BYJU'S EXAM PREP

Mechanical Engineering

Strength of Materials

Short Notes

IMPORTANT FORMULAS TO REMEMBER

CHAPTER 1: STRESS AND STRAIN

Stress: The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the load of force.

Units of stress: N/m^2 or Pa.

- 1 Pa = 1 N/m^2
- 1 MPa = 10^6 Pa and 1 GPa = 10^9 Pa

Stress

- **Engineering stress or Nominal stress or Conventional stress**
  - **Conventional stress**
  - $\sigma_0 = \frac{P}{A_0}$
  - **A_0 is area of section before loading.**
- **Actual stress or True stress**
  - $\sigma_A = \frac{P}{A}$
  - **A is area of section after application of load**

Normal Stress: Either tensile or compressive in nature. Its magnitude is given by:

$$\boxed{\sigma = \frac{P}{A}}$$

The tensile forces are termed as (+ve) while the compressive forces are termed as negative (-ve).

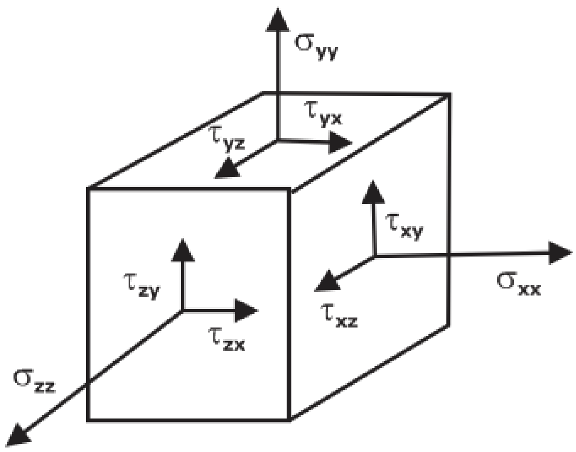
Shear Stress: Stress produced due to forces which are parallel to the area concerned, is called shear stress.

$$\boxed{\tau = \frac{\text{Shear resistance}}{\text{Shear area}} = \frac{P_s}{A}}$$

True stress: It is the applied load divided by the instantaneous cross-sectional area (the changing area with respect to time) of the specimen at that load.

$$\boxed{\text{True stress}(\sigma_T) = \frac{\text{Load (P)}}{\text{Instantenous area(A)}}}$$

Cartesian co-ordinate system:



Stress tensor is given by:

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

σ_{xx}, σ_{yy} and σ_{zz} are the tensile stresses on the x, y and z faces while various shear stress component in a 3-dimensional body is as follows $\tau_{xy}, \tau_{yx}, \tau_{yz}, \tau_{zy}, \tau_{zx}, \tau_{xz}$. Here, first subscript indicates the direction of the normal to the surface while second subscript indicates the direction of the stress.

For conditions of equilibrium:

$$\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{zx} = \tau_{xy}$$

Strain: It is the ratio of change of dimension of the body to the original dimension is known as strain. It is dimensionless.

Strain may be of following types:

Normal strain: $\boxed{\text{strain(e)} = \frac{\text{change in length}}{\text{Original lenth}} = \frac{\Delta L}{L}}$

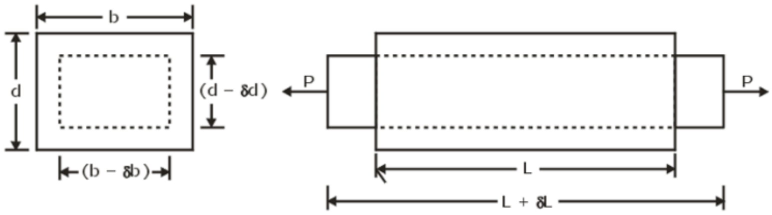
True strain: It equals to the natural log of the quotient of instantaneous length over the original length.

$$\boxed{\varepsilon_T = \ln\left(\frac{L}{L_i}\right) = \ln(1+e)}$$

Shear strain: $\boxed{\gamma = \frac{\tau}{G}}$

Where γ is the shear strain and G is the modulus of rigidity.

Lateral strain:



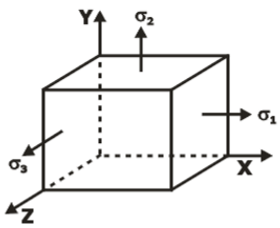
$$\boxed{\text{Lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}}$$

Poisson's ratio: The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio and it is generally denoted by μ. Hence mathematically:

Poisson's ratio: $\boxed{\mu = -\frac{\text{Lateral strain}}{\text{Longitudinal strain}}}$

Material	**μ**
Cork	Zero
Concrete	0.1 to 0.2
Metals	$\frac{1}{4}$ to $\frac{1}{3}$
Rubber, Clay, Paraffin	0.5 → Behaves like perfect plastic material

Three-Dimensional Stress System:



Total strain in the direction of x due to stresses σ_1, σ_2 and σ_3 is:

$$\boxed{e_1 = \frac{\sigma_1}{E} - \mu\left(\frac{\sigma_2}{E} + \frac{\sigma_3}{E}\right)}$$

Similarly, total strain in the direction of y due to stresses σ_1, σ_2 and σ_3 is:

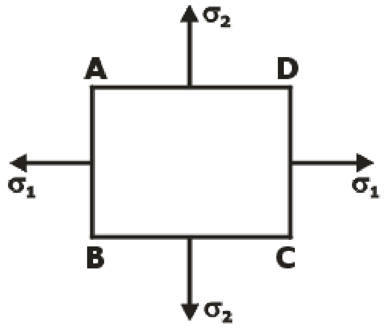
$$\boxed{e_2 = \frac{\sigma_2}{E} - \mu\left(\frac{\sigma_1}{E} + \frac{\sigma_3}{E}\right)}$$

and total strain in the direction of z due to stress σ_1, σ_2 and σ_3 is:

$$\boxed{e_3 = \frac{\sigma_3}{E} - \mu\left(\frac{\sigma_1}{E} + \frac{\sigma_2}{E}\right)}$$

Relationship between stress and strain:

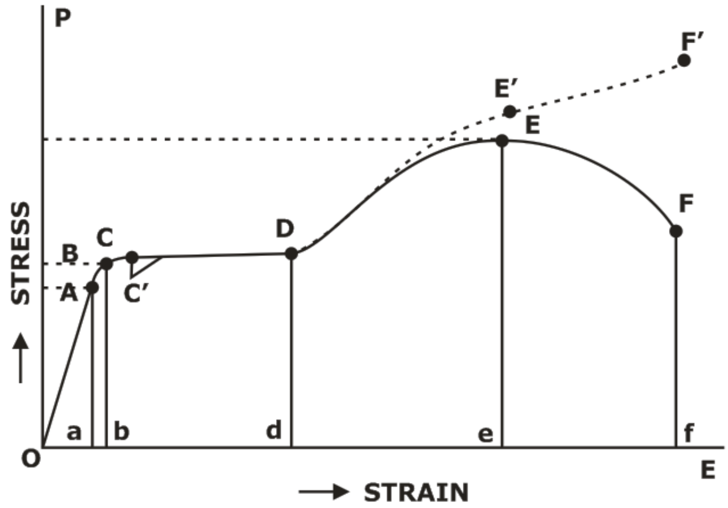
Consider a two-dimensional figure ABCD, subjected to two mutually perpendicular stresses σ_1 and σ_2.



Let, e_1 = Total strain in x-direction and e_2 = Total strain in y-direction.

Thus: $\boxed{e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}}$ and $\boxed{e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}}$

Stress-strain curve: The stress strain curve for mild steel is shown in figure.



A = Proportional Limit — Oa = Linear Deformation

B = Elastic Limit — Ob = Elastic Deformation

C = Yield Point — bd = Perfect Plastic Yielding

C′ = Lower Yield Point — de = Strain hardening

E = Ultimate Strength — ef = Necking

F = Rupture Strength/ Fracture strength

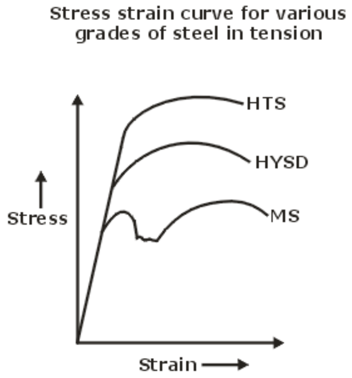
The slope of stress-strain curve is called the young's modulus of elasticity (E).

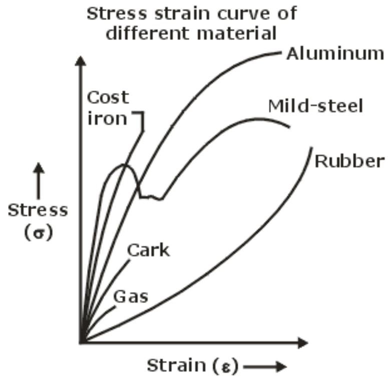
Slope of stress-strain curve: $\boxed{E = \frac{\sigma}{\varepsilon}}$

i.e., $\sigma = \varepsilon.E$

This equation is known as Hooke’s law. Thus, the modulus of elasticity (E) is the constant of proportionality which is defined as the intensity of stress that causes unit strain.

Stress strain curve for different materials:





$$E_{\text{cast iron}} = \frac{1}{2} E_{\text{Steel}} \quad \text{and} \quad E_{\text{Eluminium}} = \frac{1}{3} E_{\text{Steel}}$$

$$E_{\text{concrete}} = \left(\frac{1}{10} \text{ to } \frac{1}{20}\right) E_{\text{steel}} \quad \text{and} \quad E_{\text{wood}} = \left(\frac{1}{8} \text{ to } \frac{1}{20}\right) E_{\text{Steel}}$$

Approximate stress-strain curves for certain materials:

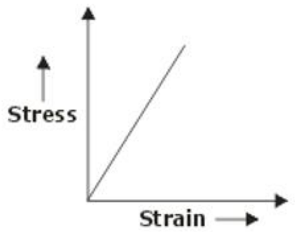


Fig. Linear-elastic curve

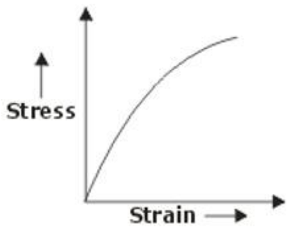


Fig. non-linear curve

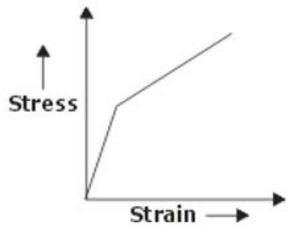


Fig. Elasto-plastic with strain hardening

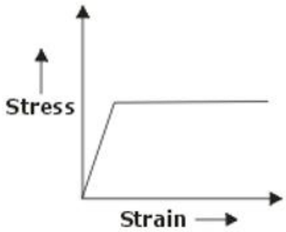


Fig. Elasto-plastic or visco-plastic

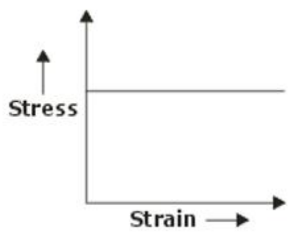


Fig. Perfectly-plastic

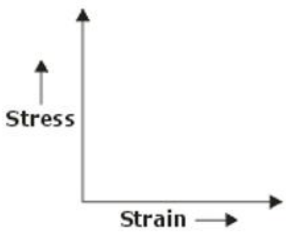


Fig. Perfectly Rigid (Ideal rigid))

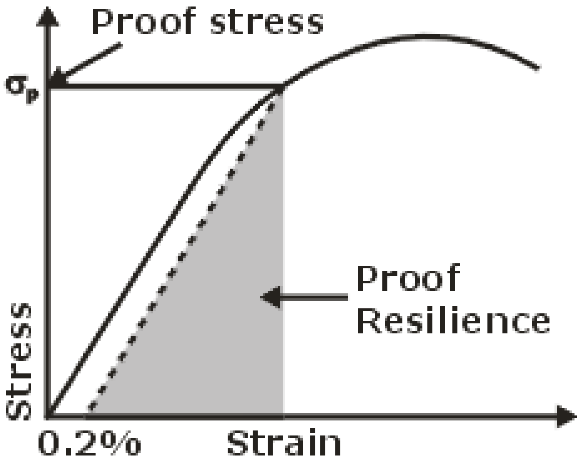
Properties of Metals:

Ductility: It is that properly of material due to which a metal piece can be drawn into wires of thin section under tensioning effect.

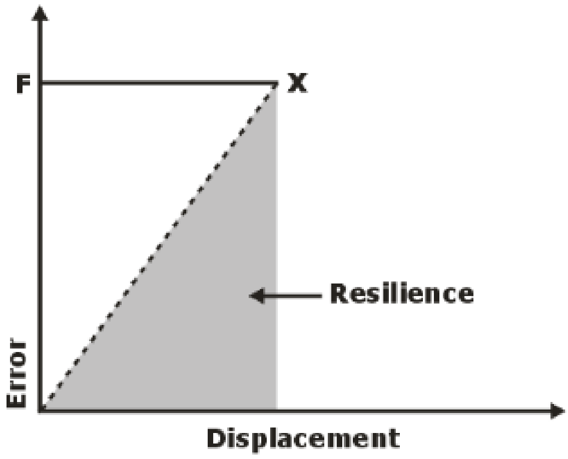
Brittleness: It is the lack of the ductility. Such metal doesn't show necking before fracture.

Malleability: It is that property of metal due to which a metal can be drawn into a thin sheet of negligible section by pressing/forging through the compression process.

Proof stress: A proof stress is a level of stress at which a material undergoes plastic deformation. It is often defined as the point when the material undergoes an amount of plastic deformation equal to 0.2 percent.

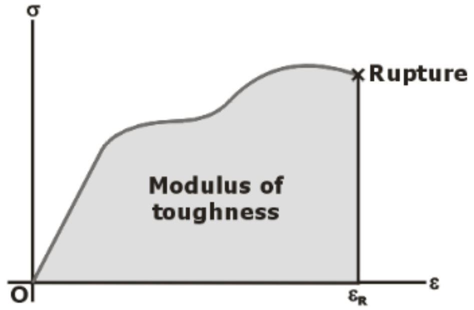


Resilience: Resilience is the ability of a material to absorb energy when it is deformed elastically and release that energy upon unloading. It is defined as the maximum energy that can be absorbed up to the elastic limit, without creating a permanent distortion. The modulus of resilience is defined as the maximum energy that can be absorbed per unit volume without creating a permanent distortion.



Toughness: It is the ability of a material to absorb energy and plastically deform without fracturing. One definition of material toughness is the amount of energy per unit volume that a material can absorb before rupturing.

Modulus of toughness: The modulus of toughness is the amount of strain energy per unit volume (i.e., strain energy density) that a material can absorb just before it fractures. The modulus of toughness is calculated as the area under the stress-strain curve up to the fracture point.

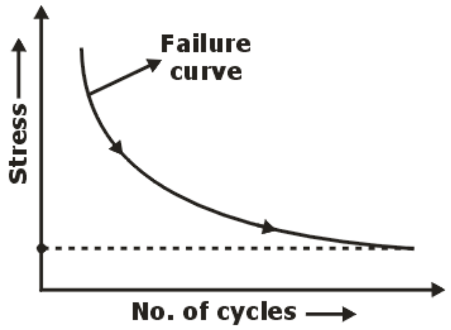


Hardness: Hardness is defined as the resistance of a material to local plastic deformation achieved from indentation of a predetermined geometry indenter onto a flat surface of metal under a predetermined load.

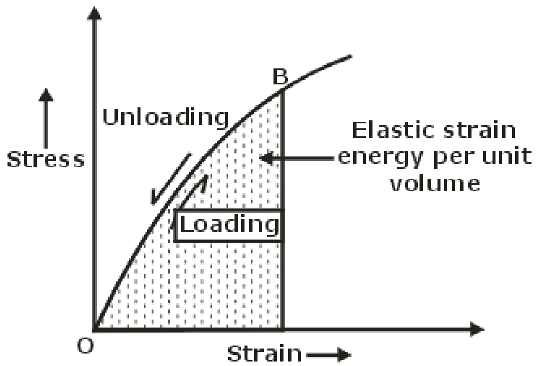
Creep: Creep (sometimes called cold flow) is the tendency of a solid material to move slowly or deform permanently under the influence of persistent mechanical stresses.

Fatigue: Fatigue strength is the highest stress that a material can withstand for a given number of cycles without breaking.

Endurance limit: An endurance or fatigue limit which is defined as the maximum stress below which the steel could presumably endure an infinite number of cycles without failure.



Elasticity: It is that property of metals due to which original dimensions will be recovered offer loading within elastic limits the stress-strain curve may be linear or non-linear.



Bulk Modulus: When a body is subjected to three mutually perpendicular like and equal direct stresses, then the ratio of direct stress to the volumetric strain is termed as bulk modulus.

$$\boxed{K = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\Delta P}{\Delta V/V}}$$

Inter relationship of Elastic Constants:

$$\boxed{E = 3K(1 - 2\mu)}$$

$$\boxed{E = 2G(1 + \mu)}$$

$$\boxed{E = \frac{9KG}{3K + G}}$$

$$\boxed{\mu = \frac{3K - 2G}{6K + 2G} = \frac{1}{m}}$$

Here, μ =Poisson's ratio and m = 1/μ.

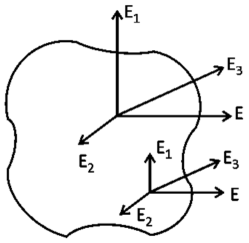
	μ	G	K
Min limit	0	$\frac{E}{2}$	$\frac{E}{3}$
Max limit	$\frac{1}{2}$	$\frac{E}{3}$	∞

For metals:

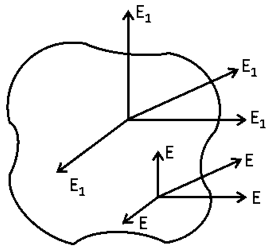
μ	G	K
$\frac{1}{4}$	0.4E	0.67E
$\frac{1}{3}$	0.375E	E

Materials based on elastic properties:

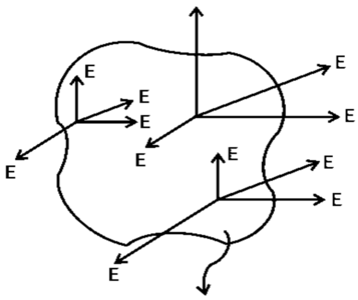
Homogeneous Material: When a material exhibits Same elastic properties at any point in a given directions than the material is known as homogenous material i.e., elastic properties are independent of location.



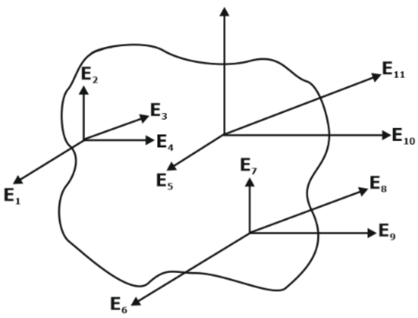
Isotropic Material: When a material exhibits Same elastic properties at any direction at a given point than the material is known as Isotropic Material i.e., elastic properties of material are independent of direction.



Homogenous & isotropic material: When a material exhibits Same elastic properties at any direction at a every point than the material is known as homogeneous Isotropic Material.



Anisotropic Material: When a material exhibits different elastic properties at every direction at a every point than the material is known as Isotropic Material i.e., they exhibit direction dependent elastic property.

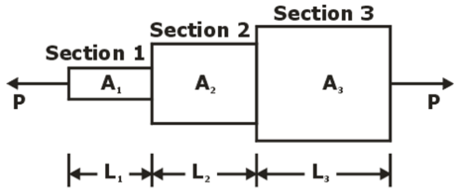


Orthotropic Material: When a material exhibits Same elastic properties at only orthogonal direction at a given point than the material is known as Orthotropic Material.

Material	No. of independent elastic constants
Isotropic	2
Orthotropic	9
Anisotropic	21

Analysis of stresses in different bars

(a). Elongation in Bars of Varying Sections:

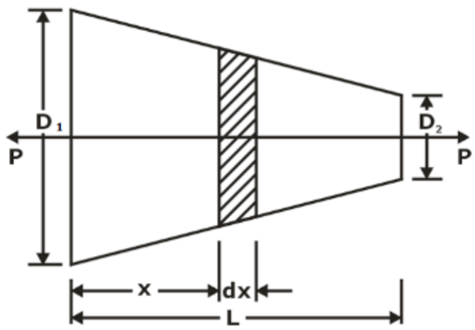


Total change in the length of the bar:

$$dL = dL_1 + dL_2 + dL_3 = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E} = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

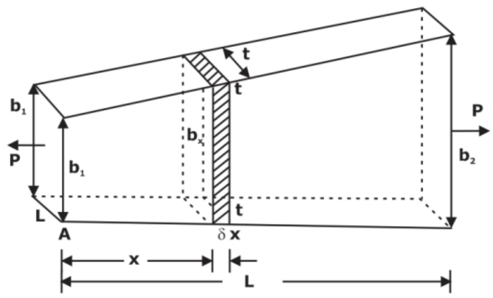
(b). Elongation of Tapering Circular Rod: A bar uniformly tapering from a diameter D_1 at one end to a diameter D_2 at the other and

L = Total length of the bar and E = Young's modulus.



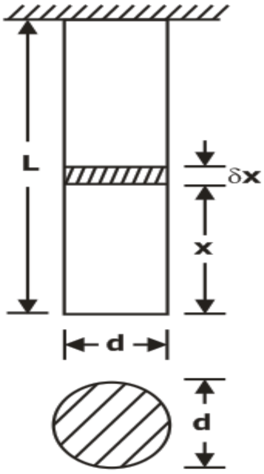
$$\Delta L = \frac{4PL}{\pi ED_1D_2}$$

(c). Elongation of Tapering Rectangular Bar



∴ Total extension: $\boxed{\Delta = \frac{PL}{(b_2 - b_1)tE}\log_e \frac{b_2}{b_1}}$

(d). Elongation of uniform section bar due to self-weight:



$$\boxed{\Delta = \frac{\lambda L^2}{2E} = \frac{\rho g L^2}{2E}}$$

Coefficient of Thermal Expansion (or contraction): The coefficient (α) is a property of the material and has a unit reciprocal of temperature change.

In SI units, α has the dimensions of either 1/K (the reciprocal of kelvins) or 1/° C (the reciprocal of degree Celsius), because the change in temperature is numerically the same in both kelvins and degrees Celsius.

Common values of α are:

Material	**Coefficient of Thermal Expansion**
Steel	10×10^{-6} to 18×10^{-6} / °C
copper	17×10^{-6} / °C
aluminium and aluminium alloys	23×10^{-6} / °C

Thermal stresses:

(a). When bar is constrained (supports unyielding):

$$\boxed{\sigma_{Th} = \frac{\Delta_{Th}E}{L} = L\alpha T \cdot \frac{E}{L} = E\alpha T} \text{ and } \boxed{(\Delta L)_{Total} = 0}$$

(b). When Supports yield: If the support yields by an amount a. In this case, the total amount of expansion checked will be $(\Delta_t - a)$. Hence the resulting temperature stress in:

$$\boxed{\sigma_{Th} = (\Delta_{Th} - a)\frac{E}{L} = (L\alpha T - a)\frac{E}{L}}$$

CHAPTER 2: SHEAR FORCE AND BENDING MOMENT DIAGRAM

Introduction: The algebraic sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force. The algebraic sum of the moments of all the forces acting to the right of left of the section is known as bending moment.

A shear force diagram is one which shows the variation of the shear force along the length of the beam. And a bending moment diagram is one which shows the variation of the bending moment along the length of the beam.

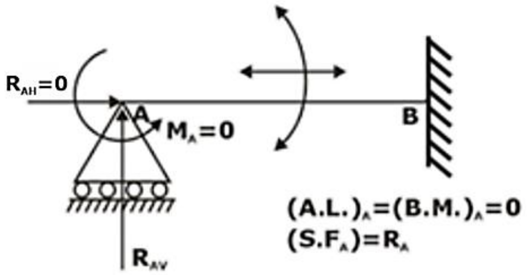
Types of supports:

(1). Simple Supports

(a). Roller Support:

Number of restricted motions by support = Number of reactions at any support

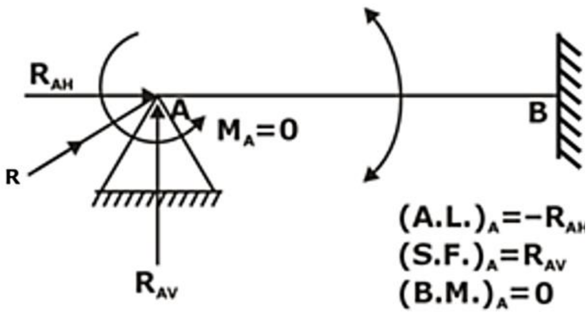
Hence, number of rection in any roller support is 1.



(b). Hinge Support (or) Pin Support:

Number of restricted motions support = Number of reactions at any support

Hence, number of rection is any hinged support is 2.

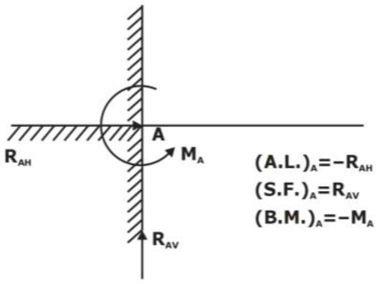


$$\boxed{R = \sqrt{(R_{AH})^2 + (R_{AV})^2}}$$

$$\boxed{\tan\theta = \frac{R_{AV}}{R_{AH}}}$$

Fixed Supports:

Clamped Supports (or) Built-in Supports:



Axial Load at any support = $\pm R_H$ at that point

Shear force at any support = $\pm R_V$ at that point

Moment reaction at any support = $\pm$ Moment reaction at that support

Sign Convention for Shear Force and Bending Moment:

Shear force: If moving from left to right, then take all upward forces as positive and downward as negative.

Left to right	Right to left
↑ (+ve)	↓ (+ve)
↓ (−ve)	↑ (−ve)

Bending moment: If moving from left to right, take clockwise moment as positive and anticlockwise as negative.

Left to right	Right to left
↻ (+ve)	↺ (+ve)
↺ (−ve)	↻ (−ve)

Points of Concern for SFD and BMD (Critical Points):

(i). Starting and end points of beam.

(ii). Point where concentrated point load or concentrated moment is acting.

(iii). Starting and end point of distributed load (UDL or UVL).

(iv). Point where SFD changes sign.

Relationship between SF, BM and load:

(i). Slope of sfd=intensity of distributed load.

$$\boxed{\frac{dV}{dx} = W_x}$$

(ii). Slope of BMD = Shear force at that section.

$$\boxed{\frac{dM}{dx} = V}$$

Loading	Shape of SFD	Shape of BMD
No load	Straight line	Inclined straight line
UDL	Inclined straight line	2° Curve
UVL	2° curve	3° curve

<table>
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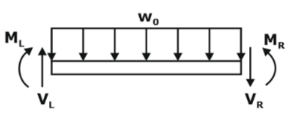
$\frac{dV}{dx} = W$</th><th>Bending Moment diagram,
$\frac{dM}{dx} = V$</th></tr>
<tr><td>
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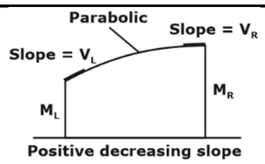
</td><td>
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• Loading is negative and constant ⇒ SF slope is negative and constant
• SF is positive decreasing ⇒ Bending moment slope is decreasing
• If load intensity is UDL ⇒ SFD is learner → BMD is parabolic
• Slope of BMD at any section is equal to SFD ordinate at that section
• Slope of SFD at any section is equal to load intensity at that section
• If load intensity is n-degree curve, SFD will be (n + 1) degree curve and BMD will be (n + 2) degree curve.

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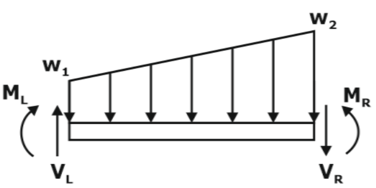
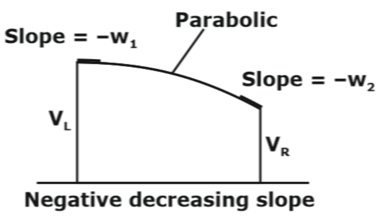
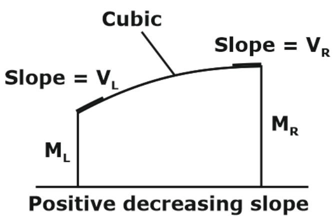
<tr><td>
</td><td>
</td><td>
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• Loading is negative and increasing ⇒ SFD slope is negative and increasing.
• SF is positive and decreasing ⇒ BMD slope is positive and decreasing.
• If load intensity is UVL (uniformly varied load) SFD is parabolic and BMD is cubic.

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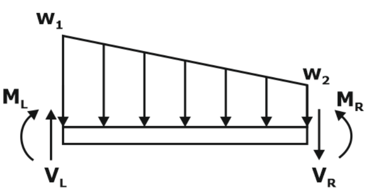
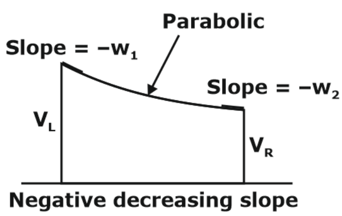
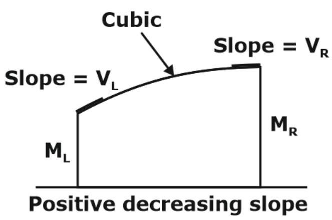
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td colspan="3">

• SF is positive and decreasing ⇒ BMD slope is positive and decreasing
• Loading is negative and decreasing ⇒ SFD slope is positive and decreasing

</td></tr>
</table>

CHAPTER 3: BENDING STRESS

Bending Theory: The following are the important assumptions in the theory of bending

1. The material of the beam is homogeneous and isotropic.
2. The value of Young's modulus of elasticity is the same in tension and compression.
3. The transverse sections which were plane before bending, remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bending into circular arcs with a common centre of curvature.
5. The radius of curvature is large compared with the dimensions of the cross-section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

$$\boxed{\frac{M}{I} = \frac{\sigma}{R} = \frac{E}{R}}$$

The equation is known as bending equation.

2. Section Modulus:

Section modulus is defined as the ratio of moment inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It is denoted by the symbol Z.

$$\boxed{Z = \frac{I}{y_{max}}}$$

I = M.O.I. about neutral axis

y_{max} = Distance of the outermost layer from the neutral axis.

$$\boxed{M = \sigma_{max} . Z}$$

In the above equation, M is the maximum bending moment (or moment of resistance offered by the section). Hence moment of resistance offered the section is maximum when section modulus Z is maximum. Hence section modulus represents the strength of the section.

Section Modulus for Various Shapes or Beam Sections:

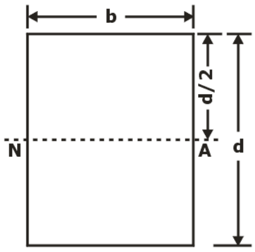
Rectangular Section:

Moment of inertia of a rectangular section about an axis passing through its C.G. (or through N.A. is given by).

$$\boxed{I = \frac{bd^3}{12}}$$

Distance of outermost layer from N.A. is given by,

$$\boxed{y_{max} = \frac{d}{2}}$$



∴ Section modulus is given by:

$$\boxed{Z = \frac{I}{y_{max}} = \frac{bd^3}{12 \times \left(\frac{d}{2}\right)} = \frac{bd^2}{6}}$$

Hollow Rectangular Section: $\boxed{Z = \frac{1}{6D}[BD^3 - bd^3]}$

Circular Section: $\boxed{Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}d^4}{\left(\frac{d}{2}\right)} = \frac{\pi}{32}d^3}$

Hollow Circular Section: $\boxed{Z = \frac{\pi}{32D}[D^4 - d^4]}$

Strength of a section: The strength of a section means the moment of resistance offered by the section and moment of resistance is given by:

Maximum stress induced $\leq$ permissible Stress.

$$\frac{M}{Z_{N.A}} \leq \sigma_{per}$$

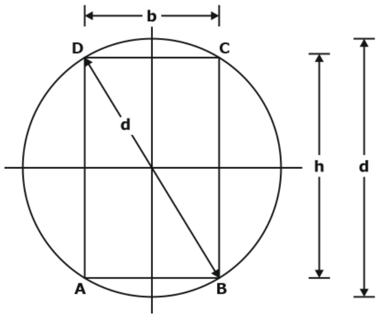
$$M \leq \{(\sigma_{per} \cdot Z_{N.A}) = M_R\}$$

$$M_R = \sigma_{per} \cdot Z_{N.A}$$

For unit radius of bend:

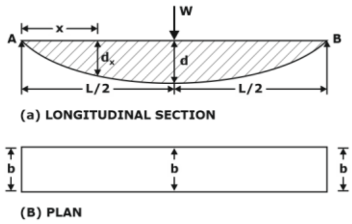
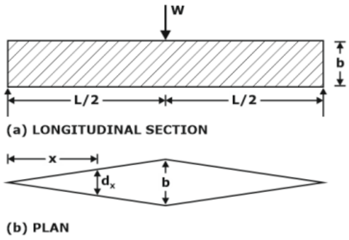
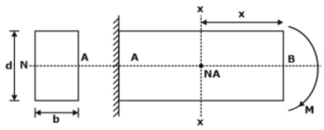
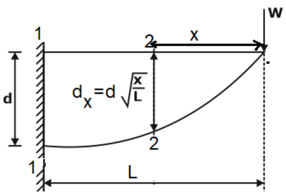
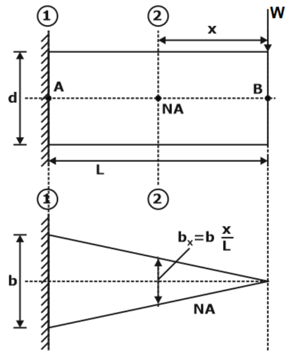
$EI_{N.A} \uparrow = M_R\ (\uparrow) = (\theta\ \&\ \delta) \downarrow =$ chances of beams failures $(\downarrow)$.

Ratio of depth of width of the strongest beam that can be cut from a circular log:



$$\boxed{\begin{array}{l} d = \sqrt{3}\,b \\ \dfrac{h}{b} = \sqrt{2} = 1.414 \end{array}}$$

Beam of uniform strength: It is the beam of uniform moment of resistance.

Constant width		Constant depth	
Simply supported $d_x = \sqrt{\dfrac{3W}{fb}}\sqrt{x} = c\sqrt{x}$	 	**Simple Supported** $b_x = \dfrac{3W}{\sigma d^2}.x = c'x$	
Cantilever beam **M=constant** $\left[(\sigma_b)_{max}\right]_{x-x} = \pm\dfrac{6M}{bd^2}$ **M=Varying** $d_x = d\sqrt{\dfrac{x}{L}}$	  	Cantilever beam $b_x = b\dfrac{x}{L}$	

CHAPTER 4: SHEAR STRESS IN BEAMS

Shear stress at a section:

The shear stress is given by:

$$\boxed{\tau = \frac{F}{Iz}.A\bar{y}}$$

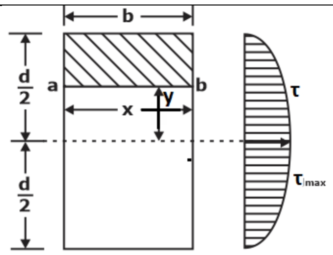
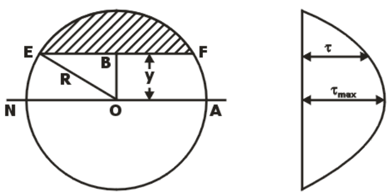
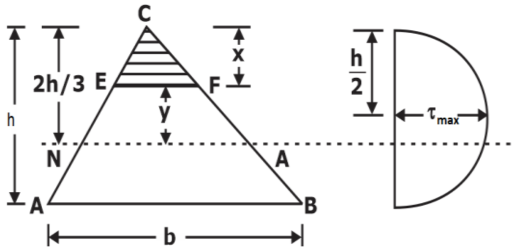
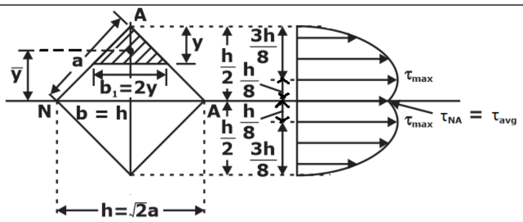
Assumptions: The above analysis is based on the following assumptions:

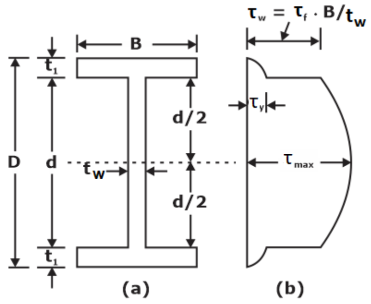
1. For all values of y, τ is uniform across the width of the cross-section, irrespective of its shape.

2. $F = \dfrac{dm}{dx}$ is derived from the assumption that bending stress varies linearly across the section and is zero at the centroid.
3. The material is homogeneous and isotropic, and the value of E is the same for tension as well as compression.

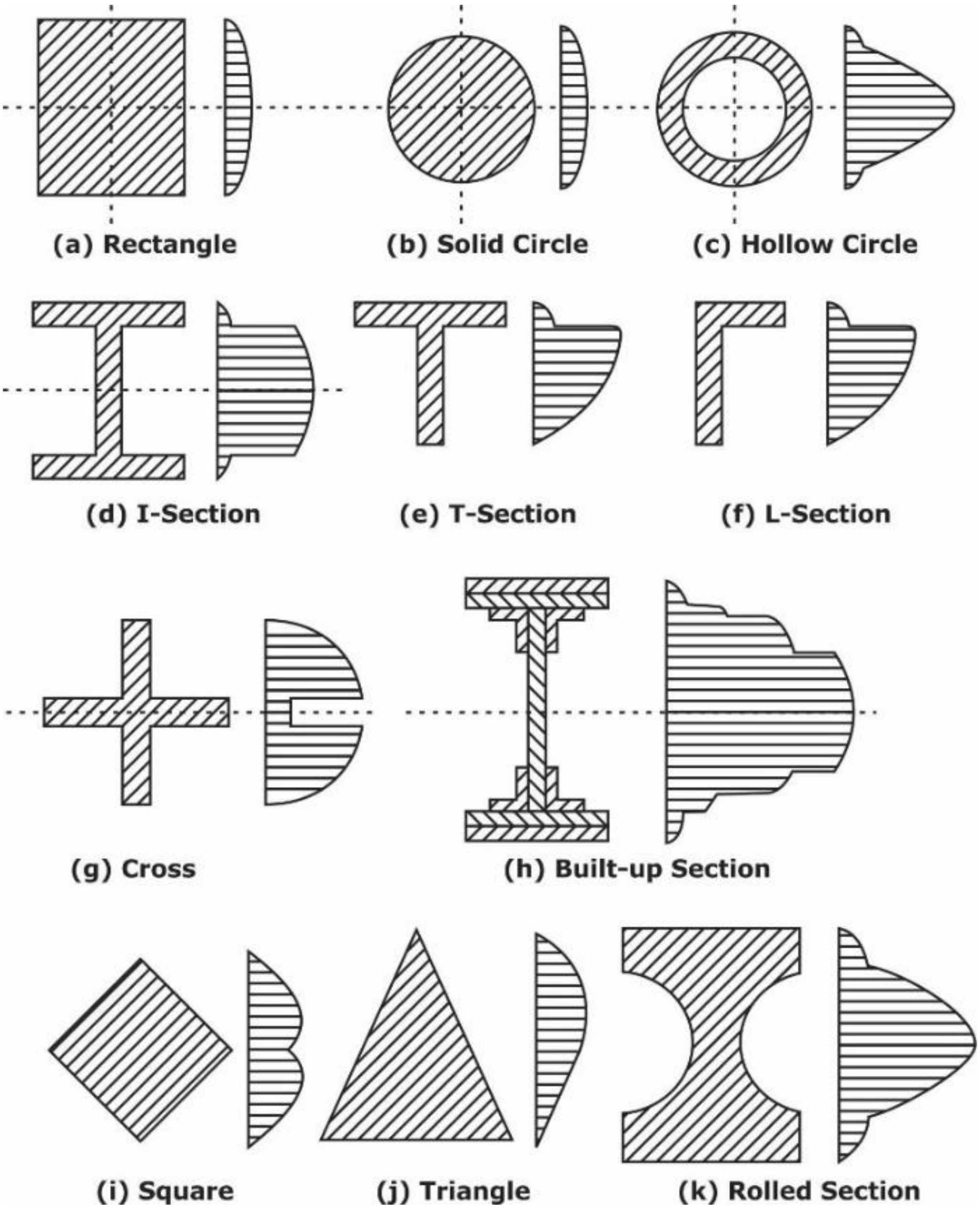
Assumption No. 1 is not strictly correct because the tangential value must be zero at the boundaries of the section. Hence it is understood that τ is the average value across the section. Hence it is understood that q is the average value across the action. Regarding assumption No 2, the stress curve is not a straight line passing through the centroid of the section.

Shear stress distribution and relations for different sections:

(a). Rectangular section: Shear stress: $\boxed{\tau = \dfrac{F}{2I}\left(\dfrac{d^2}{2} - y^2\right)}$ $\therefore \boxed{\dfrac{\tau_{max}}{\tau_{mean}} = \dfrac{3}{2} = 1.5}$	
(b). Circular section: $\boxed{\tau = \dfrac{F}{12I}\{4(r^2 - y^2)\} = \dfrac{F}{3I}(r^2 - y^2)}$ $\boxed{\tau_{max} = \dfrac{4}{3}\tau_{avg}}$	
(c). Triangular section: $\boxed{\tau = \dfrac{F}{3I}(xh - x^2)}$ $\boxed{\therefore \tau_{N.A.} = \dfrac{8}{3}\cdot\dfrac{F}{bh} = \dfrac{4}{3}\dfrac{F}{\left(\dfrac{1}{2}bh\right)} = \dfrac{4}{3}\tau_{avg}}$ $\boxed{\tau_{max} = \dfrac{3F}{bh} = \dfrac{3}{2}\dfrac{F}{\left(\dfrac{1}{2}bh\right)} = \dfrac{3}{2}\tau_{avg}}$	 (a) (b)
(d). Square Diagonal Section: $\boxed{\tau = \dfrac{4Fy}{h^4}\left[3h - 4y\right]}$ $\boxed{\tau_{N.A.} = \dfrac{2F}{(\sqrt{2}\,a)^2} = \dfrac{F}{a^2} = \tau_{avg}}$	

$\boxed{\tau_{max} = \dfrac{9F}{2\times(\sqrt{2}a)^2} = \dfrac{9F}{8a^2} = \dfrac{9}{8}\tau_{avg}}$	
(e). I beam: $\boxed{\tau_{flange} = \dfrac{F}{2I}\left(\dfrac{D^2}{4} - y^2\right)}$ $\boxed{\tau = \dfrac{F}{8I}\dfrac{B}{t_w}(D^2 - d^2) + \dfrac{F}{8I}(d^2 - 4y^2)}$ $\boxed{F_{web} = \dfrac{t_w.d}{3}(2\tau_{max} + \tau_{min})}$	

Shear stress distribution over other sections:



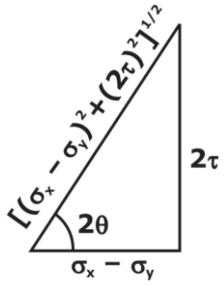
CHAPTER 5: PRINCIPAL STRESSES

Principal stresses:

Three mutually perpendicular planes, on each of which the resultant stress is wholly normal. These are known as principal planes and the normal stress across these planes, as principal stresses.

$$\boxed{\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y}}$$

Which provides two values of 2θ differing by 180° or two values of θ differing by 90°. Thus, the two principal planes are perpendicular to each other.



Principal stresses are given by:

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

Maximum (principal) shear stresses:

For maximum value of τ_θ: $\tan 2\theta = -\frac{\sigma_x - \sigma_y}{2\tau}$

Maximum shear stress: $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$

Principal planes are given by:

$$\tan 2\theta_p = \frac{2\tau}{\sigma_x - \sigma_y}$$

and planes of maximum shear stress:

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau}$$

Multiplying the two:

$\tan 2\theta_p . \tan 2\theta_s = -1$

which means $2\theta_s = 2\theta_p + 90°$

i.e. $\theta_s = \theta_p + 45°$.

This indicates that the planes of maximum shear stress lie at 45° to the planes of principal axes.

The maximum value of shear stress lies in the planes at 45° to the principal planes:

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

Normal stress on the planes of maximum shear stress:

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y)$$

Plane Stress Condition: It is used by the designer for designing thin sheets. In this condition, stress in one of the directions is neglected but strain is assumed to exist in that direction.

Plane strain condition: In plane strain condition designer assumes strain in one of the directions is zero but for preventing this deformation stress is required in that direction. Designing of thick plate, thick pressure vessel designers use this method.

Strain Rosette: A strain gauge rosette is a term for an arrangement of two or more strain gauge that are positioned closely to measure strains along different directions of the component under evaluation.

Mohr's circle:

Let CR and CS represent two perpendicular planes BD and AB respectively so that OL= σ_x, OM= σ_y and LR and MS each equal to τ in the clockwise and counter-clockwise directions respectively (Fig.9). Now if it is desired to find stresses on an inclined plane at angle θ clockwise with plane BD, a radial line CP may be drawn at angle 2θ in the clockwise direction with CR. Then ON and NP will represent the direct and shear components respectively on the plane AD and the resultant is given by OP.

Thus, the procedure may be summarised as follows:

☐ Take OL and OM as the direct components of the two perpendicular stresses σ_x and σ_y.

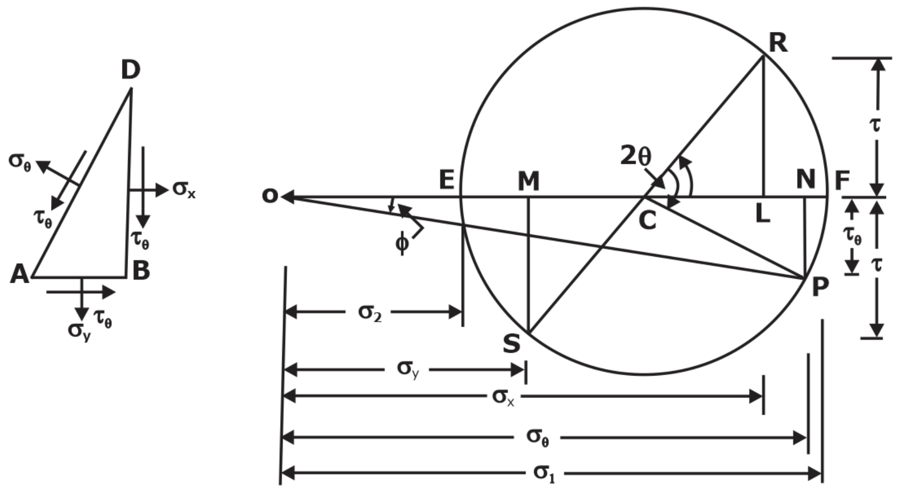
☐ At L and M draw ⊥ LR and MS on the x-axis each equal to τ using the same scale as for the direct stresses. For the stress system shown in Fig.8, LR is taken upwards as the direction on plane BD is clockwise and MS downwards as the direction on plane AB is counter clockwise.

☐ Bisect LM at C and draw a circle with C as centre and radius equal to CR (= CS).

☐ Rotate the radial line CR through angle 2θ in the clockwise direction if θ is taken clockwise and let it take the position CP.

☐ Draw NP ⊥ on x-axis. Join OP.

It can be proved that ON and NP represent the normal and the shear stress components on the inclined plane AD.



$$\boxed{\text{Major principal stress} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

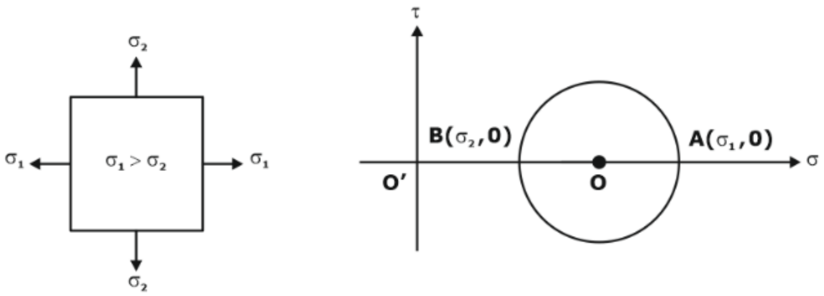
$$\boxed{\text{Minor principal stress} = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

Radius of the circle: $\boxed{R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}}$

Centre of Mohr's circle: $\boxed{C = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)}$

Mohr's circle different cases:

(a). When both the stresses are tensile in nature and no shear stress:

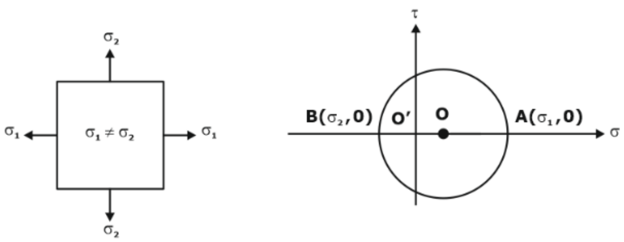


Thin cylinder stress case.

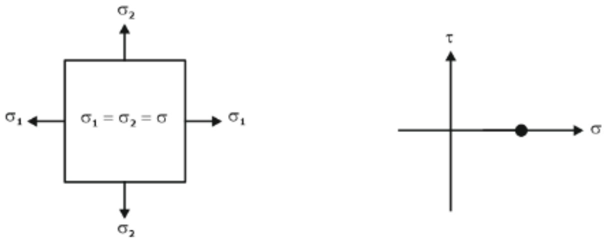
$$\boxed{\text{Hoop stress}: \sigma_h = \frac{PD}{2t} \quad \text{(tensile)}}$$

$$\boxed{\text{Longitudinal stress}: \sigma_l = \frac{PD}{4t} \quad \text{(tensile)}}$$

(b). When one is tensile and other is compressive in nature and no shear stress:

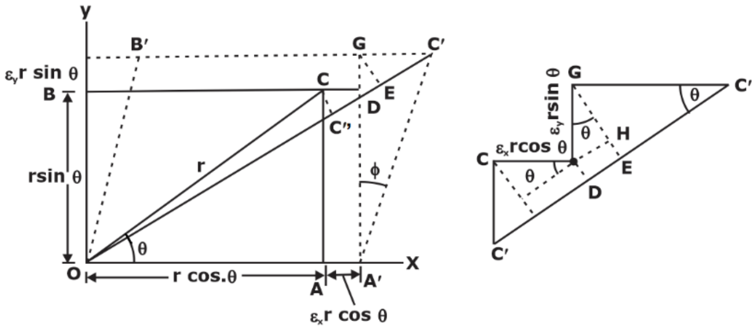


(c). When both the stresses are tensile in nature and are equal: Spherical pressure vessel case.



$$\boxed{\sigma_h = \sigma_l = \frac{PD}{4t} \quad \text{(tensile)}}$$

Strain analysis:



$$\boxed{\varepsilon_\theta = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_y - \varepsilon_y)\cos 2\theta + \frac{1}{2}\varphi \sin 2\theta}$$

$$\boxed{\gamma = -\frac{1}{2}(\varepsilon_x - \varepsilon_y)\sin 2\theta + \varphi \sin^2\theta}$$

Compare the results with bi-axial and shear stresses conditions:

☐ In a linear system: $\varepsilon_\theta = \varepsilon_x.\cos^2\theta$ or $\varepsilon_x\left(\frac{1+\cos 2\theta}{2}\right)$.

☐ In a pure shear system and for θ = 45°, $\varepsilon_{45°} = \varphi/2$

Principal strains:

The maximum and the minimum values of strains on any plane at a point are known as the principal strains and the corresponding planes as the principal planes for strains.

$$\boxed{\tan 2\theta = \frac{\varphi}{\sigma_x - \sigma_y}}$$

For shear strain to be maximum or minimum:

$$\boxed{\tan 2\theta = -\frac{\varepsilon_x - \varepsilon_y}{\varphi}}$$

The planes of maximum shear strain are inclined at 45° to the planes of maximum shear strain as in case of maximum shear stress.

Sum of direct strains on two mutually perpendicular planes:

$$\boxed{\varepsilon_\theta + \varepsilon_{\theta+90^\circ} = \varepsilon_x + \varepsilon_y}$$

Relation between principal stress and strain:

$$\boxed{\sigma_2 = \frac{E(\mu\varepsilon_1 + \varepsilon_2)}{1-\mu^2}} \text{ and } \boxed{\sigma_1 = \frac{E(\mu\varepsilon_2 + \varepsilon_1)}{1-\mu^2}}$$

Types of strain rosette:

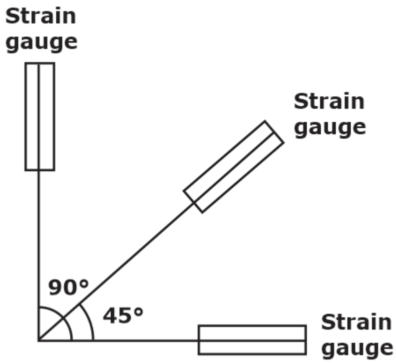
Let ϵ_x and ϵ_y be the linear strains in x and y directions and φ be the shear strain at the point under consideration. Then linear strains in any three arbitrary chosen directions at angles θ_1, θ_2 and $\theta_3$made with the x-axis will be:

$$\varepsilon_{\theta_1} = \varepsilon_x.\cos^2\theta_1 + \varepsilon_y.\sin^2\theta_1 + \varphi.\sin\theta_1.\cos\theta_1$$

$$\varepsilon_{\theta_2} = \varepsilon_x.\cos^2\theta_2 + \varepsilon_y.\sin^2\theta_2 + \varphi.\sin\theta_2.\cos\theta_2$$

$$\varepsilon_{\theta_3} = \varepsilon_x.\cos^2\theta_3 + \varepsilon_y.\sin^2\theta_3 + \varphi.\sin\theta_3.\cos\theta_3$$

Rectangular strain Rosette:



$\theta_1 = \theta$ °, θ_2=45° and θ_2 =90°

The above equations can be written:

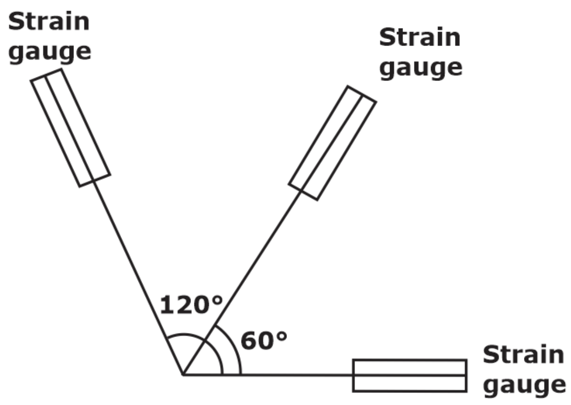
$\epsilon_{0^\circ} = \epsilon_x$ and $\epsilon_{90^\circ} = \epsilon_y$

$$\varepsilon_{45^\circ} = \frac{1}{2}(\varepsilon_x + \varepsilon_y + \varphi)$$

From which:

$\epsilon_x = \epsilon_{0^\circ}$, $\epsilon_y = \epsilon_{90^\circ}$ and $\Phi = 2\epsilon_{45^\circ} - (\epsilon_x + \epsilon_y)$

Delta strain rosette:



$\epsilon_x = \epsilon_{0°}$ and $\boxed{\varepsilon_y = \frac{1}{3}(2\varepsilon_{60°} + 2\varepsilon_{120°} - \varepsilon_{0°})}$

and $\boxed{\varphi = \frac{2}{\sqrt{3}}(\varepsilon_{60°} - \varepsilon_{120°})}$

Strain tensor: Strain tensor is used to define the state of strain at a point (i.e., different strains developed on three mutual perpendicular planes, passing through a point.

$\epsilon \rightarrow$ [Normal Strain]

$\tau \rightarrow$ [Shear strain]

$$\underset{\substack{\downarrow \\ \text{Strain} \\ \text{Tensor} \\ \text{at a point} \\ \text{in} \\ \text{3D.}}}{[\varepsilon]_{3D}} = \begin{bmatrix} \varepsilon_{xx} & \frac{\phi_{xy}}{2} & \frac{\phi_{XZ}}{2} \\ \frac{\phi_{xy}}{2} & \varepsilon_{yy} & \frac{\phi_{YZ}}{2} \\ \frac{\phi_{xz}}{2} & \frac{\phi_{YZ}}{2} & \varepsilon_{Zz} \end{bmatrix}_{3\times 3}$$

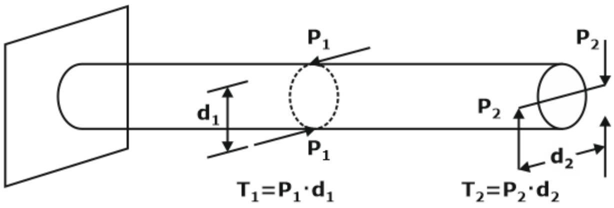
For the 2-D:

$$[\varepsilon]_{2D} = \begin{bmatrix} \varepsilon_{xx} & \frac{\phi_{xy}}{2} \\ \frac{\phi_{xy}}{2} & \varepsilon_{yy} \end{bmatrix}$$

Let ϵ, φ are the normal & shear strain or an oblique plane passing through a point under bi-axial state of strain.

CHAPTER 6: TORSION AND SPRINGS

Torsion: A shaft is said to be under pure torsion when it is subjected to two equal & opposite couples in a plane perpendicular to the longitudinal axis of the shaft.



Sign convention:

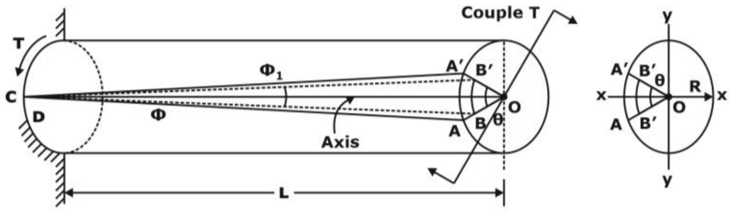
Right hand thumb rule → If right hand fingers represent direction of torque applied and thumb is pointing towards the section, the torque is taken positive.

Assumptions:

The following assumptions have been made in developing the equations for stresses and deformations in a bar subjected to pure torsion.

1. Shaft is loaded with twisting couples in planes that are perpendicular to the axis of the shaft.
2. Torsion is uniform along the length i.e., all normal cross-section which are the same axial distance suffer equal relative rotations.
3. Circular sections remain circular. Thus, radii remain straight after torsion.
4. Plane normal sections of shaft remain plane after twisting, i.e., no warping or distortion of parallel planes normal to the axis of the shaft takes place.
5. Stress is proportional to strain, i.e., stresses do not exceed proportional limit.
6. Material is homogenous and isotropic.

Torsion formula:



$$\boxed{\frac{\tau_r}{r} = \frac{\tau}{R} = \frac{T_R}{J} = \frac{G\theta}{L}}$$

Where:

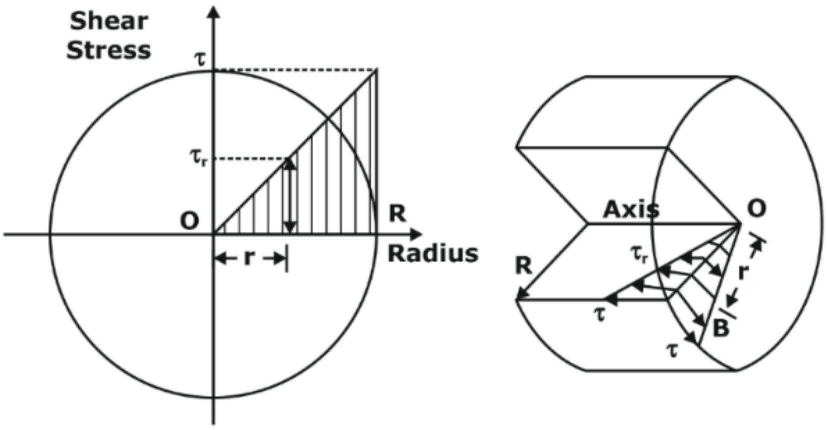
θ = twist angle in radians

τ = shear stress developed in the material

T = torque applied

G = modulus of rigidity

I_P = Polar moment of inertia

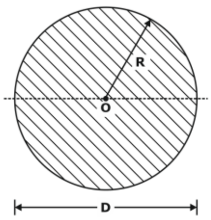
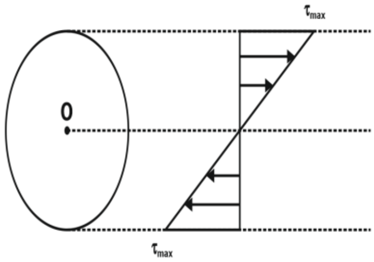
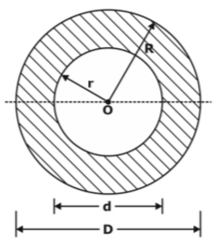
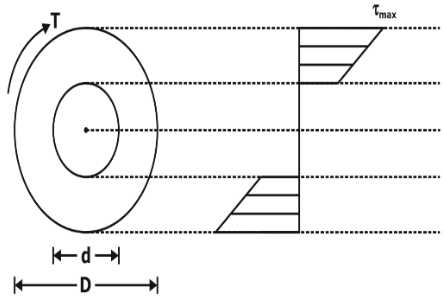


(a) (b)

Sign convention:

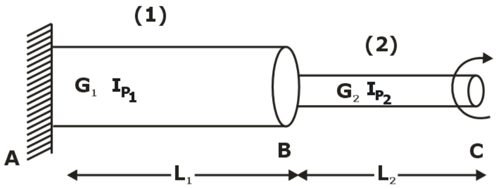
Right hand thumb rule → If right hand fingers represent direction of torque applied and thumb is pointing towards the section, the torque is taken positive.

Polar moment of inertia:

Cross section	**Polar moment of Inertia and maximum torque**	**Shear stress variation**
Solid circular shaft 	$I_{zz}=\frac{\pi}{32}D^4$ $Z_P=\frac{J}{R}=\frac{J}{D/2}=\frac{\pi}{16}D^3$ $T=\tau\times\frac{\pi}{2}\times\frac{D^3}{8}=\frac{\pi}{16}\tau D^3$	
Hollow circular shaft 	$I_{zz}=\frac{\pi}{32}\left(D^4-d^4\right)$ $Z_p=\frac{\pi}{16}\left(\frac{D^4-d^4}{D}\right)$ $T=\tau\frac{\pi}{16}\left[\frac{D_0^4-D_i^4}{D_0}\right]$	

Connection of shafts:

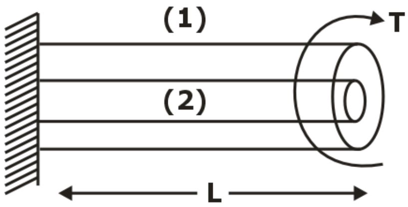
(i). Series connection:



Here,

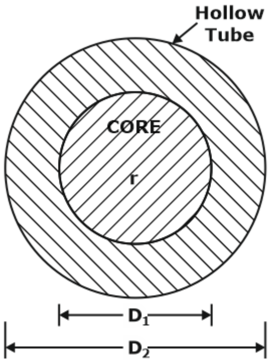
$$\boxed{\begin{aligned} T_1 &= T_2 = T \\ \theta_{AC} &= \theta_{AB} + \theta_{BC} \end{aligned}}$$

(ii). Parallel connection:



Here, $$\boxed{\begin{aligned} & T_1 + T_2 = T \\ & \theta_1 = \theta_2 \Rightarrow \frac{T_1 L_1}{G_1 J_1} = \frac{T_2 L_2}{G_2 J_2} \end{aligned}}$$

Shaft of two materials: composite shaft:



Torques: $\boxed{T_1 = T\left(\frac{G_1 J_1}{G_1 J_1 + G_2 J_2}\right)}$ and $\boxed{T_2 = T\left(\frac{G_2 J_2}{G_1 J_1 + G_2 J_2}\right)}$

Twist angle: $\boxed{\theta = \frac{TL}{G_1 J_1 + G_2 J_2}}$

Ratio of shear stresses: $\boxed{\frac{\tau_1}{\tau_2} = \frac{T_1 D_1}{T_2 D_2} \cdot \frac{J_2}{J_1} = \frac{G_1}{G_2} \cdot \frac{D_1}{D_2}}$

Strain energy due to torsion:

Total strain energy:

$U = \frac{1}{2} T.\theta = \text{Area under } T - \theta \text{ diagram}$

$$\boxed{U = \frac{\tau^2}{4G} \times \text{Volume of the shaft}}$$

For hollow shaft:

$$\boxed{U = \frac{\tau^2}{4G}\left(\frac{D^2 + d^2}{D^2}\right) \times \text{Volume of the shaft}}$$

For the thin tube: $D \approx d$

$$\boxed{U \approx \frac{\tau^2}{2G} \times \text{Volume of the shaft}}$$

Torsional stiffness (K_t): Torsional stiffness is defined as the amount of torque or twisting couple required to produce a twist of unit radian. And it represented by 'K'. $\boxed{K_t = \frac{T}{\theta}}$

For a given twisting couple 'T' $K_t = \frac{GJ}{L} \uparrow \Rightarrow \theta \downarrow \Rightarrow \varphi \downarrow \Rightarrow \gamma \downarrow \tau_{induced} \downarrow \Rightarrow \text{Chances of torsional failure} \downarrow$

Torsional Stiffness is used to compare angular twist of two different shafts which are subjected to same twisting moment whereas Torsional Rigidity is used to compare angular twist of two different shafts which are having same length and subjected to same torque.

Combined bending and torsion: Consider any point on the cross-section of as shaft.

Let, T = Torque at the section

D = Diameter of the shaft

M = B.M. at the section

Major principal stress: $\boxed{\sigma_1 = \frac{16T}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)}$

Minor principal stress: $\boxed{\sigma_2 = \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)}$

Maximum shear stress

$$\tau_{max} = \frac{\text{Major principal stress} - \text{Minor principal stress}}{2}$$

$$\boxed{\tau_{max} = \frac{16}{\pi D^3}\left(\sqrt{M^2 + T^2}\right)}$$

For a hollow shaft:

Major principal stress: $\sigma_1 = \frac{16D_0}{\pi\left[D_0^4 - D_i^4\right]}\left(M + \sqrt{M^2 + T^2}\right)$

Minor principal stress: $\sigma_2 = \frac{16D_0}{\pi\left[D_0^4 - D_i^4\right]}\left(M - \sqrt{M^2 + T^2}\right)$

Maximum shear stress: $\tau_{max} = \frac{16D_0}{\pi\left[D_0^4 - D_i^4\right]}\left(\sqrt{M^2 + T^2}\right)$

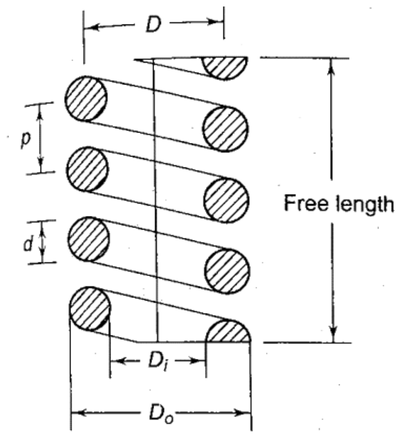
SPRINGS

A spring is defined as an elastic machine element, which deflects under the action of the load and returns to its original shape when the load is removed.

Springs are classified according to their shape.

- The spring can be a helical coil of a wire, a piece of stamping or a flat wound-up strip.
- The most popular type of spring is helical spring.
- The helical spring is made from a wire, usually of usually of circular cross-section, which is bent in the form of a helix.
- There are two types of helical springs – compression spring and extension spring.

Helical spring:



$\boxed{\text{Solid length} = N_t d}$

Where

N_t = total number of coils

$\boxed{\text{Total gap} = (N_t - 1) \times \text{Gap between adjacent coils}}$

Spring deflection:

Assumptions:

(1) The Bending & shear effects may be neglected.

(2) For the purpose of derivation of formula, the helix angle is considered to be so small that it may be neglected.

Spring deflection: $\boxed{\delta = \frac{8W.D^3n}{G.d^4}}$

Where, $W =$ axial load

$D =$ mean coil diameter

$d =$ diameter of spring wire

$n =$ number of active coils

spring index : $\boxed{C = \frac{D}{d}}$ (For circular wires)

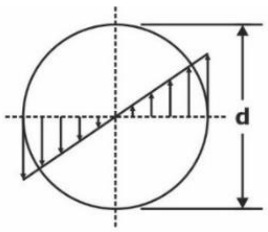
$l =$ length of spring wire

$G =$ modulus of rigidity

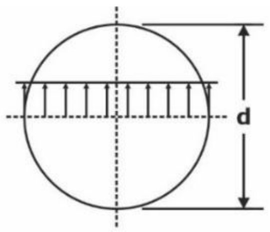
Spring stiffness: $\boxed{k = \frac{G.d^4}{8.D^3n} = \frac{G.d}{8.C^3n}}$

Shear stress: $\boxed{\tau_{max} = \frac{8WD}{\pi d^3}\left(1 + \frac{d}{2D}\right) = \frac{8WD}{\pi d^3}\left(1 + \frac{1}{2C}\right)}$

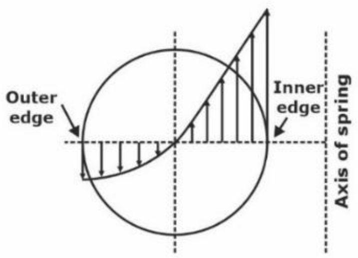
$\boxed{k_{sh} = \left(1 + \frac{d}{2D}\right) = \text{shear stress correction factor}}$



(a) Torsional shear stress diagram.



(b) Direct shear stress diagram.



(c) Resultant torsional shear and direct shear stress diagram.

Wahl's correction factor:

$\boxed{K_w = \frac{4c-1}{4c-4} + \frac{0.615}{c}}$

If we take into account, the Wahl's factor than the formula for the shear stress becomes

$$\boxed{\tau_{max} = \frac{8WD}{\pi d^3} K_w}$$

The Wahl's stress factor (K) may be considered as composed of two sub-factors, K_{sh} and K_C, such that

$$\boxed{K_w = K_{sh} \times K_c}$$

where K_{sh} = Stress factor due to shear, and

K_c = Stress concentration factor due to curvature.

Strain Energy: The strain energy is defined as the energy which is stored within a material when the work has been done on the material.

$U = \frac{T^2 L}{2GJ}$ and $L = \pi Dn$

So, after substitution:

$$\boxed{U = \frac{16T^2 Dn}{G.d^4}}$$

Connection of springs:

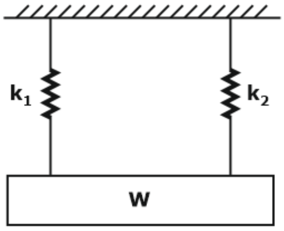
Series connection:

$$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}}$$

For n number of springs:

$$\boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \ldots \frac{1}{K_n}}$$

Spring in parallel:



$\boxed{k = k_1 + k_2}$

∴ For n number of springs

$\boxed{K_{eq} = K_1 + K_2 + K_3 + \ \ldots \ + K_n}$

equivalent stiffness of the spring in parallel connection is the sum of the stiffness of the individual spring.

CHAPTER 7: THEORIES OF FAILURE

Maximum principal stress theory: According to this theory, the failure of a material will occur when the maximum principal tensile stress (σ_1) reaches the value of the maximum stress at the elastic limit in simple tension or the minimum principal stress reaches the value of the maximum stress at the elastic limit in simple compression.

This is the simplest and oldest theory of failure and is known as Rankine's theory. If the maximum principal stress (σ_1) is the design criterion, then maximum principal stress must not exceed the permissible stress (σ_t) for the given material.

Hence, $\sigma_1 = \sigma_t$

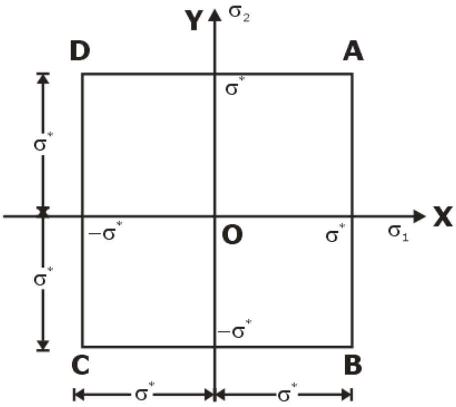
Where, σ_t = permissible stress and is given by:

$\sigma_t = \sigma_t^* \ / \text{ safety factor}$

(a). Maximum Principal Stress Theory [M.P.S.T.] or Rankine's Theory:

Condition for failure: $\sigma_1 > S_{yt}$ or S_{ut}

Condition for Safe Design: $\sigma_1 \le S_{yt}$ or S_{ut}



If Factor of safety is taken into consideration, then: $\sigma_1 \leq \frac{S_{yt}}{N}$ or $\frac{S_{ut}}{N}$

Where N is the Factor of safety.

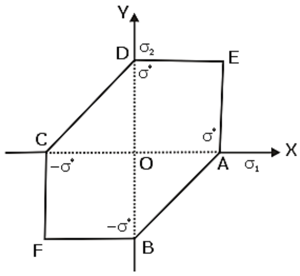
- Best theory of failure for brittle materials, under any state of stress condition.
- It is suitable for ductile materials under uniaxial state of stress condition, Biaxial state of stress condition when σ_1, σ_2 are like in nature and hydrostatic state of stress.

(b). Maximum Shear Stress Theory [M.S.S.T.]:

Under Triaxial State of Stress

$$\text{Absolute } \tau_{max} = \text{max of} \left| \frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2 - \sigma_3}{2}, \frac{\sigma_3 - \sigma_1}{2} \right|$$

$$\text{max of} \left| \frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2 - \sigma_3}{2}, \frac{\sigma_3 - \sigma_1}{2} \right| \leq \frac{S_{yt}}{2N}$$



For Biaxial State of Stress: $\sigma_3 = 0$

$$\boxed{\text{max of} \left| \frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2}{2}, \frac{\sigma_1}{2} \right| \leq \frac{S_{yt}}{2N}}$$

Since $\sigma_1 > \sigma_2$, thus we can remove σ_2 from the above expression. So, for biaxial state of stress:

$$\text{max of} \left| \frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_1}{2} \right| \leq \frac{S_{yt}}{2N}$$

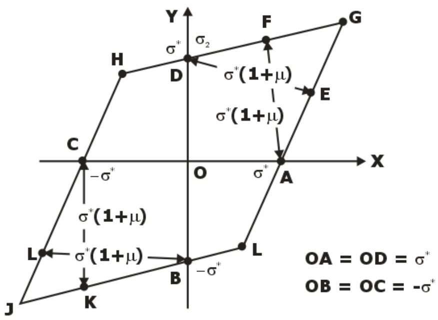
$$\text{Absolute } \tau_{max} > \frac{S_{ys}}{N} \text{ or } \frac{S_{yt}}{2N}$$

Since for brittle material: $S_{yt} \approx S_{ut}$

$$\text{Absolute } \tau_{max} > \frac{S_{yt}}{2N} \text{ or } \frac{S_{ut}}{2N}$$

It gives over safe design for ductile materials [Since it gives safe and uneconomic design].

(c). Maximum Principal Strain Theory [M.P.St.T.]: According to this theory, when Principal strain developed at a critical point under combined loading conditions exceeds yield strain under tension test, failure is likely to occur.



Maximum normal Strain > strain at yield point in tension test

$$\varepsilon_1 > \varepsilon_{YP}$$

Condition for Safe design

$$\varepsilon_1 \le \varepsilon_{YP} = \frac{S_{yt}}{E}$$

$$\boxed{\sigma_1 - \mu(\sigma_2 + \sigma_3) \le \frac{S_{yt}}{N}}$$

Under biaxial state of stress: $\sigma_3 = 0$

$$\boxed{\sigma_1 - \mu\sigma_2 \le \frac{S_{yt}}{N}}$$

(d). Total strain energy theory:

According to this theory, when total strain energy per unit volume at a critical point under combined loading conditions exceeds yield strength in tension test, failure is likely to occur.

Condition for failure

$$\boxed{\text{Total Strain energy / Volume} > \frac{\text{Strain Energy at yield point}}{\text{Volume}}}$$

Condition for Safe Design

$$\boxed{\text{Total Strain energy / Volume} \le \frac{\text{Strain Energy at yield point}}{\text{Volume}}}$$

$$\boxed{\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right) - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right) \le \left(\frac{S_{yt}}{N}\right)^2}$$

Under biaxial state of stress, $\sigma_3 = 0$

$$\boxed{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \le \left(\frac{S_{yt}}{N}\right)^2}$$

It represents an equation of ellipse

whose semi-major axis = $\dfrac{S_{yt}}{\sqrt{1-\mu}}$ and semi-minor axis = $\dfrac{S_{yt}}{\sqrt{1+\mu}}$

(e). Maximum distortion energy theory: According to this theory, when maximum distortion energy developed at a critical point under combined loading conditions exceeds the yield strength in tension test, failure is likely to occur.

$$\text{Deformation Energy / volume} = \text{Total strain energy / Volume} - \text{maximum distortion energy / Volume}$$

$$\boxed{\text{Defomation Energy / volume} = \frac{1+\mu}{6E}\left((\sigma_2 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)}$$

Under biaxial state of stress condition, $\sigma_3 = 0$

$$\boxed{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \le \left(\frac{S_{yt}}{N}\right)^2}$$

Which is an equation of an ellipse. Whose semi major axis = $\sqrt{2}S_{yt}$ and semi minor axis = $\sqrt{\frac{2}{3}}S_{yt}$.

It is the best theory of failure for Ductile materials [Since it gives safe and economic design].

CHAPTER 8: COLUMNS

Introduction:

Column:

☐ These are the vertical slender members subjected to an axial compressive load is called a column and used to carry loads of beams, slabs etc. stanchions are steel columns made of rolled steel sections, commonly used in buildings.

The failure of columns is due to the buckling at loads considerably less than those required to cause failure by crushing.

Struts: The term strut is commonly used for compression member in a roof truss, it may either be in vertical position or in inclined position.

Note.1: A compression member is generally considered to be a column when its unsupported length is more than 10 times its least lateral dimension.

Types of Columns

i. Short Columns: Short columns fail under direct compression, also called as crushing (at ultimate strength).

ii. Long Columns: Long columns fail at loads considerably lower than those required to cause crushing due to elastic instability, also called as buckling. The ***buckling behaviour is explained using Euler's theory.***

iii. Intermediate Columns: Intermediate columns **fail by a combination of crushing and buckling**. This behaviour is complex, and several semi-empirical formulations are used for ***analysis such as Rankine's formula***.

Slenderness ratio: The buckling tendency of a column varies with the ratio of the length to least lateral dimension. The ratio is known is known slenderness ratio. It is given by the following relation:

$$\boxed{\text{Slenderness ratio } (S) = \frac{L_e}{K} = \frac{\text{Effective length of member}}{\text{Least radius of gyration}}}$$

Its numerical value indicates whether the member falls into the class of columns or struts.

S.No.	Type of column	Slenderness ratio
1	Short Columns	0-40
2	Intermediate columns	40-125
3	Long Columns	>125

Radius of gyration: It is defined as the distance from the axis of rotation to a point where the total mass of the body is supposed to be concentrated so that the moment of inertia about the axis may remain the same.

The radius of gyration of a section is given by:

$$\boxed{K = \sqrt{\frac{I}{A}}}$$

Euler's theory: The struts which fails by buckling can be analysed by Euler's theory.

Assumptions of Euler's theory: The Euler's theory is based on the following assumptions:

(i). Axis of the column is perfectly straight when unloaded.

(ii). The line of thrust coincides exactly with the unstrained axis of the column.

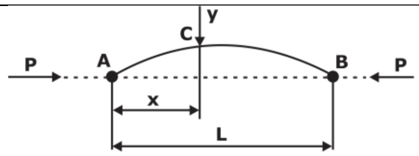
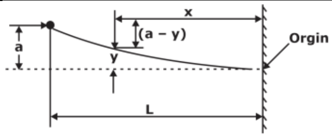
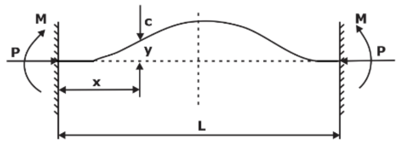
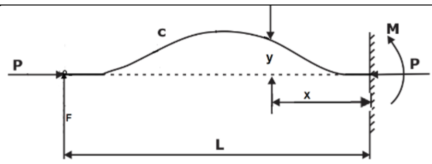
(iii). Flexural rigidity EI is uniform.

(iv) Material is isotropic and homogeneous.

(v). The buckling value of P = PE is assumed to obtain for all degrees of flexure.

Usually, the two assumptions are not really realised in practice. The column may have initial curvature, or crookedness. The theory, therefore, refers to an ideal column and not to a real one.

Equivalent length (L_e): The Effective length L_e for any column is the length of the equivalent pinned end column, i.e., it is the pinned end column having a deflection curve that exactly matches all part of deflection curve of an original column.

Condition and effective length	Diagram
a. Strut with pinned ends: $\boxed{P_e = \frac{\pi^2 EI}{L^2}}$ and $\boxed{L_e = L}$	
b. One end fixed and the other free: $\boxed{P_e = \frac{\pi^2 EI}{4L^2}}$ and $\boxed{L_e = 2L}$	
c. Strut with fixed ends: $\boxed{P_e = \frac{4\pi^2 EI}{L^2}}$ and $\boxed{L_e = L/2}$	
d. **One and fixed and other hinged:** $\boxed{P_e = \frac{2\pi^2 EI}{L^2}}$ and $\boxed{L_e = \frac{L}{\sqrt{2}}}$	

Rankine's Formula: Rankine proposed an empirical formula for columns which coven all Lasts ranging from very short to very long struts.

$$\frac{1}{P_r} = \frac{1}{P_c} + \frac{1}{P_E}$$

Where

$P_c = \sigma_C . A$ = ultimate load for a strut.

$P_E = \frac{\pi^2 EI}{L^2}$ = Eulerian crippling load for the standard case.

In the above relation, $\frac{1}{P_c}$ is constant for a material.

$$\boxed{P_r = \frac{\sigma_c . A}{1 + \left(\frac{\sigma_c}{\pi^2 E}\right)\left(\frac{L}{k}\right)^2} = \frac{\sigma_c . A}{1 + a\left(\frac{L}{k}\right)^2}}$$

Above equation is the Rankine's formula for the standard case of column hinged at ends.

Where Rankine's constant: $\boxed{a = \frac{\sigma_c}{\pi^2 E}}$

Values of σ_c, and a for the materials commonly used for columns and struts:

Material	σ_c		a
	Kg/cm²	N/mm²	(For hinged ends)
1. Wrought Iron.	2550	255	1/9000
2. Cast Iron	5670	567	1/1400
3.Mild steel	3300	330	1/7500
4.Strong timber	500	50	1/750

Combined bending and direct stresses (Eccentric Loading):

A column subjected by a compressive load P whose line of action is at a distance of 'e' from the axis of the column. Here 'e' is known as eccentricity of the load. The eccentric load will cause direct stress and bending stress.

CHAPTER 9: THIN AND THICK SHELLS

Introduction: Thin pressure vessel is defined as a closed cylindrical or spherical container designed to hold or store fluids at a pressure substantially different from ambient pressure.

Shape of Shell
- Spherical Pressure Vessels
- Cylindrical Pressure Vessels

Pressure Vessels
- Thin Pressure Vessels $[D/t \geq 20]$
- Thick Pressure Vessels $[D/t < 20]$

Common examples of pressure vessels are steam boilers, reservoirs, tanks, working chambers of engines, gas cylinders etc.

Thin cylindrical shell subject to internal pressure:

Assumptions:

(i). Stresses are assumed to be distributed uniformly.

(ii). Area is calculated considering the pressure vessel as thin.

(iii). Radial stresses are neglected.

(iv). Biaxial state of stress is assumed to be applicable.

Circumferential stress or Hoop stress: $\boxed{\sigma_H = \frac{Pd}{2t}}$

Longitudinal stress (or axial stress) σ_L: $\boxed{\sigma_L = \frac{Pd}{4t}}$

From here, we can say that

$$\boxed{\sigma_L = \frac{\sigma_H}{2}}$$

Thus, ***the magnitude of the longitudinal stress is one half of the circumferential stress***, both the stresses being of tensile nature.

Note:

The radial stress is negligible as compare to the axial stress and hoop stress. Hence the third stress is neglected.

Maximum shear stress in the plane of σ_H and σ_L :

$$\boxed{\tau_{max} = \frac{\sigma_H - \sigma_L}{2} = \frac{1}{2}\left(\frac{Pd}{2t} - \frac{Pd}{4t}\right) = \frac{Pd}{8t}}$$

$$\boxed{\text{Absolute } \tau_{max} = \frac{Pd}{4t}}$$

Strain:

Hoop strain or Circumferential strain: $\boxed{\varepsilon_c = \frac{\Delta d}{d} = \frac{Pd}{4tE}[2 - \mu]}$

Longitudinal Strain or axial strain: $\boxed{\varepsilon_L = \frac{\Delta L}{L} = \frac{Pd}{4tE}[1 - 2\mu]}$

Note:

The ratio of circumferential strain and longitudinal strain under given loading condition is

$$\boxed{\frac{\text{circumeferential strain}(\varepsilon_c)}{\text{longitudinal strain}(\varepsilon_L)} = \frac{\frac{Pd}{4tE}(2-\mu)}{\frac{Pd}{4tE}(1-2\mu)} = \frac{(2-\mu)}{(1-2\mu)}}$$

Volumetric Strain or Change in the Internal Volume:

$$\boxed{\text{volumetric strain} = \text{longitudinal strain} + 2 \times \text{circumferential strain}}$$

Volumetric Strain: $\boxed{\varepsilon_v = \frac{\Delta V}{V} = \frac{Pd}{4tE}[5 - 4\mu]}$

Thin spherical shells under internal pressure:

Hoop stress: $\boxed{\sigma_h = \frac{Pd}{4t'}}$

And longitudinal stress: $\boxed{\sigma_L = \frac{Pd}{4t'}}$

In plane shear stress in a spherical pressure vessel:

$$\boxed{\text{Maximum shear stress} = \frac{\sigma_1 - \sigma_2}{2} = \frac{P \times d}{4t} - \frac{P \times d}{4t} = 0}$$

$$\boxed{\text{Absolute } \tau_{max} = \frac{Pd}{8t}}$$

$$\boxed{\varepsilon_c = \varepsilon_L = \frac{Pd}{4tE}(1-\mu)}$$

Volumetric strain: $\boxed{\varepsilon_V = \frac{dV}{V} = \frac{3Pd}{4tE}(1-\mu)}$.

Volumetric strain in spherical shell is thrice of the longitudinal strain or hoop strain.

Thin spherical shells under external pressure:

$$\boxed{\varepsilon_V = \frac{dV}{V} = -\frac{3Pd}{4tE}(1-\mu)}.$$

Thin cylinder with hemispherical ends:

If there is no distortion of the junction of hemispherical ends with cylindrical ends under pressure, then:

$$\boxed{\frac{t'}{t} = \frac{1-\mu}{2-\mu}}$$

Thus, the thickness of cylindrical wall must be greater than thickness of hemispherical ends.

Strengthening of cylindrical pressure vessel:

Wire Winding: A tube can be strengthened against the internal pressure by winding it with wire under tension and putting the tube wall in compression. Since the vessel is in compression, as the pressure is applied, the resultant hoop stress produced is much less as it would have been in the absence of wire.

CHAPTER 10: DEFLECTION OF BEAMS

Introduction: The effect of bending results in the deflection of the beam. This is the stiffness aspect of the beam. For design purpose, a beam should be so designed that it has adequate stiffness so that the deflections are within the permissible limits.

Differential equation of the deflection curve of beam:

$$\boxed{\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d\theta}{dx} = -\frac{M}{EI}}$$

This equation can be integrated in each particular case to find the angle of rotation θ (usually called the slope) or the deflection y provided the bending moment M is known.

Sign Conventions: The following sign conventions are adopted:

(i) x is positive when measured towards the right.

(ii) y is positive when measured downwards.

(iii) θ is positive when the rotation is clockwise from the x-axis.

(iv) M is positive when sagging, i.e., when it produces compression in the upper portion of the beam.

Methods of determining deflection of beams:

(a). Double integration method: $\boxed{EI\frac{dy}{dx} = EI.\theta = -\int M}$

(b). Area moment method (Mohr's method): This method utilizes the properties of the area of the bending moment diagram and the moment of that area.

Mohr's first theorem: 1 The angle θ_B^A between the tangent of the deflection curve of two points A and B is equal to the negative area of M/EI diagram between the points.

Mohr's Second Theorem: The deviation of B from tangent at A is equal to the negative of the statical moment (or the first moment) with respect to B, of the $\frac{M}{EI}$ diagram area between A and B.

Note:

(i). Non- zero slope cross-section should be a cross-section where slope & deflection are to be determined.

(ii). $\bar{X}$- should be measured from origin (i.e., Non-zero slope cross-section).

(c). Conjugate beam method (method of elastic weights): A conjugate beam is an imaginary secondary beam, which when loaded with the M/EI diagram of the real beam, yield directly the slope and deflection of the real beam in the form of shear force and bending moment of the conjugate beam.

Preposition 1: Similarity between S.F. of beam and slope of real beam.

$$\boxed{F_x = \int \frac{d^2y}{dx^2}dx = \frac{dy}{dx} = \text{slope}}$$

Preposition 2: Similarity between B.M. of conjugate beam and deflection of real Shear force.

$$\boxed{M_x' = \int_0^x \frac{dy}{dx}dx = y = \text{deflection}}$$

Rules	Existing support condition of actual beam	Corresponding support condition for the conjugate beam
Rule -1	Fixed end	Free end
Rule -2	Free end	Fixed end
Rule -3	Simple support at the end	Simple support at the end
Rule -4	Simple support not at the end	Unsupported hinge
Rule -5	Unsupported hinge	Simple support

(d). Castigliano's first theorem (deflection from strain energy):

It may be observed that though the differentiation with respect to load may be carried out before or after the integration, the calculations are simplified if the differentiation is carried out before the integration.

In case of a beam, $U = \int_0^l \frac{M^2.dx}{2EI}$

Deflection: $\boxed{\delta_i = \frac{\partial U}{\partial W_i} = \frac{1}{EI}\int_0^l M.\frac{\partial M}{\partial W_i}dx}$

The slope θ_i of a beam at a point can also be obtained by applying a virtual couple M_i at the point and putting the same to zero before integration.

$$\boxed{\theta_i = \frac{\partial U}{\partial M_i} = \int_0^l \frac{M}{EI}\frac{M}{\partial M_i}dx = \frac{1}{EI}\int_0^l M.\frac{\partial M}{\partial W_i}dx}$$

(e). Maxwell's reciprocal deflection theorem: "The deflection of any point P resulting from application of a load at any other point Q is the same as the deflection of Q resulting from the application of the same load at P".

Deflections and slopes of different beams under different loadings:

Case (1): Deflection and slope under constant bending moment M:

$\boxed{\theta_{max} = \left[\frac{1}{K_1}\right]\left[\frac{ML}{EI_{N.A.}}\right]}$, and $\boxed{y_{max} = \left[\frac{1}{K_2}\right]\left[\frac{ML^2}{EI_{N.A.}}\right]}$

Where K_1 and K_2 are constants.

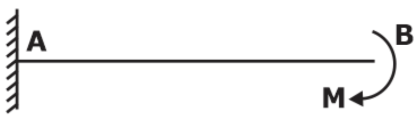
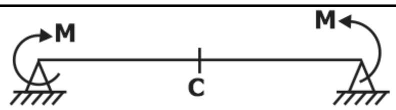
Case (2): In presence of concentrated point loads:

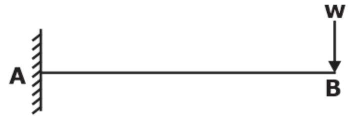
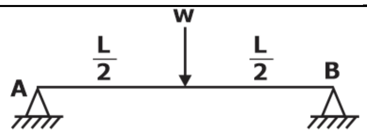
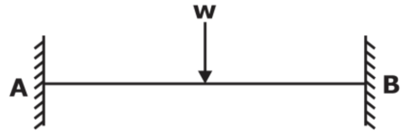
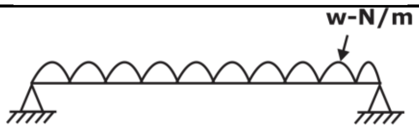
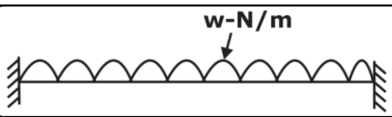
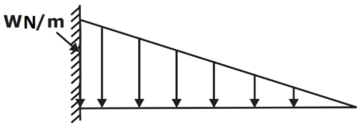
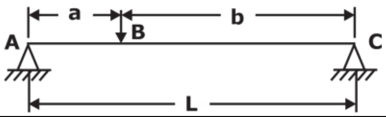
$\boxed{\theta_{max} = \left[\frac{1}{K_1}\right]\left[\frac{WL^2}{EI_{N.A.}}\right]}$ and $\boxed{y_{max} = \left[\frac{1}{K_2}\right]\left[\frac{WL^3}{EI_{N.A.}}\right]}$

Case (3): In presence of distributed loads:

$\boxed{Q_{max} = \left[\frac{1}{K_1}\right]\left[\frac{wL^3}{EI_{N.A.}}\right]}$, and $\boxed{y_{max} = \left[\frac{1}{K_2}\right]\left[\frac{wL^4}{EI_{N.A.}}\right]}$

$\boxed{y_{max} = \theta_{max}\left[\frac{k_1}{k_2}\right]L}$

S. No.	Type of beam	K_1	K_2
1		1	2
2		2	8

3		2	3
4		16	48
5		-	192
6		6	8
7		24	$\frac{384}{5}$
8		-	384
9		24	30
10		$\theta_B = \frac{Wb}{3EI_{N.A.}}[a^2 - ab]$ (Not maximum)	$\boxed{Y_B = \frac{wa^2b^2}{3EI_{N.A.}L}}$

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