

# Electrical Engineering

## ► Power System

SHORT NOTES



# POWER SYSTEM (FORMULA NOTES)

## 1. PER UNIT SYSTEM

$$\text{Quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

$$\text{Quantity in percent} = \frac{\text{actual quantity}}{\text{base value of quantity}} \times 100$$

### Advantages of pu system:

- Network analysis is made simple since all impedances of a given equivalent circuit can directly be added together regardless of the system voltages.
- It eliminates the  $\sqrt{3}$  multiplications and divisions that are required while dealing with balanced three-phase systems.
- Differences in operating characteristics of many electrical apparatus can easily be estimated by comparison of their per unit values.

(i) Base value of Impedance  $Z_B = \frac{(kV_B)^2}{(MVA)_B}$

Where,  $kV_B$  = voltage base in kilovolts,

$(MVA)_B$  = volt-ampere base in MVA

$$Z_{pu} = \frac{(Z_{\text{Actual value}})}{(kV_B)^2} MVA_B$$

Similarly,

$$I_{p.u.} = \frac{I_{\text{actual value}}}{I_{\text{Base}}} \quad \text{or} \quad V_{p.u.} = \frac{V_{\text{Actual value}}}{V_{\text{Base}}}$$

$$\text{Change of base: } Z_{pu(\text{new})} = Z_{pu(\text{old})} \left[ \frac{MVA_{B(\text{new})}}{MVA_{B(\text{old})}} \right] \left[ \frac{kV_{B(\text{old})}}{kV_{B(\text{new})}} \right]^2$$

## 2. POWER TRANSMISSION

Although copper has a much higher conductivity, aluminum conductors are normally used for overhead transmission lines. An aluminum conductor is lighter, cheaper, and has a larger diameter than a copper conductor of the same resistance.

### Conductor types:

S.No	Property	Solid conductor	Stranded Conductor (AAC)	Composite Stranded conductor (ACSR)
1	Skin effect	very high	Very less	Moderate
2	Mechanical strength	Very high	Very less	moderate

### Note Points:

- Bundled conductors is used to reduce corona loss
- Number of small conductors in composite structure  $N=3n^2+3n+1$ , where n is no. of layers
- Effective diameter of composite conductor  $D=(2n+1)d$ , d is diameter of small conductor.
- Skin effect depends on
  - Frequency
  - Permeability
  - Conductivity
  - Radius of the conductor
- Skin depth has inverse relation with skin effect

### Inductance of the Transmission line:

#### (i) Inductance of a Conductor due to Internal Flux:

A current carrying conductor whose cross section is of a long cylindrical structure with radius 'r' metres and carrying a current 'I' Amperes then

$$\psi_{int} = \int_0^r \frac{\mu I x^3}{2\pi r^4} = \frac{\mu I}{8\pi} \quad \text{WbT/m}$$

$$L_{int} = \int_0^r \frac{\mu I x^3}{2\pi r^4} = \frac{\mu}{8\pi} \quad \text{H/m}$$

If relative permeability  $\mu_r = 1$ , then permeability  $\mu$  which is equal to  $\mu_0\mu_r$  is given by

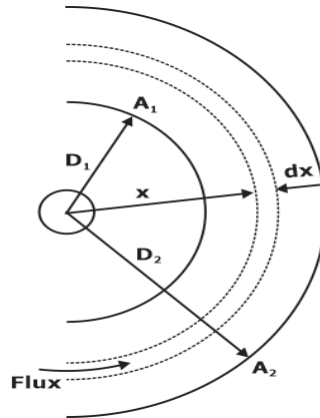
$$\mu = \mu_0\mu_r = 4\pi \times 10^{-7} \quad \text{H/m}$$

$$\text{Hence, } \psi_{int} = \frac{I}{2} \times 10^{-7} \quad \text{WbT/m}$$

$$L_{int} = \frac{1}{2} \times 10^{-7} \quad \text{H/m}$$

#### (ii) Inductance of a Conductor due to External Flux:

A same conductor with cross section of a conductor be as radius 'r' metres and carrying a current 'I' Amperes

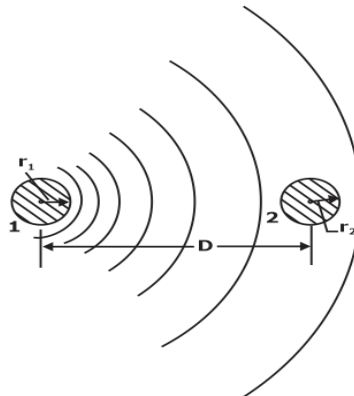


The inductance due to the external flux linkages included between A1 and A2 is

$$L_{12} = 2 \times 10^{-7} \ln \frac{D_2}{D_1} \text{ H/m}$$

### (iii) Inductance of a single-phase, two-wire system

Consider a single-phase circuit of two parallel conductors of radii  $r_1$  and  $r_2$  metres, separated by a distance 'D' metres, as shown in figure below, one conductor is the return circuit for the other.



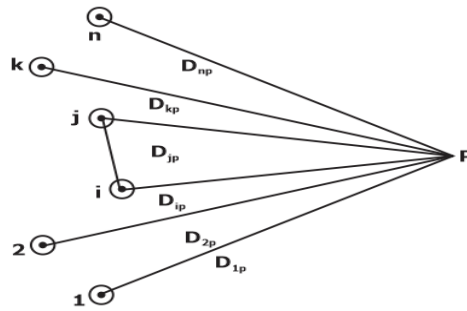
The total inductance of the circuit taking  $r_1' = r_2' = r'$  can be written as,

$$L = L_1 + L_2 = 4 \times 10^{-7} \ln \frac{D}{r'} \text{ H/m}$$

where  $r_1' = 0.7788r_1$  is the geometric mean radius (GMR) or the self-geometric mean distance (self-GMD) of a solid round conductor.

### (iv) Flux linkages of one conductor in a group of conductors

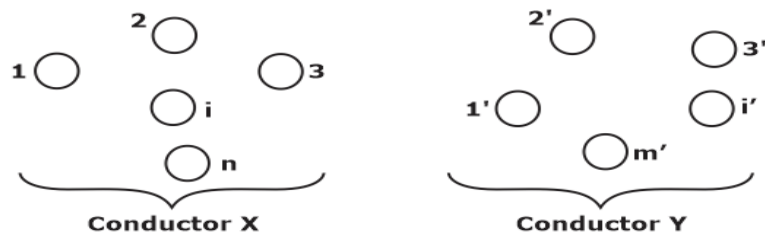
Consider a group of 'n' conductors, as shown in figure below, carrying phasor currents  $I_1, I_2, \dots, I_n$ , whose sum equals zero.



$$\psi_i = 2 \times 10^{-7} \left( I_1 \ln \frac{1}{D_{i1}} + I_2 \ln \frac{1}{D_{i2}} + \dots + I_i \ln \frac{1}{r'_i} + \dots + I_n \ln \frac{1}{D_{in}} \right)$$

#### (v) Inductance of composite conductor lines

Consider a single-phase line composed of conductor X, having 'n' identical parallel filaments, and conductor Y, which is the return circuit for the current in conductor X, having 'm' identical parallel filaments, as shown in figure below.



The inductance of conductor X, therefore, given as

$$L_x = 2 \times 10^{-7} \ln \frac{\left[ (D_{11'} D_{12'} D_{13'} \dots D_{1m'}) (D_{21'} D_{22'} D_{23'} \dots D_{2m'}) \dots \right]^{1/mn}}{\left[ (D_{i1} D_{i2} D_{i3} \dots D_{im'}) \dots (D_{n1} D_{n2} D_{n3} \dots D_{nm'}) \right]^{1/n^2}} \text{ H/m}$$

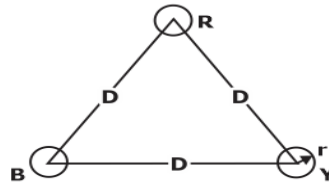
#### Bundled Conductors:

For EHV and UHV lines, bundled conductors having two or more conductors per phase are used. The use of bundled conductors reduces corona loss and decreases the reactance of the transmission line. The reduction of reactance results in increased GMR of the bundle.

#### Inductance of Three Phase lines:

##### (i) Symmetrical Spacing:

Consider an arrangement of equilaterally spaced conductors in a three-phase circuit which is shown in figure below,

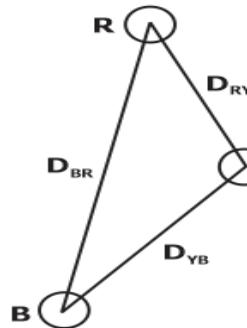


Each conductor has a radius 'r' metre and the spacing between the conductors is 'D' metres. Assume that the neutral wire is not present, hence,  $I_R + I_Y + I_B = 0$ .

$L_R = \frac{\Psi_R}{I_R} = 2 \times 10^{-7} \ln \frac{D}{r}$  H/m . The values of inductances for conductor Y & B are the same because of symmetry.

### (ii) Unsymmetrical Spacing:

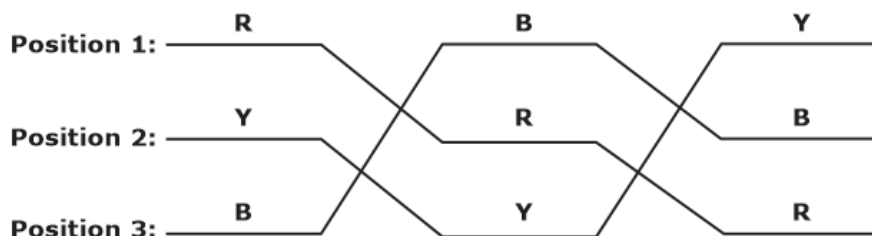
Consider an arrangement of conductors in a three-phase circuit (with unsymmetrical spacing) as shown in figure below,



Unsymmetrical spacing of the phase conductors of a three-phase line causes the flux linkages and inductance of each phase to be different and results in an unbalanced circuit.

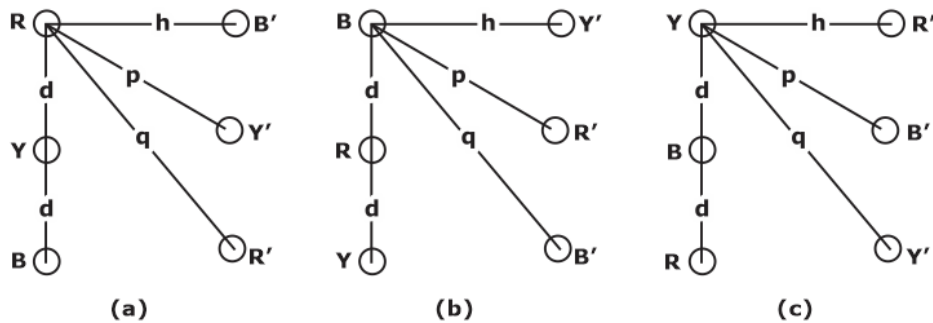
**Transposition:** The three phases can be balanced by exchanging the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of conductor positions is called transposition.

For a complete transmission cycle, the transposed lines are shown in figure below, where conductors are designated as R, Y, and B, and the positions occupied are numbered 1, 2, and 3, respectively.



### Inductance of double circuit lines:

Consider an arrangement of conductors in a double circuit three phase transposed line as shown in figure below, keeping each phase at a different place i.e. Phase R in position 1 (a), Phase R in position 2 (b), and Phase R in position 3 (c).



The conductors R, Y, B belong to one circuit while conductors R', Y', B' belong to another circuit. Inductance of each phase is given by

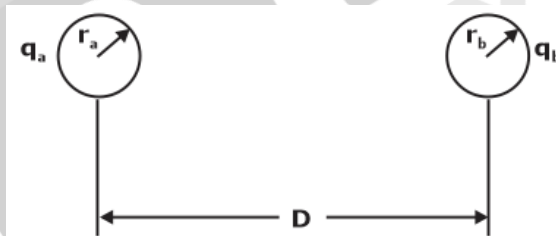
$$L = 2 \times 10^{-7} \ln \left[ 2^{1/6} \left( \frac{D}{r'} \right)^{1/6} \left( \frac{p}{q} \right)^{1/3} \right] \text{ H/m}$$

### Capacitance of the transmission line

#### (i) Capacitance of a Two-conductor Line:

Consider an arrangement of a two-conductor line connected to a single-phase ac supply as shown in figure below. The radii of the conductors are  $r_a$  and  $r_b$ , and the charges on the conductors are  $q_a$  and  $q_b$ , respectively. The distance between the two conductors is  $D$ .

**Assumption:** The ground is far away from the conductors and the charge distribution over the surface of the conductors is uniform.

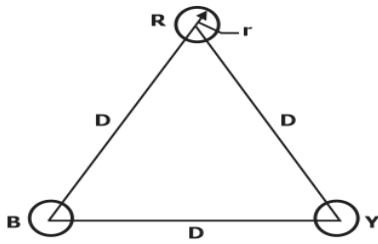


The capacitance  $C_{ab}$  between the conductors is given by

$$C_{ab} = \frac{q_a}{V_{ab}} = \frac{2\pi\epsilon}{\ln\left(\frac{D^2}{r_a r_b}\right)} = \frac{\pi\epsilon}{\ln\left(\frac{D}{\sqrt{r_a r_b}}\right)} \text{ F/m}$$

#### (ii) Capacitance of a Three-phase Line with Equilateral Spacing:

Consider a three-phase line with the conductors equilaterally spaced as shown in figure below. The three-phase conductors R, Y, and B have equal cross sections. Let the distance between the conductors be  $D$ , and the charges on the conductors be  $q_R$ ,  $q_Y$ , and  $q_B$  respectively.



The capacitance to neutral  $C_n$ , therefore, is

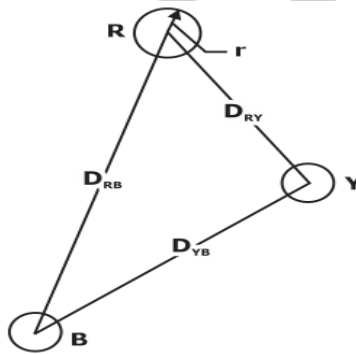
$$C_n = \frac{q_R}{V_{RN}} = \frac{2\pi\epsilon}{\ln \frac{D}{r}} \text{ F/m}$$

The charging current per phase is

$$I_{ch} = j\omega C_n V_{RN} = j\omega \times \frac{2\pi\epsilon}{\ln \frac{D}{r}} \times V_{RN} \text{ A/m}$$

### (iii) Capacitance of a Three-phase Line with Unsymmetrical:

Consider an arrangement of a three-phase, fully transposed line with three identical conductors of radius 'r' with unequal spacing as shown in figure below,



Hence, capacitance to neutral  $C_n$

$$C_n = \frac{q_R}{V_{RN}} = \frac{2\pi\epsilon}{\ln \frac{D_{eq}}{r}} \text{ F/m}$$

$$= \frac{0.02412}{\ln \frac{D_{eq}}{r}} \mu\text{F/km}$$

Also, charging current is given as

$$I_{ch} = j\omega C_n V_{RN} = j\omega \left( \frac{2\pi\epsilon}{\ln \frac{D_{eq}}{r}} \right) \times V_{RN} \text{ A/m}$$

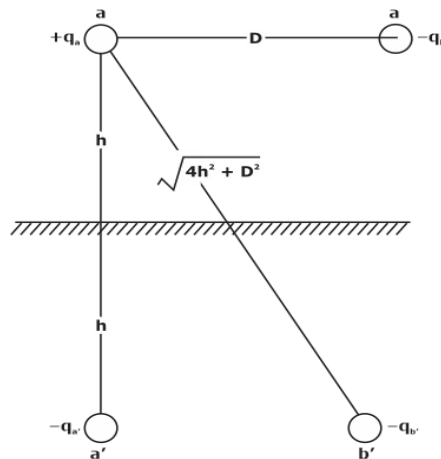
Where,

$$D_{eq} = \sqrt[3]{D_{RY} D_{YB} D_{BR}}$$

### (iv) Capacitance of a Single-phase Line with effect of earth:



Consider a single-phase charged line in the presence of the ground as shown in figure below.



Capacitance of the single-phase line, including the effect of earth, is given by

$$C_{ab} = \frac{\pi\epsilon}{\ln \frac{D}{r\sqrt{1 + \frac{D^2}{4h^2}}}} \text{ F/m}$$

Line-to-neutral capacitance of the line, is given as

$$C_n = \frac{2\pi\epsilon}{\ln \frac{D}{r\sqrt{1 + \frac{D^2}{4h^2}}}} \text{ F/m}$$

### 3. PERFORMANCE OF LINES

Performance of lines is meant for the determination of efficiency and regulation of lines.

(i) The efficiency of lines is defined as

$$\% \text{ efficiency} = \frac{\text{Power delivered at the receiving end}}{\text{Power sent from sending end}} \times 100$$

$$\% \text{ efficiency} = \frac{\text{Power delivered at the receiving end}}{\text{Power delivered at the receiving end} + \text{losses}} \times 100$$

(ii) Regulation of lines is defined as a ratio of difference between no load and full load voltage to full load voltage.

$$\text{Regulation up} = \frac{V_r' - V_r}{V_r}$$

$$\text{Regulation down} = \frac{V_r' - V_r}{V_r}, \text{ we use Regulation UP in Transmission lines}$$

Where  $V_r'$  is the receiving end voltage under no-load condition and  $V_r$  is the Receiving end voltage under full load condition.

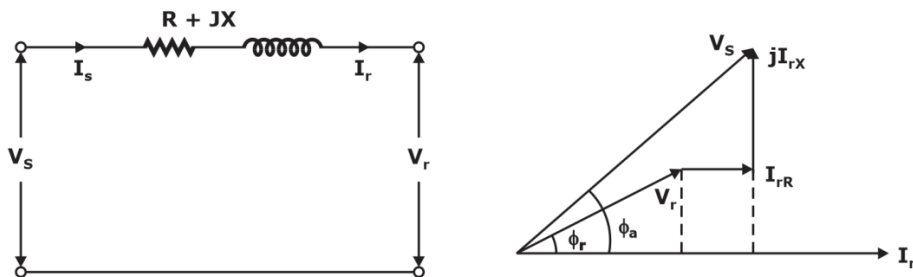
#### Parameters Table:

S. No.	Line Description	R	L	$X_L$	C	$X_C$
1.	Length Increases	Increases	Increases	Increases	Increases	Decreases
2.	Distance of separation	No change	Increases	Increases	Decreases	Increases
3.	Radius of conductor	Decreases	Decreases	Decreases	Increases	Decreases
4.	Symmetrical spacing	Does not depend	Decreases	Decreases	Increases	Decreases
5.	Unsymmetrical spacing	Does not depend	Increases	Increases	Decreases	Increases
6.	Effect of earth is taken into account	No change	No change	No change	Increases	Decreases
7.	Height of the conductor increases	No change	No change	No change	Decreases	Increases

#### Short Transmission Line:

- Based on frequency and length short line defined as:  $f < 4000\text{Hz}$  – Km
- Based on Voltage short line defined as:  $V = 0-20\text{kV}$

The equivalent circuit and vector diagram for a short transmission line are shown in fig.



By the phasor diagram

$$|V_s| \cos(\phi_a - \phi_r) = |V_r| + I_r Z \cos(\theta - \phi_r)$$

Where,

$\phi_a$  is sending end power factor

$\phi_r$  is receiving end power factor and  $\theta$  is impedance angle.

For short Transmission line,  $\phi_a - \phi_r$  is low so we can approximate equation like

$$|V_s| \approx |V_r| + I_r Z \cos(\theta - \phi_r)$$

The receiving end voltage under no load is the same as the sending end voltage under full load condition.

$$\% \text{ regulation} = \frac{V_s - V_r}{V_r} \times 100 = \left( \frac{I_r R}{V_r} \cos \phi_r + \frac{I_r X}{V_r} \sin \phi_r \right) \times 100$$

$$\text{Regulation per unit} = \frac{I_r R}{V_r} \cos \phi_r \pm \frac{I_r X}{V_r} \sin \phi_r = R \cos \phi_r \pm X \sin \phi_r$$

+ for lagging and - for leading

- Regulation is positive for lagging power factor and positive, negative or zero for leading power factors.
- Maximum regulation occurs at  $\theta = \phi_r$
- Zero regulation occurs when  $\theta + \phi_r = \frac{\pi}{2}$

ABCD Parameters equations:

$$V_s = AV_r + BI_r$$

$$I_s = CV_r + DI_r$$

- The constants for short transmission lines are,

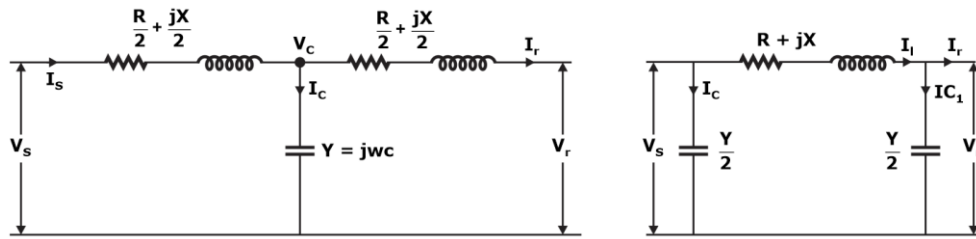
$$A = 1, B = Z, C = 0, D = 1$$

- $\% \text{ regulation} = \frac{\left| \frac{V_s}{A} \right| - V_r}{V_r} \times 100$

- $\% \eta = \frac{\text{Power received at the receiving end}}{\text{Power received per at the receiving end} + \text{losses}} \times 100$

### Medium Transmission Line:

- Based on frequency and length short line defined as:  $4000 < l f < 12000 \text{ Hz-Km}$
- Based on Voltage short line defined as:  $V = 20-100 \text{ kV}$
- The two configurations are known as nominal  $\pi$ -T and nominal  $\pi$ -T respectively.



A, B, C, D constant for nominal – T

$$A = 1 + \frac{YZ}{2}$$

$$B = Z \left( 1 + \frac{YZ}{4} \right)$$

$$C = Y$$

$$D = \left( 1 + \frac{YZ}{2} \right)$$

A, B, C, D constants for nominal –  $\pi$

$$A = 1 + \frac{YZ}{2}$$

$$B = Z$$

$$C = Y \left( 1 + \frac{YZ}{4} \right)$$

$$D = \left( 1 + \frac{YZ}{2} \right)$$

### Long Transmission Line:

- Based on frequency and length short line defined as:  $f > 12000\text{Hz} - \text{Km}$
- Based on Voltage short line defined as:  $V > 100\text{KV}$
- ABCD equations for long transmission line

$$V_s = V_r \cosh \gamma l + I_r Z_C \sinh \gamma l$$

$$I_s = V_r \frac{\sinh \gamma l}{Z_C} + I_r \cosh \gamma l$$

- The propagation constant  $\gamma = \alpha + i\beta$ ; the real part is known as attenuation constant and the quadrature component  $\beta$  the phase constant and is measured in radians per unit length.
- For loss less line  $\gamma = i\beta$
- $\gamma = \sqrt{ZY}$  and  $\beta = \omega\sqrt{LC}$
- Wavelength  $\lambda = \frac{2\pi}{\beta}$
- Since the equivalent circuit model of both the short- and medium length lines are passive, linear, bilateral, and time invariant,  $AD - BC = 1$
- Approximate ABCD of long Transmission Line:

$$A=D=\left(1+\frac{ZY}{2}\right)$$

$$B=Z\left(1+\frac{ZY}{6}\right)$$

$$C=Y\left(1+\frac{ZY}{6}\right)$$

- A, B, C, D constant for Equivalent – T

$$A=1+\frac{Y'Z'}{2}$$

$$B=Z'\left(1+\frac{Z'Y'}{4}\right)$$

$$C=Y'$$

$$D=\left(1+\frac{Y'Z'}{2}\right)$$

Where,  $Y' = \frac{1}{Z_c} \sinh \gamma l = \frac{i}{Z_c} \sin \beta l$  and  $\frac{Z'}{2} = Z_c \left( \frac{\cosh \gamma l - 1}{\sinh \gamma l} \right) = -iZ_c \left( \frac{\cos \beta l - 1}{\sin \beta l} \right)$

- A, B, C, D constants for Equivalent –  $\pi$

$$A=1+\frac{Y'Z'}{2}$$

$$B=Z'$$

$$C=Y'\left(1+\frac{Y'Z'}{4}\right)$$

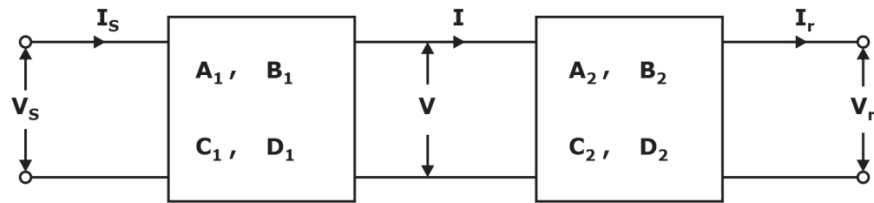
$$D=\left(1+\frac{Y'Z'}{2}\right)$$

Where,

$$\frac{Y'}{2} = \left( \frac{\cosh \gamma l - 1}{Z_c \sinh \gamma l} \right) = \frac{-i \cos \beta l - 1}{Z_c \sin \beta l}$$

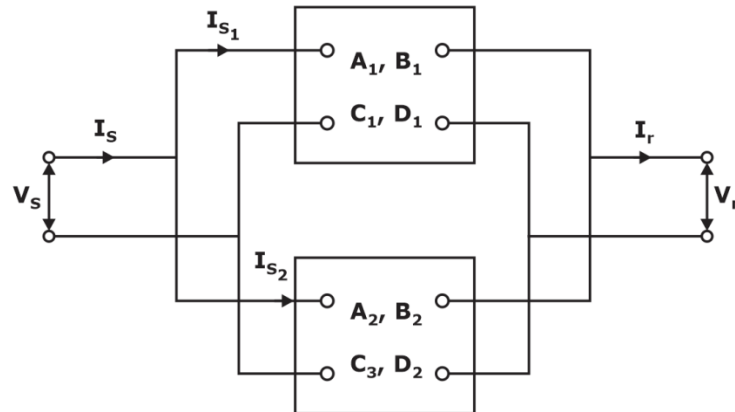
$$Z' = Z_c \sinh \gamma l = iZ_c \sin \beta l$$

### Constants for Two networks in Tandem



$$\text{Equivalent} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

### Constants for networks in parallel



$$\text{Equivalent Single Network Parameters} \left\{ \begin{array}{l} A = \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} \\ B = \frac{B_1 \cdot B_2}{B_1 + B_2} \\ A = D = \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} = \frac{D_1 B_2 + D_2 B_1}{B_1 + B_2} \\ C = C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} \end{array} \right.$$

### Surge impedance loading of lines:

- Surge impedance loading (SIL) is the power delivered at rated voltage by a lossless line to a load resistance whose value is equal to the surge impedance  $Z_c = \sqrt{Z_{oc} Z_{sc}} = \sqrt{\frac{L}{C}}$ .
- For a lossless line,  $Z_c$  is purely resistive.
- At the rated line voltage, the surge impedance loading of the line is given by

$$\text{SIL} = \frac{V_{R(\text{rated})}^2}{Z_c}$$

where rated voltage is used for a single-phase line and rated line-to-line voltage is used for a three-phase line.

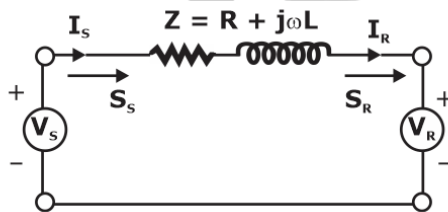
- The surge impedance loading of the line in itself is not a measure of the maximum power that can be delivered over the line.

- To utilize the transmission line effectively, loading is greater than SIL will be economical on overhead line but for stability reasons always loading is less than maximum power transfer capability ( $P_{\max}$ ).
- Loading on the underground cable done based on thermal stability. So loading is less than SIL is economical in Underground cable

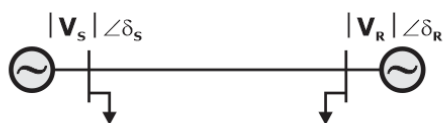
Loading = SIL	Loading > SIL	Loading < SIL
$Z_L = Z_C$	$Z_L < Z_C$	$Z_L > Z_C$
$ V_r  =  V_s $	$ V_r  <  V_s $	$ V_r  >  V_s $
Neither sink nor source of Q	Transmission line is sink of Q	Transmission line is source of Q

### Complex power transmission in short-length line

Figure shown below represents a per-phase circuit of a short transmission line.



(a) Per-phase circuit diagram for a short-length line



(b) Equivalent circuit for a short-length line

$$P_s = \frac{|V_s|^2}{|Z|} \cos \theta - \frac{|V_s||V_r|}{|Z|} \cos(\theta + \delta)$$

$$Q_s = \frac{|V_s|^2}{|Z|} \sin \theta - \frac{|V_s||V_r|}{|Z|} \sin(\theta + \delta)$$

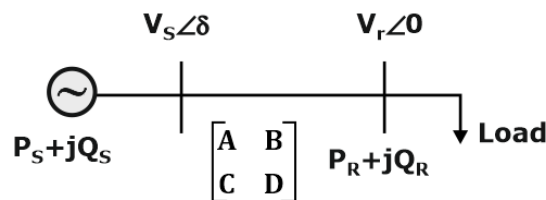
where  $P_s$  and  $Q_s$  are the active and reactive power flows, respectively, at the sending end and  $\theta$  is impedance angle

$$P_R = -\frac{|V_R|^2}{|Z|} \cos \theta + \frac{|V_R||V_S|}{|Z|} \cos(\theta - \delta)$$

$$Q_R = -\frac{|V_R|^2}{|Z|} \sin \theta + \frac{|V_R||V_S|}{|Z|} \sin(\theta - \delta)$$

$P_R$  and  $Q_R$  are the active and reactive power flows, respectively, at the receiving end

### Power Equations in ABCD Parameters



Real and Reactive powers at receiving end is  $P_R$  and  $Q_R$

$$P_R = \frac{|V_S||V_R|}{|B|} \cos(\beta - \delta) - \frac{|A|}{|B|} |V_R|^2 \cos(\beta - \alpha)$$

$$Q_R = \frac{|V_S||V_R|}{|B|} \sin(\beta - \delta) - \frac{|A|}{|B|} |V_R|^2 \sin(\beta - \alpha)$$

$$\left. \begin{aligned} A &= |A| \angle \alpha \\ B &= |B| \angle \beta \\ C &= |C| \angle \gamma \\ D &= |D| \angle \Delta \end{aligned} \right\} \text{(ABCD Parameters)}$$

### Condition for maximum power transfer

- Maximum at  $\beta = \delta$

$$P_{\max} = \frac{|V_S||V_R|}{|B|} - \frac{|A|}{|B|} |V_R|^2 \cos(\beta - \alpha)$$

$$Q_{R,\max} = -\frac{|A|}{|B|} |V_R|^2 \sin(\beta - \alpha)$$

- Always active power flows from leading voltage bus to lagging voltage bus provided that system is loss less
- Always reactive power flows from higher magnitude voltage terminal to lower magnitude voltage terminal provided that both voltages are taken on same reference.



#### 4. VOLTAGE CONTROL DEVICES AND APPLICATIONS

Static voltage control Device	Application
Shunt Capacitor	i) Power factor correction Device ii) Used at the places where under voltage occurs
Shunt Reactor	i) Practically Ferranti effect controller ii) Used to reduce voltage magnitude at the buses where overvoltage occurs under steady state operation iii) Improves power factor if load is leading type, but practically most loads are lagging type in system
Series Capacitance	i) Used to rise in voltage magnitude when load is lagging type and reduce in voltage magnitude when load is leading type ii) Steady state stability improved
Series Reactor	Practically used as fault current limiter as smoothing reactor in HVDC & PE circuits
<b>Dynamic voltage control Devices</b>	
Synchronous condenser	Over excited synchronous motor under no load condition. It is similar to shunt capacitor
Synchronous Coil	Under excited synchronous motor under no load condition. It is similar to Shunt Reactor
Synchronous Phase modifier	It is a synchronous motor that runs without mechanical load. It can be used to vary power factor by altering excitation

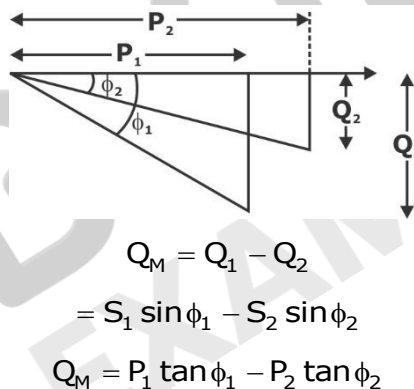
- Rating of Shunt capacitor bank to boost up voltage  $Q_{sh, cap} = \frac{(\Delta V_c) |V_s|}{X_{thvnnin}}$
- Rating of Shunt reactor bank to reduce the voltage  $Q_{sh, reactor} = \frac{(\Delta V_L) |V_s|}{X_{thvnnin}}$
- For same voltage magnitude rise rating of shunt capacitor is more than the series capacitor
- Degree of compensation: The characteristic impedance of uncompensated line is given by

$$Z_c = \sqrt{\frac{X_{line}}{B_{line}}}$$

If we add a series capacitive reactance with line and shunt inductive reactance at load then the characteristic impedance of compensated line is given by

$$Z_c = \sqrt{\frac{X_{line} - X_c}{B_{line} - B_L}}$$

- By synchronous phase modifier, the power factor got improved due to reactive power injected by the machine.



### 5. REFLECTION AND REFRACTION OF WAVES

- Forward current and voltage are in same phase and backward current and voltage are in opposite phase

Forward voltage =  $Z_c$  (Forward current)

Backward voltage =  $-Z_c$  (Backward current)

- Incident + Reflection = Refraction
- $Z_L$  and  $Z_C$  are load and characteristic impedance then

Coefficient of Voltage refraction is  $V_{refraction} = \frac{2Z_L}{Z_L + Z_C}$

Coefficient of Voltage reflection is  $V_{reflection} = \frac{Z_L - Z_C}{Z_L + Z_C}$

Coefficient of Current refraction  $I_{\text{refraction}} = \frac{2Z_C}{Z_L + Z_C}$

Coefficient of Current reflection  $I_{\text{reflection}} = -\left(\frac{Z_L - Z_C}{Z_L + Z_C}\right)$

- If the load is inductor, then

Refracted voltage equation is  $V(t) = 2Ve^{\left(\frac{-Z_C t}{L}\right)}$

$$i(t) = -\frac{2V}{Z_C} e^{\left(\frac{-Z_C t}{L}\right)}$$

Refracted current equation is

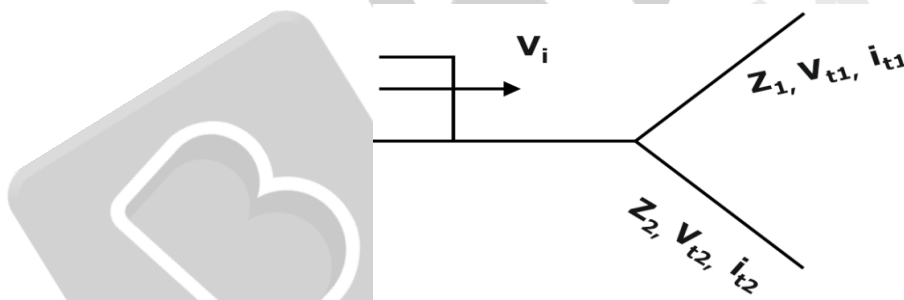
- If the load is capacitor, then

Refracted voltage equation is  $V(t) = 2V \left(1 - e^{\left(\frac{-t}{CZ_C}\right)}\right)$

$$i(t) = \frac{2V}{R} e^{\left(\frac{-t}{CZ_C}\right)}$$

Refracted current equation

#### Forked Line:



Line terminated with two different impedances ( $Z_1$  and  $Z_2$ ) which are in parallel. So combined  $Z_L$  is parallel combination of  $Z_1$  and  $Z_2$ .

In above formulas  $Z_L$  is replaced by new  $Z_L$  i.e.,  $Z_L = \frac{Z_1 Z_2}{Z_1 + Z_2}$

- Highest switching voltage experienced by the line for ac supply is  $2V_{\text{Phase}}$  (Peak value)
- In general switching overvoltage in the range of 2.5p.u -3.5p. u
- If surge propagates from Overhead (high impedance) line to cable (low impedance) then
  1. Surge voltage magnitude decreases
  2. Surge current Increases

3. Rate of rise of surge voltage (Steepness) decreases
  4. Steepness in surge current increases
- If surge propagates from cable (low impedance) line to overhead (high impedance) then
    1. Surge voltage magnitude increases
    2. Surge current decreases
    3. Rate of rise of surge voltage (Steepness) increases
    4. Steepness in surge current decreases
  - Velocity of wave propagation in Overhead line =  $3 \times 10^8$  m/sec
  - Velocity of wave propagation in cable

$$v = \frac{3 \times 10^8}{\sqrt{\epsilon_r}} \text{ m/sec as } \epsilon_r > 1 \text{ Velocity of wave in cable is less than overhead line}$$

Coefficient	Open circuit line ( $Z_L = \infty$ )	Short circuit line ( $Z_L = 0$ )	Flat line ( $Z_L = Z_C$ )
V refraction	2	0	1
V reflection	1	-1	0
I refraction	0	2	1
I reflection	-1	1	0

## 6. INSULATORS

**Pin type insulators:** Pin type insulators are used for transmission and distribution of electric power at voltages up to 33 kV.

**Suspension type insulators:** Generally, it is made up of porcelain material that includes single or a string of insulating discs hanged over a tower. It operates at above 33KV and overcomes the limitation of pin type insulator

**Strain insulators:** When there is a dead end of the line or there is sharp curve, the line is subjected to greater tension. Therefore, strain insulators are used

**Shackle insulators:** Shackle type insulators used for low voltage distribution lines and can be used either in a horizontal position or in a vertical position.

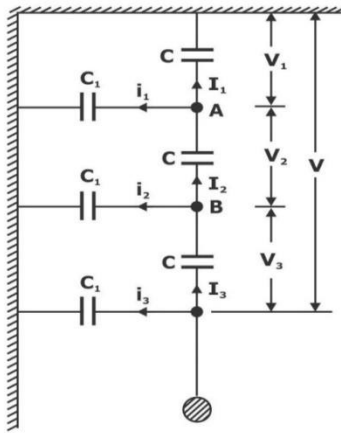
### String Efficiency:

The ratio of voltage across the whole string to the product of number of discs and the voltage across the disc nearest to the conductor is known as string efficiency,

$$\text{String efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to conductor}}$$

Where  $n$  = number of discs in the string.

An equivalent circuit for a 3-disc string, with self-capacitance of each disc is 'C' and shunt capacitance  $C_1$  is some fraction K of self-capacitance i.e.,  $C_1 = KC$ .



Voltage across top unit,

$$V_1 = \frac{V}{(1+K)(3+K)}$$

Voltage across second unit from top,  $V_2 = V_1 (1 + K)$

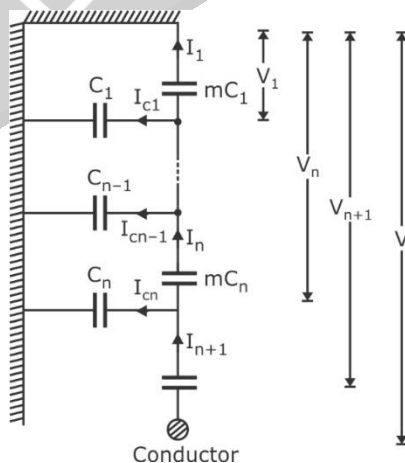
Voltage across third unit from top,  $V_3 = V_1 (1 + 3K + K^2)$

$$\% \text{ string efficiency} = \frac{\text{Voltage across string}}{n \times \text{Voltage across disc nearest to conductor}} \times 100 = \frac{V}{3 \times V_3} \times 100$$

### Methods of Improving String Efficiency:

The various methods for improving string efficiency are as follows:

1. By using longer cross-arms
2. By grading the insulators



If discs of different capacitances are used such that the product of their capacitive reactance and voltage across the disc is same so that the current flowing through the respective unit is same.

At junction n

$$I_{n+1} = I_n + I_{cn}$$

$$V\omega C_{n+1} = V\omega C_n + nV\omega C$$

$$C_{n+1} = C_n + nC$$

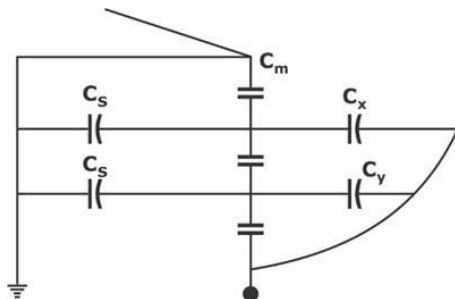
It shows as we move towards power conductor, capacitance increases.

To carry out this method, units of different capacities are required, which is uneconomical and impractical.

Hence, this method is used only for high voltage lines.

### 3. By using a guard ring or static shielding:

In this, the leakage current between the pin to the tower is cancelled out by the method of guard ring



In a n disc insulator,

$$\text{Capacitance of } p^{\text{th}} \text{ metal link to guard wire } C_p = \left( \frac{p}{n-p} \right) C_s \text{ or } C_p = \left( \frac{p}{n-p} \right) kC_m$$

Where  $C_s$  = shunt capacitance

### 7. CORONA

The power loss due to corona is given by:

$$P = 242.2 \left( \frac{f+25}{\delta} \right) \sqrt{\frac{r}{d}} (V - V_c)^2 \times 10^{-5} \text{ kW/km/phase}$$

Where  $f$  = supply frequency in Hz

$V$  = phase-neutral voltage (r.m.s.)

$V_c$  = critical disruptive voltage (r.m.s.) per phase

Methods of reducing corona loss:

- (i) By increasing conductor size
- (ii) By increasing conductor spacing

### Critical disruptive voltage:

It is the minimum voltage required between the conductors to start the ionization of air.

Conductors of radii ' $r$ ' cm and spaced ' $d$ ' cm apart. If ' $V$ ' is the phase-neutral potential, then potential gradient at the conductor surface is given by

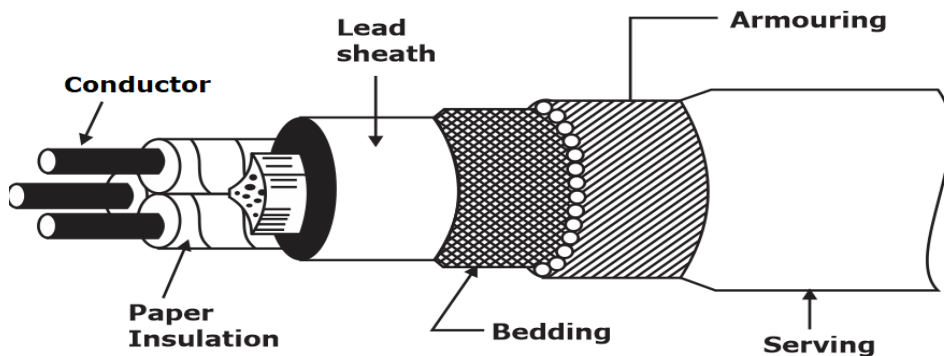
$$g = \frac{V}{r \log_e \frac{d}{r}} \text{ volts/cm}$$

In order that corona is formed, the value of 'g' must be made equal to the breakdown strength of air. The breakdown strength of air at 76 cm pressure and temperature of 25°C is 30 kV/cm (max) or 21.2 kV/cm (rms.)

## 8. UNDERGROUND CABLES

### Construction of Cables:

Figure below shows the general construction of a 3-conductor cable:



The various parts of the cable are as follows:

**Insulation:** Each core or conductor is provided with a suitable thickness of insulation, the thickness of layer depends upon the voltage which cable needs to withstand. The commonly used materials for insulation are impregnated paper, varnished cambric, or rubber mineral compound.

**Metallic sheath:** In order to protect the cable from moisture, gases, or any other damaging liquids (such as acids or alkalis) in the soil and atmosphere, a metallic sheath of lead or aluminium is provided over the insulation.

**Bedding:** It is a paper tape compound with fibrous material (like jute, cotton, hessian cloth etc). The purpose of bedding is to protect the metallic sheath against corrosion and mechanical injury.

**Armouring:** Its purpose is to protect the cable from mechanical injury while laying it and during the course of handling.

**Serving:** In order to protect armouring from atmospheric conditions, a layer of fibrous material (like jute, PVC) similar to bedding is provided over the armouring this is known as serving.

### Properties:

1. High insulation resistance to avoid leakage current.
2. High dielectric strength to avoid electrical breakdown of the cable.
3. High mechanical strength to withstand the mechanical handling of cables.
4. Non-hygroscopic i.e., it should not absorb moisture from air or soil.
5. Non-inflammable.

**Insulation resistance:** Insulation resistance of a cable is inversely proportional to its length. In other words, if the cable length increases, its insulation resistance decreases and vice-versa.

$$R_{ins} = \frac{\rho}{2\pi l} \ln\left(\frac{R}{r}\right) \text{ ohm}$$

R = internal radius of the sheath

r = radius of the conductor

- Most economical conductor size in a cable:  $r = \frac{R}{2.718}$

And the value of  $g_{max}$  under this condition is  $g_{max} = \frac{2V}{d}$  volts/m

Where R internal sheath radius and r is core radius

- Capacitance of 3-core cables:

Per phase capacitance is given by  $C_N = C_S + 3C_C$

$C_S$  is conductor-sheath capacitances and  $C_C$  is the capacitance between conductors

Charging current: Let  $V_P$  is the phase voltage then charging current  $I_{charging} = 2\pi f V_P C_N$

- Any two cores are connected to sheath and measure capacitance between third core and sheath gives capacitance =  $C_S + 2C_C$
- Bunch all core together and measure capacitance between any one core to sheath results capacitance =  $3C_S$
- Connect any one core to sheath and measure capacitance between remaining two core gives capacitance =  $\frac{C_S + 3C_C}{2} = \frac{C_N}{2}$
- In capacitance grading, different dielectrics are placed between core and sheath in such a way that dielectric with high permittivity is closer to core.

If  $\epsilon_1 > \epsilon_2 > \epsilon_3$ . Then  $\epsilon_1 d = \epsilon_2 d_1 = \epsilon_3 d_2$

Where d is diameter of core,  $d_1$  is diameter of first dielectric and so on.

Total voltage,

$$V = \frac{g_{max}}{2} \left[ d \log_e \frac{d_1}{d} + d_1 \log_e \frac{d_2}{d_1} + d_2 \log_e \frac{D}{d_2} \right]$$

- In interseath grading, homogenous dielectric is used and divided into various layer by placing metallic sheath between core and lead sheath.
- Due to the insulation resistance power loss occurs.

$$P_d = \omega C V^2 \delta, \text{ Where } \delta \text{ is the loss angle}$$

## 9. ECONOMIC LOAD DISPATCH

- The fuel cost of generator 'i' is represented as a quadratic function of real power generation,

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2$$



- The incremental fuel-cost  $\frac{dC_i}{dP_i} = 2\gamma_i P_i + \beta_i$ . Let  $\frac{dC_i}{dP_i} = \lambda$  and for economic load dispatch power plant should have equal incremental cost

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \frac{dC_i}{dP_i} = \lambda \quad i = 1, 2, \dots, n,$$

$C_i$  is the production cost of  $i^{\text{th}}$  plant,  $P_i$  is the generation of  $i^{\text{th}}$  plant

- Economic Dispatch including losses: The total transmission loss is a quadratic function of the generator power outputs, which is given as

$$P_L = \sum_{i=1}^n \sum_{j=1}^n P_i B_{ij} P_j$$

The coefficients  $B_{ij}$  are called loss coefficients or B-coefficients.

The economic dispatching problem is used to minimize the overall generating cost  $C_i$ , which is the function of plant output

Incremental cost of 'i' th unit

$$\frac{dC_i}{dP_i} = \lambda \left( 1 - \frac{\partial P_L}{\partial P_i} \right) \quad i = 1, 2, \dots, n$$

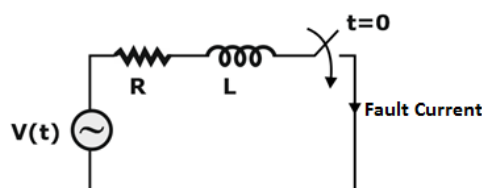
$$L_i \frac{dC_i}{dP_i} = \lambda \quad i = 1, 2, \dots, n$$

Where  $n$  is the total number of dispatchable generating plants.  $L_i$  is known as the penalty factor of plant 'i' and is given by

$$L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_i}}; \text{ Where } \frac{\partial P_L}{\partial P_i} = \text{Incremental transmission loss}$$

## 10. FAULT ANALYSIS

### Transient on Transmission line



$$V(t) = V_m \sin(\omega t + \alpha)$$

$$i(t) = i_{ss} + i_{transient} = \frac{V_m}{Z} \sin(\omega t + \alpha - \theta) + i(0)e^{-t/\tau}$$

- At  $t = 0$ ,  $i = 0$

$$i(t) = \frac{V_m}{Z} \sin(\omega t + \alpha - \theta) - \frac{V_m}{Z} \sin(\alpha - \theta) e^{-t/\tau}$$

- At  $t = t_0$ ,  $i = 0$ , then

$$i(t) = \frac{V_m}{Z} \sin(\omega t_0 + \alpha - \theta) - \frac{V_m}{Z} \underbrace{\sin(\omega t_0 + \alpha - \theta) e^{-(t-t_0)/\tau}}_{(\text{transient term})}$$

In general,  $\alpha = 0$  (Most of the numerical) if given we need to consider

For no-transients  $\Rightarrow \omega t_0 - \theta = 0$ ;  $t_0 = \theta / \omega$

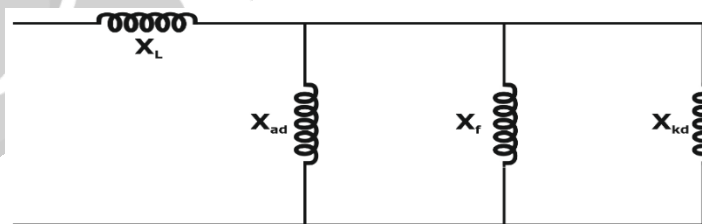
- Symmetrical RMS current (Steady state)  $I_{\text{symmetrical}} = \frac{V_m}{\sqrt{2} |Z|} = \frac{V_{\text{rms}}}{|Z|}$
- Asymmetrical RMS current ( $I_{\text{asy}}$ ):  $I_{\text{asymmetrical}} = \sqrt{\left(\frac{V_m}{\sqrt{2} Z}\right)^2 + \left(\frac{V_m}{Z} \sin(\theta - \alpha) e^{-t/\tau}\right)^2}$
- Maximum Momentary current ( $i_{\text{mm}}$ ): Maximum at  $\omega t + \alpha - \theta = \pi/2$  and generally  $\tau$  is high.

$$\text{so } e^{-t/\tau} \approx 1 ; \therefore i_{\text{mm}} = \frac{V_m}{Z} - \frac{V_m}{Z} \sin(\alpha - \theta)$$

$$\theta \approx 90^\circ, \text{ as } x_L \gg R ; \text{ equation becomes } i_{\text{mm}} = \frac{V_m}{Z} (1 + \cos \alpha)$$

$$\text{If } \alpha = 0^\circ \text{ in source voltage, } i_{\text{mm}} = \frac{2V_m}{Z}$$

- Equivalent circuit model of synchronous machine in sub-transient, transient and steady state period is analyzed by below figure: Where  $X_{ad}$ ,  $X_r$ ,  $X_{kd}$  are reactances due to armature reaction, field winding and damper winding respectively.  $X_L$  is the leakage reactance



Sub transient period:  $X_{ad}$ ,  $X_r$ ,  $X_{kd}$  three reactance's will be there along with  $X_L$

Transient period:  $X_{ad}$ ,  $X_r$  will be there along with  $X_L$

Steady state period: only  $X_{ad}$  will be there along with  $X_L$

- The sequence voltage can also be expressed in terms of phase voltages and vice versa as

given in matrix form.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \text{ and } \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

The above equations can be expressed also in terms of phase and sequence currents as

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad \text{And} \quad \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

- Sequence Impedances of Transmission Lines:  $Z_s$  is self-impedance and  $Z_m$  is mutual impedances of transmission line then

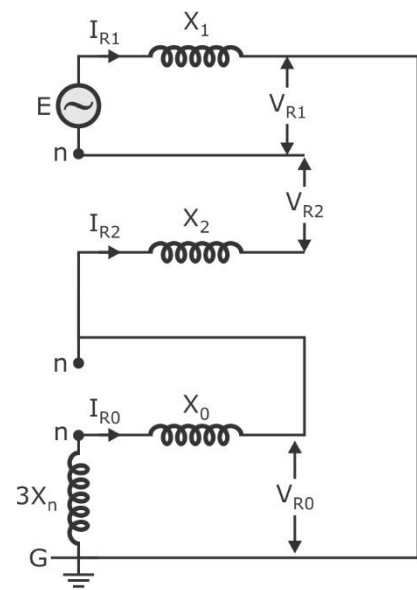
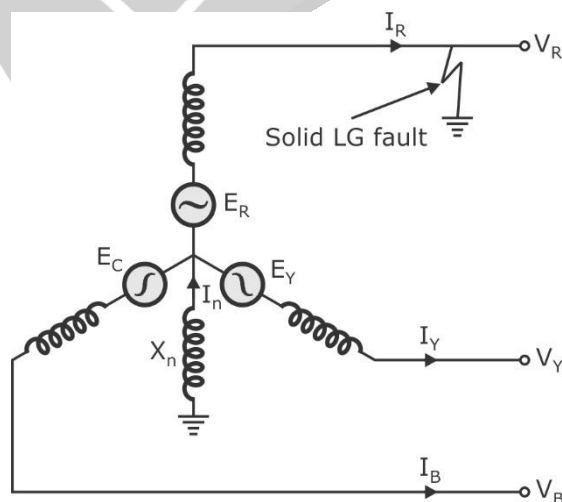
1. Zero-sequence impedance, i.e.,  $Z_0 = Z_s + 2Z_m$
2. Positive-sequence impedance, i.e.,  $Z_1 = Z_s - Z_m$
3. Negative-sequence impedance, i.e.,  $Z_2 = Z_s - Z_m$

- Symmetrical Fault or 3phase fault: A three phase bolted fault describes the condition where the three conductors are physically held together with zero impedance between them, just as if they were bolted together.

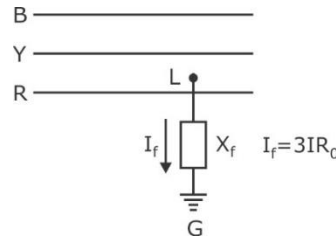
$$\text{Fault current } I_f = I_{a1} = \frac{E_a}{Z_{1eq} + Z_f}$$

### Unsymmetrical faults:

#### (i) LG Fault:



- $I_{R0} = I_{R1} = I_{R2} = \frac{E}{X_1 + X_2 + X_0 + 3X_n}$  and  $I_f = 3I_{R0} = 3I_{R1} = 3I_{R2}$
- **Special case:** If the LG fault takes place with fault impedance/reactance ( $X_f$ ) as shown below. Then

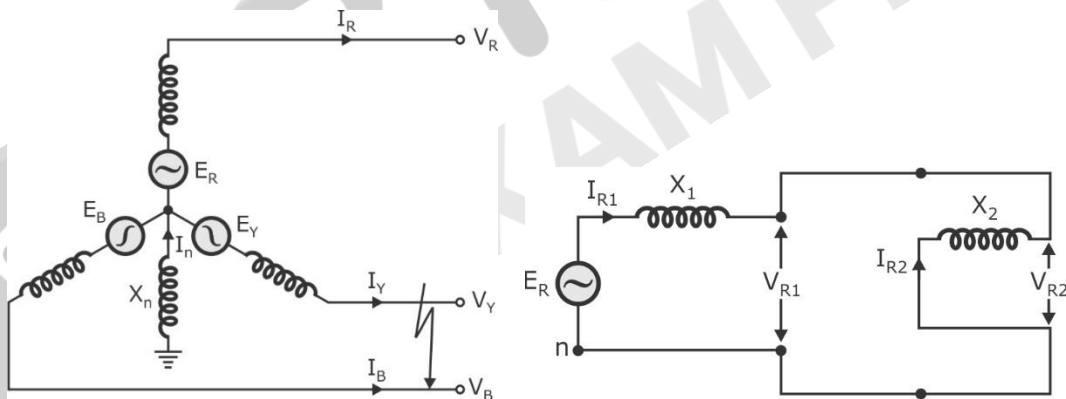


Sequence current relations will not change, but sequence voltage relations will

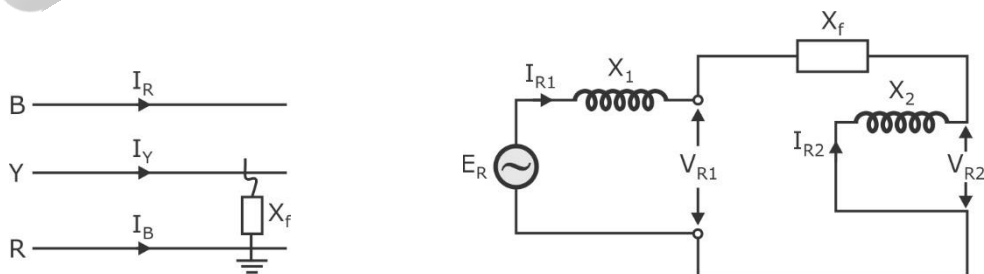
change.  $I_{R0} = I_{R1} = I_{R2} = \frac{E_R}{X_1 + X_2 + X_0 + 3X_n + 3X_f}$

**(ii) LL fault:**

- $I_f = -j\sqrt{3} I_{R1}$  or  $j\sqrt{3} I_{R2}$
- $I_{R1} = \frac{E}{X_1 + X_2} = -I_{R2}$

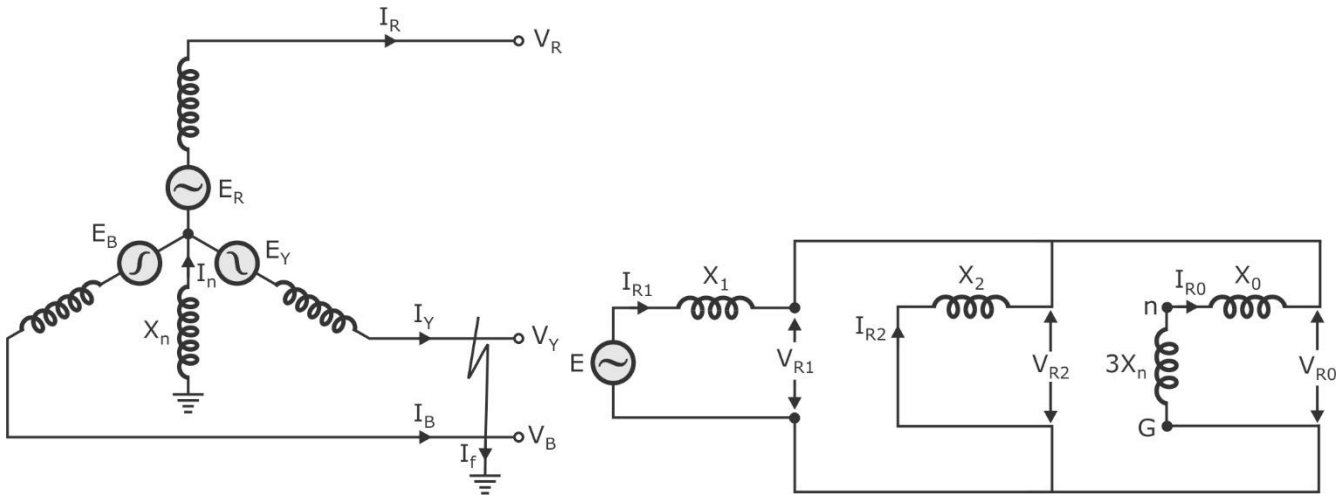


- **Special case:** If the L-L fault takes place with a fault impedance/ reactance ( $X_f$ ) as shown in figure below.



Then,  $I_{R1} = -I_{R2} = \frac{E_R}{X_1 + X_2 + X_f}$  and  $I_f = -j\sqrt{3} I_{R1}$

**(iii) LLG Fault:**



- From ohm's law,

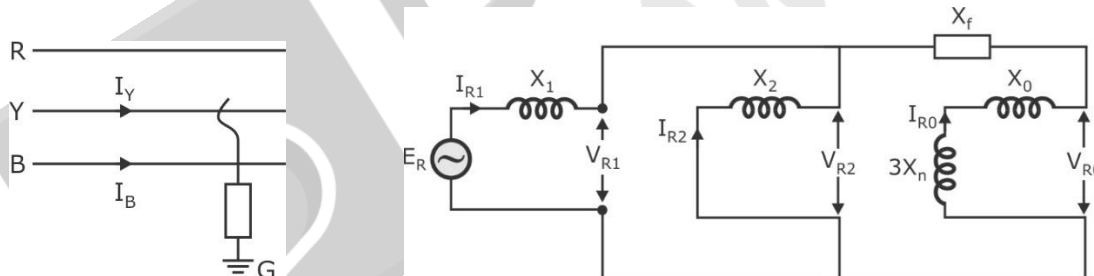
$$I_{R1} = \frac{E}{X_1 + [X_2] [X_0 + 3X_n]}$$

$$I_{R2} = -I_{R1} \cdot \frac{X_0 + 3X_n}{X_2 + X_0 + 3X_n}$$

$$I_{R0} = -I_{R1} \cdot \frac{X_2}{X_2 + X_0 + 3X_n}$$

$$I_f = 3I_{R0}.$$

**Special case:** If the LLG fault takes place with a fault impedance/reactance ( $X_f$ ) as shown below.



- From ohm's law.

$$I_{R1} = \frac{E}{X_1 + [X_2 \parallel (X_0 + 3X_n + 3X_f)]}$$

$$I_{R0} = -I_{R1} \frac{X_2}{X_2 + X_0 + 3X_n + 3X_f}$$

$$I_f = I_Y + I_B = 3I_{R0}$$

#### Note Points:

- LLG fault with isolated neutral grounding is considered as LL Fault.
- LG fault with isolated grounding results to arcing grounds due to capacitance effect, to reduce that Peterson coil is employed
- Short Circuit MVA: Power delivered during fault

$$SC \text{ MVA (p.u.)} = E_{a1}(\text{p.u.}) \times I_{a1}(\text{p.u.})$$

SC MVA (Actual) = SC MVA (p.u)  $\times$  BASE MVA, Use the same for all faults

- An open circuit fault result in
  - (i) Increase in phase voltage
  - (ii) Decrease in current
  - (iii) Increase in power factor
- A short circuit fault result in
  - (i) Increase in current
  - (ii) Decrease in voltage
  - (iii) Decrease in power factor
- LG fault is more frequent fault and is due to flashover of insulator or falling of tree branches

## 11. POWER SYSTEM STABILITY

1. **Steady state stability:** Refers to small disturbances

- Condition for stability:  $\frac{\partial p_e}{\partial \delta} > 0$
- Steady state stability is improved by using bundle conductors, series capacitor, double circuit line, SVC (static VAR compensation)
- for small disturbances, the oscillations frequency is given by

$$\omega = \sqrt{\frac{\left(\frac{\partial p_e}{\partial \delta}\right)_{\delta=\delta_0}}{M}} \text{ rad/s}$$

2. **Transient Stability:** refers to large disturbance due to short circuit faults. The following are the methods to improve transient stability

- By minimizing fault severity and duration
- Increase the restoring synchronizing forces
- Reduce the accelerating torque through prime-mover control
- Reduce the accelerating torque by applying artificial load
- Earthing of transformer neutral through resistance or reactance
- Series compensation of lines

3. **Swing equation:**

$$M \frac{d^2 \delta}{dt^2} = P_a = P_s - P_e$$

Where,  $M = J_m \omega_m$  is the angular momentum of the rotor and is called the inertia constant.

$P_s$  is the mechanical power input to the prime mover.

$P_e$  is the electrical power output.

$P_a$  is the accelerating power which accounts for any unbalance between  $P_i$  and  $P_o$  in watts.

4. **Inertia Constant:**

- The kinetic energy stored in rotating parts of synchronous machine is expressed as

$$K.E = \frac{1}{2} I \omega_s^2 \times 10^{-6} \text{ MJ}$$

- Now, the inertia constant,  $H = \frac{\text{Kinetic Energy stored}}{\text{Machine rating}} = \frac{\frac{1}{2} I \omega_s^2}{S}$

$$2HS = I \omega_s^2 = M \omega \quad (\because M = I \omega)$$

$$M = \frac{2HS}{2\pi f} = \frac{HS}{\pi f} \text{ MJ-sec/elec-rad}$$

Here S is the three-phase rating of the machine (in MVA).

### **5. Alternators in parallel:**

- Coherently then  $\delta_1 = \delta_2 = \delta$

In Coherent operation, there is no real power exchange between the systems  $M_{eq} = M_1 + M_2$

$$\frac{G_{eq} H_{eq}}{\pi f} = \frac{G_1 H_1}{\pi f} + \frac{G_2 H_2}{\pi f}$$

$$H_{eq} = \frac{G_1 H_1}{G_{eq}} + \frac{G_2 H_2}{G_{eq}}; G_{eq} \text{ is common base}$$

- In non-coherent Operation:  $\delta_1 \neq \delta_2$

One machine will deliver power and other machine will absorb power i.e., one as generator and one is motor.

$$\text{Here } M_{eq} = \frac{M_2 P_{m1} - M_1 P_{m2}}{M_1 + M_2} \text{ where } P_{m1} \text{ and } P_{m2} \text{ are mechanical powers of both machines}$$

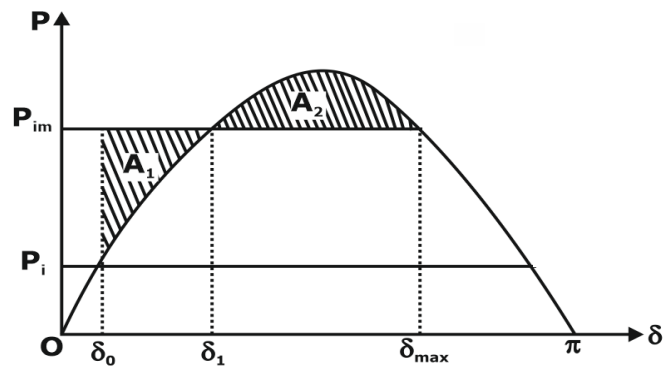
### **6. Equal Area Criteria:**

Applicable only for single machine system. By using this we do the transient stability analysis without using the swing equation.

$$\Delta \omega = \left( \frac{d\delta}{dt} \right) = \sqrt{\frac{2}{M} \int_{\delta_0}^{\delta} P_a d\delta}$$

$$\Delta \omega = \frac{d\delta}{dt} = 0 \quad \text{when} \quad \int_{\delta_0}^{\delta_m} P_a d\delta = 0$$

#### **(i) Sudden Increase in Power Input:**



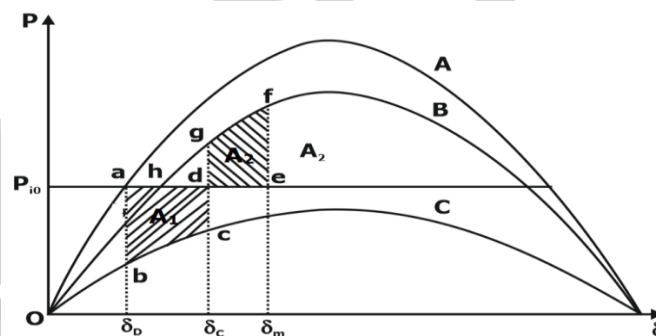
For equal area criteria  $A_1 = A_2$

$$P_{im} \times (\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{max} \sin \delta d\delta = \int_{\delta_1}^{\delta_{max}} P_{max} \sin \delta d\delta - P_i (\delta_{max} - \delta_1)$$

Where  $\delta_{max} = 180^\circ - \delta_0$

Critical fault clearing time ( $t_{cr}$ ):  $t_{cr} = \sqrt{\frac{2M(\delta_{cr} - \delta_0)}{P_i}}$ ;  $\delta_{cr}$  = Critical clearing angle

## (ii) Three-phase Fault at the Middle of One Line of the Double Circuit Line:



Critical clearing angle can be calculated by the given formula

$$\cos \delta_{cr} = \frac{P_{io} (\delta_{max} - \delta_0) + P_{maxB} \cos \delta_{max} - P_{maxC} \cos \delta_0}{P_{maxB} - P_{maxC}}$$

## 12. LOAD FLOW STUDIES

Bus	Known	Unknown
Slack bus/swing bus/ reference bus	$ V , \delta$ $V \angle \delta = 1 \angle 0$	$P, Q$
Generator Bus/PV Bus	$P, V$	$Q, \delta$ Unknown $Q_{g,min} < Q_g < Q_{g,max}$ if it is out of limits PV bus is considered as PQ bus



Load Bus/PQ bus	P, Q	$ V , \delta$
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### Important Points:

- 1) PQ Bus: Load Bus  
Fixed Capacitor bus/ Fixed Inductor Bus
  - 2) PV Bus : Damper Bus  
Reactive power support Bus  
Voltage controlled Bus/ Variable shunt capacitor Bus  
Variable capacitor/Inductor Bus  
Generator Bus
- Size of Jacobian Matrix:  $(2PQ+PV) \times (2PQ+PV)$
  - Slack Bus is nothing but one of the generator Bus which can supply the line losses totally by sharing the demand also

### Z-Bus Construction:

- By means of an impedance  $Z_b$ , a new bus is added to the reference bus then the order of z-bus matrix is increased by one by adding column and row of zeros with the diagonal element  $Z_b$
- If a new bus is added to the existing bus K then order will increase by one by adding a row and column identical to row k and column k and diagonal element is  $Z_{kk} + Z_b$
- If impedance  $Z_b$  is added between reference bus and existing  $k^{\text{th}}$  bus then new Z-bus matrix is

$$[Z^{\text{new}}] = [Z^{\text{old}}] - XX^T\beta$$

$X = K^{\text{th}}$  Column matrix

$$\beta = \frac{1}{Z_{kk} + Z_b}$$

- If impedance is added between two existing bus(j and k)

$$[Z^{\text{new}}] = [Z^{\text{old}}] - XX^T\beta$$

$X = j^{\text{th}}$  column -  $k^{\text{th}}$  Column matrix

$$\beta = \frac{1}{Z_{kk} + Z_{jj} - 2Z_{jk} + Z_b}$$

### Y-BUS:

- 1) Symmetric matrix (means upper and lower triangle matrices are same)
- 2) Sum of admittances connected to that specific bus will be the diagonal elements of Y-Bus
- 3) Negative of admittance connected between any two bus gives off diagonal elements of Y-Bus
- 4) Sparse Matrix (Most elements are Zero):

$$\% \text{ Sparsity} = \frac{\text{Number of zero elements}}{\text{Total number of elements}}$$

Number of Non-Zero elements = (Number of transmission lines) x 2 + No. of Bus

5) Both Y-bus and Z-bus are symmetrical in nature

Comparison of load flow methods:

Newton Raphson Method	Gauss –seidel method
Time taken for each iteration is high	Time taken for each iteration is less
Convergence is independent on choice of slack bus	Convergence depends on choice of slack bus
Applicable for larger power systems	Applicable for small power systems

### 13. POWER SYSTEM PROTECTION

- Restriking voltage in a circuit is given by  $e_{RV} = E_m \left( 1 - \cos \left( \frac{t}{\sqrt{LC}} \right) \right)$
- Rate of rise of recovery voltage RRRV =  $\frac{E_m}{\sqrt{LC}} \sin \left( \frac{t}{\sqrt{LC}} \right)$
- The average RRRV =  $\frac{2E_m}{\pi\sqrt{LC}}$
- Prospective voltage develop across circuit breaker during current chopping is  $v = i \sqrt{\frac{L}{C}}$
- The value of resistor required to be connected across the circuit breaker contacts which will give no transient oscillation (critical damping), is  $R = 0.5 \sqrt{\frac{L}{C}}$

Where L, C are the inductance and capacitance up to the circuit breaker

- Natural frequency of oscillations,  $f_n = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$
- Frequency of damped oscillation,  $f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$
- Symmetrical breaking current = rms value of ac component =  $\frac{X}{\sqrt{2}}$
- Asymmetrical breaking current = rms value of total current =  $\sqrt{\left( \frac{X}{\sqrt{2}} \right)^2 + Y^2}$

Where X = maximum value of ac component

Y = dc component

- Breaking capacity =  $\sqrt{3} \times V \times I \times 10^{-6}$  MVA
- Making capacity = 2.55 × symmetrical breaking capacity.

- The universal relay torque equation is given as follows  $T = K_1 I^2 + K_2 V^2 + K_3 VI (\theta - \tau) + K$

### Important note points in protection:

- Isolator is a no load (off load) switch.
- Electromagnetic attraction relays are used for both ac and dc.
- Electromagnetic relay has limited application for ac protection because pulsating force produce in ac application may damage the relay contact.
- Induction relays are used only for ac.
- For an impedance relay to operate:

$$\left| \frac{V}{I} \right| < |Z_{\text{relaysetting}}| . \text{ Impedance relay is used for protection of medium transmission lines}$$

- For a reactance relay to operate:

$$|X| < \text{constant value} .$$

Reactance relay used for short transmission lines

- For a mho relay to operate:

$$\left| \frac{V}{I} \right| < |K \cos(\theta - \tau)|$$

$\tau$  is maximum torque angle and  $\theta$  is impedance angle

Mho relay is used for long transmission lines protection, and it is least affected by power surges

- Over current relay operates when  $I_{\text{coil}} > I_{\text{pickup}}$
- Current Setting (C.S): It is defined as ratio of pickup current to the relay current setting.

$$C.S = \frac{I_{\text{pickup}}}{I_{\text{relay}}} \times 100$$

- Plug setting multiplier (PSM) =  $\frac{\text{Secondary C.T current}}{\text{Pickup current}} = \frac{\text{fault current}}{\text{CT ratio} \times \text{Pickup current}}$

PSM > 1: Relay operates

PSM < 1: Relay does not operate

PSM = 1: Relay on threshold

- Actual relay operating time = Time corresponding to PSM  $\times$  Time setting multiplier (TMS)
- A relay is generally provided with control to adjust the time of operating this adjustment is known as TMS.
- Instantaneous over current relay operating time ( $t_{op}$ ) = 0.1 sec.
- Definite time overcurrent relay  $t_{op} = (\text{TMS}) \times t_{\text{rating}}$
- IDMT over current relay  $t_{op} = \frac{0.14 \times \text{TMS}}{\text{PSM}^{0.02} - 1}$
- Very inverse over current relay  $t_{op} = \frac{13.5 \times \text{TMS}}{\text{PSM} - 1}$

- Extremely inverse OC relay  $t_{op} = \frac{80 \times TMS}{PSM^2 - 1}$
- Buchholz relays have the capability of anticipating the possible major fault in transformer.

- CT configurations:

	Power transformer connections	CT's connects in differential protection
1.	Y/ $\Delta$	$\Delta$ /Y
2.	$\Delta$ /Y	Y/ $\Delta$
3.	Y/Y	$\Delta$ / $\Delta$
4.	$\Delta$ / $\Delta$	Y/Y

- Different protection schemes:

Rotor protection	Rotor earth fault relay
Unbalanced loading	Negative sequence
Over speed protection	Watt metric relay
Failure of prime mover	Watt metric relay with directional characteristics
Loss of excitation	Offset mho Split phase relaying relay.
Stator protection	Differential relay
Interturn faults	Split phase relaying
Phase to phase and phase to ground fault	Differential protection phase to ground fault
Power transformer, Alternator	Differential relaying
Transmission lines	Distance relaying

- Biased differential relay operates when

$$r |I_1 - I_2| - r \frac{N_r}{N_o} \left| \frac{I_1 + I_2}{2} \right| > |I_{pickup}|$$

$r$  = CT ratio

$N_r$  = number of turns in restraining coil

$N_o$  = number of turns in operating coil

$\frac{N_r}{N_o}$  = Bias of differential relay

$$\% \text{ Bias} = \frac{\frac{i_1 - i_2}{i_1 + i_2}}{2} \times 100$$

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