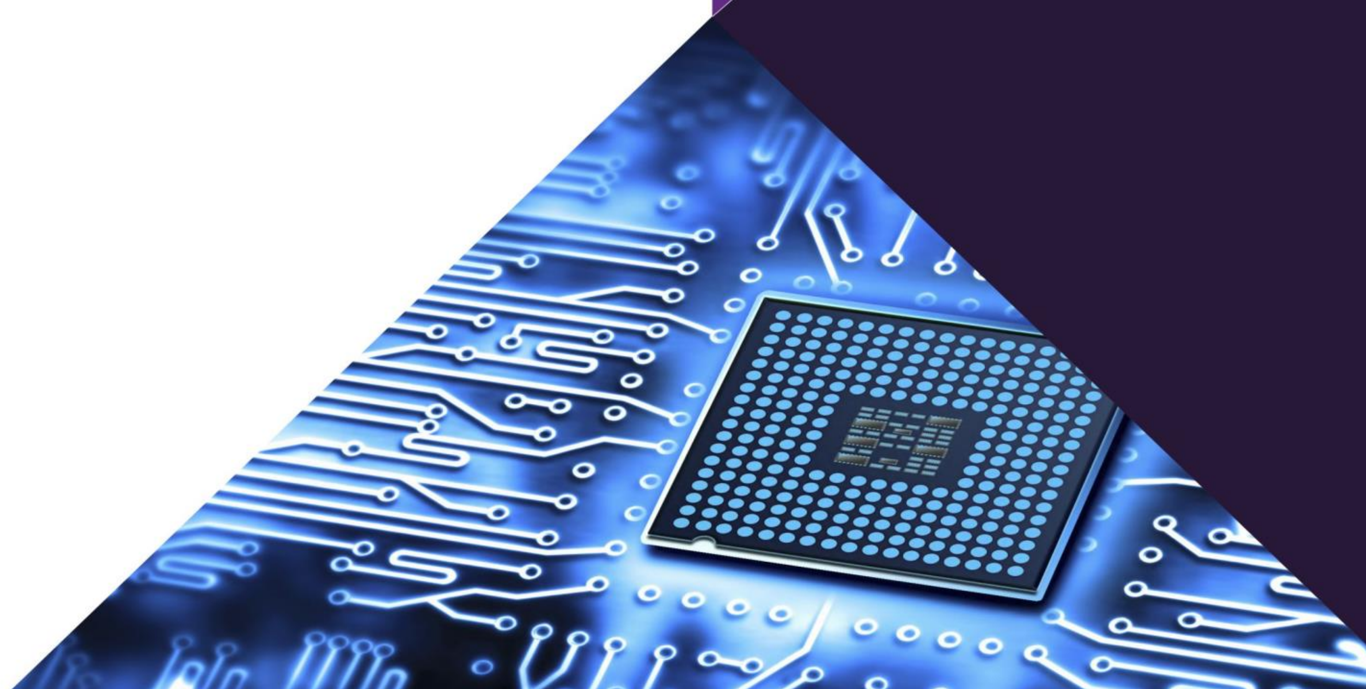
GATE 2022

Electronics & Comm. Engineering

Questions & Solutions



Section-A (General Aptitude)

1. Mr. X speaks ________ Japanese ________ Chinese.

A. neither/or

B. either/nor

C. neither/nor

D. also/but

Ans. C

Sol. Conjunctions are:

— Neither-nor

— Either-or

— Not only-but also

— Whether-or

— Both-and

2. A sum of money is to be distributed among P, Q, R, and S in the proportion 5 : 2 : 4 : 3, respectively. If R gets ₹ 1000 more than S, what is the share of Q (in ₹)?

A. 500

B. 1000

C. 1500

D. 2000

Ans. D

Sol. P : Q : R : S = 5 : 2 : 4 : 3

Money of P = 5x

Money of Q = 2x

Money of R = 4x

Money of S = 3x

Money of R = 1000 + Money of S

i.e. 4x = 1000 + 3x

x = 1000

Now, Money of Q = 2x

= 2000

3. A trapezium has vertices marked as P, Q, R and S (in that order anticlockwise). The side PQ is parallel to side SR. Further, it is given that, PQ = 11 cm, QR = 4 cm, RS = 6 cm and SP = 3 cm. What is the shortest distance between PQ and SR (in cm)?

A. 1.80

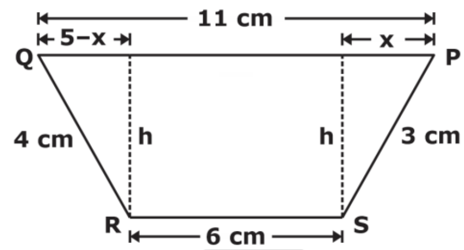
B. 2.40

C. 4.20

D. 5.76

Ans. B

Sol.



There, h = h

$\sqrt{4^2-(5-x)^2} = \sqrt{3^2-x^2}$

$\sqrt{16-(5-x)^2} = \sqrt{9-x^2}$

$16-(5-x)^2 = 9-x^2$

$16-25+10x-x^2 = 9-x^2$

10x = 18

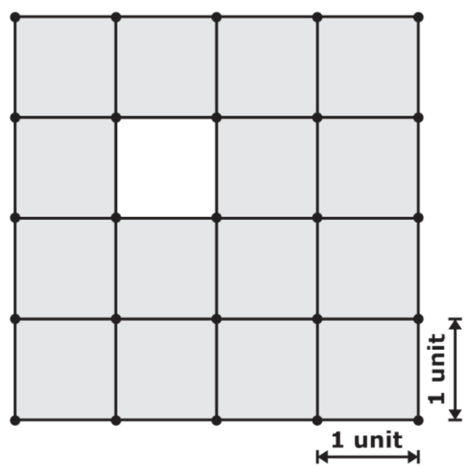
x = 1.8 cm

Now, $h = \sqrt{3^2-1.8^2}$

$h = \sqrt{9-3.24}$

h = 2.4 cm

4. The figure shows a grid formed by a collection of unit squares. The unshaded unit square in the grid represents a hole.



What is the maximum number of squares without a "hole in the interior" that can be formed within the 4 × 4 grid using the unit squares as building blocks?

A. 15

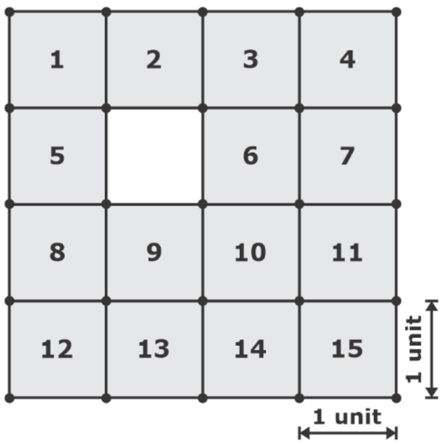
B. 20

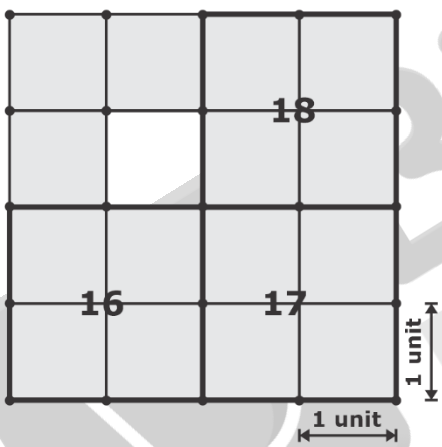
C. 21

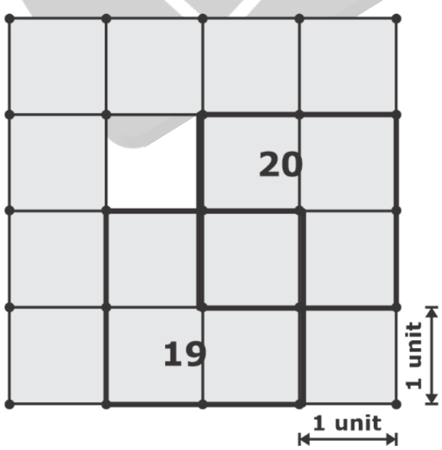
D. 26

Ans. B

Sol.

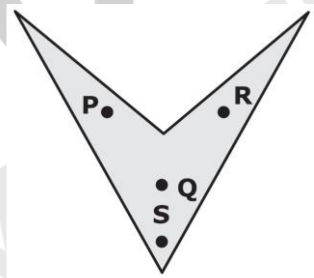






Now total number of square without hole = 20

5. An art gallery engages a security guard to ensure that the items displayed are protected. The diagram below represents the plan of the gallery where the boundary walls are opaque. The location the security guard posted is identified such that all the inner space (shaded region in the plan) of the gallery is within the line of sight of the security guard.
If the security guard does not move around the posted location and has a 360º view, which one of the following correctly represents the set of ALL possible locations among the locations P, Q, R and S, where the security guard can be posted to watch over the entire inner space of the gallery.



A. P and Q

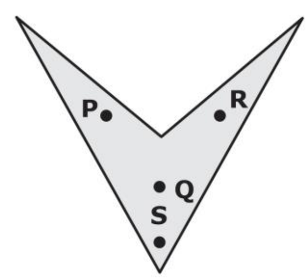
B. Q

C. Q and S

D. R and S

Ans. C

Sol.



At the position P guard can't visible R.
Similarly the position R guard can't visible P. but at the position Q and S, the security guard can posted to watch over the entire inner space of the gallery.

6. Mosquitoes pose a threat to human health. Controlling mosquitoes using chemicals may have undesired consequences. In Florida, authorities have used genetically modified mosquitoes to control the overall mosquito population. It remains to be seen if this novel approach has unforeseen consequences.

Which one of the following is the correct logical inference based on the information in the above passage?

A. Using chemicals to kill mosquitoes is better than using genetically modified mosquitoes because genetic engineering is dangerous

B. Using genetically modified mosquitoes is better than using chemicals to kill mosquitoes because they do not have any side effects

C. Both using genetically modified mosquitoes and chemicals have undesired consequences and can be dangerous

D. Using chemicals to kill mosquitoes may have undesired consequences but it is not clear if using genetically modified mosquitoes has any negative consequence

Ans. D

Sol. On the following information this statement is correct.

Using chemicals to kill mosquitoes may have undesired consequences but it is not clear if using genetically modified mosquitoes has any negative consequence

7. Consider the following inequalities.

(i) $2x - 1 > 7$

(ii) $2x - 9 < 1$

Which one of the following expressions below satisfies the above two inequalities?

A. $x \le -4$

B. $-4 < x \le 4$

C. $4 < x < 4$

D. $x \ge 5$

Ans. C

Sol. (i) $2x - 1 > 7$

$x > 4$ ...(i)

(ii) $2x - 9 < 1$

$x < 5$ ...(ii)

From (i) and (ii)

$4 < x < 5$

8. Four points P(0, 1), Q(0, − 3), R(− 2, − 1), and S(2, − 1) represent the vertices of a quadrilateral. What is the area enclosed by the quadrilateral?

A. 4

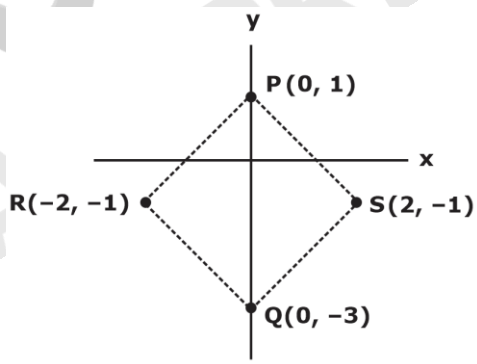
B. $4\sqrt{2}$

C. 8

D. $8\sqrt{2}$

Ans. C

Sol.



Length of PS = $\sqrt{(2-0)^2 + (-1-1)^2} = \sqrt{8}$

Length of SQ = $\sqrt{4+4} = \sqrt{8}$

Length of QR = $\sqrt{4+4} = \sqrt{8}$

Length of RP = $\sqrt{4+4} = \sqrt{8}$

Length of RS = $\sqrt{16+0} = 4$

Length of PQ = $\sqrt{0+16} = 4$

Here, Length of PQ= Length of RS

Hence, PQRS is square

Area under PQRS = $(\sqrt{8})^2$

Area under PQRS = 8

9. In a class of five students P, Q, R, S and T, only one student is known to have copied in the exam. The disciplinary committee has investigated the situation and recorded the statements from the students as given below.

Statement of P: R has copied in the exam.

Statement of Q: S has copied in the exam.

Statement of R: P did not copy in the exam.

Statement of S: Only one of us is telling the truth.

Statement of T: R is telling the truth.

The investigating team had authentic information that S never lies.

Based on the information given above, the person who has copied in the exam is

A. R

B. P

C. Q

D. T

Ans. B

Sol. **Statement of P:** R has copied in the exam.

Statement of Q: S has copied in the exam.

Statement of R: P did not copy in the exam.

Statement of S: only one of us is telling the truth.

Statement of T: R is telling the truth.

The investigating team had authentic intronization that S never lies.

On the following intronization.

If S never lies so only one of us is telling truth. statement of T is true so, statement of P, Q and R is telling false. So, only P copy in the exam.

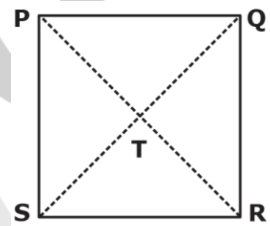
10. Consider the following square with the four corners and the center marked as P, Q, R, S and T respectively.

Let X, Y and Z represent the following operations:

X: rotation of the square by 180 degree with respect to the S-Q axis.

Y: rotation of the square by 180 degree with respect to the P-R axis.

Z: rotation of the square by 90 degree clockwise with respect to the axis perpendicular, going into the screen and passing through the point T.



Consider the following three distinct sequences of operation (which are applied in the left to right order).

(1) XYZZ

(2) XY

(3) ZZZZ

Which one of the following statements is correct as per the information provided above?

A. The sequence of operations (1) and (2) are equivalent

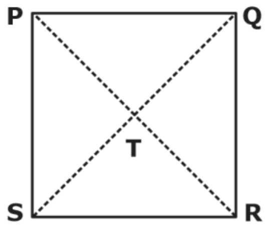
B. The sequence of operations (1) and (3) are equivalent

C. The sequence of operations (2) and (3) are equivalent

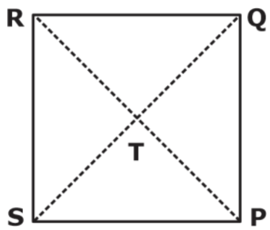
D. The sequence of operations (1), (2) and (3) are equivalent

Ans. B

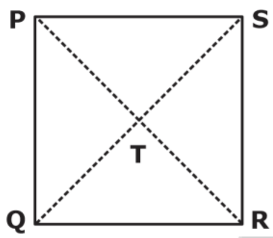
Sol.



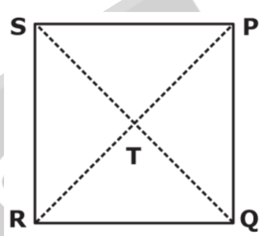
x ⇒ S – Q axis (180°)



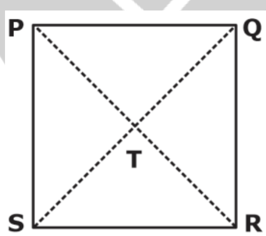
y ⇒ P – R axis (180°)



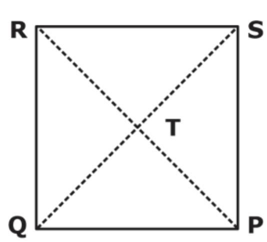
z ⇒ 90° clockwise at point T



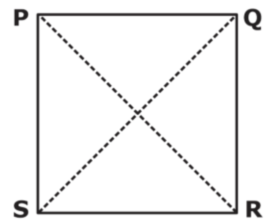
1. XYZZ



2. XY



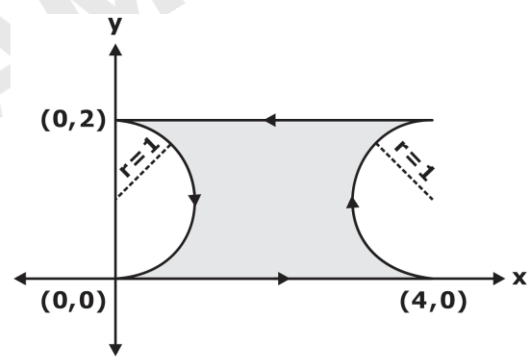
3. ZZZZ



Hence on the following the sequence of operation (1) and (2) are equivalent.

Section-B (Technical)

11. Consider the two-dimensional vector field $\vec{F}(x,y) = x\vec{i} + y\vec{j}$, where $\vec{i}$ and $\vec{j}$ denote the unit vectors along the x–axis and the y–axis, respectively. A contour C in the x–y plane, as shown in the figure, is composed of two horizontal lines connected at the two ends by two semicircular arcs of unit radius. The contour is traversed in the counter-clockwise sense. The value of the closed path integral $\oint_C \vec{F}(x,y)\cdot(dx\,\vec{i} + dy\,\vec{j})$ is _______.



A. 0

B. 1

C. $8 + 2\pi$

D. – 1

Ans. A

Sol. $\vec{F}(x,y) = x\hat{i} + y\hat{j}$

$$\oint \vec{F}(x,y)\cdot(dx\hat{i} + dy\hat{j}) = \oint \vec{F}(x,y)\cdot d\vec{\ell}$$

Apply stokes theorem

$$\oint_C F(x,y)\cdot d\ell = \int_S \nabla \times \vec{F}\cdot d\vec{s}$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & 0 \end{vmatrix}$$

$$\nabla \times \vec{F} = 0$$

$$\oint_C (\nabla \times \vec{F})\, ds = \int 0\, ds = 0$$

12. Consider a system of linear equations Ax = b, where

$$A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix},\ b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

This system of equations admits __________.

A. a unique solution for x

B. infinitely many solutions for x

C. no solutions for x

D. exactly two solutions for x

Ans. C

Sol. $A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix},\ b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$$[A : b] = \left[\begin{array}{ccc:c} 1 & -\sqrt{2} & 3 & 1 \\ -1 & \sqrt{2} & -3 & 3 \end{array}\right]$$

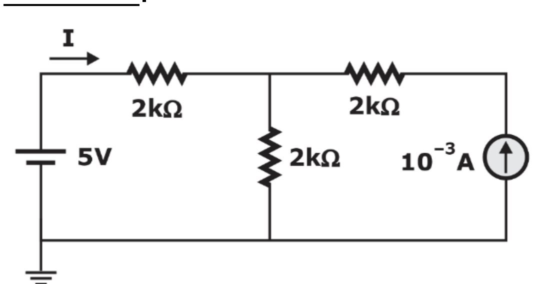
$R_2 \rightarrow R_2 + R_1$

$$[A : b] = \left[\begin{array}{ccc:c} 1 & -\sqrt{2} & 3 & 1 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

Here, Rank of A ≠ Rank of [A : b]

So, system is inconsistence so system have no solution.

13. The current I in the circuit shown is _________.



A. 1.25×10^{-3}A

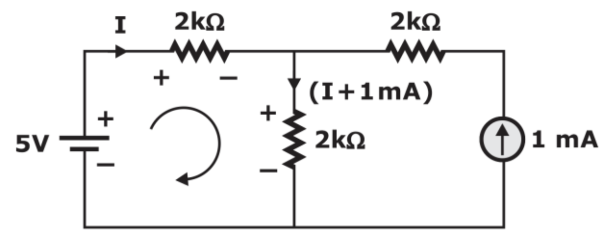
B. 0.75×10^{-3}A

C. -0.5×10^{-3}A

D. 1.16×10^{-3}A

Ans. B

Sol.



By KVL,

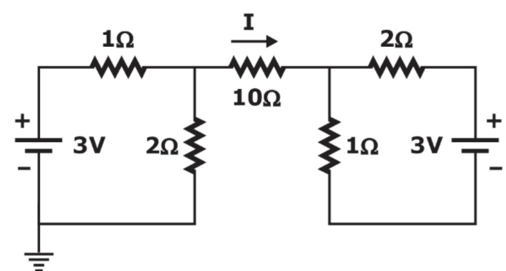
$+5 - I \times 2k - (I + 1mA)\, 2k = 0$

$5 - I\,(4k) - 2 = 0$

$I = \frac{3}{4}$ mA

= 0.75 mA

14. Consider the circuit shown in the figure. The current I flowing through the 10Ω resistor is __________.



A. 1 A

B. 0 A

C. 0.1 A

D. – 0.1 A

Ans. B

Sol. As there is no return path for the current, I will be zero

15. The Fourier transform $X(j\omega)$ of the signal

$$x(t) = \frac{t}{(1+t^2)^2} \text{ is \_\_\_\_\_\_\_.}$$

A. $\frac{\pi}{2j}\omega e^{-|\omega|}$

B. $\frac{\pi}{2}\omega e^{-|\omega|}$

C. $\frac{\pi}{2j}e^{-|\omega|}$

D. $\frac{\pi}{2}e^{-|\omega|}$

Ans. A

Sol. Consider, $x(t) = e^{-|t|}$

By taking Fourier transform,

$$X(j\omega) = \frac{2}{1+\omega^2}$$

$$e^{-|t|} \xleftrightarrow{\text{F.T.}} \frac{2}{1+\omega^2}$$

By differentiation in frequency domain property,

$$t\,x(t) \xleftrightarrow{\text{F.T.}} j\frac{d}{d\omega}X(\omega)$$

$$te^{-|t|} \xleftrightarrow{\text{F.T.}} j\left[\frac{d}{d\omega}\left(\frac{2}{1+\omega^2}\right)\right]$$

$$te^{-|t|} \xleftrightarrow{\text{F.T.}} \frac{-4j\omega}{(1+\omega^2)^2}$$

Apply duality property,

$$\frac{-4jt}{(1+t^2)^2} \xleftrightarrow{\text{F.T.}} 2\pi(-\omega)e^{-|-\omega|}$$

$$\frac{t}{(1+t^2)^2} \xleftrightarrow{\text{F.T.}} \frac{-2\pi\omega e^{-|\omega|}}{-4j}$$

$$\frac{t}{(1+t^2)^2} \xleftrightarrow{\text{F.T.}} \frac{\pi}{2j}\omega e^{-|\omega|}$$

16. Consider a long rectangular bar of direct bandgap p-type semiconductor. The equilibrium hole density is 10^{17} cm^{-3} and the intrinsic carrier concentration is 10^{10} cm^{-3}. Electron and hole diffusion lengths are 2 μm and 1 μm, respectively.

The left side of the bar (x = 0) is uniformly illuminated with a laser having photon energy greater than the bandgap of the semiconductor. Excess electron-hole pairs are generated ONLY at x = 0 because of the laser. The steady state electron density at x = 0 is 10^{14} cm^{-3} due to laser illumination. Under these conditions and ignoring electric field, the closest approximation (among the given options) of the steady state electron density at x = 2 μm, is ____________.

A. 0.37×10^{14} cm^{-3}

B. 0.63×10^{13} cm^{-3}

C. 3.7×10^{14} cm^{-3}

D. 10^{3} cm^{-3}

Ans. A

Sol. It is given, in P-type

Equilibrium hole density, $P_0 = 10^{17}$ / cm^3

Intrinsic carrier concentration,

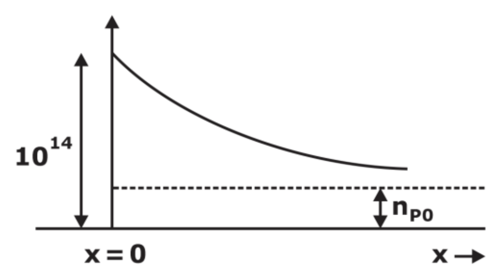
$n_i = 10^{10}$ / cm^3

Electron diffusion length, $L_n = 2\,\mu$m

Hole diffusion length, $L_p = 1\,\mu$m

steady state electron density at x = 0 is,

$n(0) = 10^{14}$ / cm^3



$$n_{PO} = \frac{n_i^2}{P_0} = \frac{(10^{10})^2}{10^{17}} = 10^3 \text{ / cm}^3$$

$$n_p(x) = n_{PO} + n'(0)e^{-x/L_n}$$

$n'(0) \rightarrow$ Excess electron concentration.

Thus, $n'(0) - n_{P0} = 10^{14} - 10^{3} = 10^{14}$/cm^3

$$n_{P(x)} = n_{P0} + n'(0)e^{-x/L_n}$$

At x = 2 μm

$n_{p(x=2\mu m)} = 10^3 + 10^{14} e^{-2/2}$

$= 10^3 + 10^{14} e^{-1}$

$\simeq 0.368 \times 10^{13}$ / cm^3

= 0.37×10^{14} /cm^3

17. In a non-degenerate bulk semiconductor with electron density n = 10^{16} cm^{-3}, the value of $E_C - E_{Fn}$ = 200 meV, where E_C and E_{Fn} denote the bottom of the conduction band energy and electron Fermi level energy, respectively. Assume thermal voltage as 26 mV and the intrinsic carrier concentration is 10^{10} cm^{-3}. For n = 0.5 × 10^{16} cm^{-3}, the closest approximation of the value of ($E_C - E_{Fn}$), among the given options, is ___________.

A. 226 meV

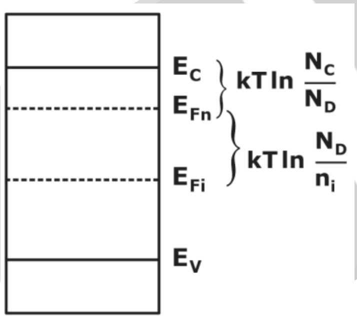
B. 174 meV

C. 218 meV

D. 182 meV

Ans. C

Sol.



n-type

Given $E_C - E_{Fn} = 200\,\text{meV}$ = 0.2 eV

We know that, $E_C - E_F = KT\ln\frac{N_C}{N_D}$

Case 1:

$n \cong N_{D1} = 10^{16}$ / cm^3

$E_C - E_{Fn1} = KT\ln\frac{N_C}{N_{D_1}} = 0.2\,\text{eV} = 200$ meV

$\frac{N_C}{N_{D_1}} = 2191.43 \Rightarrow N_C = 2191.43 \times 10^{16}$

Case 2:

$n \cong N_{D2} = 0.5 \times 10^{16}$ / cm^3

$E_C - E_{Fn2} = kT\ln\frac{N_C}{N_{D_2}}$

$= kT\ln\frac{2191.43 \times 10^{16}}{0.5 \times 10^{16}}$

$= 26\ln(\frac{2191.43}{0.5})$

$= 218$ meV

Other Method:

Let, $E_C - E_{Fn1} = kT\ln\frac{N_C}{N_{D_1}}$ ...(i)

$E_C - E_{Fn_2} = kT\ln\frac{N_C}{N_{D_2}}$ ...(ii)

(ii) – (i)

$\left(E_C - E_{Fn_2}\right) - \left(E_C - E_{Fn_1}\right) = kT\ln\frac{N_C}{N_{D_1}} \cdot \frac{N_{D_1}}{N_C}$

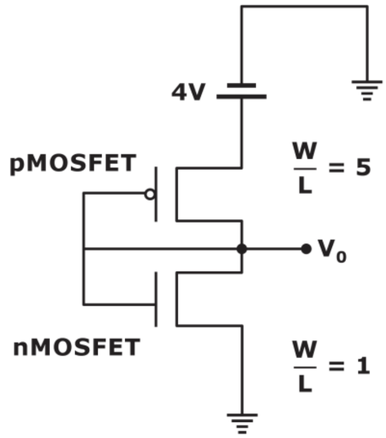
$E_C - E_{Fn_2} - 200 = kT\ln\frac{10^{16}}{0.5 \times 10^{16}}$

$E_C - E_{Fn_2} = 200 + kT\ln 2$

$= 200 + 26 \times \ln(2)$

$\cong 218$ meV

18. Consider the CMOS circuit shown in the figure (substrates are connected to their respective sources). The gate width (W) to gate length (L) ratios $\left(\frac{W}{L}\right)$ of the transistors are as shown. Both the transistors have the same gate oxide capacitance per unit area. For the pMOSFET, the threshold voltage is −1 V and the mobility of holes is $40\frac{cm^2}{V.s}$. For the nMOSFET, the threshold voltage is 1 V and the mobility of electrons is $300\frac{cm^2}{V.s}$. The steady state output voltage V_0 is _________.



A. equal to 0 V
B. more than 2 V
C. less than 2 V
D. equal to 2 V

Ans. C

Sol. Given,

$I_D = I_{Dn} = I_{DP}$

$V_{DS} \geq V_{GS} - V_t$, always true.

From the given diagram $V_G = V_D$ hence both MOSFET are in saturation

$$I_{Dn} = \frac{1}{2}\mu_n C_{OX}\left(\frac{W}{L}\right)_n (V_{GS} - V_t)^2$$

$$I_{Dp} = \frac{1}{2}\mu_P C_{OX}\left(\frac{W}{L}\right)_P (V_{GS} - |V_t|)^2$$

$I_D = I_{Dn} = I_{Dp}$

$$\frac{1}{2}\mu_n C_{OX}\left(\frac{W}{L}\right)_n (V_{GS} - V_t)^2 = \frac{1}{2}\mu_P C_{OX}\left(\frac{W}{L}\right)_P (V_{GS} - |V_t|)^2$$

$$\mu_n C_{OX}\left(\frac{W}{L}\right)_n (V_0 - V_S - V_t)^2 = \mu_P C_{OX}\left(\frac{W}{L}\right)_P (V_S - V_0 - |V_t|)^2$$

$$300 \times 1(V_0 - 0 - 1)^2 = 40 \times 5(4 - V_0 - 1)^2$$

$$3(V_0 - 1)^2 = 2(3 - V_0)^2$$

$$3V_0^2 + 3 - 6V_0 = 18 + 2V_0^2 - 12V_0$$

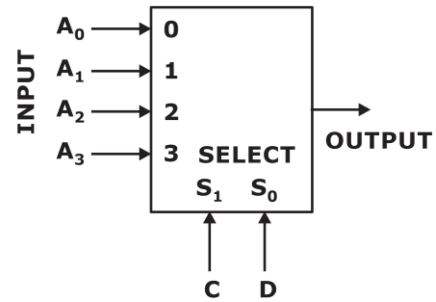
$$V_0^2 + 6V_0 - 15 = 0$$

$$V_0 = \frac{-6 \pm \sqrt{36 + 60}}{2} = 1.89V, \quad -789V$$

At V_0 = 1.89V, both MOSFET will be in saturation.

Hence, the correct option is (C).

19. Consider the 2-bit multiplexer (MUX) shown in the figure. For OUTPUT to be the XOR of C and D, the values for A_0, A_1, A_2 and A_3 are ____________.



A. $A_0 = 0, A_1 = 0, A_2 = 1, A_3 = 1$
B. $A_0 = 1, A_1 = 0, A_2 = 1, A_3 = 0$
C. $A_0 = 0, A_1 = 1, A_2 = 1, A_3 = 0$
D. $A_0 = 1, A_1 = 1, A_2 = 0, A_3 = 0$

Ans. C

Sol. Table 1:

CD	$C \oplus D$
00	0
01	1
10	1
11	0

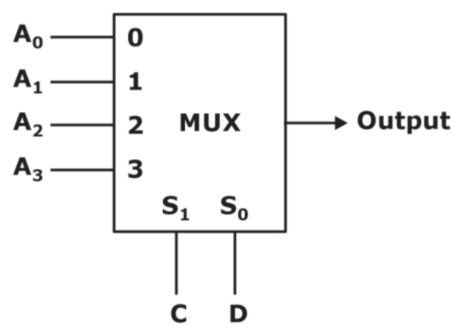


Table 2:

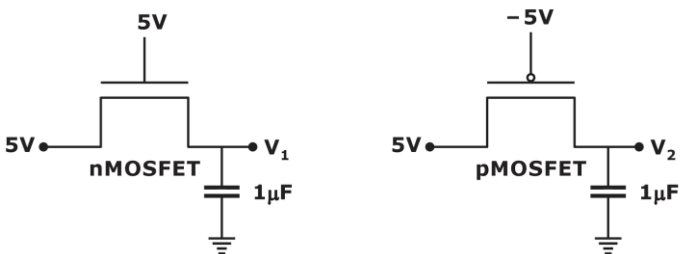
CD	Output
00	A_0
01	A_1
10	A_2
11	A_3

From table (1) and (2) we can see that

$A_0 = 0, A_1 = 1, A_2 = 1, A_3 = 0$

Correct option is (C).

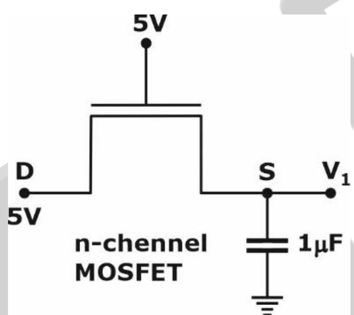
20. The ideal long channel nMOSFET and pMOSFET devices shown in the circuits have threshold voltages of 1 V and −1 V, respectively. The MOSFET substrates are connected to their respective sources. Ignore leakage currents and assume that the capacitors are initially discharged. For the applied voltages as shown, the steady state voltages are __________.



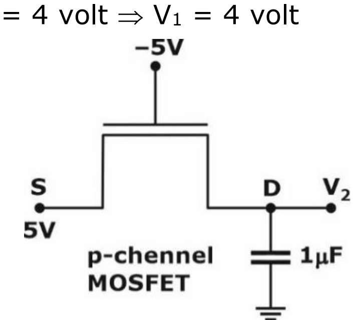
A. V_1 = 5V, V_2 = 5V
B. V_1 = 5V, V_2 = 4V
C. V_1 = 4V, V_2 = 5V
D. V_1 = 4V, V_2 = − 5V

Ans. C

Sol.

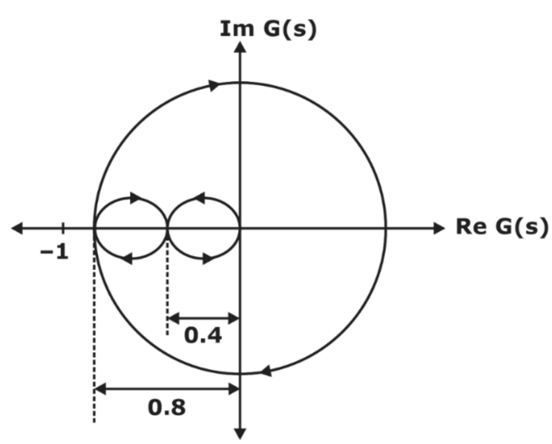


$V_{GS} = V_{DS} \Rightarrow$ MOSFET is in saturation
Channel formation $V_{GS} > V_t$
$= 5 - V_C > 1$
If capacitor charges than V_C will increase
at V_C = 4 volt V_{GS} = 1 volt
If V_C > 4 volt channel will deplete out
So V_C = 4 volt $\Rightarrow V_1$ = 4 volt



$V_{GS} = -5 - 5 = -10$
$V_{GS} < -1 \Rightarrow$ channel is formed
$V_{DS} < V_{GS} - V_t \Rightarrow$ for saturation
$V_D < V_G - V_t$
$V_C < -5 + 1$
$V_C < -4 \Rightarrow$ MOSFET will not work in saturation
MOSFET will work in linear region (always) and current will flow from higher potential to lower potential
So V_2 = 5 volt
Option C is correct

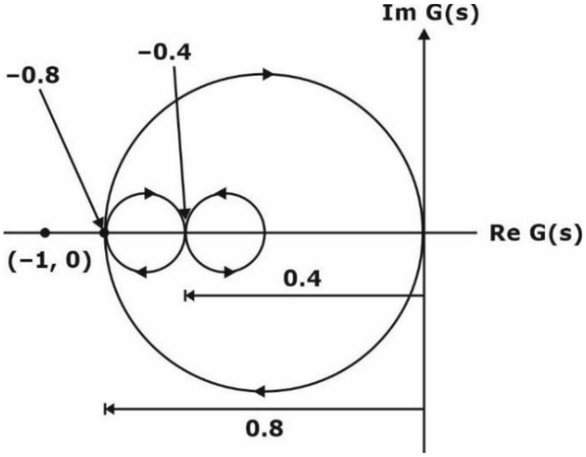
21. Consider a closed-loop control system with unity negative feedback and KG(S) in the forward path, where the gain K = 2. The complete Nyquist plot of the transfer function G(s) is shown in the figure. Note that the Nyquist contour has been chosen to have the clockwise sense. Assume G(s) has no poles on the closed right-half of the complex plane. The number of poles of the closed-loop transfer function in the closed right-half of the complex plane is _________.



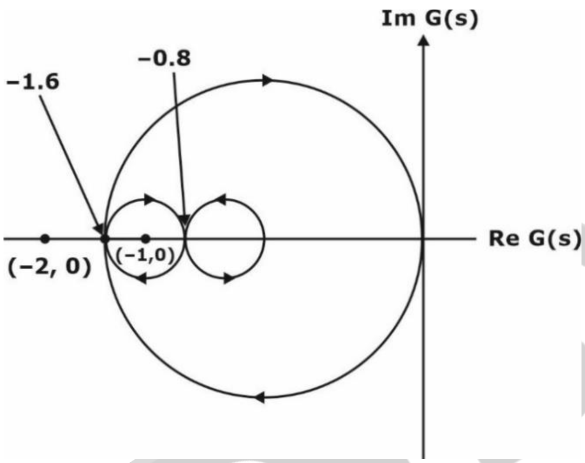
A. 0
B. 1
C. 2
D. 3

Ans. C

Sol. For, K = 1



For K = 2, the plot will be



N = No. of encirclement about (− 1, 0) in anticlockwise

P = Total number of open loop poles, in the right-hand side

Z = P − N

N = − 2, P = 0

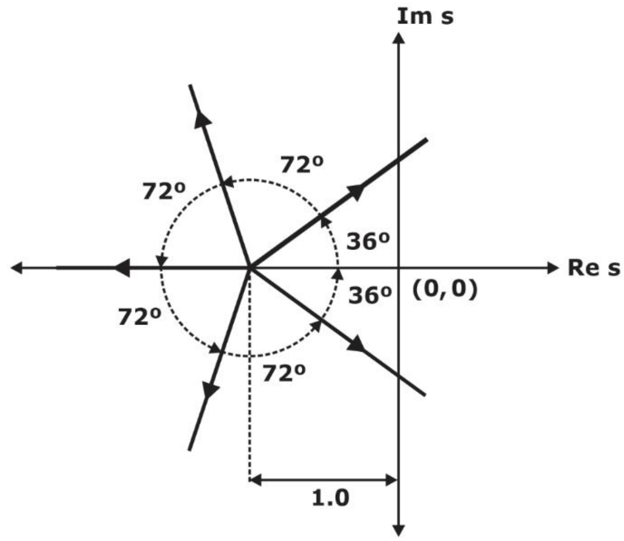
Z = 0 − (− 2) = 2

Z = 2

Two poles lie in right hand side

22. The root-locus plot of a closed-loop system with unity negative feedback and transfer function KG(S) in the forward path is shown in the figure. Note that K is varied from 0 to ∞.

Select the transfer function G(S) that results in the root-locus plot of the closed-loop system, as shown in the figure.



A. $G(s) = \dfrac{1}{(s+1)^5}$

B. $G(s) = \dfrac{1}{s^5+1}$

C. $G(s) = \dfrac{s-1}{(s+1)^6}$

D. $G(s) = \dfrac{s+1}{s^6+1}$

Ans. A

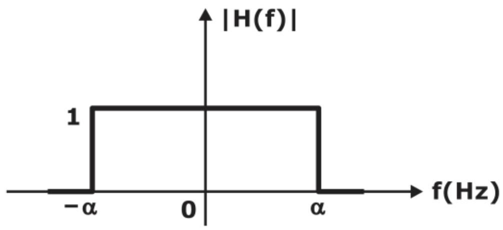
Sol. As the root locus shows 5 arms going to infinity thus, we can say that the root locus consists of 5 poles, all placed at s = −1 as seen from the root locus diagram as we can see from the options that only A satisfies that condition.

23. The frequency response H(f) of a linear time-invariant system has magnitude as shown in the figure.

Statement I: The system is necessarily a pure delay system for inputs which are bandlimited to $-\alpha \le f \le \alpha$.

Statement II: For any wide-sense stationary input process with power spectral density $S_X(f)$, the output power spectral density $S_Y(f)$ obeys $S_Y(f) = S_X(f)$ for $-\alpha \le f \le \alpha$.

Which one of the following combinations is true?



A. Statement I is correct, Statement II is correct

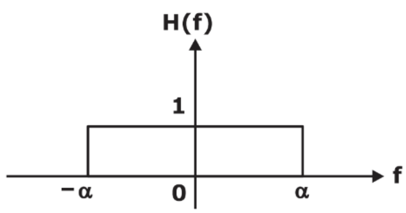
B. Statement I is correct, Statement II is incorrect

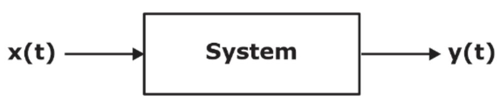
C. Statement I is incorrect, Statement II is correct

D. Statement I is incorrect, Statement II is incorrect

Ans. C

Sol.





For a delay system, the input and output can be represented as

$$y(t) = x(t - T_0)$$

$$Y(f) = e^{-j\omega T_0} X(f)$$

$$H(f) = e^{-j\omega T_0}$$

For a delay system, H(f) should be constant for all frequencies.

In the given system, H(f) is constant only for $-\alpha \le f \le \alpha$.

Hence, Statement 1 is not correct

$$S_Y(f) = |H(f)|^2 S_X(f)$$

Since, $|H(f)|^2 = 1$

$$S_Y(f) = S_X(f) \quad \text{for } -\alpha \le f \le \alpha$$

Statement 2 is correct.

Hence option C is correct.

24. In a circuit, there is a series connection of an ideal resistor and an ideal capacitor. The conduction current (in Amperes) through the resistor is $2\sin\left(t + \frac{\pi}{2}\right)$. The displacement current (in Amperes) through the capacitor is __________.

A. $2\sin(t)$

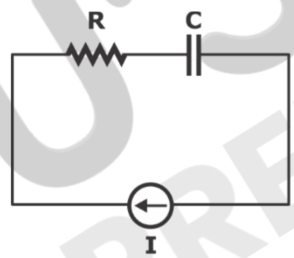
B. $2\sin(t + \pi)$

C. $2\sin\left(t + \frac{\pi}{2}\right)$

D. 0

Ans. C

Sol.



$$I = 2\sin\left(t + \frac{\pi}{2}\right)$$

$$Q = \int I dt = 2\int \sin\left(t + \frac{\pi}{2}\right) dt$$

$$Q = -2\cos\left(t + \frac{\pi}{2}\right)$$

$$Q = 2\cos\left(t - \frac{\pi}{2}\right) \quad \text{...(i)}$$

$$\sigma = \frac{Q}{A} = \frac{2}{A}\cos\left(t - \frac{\pi}{2}\right)$$

$$E = \frac{\sigma}{\varepsilon} = \frac{2}{A\varepsilon}\cos\left(t - \frac{\pi}{2}\right)$$

$$D = \frac{2}{A}\cos\left(t - \frac{\pi}{2}\right)$$

$$\frac{\delta D}{\delta t} = \frac{-2}{A}\sin\left(t - \frac{\pi}{2}\right)$$

$$J_D = \frac{\delta D}{\delta t} = \frac{2}{A}\sin\left(t + \frac{\pi}{2}\right)$$

$$\boxed{I_d = 2\sin\left(t + \frac{\pi}{2}\right) \text{ Amp}}$$

25. Consider the following partial differential equation (PDE)

$$a\frac{\partial^2 f(x,y)}{\partial x^2}+b\frac{\partial^2 f(x,y)}{\partial y^2}=f(x,y)$$

where a and b are distinct positive real numbers. Select the combination(s) of values of the real parameters ξ and η such that $f(x, y) = e^{(\xi x+\eta y)}$ is a solution of the given PDE.

A. $\xi=\frac{1}{\sqrt{2a}},\ \eta=\frac{1}{\sqrt{2b}}$

B. $\xi=\frac{1}{\sqrt{a}},\ \eta=0$

C. $\xi=0,\ \eta=0$

D. $\xi=\frac{1}{\sqrt{a}},\ \eta=\frac{1}{\sqrt{b}}$

Ans. A, B

Sol. $a\frac{\partial^2 f(x,y)}{\partial x^2}+b\frac{\partial^2 f(x,y)}{\partial y^2}=f(x,y)$ ...(A)

$f(x,y)=e^{\xi x+\eta y}$

Now, $\frac{\partial^2 f(x,y)}{\partial x^2}=\xi^2 e^{(\xi x+\eta y)}$ ...(i)

$\frac{\partial^2 f(x,y)}{\partial y^2}=\eta^2 e^{(\xi x+\eta y)}$ ...(ii)

Put the equation (i) and (ii) is equation (A) to get

$a\cdot\xi^2 e^{(\xi x+\eta y)}+b\eta^2 e^{(\xi x+\eta y)}=e^{(\xi x+\eta y)}$

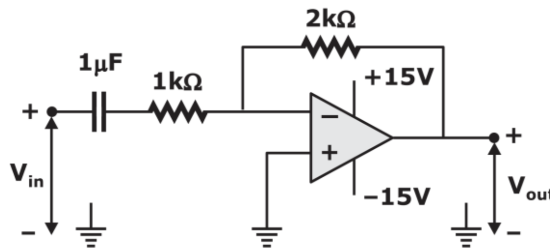
$a\xi^2+b\eta^2=1$ ...(B)

For the given option, only

A. $\xi=\frac{1}{\sqrt{2a}},\eta=\frac{1}{\sqrt{2b}}$ satisfies the equation B.

B. $\xi=\frac{1}{\sqrt{a}},\eta=0$ also satisfies the equation B.

26. An ideal OPAMP circuit with a sinusoidal input is shown in the figure. The 3 dB frequency is the frequency at which the magnitude of the voltage gain decreases by 3 dB from the maximum value. Which of the options is/are correct?



A. The circuit is a low pass filter.

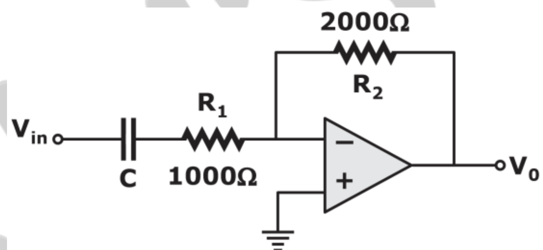
B. The circuit is a high pass filter.

C. The 3 dB frequency is 1000 rad/s.

D. The 3 dB frequency is 1000/3 rad/s.

Ans. B, C

Sol. Given circuit is shown below



$$\left|\frac{V_0}{V_{in}}\right|=\frac{sCR_2}{1+sCR_1}$$

From transfer fraction it is a high pass filter

$$\omega_{3\,dB}=\frac{1}{R_1C}=\frac{1}{1000\times 1\mu F}=1000\ \text{rad/sec.}$$

Hence, the correct option are (B) and (C).

27. Select the Boolean function(s) equivalent to x + yz, where x, y and z are Boolean variables, and + denotes logical OR operation.

A. x + z + xy

B. (x + y) (x + z)

C. x + xy + yz

D. x + xz + xy

Ans. B, C

Sol. We need to check for the correct answer from the given options

option (A):

x + z + xy= x + z

So, option (A) is incorrect

option (B):

(x + y) (x + z) = x + xz + yx + yz= x + yx + yz= x + yz

So, option (B) is correct

option (C):

x + xy + yz= x + yz

So, option (C) is correct

option (D):

x + xz + xy= x + xy= x(1 + y) = x

So, option (D) is incorrect

Hence, the correct option are (B) and (C).

28. Select the correct statement(s) regarding CMOS implementation of NOT gates.

A. Noise Margin High (NM_H) is always equal to the Noise Margin Low (NM_L), irrespective of the sizing of transistors.

B. Dynamic power consumption during switching is zero.

C. For a logical high input under steady state, the nMOSFET is in the linear regime of operation.

D. Mobility of electrons never influences the switching speed of the NOT gate.

Ans. C

Sol. For a logical high input under steady state, the nMOSFET is in the linear regime of operation

29. Let H(X) denote the entropy of a discrete random variable X taking K possible distinct real values. Which of the following statements is/are necessarily true?

A. $H(X) \le \log_2 K$ bits

B. $H(X) \le H(2X)$

C. $H(X) \le H(X^2)$

D. $H(X) \le H(2^X)$

Ans. A, B, D

Sol. Given that X is a discrete random variable taking K possible distinct real values. If 'K' symbols having equal probability then entropy will be maximum $H(X)_{max} = \log_2 K$. If symbols having different probabilities then $H(X) < \log_2 K$

So, that, $H(X) \le \log_2 K$

Option (A) is correct

Given option (b), $H(X) \le H(2X)$

Let,

$X \in \{x_i\}$	$-1\ 0\ 1$
$P_X(x_i)$	$\frac{1}{4}\ \frac{1}{2}\ \frac{1}{4}$

$$H(X) = \sum_i P_X(x_i)\log_2 \frac{1}{P_X(x_i)}$$

$$= \frac{1}{4}\log_2 4 + \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4$$

$$H(X) = 1.5\frac{\text{bits}}{\text{symbol}}$$

Let, Y = 2X

X	Y	P(Y)
–1	–2	1/4
0	0	1/2
1	2	1/4

$Y \in \{y_i\}$	$-2\ 0\ 2$
$P_Y(y_i)$	$\frac{1}{4}\ \frac{1}{2}\ \frac{1}{4}$

$$H(2X) = H(Y) = \sum_i P_Y(y_i)\log_2 \frac{1}{P_Y(y_i)} = 1.5\frac{\text{bits}}{\text{symbol}}$$

For Y = 2X, distant X values results in distinct 'Y' values so that H(X) = H(Y).

So, option (B) is true i.e., H(X) = H(2X)

Given option (C), $H(X) \le H(X^2)$;

Let $Y = X^2$

X	Y	P(Y)
–1	1	1/4
0	0	1/2
1	1	1/4

$Y \in (y_i)$	$0\ 1$
$P(Y = y_i)$	$\frac{1}{2}\ \frac{1}{2}$

$$H(X^2) = H(Y) = \sum_i P_Y(y_i)\log_2 \frac{1}{P_Y(y_i)}$$

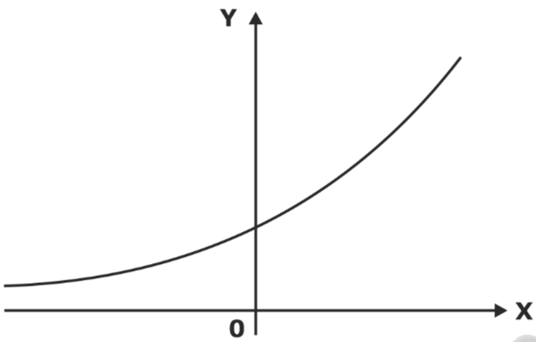
$$= \frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2 = 1\frac{\text{bit}}{\text{symbol}}$$

Whereas, $H(X) = 1.5\frac{\text{bits}}{\text{symbol}}$

Option (c) is incorrect.

Given option (d) $H(X) \le H(2^X)$

Let $Y = 2^X$



Here distinct 'X' values results in distinct 'Y' values.

So that, $H(X) = H(Y)$

i.e. $H(X) = H(2^X)$

Option (D) is true

30. Consider the following wave equation,

$$\frac{\partial^2 f(x,t)}{\partial t^2} = 10000\frac{\partial^2 f(x,t)}{\partial x^2}$$

Which of the given options is/are solution(s) to the given wave equation?

A. $f(x,t) = e^{-(x-100t)^2} + e^{-(x+100t)^2}$

B. $f(x,t) = e^{-(x-100t)} + 0.5e^{-(x+1000t)}$

C. $f(x,t) = e^{-(x-100t)} + \sin(x+100t)$

D. $f(x,t) = e^{j100\pi(-100x+t)} + e^{j100\pi(100x+t)}$

Ans. A, C

Sol. The wave equation is,

$$\frac{\partial^2 F}{\partial x^2} = \frac{1}{V^2}\times\frac{\partial^2 F}{\partial t^2} \quad \dots (1)$$

$$\frac{\partial^2 F}{\partial t^2} = V^2\frac{\partial^2 F}{\partial x^2}$$

$V^2 = 10000$

$V = 1000$ m/sec.

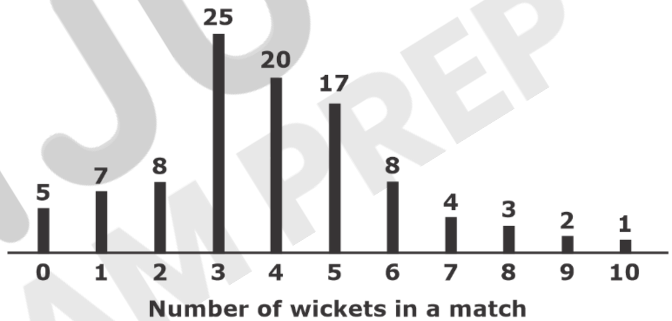
The general solution of equation (1) is,

$F(x,t) = f(Vt \mp x)$

Option (A) and (C) satisfy the general solution of wave equation.

Hence, the correct options are (A) and (C).

31. The bar graph shows the frequency of the number of wickets taken in a match by a bowler in her career. For example, in 17 of her matches, the bowler has taken 5 wickets each. The median number of wickets taken by the bowler in a match is _____ (rounded off to one decimal place).



Ans. (4.0 to 4.0)

Sol. In this bar graph

frequency number	No. of wickets	Cumulative Frequency
5	0	5
7	1	12
8	2	20
25	3	45
20	4	65
17	5	82
8	6	90
4	7	94
3	8	91
2	9	99
1	10	100

Total number of frequency = n

n = 5 + 7 + 8 + 25 + 20 + 17 + 8 + 4 + 3 + 8 + 0

n = 100

$\frac{n}{2} = 50$

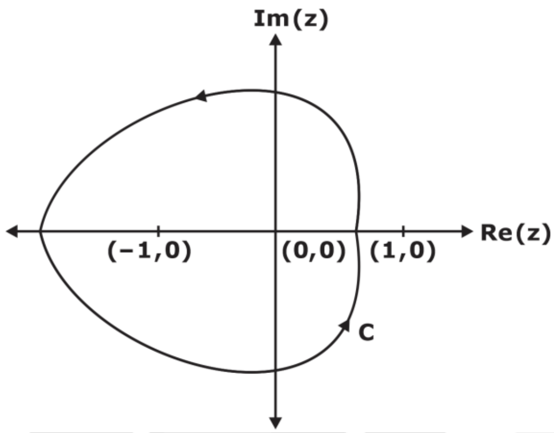
For n= 50 lies in frequency number is 20 and number of the wicket is 4.

Hence, median number of wickets taken by the bowler is = 4

32. A simple closed path C in the complex plane is shown in the figure. If

$$\oint_C \frac{2^z}{Z^2-1}dz = -i\pi A$$

Where $i=\sqrt{-1}$, then the value of A is _______ (rounded off to two decimal places).



Ans. (0.50 to 0.50)

Sol. $\oint \frac{2^z}{z^2-1}dx = -i\pi A$

LHS $\oint_C \frac{2^z}{z^2-1}dz = \frac{1}{2}\oint_C \frac{2^z}{z^2-1}dz - \frac{1}{2}\oint_C \frac{2^z}{z^2-1}dz$

For pole z = 1 does not lie inside the close path counter so apply cauchy's integral theorem

$\frac{1}{2}\oint \frac{2^z}{z-1}dz = 0$

Z = −1 lie inside the close path C. So,

$-\frac{1}{2}\oint \frac{2^z}{z+1}dz = -\frac{1}{2}\times 2\pi i \times 2^{-1} = \frac{-1}{2}\pi i$

$\oint_C \frac{2^z}{z^2-1}dz = \frac{1}{2}\oint_C \frac{2^z}{z^2-1}dz - \frac{1}{2}\oint_C \frac{2^z}{z^2-1}dz = 0 + \frac{-1}{2}\pi i$

A = 0.5

33. Let $x_1(t) = e^{-t}u(t)$ and $x_2(t) = u(t) - u(t-2)$, where u(.) denotes the unit step function. If y(t) denotes the convolution of $x_1(t)$ and $x_2(t)$, then $\lim_{t\to\infty} y(t)$ = __________. (Rounded off to one decimal place).

Ans. (0.0 to 0.0)

Sol. $x_1(t) = e^{-t}u(t)$

$x_2(t) = u(t) - u(t-2)$

$y(t) = x_1(t) * x_2(t)$

By applying Laplace transform

$$Y(s) = X_1(s)\cdot X_2(s) = \frac{1}{(s+1)}\frac{1-e^{-2s}}{s}$$

By applying final value theorem,

$$y(t)\big|_{t=\infty} = \lim_{s\to 0} sY(s) = \lim_{s\to 0}\left(\frac{1-e^{-2s}}{s+1}\right)$$

= 0

Alternate Method:

$y(t) = x_1(t) * x_2(t)$

$= e^{-t}u(t) * [u(t) - u(t-2)]$

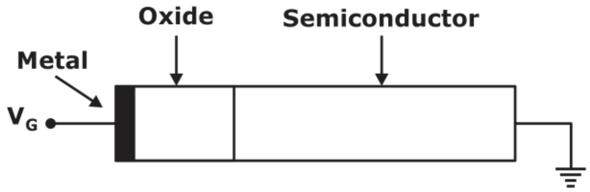
$= e^{-t}u(t) * u(t) - e^{-t}u(t) * u(t-2)$

$= \int_{-\infty}^{t} e^{-t}u(t)dt - \int_{-\infty}^{t-2} e^{-t}u(t)dt$ [u(t) is the impulse response of an integrator]

$y(t) = [1 - e^{-t}]u(t) - [1 - e^{-(t-2)}]u(t-2)$

$y(\infty) = [1 - 0]1 - [1 - 0]1 = 0$

34. An ideal MOS capacitor (p-type semiconductor) is shown in the figure. The MOS capacitor is under strong inversion with V_G = 2V. The corresponding inversion charge density (Q_{IN}) is 2.2 μC/cm^2. Assume oxide capacitance per unit area as C_{OX} = 1.7 μF/cm^2. For V_G = 4 V, the value of Q_{IN} is _______ μC/cm^2 (rounded off to one decimal place).



Ans. (5.5 to 5.7)

Sol. It is known that, the inverse charge is related as,

$Q_{inV_1} = C_{OX}\left[V_{G_1} - V_t\right] = C_{OX}V_{G_1} - C_{OX}V_t$ .... (i)

$Q_{inV_2} = C_{OX}\left[V_{G_2} - V_t\right] = C_{OX}V_{G_2} - C_{OX}V_t$ .... (ii)

By subtracting (ii) from (i), we get

$Q_{inV_2} - Q_{inV_1} = C_{OX}\left(V_{G_2} - V_{G_1}\right)$

= 2.2 μC/cm^2 + 1.7 μF/cm^2 × [4-2]V

= 2.2 μC/cm^2+ 3.4 μC/cm^2

= 5.6 μC/cm^2

35. A symbol stream contains alternate QPSK and 16-QAM symbols. If symbols from this stream are transmitted at the rate of 1 mega-symbols per second, the raw (uncoded) data rate is ________ mega-bits per second (rounded off to one decimal place).

Ans. (2.99 to 3.01)

Sol. QPSK and 16 QAM

Symbol rate = 1 M symbol/sec.

If QPSK ⇒ bit-rate = Rs × n

= symbol rate × No. of bits per symbol

= 1 M × 2 = 2 Mbps

If 16-QAM ⇒ bit rate = 1 M × 4 = 4 Mbps

∴ QPSK and 16-QAM are used alternately

Effective data rate = Avg. of 2 Mbps & 4 Mbps

= 3 Mbps

36. The function $f(x) = 8\log_e x - x_e^2 + 3$ attains its minimum over the interval [1, e] at x = _______.

(Here $\log_e$ x is the natural logarithm of x)

A. 2

B. 1

C. e

D. $\frac{1+e}{2}$

Ans. B

Sol. Here, $f(x) = 8\log_e x - x_e^2 + 3$

$f'(x) = \frac{8}{x} - 2x$

f′(0) = 0 to get the extrema value of x.

$\frac{8}{x} - 2x = 0$

$8 - 2x^2 = 0$

$x^2 = 4$

x = ± 2

$f''(x) = -\frac{8}{x^2} - 2$

For x = ± 2 f″ (x) = – 4

So, f″(x) < 0

∴ f(x) is maximum

∴ Minimum do f(x) in [1, e] is

f(1) = 2

f€ = 8 – e^2 + 3 = 3.61

$f(2) = 8\log_e 2 - 2^2 + 3 = 4.54$

Minimum value of f(x) occur at x = 1

37. Let α, β be two non-zero real numbers and v_1, v_2 be two non-zero real vectors of size 3 × 1. Suppose that v_1 and v_2 satisfy $v_1^T v_2 = 0$, $v_1^T v_1 = 1$, and $v_2^T v_2 = 1$. Let A be then 3 × 3 matrix given by:

$A = \alpha v_1 v_1^T + \beta v_2 v_2^T$

The eigenvalues of A are ___________.

A. 0, α, β

B. 0, α + β, α – β

C. 0, $\frac{\alpha+\beta}{2}$, $\sqrt{\alpha\beta}$

D. 0, 0, $\sqrt{\alpha^2+\beta^2}$

Ans. A

Sol. Given,

Size of V_1 and $V_2 = 3 \times 1$

$V_1^T \cdot V_2 = 0,\ V_1^T V_1 = V_2^T V_2 = 1$

$A = \alpha V_1 V_1^T + \beta V_2 V_2^T$

$AV_1 = \alpha V_1 V_1^T \cdot V_1 + \beta V_2 V_2^T \cdot V_1$

$AV_1 = \alpha V_1 (1) + \beta V_2 (0)$

$AV_1 = \alpha V_1 \quad \Rightarrow \quad Ax = \lambda x$

Here, $\lambda_1 = \alpha$

Now again,

$AV_2 = \alpha V_1 V_1^T \cdot V_2 + \beta V_2 V_2^T \cdot V_2$

$AV_2 = \alpha V_1 (0) + \beta V_2 (1)$

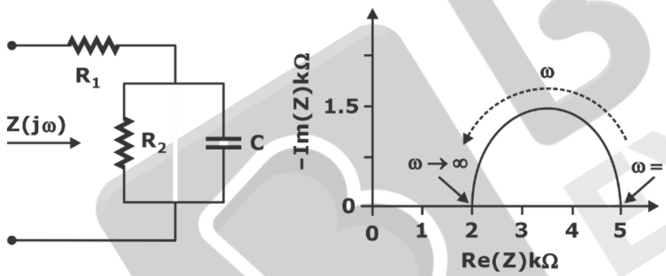
$AV_2 = \beta V_2 \quad \Rightarrow \quad Ax = \lambda x$

$\lambda_2 = \beta$

So, our given value = α, β.

Hence, Eigen value of the matrix A is 0, α, and β.

38. For the circuit shown, the locus of the impedance $Z(j\omega)$ is plotted as ω increases from zero to infinity. The values of R_1 and R_2 are:



A. $R_1 = 2k\Omega$, $R_2 = 3k\Omega$

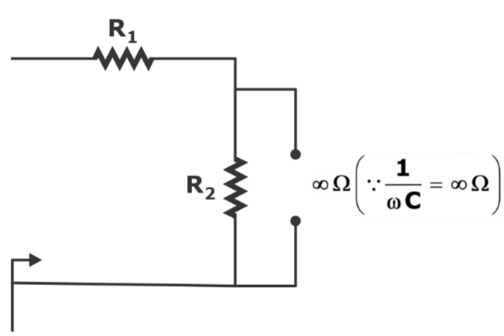
B. $R_1 = 5k\Omega$, $R_2 = 2k\Omega$

C. $R_1 = 5k\Omega$, $R_2 = 2.5k\Omega$

D. $R_1 = 2k\Omega$, $R_2 = 5k\Omega$

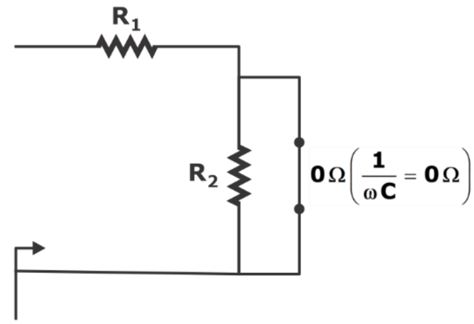
Ans. A

Sol. **At, ω = 0 rad/sec.**



$Z = R_1 + R_2 = 5k\Omega$

At, $\omega = \infty$ rad/sec.



$Z = R_1 = 2k\Omega$

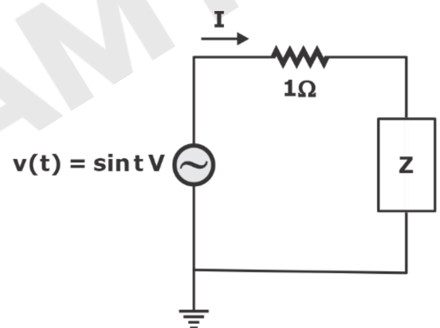
$\therefore R_2 = 5k\Omega - 2k\Omega$

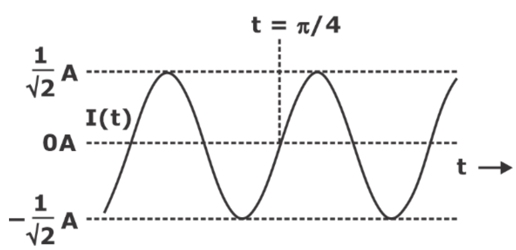
$= 3k\Omega$

$R_1 = 2k\Omega$

$R_2 = 3k\Omega$

39. Consider the circuit shown in the figure with input V(t) in volts. The sinusoidal steady-state current I(t) flowing through the circuit is shown graphically (where t is in seconds). The circuit element Z can be _________.





A. a capacitor of 1 F

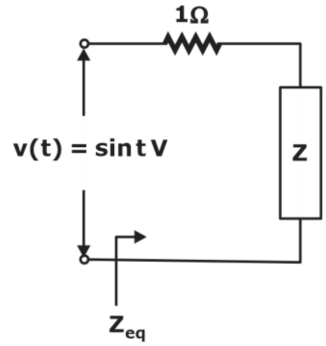
B. an inductor of 1 H

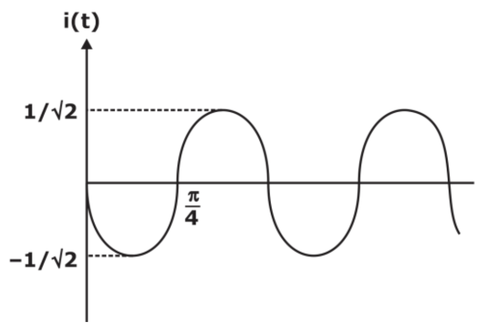
C. a capacitor of $\sqrt{3}$ F

D. an inductor of $\sqrt{3}$ H

Ans. B

Sol.





i(t) can be taken as $\frac{1}{\sqrt{2}}\sin(t-45^{o})$

$$Z_{eq}=\frac{v(t)}{i(t)}=\frac{\sin t}{\frac{1}{\sqrt{2}}\sin(t-45^{o})}=\frac{1\angle 0^\circ}{\frac{1}{\sqrt{2}}(-45^\circ)}$$

$$Z_{eq}=\sqrt{2}\angle 45^\circ$$

$$|Z|=\sqrt{R^2+X^2}$$

$$\left(\sqrt{2}\right)^2=1^2+X^2$$

X = 1

ωL = 1

L = 1 H

40. Consider an ideal long channel nMOSFET (enhancement-mode) with gate length 10 μm and width 100 μm. The product of electron mobility (μ_n) and oxide capacitance per unit area (C_{ox}) is $\mu_n C_{ox}$ = 1 mA/V². The threshold voltage of the transistor is 1 V. For a gate-to-source voltage V_{GS} = [2 – sin(2t)]V and drain-to source voltage V_{DS} = 1 V (substrate connected to the source), the maximum value of the drain-to-source current is ________.

A. 40 mA

B. 20 mA

C. 15 mA

D. 5 mA

Ans. C

Sol. Given parameter are as, L = 10 μm, W = 100 μm, $\mu_n c_{ox}=1\,mA/V^2$, V_t = 1V, $V_{DS}=1V$

Gate to source voltage is, $V_{GS}=2-\sin 2t$

$$V_{GS\ min.}=2-1=1V=V_T \qquad ...(1)$$

$$V_{GS\ max.}=2-(-1)=3V>V_T \qquad ...(2)$$

From (1),(2) it is clear that, MOSFET is always ON for all values of V_{GS}.

For

At V_{GS} = $V_{GS,max}$

$$V_{GS\ max.}-V_T=3-1=2$$

V_{DS} < $V_{GS\ max.}$ – V_T

Then MOSFET is in triode region,

$$I_{DS}=K_n\frac{W}{L}\left[(V_{GS\ max.}-V_t)V_{DS}-\frac{1}{2}V_{DS}^2\right]$$

$$=1\times\frac{100}{10}\left[2\times1-\frac{1}{2}\times1\right]$$

$$=10[2-0.5]=15\,mA$$

At, Boundary of saturation and Triode

$$V_{DS}=V_{GS}-V_T$$

$$V_{DS}=1=V_{GS}-1$$

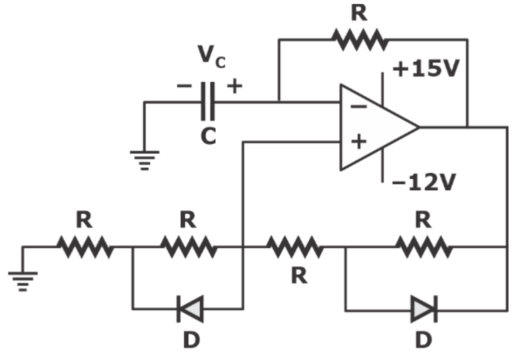
$$V_{GS}=2V \qquad (\because\ \sin 2t=0)$$

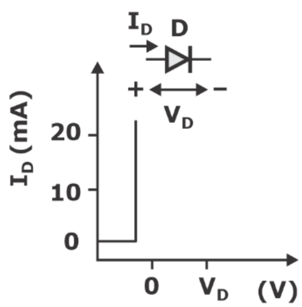
$$I_{DS}=\frac{1}{2}k_n\frac{W}{L}[V_{GS}-V_t]^2$$

$$=\frac{1}{2}\times1\times1\frac{100}{10}[2-1]^2$$

= 5 mA

41. For the following circuit with an ideal OPAMP, the difference between the maximum and the minimum values of the capacitor voltage (V_C) is _______.





A. 15 V

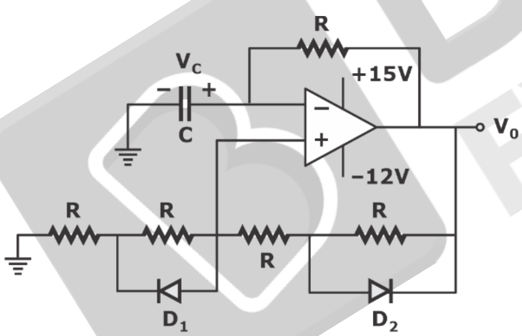
B. 27 V

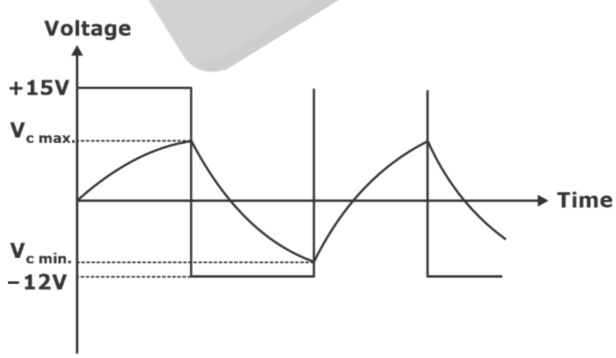
C. 13 V

D. 14 V

Ans. C

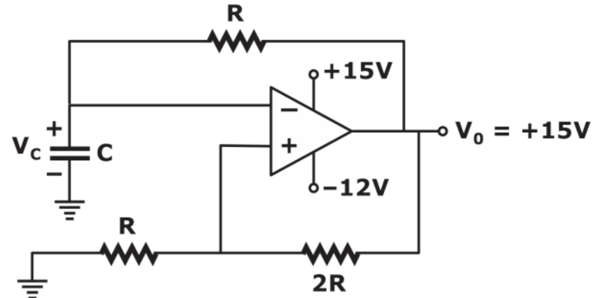
Sol.





Case 1: When V_0 = 15V

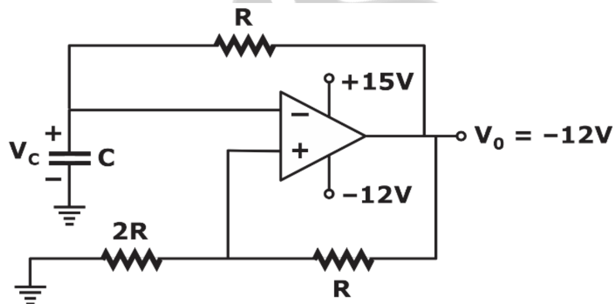
D_1 is ON, D_2 is OFF



$$V_{c\,max.} = \frac{15 \times R}{3R} = 5V$$

Case 2: When V_0 = – 12V

D_1 is OFF, D_2 is ON

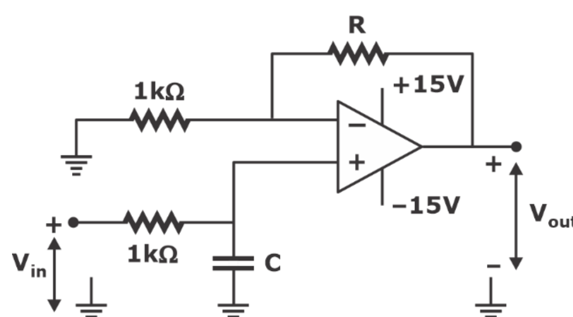


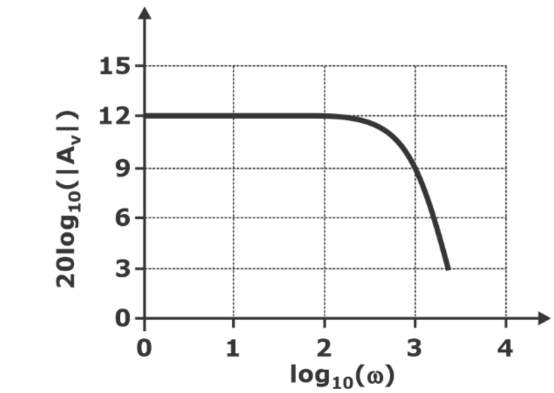
$$V_{c\,min.} = \frac{-12 \times 2R}{3R} = -8V$$

Thus, $V_{c\,max.} - V_{c\,min.} = 5 - (-8) = 13V$

Hence, the correct option is (C).

42. A circuit with an ideal OPAMP is shown. The Bode plot for the magnitude (in dB) of the gain transfer function $\left(A_V(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right)$ of the circuit is also provided (here, ω is the angular frequency in rad/s). The values of R and C are _______.





A. R = 3kΩ, C = 1 μF

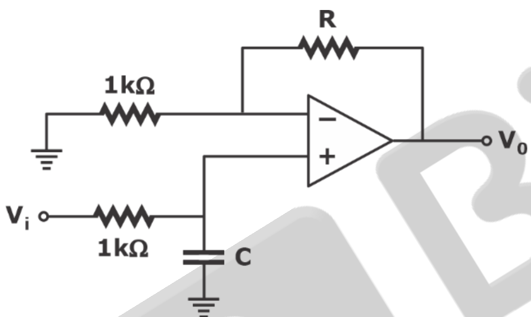
B. R = 1kΩ, C = 3 μF

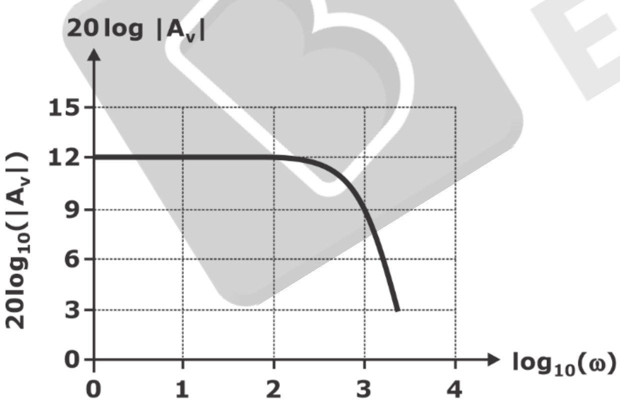
C. R = 4kΩ, C = 1 μF

D. R = 3kΩ, C = 2 μF

Ans. A

Sol. The OP-Amp Circuit is shown below





$$20\log|A_V|_{max.} = 12$$

$$|A_V|_{max.} \cong 4$$

$$1 + \frac{R_2}{R_1} = 4$$

$R_2 = 3R_1$

R = 3 × 1kΩ = 3kΩ

$$\log_{10}\omega_c = 3$$

ω_c = 1000 rad/sec.

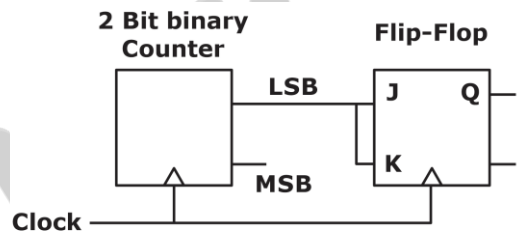
$$\omega_c = \frac{1}{RC}$$

$$1000 = \frac{1}{1000 \times C}$$

C = 1μF

Hence, the correct option is (A)

43. For the circuit shown, the clock frequency is f_0 and the duty cycle is 25%. For the signal at the Q output of the Flip-Flop, ________.



A. frequency is $\frac{f_0}{4}$ and duty cycle is 50%

B. frequency is $\frac{f_0}{4}$ and duty cycle is 25%

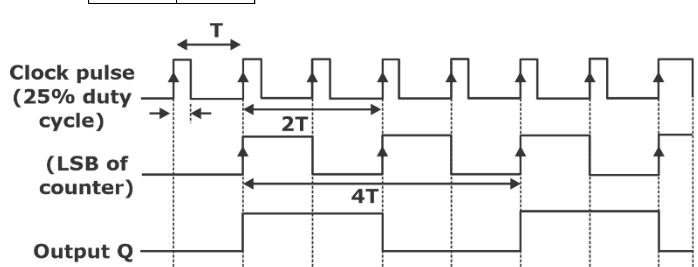
C. frequency is $\frac{f_0}{2}$ and duty cycle is 50%

D. frequency is f_0 and duty cycle is 25%

Ans. A

Sol. 2-bit binary count is shown below,

MSB	LSB
0	0
0	1
1	0
1	1



From the output Q we can observe that output frequency is $f_0/4$ and duty cycle is 50%

Hence, the correct option is (C).

44. Consider an even polynomial $p(s)$ given by

$$p(s) = s^4 + 5s^2 + 4 + K$$

Where K is an unknown real parameter, the complete range of K for which p(s) has all its roots on the imaginary axis is _________.

A. $-4 \le K \le \frac{9}{4}$

B. $-3 \le K \le \frac{9}{2}$

C. $-6 \le K \le \frac{5}{4}$

D. $-5 \le K \le 0$

Ans. A

Sol. Given,

$$p(s) = s^4 + 5s^2 + (4 + K)$$

Routh's Table:

$$\begin{array}{c|ccc} s^4 & 1 & 5 & (4+K) \\ s^3 & 0 & 0 & \\ s^2 & & & \\ s^1 & & & \\ s^0 & & & \end{array}$$

Complete s^3 is zero,

$$A(s) = s^4 + 5s^2 + (4 + K)$$

$$\frac{dA(s)}{ds} = 4s^3 + 10s + 0$$

$$\begin{array}{c|ccc} s^4 & 1 & 5 & (4+K) \\ s^3 & 4 & 10 & 0 \\ s^2 & \frac{10}{4} & (4+K) & \\ s^1 & \frac{25-4(4+K)}{5/2} & & \\ s^0 & 4+K & & \end{array}$$

1^{st} column of the R-H table must be all positive

i.e., $\frac{25-4(4+K)}{5/2} > 0 \Rightarrow K < \frac{9}{4}$

$4 + K > 0 \Rightarrow K > -4$

Range of K: $-4 < K < \frac{9}{4}$

45. Consider the following series:

$$\sum_{n=1}^{\infty} \frac{n^d}{c^n}$$

For which of the following combinations of c, d values does this series converge?

A. c = 1, d = – 1

B. c = 2, d = – 1

C. c = 0.5, d = – 10

D. c = 1, d = – 2

Ans. B, D

Sol. Here, $\sum_{n=1}^{\infty} \frac{n^d}{c^n}$

(A)

C = 1, d = -1

$$\sum_{n=1}^{\infty} \frac{n^{-1}}{1^n} = \sum_{n=1}^{\infty} \frac{1}{n} = \text{Divergent}$$

(B)

C = 2, d = – 1

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

By ratio test

$$\lim_{x\to\infty} \frac{U_{n+1}}{U_n} = \lim_{x\to\infty} \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n} = \lim_{n\to\infty} \frac{1}{2}\left(\frac{n+1}{n}\right)$$

$$= \frac{1}{2} < 1$$

So, $\sum U_n$ is convergent.

(C)

C = 0.5, d = 10

$$\sum_{n=1}^{\infty} \frac{n^{10}}{0.5^n} = \sum_{n=1}^{\infty} 2^n \cdot n^{10} = \sum_{n=1}^{\infty} U_n$$

Apply ratio test

$$\lim_{x\to\infty} \frac{U_{n+1}}{U_n} = \lim_{x\to\infty} \frac{n^{n+1}(n+1)^{10}}{2^n n^{10}} \lim_{n\to\infty} 2\left(1+\frac{1}{n}\right)$$

= 2

Her 2 > 1 → $\sum U_n$ is divergent.

(D)

C = 1, d = – 2

$$\sum_{n=1}^{\infty} \frac{n^{-2}}{1^n} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$\sum U_n$ is convergent

46. The outputs of four systems $(S_1, S_2, S_3 \text{ and } S_4)$ corresponding to the input signal sin(t), for all time t, are shown in the figure.

Based on the given information, which of the four systems is/are definitely NOT LTI (linear and time-invariant)?

$$\sin(t) \rightarrow \boxed{S_1} \rightarrow \sin(-t)$$

$$\sin(t) \rightarrow \boxed{S_2} \rightarrow \sin(t+1)$$

$$\sin(t) \rightarrow \boxed{S_3} \rightarrow \sin(2t)$$

$$\sin(t) \rightarrow \boxed{S_4} \rightarrow \sin^2(t)$$

A. S_1

B. S_2

C. S_3

D. S_4

Ans. C, D

Sol. $\sin t \rightarrow \boxed{S_1} \rightarrow \sin(-t) = -\sin(t)$

$$\sin t \rightarrow \boxed{S_2} \rightarrow \sin(t+1)$$

$$\sin t \rightarrow \boxed{S_3} \rightarrow \sin(2t)$$

$$\sin t \rightarrow \boxed{S_4} \rightarrow \sin^2(t) = \frac{1-\cos 2t}{2}$$

S_3 and S_4 are definitely not LTI as input and output sinusoidal frequencies are different.

47. Select the CORRECT statement(s) regarding semiconductor devices.

A. Electrons and holes are of equal density in an intrinsic semiconductor at equilibrium.

B. Collector region is generally more heavily doped than Base region in a BJT.

C. Total current is spatially constant in a two terminal electronic device in dark under steady state condition.

D. Mobility of electrons always increases with temperature in Silicon beyond 300 K.

Ans. A, C

Sol. Electrons and holes are of equal density in an intrinsic semiconductor at equilibrium.

i.e., $n = p = n_i$

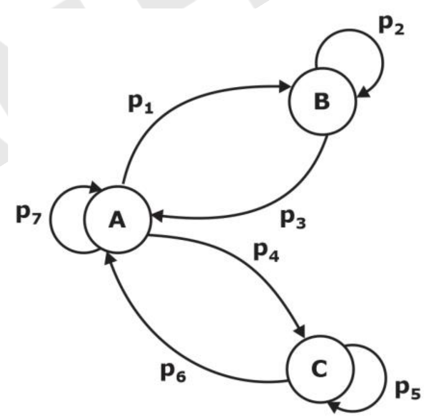
(i) Collector region is generally lightly heavily doped than Base region in a BJT.

(ii) Total current is spatially constant in a two terminal electronic device in dark under steady state condition.

(iii) Mobility of electrons always won't increase with temperature in Silicon beyond 300 K.

Mobility will start to decrease after some temperature beyond 300K

48. A state transition diagram with states A, B and C, and transition probabilities $p_1, p_2, \ldots p_7$ is shown in the figure (e.g., p_1 denotes the probability of transition from state A to B). For this state diagram, select the statement(s) which is/are universally true.



A. $p_2 + p_3 = p_5 + p_6$

B. $p_1 + p_3 = p_4 + p_6$

C. $p_1 + p_4 + p_7 = 1$

D. $p_2 + p_5 + p_7 = 1$

Ans. A, C

Sol. **Case 1:**

If present state (P.S) = A

Next state (N.S) is either A or B or C

So, P.S (A) to NS (A or B or C) is,

P_7, P_1 and P_4

$P_1 + P_4 + P_7 = 1$

Case 2:

If P.S is B

N.S = A or B

So, present state to next state probability is

$P_2 + P_3 = 1$

Case 3:

P.S = C and then N.S = A or C

So, present state to next state probability is,

$P_5 + P_6 = 1$

Then, $P_2 + P_3 = P_5 + P_6$

Hence, the correct options are (A) and (C).

49. Consider a Boolean gate (D) where the output Y is related to the inputs A and B as, $Y = A + \overline{B}$, where + denotes logical OR operation. The Boolean inputs ‘0’ and ‘1’ are also available separately. Using instances of only D gates and inputs ‘0’ and ‘1’, _______ (select the correct option(s)).

A. NAND logic can be implemented

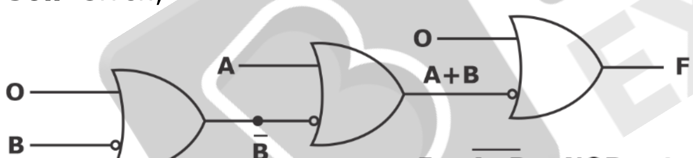
B. OR logic cannot be implemented

C. NOR logic can be implemented

D. AND logic cannot be implemented

Ans. A, C

Sol. Given,



F = $\overline{A+B}$ = NOR gate

We have implemented NOR gate by using the logic gate provide in question, and we know that NOR gate is an universal gate hence any logic can be implemented by given logic $Y = A + \overline{B}$.

Hence, the correct option are (A) and (C).

50. Two linear time-invariant systems with transfer functions

$$G_1(s) = \frac{10}{s^2 + s + 1} \text{ and } G_2(s) = \frac{10}{s^2 + s\sqrt{10} + 10}$$

have unit step responses $y_1(t)$ and $y_2(t)$, respectively. Which of the following statements is/are true?

A. $y_1(t)$ and $y_2(t)$ have the same percentage peak overshoot.

B. $y_1(t)$ and $y_2(t)$ have the same steady-state value.

C. $y_1(t)$ and $y_2(t)$ have the same damped frequency of oscillation.

D. $y_1(t)$ and $y_2(t)$ have the same 2% settling time.

Ans. A

Sol. For system $G_1(s) = \dfrac{10}{s^2 + s + 1}$

Characteristics equation,

$s^2 + s + 1 = 0$

The standard characteristics equation is

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

On comparing,

$$\omega_n = 1, \ \xi = \frac{1}{2} = 0.5$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 1\sqrt{1-(0.5)^2} = 0.866$$

Settling time, $t_s = \dfrac{4}{\xi\omega_n} = 8$ s.

Steady-state error,

$$e_{ss} = \lim_{s\to 0} \frac{s\left(\frac{1}{s}\right)\cdot 10}{s^2 + s + 1} = 10$$

$e_{ss} = 10$

For system, $G_2(s) = \dfrac{10}{s^2 + \sqrt{10}\,s + 10}$

Characteristics equation,

$s^2 + \sqrt{10}\,s + 10 = 0$

Standard characteristics equation,

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

On comparing,

$\omega_n^2 = 10 \ \Rightarrow \ \omega_n = \sqrt{10}$

$2\xi\omega_n = \sqrt{10} \Rightarrow \xi = 0.5$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{10}\sqrt{1-(0.5)^2} = 2.739$$

Setline time, $t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.5\sqrt{10}} = \frac{8}{\sqrt{10}} = 2.535$

Steady-state error $e_{ss} = \lim_{s\to 0} \frac{s\left(\frac{1}{s}\right)\cdot 10}{s^2 + \sqrt{10}\,s + 10} = 1$

$e_{ss} = 1$

Since 'ξ' value for both the system is the same. So the percentage peak overshoot for both systems is the same.

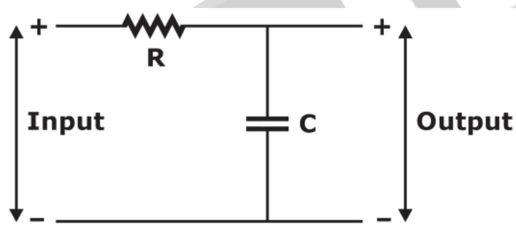
So, option (A) is correct.

51. Consider an FM broadcast that employs the pre-emphasis filter with frequency response

$H_{pe}(\omega) = 1 + \frac{j\omega}{\omega_0}$

where $\omega_0 = 10^4$ rad/sec.

For the network shown in the figure to act as a corresponding de-emphasis filter, the appropriate pair(s) of (R, C) values is/are ________.



A. R = 1kΩ, C = 0.1 μF

B. R = 2kΩ, C = 1 μF

C. R = 1kΩ, C = 2 μF

D. R = 2kΩ, C = 0.5 μF

Ans. A

Sol. $H_{Pe}(\omega) = 1 + \frac{j\omega}{\omega_0}$

$\because \omega_0 = 10^4$

$H_{De}(\omega) = \frac{1/j\omega C}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$

$|H_{Pe}(\omega)||H_{De}(\omega)| = \text{Constant}$

$\Rightarrow \left(1 + \frac{\omega^2}{\omega_0^2}\right)\left(\frac{1}{1 + \omega^2R^2C^2}\right) = \text{Constant}$

If $\frac{\omega^2}{\omega_0^2} = \omega^2R^2C^2$

$\Rightarrow \omega_0^2 = \frac{1}{R^2C^2}$

$\Rightarrow \omega_0 = \frac{1}{RC}$

$\Rightarrow RC = 10^{-4}$

Only option A satisfies.

52. A waveguide consists of two infinite parallel plates (perfect conductors) at a separation of 10^{-4} cm, with air as the dielectric. Assume the speed of light in air to be 3×10^8 m/s. The frequency/frequencies of TM waves which can propagate in this waveguide is/are ________.

A. 6×10^{15} Hz

B. 0.5×10^{12} Hz

C. 8×10^{14} Hz

D. 1×10^{13} Hz

Ans. A, B, C, D

Sol. For parallel plate wave guide, we have TM0 mode as lowest mode

$f_c = \frac{mc}{2a}\left[m_{min} = 0\right]$

$f_c = 0$

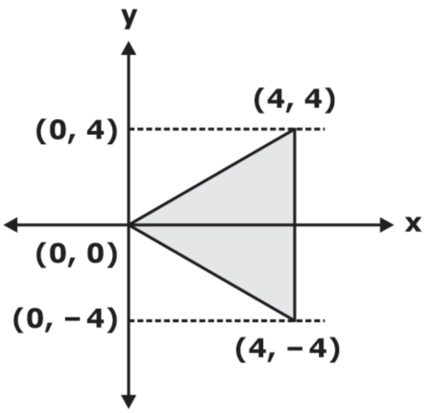
Hence all the given frequencies can be propagated through this waveguide.

NOTE: Answer key provided by IIT Kharagpur is (A, C), they have considered TM0 mode as TEM mode and have taken TM1 mode as lowest mode for TM, but ultimately TEM mode is TM mode only so this question can be challenged.

53. The value of the integral

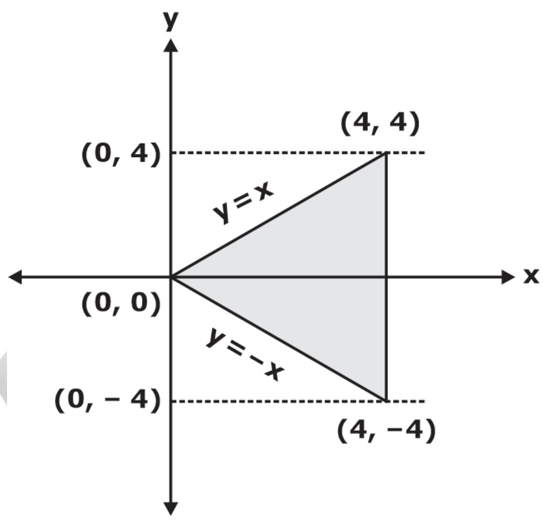
$$\iint_D 3\left(x^2+y^2\right)dxdy,$$

where D is the shaded triangular region shown in the diagram, is ______ (rounded off to the nearest integer).



Ans. (512 to 512)

Sol.



$$I=\iint_D 3\left(x^2+y^2\right)dxdy$$

$$I=\int_{x=0}^{4}\int_{y=-x}^{x} 3\left(x^2+y^2\right)dydx$$

$$I=3\int_{x=0}^{4}\left(x^2y+\frac{y^3}{3}\right)_{-x}^{x}dx=3\int_{x=0}^{4}\left(2x^3+\frac{2x^3}{3}\right)dx$$

$$I=3\int_0^4\frac{8}{3}x^3dx=8\int_0^4 x^3dx$$

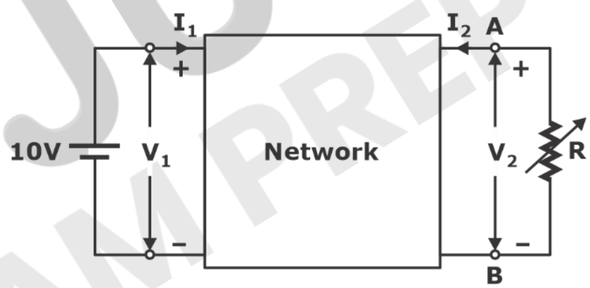
$$I=8\left[\frac{x^4}{4}\right]_0^4=2\left[4^4\right]$$

$I = 2 \times 256$

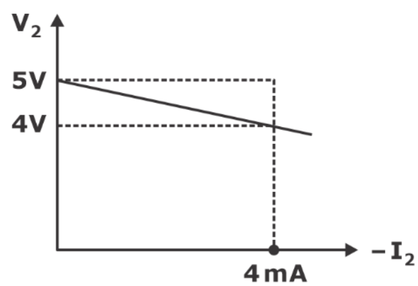
$= 512$

54. A linear 2-port network is shown in Fig. (a). An ideal DC voltage source of 10 V is connected across Port 1. A variable resistance R is connected across Port 2. As R is varied, the measured voltage and current at Port 2 is shown in Fig. (b) as a V_2 versus $-I_2$ plot. Note that for V_2 = 5V, I_2 = 0 mA, and for V_2 = 4V, I_2 = − 4 mA.

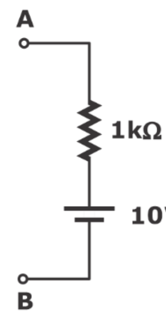
When the variable resistance R at Port 2 is replaced by the load shown in Fig. (c), the current I_2 is ________ mA (rounded off to one decimal place).



(a)



(b)



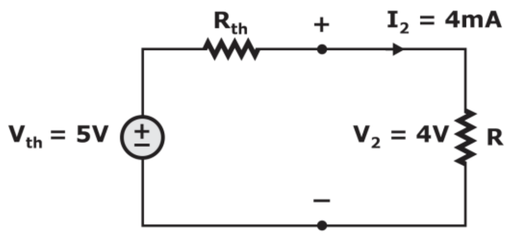
(c)

Ans. (3.9 to 4.1)

Sol. From the graph

When, $I_2 = 0$; $V_2 = 5V = V_{th}$

$I_2 = 4$ mA; $V_2 = 4V$



$$I_2 = \frac{5}{R_{th} + R}$$

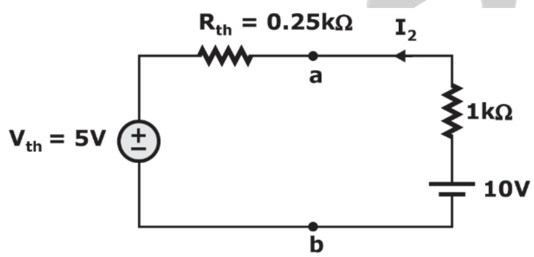
$$(or) = \frac{V_{th} - V_2}{R_{th}}$$

$$4\,mA = \frac{5-4}{R_{th}}$$

$$R_{th} = \frac{1}{4}k\,\Omega$$

$= 0.25$ kΩ

If the voltage source is connected across ab



$$I_2 = \frac{10-5}{1.25k} = 4 \text{ mA}$$

55. For a vector $\bar{x} = [x[0],\ x[1], \ldots\ldots, x[7]]$, the 8-point discrete Fourier transform (DFT) is denoted by $\bar{X} = DFT(\bar{x}) = [X[0],\ X[1], \ldots\ldots, X[7]]$, where

$$X(k) = \sum_{n=0}^{7} x[n]\exp\left(-j\frac{2\pi}{8}nk\right)$$

Here, $j = \sqrt{-1}$. If $\bar{x} = [1,0,0,0,2,0,0,0]$ and $\bar{y} = DFT(DFT\,\bar{x})$, then the value of y[0] is ________ (rounded off to one decimal place).

Ans. (7.9 to 8.1)

Sol. Given,

$$\bar{x} = \{x[0],\ x[1],\ \ldots\ldots\ldots\ldots\ldots,\ x[7]\}$$

$$\bar{X} = DFT\{\bar{x}\} = \{X[0],\ X[1],\ \ldots\ldots\ldots\ldots\ldots,\ X[7]\}$$

$$\bar{x} = \{1,0,0,0,2,0,0,0\}$$

$$\bar{g} = \{1,0,2,0\} \xrightarrow[DFT]{4pt} \bar{G} = \{3,-1,3,-1\}$$

$$\bar{G} = T.\bar{g} \Rightarrow \bar{G} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} = \{3,\ -1,\ 3, -1\}$$

We know that,

Up sampling (interpolation) in time domain corresponds to replication in DFT domain.

$$\therefore\ \bar{x} = \bar{g}\left\{\frac{n}{2}\right\} = \{1,0,0,0,2,0,0,0\}$$

$$\therefore\ \bar{X} = \bar{G}\left\{\frac{n}{2}\right\} = DFT\{\bar{x}\} = \{3,-1,3,-1,3,-1,3,-1\}$$

$$\bar{Y} = DFT\{DFT\ \bar{x}\}$$

$$\bar{Y} = DFT\{\bar{X}\} = DFT\{3,-1,3,-1,3,-1,3,-1\}$$

∴ By central ordinate prop. of DFT

$$y[0] = 3 + (-1) + 3 + (-1) + 3 + (-1) + 3 + (-1)$$

$= 8$

Alternate Method:

$x(n) \xrightarrow{DFT} X(k) = DFT[x(n)]$

$X(n) \xrightarrow{DFT} Nx(-k)\ \ DFT[X(n)]$

[Using duality property]

$$X[n] \to N \cdot x(-k) \to DFT[X[n]] = DFT[DFTx[n]]$$

=Nx[-k]

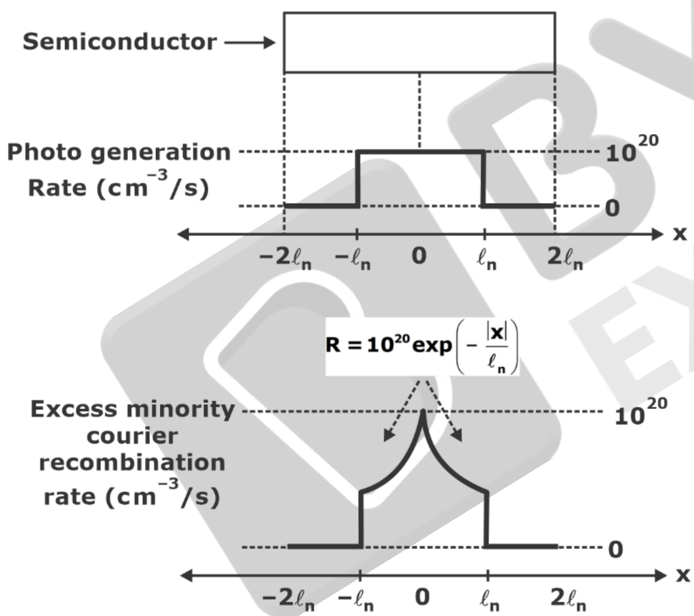
$$\boxed{D.F.T\ [D.F.T[x(n)]] = Nx(-n)} = N \cdot x(-k)$$

$$= 8 \cdot \{\underset{\uparrow}{1},\ 0,\ 0,\ 0,\ 2,\ 0,\ 0,\ 0\}$$

$$DFT[DFT(\bar{x})] = \{\underset{\uparrow}{8},\ 0,\ 0,\ 0,\ 16,\ 0,\ 0,\ 0\}$$

$$\Rightarrow y(0) = 8$$

56. A p-type semiconductor with zero electric field is under illumination (low level injection) in steady state condition. Excess minority carrier density is zero at $x = \pm 2\ell_n$, where $\ell_n = 10^{-4}$ cm is the diffusion length of electrons. Assume electronic charge, q = −1.6 × 10^{-19}C. The profiles of photo-generation rate of carriers and the recombination rate of excess minority carriers (R) are shown. Under these conditions, the magnitude of the current density due to the photo-generated electrons at $x = +2\ell_n$ is ________ mA/cm^2 (rounded off to two decimal places).



Ans. (0.57 to 0.61)

Sol. $\delta n(x) = R\tau_n = 10^{20} e^{-|x|/l_n}\tau_n$

$\delta n(ln) = 1020 e^{-1}\tau_n$ ...(i)

$l_n \le x \le 2l_n$

Continuity equation in steady state,

$$D_n \frac{\partial^2 \delta n}{\delta x^2} + G - R = 0$$

Since, $\left.\begin{matrix} G = 0 \\ R = 0 \end{matrix}\right\} l_n \le x \le 2l_n$

$$\therefore D_n \frac{\partial^2 \delta n}{\partial x^2} = 0$$

Whose solution is, $\delta n(x) = Ax + B$

Since at x = 2l$_n$:

$\delta n(2l_n) = 0$ (given)

$0 = A(2l_n) + B$

$$A = -\frac{B}{2l_n}$$

$$\therefore \delta n(x) = -\frac{B}{2l_n}x + B = B\left(1 - \frac{x}{2l_n}\right) \quad \text{...(ii)}$$

$\therefore$ At $x = l_n$: equation (i) = equation (ii)

$$10^{20} e^{-1}\tau_n = B\left(1 - \frac{l_n}{2l_n}\right)$$

$$\therefore B = 2 \times 10^{20} e^{-1}\tau_n$$

$$\therefore \delta n(x) = 2 \times 10^{20} e^{-1}\tau_n\left(1 - \frac{x}{2l_n}\right) \quad l_n \le x \le 2l_n$$

$\therefore$ Electron diffusion current density:

$$|J_n|_{diff} = qD_n\frac{dn}{dx} = qD_n \times 2 \times 10^{20} \times e^{-1} \times \tau_n\left(0 - \frac{1}{2l_n}\right)$$

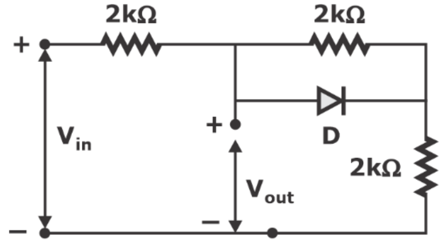
$$= \frac{1.6 \times 10^{-19} \times l_n^2 \times 2 \times 10^{20} \times e^{-1}}{2l_n}$$

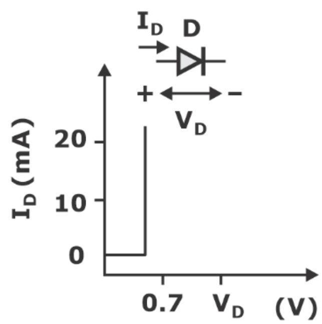
$$= 1.6 \times 10^{-19} \times l_n \times 10^{20} \times e^{-1}$$

$$= 1.6 \times 10^{1} \times 1 \times 10^{-4} \times e^{-1} \qquad (l_n = 10^{-4}\text{cm})$$

= 0.588 mA/cm^2 = 0.59

57. A circuit and the characteristics of the diode (D) in it are shown. The ratio of the minimum to the maximum small signal voltage gain $\frac{\partial V_{out}}{\partial V_{in}}$ in _______ (rounded off to two decimal places).





Ans. (0.70 to 0.80)

Sol. Given circuit is shown below,

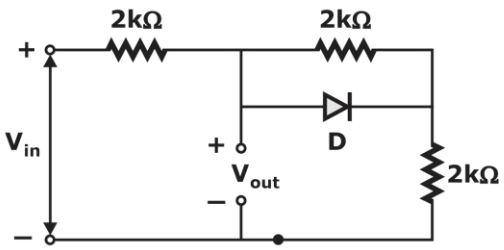


Figure (a)

And diode characteristics is,

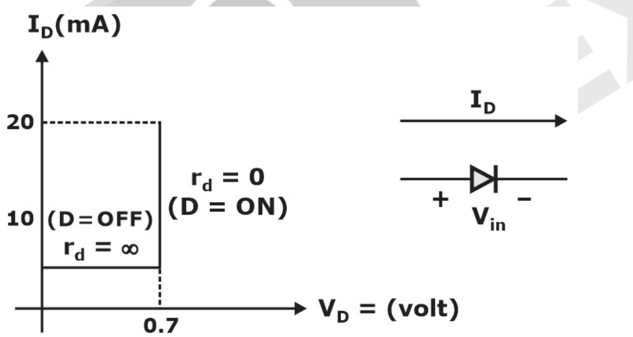
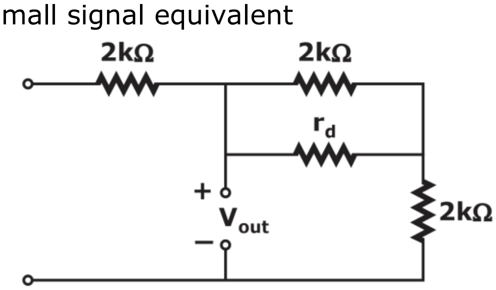


Figure (b)

Replacing the circuit in figure (a) with the small signal equivalent



Case 1: When diode ON

As r_d(ON) = 0, the 2kΩ resistor in parallel to the diode becomes short circuit.

$$\therefore\ V_{out} = \frac{V_{input} \times 2}{4} = \frac{V_{input}}{2}$$

$$\therefore\ \left.\frac{\partial V_{out}}{\partial V_{in}}\right|_{max.} = \frac{1}{2}$$

Case 2: When diode OFF

As r_d(OFF) = infinite, the equivalent resistance will 2kΩ + 2kΩ + 2kΩ = 6kΩ

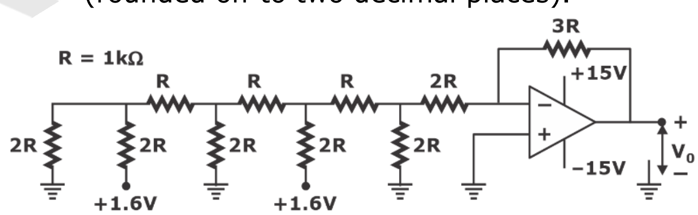
$$\therefore\ V_{out} = \frac{V_{input} \times 4}{2+2+2} = \frac{2V_{input}}{3}$$

$$\therefore\ \left.\frac{\partial V_{out}}{\partial V_{in}}\right|_{min.} = \frac{2}{3}$$

$$\therefore\ \frac{\left.\frac{\partial V_{out}}{\partial V_{in}}\right|_{min.}}{\left.\frac{\partial V_{out}}{\partial V_{in}}\right|_{max.}} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \times \frac{3}{2} = 0.75$$

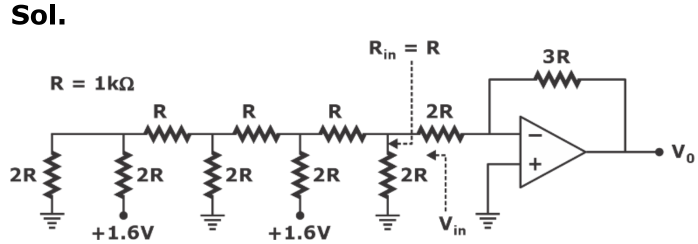
Hence, Correct answer is 0.75

58. Consider the circuit shown with an ideal OPAMP. The output voltage V_0 is ________ V (rounded off to two decimal places).



Ans. (–0.55 to –0.45)

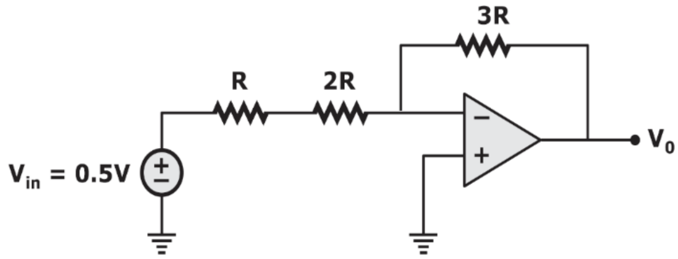
Sol.



$$V_{in} = \frac{2^0 b_0 \times 1.6 + 2^1 b_1 \times 1.6 + 2^2 b_2 \times 1.6 + 2^3 b_3 \times 1.6}{2^4}$$

$$= \frac{(1 \times 1.6V) + (4 \times 1.6V)}{16} = 0.5V$$

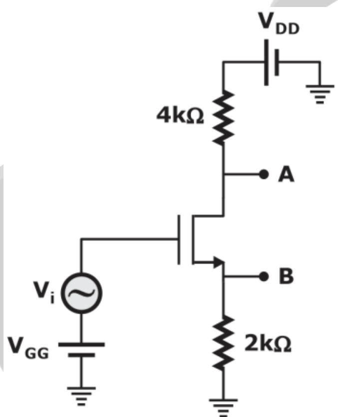
Now we get,



$$V_0 = \frac{-3R}{3R} \times 0.5 = -0.5V$$

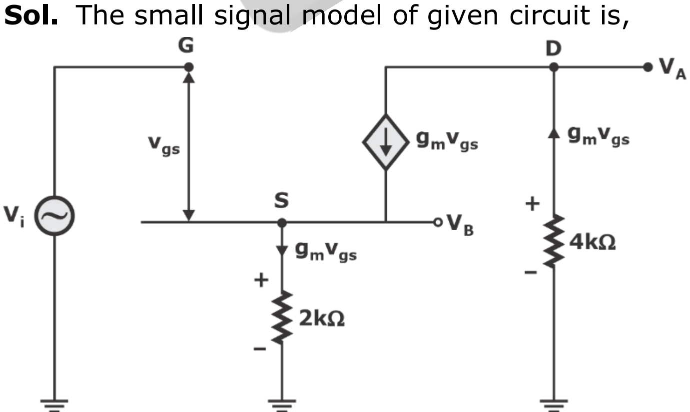
Hence, the correct answer is – 0.5V.

59. Consider the circuit shown with an ideal long channel nMOSFET (enhancement mode, substrate is connected to the source). The transistor is appropriately biased in the saturation region with V_{GG} and V_{DD} such that it acts as a linear amplifier. v_i is the small-signal ac input voltage. V_A and v_B represent the small-signal voltages at the nodes A and B, respectively. The value of $\frac{V_A}{V_B}$ is ______ (rounded off to one decimal place).



Ans. (–2.1 to –1.9)

Sol. The small signal model of given circuit is,



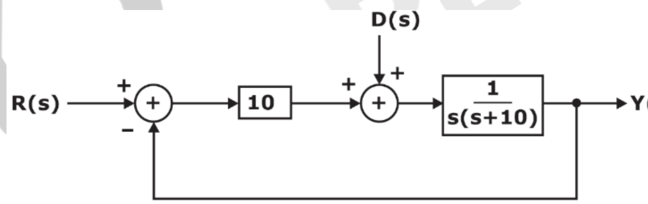
$$V_A = -g_m v_{gx} \times 4k\Omega$$

$$V_B = +g_m v_{gx} \times 2k\Omega$$

$$\frac{V_A}{V_B} = -2$$

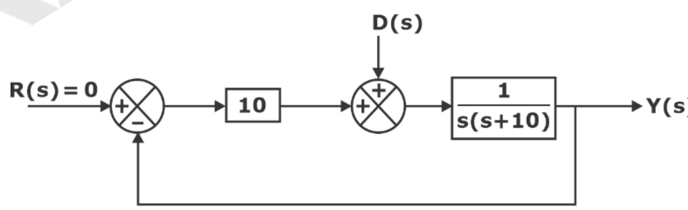
Hence, the correct answer is (–2).

60. The block diagram of a closed-loop control system is shown in the figure. R(s), Y(s), and D(s) are the Laplace transforms of the time-domain signals r(t), y(t), and d(t), respectively. Let the error signal be defined as $e(t)$ = r(t) – y(t). Assuming the reference input r(t) = 0 for all t, the steady-state error $e(\infty)$, due to a unit step disturbance d(t), is ______ (rounded off to two decimal places).



Ans. (–0.11 to –0.09)

Sol.



$$G_1(s) = 10, \quad G_2(s) = \frac{1}{s(s+10)}$$

$$\frac{E(s)}{D(s)} = \frac{-G_2(s)}{1 + G_1(s)G_2(s)}$$

$$= \frac{\frac{-1}{s(s+10)}}{1 + \left(10 \times \frac{1}{s(s+10)}\right)}$$

$$= \frac{-1}{s^2 + 10s + 10}$$

$D(s) = \frac{1}{s}$ (Given in the question)

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

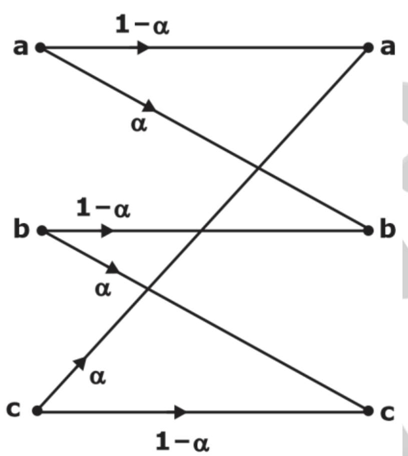
$$e_{ss} = \lim_{s\to 0} \frac{s\left(\frac{1}{s}\right)(-1)}{s^2+10s+10}$$

$$e_{ss} = \frac{-1}{10}$$

$$e_{ss} = -0.1$$

61. The transition diagram of a discrete memoryless channel with three input symbols and three output symbols is shown in the figure. The transition probabilities are as marked.

The parameter α lies in the interval [0.25, 1]. The value of α for which the capacity of this channel is maximized, is ________ (rounded off to two decimal places).



Ans. (1.00 to 1.00)

Sol. $I(X,Y) = H(X) + H(Y) - H(X,Y)$

$$I(x,y) = \log_2 3 + \log_2 3 + \alpha\log\frac{\alpha}{3} + (1-\alpha)\log\left(\frac{1-\alpha}{3}\right)$$

$$\frac{d}{d\alpha}I(x,y) = 0 + 0 + \log\frac{\alpha}{3} + \alpha\times\frac{1}{\frac{\alpha}{3}}\times\frac{1}{3} +$$

$$(-1)\log\left(\frac{1-\alpha}{3}\right) + (1-\alpha)\frac{1}{\frac{1-\alpha}{3}}\left(\frac{-1}{3}\right) = 0$$

$$\Rightarrow \quad \log\frac{\alpha}{3} + 1 - \log\left(\frac{1-\alpha}{3}\right) - 1 = 0$$

$$\Rightarrow \quad \frac{\alpha}{3} = \frac{1-\alpha}{3} \quad \Rightarrow \quad \alpha = 0.5$$

$$\frac{d^2}{d\alpha^2}I(x,y) = \frac{1}{\frac{\alpha}{3}}\times\frac{1}{3} - \frac{1}{\frac{1-\alpha}{3}}\times\frac{-1}{3}$$

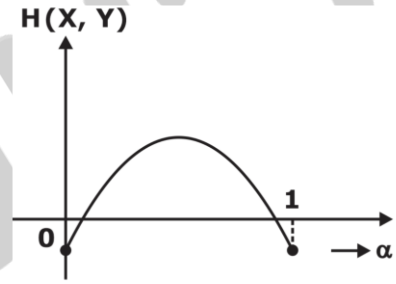
$$\frac{d^2}{d\alpha^2}I(x,y) = \frac{1}{\alpha} + \frac{1}{1-\alpha}$$

$$\left.\frac{d^2}{d\alpha^2}I(x,y)\right|_{\alpha=0.5} = \frac{1}{0.5} + \frac{1}{0.5} = 4 > 0$$

$\Rightarrow$ At α = 0.5, we have minimum I(x, y)

$$H(x,y) = \sum_{i=0}^{2}\sum_{j=0}^{2} -P(x_i, y_j)\cdot\log P(x_i, y_j)$$

$$H(x,y) = -\left(\frac{1-\alpha}{3}\right)\log\left(\frac{1-\alpha}{3}\right)\times 3 - \frac{\alpha}{3}\times\log\frac{\alpha}{3}\times 3$$



At, a = 0, 1

$\Rightarrow$ H (x, y) is minimum

$\Rightarrow$ I(x, y) is maximum

But a interval is [0.25, 1]

Hence a = 1

62. Consider communication over a memoryless binary symmetric channel using a (7, 4) Hamming code. Each transmitted bit is received correctly with probability (1 − ϵ), and flipped with probability ϵ. For each codeword transmission, the receiver performs minimum Hamming distance decoding, and correctly decodes the message bits if and only if the channel introduces at most one bit error.

For ϵ = 0.1, the probability that a transmitted codeword is decoded correctly is _______ (rounded off to two decimal places).

Ans. (0.84 to 0.86)

Sol. Probability of bit error = ∈

Message is decoded correctly only if the channel introduces max. 1 error.

Probability of correct decoding = Probability of atmost 1 bit error

= Probability of no. error + Probability of 1 error

$= (1-\epsilon)^7 + {}^7C_1 \times (1-\epsilon)^6 \cdot \epsilon^1$

If $\epsilon = 0.1$ ⇒ Probability of correct decoding

$= 0.9^7 + 7 \times 0.9^6 \times 0.1$

= 0.85

63. Consider a channel over which either symbol x_A or symbol x_B is transmitted. Let the output of the channel Y be the input to a maximum likelihood (ML) detector at the receiver. The conditional probability density functions for Y given x_A and x_B are:

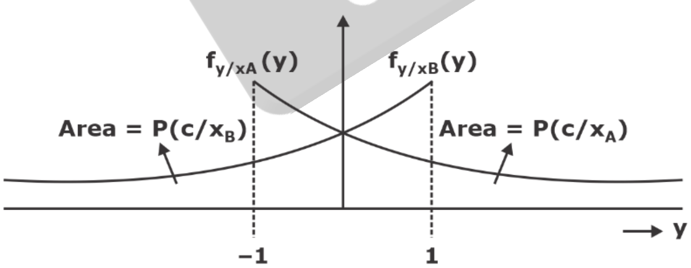
$f_{Y|x_A}(y) = e^{-(y+1)}u(y+1)$

$f_{Y|x_B}(y) = e^{(y-1)}(1 - u(y-1))$

where $u(\cdot)$ is the standard unit step function. The probability of symbol error for this system is __________ (rounded off to two decimal places).

Ans. *

Sol.



∴ ML criteria is used, threshold voltage = 0V

$$P_e = P\left(\frac{e}{x_A}\right) \times \frac{1}{2} + P\left(\frac{e}{x_B}\right) \times \frac{1}{2}$$

$$P_e = \frac{1}{2}\int_0^{\infty} e^{-(y+1)}dy + \frac{1}{2}\int_{-\infty}^{0} e^{(y-1)}dy$$

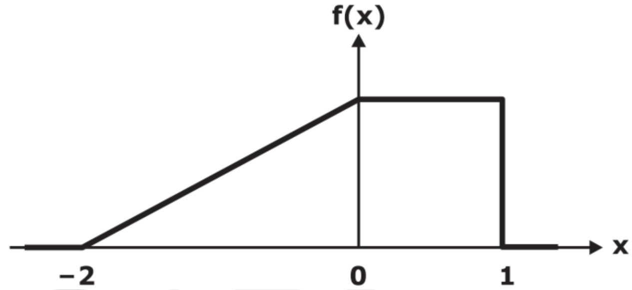
$$= \frac{1}{2e}[1 - 0 + 0 + 1]$$

$$= \frac{2}{2e} = \frac{1}{e} = 0.367$$

Probability of symbol error = e^{-1} = 0.367

Note: Answer key provided by IIT Kharagpur is (0.22 – 0.25) so this question can be challenged.

64. Consider a real valued source whose samples are independent and identically distributed random variables with the probability density function, f(x), as shown in the figure.



Consider a 1 bit quantizer that maps positive samples to value α and others to value β. If α* and β* are the respective choices for α and β that minimize the mean square quantization error, then (α* − β*) = __________ (rounded off to two decimal places).

Ans. (1.15 to 1.18)

Sol. Quantization noise power for positive values of x = $E[(x-\alpha)^2]$

$$= \int_0^1 (x-\alpha)^2 \cdot \frac{1}{2} \cdot dx$$

$$= \frac{(x-\alpha)^3}{3} \cdot \frac{1}{2}\Big|_0^1$$

$$= \frac{1}{6}[(1-\alpha)^3 - (-\alpha)^3]$$

$$= \frac{1}{6}[(1-\alpha)^3 + \alpha^3]$$

$$= \frac{1}{6}[1 - \alpha^3 + 3\alpha^2 - 3\alpha + \alpha^3]$$

$$QNP = \frac{1}{6}[1 + 3\alpha^2 - 3\alpha]$$

$$\frac{d}{d\alpha}QNP = 0 + 6\alpha - 3 = 0$$

$$\alpha = \frac{1}{2}$$

$$\alpha^* = \frac{1}{2}$$

Q.N.P for negative value of x = $E\left[(x-\beta)^2\right]$

$$= \int_{-2}^{0} (x-\beta)^2\left(\frac{1}{4}x + \frac{1}{2}\right)dx$$

$$= \frac{1}{4}\int_{-2}^{0} (x-\beta)^2\, x\, dx + \frac{1}{2}\int_{-2}^{0} (x-\beta)^2\, dx$$

$$= \frac{1}{4}\left[\int_{-2}^{0} (x^2 + \beta^2 - 2\beta x)\, x\, dx\right] + \frac{1}{2}\left[\int_{-2}^{0} (x^2 + \beta^2 - 2\beta x)dx\right]$$

$$= \frac{1}{4}\left[\int_{-2}^{0} (x^3 + \beta^2 x - 2\beta x^2)\, dx\right] + \frac{1}{2}\left[\frac{(x^3)_{-2}^{0}}{3} + \beta^2(2) - \beta(x^2)_{-2}^{0}\right]$$

$$= \frac{1}{4}\left[\frac{(x^4)_{-2}^{0}}{4} + \frac{\beta^2}{2}(x^2)_{-2}^{0} - \frac{2}{3}\beta(x^3)_{-2}^{0}\right] + \frac{1}{2}\left[\frac{8}{3} + 2\beta^2 + 4\beta\right]$$

$$QNP = \frac{1}{4}\left[-4 - 2\beta^2 - \frac{16}{3}\beta\right] + \frac{1}{2}\left[\frac{8}{3} + 2\beta^2 + 4\beta\right]$$

$$\frac{d}{d\beta}QNP = -4\beta - \frac{16}{3} + 2\beta + 4 = 0$$

$$2\beta = 4 - \frac{16}{3}$$

$$2\beta = \frac{-4}{3}$$

$$\beta = \frac{-2}{3}$$

$$\Rightarrow \beta^* = \frac{-2}{3} = -0.667$$

$$\alpha^* - \beta^* = 0.5 - (-0.667)$$
$$= 1.167$$

65. In an electrostatic field, the electric displacement density vector, $\vec{D}$, is given by $\vec{D}(x,y,z) = \left(x^3\vec{i} + y^3\vec{j} + xy^2\vec{k}\right)C/m^2$ where $\vec{i}$, $\vec{j}$, $\vec{k}$ are the unit vectors along x-axis, y-axis, and z-axis, respectively. Consider a cubical region R centered at the origin with each side of length 1 m, and vertices at (± 0.5 m, ± 0.5 m, ± 0.5 m). The electric charge enclosed within R is _______ C (rounded off to two decimal places).

Ans. (0.48 to 0.52)

Sol. From Maxwell's equation,

$$\Delta \cdot D = \rho_v$$

$$\rho_v = \frac{d}{dx}x^3 + \frac{d}{dy}y^3 + 0$$

$$\rho_v = 3x^2 + 3y^2$$

We know that $\frac{dQ}{dV} = \rho_v$

$$Q = \iiint \rho_v dV$$

$$Q = \iiint\left(3x^2 + 3y^2\right)dxdydz$$

$$Q = 3\left[\frac{x^3}{3}\right]_{-0.5}^{0.5} + 3\left[\frac{x^3}{3}\right]_{-0.5}^{0.5}$$

Q = 2 × 0.125 + 2 × 0.125

Q = 0.5C

Hence, the correct answer is 0.5.

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