Important Trigonometric Identities

- **Pythagorean Identities**
  - $\sin^2 \theta + \cos^2 \theta = 1$
  - $\tan^2 \theta + 1 = \sec^2 \theta$
  - $\cot^2 \theta + 1 = \mathrm{cosec}^2 \theta$
- **Negative of a Function**
  - sin (−x) = −sin x
  - cos (−x) = cos x
  - tan (−x) = −tan x
  - cosec (−x) = −cosec x
  - sec (−x) = sec x
  - cot (−x) = −cot x
- **If A + B = 90°, then**
  - Sin A = Cos B
  - $\mathrm{Sin}^2A + \mathrm{Sin}^2B = \mathrm{Cos}^2A + \mathrm{Cos}^2B = 1$
  - Tan A = Cot B
  - Sec A = Cosec B

Example:
If tan (x+y) tan (x-y) = 1, then find tan (2x/3).
Solution:
Tan A = Cot B, Tan A*Tan B = 1
So, A +B = 90°
(x+y) + (x-y) = 90°, 2x = 90°, x = 45°
Tan (2x/3) = tan 30° = 1/√3

- **If A − B = 90°, (A › B), then**
  - Sin A = Cos B
  - Cos A = − Sin B
  - Tan A = − Cot B
- **If A ± B = 180°, then**
  - Sin A = Sin B
  - Cos A = − Cos B
- **If A + B = 180°, then** tan A = − tan B
- **If A − B = 180°, then** tan A = tan B

Example:
Find the value of tan 80° + tan 100°.
Solution:
Since 80 + 100 = 180
Therefore, tan 80° + tan 100° = 1

- **If A + B + C = 180°, then**
  - Tan A + Tan B +Tan C = Tan A * Tan B *Tan C
  - sin θ * sin 2θ * sin 4θ = 1/4 sin 3θ
  - cos θ * cos 2θ * cos 4θ = 1/4 cos 3θ

Example:
What is the value of cos 20° cos 40° cos 60° cos 80°?
Solution: We know cos θ * cos 2θ * cos 4θ = 1/4 cos 3θ
Now, (cos 20° cos 40° cos 80°) cos 60°
1/4 (Cos 3*20) * cos 60°
1/4 Cos^2 60° = 1/4 * $(1/2)^2$ = 1/16

Example:
If 4 sin θ + 3 cos θ =2, then find the value of 4 cos θ – 3 sin θ.
Solution:
Let 2 cos θ – 3 sin θ = x
By using formulae $a^2 + b^2 = m^2 + n^2$
$4^2 + 3^2 = 2^2 + x^2$
$16 + 9 = 4 + x^2$
x= √21

- **If sin θ + cos θ = p & cosec θ – sec θ = q, then** p – (1/p) = 2/q

Example:
If sin θ + cos θ = 2, then find the value of cosec θ – sec θ.
Solution:
By using formulae:
p – (1/p) = 2/q
2-(1/2) = 3/2 = 2/q
q = 4/3 or cosec θ – sec θ = 4/3

- **If a cot θ + b cosec θ = m & a cosec θ + b cot θ = n, then** $b^2 - a^2 = m^2 - n^2$
- **If cot θ + cos θ = x & cot θ – cos θ = y, then** $x^2 - y^2 = 4\sqrt{xy}$
- **If tan θ + sin θ = x & tan θ – sin θ = y, then** $x^2 - y^2 = 4\sqrt{xy}$
- **If** $y = a^2 \sin^2 x + b^2 \text{cosec}^2 x + c$, or
  $y = a^2 \cos^2 x + b^2 \sec^2 x + c$, or
  $y = a^2 \tan^2 x + b^2 \cot^2 x + c$, or

  then,
  - $y_{min} = 2ab + c$
  - y_{max} = not defined

Example:
If y = 9 $\sin^2$ x + 16 cosec^2 x +4 then find y_{min}.
Solution:
For, y min = 2* √9 * √16 + 4
= 2*3*4 + 20 = 24 + 4 = 28

- **If** $y = a \sin x + b \cos x + c$
  $y = a \tan x + b \cot x + c$
  $y = a \sec x + b \operatorname{cosec} x + c$

  then,
  - $y_{min} = + [\sqrt{(a^2+b^2)}] + c$
  - $y_{max} = - [\sqrt{(a^2+b^2)}] + c$

Example:

If $y = 1/ (12\sin x + 5 \cos x + 20)$ then find y_{max}.

Solution:

For, y max = 1/x min

$= 1/- (\sqrt{12^2 + 5^2}) + 20 = 1/ (-13+20) = 1/7$

- **For $\sin^2 \theta$,**
  - maxima value = 1
  - minima value = 0
- **For $\cos^2 \theta$,**
  - maxima value = 1
  - minima value = 0

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