
Percentage of consolidation when effective stress at P is 175 kPa
$=\frac{175-125}{225-125} \times 100$
$=50 \%$
56. A hydraulic jump takes place in a 6 m wide rectangular channel at a point where the upstream depth is 0.5 m (just before the jump). If the discharge in the channel is 30 $\mathrm{m} 3 / \mathrm{s}$ and the energy loss in the jump is 1.6 $m$, then the Froude number computed at the end of the jump is $\qquad$ . (round off to two decimal places)
(Consider the acceleration due to gravity as $10 \mathrm{~m} / \mathrm{s}^{2}$.)
[NAT: 2 Marks]
Ans. (0.30-0.41)
Sol.

discharge $Q=30 \mathrm{~m}^{3} / \mathrm{sec}$
Width $(B)=6 \mathrm{~m}$
Energy loss ( $\mathrm{E}_{\mathrm{L}}$ ) $=1.6 \mathrm{~m}$
Pre-jump depth $=\mathrm{y}_{1}=0.5 \mathrm{~m}$
$E_{L}=\frac{\left(y_{2}-y_{1}\right)^{3}}{4 y_{1} y_{2}}$
$1.6=\frac{\left(y_{2}-0.5\right)^{3}}{4 \times 0.5 \times y_{2}}$
$3.2 y_{2}=y_{2}^{3}-3 y_{2}^{2} \times 0.5+3 y_{2}(0.5)^{2}-0.5^{3}$
$\Rightarrow y_{2}^{3}-1.5 y_{2}^{2}-2.45 y_{2}-0.125=0$
$Y_{2}=2.5 m,-0.0527 m-0.947 m$
$\Rightarrow y_{2}=2.5 \mathrm{~m}$

Froude number after jump ( $\mathrm{Fr}_{2}$ )

$$
\begin{aligned}
& \mathrm{Fr}_{2}=\frac{\mathrm{V}_{2}}{\sqrt{\mathrm{gy}_{2}}} \\
& \mathrm{~V}_{2}=\frac{\mathrm{Q}}{\mathrm{By}_{2}}=\frac{30}{6 \times 2.5} \\
& \mathrm{~V}_{2}=2 \mathrm{~m} / \mathrm{sec} . \\
& \Rightarrow \mathrm{Fr}_{2}=\frac{2}{\sqrt{9.81 \times 2.5}} \\
& \mathrm{Fr}_{2}=0.4038 \\
& \mathrm{Fr}_{2}=0.40
\end{aligned}
$$

57. A pump with an efficiency of $80 \%$ is used to draw groundwater from a well for irrigating a flat field of area 108 hectares. The base period and delta for paddy crop on this field are 120 days and 144 cm , respectively. Water application efficiency in the field is $80 \%$. The lowest level of water in the well is 10 m below the ground. The minimum required horse power (h.p.) of the pump is $\qquad$ . (round off to two decimal places)
(Consider 1 h.p. = 746 W ; unit weight of water $=9810 \mathrm{~N} / \mathrm{m}^{3}$ )
[NAT: 2 Marks]
Ans. (30-32)
Sol. Base period $(B)=120$ days
Delta $(\Delta)=144 \mathrm{~cm}$
Duty (D) $=\frac{864 B}{\Delta}=\frac{864 \times 120}{144}$

$$
=720 \text { ha/cumecs }
$$

Discharge, $Q=\frac{\text { Area }}{\text { Duty }}=\frac{108}{720}=0.15 \mathrm{~m}^{3} / \mathrm{s}$
$Q_{\text {applied }}=\frac{Q}{\eta_{\mathrm{a}}}=\frac{0.15}{0.8}=\frac{3}{16} \mathrm{~m}^{3} / \mathrm{s}$
Power required $=\gamma_{w}$ Qapplied $H$
$=9810 \times \frac{3}{16} \times 10$
$=18400 \mathrm{~W}$
$=24.66 \mathrm{HP}$

Horse power of pump $=\frac{\text { Power required }}{\eta_{\text {pump }}}$

$$
=\frac{24.66}{0.8}=30.83 \mathrm{HP}
$$

58. Two discrete spherical particles ( P and Q ) of equal mass density are independently released in water. Particle $P$ and particle Q have diameters of 0.5 mm and 1.0 mm , respectively. Assume Stokes' law is valid. The drag force on particle Q will be $\qquad$ times the drag force on particle P. (round off to the nearest integer)
[NAT: 2 Marks]
Ans. (8-8)

## Sol.



Drag force ( $\mathrm{FD}_{\mathrm{D}}$ ) = wt. of the sphere (W) Buoyant force ( $\mathrm{F}_{\mathrm{B}}$ )
$F_{D}=\rho_{\text {sphere }} \vee g-\rho_{\text {water }} \vee g$
$=\left(\rho_{\text {sphere }}-\rho_{\text {water }}\right) g \times \frac{4}{3} \pi r^{3}$
$F_{D} \propto r^{3}$
$\Rightarrow F_{D} \propto D^{3}$
$\frac{F_{D, Q}}{F_{D, P}}=\left(\frac{D_{Q}}{D_{P}}\right)^{3}=\left(\frac{1}{0.5}\right)^{3}=8$
$F_{\mathrm{D}, \mathrm{Q}}=8 \mathrm{~F}_{\mathrm{D}, \mathrm{P}}$
59. At a municipal waste handling facility, 30 metric ton mixture of food waste, yard waste, and paper waste was available. The moisture content of this mixture was found to be $10 \%$. The ideal moisture content for composting this mixture is $50 \%$. The amount of water to be added to this mixture to bring its moisture content to the ideal condition is $\qquad$ metric ton. (in integer)
[NAT: 2 Marks]
Ans. (24-24)
Sol. MSW $=30$ metric ton
Moisture content $=10 \%$
Amount of water present
$=\frac{10}{100} \times 30=3$ metric ton
Amount of solids present
$=30-3$
$=27$ metric ton
Ideal moisture content $=50 \%$
Amount of solids will not change.
Let the amount of water present in the ideal sample be $W_{w}$.
Then, $\frac{W_{w}}{27+W_{w}} \times 100=50$
$\mathrm{W}_{\mathrm{w}}=27$ metric ton
Amount of water required to be added to bring to ideal condition
= 27-3
$=24$ metric ton
60. A sewage treatment plant receives sewage at a flow rate of $5000 \mathrm{~m}^{3} /$ day. The total suspended solids (TSS) concentration in the sewage at the inlet of primary clarifier is $200 \mathrm{mg} / \mathrm{L}$. After the primary treatment, the

TSS concentration in sewage is reduced by $60 \%$. The sludge from the primary clarifier contains $2 \%$ solids concentration. Subsequently, the sludge is subjected to gravity thickening process to achieve a solids concentration of $6 \%$. Assume that the density of sludge, before and after thickening, is $1000 \mathrm{~kg} / \mathrm{m}^{3}$. The daily volume of the thickened sludge (in $\mathrm{m}^{3} /$ day) will be $\qquad$ . (round off to the nearest integer)
[NAT: $\mathbf{2}$ Marks]
Ans. (10-10)
Sol.


## solids settled in tank

Inflow rate of sewage $=5000 \mathrm{~m}^{3} /$ day
Concentration of sewage at inlet $=200$ mg/L
Amount of total suspended solids at inlet
$=5000 \times 10^{3}$ lit/day $\times 200 \times 10^{-6} \mathrm{~kg} / \mathrm{l}$
$=1000 \mathrm{~kg} /$ day
Amount of solids settled in tank
$=\frac{60}{100} \times 1000 \mathrm{~kg} /$ day $=600 \mathrm{~kg} /$ day
Given: Solids concentration is 2 \%
If the total weight of the sludge is $\mathrm{W} \mathrm{kg} /$ day
Then, $\frac{2}{100} \times W=600$
$\mathrm{W}=30000 \mathrm{~kg} /$ day
Moisture content before thickening = $98 \%$

Moisture content after thickening= $94 \%$
Amount of solids before and after thickening remains the same.

Hence, $600=W^{\prime}(1-0.94)$
where, W ': weight of sludge after thickening
$\mathrm{W}^{\prime}=10000 \mathrm{~kg} /$ day
Daily volume of thickened sludge
$=\frac{10000}{1000}=10 \mathrm{~m}^{3}$
61. A sample of air analyzed at $25^{\circ} \mathrm{C}$ and 1 atm pressure is reported to contain 0.04 ppm of $\mathrm{SO}_{2}$. Atomic mass of $\mathrm{S}=32, \mathrm{O}=16$.

The equivalent $\mathrm{SO}_{2}$ concentration (in $\mu \mathrm{g} / \mathrm{m}^{3}$ ) will be $\qquad$ . (round off to the nearest integer)
[NAT: 2 Marks]
Ans. (102-108)
Sol. From combined gas law equation, we have:
$\frac{P_{1} V_{1}}{T_{1}}=\frac{P_{2} V_{2}}{T_{2}}$
Since, 1 mol of $\mathrm{SO}_{2}$ at STP $\left(0^{\circ} \mathrm{C}, 1\right.$ atm $)$ occupies vol. of 22.4 lit
$\mathrm{P}_{1}=1 \mathrm{~atm}$
$V_{1}=22.4$ lit
$\mathrm{T}_{1}=0^{\circ} \mathrm{C}=273 \mathrm{k}$
$\mathrm{P}_{2}=1 \mathrm{~atm}$
$\mathrm{T}_{2}=25^{\circ} \mathrm{C}=298 \mathrm{k}$
$\Rightarrow \frac{1 \times 22.4}{273}=\frac{1 \times \mathrm{V}_{2}}{298}$
$V_{2}=24.45$ lit
Given that air contains 0.04 ppm of $\mathrm{SO}_{2}$
i.e. in $10^{6} \mathrm{~m}^{3}$ of air amount of $\mathrm{SO}_{2}$ present is $0.04 \mathrm{~m}^{3}$ (or 40 lit )
Moles of $\mathrm{SO}_{2}$ in $10^{6} \mathrm{~m}^{3}$ of air $=40 / 24.45$
Weight of $\mathrm{SO}_{2}$ in $10^{6} \mathrm{~m}^{3}$ of air
$=\frac{40}{24.45} \times 64 \mathrm{gm}$
$=104.7 \mathrm{~g}$
Equivalent $\mathrm{SO}_{2}$ concentration
$=\frac{104.7}{10^{6}} \mathrm{gm} / \mathrm{m}^{3}$
$\approx 105 \mu \mathrm{~g} / \mathrm{m}^{3}$
62. A parabolic vertical crest curve connects two road segments with grades $+1.0 \%$ and $-2.0 \%$. If a 200 m stopping sight distance is needed for a driver at a height of 1.2 m to avoid an obstacle of height 0.15 m , then the minimum curve length should be
$\qquad$ m. (round off to the nearest integer)
[NAT: 2 Marks]
Ans. (270-275)
Sol.
$\mathrm{n}_{1}=+1$
$n_{2}=-2 \%$
$\mathrm{N}=\mathrm{n}_{1}-\mathrm{n}_{2}$
$\mathrm{N}=1-(-2)=+3 \%$
SSD $=200 \mathrm{~m}$


So, it's a submit curve
L > SSD or OSD
The general equation for length $L$ of the parabolic curve is given by:
$L=\frac{N S^{2}}{(\sqrt{2 \mathrm{H}}+\sqrt{2 \mathrm{~h}})^{2}}$
where, $L$ = length of summit curve, $m$
$\mathrm{S}=$ sight distance
$\mathrm{N}=$ Deviation angle, equal to the algebraic difference in grades, radians or tangent of deviation angle.
$H=$ height of eye level of the driver above roadway surface, $m$
$h=$ height of an object above the road surface, m
For SSD ( $\mathrm{H}=1.2$ and $\mathrm{h}=0.15 \mathrm{~m}$ )
$\mathrm{L}=\frac{\mathrm{NS}^{2}}{4.4}$
$\mathrm{L}=\frac{3}{100} \times \frac{(200)^{2}}{4.4}$
$\mathrm{L}=272.72 \mathrm{~m}>200 \mathrm{~m}$ Hence OK $\mathrm{L} \simeq 273 \mathrm{~m}$
63. Assuming that traffic on a highway obeys the Greenshields model, the speed of a shockwave between two traffic streams (P) and $(Q)$ as shown in the schematic is kmph. (in integer)

## Direction of Traffic

(P) Flow $=1200$ vehicles

Speed $=60 \mathrm{kmph}$
(Q) Flow = 1800 vehicles/hour

Speed $=30 \mathrm{kmph}$
[NAT: 2 Marks]
Ans. (15-15)
Sol.

## Direction of Traffic

(P) Flow = 1200 vehicles

Speed $=60 \mathrm{kmph}$
(Q) Flow = 1800 vehicles/hour

Speed $=30 \mathrm{kmph}$
Speed $=\frac{\partial q}{\partial k}=\frac{q_{Q}-k_{p}}{k_{Q}-k_{p}}$
$\partial \mathbf{q}=$ change in flow
$\partial \mathrm{K}=$ change in density
$q_{Q}=1800 \mathrm{veh} / \mathrm{h}$
$\mathrm{q}_{\mathrm{P}}=1800 \mathrm{veh} / \mathrm{h}$
$\mathrm{q}_{\mathrm{p}}=1200 \mathrm{veh} / \mathrm{h}$
$\mathrm{k}_{\mathrm{Q}}=\frac{\mathrm{q}_{\mathrm{Q}}}{\mathrm{v}_{\mathrm{Q}}}=\frac{1800}{30}=60 \mathrm{veh} / \mathrm{km}$
$\mathrm{k}_{\mathrm{p}}=\frac{1200}{60}=20 \mathrm{veh} / \mathrm{cm}$
Speed $=\frac{1800-1200}{60-20}=15 \mathrm{kmph}$
64. It is given that an aggregate mix has 260 grams of coarse aggregates and 240 grams of fine aggregates. The specific gravities of the coarse and fine aggregates are 2.6 and 2.4, respectively. The bulk specific gravity of the mix is 2.3 .

The percentage air voids in the mix is
$\qquad$ . (round off to the nearest
integer)
[NAT: $\mathbf{2}$ Marks]
Ans. (8-8)
Sol. $G_{m}=2.3$
$\mathrm{G}_{C A}=2.6$
$\mathrm{G}_{F A}=2.4$
$\mathrm{~W}_{C A}=260 \mathrm{gm}$
$\mathrm{W}_{\mathrm{FA}}=240 \mathrm{gm}$
$\mathrm{G}_{\mathrm{t}}=\frac{\mathrm{W}_{\mathrm{FA}}+\mathrm{W}_{\mathrm{CA}}}{\frac{\mathrm{W}_{F A}}{\mathrm{G}_{\text {FA }}}+\frac{\mathrm{W}_{\mathrm{CA}}}{\mathrm{G}_{\mathrm{CA}}}}=\frac{260+240}{\left(\frac{260}{2.6}\right)+\left(\frac{240}{2.4}\right)}=2.5$
\% air void
$=\frac{G_{t}-G_{m}}{G_{t}} \times 100=\frac{2.5-2.3}{2.5} \times 100=8 \%$
65. The lane configuration with lane volumes in vehicles per hour of a four-arm signalized intersection is shown in the figure. There are only two phases: the first phase is for the East-West and the West-East through movements, and the second phase is for the North-South and the South-North through movements. There are no turning movements. Assume that the saturation flow is 1800 vehicles per hour per lane for each lane and the total lost time for the first and the second phases together is 9 seconds.
[NAT: 2 Marks]


The optimum cycle length (in seconds), as per the Webster's method, is $\qquad$ . (round off to the nearest integer)

Ans. (37-37)
Sol.


| $504 \longrightarrow$ |
| :---: |
| $504 \longrightarrow$ |



$$
\mathrm{qs}=1800 \mathrm{veh} / \mathrm{h}
$$

For N -S
$\mathrm{y}_{\mathrm{N}-\mathrm{S}}=\frac{396}{180}=0.22$
For E-W
$y_{E w}=\frac{504}{1800}=0.28$
$\mathrm{Co}=\frac{1.5 \mathrm{~L}+5}{1-\mathrm{y}}=\frac{1.5 \times 9+5}{1-(0.22+0.20)}=37 \mathrm{sec}$

