GATE/ESE

Civil Engineering

Structural Analysis

Important Formula Notes

IMPORTANT FORMULAS ON STRUCTURAL ANALYSIS

CHAPTER 1: INDETERMINACY & STABILITY OF A STRUCTURE

1. External Indeterminacy

Mathematically, external indeterminacy can be expressed as follows.

$$\boxed{D_{Sc} = r - s}$$

Where,

r = total number of unknown support reactions.

S = total number of equilibrium equations available.

S=3 (For 2D structure) and 6 (For 3D structure)

2. Internal Indeterminacy

Case 1: Beam

There is no internal indeterminacy for beams because if we know the support reactions, we can find the axial force, shear force and bending moment at any section in the beam.

Case 2: Trusses

The internal indeterminacy for the trusses can be determined by following expression.

$$D_{Si} = m - (2j - 3);\ \text{for plane truss}$$

$$D_{Si} = m - (3j - 6);\ \text{for space truss}$$

Where,

m = number of members

j = number of joints

3. KINEMATIC INDETERMINACY (D_K):

It is defined as the number of independent displacements at all joints in a structure. Displacements are counted always only at the joints. Displacement includes slopes and deflection. Wherever the cross-section area, changes or material changes then it is treated as a joint in any structure.

The kinematic indeterminacy can be determined for various cases as follows.

Case 1: Beams

Example:

→ Displacement at A and B in x-direction is zero

→ Displacement at A and B y-direction is zero

→ Rotation at A and B is zero

∴ Degree of freedom = D_k = 0

D_k(inextensible) = D_k (extensible) – Number of independent displacements prevented.

Note: It not given in the question, then assume that members are extensible.

Example:

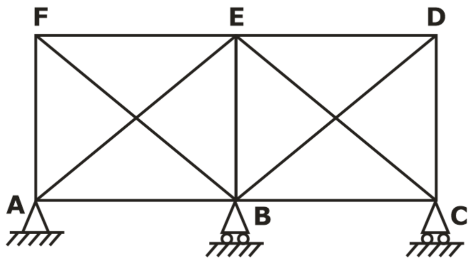
Sol.

Degree of freedom D_K = 2×3-5+4 (Due to internal Hinge) = 5

Ignoring axial deformation, D_K = 5-2 =3

Case 2: Truss

At each joint in a truss number of independent displacements are only two (horizontal and vertical displacement). Rotation of a member in a truss is not considered because it implies that the member buckled. Rigid body rotation is not counted because it is not unknown.



D_k at A = 0

D_k at B = 1

D_k at C = 1

D_k at D = 2

D_k at E = 2

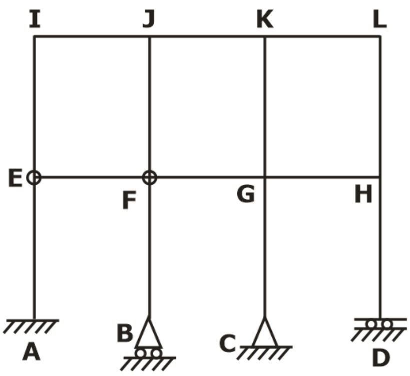
D_k at F = 2

So, degree of freedom = 0 + 1 + 1 + 2 + 2 + 2 = 8

Case 3: Frames

(i) Count only one rotation for all members meeting at a rigid joint.

(ii) Count rotation of all members meeting at a pin joint.



D_k at A = 0

D_k at B = 2

D_k at C = 1

D_k at D = 1

D_k at E = 5

D_k at F = 6

D_k at G = 3

D_k at H = 3

D_k at I = 3

D_k at J = 3

D_k at K = 3

D_k at L = 3

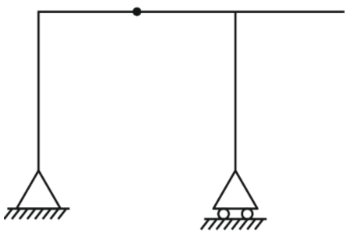
∴ D_k when extensible = 0 + 2 + 1 + 1 + 5 + 6 + 3 + 3 + 3 + 3 + 3 + 3 = 33 degree

D_k when inextensible

= D_k(extensible) – Number of independent displacements prevented.

= 33 – 14 = 29 degrees.

Example:



Sol.

Total degree of freedom D_k= 3×5-3 +4 (Due to internal Hinge) = 16

If members are considered inextensible then, D_k = 16-5=8

4. STABILITY OF STRUCTURE

The stability of structure includes external stability and internal stability. The external stability deals with support reaction and internal stability deals within the structure.

4.1. External Stability

Minimum number of reactions required for a structure to be stable externally is 3. These three reactions must be non-concurrent and non-parallel.

If three reactions are parallel then rigid body translation take place. If they are concurrent, then rigid body rotation takes place.

If the structure becomes unstable due to the improper arrangement of three reactions, then it is known as geometrically unstable structure.

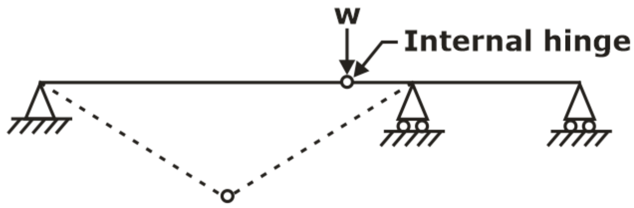
If structure becomes unstable due to less than 3 support reactions, then it is called statically unstable structure.

4.2. Internal Stability

Internal stability of various cases is explained through the following examples:

Case 1: Beams

→ Internal floating hinge



The above structure is internally unstable.

Case 2: Trusses

In case of trusses if following condition exist then it is classified as unstable truss.

$$\boxed{m < (2j - 3)}$$

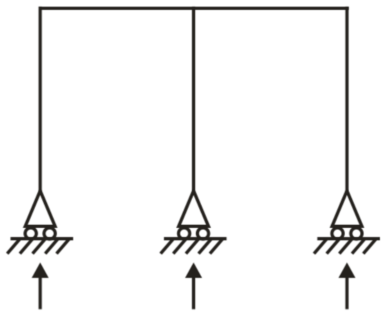
Where,

m = number of members in truss structure.

j = number of joints in truss structure.

Case 3: Frames

If reactions are parallel to each other, then the frame structure will be termed as unstable.



The above shown structure is unstable due to presence of reactions which are parallel.

CHAPTER 2: ANALYSIS OF A TRUSS

Definition: A truss is an assembly of beams or other elements that creates a rigid structure. In engineering, a truss is a structure that consists of two force members only, where the members are organized so that the whole assembly should behave as a single object.

1. ASSUMPTIONS USED IN TRUSS ANALYSIS

(i) Members of the truss will be subjected to axial force only.

(ii) Members are initially straight and load is acting only on joints

(iii) All joints can be assumed as frictionless hinges.

(iv) All the members of truss are assumed in the same plane called the middle plane of truss.

2. ANALYSIS OF STATICALLY DETERMINATE AND STABLE TRUSSES

There are two methods of analysis for statically determinate and stable trusses.

(i) Method of Section

(ii) Method of Joints

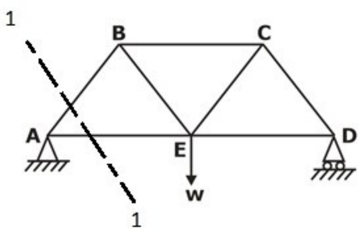
Methods of section

The advantage of this method is that, force in an intermediate member can be found directly without finding force in any other members. Equilibrium of sections of truss is considered in method of section. The procedure of this method comprises of following steps.

(i) Determine the value of support reaction.

(ii) Cut the member under consideration by a section (1)-(1) and consider equilibrium of either left hand side of (1)-(1) or R.H.S. of (1)-(1) and use $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$ to find unknown forces in members.

(iii) Cut the member such that entire truss is divided into two separate parts.



(iv) Preferably, don't cut more than 3 members (because, in method of section, we have 3 equilibrium equation which are $\Sigma F_x = 0$, $\Sigma F_y = 0$ $\Sigma M = 0$ with 3 equilibrium equation we can easily find 3 unknown values.

(v) Cut the member such that all the cut members do not meet at one joint. If they meet at one joint, $\Sigma M = 0$ becomes useless equation and it becomes method of joints problem.

Methods of Joints

The free body diagram of any joint is a concurrent force system in which the summation of moment will be no help. Recall that only two equilibrium equations can be written as $\Sigma F_x = 0$ and $\Sigma F_y = 0$. This means that to solve completely for the forces acting on a joint, we must select a joint with not more than two unknown forces involved. This can be started by selecting a joint acted on by only two members. We can assume any unknown member to be either tension or compression. If negative value is obtained, this means that the force is opposite in action to that of assumed direction. One the forces in on joint are determined their effect on adjacent joints are known. We then continue solving on successive joints until all members have been found.

3. ZERO FORCE MEMBERS

Zero force members in a truss are members which do not have any force in them. There are two rules that may be used to find zero force members in a truss. They are as follows.

Case 1: At a two-member joint which are not parallel and there are no other external loads or reaction at the joint then both members are zero force members.

Case 2: At a three-member joint, if two of those members are parallel and there are no other external loads (or reaction) at the joint then the member that is not parallel is a zero-force member.

CHAPTER 3: METHODS OF STRUCTURAL ANALYSIS

- **Force Method:** The force method of analysis, also known as the **method of consistent deformation**, uses equilibrium equations and compatibility conditions to determine the unknowns in statically indeterminate structures. This means that there is one reaction force that can be removed without jeopardizing the stability of the structure.
- **Types of Force Method:**
  Castigliano's Method or Strain Energy Method
  Virtual work method
  Column Analogy Method
  Flexibility Matrix Method
- **Displacement Method:** In the displacement method of analysis, the primary unknowns are the displacements. In this method, first **force -displacement relations are computed and subsequently equations** are written satisfying the equilibrium conditions of the structure.
- **Types of Displacement Method:**
  Slope Deflection Method.
  Moment Distribution Method.
  Direct Stiffness Method.
- **Approximate Methods:**
  Portal Method.
  Cantilever Method.
  Points of Inflection Method.
  Kani's Method.

CHAPTER 4: FORCE METHOD OF ANALYSIS

1. Energy methods:

Energy methods are based on linear elastic behaviour of material and conservation of energy i.e. work done by external forces is equal to the energy stored in the structure under the load. Strain Energy in various cases is given by following expressions.

In Axial tension or compression, $U = \frac{P^2L}{2AE}$

In Bending, $U = \frac{M^2L}{2EI}$

In Torsion, $U = \frac{T^2L}{2GJ}$

2. Castigliano's Method

As per Castigliano's theory

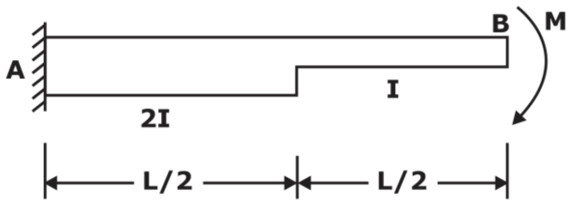
$$\Delta = \frac{\partial U}{\partial P}$$

And,

$$\theta = \frac{\partial U}{\partial M}$$

This relation can also be used in finding deflection in the beams as explained in the following example.

Example: Find rotation and deflection at free end in the beam shown in the figure below:



(a) Rotation at the free end:

Bending Moment at a distance x from the free end $M_X = -M$

So, the strain energy stored in the beam

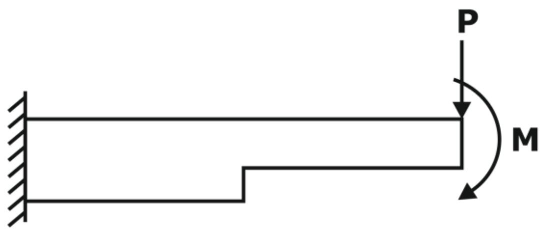
$$U = \int_0^{L/2} \frac{M^2dx}{2EI} + \int_{L/2}^{L} \frac{M^2dx}{4EI}$$

So, rotation at the free end,

$$\theta_B = \frac{\partial U}{\partial M} = \int_0^{L/2} \frac{Mdx}{EI} + \int_{L/2}^{L} \frac{Mdx}{2EI} = \frac{3ML}{4EI}$$

(b) Deflection at free end:

Applying vertical load P at the free end



Bending Moment at a distance x from free end $M_X = -M - Px$

So, Strain energy stored in the beam

$$U = \int_0^{L/2} \frac{(-M - Px)^2 dx}{2EI} + \int_{L/2}^{L} \frac{(-M - Px)^2 dx}{4EI}$$

So, Deflection at free end,

$$\Delta_B = \left.\frac{\partial U}{\partial P}\right|_{P=0} = \left. \int_0^{L/2} \frac{(M + Px)xdx}{EI} + \int_{L/2}^{L} \frac{(M + Px)xdx}{2EI} \right|_{P=0}$$

$$\Rightarrow \Delta_B = \int_0^{L/2} \frac{Mxdx}{EI} + \int_{L/2}^{L} \frac{Mxdx}{2EI} = \frac{5ML^2}{16EI}$$

3. Unit Load Method

Deflection at a point as per unit load method is given by

$$\Delta = \int \frac{M_x m_x dx}{EI}$$

Where,

M_x is the bending moment due to external loading.

m_x is the bending moment due to virtual unit load.

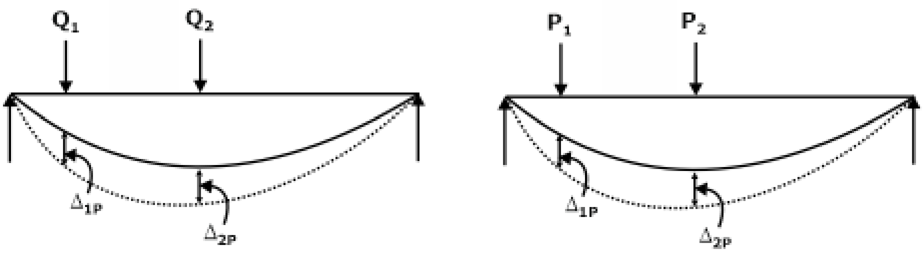
EI is the flexural rigidity of the beam.

The application of unit load method is explained using the example given below.

4. Maxwell Law of Reciprocal Theorem

This law states that in a linearly elastic structure, the deflection at any point A due to loading at some point B will be equal to deflection at B due to loading at A.

Betti's Theorem: This is a generalised case of Maxwell reciprocal theorem. As per this theorem the virtual work done by P system of forces in going through the deformation of Q system of forces is equal to virtual work done by Q system of forces in going through the deformation of P systems of forces.



Virtual work done by P system of forces due to the displacements caused by Q system of forces = $P_1\delta_{1Q} + P_2\delta_{2Q}$

Similarly,

Virtual work done by Q system of forces due to the displacements caused by P system of forces = $Q_1\Delta_{1P} + Q_2\Delta_{2P}$

As per Maxwell-Betti's Theorem

$$P_1\delta_{1Q} + P_2\delta_{2Q} = Q_1\Delta_{1P} + Q_2\Delta_{2P}$$

5. THEOREM OF LEAST WORK

This is a special case of Castigliano's theorem. This theorem states that for any statically indeterminant structure, the redundant should be such that strain energy of the system is minimum.

Thus,

$$\frac{\partial U}{\partial R} = 0$$

Where,

U = Strain energy stored in the system

R = Redundant force

6. DEFLECTION OF STATICALLY DETERMINATE TRUSSES

Two methods mainly used to calculate deflection in trusses are

(i) Castigliano's Method

(ii) Unit load method

6.1. Castigliano's Method

For getting the deflection in case of truss, there are two theorems. According to these theorem deflection and slope can be determined as follows.

(i) Castigliano's I^st theorem:

$$\boxed{W = \frac{\partial U}{\partial \delta}}$$

Here,

w = load

∂u = change in strain energy

∂δ = variation in deflection.

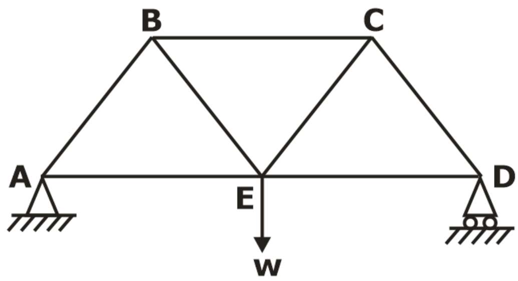
(ii) Castigliano's II^nd theorem

It states, that the first partial derivative of total strain energy with respect to a load at any point in the structure gives deflection at that point in the direction of load.

$$\boxed{\delta = \frac{\partial U}{\partial P}} \quad \boxed{\theta = \frac{\partial U}{\partial M}}$$

Application of Castigliano's theorem:

(i) To find absolute deflection of a joint in a truss.



U = Strain energy in all members

$$U = \frac{\Sigma P^2 l}{2AE}$$

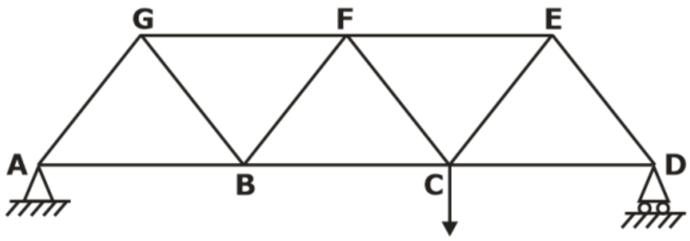
Where, P_1, P_2 … P_n = force in members due to applied load w.

and l_1, l_2 …. L_n = length of each member.

From Castigliano's II theorem

$$\Rightarrow \quad \delta_E = \frac{\partial U}{\partial W} = \Sigma \frac{2P \frac{\partial P}{\partial W} l}{2AE} = \Sigma \frac{Pkl}{AE}$$

Where, k = k_1, k_2 = $\frac{\partial P_1}{\partial w}, \frac{\partial P_2}{\partial w}$ = force in all members due to unit load applied at a point where we have to find deflection (δ).



If we want to find relative displacement of any two joints B and E, apply unit loads at B and E in the direction BE. Find forces in all members due to this load then relative displacement of two joints B and E is

$$\boxed{\delta_{BE} = \Sigma \frac{Pkl}{AE}}$$

Where,

P = P_1, P_2 etc forces in all member due to applied loads unit loads

K = forces in all members due to unit loads applied at B and E.

If we want to find rotation of any member FG, apply unit couple at G and F (these two forces form unit couple i.e., 1/a × a = 1). Find forces in all members due to these two loads, then rotation of member is given as

$$\boxed{\theta_{GF} = \Sigma \frac{Pkl}{AE}}$$

Where, P_1, P_2 … P_n = force in all members.

k = forces in all member due to unit couple applied at G and F.

6.2. Unit Load Method

This method is based on method of virtual work. From virtual work principle external work done on a body is equal to internal work done by the body.

If a unit virtual load produces internal stresses u_i in the member and the real displacement of the i^{th} member is dl_i then the internal virtual work done is equal to $\Sigma u_i dl_i$.

If the virtual load at any point is 1and the displacement at that point due to external forces is Δ then,

$$1 \times \Delta = \Sigma u_i dl_i$$

(i) Due to external loading:

Deflection of truss due to external loading is given by

$$\Delta = \sum \frac{PuL}{AE}$$

Where,

P = Force in the member due to external loading

u = Force in the member due to unit load applied in the direction at the point where deflection is to be calculated after removal of external loading

L = Length of the member

AE = Axial rigidity of the member

(ii) Temperature Change Case:

Due to temperature change,

$$dl_i = l_i \alpha_i \Delta T_i$$

So,

$$\Delta = \Sigma u_i \, l_i \alpha_i \Delta T_i$$

Where,

Δ = Deflection of truss due to temperature change

u_i = Forces in the member due to unit load at the point where deflection is to be computed.

l_i = length of the member

α_i = Coefficient of linear expansion

T_i = Temperature increment

(iii) Fabrication Error (Lack of Fit Case): If the member is shorter or longer in length, it will induce stresses in truss. In this case, the joint deflection is calculated as

$$\Delta = \Sigma u_i \, dL_i$$

Where,

Δ = Deflection of truss due to fabrication error

u_i = Forces in the member due to unit load at the point where deflection is to be computed.

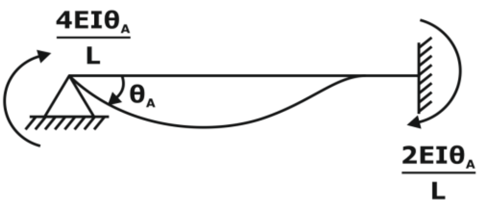
dl_i = Fabrication error

CHAPTER 5: DISPLACEMENT METHOD OF ANALYSIS (SLOPE DEFLECTION METHOD)

Definition: Slope deflection method is a classical method which is very useful in analysis of indeterminate structure like continuous beams and plane frames. In this method the unknown loads are written in terms of the displacement by using the load-displacement relations, then these equations are solved for the displacements using the joint equilibrium conditions. After computation of displacements, loads are computed using load displacement relations.

Important Results:

(i) Angular Displacement at A:

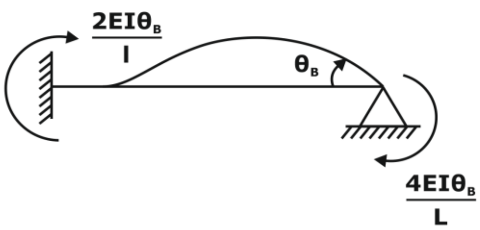


The load displacement relations are

$$M_{AB} = \frac{4EI}{L}\theta_A$$

$$M_{BA} = \frac{2EI}{L}\theta_A$$

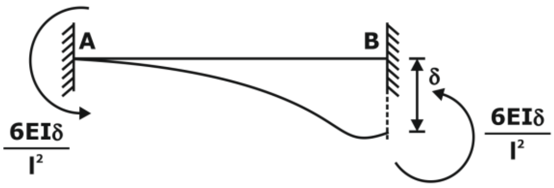
(ii) Angular Displacement at B:



$$M_{AB} = \frac{2EI}{L}\theta_B$$

$$M_{BA} = \frac{4EI}{L}\theta_B$$

(iii) Relative linear displacement, δ:



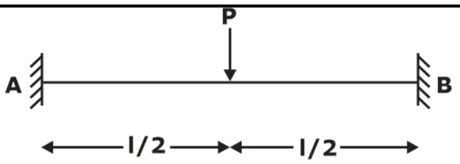
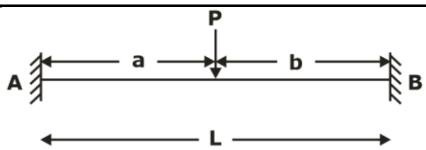
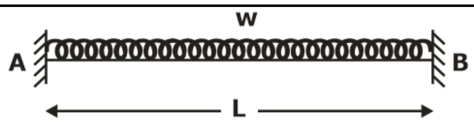
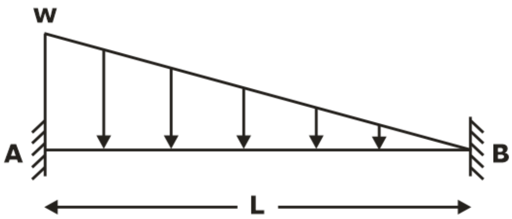
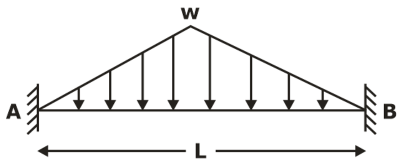
$$M_{AB} = M_{BA} = -\frac{6EI\delta}{L^2}$$

Sign Convention:

(i) Clockwise moment is taken as positive.

(ii) If δ gives clockwise rotation to member, it is considered as positive.

FIXED END MOMENTS FOR SOME STANDARD CASES

Beam	Fixed End Moments
	$M_A = M_B = \frac{Pl}{8}$
	$M_A = \frac{Pb^2a}{L}$ $\quad M_B = \frac{Pa^2b}{L}$
	$M_A = \frac{wL^2}{12}$ $\quad M_B = \frac{wL^2}{12}$
	$M_A = \frac{wL^2}{20}$ $\quad M_B = \frac{wL^2}{30}$
	$M_A = \frac{6EI\Delta}{L^2}$ $\quad M_B = \frac{6EI\Delta}{L^2}$
	$M_A = \frac{5wL^2}{96}$ $\quad M_B = \frac{5wL^2}{96}$
	$M_A = \frac{3EI\Delta}{L^2}$, $M_B = 0$

Slope Deflection Equations:

$$M_{AB} = M_{FAB} + \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B - \frac{6EI\delta}{L^2}$$

$$\Rightarrow M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

And,

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\theta_A + \frac{4EI}{L}\theta_B - \frac{6EI\delta}{L^2}$$

$$\Rightarrow M_{BA} = M_{FBA} + \frac{2EI}{L}\left(\theta_A + 2\theta_B - \frac{3\delta}{L}\right)$$

CHAPTER 6: DISPLACEMENT METHOD OF ANALYSIS (MOMENT DISTRIBUTION METHOD)

1. INTRODUCTION

It is a displacement method for analysis of statically indeterminate beams and frames developed by Hardy Cross. The method only accounts for flexural effects and ignores axial and shear effects. In this method it is assumed in the beginning that all joints of the structure are fixed. Then by locking and unlocking each joint in succession, the internal moments are distributed such that each joint attains its final position.

2. IMPORTANT DEFINITIONS

2.1. Stiffness Factor

Stiffness factor can be defined as the moment required to produce unit rotation in the beam. Stiffness factor for various cases is defined as follows.

Case 1: Far end is fixed

$$\text{Stiffness factor} = s = \frac{4EI}{l}$$

Case 2: Far end is hinged

$$Stiffness\ factor = s = \frac{3EI}{l}$$

2.2. Relative stiffness (k)

Relative stiffness is the relative value of the stiffness factor. It is value for various cases can be expressed as follows.

Case 1: Far end is fixed

$$k = \frac{I}{L}$$

Case 2: Far end is hinged

$$k = \frac{3}{4}\frac{I}{L}$$

Case 3: Far end is free

$$K = 0$$

2.3. Distribution Factor

It is the ratio in which the applied moment is distributed to various members meeting at a rigid point. Sum of distribution factor of all members meeting at a rigid joint is one. If far end is free, its D, k and distribution factor is zero.

$$DF = \frac{K}{\Sigma K}$$

Where,

K = Relative stiffness of the member

∑K = Summation of relative stiffness of all members meeting at a joint

2.4. Carry over moment

It is the moment developed at one end due to applied moment at the other end. It is developed to make the slope zero. It is exerted by the fixed support on the beam. It is developed to make slope zero not to keep the structure in equilibrium. Various case for carry over moment are as follows.

Case 1: Far end is fixed

$$\text{COM} = \frac{M}{2}$$

Case 2: Far end is hinged

$$\text{COM} = 0$$

2.5. Carry Over Factor

Carry over factor can be defined as the ratio of carry over moment and applied moment. Carry over factor for various cases can be given as follows.

Case 1: Far end is fixed

$$\text{COF} = \frac{\frac{M}{2}}{M} = \frac{1}{2}$$

Case 2: Far end is hinged

$$COF = \frac{0}{M} = 0$$

3. ANALYSIS OF BEAMS USING MOMENT DISTRIBUTION METHOD

3.1. Sign convention

(i) Clockwise end moments and clockwise rotations are taken as positive. Anti-clockwise end moments and anti-clockwise rotations are taken as negative.

(ii) Bending moment which is sagging in nature are taken as positive and hogging bending moment is taken as negative.

3.2. Procedure

Step 1: Find out Distribution factors and fixed end moments.

Step 2: Assume all joints to be initially locked. Then Determine the moment needed to bring each joint in equilibrium. Release the joints and distribute the counterbalancing moment into the connecting span at each joint. Carry these moments in each span over to its other end.

Repeat the same cycle until the moment equilibrium at the joint achieved.

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CHAPTER 7: ARCHES & CABLES

1. TYPES OF ARCHES

There are three types of arches depending upon the number of hinges provided.

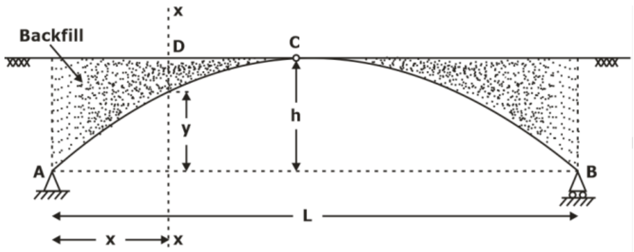
(i) Three hinged arch (Determinate)

(ii) Two hinged arch (Indeterminate to 1 degree)

(iii) Fixed arch (Indeterminate to 3 degree)

1.1. Three hinged Arch

The three hinged arches are statically determinate structure as equations of equilibrium alone are sufficient to find all the unknown quantities.



Circular Arch:

From the properly of a circle the radius r of the circular arch of span L and rise h may be found as

$$\frac{L}{2} \times \frac{L}{2} = h(2R - h)$$

$$\Rightarrow \boxed{R = \frac{L^2}{8h} + \frac{h}{2}}$$

Taking origin at A, the coordinates of any pont d on the arch may be defined as

$$x = \left[\frac{L}{2} - R\sin\theta\right]$$

$$y = R\cos\theta - (R - h)$$

$$\Rightarrow \boxed{y = h - R(1 - \cos\theta)}$$

Parabolic Arch:

Taking spring point as the origin, its equation is given by

$$\boxed{y = \frac{4h_x}{L^2}(L - x)}$$

Bending moment at the section X-X

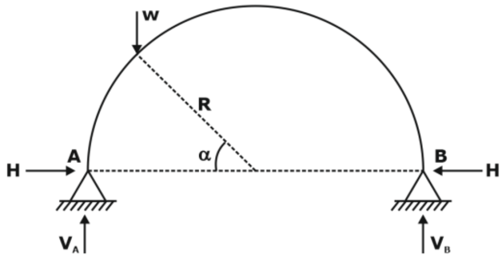
$$BM_{X-X} = +V_A \times x - H_A \times y$$

$$\Rightarrow BM_{X-X} = \text{Beam moment} - \text{H-moment}$$

When compared with a beam of similar span, bending moment at any section in a three hinged arch is less by an amount of 'H×y' or moment dur to horizontal force.

2. Two hinged arches

A two hinged arch is an indeterminate arch. The horizontal thrust is determined using Castigliano's theorem of least energy.



Two hinged circular arch

Assuming the redundant to be H, As per Castigliano's theorem

$$\frac{\partial U}{\partial H} = 0$$

Which gives the following condition

$$H = \frac{\int_0^l \frac{M_x y dx}{EI_C}}{\int_0^l \frac{y^2 dx}{EI_C}}$$

Where,

M_x = beam moment at any section x – x

I_C = Moment of inertia of the cross section of the arch at the crown.

(i) Horizontal Thrust in case of circular arch subjected to point load

$$H = \frac{W}{\pi} \sin^2 \alpha$$

(ii) Horizontal Thrust in case of circular arch subjected to UDL

$$H = \frac{4}{3} \frac{wR}{\pi}$$

(iii) Horizontal Thrust in case of parabolic arch subjected to a point load at centre

$$H = \frac{25}{128} \frac{wL}{H}$$

(iv) Horizontal Thrust in case of parabolic arch subjected to a UDL

$$H = \frac{wl^2}{8h}$$

If there is rib shortening, temperature rise by t°C and yielding of supports then horizontal thrust is given by

$$H = \frac{\int \frac{M_x y dx}{EI_C} + \alpha tl}{\int \frac{y^2 dx}{EI_C} + \frac{l}{AE} + k}$$

Where,

αtl = due to increase in temperature l/AE= due to rib shortening

K = yielding of support/unit horizontal thrust.

In a two hinged parabolic arch as the temperature increase, horizontal thrust increases. If the effect of rib shortening and yielding of support are considered, then horizontal thrust decreases.

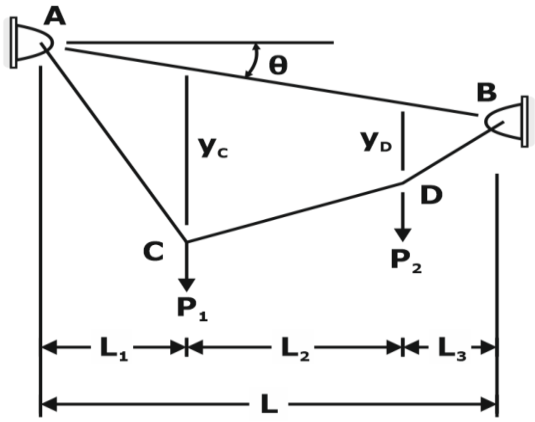
3. CABLES:

Suspension Cables are generally used to support suspension bridges, roofs and cable car system. They are used to transmit load from one structure to another. Cables are deformable structure so they can undergo change in shape according to externally applied load. Thus, bending moment and shear force at every point in the cable is zero.

3.1. CABLES SUBJECTED TO CONCENTRATED LOADS:

When cables are subjected to concentrated loads, it may take shape of several straight-line members subjected to tension. The shape cable takes is known as funicular polygon. The cable will always be subjected to pure tensile forces having the funicular shape of load.

Consider the cable subjected to concentrated loads P_1 and P_2 as shown in the figure. Due to the loading, cable assumes the shape ACDB.



The tension at each member can be easily determined using the equation of equilibrium at each joint and the geometry of the structure.

3.2 CABLES SUBJECTED TO UNIFORMLY DISTRIBUTED LOAD:

The tension in the cable at any distance x will be

$$T = \sqrt{(F_H)^2 + (w_0 x)^2}$$

$$\Rightarrow T = \sqrt{\left(\frac{w_0 L^2}{2h}\right)^2 + (w_0 x)^2}$$

Tension will be maximum when x is maximum i.e., x = L.

$$T_{max} = w_0 L\sqrt{1 + \left(\frac{L}{2h}\right)^2}$$

CHAPTER 8: INFLUENCE LINE DIAGRAM

1. MULLER BRESLAU PRINCIPLE:

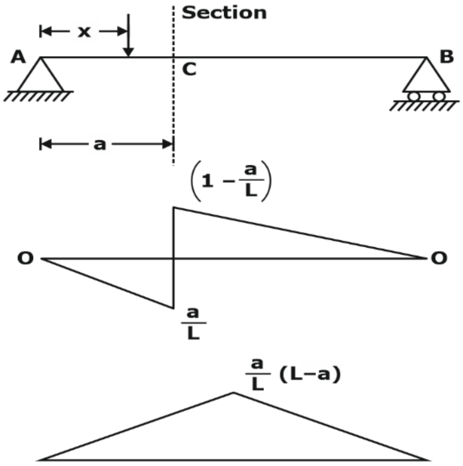
As per this principle, "If an internal stress component or reaction component is considered to act through and tends to deflect a structure than the deflected shape of the structure will be the influence line for the stress or reaction component to some scale."

Note: This Principle gives for quantitative and qualitative deflected shape for determinate structure and qualitative deflected shape for indeterminate structures.

2. MAXIMUM SHEAR FORCE AND BENDING MOMENT FOR A BEAM SUBJECTED TO MOVING LOADS

2.1. Due to Single Point Load

The influence line diagram for a single point load for Shear force and Bending moment is



So, bending moment will be maximum if the load is at the section. For maximum negative shear force the load should be just to the left of section and for maximum positive shear force the load should be just right to the section.

Absolute Maximum shear force and Bending Moment:

Absolute maximum shear force:

For absolute maximum negative shear force, x/L should be maximum. Thus, for absolute maximum negative shear force the value of x would be L i.e. at support B. For absolute maximum positive shear force, $\left(1-\frac{x}{L}\right)$ should be maximum. Thus, for absolute maximum positive shear force the value of x would be zero i.e. at support A.

For absolute maximum bending moment:

$$M_x = \frac{Wx(L-x)}{L}$$

$$\frac{dM_X}{dx} = \frac{W(L-2x)}{L} = 0$$

Thus, absolute maximum bending moment will occur at mid span in case of point load.

2.2. Due to Uniformly Distributed Load Longer than span

Maximum negative shear force occurs when the load covers portion AC only and maximum positive shear force occurs when the load covers the portion CB only. Maximum bending moment at any section will be due to UDL covering the entire span.

Absolute Maximum Value of SF and BM anywhere in the span:

For Absolute maximum Shear Force:

On observing the influence line diagram for Shear force, it is clear that maximum negative shear force will occurs at support B, when the UDL covers the entire span and maximum positive shear force will occurs at support A, when the UDL covers the entire span.

For Absolute maximum bending moment:

Maximum bending moment at any section,

$$M_x = \frac{1}{2} \times w \times \frac{x(L-x)}{L} \times L$$

$$\frac{dM_x}{dx} = \frac{w(L-2x)}{2} = 0$$

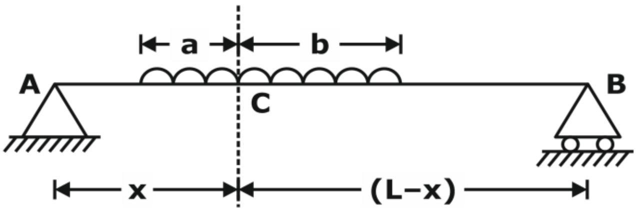
$$\Rightarrow x = \frac{L}{2}$$

Thus, absolute maximum bending moment will occur at mid span in case of UDL larger than the span.

2.3. UDL shorter than the span

For UDL shorter than the span, maximum negative shear force will take place when entire UDL is just left of the section and maximum positive shear force will take place when entire UDL is just right to the section.

For maximum bending moment at C



Load should be placed such that

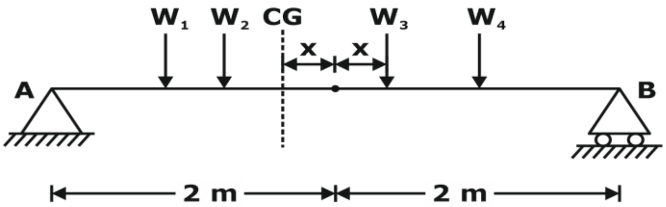
$$\frac{a}{b} = \frac{x}{L-x}$$

2.4. Due to Train of Concentrated Loads

Maximum Bending Moment at a section: Due to train of concentrated loads maximum bending moment will occur at the section if the loads are placed such that the average loading to the left of the section is equal to average loading to the right of the section.

Maximum Bending Moment under a wheel load: Maximum bending moment under a wheel load occurs if the loads are placed such that the load and the resultant of the loading is equidistant from the centre of span.

Maximum bending moment will occur under W_3 if the loads are placed as shown below.



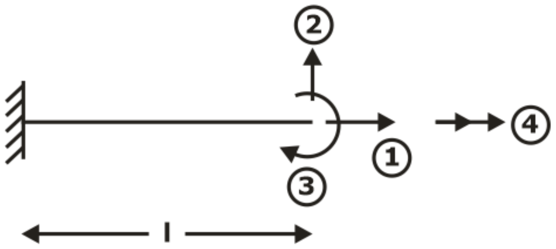
CHAPTER 9: MATRIX METHOD OF ANALYSIS

1. DISPLACEMENT METHOD/STIFFNESS METHOD:

In this method displacements at the joints are taken as unknowns and equation are expressed in terms of these unknown displacement. Additional joint equilibrium equations are developed to find the unknown displacement. This method is suitable when the Kinematic indeterminacy is less than the static indeterminacy.

1.1. Stiffness (k)

It is the load required to produce unit displacement. Stiffness for various cases are as follows.



(1) Axial stiffness (k_{11}) = $\frac{AE}{l}$

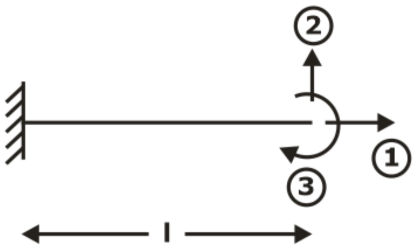
(2) Transverse stiffness (k_{22}) = $\frac{12EI}{l^3}$

(3) Flexural stiffness (k_{33}) = $\frac{4EI}{l}$

(4) Torsional stiffness (k_{44}) = $\frac{GJ}{l}$

1.2. Procedure to Construct Stiffness Matrix

To get first column of stiffness matrix, fix all the coordinates and give unit displacement at the 1st coordinate and find forces developed at all other coordinates similarly to get the second column of stiffness matric apply unit displacement at coordinate 2 and find forces at all coordinates.



The cantilever beam shown in the figure above will be subjected to three displacements (1), (2) and (3).

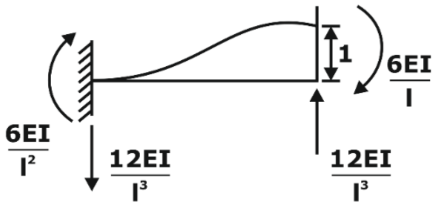
When the unit displacement is given in direction of (1) i.e., horizontal deflection only,

K_{11} = Force at (1) due to unit displacement at (1) = $\frac{AE}{L}$

K_{21} = Force at (2) due to unit displacement at (1) = 0

K_{31} = Force at (3) due to unit displacement at (1) = 0

When the unit displacement is given in direction of (2) i.e., vertical deflection only,

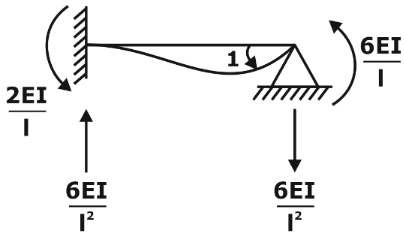


K_{12} = Force at (1) due to unit displacement at (2) = 0

K_{22} = Force at (2) due to unit displacement at (2) = $\frac{2EI}{l^3}$

K_{32} = Force at (3) due to unit displacement at (2) = $-\frac{6EI}{l^2}$

When the unit displacement is given in direction of (3) i.e., rotation only,



K_{13} = Force at (1) due to unit displacement at (3) = 0

K_{23} = Force at (2) due to unit displacement at (3) = $-\frac{6EI}{l^2}$

K_{33} = Force at (3) due to unit displacement at (3) = $\frac{4EI}{l}$

So, the stiffness matrix is

$$K = \begin{bmatrix} \frac{AE}{L} & 0 & 0 \\ 0 & \frac{2EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & -\frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix}$$

2. FLEXIBILITY MATRIX METHOD:

In this method, forces are taken as unknown and equations are expressed in terms of these forces. Additional equation called compatibility condition are developed to find all the unknown forces. This method is suitable when the static indeterminacy is less than kinematic indeterminacy.

2.1. Flexibility (δ)

Flexibility is defined as the displacement produced due to unit force. It is the inverse of stiffness. Flexibility for various cases are as follows

(a) Axial flexibility $= \frac{1}{\frac{AE}{l}} = \frac{l}{AE}$

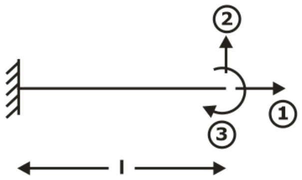
(b) Transverse flexibility $= \frac{1}{\frac{AE}{l^3}} = \frac{l^3}{12AE}$

(c) Flexural flexibility $= \frac{1}{\frac{4AE}{l}} = \frac{l}{4EI}$

(d) Torsional flexibility $= \frac{1}{\frac{GJ}{l}} = \frac{l}{GJ}$

2.2. Procedure to construct Flexibility Matrix

To get the first column of flexibility matrix, apply unit force at coordinate (1) and find displacement at all coordinates in the released structure. Similarly, to get II column of the flexibility matrix apply unit force at coordinate (2) and find displacement at all coordinates in the released structure.



The cantilever beam shown in the figure above is subjected unit forces in three directions.

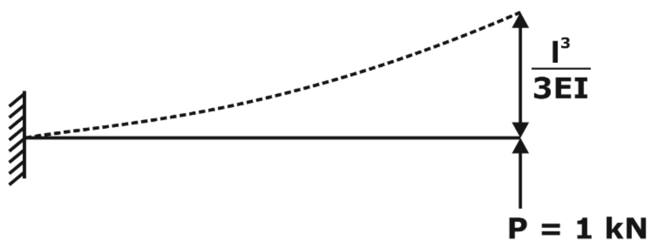
When the unit force is applied in direction of (1)

δ_{11} = displacement at coordinate (1) due to unit load at coordinate (1) $= \frac{l}{AE}$

δ_{21} = displacement at coordinate (2) due to unit load at coordinate (1) = 0

δ_{31} = displacement at coordinate (3) due to unit load at coordinate (1) = 0

When the unit load is applied in the direction of (2)

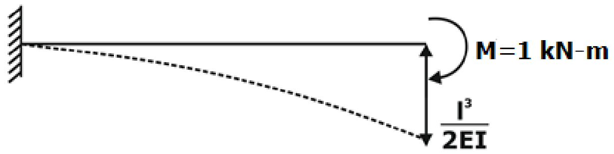


δ_{12} = displacement at coordinate (1) due to unit load at coordinate (2) = 0

δ_{22} = displacement at coordinate (2) due to unit load at coordinate (2) $= \frac{l^3}{3EI}$

δ_{32} = displacement at coordinate (3) due to unit load at coordinate (2) $= -\frac{l^2}{2EI}$

When the unit load is applied in the direction of (3)



δ_{13} = displacement at coordinate (1) due to unit load at coordinate (3) = 0

δ_{23} = displacement at coordinate (2) due to unit load at coordinate (3) $= -\frac{l^2}{2EI}$

δ_{33} = displacement at coordinate (3) due to unit load at coordinate (3) $= \frac{l}{EI}$

So, the flexibility matrix is

$$[\delta] = \begin{bmatrix} \frac{l}{AE} & 0 & 0 \\ 0 & \frac{l^3}{3EI} & -\frac{l^2}{2EI} \\ 0 & -\frac{l^2}{2EI} & \frac{l}{EI} \end{bmatrix}$$

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